

# On the Stability and Efficiency of Decentralized Matching Processes

Nadia Burani

Departament d'Economia i d'Història Econòmica

Universitat Autònoma de Barcelona

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Supervisor: Clara Ponsatí Obiols

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# Chapter 1

## Introduction

The term matching refers to the process which brings individuals together to form distinguishable entities with some common purpose that no agent alone can accomplish.

As such, the matching process is not restricted to situations in which agents belong from the outset to one of two disjoint sets (workers being assigned to firms, men getting paired with women) nor does it pertain only to relations which are strictly bilateral in nature (buyers trading with sellers).

Rather, matching is conceived as a pervasive phenomenon which arises in all economic, social and political situations where individuals carry out activities as groups or coalitions. Agents organizing themselves in firms for production purposes, clubs being formed for consumption purposes, citizens relying upon local communities for the provision of public goods are all examples that fit the above definition.

In spite of the inherent versatility of matching problems, the following essential features are of interest.

First, matching is considered as a decentralized process which takes place voluntarily. This amounts to requiring that each agent be individually rational, namely that he only participates in acceptable matches which yield at least the value of being unmatched.

Secondly, it is assumed that there always exist substitution possibilities, in the sense that no individual agent is an essential member of any match or coalition. This of course does not prevent to consider heterogeneous agents who are endowed with unique characteristics.

Finally, attention is restricted to fixed schemes, or sharing rules, according to which the value generated within a group of matched agents has to be divided among its members (in the event that such a value be transferable at all).

Within this framework, there are two natural issues that are worth addressing.

First of all, there is the question of stability. Given the "rules of the game" which describe the set of agents, the matching technology (i.e. the way in which agents can get together), the value that each possible coalition can achieve, and the division scheme applied to such value, which coalitions will form

or alternatively, which matches are consummated? Is it possible to predict how individuals will divide themselves in groups in such a way that the resulting coalition structure is immune to either unilateral deviations or deviations by subgroups of players? Thus one would want to rule out situations characterized by a constant motion of individuals who keep forming and disrupting the groups they belong to.

Secondly, there is the question of efficiency. Is a stable matching or a stable coalition structure efficient in any meaningful sense?

## 1.1 One-Sided Matching with Separable Preferences

In Chapter 2, a one-sided matching model is analyzed where no restrictions are imposed as to the set of admissible matches. Indeed, all possible groups of individuals can potentially form. Under this assumption, the matching process coincides with a special class of coalition formation games that are known in the literature as hedonic games. Hedonic games are such that alternatives available to coalitions are assumed away and the (non-transferable) utility accruing to each individual is completely determined by the composition of the coalition which he belongs to. The choice of such a framework allows to disentangle the issue of coalition formation from the problem of division of the coalition's value, and thus to focus only on the former.

Cooperative games in coalitional structure provide the natural framework for the formal analysis of environments in which individuals choose to partition themselves into mutually exclusive and exhaustive coalitions. From this cooperative viewpoint, the issue of stability reduces mainly to the question of non-emptiness of the core of a coalition structure. A coalition structure is said to be core-stable or is in the core of a coalition structure if there does not exist a group of agents that can get together and choose a feasible alternative which is strictly more satisfactory than the alternative they are entitled to under the given coalition structure.

Despite the simplicity of the model, the existence of (core) stable coalition structures proves to be a major problem. Therefore, restricted preference domains need to be considered in order to ensure positive results. Attention is initially focused on symmetric additively separable preferences. We introduce a decomposition of the vectors representing such preferences into two components, called the cardinal component and the alternating component. It turns out that these components have opposite implications for stability. If agents' preferences are purely alternating, the core of a coalition structure might be empty. But if agents' preferences are purely cardinal then stability is guaranteed. In particular, a coalition structure which is both core-stable and Nash-stable (such that no individual unilaterally has incentive to leave the current coalition in order to join another already existing group) always exists, which is almost unique.

Implicit in purely cardinal preferences are individual weights that can be assigned to each agent

in society. Such individual weights or characteristics induce a common ranking of individuals because players with higher weights are unanimously preferred to players with lower weights. This feature of agents' preferences enables us to prove that precisely the same existence results hold under much weaker requirements: neither symmetry nor additive representability need to be imposed on agents' preferences. Moreover, although a common ranking of individuals is the starting point, we end up with preference orderings over coalitions that are not common to all agents. This contrasts with Farrell and Scotchmer (1988) and Banerjee, Konishi and Sönmez (1998) where a common ranking of coalitions is imposed in order to get non-emptiness of the core of a coalition structure.

The issue of the efficiency of an equilibrium coalition structure is already subsumed by the fact that core-stability is the solution concept considered. Indeed, any core allocation is also Pareto efficient.

## 1.2 Collusion in Decentralized Markets with Random-Matching and Bargaining

Chapter 3 studies the problem of cartel formation in the context of decentralized exchange economies à la Rubinstein and Wolinsky (1985). The questions we are interested in are the following. When do cartels emerge on the two sides of the market? What are the sizes of these cartels? Which is the extent of the inefficiencies (measured by the reduction in trade) brought about by the formation of cartels on the two sides of the market? How is bilateral collusion limited by the potential instability of cartel activities?

The model developed to tackle these issues features a market where there is a continuum of identical sellers, owning one unit of an indivisible commodity each, and a continuum of identical buyers, who want to purchase the indivisible good in exchange for money. The market operates for ...nitely many rounds, with no entry of new traders after the ...rst round. At each point in time, buyers and sellers are matched in pairs by a random device and pairs bargain over the surplus generated by the indivisible good. The bargaining mechanism considered consists in an ultimatum game, where a fair lottery determines (for all matches) which type of agent is the proposer and which is the responder. Equilibrium prices at the different rounds of trade depend on the relative measures of buyers and sellers that are active in the market at those rounds. Agents in the short side of the market are able to apportion a bigger fraction of the surplus generated by trade.

Therefore, there is room for collusion among agents on the same side of the market, and cartels might endogenously emerge. It is assumed that only one cartel can be formed on each side of the market, and that outsiders always participate in trade. Cartels simultaneously set the quantity to be supplied or demanded by their members. To this end, each cartel enforces the exclusion of some traders from the market and compensates them for withdrawal. Active cartel members play the random-matching,

bilateral bargaining game just described.

For any given initial measures of buyers and sellers, and any given levels of cartel memberships, an equilibrium market outcome might be identified, which specifies the total amount of trade carried out. In general, non-members benefit from the formation of a cartel because they trade at the same price as cartel members, but do not have to compensate inactive cartel members. This free-riding problem greatly limits the extent to which cartels can effectively reduce trade while expecting to maintain their memberships.

A cartel is said to be stable when no independent trader has incentive to join the cartel and when no cartel member has incentive to leave the cartel. Since agents are atomistic and the decision of a single individual does not influence the overall market outcome, it is always profitable for a single cartel member to leave the cartel. We circumvent this problem by first considering deviations on the part of a coalition of cartel members with positive measure, and then taking the limit as such measure goes to zero.

In this sense, stable market outcomes always exist and they can be grouped into two different categories. On the one hand, there are situations where at least one cartel actively restrains trade and where the level of participation in the market is balanced on both sides, regardless of the potential sizes of supply and demand. Such stable market outcomes might be inefficient when both cartels are active, because not all gains from trade are apportioned. The second type of stable market outcomes is such that at most one cartel (more likely the one that forms on the long side of the market) is active, which reduces its participation in the market so as to slightly undercut the opponents'. In this situation, the market outcome is almost efficient because the reduction in the quantity traded is very small.

### 1.3 Matching, Search, and Intermediation with Two-Sided Heterogeneity

In Chapter 4, the role played by intermediation is analyzed in the context of a model of two-sided search with heterogeneous agents.

The basic structure of the random-matching and bargaining framework considered here is similar to the one introduced in Chapter 3, but departs from it in some important aspects. First, it is assumed that both buyers and sellers differ with respect to their reservation values for the indivisible good. Agents' types are observable only within a match. Secondly, the bargaining mechanism employed avoids strategic considerations. Indeed, the bargaining outcome will be either an agreement on a price that divides the net surplus associated to the match according to the symmetric Nash Bargaining solution, or disagreement.

When the identities of potential trading partners are not known with certainty *ex ante*, widespread externalities arise that might be the source of inefficiencies. The agents' willingness to consummate a

given match affects the distribution of unmatched types, and thus the agents' matching opportunities and their expected utilities. Moreover, being matched precludes further search and thus each individual faces a trade-off between the opportunity cost of abstaining from search, and the benefits associated with concluding a transaction immediately. Such trade-off is the main source of delay in trade and represents the cost of search.

In this environment, a stable outcome is represented by what is called a stationary search equilibrium. It is such that the agents' acceptance decisions maximize their discounted expected unmatched values, and such that the distributions of unmatched types (and hence the prices at which trade occurs) are constant through time. The existence of sorting externalities might cause inexistence or multiplicity of equilibria to arise. It can be shown that, for some parameter ranges, some matched pairs of agents systematically disagree and remain on the market. Since waiting implies a loss in overall utility (because of discounting), then some equilibria are inefficient from a utilitarian perspective.

In the second part of the paper we consider how this picture is altered when the agents also have the option of trading through a monopolistic intermediary. The objective is to show that in search markets with two-sided heterogeneity intermediation is a profitable activity and thus that there is an incentive for intermediation to arise endogenously.

At each period, an intermediary posts fixed bid and ask prices, at which he can conclude large volumes of trade. Such prices are publicly observable by the whole market. Then, buyers and sellers simultaneously have to decide which market to trade in. Again, these participation decisions are the source of externalities because each agent's participation decision affects both the expected discounted utility of searching in the decentralized market and the value of trading through the middleman of all other traders. In this context, we identify the possible stationary equilibria with intermediation, namely situations where buyers and sellers adopt optimal participation decisions given the participation policy of all other agents, where the middleman sets bid and ask prices in a profit-maximizing way, and where the market is in a stationary state.

The presence of a middleman in the market narrows the set of agents' types who search. Intuitively, traders who generate small gains from trade should prefer to trade in the search market rather than sustain the transaction costs implicit in the bid-ask spread. Conversely, agents who generate high gains from trade should prefer intermediation because it eliminates the likelihood of disagreement in bargaining and the risk of matching unsuitable partners. In particular, it is shown that the existence of intermediation as an alternative trading mechanism might induce separation between agents' types. In this circumstance, the presence of the middleman is beneficial because it enhances the total volume of trade and eliminates delay.