

## Chapter 3

# Countervailing Power? Collusion in Markets with Decentralized Trade

### 3.1 Introduction

Collective incentives to restrict trade have long been acknowledged as a prevalent phenomenon in markets. The inherent instability of cooperative agreements attempting to exploit such incentives has also been extensively discussed in the literature.

A common feature of the models that address these issues consists in assuming that collusive practices arise only on one side of the market. For instance, particular emphasis has been given to the formation of cartels by oligopolistic firms that face price-taking consumers.<sup>1</sup> More recently, developments in auction theory have addressed the issue of collusion among numerous buyers facing a single seller.<sup>2</sup>

When traders on both sides of a market behave strategically, both sides can in principle form cartels with the purpose of enhancing their collective market power with respect to the opponent's. Is it possible that cartels emerge and persist on the two sides of a market? Is it possible for collusion to be a desirable phenomenon in this context?

These questions were raised long ago by Galbraith (1952), who claimed positive answers, in his theory of countervailing power. Galbraith asserted that "in the competitive model, the power of the firm as a seller is checked or circumscribed by the competitor who offers, or threatens to offer, a better bargain. The role of the buyer on the other side of such market is essentially a passive one. However, (...) in the typical modern market of few sellers, the active restraint is provided not by the competitor but from

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<sup>1</sup>See d'Aspremont et al. (1983) and Donsimoni, Economides and Polemarchakis (1986) for a characterization of stable cartels in the context of oligopolistic markets.

<sup>2</sup>See Mac Afee and Mac Millan (1992).

the other side of the market by strong buyers".<sup>3</sup> Thus, the existence of market power on one side of the market would create an incentive for the other side to organize another position of power neutralizing the former. Countervailing power was seen behind the emergence of labor unions: "One ...nds the strongest labor unions in the United States where markets are served by strong corporations. And it is not an accident that the large automobile, steel, electrical, farm machinery companies all bargain with powerful unions. It is the strength of the corporations in these industries that made it necessary for workers to develop the protection of countervailing power".<sup>4</sup> The retail market offered another example of the operation of countervailing power. The great development of department-store chains, food chains, mail order houses was interpreted as the countervailing response of retailers, on consumers' behalf, to sellers' previously established positions of power.

Galbraith's claims were not sustained with a rigorous model. And the empirical evidence is somewhat controversial.<sup>5</sup> Yet, the preceding descriptions are suggestive and, despite their formal shortcomings, Galbraith's arguments had great impact on the development of economic policies in the second half of the 20th century. Fifty years later, very little research has formally addressed the problem,<sup>6</sup> and it is still unclear whether the theory of countervailing power can stand a game-theoretic scrutiny. The present work aims at contributing to such analysis.

In this paper, we examine the problem of bilateral cartel formation in the context of decentralized exchange economies à la Rubinstein and Wolinsky (1985), where a continuum of identical buyers confront a continuum of identical sellers, and where buyers and sellers are randomly matched in pairs and bargain a price to exchange one unit of an indivisible good. In markets that remain stationary at all rounds of trade (as in Rubinstein and Wolinsky [1985] where, at each round, new traders enter exactly in the same measure as satisfied traders exit), the advantage of the short side of the market is not sufficient to create collective incentives on either side to exclude some traders from the market.<sup>7</sup> However, such incentives do exist, and can be strong, in a market that does not remain stationary as it clears over several rounds of trade. Consequently, our analysis is carried out within environments where the relative measure of buyers to sellers changes across the different rounds of trade.

The market operates for ...nitely many rounds, with no entry of new traders after the ...rst round.<sup>8</sup> At each round, buyers and sellers search for a trading partner and, if they ...nd a match, they bargain over the price at which to transact. If they reach an agreement they trade and exit the market, otherwise they search again in the following round. Equilibrium prices at the different rounds of trade depend on

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<sup>3</sup>Galbraith (1952), p. 113.

<sup>4</sup>Galbraith (1952), p. 114-115.

<sup>5</sup>See Scherer and Ross (1990), ch. 14, and references therein.

<sup>6</sup>Bloch and Ghosal (2000), discussed below, represents the notable exception.

<sup>7</sup>See Bloch and Ghosal (2000) for a proof of this claim.

<sup>8</sup>The preceding description is best interpreted as the stage game of a repeated game, in which buyers and sellers repeatedly want to buy and sell, respectively, one unit of a good that perishes after two rounds of trade.

the relative measures of buyers and sellers that are active in the market at those rounds. Agents in the short side of the market are able to apportion a bigger fraction of the surplus generated by trade.

Cartels are coalitions aiming to increase the collective benefits of their members, all of whom trade in the same side of the market. Actual cartels do as much as they can to restrict entry and they have a major impact both on the search and on the bargaining patterns of traders. Cartels may turn a market with decentralized trade into a market with centralized trade, substantially altering the process of price formation. In our model, however, cartels are endowed with much weaker prerogatives. We will assume that the only instrument that cartels have at their disposal is self-restraint in the market participation of their members. Thus, each cartel chooses how many members, if any, to withdraw from the market (i.e. it sets the total quantity that will be supplied or demanded) and it redistributes its total payoff in order to compensate inactive members for their abstention.<sup>9</sup> Even when cartels are active, prices are set à la Rubinstein and Wolinsky (1985). These assumptions make our analysis tractable and our results independent of ad hoc assumptions concerning the process of price formation. Moreover, we suppose that only one cartel can form on each side of the market and that a cartel might not have control over the whole population on its side. Outsiders of the cartel always participate in the market.

Given the potential measures of buyers and sellers, and the levels of cartel memberships, we suppose that the two cartels play a non-cooperative game where the quantities supplied and demanded are set simultaneously. Non-members generally benefit from the formation of a cartel: they trade the indivisible good at the same price as cartel members, but do not have to compensate inactive cartel members. This free-riding problem greatly limits the extent to which cartels can effectively reduce trade while expecting to maintain their memberships. Consequently, not all the market outcomes attained as equilibria of the quantity-setting game are equally relevant. A natural criterion for selecting among equilibria of the quantity-setting game consists in requiring that they be supported by stable levels of cartel memberships. Stable market outcomes are then profiles of cartel memberships and an associated equilibrium of the quantity-setting game at which memberships are stable, in that cartel sizes do not trigger defections.

We prove that the set of stable market outcomes is non-empty, and we provide its full characterization. Stable market outcomes can be of two different types.

The first type is such that at least one cartel actively restrains trade and such that the level of participation in the market is balanced on both sides, regardless of the potential sizes of supply and demand. Market outcomes might be inefficient when both cartels are active, because not all gains from trade are apportioned. Thus, using Galbraith's terminology, both sides exercise countervailing power. But when only one cartel (the one that forms in the long side of the market) is active, only one side of the market exercises countervailing power, restraining its participation up to the point at which supply

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<sup>9</sup>In actual markets, the supply or demand of each individual is divisible and thus limitations in the days or hours of operation of cartel members (as those adopted by retailers or professional associations) have exactly the same effect.

and demand coincide. Consequently, this type of stable market outcome results in an efficient allocation and the effect of countervailing power is limited to a redistribution of the total surplus.

The second type of stable market outcomes is such that only one cartel (more likely the one that forms on the long side of the market) is active which reduces its participation in the market so as to slightly undercut the opponents'. In this situation, only one side of the market exercises countervailing power and the total surplus is redistributed in favor of this side. The market outcome is not efficient, but the reduction in the quantity traded with respect to its potential total volume is not very significant.

Our paper owes much to Bloch and Ghosal (2000) that precedes us in addressing the issue of cartel formation in the context of an exchange economy with bilateral trade and bargaining. Bloch and Ghosal (2000) considers the formation of cartels of buyers or sellers in markets with an equal and finite number of buyers and sellers. They show that cartels might be active on both sides of the market, but active cartels never withdraw more than one trader. Although there are many apparent differences between our work and that of Bloch and Ghosal, our results are closely related and, we believe, complementary to theirs. The substantial difference of our model can be ascribed to our assumption that cartels set continuous quantities. The continuum assumption, although debatable from a descriptive point of view (after all, bilateral collusion seems more likely in markets with small numbers of traders on each side), is crucial to attain a tractable analysis, and permits to analyze two-sided cartel activity in markets that are ex ante unbalanced, a case that is not addressed by Bloch and Ghosal (2000).

The rest of the paper is organized as follows. The basic model of decentralized trade is presented in Section 3.2. In Section 3.3 the non-cooperative game played by the cartels is described, and the notions of cartel stability are introduced. The equilibria of the quantity-setting game played by the cartels are characterized in Section 3.4. Stable market outcomes are described in Section 3.5.

## 3.2 Decentralized Trade

Consider a market where a continuum of identical sellers face a continuum of identical buyers. Each seller owns one unit of the same indivisible good and his valuation of the good is normalized at zero, and each buyer's valuation of the indivisible good is normalized at one.

The market operates for two trading rounds  $t = 1; 2$ .<sup>10</sup> It is assumed that a measure  $b$  of buyers and a measure  $s$  of sellers enter the market in the first round, and that the market is potentially unbalanced. No new agents enter after the first round. In each round, buyers and sellers are randomly matched in pairs and each pair bargains over the price at which the indivisible good has to be exchanged. The matching mechanism has non-negligible frictions: at round  $t$ ; traders on the short side of the market

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<sup>10</sup>This assumption is made for tractability. The results can be generalized to the case in which the market operates for many trading rounds:

...nd a trading partner only with probability  $\mu_t \in (0, 1)$ ; whereas their counterparts are matched with probability  $\frac{\min\{s_t, b_t\}}{\max\{s_t, b_t\}}$ ; where  $s_t$  and  $b_t$  are the measures of active sellers and buyers at round  $t$ :

Buyer-seller matches bargain a price to trade the indivisible object. At either round of trade, the bargaining game consists in an ultimatum game. Namely, a fair lottery selects one of the parties to propose a partition of the surplus; the other party responds by accepting the offer or rejecting it. Upon acceptance, the agents trade and leave the market. Upon rejection in the first period, the match breaks and the agents return to the market and search for other partners in the second round of trade. A rejection in the second round implies that, for the given match, the game ends without trade. The payoffs for trading at price  $p \in [0, 1]$  at either round are  $p$  to the seller and  $1 - p$  to the buyer. The utility associated with no trade is zero.

At the unique subgame perfect equilibrium of the bargaining game, the proposer offers the responder a share equal to the latter's expected value of returning to the market and the responder accepts. In other words, the responder is given his outside option at that period and the proposer gets the residual surplus. Consequently, there exists a unique market equilibrium such that, at each round, all pairs of traders immediately agree on the same price.<sup>11</sup> Since the populations of buyers and sellers might change from one round to the next, bargaining pairs face varying outside options, that are endogenously determined by the expected present value of returning to the market the following round.

Outside options in the last round are 0; independently of which type of agents are the short side of the market. Substituting backwards yields expressions for outside options at round  $t = 1$ : Suppose that sellers are initially the short side of the market, i.e.  $s < b$ : In this situation, each buyer has an outside option which is worth  $\frac{\mu_2}{2}$  (where  $\mu_t$  denotes the probability of finding a match in round  $t$ ), each seller's option is  $\frac{s}{2}$  and the present discounted value of participating in the two-rounds market is

$$V_S = \frac{(4 - \mu_1 - \mu_2)}{4} \quad (3.1)$$

for sellers and

$$V_B = \frac{(\mu_1(2 - \mu_1) + (2 - \mu_1)\mu_2)}{4} \quad (3.2)$$

for buyers. Note that both  $V_S$  and  $V_B$  depend on the initial measures of buyers  $b$  and sellers  $s$  through the matching probabilities  $\mu_1 = \frac{s}{b}$  and  $\mu_2 = \frac{(1 - \mu_1)s}{b - \mu_1 s}$ : Substituting this fact into (3.1) and (3.2) one obtains

$$V_S(b; s) = \frac{(4 - \mu_1 - \mu_2)(b - \mu_1 s)}{4(b - \mu_1 s)} \quad \text{and} \quad V_B(b; s) = \frac{\mu_1(2 - \mu_1)(b - \mu_1 s) + (2 - \mu_1)\mu_2}{4(b - \mu_1 s)}$$

<sup>11</sup>The assumption that traders use a (random proposer) ultimatum game is made for expositional clarity. Other bargaining games yield the same market equilibrium. See Ponsati (2000) for a proof that under infinite horizon alternating offers bargaining, ultimatum strategies can still prevail and yield a unique market equilibrium as well.

respectively. Symmetric expected values can be computed if  $b < s$ .

Thus, in general, the individual expected gains to a trader of type X facing agents of type Y, when the initial measure of agents of his type is  $x$  and the initial measure of his counterparts is  $y$ ; can be written as

$$u_X(x; y) = \begin{cases} u_X^S(x; y); & \text{if } x < y \\ u_X^L(x; y); & \text{otherwise} \end{cases}$$

where the superscripts S and L stand for short and long side of the market respectively, and where

$$u_X^S(x; y) = \frac{(4 - \alpha)(y - \alpha)x + (1 - \alpha)x}{4(y - \alpha)}$$

and

$$u_X^L(x; y) = \frac{y(3 - 2\alpha)(x - \alpha) + (1 - \alpha)x}{4(x - \alpha)}$$

As for the collective expected gains of traders on one side of the market, there might exist collective incentives to exclude some agents from trade. This occurs because buyers' (sellers') collective utility generally increases when the measure of buyers (sellers) trading on the market is reduced, given the measure of active sellers (buyers).

In a market where the initial measures of traders is  $(x; y)$ ; the collective expected payoff to the  $x$  traders that enter the market initially and face a measure  $y$  of trading counterparts is simply  $u_X(x; y)x$ : When the group  $x$  of traders are the long side of the market, that is when  $x > y$ ; their joint utility is

$$u_X^L(x; y)x = \frac{y(3 - 2\alpha)(x - \alpha) + (1 - \alpha)x}{4(x - \alpha)}$$

whose derivative with respect to  $x$  takes value

$$\frac{\partial}{\partial x} u_X^L(x; y)x = -\frac{(1 - \alpha)^2 y^2}{4(x - \alpha)^2} < 0$$

It immediately follows that a decrease in the measure of active traders is always collectively beneficial for the long side of the market. Conversely, when  $x < y$ ; the collective utility of the agents in the short side of the market is

$$u_X^S(x; y)x = \frac{x(4 - \alpha)(y - \alpha) + (1 - \alpha)x}{4(y - \alpha)}$$

It is easily checked that  $u_X^S(x; y)x$  is a strictly concave function in all its domain, which reaches a maximum at

$$b(y) = y \frac{(5 - 2\alpha) + \frac{\alpha(1 - \alpha)(5 - 2\alpha)}{4}}{(5 - 2\alpha)}$$

with  $b(y) < y$  if and only if

$$\frac{7 - \alpha}{4} = 0.71922 < \alpha \tag{3.3}$$

For further reference, we will say that market frictions are high when condition (3.3) is met.

We conclude that collective incentives to restrict market participation are present, and they might exist even for the short side of the market. It seems therefore natural to analyze whether coalitions that attempt to restrict trade are sustainable or not, and what impact they have on market performance. We address these issues in the following sections.

### 3.3 Cartel Games

Assume that one and only one cartel can form on each side of the market. A measure  $\alpha$  of buyers and a measure  $\beta$  of sellers, with  $0 < \alpha, \beta < 1$ ; operate as independent traders and remain active in the market as long as they have trade to carry out. The two cartels control the remaining traders, namely  $(1 - \alpha)b$  buyers are members of the buyers' cartel and the sellers' cartel size is  $(1 - \beta)s$ .

Cartels play a quantity-setting game where each cartel chooses its participation level in the market as a function of the cartel sizes. That is, each cartel may withdraw some measure of its members from the market, and the aggregate cartel payoffs are redistributed to compensate inactive agents for their abstention. It is assumed that each cartel can enforce the exclusion of traders, i.e. its members cannot sneak in the market when they have been ordered to stay out and they cannot organize parallel trade of the excluded quantities.

The buyers' cartel sets the measure of its active members, i.e. it sets its demand  $q_b$ , where  $q_b \in [0; (1 - \alpha)b]$ ; and the sellers' cartel sets its supply  $q_s \in [0; (1 - \beta)s]$ : Since non-members always participate in trade, the total market demand is given by  $\bar{q} = q_b + \alpha b$  and the total market supply is equal to  $\bar{q} = q_s + \beta s$ : Therefore, it is as if the buyers' cartel sets market demand  $\bar{q} \in [\alpha b; b]$  and the sellers' cartel chooses market supply  $\bar{q} \in [\beta s; s]$ :

For each profile of cartel memberships  $(\alpha, \beta)$ , the payoffs of the quantity-setting game  $G^{\alpha, \beta}$ ; where both cartels act simultaneously, can be expressed as functions of market demand and supply as

$$B(\bar{q}; q_s) = \begin{cases} (\bar{q} - \alpha b) \frac{1}{4} \bar{q} & \text{if } \bar{q} \in [\alpha b; q_s] \\ (\bar{q} - \alpha b) \frac{1}{4} q_s & \text{if } \bar{q} \in [q_s; b] \end{cases}$$

for buyers' cartel, and as

$$S(\bar{q}; q_b) = \begin{cases} (q_b - \beta s) \frac{1}{4} \bar{q} & \text{if } q_b \in [\beta s; \bar{q}] \\ (q_b - \beta s) \frac{1}{4} q_b & \text{if } q_b \in [\bar{q}; s] \end{cases}$$

for sellers' cartel.

Let  $q_b^*(\alpha, \beta)$  and  $q_s^*(\alpha, \beta)$  denote the best responses of the sellers' and buyers' cartels, respectively, in  $G^{\alpha, \beta}$ : If necessary, we will use the notation  $q_b^{\alpha, \beta}(\alpha, \beta)$  and  $q_s^{\alpha, \beta}(\alpha, \beta)$  to make clear that the best responses depend

on a particular pair of cartel sizes  $(\theta; \frac{1}{2})$ . Similarly, the payoff functions will be denoted by  $B^{\theta; \frac{1}{2}}(\bar{q}; \frac{3}{4})$  and  $S^{\theta; \frac{1}{2}}(\bar{q}; \frac{3}{4})$ .

A market outcome is a profile  $(\theta; \frac{1}{2}; \bar{q}; \frac{3}{4})$  such that  $(\bar{q}; \frac{3}{4})$  is a Nash equilibrium of  $G^{\theta; \frac{1}{2}}$ , that is  $\frac{3}{4} = \frac{3}{4}(\bar{q})$  and  $\bar{q} = \bar{q}(\frac{3}{4})$  for given cartel sizes.

A market outcome is a reasonable prediction for the market operation only when cartels can be expected to maintain their membership levels. Cartels might not preserve their memberships either because members wish to defect and become free traders or because free traders wish to join the cartel. In the present model, since cartels generate positive externalities that benefit outsiders on the same side of the market, the incentives of insiders to defect from the cartel may strongly undermine the stability a market outcome, whereas the incentives of outsiders to join the cartel will not be a concern.

With a continuum of non-atomic agents, the defection of a small single agent has a negligible impact on market outcomes, therefore the notion of stability must be weaker than that of immunity from unilateral deviations. Our notion of cartel stability postulates the absence of incentives to deviate by coalitions of strictly positive measure. Given a market outcome  $(\theta; \frac{1}{2}; \bar{q}; \frac{3}{4})$  we will say that an  $\epsilon$ -coalition, (that is a measure  $\epsilon > 0$  of either cartel members or outsiders), of sellers in the cartel benefits from defecting the cartel if and only if

$$\frac{1}{4}S(\bar{q}; \frac{3}{4}^{\theta; \frac{1}{2} + \epsilon}(\bar{q})) > \frac{1}{4}S(\bar{q}; \frac{3}{4}):$$

Similarly, an  $\epsilon$ -coalition of buyers in the cartel benefits from a defection if and only if

$$\frac{1}{4}B(\bar{q}^{\theta; \frac{1}{2} + \epsilon}(\frac{3}{4}); \frac{3}{4}) > \frac{1}{4}B(\bar{q}; \frac{3}{4}):$$

The conditions under which an  $\epsilon$ -coalition of free traders benefits from joining the cartel can be given analogously.

Observe that, in assessing whether an  $\epsilon$ -coalitional defection is profitable or not at the market outcome  $(\theta; \frac{1}{2}; \bar{q}; \frac{3}{4})$ , the quantity supplied or demanded by the other side of the market is taken as given. Upon defection, it is assumed that the affected cartel continues to respond optimally to such quantity. The idea behind this requirement is that, while a cartel might be aware of its members' defections and can react appropriately to them, it does not observe if its opponent suffers a defection.

A cartel is  $\epsilon$ -stable at  $(\theta; \frac{1}{2}; \bar{q}; \frac{3}{4})$  if and only if no positive but arbitrarily small measure  $\epsilon$  of its members (non-members) benefits from defecting (joining) the cartel. A profile  $(\theta; \frac{1}{2}; \bar{q}; \frac{3}{4})$  is an  $\epsilon$ -stable market outcome if and only if: (i) it is a market outcome, and (ii) both cartels are  $\epsilon$ -stable:

We will say that the profile  $(\theta; \frac{1}{2}; \bar{q}; \frac{3}{4})$  is a stable market outcome if it is an  $\epsilon$ -stable market outcome for all  $\epsilon > 0$ :

The remainder of the paper is devoted to proving that the sets of  $\epsilon$ -stable and stable market outcomes are non-empty, and to its characterization. This is done in two steps. In the first step, market outcomes

for generic cartel sizes ( $\theta; \frac{1}{2}$ ) are characterized. In the second step, the constraints of stability at the market outcomes are explored, providing the desired characterization and the proof of existence.

### 3.4 Market Outcomes

At this stage, we will explore the properties and existence of market outcomes, that is of Nash equilibria of the quantity-setting game with given measures of cartel memberships.

The following assertions about the properties of the buyers' cartel payoff, i.e.  $B(\bar{x}; \frac{3}{4})$ , are useful to gain some intuition about the results. The proofs are straightforward and will be omitted for the sake of brevity. Similar claims hold for  $S(\bar{x}; \frac{3}{4})$ :

**Claim 42** The payoff function  $B(\bar{x}; \frac{3}{4})$  is continuous at all  $(\bar{x}; \frac{3}{4})$ ; since  $\frac{1}{4}L(x; x) = \frac{1}{4}S(x; x)$  for all  $(x; x)$ .

**Claim 43** The payoff function  $B(\bar{x}; \frac{3}{4})$  is strictly increasing at  $\bar{x} = \theta b$ ; therefore the buyers' cartel always demands positive quantities.

**Claim 44** If total demand  $\bar{x}$  is such that  $\bar{x} > \frac{3}{4}$  and  $B(\bar{x}; \frac{3}{4}) \leq B(\bar{x}^0; \frac{3}{4})$  for all  $\bar{x}^0 > \frac{3}{4}$ ; then  $\bar{x} = b$ :

If total demand  $\bar{x}$  is such that  $\bar{x} < \frac{3}{4}$  and  $B(\bar{x}; \frac{3}{4}) \leq B(\bar{x}^0; \frac{3}{4})$  for all  $\bar{x}^0 < \frac{3}{4}$ ; then  $\bar{x}$  solves

$$\frac{\partial}{\partial \bar{x}} B(\bar{x}; \theta b) \frac{1}{4} S(\bar{x}; \frac{3}{4}) = 0 \quad (3.4)$$

Taking into account the above claims, Example 45 considers the different possible shapes of the payoff function  $B(\bar{x}; \frac{3}{4})$ .

**Example 45** Let market frictions be equal to  $\phi = \frac{3}{4}$  and take  $\frac{3}{4} = \frac{1}{2}$ : (i) Set  $\theta = \frac{1}{4}$  in which case the buyers' cartel payoff, as a function of total demand  $\bar{x}$ ; is equal to

$$B_{\frac{1}{4}; \frac{1}{2}}(\bar{x}; \frac{1}{2}) = \begin{cases} < \frac{3}{4} \bar{x} - \frac{1}{4} \frac{13\bar{x} - 21}{2\bar{x} - 3} & \text{if } 0 < \bar{x} < \frac{1}{2} \\ \frac{3}{32} \bar{x} - \frac{1}{4} \frac{7\bar{x} - 9}{-(\bar{x} - \frac{3}{8})} & \text{if } \frac{1}{2} < \bar{x} < 1 \end{cases}$$

and is represented by Figure 3.1. It is easy to see that the value of  $\bar{x}$  that maximizes the above payoff function is  $\bar{x} = 1$ : (ii) Suppose that the fraction of independent buyers is  $\theta = \frac{1}{8}$ ; whereby the buyers' cartel profits become

$$B_{\frac{1}{8}; \frac{1}{2}}(\bar{x}; \frac{1}{2}) = \begin{cases} < \frac{3}{4} \bar{x} - \frac{1}{8} \frac{13\bar{x} - 21}{2\bar{x} - 3} & \text{if } 0 < \bar{x} < \frac{1}{2} \\ \frac{3}{32} \bar{x} - \frac{1}{8} \frac{7\bar{x} - 9}{-(\bar{x} - \frac{3}{8})} & \text{if } \frac{1}{2} < \bar{x} < 1 \end{cases} ;$$

whose graphical representation is as in Figure 3.2. In this case, the buyers' cartel payoff is maximal when  $\bar{x} = \frac{3}{4} = \frac{1}{2}$ : (iii) When  $\theta = \frac{1}{20}$  the buyers' cartel profit is maximized when total demand  $\bar{x}$  solves condition (3.4) and is such that  $\bar{x} < \frac{3}{4}$  (namely,  $\bar{x} = 0.4953$ ); as Figure 3.3 shows.

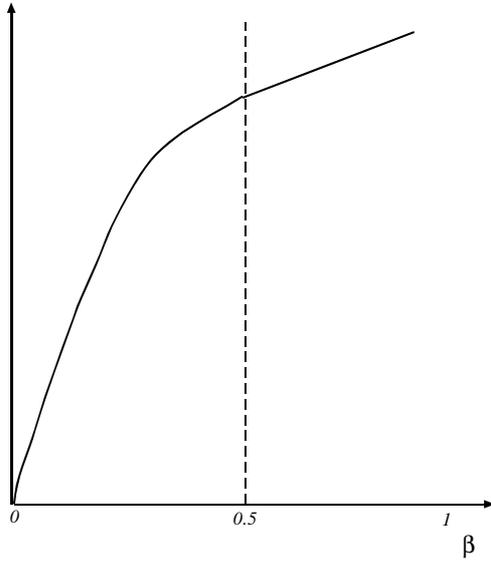


Figure 3.1: Case (i) in Example 45

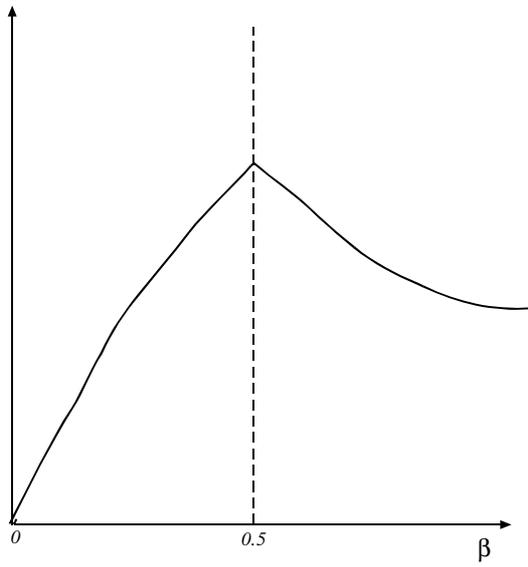


Figure 3.2: Case (ii) in Example 45

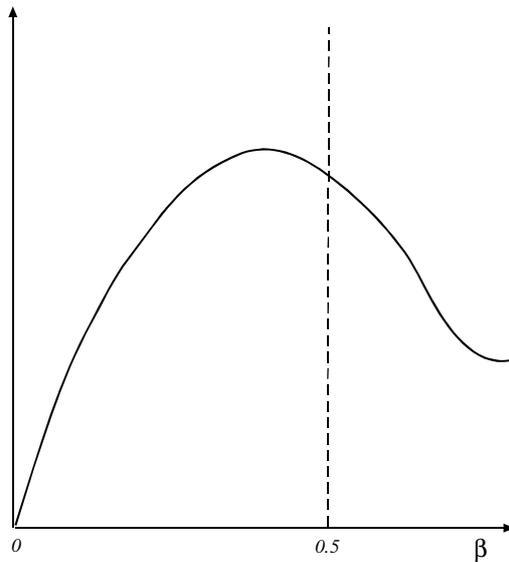


Figure 3.3: Case (iii) in Example 45

Therefore the best reply of the buyers' cartel to any level of market supply  $s$ , i.e.  $\bar{p}(s)$ ; is one of the following three: (i) not to withdraw any members and set  $\bar{p}(s) = b$ ; (ii) to set demand exactly equal to supply  $\bar{p}(s) = s$ , or (iii) to set demand below supply, i.e.  $\bar{p}(s) < s$ , at a level satisfying condition (3.4). Why? Suppose first that (iii) is not a payoff-maximizing solution and that the cartel's decision is based on the comparison between its payoffs at (i) and (ii); which are

$$B(b; s) = b(1 - \frac{s}{b}) \quad \text{and} \quad B(s; s) = (s - b) \frac{s}{b}$$

It is easy to check that  $B(b; s) > B(s; s)$  if and only if

$$s > \frac{2(2 - \theta)b}{(1 + \theta)(3 - 2\theta)} \equiv s_1;$$

where  $s_1 < s$  if and only if

$$\theta < \frac{\frac{s}{b}}{2(2 - \theta) - \frac{s}{b}(3 - 2\theta)} \equiv \theta_1;$$

Let us now address the case in which (iii) is indeed the payoff-maximizing decision. Observe that a solution to (3.4) satisfying  $\bar{p} < s$  cannot exist when  $\theta$  is too small, namely when market frictions are high. Indeed, condition (3.4) yields

$$b(\bar{p}) = \frac{\bar{p}}{\frac{s(5 - 2\theta)}{(1 - \theta)(5 - 2\theta)} - \frac{s(1 - \theta)(5 - 2\theta)(\bar{p} - b)}{(5 - 2\theta)}}; \quad (3.5)$$

where  $b(\bar{p}) < s$  never holds under high market frictions.<sup>12</sup> Otherwise,  $b(\bar{p}) < s$  is satisfied if and only

<sup>12</sup>This involves some tedious, but otherwise straightforward algebra.

if

$$\frac{3}{4} > \frac{\theta \cdot b}{(1_i (5_i 2^\circ)(1_i \circ))} \cdot \frac{3}{4}_2;$$

with  $\frac{3}{4}_2 < s$  if and only if

$$\theta < \frac{(1_i (5_i 2^\circ)(1_i \circ)) \frac{3}{4}_2}{b} \cdot \theta_2;$$

Finally observe that  $\frac{3}{4}_1 \cdot \frac{3}{4}_2$  if and only if

$$\theta > \frac{2(2_i \circ)(1_i (5_i 2^\circ)(1_i \circ))_i \circ^2}{\circ^2(3_i 2^\circ)} \cdot \theta;$$

with  $\theta > 0$  if and only if

$$\circ > 0.78203 \cdot \theta; \tag{3.6}$$

When condition (3.6) holds, we will say that market frictions are low, and market frictions will be called intermediate whenever  $\theta < \theta \cdot \theta$ . For low market frictions and for  $\theta > \theta$ , the payoff-maximizing decision of buyers' cartel cannot be (ii), and the cartel's optimal choice will thus be based on the comparison between its payoffs at (i) and (iii): In particular, let

$$\frac{3}{4}_3 \cdot \text{solution to } B(b; \frac{3}{4}) = B(\mathbf{b}(\frac{3}{4}); \frac{3}{4});$$

where  $\frac{3}{4}_2 < \frac{3}{4}_3 < \frac{3}{4}_1$  always holds for  $\theta > \theta$  and  $\theta < \theta$ .<sup>13</sup> Moreover, let us de...ne

$$\theta_3 \cdot \text{solution to } \frac{3}{4}_3 = s;$$

with  $\frac{3}{4}_3 < s$  if and only if  $\theta < \theta_3$ : Then (iii) represents the cartel's choice for  $\frac{3}{4} > \frac{3}{4}_3$ ; being  $B(b; \frac{3}{4}) \cdot B(\mathbf{b}(\frac{3}{4}); \frac{3}{4})$  in this case, whereas (i) is the cartel's payoff-maximizing solution for  $\frac{3}{4} < \frac{3}{4}_3$ :

The optimal decisions of the sellers' cartel are characterized analogously, with  $\bar{1}, \bar{2}, \bar{3}$ ; and  $\frac{1}{2}_1; \frac{1}{2}_2, \frac{1}{2}; \frac{1}{2}_3$  de...ned symmetrically. This completes the proof of Lemma 46 that follows.

**Lemma 46** Consider the game  $G^{\theta; \frac{1}{2}}$  with  $\theta; \frac{1}{2} > 0$ ; and let  $s \cdot b = 1$ .

(a) When market frictions are high, the best reply function of buyers' cartel is

$$\bar{1}(\frac{3}{4}) = \begin{cases} < 1 & \text{if } \frac{3}{4} < \min \{ \frac{3}{4}_1; sg \} \\ \frac{3}{4} & \text{if } \frac{3}{4} > \min \{ \frac{3}{4}_1; sg \} \end{cases} \tag{a}$$

and similarly the best reply of sellers' cartel is

$$\frac{3}{4}(\bar{1}) = \begin{cases} < s & \text{if } \bar{1} < \min \{ \bar{1}_1; 1g \} \\ \min \{ \bar{1}_1; sg \} & \text{if } \bar{1} > \min \{ \bar{1}_1; 1g \} \end{cases};$$

<sup>13</sup>We omit the analytical expression for  $\frac{3}{4}_3$ , as it is uninformatively complicated, being  $\frac{3}{4}_3$  one of the roots of a fourth-degree polynomial equation.

(b) When market frictions are intermediate, or when market frictions are low and the fraction of independent buyers is such that  $\theta > \frac{1}{2}$ ; the best reply function of buyers' cartel is

$$r(\frac{3}{4}) = \begin{cases} 1 & \text{if } \frac{3}{4} < \min f_{\frac{3}{4}1}; sg \\ \frac{3}{4} & \text{if } \min f_{\frac{3}{4}1}; sg < \frac{3}{4} < \min f_{\frac{3}{4}2}; sg \\ b(\frac{3}{4}) & \text{if } \frac{3}{4} \geq \min f_{\frac{3}{4}2}; sg \end{cases} \quad (b)$$

and similarly (for  $\frac{1}{2} \leq \frac{1}{2}$  when  $\theta > \frac{1}{2}$ ); the sellers' cartel best reply function is

$$\frac{3}{4}(r) = \begin{cases} s & \text{if } r < \min f_{-1}^{-}; 1g \\ \min f_{\frac{1}{2}}^{-}; sg & \text{if } \min f_{-1}^{-}; 1g \cdot r < \min f_{-2}^{-}; 1g \\ \min f_{\frac{1}{2}}^{-}; sg & \text{if } r \geq \min f_{-2}^{-}; 1g \end{cases}$$

(c) When market frictions are low and the fraction of independent buyers is such that  $0 < \theta < \frac{1}{2}$ ; the best reply function of buyers' cartel consists in

$$r(\frac{3}{4}) = \begin{cases} < 1 & \text{if } \frac{3}{4} < \min f_{\frac{3}{4}3}; sg \\ : b(\frac{3}{4}) & \text{if } \frac{3}{4} \geq \min f_{\frac{3}{4}3}; sg \end{cases} \quad (c)$$

and similarly (when  $0 < \frac{1}{2} < \frac{1}{2}$ ) the sellers' cartel best reply function is

$$\frac{3}{4}(r) = \begin{cases} < s & \text{if } r < \min f_{-3}^{-}; 1g \\ : \min f_{\frac{1}{2}}^{-}; sg & \text{if } r \geq \min f_{-3}^{-}; 1g \end{cases}$$

**Remark 47** Consider the case in which market frictions are intermediate. Notice that  $\min f_{\frac{3}{4}1}; sg = \frac{3}{4}_1$  and  $\min f_{\frac{3}{4}2}; sg = \frac{3}{4}_2$  hold if and only if  $\theta > \theta_1$  and  $\theta > \theta_2$  respectively, with  $\theta_1 > \theta_2$  for  $\theta > \frac{1}{2}$ . Hence, if  $\theta_2 < \theta < \theta_1$ ; the buyers' cartel best reply function (b) reduces to (a), whereas if  $\theta > \theta_1$  buyers' best response is  $r(\frac{3}{4}) = 1$  for all  $\frac{3}{4}$ , independently of the level of market frictions.

Example 48 below displays buyers' cartel reaction functions under the different scenarios contemplated in Lemma 46.

**Example 48** Let  $s = \frac{4}{5}$  be the ex ante measure of sellers and let  $\theta = \frac{1}{10}$  be the fraction of independent buyers: (a) Suppose first that market frictions are low and set  $\theta = \frac{1}{2}$ : Then  $\frac{3}{4}_1 = \frac{1}{2} < s$  and buyers' cartel reaction function is

$$r(\frac{3}{4}) = \begin{cases} < 1 & \text{if } \frac{3}{4} < \frac{1}{2} \\ : \frac{3}{4} & \text{if } \frac{1}{2} \leq \frac{3}{4} \leq \frac{4}{5} \end{cases} \quad (a)$$

whose graph is displayed in Figure 3.4. Note that, for  $\theta = \frac{1}{2}$  and  $s = \frac{4}{5}$ ; any proportion of independent buyers  $\theta > \frac{2}{11}$  would yield  $r(\frac{3}{4}) = 1$  as the cartel's reaction function: (b) Let us now consider intermediate

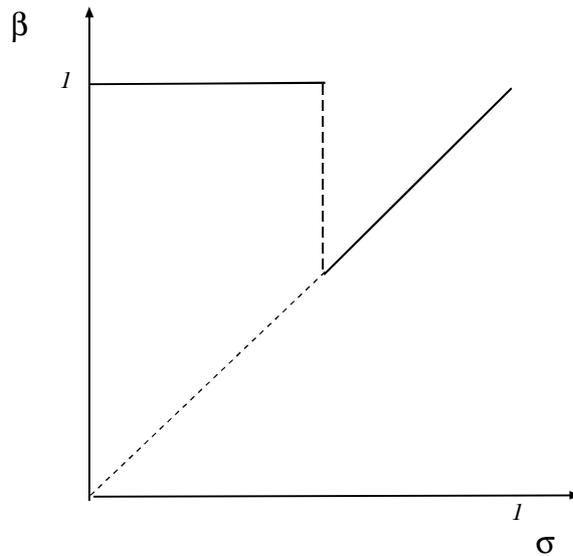


Figure 3.4: Case (a) in Example 48

market frictions and set  $\theta = \frac{3}{4}$ : In this situation,  $\frac{3}{4}_1 = \frac{20}{69}$  and  $\frac{3}{4}_2 = \frac{3}{5} < s$ ; whereby the buyers' cartel reaction function has expression

$$r(\frac{3}{4}) = \begin{cases} 1 & \text{if } \frac{3}{4} < \frac{20}{69} \\ \frac{3}{4} & \text{if } \frac{20}{69} \cdot \frac{3}{4} \cdot \frac{3}{5} ; \\ \frac{4}{3} \frac{3}{4} i \frac{2}{21} & \text{if } \frac{3}{5} \cdot \frac{3}{4} \cdot \frac{4}{5} \end{cases} \quad (b)$$

which is represented by Figure 3.5. Note that, for all  $\frac{2}{15} \cdot \theta < \frac{3}{8}$ ; the above reaction function will have the same functional form as (a). And for  $\theta > \frac{3}{8}$  the reaction function is  $r(\frac{3}{4}) = 1$  independently of the total supply. (c) Finally consider the case in which market frictions are low and set  $\theta = \frac{5}{6}$ ; whereby  $\theta = \frac{37}{100} > \frac{1}{10}$  and  $\frac{3}{4}_1 = \frac{21}{85} > \frac{3}{16} = \frac{3}{16}$ : The point at which buyers' cartel is indifferent between being inactive and playing  $r(\frac{3}{4})$  can be approximated to  $\frac{3}{4}_3 = 0.2456$ ; hence the cartel's reaction function can be written as

$$r(\frac{3}{4}) = \begin{cases} 1 & \text{if } \frac{3}{4} < \frac{3}{4}_3 \cdot 0.2456 \\ \frac{6}{5} \frac{3}{4} i \frac{3}{25} & \text{if } \frac{3}{4}_3 \cdot \frac{3}{4} \cdot \frac{4}{5} \end{cases} \quad (c)$$

and represented as in Figure 3.6.

The best response functions characterized in Lemma 46 generate two distinct types of market outcomes. On the one hand, the best responses may overlap for a non-empty interval along the diagonal. In this case, all market outcomes yield a perfect match in the measures of active buyers and sellers, and at least one cartel actively restrains trade. This symmetric trade scenario is always attained when

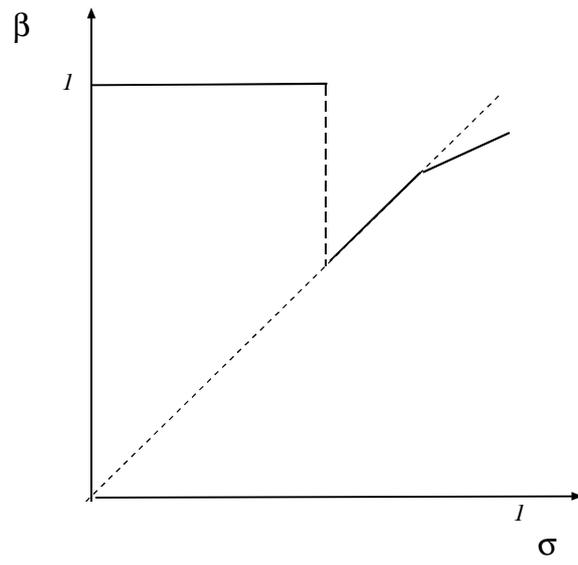


Figure 3.5: Case (b) in Example 48

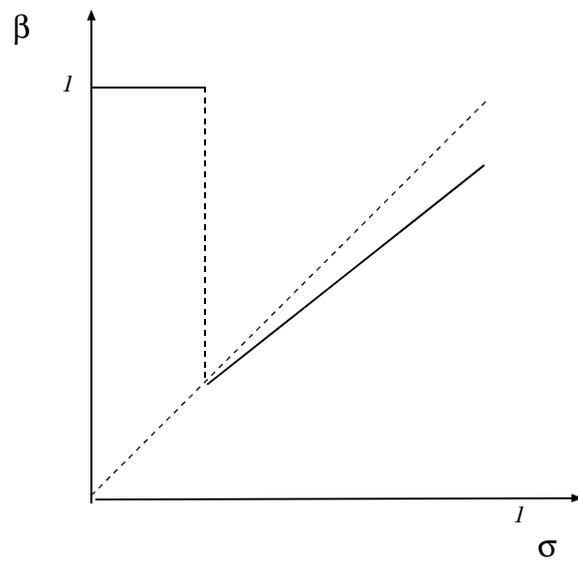


Figure 3.6: Case (c) in Example 48

market frictions are high and  $\frac{3}{4}_1 \cdot s$  holds; otherwise it prevails only if both  $\frac{3}{4}_1 \cdot \frac{3}{4}_2$  and  $\bar{c}_1 \cdot \bar{c}_2$  hold (equivalently only if  $\bar{c} \leq \underline{c}$  and  $\frac{1}{2} \leq \underline{1/2}$ ; which is always the case when market frictions are intermediate) and if and only if

$$q \leq \min \{ \max \{ f^{-1}_1; \frac{3}{4}_1 g \}; s g \} \cdot \min \{ f^{-1}_2; \frac{3}{4}_2; s g \} \bar{q} \quad (3.7)$$

On the other hand, when the best response functions do not overlap along the diagonal (which is always the case when market frictions are low and either  $0 \cdot \bar{c} < \underline{c}$ ; or  $0 \cdot \frac{1}{2} < \underline{1/2}$ , or both are satisfied), they may intersect at the boundaries. This yields a unique asymmetric market outcome at which one cartel is completely inactive; the other cartel may reduce its market participation so that it slightly undercuts the opponent's, or it may also remain inactive.

The relative cartel memberships, namely  $(1 - \bar{c})b$  and  $(1 - \frac{1}{2})s$ ; determine whether the Nash equilibrium of the game  $G^{\bar{c}; \frac{1}{2}}$  exists and if so whether it is a symmetric market outcome or an asymmetric one. In particular, if membership levels are close to each other and sufficiently high, then condition (3.7) holds and symmetric market outcomes, with at least one cartel restraining its market participation, are attained. Conversely, if cartel memberships are sufficiently different, a unique Nash equilibrium prevails in which the quantities traded are asymmetric, because only the larger cartel restrains trade. Finally, for intermediate cases the game  $G^{\bar{c}; \frac{1}{2}}$  might not have a Nash equilibrium. Of course, when cartel sizes  $(\bar{c}; \frac{1}{2})$  do not support a market outcome, then no profile  $(\bar{c}; \frac{1}{2}; \bar{c}; \frac{1}{4})$  can be an  $\pi$ -stable market outcome (neither a stable market outcome).

The proposition below characterizes the Nash equilibria of the game  $G^{\bar{c}; \frac{1}{2}}$ , and states the necessary and sufficient conditions for their existence.

**Proposition 49** Let  $s \cdot b = 1$ : (i) If market frictions are high, the strategy pair  $(\bar{c}; \frac{3}{4}) = (q; q)$  is a Nash equilibrium of the game  $G^{\bar{c}; \frac{1}{2}}$  if and only if  $\frac{3}{4}_1 \cdot s$ ; where  $q$  is any quantity such that  $q \geq [\min \{ \max \{ f^{-1}_1; \frac{3}{4}_1 g \}; s g \}]$ ; the strategy pair  $(\bar{c}; \frac{3}{4}) = (1; s)$  is the unique Nash equilibrium if  $\frac{3}{4}_1 > s$ : (ii) If market frictions are not high, the strategy pair  $(\bar{c}; \frac{3}{4}) = (q; q)$  is a Nash equilibrium of  $G^{\bar{c}; \frac{1}{2}}$  only if  $\bar{c} \leq \underline{c}$  and  $\frac{1}{2} \leq \underline{1/2}$  (equivalently only if  $\frac{3}{4}_1 \cdot \frac{3}{4}_2$  and  $\bar{c}_1 \cdot \bar{c}_2$ ) both hold and if and only if (3.7) is satisfied, with  $q$  being any quantity such that  $q \geq \frac{f}{g} \bar{q}$ . When  $\bar{c} \leq \underline{c}$  and  $\frac{1}{2} \leq \underline{1/2}$  but (3.7) fails or when either  $\bar{c} < \underline{c}$  or  $\frac{1}{2} < \underline{1/2}$ , the unique Nash equilibrium is

$$(\bar{c}; \frac{3}{4}) = \begin{cases} \bar{b}(s); s & \text{if } \max \{ f^{-1}_2; \frac{3}{4}_2 g \} \cdot s \text{ and } \bar{b}(s) < \min \{ f^{-1}_1; \bar{c}_3 g \} \\ (1; \bar{b}(1)) & \text{if } \max \{ f^{-1}_2; \bar{c}_3 g \} \cdot s \text{ and } \bar{b}(1) < \min \{ f^{-1}_1; \frac{3}{4}_3; s g \} \\ (1; s) & \text{if } s < \min \{ f^{-1}_1; \frac{3}{4}_3 g \} \text{ and } \bar{b}(1) \leq s: \end{cases}$$

Otherwise, no Nash equilibrium exists.

Proof. The proof follows straightforwardly from inspection of the best response functions.<sup>14</sup> ■

Remark 50 Observe that when market frictions are not high and each cartel controls all traders on its side of the market, i.e.  $\alpha = \beta = 0$ ; the unique Nash equilibrium is  $(\bar{q}; \bar{q}) = (0; 0)$ :

Also observe that, when market frictions are not high and  $\beta > \beta_2$ ; the function  $\mathbf{b}(\beta)$  is strictly increasing in  $\beta$  and such that  $\mathbf{b}(\beta) < \beta$ : Therefore  $\mathbf{b}(s) \cdot s$  is always satisfied for  $\max \beta_2; \beta_3 \cdot s$ : Conversely, the condition  $\mathbf{b}(1) < s$  is satisfied if and only if

$$\frac{1}{2} < \frac{s^\circ (5_i 2^\circ)(2_i s^\circ)_i (4_i \circ)}{(1_i \circ)_i s^\circ} = \bar{\eta} ;$$

where  $\bar{\eta} > 0$  if and only if

$$s > \frac{(5_i 2^\circ)_i \rho (1_i \circ)_i (5_i 2^\circ)}{\circ (5_i 2^\circ)} \cdot \bar{\eta} :$$

Under high market frictions, Nash equilibria always exist. If  $\max \beta_1; \beta_1 g < s$  (or equivalently if  $\alpha < \alpha_1$  and  $\frac{1}{2} < \frac{1}{2}_1$ ), market outcomes with active cartels and symmetric market participation, that is equilibria such that  $(\bar{q}; \bar{q}) = (q; q)$  with  $q < s$ ; prevail. Otherwise, if  $\beta_1 = s$ ; only the buyers' cartel is active, or if  $\beta_1 > s$ ; neither cartel is active.

Under intermediate market frictions, both symmetric and asymmetric market outcomes might be attained, but it might also be the case that no Nash equilibrium of the game  $G^{\alpha; \beta}$  exists.

Finally, under high market frictions and when either  $0 < \alpha < \alpha_1$  or  $0 < \frac{1}{2} < \frac{1}{2}_1$  (or both) hold, if an equilibrium exists at all, it is represented by an asymmetric market outcome with at most one active cartel. Example 51 considers the range of market outcomes under intermediate market frictions.

Example 51 Let  $s = \frac{4}{5}$  be the ex ante measure of sellers and let  $\alpha = \frac{3}{4}$ : (i) Assume first that the proportions of independent buyers and sellers are such that  $\alpha = \frac{1}{10}$  and  $\frac{1}{2} = \frac{10}{77}$  respectively, whereby  $\beta_1 = \beta_1 = \frac{20}{69}$  and  $\beta_2 = \frac{3}{5} < \beta_2 = \frac{48}{77}$ . The game  $G^{\frac{1}{10}; \frac{10}{77}}$  is such that all pairs  $(\bar{q}; \bar{q}) = (q; q)$  with  $q \in [\frac{20}{69}; \frac{3}{5}]$  represent equilibrium outcomes. This result is shown in Figure 3.7. (ii) Assume now that the proportions of independent buyers and sellers are such that  $\alpha = \frac{1}{30}$  and  $\frac{1}{2} = \frac{27}{50}$  respectively, whereby  $\beta_1 = \frac{20}{189}$ ;  $\beta_2 = \frac{1}{5}$  whereas  $\beta_1 = \frac{144}{181}$  and  $\beta_2 = \frac{324}{125} > s$ : The unique equilibrium of game  $G^{\frac{1}{30}; \frac{27}{50}}$  is then given by the pair  $(\bar{q}; \bar{q}) = (\mathbf{b}(s); s) = (\mathbf{b}(s); s) = (\frac{112; 2}{105}; \frac{4}{5})$  and is shown in Figure 3.8. (iii) Finally suppose that the measure of independent buyers and sellers are  $\alpha = \frac{1}{20}$  and  $\frac{1}{2} = \frac{1}{6}$  respectively, whereby  $\beta_1 = \frac{20}{129}$ ;  $\beta_2 = \frac{3}{10}$  whereas  $\beta_1 = \frac{16}{45} > \beta_2$  and  $\beta_2 = \frac{4}{5} = s$ : It is straightforward to check that the game  $G^{\frac{1}{20}; \frac{1}{6}}$  has no Nash equilibrium, as shown in Figure 3.9.

<sup>14</sup>See Example 51 below.

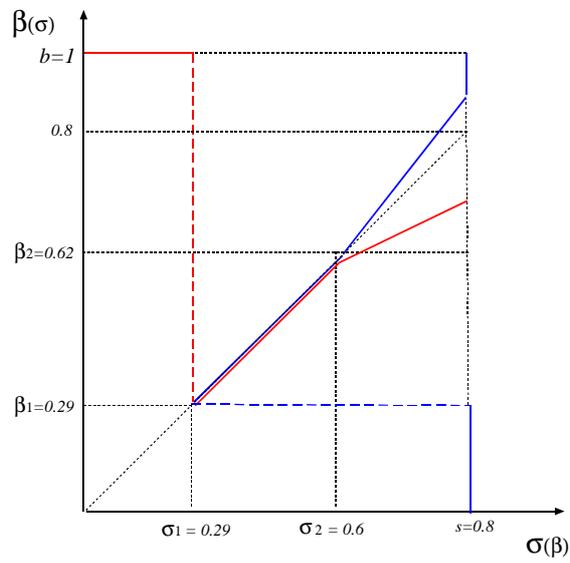


Figure 3.7: Case (i) in Example 51: Multiplicity of equilibria

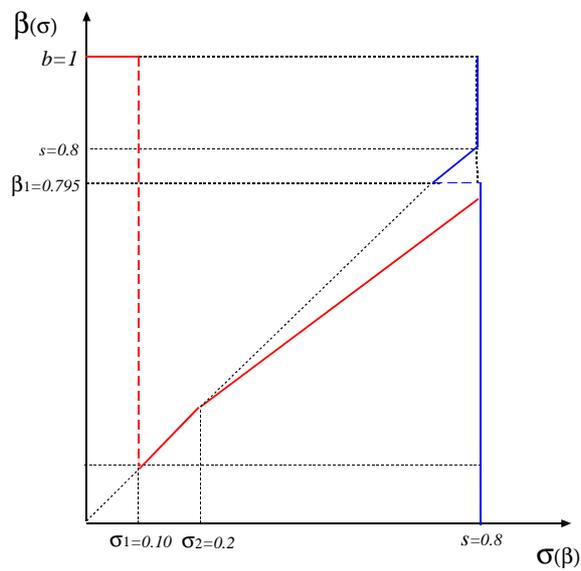


Figure 3.8: Case (ii) in Example 51: Uniqueness of the equilibrium

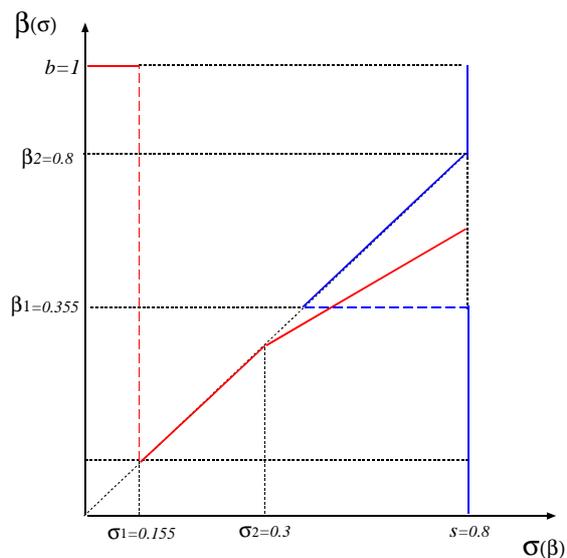


Figure 3.9: Case (iii) in Example 51: Non-existence of an equilibrium

### 3.5 Stability

Depending on the relative magnitudes of the cartel memberships  $(1 - \alpha)b$  and  $(1 - \alpha)s$ , a broad set of Nash equilibria might exist. But not all market outcomes are equally relevant, because some of them cannot support stable levels of cartel memberships.

Within the set of market outcomes yielding restricted trade, only a few can be supported by  $\beta$ -stable levels of cartel memberships. Indeed, given a profile  $(\alpha; \beta; \gamma)$ , there is at most one market outcome that supports its membership levels. It is shown that, for a wide set of parameter configurations, there exist  $\beta$ -stable (and stable) market outcomes with at least one active cartel. At these market outcomes, the quantities traded must be symmetric, regardless of the potential measures of traders on each side of the market. If frictions are not high,  $\beta$ -stable (and stable) market outcomes where at most one cartel is active exist as well. At these outcomes, the active cartel is the one with greater membership and it reduces its supply or demand so as to slightly undercut the counterpart's (unconstrained) market participation.

**Proposition 52** Let  $s \cdot b = 1$ : (i) An  $\beta$ -stable market outcome with at least one active cartel exists only if  $\alpha \leq \underline{\alpha}$  and  $\beta \leq \underline{\beta}$  hold (equivalently only if  $\gamma_1 \leq \gamma_2$  and  $\beta_1 \leq \beta_2$ ).<sup>15</sup> The profile  $(\alpha; \beta; \gamma)$  with  $\beta = \gamma \cdot s$ ; is an  $\beta$ -stable market outcome if and only if  $\alpha \leq \underline{\alpha}$ ,  $\beta \leq \underline{\beta}$ ; and  $\beta = \beta_1 = \gamma_1 = \gamma_2$ : (ii) An  $\beta$ -stable market outcome with at most one active cartel exists only if frictions are not high, and it consists in either  $(\alpha; \beta; \mathbf{b}(s); s)$  or  $(\alpha; \beta; 1; \mathbf{b}(1))$ : The market outcome  $(\alpha; \beta; \mathbf{b}(s); s)$  is  $\beta$ -stable

<sup>15</sup>This requirement is always satisfied when market frictions are not low, i.e. when  $\alpha \leq \alpha^*$  (see page 50 for further reference).

if and only if: (a)  $\frac{1}{2} > \max f_{\frac{1}{2}}; \frac{1}{2}g$  and  $\bar{p} = \bar{p}_2$ ; where  $s_i < b(s) < s$ ; or (b)  $\frac{1}{2} > \max f_{\frac{1}{2}}; \frac{1}{2}g$  and  $\bar{p} = \bar{p}_3$ : The market outcome  $(\bar{p}; \frac{1}{2}; 1; \bar{p}(1))$  is  $\pi$ -stable only if  $\bar{p}(1) < s$  or equivalently only if  $\frac{1}{2} < \bar{p}$  and if and only if: (a)  $\bar{p} > \max f_{\bar{p}_1}; \bar{p}_3g$  and  $\frac{1}{2} = \frac{1}{2}_i < \bar{p}$  (in this case  $s_i < \bar{p}(1) < s$  for  $s \in (s^*, 1]$ ; with  $s^*$  solving  $\frac{1}{2}_i < \bar{p}$ ); or (b)  $\bar{p} > \max f_{\bar{p}_1}; \bar{p}_3g$  and  $\frac{1}{2} = \frac{1}{2}_3 < \bar{p}$ :

Proof. See Appendix A.1. ■

Remark 53 Note that  $\bar{p} = \bar{p}_1 = \frac{3}{4} = \frac{3}{4}$  is equivalent to

$$\bar{p} = \frac{\frac{1}{2}s}{1 + \frac{1}{2}(3i - 2^s)(1 - s)} : \quad (3.8)$$

Observe that an  $\pi$ -stable market outcome where only the sellers' cartel is active might exist when the market is balanced, i.e. if  $s = b = 1$ ; or only if the total measure of sellers  $s$  is still smaller than the total measure of buyers  $b$ , but by an infinitesimal amount. In this case the sellers' cartel is active but only marginally so.

Also note that asymmetric  $\pi$ -stable market outcomes of type (a) do not survive in the limiting situation where  $\pi \rightarrow 0$ : A characterization of the set of stable market outcomes is presented in the next proposition, whose proof is immediate and therefore omitted.

Proposition 54 Let  $s \cdot b = 1$ : (i) The profiles  $(\bar{p}; \frac{1}{2}; \bar{p}; \frac{3}{4})$ ; where  $\bar{p} < \bar{p}_1$ ,  $\frac{1}{2} < \frac{1}{2}_1$  and  $\bar{p} = \bar{p}_1 = \frac{3}{4} = \frac{3}{4}$  are the only symmetric stable market outcomes. (ii) The profiles  $(\bar{p}; \frac{1}{2}; b(s); s)$ ; where  $\frac{1}{2} > \max f_{\frac{1}{2}}; \frac{1}{2}g$  and  $\bar{p} = \bar{p}_3$ ; and the profiles  $(\bar{p}; \frac{1}{2}; 1; \bar{p}(1))$ ; where  $\bar{p} > \max f_{\bar{p}_1}; \bar{p}_3g$  and  $\frac{1}{2} = \frac{1}{2}_3 < \bar{p}$ ; are the only asymmetric stable market outcomes:

An immediate consequence of Proposition 54 is that stable market outcomes with two active cartels always exist when market frictions are either high or intermediate. However, these market outcomes might be inefficient.

Corollary 55 Let  $s \cdot b = 1$ : The set of symmetric stable market outcomes includes inefficient profiles  $(\bar{p}; \frac{1}{2}; \bar{p}; \frac{3}{4})$ ; such that  $\bar{p} < \bar{p}_1$ ,  $\frac{1}{2} < \frac{1}{2}_1$ ; and  $0 < \bar{p}_1 = \frac{3}{4} < s$ : At these profiles both cartels restrain trade.

Observe that the degree of inefficiency at the symmetric stable market outcomes described in Corollary 55 is not bounded. Indeed, condition (3.8) is compatible with symmetric stable market outcomes in which the cartels control more and more traders and force very limited market participation. In the limit, the market is driven to the collapse, given that the profile  $(0; 0; 0; 0)$  represents a stable symmetric market outcome for any level of market frictions: Nonetheless, Proposition 54 does not rule out symmetric market outcomes at which  $\bar{p} = \bar{p}_1 = \frac{3}{4} = \frac{3}{4} = s$ ; thus the following holds.

**Corollary 56** Let  $s \cdot b = 1$ : The set of stable market outcomes includes an efficient profile  $(\theta; \frac{1}{2}; s; s)$ ; where  $\theta = \theta_1$  and  $\frac{1}{2} > \max\{f\frac{1}{2}; \frac{1}{2}g\}$ ; which is such that the sellers' cartel is not active and the buyers' is active only to assure that demand equals supply.

When  $s < b = 1$ ; at the stable market outcomes described in Corollary 56, symmetric trade is achieved with the buyers' cartel withdrawing a positive measure of its members. Since cartel activity does not decrease trade, efficiency is not affected. Despite this fact, the distribution of surplus between buyers and sellers is substantially altered in favor of buyers.

As long as  $s < b = 1$ ; at the symmetric stable market outcomes the sellers' cartel always withdraws from the market fewer traders than the buyers' cartel does. Does the buyers' cartel necessarily control more traders than the sellers? Does it need to encompass a relatively higher proportion of traders? These questions are answered in the following corollary.

**Corollary 57** Let  $s < b = 1$ : At a stable symmetric market outcome the buyers' cartel is such that: (i) it controls more traders than the sellers' cartel, i.e.  $(1 - \theta) > s(1 - \frac{1}{2})$ ; (ii) it always controls a larger fraction of the total population on its side than the sellers' cartel, i.e.  $(1 - \theta) > (1 - \frac{1}{2})$ :

**Proof.** Taking into account condition (3.8), the inequality  $s(1 - \frac{1}{2}) < (1 - \theta)$  holds if and only if

$$s(1 - \frac{1}{2}) < 1 - \frac{\frac{1}{2}s}{1 + \frac{1}{2}(3 - 2^s)(1 - s)} ; \tag{3.9}$$

and it is straightforward to check that (3.9) holds for all  $\frac{1}{2}$  and  $s$ : Inequality  $(1 - \theta) > (1 - \frac{1}{2})$  also holds for all  $s < 1$ : ■

When  $s < b = 1$  and when the only stable asymmetric market outcome consists in a profile such as  $(\theta; \frac{1}{2}; b(s); s)$ , then only the buyers' cartel is active and it slightly undercuts sellers' participation level  $s$ : Even though the buyers' cartel actually withdraws a non-negligible measure of its members, efficiency is only marginally affected since the level of realized trade decreases only by a small amount. The distribution of surplus between buyers and sellers is again substantially altered in favor of buyers.

We may now summarize our results. There are stable market outcomes where both sides exercise countervailing power. In this case, countervailing power might be the source of severe market inefficiency. However, in markets that are not balanced ex ante (maybe because the short side has effective means to prevent entry without compensation to the excluded traders) there exist stable outcomes at which the exercise of countervailing power by the long side affects the distribution of surplus without damaging efficiency. We find that these market outcomes (partially) vindicate Galbraith's claims that countervailing power plays a desirable role in some markets.