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Ex-post Real Interest Rates versus Ex-ante Real Rates: a CCAPM Approach

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Abstract

We use an integrated framework based on the CCAPM to jointly estimate ex-ante real interest rates, (bounds on) inflation risk premia and (bounds on) agents' inflation expectation errors. Using the Spanish economy as a natural case study, we illustrate that, for different preference specifications, 1-year ex-ante real interest rates exhibit a very low correlation with the 1-year ex-post rate. According to our results, the difference between both real rates seems to be mainly explained in terms of agents' inflation expectation errors, while the inflation premia play a minor role.

Key words: Inflation expectation errors, inflation risk premia, preference specifications.
JEL numbers: E43, G12.

Resumen

Usando el marco proporcionado por el modelo CCAPM podemos estimar conjuntamente tipos de interés reales *ex-ante*, y cotas a las expectativas de inflación y primas de riesgo inflacionista. Utilizamos la economía española para ilustrar la baja correlación existente entre los tipos reales *ex-ante* a un año y sus correspondientes tipos *ex-post*. Nuestros resultados confirman la importancia de las expectativas de inflación para explicar tales diferencias.

Palabras clave: errores de expectativas de inflación, prima de riesgo inflacionista, preferencias.

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1. Introduction

Real interest rates are a critical variable in economics. They measure the real cost of capital, thus playing a crucial role in determining long-run output growth. Moreover, real as opposed to nominal interest rates determine agents' consumption and investment decisions. Thus, when central banks steer (short-term) nominal interest rates in order to attain monetary policy targets, disentangling the relationships between nominal interest rates, real interest rates and inflation expectations is crucial to understand the relative importance of different channels in the monetary policy transmission mechanism.

Unfortunately, real interest rates are non-observable and are usually proxied by the so-called ex-post real interest rates, i.e., the difference between the nominal interest rate and the ex-post observed inflation rate. As is well known, however, ex-post rates include two disturbing components which can render them a misleading proxy for non-observable ex-ante real interest rates: inflation risk premia and agents' inflation expectation errors. Like ex-ante real interest rates, these two variables are also non-observable.

There are reasons to think that both disturbances are probably negligible. First, inflation premia can hardly be relevant if the inflation rate is not very volatile. And second, if agents are rational when forming their inflation expectations, the expectation error should be zero on average. Although it is still an open question, this view has been recently challenged in the literature, particularly in relation to the magnitude of the inflation expectation error. Thns, a series of papers have found that, due to informational or to (monetary policy) credibility problems, inflation rates can be successfully characterised by switching-regime models à la Hamilton, not only in high-inflation countries like Argentina, Israel or Mexico (see Kaminsky and Leiderman, 1996) but also in countries whose inflation rates are lower and more stable like the US (Evans and Lewis, 1995) or Canada (Bank of Canada, 1996). These switching-regime models produce inflation expectation errors which have zero-mean ex-ante but, ex-post, can show a non-zero mean. Similarly, according to King (1996) if agents do not immediately learn about central bank behaviour, disinflationary processes will probably be characterised by inflation targets (and, therefore, by actual inflation) below agents' inflation expectations. Lasting inflation expectation errors are also predicted by models à la Backus-Driffill (1985) where central bankers face credibility problems and need time to build their anti-inflationary reputation.

In this paper we use an integrated framework that provides a joint estimate of the ex-ante real interest rate, (bounds on) inflation premia and (bounds on) inflation expectation errors, on the basis of the Consumption Capital Asset Pricing Model (CCAPM). This framework allows, first, for a comparison between ex-ante and ex-post real interest rates to analyse to what extent the latter are a good proxy for the former. Second, relative to other alternative methods that estimate ex-ante real interest rates, this approach also offers a simultaneous estimate of inflation premia and inflation expectation errors, thus providing an explanation of the discrepancies between both interest rates. This ability to analyse the three variables jointly is one of the advantages of this method. An additional advantage of our approach lies in its data requirements. All the results are obtained using information on non-durable consumption alone which, thus, acts as a sufficient statistic for the three non-observable variables.

Most of the existing empirical evidence is not generally favourable to the CCAPM (see, for instance, Sentana 1993). Yet such evidence, is usually based on a joint test of the CCAPM and the isoelastic preference assumption for data on the US economy. When other preference specifications are considered, however, the evidence is less clear-cut (see Cochrane and Hansen, 1992; and Campbell and Cochrane, 1995). The same applies when other economies are analysed (see Ayuso, 1996 for Spanish evidence, or Bakshi and Naka, 1997 for Japanese data).

In this paper, we address this issue by considering different preference specifications. Thus, to obtain the non-observable ex-ante real interest, we generate different ex-ante rates under a wide range for the most important features (say, deep parameters) in agents' utility functions: time preference, risk aversion, intertemporal substitution, habits and keeping-up-with-the-Joneses effects. As usual in any calibration analysis, we then look at the correlation of the ex-post real rates and these (model-) generated ex-ante real rates. This allows us to analyse to what extent ex-post real interest rates are a good proxy for our model-depending ex-ante ones. Moreover, for each preference specification we also build the corresponding bounds on inflation expectations and risk promia. This allows us to discuss the sources of discrepancy between the ex-post real rates and the ex-ante rates.

We choose the Spanish economy as our case study. As can be seen from Figure 1a, the Spanish inflation rate has showed a notably changing pattern in the last 20 years. In 1979, the inflation rate was higher than 15%. By 1996, it had decreased by more than 10 points although there has been some episodes of growing inflation as that around 1988. This quick disinflation process, sprinkled with some upward movements, opens the door to the existence of non-negligible inflation premia andlor inflation expectation errors. Regarding the empirical evidence on the CCAPM performance for Spanish data, Ayuso (1996) assumed isoelastic preferences and focusing on the inflation uncertainty as the main source of risk, he found results favourable to the CCAPM. He used monthly data spanning the period 1985-1995. On the contrary, using quarterly data from 1972 to 1994, Rodríguez (1997) rejects the time-separability assumption and points towards habits as a more appropriate specification. On the other hand, Rubio (1996) showed that the conclusions on the CCAPM performance for the Spanish economy based on the Hansen-Jagannathan (1991) bounds are rather sensitive to the uncertainty involved in the estimates of such bounds. Therefore, our approach regarding agents' utility functions seems to be particularly appropriate in this case.

The paper is organised as follows. Section 2 briefly states the main theoretical relationships among ex-ante real interest rates, ex-post real interest rates, inflation risk premium and inflation expectation errors, in the CCAPM framework. Section 3 presents the empirical results, which show that Spanish ex-post real interest rates are a bad proxy for the ex-ante rates, the correlation between them never reaching .50. This different behaviour seems to be mainly due to the existence of inflation expectation errors, inflation premia playing a minor role. Finally, Section 4 summarises the main conclusions of the paper.

2. The theoretical relationship between nominal interest rates, real interest rates, inflation premia and inflation expectations

2.1. The model

As stated before, ex-post real interest rates are obtained as the difference between nominal interest rates and the ex-post observed inflation rate. In this Section, we obtain the theoretical relationship between nominal interest rates, real interest rates, inflation promia and inflation expectations in the framework of the conventional intertemporal frictionless asset pricing model originally proposed by Lucas (1978).

Thus, suppose that the economy is populated by many households having identical preferences about future consumption. Households choose the composition of their portfolios that maximise the expected utility of the infinite path of future contingent consumption and their only source of wealth is, precisely, the return on this portfolio. Financial markets are assumed to be frictionless and financial assets in this economy mature one period ahead.

Under these conditions, at each t, households solve the following problem

$$Max \;\; E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k})$$

subject to the following set of restrictions

$$C_s + \sum_{j=1}^{J} W_s^j = \sum_{j=1}^{J} \frac{P_{s-1}}{P_s} W_{s-1}^j i_{s-1}^j; \quad s \geq t.$$

where β is a parameter of time preference; C_s is the real consumption of the household at s; W_s^j is the real amount invested at s in the financial asset j, whose nominal return —including the principal returned— is i_s^j; P_s is the general level of prices at s; and, lastly, J is the number of assets in this economy.

To the purpose of this paper, we are interested in two of the J assets in the economy. First, we consider a 1-period default-free zero-coupon bond. Its riskless *nominal* return —i.e. known at t— is denoted by i_t. Second, we also consider a perfectly indexed 1-period default-free zero-coupon bond. Its riskless *real* return, known at t, is denoted by r_t. The latter can be understood as a «real bond» that returns units of consumption at its maturity date. It is straightforward to see that the first-order conditions of the household problem yield the following set of equilibrinm equations relating zero-coupon bond interest rates (nominal and real), consumption and inflation expectations:

$$E_t[MRS_{t+1}]r_t = 1, \quad \forall t. \tag{1.a}$$

$$E_t[MRS_{t+1}\pi_{t+1}^{-1}]i_t = 1, \quad \forall t. \tag{1.b}$$

where

$$MRS_{t+1} = \beta\frac{\partial U(C_{t+1})/\partial C_{t+1}}{\partial U(C_t)/C_t}, \;\; \pi_{t+1} = \frac{P_{t+1}}{P_t}, \;\; \forall t,$$

and E_t represents the expectation operator conditional on information available at t.

Under the existence of a representative household in the economy, these first-order conditions are satisfied for per-capita consurnption.[1] Thus, equations [1.a] and [1.b] impose statistical restrictions on the comovements between interest rates (nominal and real), expected inflation and per-capita consumption. Equations [1.a] and [1.b] yield:

$$\frac{1}{i_t} = \frac{1}{r_t}E_t[\pi_{t+1}^{-1}] + COV_t[MRS_{t+1}\pi_{t+1}^{-1}], \quad \forall t. \tag{2}$$

Thus, this model provides a general framework to relate nominal and real interest rates which is broadly consistent with the well-known Fisher equation. Equation [2] states that the nominal interest rate is positively related to the real interest rate and the expected inflation as in the conventional Fisher equation. But, the conditional covariance between the ratio of marginal utilities and inflation also enters into this relationship. In fact, this term can be interpreted as an inflation risk premium: if the covariance term is negative (positive), the nominal bonds provide a poor (good) hedge against unanticipated consumption changes-and households require higher (lower) nominal interest rates.

This inflation premium acts as a first potential wedge between ex-ante and ex-post observed real rates. But even if the inflation premium is nil, there might still be another wedge if agents do not perfectly forecast the future inflation. The difference between the expected inflation rate and the observed one, i.e. the expectation error, will also contaminate the ex-post real interest rate.

In the following Sections we exploit equations [1.a], [1.b] and [2] to estimate ex-ante real interest rates, inflation premia and inflation expectation errors.

2.1.a. Real interest rates

Real interest rates can be easily estimated according to equation [1.a]:

$$r_t = \frac{1}{E_t(MRS_{t+1})}, \quad \forall t \tag{3}$$

1. Notice that allowing for many homogeneous agents is a sufficient but not a necessary condition for the representative agent assumption. A necessary condition is the existence of complete markets (see Constantinides, 1982).

Observe that, in order to estimate *ex-ante* real interest rates, we only need information on the stochastic discount factor MRS_{t+1}, i.e. information on preferences and per-capita consumption.

2.1.b. Bounds for inflation expectations and for inflation premia

Estimating inflation expectations and inflation premia is not so straightforward. Nevertheless, it is still possible to derive bounds for the inflation expectations through the implied bounds for the risk premium term in equation [2], as in Ireland (1996).

Since the risk premium is a covariance, the following relation holds:

$$\varrho_t[MRS_{t+1}\pi_{t+1}^{-1}] = \frac{Cov_t[MRS_{t+1}\pi_{t+1}^{-1}]}{\sigma_t[MRS_{t+1}]\sigma_t[\pi_{t+1}^{-1}]}$$

where ϱ_t and σ_t denote conditional correlation and conditional standard deviation, respectively. But the correlation coefficient must lie inside the interval [–1, 1]. Thus:

$$\left|Cov_t[MRS_{t+1}\pi_{t+1}^{-1}]\right| \leq \sigma_t(MRS_{t+1})\sigma_t(\pi_{t+1}^{-1})\,. \qquad [4]$$

Under an additional assumption on inflation volatility, namely:

$$\sigma_t[\pi_{t+1}^{-1}] \leq E_t[\pi_{t+1}^{-1}]\,. \qquad [5]$$

we can eliminate inflation from equation [4].[2] Thus, equations [1.a], [2], [4] and [5] yield

$$\begin{aligned} i_t\{E_t[MRS_{t+1}] - \sigma_t[MRS_{t+1}]\} &\leq \left\{E_t\left[\frac{1}{\pi_{t+1}}\right]\right\}^{-1} \leq \\ &\leq \{E_t([MRS_{t+1}] + \sigma_t[MRS_{t+1}])\}i_t \end{aligned} \qquad [6]$$

Since $\{E_t(1/\pi_{t+1})\}^{-1} \approx E_t(\pi_{t+1})$, equation [6] defines bounds on expected inflation that can be compared with the observed inflation, thus providing bounds on the expectation errors. Observe that, as in the case of the real interest rates, all we need to estimate those bounds is data on per capita consumption and agents' preferences. Moreover, the width of the band for expected inflation also provides information on the magnitude of the inflation premium. Observe that by using equation [3], equation [6] can be rewritten as

2. Estimates of an AR-ARCH model for Spanish data show that the conditional standard deviation in equation [5] has never been above 0.01 times the corresponding conditional expectation during the period we analyse. The assumption, thus, does not seem to be too restrictive.

$$\frac{i_t}{r_t} - i_t\ \sigma_t[MRS_{t+1}] \leq \left\{ E_t\left[\frac{1}{\pi_{t+1}}\right]\right\}^{-1} \leq$$

$$\leq \frac{i_t}{r_t} + i_t\ \sigma_t[MRS_{t+1}]$$

and i_t/r_t is the inflation expectation estimate provided the inflation premium is zero. Therefore, equation [6] yields a band for the inflation expectations centred around the inflation expectation under risk neutrality, (half) the width of that band being the (absolute) maximum value for the inflation premium.

2.2. Investor preferences

In the previous Section we have shown that real interest rates and bounds for inflation expectations and inflation premia can be estimated from data on the marginal rate of substitution. But $MRS_{t,t+1}$ depends on consumption dynamics through the household preference specifications. In this Section we consider four different utility functions commonly used in the asset-pricing literature, that will be used later in the empirical part of the paper.[3] The first one is the well-known isoelastic utility function. The other three functions encompass the isoelastic as a particular case.

2.2.a. Constant relative risk aversion: the isoelastic utility function

The most usual utility function in the financial literature is the isoelastic utility function:

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}, \quad \gamma \neq 1\ .$$

As usual, $\gamma = 1$ implies the log utility function. In this case, preferences are not time-dependent and γ measures both the household degree of (constant) relative risk aversion and the inverse of the (constant) elasticity of intertemporal substitution between future and current consumption. It is easy to prove that, in this case

$$MRS_{t+1} = \beta(g_{t+1})^{-\gamma} \qquad [7]$$

where $g_{t+1} = C_{t+1}/C_t$

2.2.b. Disentangling between risk aversion and intertemporal substitution: the generalised isoelastic preferences

As noted before, the isoelastic utility function restricts the coefficient of relative risk aversion to the inverse of the elasticity of intertemporal substitution. The generalized isoelastic preferences analysed in Epstein & Zin (1989); Weil (1990)

3. See, for instance, Cochrane and Hansen (1992) and Kocherlakota (1996).

allow such an assumption to be relaxed. These preferences can be represented recursively as

$$U_t = \left\{ C_t^{1-\varrho} + \beta [E_t(U_{t+1}^{1-\gamma})]^{\frac{1-\varrho}{1-\gamma}} \right\}^{\frac{1}{1-\varrho}}$$

where $U_t = U(C_t)$, γ measures, as before, (constant) household relative risk aversion and $1/\varrho$ is the (constant) elasticity of substitution between future and current consumption. Observe that this expression reduces to the isoelastic case if $\gamma = \varrho$.

The existence of time dependence in preferences induces higher dynamics in the marginal rate of substitution. Following Kocherlakota (1990), it takes the following form:

$$MRS_{t+1} = \beta\, [E_t(U_{t+1}^{1-\gamma})]^{\frac{\gamma-\varrho}{1-\gamma}} U_{t+1}^{\varrho-\gamma}\; g_{t+1}^{-\varrho} \qquad [8.a]$$

Notice that MRS_{t+1} depends on non-observable variables (U_{t+1} and its expected value). However, if consumption growth evolves over time independently and identically distributed (say, iid), then utility at t can be expressed as a time-independent constant proportion of consumption growth at t and, therefore, equation [8.a] simplifies to

$$MRS_{t+1} = \beta\, [E_t(g_{t+1}^{1-\gamma})]^{\frac{\gamma-\varrho}{1-\gamma}} g_{t+1}^{-\gamma} \qquad [8.b]$$

where all of the variables are (ex-post) observable.

Strictly speaking, the iid assumption is usually rejected (see, for instance, Deaton, 1993). However, even being formally rejected, the assumption can still be useful if the induced error is empirically small. In that these preferences encompass the isoelastic case, we can evaluate such an error simply by comparing the different results provided by equation [7] and equation [8.b] constraining ϱ to be equal to γ. Should the error be small, we could still exploit [8.b] despite the formal rejection of the iid assumption.

2.2.c. Time non-separabilities: habit formation or durability

A different way of introducing time dependence in preferences is in terms of the existence of habits or durability in household decisions. Following Constantinides (1990) and Heaton (1993), a parsimonious way of capturing these effects is to specify the following utility function:

$$U_t = \frac{(C_t - \lambda C_{t-1})^{1-\gamma}}{1-\gamma}, \qquad \gamma \neq 1$$

where λ measures the degree of habits or durability and, for U_t to be well defined, $\lambda > Min\{g_t\}$ *is* assumed. Notice that $\lambda > 0$ implies that current utility is a negative function of past consumption, so there are habits present (it takes more consumption today to make an investor happier if he consumed more yesterday). We can generate durability by making $\lambda > 0$ and, by setting $\lambda = 0$, that expression reduces to the isoelastic utility function. Notice also that the degree of relative risk aversion is not constant but a function of γ, λ and g_t

After some algebraic manipulation we can obtain the corresponding marginal rate of substitution:

$$MRS_{t+1} = \beta \frac{(g_{t+1}g_t - \lambda g_t)^{-\gamma} - \lambda\beta(g_{t+1}g_t)^{-\gamma}(g_{t+2}-\lambda)^{-\gamma}}{(g_t-\lambda)^{-\gamma} - \lambda\beta g_t^{-\gamma}(g_{t+1}-\lambda)^{-\gamma}} \quad [9]$$

2.2.d. Relative consumption effects

The last extension of household preferences we consider is the presence of externalities in the utility functions. In particular, following Abel (1990) or Galí (1994), household preferences do not only depend on own consumption but also on the aggregate (economy-wide) level of consumption. A simple way of capturing this aspect of preferences has been proposed by Abel (1990), who assumes the following utility function:

$$U_t = \frac{c_t^{1-\gamma}}{1-\gamma} C_{t-1}^{\phi}$$

where c_t is individual consumption at t, as oppo sed to per-capita consumption in the economy C_t. The parameter ϕ measures the dependence of the individual utility on the general level of consumption in the economy. A positive value of ϕ implies that the individual is altruistic in that the more the society consumes the better off he is. On the contrary, a negative value means that the household is invidious. Finally, the utility function reduces to the isoelastic case when $\phi = 0$.

Evaluated in equilibrium, where individual and per-capita consumption coincide, the marginal rate of substitution takes the following form:

$$MRS_{t+1} = \beta\, [g_{t+1}]^{-\gamma}\, [g_t]^{\phi} \quad [10]$$

3. Empirical results

As previously stated, the Spanish economy offers a good example of a markedly changing pattern for the inflation rate, thus being a natural case study. The strategy we propose to analyse the potential error when ex-post real interest rates are used instead of ex-ante real interest rates is the following. As a starting point, given that the isoelastic case can always be seen as a particular case of the other specifications, we use the estimates for the isoelastic case reported by Ayuso (1996). In this paper, moreover, inflation uncertainty is explicitly accounted for, which is particularly interesting given our goals. Thus, given λ and β estimates and per-capita consumption data, we compute the ex-post marginal rate of substitution corresponding to the isoelastic utility function.[4] Then, we estimate an ARMA univariate model for that (ex-post) observed marginal rate of substitution. This ARMA model provides estimates of $E_t(MRS_{t+1})$. Given these estimates, equation [3] is used to recover the ex-ante real interest rate.

Next, we depart from this benchmark case by considering different values for γ in equation [7] and for the remaining parameters in equations [8.b] —the inverse of the elasticity of intertemporal substitution ϱ—, [9] —the habit parameter λ— and [10] the relative consumption effect ϕ.[5] Again, in each case, a new ex-post MRS_{t+1}, i.e. a new discount factor in terms of Hansen and Jagannathan (1991), is built and a new univariate model is estimated.[6] Thus we obtain a sensible range for ex-ante real interest rate under different preference specifications and their corresponding correlations with the ex-post real rate.

As commented in the Introduction, our main purpose is to analyse whether the ex-post real rate is a good proxy for the non-observable ex-ante real rate. Because the correlation between two series constitutes a simple basic statistics to check whether both series behave in a sirnilar way, we compute the correlation between the ex-post real rate and any of our model-generated ex-ante real rates. Should the former be a good proxy, this correlation would be high for, at least, any of our parameterizations.[7]

Finally, in order to analyse whether (potential) differences in the pattern of ex-ante and ex-post real rates are due to inflation premia or to inflation expectation errors, we assume that the conditional standard deviation of the marginal rate of substitution is constant —as will be cornmented later, we use quarterly data, for which homoscedasticity is not a too restrictive assumption. Thus, each univariate model for the ex post marginal rate of substitution also provides an estimate of the conditional standard deviation $\sigma_t(MRS_{t+1})$ which is used in equation [4] to build bounds on inflation expectations.

Regarding data, Blinder (1994) and Kashyap (1995) show evidence on short-run price stickiness. According to them, overall evidence on price-setting suggests that price-setters move prices once a year. Therefore, 1 year is considered as the empirical definition of the theoretical concept of «1-period». Therefore, we aim at estimating 1-year real interest rates and at bounding inflation expectation errors and inflation premia to such horizon. However, just to increase the number of observations we exploit the quarterly per-capita private non-durable consumption in Spain from 1970: I to 1995: IV, provided by the *Instituto Nacional de Estadística* (see Figure 1b).[8] Moreover, we also use the

4. Contrary to Ayuso (1996) we do not make the log transformation of the MRS which would not be possible for equations [8.b] or [9].
5. Notice that since β is a scale factor in MRS_{t+1}, it does not affect the correlation between ex-post and ex-ante real interest rates.
6. Notice that in equation [8.b] $E_t(g_{t+1}^{1-\gamma})$ is not observable. In order to prevent it from collapsing to equation [7], we use the unconditional mean instead of the observed $g_{t+1}^{1-\gamma}$.
7. Provided, of course, that the assumptions behind our theoretical approach hold. On the other hand, it can be easily shown that the differences between the means of both series, that are not taken into account when we look at correlations, can be corrected by changing β.
8. We face a data overlapping problem that is solved by including the corresponding moving average terms in the univariate models.

Figure 1a. Spanish inflation rate (%).

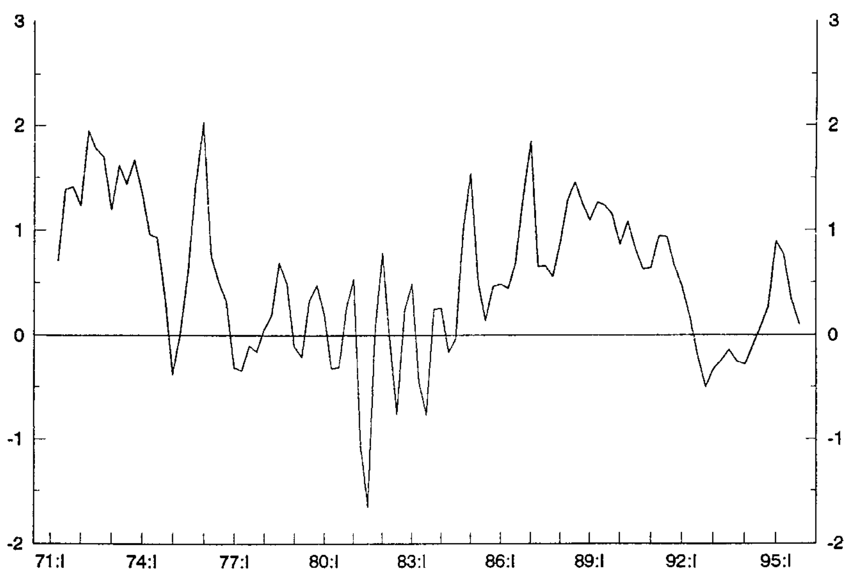


Figure 1b. Spanish (quarterly log) consumption growth (%).

Summary statistics: Mean = .51%
Std. Dev. = .71%
1st order autocorrelation (s.e.) = .73 (.10)
4th order autocorrelation (s.e.) = .47 (.10)

1-year nominal interest in the domestic interbank market. Although this series is the longest available, it only covers the period 1979: III - 1995: IV. Therefore, the whole period is used to estimate the univariate model for MRS_{t+1}, but real interest rates and bounds are estimated only since 1979: III.

3.1. Ex-ante real interest rates

As commented before, in a recent paper, Ayuso (1996) followed Hansen and Singleton (1982) and estimated a CCAPM with isoelastic preferences for monthly data on the Spanish economy spanning the period 1985-1995. In that paper, inflation is the only source of risk, what is a desirable feature given our main goal. According to his results, the data do not reject that specification of the model, γ and β are estimated to be equal to .216 and .996 —for monthly data, i.e., .984 for quarterly data—, respectively. Figure 2.a shows the corresponding 1-year ex-ante real interest rate, as well as the *ex-post* rate.[9] As can be seen, both series look rather different, the correlation between them only being .381 (see Table 1). The ex-post interest rate is higher (more than 70 basis points in average) and very much volatile. Moreover, and more importantly, the difference between them is far from the usual random zero-mean case. Thus, it remains quantitatively important (more than 2 percentage points) during long periods.

This low degree of correlation between ex-ante and ex-post real interest might be due to the chosen value for γ. Thus, before considering a more general class of preferences, a higher degree of risk aversion is contemplated. Table 1 shows how that correlation changes when the risk aversion degree increases to 1, 2 or 5.[10] As can be seen, considering those different degrees of risk aversion provides a slightly lower correlation between ex-ante and ex-post 1-year real interest rates. Moreover, as Figure 2.a shows, increasing γ beyond 1 notably multiply the volatility of the ex-ante real rate and boosts its average —the usual effects highlighted by the well known riskless rate puzzle.

Regarding a more general class of preferences, estimating the relevant parameters is beyond the scope of this paper.[11] We prefer to develop the following sensitivity analysis. Given that the isoelastic utility function can always be seen as a particular case of any of the remaining preferences, we build upon a sensible range for ϱ, λ and γ around the isoelastic preferences. In all cases, the values for β and γ are those in Ayuso (1996), our benchmark case.[12] The parameter ranges are chosen as follows.

9. Here, as for the remaining univariate models, 4 autoregressive terms, jointly with the 3 moving average terms due to the overlapping problem, provide a good fit of the data, never showing significant residual autocorrelation and providing R^2 between 40% and 95%.
10. See also Alonso and Ayuso (1996) for a survey on y estimates in Spain in a CAPM framework.
11. This goal is pursued by Rodríguez (1997).
12. There are arguments that support the view that γ estimates under isoelastic preferences should be taken as estimates of the degree of relative risk aversion better than as estimates of the inverse of the elasticity of intertemporal substitution. See, for instance, Kocherlakota (1990).

Table 1. Correlations between ex-ante 1-year real interest rates and the ex-post 1-year real rate.

Preference	Specification	Correlation
Isoelastic	$\gamma =$ .216	.381
	1	.380
	2	.379
	5	.378
Epstein-Zin $\sigma = \gamma\ (1+t)$	$t =$ -.95	.329
	-.05	.329
	.05	.329
	.95	.329
Habits	$\lambda =$ -.8	.376
	-.2	.461
	.2	.495
	.8	-.274
Relative Consumption Effects	$\phi =$ -.75	.443
	-.25	.423
	.25	.347
	.75	-.482

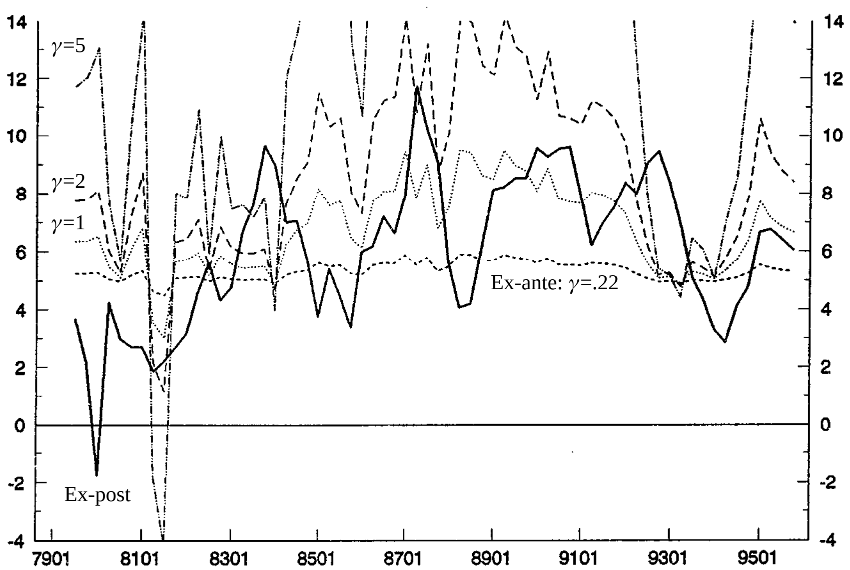
First, ϱ is allowed to vary +5% and +95% around its value under the isoelastic assumption -i.e. around the benchmark value of .216.[13] This implies that the elasticity of intertemporal substitution varies between 2 and more than 50. Second, as commented in Section 2.2.c., λ has to be below the minimum rate of consumption growth which, in our case, is about .9. Thus, four different values for λ are considered, allowing for both, habits and durability:[14] .8, .2, -.2 and -.8. Finally, regarding ϕ we bound it in such a way that the relative weight of the social consumption in the individual utility function is lower than that of the own individual consumption. Thus, we have considered four alternative values: .75, .25, -.25 and -.75.

Table 1 shows the different correlations between ex-ante and ex-post real interest rates. Figures 2.b, 2.c and 2.d show the 1-year ex-ante real interest rates corresponding to each preference specification and each parameter value.[15] A number of remarks are in order.

First of all, Table 1 clearly shows that none of the preference specifications is able to significantly improve the results of our benchmark. Thus, the maximum correlation is .495 and corresponds to presence of a small degree of habits (λ = .2)

13. That is, $\varrho = \gamma(1+t)$, t = -.95, -.05, .05 and .95.
14. Strictly speaking, we are using non-durable consumption data, thus durability should not be considered. However, in order to keep symmetry, we also allow for negative λ values. Intermediate values for λ have also be considered without any change in our conclusions.
15. In a previous version of the paper, circulated as Banco de España Working Paper 9633, a wider range of parameters is considered, without any major change in the final conclusions.

a) Isoelastic preferences.



b) Epstein-Zin preferences.
$\varrho = \gamma(1+t)$

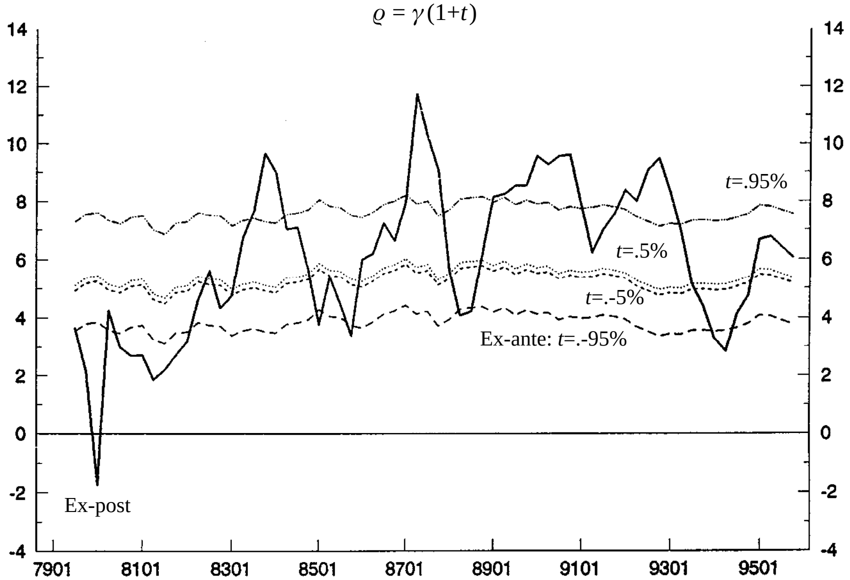
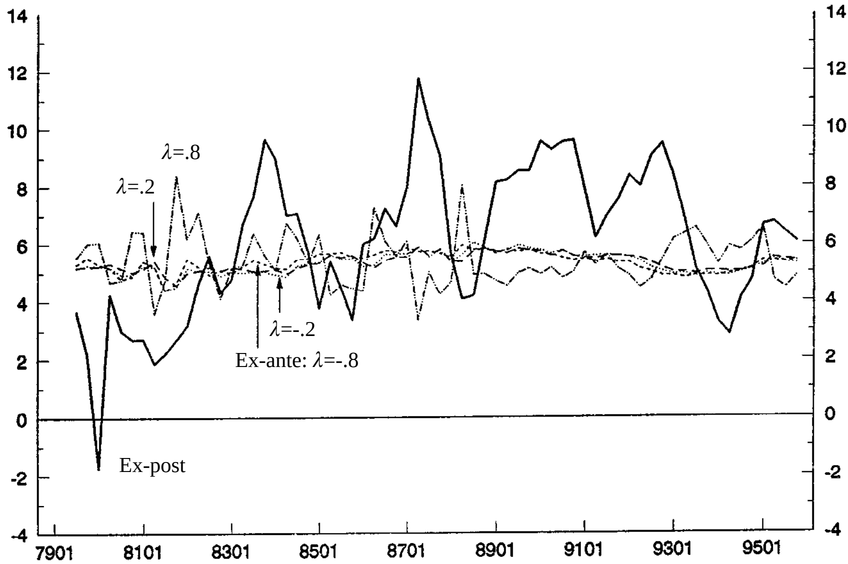


Figure 2. 1-year real interest rates.

c) Habits.



d) Relative consumption effects.

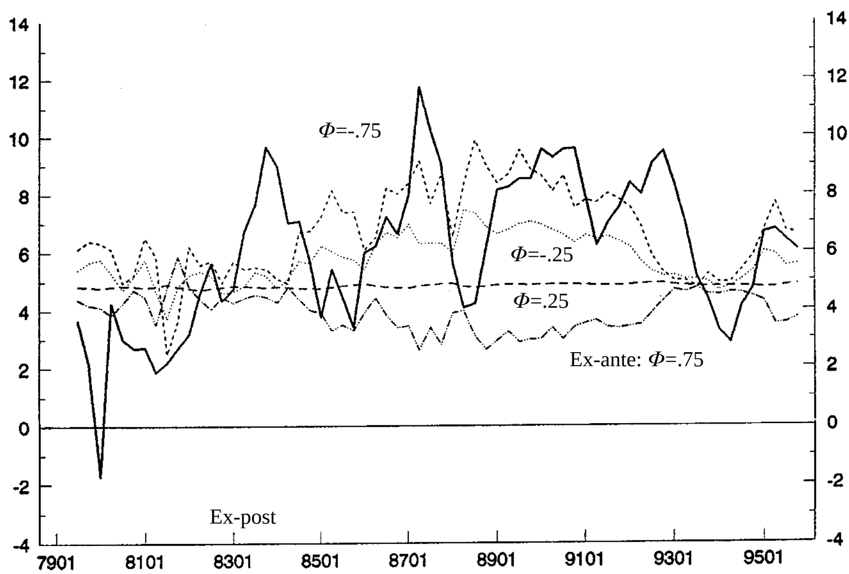


Figure 2. 1-year real interest rates (cont.).

in agent's preferences. Relative consumption effects also increase that correlation but only to .443, and the generalised isoelastic preferences provide worse results than the benchmark, reaching a maximum correlation of .329. Notice that, in any case, all those correlations are not far away from the .381 record corresponding to the benchmark.

Looking now at Figure 2.b, it shows that disentangling between relative risk aversion and (the inverse of) elasticity of intertemporal substitution mainly produces a scale effect: the higher ϱ, the higher the average real interest rates without any remarkable change in its variability.[16] Therefore, generalising the isoelastic preferences as proposed by Epstein & Zin (1989) or Weil (1990) does not provide an ex-ante real interest rate closer to the ex-post rate than that estimated under isoelasticity.[17]

Regarding Figure 2.c, when habits are important —that is when λ is very close to its upper limit— the ex-post real interest rate is markedly different from that of the benchmark. But in this case, the correlation between ex-ante and ex-post real interest rates becomes negative (see Table 1). Note that, in all cases, the ex-ante real interest rates are lower than the ex-post one (around 50 basis points, in average) and differences are also remarkable (about 2 percentage points) during long periods, as, for example, between 1989 and 1993.

Finally, Figure 2.d offers a somehow different picture. For negative ϕ values, estimated real interest rate patterns look rather different from that corresponding to the benchmark. Nevertheless, their correlations are never above .44 and the difference between them and the ex-post real interest rate is still quantitatively important over long periods. Note, for example, that these ex-ante real rates are about 2 percentage points above the *ex-post* one during the first four years in the sample.

To summarise, there is a very low correlation between 1-year ex-ante and ex-post real interest rates, independently of the chosen preference specification. The next step is to investigate whether the differences between both series may be mainly explained in terms of inflation expectation errors or in terms of inflation premia. We deliver this analysis to the following Section.

3.2. Inflation expectation errors and inflation risk premium

The different behaviour of *ex-post* and *ex-ante* real interest rates depends on the evolution of two unknown variables: inflation expectation errors and inflation risk premia. As shown in Section 2, the choice of the parameters determining the

16. Our real interest rate estimates under [8.b] when p is constrained to be equal to y do not significantly differ from those coming from the isoelastic case. Thus, the iid assumption behind going from [8.a] to [8.b] does not seem to be empirically relevant.
17. This result is consistent with those in Cochrane and Hansen (1992), who found that most of the ability of this model to provide a more volatile discount factor (the inverse of the real interest rate) does not depend on consumption dynamics.

stochastic discount factor (i.e. MRS_{t+1}) crucially affects the joint evolution of these two variables. In this Section, we focus on how such a choice deterrnines this evolution.

Our strategy is similar to that used in the previous Section. We first discuss how 1-year inflation expectations and risk premia behave in our benchmark case. Then, we will turn, as a robustness check, to analyse whether alternative preference specifications change the main results.

Thus, in Figure 3.a we plot upper and lower bounds for the inflation expectations under the isoelastic specification. These bounds are jointly plotted with the observed inflation. Accordingly, the differences between the binding bound and the observed inflation reflect (minimum) agents' expectation errors.

A first glance to Figure 3.a allows us to infer that in the benchmark case inflation expectations bounds are very closely related and move together. This indicates that risk premium terms are quantitatively small and rather constant for the entire sample period. In this case, what mainly seems to determine the differences between ex-post and ex-ante real interest rates are the inflation expectation errors.

This incapacity of the model to generate inflation premia able to reconcile ex-ante and ex-post real rates may be seen as another example of the well known equity premium puzzle in a context in which inflation is the only source of risk, and agents' inflation expectation errors follow the standard white-noise process.

However, in this case, the puzzle could be settle if we depart from the white-noise assumption. The downward trend in inflation shown in Figure la could reflect the behaviour of two different components: a permanent change in the mean of the inflation rate that has been consolidated in the latter part of our sample period, and a series of fluctuating transitory inflation swings. Thus, since 1992 it appears that the inflation rate has been around a new level below the 5.5% mean level for the period 1988-1991, or the 10% mean level for the period 1979-1986. Accordingly, the way in which inflation evolves over time will depend upon the persistence of the transitory shocks, i.e. the way in which inflation returns to its mean value after a shock, and the process describing how the inflation mean switches. Bearing this in mind may help to understand how inflation expectation may have evolved along this period.

According to Chart 3.a.1, from 1979 to 1982 agents' inflation expectations were lower than observed inflation. From 1983 to 1986 observed inflation and inflation expectations were more closely related, although in the first part of this period inflation expectations were higher than actual inflation. This behaviour changed during 1984 and 1985. Overall, inflation expectations fell sharply during the period 1983-1985, and did not return to the high levels of the 1979-1982 period. Nevertheless, from 1987 to 1992 (with the exception of 1988) agents' expectations were higher than observed inflation. But again, from 1993, inflation expectations decreased considerably.

Comparing observed inflation to inflation expectations, it seems that it takes some time for the new pattern of inflation to be understood and incorporated into agents' inflation expectations. That is to say, inflation expectations may reflect movements in the inflation trend with some delay. The episode from 1989 to 1992 can illustrate this point. In this period, expected inflation tended (on average) to overpredict observed inflation in that it did not immediately react to its observed decreasing path. Thus, agents seemed not to be convinced that the inflation reduction would last. In other words, they needed «some» time to believe that the course of inflation reflected a permanent change. To make this situation compatible with the rational expectations hypothesis, we can argue that a sort of «peso-problem» is behind such a behaviour: during the overpredicting 1989-1992 period, agents expectations were swayed by the feeling that movements in observed inflation would not be consolidated in subsequent periods.

These comments about inflation expectation errors and risk premia are based, however, upon the hypothesis of an isoelastic utility function with a lower degree of risk aversion. As a matter of robustness, the remaining plots in Figure 3 illustrate the sensitivity of inflation expectations and risk premia when we consider a higher risk aversion, a higher elasticity of intertemporal substitution, the existence of habit formation in agents' utility functions, and the presence of relative consumption effects.

As could be expected, a higher degree of relative risk aversion markedly widens the expected inflation bounds, also rendering ex-ante real rates higher and markedly more volatile than in our benchmark (see Figure 2). On the contrary, Figure 3.b shows that increasing the elasticity of intertemporal substitution does not affect the risk premia, nor eliminates the peso problem affecting inflation expectations. The Epstein-Zin preferences just induce almost parallel movements in the bounds, so leaving unchanged the main message obtained from the isoelastic specification. From Figure 3.c we can infer that allowing for habit formation slightly increaces risk premium, without affecting the peso problem. Finally, Figure 3.d shows that when we allow for a markedly relative consumption effect, the risk premium is increased (although it is still constant over time) and the peso problem is less evident.

As to the implications for the analysis of the wider expectation bands we find in some cases, a series of comments are in order. Notice that our approach does not allow for directly estimating inflation expectations. On the contrary, we are able to bound them. It should be clear that the wider the bands, the lower their informational content when the observed inflation is inside them. Whenever the observed inflation rate is outside the bounds we can infer that inflation expectation errors play an important role. However, if inflation is inside the band, our conclusions will depend on the width of such a band. If the band is narrow enough, we can conclude that inflation expectation errors are small. Because it is only a bound for inflation expectations, if the band is too wide and contains the observed inflation inside, nothing can be said about the relative importance of each wedge between ex-ante and ex-post real interest rates.[18]

All in all, our results point to the inflation expectation errors as the main factor to explain why ex-ante and ex-post real interest rates are so different. This result is in line with those in several papers that have detected relatively long periods in

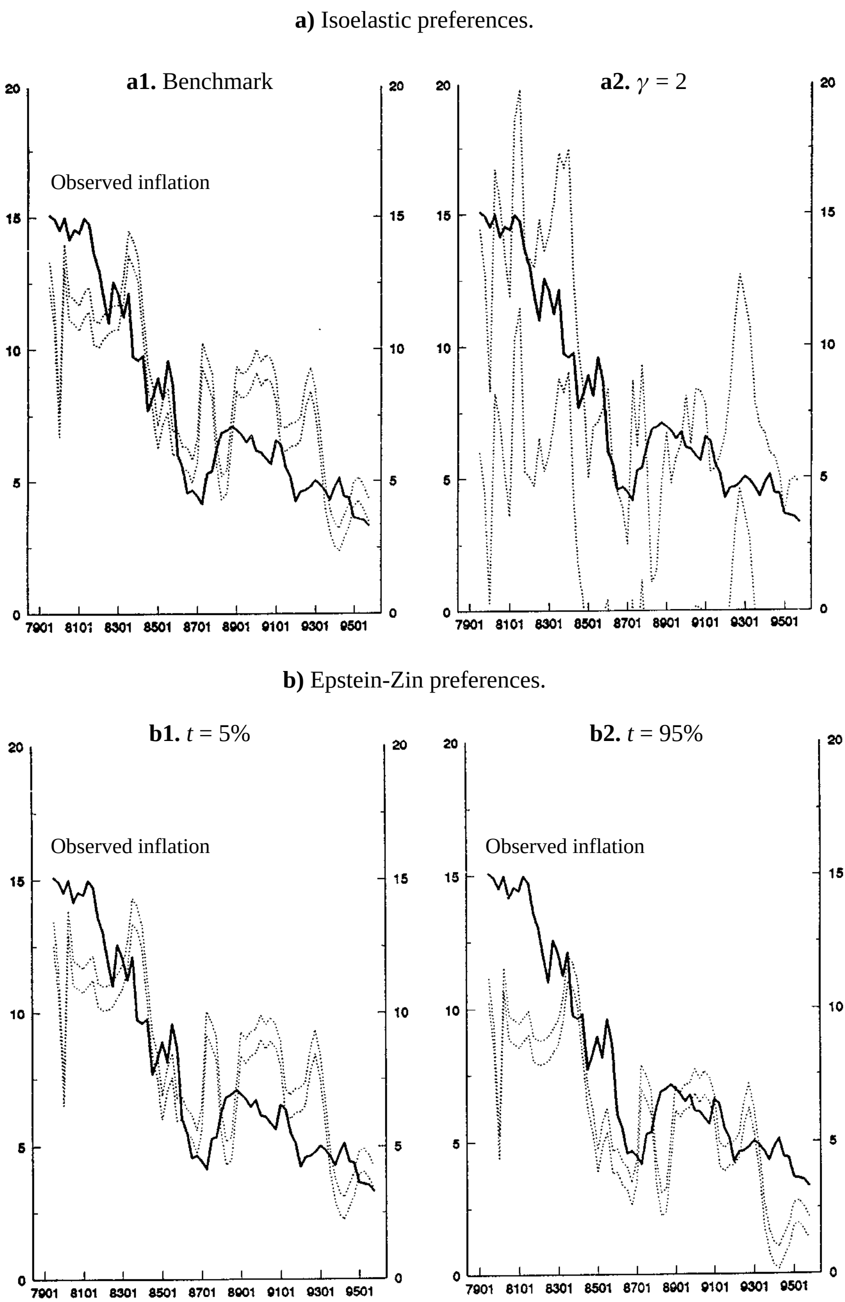
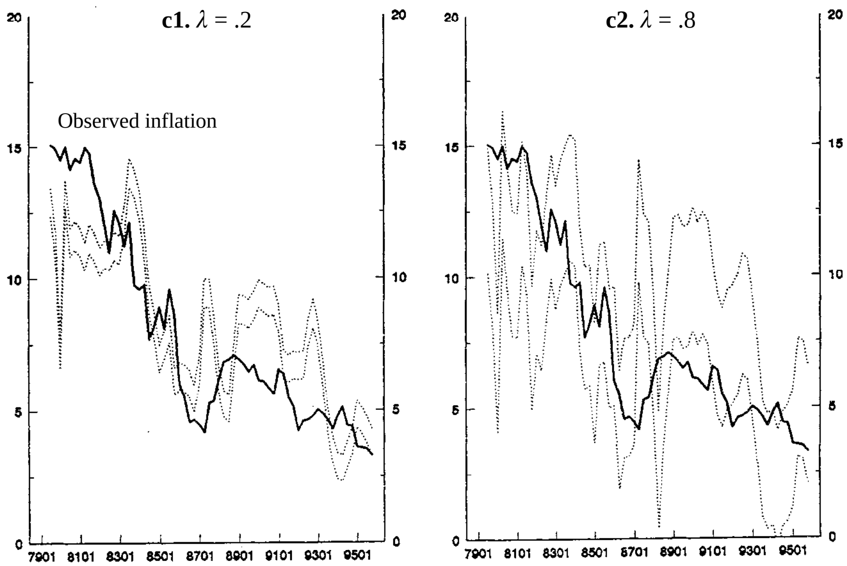


Figure 3. Bounds on inflation expectations.

c) Habits.



d) Relative consumption effects.

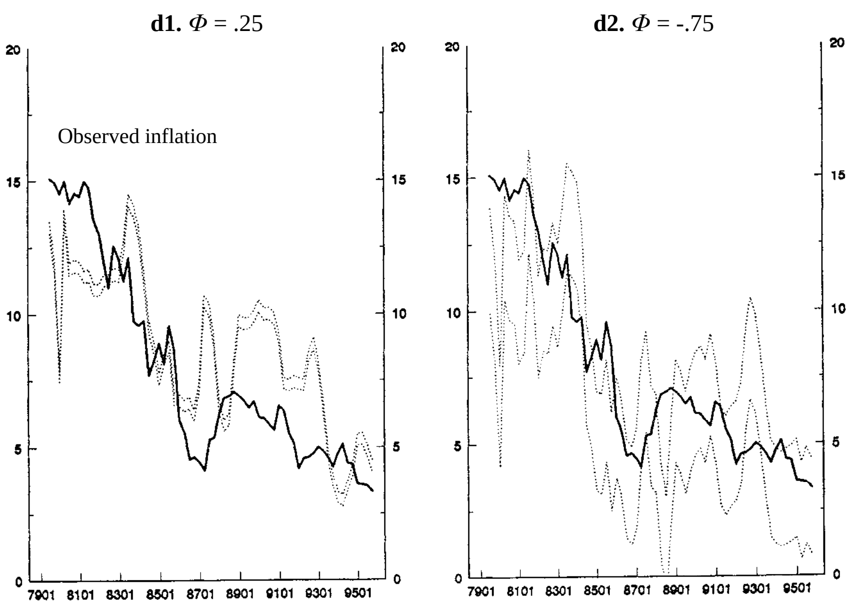


Figure 3. Bounds on inflation expectations (cont.).

which agents fail to correctly forecast inflation and, moreover, offered a theoretical framework in which those expectation errors do not imply irrational agents. Thus, for the UK, King (1996) also shows relatively long inflation overprediction periods since 1982 which are justified in terms of a disinflationary process in which both, agents and the central bank do not immediately learn about each other's reactions to their decisions. For the US economy, the existence of lasting overprediction episodes has been recently stressed from a different perspective by Ceccheti (1996) and Watson (1996). Finally, overpredicting periods during disinflationary processes have also been detected in Canada (Ragan, 1995), New-Zealand (Walsh, 1996) or Denmark (Christensen, 1996), among others.

4. Conclusions

The non-observable ex-ante real interest rates are usually proxied by the so-called ex-post real interest rates, i.e., the difference between the nominal interest rate and the ex-post observed inflation rate. Ex-post rates, however, can be a misleading proxy if nominal interest rates include a non-negligible inflation risk premium or agents' inflation expectation errors are, ex-post, far from the usual zero-mean white-noise case.

In this paper we have shown how the CCAPM provides an integrated framework to jointly estimate the ex-ante real interest rate, (bounds on) inflation premium and (bounds on) expectation errors. Thus, this framework allows not only a comparison of ex-ante and ex-post real rates, but also provides an explanation of their discrepancies.

Using the Spanish economy as a natural case study, and considering alternative preference specifications, we have illustrated that ex-post real rates are very different from any of our model-generated ex-ante rates, the correlation between them never being higher than .50. Within the CCAPM framework, this discrepancy seems to be mainly explained in terms of agents' inflation expectation errors, the inflation premia playing a minor role. Although this is still an open question in the literature, the existence of relatively lasting inflation expectation errors is in line with several different recent works that have detected relatively long periods in which agents fail to correctly forecast inflation and, moreover, offered a theoretical framework in which those expectation errors do not imply irrational agents. These works emphasise the role of information problems and learning processes in the search for an optimal monetary policy.

18. At this noint, the main message emerging from the benchmark can be reinforced by the available direct estimates of inflation premia. Thus, Alonso and Ayuso (1996) estimated inflation premia below 40 basis points for Spain; Söderlind (1995) found inflation premia below 30 basis points for the American economy and, finally, Levind and Copeland (1993) estimated the sum of the inflation premium and the (always negative) Jensen inequality term to be around -.16 basis points in the UK.

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