Medical image registration based on a creaseness measure

A dissertation submitted by David Lloret i Villalonga at Universitat Autònoma de Barcelona to fulfil the degree of Doctor en Informática.

Bellaterra, November 19, 2001
a la Maria Dolors i en Josep
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Abstract

Today many advances in medicine are motivated by the emergence of new acquisition devices and the increasing capabilities of computers. Processing the generated digital information represents an engineering challenge: volume images, video sequences and new modalities are combined in order to provide new diagnosis tools for clinicians.

This thesis is concerned with the automatic alignment of medical images. When several images of the same patient are taken, inevitably some sort of misalignment will occur, causing features in the images will not be shown at the same location. But, for the images to be compared point-wise, it is necessary to establish some geometric correspondence between them. This task, when done manually, is lengthy, user-dependent and prone to errors, and therefore automatic alignment algorithms are necessary.

We propose a generic algorithm applicable to images having a particular requisite: they must depict crest or valley-like features. That is possible if we model the images as sampled scalar functions, being the intensity or gray level the scalar magnitude. These features are extracted automatically by means of a differential operator sensible to these particular shapes, and designed to be invariant to translations and rotations, and robust against noise and low contrast. Examples of detectable features are the bone issue in volume CT and MR, the vessels in ophthalmologic images and sulci in ecographies.

The output of the operator will depict relevant landmarks which, since they appear in all the images, can be used as the reference for the alignment algorithm. The operator is specially useful for images that can not be directly compared because they are too different, either because they belong to different modalities, or because the features contained in the original image have been altered.

Once the features have been extracted, one image has to be iteratively transformed until an alignment assessment function reaches a maximum. But this simple iterative scheme works only for almost identical images with small misregistration values. Real data requires to sample the function at a coarse step, and the best way to do it is by means of a hierarchical approach. Exhaustive, costly search is done only for levels where transformations are fast to compute, and results are refined through the hierarchical pyramid until a single transformation is selected.

We have made full reports of the performance of this algorithm for several modalities and conditions. Firstly, we have applied it to CT to MR volume image registration. We designed and ran several experiments to test its robustness under severe misregistrations, and compared favourably its results to those of a mutual infor-
mation based method. Then, we participated in a project to evaluate the accuracy against a golden standard, for a database of about one hundred pairs of images, whose results ranked us to be very accurate for some modality pairs.

A different medical subject was the registration of ophthalmologic images. In this modality, 2–D images with a high rate of noise and varying contrast must be aligned to correct for the involuntary movement or blinking of the eye. Our algorithm worked better and more generally than previous papers in literature, and could be applied also to long sequences of SLO video images. We performed exhaustive tests to permit a fast and robust convergence which contributed, in collaboration with another research group, to set a real medical application already working in a hospital.

Finally, we explored several registration issues in the area of intra-operative ecographies. After designing a system to grab and locate the ecography transducer, we started the experiments with an in vitro human brain. We could compound a volume with the acquired B-frames and register it accurately to an MR volume (3D–3D), and also register each individual B-scan to the corresponding area in the volume (2D–3D). The registration algorithm for the latter case followed the same general scheme as the others.
Molts dels avanços de la medicina actual són possible per l’aparició de noves modalitats d’imatges mèdiques i ordinadors cada vegada més potents. El processament d’alts volums d’informació digital (imatges 3D i seqüències de vídeo) és un repte per l’enginyeria informàtica i origina noves eines de diagnosi mèdica.

El tema principal d’aquesta tesi és la posada en corresponentia automàtica d’imatges mèdiques. Quan es prenen diverses imatges del mateix pacient, és inevitable que aquestes no estiguin en línia, és a dir, que els seus continguts no tinguin les mateixes coordenades espacials. Malgrat això, per tal de comparar les imatges, cal establir entre elles alguna mena de corresponentia geomètrica. Aquesta feina, si es fa manualment, és llarga, depenent de l’usuari i susceptible a errors, i per tant és convenient fer-la d’una manera automàtica.

Proposem un algorisme genèric aplicable a imatges amb un requisit determinat: han de contenir trets que tinguin forma de cresta o de vall. Això és possible si modelem les imatges com a funcions mostrejades, on la intensitat o nivell de gris esdevé la magnitud escalar. Aquests trets són extrets automàticament per mitjà d’un operador diferencial, sensible a aquest tipus de característiques i dissenyat per ser invariant a translacions i rotacions, i per tractar imatges sorolloses i amb poc contrast. En són un exemple les tomografies i ressonànies magnètiques de crani, els vasos sanguinis en oftalmologies i els solcs cerebrals en ecografies.

L’operador de crestes extreu marques comunes a totes les imatges, que fem servir com a referència per alinear-les. L’operador és útil sobretot per imatges massa diferents per ser comparades, sigui perquè són de diferents modalitats, sigui perquè el teixit examinat ha canviat entre les adquisicions.

Un cop extretes les marques, una de les imatges es transforma iterativament fins que la funció d’alineament assoleix un màxim. Aquest esquema simple, però, funciona només pels casos més triviais de parelles d’imatges. Per casos reals, cal mostregar la funció en intervals significatius, i una de les maneres de fer-ho eficientment és per mitjà d’un model jerarquic. La cerca inicial, exhaustiva, es fa doncs només en l’últim nivell, on el càlcul de les transformacions és més ràpid, i els resultats parciaus es passen d’un nivell al següent fins que en queda només un, el de major valor.

Hem fet un estudi exhaustiu de les prestacions del nostre algorisme per diverses modalitats i condicions. En primer lloc, l’hem aplicat a la posada en corresponentia de tomografies (TAC) i ressonànies 3D. En un experiment, l’hem posat a prova davant d’imatges amb un desalineament alt i conegut, i els seus resultats han estat comparables als d’un altre algorisme de referència basat en informació mútua. Així

Resum
mateix, hem participat en un projecte d’avaluació extern sobre una base de dades d’un centenar de parells d’imatges, que ha establert la seva precisió per una modalitat determinada com la millor , i acceptable per les altres.

Un altre camp d’aplicació són les imatges oftalmològiques. En aquesta modalitat les imatges, amb molt de soroll i contrast canviant, els canvis d’una imatge a l’altra poden arribar a ser molt grans. El nostre algorisme funciona més ràpid i és més genèric que els que existeixen en la literatura, i, a més, pot ser aplicat a una modalitat, les seqüències d’imatges SLO, encara més difícilment. Hem executat bateries de tests per refinarn la velocitat i robustesa, la qual cosa ens ha permès incorporar-lo amb fiabilitat a una aplicació mèdica que es troba ja en funcionament en un hospital.

Per acabar, hem explorat diversos problemes de posada en correspondència que apareixen en l’àrea d’ecografies intraoperatives. Per fer-ho ens ha calgut, en primer lloc, construir un sistema capaç d’adquirir imatges de vídeo i de localitzar la posició del capçal de l’ecògraf, tot en temps real. Hem fet els experiments amb un cervell humà in vitro; hem composat una imatge ecogràfica volumètrica i l’hem posat en correspondència amb una resonància del mateix òrgan, fent servir el nostre algorisme genèric. A més, hem ampliat aquest esquema per alinear les ecografies individuals 2D en una imatge 3D. En tots els casos els resultats han estat satisfactoris, i són el primer pas d’un algorisme que permetrà mesurar la deformació que experimenta el cervell durant una operació quirúrgica.
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