Revista Española de Economía, Vol. 15, nº 3, 1998 **355-377**

A Simple Model of Recurrent Hyperinflation*

Jesús Vázquez

Universidad del País Vasco.

Recibido: enero de 1998
Aceptado: julio de 1998

Abstract

The model developed in this paper builds on Cagan's demand for money by considering that a share of government expenditure is spent on servicing foreign debt obligations and fixed in foreign currency. Given this fiscal policy the choice of the depreciation rate implies a passive money supply rule. We show in this model that two types of high-inflation path exist. One type is driven by crawling peg rules of the official exchange rate and is characterized by the saddle-stable path dynamics. The other type characterizes hyperinflationary dynamics. This latter type is a consequence of the global dynamics of the model. The existence of these two types of path can generate recurrent hyperinflation. Moreover, hyperinflation can arise independently of government spending.

Key words: Crawling peg rules, government debt, global and local dynamics.
JEL Classification: E31, F31.

Resumen. *Un modelo sencillo de hiperinflaciones recurrentes*

A partir de la función de demanda de dinero de Cagan, el modelo desarrollado en este artículo considera que una fracción del gasto gubernamental se destina al pago de intereses asociado a la deuda externa y que dicho pago es constante medido en unidades de la moneda extranjera. Dada esta política fiscal, la elección de una tasa de depreciación del tipo de cambio implica una política monetaria pasiva. En este modelo, mostramos que dos tipos de sendas de alta inflación existen. Un tipo de estas sendas se deriva de la adopción de reglas sobre la tasa de depreciación del tipo de cambio oficial, conocidas como *crawling peg*

* I am grateful for helpful conversations and comments from Carlos Borondo, Cruz Ángel Echevarría, María José Gutiérrez, Albert Marcet, two anonymous referees and participants in seminars at ESADE (Barcelona), XX Simposio de Análisis Económico (Universitat Autònoma de Barcelona), 95 Atlantic Economic Conference (Vien) and Universidad de Valladolid on previous drafts. A previous version of this paper was circulated under the title «Government Foreign Debt, Crawling Peg Rules and Hyperinflation: The Peruvian Case». Financial support from Ministerio de Educación and Universidad del País Vasco (Spain) through projects DGICYT PS95-0110 and UPV 035.321-HB067/96 is gratefully acknowledged.

rules. Este tipo de sendas están caracterizadas por una dinámica de punto de silla. El otro tipo de sendas caracterizan dinámicas hiperinflacionarias. Este último tipo es una consecuencia de la dinámica global del modelo. La coexistencia de estos dos tipos de sendas puede generar hiperinflaciones recurrentes. Además, las hiperinflaciones pueden surgir independientemente de variaciones del gasto del gobierno.

Palabras clave: reglas *crawling peg*, deuda pública, dinámica global y local.

1. Introduction

Many Latin American economies have suffered recurrent hyperinflationary episodes (e.g. Argentina, Bolivia, Brazil and Peru) over the last decade. The observed evolution of inflation in these economies alternates relatively long periods of moderate and steady inflation with a few short periods of hyperinflation. The evolution of inflation for Peru is shown in Figure 1. Similar inflation patterns for Argentina and Brazil are shown by Mondino, Sturzenegger and Tommasi (1996). Moreover, recurrent hyperinflation is quite remarkable because the peaks of the inflation rate seem to be independent of any strong changes in seigniorage.[1]

Hyperinflationary episodes are widely understood to be unstable processes where inflation speeds up and real money balances fall. The conventional view of hyperinflation, as stated clearly by Cagan (1956) and expressed by many authors (for instance, Sachs and Larrain (1993: 742-744)), is that hyperinflation arises when required seignorage exceeds its maximum steady state level and when expectations or real money holdings are adaptive.[2] However, the conventional view fails to explain the existence of recurrent hyperinflation which is not caused by strong movements in the level of seigniorage.

Recently, Nicolini and Marcet (1996) and Mondino et al. (1996) have developed hyperinflationary models which accounts for the observations of recurrent hyperinflation. The key feature in Nicolini and Marcet's model is that agents are endowed with a type of bounded rationality whereas the relevant assumption in Mondino et al.'s model is the introduction of financial adaptation to new financial products when those assets start to compete with *old* depreciating domestic money.

This paper develops a perfect foresight inflationary finance model which provides an alternative explanation of the recurrent hyperinflation phenomena observed during the 1980's. The model proposed in this paper is a slight variation on the model considered in the traditional inflationary finance literature (for instance, Evans and Yarrow (1981), Bruno (1989) and Bruno and Fischer (1990)). The model builds on Cagan's demand for money, but, in addition, the model assumes, first, that a portion of government spending is devoted to the

1. See Bruno et al. (1988) and (1991) for details. Nicolini and Marcet (1996) give a neat exposition of the stylized facts during recurrent hyperinflation.
2. See Vázquez (1998) for an alternative view of hyperinflation and for a review of different attempts made in the literature to characterize the explosive evolution during hyperinflationary episodes.

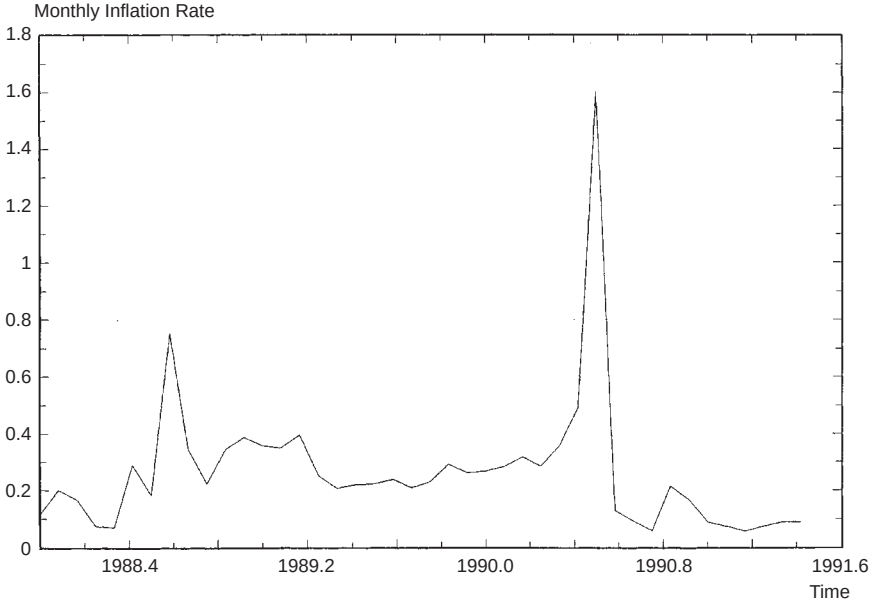

Figure 1. Peruvian Hyperinflation. **Source:** IMF Statistics.

service of foreign debt and fixed in foreign currency. Second, the existence of crawling peg rules for the exchange rate, which are supported by rationing the foreign exchange market. This rationing typically results in the creation of a black foreign exchange market. We believe that these additional features, which are common in many Latin American countries suffering endemic high-inflation processes, are much relevant to understand recurrent hyperinflation.[3]

Given a level of government spending, our model exhibits two types of high-inflation dynamics. One type is characterized by the local dynamics around the unique steady state of the model. The steady state is saddle-point stable. Therefore, the dynamics of inflation are described in this case by the unique stable manifold converging to the steady state. The steady state rate of inflation under this type of dynamics is determined by the policy choice of the rate of crawl of the exchange rate. The other type of high-inflation dynamics is characterized by the global dynamics of the model. We identify these dynamics as hyperinflationary dynamics. Along these hyperinflationary paths inflation increases, real

3. The German government also had to pay war reparations after World War I, leading to huge budget deficits and hyperinflation. An important difference between Latin American hyperinflationary episodes and German hyperinflation was that the German government did not ration the foreign exchange market. See Bresciani-Turroni (1937) for more details on how the foreign exchange market worked during German hyperinflation.

money balances fall and the official depreciation rate is fixed. These hyperinflationary dynamics lead the economy to a hyperinflationary plateau which is *globally* stable. Nevertheless, the hyperinflationary plateau might not be reached for two reasons. First, the black market premium is increasing rapidly along hyperinflationary paths because the black market depreciation rate increases with inflation whereas the official rate stays constant. Thus the proportion of private exports being smuggled will increase, reducing the net tax transfer from the private sector to the government, which will lead to an increase of the official depreciation rate (or equivalently a big devaluation) in order to reduce the black market premium. This policy can ultimately act as an anchor for inflation in order to drive the economy to the local stable manifold, where inflation, official and black depreciation rates remain close to each other, implying a more stable black market premium which leads to a more stable net tax on exports. Second, since domestic price is a jump-variable, changes in beliefs can drive the economy from a hyperinflationary path to the stable manifold and vice versa, generating recurrent hyperinflation.

The remainder of the paper is organized as follows. Section 2 introduces a slight variation on the traditional inflationary finance model which builds on Cagan's demand for money. In Section 3, we show how the existence of both local and global dynamics of inflation helps to understand recurrent hyperinflation. The results are illustrated by the Peruvian hyperinflation of 1988-1990. Finally, Section 4 concludes.

2. The Inflationary Finance Model

In this section we develop a model built on Cagan's model, which considers dual foreign exchange markets. There is no uncertainty. The money market equilibrium condition is given by

$$\ln(m(t)) = \gamma - \lambda\pi(t)\,, \qquad [1]$$

where $\ln(m)$ denotes the log of real money balances, π is the expected rate of inflation, γ and λ are constant positive parameters. The right-hand side of [1] is a continuous time version of Cagan's demand for money which establishes that the demand for real money balances is inversely related to expected inflation.[4] We assume perfect foresight, that is, actual and expected rates of inflation are equal.

Moreover, we assume that a share of government spending is made on the payments associated with service obligations of foreign debt, and fixed in foreign currency. We assume that government spending is entirely financed by issuing domestic money according to the following expression

$$\dot{M} = ed + Pg\,, \qquad [2]$$

4. Appendix 1 provides a microeconomic foundation of Cagan's demand for money. This microeconomic foundation is considered below, first, to characterize the hyperinflationary dynamics and second, to rule out hyperdeflationary paths.

where M is the nominal level of money supply, e is the official exchange rate, d is the government expenditure devoted to the service of foreign debt (fixed in dollars), P is the domestic price level and g is the government spending made on consumption goods. As usual a dot on a variable denotes its time derivative. Both d and g are assumed to be constants to show the ability of the model to generate recurrent hyperinflation even when the level of government expenditure and the service obligations of foreign debt remain constant. With this assumption we do not intend to claim that changes in government spending cannot contribute to explaining hyperinflation in practice. Rather, we are aiming to suggest a mechanism which can amplify the effects of small changes of government spending in inflation.

Following Kharas and Pinto (1989) and Pinto (1991) we have in mind the existence of a private export/import sector which is not modeled in this paper for the sake of simplicity. Private residents are expected to surrender all their export receipts, which are nominated in foreign currency, to the Central Bank for domestic currency at the official exchange rate because they are not allowed to hold foreign currency. However, they do so illegally. Therefore, there is currency substitution with residents holding domestic and foreign currencies. In the black market, the (black) foreign exchange rate is $f \geq e$, with smuggled exports flowing into this market to obtain (illegally) higher returns.

In the official exchange rate market, the official exchange rate e is managed by the government. Rationing in this official market works as follows. A certain amount of surrendered private export receipts are set aside for paying the service of foreign debt. The rest of surrendered export receipts are allocated to the private sector by allowing some types of imports to be made eligible for foreign exchange at the subsidized official exchange rate. Official reserves (fixed in dollars) are then protected and remain constant The rationing in the official market induces a black market. Thus the remainder of private export receipts are smuggled into the black market at rate f.[5]

A government may have many incentives for rationing the foreign exchange market during a high-inflation episode. One possibility is that the rationing of the foreign exchange market works as a way of reducing the pressure on depreciating domestic money. Another possibility is that the black market premium generates an implicit net tax transfer from the private export sector to itself. Another possibility is that the government can subsidize some categories of imports by made them eligible for foreign exchange at the highly subsidized official exchange rate.[6]

5. In this paper we omit for the sake of simplicity the demand for money by private exporters. Alternatively, we can assume that the total demand for money in the economy is given by

$$\ln(m) = \gamma - \lambda\pi - \theta\hat{f} \qquad [1']$$

instead of equation [1], in order to capture the idea that demand for money by some agents depends on the black market depreciation rate. Equation [1'] is similar to the money demand suggested by Abel, Dornbusch, Huizinga and Marcus (1979) which reflects that foreign assets and real assets can both be substitutes for domestic money. This money demand specification was tested by Vázquez (1995) using data from the German hyperinflation. As shown in Appendix 2, replacing [1] by [1'] has no effect on the dynamic analysis of the model.

6. For instance, Lago (1991: 302) reports that 72 percent of total peruvian imports were given access to official market during Peruvian hyperinflation. Lago claims that this policy was aiming to prompt a quick recovery by boosting import levels.

In order to close the model we need to specify how the black market depreciation rate is determined. Following Culbertson (1975) we assume that the black market depreciation rate $\hat{f} = \frac{\dot{f}}{f}$ is simply a linear function of domestic inflation π and the official market depreciation rate $z = \frac{\dot{e}}{e}$.[7] The idea behind this assumption is that a free (black) foreign exchange rate is largely determined by the purchasing power of domestic currency relative to the purchasing power of foreign currency. Since the foreign inflation can be considered nearly constant when analyzing countries involved in hyperinflationary process, the rate of depreciation of the black foreign exchange rate is then mainly determined by the rate of domestic inflation.

In Kharas and Pinto (1989) domestic inflation is determined by the production sector, being a weighted average of official market depreciation rate and black market depreciation rate whereas the black depreciation rate is determined by the portfolio balance condition. In contrast, domestic inflation in our model, which builds on Cagan's demand for money, is determined by money creation, and the black market depreciation rate is mainly caused by inflation. In other words, our model follows a quantity theory of money approach whereas Kharas and Pinto (1989) follow a portfolio balance approach. Appendix 3 shows evidence that causality ran from inflation rate to black market depreciation rate during Peruvian hyperinflation. This empirical evidence supports the approach followed in this paper.

3. Hyperinflationary Dynamics

In this section, we characterize the two types of hyperinflationary dynamics described by the model. Let us denote $\frac{M}{e}$ by m_e. Using this definition we have that

$$m = m_e s, \qquad [3]$$

where $s = \frac{e}{P}$ denotes the official real exchange rate. Implicitly we have normalized the level of foreign prices to one. Further, differentiating $m_e = \frac{M}{e}$ with respect to time and using [2] we obtain

$$\dot{m}_e = d + \frac{g}{s} - m_e z\,. \qquad [4]$$

7. The black market depreciation rate depends, of course, upon other many factors such as the variation of the level of reserves, changes in the penalty structure associated with black market activities and so on. Culbertson (1975) provides a theoretical framework for assessing the main determinants of black market exchange rates. As in Kharas and Pinto (1989), we are implicitly assuming that there is no net accumulation of reserves or changes in those other factors affecting black market exchange rate.

Equation [4] can be viewed as a passive monetary rule determined by the choice of d, g and z. The choice of the rate of crawl for the official exchange rate z has been common in Latin America (called «tablitas») as reported by many authors (Kharas and Pinto (1989), Pinto (1991), Lago (1991) among others). The government choice of the rate of crawl for the official exchange rate z can be due to three reasons. First, as mentioned above, management of the exchange rate generates an implicit tax on exports. Second, the government at some point in time could have incentives to reduce the black market premium ϕ. These incentives appear because for sufficiently high ϕ the private export receipts being smuggled can increase so much that the tax transfer from the private sector to the government induced by the rationing decreases. Finally, the government can change the subsidizes of the imports eligible for official foreign exchange rate by varying z.

From the definition of the official real exchange rate we have that

$$\dot{s} = s(z-\pi)\,. \qquad [5]$$

The money demand function, equation [1], can be written as

$$\ln(m_e) = \gamma - \ln(s) - \lambda\pi\,.$$

Substituting this expression in [5] we obtain

$$\dot{s} = \frac{s}{\lambda}[\lambda z + \ln(m_e) + \ln(s) - \gamma]\,. \qquad [6]$$

Equations [4] and [6] form a pair of differential equations describing the dynamics of the economy. Figure 2 summarizes these dynamics. Appendix 2 shows that for a given choice of the rate of crawl of the official exchange rate, z, there is a unique steady state which is saddle-point stable.[8] Moreover, Appendix 2 shows that the slope of $\dot{m}_e = 0$ is greater than the slope of $\dot{s} = 0$ and both slopes are negative as drawn in Figure 2.

This model displays two sources of high-inflation dynamics. First, as in Kharas and Pinto's model, based on local dynamic analysis, our model displays a continuum of saddle-point stable equilibria, each of them corresponding to alternative policy choices of z. According to equation [5], an increase of z drives the steady-state inflation rate to a higher level. Eventually for high levels of z the economy will be evolving in a high-inflation scenario. Second, for a given level of z the global dynamics of the system characterize paths asymptotically tending to m_e-axis.[9] Along this type of path, characterized by the global dynamics of the model, s goes to zero and m_e goes to infinity. Since real money balances by definition are deter-

8. Appendix 2 shows that the determinant of the Jacobian associated with the linear approximation of the system is $-\frac{d}{\lambda m_e}$, which is always negative.

9. As pointed out by George and Oxley (1991) ignoring global dynamics and focusing on local dynamics could yield misleading conclusions. The analysis of our model provides, as a by-product, an example of the consequences of ignoring global dynamics.

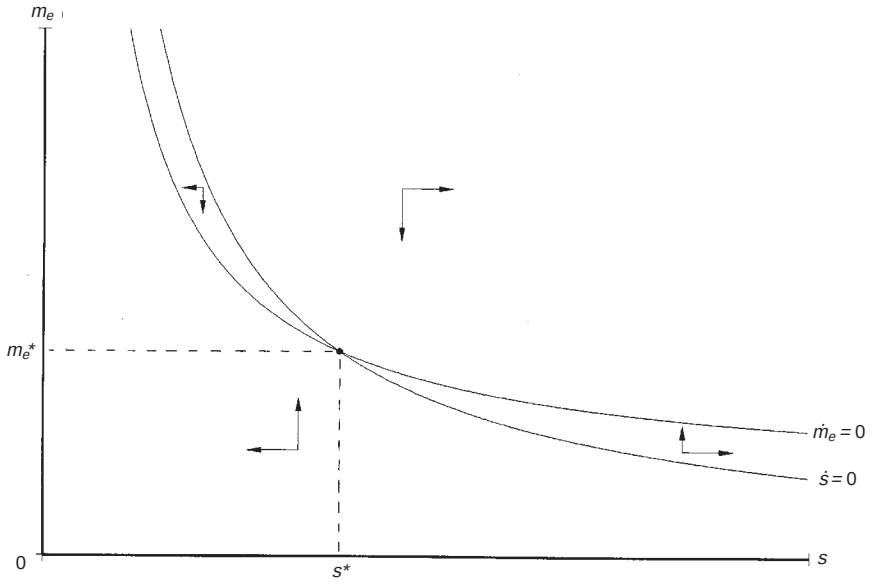

Figure 2. Phase Diagram.

mined by $m = m_e s$, what happens with inflation along these paths? The precise characterization of the global dynamics of the system of equations [4] and [6] can only be undertaken numerically.[10] Numerical simulations of the global dynamics of the model show that starting at any point located to the left of the saddle-stable manifold inflation reaches a hyperinflationary plateau and real balances m approach zero as shown by Figures 3 and 4, respectively.[11] Numerical simulations also provide a picture of the relative size of inflation along hyperinflationary paths compared to the rate of inflation along the stable manifold, i.e, in the vicinity of the saddle-point steady state. More specific, using the parameter values displayed in footnote 10 the monthly inflation rate at the hyperinflationary plateau is 271.2% whereas the inflation rate at the saddle-point stable steady state is 14%.

10. We used the software package SIMGAUSS V.20 and the Graphic software package, both written in GAUSS V3.01, to derive the numerical solutions of the differential equation system and Figure 2, respectively. The parameter values used to carry out the numerical simulations and to draw Figure 2 are the following: $\lambda = 1.33$, $g = 0.2$, $d = 0.02$, $z = 0.14$ and $\gamma = 1$. Appendix 4 provides an explanation of how the parameter values were chosen. In Appendix 4, we have also performed a sensibility analysis to see whether alternative values for the parameters give reasonable values of the rate of inflation at the hyperinflationary plateau. Notice that the existence of the two types of hyperinflationary dynamics does not depend on the parameter values chosen provided that the unique steady state exists. The computer programs are available from the author upon request.
11. Global dynamics also characterize hyperdeflationary paths. Along this type of path the economy tends to travel along the locus $\dot{m}_e = 0$ which implies that $m = m_e s$ tends to infinity (see the phase diagram displayed in Figure 2). As shown in Appendix 1 these hyperdeflationary paths are not feasible because they violate the transversality condition of the underlying household optimization problem, equation [A.7].

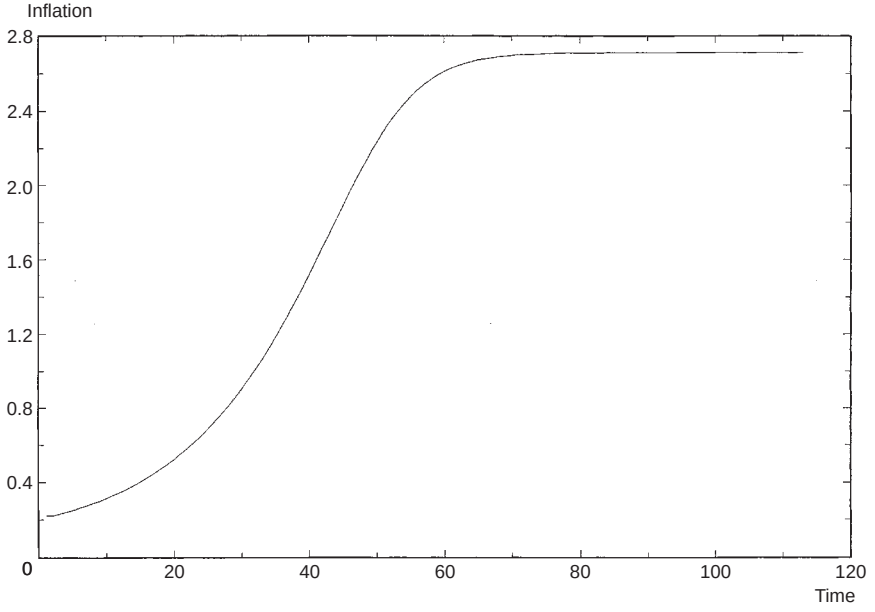

Figure 3. Evolution of Simulated Inflation.

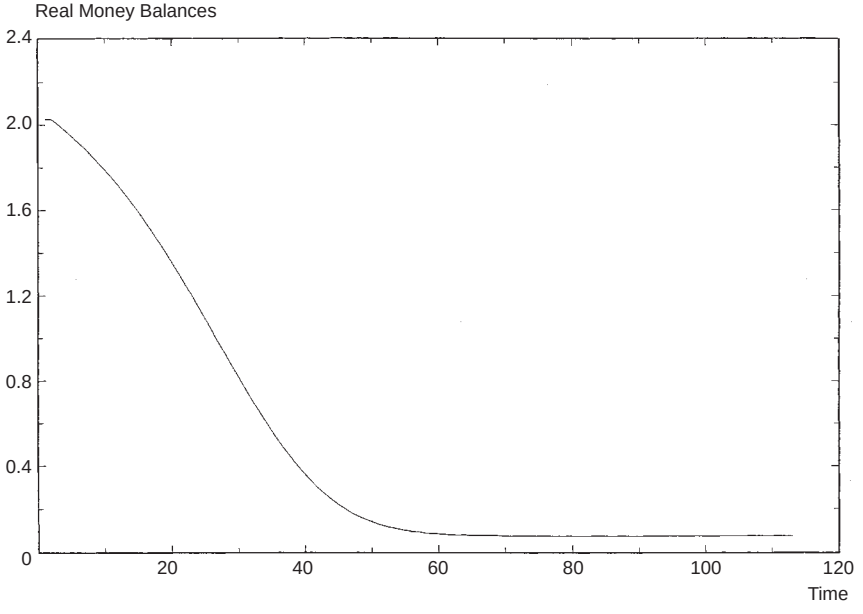

Figure 4. Evolution of Simulated Real Money Balances.

An explanation of the existence of this hyperinflationary plateau is quite simple. Notice that the first-order differential equation characterizing real money balances in our model is given by

$$\dot{m} = \frac{\dot{M}}{P} - m\pi = \frac{e}{P}d + g - m\pi \,, \qquad [7]$$

where the second equality is obtained using the government's budget constraint (equation [2]). Along the hyperinflationary path, characterized by the global dynamics of the system, the rate of inflation accelerates whereas the rate of depreciation of the official exchange rate holds constant. The real exchange rate ($s = \frac{e}{P}$) thus tends to zero, which implies that the portion of seignorage devoted to finance the service of foreign debt ($\frac{e}{P}d$) vanishes. In the limit, real money balance dynamics are then determined by government spending g. Formally, equation [7] can be rewritten as

$$\dot{m} = g + \frac{m}{\lambda}[\ln(m) - \lambda] \,, \qquad [8]$$

where we are using equation [1]. Equation [8] characterizes the *degenerated* dynamics of our model when s tends to zero. The dynamics of inflation described by this equation are well known in the literature and correspond to the case where seignorage is entirely devoted to finance government spending. These dynamics are derived in Appendix 1 for the sake of completeness. These dynamics imply the possibility of dual equilibria and the existence of a high-inflation trap. We can identify the high-inflation steady state (point A in Figure 1A) with the hyperinflationary plateau. A well known feature of these dynamics is that the rate of inflation associated with the high-inflation steady state is inversely related to government spending. It is straightforward to show that, given the choice of parameter values, at the high-inflation steady state we have $\pi_A = 2.712$ and $m_A = 0.071$, which coincides with the values of inflation and real money balances at the hyperinflationary plateau obtained by numerical simulation as shown in Figures 3 and 4, respectively. However, it must be noticed that the low-inflation steady state (point B in Figure 1A) associated with the *degenerated* model does not correspond to the saddle-stable steady state in our model. A simple way of seeing this is to notice that the rate of inflation associated with the saddle-stable steady state is determined only by the rate of depreciation in the official market z, whereas the low-inflation steady state in the *degenerated* system depends upon g, λ and γ. Therefore, our model generates a high-inflation trap because, given a level of government spending the low-inflation steady state is globally unstable (locally saddle-path stable) whereas the hyperinflationary plateau (the high-inflation steady state in the *degenerated* model) is globally stable.

A remarkable feature of our model is that hyperinflation is generated even when seignorage decreases. To see this notice that

$$\dot{m} = \frac{\dot{M}}{P} - m\pi = \frac{ed}{P} + g - m\pi \,,$$

where seignorage is equal to $\frac{ed}{P}+g$. Seignorage thus decreases along hyperinflationary paths because the rate of inflation is greater than the official rate of depreciation under this type of dynamics, that is, the ratio $\frac{e}{P}$ decreases. This result challenges the conventional view of hyperinflation expressed by many authors, which states that hyperinflation arises when seignorage exceeds a threshold level.

The hyperinflationary plateau, however, might not be reached because along hyperinflationary paths the black market premium rapidly increases, which forces the government to raise the official depreciation rate in order to avoid the massive smuggling of private export receipts. This crawling peg rule can drive the economy to the saddle-stable manifold, which implies a rate of inflation close to the official depreciation rate. This mechanism is not the only one working in this economy. Since domestic price is a jump variable, changes in beliefs can drive the economy from a hyperinflationary path to the high-inflation stable manifold and vice versa. The interaction of these two mechanisms can generate recurrent hyperinflation, which is not directly related to large changes in government spending. A key feature to understand why expectations can be highly volatile and then why high-inflation and hyperinflationary episodes follow one another in some Latin American countries is the erratic economic policy measures implemented by their governments, which are mainly driven by short-run developments.[12] The following paragraph taken from Lago (1991: 302) illustrates this point for the Peruvian case:

> […] (policy) measures followed a «zig-zag» pattern, with later measures running in opposite direction to earlier ones in an attempt to correct their destabilizing effects. The initial new policies included, inter alia, introduction of a crawling peg (at a continuously fine-tuned rate), monthly wage increases, sporadic public price adjustments,… Since July 1989, the Central Bank started to accomodate again government credit requests… The free market exchange rate, however, very soon reflected expansionary financial policies. By the end of the year the spread over the official rate had surpassed 100 percent. At this point the authorities switched into a containment effort aimed at avoiding a further explosion of the exchange rate and prices before the forthcoming elections of 8 April 1990. As part of this strategy, the Central Bank start to provide dollar loans to exporters and to sell promissory notes to importers in an attempt to depress the free market exchange rate. The Central Bank's financing of GDP was small, about 4.3 percent of GDP … But, as often happens with high inflation processes, the pressure of just of a little Central Bank financing on a tiny financial system was enough to sustain hyperinflation.

Based on our model, can we say anything about which alternative sources of high-inflation dynamics were acting primarily during the period May 1988 - August 1990 in Peruvian economy? In analyzing Peruvian high-inflation episodes

12. Theorists have tried for a long time a better understanding of the motives and short-run constraints facing the inflating governments. As stated by Cukierman (1988), a plausible hypothesis is that, although policy errors can be a part of the story, a substantial part of inflating policies can be understood as deliberate policy choices made by rational governments with a high discount rate.

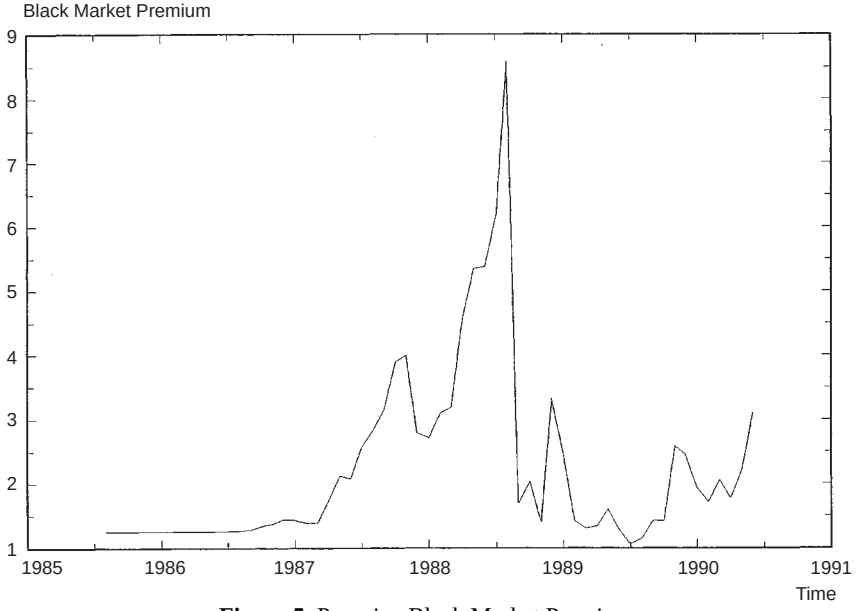

Figure 5. Peruvian Black Market Premium.
Source: Lago (1991: 299-301).

we understand that it is useful to distinguish between three periods: first, the period from May 1988 to September 1988, second, the period from September 1988 to June 1990, and finally the period from June 1990 to August 1990. During the first and third periods Peruvian inflation was running into hyperinflation, reaching huge inflation rates in September 1988 and August 1990, respectively. On the one hand, in September 1988 the so-called «zero plan» was implemented (see Lago (1991: 294) for details). This plan mainly resulted in a big credit crunch which temporarily stopped hyperinflation and brought the economy to a high-inflation scenario with a monthly inflation rate close to 35%. On the other hand, in August 1990 another stabilization plan was implemented. This latter plan meant a successful policy regime switch in the sense that it was able to stop hyperinflation permanently.

Figure 5 shows the black market premium along the hyperinflationary path from May 1988 to September 1988. During that period the rate of inflation and the black market premium sped up,[13] whereas the depreciation rate of the official market was almost zero (see Figure 6) supporting the hypothesis that Peruvian hyperinflation dynamics were driven by global dynamics rather than local dynamics.

Figure 7 shows that between September 1988 and June 1990 inflation, depreciation rate in the official market and depreciation rate in the black market were mov-

13. Unfortunately, the black market observations reported by Lago (1991) stop at June 1990, thus we cannot show evidence of the evolution of those variables during the third period considered: June-August 1990. Figures 5, 6 and 7 show growth rates as monthly percentage increase.

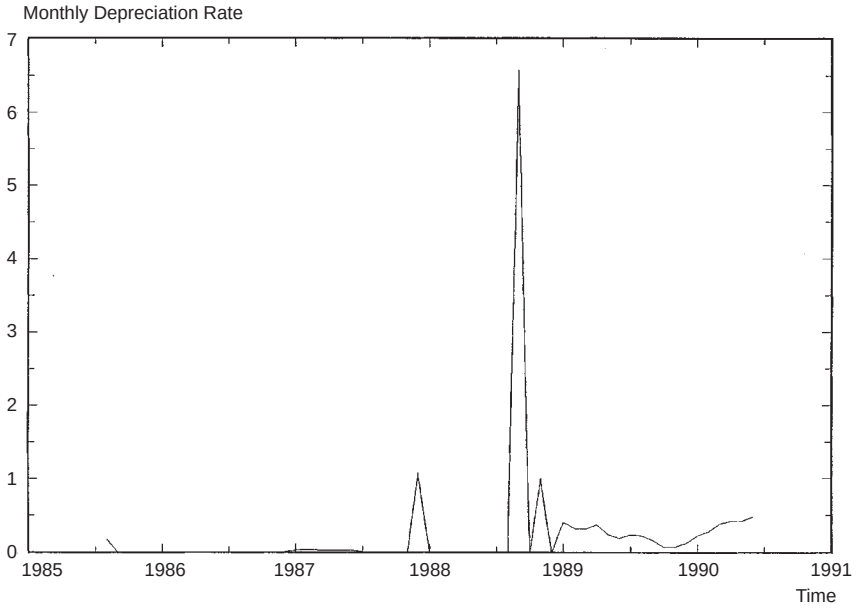

Figure 6. Peruvian Depreciation Rate in the Official Market.
Source: Lago (1991: 299-301).

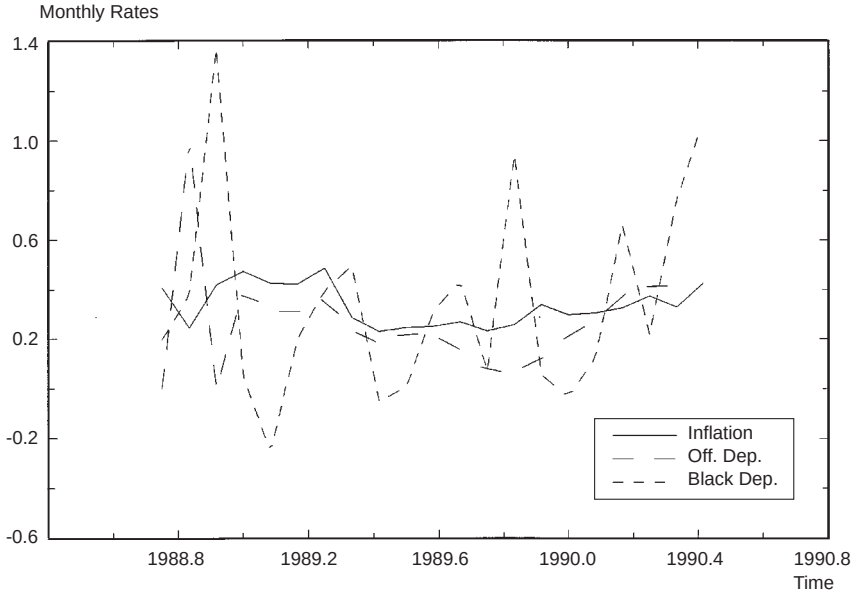

Figure 7. Peruvian Inflation and Depreciation Rates.
Source: Lago (1991: 299-301).

ing roughly together suggesting that the Peruvian economy was jumping from one stable manifold to another depending on which crawling peg rule was chosen for the official foreign exchange market. In other words, the Peruvian economy seems to have been driven by local dynamics from September 1988 to June 1990.

4. Conclusions

This paper investigates recurrent hyperinflationary dynamics by analyzing a slightly different version of the perfect foresight inflationary finance model considered in the literature. Following this literature, we have also adopted Cagan's demand for money but, in addition, we have considered that a share of government expenditure is spent on payments associated with the service obligations of government foreign debt. Moreover, we have assumed that the government manages the depreciation of the official nominal exchange rate, which is supported by rationing the foreign exchange market.

A salient feature of our model is that can generate two types of inflationary dynamics, implying the possibility of recurrent hyperinflation which is consistent with a given level of government spending. This result challenges the conventional view of hyperinflation, which establishes that hyperinflation arises when seignorage or government expenditure exceeds a threshold level. This result, however, should not lead to the conclusion that small changes in government spending cannot contribute to explain hyperinflations in practice. Rather, the existence of two types of inflationary dynamics should be interpreted as a mechanism by which inflation can overreact because of small changes in government spending in an inflationary economy. This feature of the model can help in the interpretation of some recurrent hyperinflationary episodes which occurred in Latin American economies in the 1980's, where inflation seem to move independently of government spending.

Finally, we understand that the explanation of recurrent hyperinflation given in this paper is not incompatible with other explanations provided in the recent literature. Rather, our interpretation should be viewed as reinforcing those other explanations.

Appendix 1

We assume a continuous time model where the economy consists of a large number of identical infinitely-lived households. There is no uncertainty. Each household has a non-produced endowment $y(t) > 0$ of the non-storable consumption good per unit of time. The representative household's utility at time 0 is

$$\int_0^\infty e^{-\beta t}[U(x(t)) + V(m(t))]dt\,, \qquad \text{[A.1]}$$

where $\beta > 0$ denotes the instantaneous discount factor, $x(t)$ and $m(t)$ are per capita consumption and real money balances, respectively; $U(.)$ and $V(.)$ are increasing

and concave functions. Money is introduced into the household's utility function as an easy way of modeling transaction costs.[14]

We define financial wealth and nominal interest rate as

$$w(t) = m(t) + b(t),$$

$$i(t) = r(t) + \pi(t),$$

respectively, where $b(t)$ denotes real per capita government debt, $r(t)$ is the real interest rate and $\pi(t)$ is the rate of inflation. The household's budget constraint is

$$\dot{w}(t) = r(t)w(t) + y(t) - [x(t) + i(t)m(t)]\,, \qquad \text{[A.2]}$$

where the term in brackets denotes full consumption, that is, the sum of consumption and costs of holding money.

The necessary and sufficient conditions for the optimum are

$$\dot{\lambda}(t) = \lambda(t)[\beta - r(t)]\,, \qquad \text{[A.3]}$$

$$\dot{w}(t) = r(t)w(t) + y(t) - x(t) - i(t)m(t)\,, \qquad \text{[A.4]}$$

$$U'(x(t)) = \lambda(t)\,, \qquad \text{[A.5]}$$

$$V'(m(t)) = \lambda(t)i(t)\,, \qquad \text{[A.6]}$$

$$\lim_{t\to\infty} e^{-\beta t}\lambda(t)w(t) = 0\,. \qquad \text{[A.7]}$$

Equation [A.6] is the demand for money. We specialize $V(m(t))$ in order to get Cagan's demand for money. Let $V(m(t))$ be defined by[15]

$$V(m(t)) = m(t)[k + 1 - \ln(m(t))]\,, \qquad \text{[A.8]}$$

where k is a constant.[16] Then equation [A.6] can be written as

$$\ln(m(t)) = k - \lambda(t)i(t)\,. \qquad \text{[A.9]}$$

Cagan (1956) argues that during hyperinflationary periods the variation in real income is negligible compared to the variation in inflation. In our model, this argument implies that $y(t)$ can be considered a constant. Moreover, if the per capita government consumption g is constant, from the equilibrium condition in the goods market, $y = x(t) + g$, then the agent's consumption is a constant. Hence, according to [A.5], $\lambda(t)$ is also a constant and then, $r(t) = \beta$. Therefore, equation [A.9] becomes

$$\ln(m(t)) = \gamma - \lambda\pi(t)\,, \qquad \text{[A.10]}$$

14. Feenstra (1986) studies the functional equivalence between introducing real money balances as an argument in the utility function and entering liquidity costs in the budget constraint.
15. Calvo and Leiderman (1992) use a similar utility function for money.
16. k has to satisfy $m(t) < e^k$ for all t in order for $V(m(t))$ to be increasing.

where $\gamma = k - \lambda\beta$. Equation [A.10] is a continuous-time non-stochastic version of Cagan's demand for money, where the semielasticity of the demand for money with respect to inflation, λ, according to [A.5], is the marginal utility of consumption.

The dynamics of the inflationary finance model based on Cagan's demand for money under perfect foresight are well known (Evans and Yarrow (1981), Bruno and Fischer (1990)). We summarize their results here. The traditional inflationary finance model assumes that high-powered money is used according to the following expression

$$\dot{M}(t) = P(t)g(t)\,. \qquad [A.11]$$

For the sake of simplicity we drop the index t from now on. The definition of real money balances and equation [A.11], imply that $\dot{m} = g - m\pi$. Substituting [A.10] in this equation we have the following first-order differential equation

$$\dot{m} = g + \frac{m}{\lambda}[\ln((m) - \gamma)]\,. \qquad [A.12]$$

Figure A.1 shows that dual steady states may co-exist for a given level of government spending whenever this level is not too high. This figure also shows that the high-inflation steady state, m_1, is stable and the low-inflation steady state, m_2, is unstable.

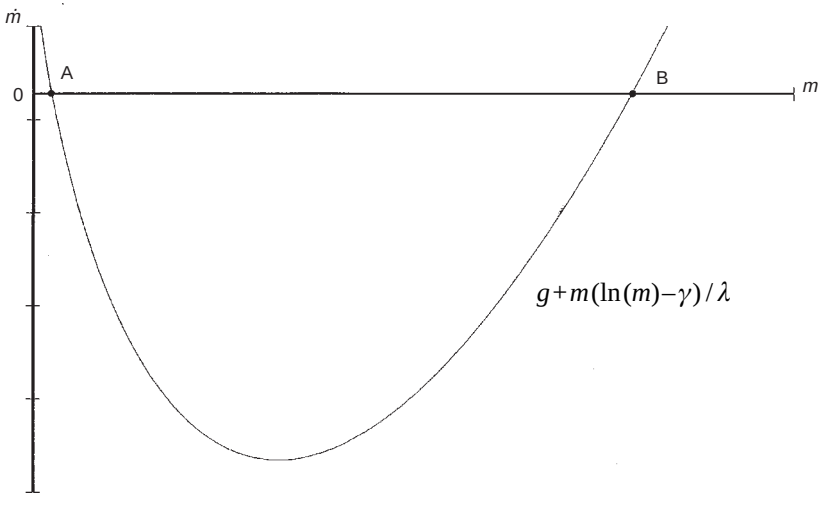

Figure A.1. Phase Diagram for the Degenerated Model.

Appendix 2

The dynamics in our model are characterized by the following pair of first-order differential equations:

$$\dot{m}_e = d + \frac{g}{s} - m_e z \,, \qquad [4]$$

$$\dot{s} = \frac{s}{\lambda}[\lambda z + \ln(m_e) + \ln(s) - \gamma] \,. \qquad [6]$$

The expression for the locus $\dot{m}_e = 0$ is given by

$$m_e = \frac{d}{z} + \frac{g}{sz} \,.$$

The expression for the locus $\dot{s} = 0$ is given by

$$m_e = \frac{e^{\gamma - \lambda z}}{s} \,.$$

Equating the last two expressions we obtain the unique steady state of the model:

$$s^* = \frac{z}{d} e^{\gamma - \lambda z} - \frac{g}{d} \,.$$

$$m_e^* = \frac{d e^{\gamma - \lambda z}}{z e^{\gamma - \lambda z} - g} \,.$$

The slope of $\dot{m}_e = 0$ is given by

$$\frac{\partial m_e}{\partial s}\Big|_{\dot{m}_e = 0} = -\frac{g}{s^2 z} \,.$$

The slope of $\dot{s} = 0$ is given by

$$\frac{\partial m_e}{\partial s}\Big|_{\dot{s} = 0} = -\frac{m_e}{s} \,.$$

Let us show that

$$\frac{\partial m_e}{\partial s}\Big|_{\dot{s} = 0} < \frac{\partial m_e}{\partial s}\Big|_{\dot{m}_e = 0} \,,$$

at the steady state. Since the steady state is unique, the last inequality must always hold, that is, the slope of $\dot{m}_e = 0$ is always greater than the slope of $\dot{s} = 0$.

Since $z > 0$ the last inequality implies that

$$z \frac{\partial m_e}{\partial s}\Big|_{\dot{s} = 0} < z \frac{\partial m_e}{\partial s}\Big|_{\dot{m}_e = 0} \,.$$

Using the expressions for the slopes we have

$$-\frac{z m_e}{s} < -\frac{g}{s^2} \,.$$

At the steady state $zm_e = d + \frac{g}{s}$. Then, the last inequality can be written as $-\frac{d}{s} < 0$, which is true.

In order to study the local dynamics of the system formed by equations [4] and [6], we linearize the system around the unique steady state (m_e^*, s^*). Formally,

$$\begin{pmatrix} \dot{m}_e \\ \dot{s} \end{pmatrix} = \begin{pmatrix} -z & -\frac{g}{s^2} \\ \frac{s}{\lambda m_e} & \frac{1}{\lambda} \end{pmatrix} \begin{pmatrix} m_e - m_e^* \\ s - s^* \end{pmatrix}.$$

The determinant of the Jacobian matrix J is

$$|J| = -\frac{z}{\lambda} + \frac{g}{\lambda m_e s} = -\frac{d}{m_e \lambda} < 0,$$

to get the last equality we use the fact that at the steady state $\dot{m}_e = 0$ or equivalently, $z = \frac{d}{m_e} + \frac{g}{s m_e}$.

Next we consider the demand for money proposed by Abel et al. (1979) to see that the dynamics of the model remain the same. The money market equilibrium condition is

$$\ln(m) = \gamma - \lambda\pi - \theta \hat{f} \ . \qquad [1']$$

We assume in the paper that black market depreciation rate is a linear combination of inflation and official depreciation rates, i.e.

$$\hat{f} = \alpha\pi + (1-\alpha)z \, .$$

Substituting this expression in [1'] we obtain

$$\ln(m) = \gamma - (\lambda + \theta\alpha)\pi - \theta(1-\alpha)z \, .$$

Rearranging,

$$\pi = \frac{1}{\lambda + \theta\alpha}[\gamma - \ln(m) - \theta(1-\alpha)z] \, .$$

Using this expression and after a little algebra we obtain

$$\dot{s} = s(z - \pi) = \frac{s}{\lambda + \theta\alpha}[(\lambda + \theta)z + \ln(m_e) + \ln(s) - \gamma] \, . \qquad [6']$$

Notice that equation [4] is independent of the assumption made on the money demand. Therefore, the dynamics associated with money demand [1'] are completely characterized by [4] and [6']. Notice that the slope of $\dot{s} = 0$ and the phase diagram do not change when considering [1'] instead of [1]. Therefore, the steady state is also unique.

In order to study the local dynamics of the system formed by equations [4] and [6'], we linearize the system around the unique steady state (m_e^{**}, s^{**}). Formally,

$$\begin{pmatrix} \dot{m}_e \\ \dot{s} \end{pmatrix} = \begin{pmatrix} -z & -\dfrac{g}{s^2} \\ \dfrac{s}{(\lambda+\theta\alpha)m_e} & \dfrac{1}{\lambda+\theta\alpha} \end{pmatrix} \begin{pmatrix} m_e - m_e^{**} \\ s - s^{**} \end{pmatrix}.$$

The determinant of the Jacobian matrix J is

$$|J| = -\frac{z}{(\lambda+\theta\alpha)} + \frac{g}{(\lambda+\theta\alpha)m_e s} = -\frac{d}{m_e(\lambda+\theta\alpha)} < 0,$$

to get the last equality we use the fact that at the steady state $\dot{m}_e = 0$ or equivalently, $z = \dfrac{d}{m_e} + \dfrac{g}{s m_e}$. Therefore, the unique steady state is also saddle-point stable using [1'] instead of [1].

Appendix 3

This appendix analyzes the causality between the rate of inflation and the black market depreciation rate by performing Granger causality tests using data from the Peruvian hyperinflation. More specifically, the VAR coefficients from a five-lag, three-variable system formed by inflation, black depreciation and official depreciation rates, i.e.

$$\pi_t = \alpha_0 + \sum_{k=1}^{5} \alpha_k \pi_{t-k} + \sum_{k=1}^{5} \beta_k f_{t-k} + \sum_{k=1}^{5} \varrho_k z_{t-k},$$

$$f_t = \gamma_0 + \sum_{k=1}^{5} \gamma_k \pi_{t-k} + \sum_{k=1}^{5} \delta_k f_{t-k} + \sum_{k=1}^{5} k_k z_{t-k},$$

$$z_t = \tau_0 + \sum_{k=1}^{5} \tau_k \pi_{t-k} + \sum_{k=1}^{5} \phi_k f_{t-k} + \sum_{k=1}^{5} \lambda_k z_{t-k},$$

is considered. To carry out this exercise we use the time series reported by Lago (1991) from July 1985 to June 1990. Augmented Dickey-Fuller tests show evidence that all three variables are integrated of order 1, i.e. $I(1)$.[17] The VAR sys-

17. Under the null hypothesis that a particular variable is $I(1)$ the augmented Dickey-Fuller statistic (ADF with drift) takes the following values: $ADF(\pi) = -1.17$, $ADF(f) = -1.13$ and $ADF(z) = -3.02$ for inflation, black depreciation and official depreciation rates, respectively. Under the null hypothesis that a particular variable is $I(2)$ the ADF with drift takes the following values $ADF(\pi) = -5.13$, $ADF(f) = -4.62$ and $ADF(z) = -5.68$. Since the critical values for a sample size of 50 reported by Fuller (1976) are $\tau_{0.01} = -3.58$ and $\tau_{0.05} = -2.93$, it can be concluded that inflation and black depreciation rates are $I(1)$ whereas for the official depreciation rate the $I(1)$ hypothesis against the stationary alternative can only be rejected at the 1% significance level.

tem is estimated using an unrestricted least square estimator (see Lütkepohl (1991: 369)). Then two null hypothesis are tested: $\beta_k = 0$ and $\gamma_k = 0$ with $k = 1, 2, \ldots, 5$. Given that the time series are $I(1)$ we use the following Wald statistic (see Lütkepohl (1991: 378-379)

$$\lambda_\omega = T(C\tilde{\alpha} - c)'(C\tilde{\Sigma}C')^{-1}(C\tilde{\alpha} - c) \rightarrow \chi^2(N),$$

where $T = 52$ is the sample size, $\tilde{\alpha}$ is the unconstrained least square estimator which is a $(pr^2 + 3)$x1 vector constructed by stacking the coefficients of the VAR system as follows[18]

$$\tilde{\alpha} = (\alpha_0, \gamma_0, \tau_0, \alpha_1, \gamma_1, \tau_1, \beta_1, \delta_1, \phi_1, \varrho_1, k_1, \lambda_1, \alpha_2, \gamma_2, \ldots, \varrho_5, k_5, \lambda_5).$$

$\tilde{\Sigma}$ is a consistent estimator of the variance-covariance matrix of $\tilde{\alpha}$. To calculate λ_ω the null linear hypothesis has to be written in matrix form as $C\tilde{\alpha} = c$. For H_0: $\beta_k = 0$ the statistic λ_ω takes the value 7.912. For H_0: $\gamma_k = 0$ the statistic λ_ω takes the value 30.545. Since $\chi_{0.05}(5) = 11.070$, it can be concluded that the data do not reject the possibility that the black depreciation rate does not cause the rate of inflation, but the data do reject the possibility that inflation does not cause the black depreciation rate.[19] In summary, the empirical evidence from Peruvian hyperinflation supports the quantity theory of money approach followed in this paper, that is, causality runs from inflation to black depreciation rate.

Table A.1. Granger causality tests for alternative VAR orders.

p	λ_ω for $\beta_k = 0$	λ_ω for $\gamma_k = 0$
1	2.2715	1.8342
2	2.6329	7.1890
3	2.8104	29.6645
4	4.8163	30.3171
5	7.9116	30.5446
6	8.2256	79.1872
7	9.7147	74.7034

18. p, r and N denote the number of lags, the dimension of the VAR system and the number of linear restrictions imposed by the null hypothesis, respectively, in this application, we have $p = 5$ $r = 3$ and $N = 5$.
19. Table A.1 shows that the results of the Granger causality tests are robust to the choice of VAR order. In particular, we have checked that the results of the Granger causality tests given above hold for the following choices of $p = 2, 3, 4, 5, 6, 7$. The reason why we have performed the tests for alternative VAR orders is that the usual criteria for VAR order selection are not conclusive in this particular case.

Appendix 4

Taking into account the empirical evidence from Peruvian hyperinflation reported by Lago (1991), this appendix explains how the parameter values used in numerical simulations were chosen. Since per capita endowment (y) is distributed in our model between private consumption (x) and government consumption, normalizing y to one, we assume that the government expenditure is 20% of the endowment. This value is close to the average of government spending as a percentage of GDP during Peruvian hyperinflation as reported by Lago (1991, Table 9.8). Therefore, in our simple model that figure implies that domestic private consumption as a percentage of GDP during Peruvian hyperinflation was around 75% and the trade balance surplus (exports minus imports) was 5% of GDP (see Lago (1991 Table 9.6 and 9.8)). The utility of consumption is assumed to be logarithmic, so we have that $\lambda = \frac{1}{0.75} = 1.33$. The value reported by Lago (1991) for debt service payments on public foreign debt as percentage of GDP is around 2%.[20] z is chosen equal to 0.14. This value is roughly the value taken by inflation during the initial periods of Peruvian hyperinflation from January to July 1988 (see Lago (1991: 304)). Since the steady state values of inflation and depreciation rates of the official and black markets are all the same, by choosing that value for z we are conjecturing that the Peruvian economy was close to the steady state during that period.

We next perform a sensibility analysis to see whether alternative values for the parameters give reasonable values of the rate of inflation at the hyperinflationary plateau. As discussed in Section 3, the rate of inflation at the plateau does not depend on the values chosen for d and z. Moreover, γ is a scale parameter measuring units of the money demand. Therefore, the sensibility analysis, which is relevant, is to analyze how alternative values for g and λ affect the rate of inflation at the hyperinflationary plateau. Table A.2 displays the results. The values of the rate of inflation at the plateau are of reasonable size when comparing with the peaks of inflation observed during hyperinflationary episodes. Moreover, as discussed in Section 3, the rate of inflation at the hyperinflationary plateau (that is, the rate of inflation at the high-inflation steady state in the *degenerated* system) is inversely related to the size of government spending g for a given value of λ.

Table A.2. Sensibility Analysis for the rate of inflation at the hyperinflationary plateau

	g = 0.005	g = 0.2	g = 0.3
λ = 1.11	5.058	3.472	2.964
λ = 1.33	4.057	2.712	2.275
λ = 1.54	3.386	2.209	1.811

20. This figure is obtained by calculating (debt service/exports) times (exports/GDP) from the figures reported in Lago (1991 Tables 9.7 and 9.6), respectively.

References

ABEL, A.; DORNBUSCH, R.; HUIZINGA, J.; MARCUS A. (1979). «Money Demand during Hyperinflation». *Journal of Monetary Economics*, 5, 97-104.

BRESCIANI-TURRONI, C. (1937). *The Economics of Inflation*. London: George Allen and Unwin.

BRUNO, M. (1989). «Econometrics and the Design of Economic Reform». *Econometrica*, 57, 275-306.

BRUNO, M.; DI TELLA, G.; DORNBUSCH, R.; FISCHER, S. (1988). *Inflation Stabilization. The Experience of Israel, Argentina, Brazil and Mexico*. Cambridge, MA: MIT Press.

BRUNO, M.; FISCHER, S. (1990). «Seigniorage, Operating Rules, and the High-Inflation Trap». *Quarterly Journal of Economics*, 105, 353-374.

BRUNO, M.; FISCHER, S.; HELPMAN, E.; LIVIATAN, N. (1991). *Lessons of Economic Stabilization and its Aftermath*. Cambridge, MA: MIT Press.

CAGAN, P. (1956). «The Monetary Dynamics of Hyperinflation». In Milton Friedman (ed.). *Studies in the Quantity Theory*. Chicago: University of Chicago Press, p. 25-117.

CALVO, G.; LEIDERMAN, L. (1992). «Optimal Inflation Tax under Precommitment: Theory and Evidence». *American Economic Review*, 82, 174-194.

CUKIERMAN, A. (1988). «Rapid Inflation - Deliberate Policy or Miscalculation?» *Carnegie-Rochester Conference on Public Policy*, 29, 11-76.

CULBERTSON, W. (1975). «Purchasing Power Parity and Black-Market Exchange Rates». *Economic Inquiry*, 13, 287-296.

EVANS, J.; YARROW, G. (1981). «Some Implications of Alternative Expectations Hypothesis in the Monetary Analysis of Hyperinflation». *Oxford Economic Papers*, 33, 61-80.

FEENSTRA, R. (1986). «Functional Equivalence between Liquidity Costs and the Utility of Money». *Journal of Monetary Economics*, 17, 271-291.

FULLER, W. (1976). *Introduction to Statistical Time Series*. USA: Wiley and Sons.

GEORGE, D.; OXLEY, L. (1991). «Fixed Money Growth Rules and the Rate of Inflation: Global versus Local Dynamics». *Scottish Journal of Political Economy*, 38, 209-226.

KHARAS, H.; PINTO, B. (1989). «Exchange Rate Rules, Black Market Premia and Fiscal Deficits: The Bolivian Hyperinflation». *Review of Economic Studies*, 56, 435-448.

LAGO, R. (1991). «The Illusion of Pursuing Redistribution Through Macropolicy: Peru's Heterodox Experience, 1985-1990». In Rudiger Dornbusch; Sebastian Edwards (eds.). *The Macroeconomics of Populism in Latin America*. Chicago: University of Chicago Press, p. 263-323.

LÜTKEPOHL, H. (1991). *Introduction to Multiple Time Series Analysis*. Berlin: Springer-Verlag.

MONDINO, G.; STURZENEGGER, F.; TOMMASI, M. (1996). «Recurrent High Inflation and Stabilization: A Dynamic Game». *International Economic Review*, 37, 981-996.

NICOLINI, J.; MARCET, A. (1996). «Recurrent Hyperinflations and Learning». Universidad Torcuato Di Tella, Working Paper 29.

PINTO, B. (1991). «Black Markets for Foreign Exchange, Real Exchange Rates and Inflation». *Journal of International Economics*, 30, 121-135.

SACHS, J.; LARRAIN, F. (1993). *Macroeconomics in the Global Economy*. USA: Prentice Hall.

VÁZQUEZ, J. (1995). «The Relative Importance of Inflation and Currency Depreciation in the Demand for Money: An Application of the Estimation by Simulation Method to the German Hyperinflation». *Investigaciones Económicas*, 19, 269-289.

— (1998). «How High Can Inflation Get During Hyperinflation? A Transaction Cost Demand for Money Approach». *European Journal of Political Economy*, 14, 433-451.