The two operators F, F^* are introduced on p. 229 and defined by the relations

$$(1) \quad F\varphi(q) = \int_{R^{I}} \overline{\phi(q,r)}\, \varphi(q)\, dq, \qquad F^*\xi(r) = \int_{R^{II}} \phi(q,r)\, \xi(r)\, dr,$$

where

$$(2) \quad \phi(q,r) = \sum_{m,n} f_{mn}\, \varphi_m(q)\, \xi_n(r)$$

and $\{\varphi_m(q)\}$, $\{\xi_n(r)\}$ are complete systems of orthonormal functions in $R^{I} \equiv \{\varphi(q)\}$, $R^{II} \equiv \{\xi(r)\}$, resp.

$F\varphi(q)$ is a point of R^{II}; $F^*\xi(r)$ is a point of R^{I}. Then F^*F, FF^* act on the elements of R^{I}, R^{II} resp. and give as results elements of R^{II}, R^{I}. At the bottom of the p. cited one reads as follows: "Da $F\varphi_m(q)$ gleich $\sum_{n}^{\infty} \overset{(A)}{\bar{f}_{mn}}\, \xi_n(r)$ ausfällt, hat F die Matrix $\{\bar{f}_{mn}\}$ [unter Verwendung der vollständigen normierten Orthogonalsysteme $\varphi_m(q)$ bzw. $\xi_n(r)$], ebenso hat F^* die Matrix $\overset{(B)}{\{f_{mn}\}}$. Also haben F^*F, FF^* die Matrizen

$$\left\{ \sum_{1}^{\infty} {}^{n}\, \bar{f}_{mn} f_{m'n} \right\} \quad \text{bzw.} \quad \left\{ \sum_{1}^{\infty} {}^{m}\, \bar{f}_{mn} f_{mn'} \right\}."$$

But because of (1) and (2) we obtain actually

$$(3) \quad F\varphi_m(q) = \sum_{m',n'} \bar{f}_{m'n'}\, \bar{\xi}_{n'}(r) \int_{R^{I}} \bar{\varphi}_{m'}(q)\, \varphi_m(q)\, dq = \sum_{n} \bar{f}_{mn}\, \bar{\xi}_n(r)$$

and this gives us the matrix of F with respect to $\{\varphi_m(q), \bar{\xi}_n(r)\}$. Similarly

$$F^*\xi_n(r) = \sum_{m',n'} f_{mn'}\, \varphi_m(q) \int_{R^{II}} \xi_n(r)\, \xi_{n'}(r)\, dr.$$

Thus we have

$$\{f_{mn}\} \neq \left\{ \sum_{n'} f_{mn'} \int \xi_n(r)\, \xi_{n'}(r) \right\},$$

if $\xi_n(r)$ are not real.

The whole reasoning assumes the ~~relevance~~ validity of (A) and (B) which contradict the results obtained above. I think it might perhaps be amended in the following way. Let us introduce the operators $F, \bar{F}, F^*$ and F' defined by the identities

$$(5)\quad F\varphi(q) = \int \overline{\phi(q,r)}\,\varphi(q)\,dq, \qquad \bar{F}\varphi(q) = \int \phi(q,r)\,\varphi(q)\,dq.$$
$$F^*\xi(r) = \int \phi(q,r)\,\xi(r)\,dr, \qquad F'\xi(r) = \int \overline{\phi(q,r)}\,\xi(r)\,dr.$$

Each of these four operators is a "vollstetige" one and F^*F, $\bar{F}F$ are definite. The matrices of the former are defined by

$$(6)\quad F\varphi_m(q) = \sum_n \bar{f}_{mn}\,\bar{\xi}_n(r), \qquad F^*\bar{\xi}_n(r) = \sum_m f_{mn}\,\varphi_m(q)$$
$$F'\xi_n(r) = \sum_m \bar{f}_{mn}\,\bar{\varphi}_m(q), \qquad \bar{F}\bar{\varphi}_m(q) = \sum_n f_{mn}\,\xi_n(r)$$

so that

$$(7)\quad (F^*F)\varphi_m(q) = \sum_n \bar{f}_{mn}\cdot F^*\bar{\xi}_n(r) = \sum_{m'}\Big(\sum_n \bar{f}_{mn} f_{m'n}\Big)\varphi_{m'}(q);$$
$$(\bar{F}F')\xi_n(r) = \sum_m \bar{f}_{mn}\cdot \bar{F}\bar{\varphi}_m(q) = \sum_{n'}\Big(\sum_m \bar{f}_{mn} f_{mn'}\Big)\xi_{n'}(r).$$

Therefore

$$(8)\quad U = F^*F, \qquad (9)\quad U = \bar{F}F^*$$

are the projections of the statiscal operator $U = P_\phi$ on $\mathfrak{R}^{I}$, $\mathfrak{R}^{II}$ resp. (8) agrees with the corresponding formula as STATED in your book, while (9) I think is quite different from that of the text. According to the former

$$U\xi(r) = \iint \phi(q,r)\,\overline{\phi(q,s)}\,\xi(s)\,ds\,dq$$

and from the latter we get

$$U\xi(r) = \iint \overline{\phi(q,r)}\,\phi(q,s)\,\xi(s)\,ds\,dq.$$

Notwithstanding, the ultimate result (p. 231)

$$\phi(q,r) = \sum_{\kappa=1}^{M} \sqrt{w_\kappa}\,\varphi_{\mu_\kappa}(q)\,\xi_{\nu_\kappa}(r)$$

holds, in spite of the modifications I propose. (the other noun is "amendment", but it is generally used in a legal sense)

En la pg. 229 introduce los dos operadores F y F^* definidos por

$$F\varphi(q) = \int \overline{\phi(q,r)}\,\varphi(q)\,dq,$$

$$F^*\xi(r) = \int \phi(q,r)\,\xi(r)\,dr.$$

~~El~~ F establece una correspondencia unívoca entre el espacio $\{\varphi(q)\}$ $\mathfrak{R}^{I}$ y el $\{\xi(r)\}$ $\mathfrak{R}^{II}$; el segundo entre el espacio $\{\xi(r)\}$ y el $\{\varphi(q)\}$. Luego F^*F está definido en $\mathfrak{R}^{I}$ y FF^* en $\mathfrak{R}^{II}$. Ahora

donde

$$\phi(q,r) = \sum_{m,n=1}^{\infty} f_{mn}\,\varphi_m(q)\,\xi_n(r).$$

$\{\varphi_m(q)\}$ es un sistema ortonormal en $\{\varphi(q)\}$ y $\xi_n(r)$ lo es en $\{\xi(r)\}$.

Pues, al final de la página se lee: Da $F\varphi_m(q)$ gleich $\sum_{n}^{\infty} \bar{f}_{mn}\,\xi_n(r)$ ausfällt, hat F ① die Matrix $\{\bar{f}_{mn}\}$ [unter Verwendung der vollständigen normierten Orthogonalsysteme $\varphi_m(q)$ bzw. $\xi_n(r)$], ebenso hat F^* die Matrix $\{f_{mn}\}$ ②. Also haben F^*F, FF^* die Matrizen

$$\left\{\sum_{n=1}^{\infty} \bar{f}_{mn}\,f_{m'n}\right\} \text{ bzw. } \left\{\sum_{n=1}^{\infty} \bar{f}_{mn}\,f_{mn'}\right\}.$$

Pero efectuando los cálculos se obtiene:

$$F\varphi_m(q) = \sum_{m',n'} \bar{f}_{m'n'}\,\bar{\xi}_{n'}(r)\int \bar{\varphi}_{m'}(q)\,\varphi_m(q)\,dq = \sum_{n} \bar{f}_{mn}\,\bar{\xi}_n(r),$$

es decir, $\bar{f}$ es la matriz de F referida a la base $\{\bar{\xi}_n(r)\}$, no a la $\{\xi_n(r)\}$, y

$$F^*\xi_n(r) = \sum_{m',n'} f_{m'n'}\,\varphi_{m'}(q)\int \xi_n(r)\,\xi_{n'}(r)\,dr$$

y evidentemente

$$f_{mn} \neq \sum_{n'} f_{mn'}\int \xi_n(r)\,\xi_{n'}(r)\,dr.$$

Todo el razonamiento siguiente se basa en ① y en ② los que, a mi parecer, no son correctos. Por esto modifiqué el razonamiento del siguiente modo: Introduzcamos los operadores F, $\bar{F}$, F^* y F' mediante las relaciones

$$F\varphi(q) = \int \overline{\phi(q,r)}\,\varphi(q)\,dq \qquad \bar{F}\varphi(q) = \int \phi(q,r)\,\varphi(q)\,dq$$

$$F^*\xi(r) = \int \phi(q,r)\,\xi(r)\,dr \qquad F'\xi(r) = \int \overline{\phi(q,r)}\,\xi(r)\,dr.$$

~~Atención~~: Todos estos operadores son completamente continuos y F^*F, $\bar{F}F'$ son definidos.

Sus matrices son:

$$F\varphi_m(q) = \sum_n \bar f_{mn}\, \bar\xi_n(r) \qquad \bar F^*\xi_n = \sum_m f_{mn}\, \varphi_m(q)$$

$$\bar F'\xi_n(r) = \sum_m \bar f_{mn}\, \bar\varphi_m(q) \qquad \bar F\bar\varphi_m(q) = \sum_n f_{mn}\, \xi_n(r)$$

~~y~~ de donde

$$(F^*F)\varphi_m(q) = \sum_n \bar f_{mn}\cdot F^*\bar\xi_n(r) = \sum_{m'}\left(\sum_n \bar f_{mn} f_{m'n}\right)\varphi_{m'}(q),$$

~~FF~~

$$(\bar F F')\xi_n(r) = \sum_m \bar f_{mn}\cdot \bar F\bar\varphi_m(q) = \sum_{n'}\left(\sum_m f_{mn} f_{mn'}\right)\xi_{n'}(r).$$

lo que demuestra que las proyecciones del operador estadístico $U = P_\phi$ ~~[illegible]~~ en R^{I} y R^{II} son

$$(8) \qquad U = F^*F, \qquad U = \bar F F'.$$

La primera fórmula coincide con la del texto, pero no la segunda. Según la fórmula de la p. 230 sería

$$U\xi(r) = \int \overline{\phi(q,r)}\,dq \int \phi(q,s)\,\xi(s)\,ds$$

y según lo que acabamos de obtener

$$U\xi(r) = \int \phi(q,r)\,dq \int \overline{\phi(q,s)}\,\xi(s)\,ds.$$

A partir de (3), modificando ligeramente el razonamiento del texto, se llega pues al mismo resultado (p. 231)

$$\phi(q,r) = \sum_{k=1}^{M} \sqrt{w_k}\,\varphi_{\alpha_k}(q)\,\xi_{\nu_k}(r).$$

on p. 229 , i.e.

to be defined by

The ~~matrix~~ operator ~~(a~~) may be expressed in terms of $\varphi_m(q)$ - functions, and is easily seen to be — We then get

Thus, calling — , we have

Let us therefore put, corresponding to (1), . . , defining in this way —

~~defining~~