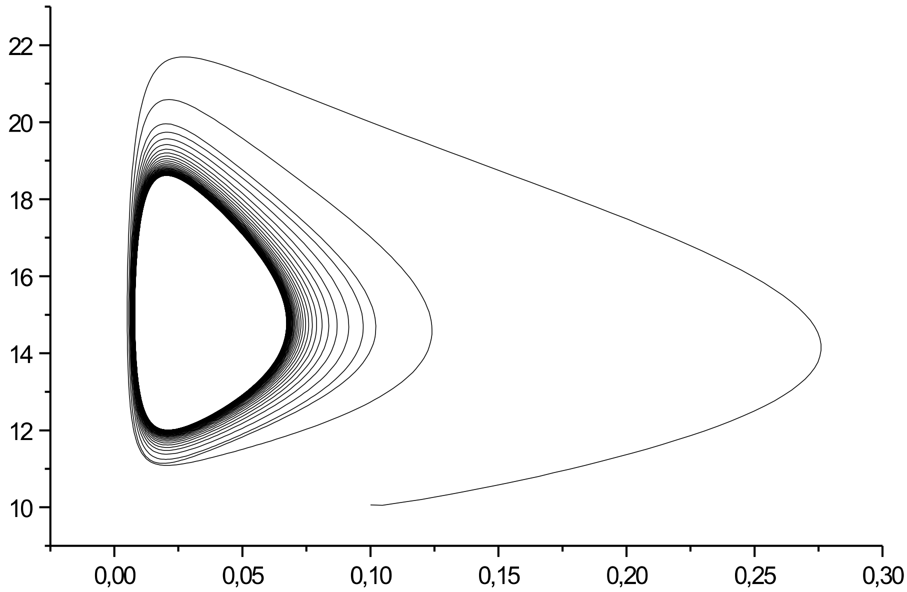

Figure 6.26: The curve (r, p) for $k_1 = 200, v_0 = 1, r_0 = 0.1, \Delta t = 0.01$ and $N = 10000$

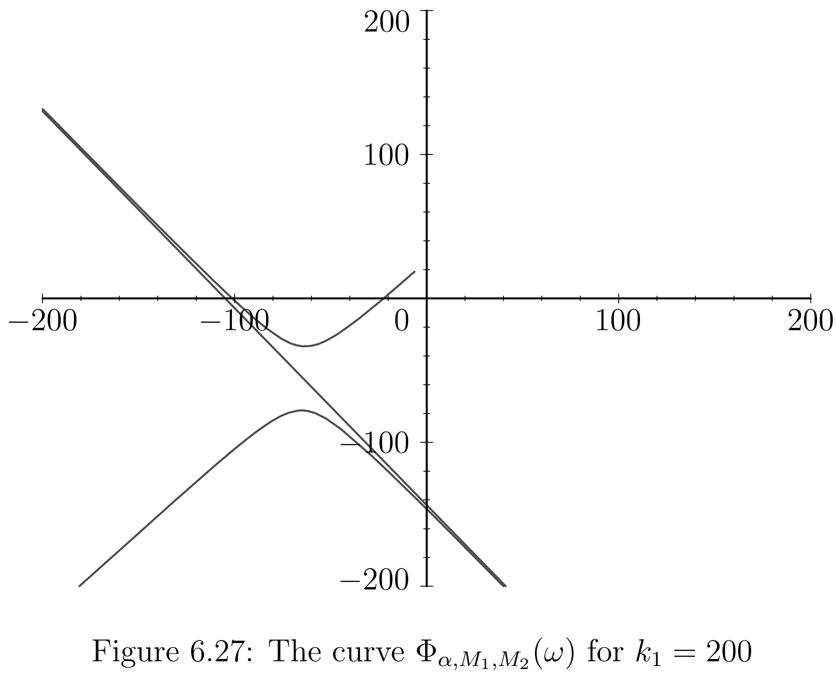

Figure 6.27: The curve $\Phi_{\alpha,M_1,M_2}(\omega)$ for $k_1 = 200$

Remarks

- Notice that in the second example, the convergence of the numerical program requires taking the steptime $\Delta t \leq 0.1$. It seems that this is a consequence of the larger value of the fertility $b(= 10)$.
- Unlike the first example where the stability of the steady state (u_e, v_e, r_e) depends only on k_2, in this one we note that the stability of this steady state does not depend on k_2 (for all k_2, the implicit scheme show that this point is stable if $k_1 < \tilde{k}_1$).
- Changing k_1 from 100 to 110, we note only a significant change for the value of M_1'. This is coherent with the method used in Sect. 4.2.

Next, for both examples we give some figures for some different values of the initial conditions. It seems from these figures that when (u_e, v_e, r_e) is stable it is moreover a global attractor.

Finally, we can note that the linear implicit program and the Fortran program seem to work very well. This is reflected especially in the curves (r, v) and (r, p) that approach the axes but never become negative.

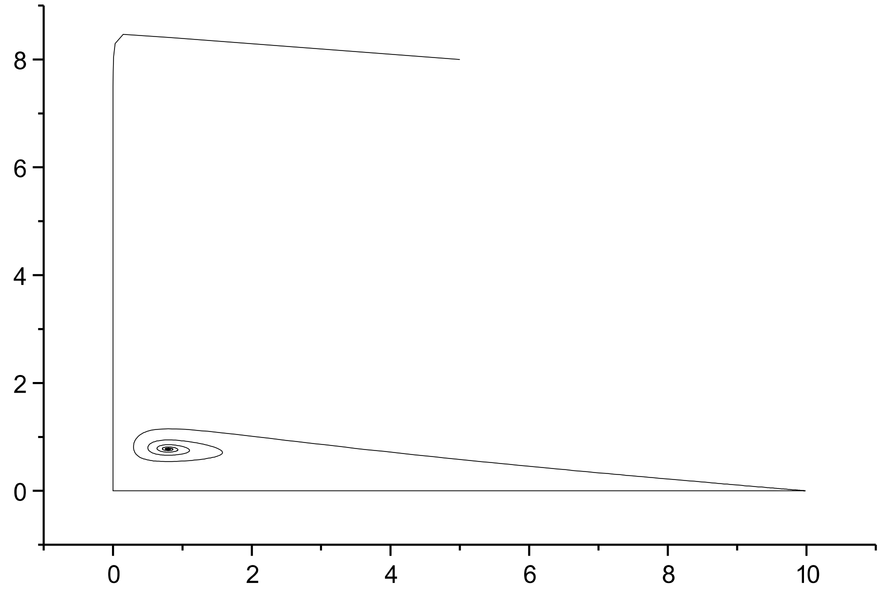

Figure 6.28: The curve (r, v) for $k_2 = 1, v_0 = 8, r_0 = 5, \Delta t = 0.1$ and $N = 3000$

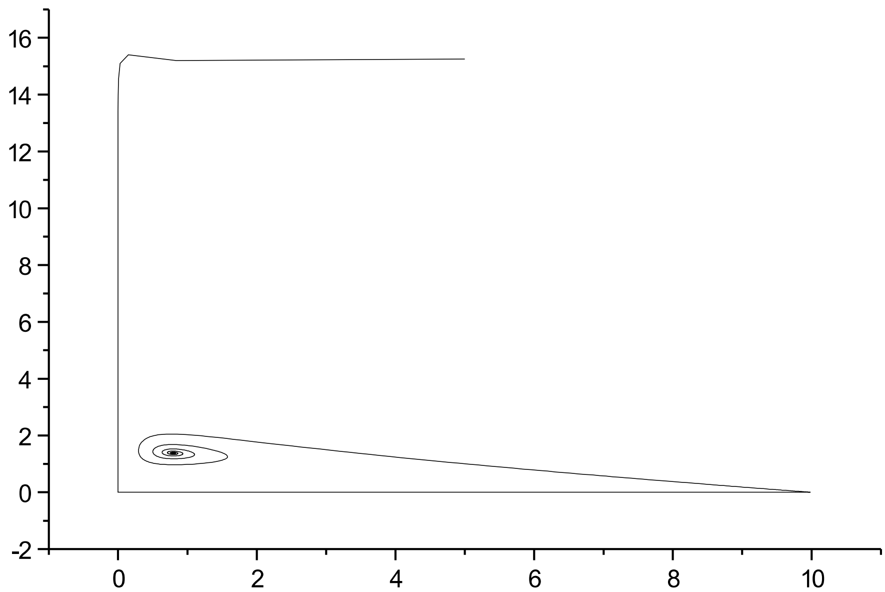

Figure 6.29: The curve (r, p) for $k_2 = 1, v_0 = 8, r_0 = 5, \Delta t = 0.1$ and $N = 3000$

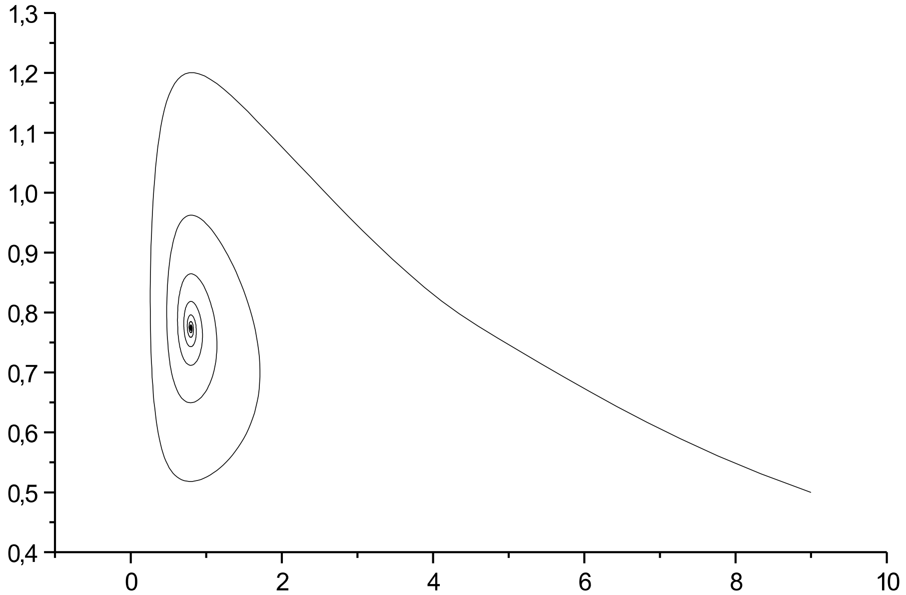

Figure 6.30: The curve (r, v) for $k_2 = 1, v_0 = 0.5, r_0 = 9, \Delta t = 0.1$ and $N = 3000$

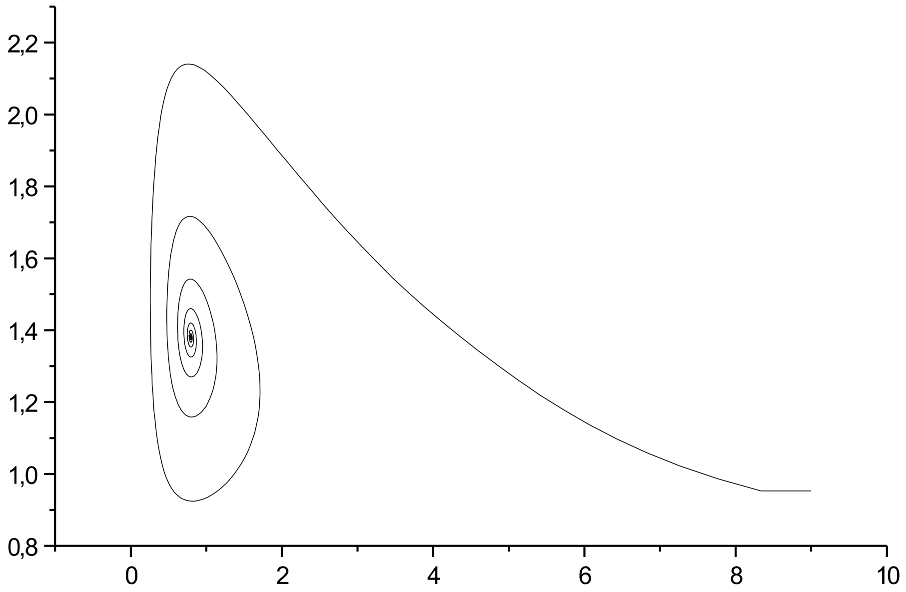

Figure 6.31: The curve (r, p) for $k_2 = 1, v_0 = 0.5, r_0 = 9, \Delta t = 0.1$ and $N = 3000$

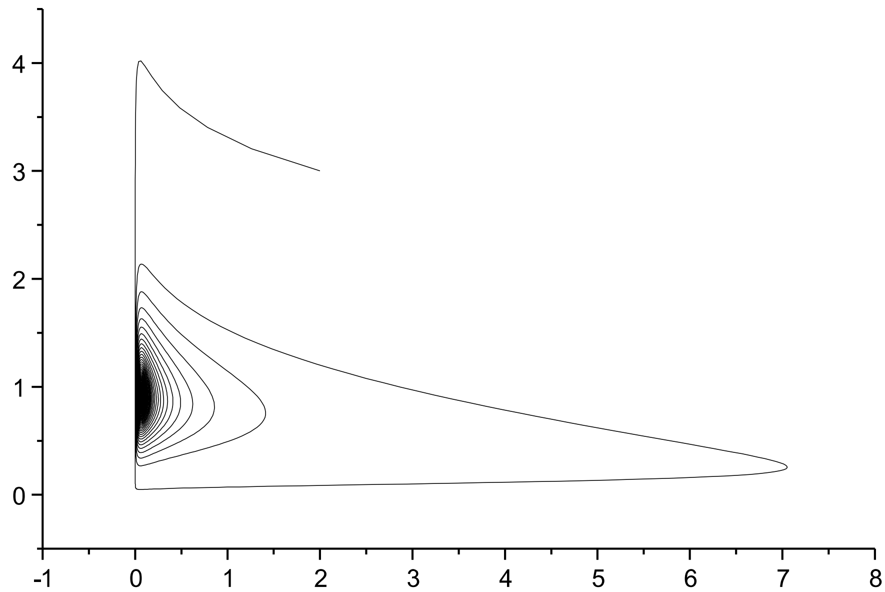

Figure 6.32: The curve (r, v) for $k_2 = 30, v_0 = 3, r_0 = 2, \Delta t = 0.1$ and $N = 20000$

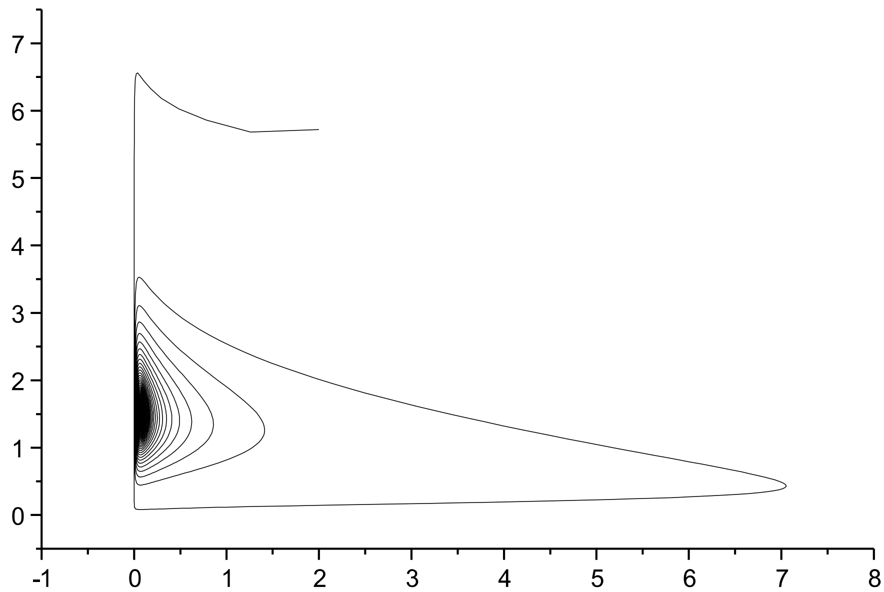

Figure 6.33: The curve (r, p) for $k_2 = 30, v_0 = 3, r_0 = 2, \Delta t = 0.1$ and $N = 20000$

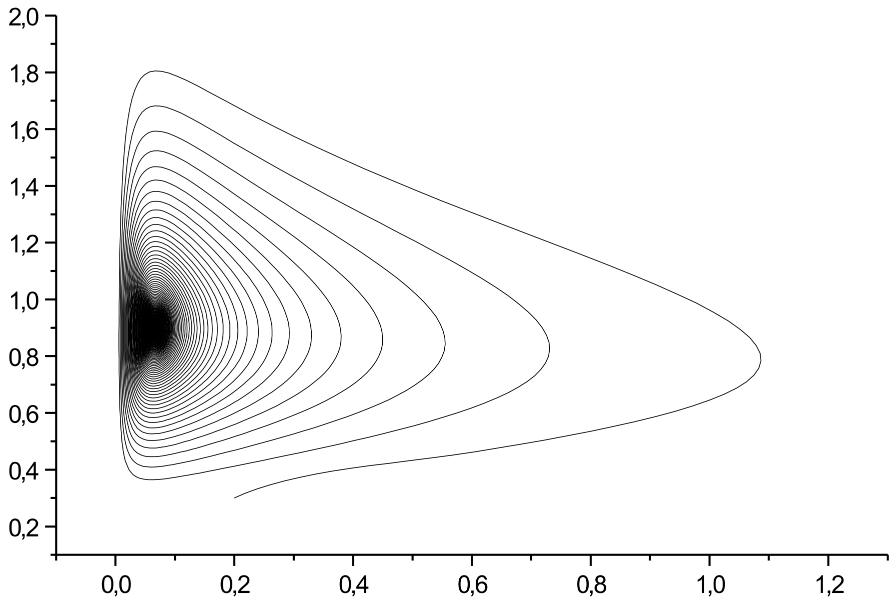

Figure 6.34: The curve (r, v) for $k_2 = 30, v_0 = 0.3, r_0 = 0.2, \Delta t = 0.1$ and $N = 20000$

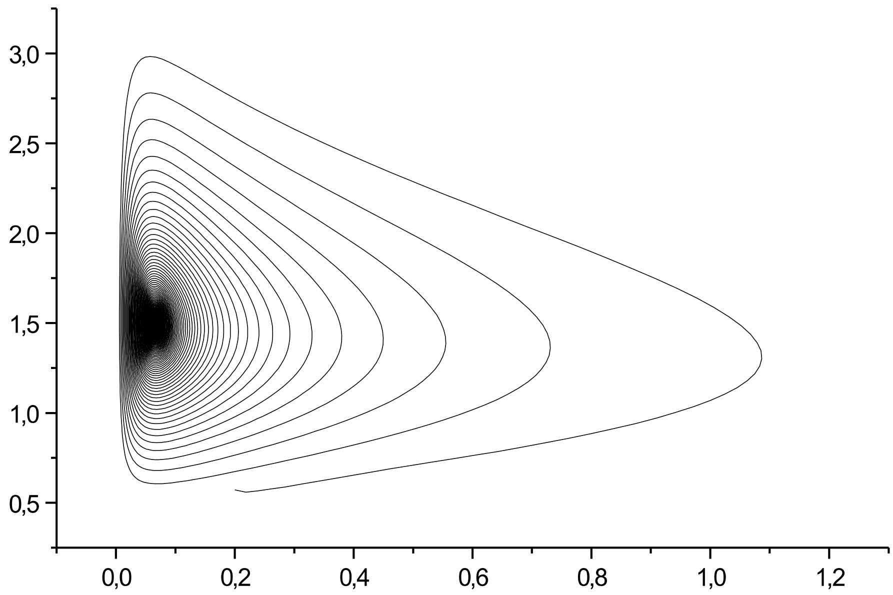

Figure 6.35: The curve (r, p) for $k_2 = 30, v_0 = 0.3, r_0 = 0.2, \Delta t = 0.1$ and $N = 20000$

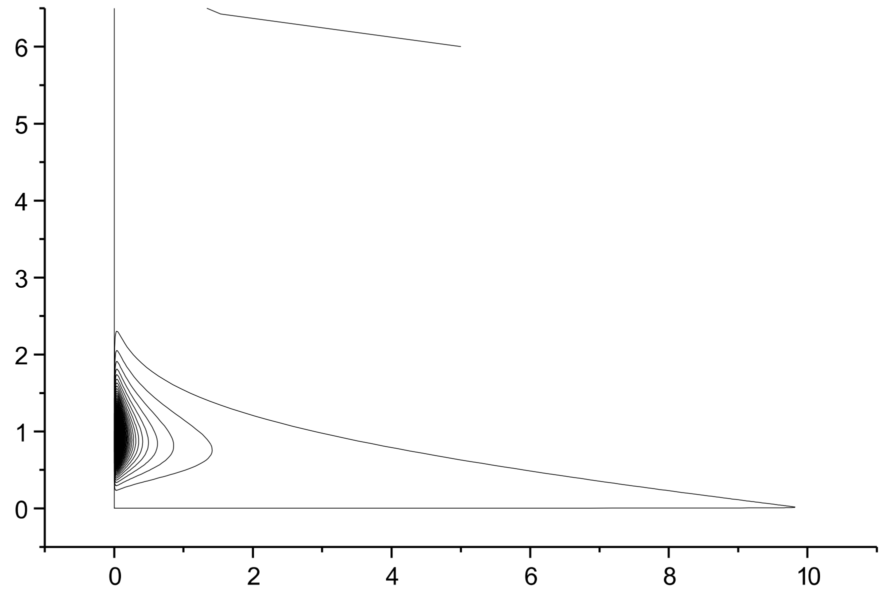

Figure 6.36: The curve (r, v) for $k_2 = 60, v_0 = 6, r_0 = 5, \Delta t = 0.1$ and $N = 100000$

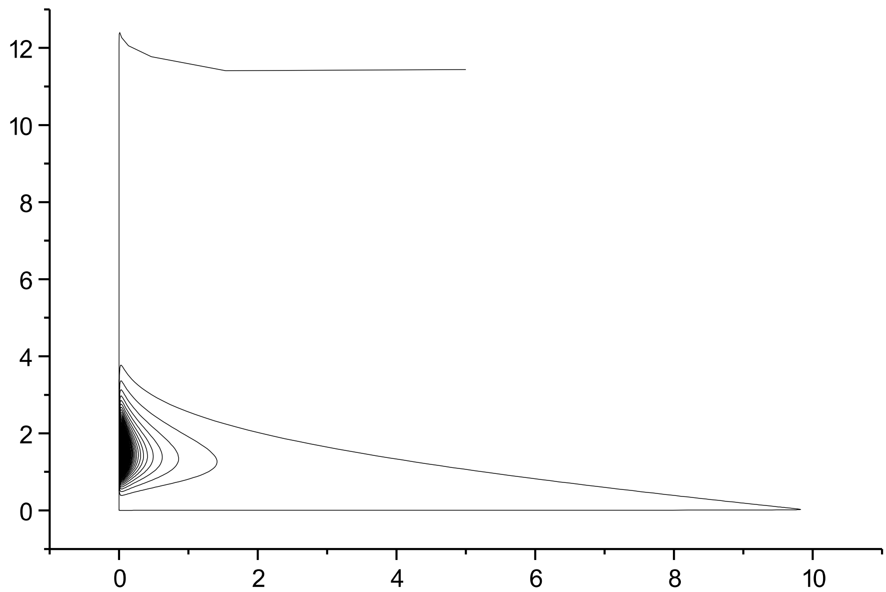

Figure 6.37: The curve (r, p) for $k_2 = 60, v_0 = 6, r_0 = 5, \Delta t = 0.1$ and $N = 100000$

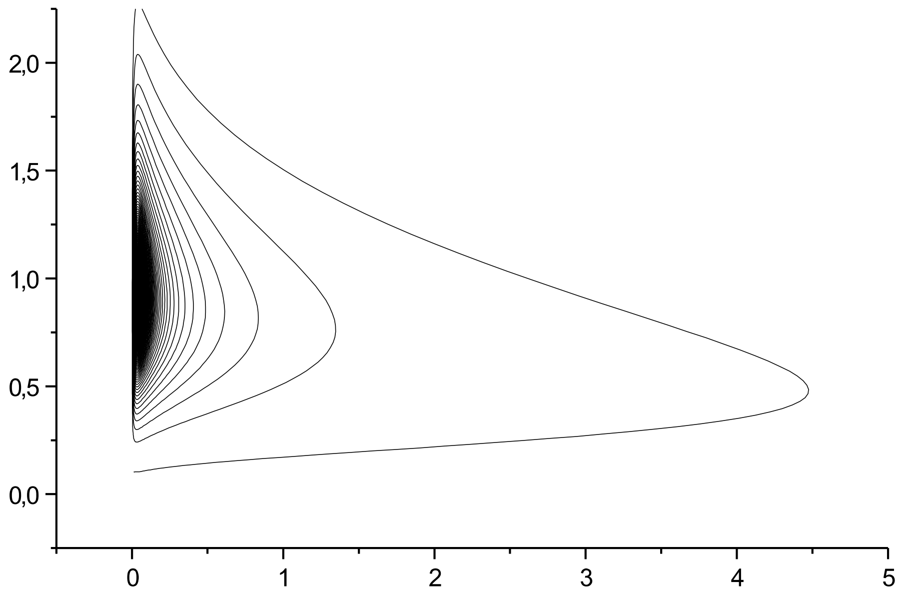

Figure 6.38: Curve (r, v) for $k_2 = 60, v_0 = 0.1, r_0 = 0.01, \Delta t = 0.1$ and $N = 100000$

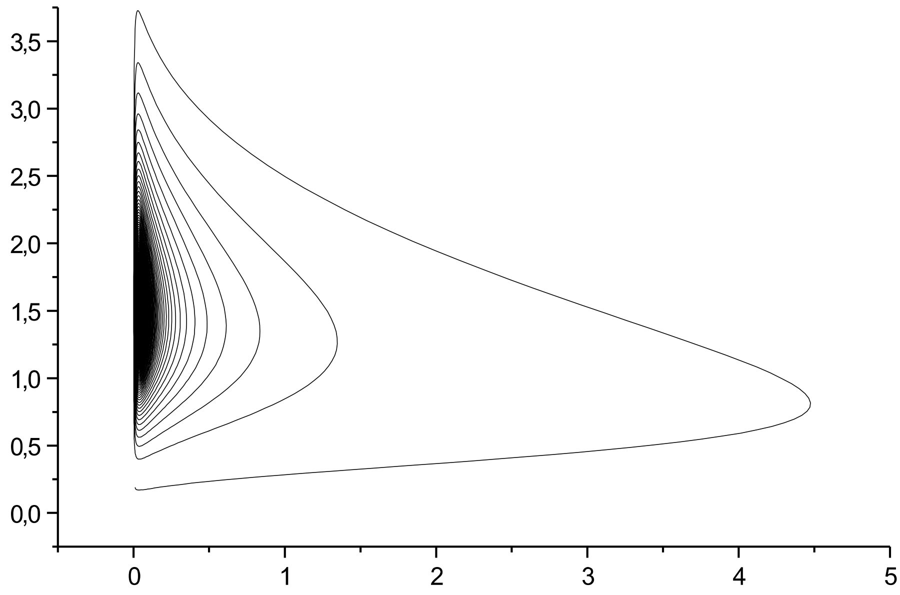

Figure 6.39: Curve (r, p) for $k_2 = 60, v_0 = 0.1, r_0 = 0.01, \Delta t = 0.1$ and $N = 100000$

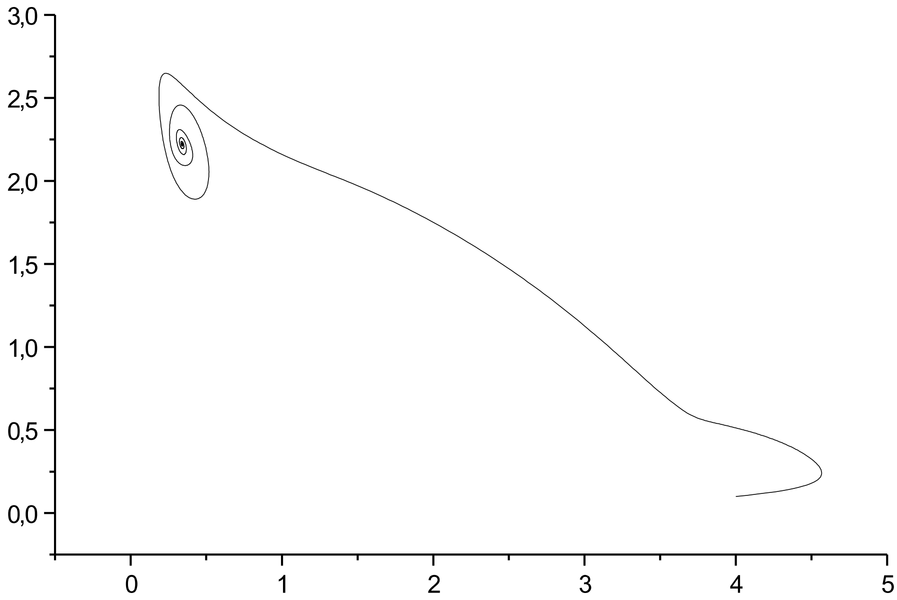

Figure 6.40: The curve (r, v) for $k_1 = 10, v_0 = 0.1, r_0 = 4, \Delta t = 0.01$ and $N = 3000$

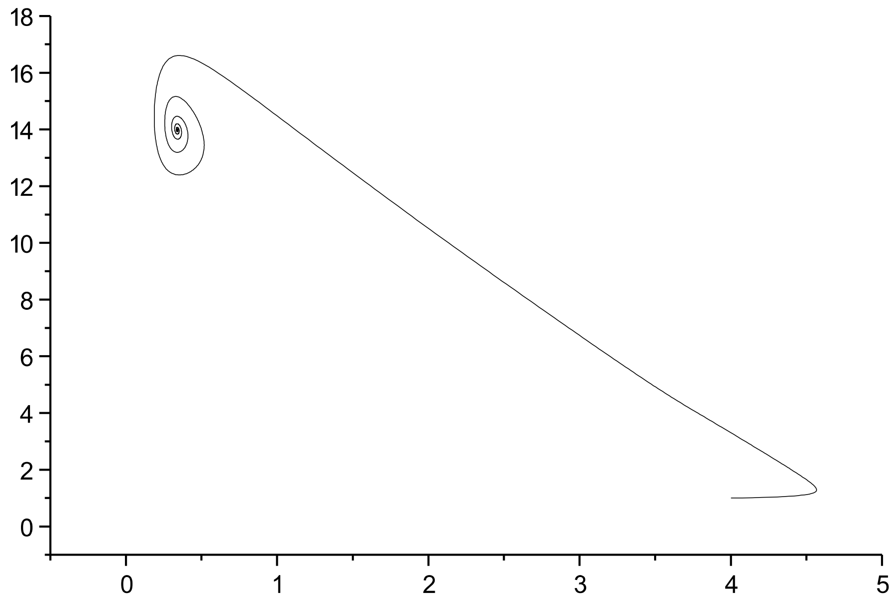

Figure 6.41: The curve (r, p) for $k_1 = 10, v_0 = 0.1, r_0 = 4, \Delta t = 0.01$ and $N = 3000$

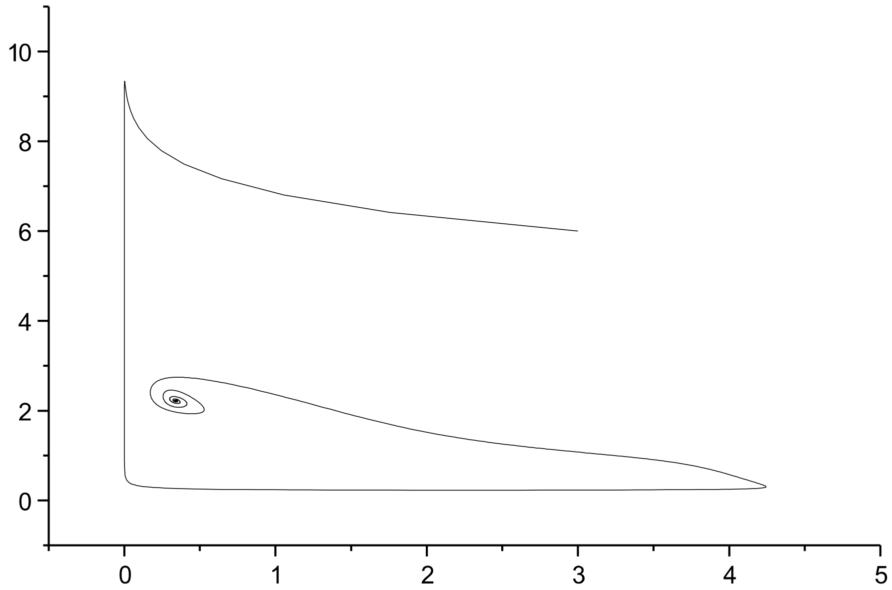

Figure 6.42: The curve (r, v) for $k_1 = 10, v_0 = 6, r_0 = 3, \Delta t = 0.01$ and $N = 3000$

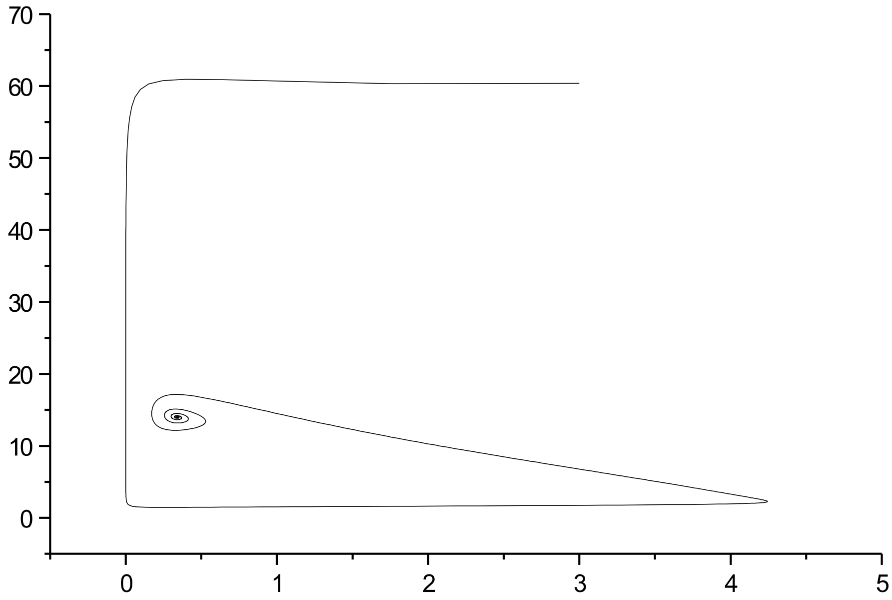

Figure 6.43: The curve (r, p) for $k_1 = 10, v_0 = 6, r_0 = 3, \Delta t = 0.01$ and $N = 3000$

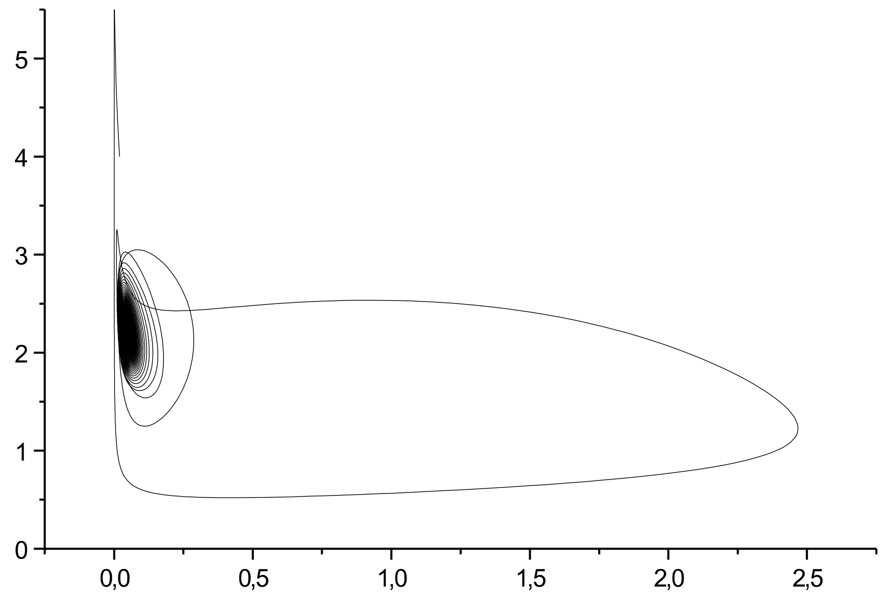

Figure 6.44: The curve (r, v) for $k_1 = 100, v_0 = 4, r_0 = 0.2, \Delta t = 0.01$ and $n = 100000$

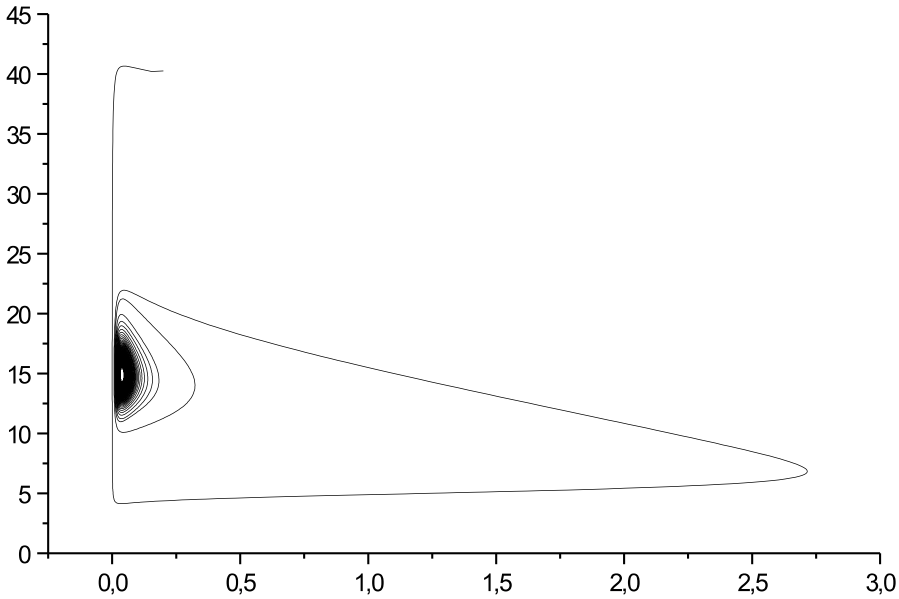

Figure 6.45: The curve (r, p) for $k_1 = 100, v_0 = 4, r_0 = 0.2, \Delta t = 0.01$ and $n = 100000$

```
C     FORTRAN PROGRAM FOR TO SOLVE LINEAR SYSTEM (4.9)
C
      EXTERNAL CALCUL,MATRIZ,VECTOR,GAUSS,SOLVE
C
      INTEGER I,J,N,M
      REAL*8 M1,M1D,M2,M2D,BB,R,V,G,GD,L1,M10,M11,M20,M21,K1,K2,RC,C1,C2
     *,C3,C,K,L,DT,DA,DV,DR,W,P
      REAL*8 MAT(100,100),B(1:100),AGE(0:97),DU(0:97),U1(0:97)
C
C
      OPEN(99,FILE ='RES.TXT')
      OPEN(98,FILE ='DATOS.DAT')
C
C
      READ(98,*) N,M,DT,L,BB,M10,M11,K1,M20,M21,K2,C1,C2,C3,RC,C,K,R,V
C
C    DA = STEPSIZE IN AGE C
      DA = L/(M-3)
        DO 1 I = 0,M-3,1
           AGE(I) = I*DA
1       CONTINUE
         DO 2000 I=0,M-3,1
            U1(I) = C*EXP(-K*AGE(I))
2000     CONTINUE C C     P = THE TOTAL POPULATION OF PREDATORS C
    P = (DA*(U1(0) + U1(M-3))/2.0D0) + v
C
        DO 3000 I = 1,M-4,1
           P = P + DA*U1(I)
3000    CONTINUE
C       WRITE(99,*) (U1(I), I = 0,M-3)
        WRITE(99,*) V,R
C     WRITE(99,*) P,R
C
          DO 2 J = 1,N, 1
```

```
C
      CALL CALCUL (M1,M1D,M2,M2D,G,GD,L1,M10,M11,M20,M21,K1,K2,RC,C1,C2,
     *C3,R,V,DA,M,U1)
C
      CALL MATRIZ (MAT,DT,DA,M1,M1D,M2,M2D,G,GD,L1,C1,C2,R,V,M,BB,U1)
C
      CALL VECTOR(M,DT,DA,M1,M2,R,V,G,L1,B,U1)
C
      CALL GAUSS(M,MAT,B)
C
      CALL SOLVE(M,MAT,B,DU,DV,DR)
C
          DO 3 I=0,M-3,1
           U1(I) = U1(I) + DU(I)
3         CONTINUE
         P = (DA*(U1(0) + U1(M-3))/2.0D0) + v
C
       DO 4000 I = 1,M-4,1
          P = P + DA*U1(I)
4000     CONTINUE

          V = V + DV
          R = R +   DR

        WRITE(99,*) V,R
C     WRITE(99,*) P,R 110     FORMAT ('0', 10(F6.3,1X))
2     CONTINUE
       W = (C1*DA*(U1(0) + U1(M-3))/2.0D0) + C2*V
C
       DO 201 I = 1,M-4,1
          W = W + C1*DA*U1(I)
201     CONTINUE C       WRITE(99,*) W C
        END
C
```

```
C
      SUBROUTINE CALCUL (M1,M1D,M2,M2D,G,GD,L1,M10,M11,M20,M21,K1,K2,RC,
     *C1,C2,C3,R,V,DA,M,U1)
C
      INTEGER I,M
      REAL*8 M1,M1D,M2,M2D,R,V,G,GD,L1,M10,M11,M20,M21,K1,K2,RC,C1,C2,C3
     &,DA
      REAL*8 U1(0:97)
C
C     M1 = DEATH RATES OF THE JUVENILES
C     M1D = DERIVATIVE OF M1

C     M2 = DEATH RATES OF THE THE ADULTS
C     M2D = DERIVATIVE OF M2
C     GD = DERIVATIVE OF
C
C     L1 = L(U,V)
C
          M1 = M11 + (M10 - M11)/(1 + K1*R)
          M1D = K1*(M11 - M10)/((1 + K1*R)**2)
          M2 = M21 + (M20 - M21)/(1 + K2*R)
          M2D = K2*(M21 - M20)/((1 + K2*R)**2)
          G = C3*(1 - R/RC)
          GD = -C3/RC
          L1 = (C1*DA*(U1(0) + U1(M-3))/2.0D0) + C2*V
C
       DO 10 I = 1,M-4,1
          L1 = L1 + C1*DA*U1(I)
10     CONTINUE
       RETURN
       END
C

      SUBROUTINE MATRIZ (MAT,DT,DA,M1,M1D,M2,M2D,G,GD,L1,C1,C2,R,V,M,BB,
```

```
     &U1)
C
C      BB = THE FERTILITY OF THE ADULTS
C      M  = EL ORDER OF MAT

C      MAT = MATRIX WHICH WE WANT TO TRIANGULATE C
        INTEGER I,J,M
        REAL*8 M1,M1D,M2,M2D,BB,R,V,G,GD,L1,C1,C2,DT,DA
        REAL*8 MAT(100,100),U1(0:97)
C
       DO 400 I=1,M,1
        DO 500 J=1,M,1
        MAT(I,J)=0.0D0
500     CONTINUE 400   CONTINUE
         MAT(1,1) = 1.0D0 + (DT*M2/2.0D0)
         MAT(2,1) = -BB
         MAT(2,2) = 1.0D0
         MAT(3,2) = -DT/(4*DA)
         MAT(M,1) = DT*C2*R/2.0D0
         MAT(M,2) = DT*DA*C1*R/4.0D0
         MAT(M,M-1)=DT*DA*C1*R/4.0D0
         MAT(M,M) = 1.0D0 + (DT*(-R*GD - G + L1)/2.0D0)
         MAT(1,M-1)=-DT/2.0D0
         MAT(1,M)=DT*V*M2D/2.0D0
         MAT(M-2,M-1)=DT/(4.0D0*DA)
         MAT(M-1,M-1)=1.0D0 + (DT*(M1*DA + 1.0D0)/(2.0D0*DA))
         MAT(M-1,M-2)=-DT/(2.0D0*DA)
         MAT(M-1,M)=DT*U1(M-3)*M1D/2.0D0
         DO 20 I = 3, M-2, 1
         MAT(M,I)=DT*DA*C1*R/2.0D0
         MAT(I,M)=DT*U1(I-2)*M1D/2.0D0
20       CONTINUE
         DO 21 I = 3, M-2, 1
         MAT(I,I) = 1.0D0 + (DT*M1/2.0D0)
```

```
21      CONTINUE
        DO 22 I= 3, M-3, 1
        MAT(I,I+1)=DT/(4.0D0*DA)
        MAT(I+1,I)=-DT/(4.0D0*DA)
22      CONTINUE
C        WRITE (99,*) ((MAT(I,J),J=1,M),I=1,M)
C       RETURN
C       END
C
      SUBROUTINE VECTOR(M,DT,DA,M1,M2,R,V,G,L1,B,U1)
C
        INTEGER I,M
        REAL*8 DT,DA,M1,M2,R,V,G,L1
        REAL*8 B(100),U1(0:97)
C
C     B = THE VECTOR SUCH THAT THE LINEAR SYSTEM MAT*X = B
C     X = UNKNOWNS
        B(1)=DT*(U1(M-3) - M2*V)
        B(2)=0.
        B(M-1)=-DT*(M1*U1(M-3) + (U1(M-3) - U1(M-4))/DA)
        B(M)=DT*R*(G-L1)
        DO 30 I=3,M-2, 1
      B(I)=-DT*(M1*U1(I-2) + (U1(I-1) - U1(I-3))/(2.0D0*DA))
30      CONTINUE
C         WRITE(99,*)(B(I), I=1,M)
        RETURN
        END
C
C
      SUBROUTINE GAUSS(M,MAT,B)
C
       INTEGER I,M
       REAL*8 MAT(100,100),B(1:100)
C     TRIANGULATION OF MAT BY GAUSSIAN ELIMINATION
```

```
      DO 40 I=2,M-2, 1
        MAT(I,I)= MAT(I,I) - MAT(I-1,I)*MAT(I,I-1)/MAT(I-1,I-1)
        MAT(I,M-1)=MAT(I,M-1) - MAT(I-1,M-1)*MAT(I,I-1)/MAT(I-1,I-1)
        MAT(I,M)=MAT(I,M) - MAT(I-1,M)*MAT(I,I-1)/MAT(I-1,I-1)
        MAT(M,I)=MAT(M,I) - MAT(I-1,I)*MAT(M,I-1)/MAT(I-1,I-1)
        MAT(M,M-1)=MAT(M,M-1) - MAT(I-1,M-1)*MAT(M,I-1)/MAT(I-1,I-1)
        MAT(M,M)=MAT(M,M)-MAT(I-1,M)*MAT(M,I-1)/MAT(I-1,I-1)
        B(I) = B(I) - B(I-1)*MAT(I,I-1)/MAT(I-1,I-1)
        B(M) = B(M) - B(I-1)*MAT(M,I-1)/MAT(I-1,I-1)
        MAT(I,I-1) = 0.
        MAT(M,I-1) = 0.
40    CONTINUE
      MAT(M-1,M-1)=MAT(M-1,M-1)-MAT(M-2,M-1)*MAT(M-1,M-2)/MAT(M-2,M-2)
      MAT(M-1,M)=MAT(M-1,M)-MAT(M-2,M)*MAT(M-1,M-2)/MAT(M-2,M-2)
      B(M-1)=B(M-1) - B(M-2)*MAT(M-1,M-2)/MAT(M-2,M-2)
      MAT(M,M-1)=MAT(M,M-1) - MAT(M-2,M-1)*MAT(M,M-2)/MAT(M-2,M-2)
      MAT(M,M)=MAT(M,M) - MAT(M-2,M)*MAT(M,M-2)/MAT(M-2,M-2)
      B(M)= B(M) - B(M-2)*MAT(M,M-2)/MAT(M-2,M-2)
      MAT(M-1,M-2)=0.
      MAT(M,M-2)=0.
      MAT(M,M)=MAT(M,M) - MAT(M-1,M)*MAT(M,M-1)/MAT(M-1,M-1)
      B(M)= B(M) - B(M-1)*MAT(M,M-1)/MAT(M-1,M-1)
      MAT(M,M-1)=0.
      RETURN
      END
C
C
     SUBROUTINE SOLVE(M,MAT,B,DU,DV,DR)
C
      INTEGER I,M
      REAL*8 DV,DR,DU(0:97),MAT(100,100),B(100)
C C   SOLUTION OF THE LINEAR SYSTEM AFTER THE TRIANGULATION OF
MAT C
      DR=B(M)/MAT(M,M)
```

```
      DU(M-3)=(B(M-1) - MAT(M-1,M)*DR)/MAT(M-1,M-1)
      DU(0)=(B(2) -  (MAT(2,M-1)*DU(M-3)) - (MAT(2,M)*DR))/MAT(2,2)
      DU(M-4)=(B(M-2)-MAT(M-2,M-1)*DU(M-3)-MAT(M-2,M)*DR)/MAT(M-2,M-2)
C      WRITE(99,*)B(1),MAT(1,M-1),DU(M-3),MAT(1,M),DR,MAT(1,1)
      DV=(B(1) - (MAT(1,M-1)*DU(M-3)) - (MAT(1,M)*DR))/MAT(1,1)
     DO 50 I=M-3,3, -1
     DU(I-2)=(B(I)-(MAT(I,I+1)*DU(I-1))-(MAT(I,M-1)*DU(M-3))-(MAT(I,M)*
    & DR))/MAT(I,I)
50      CONTINUE
      RETURN
      END
```

Concluding remarks

The dynamics of many populations depend in an essential way on the life cycle stages of individuals that form the population. In this thesis a semilinear equation is considered and analyzed for a nonlinear age-structured population model with two stages (juveniles structured by age and adults unstructured), which feeds on a single resource (unstructured). The model is continuous and leads to three differential equations, with a nonlinearity in the equation for the resource. The first and the second equations describe, respectively, the dynamics of the juvenile class and the adult class which depend on one type of resource via their death rates. The third equation describes the dynamics of the resource. The realization of the model is a nonlinear partial differential equation coupled to two ordinary differential equations. The thesis is divided into six chapters.

In the first chapter we construct Problem (2.1) and, in order to motivate this problem, we make some comments about the maturation age l from the evolutionary point of view. The simplest biologically significant hypothesis is to assume that the adult fertility is an increasing function of the maturation age, i.e. $b = b(l)$, vanishing at 0. The main result of this chapter is Proposition 1.1 which concludes that there is a unique E.S.S. $\hat{l}$ if and only if $b(l) > \nu$ for some l and $\lambda(\hat{l})$ belongs to the interval $(m(\infty), m(0))$.

In the second chapter we analyze Problem (2.1) without distinguishing between the nonuniform mortality case and the uniform case. We show that the solutions of the linear part of Problem (2.1) are C^1 functions of time and give rise to a linear semigroup. Using the theory of semilinear equations in Banach spaces (see [65]) this chapter also deals with the existence and uniqueness of the solutions which evolve with time in $L^1[0,l] \times \mathbb{R}^2$. In order to assure the global existence we prove the positivity of the solutions. Theorem 2.7 studies the equilibria of Problem 2.1. The trivial equilibrium $(0,0,0)$ and the stationary solution without predators $(0,0,r_c)$ are always equilibrium points. If $b = m_2(0)e^{\int_0^l m_1(a,0)\,da}$ then $\left(bve^{-\int_0^a m_1(a',0)\,da'}, v, 0\right)$ are equilibrium points for any $v > 0$. Whenever $b \in \left(m_2(r_c)e^{\int_0^l m_1(a,r_c)\,da}, m_2(0)e^{\int_0^l m_1(a,0)\,da}\right)$ there is a unique nontrivial equilibrium (u_e, v_e, r_e). Moreover $0 < r_e < r_c$. In particular, the condition for the existence of the coexistence equilibrium is equivalent to

$$\frac{be^{-\int_0^l m_1(a,0)\,da}}{m_2(0)} < 1 < \frac{be^{-\int_0^l m_1(a,r_c)\,da}}{m_2(r_c)}.$$

Let us notice that if one considers the (linear) equations for the consumer populations assuming a constant amount of resources r, then the so-called reproduction number R_0, i.e. the expected number of offspring of an individual along its whole life, takes the value $\frac{be^{-\int_0^l m_1(a,r)\,da}}{m_2(r)}$ (since $e^{-\int_0^l m_1(a,r)\,da}$ is the probability of surviving until becoming an adult whereas $\frac{1}{m_2(r)}$ is the expected value of the length of the life as adult). So, not surprisingly, the condition above can be rephrased by saying that an equilibrium with non-trivial consumer population does exists if and only if the reproduction number (of the consumers) is bigger than one for a prey population number equal to the carrying capacity (so then the environment can sustain both the prey and, indirectly, the predators) and it is smaller than one when there are no resources (this is biologically clear and, in fact, values of the fertility modulus b not fulfilling this hypothesis lack any biological sense).

The third chapter proves results about the asymptotic behaviour of the solutions of System (2.1) in the case of uniform increase of mortality, i.e. when the death rates of juveniles and adults differ by a constant: $m_2(r) = \nu + m_1(r)$. It establishes these results by first using that the linear part of the equations for the predators generates a strongly continuous linear semigroup and that the complete system can be reduced to a two dimensional system of nonlinear ordinary differential equations with time-dependent coefficients. Time-dependence is introduced by the solution of the linear equation for the predators. The study concentrates on the two-dimensional system. A distinctive feature of this work is how we cope with time-dependence. In fact, the coefficients of the equation are asymptotically constant. A crucial result in that respect is Proposition 3.4 showing that all the solutions are bounded, together with Theorem 3.4 which states that the semigroup is eventually compact. The two results together allow us to restrict the study to the omega-limit set of the solutions, where the equation reduces to a time-dependent two dimensional system of ordinary differential equations. This part of the work is in the line of a steadily developing research about infinite dimensional systems which asymptotically reduce to finite dimensional systems. Amongst people who contributed to this line of research, we can quote, with no claim for being exhaustive, the pioneer papers by Markus and the much more recent work by K. Mischaikow, H.L. Smith and H.R. Thieme [62, 81], O. Arino and M. Kimmel [5] and O. Arino and M. Pituk [9].

Theorem 3.11 describes completely the dynamics of System 2.1. The results depend on the number λ^*, which is the dominant eigenvalue of the operator A. This number can be thought of as a measure of the fitness of the population when there is no restriction for food.

In particular, if $\lambda^* \leq m_1(r_c)$, which is equivalent to the adult fertility b being less than or equal to $m_2(r_c)e^{m_1(r_c)l}$, then there is no coexistence equilibrium and the consumer population becomes extinct whereas the resource amount tends to the environmental capacity r_c. On the other hand, if $\lambda^* \geq m_1(0)$, i.e., if $b \geq m_2(0)e^{m_1(0)l}$, then there is no coexistence equilibrium but now the solutions are unbounded.

Finally, the most biologically significant case arises if $m_1(r_c) < \lambda^* < m_1(0)$, i.e., if the adult fertility b belongs to the interval $\left(m_2(r_c)e^{m_1(r_c)l}, m_2(0)e^{m_1(0)l}\right)$. In this case a coexistence equilibrium exists which is a global attractor if either $g(r) \equiv r$, or $G'(r_e) < 0$ and the limit of the asymptotically autonomous ordinary differential system has not any periodic orbit. We note that in the case $\tilde{g}(r) \neq r$ and $G'(r_e) > 0$ and if the periodic orbits of the limit of the asymptotically autonomous ordinary differential system are all isolated, then any $\omega-$limit set of a non-stationary solution is a periodic orbit.

The first goal of Chapter 4 has been to exploit the semilinear formulation in the treatment of an age-structured population dynamics model, mainly from the viewpoint of the asymptotic behaviour of the solutions in the nonuniform mortality rates case.

In the case when m_1 depends only on r and $g(r) \equiv r$, the stability/instability of the equilibria of System 2.1 is studied almost completely using the characteristic equation. In particular the coexistence equilibrium, whenever it exists, turns out to be always asymptotically stable if $m_1'(r_e) = m_2'(r_e)$ (see Theorem 4.6). This is a (local) generalization of the results of Theorem 3.11 where the coexistence equilibrium is shown to be a global attractor assuming $m_1(r) \equiv m_2(r) + \nu$. To go further in the analysis of the characteristic equation obtained from the linearization at the coexistence equilibrium, the operator $L(u, v)$, measuring the relative weight of the consumers in terms of predation pressure, is taken to be equal to the total population of predators. This permits an explicit computation of the stability curve (4.13) in the parameter plane. Crossing this curve generates a loss of stability of the coexistence equilibrium point via a Hopf bifurcation. Moreover, the stability curve shows, for instance, that a small value of v_e, i.e., a coexistence equilibrium close to the non-coexistence one $(0, 0, r_c)$ (see Section 4.1), ensures stability except for very large values of $|m_i'(r_e)|, i = 1, 2$. This is an extension of (local) Theorem 4.5 showing that small equilibrium predator populations tend (in our model) to be stable. Moreover, the stability curve keeps away from the diagonal $m_1'(r_e) = m_2'(r_e)$ showing that the coexistence equilibrium point can become unstable only when the death rates of the juveniles and the adults react in a noticeably different way to (local) changes in the amount of resources. *A biological interpretation of this fact is that a destabilizing mechanism arises when young and adults are sufficiently different with respect to their relationship to resources.*

On the other hand, Proposition 4.2 states that the coexistence equilibrium is stable for $|\alpha| = |g'(r_e)|r_e$ sufficiently large. Moreover, the size of the stability regions tend to increase when $|\alpha|$ increases and the other parameters remain fixed (see Figure 4.4 and Figure 4.5). This can be interpreted biologically as a stabilizing mechanism: a larger sensitivity of the resources relative growth rate $g(r)$ to changes in the amount of resources causes an increase of the stability of the coexistence equilibrium point. Notice that this is a generalization of the same property held by the internal dynamics of the (logistic) equation for the amount of the resources.

Proposition 4.3 (see also Figure 4.1 and Figure 4.3) shows that if $m_2(r_e)$ and $|m_2'(r_e)|$ are both small, i.e. the adults death rate is small and not very sensitive to the amount of the resources, then the coexistence equilibrium is stable.

With respect to the global dynamics, in some biologically relevant cases, the asymptotic extinction of the consumer species is proven (Theorem 4.2) and in others, existence of a compact global attractor, containing a coexistence equilibrium, is established.

The key point in proving dissipativeness property of the model lies in the sensitivity of the adult death rate m_2 to the resource level. More precisely, the results on existence of the global attractor apply to species with a birth rate ranging from 0 up to the value of the adult death rate when there are no resources ($m_2(0)$) (see Theorem 4.11). In most cases one expects that $m_2(0)$ be very large compared to the death rate when there are resources an infinite amount of resources ($m_2(\infty)$), so that Theorem 4.12 works for the values of the birth rate b that make biological sense.

Nevertheless, in some species with adult populations having little need of resources, for instance lepidopterous insects with short living imagos (e.g. silkworms), the adult death rate in starvation conditions ($m_2(0)$) may not be much larger than the same rate with infinite amount of resources ($m_2(\infty)$). In these cases, the existence of a compact global attractor remains unsolved for a large range of values of the birth rate. This is perhaps not only a technical difficulty. In fact, very large birth rates, namely, those larger than $m_2(0)e^{m1(0)l}$, give rise to unbounded solutions, obviously lacking biological relevance.

Removing the hypothesis of uniform increase of mortality destroys the algebraic structure of the system in the sense that the right hand side of the equation for u and v is no longer a linear operator plus a scalar multiple of the identity operator. This prevents from obtaining the solutions in the form of a real function times a linear semigroup. So in the general case with death rates $m_1(a,r)$ and $m_2(r)$ the knowledge of the global dynamics seems to be very difficult. Nevertheless, a perturbation study starting from the "uniform increase of mortality" case is still possible and it is undertaken in Chapter 5. The perturbation of Problem 2.1 is made through in the perturbation of the death rate of the juveniles by a function $\varepsilon(a,r)$ which depends on the age and the amount of the resources. The goal of this chapter is stated in Theorem 5.3 showing that the coexistence equilibrium point $(u_\varepsilon, v_\varepsilon, r_\varepsilon)$ of Problem (5.5) is asymptotically stable when the norm of the perturbation is bounded by a constant which depends on the parameters of our problem.

Finally, a numerical algorithm is presented in Chapter 6 for the solutions of Problem (2.1) when a coexistence equilibrium point exists. This numerical implicit method leads to a linear system defined by a tridiagonal matrix. The numerical solving is based on Gaussian elimination, and uses Fortran language. The results show that this program works very well. We show that the coexistence equilibrium point can lose its stability via Hopf bifurcations. The computations show that the Hopf bifurcations are supercritical (see the graphs).

I hope that the thesis will be a worthwhile contribution to the subject of structured population models.

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