

Degree programme	Type	Course
Applied Statistics	OB	2

Contact lecturer

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Teaching staff

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Group languages

You can consult this information at the [end](#) of the document.

Prerequisites

Elementary probability. Real random variables. Differential and integral calculation. Elemental algebra: vector spaces and determinants.

Objectives

The main objective of this course is the study of probability distributions of random vectors, both discrete and continuous. The course examines the main characteristics of joint distributions, including the mean vector, the variance-covariance matrix, and marginal and conditional distributions.

As a representative discrete model, the multinomial distribution is studied in detail. In the continuous setting, particular emphasis is placed on the multivariate normal distribution. This distribution plays a central role in modern statistics and has numerous applications in linear model theory, multivariate analysis, and statistical decision theory.

The study of continuous multivariate distributions requires the use of differential and integral calculus for functions of several variables. For this reason, the course includes a review of the fundamental concepts and techniques in this area, focusing on their applications to probability and statistics.

Finally, the course introduces the theory of copulas and examines some of their applications in modeling dependence among random variables.

Learning outcomes

- KM10 (Describe the characteristics of the distribution and density functions of random variables.)
Describe the characteristics of the distribution and density functions of random variables.

Contents

1. Random Vectors. Multivariate Distributions.

1.1 Introduction to Bivariate Distributions.

1.2 Joint Probability Functions and Marginal Probability Functions.

1.3 Continuous Case. Joint Density Functions and Marginal Density Functions.

1.4 Joint Distribution Function.

1.5 Multivariate Distributions. The Multinomial Distribution.

1.6 Functions of Two or More Random Variables.

2. Independence and Conditioning.

2.1 Independence of Random Variables.

2.2 Conditional Distributions in the Discrete and Absolutely Continuous Cases.

3. Expectation and Other Numerical Characteristics.

3.1 Expectation of a Function of a Random Vector.

3.2 Covariance and Correlation Coefficient.

3.3 Conditional Expectation.

3.4 Moment-Generating Function.

3.5 Numerical Characteristics of the Multinomial Distribution.

4. The Multivariate Normal Distribution. Distributions Related to the Normal Distribution.

4.1 Study of the Bivariate Normal Distribution.

4.2 The Multivariate Normal Distribution.

4.3 Distributions Related to the Normal Distribution: Chi-Square, Student's t , and Fisher-Snedecor F Distributions.

4.4 Fisher's Theorem. Cochran's Theorem.

4.5 Relationship Between the Multinomial Distribution and the Multivariate Normal Distribution.

5. Copulas of Random Variables.

5.1 Introduction.

5.2 Inversion Method.

5.3 Mathematical Definition of Copulas.

5.4 Sklar's Theorem.

5.5 Extreme Copulas.

5.6 Fréchet-Hoeffding Theorem (Bivariate Case).

5.7 FGM Family.

5.8 Dependence Measures Based on Copulas.

5.9 Copulas in Higher Dimensions.

Learning activities and methodology

Title	Hours	ECTS	Learning outcomes
Exam preparation	20	0.8	
Computer laboratory practical sessions (with Maxima and R) and seminars	12	0.48	
Practical lessons about problems and exercises	12	0.48	
Theory lessons	25	1	
Personal work (exercices and theory)	61	2.44	KM10

This is a one-semester course that includes, on average, two hours of lectures and one hour of problem-solving classes per week of face-to-face instruction. In addition, there will be three seminar sessions of two hours each and three two-hour laboratory sessions.

The introduction of concepts in the theory classes is essential for students to understand and grasp the foundations of probability theory introduced in this course. Knowledge of the notions presented in the theory lectures, the statements of propositions and theorems, as well as examples of applications, is indispensable for students to be able to solve the problems presented in the practical sessions using similar methods. The structure of definition-theorem-proof-application will be followed, as it enables students to understand and follow the reasoning behind the mathematical theory being taught. At the same time, it allows them to see and comprehend the role played by the different elements involved in proving new mathematical results, as well as the assumptions that need to be imposed. Naturally, the course aims to foster a critical attitude toward any mathematical statement, as well as the intuition necessary to assess the appropriateness of various mathematical models to a wide range of real-world situations (physical, biological, economic, etc.), through applied problems where modeling plays a central role.

In the problem-solving classes, practical problems will be addressed. Special attention will also be paid to the students' oral and written communication. Furthermore, in the seminar and lab sessions, students will work-under the guidance of the instructor-on practical situations related to the content covered in the theory classes. These sessions will also allow both the instructor and the student to monitor the progress made in understanding the concepts and methods introduced in the theoretical component of the course.

Annotation: within the schedule set by the centre or degree programme, 15 minutes of one class will be reserved for students to evaluate their lecturers and their courses or modules through questionnaires.

Assessment

Continuous assessment activities

Title	Weight	Hours	ECTS	Learning outcomes
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Wooclap	10%	2	0.08	KM10
Second midterm exam	40%	4	0.16	KM10
Retake exam	80%	4	0.16	KM10
Lab reports	10%	6	0.24	KM10
First midterm exam	40%	4	0.16	KM10

Continuous Assessment

Two midterm examinations will be held: the first in the middle of the semester and the second at the end of the semester. Each examination will account for 40% of the final grade.

Three laboratory sessions will be carried out throughout the course. Students will be required to submit a report for each laboratory session. The combined grade for these reports will account for 10% of the final grade.

At the end of each of the twelve theory sessions, a Wooclap activity will be conducted. The score for each activity will be the percentage of correct answers. The grade corresponding to these activities will be computed as the average of the nine highest scores obtained during the course and will account for 10% of the final grade.

Single Assessment

Students opting for the single assessment system will complete the following activities on the date scheduled for the final examination:

- A theory and problem-solving examination, accounting for 80% of the final grade.
- A practical examination, accounting for 10% of the final grade.
- A Wooclap activity, accounting for 10% of the final grade.

Bibliography

- Cuadras, C. M. (1995). *Problemas de probabilidades y estadística. Vol. 1: Probabilidades*. EUB.
- Johnson, R. A., & Wichern, D. W. (2018). *Applied Multivariate Statistical Analysis* (6th ed.). Pearson.
- Julià, O., Márquez-Carreras, D., Rovira, C., & Sarrà, M. (2005). *Probabilitats: problemes i més problemes*. Publicacions i Edicions de la Universitat de Barcelona.
- Nelsen, R. B. (2006). *An Introduction to Copulas* (2nd ed.). Springer.
- Ross, S. M. (2019). *A First Course in Probability* (10th ed.). Pearson.
- Spiegel, M. R., Schiller, J. J., & Srinivasan, R. A. (2014). *Probability and Statistics* (4th ed.). McGraw-Hill Education.

Software

- R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. <https://www.r-project.org/>
- wxMaxima. *Graphical User Interface for the Computer Algebra System Maxima*. <https://wxmaxima-developers.github.io/wxmaxima/>

Course groups and languages

The information provided is provisional until November 30. After this date, you will be able to consult the language of each group through this [link](#). To access the information, you will need to enter the course CODE

Type of teaching	Group	Language	Semester	Shift
(TE) Theory	1	Catalan	first semester	afternoon
(PAUL) Classroom practices	1	Catalan	first semester	afternoon
(PLAB) Practical laboratories	1	Catalan	first semester	afternoon
(PLAB) Practical laboratories	2	Catalan	first semester	afternoon