Adaptive dynamics in an infinite dimensional setting

Sílvia Cuadrado

In this thesis we study selection-mutation models for the density of individuals with respect to a phenotypic evolutionary variable. This approach provides a dynamical system in strictu sensu for the complete description of biological evolution: random variability plus natural selection.
The relationship between the equilibrium densities and the evolutionarily stable strategies (the steady states of the evolutionary dynamics in the sense that a "resident" population with this value of the evolutionary variable cannot be invaded by "mutants" with a different value) is studied.
More precisely, these equilibrium densities are functions of the evolutionary variable that tend to concentrate at the evolutionarily stable value of the variable when the mutation rate tends to zero.
The technics mainly involve positive semigroup theory and infinite dimensional Perron-Frobenius theorems and take benefit of the special form of the evolution equation, namely, $u_t = A(E(u))u$, where $A(E)$ is the generator of an analytic irreducible positive (linear) semigroup and the nonlinear interaction takes place through a finite dimensional valued "environment" E.
Studying the spectrum of the linearized operator at the equilibrium density of the evolution equation we obtain some results about stability.
The convergence results for the evolution equation are applied to three examples: two versions of an integro-differential equations model for the distribution of individuals with respect to the age at maturity, being this age at maturity the evolutionary variable, and a predator prey model for the density of predators with respect to their index of activity during daytime.