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SMOOTHNESS OF CAUCHY RIEMANN MAPS FOR A CLASS OF REAL HYPERSURFACES

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Abstract

We study the regularity problem for Cauchy Riemann maps between hypersurfaces in $\mathbb{C}^n$. We prove that a continuous Cauchy Riemann map between two smooth $\mathcal{C}^\infty$ pseudoconvex decoupled hypersurfaces of finite D'Angelo type is of class $\mathcal{C}^\infty$.

Introduction

Many classical problems in complex analysis rely on the boundary behavior of holomorphic maps and, as a consequence, on the regularity of Cauchy Riemann maps between real hypersurfaces. Only partial results have been obtained when the hypersurfaces are not assumed real analytic: the smoothness of a continuous Cauchy Riemann (CR) map between smooth real hypersurfaces was proved, for instance, in [**17**] for strictly pseudoconvex hypersurfaces, in [**11**] for pseudoconvex hypersurfaces of finite D'Angelo type in $\mathbb{C}^2$, and in [**8**] for a Lipschitz CR map between convex hypersurfaces of finite D'Angelo type. It is natural to study decoupled hypersurfaces (or domains) to understand the link between complex dimension two and higher complex dimensions: for such domains J. D. McNeal [**16**] gave estimates on Bergman, Caratheodory and Kobayashi metrics, J. E. Fornæss and J. D. McNeal [**14**] constructed local peak holomorphic functions at boundary points, and D. C. Chang and S. Grellier [**7**] gave properties of the Szegö projection under an additive global assumption.

In this paper we prove the following local result:

Theorem 1. *Let Γ_1 and Γ_2 be two $\mathcal{C}^\infty$ pseudoconvex real hypersurfaces in $\mathbb{C}^{n+1}$, containing the origin 0, and let f be a continuous non constant CR map from Γ_1 to Γ_2, satisfying $f(0) = 0$. If Γ_1 and Γ_2 are decoupled, of finite D'Angelo type at the origin, then f is a smooth $\mathcal{C}^\infty$ locally finite map near the origin.*

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According to a result of S. Bell and D. Catlin [**4**], it is sufficient to prove that the set $f^{-1}(f(0))$ is compact near the origin i.e. that for any neighborhood U of 0 the set $f^{-1}(f(0)) \cap U$ is relatively compact in U. Using a dilation we reduce the study of the regularity of f to the study of the boundary behavior of a holomorphic map F between rigid algebraic domains D_1 and D_2. This map is the limit of some holomorphic maps F^ν. The algebraicity of domains D_1 and D_2 gives some information on the map F, detailed in Proposition 1.2. Moreover one can prove that if the dilated maps F^ν satisfy uniform Hölder estimates then the non compactness of the set $f^{-1}(f(0))$ implies the non compactness of $F^{-1}(F(0))$ (the origin is mapped to the origin by the scaling process applied to Γ_1). Such Hölder estimates may be obtained when the sequence $(F^\nu(0))_\nu$ is bounded; we prove this property on $(F^\nu(0))_\nu$, and consequently Theorem 1, as soon as the hypersurface ∂D_1 is not spherical, i.e. not locally biholomorphic to a sphere at a strictly pseudoconvex point. This relies on a classification of vector fields tangent to a weighted homogeneous rigid polynomial hypersurface, given by E. Bedford and S. Pinchuk [**3**], and on a precise study of the restrictions of F to some subvarieties of D_1 (Sections 2 and 3). If ∂D_1 is spherical it may happen that $\lim_{\substack{z\in D_1 \\ z\to 0}} |F(z)| = \infty$. Using the characterization of spherical rigid hypersurfaces, given by A. V. Isaev [**15**] and obtained in the spirit of the Chern-Moser theory, we describe F in terms of a correspondence of the unit ball. This proves the compactness of $F^{-1}(\infty)$ in ∂D_1 and ends the proof of Theorem 1 (Section 4). We note that these techniques also provide a modified proof of Theorem 0.1 in [**11**].

1. Reduction of the problem

Decoupled hypersurfaces are defined as follows:

Definition 1.1. A hypersurface Γ in $\mathbb{C}^{n+1}$ containing the origin 0 is decoupled at 0 if there are a neighborhood U of 0, holomorphic coordinates $(z_0, z) = (z_0, z_1, \dots, z_n)$ centered at 0 and a defining function r such that:

$$\Gamma = \left\{ (z_0, z) \in U : r(z_0, z) = \operatorname{Im} z_0 + \sum_{j=1}^{n} f_j(z_j, \overline{z}_j) = 0 \right\}$$

where f_j is a real function for every $j = 1, \dots, n$.

Let $\Gamma_1 = \{(z_0, z) \in U : r_1(z_0, z) = 0\}$ and $\Gamma_2 = \{(z_0, z) \in U : r_2(z_0, z) = 0\}$ be two smooth $\mathcal{C}^\infty$ decoupled pseudoconvex hypersurfaces in $\mathbb{C}^{n+1}$, of finite type $2m$ and $2k$ respectively (in the sense of D'Angelo [**12**]) at the origin. We may assume that the functions f_j^1 and f_j^2, in the expansions of r_1 and r_2, are subharmonic functions of class $\mathcal{C}^\infty$, without harmonic terms, vanishing at order less than or equal to $2m$ and $2k$ respectively. If f is a continuous CR map from Γ_1 to Γ_2 then according to [**4**] we have:

(i) f extends locally to a holomorphic map (still called f) from the pseudoconvex side Ω_1 of Γ_1 to the pseudoconvex side Ω_2 of Γ_2,
(ii) the extension f is continuous up to Γ_1 with $f(\Gamma_1) \subset \Gamma_2$.

Since the order of contact between the $(j+1)$th coordinate complex line and Γ_1 at the origin is less than or equal to $2m$ we may write for every $1 \leq j \leq n$: $f_j^1(z_j, \overline{z_j}) = H_j(z_j, \overline{z_j}) + R_j(z_j, \overline{z_j})$ where H_j is a homogeneous polynomial of degree $2m_j \leq 2m$ and R_j denotes terms of larger degree. If $(p^\nu)_\nu$ is a sequence of points in Ω_1 converging to 0 then $(f(p^\nu))_\nu = (q^\nu)_\nu$ converges to 0 by (ii). We may assume that there is a unique point $z^\nu \in \Gamma_2$ such that $\mathrm{dist}(q^\nu, \Gamma_2) = |q^\nu - z^\nu| = \delta^\nu$. Let $L_0, \ldots, L_n$ be the vector fields defined by $L_0 = \dfrac{\partial}{\partial z_0}$ and for $1 \leq j \leq n$, $L_j = \dfrac{1}{2i}\dfrac{\partial}{\partial z_j} - \dfrac{\partial r_2}{\partial z_j}\dfrac{\partial}{\partial z_0}$. We note that $\{L_0 - \overline{L_0}, L_1, \ldots, L_n, \overline{L_1}, \ldots, \overline{L_n}\}$ span the real tangent space to Γ_2 and $\{L_1, \ldots, L_n\}$ span $\mathbb{C}T^{1,0}(\Gamma_2)$. For every ν, let $\mathcal{L}_{s,t}^j r_2(z^\nu)$ be the commutator of L_j, $\overline{L_j}$ of length $s-1$ in z_j and $t-1$ in $\overline{z_j}$ at z^ν. Since Ω_2 is of finite type we can set:

$$M_j = \inf\{m_j \text{ such that } \mathcal{L}_{s,t}^j r_2(z^\nu) \neq 0 \text{ for some } s, t \text{ such that } s + t = m_j\},$$

$$C_l^j(z^\nu) = \sup\{|\mathcal{L}_{s,t}^j r_2(z^\nu)| : s + t = l\},$$

$$\tau_j^\nu = \inf\{(\delta^\nu / C_l^j(z^\nu))^{1/l} : 2 \leq l \leq M_j\}$$

and define the dilation:

$$\Lambda_2^\nu(z_0, z) = (z_0/\delta^\nu, z_1/\tau_1^\nu, \ldots, z_n/\tau_n^\nu).$$

Let us consider automorphisms U^ν of $\mathbb{C}^{n+1}$ converging to the identity with $U^\nu(q^\nu) = (-\delta^\nu, 0)$. We may choose U^ν such that $r_2 \circ (U^\nu)^{-1}$ is decoupled and has no harmonic terms in $z_1, \ldots, z_n$.

If $p^\nu = (-\varepsilon_\nu, 0)$ is on the real inward normal to Ω_1 at the origin and Λ_1^ν is the dilation $\Lambda_1^\nu(z_0, z) = ((\varepsilon_\nu)^{-1} z_0, (\varepsilon_\nu)^{-1/2m_1} z_1, \ldots, (\varepsilon_\nu)^{-1/2m_n} z_n)$ we may consider the family of maps $F^\nu = \Lambda_2^\nu \circ U^\nu \circ f \circ (\Lambda_1^\nu)^{-1}$. Without any restriction we may assume that each map F^ν is defined on Ω_1^ν with values in Ω_2^ν where $\Omega_1^\nu = \{(z_0, z) \in \Lambda_1^\nu(U) : r_1((\Lambda_1^\nu)^{-1}(z_0, z)) < 0\}$ and $\Omega_2^\nu = \{(z_0, z) \in \Lambda_2^\nu(U) : r_2^\nu(z_0, z) = r_2((U^\nu)^{-1} \circ (\Lambda_2^\nu)^{-1}(z_0, z)) < 0\}$.

We obtain after extraction of subsequences:

Proposition 1.1.

(i) *The sequence $(\Omega_1^\nu)_\nu$ converges, in the Hausdorff convergence, to the domain $D_1 = \{(z_0, z) \in \mathbb{C}^{n+1} : \operatorname{Im} z_0 + \sum_{j=1}^n H_j(z_j, \overline{z}_j) < 0\}$.*

(ii) *The sequence $(\Omega_2^\nu)_\nu$ converges to $D_2 = \{(z_0, z) \in \mathbb{C}^{n+1} : \operatorname{Im} z_0 + \sum_{j=1}^n P_j(z_j, \overline{z}_j) < 0\}$ where P_j is a non harmonic subharmonic polynomial of degree less than or equal to $2k$.*

(iii) *The family $(F^\nu)_\nu$ converges uniformly on compact subsets of D_1 to a holomorphic map F from D_1 to D_2.*

Proof of Proposition 1.1: The expression of the dilation Λ_1^ν gives part (i).

Part (ii): by the definition of τ_j^ν there is a real positive constant C such that for every sufficiently large ν and every $j \geq 2$, $\tau_j^\nu \lesssim (\delta^\nu)^{1/2k}$; the sequence $(r_2((U^\nu)^{-1} \circ (\Lambda_2^\nu)^{-1}))_\nu$ converges to a function $(z_0, z) \mapsto \operatorname{Im} z_0 + \sum_{j=1}^n P_j(z_j, \overline{z_j})$ where P_j is a subharmonic polynomial of degree less than or equal to $2k$, without harmonic term.

Part (iii) was proved by F. Berteloot [**5**] in $\mathbb{C}^2$: let $(h^k = (h_0^k, \ldots, h_n^k))_k$ be a sequence of holomorphic maps from the unit disc Δ to Ω_2^ν such that $(h^k(0))_k$ is relatively compact in D_2. According to the Gauss formula we have for every fixed $1 \leq j \leq n$:

$$\int_0^1 \frac{dt}{t} \int_{\Delta_t} \frac{\partial^2 (r_2^\nu \circ \tilde{h}^k)}{\partial z_j \partial \bar{z}_j} = \frac{1}{2\pi} \int_0^{2\pi} r_2^\nu \circ \tilde{h}^k(e^{i\theta}) d\theta - r_2^\nu \circ \tilde{h}^k(0)$$

where $\tilde{h}^k = (h_1^k, \ldots, h_n^k)$ and $\Delta_t = \{\lambda \in \mathbb{C} : |\lambda| < t\}$.

Using the expression of r_2^ν we get:

$$\int_0^1 \frac{dt}{t} \int_{\Delta_t} \frac{\partial^2 (r_2^\nu \circ \tilde{h}^k)}{\partial z_j \partial \bar{z}_j} = \int_0^1 \frac{dt}{t} \int_{\Delta_t} \frac{\partial^2 ((r_2^\nu)_j \circ h_j^k)}{\partial z_j \partial \bar{z}_j}$$

where $(r_2^\nu)_j$ is the restriction of r_2^ν to the jth complex coordinate axis.

However for every $0 < r < 1$ we have:

$$\int_0^1 \frac{dt}{t} \int_{\Delta_t} \frac{\partial^2 ((r_2^\nu)_j \circ h_j^k)}{\partial z_j \partial \bar{z}_j} \geq \int_r^1 \int_{\Delta_r} \frac{\partial^2 ((r_2^\nu)_j \circ h_j^k)}{\partial z_j \partial \bar{z}_j} \geq \int_{h_j^k(\Delta_r)} \frac{\partial^2 (r_2^\nu)_j}{\partial z_j \partial \bar{z}_j}.$$

Hence, since $r_2^\nu \circ \tilde{h}^k(e^{i\theta}) \leq -\operatorname{Re} h_0^k(e^{i\theta})$, we obtain:

$$\int_{h_j^k(\Delta_r)} \frac{\partial^2 (r_2^\nu)_j}{\partial z_j \partial \bar{z}_j} \leq -\operatorname{Re} h_0^k(0) - r_2^\nu \circ \tilde{h}^k(0).$$

The term $-\operatorname{Re} h_0^k(0) - r_2^\nu \circ \tilde{h}^k(0)$ being bounded from above independently of k, the sequence $(h_j^k)_k$ converges uniformly on compact subsets of Δ to a holomorphic map h from Δ to $\overline{D_2}$. Since $(h^k(0))_k$ is relatively compact in D_2 we have the inclusion $h(\Delta) \subset D_2$. By covering the unit ball of $\mathbb{C}^{n+1}$ by disks we get the same convergence for any sequence $(h^k)_k$ of holomorphic maps defined on the unit ball and also for any sequence of holomorphic maps defined on D_1. This proves part (iii). □

It happens that the map F has some intrinsic properties coming mainly from the rigidity of the domains D_1 and D_2. For instance we know by [**18**] that if, in our situation, there is a sequence $(z^\nu)_\nu$ of points in D_1 converging to a point z^∞ in ∂D_1 such that the sequence $(F(z^\nu))_\nu$ converges to a point w^∞ in ∂D_2 then F extends continuously to a neighborhood of z^∞ in $\overline{D_1}$ and we can set $F(z^\infty) = w^\infty$. In that case we say that $F(z^\infty)$ is a finite point in ∂D_2.

Proposition 1.2.

(i) *The map F is locally proper i.e. F is proper from $D_1 \cap B(0,r)$ to $F(D_1 \cap B(0,r))$ for every real positive number r.*
(ii) *If $F(0)$ is a finite point in ∂D_2 then the set $F^{-1}(F(0))$ is finite near 0 in ∂D_1.*
(iii) *There exist strictly pseudoconvex points p in ∂D_1 and q in ∂D_2 such that F extends to a biholomorphism in a neighborhood of p with $F(p) = q$.*
(iv) *F is algebraic.*

The tools used to prove Proposition 1.2, developed in different papers and valid for decoupled domains, are the existence of peak plurisubharmonic functions with algebraic growth (constructed in [**13**]) implying an equivalence distance property for f (part (i)), and the existence of peak holomorphic functions at infinity for rigid domains, given by [**2**] (part (iii)). Part (ii) uses the transversality of the Segre varieties, satisfied in any dimension and part (iv) is an immediate consequence of part (iii) by [**19**]. The complete proof of Proposition 1.2 is given by Proposition 1.5 and Lemma 2.2 of [**8**].

Without changing the properties of D_2, we may assume that $q = 0$.

The map F inherits some properties from f, since the family $(F^\nu)_\nu$ satisfies the following uniform properties:

Proposition 1.3. *Let a be a point in ∂D_1 and $(a^\nu)_\nu$ a sequence of points in $\partial\Omega_1^\nu$, converging to a.*

(i) *If $(F^\nu(a^\nu))_\nu$ is bounded then there exists a neighborhood U of a in $\mathbb{C}^{n+1}$ and a real positive constant C such that for every positive integer ν and every z, z' in $\Omega_1^\nu \cap U$ we get:*

$$|F^\nu(z) - F^\nu(z')| \leq C|z - z'|^{(1/2k)}.$$

(ii) *If $\lim_{\nu\to\infty} |F^\nu(a^\nu)| = +\infty$ then $\lim\limits_{\substack{z\in\overline{D}_1 \\ z\to a}} |F(z)| = +\infty$.*

Proof: See Propositions 4.1 and 5.1 of [**10**]. □

We deduce from Propositions 1.2 and 1.3 the following result:

Proposition 1.4. *If f satisfies the assumptions of Theorem 1 then the set $f^{-1}(f(0))$ is compact near the origin in Γ_1 and f is a $\mathcal{C}^\infty$ locally finite map at the origin under one of the following conditions:*

(i) *$F(0)$ is a finite point in ∂D_2.*

(ii) *$\lim\limits_{\substack{z\in\overline{D}_1 \\ z\to 0}} |F(z)| = +\infty$ and $F^{-1}(\infty)$ is a finite set in ∂D_1.*

Proof of Proposition 1.4: Assume by contradiction that the set $f^{-1}(f(0))$ is not compact near the origin. Then there is, for every sufficiently small real positive number r, a point in $f^{-1}(f(0))$ of modulus r. Using the expression of the dilation we can find for every ν, k larger than or equal to one a point a_k^ν in $\partial\Omega_1^\nu$ satisfying: $F^\nu(a_k^\nu) = F^\nu(0)$, $|a_k^\nu| = \frac{1}{k}$. We may assume that for every k the sequence $(a_k^\nu)_\nu$ converges to a point a_k in ∂D_1 with modulus $1/k$: the sequence $(a_k)_k$ converges to 0 in ∂D_1.

Part (i): by the assumption and Proposition 1.3 part (ii) the sequence $(F^\nu(0))_\nu$ is bounded. Then by Proposition 1.3 part (i) and Proposition 1.1 part (iii) this sequence converges to $F(0)$. The condition $F(a_k) = F(0)$ contradicts Proposition 1.2 part (ii).

Part (ii): the non compactness of $f^{-1}(f(0))$ implies that $\lim_{\nu\to\infty} |F^\nu(a_k^\nu)| = +\infty$ for every k and thus by Proposition 1.3 part (ii) that the point a_k belongs to $F^{-1}(\infty)$ for every k: this contradicts the finiteness of $F^{-1}(\infty)$. The smoothness of f is then given by [**4**]. □

According to Proposition 1.4 one needs to answer the two following questions to prove Theorem 1:

1. When is $F(0)$ a finite point in ∂D_2?
2. What is the structure of the set $F^{-1}(\infty)$ if $\lim\limits_{\substack{z\in D_1 \\ z\to 0}} |F(z)| = +\infty$?

In the next section we describe the CR infinitesimal automorphisms of ∂D_1. We use this description in Section 3 to answer question 1. The answer to question 2 is given in Section 4.

2. Classification of vector fields

Since we deal with decoupled real hypersurfaces we start this section with some results on real hypersurfaces in complex dimension two. Let H be a real subharmonic non harmonic homogeneous polynomial defined in $\mathbb{C}$, and Γ be the real hypersurface defined by: $\Gamma = \{(z_0, z_1) \in \mathbb{C}^2 : \operatorname{Im} z_0 + H(z_1, \overline{z_1}) = 0\}$. The real dimension of the real vector space of CR infinitesimal automorphisms at a strictly pseudoconvex point of Γ is equal to 2, 3 or 8 according to a result of E. Cartan [**6**]. This equals 8 if Γ is spherical, in which case we may assume that $H(z_1, \overline{z_1}) = |z_1|^{2m}$, and 3 if Γ is a tube hypersurface (and $H(z_1, \overline{z_1}) = (\operatorname{Im} z_1)^{2m}$).

We assume in this section that the hypersurface ∂D_1 has at least one non spherical direction at point p given by Proposition 1.2. This means that if we write $p = (p_0, \dots, p_n)$ then there is an integer l satisfying $0 \le l \le n-1$ such that for every integer j larger than l the hypersurface $\Gamma_j = \{(z_0, z_j) \in \mathbb{C}^2 : \operatorname{Im} z_0 + H_j(z_j, \overline{z_j}) = -\sum_{k \neq j} H_k(p_k, \overline{p_k}\}$ is not biholomorphic to the unit sphere at (p_0, p_j). In the following l denotes the smallest integer satisfying this condition and we remark that the equality $l = 0$ means that ∂D_1 is a spherical hypersurface in $\mathbb{C}^{n+1}$. The infinitesimal CR automorphisms of ∂D_1 are then given by the following proposition:

Proposition 2.1. *Let p be given by Proposition 1.2 and let $X = \sum_{j=0}^{n} X_j \frac{\partial}{\partial z_j}$ be a CR infinitesimal automorphism at p.*

- ***Case 1:*** $l = 0$.

There exist $(n+2)$ real constants $a_0, b_0, b_1, \dots, b_n$ such that:

$$\begin{cases} X_0(z_0, z) = a_0 + b_0 z_0 + 2\sum_{k=1}^{n} m_k b_k z_k^{2m_k - 1} & \\ X_j(z_0, z) = 2m_j b_0 z_j - \frac{i}{2} b_j & \forall\, j \ge 1 \end{cases}.$$

- ***Case 2:*** $l > 0$.

There exist $(n+2)$ real constants $a_0, b_0, b_1, \dots, b_n$ and $l(l-1)$ complex constants c_{jk} defined for $1 \le j, k \le l$ with $j \neq k$ such that:

$$\begin{cases} X_0(z_0, z) = a_0 + b_0 z_0 + 2\sum_{k=l+1}^{n} m_k b_k z_k^{2m_k - 1} & \\ X_j(z_0, z) = \frac{i}{2}\Big(b_j z_j + \frac{1}{z_j^{m_j - 1}} \sum_{\substack{k=1 \\ k \neq j}}^{l} c_{jk} z_k^{m_k}\Big) & \forall\, 1 \le j \le l \\ X_j(z_0, z) = 2m_j b_0 z_j - \frac{i}{2} b_j & \forall\, j \ge l+1 \end{cases}.$$

Proof of Proposition 2.1: Since case 1 may be considered as a special case of case 2, let us study case 2. We may assume that for every integer k between 1 and l the polynomial H_k is defined for z_k in $\mathbb{C}$ by: $H_k(z_k,\overline{z_k}) = |z_k|^{2m_k}$. Let j be an integer between $l+1$ and n. Fixing all the variables $z_k = p_k$ for k different from j $(1 \leq k \leq n)$ the vector field

$$X^j(z_0,z_j) = \left(X_0({}^jz) - 2i \sum_{\substack{k \neq j \\ k=1}}^{n} \frac{\partial H_k}{\partial z_k}(p_k,\overline{p_k}) X_k({}^jz) \right) \frac{\partial}{\partial z_0} + X_j({}^jz)\frac{\partial}{\partial z_j},$$

where ${}^jz = (z_0,p_1,\dots,p_{j-1},z_j,p_{j+1},\dots,p_n)$, is tangent to the hypersurface

$$S^j = \left\{ (z_0,z_j) \in \mathbb{C}^2 : \operatorname{Im} z_0 + H_j(z_j,\overline{z_j}) = -\sum_{k\neq j} H_k(p_k,\overline{p_k}) \right\}$$

at (p_0,p_j). As S^j is not spherical, every local tangent vector field extends to a global one acording to [**6**]. Then, because of the homogeneity of S^j, each homogeneous part in the expansion of X^j is also a CR infinitesimal automorphism for S^j. Since S^j is of finite type the real vector space of such vector fields has a finite dimension; thus X^j is a polynomial vector field which means that all the coefficients of X^j are polynomial. Hence the vector field $\tilde{X}$ defined for $(z_0,z_{l+1},\dots,z_n)$ in a neighborhood of $(p_0,p_{l+1},\dots,p_n)$ in $\mathbb{C}^{n-l+1}$ by:

$$\tilde{X}(z_0,z_{l+1},\dots,z_n) = \left(X_0(z^l) - 2i\sum_{k=1}^{l} \frac{\partial H_k}{\partial z_k}(p_k,\overline{p_k})X_k(z^l) \right)\frac{\partial}{\partial z_0} + \sum_{k=l+1}^{n} X_k(z^l)\frac{\partial}{\partial z_k},$$

where $z^l = (z_0,p_1,\dots,p_l,z_{l+1},\dots,z_n)$, is polynomial.

Let us expand $\tilde{X}$ in homogeneous vector fields with respect to the weights $2m_{l+1},\dots,2m_n$ (we recall that $2m_j$ is the degree of polynomial H_j given by Proposition 1.1) with the convention that $\partial/\partial z_j$ has weight $-1/2m_j$. The classification of homogeneous vector fields Q tangent to $\tilde{S} = \{(z_0,z_{l+1},\dots,z_n) \in \mathbb{C}^{n-l+1} : \operatorname{Im} z_0 + \sum_{j=l+1}^{n} H_j(z_j,\overline{z_j}) = -\sum_{j=1}^{l} |p_j|^{2m_j}\}$ is given by [**3**] (Lemmas 2.7, 3.4, 3.5). Since there is no spherical direction in $\tilde{S}$ the admissible weights for these vector fields are -1 (corresponding to the translation $\frac{\partial}{\partial z_0}$), $-1/2m_j$ (with $l+1 \leq j \leq n$)

and 0. Moreover the only CR infinitesimal automorphism of weight 0 corresponds to the dilation:

$$Q(z_0,z) = z_0\frac{\partial}{\partial z_0} + \sum_{k=1}^{n}\frac{1}{2m_k}z_k\frac{\partial}{\partial z_k}.$$

A straightforward computation based on Lemma 2.7 of [**3**] gives the form of the homogeneous CR infinitesimal automorphisms of weight $-1/2m_j$ and consequently of the CR infinitesimal automorphisms of weight $wt(Q)$ with $-1 < wt(Q) < 0$:

$$Q(z_0,z) = \sum_{k=1}^{n} b_k\left(2m_k z_k^{2m_k-1}\frac{\partial}{\partial z_0} - \frac{i}{2}\frac{\partial}{\partial z_k}\right), \quad b_k\in\mathbb{R}.$$

Thus we have $\tilde{X} = \tilde{X}_0\frac{\partial}{\partial z_0} + \sum_{j=l+1}^{n}\tilde{X}_j\frac{\partial}{\partial z_j}$ with:

$$\begin{cases}\tilde{X}_0(z_0,z_{l+1},\dots,z_n) = a_0 + b_0z_0 + 2\displaystyle\sum_{k=l+1}^{n} m_kb_kz_k^{2m_k-1}\\ \tilde{X}_j(z_0,z_{l+1},\dots,z_n) = 2m_jb_0z_j - \frac{i}{2}b_j, \qquad \forall\, l+1\le j\le n\end{cases}$$

where $a_0, b_0, b_{l+1},\dots,b_n$ are real analytic real functions of the variables $z_1,\dots,z_l$, defined locally at $(p_1,\dots,p_l)$. Thus the vector field X satisfies the following system:

$$\text{(S)}\quad\begin{cases}X_0(z_0,z) - 2i\sum_{k=1}^{l}(m_k\overline{z}_k^{m_k}z_k^{m_k-1})X_k(z_0,z)\\ \qquad\qquad = a_0 + b_0z_0 + 2\sum_{k=l+1}^{n} m_kb_kz_k^{2m_k-1}\\ X_j(z_0,z) = 2m_jb_0z_j - \frac{i}{2}b_j, \qquad \forall\, l+1\le j\le n\end{cases}$$

for (z_0,z) close to p. Since the vector field X is holomorphic it follows from the second equation of system (S) that $b_0, b_{l+1},\dots,b_n$ are real constants.

Let us fix an integer j such that $1\le j\le l$.

Differentiating the first equation of system (S) with respect to $\overline{z_j}$ we have:

$$\text{(1)}\qquad -2im_j^2\overline{z}_j^{m_j-1}z_j^{m_j-1}X_j = \overline{\partial}_ja_0.$$

Since X_j is holomorphic there exist:

- A holomorphic function f_j of the variables $z_1,\dots,z_l$ such that $f_{j\,|\{z_j=0\}} = 0$.
- A holomorphic function $\tilde{f}_j$ depending on the variables z_k for $1\le k\le l$, $k\ne j$.

- A real analytic function g_j not depending on $\overline{z}_j$ such that $a_0 = \overline{z_j}^{m_j}(f_j + \tilde{f}_j) + g_j$.

Since a_0 is a real function and f_j is holomorphic we have $f_j(z_1,\ldots,z_l) = \alpha_j z_j^{m_j}$ where α_j is a real constant and $g_j = z_j^{m_j}\overline{\tilde{f}_j} + \tilde{g}_j$ where $\tilde{g}_j$ is a real analytic real function depending neither on z_j nor on $\overline{z_j}$. Then $a_0 = \alpha_j|z_j|^{2m_j} + (\overline{z_j}^{m_j}\tilde{f}_j + z_j^{m_j}\overline{\tilde{f}_j}) + \tilde{g}_j$. Replacing $X_1,\ldots,X_l$ in terms of $\overline{\partial}_j a_0$ (see equation (1)) in the first equation of system (S) we obtain the following equality:

$$X_0(z_0,z) = -\sum_{\substack{k=1\\k\neq j}}^{l}(\alpha_k|z_k|^{2m_k} + \overline{z}_k^{m_k}\tilde{f}_k) + z_j^{m_j}\overline{\tilde{f}}_j + \tilde{g}_j + b_0 z_0 + 2\sum_{k=l+1}^{n} m_k b_k z_k^{2m_k-1}$$

and adding these l equalities for $1 \leq j \leq l$:

$$lX_0 = -(l-1)\sum_{k=1}^{l}(\alpha_k|z_k|^{2m_k}) - l\sum_{k=1}^{l}\overline{z}_k^{m_k}\tilde{f}_k + \sum_{k=1}^{l}(\overline{z}_k^{m_k}\tilde{f}_k + z_k^{m_k}\overline{\tilde{f}}_k) + \sum_{k=1}^{l}\tilde{g}_k + lb_0 z_0 + 2l\sum_{k=l+1}^{n} m_k b_k z_k^{2m_k-1}.$$

Let us identify the holomorphic terms in this expression. Since $\sum_{k=1}^{l}\overline{z}_k^{m_k}\tilde{f}_k$ is not holomorphic, this is a real function and we have:

$$\begin{cases} lX_0 = a + lbz_0 + 2l\sum_{k=l+1}^{n} m_k b_k z_k^{2m_k-1} \\ \sum_{k=1}^{l}\tilde{g}_k = (l-1)\sum_{k=1}^{l}(\alpha_k|z_k|^{2m_k}) + l\sum_{k=1}^{l}\overline{z}_k^{m_k}\tilde{f}_k \\ \qquad + \sum_{k=1}^{l}(\overline{z}_k^{m_k}\tilde{f}_k + z_k^{m_k}\overline{\tilde{f}}_k) \end{cases}.$$

Consequently $\sum_{k=1}^{l}\overline{z}_k^{m_k}\tilde{f}_k = \sum_{k=1}^{l} z_k^{m_k}\overline{\tilde{f}_k}$ and for every $1 \leq j \leq l$:

$$\tilde{f}_j = \sum_{k\neq j} z_k^{m_k}\overline{\left(\frac{\partial_j \tilde{f}_k}{m_j z_j^{m_j-1}}\right)}.$$

Since $\tilde{f}_j$ is holomorphic in z_k for $1 \leq k \leq l$, $k \neq j$ it follows that $\overline{z_k^{m_k}\left(\frac{\partial_j \tilde{f}_k}{m_j z_j^{m_j-1}}\right)}$ is holomorphic: there is a complex constant a_{jk} such that $\partial_j \tilde{f}_k = \overline{a_{jk}} m_j z_j^{m_j-1}$ or:

$$\tilde{f}_j = \sum_{k \neq j} c_{jk} z_k^{m_k}.$$

It is then sufficient to set $b_j = \alpha_j / m_j$ and $c_{jk} = a_{jk}/m_j$ for $1 \leq j \leq l$, $1 \leq k \leq l$, $k \neq j$. □

3. Classification of maps from D_1 to D_2

We recall that the limit map F obtained in Section 1 is locally proper according to Proposition 1.2 part (i). We assume in this section that ∂D_1 has at least one non spherical direction and we use the description of CR infinitesimal automorphisms given by Proposition 2.1 to obtain the following:

Proposition 3.1. *$F(0)$ is a finite point in ∂D_2.*

Proof of Proposition 3.1: According to Proposition 1.2 parts (iii)–(iv) the map $G = F^{-1}$, locally defined at 0 with $G(0) = p$, is an algebraic map. Since the vector field $G_\star(\frac{\partial}{\partial z_0})$ is a CR infinitesimal automorphism at p the map G satisfies:

- In case 1 of Proposition 2.1:

$$\text{(S1)} \quad \begin{cases} \dfrac{\partial G_0}{\partial z_0} = a_0 + b_0 G_0 + 2\sum_{k=1}^{n} m_k b_k G_k^{2m_k - 1} & \\ \dfrac{\partial G_j}{\partial z_0} = 2 m_j b_j G_j - \frac{i}{2} b_j & \forall\, j \in \{1, \ldots, n\} \end{cases}.$$

- In case 2 of Proposition 2.1:

$$\text{(S2)} \quad \begin{cases} \dfrac{\partial G_0}{\partial z_0} = a_0 + b_0 G_0 + 2\sum_{k=l+1}^{n} m_k b_k G_k^{2m_k - 1} & \\ \dfrac{\partial G_j}{\partial z_0} = \frac{i}{2}\left(b_j G_j + \sum_{\substack{k=1 \\ k \neq j}}^{l} c_{jk} \dfrac{G_k^{m_k}}{G_j^{m_j-1}}\right) & \forall\, j \in \{1, \ldots, l\} \\ \dfrac{\partial G_j}{\partial z_0} = 2 m_j b_0 G_j - \frac{i}{2} b_j & \forall\, j \in \{l+1, \ldots, n\} \end{cases}.$$

Since system (S1) can be considered as a subsystem of system (S2) we will focus on the resolution of system (S2). The map G being algebraic the resolution of the equation $\dfrac{\partial G_j}{\partial z_0} = 2m_j b_0 G_j - \dfrac{i}{2} b_j$ implies that $b_0 = 0$. Moreover there exist functions $\tilde{G}_0, \ldots, \tilde{G}_n, \tilde{G}_{jk}$ $(1 \leq j \leq l, 0 \leq k \leq l-1)$, holomorphic in a neighborhood U' of 0 in $\mathbb{C}^n$, such that:

$$(S3) \begin{cases} G_0(z_0, z) = az_0 & \\ \qquad +2i \sum_{l+1}^n (\frac{-i}{2} b_j z_0 + \tilde{G}_j(z))^{2m_j} + \tilde{G}_0(z) & \\ (G_j(z_0, z))^{m_j} = \sum_{k=0}^{l-1} \tilde{G}_{jk}(z) z_0^k & \forall j \in \{1, \ldots, l\} \\ G_j(z_0, z) = \frac{-i}{2} b_j w + \tilde{G}_j(z) & \forall j \in \{l+1, \ldots, n\} \end{cases}.$$

Let us write for every integer j in $\{1, \ldots, n\}$ $H_j(z_j + p_j, \overline{z}_j + \overline{p}_j) = Q_j(z_j, \overline{z}_j) + \operatorname{Im}(S_j(z_j, \overline{z}_j)) + H_j(p_j, \overline{p}_j)$ where Q_j is a real subharmonic polynomial without harmonic term and S_j is a holomorphic polynomial without constant term. If T is the transformation:

$$T(z_0, z) = \left(z_0 - p_0 + \sum_{j=1}^n S_j(z_j - p_j, \overline{z}_j - \overline{p}_j), z_1 - p_1, \ldots, z_n - p_n \right)$$

and D_1' is the domain $D_1' = \{(z_0, z) \in \mathbb{C}^{n+1} : \operatorname{Im} z_0 + \sum_{j=1}^n Q_j(z_j, \overline{z}_j) < 0\}$ then we have the equivalence:

$$(z_0, z) \in D_1 \Leftrightarrow T(z_0, z) \in D_1'.$$

Let us denote $T \circ G = g = (g_0, \ldots, g_n)$. By assumption $g(0) = 0$. For every z^0 in U' the function g is well defined on the half plane $\Lambda_{z^0} = \{(z_0, z) \in \mathbb{C}^{n+1} : \operatorname{Im} z_0 < -\sum_{j=1}^n P_j(z_j^0, \overline{z_j^0}),\ z = z^0\}$ and $(F \circ T^{-1}) \circ g$ is the identity in a neighborhood of the origin on Λ_{z^0}.

Assume that there is a point (z_0^0, z^0) on Λ_{z^0} and a path γ in Λ_{z^0} connecting 0 and (z_0^0, z^0) (i.e. $\gamma(0) = 0$, $\gamma(1) = (z_0^0, z^0)$) such that $g(\gamma(t))$ belongs to D_1' for t in $]0, 1[$ and $\lim_{t \to 1} g(\gamma(t)) \in \partial D_1'$. Since $(F \circ T^{-1}) \circ g = \mathrm{id}$ in a neighborhood of $\gamma(]0, 1[)$ we may assume that $g(\gamma(t))$ converges to a finite point in $\partial D_1'$ when t converges to 1. Its image by the local proper map $F \circ T^{-1}$ (see Proposition 1.2 (i)) is (z_0^0, z^0): this is a contradiction and thus the set $g(\Lambda_{z^0})$ is contained in D_1'.

Assume now that either $\tilde{G}_{jk}(z^0) \neq 0$ for some (j, k) with $1 \leq j \leq l$, $0 \leq k \leq l-1$ or $b_j \neq 0$ for some j with $l+1 \leq j \leq n$. According to system (S3) the restriction of g to Λ_{z^0} is a polynomial map of the variable z_0 and the restriction of the function $\operatorname{Im} g_0 + \sum_{j=1}^n Q_j(g_j, \overline{g}_j)$ to Λ_{z^0} is a negative subharmonic polynomial function. Its weighted homogeneous part of largest degree, given by the terms of largest degree of the polynomial

$$2\operatorname{Im}\sum_{j=l+1}^{n}\left(\frac{-i}{2}b_j z_0\right)^{2m_j}+\sum_{j=1}^{l}\left|\sum_{k=0}^{l-1}\tilde{G}_{jk}(z^0)z_0^k\right|^{2m_j} + \sum_{j=l+1}^{n}H_j\left(\frac{-i}{2}b_j z_0, \frac{i}{2}b_j\overline{z_0}\right),$$

is a subharmonic non harmonic function on Λ_{z^0}. By Lemma 1.2 of [**2**] this is positive in some directions. Since its limit at infinity in these directions is infinite it follows that $\operatorname{Im} g_0+\sum Q_j(g_j,\overline{g}_j)$ is not a negative function on Λ_{z^0}: this is a contradiction.

Consequently $b_j = 0$ for $l+1 \leq j \leq n$ and $\tilde{G}_{jk}$ is identically zero on U' for $1 \leq j \leq l$, $0 \leq k \leq l-1$. Then $g(z_0,z) = (az_0 + h_0(z), h_1(z),\dots,h_n(z))$, where $h_0,h_1,\dots,h_n$ are holomorphic functions defined locally at 0 in $\mathbb{C}^n$. Moreover there is a real analytic function λ defined in a neighborhood of 0 in $\mathbb{C}^{n+1}$ with $\lambda(0)=0$ such that we get locally at 0:

$$\operatorname{Im} g_0(z_0,z)+\sum_{j=1}^{n}Q_j(g_j(z_j),\overline{g}_j(z_j)) = \lambda(z_0,z)\left(\operatorname{Im} z_0+\sum_{j=1}^{n}P_j(z_j,\overline{z}_j)\right).$$

By setting $z_0=0$ in this equation we obtain, since $h_j(0)=0$ and Q_j, P_j have no harmonic terms, that h_0 is identically zero. Thus we get: $g(z_0,z)=(az_0,h_1(z),\dots,h_n(z))$, with $h_j(0)=0$. According to the implicit function theorem, there is a complex constant a' and holomorphic functions $\tilde{f}_1,\dots,\tilde{f}_n$, locally defined at the origin, such that:

$$(F\circ T^{-1})(z_0,z)=(a'z_0,\tilde{f}_1(z),\dots,\tilde{f}_n(z)).$$

Since $F\circ T^{-1}$ is globally defined on D_1', the functions $\tilde{f}_1,\dots,\tilde{f}_n$ are holomorphic on $\mathbb{C}^n$. For every $z=(z_1,\dots,z_n)$ in $\mathbb{C}^n$, the point $(-i\sum_{j=1}^n Q_j(z_j,\overline{z_j}),z)$ belongs to $\partial D_1'$ and so the point $F\circ T^{-1}((-i\sum_{j=1}^n Q_j(z_j,\overline{z_j}),z))$ belongs to ∂D_2. Thus we have for $z=(z_1,\dots,z_n)$ in $\mathbb{C}^n$:

$$\sum_{j=1}^{n}P_j(\tilde{f}_j(z),\overline{\tilde{f}_j(z)}) = \operatorname{Im}(a')\sum_{j=1}^{n}Q_j(z_j,\overline{z_j}).$$

Let k be an integer between 1 and n and, for every l different from k, z_l^0 be a fixed complex number. If we set $z^k=(z_1^0,\dots,z_{k-1}^0,z_k,z_{k+1}^0,\dots,z_n^0)$ then, for every integer j in $\{1,\dots,n\}$, the function $z_k\mapsto P_j(\tilde{f}_j(z^k),\overline{\tilde{f}_j(z^k)})$

is subharmonic on $\mathbb{C}$ and its Laplacian has an algebraic growth; this is a subharmonic polynomial. So there are an integer k_0 and complex constants $\alpha_{s,t}$, defined for (s,t) with $1 \leq s,t \leq k_0$, such that:

$$P_j(\tilde{f}_j(z), \overline{\tilde{f}_j(z)}) = \sum_{1\leq s,t\leq k_0} \alpha_{s,t} z^s \overline{z}^t. \tag{2}$$

If $P_j(z_j, \overline{z_j}) = \sum_{1\leq s,t\leq 2k} a_{s,t} z_j^s \overline{z_j}^t$ a polarization of equation (2) implies for (z_k, ζ) in $\mathbb{C}^2$:

$$\sum_{1\leq s,t\leq 2k} a_{s,t}(\tilde{f}_j(z^k))^s(\overline{\tilde{f}_j}(\overline{\zeta}^k))^t = \sum_{1\leq s,t\leq k_0} \alpha_{s,t} z_k^s \zeta^t$$

with $\zeta^k = (z_1^0, \ldots, z_{k-1}^0, \zeta, z_{k+1}^0, \ldots, z_n^0)$.

Hence $\displaystyle\sum_{1\leq s,t\leq 2k} a_{s,t} \frac{d^{k_0+1}}{dz^{k_0+1}}((\tilde{f}_j(z^k))^s)(\overline{\tilde{f}_j}(\overline{\zeta}^k))^t$ is identically zero on $\mathbb{C}^2$. Considering this as a polynomial in $\overline{\tilde{f}_j}(\overline{\zeta}^k)$ and since P_j is not identically zero, there is an integer s between 1 and $2k$ such that $\dfrac{d^{k_0+1}}{dz^{k_0+1}}((\tilde{f}_j(z^k))^s)$ is identically zero on $\mathbb{C}$: the map $z_k \mapsto (\tilde{f}_j(z^k))^s$ is a polynomial. Hence the holomorphic function $z_k \mapsto \tilde{f}_j(z^k)$, globally defined on $\mathbb{C}$, is polynomial and thus the map $F \circ T$ is polynomial. Finally the point $F(0) = (F \circ T)(T^{-1}(0))$ is a finite point in ∂D_2. This proves Proposition 3.1. □

4. The end of the proof of Theorem 1

We proved in Section 3 that if ∂D_1 has at least one non spherical direction then $F(0)$ is a finite point. Theorem 1 is then a consequence of Proposition 1.4. Assume now that the hypersurface ∂D_1 is spherical at point p. We may write: $D_1 = \{(z_0, z) \in \mathbb{C}^{n+1} : \operatorname{Im} z_0 + \sum_{j=1}^n |z_j|^{2m_j} < 0\}$. By Proposition 1.2 part (iii) the domain D_2 is spherical at the origin. Hence by a work of A. V. Isaev [**15**] the polynomial $P = \sum_{j=1}^n P_j$ satisfies a system of partial differential equations:

$$\frac{\partial^2 P}{\partial z_j \partial z_k} = \sum_{l=1}^n \frac{\partial P}{\partial z_l}\left(E^l \frac{\partial P}{\partial z_j}\frac{\partial P}{\partial z_k} + D_j^l \frac{\partial P}{\partial z_k} + D_k^l \frac{\partial P}{\partial z_j} + C_{jk}^l\right) + H_{jk}$$

where E^l, D_j^l, C_{jk}^l and H_{jk} are holomorphic functions defined in a neighborhood of the origin. It follows by setting $z_k = 0$ for $k \neq j$ that the polynomial P_j satisfies an equivalent system and the domain $\{(z_0, z_j) \in \mathbb{C}^2 : \operatorname{Im} z_0 + P_j(z_j, \overline{z}_j) < 0\}$ is also spherical in $\mathbb{C}^2$ for every j in $\{1, \ldots, n\}$ by Proposition 2.2 of [**15**]. Since P_j has no pure anti-holomorphic term we obtain by a comparison of the terms of maximal degree in $\overline{z_j}$ that $E^j = D^j_j = H_{jj} = 0$. Thus P_j satisfies the following system in $\mathbb{C}^2$:

$$\frac{\partial^2 P_j}{\partial z_j^2}(z_j, \overline{z_j}) = C^j_{jj}(z_j)\frac{\partial P_j}{\partial z_j}(z_j, \overline{z_j})$$

where C^j_{jj} is a holomorphic function. An integration shows that there exists a holomorphic polynomial h_j such that $P_j(z_j, \overline{z_j}) = |h_j(z_j)|^2$. Since D_1 is biholomorphic to the ellipsoïd $\{(z_0, z) \in \mathbb{C}^{n+1} : |z_0|^2 + \sum_{j=1}^n |z_j|^{2m_j} < 1\}$ one can prove as in Proposition 3.1 of [**8**] that the map F is a proper map from D_1 to D_2. Thus if g^1 and g^2 are the maps defined in $\mathbb{C}^{n+1}$ by $g^1 \colon (z_0, z) \mapsto (z_0, z_1^{m_1}, \ldots, z_n^{m_n})$ and $g^2 \colon (z_0, z) \mapsto (z_0, h_1(z_1), \ldots, h_n(z_n))$, there is a proper holomorphic auto correspondence $\tilde{F}$ of the unbounded representation $\mathbb{H}$ of the unit ball of $\mathbb{C}^{n+1}$ such that: $\tilde{F} \circ g^1 = g^2 \circ F$ ([**1**]). Every component of its graph is the graph of an automorphism of $\mathbb{H}$. Since $F^{-1}(\infty)$ is contained in $(\tilde{F} \circ g^1)^{-1}(\infty)$ and every automorphism of $\mathbb{H}$ extends to a homeomorphism from $\overline{\mathbb{H}} \cup \{\infty\}$ onto $\overline{\mathbb{H}} \cup \{\infty\}$ we obtain that $F^{-1}(\infty)$ is compact in ∂D_1. Theorem 1 is then given by Proposition 1.4 condition (ii). □

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