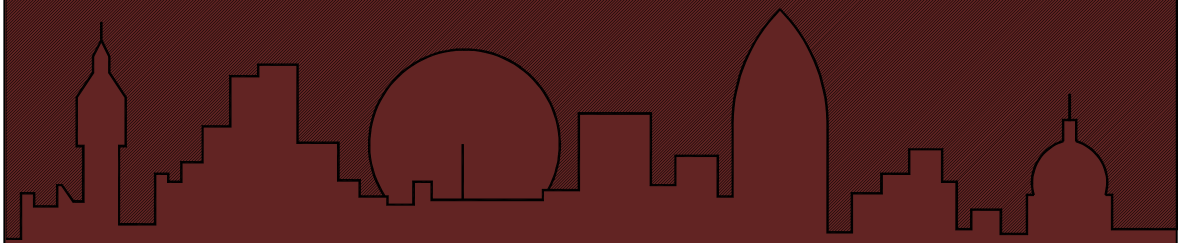

Triangulate the world

TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

Anna Vera Llistar

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School mentor: **CARLOTA PETIT OLIVELLA**

BY NC ND

TED Master's Degree, 2011

Student's book

Acknowledgements

Special thanks to Carlota Petit and Oriol Pallarés for their guidance throughout the development of this unit. Their support, patience, dedication and experience have made this material possible. Thanks also to IES Salvador Espriu for a wonderful internship.

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LET'S GET PREPARED

In this unit you will experiment, observe, deduce and prove everything related to triangles. There will be a connection between the mathematical concepts and the present architecture in our lives.

By working with this unit you will be able to:

- Get familiar with geometry.
- Classify geometrical figures.
- Recognize triangles in buildings and in London in particular.
- Work with triangles in real-life situations.
- Use shadows as useful tools for measurement.
- Use some internet resources.

The teacher expects you to:

- Understand geometrical figures.
- Recognize figures in architecture.
- Follow the lessons and participate actively.
- Use English for oral and written skills at all time.
- Participate in group work.
- Be creative.

How are you going to be evaluated?

- You will have to work on **several tasks**. Some will be done in class and others at home.
- **Team work** and **individual work** is going to be assessed, as well as **participation** in class.
- You are going to help the teacher **evaluate** your classmates' work.
- A final **exam** when the unit is finished will be taken to consolidate knowledge and skills.

Image 1. London's skyline

ENJOY THE TRIP AROUND THE WORLD!

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LESSON-BY-LESSON OVERVIEW

SESSION	ACTIVITIES	INTERACTION	SKILLS	ICT	COMMENTS
1	**Introduction to Ch 1: Architecture.** Video “How much does your building weigh, Mr Foster?	T - Class		X	
	Video in Moodle: “Welcome to architecture”	-		X	More immersion into architecture
	Discussion: What does an architect do?	SS - SS			
	Introduction to Ch 2: Polygons Review concepts from previous year	SS - SS			
2	Warm-up: Homework correction	SS - Class			
	Areas and perimeters of polygons	T - Class			
	Introduction to Ch 3: Triangles. Classification and characteristics. Proofs	T – Class SS - SS			
3	Warm-up: Homework correction	T - Class			
	Recognition of polygons in buildings	SS - SS			Real buildings from all over the world
	Proofs: Properties of triangles proved by students	SS - SS			
	Peer-assessment on proofs	SS – SS			
4	Warm-up: Homework correction	SS - Class			
	Introduction to Ch 6: Project Explanation of the unit’s project	T - Class		X	
	Proofs: Properties of triangles proved by students	SS - SS			
	Peer-assessment on proofs	SS – SS			

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SESSION	ACTIVITIES	INTERACTION	SKILLS	ICT	COMMENTS
5	Warm-up: Homework correction	SS - Class			
	Centers of triangles	T - Class			Centers studied in the Art class
	Testing what we know! Groups prepare word problems to test other groups	SS – SS			
	Peer-assessment on word problems	SS – SS			
	Proof Quiz: Properties of triangles	-			
6	Warm-up: Homework correction	SS - Class			
	Introduction to Ch 4: Pythagorean Theorem. Video "Pythagoras was a square"	SS - Class		X	Several geometrical interpretations of the theorem
	Discovering the geometrical and algebraic interpretation of the theorem	SS – SS T - Class			Experimenting with triangles and squares
7	Warm-up: Homework correction	SS - Class			
	The importance of the theorem	T - Class			
	Finding out if specific triangles are right-angled triangles.	SS - SS			
	Practice word problems: Let's practice across culture!	SS - SS			Real buildings from all over the world
8	Warm-up: Homework correction	SS - class			
	Introduction to Ch 5: Thales	SS - SS			
	Thales Theorem and its uses	T - Class			
	Word-problem dictation	-		X	A Voki will dictate word problems on Thales and proportions

SESSION	ACTIVITIES	INTERACTION	SKILLS	ICT	COMMENTS
9	Warm-up: Homework correction	SS - class			
	Introduction to Ch 7: Final thought. Video "Andrea Palladio and Marcus du Sautoy"	SS - Class		X	An example of the relationship between math and architecture
	Who's who? What do they have in common? Class debate	SS - SS			
	Unit review and practice	SS – SS		X	Also, architectural review of all the buildings in London seen throughout the unit and others to visit in the trip
10	Unit Exam	-			

1.WELCOME TO ARCHITECTURE

To get started with this unit, we are going to warm up watching a video of a famous English architect, whose buildings you will be visiting when travelling to London in May.

Jofre Sendrós 2011

Eliana Manzano 2011

Video: How much does your building weigh, Mr. Foster?

http://www.youtube.com/watch?v=9740R6Kd48I

1.1 INTRODUCTION TO ARCHITECTURE

Take a close look to the art of designing spaces for human beings. Hopefully the videos in this unit will open your minds towards the amazing world of architecture and the geometry needed to make spaces become true.

Instructions:

1. The following video is uploaded in Moodle.
2. Watch the video at home.
3. Reflect and think about what you have seen.
4. Try to identify some of the buildings.
5. Answer the questions in the next page.

Image 2. Petronas Towers. Kuala Lumpur, Malaysia Pelli and Cerico

Image 3. Agbar Tower, Barcelona Jean Nouvel

Jofre Sendrós 2011

Eliana Manzano 2011

Video: Welcome to architecture!

http://www.youtube.com/watch?v=Lk2cMR0fw_w&feature=related

Questions:

1. Have you recognized any building in Barcelona? Which one?

....

2. What geometrical figures have you identified?

....

3. Apart from the buildings you have seen, do you have any favorite building? Why?

....

4. Of all the buildings shown, which ones belong to **modern architecture***?

....

5. Of all the buildings shown, which ones belong to **contemporary architecture****?

....

**Architectural Tip – Modern Architecture?*

- *20th Century* - *"Form follows function"*
- *Simple forms* - *"The house is a machine to live in"*
- *No ornament* - *New materials: iron,. Steel, glass*

Image 4. Guggenheim Museum in Bilbao, Spain.
Frank Gehry

Image 5. Farnsworth House in Illinois, USA
Mies van der Rohe

***Architectural Tip – Contemporary Architecture?*

- *21st Century*
- *Material used with technology and geometry*
- *The spectacular architecture nowadays*

CC BY NC ND

1.2 WHAT DOES AN ARCHITECT DO?

Now that you have an idea of what the word **architecture** refers to, try to describe what an architect does.

1. Discuss, with the members of your group, what you know about architecture and what an architect does, and complete the blanks below.

Discussion Tips

Do they only draw?
Do they only build houses?
Are most architects men?
What are their abilities?
Are they wealthy (=rich)?

Image 6. Solomon R. Guggenheim Museum, in New York. Frank Lloyd Wright

Image 7. Top of the Rock Building, in New York

2. Write as much as you know about this topic. Be prepared to share your ideas with the rest of the groups! One is done for you as a model.

Architects look at the materials used in a building

If you want to know more about the building in the picture on the left, watch and enjoy:
http://www.architects-talk.com/2011/03/architecture-tour-solomon-r-guggenheim.html

We are now ready to start the unit!

2. POLYGONS

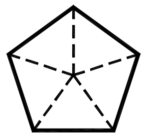
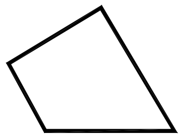
2.1 REVIEW

As you worked with polygons last year, let's just review the main concepts.

Fill in the blanks to obtain definitions.

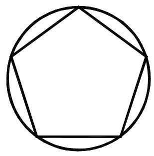
1. A .... is a closed plane figure with sides and angles.

   Polygons can be .... or ....

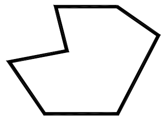
   
   

2. In a .... polygon all of its angles have the .... measure and all of its sides the same length.

   We can always .... them in a circle.

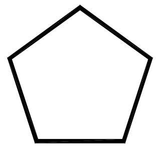
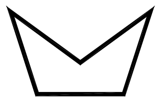
   

3. In an .... polygon the angles and the sides have different measures.

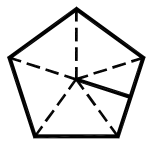
   

4. An irregular polygon can be convex or concave, depending on its angles.

   In .... polygons all interior angles are less than 180º, while .... polygons have at least one interior angle greater than 180º.

5. The .... is a line from the center of the regular polygon to the middle of one of the sides. The apothem divides the triangles which conform the polygon into other triangles.

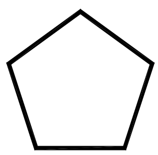
   

6. The .... of a regular polygon equals half of the perimeter multiplied by the apothem.

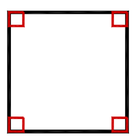
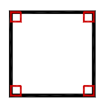
   $$\boxed{A = \frac{ap \cdot P}{2}}$$

7. The .... of a polygon is the distance outside of the polygon, the sum of the length of each side.

   In .... polygons the perimeter can also be found by multiplying the number of sides by the length of one side.

8. .... are polygons which have all corresponding angles congruent and all corresponding sides proportional (remember unit 3).

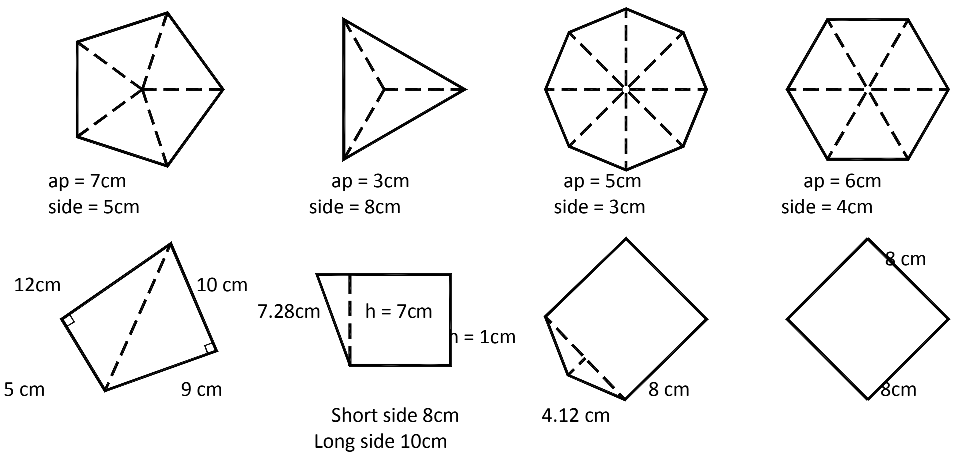
   
   

Below are the missing words in the blanks. Some of them may be used more than once.

2.2 PRACTICE: AREAS AND PERIMETERS

1. Take a look at these figures and classify them as **regular** or **irregular** polygons.

Then, calculate the **area** and **perimeter** of each figure in your **notebook**.



2. What is the area of a parallelogram with bases 10cm and height 5cm?

> *Hint*
> *Step 1. Draw the figure.*
> *Step 2. Think about areas.*
> *Step 3. Calculate*

Maria Morales 2011

3. f the length of an apothem of a pentagon is 3cm, what is the shortest length from its center to one side?

4. Can you recognize polygons in the pictures below? Which ones?

Image 8. 30St Mary Axe, in London. Foster and Partners

Image 9. Stockholm's Public Library, in Sweden Erik Gunnar Asplund

Image 10. Catalunya National Theater, in Barcelona. Ricardo Bofill

Image 11. Hearst Tower, in New York. Foster and Partners

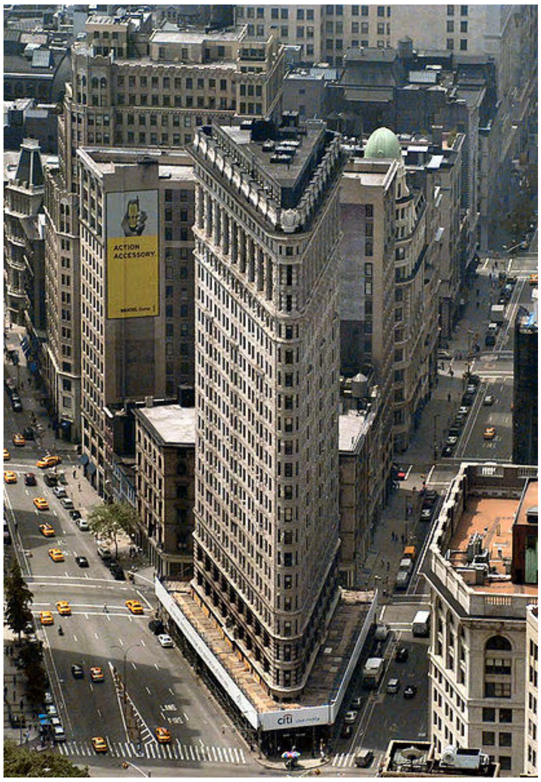


Image 12. Flatiron Building, in New York Daniel Burnham

BY NC ND

2.3 THE "STAR" OF POLYGONS

a) If you had to choose a polygon, which one would you think of? Discuss and then draw a polygon with the members of your group, so that you can answer the following questions:

- Can your polygon be triangulated?
- Can your polygon be quadrangulated?

Image 13

Choose the best drawing, show to the rest of the class using the whiteboard and reason your choice.

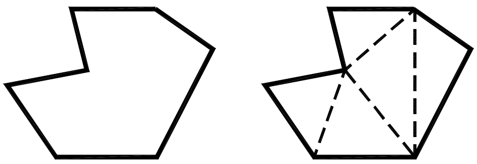
Discussion Tips
If it has diagonals …
If it doesn't have diagonals
Depending on the sides …

Language Tip
We have decided that …
We think that …
The "star" of polygons is …

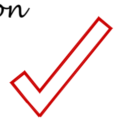
b) What do triangles have that make them so important?

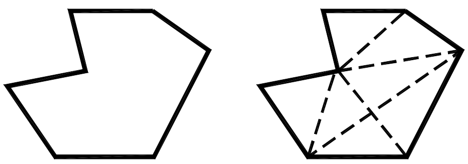
The **triangulation** of polygons refers to the decomposition of the polygon into **triangles**.
We draw **diagonals** to divide the polygon in triangles. Triangulation reduces complex shapes into simple shapes. The polygon becomes an addition of triangles. Any polygon admits a triangulation. If the polygon has .... **sides**, then the polygon has .... **triangles.**

Look at the two examples below:

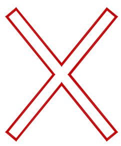


We can triangulate the seven-sided irregular polygon into 5 triangles. This means that if the polygon has 7 sides, then the polygon has 7 -2 triangles.





On the left, the seven-sided irregular polygon is divided into 4 triangles. This cannot be considered triangulation because the triangles are not obtained with the diagonals.



As all polygons can be triangulated, we can work with triangles to know more about polygons.

BY NC ND

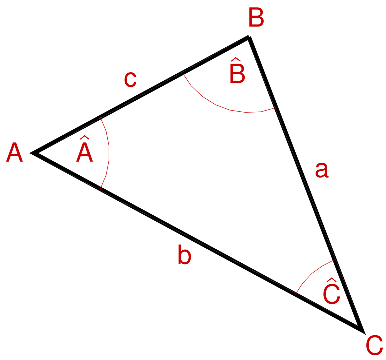
3. TRIANGLES

From now on we are going to talk about triangles, the "star" of polygons.


Laura Serrano 2011

Do you remember?

A triangle is a polygon with 3 sides.



1. Triangles have vertices, sides and angles.
2. We represent them this way:

- ✔ Vertices: Capital letter A, B, C
- ✔ Sides: Lower case a, b, c
- ✔ Angles: Capital letters with Â, B, C

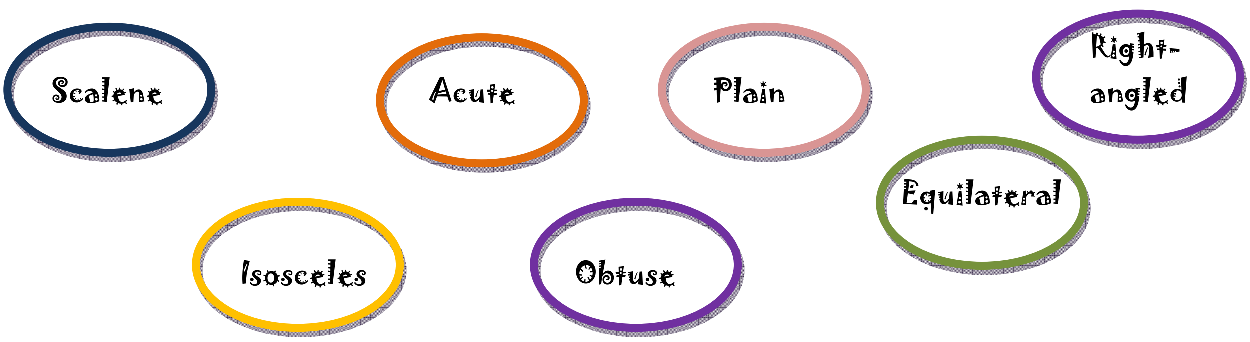
When we write about a triangle, we must write the letters of the vertices with a small triangle on top, as $\overset{\triangle}{ABC}$.

3.1 CLASSIFICATION OF TRIANGLES

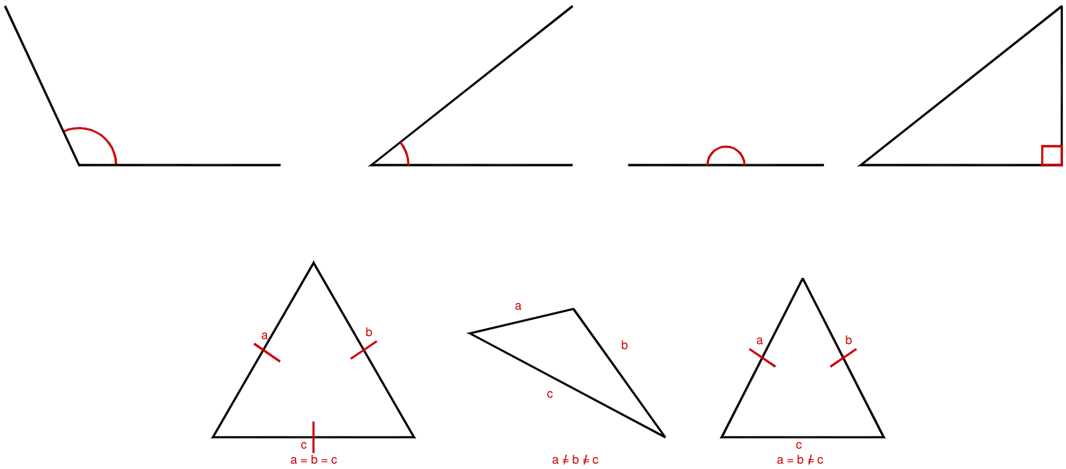
Triangles are classified according to their sides and angles.

1. Match the characteristics below with the corresponding names.

- ✔ **3 sides are different and 3 different angles**
- ✔ **no angle = 180º**
- ✔ **angle > 90º**
- ✔ **angle < 90º**
- ✔ **2 sides are the same and 2 same angles**
- ✔ **3 sides are the same and 3 same angles**
- ✔ **angle = 90º**



2. Label and classify each of the following triangles according to their sides and angles:

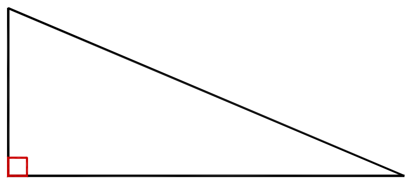


3. The longest side in a right- angled triangle is called **hypotenuse.**

The adjacent sides to the angle are called **legs.**

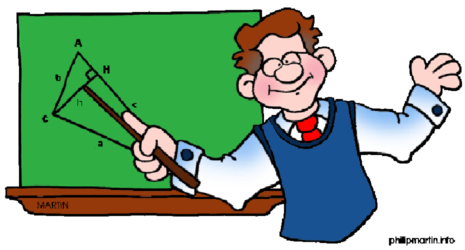
There is always a hypotenuse in all right-angled triangles.

Label the right-angled triangle.



Did you know...

Hypotenuse comes from the Greek *hupoteinousa, Hupo* meaning the opposite; opposite to the right angle.



4. TRUE OR FALSE? Decide the correct answer for the following statements:

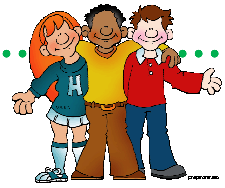
T F In a triangle, sides and angles are opposite to each other.

T F The largest angle is opposite to the shortest side.

T F The middle-sized angle is next to the middle-sized side.

T F The smallest-sized angle is opposite to the smallest angle.

MATH TIP - LET'S ALL PROVE TOGETHER



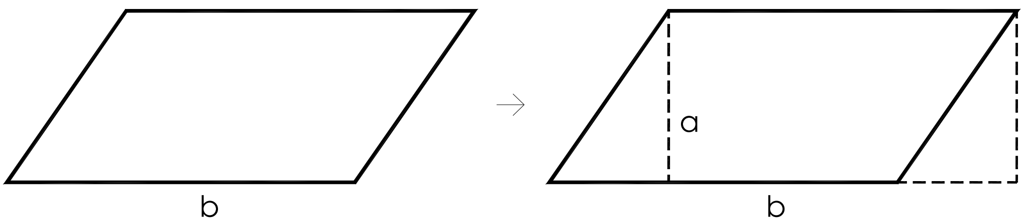
In this unit we will have to understand and deduce **mathematical proofs**.

1. You will **work in teams**. Assign yourselves a number from 1 to 4. A student from each group will be responsible for the **explanation of a proof** to the rest of the members of the group.
2. The teacher will provide the "expert" student with a card with the necessary information.

Here is a proof done for you as an **example**:

Statement: The area of a parallelogram is the length of the base multiplied by the height.

Proof: We make a parallelogram become a rectangle as shown in the drawing below:



1. We remove the triangle of one side and put it to the other side.

2. We obtain a rectangle.

3. Since we know the area of a rectangle is b x a, *then we can say that the area of a parallelogram is also* b x a. *This is what you do when you prove it.*

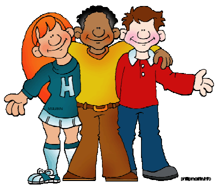
Q.E.D

Did you know…

The abbreviation Q.E.D. on the right refers to the Latin *quod erat demonstrandum*, meaning "that which was to be proved".
It is written after a mathematical proof.

3.2 PROPERTIES OF TRIANGLES

As a good "star", triangles have a lot of beautiful properties. Let's all prove together!



Property 1: The angles of a triangle

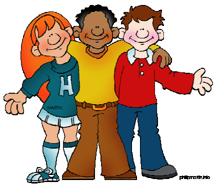
Statement: The three interior angles in a triangle always add up to180º.

The Proof (listen to the expert)

Self-assessment chart Proofs 1 and 2 **Check your answer**

	PROOF 1		PROOF 2	
Did you understand the statement?				
Did you understand the proof?				
Was your classmate clear?				
Indicate your level of satisfaction				

BY NC ND

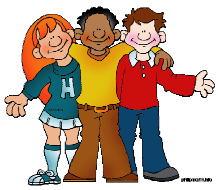


Property 2 : The angles of a polygon

Statement: The addition of the interior angles in a polygon is 180 · (n-2).

Math tip: Start with the simplest polygon and find a pattern!

The Proof (listen to the expert)



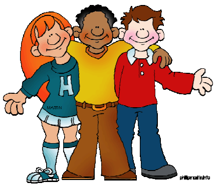
Property 3 : Triangle inequality

Statement: The addition or subtraction of the length of two sides of a triangle is always greater than (addition) or less than (subtraction) the length of the other side.

The Proof (listen to the expert)

Self-assessment chart Proofs 3 and 4 **Check your answer**

	PROOF 3		PROOF 4	
Did you understand the statement?				
Did you understand the proof?				
Was your classmate clear?				
Indicate your level of satisfaction				



Property 4 : Area

Statement: The area of a triangle is the base of the triangle multiplied by the height of the triangle and divided into two parts.

$$A = \frac{b \cdot h}{2}$$

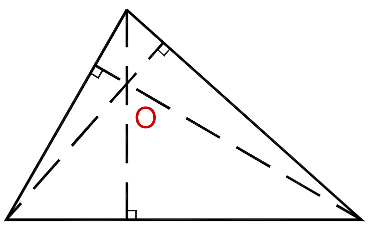
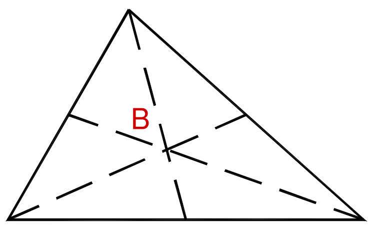
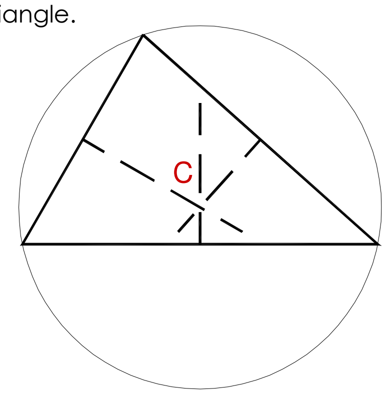
The Proof (listen to the expert)

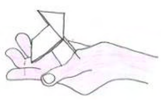
Remember that triangles are also studied in other subjects as Art or in Technology.

Here is a brief explanation of the **3 different centers** according to their characteristics:

orthocenter, barycenter or **centroid** and **circumcenter**.

Let's look at each:

Orthocenter	Barycenter	Circumcenter
Draw the height from each vertex. The height is defined as the perpendicular segment between a vertex and its opposite side. The 3 heights will cross at the **orthocenter**. (ortho=perpendicular)	Draw a line from the midpoint of each side to its opposite vertex. This line is defined as median. The 3 medians will cross at the **barycenter**. This is the center of gravity of the triangle.	Draw a line perpendicular to each side from its midpoint. It is also called line bisector, since it splits the side into 2 segments of equal length. The 3 perpendicular lines will cross at the **circumcenter**. The triangle will be inscribed in a circle, or in other words, a circle will circumscribe the triangle.
		



Eva Carrillo
2011

Now it's your turn!   You must **hand in** the following:

Draw a triangle with sides 7 cm, 8 cm and 12 cm.

1. Draw its heights and find its orthocenter.
2. Draw its medians and find its barycenter.
3. Find its circumcenter and draw the circumscribed circle.

3.3 PRACTICE

Solve the problems in your notebook.

Hint
Step 1. Draw the figure.
Step 2. Think about triangles.
Step 3. Calculate / Explain

1. How many right angles can there be in a triangle?

2. How many obtuse angles can a triangle have?

3. A scalene triangle has no sides the same length and no angles the same measure. Why is it then still considered a triangle?

Maria Morales 2011

4. Draw an equilateral triangle knowing that one of its angles is 70º.

5. How much do the angles of a triangle sum up to? And the angles of a regular polygon? Explain your answer.

6. If a polygon has 7 sides, what is the sum of all of its angles?

7. An irregular polygon has 13 sides. How many triangles does it contain?

8. a) Calculate the sum of the angles inside a quadrilateral and give the measure of each angle in the polygon.

   b) Do the same with an octagon.

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TESTING WHAT WE KNOW!

1. The class is divided into 7 teams. Each team must prepare a word problem about triangles to test another team in the classroom.
2. Each team must prepare the solution to the word problem to be able to correct it.
3. Teams will exchange the word problems (without the solutions!)
4. Teams must solve the word problems and must complete the chart on the next page.

An example has been done for you:

1. *Draw a triangle with sides a, b and c and label the corresponding angles.*
2. ....

Work on your word problem here

Peer-assessment chart

The team which has prepared and corrected the word problem is:

The team which has solved the word problem is:

	1	2	3	4	THE GRADE OBTAINED IS:
MATHEMATICAL TERMINOLOGY (words)	No use of mathematical terminology	Very little use of mathematical terminology	Some mathematical terminology used	All written with mathematical terminology and expressions	
GRAMMAR AND SPELLING	A great amount of grammar and spelling mistakes	More than two grammar and spelling mistakes	Only one or two grammar and spelling mistakes	No grammar and spelling mistakes	
NEATNESS	Illegible writing, very messy	Legible writing but messy	Legible writing and not messy	Nice handwriting and neat	
CONTENT	Students have made no effort and the word problem makes no sense	Students prepare a simple word problem with no relation to the previous content	The word problem is related to the previous content	Students have come up with a good word problem	
				T OTAL SCORE	points

Conclusions about triangles:

At most, there can be only .... right angle and .... obtuse angle in a triangle.

All .... triangles have the same angles and these are always 60º.

Moreover, …

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4. PYTHAGOREAN THEOREM

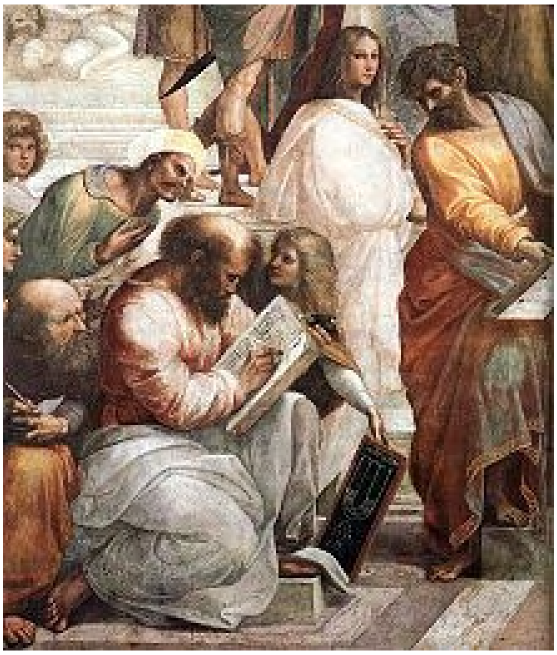


Image 14. Pythagoras

4.1 HIS LIFE

Pythagoras was a Greek mathematician and philosopher who was born in Samos Island in c. 570 BC. He was responsible for mathematical and scientific discoveries, one of them the theorem named after him.

Pythagoras also worked on music, astronomy, and medicine, but was very interested in religion. He was said to have practiced divination and prophecy.

He died in Metapontum - c. 495 BC at the age of 75.

In the picture, Pythagoras is in the center with the book, teaching music in The School of Athens.

4.2 "PYTHAGORAS WAS A SQUARE", brainstorming ideas

We are going to watch a video about Pythagoras and several geometrical interpretations of the Pythagorean Theorem.

Jofre Sendrós 2011

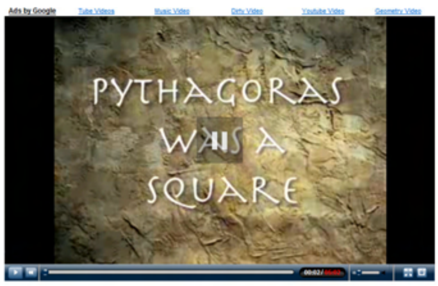
Eliana Manzano 2011

Video: *Pythagoras was a square.*

http://www.teachertube.com/viewVideo.php?video_id=23675

The purpose of this task is to start having **ideas** by **understanding** and **thinking** about the **geometrical interpretations** shown.

Then, fill in the blanks and answer the questions.



1. **Pythagoras of Samos lived …**

… between 570 and 490 B.C. and was a ....mathematician known as the ……………….. of ....

He never wrote a book or did a two-column ....

Pythagoras was known as an .... in the fate of...................., and used .... to explain the numbers in .... He identified the relationship of .... and ...., and his most famous .... may have been done by his students.

Pythagoras stated that "The area of the square drawn on the .... of a right triangle is .... to the sum of the .... drawn on the legs".

But why squares?

2. How may geometrical interpretations of the Pythagorean Theorem have you identified? Name them.

....

3. What is the problem of using similar rectangles to interpret the theorem?

....

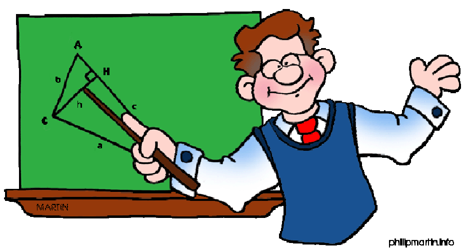
4. Can we come up with a true statement by using quarter circles?

....

Pythagoras had squares. You have algebra. Use it!

Language Tip

We have identified …
The theorem can be explained…
There are several …
If we use…



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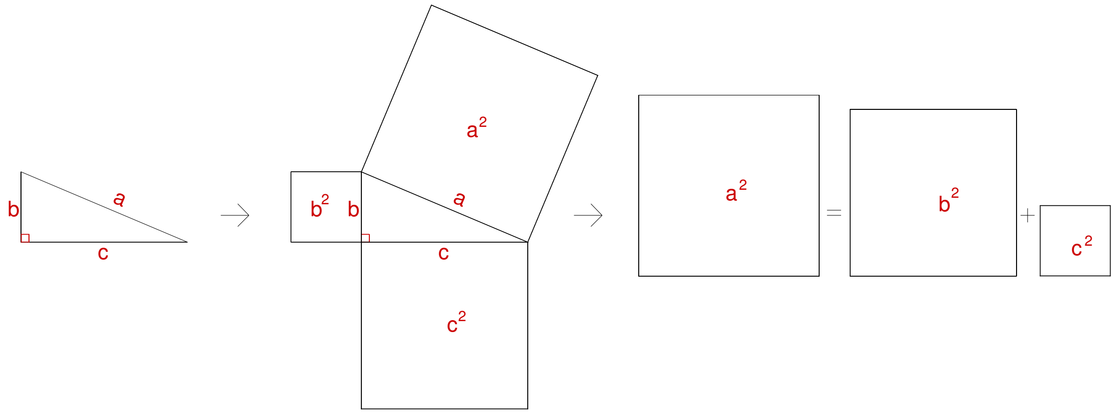
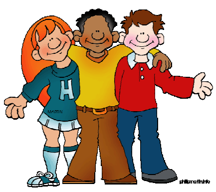
4.3 PYTHAGOREAN THEOREM

As we have watched in the video, there is a relation between the legs of a right-angled triangle and the hypotenuse. Let's take a look at the **geometrical interpretation** using squares:



What can we say about any right-angled triangle?

Can you establish a relation in the form of an equality for the geometrical interpretation? Which one?



LET'S ALL PROVE TOGETHER

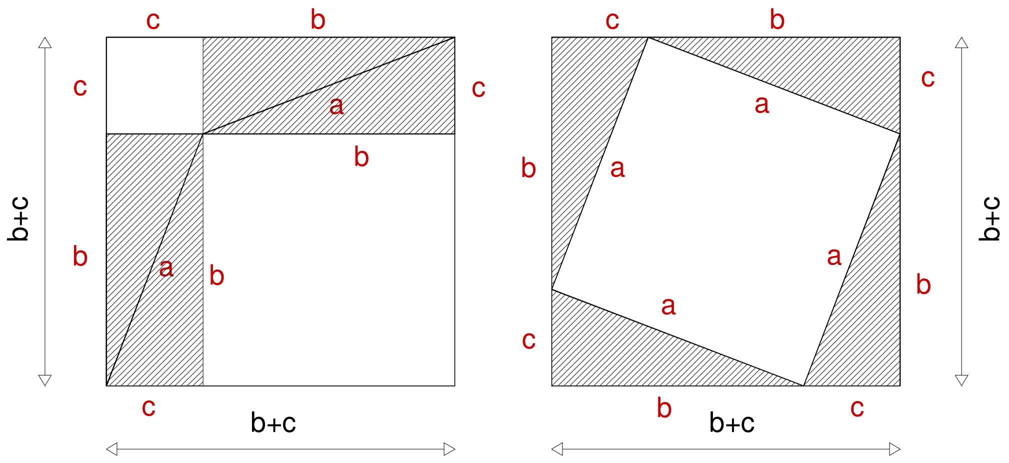
Statement: the Pythagorean Theorem states that in any right-angled triangle, the area of the square with side the hypotenuse is equal to the addition of the areas of the squares of the legs.

Follow the steps:

1. Draw and cut **two squares**, one with sides 6cm and the other with sides 8cm.
2. Draw and cut **8 right-angled triangles** with hypotenuse 10cm and legs 6cm and 8cm.
3. **Arrange them** in two different ways so that you get **two big squares with sides 6+8cm.**
4. Each square with sides 6+8cm contains 4 right-angled triangles with hypotenuse 10cm and legs 6cm and 8cm.
5. Can you see any **empty squares** inside the big squares with sides 6+8cm?
6. What **areas** do they have?
7. As the two initial squares and triangles were the same, do we have an **equality**?

Proof 1:

Geometrically



Q.E.D

Proof 2:

Algebraically

$$c^2 + b^2 + 4\,\frac{(c \cdot b)}{2} = a^2 + 4\,\frac{(c \cdot b)}{2}$$

$$c^2 + b^2 = a^2$$

Q.E.D

TRUE OR FALSE? Decide the correct answer for the following sentences:

T F The theorem helps you find the missing leg of a triangle.

T F This theorem is true for all triangles, no matter the angles.

T F We use the theorem to find the length of the hypotenuse.

T F The theorem can be used to find the diagonal of a square.

T F The theorem can be used to find the diagonal of a rectangle.

T F The theorem can be proved with quarter circles.

T F The theorem can be proved with any geometrical figure.

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4.4 THE IMPORTANCE OF THE THEOREM

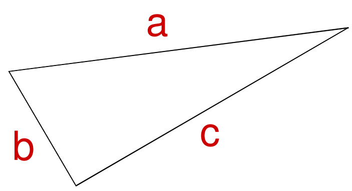
1. To check that the equality is true for all right- angled triangles.

Pythagoras claimed that the equality is always true for all right triangles, but how do we know a triangle is a right-angled triangle?

To find this out, we are going to do a **running dictation**. In your groups, number yourselves from 1 to 4. You will be called by your number and will be dictated the measures of each side of a triangle.

Get ready to **run, listen, write,** and **calculate together!**

	a	b	c	Solution
Example	5	3	4	$c^2 + b^2 = a^2$ IT IS A RIGHT-ANGLED TRIANGLE
Student 1				
Student 2				
Student 3				
Student 4				



What have you found out?..

Do all the triangles have the same solutions? ……………………………………………………………….

How many solutions can we have?...

2. To find the length of the hypotenuse.

If we have the length of 2 legs of a triangle, we can find the length of the hypotenuse.

As we know that $a^2 = b^2 + c^2$, we have an equation to solve.

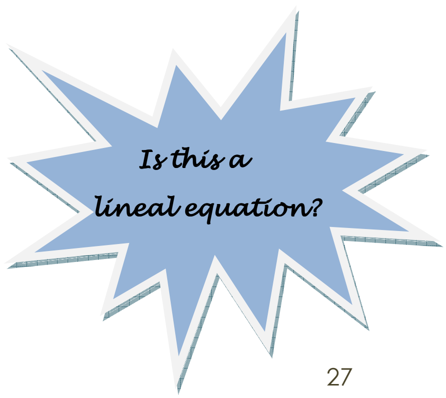
For example:

For a =?, b= 3, and c= 4

$a^2 = b^2 + c^2$

$a^2 = 9 + 16$

$a^2 = 25$ $\boxed{a = 5}$



3. To find the length of a side.

If we have the length of the hypotenuse and the length of a leg, <u>we can find the length of the missing leg.</u>

As we know that $a^2 = b^2 + c^2$, we have an equation to solve.

For example:

For a =12, b= 8, and c= ?

$$a^2 = b^2 + c^2$$

$$144 = 64 + c^2$$

$$c^2 = 144 - 64$$

$$c^2 = 80 \qquad \boxed{c = \sqrt{80}}$$

BY NC ND

4.5 LET'S PRACTICE ACROSS CULTURE

In the following pages you will find a compilation of buildings. There is a brief explanation of each one. You must recognize the existing triangles and solve the word problem with the information given using everything you know about triangles.

The **Forum Building** was built in Barcelona for the 2004 Universal Forum of Cultures in an area of expansion of the city towards the coast.

The designers of this building are the Swiss architects Herzog & de Meuron.

The triangular shape measures 180 meters on each side and 25 meters in height.

Image 15. Forum Building in Barcelona. Herzog & de Meuron

TASK Classify the triangle according to the sides and the angles of the building.

Language Tip

The figure is a …
Each side of the …
It is named …
Measures … Because … So …

Hint

Step 1. Draw the figure
Step 2. Think about triangles
Step 3. State the equation
Step 4. Calculate / Explain

In my building's entrance there is a small staircase to reach the level where the elevators are. It is a problem for wheelchairs and baby cars, so all the neighbours have decided to substitute the stairs for a ramp.

Each step measures 18cm height and 28 cm horizontal.

TASK What will be the ramp's length?

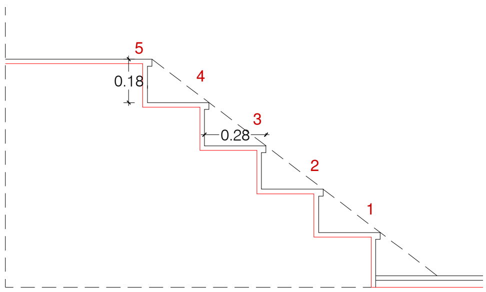


Image 16.
Turning Torso in Malmö, Sweden. Santiago Calatrava

The Spanish architect Santiago Calatrava is responsible for the design of the highest residential building in Sweden named **Turning Torso** and located in Malmö.

Triangles are present for structural purposes, as they reinforce and make rigid the structure as a whole.

The building is composed by 9 rotatory cubes, from bottom to top, with 9 corresponding structural triangles on each façade.

TASK Let's assume the sides of each triangle measure 24 meters the longest side, and 20 meters and 18 meters the other sides. Are the triangles right- angled? How do you know?

Big Ben is a Gothic clock tower built in London in 1858.

It is the clock tower of the Palace of Westminster. The height of the tower is about 96 meters.

TASK If you are standing 5m from the tower, what will be the distance from your feet to the top of the tower?

Image 17. Big Ben in London. Charles Barry

Image 18. The tower

The **Petronas Twin Towers** in Kuala Lumpur (Malaysia) were designed by the Argentine architects Cesar Pelli and Djay Cerico and finished in 1998. The towers have 88 floors and in floors 41 and 42 there is a sky bridge of 58 meters long and 2 floors that connects the two towers. Approximately, the structural brace of each hypotenuse measures 42m.

Image 19. Petronas Towers
Kuala Lumpur, Malaysia
Pelli and Cerico

Image 20. The two-floor sky bridge

TASK If the height of each floor is 4m, how many floors does the side of the triangle occupy?

One of the greatest planning interventions in urbanism (= planning cities and territory) is the **Eixample in Barcelona**. Designed by architect Ildefonso Cerdà in 1859 to organize the city, it consists of residential blocks conforming a grid of 113,3m x 113,3m.

Cerdà stipulated that streets would measure 20m, the maximum height of buildings 16 to 20m and the importance of the free outdoor space in the interior of each block.

Image 21. Eixample in Barcelona. Ildefonso Cerdà

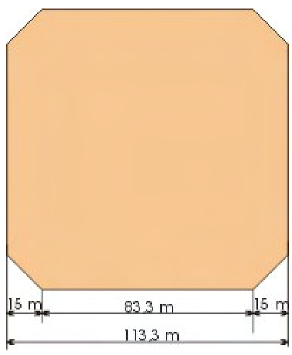


Image 22

TASK If each block has a corner (*xamfrà*) of 15m per side,

Can you figure out what does the hypotenuse of the corner measure?
Calculate the area and the perimeter of each block of buildings.

Image 23. Louvre Museum, in Paris. I.M. Pei

Image 24. Inverted Pyramid in the Louvre Museum.

This is the main entrance to the **Louvre Museum** in Paris, France.

Designed by the architect I.M. Pei, the entrance pyramid was inaugurated in 1989.

There are two pyramids, the exterior entrance on the left and the inverse interior pyramid in the lobby, from which the visitor can observe the palace facades reflected on it.

The height of the pyramid is 20.6 meters and its square base has sides of 35 meters.

TASK What is the length of the edge between two sides, from the ground to the top of the pyramid? Hint: you will need to use a famous theorem twice.

Did you know…

Nowadays new buildings must be built prepared to generate electricity using solar energy, so we can see these elements (solar or photovoltaic pannels on roofs).

The picture on the right is the photovoltaic structure in the Forum area in Barcelona. It is designed by two Spanish architects: Elias Torres and José Antonio Martínez Lapeña. It has an area of 3.780 m² and an inclination angle of 35°.Its highest point is 50m above the sea level and it consists of 2.668 pannels.

Image 25. Forum's photovoltaic structure

5. THALES' AND PROPORTIONS

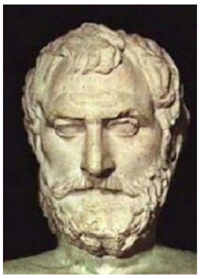


Image 26. Thales

5.1 Who was Thales of Miletus?

1. Below are some sentences about Thales you must read.
2. With the members of your group, discuss and organize the correct order of the sentences.
3. Then, write a brief summary.

- Thales used similar triangles and right angles.
- Thanks to Thales, we can measure the height of a building using shadows and objects.
- He was mainly interested in geometrical figures and was the first one to proclaim angles as mathematical measures.
- Thales of Miletus (c. 624 BC. – c. 546) was a philosopher and mathematician born in the Greek city of Miletus, which is now Turkey.
- Thales measured the height of the pyramids using shadows and his own height.
- A theorem is named after him.
- It is about proportions and similarities.
- He was known for his innovative use of Geometry.

Language Tip

Thales of Miletus was…
He was interested…
He worked with…
He was known for…

A brief summary

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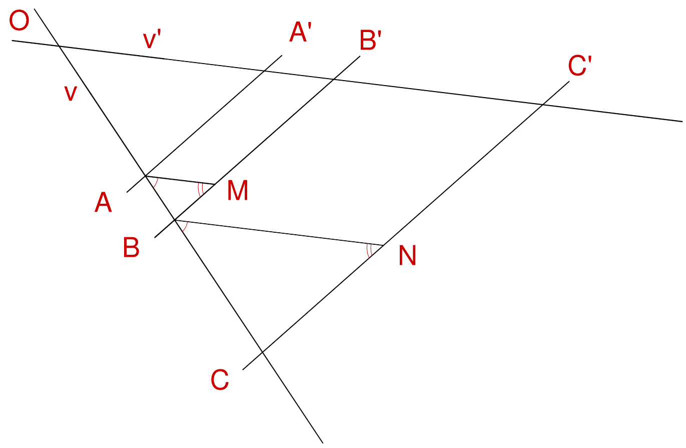
5.2 THE THEOREM

Statement: Given two straight lines v and v' that cross at point O, and that are intersected by parallel segments AA', BB', CC'.

We can say:



1. $\frac{AB}{AC} = \frac{A'B'}{A'C'}$
2. If AM is parallel to A'B' then AM = A'B'

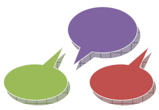
   If BN is parallel to B'C' then BN = B'C'

Let's think about the theorem:

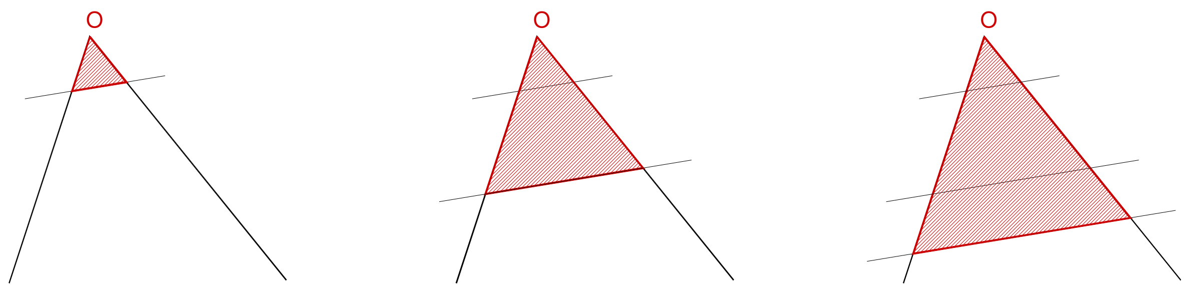
The parallel segments AMB and BNC form two ……………These are named triangle …………… and triangle…………… As all their segments are parallel, the triangles are considered **similar**: the 3 angles are equal and the 3 corresponding sides are parallel to each other.

$$\frac{AM}{BN} = \frac{AB}{BC} = \frac{BM}{CN}$$

Let's think a little bit more:



What do you think is the relation between the red triangles in the three pictures? Discuss with the members of your group and …



Discussion Tips

Do they look the same?

Do they have something in common ?

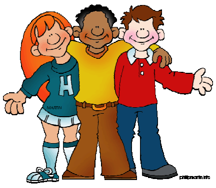
Is there any relation between them?

…Soon you'll find out!

5.3 THE USE OF THE THALES THEOREM

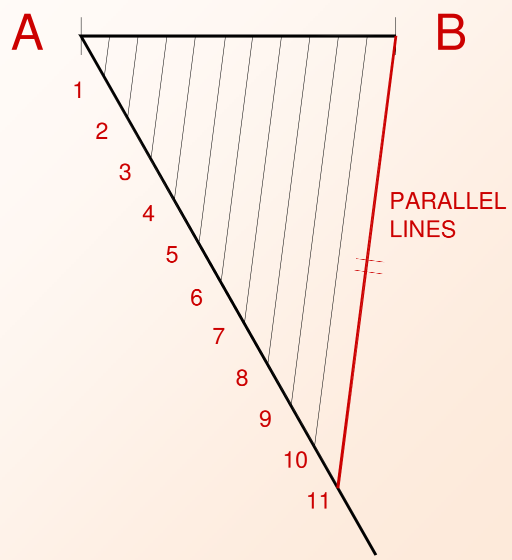
1. To divide a segment into equal parts:

We have segment AB and we want to divide it into 11 equal parts. We do not know the length of segment AB, so we use the Thales' Theorem to make the division.



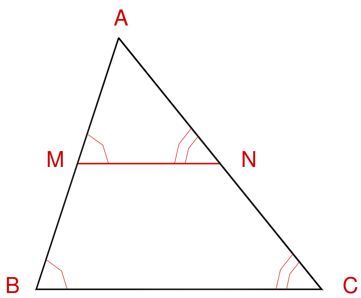
In your groups, one student will dictate the steps to follow and the others will take notes. Try to divide the segment into equal parts!

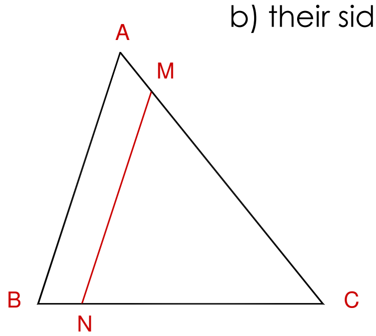
a) Draw a line from one of the ends of the segment, in the direction you want.
b) As you have decided the line's length, divide the line into 11 equal parts.
c) Join the last point with the end of the segment.
d) Draw parallel lines and the segment AB will be equally divided.



2. For proportions and similarities:

Triangles can be placed in the Thales' position → They are named **SIMILAR TRIANGLES** ("semblants" in Catalan). Two triangles are similar if: a) they have equal angles
b) their sides are proportional



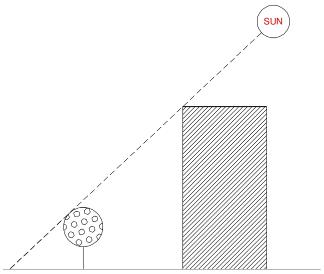


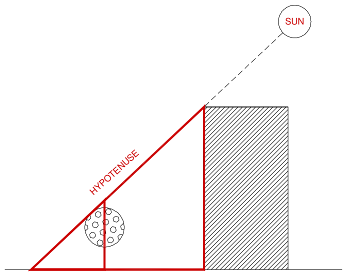
We can draw more parallel lines and obtain more similar triangles.

We use **shadows and proportionality** to find the length of objects, height of buildings, persons, etc.

The **rays of sunlight form a triangle** with the ground and the line of our object.

The sunlight becomes the hypotenuse of a right-angled triangle (if the ground is straight and forms a 90º angle).





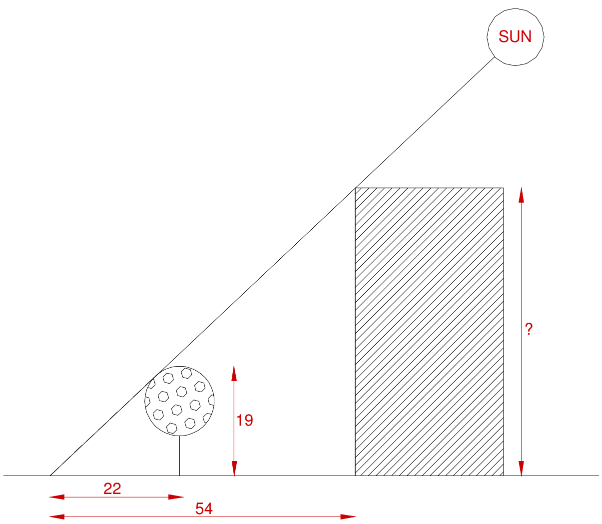
For example:

We want to know the height of a building.

1. We measure the **real height** of an object we have and the length of its **shadow projected** on the ground.
2. We measure the length of the building's shadow projected on the ground.
3. We can establish a **proportion equality** between the object, its shadow, the building's shadow and the height we want to know (x). Therefore:

$$\frac{\text{shadow of the object}}{\text{real length of the object}} = \frac{\text{shadow of the building}}{\text{x (variable: real height of building)}}$$

The example with numbers:



Let's imagine…

The shadow of the tree is 22m

The real height of the tree is 19m

The shadow of the building is 54m

The real height of the building is unknown (x)

$$\frac{22}{19} = \frac{54}{x}$$

$$22x = 19 \cdot 54$$

$$x = \frac{1026}{22}$$

$$x = 46{,}64 \text{ m}$$

The height of the building is 46,64m

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5.4 PRACTICE

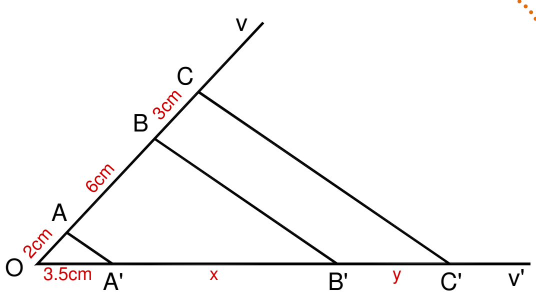
In this section, you are going to practice by doing some exercises.

Instructions:

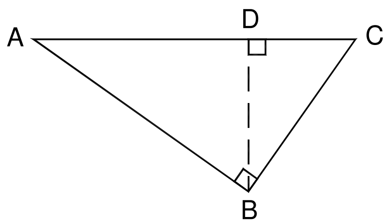
1. Watch the video and follow the instructions given.
2. Be prepared to write each word problem in the spaces provided below.
3. Check what you have written with the student sitting next to you.
4. You are ready to solve the problems. Draw and calculate.

1. Draw a …

2. Calculate …



3. Are the …



4. The man …


Image 27

5. A rectangle …

6. PROJECT - ANCIENT TRIANGLES

Below are some buildings with triangles belonging to ancient architecture.
Time has gone by but geometry remains… Take a look and then focus on the list of buildings you are going to work with.
The purpose of this task is to discover, find out, and think about the use of geometrical figures in the architecture of Turkey, Greece, Egypt, Italy, and China. Moodle and Wikispaces will be used to share the projects with your classmates.

Architectural Tip - Ancient Architecture?

- *Ancient civilizations*
- *The divine and the supernatural*
- *Symbolism: power, ruler*
- *Ideals*

Instructions:

1. The class will be **working in groups,** the classmates you are already seated with. Each group will be assigned a building belonging to **ancient architecture.**
2. You will have to:
   - ✔ recognize the triangles present.
   - ✔ gather information about its **historical approximation.**
   - ✔ calculate the **area** and the **perimeter** of the main triangles with the data obtained in your research.
3. Pictures, videos, images, ideas, drawings, measurements… will have to be searched.
4. Once you have all the materials, you will have to create and prepare a presentation using SlideShare and Audacity, which will be uploaded in the Wikispace provided for this class in Moodle.
5. Do you know what **SlideShare** and **Audacity** are about?

SlideShare is a good way to share presentations.

Audacity is an audio editor and recorder.

Here are some suggested steps to follow:

1. You must first prepare a power point presentation.
2. Go to http://www.slideshare.net/ and sign in.
3. Click **Upload**. Then click **Upload publicly**.
4. Upload your power point presentation.
5. Now you must prepare an **MP3 file** using Audacity.
6. Download the program and the MP3 converter .
7. Record your voice clicking on the "dot" icon.
8. Go back to SlideShare and upload the recording by editing the presentation (edit).
7. Then click on "create slidecast".
8. Upload the MP3 file
9. Check if the voice belongs to the correct slide.

In class you are going to watch an example done for you using **SlideShare** and **Audacity**:

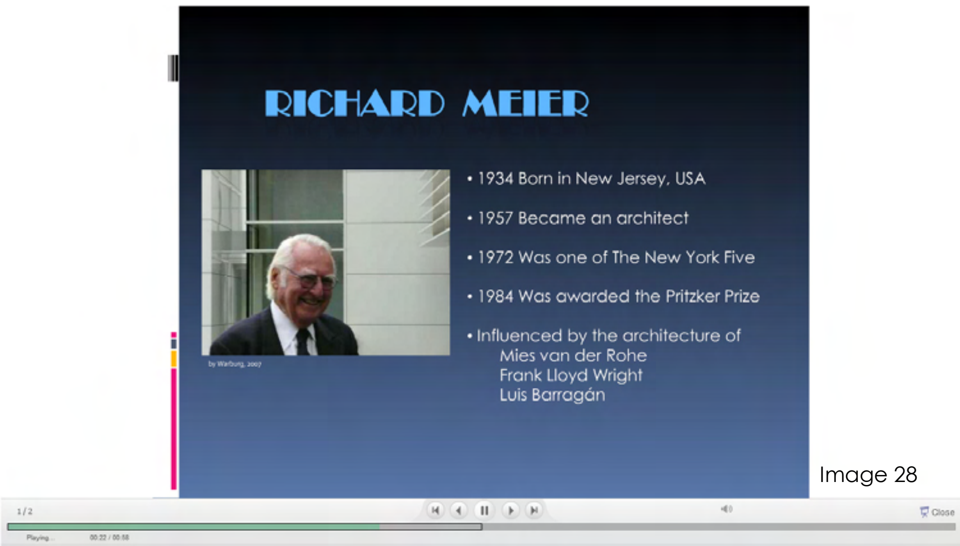


Image 28

Here are some ideas of what your projects should contain:

Pictures | Explanations with writings | Explanations with voice | Drawings

THE LIST OF BUILDINGS

Team 1. Temple of Venus and Rome in Italy.

Team 2. Parthenon. Athens, Greece.

Team 3. The Pyramids at Gizah, Egypt.

Team 4. Obelisk of Pharaoh Senusret I in Cairo, Egypt.

Team 5. The Beehive Houses in Haran, Turkey.

Team 6. Temple of Heaven. Beijing, China.

Team 7. Roman theatre of Aspendos, Turkey.

Language Tip

This building is located ...

We have found that

The triangles found are...

We have read about...

Try matching your building's name with one of the pictures below.

Image 29

Image 30

Image 31

Image 32

Image 33

Image 34

Image 35

The projects will be **evaluated** according to the **information** obtained, the use of **language** to explain in your own words, the **ideas** developed, the **mathematical processes**, and **creativity**.

Each group will be presenting the project in class, so try to do your best and have fun!
You will evaluate other groups' project:

1. You will go to Moodle, to this unit's Wiki where everyone will have uploaded their projects and take a look at all of them.
2. Then, you will evaluate each project according to the rubric below:

Peer-assessment chart **- Circle the best mark for each**

	The project is organized, with a logical order	The project has a good and appropriate title	The information given is of good quality, with details of the building	All grammar and spelling are correct	Nice pictures and nice presentation layout	**Total score obtained:**
GROUP 1	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
GROUP 2	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
GROUP 3	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
GROUP 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
GROUP 5	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
GROUP 6	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
GROUP 7	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	

EXTRA CREDIT

Take a look to the modern ways of using triangles nowadays in the pictures below. 2000 years have gone by since the ancient buildings, so…get inspired and design the building of the year 4000! What would it look like? Use geometrical tools, of course!

My building of the year 4000

Image 36. City Hall, in London.
Foster and Partners

Image 37. The Millennium Dome, in London.
Richard Rogers

7. FINAL THOUGHT

The end of this unit is here…

Let's watch a video of the well-known English mathematician **Marcus du Sautoy** in one of the famous Italian architect **Palladio**'s buildings. Who's who? Class discussion.

Image 38

ANDREA PALLADIO

An Italian architect born in 1508 who deeply studied the Roman architecture. He designed palaces, churches and villas in the High Renaissance, but was especially known for his "palladian villas", mostly located in the city of Vicenza. His architecture approached (got close to) culture, as his buildings expressed the people's position in the social order of the time.
He wrote *The Four Books of Architecture,* with drawings and rules to follow.
Some of his work is now considered part of the World's Heritage Site. Below are two of the many villas he designed.

Image 39. Villa Rotonda

Image 40. Villa Godi

Discussion Tips

Who was Palladio?
Who is Marcus du Sautoy?
What do they have in common?
What is the video about?
Where is the video recorded?

Jofre Sendrós 2011

Eliana Manzano 2011

VIDEO: http://news.bbc.co.uk/2/hi/programmes/newsnight/review/7865508.stm

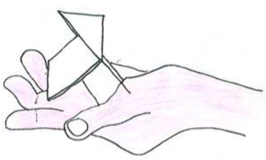
MARCUS DU SAUTOY

English scientist and mathematician born in 1965. Known for making mathematics popular, du Sautoy won, in 2001, the Berwick Prize of the London Mathematical Society, awarded to the best mathematical research under 40 years old.
He has published a great amount of articles and books on mathematics, about nature, symmetry, and music in relation to mathematics.

Key: the word problem-dictation from the video on page 37.

1. Draw a segment of 4.5 cm and divide it into 13 equal parts.
2. Calculate the values of x and y in the drawing.
3. Are the triangles ABD and BDC similar to triangle ABC? Why?
4. The man in the picture is going to enter a store in the Soho District in New York City.
   His height is 1.82 m. Calculate the height of the door.
5. A rectangle has sides 10 cm x 25 cm. A smaller and similar rectangle has its short side measuring 8 cm.
   What is the similarity ratio of the rectangles? And, what does the long side of the small rectangle measure?

Special thanks to the students in 2nd of ESO, authors of these pictures:



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