CHAPTER 4: Nonlinear stochastic trends and economic fluctuations

1 Introduction

Many nonstationary variables, even though may behave separately in the short-run, present a closely related long-run pattern. Engle and Ganger (1987) describe these variables as being in a long-run equilibrium, in the sense that a stationary linear combination of their levels behaves as an attractor. Thus, while most of the time the system is out of equilibrium, economic forces such as a market mechanism or government intervention, tend to correct these equilibrium errors.

The main drawback of this seminal linear model is that it implicitly imposes symmetry in the strength on which the economy tends to push the system back toward equilibrium. That is, positive and negative deviations from the attractor are restricted to have the same dynamics. We think that there are economic reasons for questioning this assumption, however.

Let us focus the analysis in certain macroeconomic variables for which positive and negative deviations from the attractor may be interpreted as expansions and recessions. The long-run pattern may then be seen as the steady state equilibrium where the business cycles movements vanish. We present market mechanisms and government policies leading to asymmetric adjustment to the equilibrium (i.e.

asymmetries in eliminating the business cycles).

On the one hand, Caballero and Hammour (1994) argue, within a creative-destruction framework, that negative effects of recessions are concentrated in less productive plants which are quickly dropped out of the market, whereas high productive firms have competitive advantages that make them more likely to survive. Hence, there are market mechanisms that move the economy from a deep recession into the attractor more aggressively that it falls from expansions.

On the other hand, the asymmetric adjustment may also be due to policy interventions. For example, stabilization policies are often characterized by discrete interventions. Within expansions, macroeconomic variables are allowed to move freely as long as they do not exceed certain bands, which implies that movements toward the equilibrium need not occur aggressively every period. However, during recessions, policy authorities may react more drastically against the adverse economic situation, accelerating the convergence toward the attractor. Even though we assume similar initiative for mitigating the effects of expansions and recessions, many authors have postulated the existence of a convex aggregate supply curve implying that monetary policy would have stronger effects during recessions. Garcia and Schaller (1996) have found empirical evidence supporting this view.

In this paper we postulate a nonlinear Vector Error Correction Model (VECM) to consider the asymmetric adjustment toward the long-run equilibrium. We allow for nonlinear behavior by introducing shifting parameters as in Hamilton (1989). We study the theoretical properties of the statistical model and we find that the idea of nonstationary variables with shifting autoregressive parameters may be related with the presence of nonlinear common stochastic trends. Thus, we use a common trends framework to implement long-run restrictions to identify and interpret the effect of structural disturbances to the different endogenous variables. In analyzing the dynamics of the Markov-switching model, we find the explicit expressions of the impulse-response functions (IRF) and the variance-decomposition (VD) analysis. Thus, we employ an alternative approach to the Dynamic Factor Regime Switching Model of Kim and Piger (2000).

Given these results, we turn to examine the short-run effect of permanent shocks to output, consumption and investment. As outlined by King, Plosser, Stock and Watson (1991), henceforth KPSW, we find that these shocks are capable of explaining more than two-thirds of the variation in output and consumption but less than two-fifths of the movements of investment at horizons over the business-cycles. Our mayor contribution is that we detect the presence of asymmetries across two regimes that we identify as expansions and recessions: the ability of these shocks to explain short-run variations depends upon the state of the cycle.

We organize the paper as follows. Section 2 provides a set of statistical relations linking the concepts of switching VECM and stochastic trends. Section 3 develops a framework for analyzing VAR models in a context that allows for state-dependent responses to shocks. Section 4 applies the methodology in investigating the asymmetric responses of output, consumption and investment to technological shocks. Concluding remarks appear in the last section.

2 Switching VECM and stochastic trends

The main aim of this study is to consider that, in a context of cointegrated variables, the strength with which the economy tends to eliminate the deviations from the long-run equilibrium may depend on the phase of the business-cycle. This leads to propose a vector of equilibrium errors following a stationary Markov Switching Vector Autoregressive (MS-VAR) specification like

$$z_t = m_{s_t} + F_{s_t}(L) z_{t-1} + e_t, \tag{1}$$

where $F_{s_t}(L) = (F_{s_t}^{(1)} + ... + F_{s_t}^{p} L^{p-1})$ and $e_t | s_t \sim N(0, V)$.

Assume that x_t is the $(n \times 1)$ vector of nonstationary variables generating these equilibrium errors. That is, the stationary errors are expressed as r linear combinations of x_t, $z_t = \beta' x_t$, being β the $(n \times r)$ cointegrating matrix. We prove in Appendix 1 that the cointegrating errors lead to the following Markov Switching VECM (MS-VECM)

$$\Delta x_t = \mu_{s_t} - \alpha_{s_t} z_{t-1} + \pi_{s_t}(L) \Delta x_{t-1} + \epsilon_t, \tag{2}$$

where $\pi_{s_t}(L) = (\pi_{s_t}^1 + ... + \pi_{s_t}^p L^{p-1})$, and $\epsilon_t | s_t \sim N(0, \Sigma)$. Note that we postulate that the strength with which the equilibrium errors are corrected (measured by the matrix α_{s_t}) vary across regimes, whereas we assume a state-independent long-run attractor (represented by the matrix β).

To complete the statistical properties of the baseline model, it is standard to assume that the varying parameters in (2) depend upon an unobservable state variable s_t that evolves according to an irreducible q-state Markov process which is defined by the transition probabilities

$$p\left(s_t = j | s_{t-1} = i, s_{t-2} = k, ..., \chi_{t-1}\right) = p\left(s_t = j | s_{t-1} = i\right) = p_{ij}, \tag{3}$$

where $i, j = 1, 2, ..., q$, and $\chi_t = (z_t, , z_{t-1}, ...)$. It is convenient to collect the transition probabilities in the $(q \times q)$ transition matrix

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{q1} \\ p_{12} & p_{22} & \cdots & p_{q2} \\ \vdots & \vdots & \ldots & \vdots \\ p_{1q} & p_{2q} & \cdots & p_{qq} \end{bmatrix}. \tag{4}$$

Finally, let us define $\xi_{t/t}$ as the $(q \times 1)$ vector whose i-th element is $P(s_t = i|\chi_t)$, and $\xi_{t+h/t}$ as $P^h \xi_{t/t}$.

The cointegrated process x_t has an alternative representation in terms of a reduced number of common nonlinear stochastic trends. To see this, we state in Appendix 2 that the stationary change in x_t will have the switching moving average representation

$$\Delta x_t = \delta_{s_t} + C_{s_{t\downarrow}}(L)\epsilon_t, \tag{5}$$

where δ_{s_t} is the conditional mean of Δx_t, and $C_{s_{t\downarrow}}(L) = (I + C^1_{s_t}L + C^2_{s_t,s_{t-1}}L^2 + C^3_{s_t,s_{t-2}}L^3 + ...)$, with $\sum_{j=1}^{\infty} j|C^j_{s_t,s_{t-(j-1)}}| < \infty$.[1]

Substituting recursively and using the relation derived in Appendix 3 and assuming $\epsilon_0 = 0$:[2]

$$C_{s_{t\downarrow}}(L) = C(1) + (1 - L)C^*_{s_{t\downarrow}}(L), \tag{6}$$

equation (5) becomes

$$x_t = x_0 + \sum_{j=1}^{t} \delta_{s_j} + C(1)\sum_{j=1}^{t} \epsilon_j + C^*_{s_{t\downarrow}}(L)\epsilon_t. \tag{7}$$

Since we have assumed stationary equilibrium errors, it should be true that $\beta' C(1) = 0$, and $\beta'\delta_{s_j} = 0$ for all $s_j = 1, ..., q$.[3] This implies that each δ_{s_j} lies in the column space of $C(1)$ and therefore can be written as $\delta_{s_j} = C(1)\rho_{s_j}$, where ρ_{s_j} is an $(n \times 1)$ vector. Thus, since cointegration implies that $rank\,[C(1)] = k = n - r$, there is

[1]What we mean with $C_{s_{t\downarrow}}$ is that parameters in $C(L)$ depends not only on s_t but also on $s_{t-1}, s_{t-2}, ...$, and what we mean with $C^j_{s_t,s_{t-(j-1)}}$ is that the matrix C^j depends on the sequence of states $s_t, s_{t-1}, ..., s_{t-(j-1)}$.

[2]Matrix $C(1)$ refers to $(I + C^1 + C^2 + ...)$, where

$$C^j = \sum_{i_0=1}^{q} ... \sum_{i_{(j-1)}=1}^{q} P(s_t = i_0, ..., s_{t-(j-1)} = i_{j-1})C^j_{s_t,s_{t-(j-1)}}.$$

Expression $(1 - L)C^*_{s_{t\downarrow}}(L)$ is equivalent to $C^*_{s_{t\downarrow}}(L) - C^*_{s_{t-1\downarrow}}(L)L$, where

$$C^{*j}_{s_t,s_{t-(j-1)}} = -C(1) + I + C^1_{s_{t-(j-1)}} + C^2_{s_{t-(j-2)},s_{t-(j-1)}} + ... + C^j_{s_t,s_{t-(j-1)}}.$$

[3]Let us consider the simpler two-states case such that the economy is in estate 1 and state 2 in t_1 and t_2 times respectively. If x_t is cointegrated, then both $\beta'\delta_1 t_1$ and $\beta'\delta_2 t_2$ should be zero.

a $(n \times r)$ matrix Γ_r^{-1} such that $C(1)\Gamma_r^{-1} = 0$ for all t. Define the $(n \times k)$ matrix $\Upsilon = C(1)\Gamma_k^{-1}$ such that the k columns of Γ_k^{-1} are orthogonal to the columns of Γ_r^{-1}. This means that $C(1)\Gamma^{-1} = \Upsilon S_k$, where $\Gamma^{-1} = \left(\Gamma_k^{-1}\Gamma_r^{-1}\right)$ and S_k is the $(k \times n)$ selection matrix $[I_k 0_{k \times r}]$.

Using these properties, expression (7) may be transformed into:

$$x_t = x_0 + \Upsilon \left\{ S_k \Gamma \sum_{j=1}^{t} \left(\rho_{s_j} + \epsilon_j\right) \right\} + C^*_{s_{t\downarrow}}(L)\epsilon_t. \tag{8}$$

To interpret the expression in curly brackets we introduce the notion of nonlinear stochastic trends, following Granger et al. (1997). Standard literature usually decompose the random walk processes with drift into the sum of a linearly deterministic trend plus the sum of persistent errors. However, in our context, the dynamics of the variables is state-dependent, which seems to be associated with trends whose "deterministic" growth is not constant along time but rather shifting among regimes. Specifically, we consider a wider class of trend-generating k dimensional vector τ_t of random walks with switching drift ϑ_{s_t} and white noise innovations φ_t:

$$\tau_t = \vartheta_{s_t} + \tau_{t-1} + \varphi_t = \sum_{j=1}^{t} \left(\vartheta_{s_j} + \varphi_j\right). \tag{9}$$

A trivial verification shows that the expression in braces appearing in (8) can be seen as common switching stochastic trends, where $\vartheta_{s_j} = S_k \Gamma \rho_{s_j}$, and $\varphi_j = S_k \Gamma \epsilon_j$. This leads to the following extension of the Stock-Watson common trend representation:

$$x_t = x_0 + \Upsilon \tau_t + C^*_{s_{t\downarrow}}(L)\epsilon_t, \tag{10}$$

with $\Upsilon\tau_t$ and $C^*_{s_{t\downarrow}}(L)\epsilon_t$ representing the (nonlinear) permanent and transitory components. We call this expression Switching Common Trends model.

3 Asymmetric responses

Up to this point, we have stated the assumptions and technical relationships that we need for extending the standard VEC analysis of economic data to a framework that allows us to introduce nonlinearities. We now try to concille these mathematical results with the idea of asymmetric responses to exogenous shocks by dividing our study into two related issues. First, we find a solution for the identification problem, that is, how structural shocks can be recovered from the MS-VAR model. Second, we investigate how these shocks are propagated through time, that is, we look for an explicit expression of the IRF and the VD analysis.

3.1 Identifying structural shocks

The analysis of the dynamic responses is not straightforward. Due to the presence of correlations among statistical errors, we need to identify the component of the shock that is not a simple reaction to other shocks, i.e., that is exogenous. A common way to deal with this problem is using enough restrictions in the estimations to recover the orthogonalized residuals from the correlated shocks. Depending on the nature of the restrictions, they are classified in contemporaneous and long-run restrictions. The former are either recursive or nonrecursive and are, by much, the most used in the traditional literature.[4] The latter are based on postulating restrictions on the matrix of long-run multipliers.[5]

We assume that the reduced-form model has a $MA(\infty)$ representation

$$\Delta x_t = \delta_{s_t} + R_{s_{t\downarrow}}(L) v_t, \tag{11}$$

with $R_{s_{t\downarrow}}(L) = I + R^1_{s_t} L + R^2_{s_t, s_{t-1}} L^2 + ...$, and v_t being independent and identically distributed with a mean of 0 and a variance of I_n. Given the information up to time t, any matrix $R^h_{s_t, s_{t-(h-1)}}$ collects the reactions of the endogenous variables at time t to one standard deviation shocks at $t-h$. This is what we call *backward-looking* responses.[6]

One of the mayor contributions in KPSW is to see that an intuitive way for identifying the system is to look for the matrix that relates structural-form and reduced-form errors that holds the restrictions of the common trends model, that is

$$\epsilon_t = \Gamma^{-1} v_t. \tag{12}$$

In our nonlinear context, we consider the flowering relationships to overcome the identification problem:

$$R_{s_{t\downarrow}}(L) = C_{s_{t\downarrow}}(L)\Gamma^{-1} = C_{s_{t\downarrow}}(L)\left(\Gamma_k^{-1}\Gamma_r^{-1}\right), \tag{13}$$

[4] All of them follow the seminal line proposed by Sims (1980).

[5] Blanchard and Quah (1989) motivate the long-run restrictions in macroeconomic predictions, whereas King et al. (1991), Warne (1993) and Gonzalo and Ng (2001) leave the data to impose the cointegrating (long-run) constrains.

[6] Within the standard linear literature, our backward-looking responses represent the actual impulse-response functions. Note that it is not necessarily true in our nonlinear context.

with $R_{s_{t\downarrow}}(1) = (\Upsilon, 0)$. This implies that shocks that occurs at time $t-h$, with h large enough, may be decomposed into permanent-effect shocks (first k elements of ϵ_t, with responses at time t equal to Υ) and transitory-effect shocks (last r elements of ϵ_t, whose effect vanishes at time t). Additionally, it is assumed that the structural shocks with permanent effects are uncorrelated with those structural shocks with only temporary effects. This reduces the identification of the structural dynamics to the identification of the matrix $\Gamma = (\Gamma_k' \Gamma_r')'$.

On the one hand, KPSW show that the $(k \times n)$ matrix Γ_k may be obtained as

$$\Gamma_k = (\Upsilon'\Upsilon)^{-1}\Upsilon' C(1), \tag{14}$$

with Υ derived from the relation $\Upsilon\Upsilon' = C(1)\Sigma C(1)$.[7] On the other hand, Warne (1993) point out that the $(r \times n)$ matrix Γ_r allowing uncorrelated permanent and transitory-effect shocks may be estimated from the relation

$$\Gamma_r = \zeta Q^{-1}\Sigma^{-1}, \tag{15}$$

where Q is the Cholesky decomposition of $\zeta'\Sigma^{-1}\zeta$, and $\zeta = \alpha\,(U\alpha)^{-1}$, with α being

[7] To identify the system it is standard to impose $k(k-1)/2$ restrictions by assuming the loading matrix to be lower triangular. However one technical difficulty emerges since the right-hand side that equation is not of full rank. KPSW suggest this problem be overcome by writing the loading matrix as $\Upsilon_0\omega$, such that $\beta'\Upsilon_0 = 0$, and estimating ω using the relation

$$\omega\omega' = (\Upsilon_0'\Upsilon_0)^{-1}\Upsilon_0 C(1)\Sigma C'(1)\Upsilon_0'(\Upsilon_0'\Upsilon_0)^{-1}.$$

the last r columns of $M^{-1}C(1)^{-1}$.[8]

What it is clearly left is to explain how to estimate Σ and $C(1)$. We try to concille the ideas of Krolkig (1996) who suggests a two-stage procedure to estimate the parameters of MS-VECM, and Warne (1993) who uses an intermediate estimation for capturing the responses to shocks in VECM. First, we test for cointegration and obtain an estimation for the cointegrating matrix β, using the standard techniques developed for linear models. Second, applying the standard EM algorithm conditional on the cointegration matrix, we estimate the so-called Markov Switching Restricted VAR(MS-RVAR) model

$$(I - B_{s_t}(L)L)\, y_t = \mu^*_{s_t} + \epsilon^*_t, \tag{16}$$

which comes from an appropriate manipulation of MS-VECM. Note that $I - B_{s_t}(L)L = (I - B^1_{s_t}L - ... - B^p_{s_t}L^p) = M\left[(I - \pi_{s_t}(L)L)\, M^{-1}D(L) + \alpha^*_{s_t}L\right]$, $\alpha^*_{s_t} = (0\alpha_{s_t})$, $\mu^*_{s_t} = M\mu_{s_t}$ and $\epsilon^*_t = M\epsilon_t$, with $\epsilon^*_t|s_t \sim N(0, \Omega)$ and $\Omega = M\Sigma M'$. Note that $y_t = D_\perp(L)Mx_t$ is stationary, which allows us to obtain the appropriate estimates of $B_{s_t}(L)$, Ω, and P.[9]

Finally, we recover the parameters of the MS-VECM from the MS-RVAR estimates. To estimate Σ we may use the relation $\Sigma = M^{-1}\Omega\,(M')^{-1}$. We propose in

[8] U is the zero-one matrix that imposes the $r(r-1)/2$ additional restrictions needed for the identification of the system. M is $[O_k\beta]'$ where O_k is orthogonal to β such that M is invertible.

[9] We consider that $D(L) = \begin{bmatrix} I_k & 0 \\ 0 & (1-L)I_r \end{bmatrix}$, and $D_\perp(L) = \begin{bmatrix} (1-L)I_k & 0 \\ 0 & I_r \end{bmatrix}$.

Appendix 4 how to estimate the moving average parameters as follows:[10]

$$C^{j+1} = M^{-1}J\Phi_{t/t-j}J'M - M^{-1}DJ\Phi_{t/t-j-1}J'M \text{ for } j > 1, \tag{17}$$

with $C^0 = I$, $C^1 = M^{-1}Jb\xi^*_{t/t}J'M - M^{-1}DM$,and $C^2 = M^{-1}J\Phi_{t/t-1}J'M - M^{-1}DJb\xi^*_{t/t}J'M$. Thus, any backward-looking responses for the difference of the variables may be estimated by $R^j = C^j\Gamma^{-1}$, whereas adding these expressions is a simple way of obtaining responses for the levels.

3.2 Propagation of shocks

In linear VAR specifications, it is standard to incorporate two additional tools that provide a summary of the system's dynamic properties: impulse response functions (IRF) and variance decomposition (VD). The former, shows the time responses of

[10] Let $D = D_\perp(1)$. Let J be the $(n \times nq)$ matrix $(I_n 0...0)$. For any $(a \times b)$ matrix W let us define the $(anp \times bnp)$ matrix $W^* = (W \otimes I_{np})$. In this way, we introduce the matrices P'^*, $\xi^*_{h/h}$, and $\widetilde{\xi}^*_{h/h-1}$, with $\widetilde{\xi}_{h/h-1}$ being the $(q \times q)$ diagonal matrix whose j-th diagonal element is $e'_j\xi_{h/h-1}/e'_1\xi_{h/h-1}$. We also consider the $(np \times np)$, $(n \times npq)$ and $(npq \times npq)$ matrix

$$B_j = \begin{pmatrix} B_j^{(1)} & \dots & B_j^{(p-1)} & B_j^{(p)} \\ I_n & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & I_n & 0 \end{pmatrix}, b = (B_1, ...B_q), \text{ and } B = \begin{pmatrix} B_1 & \dots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \dots & B_q \end{pmatrix}.$$

Finally, let us define the $(npq \times npq)$ matrix

$$\Phi_{t/t-j} = b\left(\widetilde{\xi}^*_{t/t-1}P'^*B\widetilde{\xi}^*_{t-1/t-2}P'^*B...\widetilde{\xi}^*_{t-(j-1)/t-j}P'^*B\right)\xi^*_{t-j/t-j}.$$

the VAR from an unitary structural shock hitting a particular series. The latter, measures the contribution of each structural disturbance to the variance of k-periods ahead forecast errors for each variable in the VAR.

In both cases, the traditional assumption of linearity is associated with the idea of responses that are independent from the past. This is difficult to reconcile with the history dependence of nonlinear switching models since, given a structural shock at time t, the j-periods ahead response of the endogenous variables depends on the sequence of states. Thus, the current response to lagged shocks, measured with $R_{s_{t\downarrow}}$ does not correspond to traditional forward-looking responses so we need to introduce a new concept for analyzing the dynamic response of the system.

To formalize this idea, we suggest a way of predicting the responses of the endogenous variables to current structural shocks in a nonlinear MS-VAR context. For this attempt, we define the $(n \times n)$ matrix $\overline{R}^j$ such that the row i, column j element is an optimal *forward-looking* estimation of the consequences of a one-unit increase in the jth variable fundamental innovation at date t for the value of the difference of the ith variable at time $t+j$. In Appendix 5, we present the following expression for this matrix

$$\overline{R}^{j+1} = \left[M^{-1}Jb\,(P^*B)^j\,\xi^*_{t+1/t}J'M - M^{-1}DJb\,(P^*B)^{j-1}\,\xi^*_{t+2/t}M\right]\Gamma^{-1}, \text{ for } j>0, \tag{18}$$

with $\overline{R}^0 = \Gamma^{-1}$, $\overline{R}^1 = \left[M^{-1}Jb\xi^*_{t+1/t}J'M - M^{-1}DM\right]\Gamma^{-1}$, and where the remaining parameters are stated in the estimation of the vector autoregressive parameters (17).

Similar forward-looking responses for the level of the endogenous variables are the sum of the sequence of matrices $\overline{R}^j$.

It is worth pointing out that other works as for example Ehrmann et al. (2000), estimate different switching IRF for each possible state under the unrealistic hypothesis that the economy remains in one of these states with probability one. On the contrary, we propose a unique estimate of the responses to structural shocks that helps us to analyze the most probable future effects of these shocks hitting the economy in a specific date. That is, the optimal forecasted response to an structural perturbance is computed as the weighted average of the estimated responses in each state, where the weights are the forecasted probabilities of being in each of these states.

3.3 Variance decomposition

Using the notation stated in Warne (1993) we define the forward-looking variance decomposition $\overline{v}^s$ as the matrix whose (i,j) element is the fraction of the variance at time $t+s$ of the *i-th* variable (either in differences or in levels) that is accounted for by *j-th* shock hitting the economy at time t, that is

$$\overline{v}^s = \left[\sum_{m=1}^{s} \overline{R}^m \overline{R}^{m\prime} \odot I_n \right]^{-1} \left[\sum_{m=1}^{s} \overline{R}^m \odot \overline{R}^m \right], \tag{19}$$

where $\odot$ refers to the Hadamard product.[11]

3.4 Inference

Impulse responses and variance decompositions are usually presented along with some indicator of their statistical reliability. We present in this section a simple way of computing confidence bands for IRF and standard errors for VD using Monte Carlo methods to infer their respective distributions.

Conditional to the state, we would ramdomly generate a large enough set of separate drawns for the parameters collected in $B_{s_t}(L)$. For each of them, we may calculate the respective IRF and VD leading to a numerical approximation of their distributions. This is the basis for adding confidence bands to the IRF estimates and for computing standard errors for VD.

4 Empirical example

In this section we consider an application to real data to illustrate the aforementioned procedures. In our empirical analysis we employ the time series used by KPSW to

[11] Alternatively we may consider the backward-looking variance decomposition. For this attempt we need the sequence of matrices v^s whose (i, j) element is the fraction of the variance at time t of the *i-th* (diferences of the) variable that is accounted for by *j-th* shock hitting the economy at time $t - s$.

gain insights by comparing results from the linear and the nonlinear frameworks.[12] Specifically, we consider the quarterly series y, c and i referred to the logarithms of per capita gross national product, per capita real consumption expenditures and per capita gross private domestic fixed investment.[13] The effective sample runs over the period 1949.1-1988.4 with previous observations being left as initial values.

The preliminary analysis of stationarity and cointegration of the series is sufficiently detailed in KPSW, so this is not treated in this paper. They detect that the three variables are nonstationary and that the number of linearly independent cointegrating vectors is two (i.e. the number of common stochastic trends is one). Based on both economic theory and econometric evidence they propose the cointegrating vectors $(-1, 1, 0)$ and $(-1, 0, 1)$.[14] This is crucial for understanding the dynamics of the model: productivity shocks move the system towards a new steady state producing *balanced* long-run responses of the series. Economic fluctuations are basically movements along the adjustment path to the new steady state.

[12] They develop both a three-variable and a six-variable model. We concentrate our study in the three-variable model to remain tractable the number of parameters to estimate within the nonlinear approach.

[13] The Citibase series used are GNP82 minus GGE82 for output, GC82 for consumption and GIF82 for investment. They are tranformed into per capita data with the series P16.

[14] In basic neoclassical model with uncertainty the great ratios consumption over output and investment over output are stationary stochastic processes along the steady-state. Moreover these models assume a common (to the logarithms of the three series) stochastic trend that allows productivity shocks to raise the expected long-run growth path.

Let us consider first the linear model in Table 1. We introduce some innovations with respect to the KPSW specification. First, they work with eight lags in regressions whereas we have found, using AIC, BIC and Hannan-Quinn that the maximum lag length should be one. Second, we estimate the parameters of the moving average expression using the RVAR model proposed in Warne (1993). Third, we compute the asymptotic standard errors for the IRF and VD instead of the simulations employed by KPSW. Finally, even when we consider the balanced growth shock restriction, we do not normalize the long-run response to 1. Thus, we obtain an estimate of 0.007 (instead of 1) for row of Υ and of 1 (instead of 0.007) for the standard deviation of φ_t.

Within the linear framework, we obtain essentially the same conclusions than KPSW. Figure 1 plots the IRF of the endogenous variables to a one standard deviation change in the technology shock together with the 95% confidence intervals. The three variables present responses over the short-run horizons that are larger than its long-run responses, reaching a peak at one year, and returning smoothly to the steady level of 0.007 after three years. However, we detect differences in the volatility of the responses According to what we expected, investment fluctuates much more dramatically than the other series whereas consumption's response is the most smooth. Additionally, Table 2 reports the fraction of forecast-error variance explained by the permanent shock for various forecast horizons along with the estimates of the asymptotic standard errors. We find that more than two-thirds of the unpredictable variation of output and consumption may be attributed to the balanced-growth shock over the business-cycles horizon. However, the permanent innovation is not able to explain more than two-fifths of the movements of investment at horizons up to six year.

Thus, as in KPSW, we have detected that permanent shocks lead not only to long-run responses but also to transitory movements of output, consumption and investment. These shocks explain a considerable fraction of the short-run variation output and consumption but it is not so clear that short-run variation of investment may be attributable to permanent shocks. This claims for an extensive study of the short-run dynamics of the responses.

Figure 2 plots the logarithms of output, consumption and investment and Figure 3 shows their respective rates of growth, together with the NBER-designated contractionary phases over the effective sample period. They point out interesting features: first, there are clear upward trends of the variables, but they are not smooth curves but are rather a sequence of upturns and downturns that are closely related to the official business-cycles phases. Second, these fluctuations are more dramatic for investment and more smooth for consumption.

The key graph of this study is presented in Figure 4. This plots the equilibrium errors, the logarithms of the consumption:output ratio $(c - y)$ and of the investment:output ratio $(i - y)$. They fluctuate around a constant mean, but the fluctuation has a particular dynamics: the broad changes of direction in the series seem to mark quite well the NBER-referenced business cycles. During recessions, the value of the first equilibrium error is usually greater than its mean, due to the smoothness of consumption that falls less than output. On the other hand, the value of the second equilibrium error declines within recessions due to the higher volatility of investment.

In order to be more confident that equilibrium errors share this business-cycles pattern, we fit for them a MS-VAR model as in (1) with lag length one. Figure 5 displays the filter and smooth probabilities of being in state 2 along with the usual shaded areas correponding to the NBER recessions. It is easy to interpret state 2 as recessions and the series plotted in this chart as probabilities of being in recessions.[15] Thus, according to our theoretical proposal, we have found evidence of a cointegrated system in which, even though the cointegrating vectors are linear, the dynamics of the equilibrium errors towards the linear steady state presents the standard business-cycles asymmetries. This claims for the MS-VECM and MS-CT specifications to consider the short-run and long-run connections among the series.

Tables 3 and 4 present the nonlinear common trends estimates leading to the following interesting features. First, permanent shocks lead to a lower long-run impact (0.005 in nonlinear models vs. 0.007 in linear models). Second, within recessions the system moves more dramatically to the stationary level than within expansions. To see that, let us consider the simple example such that the equilibrium

[15] The accuracy of this indicator at signaling the NBER recessions presents three exceptions: shows two false recession signals at 1951.1 and 1967.1 and anticipates the first 1950's recession.

errors $c_t - y_t$ and $i_t - y_t$ are both equal to 1 at time t. Due exclusively to the long-run adjustment (i.e. $-\alpha_{s_t}\beta' x_t$ in the MS-VECM specification), the rates of growth of output, consumption and investment at time $t+1$ would be 0.08, -0.07 and -0.15 within expansions and 0.66, 0.13 and 0.48 within recessions.

The backward-looking responses to permanent shocks are investigated in Figure 6. This reveals that responses to permanent shocks depend upon the *age* of the shock. The response in 1984.4 is maximum for shocks hitting the system in 1988.4 − 1988.2, is declining for shocks produced between 1988.1 and 1986.2, and reach the long-run level for shocks occurring before 1986. Interestingly, Table 5 shows that shocks produced six year prior to 1988.4 are able to explain almost the totality of the variation in output and consumption and the half of the variation in investment.

As we have stated in Section 3, even though the backward-looking analysis of the responses is the nonlinear version of KPSW study, it may be of interest the study of the dynamic effects of *permanent* shocks hitting the economy in 1988.4. A notable feature of the forward-looking responses to permanent shocks is that the variables do not necessarily have balanced responses. Figure 7 shows again the smoothness of consumption, the volatility of investment, the hump-shaped responses peacking at three quartets and declining after until reaching the long-run response within three years. However, the long-run responses of output, consumption and investment are 0.008, 0.005 and 0.007 respectively. Additionally, Table 6 reveals that the fractions of the short-run forecasted-error attributed to such shocks are higher for output and similar for consumption and investment to those found by KPSW.

Finally, we investigate the possible state-dependence of the short-run effects of permanent shocks. We approach this problem by computing the responses of the series to permanent shocks under the assumption that these shock occurs in 1983.1, just after almost four years of official recessions. With respect to the effects of the shock in 1988.4 (after 5 years of expansions) consumption is the most affected: short run responses due to the permanent shock are more smooth (Figure 8) and this shock account for a smaller fraction of the total variability of consumption at the six-year horizon (Table 7).

5 Conclusion

In this paper, we present both theoretical and empirical reasons to believe that even though equilibrium errors fluctuate around a linear long-run attractor, the dynamic adjustment toward the attractor may be nonlinear. Specifically, we consider a Markov-switching structure in developing a nonlinear stochastic trends model that allows us to connect the short-run and long-run interactions among a set of variables.

For gaining insights in comparison, we apply the model to the data used by King et al. (1991) obtaining two main results. First, we find that the strength to vanish short-run deviations from the long-run equilibrium is higher during recessions. Second, we detect evidence of asymmetric adjustment dynamics to permanent shocks: a shock hitting the economy within expansions is capable of explain larger fraction of the variability of consumption than the same shock occurring just after a deep recession.

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