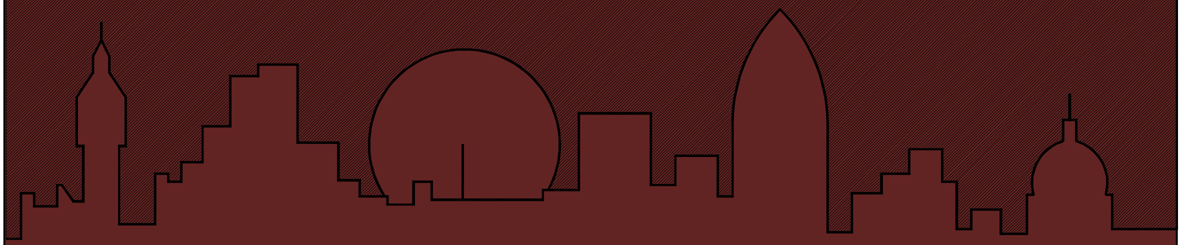

Triangulate the world

TRIANGLES, PYTHAGORAS AND THALES THEOREMS RELATED TO ARCHITECTURE

Anna Vera Llistar

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School mentor: **CARLOTA PETIT OLIVELLA**

UAB

TED Master's Degree, 2011

Teacher's book

Acknowledgements

Special thanks to Carlota Petit and Oriol Pallarés for their guidance throughout the development of this unit. Their support, patience, dedication and experience have made this material possible. Thanks also to IES Salvador Espriu for a wonderful internship.

TABLE OF CONTENTS

CLIL PLANNING TEMPLATE

UNIT TITLE: Triangulate the world

AUTHOR: Anna Vera Llistar

CLASS/AGE: 2nd of ESO / 13-14 years old

SUBJECTS INVOLVED: English and Math (English and Math teacher)

NUMBER OF LESSONS: 10 sessions of 55-60 minutes

COE LEVEL: Recommended for A2

INTRODUCTION TO THE TASK

In this unit students will experiment, observe, deduce and prove everything related to triangles. There will be a connection between the mathematical concepts and the present architecture in our lives.

MAIN GOALS AND COMPETENCES

By the end of this unit, the students will be able to:

- Value mathematics as part of our culture, from history to nowadays.
- Recognize figures in our environment: recognition of geometry in architecture.
- Visualize and use geometric models to solve problems: work with triangles in real-life situations.
- Use shadows as useful tools for measurement.
- Prove, reason and reach conclusions of mathematical concepts.
- Connect the concepts in the Math class with the contents in other subjects as Art and Technology.
- Use different languages, as verbal, numerical, graphic and algebraic language to identify and represent relations between variables.
- Search information and use ICTs to make presentations of final products.
- Communicate and organize the final products presented.
- Participate actively in oral, written and audiovisual interactions, using English as the vehicular language at all time.
- Express and comprehend oral, written and audiovisual messages.
- Work collaboratively and cooperatively.

CLIL~SI UAB

TOPIC RELATED CONTENTS

MAIN TARGET KNOWLEDGE

1. Introduction to the concept of architecture: ancient, modern and contemporary.
2. Review on polygons.
3. Analyze triangles: classification, characteristics, and properties.
4. Pythagorean Theorem and its application.
5. Thales Theorem and its application.
6. Historical contexts: proportions and geometry in Italy, Greece, Egypt, Turkey, and China.
7. Relationship between

TOPIC RELATED CONTENTS

MAIN TARGET SKILLS

1. Understanding English speeches and main concepts in short videos.
2. Identifying the architecture in the city they live.
3. Discussing ideas in group about previous or acquired knowledge.
4. Expressing and giving opinions on topics.
5. Recognizing geometry in buildings and solving problems.
6. Proving the properties of triangles by becoming experts in their group.
7. Being able to reflect on statements and proofs, for self and peer-assessment.
8. Drawing geometrical figures to solve problems.
9. Formulate word problems working in teams to test other teams
10. Understanding geometrical interpretations of theorems.
11. Organizing and summarizing short texts.
12. Producing an organized and coherent presentation using ICT resources and oral expression.

CONTENT OBLIGATORY LANGUAGE

TERMINOLOGY

TOPIC SPECIFIC

- Ancient, Modern and Contemporary architecture.
- Regular and irregular polygons. Convex, concave, apothem, area, perimeter, triangulation, diagonals.
- Triangles, vertices, angles, sides, legs, hypotenuse, orthocenter, barycenter, centroid, circumcenter.
- Pythagorean Theorem, proof, statement, geometrical interpretation.
- Thales Theorem, shadows, similarities, and proportionality.

GENERAL ACADEMIC

- We have decided that…
- We think that…
- We have identified..
- The theorem can be explained…
- There are several interpretations…
- If we use…
- The figure is a…
- Each side of the…
- It is named…
- Measures…because…so…
- Thales of Miletus was…
- He was interested in…
- He worked with…
- He was known for…
- We can conclude that…
- As conclusion

DISCOURSE GENRE / TEXT TYPE

TO UNDERSTAND

- Videos in English on math concepts
- Mathematical operations
- Descriptive texts about buildings and mathematicians
- Discussion tips and language tips

TO GENERATE

- Discussion between peers
- Summaries
- Oral mathematical discourses as:
  - · experiment
  - · deduce
  - · prove
  - · relate
  - · explain
  - · define
  - · classify
  - · compare
  - · conclude

TO NEGOTIATE

- Meaning between peers
- Knowledge to develop skills

PRAGMATIC STRATEGIES

Language is used in a social context

- Group discussions
- Group collaborative work
- Group productions

SOCIAL & CULTURAL VALUES; PERSONAL & EMOTIONAL DEVELOPMENT

- Understanding the existing relationship between math and architecture, from past to present times.
- Valuing the different landmarks around the world.
- Valuing the architecture in their city.
- Understanding different cultures.
- Giving importance to the use of mathematics in real –life situations.
- Working collaboratively to solve problems.
- Approach the world of math from a different point of view.

SUMMATIVE ASSESSMENT

(with formative value)

TASKS

1. Active participation in class
2. Questions on videos
3. Group discussions
4. Recognition of geometrical figures in buildings
5. Proofs of triangle properties
6. Word problem dictation and real-life situations
7. Project: Group generated presentation using ICTs with images, drawings and voice.
8. Unit Exam

SUMMATIVE ASSESSMENT

(with formative value)

ASSESSMENT CRITERIA

Catalan Curriculum assessment criteria for English (E) and Math (M)

1. Positives-negatives list
   Criteria: E1, E2, E5
2. Checklist
   Criteria: E2
3. Gradebook
   Criteria: E1
4. Gradebook
   Criteria: M3, M8, E10
5. Quiz
   Criteria: E3, M4
6. Checklist
   Criteria E2, M6
7. Rubric
   Criteria: E5, E7, M4, M6, M8, M9
8. Exam
   Criteria: E3, M4, M6, M8, M9

MATERIALS and RESOURCES

MATERIALS

Booklet, worksheets, proof expert cards, running dictation cards, PDF file for digital board, geometric figures for Pythagoras, and unit test.

RESOURCES

Computer, digital board, black board, Internet connection, Youtube.com, Teachertube.com, Architects-talk, News.BBC.co.uk, Voki.com, Slideshare.com, Audacity.com, wordle.net, Moodle.

COMMENTS

This unit has been especially designed for a 2nd of ESO Math CLIL class. Although it has been implemented by one student-teacher, it is also suitable for Tandem Teaching with at least one Math teacher.

In this unit there is a connection between the mathematical concepts and the present architecture in our lives. As the group will be travelling to London in a school trip, many of the images are from the architecture students will be able to recognize and enjoy.

Among the tasks in the unit, students will have to present at the end, final projects in groups and take a unit exam, to consolidate knowledge and skills.

REFERENCES

ACKNOWLEDGEMENTS

Special thanks to Carlota Petit and Oriol Pallarés for their guidance throughout the development of this unit. Their support, patience, dedication and experience have made this material possible. Thanks also to IES Salvador Espriu for a wonderful internship.

CHECKED MATERIALS

- Check Annex 1 for image sources.

Some of the images are from pictures taken by the author of this unit.

The rest of them have been taken from Wikipedia, Wikimedia Commons. Files licensed under the Creative Commons Attribution- Share Alike, or in the Public Domain.

Free Clip Arts by Phillip Martin.

-Visited websites: Youtube.com, Teachertube.com, Architects-talk, News.BBC.co.uk, Voki.com, Slideshare.com, Audacity.com, wordle.net, Moodle IES Salvador Espriu, Phillipmartin.com.

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ISBN: 978-84-692-7938-0

KEY COMPETENCES

1. Communication in the mother tongue

As this unit is designed for a Math CLIL class, it establishes that all communication must be done in L2. Code-switching may only be used in particular cases to avoid misunderstanding of mathematical concepts.

2. Communication in foreign languages

As the unit is for a CLIL class, it makes special attention to the acquisition of the L2 language. Most of the tasks are done in group work, and the unit contains language tips, discussion tips to help students interact with their peers, videos in the target language with no subtitles, and the use of ICT techniques to record themselves. The unit will contribute in making the student aware of specific mathematical language, necessary for the development of sciences and to cope with daily situations. The target language will be used for oral and written expressions at all times to describe concepts, processes, reasoning, proofs, and for communication in general.

3. Mathematical competences and basic competences in science and technology

In this unit students will experiment, observe, deduce and prove everything related to triangles. The architecture present in the unit will help students relate concepts to real-life situations, formulate and solve math problems. By working in teams, students communicate and interact using the mathematical language.

4. Digital competence

Computers are used throughout the unit to use Moodle. Videos are presented in class and uploaded in Moodle. ICTs are used for the final project, especially power point presentations and websites as Slideshare.com and Audacity.com.
Calculators are also used to help students with calculus.

5. Learning to learn

Concepts are related from one year to another or with other subjects. It is clearly specified throughout the unit and some reviews are made to go over material as an introduction to new concepts.
Discussions about topics are prepared so that students become critical and develop their own point of view and decisions.

6. Social and civic competences

By working in groups cooperatively in some of the tasks in this unit, students must respect and accept the ideas of their mates in the process of solving mathematical problems.

7. Sense of initiative and entrepreneurship

In this unit students are able to work in groups and individually. Some of the practice activities are to be done autonomously, so that the students make his/her own decisions and become self-confident. A unit test at the end will be done to consolidate knowledge and skills.

8. Cultural awareness and expression

The unit is related to architecture, so that students know how to visualize and understand the geometrical figures in buildings from all the world and use the mathematical thought to solve problems.

LESSON-BY-LESSON OVERVIEW

SESSION	ACTIVITIES	TIMING	INTERACTION	SKILLS	ICT	ASSESSED	COMMENTS
1	**Introduction to Ch 1: Architecture.** Video "How much does your building weigh, Mr Foster?	10'	T - Class	👂 🗣	X	X	
	Video in Moodle: "Welcome to architecture"		-	👂 ✍	X	X	More immersion into architecture
	Discussion: What does an architect do?	10'	SS - SS	💬	X		
	Introduction to Ch 2: Polygons Review concepts from previous year	15'	SS - SS	📖 ✍			
2	Warm-up: Homework correction	10'	SS - Class	🗣			
	Areas and perimeters of polygons	10'	T - Class	👂 ✍			
	Introduction to Ch 3: Triangles. Classification and characteristics. Proofs	25'	T – Class SS - SS	👂 🗣 ✍			
3	Warm-up: Homework correction	5'	T - Class	👂			
	Recognition of polygons in buildings	20'	SS - SS	💬		X	
	Proofs: Properties of triangles proved by students	25'	SS - SS	🗣 ✍			
	Peer-assessment on proofs	2'	SS - SS	💬		X	
4	Warm-up: Homework correction	10'	SS - Class	🗣			
	Proofs: Properties of triangles proved by students	25'	SS - SS	🗣 ✍			
	Peer-assessment on proofs	2'	SS - SS	💬		X	
	Introduction to Ch 6: Project Explanation of the unit's project	10'	T - Class	👂	X	X	The teacher models with an example using Slideshare

SESSION	ACTIVITIES	TIMING	INTERACTION	SKILLS	ICT	ASSESSED	COMMENTS
5	Warm-up: Homework correction	10'	SS - Class				
	Centers of triangles	15'	T - Class				Centers studied in the Art class
	Testing what we know! Groups prepare word problems to test other groups	20'	SS - SS				
	Peer-assessment on word problems	2'	SS - SS			X	
	Proof Quiz: Properties of triangles	10'	-			X	
6	Warm-up: Homework correction	10'	SS - Class				
	Introduction to Ch 4: Pythagorean Theorem. Video "Pythagoras was a square"	10'	SS - Class		X	X	Several geometrical interpretations of the theorem
	Discovering the geometrical and algebraic interpretation of the theorem	25'	SS - SS T-Class				
7	Warm-up: Homework correction	10'	SS - Class				
	The importance of the theorem	15'	T - Class				
	Finding out if specific triangles are right-angled triangles.	10'	SS - SS				
	Practice word problems: Let's practice across culture!	10'	SS - SS				
8	Warm-up: Homework correction	10'	SS - class				
	Introduction to Ch 5: Thales	10'	SS - SS			X	
	Thales Theorem and its uses	25'	T - Class				
	Word-problem dictation	5'	-		X	X	Voki

SESSION	ACTIVITIES	TIMING	INTERACTION	SKILLS	ICT	ASSESSED	COMMENTS
9	Warm-up: Homework correction	10'	SS - class				
	Introduction to Ch 7: Final thought. Video "Andrea Palladio and Marcus du Sautoy"	10'	SS - Class		X	X	
	Who's who? What do they have in common? Class debate	10'	SS - SS			X	
	Unit review and practice	20'	SS - SS		X	X	Also, architectural review of all the buildings in London seen throughout the unit and others to visit in the trip
10	**Unit Exam**	55'	-			X	

INSTRUCTIONS FOR IMPLEMENTATION AND ANSWER KEY

In this unit there will be a connection between the mathematical concepts and the present architecture in our lives. Therefore, it is important to start by explaining the meaning of the cover and the title of the unit.
Title: Triangulate the world.
Students will know about triangles and will practice with buildings from all over the world.
Cover: London's skyline.

LESSON 1

Introduction to architecture

As the group will be travelling to London in May, the lesson starts with a warm-up activity to know who Sir Norman Foster is and the students' previous knowledge on architecture.
The teacher shows a video: How much does your building weigh, Mr. Foster?
http://www.youtube.com/watch?v=9740R6Kd48I

The video is short and can be played twice. Students should take notes.
After watching it, the teacher starts a class discussion, to share opinions and to get an idea of how much do students know about the topic.

Suggested homework: Students should watch a video which is uploaded in Moodle at home, and should answer the questions on page 5 in the SB. Architectural tips are given to help students understand the video:
http://www.youtube.com/watch?v=Lk2cMR0fw_w&feature=related

What does an architect do?

Students get in groups of 4 and try to describe what an architect does. There is a discussion tip box to help students think about different aspects to talk about.
Groups must hand in a brief summary of their ideas.
All the class fills in the blanks on page 7 in the SB, and the teacher writes ideas on the board.

Polygon review

As students have worked with polygons the previous year, this is just a review of the main concepts.

Answer key

1. *Polygon/regular/irregular*
2. *Regular/same/inscribe*
3. *Irregular*
4. *Convex / concave*
5. *Apothem*
6. *Area*
7. *Perimeter/regular*
8. *Similar polygons*

LESSON 2

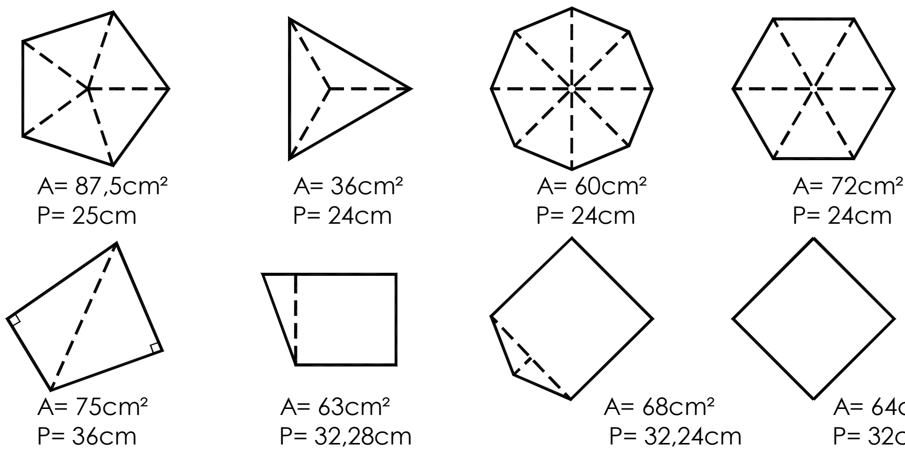
Warm-up with homework correction from previous lesson.

a) Students hand in questions on video watched at home. The teacher goes over the questions and students should participate actively, especially when discussing what buildings are located in the city of Barcelona.
b) Polygon review: teacher allocates turns so that students read out loud and give the correct answer for each blank.
   The teacher should go over the important concepts as apothems, area…

Practice: Areas and perimeters

Students first classify the figures as regular or irregular. Then, they calculate the area and perimeter of each figure.
The teacher helps the class calculate two figures together.

Answer key



The "star" of polygons

Students get in groups of 4 to discuss and draw any polygon in the whiteboards.
The teacher asks the groups to form triangles inside their polygons. There is a discussion tip box and a language tip box to help students express their ideas and thoughts.
The teacher asks all the whiteboards with all the different polygons to be shared.
The teacher introduces the triangulation of polygons and asks the groups to triangulate again, this time using diagonals.
An example is given for students on page 11 in the SB.
Students conclude the triangulation of polygons and fill in the blanks on page 8:

Answer key

n sides / n-2 triangles

Triangles. Classification

The teacher draws a triangle on the board and with the help of the students, writes the vertices, sides and angles.

Then, the teacher allows the students to match and classify triangles according to their sides and angles.

Answer key

Scalene:	*3 sides are different and 3 different angles*
Acute:	*angle < 90º*
Plain:	*no angle = 180º*
Right-angled:	*angle = 90º*
Isosceles:	*2 sides are the same and 2 same angles*
Obtuse:	*angle > 90º*
Equilateral:	*3 sides are the same and 3 same angles*

The term hypotenuse is introduced by the teacher, who emphasizes the need of a right angle. There is a Did you know… box to help students understand the word. The teacher asks the students to label a right-angled triangle (hypotenuse and legs).

Students are now able to complete the True or False statements:

Answer key

True or False?

(T)	F	*In a triangle, sides and angles are opposite to each other.*
T	(F)	*The largest angle is opposite to the shortest side.*
T	(F)	*The middle-sized angle is next to the middle-sized side.*
(T)	F	*The smallest-sized angle is opposite to the smallest angle.*

MATH TIP – LET'S ALL PROVE TOGETHER

The teacher models a mathematical proof with an example on page 14 in the SB.

Students should be aware of the need of a statement, the development of the proof and the # or Q.E.D. at the end.

Students get together in groups of 4. They assign themselves a number from 1 to 4, and will be responsible for the explanation of a proof to the rest of the members of the group.

The teacher provides the "expert" student in each group with a card with the necessary information (see Annex 2 Materials needed for implementation).

LESSON 3

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students' needs.

Recognition of polygons in buildings
Students get together in groups of 4. Each group has a paper slip for each of the buildings on page 10 in the SB. They must list and/or draw all the polygons they can find in the pictures.
The teacher collects all the slips and organizes them according to buildings, sharing the results with the class so that students can learn from their peers.
(See Annex 2 Materials needed for implementation).

Properties of triangles
The properties of triangles will be proved by the "expert" student in each group. He/She will be responsible for the explanation to the rest of the group.
The teacher has a private conversation with all the experts to make sure they have all understood the proof and to reinforce the idea that they are now the teachers.

Property 1: The angles of a triangle.
Statement: The three interior angles in a triangle always add up to 180º.

Property 2: The angles of a polygon.
Statement: The addition of the interior angles in a polygon is 180(n-2).

After the proofs have been explained, each student should reflect on what he/she has understood and fill in the self-assessment chart on proofs 1 and 2 on page 15 in the SB
(1 minute).

The teacher will give each group a paper slip. It is a rubric for peer-assessment. Each expert student will be graded according to a rubric and will receive the slip with the detailed parts of the assessment for future improvement (see Annex 2 Materials needed for implementation).

LESSON 4

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students' needs.

Properties of triangles

The properties of triangles will be proved by the "expert" student in each group. He/She will be responsible for the explanation to the rest of the group.
The teacher has a private conversation with all the experts to make sure they have all understood the proof and to reinforce the idea that they are now the teachers.

Property 3: Triangle inequality.
Statement: The addition or subtraction of the length of two sides of a triangle is always greater than (addition) or less than (subtraction) the length of the other side.

Property 4: Area.
Statement: The area of a triangle is the base of the triangle multiplied by the height of the triangle and divided into two parts.

After the proofs have been explained, each student should reflect on what he/she has understood and fill in the self-assessment chart on proofs 3 and 4 on page 17 in the SB (1 minute).

The teacher will give each group a paper slip. It is a rubric for peer-assessment. Each expert student will be graded according to a rubric and will receive the slip with the detailed parts of the assessment for future improvement (see Annex 3 Materials needed for implementation).

Introduction to the PROJECT- Ancient Triangles

At this stage of the unit where students know quite a bit on triangles, the teacher should explain what the Project will consist of, so that students know what is going to be expected from them. The deadline date should be announced, as the project will be assessed and included in the unit's final evaluation.

The purpose of this task is to make students discover, find out, and think about the use of geometrical figures in the architecture of Turkey, Greece, Egypt, Italy, and China.
ICTs (school's Moodle, Wikispaces, Audacity and SlideShare) will be used to share the projects within classmates.

Students get together in groups of 4. They are assigned a building belonging to ancient architecture which students must match to the pictures on page 41 in the SB. There is an architectural tip box to help students understand the meaning of ancient architecture.

The teacher goes over the instructions on page 39 in the SB with all the class to make sure everything is clear. Although SlideShare and Audacity are introduced to students on page 40 in the SB with defined steps to follow, the teacher models with an example of a presentation in which these ICT resources are used.

After all the projects are uploaded in Moodle, the teacher asks to look at other groups' presentations and evaluate them according to the rubric on page 42 in the SB. The rubrics are collected by the teacher so that he/she can take into account the students' opinions and make sure that all projects have been shared.

LESSON 5

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students' needs.

Centers of triangles
The teacher explains 3 different centers of triangles according to their characteristics: <u>orthocenter, barycenter or centroid, and circumcenter.</u>
The teacher elicits previous knowledge, as this topic is also studied in other subjects as Art or Technology.
Students should participate in the process of finding out the center of a triangle with heights, medians and midpoints. The teacher helps in the process.
Students can draw, with the help of a compass, the triangle specified on page 19 in the SB to find its orthocenter, barycenter and circumcenter and hand it in to the teacher for correction.

Testing what we know!
Students get together in groups of 4. They must prepare a word problem about triangles to test another team in the classroom. An example is provided on page 21 in the SB.

Teams will exchange the word problems, must solve them and complete the peer-assessment chart on the following page.

At the end of the activity, the teacher asks a student to read the conclusions about triangles filling in the blanks:

Answer key

One
One
Equilateral

Proof Quiz
As students have been "experts" proving the properties of triangles, they must take a 10' quiz to consolidate knowledge. Each student was responsible for the explanation of a proof to the rest of the group, but they are also responsible to know about the proofs explained to them.
The teacher should make sure that each student is asked to prove a different property of triangles than the one proved as an "expert".
(See Annex 2 Materials needed for implementation).

Important!
The teacher asks students to bring the materials specified on page 25 in the SB (Follow the steps…) for the next lesson.

LESSON 6

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students' needs.

Pythagoras was a square
To introduce Pythagoras to students, the teacher shows the following video:
Pythagoras was a square http://www.teachertube.com/viewVideo.php?video_id=23675

The purpose of this task is to make students think about the geometrical interpretations shown in the video. They should watch, listen and then fill in the blanks and answer the questions on page 24 in the SB.
As the geometrical interpretations are not easy to follow at first sight, the teacher can upload the video in Moodle so that students can watch it again at home as many times as needed to understand and answer the questions.
There is a language tip box to help students answer the questions with complete sentences.

Answer key

1. *Greek*
Father / numbers
Proof
Expert / souls / ratios
Music / perfect squares
Square roots / theorem
Hypotenuse
Equal / squares

2. *Three interpretations: squares, rectangles, quarter circles*

3. *The problem of using similar rectangles to interpret the theorem is that there is more information needed; the measurement of the two sides.*

4. *Yes, we can come up with a true statement by using quarter circles.*

Pythagorean Theorem - Let's all prove together
Students get together in groups of 4, all of them with the squares and right-angled triangles students were asked to bring in the previous lesson. The teacher asks the class to arrange the 2 squares and the 8 triangles in two different ways according to the steps specified on page 25 in the SB.
The purpose of this task is to prove the Pythagorean Theorem statement, both geometrically and algebraically.
The teacher does the same with his/her squares and triangles on the board, using blue-tack to hold them up so that all the class can see: the geometrical interpretation is proved.
The teacher goes a step further and asks students to prove the algebraic interpretation of the theorem from what they have arranged.

LESSON 7

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students' needs.

The importance of the theorem

The teacher explains why the theorem is important.

1. To check that the equality is true for all right-angled triangles.
2. To find the length of the hypotenuse.
3. To find the length of a side.

Although examples are given for each, a running dictation has been prepared to make sure students know how to use the theorem.
(In a running dictation students work in groups and each member runs to listen to information which is being dictated. Then, the student goes back to the group and all members work collaboratively.)

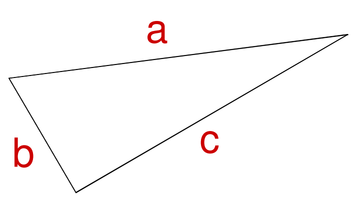
Students get together in groups of 4, and number themselves from 1 to 4. The teacher calls them by their numbers randomly and dictates the measures of each side of a triangle.
(See Annex 2 Materials needed for implementation).

A chart is prepared on page 25 in the student's book to fill in, with an example given.

When the chart is completed, students are able to answer the questions below it.

Answer key

	a	b	c	Solution
Example	5	3	4	$c^2 + b^2 = a^2$ IT IS A RIGHT-ANGLED TRIANGLE
Student 1	26	24	10	676 = 676 YES
Student 2	20	17	19	400 = 650 NO
Student 3	17	8	15	289 = 289 YES
Student 4	30	20	40	900 = 2000 NO



Answer key

True or False page 24

(T)	F	*The theorem helps you find the missing leg of a triangle.*
T	(F)	*This theorem is true for all triangles, no matter the angles.*
(T)	F	*We use the theorem to find the length of the hypotenuse.*
(T)	F	*The theorem can be used to find the diagonal of a square.*
(T)	F	*The theorem can be used to find the diagonal of a rectangle.*
(T)	F	*The theorem can be proved with quarter circles.*
T	(F)	*The theorem can be proved with any geometrical figure.*

Let's practice across culture!

In this section seven word problems are prepared to practice using the Pythagorean Theorem in real buildings from all over the world. A language tip box and a hint box are present to help the student solve the word problem and explain it to the rest of the class.
Students can practice by working in groups or individually.

Answer key

Forum Building: *It is an equilateral triangle, as each of its sides measures 180m.*

Staircase: *The ramp's length will be 1,66m.*

Turning Torso: *576 = 724 NO, the triangles are not right-angled.*

Big Ben: *The distance from the feet to the top of the tower will be 96,13m.*

Petronas Twin Towers: *The missing side of the triangle measures 30,38m and it occupies 7,6 floors.*

Eixample in Barcelona: *The hypotenuse of the corner measures 21,22m.*
The area of each block is 12.386,89m².
The perimeter of each block is 418,08m.

Louvre Museum: *The theorem is used twice.*
The length of the edge between two sides is 32,2m.

LESSON 8

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students' needs.

Who was Thales of Miletus?
Students get together in groups of 4 and read some sentences about Thales of Miletus.
They can discuss and organize the sentences to place them in the correct order.
Then, with the help of a language tip box, students write a brief summary together. The teacher collects one per working group.

The theorem
The teacher explains the theorem starting with the statement and drawing it.
Students should be able to fill in the blanks about similar triangles.
A discussion tip box is provided to help students share their ideas and opinions on three pictures of similar triangles.

Answer key

Triangles
Triangle AMB and triangle BNC

The use of the Thales Theorem
The teacher elicits previous knowledge, as this topic is also studied in other subjects as Art or Technology.
The teacher explains why the theorem is important.

1. To divide a segment into equal parts.
2. For proportions and similarities. Triangles placed in the Thales' position.
3. Shadows and proportionality to find the length of objects, buildings…

The teacher explains the three concepts, gives an example of each and asks the students to draw a segment and divide it into equal parts without using a ruler; only a compass.

The teacher goes over the examples written on page 35 on rays of sunlight forming a triangle. Although the example is clear, the teacher prepares a real example in the classroom using string and a measuring tape:
What is the classroom's height?
Students imagine the height cannot be measured, so a string will be used to find out. The string hangs from the ceiling and is pulled forming a triangle. A student will be inside the triangle formed. The procedure is the same as if there was sunlight.

Practice – Voki

The word problems in this section have been thought to be dictated by a Voki character. The idea is that students first have a dictation and then solve the problems, so the teacher can also dictate the following questions:

1. Draw a segment of 4.5 cm and divide it into 13 equal parts.
2. Calculate the values of x and y in the drawing.
3. Are the triangles ABD and BDC similar to triangle ABC? Why?
4. The man in the picture is going to enter a store in the Soho District in New York City. His height is 1.82 m. Calculate the height of the door.
5. A rectangle has sides 10 cm x 25 cm. A smaller and similar rectangle has its short side measuring 8 cm. What is the similarity ratio of the rectangles? And, what does the long side of the small rectangle measure?

Answer key

1. *Draw…*
2. *x = 10,5cm / y = 5,25cm*
3. *Triangles ADB and ABC have both a 90º angle and the common angle A.*
   Triangles BDC and ABC have both a 90º angle and the common angle C.
   Therefore, triangles BDC and ADB are similar to triangle ABC.
4. *The door's height is 3,2m.*
5. *The similarity ratio of the rectangles is 1,25.*
   The long side of the small rectangle measures 20cm.

LESSON 9

Warm-up with homework correction from previous lesson.
Students write practice exercises on the black board and explain the process used to the class. The teacher supervises and explains/corrects when necessary, depending on the importance of the math problem or the students' needs.

Final thought
As mathematics have been related to architecture throughout the unit, the teacher will show a video of the well-known English mathematician Marcus du Sautoy in one of the famous Italian architect Palladio's buildings.

The teacher will ask a student to read about Andrea Palladio and another student to read about Marcus du Sautoy.
A discussion tip box is prepared to help students start with a class discussion. To make students talk, the teacher asks the students to divide themselves into two big groups, one on the side of mathematicians and the other on the side of architects, as a general class debate.

After that, a quick review of all the important mathematical concepts seen throughout the unit with practice.
And after that, a quick review of all the important buildings to visit in London during the school trip.

LESSON 10

Unit Exam (See Annex 2 Materials needed for implementation).

Answer key

1. a) Possible answers: *areas, volumes, perimeters, calculus, drawings, polygons…*

b) It is an open question, as it depends on students' opinion.

2. Triangulation of polygons

The triangulation of polygons refers to the decomposition of the polygon into triangles. We draw diagonals to divide the polygon in triangles. Triangulation reduces complex shapes into simple shapes. The polygon becomes an addition of triangles. Any polygon admits a triangulation. If the polygon has n sides, then the polygon has n-2 triangles.

3. Hypotenuse

The longest side in a right- angled triangle is called hypotenuse.

There is always a hypotenuse in all right-angled triangles.

Hypotenuse comes from the Greek hupoteinousa, Hupo meaning the opposite; opposite to the right angle.

4. Answers: *Right angle / Obtuse / Acute / Isosceles*

5. Prove that the addition of the interior angles of a triangle is 180º.

Algebraic explanation

Given any triangle, label the 3 angles (A;B;C) and draw 3 auxiliary lines:

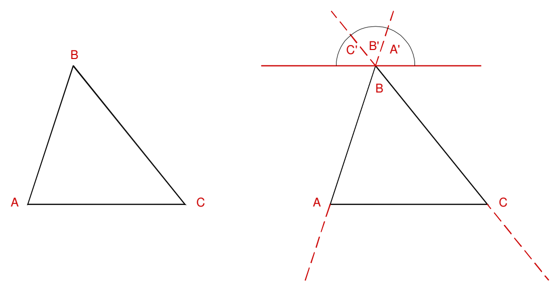
a line parallel to the base of the triangle and elongate the other 2 sides.

3 new angles A', B', C' are obtained.

Notice that <A and >A' and <C and <C' are congruent angles (alternate),

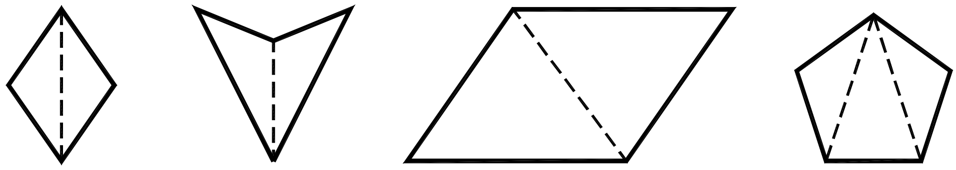
So <A' + <B' + <C' form a plain angle. We know a plain angle is 180º.

Therefore, we can substitute and say that ***<A + <B + <C = 180º***



Q.E.D

6.



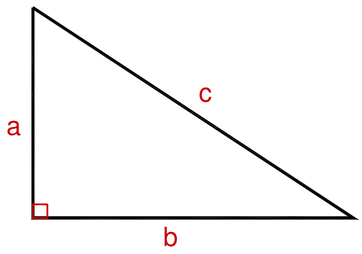
Σ = 2 · 180º = 360º   Σ = 2 · 180º = 360º   Σ = 2 · 180º = 360º   Σ = 3 · 180º = 540º

The formula used is:

Σ angles in a polygon with n sides = (n-2) · 180

7.

a	b	c
a=4	**3**	c=5
a=3	b=5	**5,83**
8,94	b=8	c=12
a=6	**8**	c=10
a=3	b=4	**5**



8. a) *The pyramid's height is 146,09m.*

b) *Marc's ladder is 6,5m long.*

Extra Credit

The interior angle of a regular pentagon is 108º.

The measure of an interior angle in a regular polygon with n sides is $\frac{(n-2)\cdot 180}{n}$

ASSESSMENT CHART

Session	Activities	Timing	Assessment tool / form	Assessment criteria	Comments
In all sessions throughout the unit	**Active participation in class**		Positives-negatives list	E1:Students participate in class discussions E2: Students should comprehend specific information given E5: Students should make use of a coherent oral expression	Teacher writes active participation in the positives-negatives list.
1	**Introduction to Ch 1: Architecture.** Video "How much does your building weigh, Mr Foster?	10'			
	Questions on video in Moodle: "Welcome to architecture"	-	Homework	E2:Students should comprehend specific information given: Recognize buildings in their city	Students hand in questions: Teacher gives a grade
	Group discussion: What does an architect do?	10'	Gradebook	E1:Students participate in class discussions ton communicate and to share points of view	Students hand in summary: Teacher gives a grade
	Introduction to Ch 2: Polygons. Review concepts from last year	15'			
2	Homework correction	10'			
	Areas and perimeters of polygons	10'			
	Introduction to Ch 3: Triangles. Classification and characteristics. Proofs	25'			

Session	Activities	Timing	Assessment tool / form	Assessment criteria	Comments
3	Homework correction	5'			
	Recognition of polygons in buildings. Group discussion	20'	Gradebook	M3: Students should share and discuss mathematical ideas M8: Students should be able to Identify geometrical figures in real buildings E10: Work collaboratively	Groups hand in analyzed slips for each building: Teacher gives a grade
	Proofs: 2 Properties of triangles proved by 2 expert students	25'			
	Peer-assessment on proofs	2'	Rubric	E9: Each working group evaluates the work done by its expert student	Students assess the expert student with a rubric and give a final grade
4	Homework correction	10'			
	Proofs: 2 Properties of triangles proved by 2 expert students	25'			
	Peer-assessment on proofs	2'	Rubric	E9: Each working group evaluates the work done by its expert student	Students assess the expert student with a rubric and give a final grade
	Introduction to Ch 6: Project Explanation of the unit's project. Teacher models the final product with a Slideshare presentation	10'	Rubric	E5: Students should make use of a coherent oral expression E7: Digital and internet resources should be used to search, organize, gather and present information M4: Students should be able to write about mathematical thoughts and ideas M6: Students should Identify and use proportionality to solve real-life situations M8: Students can Identify geometrical figures in	Teacher gives a grade following the rubric's results

Session	Activities	Timing	Assessment tool / form	Assessment criteria	Comments
				non-mathematical contexts M9: Students must be able to calculate lengths, areas and volumes and should express the result with adequate measuring units.	
5	Homework correction	10’			
	Centers of triangles	15’			
	Team work: Testing what we know! Groups prepare a word problem to test other groups	20’			
	Peer-assessment on word problems	2’	Rubric	M3: Students should share mathematical ideas working in groups. E9:Each working group evaluates the work done by other groups E10: Work collaboratively	Students assess the work done by another group
	Proof Quiz: Properties of triangles	10’	Quiz	E3: Students should understand the specific questions written M4: Students can express their thoughts and ideas on the topic	Teacher writes quiz grades in the gradebook
6	Homework correction	10’			
	Introduction to Ch 4: Pythagorean Theorem. Video “Pythagoras was a square”	10’	Positives-negatives list	E1: Students participate in class discussions to communicate and to share points of view	Teacher writes active participation in the positives-negatives list

Session	Activities	Timing	Assessment tool / form	Assessment criteria	Comments
	Questions on video in Moodle: “Pythagoras was a square”	-	Homework	E2: Students should comprehend specific information given	Students hand in questions: Teacher gives a grade
	Discovering the geometrical and algebraic interpretation of the theorem	20’			
7	Homework correction	10’			
	The importance of the theorem	15’			
	Running dictation to find out if specific triangles are right-angled triangles	10’			
	Practice: Let’s practice across culture!	10’			
8	Homework correction	10’			
	Introduction to Ch 5: Thales. Group discussion to organize sentences and produce a brief summary	10’	Gradebook	E1: Students should participate in class discussions to communicate and to share points of view	Students hand in a summary: Teacher gives a grade
	Thales Theorem and its uses	20’			
	Word-problem dictation (Voki)	5’	Homework	M6: Students should Identify and use proportionality to solve real-life situations E2: Students should comprehend specific information	Students hand solved word problems: Teacher gives a grade

Session	Activities	Timing	Assessment tool / form	Assessment criteria	Comments
9	Homework correction	10’			
	Introduction to Ch 7: Final thought. Video “Andrea Palladio and Marcus du Sautoy”	10’	Positives-negatives list	E1:Students participate in class discussions to communicate and to share points of view	Teacher writes active participation in the positives-negatives list
	Who’s who? What do they have in common? Class debate	10’	Gradebook	E1:Students participate in class discussions to communicate and to share points of view	Students hand in discussion conclusion: Teacher gives a grade
	Unit review and practice	20’			Also, architectural review of all the buildings in London seen throughout the unit and others
10	Unit Exam	55’		E3: Students should understand the specific questions written M4: Students can express their thoughts and ideas on the topic M6: Students should Identify and use proportionality to solve real-life situations M8: Students can Identify geometrical figures in non-mathematical contexts M9: Students must be able to calculate lengths, areas and volumes and should express the result with adequate measuring units.	Teacher writes exam grades in the gradebook

ANNEXES

ANNEX 1. IMAGE SOURCES

Image 1, Student's book, page 1:
Iliff, David. "City of London skyline from London City Hall - Oct 2008.jpg". Wikipedia.org,
http://en.wikipedia.org/wiki/File:City_of_London_skyline_from_London_City_Hall_-_Oct_2008.jpg
(last accessed March 12, 2011)
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Image 2, Student's book, page 5:
User: Someformofhuman. "Petronas Panorama II". Wikipedia.org,
http://en.wikipedia.org/wiki/File:Petronas_Panorama_II.jpg (last accessed April 17th, 2011)
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Image 3, Student's book, page 5:
Alexander Z. "Torre Agbar, Barcelona". Wikipedia.org,
http://es.wikipedia.org/wiki/Archivo:Barcelona_Torre_Agbar_01.jpg (last accessed April 17, 2011).
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Image 4, Student's book, page 6:
User: MykReeve. "Guggenheim Museum Bilbao". Wikipedia.org,
http://es.wikipedia.org/wiki/Archivo:Guggenheim-bilbao-jan05.jpg (last accessed March 6, 2011).
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Image 5, Student's book, page 6:
Flickr User: tinyfroglet. "Farnsworth House (Illinois) by Mies van der Rohe". Wikipedia.org,
http://es.wikipedia.org/wiki/Archivo:FarnsworthHouse-Mies-1.jpg (last accessed March 6, 2011).
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Image 6, Student's book, page 7:
Vera Llistar, Anna. Solomon R. Guggenheim Museum, in New York. Photograph by the author.

Image 7, Student's book, page 7:
Vera Llistar, Anna. Top of the Rock Building, in New York. Photograph by the author.

Image 8, Student's book, page 10:
Pingstone, Adrian. "The base of the Swiss Re Tower.". Wikipedia.org,
http://es.wikipedia.org/wiki/Archivo:Swiss.re.arp.750pix.jpg (last accessed March 6, 2011).
Photograph courtesy of Adrian Pingstone.

Image 9, Student's book, page 10:
Ribbefjord, Andreas. "Stockholms-stadsbibliotek-2003-04-14.jpg". Wikipedia.org, http://en.wikipedia.org/wiki/File:Stockholms-stadsbibliotek-2003-04-14.jpg (last accessed February 20, 2011). This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported (http://creativecommons.org/licenses/by-sa/3.0/deed.en) license.

Image 10, Student's book, page 10:
Larripa, Sergi. "Catalunya National Theater, in Barcelona". Wikipedia.org, http://commons.wikimedia.org/wiki/File:050529_Barcelona_028.jpg (last accessed February 20, 2011). This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported (http://creativecommons.org/licenses/by-sa/3.0/deed.en) license.

Image 11, Student's book, page 10:
Vera Llistar, Anna. Hearst Tower, in New York**.** Photograph by the author.

Image 12, Student's book, page 10:
User: Lorax. "The Flatiron Building in New York City". Wikipedia.org, http://es.wikipedia.org/wiki/Archivo:Flatiron_crop_20040522_114306_1.jpg (last accessed February 20, 2011). This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Generic (http://creativecommons.org/licenses/by-sa/3.0/deed.es) license.

Image 13, Student's book, page 11:
Vera Llistar, Anna. IES Salvador Espriu, Barcelona. Photograph by the author.

Image 14, Student's book, page 23:
User: Jacobolus. "Detail of The School of Athens by Raffaello Sanzio, 1509, depicting Pythagoras, writing in foreground, with Averroes, Hypatia of Alexandria, and Parmenides behind him". Wikipedia.org, http://en.wikipedia.org/wiki/File:Sanzio_01_Pythagoras.jpg (last accessed March 6, 2011).
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Flickr User: ILESH. "Edifici Fòrum a Barcelona, vist de s de l'Hotel AC". Wikipedia.org, http://es.wikipedia.org/wiki/Archivo:Edifici_F%C3%B2rum_Barcelona_Catalonia.jpg (last accessed March 7, 2011). This image is licensed under the Creative Commons Attribution 2.0 Generic (http://creativecommons.org/licenses/by/2.0/deed.es) license.

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Image 20, Student's book, page 31:
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http://ca.wikipedia.org/wiki/Fitxer:Eixample_aire.jpg (last accessed March 7, 2011).
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Image 22, Student's book, page 31:
User: Xavigivax. "Mides de les illes de l'Eixample de Barcelona". Wikipedia.org,
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Image 23, Student's book, page 32:
User: Demeester. "Place du Carrousel in Paris, France". Wikipedia.org,
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Image 24, Student's book, page 32:
User: Arnaud 25. "Pyramide inversée du Louvre - Paris". Wikipedia.org, http://pt.wikipedia.org/wiki/Ficheiro:Pyramide_invers%C3%A9e_du_Louvre.JPG (last accessed March 7, 2011). Photograph courtesy of user Arnaud 25.

Image 25, Student's book, page 32:
User: 1997. "Placa Fotovoltàica al Fòrum,Barcelona,Catalunya". Wikipedia.org, http://es.wikipedia.org/wiki/Archivo:BarcelonaFotovoltaica.JPG (last accessed March 7, 2011). Photograph courtesy of user 1997.

Image 26, Student's book, page 33:
User: Odysses. "marble statue of Thales". Wikipedia.org, http://en.wikipedia.org/wiki/File:Thales-04.jpg (last accessed March 8, 2011). Photograph courtesy of user Odysses.

Image 27, Student's book, page 38:
Vera Llistar, Anna. The Soho District, in New York. Photograph by the author.

Image 28, Student's book, page 40:
User: Warburg. "Richard Meier, architect". Wikipedia.org, http://commons.wikimedia.org/wiki/File:Meier_001.jpg (last accessed March 30, 2011)
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User: Sebi. "Obélisque de Sésostris Ier - XIIe dynastie - Moyen Empire". Wikipedia.org, http://en.wikipedia.org/wiki/File:H%C3%A9liopolis200501.JPG (last accessed March 8, 2011). Photograph courtesy of user Sebi.

Image 30, Student's book, page 41:
Liberato, Ricardo. "All Gizah Pyramids in one shot". Wikipedia.org, http://es.wikipedia.org/wiki/Archivo:All_Gizah_Pyramids-2.jpg (last accessed March 7, 2011).
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Fong, Charlie. "The Hall of Prayer for Good Harvest at the Temple of Heaven in Beijing". Wikipedia.org, http://en.wikipedia.org/wiki/File:Hall_of_Prayer_for_Good_Harvest.JPG (last accessed March 8, 2011).
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Vera Llistar, Anna. Parthenon, in Athens, Greece. Photograph by the author.

Image 35, Student's book, page 41:
User: Acarvin. "Traditional mud brick houses shaped like beehives, in the ancient village of Harran, Turkey". Wikipedia.org,
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Image 36, Student's book, page 43:
User: Padraig. "City Hall London UK taken from the Walkway above Tower bridge". Wikipedia.org,
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Image 39, Student's book, page 44:
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Clip Arts, Student's book, pages 13, 14, 15, 16, 17, 18, 21, 22, 24, 25, 27, 35, 39, 42:
Martin, Phillip. "Free Clip Art by Phillip Martin"
http://www.phillipmartin.info/clipart/homepage.htm (last accessed March 8, 2011).
(See the attached mail from the author).

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Photograph courtesy of Adrian Pingstone.

Ribbefjord, Andreas. "Stockholms-stadsbibliotek-2003-04-14.jpg". Wikipedia.org,
http://en.wikipedia.org/wiki/File:Stockholms-stadsbibliotek-2003-04-14.jpg (last accessed February 20, 2011).
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported (http://creativecommons.org/licenses/by-sa/3.0/deed.en) license.

Larripa, Sergi. "Catalunya National Theater, in Barcelona". Wikipedia.org,
http://commons.wikimedia.org/wiki/File:050529_Barcelona_028.jpg (last accessed February 20, 2011).
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported (http://creativecommons.org/licenses/by-sa/3.0/deed.en) license.

Images in Teacher's book, page 40:
Vera Llistar, Anna. Hearst Tower, in New York**.** Photograph by the author.

User: Lorax. "The Flatiron Building in New York City". Wikipedia.org,
http://es.wikipedia.org/wiki/Archivo:Flatiron_crop_20040522_114306_1.jpg (last accessed February 20, 2011).
This image is licensed under the Creative Commons Attribution-Share Alike 3.0 Generic (http://creativecommons.org/licenses/by-sa/3.0/deed.es) license.

Clip Arts, Teacher's book, pages 41, 42, 43, 44, 45, 46, 49, 50:
Martin, Phillip. "Free Clip Art by Philli, p Martin"
http://www.phillipmartin.info/clipart/homepage.htm (last accessed March 8, 2011).
(See the attached mail from the author).

ANNEX 2. MATERIALS FOR IMPLEMENTATION

Recognition of polygons in buildings

Let's all prove together. Expert cards

Let's all prove together. Peer- assessment cards.

Proof Quiz

Pythagoras Running Dictation

Unit Exam

Recognition of polygons in buildings

Can you recognize polygons in this picture? Which ones?

✓ ...

✓

✓

✓

✓

30St Mary Axe, in London. Foster and Partners

Can you recognize polygons in this picture? Which ones?

✓ ...

✓

✓

✓

✓

Stockholm's Public Library, in Sweden. Erik Gunnar Asplund

Can you recognize polygons in this picture? Which ones?

✓ ...

✓

✓

✓

✓

Catalunya National Theater, in Barcelona. Ricardo Bofill

Can you recognize polygons in this picture? Which ones?

- ✓ ...
- ✓
- ✓
- ✓
- ✓

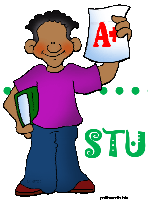
Hearst Tower, in New York. Foster and Partners

Can you recognize polygons in this picture? Which ones?

- ✓ ...
- ✓
- ✓
- ✓
- ✓

Flatiron Building, in New York. Daniel Burnham

LET'S ALL PROVE TOGETHER — EXPERT CARD — STUDENT 1



Property 1: The angles of a triangle

Statement: The three interior angles in a triangle always add up to 180º.

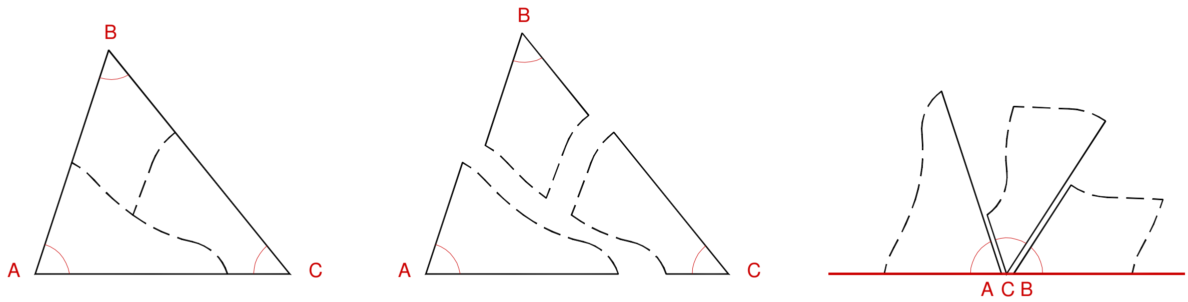
Proof: The proof can be done in two ways:

1. Using a physical model.

   Given a physical scalene (a non-specific triangle) triangle in a piece of paper, label and then tear up the 3 angles.

   Place the 3 different angles next to each other. They form a straight line.

   We know a straight line is 180º.



<A + <B + <C = 180º

2. Algebraic explanation

Q.E.D.

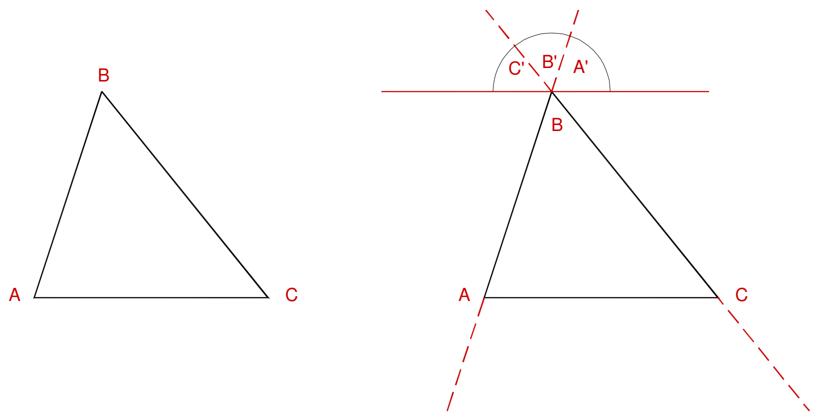
   Given any triangle, label the 3 angles (A;B;C) and draw 3 auxiliary lines: a line parallel to the base of the triangle and elongate the other 2 sides.

   3 new angles A', B', C' are obtained.

   Notice that <A and >A' and <C and <C' are congruent angles (alternate),

   So <A' + <B' + <C' form a plain angle. We know a plain angle is 180º.

   Therefore, we can substitute and say that **<A + <B + <C = 180º**



Q.E.D

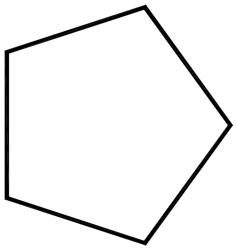
LET'S ALL PROVE TOGETHER — EXPERT CARD — STUDENT 2

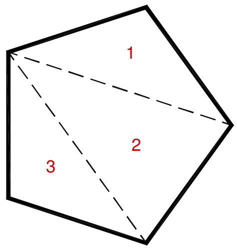
Property 2: The angles of a polygon

Statement: The addition of the interior angles in a polygon is $180^0 \cdot (n\text{-}2)$

Proof:

1. We choose a pentagon (**5 sides**) as our polygon and we triangulate it.
   (Remember that triangulation can only be done with diagonals).
2. We have obtained **3 triangles**.

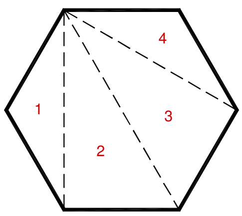


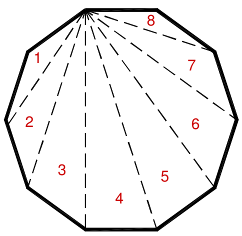


3. As we know, the three interior angles in a triangle always add up to **180º**.

   The angles in △ 1 = 180º
   The angles in △ 2 = 180º
   The angles in △ 3 = 180º
   540º

   We do the same with a hexagon and a decagon.

1 



2

The number of triangles in polygon 1 = 4

The number of triangles in polygon 2 = 8

We can find a pattern:

We will always obtain 2 triangles less than the sides of the polygon.

(n sides - 2) · 180 (the angles inside a triangle) = 540º (Σ angles in a polygon)

(n-2) · 180 = Σ angles in a polygon with n sides

Q.E.D

LET'S ALL PROVE TOGETHER   EXPERT CARD   STUDENT 3

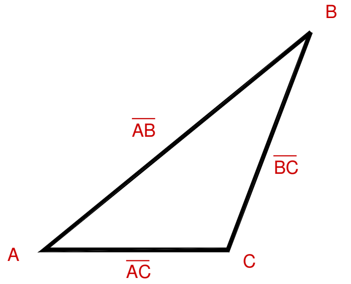
Property 3: Triangle inequality

Statement: The **addition** of the length of two sides of a triangle is always greater than the length of the other side.
The **subtraction** of the length of two sides of a triangle is always less than the length of the other side.

Proof:

Let's check for AB = 40, BC = 27, AC = 21

There is a special relationship between the 3 sides in any triangle:



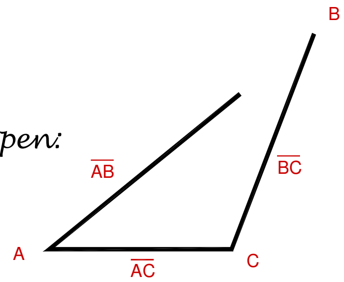
Addition

$AB + AC > BC \rightarrow 40 + 21 > 27$

$AB + BC > AC \rightarrow 40 + 27 > 21$

$BC + AC > AB \rightarrow 27 + 21 > 40$

Moreover, with < it cannot be possible. This would happen:



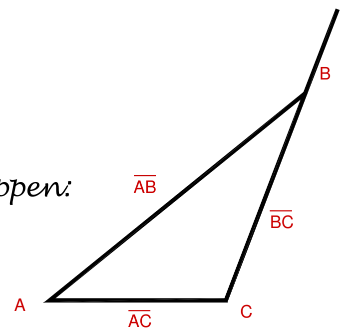
Subtraction

$AB - AC < BC \rightarrow 40 - 21 < 27$

$AB - BC < AC \rightarrow 40 - 27 < 21$

$BC - AC < AB \rightarrow 27 - 21 < 40$

Moreover, with > it cannot be possible. This would happen:

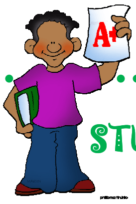


All these are inequalities, **>** or **<**.

If they were true with an equal sign **=**, we would obtain a straight line.

This is the **border case**. What area does the triangle have?

Q.E.D



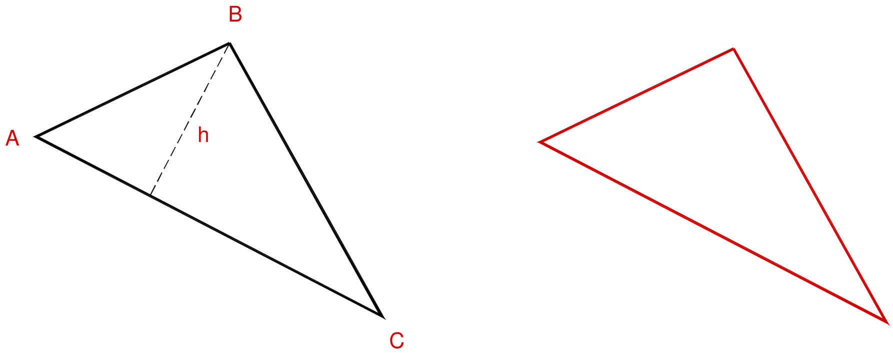
LET'S ALL PROVE TOGETHER EXPERT CARD STUDENT 4

Property : Area

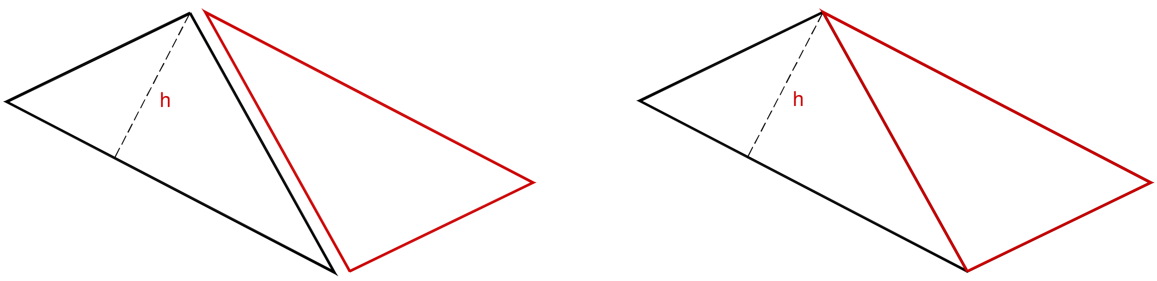
Statement: The area of a triangle is the base of the triangle multiplied by the height of the triangle and divided into two parts.

Proof:

We have two triangles that are exactly the same.



If we place them in the position shown in the drawing below, we obtain a parallelogram with the same base and height as the triangles.



We know the area of a parallelogram is the length of the base multiplied by the height: $A = b \cdot h$

This parallelogram has been formed with two same triangles, therefore, the area of each triangle is the area of the parallelogram divided by 2.

$$A = \frac{b \cdot h}{2}$$

Q.E.D

Peer-assessment on proofs

Student 1

5	10	THE GRADE OBTAINED
Explanation is not clear at all	Explanation is very clear	
No use of mathematical terminology	Some mathematical terminology used	
Examples not used to explain the proof	Examples used to explain the proof	
The expert student does not make sure the members of the group understand the proof	The expert student makes sure the members of the group understand the proof	
	TOTAL SCORE	points

Peer-assessment on proofs

Student 2

5	10	THE GRADE OBTAINED
Explanation is not clear at all	Explanation is very clear	
No use of mathematical terminology	Some mathematical terminology used	
Examples not used to explain the proof	Examples used to explain the proof	
The expert student does not make sure the members of the group understand the proof	The expert student makes sure the members of the group understand the proof	
	TOTAL SCORE	points

Peer-assessment on proofs

Student 3

5	10	THE GRADE OBTAINED
Explanation is not clear at all	Explanation is very clear	
No use of mathematical terminology	Some mathematical terminology used	
Examples not used to explain the proof	Examples used to explain the proof	
The expert student does not make sure the members of the group understand the proof	The expert student makes sure the members of the group understand the proof	
	TOTAL SCORE	points

Peer-assessment on proofs

Student 4

5	10	THE GRADE OBTAINED
Explanation is not clear at all	Explanation is very clear	
No use of mathematical terminology	Some mathematical terminology used	
Examples not used to explain the proof	Examples used to explain the proof	
The expert student does not make sure the members of the group understand the proof	The expert student makes sure the members of the group understand the proof	
	TOTAL SCORE	points

NAME: ____**DATE**: _____________

10' QUIZ: Properties of triangles

Prove the following property of triangles:

Statement: The three interior angles in a triangle always add up to 180º.

NAME: ____**DATE**: _____________

10' QUIZ: Properties of triangles

Prove the following property of triangles:

Statement: The addition of the interior angles in a polygon is 180 (n-2).

NAME: ____**DATE**: _____________

10' QUIZ: Properties of triangles

Prove the following property of triangles:

Statement: The addition or subtraction of the length of two sides of a triangle is always greater than (addition) or less than (subtraction) the length of the other side.

NAME: ____**DATE**: _____________

10' QUIZ: Properties of triangles

Prove the following property of triangles:

Statement: The area of a triangle is the base of the triangle multiplied by the height of the triangle and divided into two parts.

RUNNING DICTATION

STUDENT 1

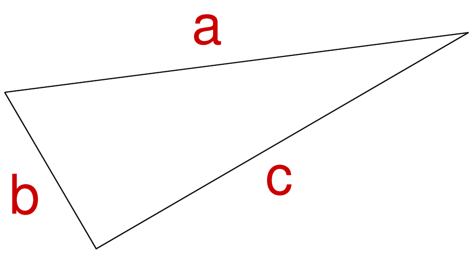
Is this a right-angled triangle?

Check if the triangle with sides a, b, and c is a right-angled triangle.

For: $a = 26$

$b = 24$

$c = 10$



RUNNING DICTATION

STUDENT 2

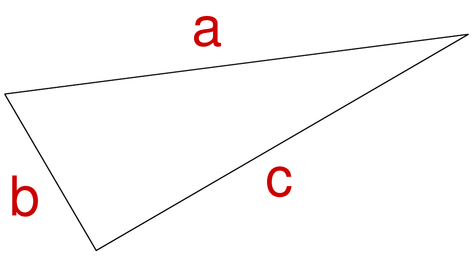
Is this a right-angled triangle?

Check if the triangle with sides a, b, and c is a right-angled triangle.

For: $a = 20$

$b = 17$

$c = 19$



RUNNING DICTATION

STUDENT 3

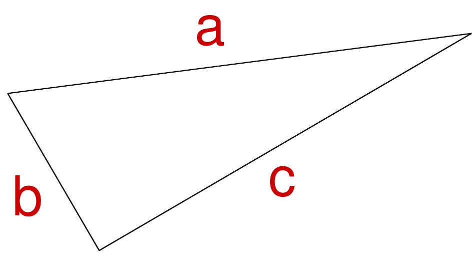
Is this a right-angled triangle?

Check if the triangle with sides a, b, and c is a right-angled triangle.

For: $a = 17$

$b = 8$

$c = 15$



RUNNING DICTATION

STUDENT 4

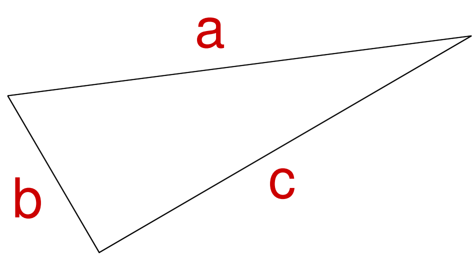
Is this a right-angled triangle?

Check if the triangle with sides a, b, and c is a right-angled triangle.

For: $a = 30$

$b = 20$

$c = 40$



NAME: ____**DATE**: ___________________

UNIT EXAM: Triangulate the world

1. About architecture and math:
   a) List five words (verbs, names, actions…) related to **architecture** and to the tasks you do in **math.** Follow the example given.

Architecture word	When do you use them in math
measure	*In maths we measure angles*
1.	
2.	
3.	
4.	
5.	

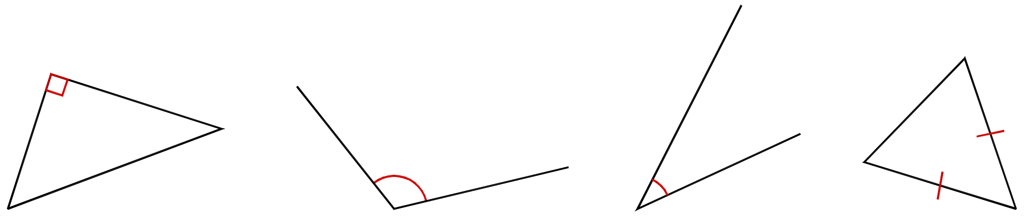
   b) From all the **buildings** you have seen throughout the unit, what is the one that you like best? Use geometrical reasons to explain.
   Where is it located?

2. Explain the **triangulation** of polygons.
   Explain <u>what</u> it is and <u>how</u> to do it. Give an example.

Hint: I hope you will use the word diagonal

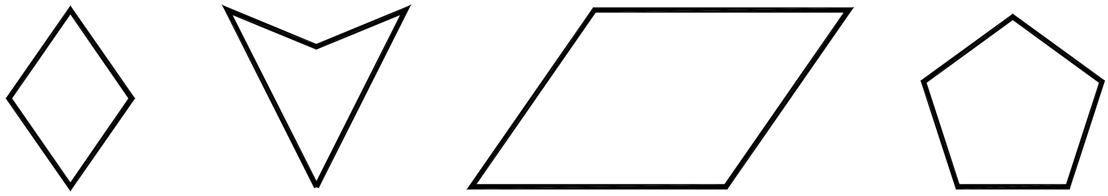
3. Define the word **hypotenuse.**

4. **Classify** and **label** the following triangles and angles:



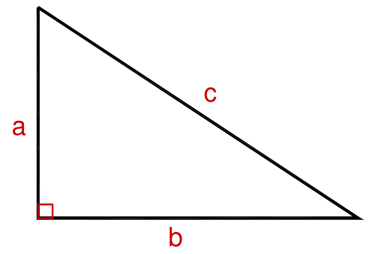
5. Prove that the addition of the interior **angles** of a triangle is 180º.

6. Triangulate each polygon to find the sum of its interior angles. Write the formula you have used.



7. Complete the following chart according to the values given for sides a, b, and c, using a famous theorem.

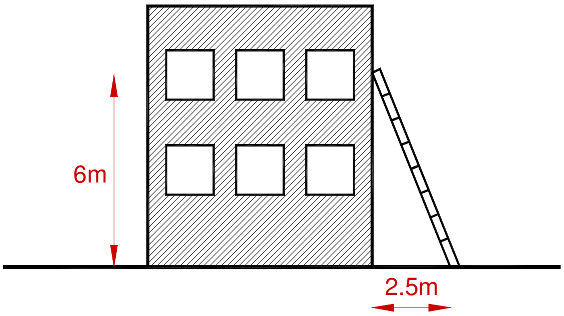
a	b	c
a=4	?	c=5
a=3	b=5	?
?	b=8	c=12
a=6	?	c=10
a=3	b=4	?



8. Solve the following word problems:
   a) Last summer Maria travelled to Egypt and visited the Great Pyramid of Giza. As it was sunny, she could observe shadows projected on the ground. If Maria is 1.60m tall and her shadow projected on the ground was of 1.15m, what is the pyramid's height if its shadow at the same time and with the same endpoint measured 105m?

b) Marc has lost his keys and tries to get inside the house through a window. He has a ladder to use, which he places 2.5m from the façade. We know the window's height is approximately 6m from the ground.

How long is Marc's ladder?



Extra Credit

Find the interior angle of a regular pentagon.

What is the formula for the measure of an interior angle in a regular polygon?

ANNEX 3. VIDEO LIST

How much does your building weigh, Mr. Foster?
http://www.youtube.com/watch?v=9740R6Kd48I
Last accessed April 24, 2011

Welcome to architecture
http://www.youtube.com/watch?v=Lk2cMR0fw_w&feature=related
Last accessed April 24, 2011

Guggenheim New York
http://www.architects-talk.com/2011/03/architecture-tour-solomon-r-guggenheim.html
Last accessed April 18, 2011

Pythagoras was a square
http://www.teachertube.com/viewVideo.php?video_id=23675
Last accessed April 12, 2011

Marcus du Sautoy
http://news.bbc.co.uk/2/hi/programmes/newsnight/review/7865508.stm
Last accessed April 18, 2011

Voki (avatar created by the author of the unit)

Question 1

http://www.voki.com/pickup.php?scid=3461147&height=267&width=200

Question 2

http://www.voki.com/pickup.php?scid=3499655&height=267&width=200

Question 3

http://www.voki.com/pickup.php?scid=3499904&height=267&width=200

Question 4

http://www.voki.com/pickup.php?scid=3499934&height=267&width=200

Question 5

http://www.voki.com/pickup.php?scid=3499952&height=267&width=200

ANNEX 4. CD-ROM

Materials for implementation
Student's Book for digital projection