	ECAL	τBF	Weak par.	MC stat.	a_1 dyn.	W dis.	TOTAL
π–π	1.35	0.25	0.09	0.36	0.	0.	1.42
π–ρ	1.42	0.21	0.04	0.26	0.	0.27	1.49
π–$\pi 2\pi^0$	3.94	0.37	0.10	0.50	3.60	0.77	5.43
π–3π	1.47	0.05	0.03	0.05	3.08	0.36	3.43
ρ–ρ	0.55	0.04	0.05	0.06	0.	2.47	2.53
ρ–$\pi 2\pi^0$	1.54	0.11	0.08	0.10	0.81	0.74	1.90
ρ–3π	3.23	0.09	0.	0.09	2.89	3.96	5.88
$\pi 2\pi^0$–$\pi 2\pi^0$	9.29	0.35	0.07	0.45	7.87	0.24	12.19
$\pi 2\pi^0$–3π	1.70	0.24	0.05	0.37	4.15	2.90	5.36
3π–3π	0.77	1.18	0.12	1.34	5.17	1.64	5.76

Table 6.7: Systematic uncertainties on the $\Re(d_\tau)$ for the different channels, and on the last column the total systematic uncertainty for each final state topology. The weak electric dipole moment is assumed dimensionless in this table, and the errors are expressed in units of 10^{-4}.

	ECAL	τBF	Weak par.	MC stat.	a_1 dyn.	W dis.	TOTAL
π–π	0.88	0.45	0.03	0.51	0.	0.	1.11
π–ρ	3.02	0.92	0.08	1.11	0.	1.50	3.67
π–$\pi 2\pi^0$	2.33	1.29	0.74	1.56	7.85	7.05	11.01
π–3π	7.61	0.24	0.14	0.40	4.39	0.96	8.85
ρ–ρ	3.16	0.06	0.04	0.37	0.	1.15	3.39
ρ–$\pi 2\pi^0$	5.18	0.09	0.06	0.09	4.59	0.31	6.93
ρ–3π	3.06	0.27	0.04	0.27	0.63	1.42	3.45
$\pi 2\pi^0$–$\pi 2\pi^0$	26.82	0.71	0.86	0.88	30.03	3.95	40.48
$\pi 2\pi^0$–3π	5.34	0.31	0.07	0.31	1.15	5.54	7.79
3π–3π	5.97	0.25	0.18	0.19	4.69	0.56	7.62

Table 6.8: Systematic uncertainties on the $\Im(d_\tau)$ for the different channels, and on the last column the total systematic uncertainty for each final state topology. The weak electric dipole moment is assumed dimensionless in this table, and the errors are expressed in units of 10^{-4}.

difference between σ and σ_{stat}.

To construct an upper limit on the absolute value of the four dipole coupling terms from the results of table 6.9, each fit value is considered to describe a Gaussian probability density function with a given mean and a width equal to the total error. The symmetric region about zero containing 95% of this probability is then quoted

Parameter	Fit value	σ	σ_{stat}	σ_{sys}
$\Re(\mu_\tau)[10^{-3}]$	0.63	0.43	0.40	0.16
$\Im(\mu_\tau)[10^{-3}]$	-0.63	0.84	0.82	0.18
$\Re(d_\tau)[10^{-18}\text{e}\cdot\text{cm}]$	-0.80	2.22	2.14	0.59
$\Im(d_\tau)[10^{-18}\text{e}\cdot\text{cm}]$	-1.58	3.93	3.77	1.11

Table 6.9: Final number of this analysis for the weak anomalous dipole moments at 68 % C.L.

as the 95% C.L. upper limit for each dipole moment. These limits are found to be

$$\begin{aligned}
|\Re(\mu_\tau)| &< 1.34 \times 10^{-3} \ (95\ \%\ \text{C.L.}) \\
|\Im(\mu_\tau)| &< 2.02 \times 10^{-3} \ (95\ \%\ \text{C.L.}) \\
|\Re(d_\tau)| &< 4.62 \times 10^{-18}\text{e cm} \ (95\ \%\ \text{C.L.}) \\
|\Im(d_\tau)| &< 8.29 \times 10^{-18}\text{e cm} \ (95\ \%\ \text{C.L.})\,.
\end{aligned}$$

The results obtained in this analysis are presented together with the previous measurements in table 6.10, splitting the errors in their statistical and systematic components.

Experiment	$\Re(\mu_\tau)[10^{-3}]$	$\Im(\mu_\tau)[10^{-3}]$
L3	$0.0 \pm 1.6 \pm 2.3$	$-1.0 \pm 3.6 \pm 4.3$
SLD	$0.26 \pm 0.99 \pm 0.75$	$-0.02 \pm 0.62 \pm 0.24$
New ALEPH(*)	$0.63 \pm 0.40 \pm 0.16$	$-0.63 \pm 0.82 \pm 0.18$
Experiment	$\Re(d_\tau)[10^{-18}\text{e}\cdot\text{cm}]$	$\Im(d_\tau)[10^{-18}\text{e}\cdot\text{cm}]$
L3	$-4.4 \pm 8.8 \pm 13.3$	-
SLD	$1.8 \pm 6.1 \pm 2.8$	$-2.6 \pm 3.5 \pm 1.3$
ALEPH	$-0.29 \pm 2.59 \pm 0.88$	-
OPAL	$0.72 \pm 2.46 \pm 0.24$	$3.5 \pm 5.7 \pm 0.8$
DELPHI	$-1.48 \pm 2.64 \pm 0.27$	$-4.4 \pm 7.7 \pm 1.3$
New ALEPH(*)	$-0.80 \pm 2.14 \pm 0.59$	$-1.58 \pm 3.77 \pm 1.11$

Table 6.10: Previous results on the weak dipole moments, extracted from refs. [3, 4], together with the results of this analysis (New ALEPH), at 68% C.L. The error is splitted in the statistical and systematic components.
(*) The new ALEPH numbers are not published yet.

The CP–violating electric dipole moment has received plenty of attention by the four LEP experiments, being measured many times from the early times of LEP.

The results we report here are extracted from refs. [3, 4]. Table 6.11 compares the 95% C.L. limits of the previous weak dipole moment measurements with this analysis. We see that our results are fairly good on the electric components. On the one hand, the limit on the real part has been improved by 8 % with respect to the most accurate numbers, and it is about 14 % better than the previous ALEPH number. On the other, the bound on the imaginary part is 5 % better than the SLD limit.

Experiment	Limit on $\lvert\Re(\mu_\tau)\rvert$	Limit on $\lvert\Im(\mu_\tau)\rvert$
L3	5.5	10.0
SLD	2.48	1.30
New ALEPH(*)	1.34	2.02
Experiment	**Limit on $\lvert\Re(d_\tau)\rvert$**	**Limit on $\lvert\Im(d_\tau)\rvert$**
L3	32.4	-
SLD	13.6	8.7
ALEPH	5.40	-
OPAL	5.04	13.1
DELPHI	5.91	17.4
New ALEPH(*)	4.62	8.29

Table 6.11: The 95% C.L. limits on the weak dipole moments of previous measurements and of this analysis (New ALEPH). The electric terms are expressed in units of $10^{-18}\mathrm{e}\cdot\mathrm{cm}$, and the magnetic terms are expressed in units of 10^{-3}. The limits of the previous measurements are extracted from the results of refs. [3,4].
(*) The new ALEPH numbers are not published yet.

The weak magnetic dipole moment has only been measured by the L3 and SLD collaborations. The 95% CL limits from their latter results are also shown in table 6.11 together with our numbers. The bound on $|\Re(\mu_\tau)|$ has been considerably improved, by 46%. However, the SLD limit for $|\Im(\mu_\tau)|$ is better by 36%.

The SLD measurements are better on the imaginary parts because the polarisation of the beams increases their sensitivity on these terms. However, for the real parts, they have the same sensitivity as LEP with less events.

With respect to the previous LEP numbers, one of the main differences of our analysis is the use of the information of all the cross–section terms. Table 6.12 shows the most sensitive observables for each of the anomalous couplings, according to the definition of eq. 2.22. In the measurement of the real parts $(A_{32})_+$ and $(A_{31})_-$ were

$\Re(\mu_\tau)$		$\Im(\mu_\tau)$		$\Re(d_\tau)$		$\Im(d_\tau)$	
$1^{st}Obs.$	$2^{nd}Obs.$	$1^{st}Obs.$	$2^{nd}Obs.$	$1^{st}Obs.$	$2^{nd}Obs.$	$1^{st}Obs.$	$2^{nd}Obs.$
$(A_{02})_+$	$(A_{32})_+$	$(A_{31})_+$	$(A_{01})_+$	$(A_{01})_-$	$(A_{31})_-$	$(A_{32})_-$	$(A_{02})_-$
	1^{st} time	1^{st} time			1^{st} time		1^{st} time

Table 6.12: The most sensitive observables for each of the tensorial couplings, showing if they are used for the first time in this analysis.

not taken into account before. And for the imaginary couplings, the new observables considered are $(A_{31})_+$ and $(A_{02})_-$. The most remarkable case is that of $\Im(\mu_\tau)$, since we use for the first time the most sensitive observable, which is $(A_{31})_+$. This last point was suggested in ref. [19].

Another relevant point in the comparison with previous LEP measurements is the size of the data sample. In this analysis we have used the π, ρ, $\pi 2\pi^0$ and 3π decay channels, in which the sensitivity to the tau direction is higher. Nevertheless, also the fully leptonic channels (e and μ decays) could be used and would certainly improve the statistical error of this measurement. In the former LEP analysis, the whole set of τ decays were used and this has to be taken into account if one wants to compare the performance of the methods used.

Therefore, we have shown the convenience of using the information of the full cross–section terms in the determination of the weak anomalous couplings, by obtaining the best world measurements of $(\Re(\mu_\tau)$, $\Re(d_\tau)$, $\Im(d_\tau))$, and by setting very competitive limits on $|\Im(\mu_\tau)|$.

Chapter 7

Summary and Conclusions

In this thesis we have set very stringent limits on the weak dipole moments of the tau lepton, using the large data sample collected by the ALEPH detector from 1990 to 1995.

The method for the extraction is based on a likelihood fit built from the full differential cross–section, obtained applying the ideas of refs. [25, 22]. The expression for the tau production has been taken from [11, 39] and is given in appendix A after some algebra. The tau decay is described by the TAUOLA formulae [21], in which the sensitivity to the tau polarisation is maximal through the use of polarimeters, the $\vec{h}$ vectors, different for each decay topology. These expressions are shown in appendix B.

The selection and particle identification make use of many tools already developed in previous ALEPH analysis [36, 34, 37, 38]. By contrast, we also require the correct reconstruction of the event observables, reducing the number of candidates by 21% and also helping in the cleaning of background considerably.

The tau flight direction is needed in the polarimeter expressions and, in principle, the information of the vertex detector could help in its determination [44, 45] if the cones intersect. On the other hand, this has been explored by one of the methods to measure the tau polarisation in ALEPH [46] and it has been shown that the improvement is small. Therefore, for events with intersecting cones, we do not try to distinguish between the two available tau directions and just average the information coming from both solutions in the fitting formula.

The detector effects and background are incorporated in the fitting formula by means of an efficiency matrix and a complete set of $smearing functions$, the D_{ij}, where we introduce the most relevant correlations between the observables of the fit. We have empirically found that the following pairs of variables are significantly correlated: (W-W^0,W^0), ($\cos\theta_h$ - $\cos\theta_h^0$, $\cos\theta_h^0$) and (ϕ_h-ϕ_h^0, $\cos\theta_h$). The shape of the smearing functions is easy to understand for the π channel, with only two particles in the final state, but it is not simple to be explained for the other decays.

The performance of the fit has then been checked with the SCOT Monte Carlo program [39], using various Monte Carlo samples generated at different values of the anomalous couplings. The reconstructed parameters are plotted versus those of the generation and a straight line fit is applied, obtaining significant deviations of the slopes from 1 in various cases. Therefore, we correct for this effect in the final results.

Afterwards, we have compared the distributions of the observables for the data and the Monte Carlo simulation. The polarimeter angular distributions of the data are consistent with those of the the Monte Carlo simulation. Thus, other possible systematic errors not treated in the analysis are under control. On the contrary, the W distributions of the data significantly differ to those of the Monte Carlo simulation, and this effect has been considered as an additional systematic uncertainty.

The systematic uncertainties have been estimated for each of the anomalous couplings and each topology separately, considering some of the most common effects of previous ALEPH analysis. This measurement is dominated by the statistical error and an overall control of the systematic uncertainties is expected from the comparison of the observables between data and Monte Carlo, as said above.

The final numbers are obtained by a combination of the individual measurements from each channel, using a covariance matrix which only accounts for the statistical correlations. However, as long as the systematic errors are small this method is correct.

On the results, it is the first time ALEPH measures $\Re(\mu_\tau)$, $\Im(\mu_\tau)$ and $\Im(d_\tau)$. $\Re(d_\tau)$ was measured several times and we improve the limit by 14%.

Comparing with other experiments, we are able to set the best world limits for both real parts of the couplings. The improvement is about 46% for $|\Re(\mu_\tau)|$, and about 8% for $|\Re(d_\tau)|$. The bound on $|\Im(d_\tau)|$ has also been improved with respect to the previous best limit, by 5%. However, SLD gives better results on $\Im(\mu_\tau)$ due to the polarisation of the beams.

The final numbers agree with the SM prediction. However, we have shown that the performance of this method is better than that of the previous approaches of the LEP collaborations by an overall reduction of the total uncertainties. It should be also noticed that we only consider the π, ρ, $\pi 2\pi^0$ and 3π channels. Thus, our statistical error would decrease if the e and μ channels were also used.

In our data sample, about 52% of the events have at least one of the taus decaying into the a_1 channel. This decay has a noticeable theoretical uncertainty, as shown in the tables of the systematic errors. Hence, a more precise theoretical knowledge of the underlying dynamic certainly will improve our numbers. However, it is also true that from the experimental side, more study on the W disagreement might also decrease our errors.

The extraction of the weak anomalous couplings by a likelihood fit was already done by the SLD collaboration [6]. Nevertheless, in obtaining these couplings we use for the first time the spin correlations between hemispheres in a likelihood fit. Mostly for the imaginary parts this point is crucial in the analysis of LEP data, since the most sensitive terms of the cross–section to $\Im(\mu_\tau)$ and $\Im(d_\tau)$ are $(A_{31})_+$ and $(A_{32})_-$ respectively.

As future prospects, we can finalize by saying that this method has broad applications in the future e^+e^- linear collisions with other final state topologies, mainly $t\bar{t}$ production.

Appendix A

Cross section explicit expression

In section 2.1.3 the total differential cross section was expressed as the sum of some matrix elements R_{ij} multiplying the τ spin components. Here, the full explicit expression for these elements is presented. It could be also extracted from ref. [11] after some algebra.

The γ interchange is assumed to happen under the SM; for the Z, both the Ze^+e^- and $Z\tau^+\tau^-$ vertex have the axial and vector couplings of the SM and for the latter are also added the tensorial couplings μ_τ and d_τ.

The chosen reference frame is that of fig. 2.1.

$$\begin{aligned}
R_{00} &= 2\sum_{i=1,2}\left[\Re(A_i)+\Re(B_i)+\Re(C_i)+\Re(D_i)\right]\\
R_{11} &= 2\sum_{i=1,2}\left[-\Re(A_i)+\Re(B_i)-\Re(C_i)+\Re(D_i)\right]\\
R_{22} &= 2\sum_{i=1,2}\left[\Re(A_i)-\Re(B_i)-\Re(C_i)+\Re(D_i)\right]\\
R_{33} &= 2\sum_{i=1,2}\left[\Re(A_i)+\Re(B_i)-\Re(C_i)-\Re(D_i)\right]\\
R_{30} &= 4\sum_{i=1,2}\left[\Re(E_i)\pm\Re(J_i)\right]\\
R_{21} &= -4\sum_{i=1,2}\left[\Im(E_i)\pm\Im(J_i)\right]\\
R_{01} &= 4\sum_{i=1,2}\left[\pm\Re(F_i)+\Re(I_i)\right]\\
R_{32} &= -4\sum_{i=1,2}\left[\pm\Im(F_i)+\Im(I_i)\right]
\end{aligned}$$

$$\begin{aligned} R_{02} &= -4\sum_{i=1,2}[\Im(G_i)\pm\Im(H_i)] \\ R_{31} &= 4\sum_{i=1,2}[\Re(G_i)\pm\Re(H_i)] \end{aligned} \tag{A.1}$$

where the $\pm$ sign refers to $R_{ab}\to R_{ba}$, and L_i ($L=A,...,J$; $i=1,2$) is the addition of three terms coming from the Z exchange, the $Z-\gamma$ interference and the γ exchange as follows:

$$L_i = |N_Z(q^2)|^2(L_i)_Z + (L_i)_{Z\gamma} + |N_\gamma(q^2)|^2(L_i)_\gamma \tag{A.2}$$

with

$$N_Z(q^2)=\frac{i4e^2}{q^2-{M_Z}^2+i\Gamma_Z q^2/M_Z} \qquad N_\gamma(q^2)=\frac{i4Q_eQ_\tau e^2}{q^2}\ . \tag{A.3}$$

$$\tag{A.4}$$

The non vanishing terms in each case are

<u>Z contribution</u>:

$$\begin{aligned}
(A_1)_Z &= (v_e{v_e}^*v_\tau{v_\tau}^*+v_e{v_e}^*\mu_\tau{\mu_\tau}^*)+\beta^2\cos^2\theta\, a_e{a_e}^*a_\tau{a_\tau}^*+2v_e{v_e}^*\Re(v_\tau{\mu_\tau}^*) \\
&+ 2\beta\cos\theta\,[\Re(v_e v_\tau {a_e}^*{a_\tau}^*)+\Re(v_e\mu_\tau{a_e}^*{a_\tau}^*)] \\
(A_2)_Z &= (a_e{a_e}^*v_\tau{v_\tau}^*+a_e{a_e}^*\mu_\tau{\mu_\tau}^*)+\beta^2\cos^2\theta\, v_e{v_e}^*a_\tau{a_\tau}^*+2a_e{a_e}^*\Re(v_\tau{\mu_\tau}^*) \\
&+ 2\beta\cos\theta\,[\Re(a_e v_\tau {v_e}^*{a_\tau}^*)+\Re(a_e\mu_\tau{v_e}^*{a_\tau}^*)] \\
(B_1)_Z &= \beta^2 v_e{v_e}^*a_\tau{a_\tau}^*+\cos^2\theta\,(a_e{a_e}^*v_\tau{v_\tau}^*+a_e{a_e}^*\mu_\tau{\mu_\tau}^*)+2\beta\cos\theta\,\Re(v_e a_\tau{a_e}^*{v_\tau}^*) \\
&+ 2\beta\cos\theta\,\Re(v_e a_\tau{a_e}^*{\mu_\tau}^*)+2\cos^2\theta\, a_e{a_e}^*\Re(v_\tau{\mu_\tau}^*) \\
(B_2)_Z &= \beta^2 a_e{a_e}^*a_\tau{a_\tau}^*+\cos^2\theta\,(v_e{v_e}^*v_\tau{v_\tau}^*+v_e{v_e}^*\mu_\tau{\mu_\tau}^*)+2\beta\cos\theta\,\Re(a_e a_\tau{v_e}^*{v_\tau}^*) \\
&+ 2\beta\cos\theta\,\Re(a_e a_\tau{v_e}^*{\mu_\tau}^*)+2\cos^2\theta\, v_e{v_e}^*\Re(v_\tau{\mu_\tau}^*) \\
(C_1)_Z &= \frac{\beta^2}{m^2}\sin^2\theta\, a_e{a_e}^*d_\tau{d_\tau}^* \\
(C_2)_Z &= \frac{\beta^2}{m^2}\sin^2\theta\, v_e{v_e}^*d_\tau{d_\tau}^* \\
(D_1)_Z &= m^2\sin^2\theta\, a_e{a_e}^*v_\tau{v_\tau}^*+\frac{1}{m^2}\sin^2\theta\, a_e{a_e}^*\mu_\tau{\mu_\tau}^*+2\sin^2\theta\, a_e{a_e}^*\Re(v_\tau{\mu_\tau}^*) \\
(D_2)_Z &= m^2\sin^2\theta\, v_e{v_e}^*v_\tau{v_\tau}^*+\frac{1}{m^2}\sin^2\theta\, v_e{v_e}^*\mu_\tau{\mu_\tau}^*+2\sin^2\theta\, v_e{v_e}^*\Re(v_\tau{\mu_\tau}^*) \\
(E_1)_Z &= \beta v_e{v_e}^*v_\tau{a_\tau}^*+\cos\theta\,(v_e v_\tau{v_\tau}^*{a_e}^*+v_e v_\tau{a_e}^*{\mu_\tau}^*)+\beta v_e{v_e}^*\mu_\tau{a_\tau}^* \\
&+ \cos\theta\,(v_e\mu_\tau{a_e}^*{v_\tau}^*+v_e\mu_\tau{\mu_\tau}^*{a_e}^*)+\beta^2\cos\theta\, a_e a_\tau{a_\tau}^*{v_e}^* \\
&+ \beta\cos^2\theta\,(a_e{a_e}^*a_\tau{v_\tau}^*+a_e{a_e}^*a_\tau{\mu_\tau}^*)
\end{aligned}$$

$$
\begin{aligned}
(E_2)_Z &= \beta a_e {a_e}^* v_\tau {a_\tau}^* + \cos\theta\,(a_e v_\tau {v_\tau}^* {v_e}^* + a_e v_\tau {v_e}^* {\mu_\tau}^*) + \beta a_e {a_e}^* \mu_\tau {a_\tau}^* \\
&+ \cos\theta\,(a_e \mu_\tau {\mu_\tau}^* {v_e}^* + a_e \mu_\tau {v_e}^* {v_\tau}^*) + \beta^2 \cos\theta\, v_e a_\tau {a_\tau}^* {a_e}^* \\
&+ \beta\cos^2\theta\,(v_e {v_e}^* a_\tau {v_\tau}^* + v_e {v_e}^* a_\tau {\mu_\tau}^*) \\
(F_1)_Z &= -\frac{\beta}{m}\sin\theta\,(v_e v_\tau {a_e}^* {d_\tau}^* + v_e \mu_\tau {a_e}^* {d_\tau}^*) - \frac{\beta^2}{m}\sin\theta\,\cos\theta\, a_e {a_e}^* a_\tau {d_\tau}^* \\
(F_2)_Z &= -\frac{\beta}{m}\sin\theta\,(a_e v_\tau {v_e}^* {d_\tau}^* + a_e \mu_\tau {v_e}^* {d_\tau}^*) - \frac{\beta^2}{m}\sin\theta\,\cos\theta\, v_e {v_e}^* a_\tau {d_\tau}^* \\
(G_1)_Z &= -im\,\sin\theta\, v_e v_\tau {v_\tau}^* {a_e}^* - i\frac{1}{m}\,\sin\theta\, v_e v_\tau {a_e}^* {\mu_\tau}^* - im\,\sin\theta\, v_e \mu_\tau {a_e}^* {v_\tau}^* \\
&- i\frac{1}{m}\,\sin\theta\, v_e \mu_\tau {\mu_\tau}^* {a_e}^* - i\beta m\,\sin\theta\,\cos\theta\, a_e {a_e}^* a_\tau {v_\tau}^* - i\frac{\beta}{m}\,\sin\theta\,\cos\theta\, a_e {a_e}^* a_\tau {\mu_\tau}^* \\
(G_2)_Z &= -im\,\sin\theta\, a_e v_\tau {v_\tau}^* {v_e}^* - i\frac{1}{m}\,\sin\theta\, a_e v_\tau {v_e}^* {\mu_\tau}^* - im\,\sin\theta\, a_e \mu_\tau {v_e}^* {v_\tau}^* \\
&- i\frac{1}{m}\,\sin\theta\, a_e \mu_\tau {\mu_\tau}^* {v_e}^* - i\beta m\,\sin\theta\,\cos\theta\, v_e {v_e}^* a_\tau {v_\tau}^* - i\frac{\beta}{m}\,\sin\theta\,\cos\theta\, v_e {v_e}^* a_\tau {\mu_\tau}^* \\
(H_1)_Z &= -\frac{\beta^2}{m}\sin\theta\, v_e a_\tau {a_e}^* {d_\tau}^* - \frac{\beta}{m}\sin\theta\,\cos\theta\,(a_e {a_e}^* v_\tau {d_\tau}^* + a_e {a_e}^* \mu_\tau {d_\tau}^*) \\
(H_2)_Z &= -\frac{\beta^2}{m}\sin\theta\, a_e a_\tau {v_e}^* {d_\tau}^* - \frac{\beta}{m}\sin\theta\,\cos\theta\,(v_e {v_e}^* v_\tau {d_\tau}^* + v_e {v_e}^* \mu_\tau {d_\tau}^*) \\
(I_1)_Z &= -i\beta m\,\sin\theta\, v_e a_\tau {a_e}^* {v_\tau}^* - i\frac{\beta}{m}\,\sin\theta\, v_e a_\tau {a_e}^* {\mu_\tau}^* - im\,\sin\theta\,\cos\theta\, a_e {a_e}^* v_\tau {v_\tau}^* \\
&- i\frac{1}{m}\,\sin\theta\,\cos\theta\, a_e {a_e}^* v_\tau {\mu_\tau}^* - im\,\sin\theta\,\cos\theta\, a_e {a_e}^* \mu_\tau {v_\tau}^* \\
&- i\frac{1}{m}\,\sin\theta\,\cos\theta\, a_e {a_e}^* \mu_\tau {\mu_\tau}^* \\
(I_2)_Z &= -i\beta m\,\sin\theta\, a_e a_\tau {v_e}^* {v_\tau}^* - i\frac{\beta}{m}\,\sin\theta\, a_e a_\tau {v_e}^* {\mu_\tau}^* - im\,\sin\theta\,\cos\theta\, v_e {v_e}^* v_\tau {v_\tau}^* \\
&- i\frac{1}{m}\,\sin\theta\,\cos\theta\, v_e {v_e}^* v_\tau {\mu_\tau}^* - im\,\sin\theta\,\cos\theta\, v_e {v_e}^* \mu_\tau {v_\tau}^* \\
&- i\frac{1}{m}\,\sin\theta\,\cos\theta\, v_e {v_e}^* \mu_\tau {\mu_\tau}^* \\
(J_1)_Z &= i\beta\sin^2\theta\, a_e {a_e}^* d_\tau {v_\tau}^* + i\frac{\beta}{m^2}\sin^2\theta\, a_e {a_e}^* d_\tau {\mu_\tau}^* \\
(J_2)_Z &= i\beta\sin^2\theta\, v_e {v_e}^* d_\tau {v_\tau}^* + i\frac{\beta}{m^2}\sin^2\theta\, v_e {v_e}^* d_\tau {\mu_\tau}^* \qquad \text{(A.5)}
\end{aligned}
$$

$\underline{Z-\gamma \text{ contribution}}$:

$$
\begin{aligned}
(A_1)_{Z\gamma} &= 2\Re\Big\{\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\,(v_e v_\tau + v_e \mu_\tau) \\
&+ \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\,\beta\cos\theta\, a_e a_\tau\Big\} \\
(B_2)_{Z\gamma} &= 2\Re\Big\{\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\,\beta\cos\theta\, a_e a_\tau \\
&+ \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\,\cos^2\theta\,(v_e v_\tau + v_e \mu_\tau)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
(D_2)_{Z\gamma} &= 2\Re\Big\{\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, m^2\sin^2\theta\, v_e v_\tau \\
&+ \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\sin^2\theta\, v_e\mu_\tau\Big\} \\
(E_1)_{Z\gamma} &= \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\,\beta v_e{}^* a_\tau{}^* \\
&+ \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big](\cos\theta\, a_e{}^* v_\tau{}^* + \cos\theta\, a_e{}^*\mu_\tau{}^*) \\
(E_2)_{Z\gamma} &= \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big](\cos\theta\, a_e v_\tau + \cos\theta\, a_e\mu_\tau) \\
&+ \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\,\beta\cos^2\theta\, v_e a_\tau \\
(F_1)_{Z\gamma} &= -\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\frac{\beta}{m}\sin\theta\, a_e{}^* d_\tau{}^* \\
(G_1)_{Z\gamma} &= -\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, im\,\sin\theta\, a_e{}^* v_\tau{}^* \\
&- \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, i\frac{1}{m}\,\sin\theta\, a_e{}^*\mu_\tau{}^* \\
(G_2)_{Z\gamma} &= -\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, im\,\sin\theta\,(a_e v_\tau + a_e\mu_\tau) \\
&- \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, i\beta m\,\sin\theta\,\cos\theta\, v_e a_\tau \\
(H_2)_{Z\gamma} &= -\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, i\frac{\beta}{m}\sin\theta\,\cos\theta\, v_e{}^* d_\tau{}^* \\
(I_2)_{Z\gamma} &= -\Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, i\beta m\,\sin\theta\, a_e a_\tau \\
&- \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, im\,\sin\theta\,\cos\theta\,(v_e v_\tau + v_e\mu_\tau) \\
&- \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, im\,\sin\theta\,\cos\theta\, v_e{}^* v_\tau{}^* \\
&- \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) + \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, i\frac{1}{m}\,\sin\theta\,\cos\theta\, v_e{}^*\mu_\tau{}^* \\
(J_2)_{Z\gamma} &= \Big[\Re(N_Z(q^2)N_\gamma^*(q^2)) - \Im(N_Z(q^2)N_\gamma^*(q^2))\Big]\, i\beta\sin^2\theta\, v_e d_\tau
\end{aligned}
\tag{A.6}
$$

γ contribution:

$$
\begin{aligned}
(A_1)_\gamma &= 1 \\
(B_2)_\gamma &= \cos^2\theta \\
(D_2)_\gamma &= m^2\sin^2\theta \\
(I_2)_\gamma &= -im\,\sin\theta\,\cos\theta
\end{aligned}
\tag{A.7}
$$

where $m = m_\tau/(q/2)$, and β is the tau velocity in the center of mass system.

Appendix B

Elements for the decay process

In section 2.2, it was shown that the decay dependent elements in the differential partial decay rate of the tau are the polarimeter vector $\vec{h}$, the spin averaged squared matrix element $|\bar{M}|^2$, and the phase space factor P_i. In this appendix, these terms are presented for each of the decay modes considered. Also, the set of independent observables $\bar{X}$ for each topology is given.

In the following subsections, the four–momenta not expressed in covariant form are supposed to be in the tau rest frame. Since in the experiment the available four–momenta are those of the laboratory system, the tau direction of flight is needed for the transformation to the tau rest frame.

B.0.1 Decay into one pion

In the decay to one pion,

$$\tau^{\pm}(p_\tau, s) \rightarrow \bar{\nu}_\tau(p_\nu)\pi^{\pm}(p_{\pi^\pm}) \ ,$$

the polarimeter $\vec{h}$ is the following:

$$\vec{h}^{\pm} \quad = \quad \mp\, m_\tau \frac{2(p_{\pi^\pm}\cdot p_\nu)\vec{p_{\pi^\pm}} - p_{\pi^\pm}^2 \vec{p}_\nu}{2(p_{\pi^\pm}\cdot p_\nu)(p_{\pi^\pm}\cdot p_\tau) - p_{\pi^\pm}^2 (p_\nu \cdot p_\tau)} \ , \tag{B.1}$$

which, after some algebra, can be expressed as

$$\vec{h}^{\pm} = \mp \frac{2\, m_\tau}{m_\tau^2 - m_\pi^2} \vec{p}_{\pi^\pm} \, . \tag{B.2}$$

The spin averaged squared matrix is

$$|\bar{M}|^2 = G_F^2 f_1^2 (m_\tau^2 - m_\pi^2) m_\tau^2 \quad \text{with} \;\; f_1 = \sqrt{2} f_\pi \cos\theta_C = 128.4 \text{ MeV}. \tag{B.3}$$

The phase space factor P_i is

$$P_\pi = (4\pi) \frac{1}{2^5 \pi^2} \frac{\lambda^{1/2}(m_\tau^2, m_\pi^2, m_\nu^2)}{m_\tau^2} \, , \tag{B.4}$$

with

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \, . \tag{B.5}$$

The set of independent variables, the $\bar{X}$ set, is formed, in this case, by the polar and azimuthal angles of the pion in the tau rest frame.

B.0.2 Decay into two pions

This decay is dominated by the ρ production. We call it also the ρ channel. The notation for the kinematic will be the following:

$$\tau^\pm(p_\tau, s) \to \bar{\nu}_\tau(p_\nu) \pi^\pm(p_{\pi^\pm}) \pi^0(p_{\pi^0}) \, .$$

The polarimeter and the $|\bar{M}|^2$ are

$$\vec{h}^{\pm} = \mp m_\tau \frac{2(q \cdot p_\nu)\vec{q} - q^2 \vec{p}_\nu}{2(q \cdot p_\nu)(q \cdot p_\tau) - q^2 (p_\nu \cdot p_\tau)}$$

$$|\bar{M}|^2 = (G_F \cos\theta_C)^2 |F(Q^2)|^2 4 \left[2(q \cdot p_\nu)(q \cdot p_\tau) - q^2 (p_\nu \cdot p_\tau) \right] \tag{B.6}$$

with the following definitions:

$$
\begin{aligned}
Q &= p_{\pi^\pm} + p_{\pi^0} \\
q &= p_{\pi^\pm} - p_{\pi^0} \\
\vec{p}_\nu &= -(\vec{p}_{\pi^\pm} + \vec{p}_{\pi^0}) \\
p_\nu(4) &= \frac{m_\tau^2 - (p_{\pi^\pm} + p_{\pi^0})^2}{2m_\tau} \, .
\end{aligned} \tag{B.7}
$$

$F(Q^2)$ is the pion form factor from ref. [49],

$$
F(w) = \frac{B_w(w^2, m_\rho, \Gamma_\rho) + \beta B_w(w^2, m_{\rho'}, \Gamma_{\rho'})}{1+\beta} \, ,
$$

with $m_\rho = 773\ MeV$, $\Gamma_\rho = 145\ MeV$, $m_{\rho'} = 1370\ MeV$, $\Gamma_{\rho'} = 510\ MeV$, $\beta = -0.145$; and $B_w(s^2, m, \Gamma)$ is the p-wave Breit-Wigner for the ρ defined as

$$
B_w(s^2, m, \Gamma) = \frac{m^2}{m^2 - s^2 - i m g_s} \, ,
$$

with

$$
g_s = \begin{cases} \frac{m\Gamma}{s} \left(\frac{\sqrt{||s^2/4 - m_\pi^2| + s^2/4 - m_\pi^2|}}{\sqrt{m^2/2 - 2m_\pi^2}} \right)^3 & \text{if } s^2 > 4m_\pi^2 \\ 0 & \text{otherwise.} \end{cases}
$$

The phase space factor P_i is

$$
\begin{aligned}
P_\rho &= (4\pi)^2 \frac{1}{2^{11}\pi^5} \frac{\lambda^{1/2}(m_\tau^2, m_\nu^2, Q^2)}{m_\tau^2} \frac{\lambda^{1/2}(Q^2, m_{\pi^0}^2, m_{\pi^\pm}^2)}{Q^2} \\
&\times \frac{(Q^2 - m_\rho^2)^2 + (m_\rho \Gamma_\rho)^2}{m_\rho \Gamma_\rho} (\alpha_{max}^2 - \alpha_{min}^2) \, ,
\end{aligned} \tag{B.8}
$$

with

$$
\alpha_{max} = m_\tau - m_\nu \quad \text{and} \quad \alpha_{min} = m_{\pi^\pm} + m_{\pi^0} \, . \tag{B.9}
$$

In this case, there are five independent variables: four angles and a sampling for the ρ resonance, which may take values between α_{min} and α_{max}.

B.0.3 Decay into three pions

This decay is dominated by the a_1 resonance and, in analogy with the ρ mode, it is called the a_1 channel. The a_1 can either decay into three charged pions or into one charged pion plus two π^0's. The notation used is

$$\tau^{\pm}(p_\tau, s) \to \bar{\nu}_\tau(p_\nu)\pi^{\pm}(p_1)\pi^{\pm}(p_2)\pi^{\mp}(p) \qquad \tau^{\pm}(p_\tau, s) \to \bar{\nu}_\tau(p_\nu)\pi^{0}(p_1)\pi^{0}(p_2)\pi^{\pm}(p) \, .$$

We will also use that $p_{a_1} = p + p_1 + p_2$; $d_i = p_{a_1} \cdot (p_i - p)$, $i = 1, 2$; $m_1^2 = p_1^2$; $m_2^2 = p_2^2$; $\tilde{m}_{\rho_1}^2 = (p + p_1)^2$; $\tilde{m}_{\rho_2}^2 = (p + p_2)^2$; $M^2 = (p_1 + p_2)^2$ and $\tilde{m}_{a_1}^2 = p_{a_1}^2$.

The polarimeter takes into account the two resonances. The expressions presented below assume that the z-axis is parallel to the a_1 direction. However, the proper orientation of the reference frame is recovered after the calculations are performed. The polarimeter is first defined in terms of the p_k and p_v vectors as

$$\vec{h}^{\pm} = \mp \frac{\vec{p}_k^{\,\pm} - \vec{p}_v}{p_k(4)^{\pm} - p_v(4)} \, , \tag{B.10}$$

with the following definition for the p_k and p_v:

$$\begin{aligned}
p_k(1)^{\pm} &= \mp \{-2p_\nu(3)\Im\,[H(2)H(4)^* - H(4)H(2)^*] \\
&+ 2p_\nu(4)\Im\,[H(2)H(3)^* - H(3)H(2)^*]\} \\
p_k(2)^{\pm} &= \mp \{-2p_\nu(4)\Im\,[H(1)H(3)^* - H(3)H(1)^*] \\
&+ 2p_\nu(3)\Im\,[H(1)H(4)^* - H(4)H(1)^*]\} \\
p_k(3)^{\pm} &= \mp 2p_\nu(4)\Im\,[H(1)H(2)^* - H(2)H(1)^*] \\
p_k(4)^{\pm} &= \mp 2p_\nu(3)\Im\,[H(1)H(2)^* - H(2)H(1)^*]
\end{aligned}$$

$$p_v = 4\Re\,\{[H(4)p_\nu(4) - H(3)p_\nu(3)]\,H^*\} - 2\Re(H \cdot H^*)p_\nu \tag{B.11}$$

where the hadronic current H depends on two vectors (P_{V_1} and (P_{V_2}),

$$\begin{aligned}
H &= \frac{2\sqrt{2}m_{a_1}^2}{3\,\Pi} B(\tilde{m}_{a_1}, m_{a_1}, \tilde{\Gamma}_{a_1})\,[F(\tilde{m}_{\rho_1})P_{V_1} + F(\tilde{m}_{\rho_2})P_{V_2}] \, , \\
\text{with} \quad \Pi &\equiv 93.3 \times 10^{-3} \, .
\end{aligned}$$

$F(Q^2)$ is the pion form factor, defined in the previous section. The vectors P_{V_i} and the $\tilde{\Gamma}_{a_1}$ are defined as

$$\begin{aligned} P_{V_i} &= p_i - p - \frac{p_{a_1} d_i}{\tilde{m}_{a_1}^2}, \\ \tilde{\Gamma}_{a_1} &= \Gamma_{a_1} \frac{G(\tilde{m}_{a_1}^2)}{G(m_{a_1}^2)}, \end{aligned}$$

where $G(q)$ is a function to introduce the energy dependence in the a_1 width, with the approximate value from ref. [49],

$$G(q) = \begin{cases} 4.1(q - 9\, m_{\pi 0}^2)^3 \left[1 - 3.3(q - 9\, m_{\pi 0}^2) + 5.8(q - 9\, m_{\pi 0}^2)^2\right] & \text{if } q < (m_\rho + m_\pi)^2 \\ q\,(1.623 + 10.38/q - 9.32/q^2 + 0.65/q^3) & \text{otherwise} . \end{cases}$$

The spin averaged squared matrix is

$$|\bar{M}|^2 = G_F^2 m_\tau (p_v(4) - p_k(4)^{\pm}) . \tag{B.12}$$

And the phase space factor P_i is

$$\begin{aligned} P_{a_1} &= (4\pi)^3 \frac{1}{2^{17}\pi^8} \frac{\lambda^{1/2}(m_\tau^2, m_\nu^2, \tilde{m}_{a_1}^2)}{m_\tau^2} \frac{\lambda^{1/2}(\tilde{m}_{a_1}^2, m_\pi^2, M^2)}{\tilde{m}_{a_1}^2} \\ &\times \frac{\lambda^{1/2}(M^2, m_2^2, m_1^2)}{M^2} \frac{(\tilde{m}_{a_1}^2 - m_{a_1}^2)^2 + (m_{a_1}\Gamma_{a_1})^2}{m_{a_1}\Gamma_{a_1}} \\ &\times (Q_{max}^2 - Q_{min}^2)(M_{2,max}^2 - M_{2,min}^2) , \end{aligned} \tag{B.13}$$

where

$$Q_{min} = m_1 + m_2 + m_\pi \qquad Q_{max} = m_\tau - m_\nu ,$$

and

$$M_{2,min} = m_1 + m_2 \qquad M_{2,max} = \tilde{m}_{a_1} - m_\pi .$$

Finally, in this mode there are eight independent variables: six angles and two more variables to sample the a_1 and the ρ resonances. The variation interval for the invariant mass of the a_1 resonance is from Q_{min} to Q_{max}, and that of the ρ resonance is from $M_{2,min}$ to $M_{2,max}$.

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