UAB
Universitat Autònoma de Barcelona

Partial Differential Equations: Modelling, Analysis and Numerical Approximation

Code: 45561
ECTS Credits: 6

2025/2026

Degree	Type	Year
Modelización para la Ciencia y la Ingeniería / Modelling for Science and Engineering	OP	1

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Teachers

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Teaching groups languages

You can view this information at the end of this document.

Prerequisites

Students should have basic knowledge of calculus, algebra and ordinary differential equations, as well as basic notions of programming.

Objectives and Contextualisation

Many phenomena that unfold in space and/or time can be modelled by means of partial differential equations. The purpose of this course is to provide the main concepts about such models as well as numerical methods for computing their solution.

Learning Outcomes

1. CA18 (Competence) Computationally implement numerical analysis techniques to roughly solve partial differential equations.
2. CA19 (Competence) Integrate partial differential equations into other modelling tools in the context of multidisciplinary projects.
3. CA19 (Competence) Integrate partial differential equations into other modelling tools in the context of multidisciplinary projects.
4. CA20 (Competence) Incorporate, using partial differential equations, sustainability and/or environmental efficiency criteria in mathematical modelling projects.

5. CA20 (Competence) Incorporate, using partial differential equations, sustainability and/or environmental efficiency criteria in mathematical modelling projects.
6. KA15 (Knowledge) Identify the mathematical analysis methods of partial differential equations.
7. KA16 (Knowledge) Recognise the role and usefulness of partial differential equations in the construction of mathematical models.
8. SA18 (Skill) Apply models based on partial differential equations to solve specific problems.
9. SA19 (Skill) Interpret the meaning and phenomenology associated with the parameters present in partial differential equations, in order to describe specific processes.
10. SA20 (Skill) Interpret the results obtained from applying a formalised model with partial differential equations.

Content

PART I: PDE MODELS AND THEIR MAIN PROPERTIES

I.0. Introduction: Examples, different types of equations.

I.1. The heat equation. The solution formula for the pure initial value problem; the Gauss kernel. Solution by means of the Fourier method in the case of a bounded interval with Dirichlet or Neumann boundary conditions. Dissipative character of the heat equation. The parabolic maximum principle.

I.2. The wave equation. The solution formula for the pure initial value problem. Solution by means of the Fourier method in the case of a bounded interval with Dirichlet or Neumann boundary conditions. Conservative character of the wave equation.

I.3. Laplace's equation with Dirichlet or Neumann boundary conditions. Variational principle. The elliptic maximum principle. The Poisson kernel. Solution by means of the Fourier method in the case of a rectangle, a circle or a sphere.

I.4. Turing's "chemical basis of morphogenesis".

I.5. Travelling-wave solutions of non-linear heat equations.

I.6. The traffic equation and scalar conservation laws. Shocks. Weak solutions. Rankine-Hugoniot and entropy conditions.

I.7. The Navier-Stokes equations.

PART II: NUMERICAL METHODS

II.1. Finite difference methods for scalar parabolic equations: Euler explicit, Euler implicit and Crank-Nicholson methods: Von Neumann stability test. Parabolic stability Courant-Friedrichs-Lewy condition. Examples.

II.2. Numerical methods for elliptic equations.

II.3. Numerical methods for scalar conservation laws: Finite difference methods in conservation form. Shock-capturing schemes. Monotone schemes: Lax-Friedrichs and upwind schemes. Convergence and stability conditions. Entropy-satisfying schemes. Examples.

Activities and Methodology

Title	Hours	ECTS	Learning Outcomes
Type: Directed			

Classes of theory and exercises	30	1.2	CA18, CA19, CA20, KA15, KA16, SA18, SA19, SA20, CA18
Type: Supervised			
Internship classes	8	0.32	CA18, CA19, CA20, KA15, KA16, SA18, SA19, SA20, CA18
Type: Autonomous			
Studies and practical work by the student.	96	3.84	CA18, CA19, CA20, KA15, KA16, SA18, SA19, SA20, CA18

The aim of the classes of theory, problems and practices is to give to the students the most basic knowledge about partial differential eauqtions and their applications.

Annotation: Within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continous Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
First partial exam	30%	4	0.16	CA18, CA19, CA20, KA15, KA16, SA18, SA19, SA20
Second partial exam	30%	4	0.16	CA18, CA19, CA20, KA15, KA16, SA18, SA19, SA20
Solution of a problem with a computer	40%	8	0.32	CA18, CA19, CA20, KA15, KA16, SA18, SA19, SA20

The assessment will consist of two partial exams and the delivery of the resolution of a problem by means of the computer.

Bibliography

Bibliography

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F. John, Partial Differential equations, vol. 1, Applied Math Sciences, Springer, (1978).

P.D. Lax, Hyperbolic systems of Conservation Laws and The Mathematical Theory of Shock Waves SIAM, 1973.

R.J. LeVeque, Finite Volume Methods for Hyperbolic problems, Cambridge University Press, 2002.

Y.Pinchover, J. Rubinstein, An Introduction to Partial Differential Equations, Cambridge 2005.

S. Salsa, Partial differential equations in action : from modelling to theory Springer, 2008.

G. Strang, Introduction to Applied Mathematics, Wellesley-Cambridge Press, (1986).

E.F. Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics: A practical Introduction, Springer-Verlag, 2009.

G.B. Whitham Linear and nonlinear Waves, Wiley-Intersciences, (1999).

Software

We leave full freedom to students to use the language that suits them best to do the numerical exercises of this course.

Groups and Languages

Please note that this information is provisional until 30 November 2025. You can check it through this link. To consult the language you will need to enter the CODE of the subject.

Name	Group	Language	Semester	Turn
(TEm) Theory (master)	1	English	second semester	afternoon