Triangulate the world

TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

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Acknowledgements

Special thanks to Carlota Petit and Oriol Pallarés for their guidance throughout the development of this unit. Their support, patience, dedication and experience have made this material possible. Thanks also to IES Salvador Espriu for a wonderful internship.
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**UNIT TITLE:** Triangulate the world  
**AUTHOR:** Anna Vera Listar  
**CLASS/AGE:** 2nd of ESO / 13-14 years old  
**SUBJECTS INVOLVED:** English and Math (English and Math teacher)  
**NUMBER OF LESSONS:** 10 sessions of 55-60 minutes  
**COE LEVEL:** Recommended for A2  

**INTRODUCTION TO THE TASK**

In this unit students will experiment, observe, deduce and prove everything related to triangles. There will be a connection between the mathematical concepts and the present architecture in our lives.

**MAIN GOALS AND COMPETENCES**

By the end of this unit, the students will be able to:

- Value mathematics as part of our culture, from history to nowadays.
- Recognize figures in our environment: recognition of geometry in architecture.
- Visualize and use geometric models to solve problems: work with triangles in real-life situations.
- Use shadows as useful tools for measurement.
- Prove, reason and reach conclusions of mathematical concepts.
- Connect the concepts in the Math class with the contents in other subjects as Art and Technology.
- Use different languages, as verbal, numerical, graphic and algebraic language to identify and represent relations between variables.
- Search information and use ICTs to make presentations of final products.
- Communicate and organize the final products presented.
- Participate actively in oral, written and audiovisual interactions, using English as the vehicular language at all time.
- Express and comprehend oral, written and audiovisual messages.
- Work collaboratively and cooperatively.
TOPIC RELATED CONTENTS

MAIN TARGET KNOWLEDGE

1. Introduction to the concept of architecture: ancient, modern and contemporary.
2. Review on polygons.
3. Analyze triangles: classification, characteristics, and properties.
4. Pythagorean Theorem and its application.
5. Thales Theorem and its application.
6. Historical contexts: proportions and geometry in Italy, Greece, Egypt, Turkey, and China.
7. Relationship between

TOPIC RELATED CONTENTS

MAIN TARGET SKILLS

1. Understanding English speeches and main concepts in short videos.
2. Identifying the architecture in the city they live.
3. Discussing ideas in group about previous or acquired knowledge.
4. Expressing and giving opinions on topics.
5. Recognizing geometry in buildings and solving problems.
6. Proving the properties of triangles by becoming experts in their group.
7. Being able to reflect on statements and proofs, for self and peer-assessment.
8. Drawing geometrical figures to solve problems.
9. Formulate word problems working in teams to test other teams
10. Understanding geometrical interpretations of theorems.
11. Organizing and summarizing short texts.
12. Producing an organized and coherent presentation using ICT resources and oral expression.
TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

TERMINOLOGY

TOPOIC SPECIFIC
- Ancient, Modern and Contemporary architecture.
- Regular and irregular polygons.
  Convex, concave, apothem, area, perimeter, triangulation, diagonals.
- Triangles, vertices, angles, sides, legs, hypotenuse, orthocenter, barycenter, centroid, circumcenter.
- Pythagorean Theorem, proof, statement, geometrical interpretation.
- Thales Theorem, shadows, similarities, and proportionality.

GENERAL ACADEMIC
- We have decided that...
- We think that...
- We have identified...
- The theorem can be explained...
- There are several interpretations...
- If we use...
- The figure is a...
- Each side of the...
- It is named...
- Measures...because...so...
- Thales of Miletus was...
- He was interested in...
- He worked with...
- He was known for...
- We can conclude that...
- As conclusion

DISCOURSE GENRE / TEXT TYPE

TO UNDERSTAND
- Videos in English on math concepts
- Mathematical operations
- Descriptive texts about buildings and mathematicians
- Discussion tips and language tips

TO GENERATE
- Discussion between peers
- Summaries
- Oral mathematical discourses as:
  - experiment
  - deduce
  - prove
  - relate
  - explain
  - define
  - classify
  - compare
  - conclude

TO NEGOTIATE
- Meaning between peers
- Knowledge to develop skills

PRAGMATIC STRATEGIES

Language is used in a social context
- Group discussions
- Group collaborative work
- Group productions
SOCIAL & CULTURAL VALUES; PERSONAL & EMOTIONAL DEVELOPMENT

- Understanding the existing relationship between math and architecture, from past to present times.
- Valuing the different landmarks around the world.
- Valuing the architecture in their city.
- Understanding different cultures.
- Giving importance to the use of mathematics in real-life situations.
- Working collaboratively to solve problems.
- Approach the world of math from a different point of view.

SUMMATIVE ASSESSMENT
(with formative value)

TASKS

1. Active participation in class
2. Questions on videos
3. Group discussions
4. Recognition of geometrical figures in buildings
5. Proofs of triangle properties
6. Word problem dictation and real-life situations
7. Project: Group generated presentation using ICTs with images, drawings and voice.
8. Unit Exam

SUMMATIVE ASSESSMENT
(with formative value)

ASSESSMENT CRITERIA

Catalan Curriculum assessment criteria for English (E) and Math (M)

1. Positives-negatives list
   Criteria: E1, E2, E5
2. Checklist
   Criteria: E2
3. Gradebook
   Criteria: E1
4. Gradebook
   Criteria: M3, M8, E10
5. Quiz
   Criteria: E3, M4
6. Checklist
   Criteria: E2, M6
7. Rubric
   Criteria: E5, E7, M4, M6, M8, M9
8. Exam
   Criteria: E3, M4, M6, M8, M9
This unit has been especially designed for a 2nd of ESO Math CLIL class. Although it has been implemented by one student-teacher, it is also suitable for Tandem Teaching with at least one Math teacher. In this unit there is a connection between the mathematical concepts and the present architecture in our lives. As the group will be travelling to London in a school trip, many of the images are from the architecture students will be able to recognize and enjoy. Among the tasks in the unit, students will have to present at the end, final projects in groups and take a unit exam, to consolidate knowledge and skills. 

MATERIALS and RESOURCES

MATERIALS
Booklet, worksheets, proof expert cards, running dictation cards, PDF file for digital board, geometric figures for Pythagoras, and unit test.

RESOURCES
ACKNOWLEDGEMENTS
Special thanks to Carlota Petit and Oriol Pellarés for their guidance throughout the development of this unit. Their support, patience, dedication and experience have made this material possible. Thanks also to IES Salvador Espriu for a wonderful internship.

CHECKED MATERIALS
- Check Annex 1 for image sources.
Some of the images are from pictures taken by the author of this unit.
The rest of them have been taken from Wikipedia, Wikimedia Commons. Files licensed under the Creative Commons Attribution-Share Alike, or in the Public Domain.
Free Clip Arts by Phillip Martin.


- Bibliography
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KEY COMPETENCES

1. Communication in the mother tongue
As this unit is designed for a Math CLIL class, it establishes that all communication must be done in L2. Code-switching may only be used in particular cases to avoid misunderstanding of mathematical concepts.

2. Communication in foreign languages
As the unit is for a CLIL class, it makes special attention to the acquisition of the L2 language. Most of the tasks are done in group work, and the unit contains language tips, discussion tips to help students interact with their peers, videos in the target language with no subtitles, and the use of ICT techniques to record themselves. The unit will contribute in making the student aware of specific mathematical language, necessary for the development of sciences and to cope with daily situations. The target language will be used for oral and written expressions at all times to describe concepts, processes, reasoning, proofs, and for communication in general.

3. Mathematical competences and basic competences in science and technology
In this unit students will experiment, observe, deduce and prove everything related to triangles. The architecture present in the unit will help students relate concepts to real-life situations, formulate and solve math problems. By working in teams, students communicate and interact using the mathematical language.

4. Digital competence
Computers are used throughout the unit to use Moodle. Videos are presented in class and uploaded in Moodle. ICTs are used for the final project, especially power point presentations and websites as Slideshare.com and Audacity.com. Calculators are also used to help students with calculus.

5. Learning to learn
Concepts are related from one year to another or with other subjects. It is clearly specified throughout the unit and some reviews are made to go over material as an introduction to new concepts. Discussions about topics are prepared so that students become critical and develop their own point of view and decisions.

6. Social and civic competences
By working in groups cooperatively in some of the tasks in this unit, students must respect and accept the ideas of their mates in the process of solving mathematical problems.

7. Sense of initiative and entrepreneurship
In this unit students are able to work in groups and individually. Some of the practice activities are to be done autonomously, so that the students make his/her own decisions and become self-confident. A unit test at the end will be done to consolidate knowledge and skills.

8. Cultural awareness and expression
The unit is related to architecture, so that students know how to visualize and understand the geometrical figures in buildings from all the world and use the mathematical thought to solve problems.
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</table>

Also, architectural review of all the buildings in London seen throughout the unit and others to visit in the trip.
INSTRUCTIONS FOR IMPLEMENTATION AND ANSWER KEY

In this unit there will be a connection between the mathematical concepts and the present architecture in our lives. Therefore, it is important to start by explaining the meaning of the cover and the title of the unit.
Title: Triangulate the world.
Students will know about triangles and will practice with buildings from all over the world.
Cover: London’s skyline.

LESSON 1
Introduction to architecture
As the group will be travelling to London in May, the lesson starts with a warm-up activity to know who Sir Norman Foster is and the students’ previous knowledge on architecture.
The teacher shows a video: How much does your building weigh, Mr. Foster?
http://www.youtube.com/watch?v=9740R6Kd48I
The video is short and can be played twice. Students should take notes. After watching it, the teacher starts a class discussion, to share opinions and to get an idea of how much do students know about the topic.

Suggested homework: Students should watch a video which is uploaded in Moodle at home, and should answer the questions on page 5 in the SB. Architectural tips are given to help students understand the video:
http://www.youtube.com/watch?v=Lk2cMR0fw_w&feature=related

What does an architect do?
Students get in groups of 4 and try to describe what an architect does. There is a discussion tip box to help students think about different aspects to talk about. Groups must hand in a brief summary of their ideas. All the class fills in the blanks on page 7 in the SB, and the teacher writes ideas on the board.

Polygon review
As students have worked with polygons the previous year, this is just a review of the main concepts.

<table>
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<th>Answer key</th>
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<td>1. Polygon/regular/irregular</td>
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<td>2. Regular/same/inscribe</td>
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<td>3. Irregular</td>
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<td>4. Convex / concave</td>
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<td>5. Apothem</td>
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<td>6. Area</td>
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<td>7. Perimeter/regular</td>
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<td>8. Similar polygons</td>
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</tbody>
</table>
LESSON 2

Warm-up with homework correction from previous lesson.

a) Students hand in questions on video watched at home. The teacher goes over the questions and students should participate actively, especially when discussing what buildings are located in the city of Barcelona.

b) Polygon review: teacher allocates turns so that students read out loud and give the correct answer for each blank.

The teacher should go over the important concepts as apothems, area...

Practice: Areas and perimeters

Students first classify the figures as regular or irregular. Then, they calculate the area and perimeter of each figure.

The teacher helps the class calculate two figures together.

Answer key

### Answer key

- **A = 87.5cm²**  
  **P = 25cm**

- **A = 36cm²**  
  **P = 24cm**

- **A = 60cm²**  
  **P = 24cm**

- **A = 72cm²**  
  **P = 24cm**

- **A = 75cm²**  
  **P = 36cm**

- **A = 63cm²**  
  **P = 32,28cm**

- **A = 68cm²**  
  **P = 32,24cm**

- **A = 64cm²**  
  **P = 32cm**

The “star” of polygons

Students get in groups of 4 to discuss and draw any polygon in the whiteboards. The teacher asks the groups to form triangles inside their polygons. There is a discussion tip box and a language tip box to help students express their ideas and thoughts.

The teacher asks all the whiteboards with all the different polygons to be shared. The teacher introduces the triangulation of polygons and asks the groups to triangulate again, this time using diagonals.

An example is given for students on page 11 in the SB.

Students conclude the triangulation of polygons and fill in the blanks on page 8:

Answer key

$n$ sides / $n-2$ triangles
Triangulate the world

TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

Triangles. Classification
The teacher draws a triangle on the board and with the help of the students, writes the vertices, sides and angles.
Then, the teacher allows the students to match and classify triangles according to their sides and angles.

**Answer key**
- Scalene: 3 sides are different and 3 different angles
- Acute: angle < 90°
- Plain: no angle = 180°
- Right-angled: angle = 90°
- Isosceles: 2 sides are the same and 2 same angles
- Obtuse: angle > 90°
- Equilateral: 3 sides are the same and 3 same angles

The term hypotenuse is introduced by the teacher, who emphasizes the need of a right angle. There is a Did you know... box to help students understand the word. The teacher asks the students to label a right-angled triangle (hypotenuse and legs).
Students are now able to complete the True or False statements:

**Answer key**

**True or False?**
- T  F  In a triangle, sides and angles are opposite to each other.
- T  F  The largest angle is opposite to the shortest side.
- T  F  The middle-sized angle is next to the middle-sized side.
- T  F  The smallest-sized angle is opposite to the smallest angle.

**MATH TIP – LET’S ALL PROVE TOGETHER**
The teacher models a mathematical proof with an example on page 14 in the SB. Students should be aware of the need of a statement, the development of the proof and the # or Q.E.D. at the end.
Students get together in groups of 4. They assign themselves a number from 1 to 4, and will be responsible for the explanation of a proof to the rest of the members of the group.
The teacher provides the “expert” student in each group with a card with the necessary information (see Annex 2 Materials needed for implementation).
**LESSON 3**

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students’ needs.

**Recognition of polygons in buildings**
Students get together in groups of 4. Each group has a paper slip for each of the buildings on page 10 in the SB. They must list and/or draw all the polygons they can find in the pictures. The teacher collects all the slips and organizes them according to buildings, sharing the results with the class so that students can learn from their peers. (See Annex 2 Materials needed for implementation).

**Properties of triangles**
The properties of triangles will be proved by the “expert” student in each group. He/She will be responsible for the explanation to the rest of the group. The teacher has a private conversation with all the experts to make sure they have all understood the proof and to reinforce the idea that they are now the teachers.

Property 1: The angles of a triangle.
Statement: The three interior angles in a triangle always add up to 180°.

Property 2: The angles of a polygon.
Statement: The addition of the interior angles in a polygon is 180(n-2).

After the proofs have been explained, each student should reflect on what he/she has understood and fill in the self-assessment chart on proofs 1 and 2 on page 15 in the SB (1 minute).

The teacher will give each group a paper slip. It is a rubric for peer-assessment. Each expert student will be graded according to a rubric and will receive the slip with the detailed parts of the assessment for future improvement (see Annex 2 Materials needed for implementation).
Lesson 4

Warm-up with homework correction from previous lesson. Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students’ needs.

Properties of triangles
The properties of triangles will be proved by the “expert” student in each group. He/She will be responsible for the explanation to the rest of the group. The teacher has a private conversation with all the experts to make sure they have all understood the proof and to reinforce the idea that they are now the teachers.

Property 3: Triangle inequality.
Statement: The addition or subtraction of the length of two sides of a triangle is always greater than (addition) or less than (subtraction) the length of the other side.

Property 4: Area.
Statement: The area of a triangle is the base of the triangle multiplied by the height of the triangle and divided into two parts.

After the proofs have been explained, each student should reflect on what he/she has understood and fill in the self-assessment chart on proofs 3 and 4 on page 17 in the SB (1 minute).

The teacher will give each group a paper slip. It is a rubric for peer-assessment. Each expert student will be graded according to a rubric and will receive the slip with the detailed parts of the assessment for future improvement (see Annex 3 Materials needed for implementation).

Introduction to the PROJECT- Ancient Triangles
At this stage of the unit where students know quite a bit on triangles, the teacher should explain what the Project will consist of, so that students know what is going to be expected from them. The deadline date should be announced, as the project will be assessed and included in the unit’s final evaluation.

The purpose of this task is to make students discover, find out, and think about the use of geometrical figures in the architecture of Turkey, Greece, Egypt, Italy, and China. ICTs (school’s Moodle, Wikispaces, Audacity and SlideShare) will be used to share the projects within classmates.

Students get together in groups of 4. They are assigned a building belonging to ancient architecture which students must match to the pictures on page 41 in the SB. There is an architectural tip box to help students understand the meaning of ancient architecture.
The teacher goes over the instructions on page 39 in the SB with all the class to make sure everything is clear. Although SlideShare and Audacity are introduced to students on page 40 in the SB with defined steps to follow, the teacher models with an example of a presentation in which these ICT resources are used.

After all the projects are uploaded in Moodle, the teacher asks to look at other groups’ presentations and evaluate them according to the rubric on page 42 in the SB. The rubrics are collected by the teacher so that he/she can take into account the students’ opinions and make sure that all projects have been shared.
LESSON 5
Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the
class. The teacher supervises and explains/corrects when necessary, depending on the
importance of the math problem or the students’ needs.

Centers of triangles
The teacher explains 3 different centers of triangles according to their characteristics:
orthocenter, barycenter or centroid, and circumcenter.
The teacher elicits previous knowledge, as this topic is also studied in other subjects as Art or
Technology.
Students should participate in the process of finding out the center of a triangle with heights,
medians and midpoints. The teacher helps in the process.
Students can draw, with the help of a compass, the triangle specified on page 19 in the SB to
find its orthocenter, barycenter and circumcenter and hand it in to the teacher for correction.

Testing what we know!
Students get together in groups of 4. They must prepare a word problem about triangles to
test another team in the classroom. An example is provided on page 21 in the SB.
Teams will exchange the word problems, must solve them and complete the peer-assessment
chart on the following page.
At the end of the activity, the teacher asks a student to read the conclusions about triangles
filling in the blanks:

Answer key
One
One
Equilateral

Proof Quiz
As students have been “experts” proving the properties of triangles, they must take a 10’ quiz
to consolidate knowledge. Each student was responsible for the explanation of a proof to the
rest of the group, but they are also responsible to know about the proofs explained to them.
The teacher should make sure that each student is asked to prove a different property of
triangles than the one proved as an “expert”.
(See Annex 2 Materials needed for implementation).

Important!
The teacher asks students to bring the materials specified on page 25 in the SB (Follow the
steps…) for the next lesson.
LESSON 6

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students’ needs.

Pythagoras was a square
To introduce Pythagoras to students, the teacher shows the following video:
Pythagoras was a square http://www.teachertube.com/viewVideo.php?video_id=23675

The purpose of this task is to make students think about the geometrical interpretations shown in the video. They should watch, listen and then fill in the blanks and answer the questions on page 24 in the SB.
As the geometrical interpretations are not easy to follow at first sight, the teacher can upload the video in Moodle so that students can watch it again at home as many times as needed to understand and answer the questions.
There is a language tip box to help students answer the questions with complete sentences.

Answer key

1. Greek
   Father / numbers
   Proof
   Expert / souls / ratios
   Music / perfect squares
   Square roots / theorem
   Hypotenuse
   Equal / squares

2. Three interpretations: squares, rectangles, quarter circles

3. The problem of using similar rectangles to interpret the theorem is that there is more information needed, the measurement of the two sides.

4. Yes, we can come up with a true statement by using quarter circles.

Pythagorean Theorem - Let’s all prove together
Students get together in groups of 4, all of them with the squares and right-angled triangles students were asked to bring in the previous lesson. The teacher asks the class to arrange the 2 squares and the 8 triangles in two different ways according to the steps specified on page 25 in the SB.
The purpose of this task is to prove the Pythagorean Theorem statement, both geometrically and algebraically.
The teacher does the same with his/her squares and triangles on the board, using blue-tack to hold them up so that all the class can see: the geometrical interpretation is proved.
The teacher goes a step further and asks students to prove the algebraic interpretation of the theorem from what they have arranged.
LESSON 7

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students’ needs.

The importance of the theorem
The teacher explains why the theorem is important.
1. To check that the equality is true for all right-angled triangles.
2. To find the length of the hypotenuse.
3. To find the length of a side.

Although examples are given for each, a running dictation has been prepared to make sure students know how to use the theorem.
(In a running dictation students work in groups and each member runs to listen to information which is being dictated. Then, the student goes back to the group and all members work collaboratively.)

Students get together in groups of 4, and number themselves from 1 to 4. The teacher calls them by their numbers randomly and dictates the measures of each side of a triangle. (See Annex 2 Materials needed for implementation).

A chart is prepared on page 25 in the student’s book to fill in, with an example given.

When the chart is completed, students are able to answer the questions below it.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>(c^2 + b^2 = a^2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IT IS A RIGHT-ANGLED TRIANGLE</td>
</tr>
<tr>
<td>Student 1</td>
<td>26</td>
<td>24</td>
<td>10</td>
<td>676 = 676</td>
</tr>
<tr>
<td>Student 2</td>
<td>20</td>
<td>17</td>
<td>19</td>
<td>400 = 650</td>
</tr>
<tr>
<td>Student 3</td>
<td>17</td>
<td>8</td>
<td>15</td>
<td>289 = 289</td>
</tr>
<tr>
<td>Student 4</td>
<td>30</td>
<td>20</td>
<td>40</td>
<td>900 = 2000</td>
</tr>
</tbody>
</table>

Answer key
True or False page 24

**T**

The theorem helps you find the missing leg of a triangle.

**F**

This theorem is true for all triangles, no matter the angles.

**T**

We use the theorem to find the length of the hypotenuse.

**F**

The theorem can be used to find the diagonal of a square.

**F**

The theorem can be used to find the diagonal of a rectangle.

**F**

The theorem can be proved with quarter circles.

**T**

The theorem can be proved with any geometrical figure.

**Let’s practice across culture!**

In this section seven word problems are prepared to practice using the Pythagorean Theorem in real buildings from all over the world. A language tip box and a hint box are present to help the student solve the word problem and explain it to the rest of the class. Students can practice by working in groups or individually.

**Answer key**

**Forum Building:**

It is an equilateral triangle, as each of its sides measures 180m.

**Staircase:**

The ramp’s length will be 1,66m.

**Turning Torso:**

576 = 724 NO, the triangles are not right-angled.

**Big Ben:**

The distance from the feet to the top of the tower will be 96,13m.

**Petronas Twin Towers:**

The missing side of the triangle measures 30,38m and it occupies 7,6 floors.

**Eixample in Barcelona:**

The hypotenuse of the corner measures 21,22m.
The area of each block is 12.386,89 m².
The perimeter of each block is 418,08m.

**Louvre Museum:**

The theorem is used twice.
The length of the edge between two sides is 32,2m.
LESSON 8

Warm-up with homework correction from previous lesson. Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students’ needs.

Who was Thales of Miletus? Students get together in groups of 4 and read some sentences about Thales of Miletus. They can discuss and organize the sentences to place them in the correct order. Then, with the help of a language tip box, students write a brief summary together. The teacher collects one per working group.

The theorem The teacher explains the theorem starting with the statement and drawing it. Students should be able to fill in the blanks about similar triangles. A discussion tip box is provided to help students share their ideas and opinions on three pictures of similar triangles.

1. To divide a segment into equal parts.
2. For proportions and similarities. Triangles placed in the Thales’ position.
3. Shadows and proportionality to find the length of objects, buildings...

The teacher explains the three concepts, gives an example of each and asks the students to draw a segment and divide it into equal parts without using a ruler; only a compass.

The teacher goes over the examples written on page 35 on rays of sunlight forming a triangle. Although the example is clear, the teacher prepares a real example in the classroom using string and a measuring tape:
What is the classroom’s height?

Students imagine the height cannot be measured, so a string will be used to find out. The string hangs from the ceiling and is pulled forming a triangle. A student will be inside the triangle formed. The procedure is the same as if there was sunlight.
**Triangulate the world**

**Triangles, Pythagoras and Thales theorems related to architecture**

**Practice – Voki**

The word problems in this section have been thought to be dictated by a Voki character. The idea is that students first have a dictation and then solve the problems, so the teacher can also dictate the following questions:

1. Draw a segment of 4.5 cm and divide it into 13 equal parts.
2. Calculate the values of x and y in the drawing.
3. Are the triangles ABD and BDC similar to triangle ABC? Why?
4. The man in the picture is going to enter a store in the Soho District in New York City. His height is 1.82 m. Calculate the height of the door.
5. A rectangle has sides 10 cm x 25 cm. A smaller and similar rectangle has its short side measuring 8 cm. What is the similarity ratio of the rectangles? And, what does the long side of the small rectangle measure?

---

**Answer key**

1. Draw...
2. \( x = 10.5\, \text{cm} / \, y = 5.25\, \text{cm} \)
3. Triangles ADB and ABC have both a 90° angle and the common angle A.
   Triangles BDC and ABC have both a 90° angle and the common angle C.
   Therefore, triangles BDC and ADB are similar to triangle ABC.
4. The door's height is 3.2 m.
5. The similarity ratio of the rectangles is 1.25.
   The long side of the small rectangle measures 20 cm.
Triangulate the world

TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

LESSON 9

Warm-up with homework correction from previous lesson. Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students’ needs.

Final thought
As mathematics have been related to architecture throughout the unit, the teacher will show a video of the well-known English mathematician Marcus du Sautoy in one of the famous Italian architect Palladio’s buildings.

The teacher will ask a student to read about Andrea Palladio and another student to read about Marcus du Sautoy. A discussion tip box is prepared to help students start with a class discussion. To make students talk, the teacher asks the students to divide themselves into two big groups, one on the side of mathematicians and the other on the side of architects, as a general class debate.

After that, a quick review of all the important mathematical concepts seen throughout the unit with practice. And after that, a quick review of all the important buildings to visit in London during the school trip.
LESSON 10

Unit Exam (See Annex 2 Materials needed for implementation).

Answer key

1. a) Possible answers: areas, volumes, perimeters, calculus, drawings, polygons...
   b) It is an open question, as it depends on students’ opinion.

2. Triangulation of polygons
   The triangulation of polygons refers to the decomposition of the polygon into triangles.
   We draw diagonals to divide the polygon in triangles. Triangulation reduces complex
   shapes into simple shapes. The polygon becomes an addition of triangles. Any polygon
   admits a triangulation. If the polygon has n sides, then the polygon has n-2 triangles.

3. Hypotenuse
   The longest side in a right-angled triangle is called hypotenuse.
   There is always a hypotenuse in all right-angled triangles.
   Hypotenuse comes from the Greek hypoteinousa, Hupo meaning the opposite; opposite to
   the right angle.

4. Answers: Right angle / Obtuse / Acute / Isosceles

5. Prove that the addition of the interior angles of a triangle is 180°.
   Algebraic explanation
   Given any triangle, label the 3 angles (A;B;C) and draw 3 auxiliary lines:
   a line parallel to the base of the triangle and elongate the other 2 sides.
   3 new angles A’, B’, C’ are obtained.
   Notice that <A and >A’ and <C and <C’ are congruent angles (alternate),
   So <A’ + <B’ + <C’ form a plain angle. We know a plain angle is 180°.
   Therefore, we can substitute and say that <A + <B + <C = 180°

Q.E.D
6. 

\[
\sum = 2 \cdot 180^\circ = 360^\circ \quad \sum = 2 \cdot 180^\circ = 360^\circ \quad \sum = 2 \cdot 180^\circ = 360^\circ \quad \sum = 3 \cdot 180^\circ = 540^\circ
\]

The formula used is: 
\[
\sum \text{ angles in a polygon with } n \text{ sides} = (n-2) \cdot 180
\]

7. 

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
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<tbody>
<tr>
<td>4</td>
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</tr>
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<td>12</td>
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<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
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</tr>
</tbody>
</table>

8. 

a) The pyramid’s height is 146.09 m.

b) Marc’s ladder is 6.5 m long.

Extra Credit

The interior angle of a regular pentagon is 108°.
The measure of an interior angle in a regular polygon with \( n \) sides is \( \frac{(n-2) \cdot 180}{n} \).
### Assessment Chart

<table>
<thead>
<tr>
<th>Session</th>
<th>Activities</th>
<th>Timing</th>
<th>Assessment tool / form</th>
<th>Assessment criteria</th>
<th>Comments</th>
</tr>
</thead>
</table>
| In all sessions throughout the unit | **Active participation in class**              |        | Positives-negatives list | E1: Students participate in class discussions  
E2: Students should comprehend specific information given  
E5: Students should make use of a coherent oral expression | Teacher writes active participation in the positives-negatives list.       |
| 1       | **Introduction to Ch 1: Architecture.**        | 10’    |                        |                                                                                      |                                                                          |
|         | Video “How much does your building weigh,      |        |                        |                                                                                      |                                                                          |
|         | Mr Foster?”                              |        |                        |                                                                                      |                                                                          |
|         | Questions on video in Moodle: “Welcome to     |        | Homework               | E2: Students should comprehend specific information given:  
Recognize buildings in their city                                       | Students hand in questions:  
Teacher gives a grade                        |
|         | architecture”                              |        |                        |                                                                                      |                                                                          |
|         | **Group discussion: What does an architect do?** | 10’    | Gradebook              | E1: Students participate in class discussions to communicate and to share points of view | Students hand in summary:  
Teacher gives a grade                        |
<p>|         | <strong>Introduction to Ch 2: Polygons.</strong>           | 15’    |                        |                                                                                      |                                                                          |
|         | Review concepts from last year               |        |                        |                                                                                      |                                                                          |
| 2       | <strong>Homework correction</strong>                      | 10’    |                        |                                                                                      |                                                                          |
|         | <strong>Areas and perimeters of polygons</strong>         | 10’    |                        |                                                                                      |                                                                          |
|         | <strong>Introduction to Ch 3: Triangles.</strong>         | 25’    |                        |                                                                                      |                                                                          |
|         | Classification and characteristics. Proofs    |        |                        |                                                                                      |                                                                          |</p>
<table>
<thead>
<tr>
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<th>Assessment criteria</th>
<th>Comments</th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>Homework correction</td>
<td>5’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Recognition of polygons in buildings. Group discussion</td>
<td>20’</td>
<td>Gradebook</td>
<td>M3: Students should share and discuss mathematical ideas M8: Students should be able to identify geometrical figures in real buildings E10: Work collaboratively</td>
<td>Groups hand in analyzed slips for each building: Teacher gives a grade</td>
</tr>
<tr>
<td></td>
<td>Proofs: 2 Properties of triangles proved by 2 expert students</td>
<td>25’</td>
<td>Gradebook</td>
<td>E9: Each working group evaluates the work done by its expert student</td>
<td>Students assess the expert student with a rubric and give a final grade</td>
</tr>
<tr>
<td></td>
<td>Peer-assessment on proofs</td>
<td>2’</td>
<td>Rubric</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Homework correction</td>
<td>10’</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Proofs: 2 Properties of triangles proved by 2 expert students</td>
<td>25’</td>
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<td>2’</td>
<td>Rubric</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Introduction to Ch 6: Project</strong></td>
<td>10’</td>
<td>Rubric</td>
<td>E5: Students should make use of a coherent oral expression E7: Digital and internet resources should be used to search, organize, gather and present information M4: Students should be able to write about mathematical thoughts and ideas M6: Students should identify and use proportionality to solve real-life situations M8: Students can identify geometrical figures in</td>
<td>Teacher gives a grade following the rubric’s results</td>
</tr>
<tr>
<td>Session</td>
<td>Activities</td>
<td>Timing</td>
<td>Assessment tool / form</td>
<td>Assessment criteria</td>
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<tr>
<td>5</td>
<td><strong>Homework correction</strong></td>
<td>10’</td>
<td></td>
<td>��non-mathematical contexts M9: Students must be able to calculate lengths, areas and volumes and should express the result with adequate measuring units.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Centers of triangles</strong></td>
<td>15’</td>
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<td>��                                                                ��</td>
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</tr>
<tr>
<td></td>
<td><strong>Team work: Testing what we know! Groups prepare a word problem to test other groups</strong></td>
<td>20’</td>
<td></td>
<td>��M3: Students should share mathematical ideas working in groups. E9: Each working group evaluates the work done by other groups E10: Work collaboratively</td>
<td>Students assess the work done by another group</td>
</tr>
<tr>
<td>5</td>
<td><strong>Peer-assessment on word problems</strong></td>
<td>2’</td>
<td>Rubric</td>
<td>��</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Proof Quiz: Properties of triangles</strong></td>
<td>10’</td>
<td>Quiz</td>
<td>��E3: Students should understand the specific questions written M4: Students can express their thoughts and ideas on the topic</td>
<td>Teacher writes quiz grades in the gradebook</td>
</tr>
<tr>
<td>6</td>
<td><strong>Homework correction</strong></td>
<td>10’</td>
<td></td>
<td>��                                                                ��                                                                ��</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Introduction to Ch 4: Pythagorean Theorem. Video “Pythagoras was a square”</strong></td>
<td>10’</td>
<td>Positives-negatives list</td>
<td>��E1: Students participate in class discussions to communicate and to share points of view</td>
<td>Teacher writes active participation in the positives-negatives list</td>
</tr>
<tr>
<td>Session</td>
<td>Activities</td>
<td>Timing</td>
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<td>---------------------------------------------------------------------------------------------------</td>
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<tr>
<td></td>
<td>Questions on video in Moodle: “Pythagoras was a square”</td>
<td>-</td>
<td>Homework</td>
<td>E2: Students should comprehend specific information given</td>
<td>Students hand in questions: Teacher gives a grade</td>
</tr>
<tr>
<td></td>
<td>Discovering the geometrical and algebraic interpretation of the theorem</td>
<td>20’</td>
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</tr>
<tr>
<td></td>
<td>Homework correction</td>
<td>10’</td>
<td></td>
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<tr>
<td>7</td>
<td>The importance of the theorem</td>
<td>15’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Running dictation to find out if specific triangles are right-angled triangles</td>
<td>10’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Practice: Let’s practice across culture!</td>
<td>10’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Homework correction</td>
<td>10’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td><strong>Introduction to Ch 5: Thales.</strong> Group discussion to organize sentences and produce a brief summary</td>
<td>10’</td>
<td>Gradebook</td>
<td>E1: Students should participate in class discussions to communicate and to share points of view</td>
<td>Students hand in a summary: Teacher gives a grade</td>
</tr>
<tr>
<td></td>
<td>Thales Theorem and its uses</td>
<td>20’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Word-problem dictation (Voki)</td>
<td>5’</td>
<td>Homework</td>
<td>M6: Students should identify and use proportionality to solve real-life situations E2: Students should comprehend specific information</td>
<td>Students hand solved word problems: Teacher gives a grade</td>
</tr>
<tr>
<td>Session</td>
<td>Activities</td>
<td>Timing</td>
<td>Assessment tool / form</td>
<td>Assessment criteria</td>
<td>Comments</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>--------</td>
<td>------------------------</td>
<td>---------------------</td>
<td>----------</td>
</tr>
<tr>
<td>9</td>
<td>Homework correction</td>
<td>10’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Introduction to Ch 7: Final thought.</strong> Video “Andrea Palladio and Marcus du Sautoy”</td>
<td>10’</td>
<td>Positives-negatives list</td>
<td>E1: Students participate in class discussions to communicate and to share points of view</td>
<td>Teacher writes active participation in the positives-negatives list</td>
</tr>
<tr>
<td>9</td>
<td>Who’s who? What do they have in common? Class debate</td>
<td>10’</td>
<td>Gradebook</td>
<td>E1: Students participate in class discussions to communicate and to share points of view</td>
<td>Students hand in discussion conclusion: Teacher gives a grade</td>
</tr>
<tr>
<td>9</td>
<td>Unit review and practice</td>
<td>20’</td>
<td></td>
<td></td>
<td>Also, architectural review of all the buildings in London seen throughout the unit and others</td>
</tr>
<tr>
<td>10</td>
<td>Unit Exam</td>
<td>55’</td>
<td></td>
<td>E3: Students should understand the specific questions written M4: Students can express their thoughts and ideas on the topic M6: Students should identify and use proportionality to solve real-life situations M8: Students can identify geometrical figures in non-mathematical contexts M9: Students must be able to calculate lengths, areas and volumes and should express the result with adequate measuring units.</td>
<td>Teacher writes exam grades in the gradebook</td>
</tr>
</tbody>
</table>
ANNEXES

Annex 1. Image sources
Annex 2. Materials needed for implementation
Annex 3. Video list
Annex 4. CD-ROM
ANNEX 1. IMAGE SOURCES

Image 1, Student’s book, page 1:
Iliff, David. “City of London skyline from London City Hall - Oct 2008.jpg”. Wikipedia.org,
(last accessed March 12, 2011)
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported
(http://creativecommons.org/licenses/by-sa/3.0/deed.en) license.

Image 2, Student’s book, page 5:
User: Someformofhuman. “Petronas Panorama II”. Wikipedia.org,
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported
(http://creativecommons.org/licenses/by-sa/3.0/deed.en), 2.5 Generic
(http://creativecommons.org/licenses/by-sa/2.5/deed.en), 2.0 Generic
(http://creativecommons.org/licenses/by-sa/2.0/deed.en) and 1.0 Generic
(http://creativecommons.org/licenses/by-sa/1.0/deed.en) license.

Image 3, Student’s book, page 5:
Alexander Z. “Torre Agbar, Barcelona”. Wikipedia.org,
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Generic
(http://creativecommons.org/licenses/by-sa/3.0/deed.es) license.

Image 4, Student’s book, page 6:
User: MykReeve. “Guggenheim Museum Bilbao”. Wikipedia.org,
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Generic
(http://creativecommons.org/licenses/by-sa/3.0/deed.es) license.

Image 5, Student’s book, page 6:
Flickr User: tinyfroglet. “Farnsworth House (Illinois) by Mies van der Rohe”. Wikipedia.org,
This image is licensed under the Creative Commons Attribution 2.0 Generic
(http://creativecommons.org/licenses/by/2.0/deed.es) license.

Image 6, Student’s book, page 7:

Image 7, Student’s book, page 7:
Vera Llistar, Anna. Top of the Rock Building, in New York. Photograph by the author.

Image 8, Student’s book, page 10:
Pingstone, Adrian. “The base of the Swiss Re Tower.”. Wikipedia.org,
Photograph courtesy of Adrian Pingstone.
Triangulate the world

TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

Image 9, Student’s book, page 10:

Image 10, Student’s book, page 10:

Image 11, Student’s book, page 10:
Vera Llistar, Anna. Hearst Tower, in New York. Photograph by the author.

Image 12, Student’s book, page 10:

Image 13, Student’s book, page 11:
Vera Llistar, Anna. IES Salvador Espriu, Barcelona. Photograph by the author.

Image 14, Student’s book, page 23:
User: Jacobolus. “Detail of The School of Athens by Raffaello Sanzio, 1509, depicting Pythagoras, writing in foreground, with Averroes, Hypatia of Alexandria, and Parmenides behind him”. Wikipedia.org, http://en.wikipedia.org/wiki/File:Sanzio_01_Pythagoras.jpg (last accessed March 6, 2011). “This image is in the public domain because its copyright has expired. This applies to Australia, the European Union and those countries with a copyright term of life of the author plus 70 years”.

Image 15, Student’s book, page 29:
Flickr User: ILLESH. “Edifici Fòrum a Barcelona, vist de s de l’Hotel AC”. Wikipedia.org, http://es.wikipedia.org/wiki/Archivo:Edifici_F%C3%B2rum_Barcelona_Catalonia.jpg (last accessed March 7, 2011). This image is licensed under the Creative Commons Attribution 2.0 Generic (http://creativecommons.org/licenses/by/2.0/deed.es) license.

Image 16, Student’s book, page 30:
Image 17, Student's book, page 30:

Image 18, Student's book, page 30:

Image 19, Student's book, page 31:
User: Someformofhuman. “Petronas Panorama II”. Wikipedia.org, http://en.wikipedia.org/wiki/File:Petronas_Panorama_II.jpg (last accessed April 17th, 2011) This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported (http://creativecommons.org/licenses/by-sa/3.0/deed.en) , 2.5 Generic (http://creativecommons.org/licenses/by-sa/2.5/deed.en) , 2.0 Generic (http://creativecommons.org/licenses/by-sa/2.0/deed.en) and 1.0 Generic (http://creativecommons.org/licenses/by-sa/1.0/deed.en) license.

Image 20, Student's book, page 31:

Image 21, Student's book, page 31:

Image 22, Student's book, page 31:

Image 23, Student's book, page 32:
Triangulate the world
TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

Image 24, Student’s book, page 32:

Image 25, Student’s book, page 32:

Image 26, Student’s book, page 33:

Image 27, Student’s book, page 38:
Vera Llistar, Anna. The Soho District, in New York. Photograph by the author.

Image 28, Student’s book, page 40:
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Image 29, Student’s book, page 41:

Image 30, Student’s book, page 41:

Image 31, Student’s book, page 41:

Image 32, Student’s book, page 41:
**Triangulate the world**

**TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE**

**Image 33, Student’s book, page 41:**
Photograph courtesy of user NoFight.

**Image 34, Student’s book, page 41:**
Vera Listar, Anna. Parthenon in Athens, Greece. Photograph by the author.

**Image 35, Student’s book, page 41:**

**Image 36, Student’s book, page 43:**
Photograph courtesy of user Padraig.

**Image 37, Student’s book, page 43:**
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**Image 38, Student’s book, page 44:**
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**Image 39, Student’s book, page 44:**
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**Image 40, Student’s book, page 44:**
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Triangulate the world
TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

Clip Arts, Student's book, pages 13, 14, 15, 16, 17, 18, 21, 22, 24, 25, 27, 35, 39, 42:
Martin, Phillip. “Free Clip Art by Phillip Martin”
(See the attached mail from the author).

Smileys, Student's book, pages 15, 17, 42:
Clker.com."Vector clip art online, royalty free & public domain"

Images in Teacher's book, page 39:
Pingstone, Adrian. “The base of the Swiss Re Tower.”. Wikipedia.org,
Photograph courtesy of Adrian Pingstone.

Ribbefjord, Andreas. “Stockholms-stadsbibliotek-2003-04-14.jpg”. Wikipedia.org,
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported
(http://creativecommons.org/licenses/by-sa/3.0/deed.en) license.

Larripa, Sergi. “Catalunya National Theater, in Barcelona”. Wikipedia.org,
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported
(http://creativecommons.org/licenses/by-sa/3.0/deed.en) license.

Images in Teacher's book, page 40:
Vera Llistar, Anna. Hearst Tower, in New York. Photograph by the author.

User: Lorax. “The Flatiron Building in New York City”. Wikipedia.org,
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Generic
(http://creativecommons.org/licenses/by-sa/3.0/deed.es) license.

Clip Arts, Teacher's book, pages 41, 42, 43, 44, 45, 46, 49, 50:
Martin, Phillip. “Free Clip Art by Phillip, p Martin”
(See the attached mail from the author).

http://creativecommons.org/licenses/by-sa/3.0/deed.es license.
ANNEX 2. MATERIALS FOR IMPLEMENTATION

Recognition of polygons in buildings
Let’s all prove together. Expert cards
Let’s all prove together. Peer-assessment cards.
Proof Quiz
Pythagoras Running Dictation
Unit Exam
Recognition of polygons in buildings

Can you recognize polygons in this picture? Which ones?

✔ ...

30S1 Mary Axe, in London. Foster and Partners

Can you recognize polygons in this picture? Which ones?

✔ ...

Stockholm’s Public Library, in Sweden. Erik Gunnar Asplund

Can you recognize polygons in this picture? Which ones?

✔ ...

Catalunya National Theater, in Barcelona. Ricardo Bofill
Can you recognize polygons in this picture? Which ones?

✓ ...
✓
✓
✓
✓
✓

Hearst Tower, in New York. Foster and Partners

Can you recognize polygons in this picture? Which ones?

✓ ...
✓
✓
✓
✓
✓

Flatiron Building, in New York. Daniel Burnham
Property 1: The angles of a triangle

Statement: The three interior angles in a triangle always add up to 180°.

Proof: The proof can be done in two ways:

1. Using a physical model.
   Given a physical scalene (a non-specific triangle) triangle in a piece of paper, label and then tear up the 3 angles. Place the 3 different angles next to each other. They form a straight line.
   We know a straight line is 180°.

   \[ \angle A + \angle B + \angle C = 180^\circ \]

2. Algebraic explanation
   Given any triangle, label the 3 angles \( A, B, C \) and draw 3 auxiliary lines: a line parallel to the base of the triangle and elongate the other 2 sides. 3 new angles \( A', B', C' \) are obtained.
   Notice that \( \angle A \) and \( \angle A' \) and \( \angle C \) and \( \angle C' \) are congruent angles (alternate), so \( \angle A' + \angle B' + \angle C' \) form a plain angle. We know a plain angle is 180°.
   Therefore, we can substitute and say that \( \angle A + \angle B + \angle C = 180^\circ \)

Q.E.D.
Property 2: The angles of a polygon

Statement: The addition of the interior angles in a polygon is $180^\circ \cdot (n-2)$

Proof:
1. We choose a pentagon (5 sides) as our polygon and we triangulate it.
   (Remember that triangulation can only be done with diagonals).
2. We have obtained 3 triangles:

   ![Triangle Triangulation Diagram]

3. As we know, the three interior angles in a triangle always add up to $180^\circ$.
   - The angles in $\triangle 1 = 180^\circ$
   - The angles in $\triangle 2 = 180^\circ$
   - The angles in $\triangle 3 = \frac{180^\circ}{540^\circ}$

   We do the same with a hexagon and a decagon:

   ![Hexagon and Decagon Triangulation Diagrams]

   - The number of triangles in polygon 1 = 4
   - The number of triangles in polygon 2 = 8

   We can find a pattern:
   We will always obtain 2 triangles less than the sides of the polygon.

   $(n \text{ sides} - 2) \cdot 180^\circ$ (the angles inside a triangle) = $540^\circ$ (the angles in a polygon)

   $(n-2) \cdot 180^\circ = \text{angles in a polygon with n sides}$

Q.E.D
**Property 3: Triangle inequality**

**Statement:** The addition of the length of two sides of a triangle is always greater than the length of the other side.

The subtraction of the length of two sides of a triangle is always less than the length of the other side.

**Proof:**
Let’s check for \( AB = 40, \ BC = 27, \ AC = 21 \)

There is a special relationship between the 3 sides in any triangle:

**Addition**
\[
\begin{align*}
AB + AC & > BC \rightarrow 40 + 21 > 27 \\
AB + BC & > AC \rightarrow 40 + 27 > 21 \\
BC + AC & > AB \rightarrow 27 + 21 > 40
\end{align*}
\]

Moreover, with \( < \) it cannot be possible. This would happen:

**Subtraction**
\[
\begin{align*}
AB - AC & < BC \rightarrow 40 - 21 < 27 \\
AB - BC & < AC \rightarrow 40 - 27 < 21 \\
BC - AC & < AB \rightarrow 27 - 21 < 40
\end{align*}
\]

Moreover, with \( > \) it cannot be possible. This would happen:

All these are inequalities, \( > \) or \( < \).

If they were true with an equal sign \( = \), we would obtain a straight line.

This is the border case. What area does the triangle have?

Q.E.D
Property: Area

Statement: The area of a triangle is the base of the triangle multiplied by the height of the triangle and divided into two parts.

Proof:

We have two triangles that are exactly the same:

If we place them in the position shown in the drawing below, we obtain a parallelogram with the same base and height as the triangles.

We know the area of a parallelogram is the length of the base multiplied by the height: \( A = b \cdot h \)

This parallelogram has been formed with two same triangles, therefore, the area of each triangle is the area of the parallelogram divided by 2.

\[
A = \frac{b \cdot h}{2}
\]

Q.E.D
### Peer-assessment on proofs

<table>
<thead>
<tr>
<th></th>
<th>Student 1</th>
<th>Student 2</th>
<th>Student 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5</strong></td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td><strong>Explanation is not clear at all</strong></td>
<td>Explanation is very clear</td>
<td>Explanation is very clear</td>
<td>Explanation is very clear</td>
</tr>
<tr>
<td><strong>No use of mathematical terminology</strong></td>
<td>Some mathematical terminology used</td>
<td>Some mathematical terminology used</td>
<td>Some mathematical terminology used</td>
</tr>
<tr>
<td><strong>Examples not used to explain the proof</strong></td>
<td>Examples used to explain the proof</td>
<td>Examples used to explain the proof</td>
<td>Examples used to explain the proof</td>
</tr>
<tr>
<td><strong>The expert student does not make sure the members of the group understand the proof</strong></td>
<td>The expert student makes sure the members of the group understand the proof</td>
<td>The expert student makes sure the members of the group understand the proof</td>
<td>The expert student makes sure the members of the group understand the proof</td>
</tr>
<tr>
<td><strong>TOTAL SCORE</strong></td>
<td><strong>points</strong></td>
<td><strong>points</strong></td>
<td><strong>points</strong></td>
</tr>
</tbody>
</table>
**Peer-assessment on proofs**

<table>
<thead>
<tr>
<th>5</th>
<th>10</th>
<th>THE GRADE OBTAINED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanation is not clear at all</td>
<td>Explanation is very clear</td>
<td></td>
</tr>
<tr>
<td>No use of mathematical terminology</td>
<td>Some mathematical terminology used</td>
<td></td>
</tr>
<tr>
<td>Examples not used to explain the proof</td>
<td>Examples used to explain the proof</td>
<td></td>
</tr>
<tr>
<td>The expert student does not make sure the members of the group understand the proof</td>
<td>The expert student makes sure the members of the group understand the proof</td>
<td></td>
</tr>
</tbody>
</table>

**TOTAL SCORE** points
Prove the following property of triangles:

Statement: The three interior angles in a triangle always add up to 180°.

10' QUIZ: Properties of triangles

Prove the following property of triangles:

Statement: The addition of the interior angles in a polygon is 180 (n-2).
10' QUIZ: Properties of triangles

Prove the following property of triangles:

Statement: The area of a triangle is the base of the triangle multiplied by the height of the triangle and divided into two parts.

Prove the following property of triangles:

Statement: The addition or subtraction of the length of two sides of a triangle is always greater than (addition) or less than (subtraction) the length of the other side.
RUNNING DICTATION

Is this a right-angled triangle?

Check if the triangle with sides \( a \), \( b \), and \( c \) is a right-angled triangle.

For: \( a = 26 \)
\( b = 24 \)
\( c = 10 \)

--

RUNNING DICTATION

Is this a right-angled triangle?

Check if the triangle with sides \( a \), \( b \), and \( c \) is a right-angled triangle.

For: \( a = 20 \)
\( b = 17 \)
\( c = 19 \)
**RUNNING DICTATION**

**STUDENT 3**

Is this a right-angled triangle?

Check if the triangle with sides $a$, $b$, and $c$ is a right-angled triangle.

For:  
$a = 17$
$b = 8$
$c = 15$

---

**RUNNING DICTATION**

**STUDENT 4**

Is this a right-angled triangle?

Check if the triangle with sides $a$, $b$, and $c$ is a right-angled triangle.

For:  
$a = 30$
$b = 20$
$c = 40$
UNIT EXAM: Triangulate the world

1. About architecture and math:
   a) List five words (verbs, names, actions...) related to architecture and to the tasks you do in math. Follow the example given.

<table>
<thead>
<tr>
<th>Architecture word</th>
<th>When do you use them in math</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. measure</td>
<td>In maths we measure angles</td>
</tr>
<tr>
<td>2.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
</tr>
</tbody>
</table>

   b) From all the buildings you have seen throughout the unit, what is the one that you like best? Use geometrical reasons to explain. Where is it located?

2. Explain the triangulation of polygons. Explain what it is and how to do it. Give an example. Hint: I hope you will use the word diagonal.

3. Define the word hypotenuse.
4. **Classify** and **label** the following triangles and angles:

![Diagrams of triangles and angles]

5. Prove that the addition of the interior **angles** of a triangle is 180°.

6. **Triangulate** each polygon to find the sum of its interior angles. Write the formula you have used.

![Diagrams of polygons]
7. Complete the following chart according to the values given for sides a, b, and c, using a famous theorem.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=4</td>
<td>?</td>
<td>c=5</td>
</tr>
<tr>
<td>a=3</td>
<td>b=5</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>b=8</td>
<td>c=12</td>
</tr>
<tr>
<td>a=6</td>
<td>?</td>
<td>c=10</td>
</tr>
<tr>
<td>a=3</td>
<td>b=4</td>
<td>?</td>
</tr>
</tbody>
</table>

8. Solve the following word problems:

a) Last summer Maria travelled to Egypt and visited the Great Pyramid of Giza. As it was sunny, she could observe shadows projected on the ground. If Maria is 1.60m tall and her shadow projected on the ground was of 1.15m, what is the pyramid’s height if its shadow at the same time and with the same endpoint measured 105m?
b) Marc has lost his keys and tries to get inside the house through a window. He has a ladder to use, which he places 2.5m from the façade. We know the window’s height is approximately 6m from the ground. How long is Marc’s ladder?

**Extra Credit**

Find the interior angle of a regular pentagon.

What is the formula for the measure of an interior angle in a regular polygon?
ANNEX 3. VIDEO LIST

How much does your building weigh, Mr. Foster?
http://www.youtube.com/watch?v=9740R6Kd48I
Last accessed April 24, 2011

Welcome to architecture
http://www.youtube.com/watch?v=Lk2cMR0fw_w&feature=related
Last accessed April 24, 2011

Guggenheim New York
http://www.architects-talk.com/2011/03/architecture-tour-solomon-r-guggenheim.html
Last accessed April 18, 2011

Pythagoras was a square
Last accessed April 12, 2011

Marcus du Sautoy
http://news.bbc.co.uk/2/hi/programmes/newsnight/review/7865508.stm
Last accessed April 18, 2011

Voki (avatar created by the author of the unit)
Question 1
Question 2
Question 3
Question 4
Question 5
ANNEX 4. CD-ROM

Materials for implementation
Student’s Book for digital projection