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HOPF ALGEBRAS AND GALOIS DESCENT

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Knus and Ojanguren say in |5| that: "some of the methods used in Galois theory and in the radical theory may be formulated in the Hopf algebras framework. This analogy led to Chase and Sweedler to define a Galois notion of object in terms of Hopf algebras which generalizes both cases. They don't formulate a corresponding theory of descent". The purpose of this notes is to develop this theory, using as Galois definition of object, the one given by Chase and Sweedler in |2|, and following the line of exposition in |5|.

We define a Galois descent data over a Hopf algebra H for a S-module M, being S an Galois H-object, as a homomorphism of algebras $h: H^* \longrightarrow End_k(M)$ satisfying a condition of "semilinearity". Starting with this definition it is obtained as a particular case, the Galois descent for groups, simply by taking. $H = kG^*$, where k is a commutative ring, G the finite group of Galois extension S of k and kG^* the dual of its group algebra. The characterization of the descended object given in section 2, is useful for several questions related to Galois H-objects studied in a different way in |2|.

In section 3, once given, H as a Hopf algebra and A as an H-module algebra, it is defined the set of cohomology of H with coefficients in A, $H^1(H,A)$ with a distinghished element, the class of the unit element of Reg(H,A). If S is a Galois H-object, then $H^1(H, Hom_k(N, S \otimes N))$ classifies the twisted forms of N over S.

All through this notes K is a fixed commutative ring with 1, each $\otimes$, *Hom*, etc. is taken over K.

1. Preliminaries

DEFINITION 1.1. A *Hopf algebra* (over k) is a k-module together with the following structures $\mu: H \otimes H \longrightarrow H$ multiplication, $\Delta: H \longrightarrow H \otimes H$ diagonalization η: k $\longrightarrow H$ unit, $\epsilon: H \longrightarrow$ k counit so that (H, μ, η) is a k-algebra, (H, Δ, ϵ) is a k-coalgebra; μ and η are homomorphisms of coalgebras, or equivalently Δ and ϵ are homomorphisms of algebras.

A Hopf algebra H is said to have *antipode* or inverse if there is a homomorphism of k-modules $\lambda: H \longrightarrow H$ such that $\mu \cdot (H \otimes \lambda) \cdot \Delta = \eta \cdot \epsilon = \mu \cdot (\lambda \otimes H) \cdot \Delta$; If $\tau: H \otimes H \longrightarrow H \otimes H$ is the map which interchanges the factors, then H is said to be *commutative (cocommutative)* if $\mu \cdot \tau = \mu$, $(\tau \cdot \Delta = \Delta)$. Moreover $\lambda: H \longrightarrow H$ verifies $\tau \cdot \Delta \cdot \tau = (\lambda \otimes \lambda) \cdot \Delta$ and $\lambda \cdot \mu \cdot \tau = \mu(\lambda \otimes \lambda)$.

DEFINITION 1.2. A *finite Hopf algebra* is a Hopf algebra H with antipode which is finitely generated and projective as K-module.

DEFINITION 1.4. Let H be a Hopf algebra, and S be an H-object, we define the algebra homomorphism. $\gamma_S: S \otimes S \longrightarrow S \otimes H$ by the formula $\gamma_S(x \otimes y) = (x \otimes 1_H) \cdot a_S(y) = \sum_{(y)} x y_{(1)} \otimes y_{(2)}$.

S will be called a *Galois H-object* if the following conditions are satisfied:

a) S is a faithfully flat K-module

b) γ_S is an isomorphism.

PROPOSITION 1.5. *If H is a finite Hopf Algebra, then $H^* = Hom(H,K)$ is also a finite Hopf algebra.* |2| (7.1).

If H is a finite Hopf algebra and S is a K-module then we have the following:

$Hom(S, S \otimes H) \cong Hom(S, Hom(H^*, S)) \cong Hom(H^* \otimes S, S)$ the first isomorphism arising from the fact that H is a finitely generated projective K-module, the second is the adjointness isomorphism. In particular if S is an H-object, we may apply this isomorphism to the element a_S of $Hom(S, S \otimes H)$ to obtain a map $b_S : H^* \otimes S \longrightarrow S$, which we shall write as $b_S(g \otimes x) = g(x) = \sum_{(x)} x_{(1)} \langle g_1, x_{(2)} \rangle$. It is easy to see that b_S is a H^*-module structure for S, |2|.

DEFINITION 1.6. Let H be Hopf algebra, a K-algebra S which is a left H-module is called a *H-module algebra* if the structure morphism (unit and multiplication) are H-module morphisms.

If S is an H-object then it is a H^*-module algebra.

DEFINITION 1.7. Let H be a Hopf algebra and A a left H-module algebra. We define an algebra $A \# H$ as follows:

$A \otimes H$ as a K-module, the multiplication is defined by setting $(x \# g) \cdot (y \# h) = \sum_{(g)} x \cdot g_{(1)}(y) \# g_{(2)} h$ (we write $x \# g$ for $x \otimes g$ when thought of as and element of $A \# H$, $x \in A$, $g \in H$). Where $g_{(1)}(y)$ is the H-module structure of A, and unit $1_A \# 1_H$.

$A \# H$ is called *the smash product* of A and H.

In A, a structure of $A \# H$-module is defined as $(x \# g)(y) = x \cdot g(y)$ $x, y \in A$, $g \in H$.

TEOREMA 1.8. *If H is a finite commutative Hopf algebra and S is any H-object, then the statements below are equivalent.*

a) *S is a Galois H-object*

b) *S is a finitely generated faithful projective K-module, and the mapping $\psi : S \# H^* \longrightarrow End(S)$ arising from the left $S \# H^*$-module structure on S, is an isomorphism of algebras* |2| Th 9.3.

2. Galois Descent

In this section H will be a finite commutative Hopf algebra, S is a Galois H-object and M is a left S-module.

DEFINITION 2.1. A K-algebra homomorphism $h : H^* \longrightarrow End(M)$, $g \longmapsto hg$ satisfying the "*semilinearity*" condition expressed by $hg(x \cdot m) = \sum_{(g)} g_{(1)}(x) \cdot hg_{(2)}(m)$ with $g \in H^*$, $x \in S$, $m \in M$, is said to be a *Galois descent data* for M over S.

PROPOSITION 2.2. *Each Galois descent data for M over S de-*

termines a faithfully projective descent data of M over S. Coversely each faithfully projective descent data determines one of Galois.

These processes are inverse to each other.

Proof. Given $h : H^* \longrightarrow End(M)$, we define

$t : End(S) \longrightarrow End(M)$

$$t : ENd(S) \xrightarrow{\psi-1} S \# H^* \xrightarrow{S\otimes h} S \otimes End(M) \xrightarrow{\theta} End(M)$$

where ψ is the algebra isomorphism of theorem (1.8) defined as $\psi(s \# g)(x) = s \cdot g(x)$ and $\theta(x \otimes f)(m) = x \cdot f(m)$ is a S-module structure for $End(M)$.

To check that t is a faithfully projective descent data amounts to seeing that t is an K-algebras homomorphism and homomorphism of left and right S-modules

t will be an K-algebra homomorphism if

$\theta \cdot (S \# h) = \theta' : S \# H^* \longrightarrow End(M)$ is an K-algebra homomorphism.

If $x, y \in S$, $g, p \in H^*$, then

$$\theta'(\sum_{(g)} x \cdot g_{(1)}(y) \# g_{(2)}p)(m) \qquad \text{by definition of } \theta'$$

$$= \sum_{(g)} x \cdot g_{(1)}(y) \cdot \left| h g_{(2)} p(m) \right| \qquad \text{because } h \text{ is a } K\text{-algebra homomorphism}$$

$$= \sum_{(g)} x \cdot g_{(1)}(y) \cdot \left| h g_{(2)}(h_p(m)) \right| \qquad \text{because of "semilinearility"}$$

$$= x \cdot hg \left| y \cdot h_p(m) \right| = \theta'(x \# g) \cdot \left| \theta(y \# p)(m) \right| \qquad \text{by definition of } \theta'$$

The verification that t is a left and right S-module homomorphism is immediate.

Conversely, given $t : End(S) \longrightarrow End(M)$ a faithfully projective descent data, we know that there exist N, a K-module, and $\ell : S \otimes N \longrightarrow M$ an S-module isomorphism |5|.

We define $h : H^* \longrightarrow End(M)$ as

(2.2.a) $hg(m) = \ell(\sum_i g(x_i) \otimes n_i)$ where $\sum_i x_i \otimes n_i = \ell^{-1}(m)$ with $m \in M$, $g \in H^*$

From the relation between ℓ and t, given $u \in End(S)$, $\forall m \in M$ we obtain that $t_u(m) = \ell(\sum_i u(x_i) \otimes n_i)$ where $\ell^{-1}(m) = \sum_i x_i \otimes n_i$ deduced from |5| the 4.3.; moreover $h = t \cdot \psi \cdot (n_S \otimes H^*)$ because it is a composition of K-algebra homomorphisms, is an K-algebra homomorphism, and

$$\sum_{(g)} g_{(1)}(x) \cdot hg_{(2)}(m)$$ by definition of h

$$=$$

$$\sum_{(g)} g_{(1)}(x) \cdot \ell \left| \sum_i g_{(2)}(x_i) \otimes n_i \right|$$ because ℓ is a S-module homomorphism

$$=$$

$$\sum_{(g)} \ell(\sum_i g_{(1)}(x) \cdot g_{(2)}(x_i) \otimes n_i)$$ because μ_S is a H-module homomorphism (1.6)

$$=$$

$$\ell \left| \sum_i g(x \cdot x_i) \otimes n_i \right| = h_g(x \cdot m)$$ by definition of h, and because ℓ is a S-module isomorphism

Then h verifies the condition of semilinearity. Both processes are trivially inverse to each other.

In |5| up to isomorphism, it is defined the K-module N such that $S \otimes N \cong M$ by $N = \{m \in M \mid tu(x \cdot m) = u(x) \cdot m \quad \forall x \in S, \forall u \in End(S)\}$

COROLARIO 2.3. *In the conditions of theorem 2.2 if* $N' = \{m \in M / hq(m) = \varepsilon(g) \cdot m \quad \forall q \in H^*\}$ *then* $N = N'$

Proof. Given $m \in N$, then for every q in H^* we obtain

$hq(m)$ (by definition of h (2.2.))

$\|$

$t\psi_{(1_S \# q)}(1_S \cdot m)$ (because $m \in N$)

$\|$

$[\psi_{(1_S \# q)}(1_S)] \cdot m$ (by definition of ψ)

$\|$

$q(1_S) \cdot m$ (because η_S is an H^*-module homomorphism)

$\|$

$\varepsilon(q) \cdot m$ then $N \in N'$

Given $m \subset N'$, then for any x in S and any u in $End(S)$ we have

$t_u(x \cdot m)$ (by definition of t (2.2))

$[\theta \cdot (S \otimes h) \cdot \psi^{-1}]_u (x \cdot m)$ (supposing $\psi^{-1}(u) = \sum_i x_i \# q_1$)

$\|$

$[\theta\, (\sum_i x_i \# hq_i)\ (x \cdot m)$ (by definition of θ (2.2))

$\|$

$\sum_i x_i \cdot hq_i(x \cdot m)$ (by semilinearity of h)

$\|$

$\sum_i x_i \sum_{(q)} q_{i_{(1)}}(x) hq_{i_{(2)}}(m)$ (because $m \in N'$)

$\|$

$\sum_i x_i \sum_{(q)} g_{i_{(1)}}(x)\ \ \varepsilon(q_{i_{(2)}}) \cdot m$ (because H^* is a Hopf algebra)

$\|$

$\sum_i x_i q_i(x) \cdot m$ (by definition of ψ)

$\|$

$\psi(\sum_i x_i \# q_i)(x) \cdot m = u(x) \cdot m$ then $N' \subset N$

PROPOSITION 2.4. *In the hipothesis of (2.2), if* M *is an* S-*algebra with structures* $\mu_M : M \otimes_S M \longrightarrow M$ *and* $\eta_M : S \longrightarrow M$ *and* *the Galois descent data verifies the conditions:*

a) $hq(m_1 \cdot m_2) = \sum_{(q)} hq_{(1)}(m_1) \cdot hq_{(2)}(m_2) \quad m_1, m_2 \in M \quad q \in H^*$

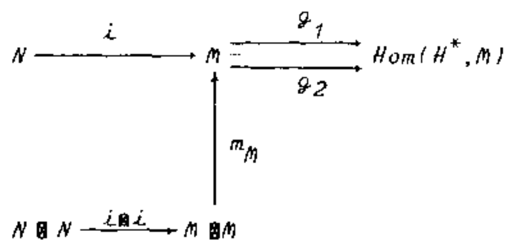
b) $hq(1_M) = \varepsilon(q) \cdot 1_M \qquad q \in H^*$

Then N *is a K-algebra and* $f : S \; N \longrightarrow M$ *is an S-algebra isomorphism.*

Conversely if N *is an K-algebra and* f *is a K-algebra isomorphism, then* h *verifies the conditions a) and b).*

Proof. Using the relation between h and f, the second part of the proposition is immediate

For the first one, we consider the diagram:



where the upper line is an equalizer by (2.3)

$$g_1(m) : H^* \longrightarrow M$$
$$\qquad\quad q \longrightarrow hq(m)$$

$$g_2(m) : H^* \longrightarrow M$$
$$\qquad\quad q \longrightarrow \varepsilon(q) \cdot m$$

$$m_M : M \otimes M \longrightarrow M \otimes_S M \xrightarrow{\mu_M} M$$

and i is the inclusion of N in M.

Moreover, for any n_1 and n_2 in N and q in H^* we have got that

$g_1[m_M(n_1 \otimes n_2)] = hq(n_1 \cdot n_2)$	by a)
$\sum_{(q)} hq_{(1)}(n_1) \cdot hq_{(2)}(n_2)$	by definition of N
$\sum_{(q)} \varepsilon(q_{(1)})n_1 \cdot \varepsilon(q_{(2)})n_2$	because H^* is a Hopf algebra
$\varepsilon(q) \cdot n_1 \cdot n_2 = g_2[m_M(n_1 \otimes n_2)]$	Then there exists
$\mu_N : N \otimes N \longrightarrow N$	The rest of the proof is easy

3. H^1 and twisted forms

In the section S indicates a commutative K-algebra.

DEFINITION 3.1. Le N be a K-module, possibly with some additional algebraic structure. *A twist form* of N for the extension S is an isomorphism class of K-modules $[B]$ with the same type of structure as N has, and such that for each representative B, there exists $f_B : S \otimes B \longrightarrow S \otimes N$ an S-module isomorphism.

DEFINITION 3.2. If B is a coalgebra, A an algebra, and $f, g \in Hom(B,A)$ then we may define f *convolution* $g = f \wedge g$ as $f \wedge g : \mu_A \cdot (f \otimes g) \cdot \Delta_B$

With this operation $Hom(B,A)$ has monoide structure whose unit element is $\eta_A \cdot \varepsilon_B$. If B is a cocommutative coalgebra and A is a commutative algebra, then $Hom(B,A)$ is a commutative monoide.

The group of invertible elements of $Hom(B,A)$ for the convolution is denoted by $Reg(B,A)$

DEFINITION 3.3. Let H be a Hopf algebra and A an H-module algebra; we consider the simplicial complex

$$Reg(K,A) \underset{\delta^1}{\overset{\delta^0}{\rightrightarrows}} Reg(H,A) \underset{\delta^2}{\overset{\delta^0}{\equiv}} Reg(H \otimes H,A) \rightrightarrows \cdots$$

$$\delta^0 : Reg(\overset{n-1}{\otimes} H,A) \longrightarrow Reg(\overset{n}{\otimes} H,A)$$

$$f \longrightarrow \psi_A \cdot (H \otimes f)$$

where ψ_A is the structure of A as H-module.

$$\delta^i : Reg(\overset{n-1}{\otimes} H,A) \longrightarrow Reg(\overset{n}{\otimes} H,A)$$

$$f \longrightarrow f \cdot (H \otimes \dots \otimes \mu_H \otimes H \otimes \dots \otimes H)$$

where μ_H is in the i-esima position

$i = 1 \dots n-1$

$$\delta^n : Reg(\overset{n-1}{\otimes} H,A) \longrightarrow Reg(\overset{n}{\otimes} H,A)$$

$$f \longrightarrow f \otimes \epsilon$$

We define a *1-cocycle* $f \in Reg(H,A)$ as that one for which $\delta^1 f = \delta^0 f \wedge \delta^2 f$ or what is the same $\forall q,p \in H$ $fpq = \sum_{(q)} (q_{(1)} (f_p)) \cdot fq_{(2)}$.

We will say that two 1-coclycles $f,g \in Reg(H,A)$ are *cohomologous* if there exists $v \in Reg(K,A)$ such that $\delta^0 v \wedge f = g \wedge \delta^1 v$.

The previous relation is an equivalence relation. The set of equivalence classes is denoted by analogy with the abelian case $H^1(H,A)$. It is a set with distinguished element the class of $\eta_A \cdot \epsilon_H$, identity element of $Reg(H,A)$

PROPOSITION 3.4. *Let H^* be a Hopf algebra and (S, b_S) a commutative H^*-module algebra. (For example if S is a Galois H-object). Then $\mathcal{E}' = Hom(N, S \otimes N) \cong End_S(S \otimes N)$ is a H-module algebra.*

Proof. We define $\eta_{\mathcal{E}'} : K \longrightarrow \mathcal{E}'$ as
$$N \longrightarrow S \otimes N$$
$$n \longrightarrow 1_S \otimes n$$

$$\mu_{\mathcal{E}'} : \mathcal{E}' \otimes \mathcal{E}' \longrightarrow \mathcal{E}'$$

$$(u \otimes v) \longrightarrow u \cdot v : N \longrightarrow S \otimes N$$

$$n \longrightarrow \sum_{i,j} x_i y_i \otimes n_{ij}$$

where $v(n) = \sum_i x_i \otimes n_i$ and $u(n_i) = \sum_j y_j \otimes n_{ij}$

$$b_{\mathcal{E}'} : H^* \otimes \mathcal{E}' \longrightarrow \mathcal{E}'$$

$$q \otimes u \longrightarrow q(u) : N \longrightarrow S \otimes N$$

$$n \longrightarrow \sum_i q(x_i) \otimes n_i$$

where $\sum_i x_i \otimes n_i = u(n)$.

It is a simple computation to check that with these morphisms so defined, $Hom(N, S \otimes N)$ is a H^*-module algebra.

To the Amitsur complex associated to the commutative K-algebra S,

$$K \longrightarrow S \underset{n_2}{\overset{n_1}{\rightrightarrows}} S \otimes S \; \substack{\longrightarrow \\ \longrightarrow \\ \longrightarrow} \; S \otimes S \otimes S \; \ldots$$

we apply the functor $Aut\_(-\otimes N)$ which associated the group $Aut_R(R\otimes N)$ to each commutative K-algebra R and we obtain

$$Aut_K(N) \xrightarrow{\partial^0} Aut_S(S\otimes N) \underset{\partial^1}{\overset{\partial^0}{\rightrightarrows}} Aut_{S\otimes S}(S\otimes S\otimes N) \underset{\partial^2}{\overset{\partial^0}{\equiv}} \cdots$$

A 1-coclycle f is defined as an element of $Aut_{S\otimes S}(S\otimes S\otimes N)$ such taht $\partial^1(f) = \partial^0 f\cdot \partial^2 f$. It is said that two 1-cocycles $f, g \in Aut_{S\otimes S}(S\otimes S\otimes N)$ are cohomologous if there exists $\nu \in Aut_S(S\otimes N)$ such that $\partial^0\nu\cdot f = g\cdot\partial^1\nu$. The previous relation is of equivalence over the set 1-coclycles. The set of cohomology classes is denoted by $H^1(S/K, Aut(-\otimes N))$ and has as distinguished element the class of the identity in $Aut_{S\otimes S}(S\otimes S\otimes N)$.

PROPOSITION 3.5. *In the hypothesis of (3.4), there are group homomorphisms* w_i *such that following diagrams*

$$\begin{array}{ccccccc}
Aut_S(S\otimes N) & \underset{\partial^1}{\overset{\partial^0}{\rightrightarrows}} & Aut_{S\otimes S}(S\otimes S\otimes N) & \underset{\partial^2}{\overset{\partial^0}{\equiv}} & Aut_{S\otimes S\otimes S}(S\otimes S\otimes S\otimes N) & \vdots\cdots \\
\downarrow w_0 & & \downarrow w_1 & & \downarrow w_2 & \\
Reg(K,\mathcal{E}') & \underset{\partial^1}{\overset{\partial^0}{\rightrightarrows}} & Reg(H^*,\mathcal{E}') & \underset{\partial^2}{\overset{\partial^0}{\equiv}} & Reg(H^*\otimes H^*,\mathcal{E}') & \vdots\cdots
\end{array}$$

are commutatives, i.e.

$$\delta_0^i\cdot w_{n-1} = w_n\cdot\partial^i$$

Proof. We define $w_n(f) : H^*\otimes\ldots\otimes H^* \longrightarrow Hom(N, S\otimes N)$

$$h_1\otimes h_2\otimes\ldots\otimes h_n \longrightarrow \theta$$

where $\theta : N \longrightarrow S \otimes N$

$$m \longrightarrow \sum_i x_i^1 \cdot h_1 [x_i^2 h_2 [\ldots x_i^{q-1} h_{n-1} [x_i^q h_n [x_i^{q+1}]] \otimes m_i$$

being $\sum_i x_i^1 \otimes x_i^2 \otimes \ldots \otimes x_i^{q+1} \otimes m_i = \mathcal{L}(1_S \otimes \ldots \otimes 1_S \otimes m)$

The proof of the proposition is not essentially different to the one given by Sweedler in |8|.

Let be a finite commutative Hopf algebra, S a Galois H-object and $\Phi : H^1(S/K, Aut\_(- \otimes N)) \to H^1(H^*, Hom(N, S \otimes N))$ the map induced by w_1. Then we have got the followings:

PROPOSITION 3.6.

Φ is a bijective map which takes the distinguished element into the distinguished element.

Proof. a) Φ is surjective.

Let $g \in Reg(H^*, Hom(N, S \otimes N))$ be a 1-cocycle, we may define then $h : H^* \longrightarrow End(S \otimes N)$

$$q \longrightarrow hq : S \otimes N \longrightarrow S \otimes N$$

$$x \otimes m \longrightarrow \sum_{(q)} \sum_i q_{(1)}(x) y_i \otimes m_i$$

being $\sum_i y_i \otimes m_i = g_{q_{(2)}}(m)$

being h defined like this, it happens that h is a Galois descent data, the proof of this assertion is easy, though it is tedious. There are then B and $\mathcal{L}_B$ such taht $\mathcal{L}_B : S \otimes B \longrightarrow S \otimes N$ is a left S-module isomorphism, h takes the form given in (2.2.a) and the 1-cocycle $h' = (\tau \otimes N) \cdot (S \otimes \mathcal{L}) \cdot (\tau \otimes B) \cdot (S \otimes \mathcal{L}^{-1}) \in Aut_{S \otimes S}(S \otimes S \otimes N)$ is associated to $\mathcal{L}$ |5|. Where τ is the application which interchanges the factors (1.1).

It follows then that $\omega_1(h')_q(m) = \sum_{i,j} y_{i_j}\, q(x_i) \otimes m_{i_j} =$
(because S is commutative and ℓ a S-module homomorphism).

$= \ell\left[\sum_i q(x_i) \otimes b_i\right]$ (because of the (2.2.a) relation between ℓ and h).

$\shortparallel$

$hq(1_S \otimes m)$ (by definition of h starting from g)

$\shortparallel$

$g_q(m)$ Q.E.D. where $\sum_i x_i \otimes b_i = \ell^{-1}(1_S \otimes m)$ and $\ell(1_S \otimes b_i) = \sum_j y_{i_j} \otimes m_{i_j}$

b) Φ is injective.

Let g and g' be two 1-cocycles is $Aut_{S\otimes S}(S \otimes S \otimes N)$ such that $\omega_1(g)$ is cohomologous to $\omega_1(g')$ in $Reg(H^*, Hom(N, S \otimes N))$

The 1-coclycles g and g' have associated to then $[B]$ and $[B']$ which are two twisted forms |5| respectively. $\omega_1(g)$ has associated as in a) a Galois descent data whose descent is B. Similarly $\omega_1(g')$ with B'. Because (2.3) B and B' are respectively determined by the equalizers

$$B \longrightarrow S \otimes B \underset{g_2^B}{\overset{g_1^B}{\rightrightarrows}} Hom(H^*, S \otimes B)$$

$$B' \longrightarrow S \otimes B' \underset{g_2^{B'}}{\overset{g_1^{B'}}{\rightrightarrows}} Hom(H^*, S \otimes B')$$

Given $\nu \in Reg(K, \mathcal{E}')$ such that $\delta^0 \nu \wedge \omega_1(q) = \omega_1(g') \wedge \delta^1 \nu$ we form $\ell : S \otimes B \longrightarrow S \otimes B'$, $\ell = \ell_{B'}^{-1} \cdot (\mu_S \otimes N) \cdot (S \otimes \nu_1) \cdot \ell_B$ where ν_1 is the homomorphism of N in $S \otimes N$ determined by $\nu \cdot \ell$ is trivially a left S-module isomorphism, simply taking into account that ℓ_B and $\ell_{B'}$ are too and that ν has an inverse in $Reg(K, \mathcal{E}')$ for the convolution.

An eassy computation gives $g_i^{B'} \cdot \mathcal{L} = Hom(H^*, \mathcal{L}) \cdot g_i^{B}$ for i = 1,2.

We have then that $B \simeq B'$ so they define the same twisted form, and g and g' turn out to be cohomologous |5|.

COROLARIO 3.7. *The twisted form of* N *over* S *are classified in the hypothesis of (3.6) by* $H^1(H^*, Hom(N, S \otimes N))$.

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