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THE SPAN AND THE STABLE SPAN OF A MANIFOLD

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Given a closed connected smooth n-dimensional manifold M, it is interesting to compare its span (i.e. the maximum number of linearly independent tangent vector fields) to its stable span (i.e. the maximum number k such that $TM \oplus \mathbb{R}$ allows $k+1$ linearly independent sections). The example of the s-sphere S^n shows that these two numbers can differ dramatically: stable span $(S^n) = n$ while span S^n is the Hurwitz-Radon number $h(n)$, e.g. $h(n) = 0$ for n even, $h(n) = 1$ for $n \equiv 1(4)$, $h(n) = 3$ for $n \equiv 3(8)$ ... It has been known since long that for general M^n often span(M) equals either the stable span of M or else the span $h(n)$ of the sphere S^n of the same dimension (see e.g. [1] for the case when M is stably parallelisable and [2] for many other cases), so it was widely believed that this should always hold. In our talk we disprove this conjecture. First we define an integer $s(M)$ $(\geq \text{span}(S^n))$ which can sometimes be calculated, e.g. $s(M) = 0$ iff n is even; $s(M) = 1$ iff $n \equiv 1(4)$ and $w_1(M)^2 = 0$; $s(M) = 2$ iff $n \equiv 1(4)$, $w_1(M)^2 \neq 0$ but $w_1(M)^2 = yw_1(M) + y^2$ for some $y \in H^1(M; \mathbb{Z}_2)$; etc. Then we use the singularity approach to vectorfield problems (see [3]) to show that most of the time $s(M)$ is the correct alternative: the span of M is equal either to $s(M)$ or to the stable span of M. As an example we exhibit an infinite family of manifolds of the form $M^n = P^r \times S^q$ such that span(M) differs from stable span (M), from span (S^n) and also from the span of the factor sphere S^q, but span(M) = s(M) = 4. We give many other counterexamples to the conjecture mentioned above, including oriented ones.

(More details are given in the last section of [3]).

References

[1] G. Bredon and A. Kosinski, Vectorfields on π-manifolds, Ann. Mat. 84 (1966), 85-90.

[2] V. Eagle, Rutgers Ph. D. thesis, 1978, (unpublished).

[3] U. Koschorke Vectorfields and other vector bundle morphisms - a singularity approach, Lect. Notes in Math., vol. 847, Springer-Verlag, 1981.

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