

Essays on Education and Economic Performance

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International Doctorate in Economic Analysis

June 2003

A mis padres

Acknowledgements

I would like to express my gratitude to those who have contributed with their help and support to the completion of this dissertation.

First, I am deeply indebted to my supervisor, Carmen Beviá for her help and encouragement along the hard process of doing this thesis. I am also very grateful to Jordi Caballé who has followed my work as a second supervisor, and to Ignacio Conde, whose interest and support were crucial at the beginning of this project.

The fourth chapter of this thesis is a joint work with Elena del Rey. I am really grateful for her collaboration in this work. This thesis has also benefited from comments and suggestions of Luis Corchón, Philippe De Donder, Íñigo Iturbe-Ormaetxe, Raquel Fernández, Omer Moav, Carmelo Rodríguez and Gilles Saint-Paul.

My thanks to the Faculty and Students of IDEA for the friendly and cooperative research environment from which my research has greatly benefited. The support of my classmates and friends there: Debrah, Jana, Jernej, Jo, Julius, Kaniska, Maritxu, Nico, Óscar and Pablo, has been essential for me during these years.

I really appreciate the hospitality of New York University, where I spent three unforgettable months as a visiting student. I am indebted to Raquel Fernández for her guidance and for all I have learnt from her. Special thanks to Anna and Luca, for their kindness and hospitality, and to Miquel, for his enthusiasm.

This thesis has been completed during my stay at Université de Toulouse 1 as a Marie Curie visiting student. I am very grateful to Gilles Saint-Paul, who has followed my work during the academic year spent in Toulouse. I also thank Aude, for all her help, and Anna and Marta, for their friendship.

Financial support from the Comissionat de Universitats i Recerca of Generalitat de Catalunya is also gratefully acknowledged.

Finally, I want to express all my gratitude and love to my family, and especially to my parents, for being always so close even in the distance, and to Carmelo, for everything.

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Chapter 1

Introduction

Economists have devoted considerable attention to understanding the role of human capital on economic performance. This is not surprising given the enormous relevance of human capital to economic well being. The underlying notion behind the concept of human capital is that individuals make investment decisions in education that allows them to acquire the skills that are relevant for the labor market. The accumulated skills resulting from these investments represent the human capital of an individual.

The empirical work on human capital has concentrated mainly on the impact of educational attainment or years of schooling on economic growth. This literature documents the existence of a positive correlation between educational attainment, especially primary education, and the average growth rate of countries.¹ Theoretical contributions to understanding the fundamental role of human capital on economic development date back to Lucas (1988), who attribute the differences in growth rates among countries to the disparities in the rates at which countries accumulate human capital over time.

In analyzing human capital and its implications for future outcomes, it has been frequently ignored where individuals' skills come from or how human capital is produced. Economic research on the determinants of education and its consequences has only recently

¹See for instance Mankiw *et al* (1992), Benhabib and Spiegel (1994) and Barro and Sala-i-Martin (1995).

attracted the interest of researchers. Formal schooling, individual abilities, family income and cultural background are some of the factors that contribute to the accumulation of human capital. Cognitive skills, measured by students' performance on standardized tests, are found to be positively related to individual earnings, productivity and economic growth. The earnings advantages to higher scores on standardized tests, after holding constant years of schooling, are documented in a number of empirical studies.²

Family background and, in particular, the level of education of the parents is also a crucial factor in determining the educational attainment of the children. There is evidence supporting the fact that the probability of attending college is higher for the children of parents with university degrees. For instance, adults of educated parents are between three and four times more likely to attain tertiary education than those with uneducated parents (see OECD (1998)). Investments in education take two forms: direct investments of the parent's time and investments in formal schooling that must be purchased in the market. The skills of parents in educating the child, and hence, the quality of the time devoted to his education, seem to be influenced by parental educational attainment.

Investments in formal education return future economic benefits in the same way as a firm investing in physical capital returns future production and income. These investments are guided first, by their expected return, which is estimated to be high: between 17 and 22 percent for lower education, 15 and 16 percent for high school, 12 and 13 percent for college and 7 percent for graduate school (Willis, 1986). The second factor affecting investment decisions in education is parental resources. The evidence is strong that family income has an important effect on the probability of staying in school, at every level of education.³

The unobservability of children's ability, moral hazard problems and the fact that de-

²The earnings advantages to higher performance is documented in Bishop (1989), Grogger and Eide (1993) and Neal and Johnson (1996). On the other hand, Hanushek and Pace (1995) find that college completion is significantly related to higher test scores at the end of high school.

³See Cameron and Heckman (1996).

isions are made by parents on behalf of their children suggest that education is a special investment good. Moreover, the inability of parents to borrow against the future human capital of their children leads to inefficient educational investments by low income families. Thus, a system where parents finance the education of their children is inefficient. Since investments in human capital depend on parental income, and income vary across parents, educational attainment will vary across children for reasons unrelated to children's characteristics, such as their cognitive abilities.

The importance of the acquisition of skills appears in its effects on the earnings of individuals and on the subsequent distribution of income in the economy. Thus, if differences in parental resources affect the future earnings of children, the inequality associated to social immobility in the earnings distribution will persist over time. As long as the marginal return from investing in education is greater for high-ability students, high persistency in economic status suggests that there is substantial underinvestment at the bottom of the income distribution. Hence, improvements in the quality of education for children from poor families will increase efficiency, since a higher correlation between children's cognitive skills and investments in education raises the accumulation of human capital.

The inexistence of a market for loans to finance educational investments, especially in primary and secondary education, may help explain why in all countries governments subsidize education. If one of the objectives of the government is to ensure equal opportunity, the subsidization of education may correct inefficiencies in the capital markets allowing low income students to have access to education at almost no cost. However, government intervention in education is not restricted to subsidize a certain amount of education. In most countries, education -even at the tertiary level- is publicly provided, although these educational services are available in the private market. This public provision may be rationalized on the basis of the external effects of education, especially at the primary and secondary levels. Thus, the government provides citizens with a minimum degree of literacy that benefits the society as a whole.

Another argument that regards education as a special good is that each student creates externalities for other students and for the educational process. The quality of the education any student obtains from school or college depends in good measure on the quality of that student's peers. Along with per-student expenditures and school's facilities, average scores on standardized tests are often used as a measure of institutional quality. The special features of the educational production make the market for education different from other markets. The customers in this market, the students, are also the input in the production of school quality. Educational institutions are publicly owned in many countries, but frequently face competition from private institutions. Some private educational institutions pursue profits while others have different goals.

This dissertation consists of three chapters exploring different aspects of education and its effects on economic performance. Chapter 2 and Chapter 3 investigate the effects of public subsidization of education on human capital accumulation and income distribution. In these chapters, I adopt the approach of modelling public education policies as the equilibrium outcome of a political process. This approach differs from policy analysis in the traditional Public Finance, which is almost entirely normative, and it instead focuses on positive issues, combining economic and political analysis. Chapter 4 analyzes the strategic interaction among educational institutions in the higher education market. The approach taken in this chapter builds on industrial organization models of competition.

The study of intergenerational mobility, that is, the correlation of economic or education status among individuals belonging to the same family, is important for economic performance. Mobility implies a higher correlation between the ability of individuals and their educational attainment and thus, since high-ability individuals obtain a higher return to educational investments, economic resources are more efficiently allocated in mobile economies. Chapter 2 investigates on the one hand, the impact of education subsidies on intergenerational mobility and on the other, the conditions under which the subsidization of education emerges as a political equilibrium. In the absence of capital

markets to finance investments in human capital, education subsidies play a key role in intergenerational mobility. We find that the political support of these subsidies depends positively on the level of development of the economy, which is measured by the proportion of educated individuals. Interestingly, the impact of income inequality on the size of the subsidy is not monotone. In our model, income inequality raises with the wage gap between educated and uneducated labor. Intuitively, a low wage gap reduces incentives to invest in education while a high level of inequality raises the conflict of interests among poor and rich individuals over the degree of subsidization of education. Hence, we obtain that the magnitude of the subsidy is higher at intermediate levels of inequality.

Government intervention in education frequently takes the form of public provision. Advocates of school choice argue that a public school system offering a uniform -and frequently low- educational quality, independently of individuals' specific needs may fail to ensure equal educational opportunities. Discontent with public schools in many countries may help explain the interest on issues related to school choice, including education vouchers. A voucher program provides students attending private schools a tax-financed payment covering all or most of the tuition charged. Chapter 3 evaluates the impact of education vouchers on the efficient sorting of students into public and private schools. Private schools offer a continuum of quality levels, while public schools provide a uniform quality, funded with proportional taxes. Students differ over family income and ability and parents choose between public and private schools. We find that a tax-minimizing voucher will be approved by majority when the level of public educational quality is sufficiently high. In the calibrated model, the equilibrium voucher entails welfare gains although leads to greater inequality. The impact of different voucher policies is also analyzed. We find that welfare gains increase with the voucher size but the impact of the magnitude of the voucher on income inequality is not monotone.

The provision of education, in contrast to other goods, is not allocated only by prices. Higher education institutions usually use exams to allocate students to schools. The

importance of this instrument varies depending on the country and institution. In many European countries, public universities set very low prices and use exams as the main guide to determine their admissions while private universities are usually of lower quality. In contrast, in the United States the majority of universities are private and non-profit institutions and both exams and prices play an important role as instruments to allocate students. Chapter 4 focuses on optimal choices of prices and exams by universities in the presence of borrowing constraints. There are two institutions, one public and one private, providing educational quality in the higher education market. Students differ in their unobservable ability and in their income endowment and they choose whether to attend a university or remain uneducated. We first compare the optimal behavior of a public monopoly, maximizing public surplus, with the optimal choices of a private monopoly, maximizing profits. We find that the private university only uses prices as allocation device, while the public institution uses exams while setting a zero price for its educational services in the presence of borrowing constraints. Next, we model competition between a public and a private university and we show that in equilibrium, the public university provides a higher quality than the one provided privately. This result may be explained by the different strategies, exams versus prices, followed by the public and the private university respectively. The use of exams allows the public university to behave as a monopoly in the higher education market and the private university attracts those students of lower ability who are not accepted at the public institution.

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Chapter 2

Education Subsidies, Income Inequality and Intergenerational Mobility

2.1 Introduction

In any society, individuals are born with different parental income, learning ability and family background. We may claim that society provides equal opportunity to its members when individuals' future success is largely unpredictable on the basis of family background. Consequently, we can judge if there is equal opportunity by looking at parents and their children to evaluate whether children' success is determined in large part by their parents' income. In the presence of imperfect capital markets to finance the acquisition of human capital, parental income becomes a crucial factor in determining future children' economic status.¹ Thus, since rich families tend to invest more in their children' education, we expect that initial differences in income across parents will affect their offspring's economic

¹For instance, it is well documented that children from rich families are more likely to attend university than children from poor families.

success. A high persistence in economic status across generations within the same family is therefore, an indicator of low intergenerational mobility.

The study of mobility is also important for human capital accumulation, since mobility implies a higher correlation between individual's ability or effort, and educational attainment. As long as the marginal return from studying is greater for talented students, a higher intergenerational mobility means that resources are allocated more efficiently and thus, the accumulation of human capital raises. Moreover, there exists some empirical evidence that mobility is positively correlated with income equality and that it is higher in more developed economies.²

This chapter investigates the impact of education subsidies on intergenerational mobility and analyzes the conditions under which the subsidization of education emerges as a political equilibrium. The novel feature of this work is precisely the endogenous determination of this education policy in a dynamic context.³ The initial income distribution plays a crucial role in the evolution of the economy because it determines the equilibrium level of subsidization of education and thus, the proportion of individuals investing in education. Thus, economies with different levels of income per capita and inequality choose different levels of education subsidies and exhibit very different patterns of mobility.

Our work is related to two different lines of research: the literature on income inequality and redistributive politics and the line of research studying the relationship between income inequality and intergenerational mobility. The first strand of literature links income inequality and economic development through the effect that income distribution has on redistribution, and its subsequent effects on economic growth. In this literature,

²Becker and Tomes (1986) survey a number of empirical studies for different countries showing a positive relationship between intergenerational mobility and income per capita. Björklund and Jäntti (1997) show that Sweden has higher income equality and intergenerational mobility than United States. Contradictory findings appear in Checchi *et al.* (1999) who show that Italy is more equal but less mobile than the US.

³Fernández and Rogerson (1995) have investigated the endogenous determination of education subsidies in a political process but their model is static.

we can distinguish two different approaches explaining the negative relationship between income inequality and economic growth supported by some data. In the approach of Persson and Tabellini (1994) and Alesina and Rodrik (1994) growth accrues from the accumulation of physical capital. Thus, the link between initial income inequality and subsequent growth is the higher taxation on investment induced by higher inequality, which is translated into a poorer median voter who pressures for more redistribution. This, in turn discourages investment and growth.

Another explanation for the negative relationship between income inequality and growth comes from the theoretical literature on human capital accumulation in the presence of credit market imperfections. According to this literature, it is the existence of decreasing returns in the accumulation of this production factor which makes redistribution toward those individuals with lower human capital endowments to increase growth. However, empirical evidence is not conclusive about the relationship between income inequality and economic development and some theoretical papers provide an explanation for these contradictory empirical findings. For instance, Perotti (1993) shows that the level of development is crucial in determining the effects that both the level of income inequality and the equilibrium level of taxation may have on economic growth. In Saint-Paul and Verdier (1993), higher inequality leads to higher growth because it induces a higher expenditure in public education, which is the source of growth in their model.

In the second strand of literature, Becker and Tomes (1986) and Loury (1981) analyze the dynamic relationship between income inequality and intergenerational mobility under imperfect capital markets, in the context of an unchanging macroeconomic environment, in which mobility does not affect the return of education. Owen and Weil (1998) investigate the interaction between economic growth and intergenerational mobility with a model that allows them to compare the degree of mobility between economies that are at different stages of the development process. While their analysis is restricted to steady state equilibria, Maoz and Moav (1999) study the dynamics of inequality and mobility along

the growth path. In both papers, the existence of complementarities between educated and uneducated labor provides a theoretical explanation, in the context of a changing economic environment, for the positive relationship between economic development, mobility and income equality supported by some empirical literature.

This work merges these two lines of research to investigate the connection between income inequality, intergenerational mobility and endogenous education policies. We present an overlapping-generations model based in Maoz and Moav (1999), in which individuals are heterogenous in their parents' income and in their innate ability to learn. Capital markets are inexistent and thus, investments in human capital depend crucially on parental income and the size of education subsidies. In our model, human capital accumulation accrues from an increase in the proportion of individuals who invest in education over time. This process is fueled by intergenerational mobility of individuals born of uneducated parents who acquire education.

The equilibrium degree of subsidization of education is the outcome of a political process. The level of education subsidies is chosen by majority voting by individuals with conflicting interests regarding the education policy. This conflict reflects socio-economic factors derived from differences across them, either in terms of income or ability, and these factors are those shaping the distribution of individuals' preferences over the magnitude of subsidization of education. Education subsidies finance partially the cost of acquiring education, which depends negatively on individuals' ability, and they are funded by proportional taxes over total income in the economy. Education subsidies are available only to those individuals who acquire education.⁴

In our model, mobility raises when the size of the subsidy is sufficiently high to allow some individuals from poor families to invest in education. In this case, education subsidies become an endogenous mechanism to generate mobility in economies stuck in

⁴In developed economies, education subsidies are intended to *partially* finance the costs of acquiring tertiary education, since primary and secondary education are completely funded publicly. However, in poor countries, they may subsidize lower education levels.

poverty traps. We find that a minimum level of development, measured by the proportion of educated individuals in the economy, is required for the political support of education subsidies. This result may be explained by the inexistence of a capital market to finance the acquisition of education. Thus, in poor economies the proportion of individuals acquiring education is low and education subsidies cannot be supported by a majority of individuals in the economy.

The level of income inequality is also a relevant variable in determining the equilibrium degree of subsidization of education. We obtain that the magnitude of the subsidy is higher at intermediate levels of income inequality. In our model, income inequality is monotone in the wage gap between educated and uneducated labor. Intuitively, a low wage gap reduces incentives to invest in education while a high level of inequality raises the conflict of interests among poor and rich individuals over the degree of subsidization of education. In both cases, the political support of this education policy decreases.

This chapter is structured as follows. Section 2.2 presents the economic environment in which individuals' economic decisions and the conditional evolution of the economy are analyzed. In Section 2.3 the political equilibrium level of education subsidies is characterized. Section 2.4 studies the effect of endogenous education subsidies in the process of development, and how both the dynamics and the steady state level of educated individuals are affected by such subsidies. Finally, in section 2.5 we present some concluding remarks.

2.2 The Model

2.2.1 Production

The economy produces a single homogeneous good that can be devoted to either consumption or investment in education. Aggregate output is given by a linear production

function of educated labor E_t , and uneducated labor U_t , in period t :

$$Y_t = w^E E_t + w^U U_t, \quad (2.1)$$

where w^E and w^U are the wages or marginal productivity of educated and uneducated labor respectively. We assume that educated individuals are more productive than uneducated individuals and the wage of educated labor is at least as twice the wage of uneducated labor, $w^E \geq 2 w^U$.⁵

We choose this simple production function in order to analyze the role of education subsidies in the degree of intergenerational mobility of the economy. For instance, with a Cobb-Douglas production function in which both labor types are complements, intergenerational mobility would be affected by labor wages and thus, it would be difficult to isolate the role of education subsidies in mobility.

Individuals supply inelastically one unit of labor, either as educated or uneducated. The total number of individuals in the economy is normalized to one and is equal to the number of individuals living in each generation. Thus, we can express the proportion of uneducated workers, $U_t = 1 - E_t$ as a function of E_t .

2.2.2 Individuals

We consider an overlapping generations economy, in which individuals live for two periods. Each individual gives birth to another in his second period of life, so population remains constant over time and is normalized to one. In their first period of life, young individuals do not work. They receive a transfer from their parents that may be used either to consume or to purchase education. Young individuals also decide by majority voting the level of education subsidies in this period.⁶ In their second period of life, individuals work

⁵The wage ratio of educated labor to uneducated labor is usually measured by the 90th-10th percentile of the wage distribution and this ratio is above 2 in OECD countries. See Machin (2002).

⁶It would be equivalent, in terms of preferred subsidies of individuals, to consider that it is the old generation the one who is choosing the subsidization of education of their children.

as either educated or uneducated laborers. They do not consume in this period since they bequeath all their labor income to their offsprings.

Young individuals differ not only in the bequest received from their parents, but also in their innate ability. Differences in abilities are represented by different education costs.⁷ The ability of a young individual i is independent of both the ability and income of his parent and it is inversely correlated with his education cost, h^i . Education costs are uniformly distributed in the interval $[\underline{h}, \bar{h}]$, where $\underline{h} > 0$.⁸

We assume that capital markets to finance investments in education are inexistent, and thus, young individuals cannot borrow against their future income to acquire education. This assumption implies that individuals' wealth in their second period of life is only their labor income. Since education is costly and borrowing is not possible, some individuals may not be able to afford an education, or they may not be willing to acquire it in order to increase first-period consumption.

Individuals derive utility from consumption in their first period of life and from transfers to their children when they are old. Old individuals obtain utility from the size of the bequest they leave to their offsprings. Therefore, altruism of the parents is based on the "joy of giving" motive.⁹ The utility function is logarithmic and equal for all individuals born in period t :

$$U_t^i(c_t^i, w_{t+1}^i) = \ln c_t^i + \ln w_{t+1}^i, \quad (2.2)$$

where c_t^i is the consumption of individual i when he is young, in period t and w_{t+1}^i is the bequest he passes to his child when he is old, in period $t + 1$.

The utility maximization problem of a young individual can be solved backwards in two stages. In the first stage, he chooses his most preferred subsidy and the equilibrium level of education subsidies is decided by majority voting. Only those individuals who

⁷Education costs summarize both monetary and opportunity costs of purchasing education. Such opportunity costs represent lost consumption in the first period.

⁸If $\underline{h} = 0$, the acquisition of education does not involve any cost for some high-ability individuals.

⁹This type of altruism also appears in Galor and Zeira (1993) and Banerjee and Newman (1993).

acquire education in period t are eligible to receive education subsidies, S_t . These subsidies are funded by proportional taxes θ_t over bequests or total first-period income, \bar{w}_t , which is the average wage in period t

$$\bar{w}_t = E_t w^E + (1 - E_t) w^U = Y_t. \quad (2.3)$$

In the second stage, individuals decide whether to acquire education or not, given the level of education subsidies, S_t and the proportional tax rate over total income, θ_t . We first solve this stage of young individuals' utility maximization problem and we leave the analysis of political decisions for the next section.

In our economy, individuals only make economic decisions in their first period of life, when they decide whether they purchase education or not, since in the second period they just bequeath their labor income, w_{t+1}^i to their offsprings. Since there are no savings, the only way for a young individual to increase his future income is by means of investing in education. The decision of acquiring education varies across young individuals, since they differ in the transfer received from their parents, w_{t+1}^i , $i = E, U$ and in their education costs, $h^i \in [\underline{h}, \bar{h}]$.

Old individuals belonging to the same type, either educated or uneducated, receive the same wage, and thus, there are only two levels of second-period income and hence, of bequests. Thus, the superindex i corresponding to the transfer a young individual receives from his parent can be replaced by his parent's type: E or U . Labor wages for educated and uneducated labor are constant over time and thus, $w_t^i = w_{t+1}^i = w^i$, $i = E, U$.

A young individual i must substitute first-period consumption for second-period income to acquire education, since he has to give up consumption in order to become educated.¹⁰ Thus, an individual i investing in education consumes $c_t^i = (1 - \theta_t)w^i + S_t - h^i$ in his first period of life and earns the educated labor wage, w^E in the second period. Conversely, if he decides not to become educated, he instead consumes, $c_t^i = (1 - \theta_t)w^i$

¹⁰Note that those individuals with education costs below the level of subsidies, $h^i < S_t$ are able to purchase education without giving out first period consumption.

and obtains the uneducated labor wage, w^U in the second period of his life. Hence, a young individual i will invest in education if and only if the indirect utility derived from purchasing education, U_t^E is higher or equal than the utility derived from not investing in human capital, U_t^U :

$$U_t^E \geq U_t^U, \quad (2.4)$$

where $U_t^E = \ln((1 - \theta_t)w^i + S_t - h^i) + \ln w^E$ and $U_t^U = \ln((1 - \theta_t)w^i) + \ln w^U$.

Condition (2.4) may be interpreted as follows: given the transfer received from his father, w^i , the level education subsidies, S_t and the tax rate, θ_t , an individual i will invest in education if and only if his education cost, h^i is small enough. This condition determines a critical education cost, \hat{h}_t^i , such that only those individuals with an education cost below this threshold, $h^i \leq \hat{h}_t^i$ acquire education:

$$\hat{h}_t^i = (1 - \theta_t)w^i \left(1 - \frac{w^U}{w^E}\right) + S_t \equiv \hat{h}_t^i(w^E, w^U, \theta_t, S_t). \quad (2.5)$$

Since \hat{h}_t^i is a continuous and differentiable function of all their components, we obtain the following derivative signs:

$$\frac{\partial \hat{h}_t^i}{\partial w^i} > 0, \quad \frac{\partial \hat{h}_t^i}{\partial \left(1 - \frac{w^U}{w^E}\right)} > 0, \quad \frac{\partial \hat{h}_t^i}{\partial \theta_t} < 0, \quad \frac{\partial \hat{h}_t^i}{\partial S_t} > 0. \quad (2.6)$$

The first of the above derivatives shows how a higher bequest, w^i increases the probability that a young individual invests in education. The second derivative means that a higher wage gap between educated and uneducated labor serves as an incentive for purchasing education.¹¹ A higher tax rate affects negatively the threshold cost, since it reduces first-period disposable income and the positive sign of the derivative of \hat{h}_t^i with

¹¹Note that an increase in the wage of uneducated labor, w^U has two effects of opposite sign on the critical cost of individuals born of uneducated parents, \hat{h}_t^U . On the one hand, an increase in w^U relaxes constraints to invest in education for these individuals and on the other, it decreases the wage ratio of educated to uneducated labor and therefore, the incentives to acquire education are lower. Note that the net effect is positive, i.e., $\frac{\partial \hat{h}_t^U}{\partial w^U} \geq 0$ since $\frac{w^E}{w^U} \geq 2$.

respect to the education subsidy shows how this subsidy reduces the cost of acquiring education.

Since there are only two income types, either educated or uneducated, we have two critical values for education costs

$$\begin{aligned}\widehat{h}_t^E &= (1 - \theta_t)w^E \left(1 - \frac{w^U}{w^E}\right) + S_t, \\ \widehat{h}_t^U &= (1 - \theta_t)w^U \left(1 - \frac{w^U}{w^E}\right) + S_t,\end{aligned}\tag{2.7}$$

where \widehat{h}_t^E and \widehat{h}_t^U are the critical cost levels for young individuals born of educated and uneducated parents respectively.

Note that old educated individuals earn more and thus, they bequeath more than uneducated individuals, and this makes the critical cost of education for their children higher or equal than the critical cost of those of uneducated parents,

$$\widehat{h}_t^E \geq \widehat{h}_t^U, \quad \forall \theta_t \in [0, 1].\tag{2.8}$$

From (2.8) it follows that there exists a positive correlation between educational attainment of the children and the level of education of the parents since the child of an educated parent is more likely to acquire education than the child of an uneducated parent.¹²

We assume throughout the paper that the following condition holds:

$$w^E - w^U > \underline{h}.\tag{A1}$$

Assumption (A1) implies that in the absence of education subsidies, some individuals born of educated parents always acquire education since their critical cost is strictly higher than the minimum cost of education, $\widehat{h}_t^E > \underline{h}$. Thus, the wage gap between educated and uneducated labor, $w^E - w^U$ must be strictly higher than the minimum cost of education, \underline{h} for some individuals born of educated parents to be willing to invest in education.

¹²This result is a direct consequence of the inexistence of capital markets in the economy and it does not require the child's ability to be correlated with the parent's education or ability.

2.2.3 The Conditional Dynamical System

We now turn to study the evolution of the economy, conditional on the level of education subsidies.¹³ The economy grows when the proportion of individuals who purchase education raises over time as the result of dynasties' mobility from one labor type to the other. *Upward mobility* takes place when some individuals born of uneducated parents purchase education and equivalently, *downward mobility* occurs when some individuals born of educated parents do not to invest in human capital. Thus, an increase in the number of educated individuals, E_t takes place when the number of upward-mobile individuals, $(1 - E_t)F(\widehat{h}_t^U)$ exceeds the number of downward-mobile individuals, $E_t(1 - F(\widehat{h}_t^E))$.

The proportion of individuals purchasing education in period t and working as educated labor in period $t + 1$, E_{t+1} is simultaneously determined by the following equations:

$$E_{t+1} = E_t F(\widehat{h}_t^E) + (1 - E_t) F(\widehat{h}_t^U), \quad (2.9)$$

$$S_t = \begin{cases} \frac{\theta_t \bar{w}_t}{E_{t+1}} & \text{if } E_{t+1} > 0, \\ 0 & \text{if } E_{t+1} = 0. \end{cases} \quad (2.10)$$

It follows from (2.9) that the number of individuals who acquire education in period t , E_{t+1} is the sum of individuals of educated parents, $E_t F(\widehat{h}_t^E)$ and individuals of uneducated parents, $(1 - E_t) F(\widehat{h}_t^U)$ who invest in education in this period, where $\widehat{h}_t^E = \widehat{h}^E(w^E, w^U, S_t, \theta_t)$ and $\widehat{h}_t^U = \widehat{h}^U(w^E, w^U, S_t, \theta_t)$ are given by (2.7).

The proportion of individuals who invest in education in period t is also determined by the level of per-student subsidies in period t , S_t , which consists of total government revenues, $\theta_t \bar{w}_t$ divided by the number of individuals purchasing education in period t , E_{t+1} .

Solving equation (2.10) for θ_t and substituting into (2.9), we obtain E_{t+1} as an implicit

¹³In this section, we consider that the level of subsidies is exogenously given and constant over time and hence, the tax rate varies over time as the economy evolves. The dynamic analysis performed in this section is conditional on a given level of education subsidies.

function of E_t and S_t :

$$E_{t+1} = E_t F(\widehat{h}^E(E_t, E_{t+1}, S_t)) + (1 - E_t) F(\widehat{h}^U(E_t, E_{t+1}, S_t)). \quad (2.11)$$

In every period t , the conditional evolution of the economy is completely characterized by the proportion of educated individuals, E_t , since education subsidies are exogenous and constant over time, $S_t = \overline{S} \in [0, S_t^{\max}]$.¹⁴

The following proposition states that E_{t+1} is uniquely determined by E_t .

Proposition 2.1. *In the range $0 < E_t < 1$ there exists a unique $E_{t+1} \in (0, 1]$, which is the solution to (2.11) for any E_t .*

Proof. First, we prove that $E_{t+1} > 0, \forall E_t \in (0, 1)$. Assume that $E_{t+1} = 0$, then $\overline{S} = 0$ and under (A1), $\widehat{h}_t^E > \underline{h}$, which implies that the right-hand side of (2.11) is strictly positive and thus, it contradicts $E_{t+1} = 0$.

Existence of a unique $E_{t+1} \in (0, 1]$ holds since the right-hand side of (2.11) is a continuous and strictly decreasing function of E_{t+1} from (2.7) and (2.10), while the left-hand side is increasing in E_{t+1} with a slope of one. ■

We assume that the initial proportion of individuals in the economy is strictly positive, $0 < E_0 < 1$, and thus, from Proposition 2.1 at any period t the number of educated individuals is strictly positive. The following proposition shows that, given the level of the subsidy \overline{S} , in economies *with* mobility an increase in the magnitude of subsidization of education raises the number of individuals acquiring education.

Proposition 2.2. *In economies with mobility, an increase in the level of education subsidies raises the number of individuals who invest in education.*

Proof. In economies with mobility, there exists either downward mobility or upward mobility, or both. We define the following function:

$$G(E_t, E_{t+1}) = E_{t+1} - E_t F(\widehat{h}^E(E_t, E_{t+1}, \overline{S})) - (1 - E_t) F(\widehat{h}^U(E_t, E_{t+1}, \overline{S})).$$

¹⁴Note that the law of motion of the economy is stable since $F(\widehat{h}_t^E) \leq 1$ and $F(\widehat{h}_t^E) \geq F(\widehat{h}_t^U)$.

We apply the implicit function theorem to compute the effect of an increase in the level of education subsidies, \bar{S} on the proportion of individuals acquiring education, i.e., $\frac{\partial E_{t+1}}{\partial \bar{S}}$. This theorem holds since G is a continuous function of E_t , E_{t+1} and \bar{S} and as follows from Proposition 2.1, there exists a unique E_{t+1} for each E_t , such that $G(E_t, E_{t+1}) = 0$.

From (2.7), we find that $\frac{\partial E_{t+1}}{\partial \bar{S}} = \frac{-\frac{\partial G}{\partial \bar{S}}}{\frac{\partial G}{\partial E_{t+1}}} > 0$, where $\frac{\partial G}{\partial \bar{S}} = \frac{-1+E_{t+1}\left(1-\frac{w^U}{w^E}\right)}{\bar{h}-\underline{h}} < 0$, since $E_{t+1} \leq 1$ and $\left(1 - \frac{w^U}{w^E}\right) < 1$ and $\frac{\partial G}{\partial E_{t+1}} > 0$ since $\frac{\partial F(\hat{h}^i)}{\partial E_{t+1}} < 0$, $i = E, U$. ■

An increase in education subsidies raises upward mobility, since it allows more individuals born of uneducated parents to acquire education and the redistributive role of subsidies makes poor families to benefit more from an increase in the level of subsidies than rich families.¹⁵

It is also of interest to understand which is the role of education subsidies in economies *without* mobility. These economies are characterized by a perfect correlation between young individuals' educational attainment and parental education or income. Hence, the probability that an individual born of a educated parent chooses to acquire education is equal to one, $F(\hat{h}_t^E) = 1$ and conversely, the probability that an individual with an uneducated parent invests in education is equal to zero, $F(\hat{h}_t^U) = 0$. Figure 2.1 shows the effects of education subsidies on critical values, \hat{h}_t^E and \hat{h}_t^U . For low levels of the subsidy, there is no mobility since $\hat{h}_t^E \geq \bar{h}$ and $\hat{h}_t^U < \underline{h}$. Upward mobility occurs when the subsidy is sufficiently high and the critical cost of individuals born of uneducated parents, \hat{h}_t^U crosses the lower limit of the cost distribution, $\hat{h}_t^U > \underline{h}$. In this case, the subsidy raises disposable income of poor families in an amount high enough to allow some children of uneducated parents to acquire education. Similarly, downward mobility exists when the critical value of the cost of education for young individuals with educated parents crosses the upper limit of the cost distribution, $\hat{h}_t^E < \bar{h}$. This situation happens when the subsidy reduces disposable income of rich families and some individuals born of educated parents

¹⁵Downward mobility raises only if the subsidy is sufficiently high to reduce disposable income of rich families.

do not invest in human capital as Figure 2.1 illustrates.

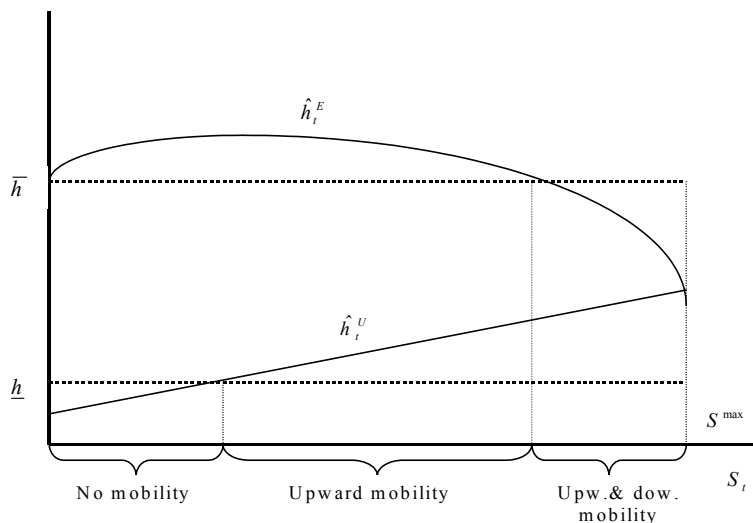


Figure 2.1: Effect of S_t on critical levels, \hat{h}_t^E and \hat{h}_t^U .

2.2.4 Conditional Steady State Equilibria

A conditional steady state equilibrium is defined as the proportion of educated individuals, E^{ss} that is invariant over time when the education subsidy is constant and exogenously given. Steady state equilibria can be characterized by the existence of intergenerational mobility of dynasties, from an education type to the other, or by no mobility. The possible laws of motion of the economy are the following:¹⁶

1. Only downward mobility: $\underline{h} < \hat{h}_t^E < \bar{h}$ and $\hat{h}_t^U < \underline{h}$ for all $t > 0$.

In such an economy, only some individuals with educated parents invest in education, $0 < F(\hat{h}_t^E) < 1$, while no individual born of uneducated parents acquire education, $F(\hat{h}_t^U) = 0$. Thus, according to (2.11), the number of educated individuals decreases

¹⁶The dynamics of the economy may be characterized by changes in the intergenerational mobility regime, due to the presence of education subsidies.

over time. The steady state of this economy is characterized by no mobility between education classes since in steady state nobody invest in education, i.e., $E^{ss} = 0$.

2. Only upward mobility: $\hat{h}_t^E \geq \bar{h}$ and $\underline{h} < \hat{h}_t^U < \bar{h}$ for all $t > 0$.

In an economy with only upward mobility, all individuals of educated parents acquire education, $F(\hat{h}_t^E) = 1$ and also some individuals of uneducated parents, $0 < F(\hat{h}_t^U) < 1$. This economy converges to a unique steady state without mobility between income types, in which all individuals acquire education, i.e., $E^{ss} = 1$.

3. Upward and Downward mobility: $\hat{h}_t^E < \bar{h}$ and $\hat{h}_t^U > \underline{h}$ for all $t > 0$.

When both types of intergenerational mobility exist, only some individuals of both types of parents acquire education, $0 < F(\hat{h}_t^i) < 1$, $i = E, U$. In this economy, the steady state level of educated individuals is $0 < E^{ss} < 1$ and it is characterized by both types of intergenerational mobility. In steady state, the number of downward-mobile individuals is equal to the number of upward mobile individuals, i.e.,

$$E^{ss}(1 - F(\hat{h}^E(E^{ss}))) = (1 - E^{ss})F(\hat{h}^U(S^{ss})).$$

4. No mobility: $\hat{h}_t^E \geq \bar{h}$ and $\hat{h}_t^U \leq \underline{h}$, for all $t > 0$.

An economy is in a *poverty trap* if there is no intergenerational mobility. Such an economy is characterized by a high level of wage inequality and thus, the wage of educated labor, w^E is so high that allows all children of educated parent to acquire education while the wage of uneducated labor, w^U is so low that no child of uneducated families invests in education. Therefore, children's educational attainment is perfectly correlated with parental income in this economy. The steady state number of educated individuals is the initial number of educated individuals at $t = 0$, $E^{ss} = E_0$.

2.3 Political Equilibrium

In this section we analyze the determination of education subsidies by majority voting. For this purpose, we first analyze individuals' preferences over education subsidies and how preferred subsidies of individuals depend both on their income and ability. We next turn to provide necessary and sufficient conditions for a majority voting equilibrium to exist in our economy. Moreover, we prove that the existence of a political equilibrium level of education subsidies is always guaranteed. Finally, we discuss how the level of development, measured by the proportion of educated individuals in the economy, E_t and the wage gap between educated and uneducated labor, $w^E - w^U$, affect the equilibrium level of subsidization of education in the economy.

We focus our analysis on economies in which the presence of education subsidies raises intergenerational mobility. We assume hereinafter that

$$\bar{h} \geq w^E - w^U \geq \underline{h} \frac{w^E}{w^U}. \quad (\text{A2})$$

Assumption (A2) implies that in the absence of education subsidies, the critical cost of individuals born of educated parents is $\hat{h}_t^E \leq \bar{h}$ and the critical cost of individuals born of uneducated parents satisfies $\hat{h}_t^U \geq \underline{h}$. This assumption means that in the absence of education subsidies, some individuals of both types of parents, educated and uneducated, acquire education.

2.3.1 Preferred Subsidy Levels

Education subsidies are intended to partially cover the cost of acquiring education and, depending on the level of the subsidy, transfers are not made to all individuals. This feature makes individuals' preferences over these subsidies different to those over a purely redistributive policy, in which a proportional tax funds equal per-capita lump-sum transfers to all individuals. In that case, individuals whose income is below the mean income prefer the maximum transfer allowed by economic resources (or equivalently, a tax rate

equal to one) while individuals with income above the mean prefer a transfer of zero. In the context of our model, this means that individuals born of uneducated parents with income, w^U would favor redistribution whereas those with educated parents and income, w^E would be opposed to redistribution.

However, since education subsidies are only transferred to those individuals who become educated, preferred subsidies of individuals crucially depend on the decision of investing in education. We have shown that this decision, given education subsidies, S_t and the tax rate used to finance them, θ_t , depends on individuals' characteristics, such as their income, w^i and their education costs, h^i . Thus, an individual i is willing to invest in education if and only if his education cost is small enough, $h^i \leq \widehat{h}_t^i \equiv \widehat{h}^i(w^E, w^U, S_t, \theta_t)$.

Using the government budget constraint, given by (2.10), and equation (2.9), we can express the tax rate, θ_t as a function of the level of education subsidies S_t and the proportion of old educated individuals in period t , E_t . Thus, we can write the condition of investing in education as a function of a single political variable, S_t :

$$h^i \leq \widehat{h}_t^i \equiv \widehat{h}^i(w^E, w^U, S_t, E_t), \quad i = E, U. \quad (2.12)$$

Alternatively, the decision of acquiring education of an individual i may be written in terms of the minimum level of education subsidies that he requires to be willing to purchase education. Solving (2.12) for S_t , we obtain that an individual i will invest in education, $U_t^E \geq U_t^U$, if and only if the level of education subsidies S_t is high enough

$$S_t \geq \widehat{S}_t^i(w^E, w^U, h^i, E_t), \quad (2.13)$$

where \widehat{S}_t^i is the individual's i critical level of education subsidies. This critical level is the subsidy that makes individual i indifferent between acquire education or remain uneducated, i.e., $U_t^E(\widehat{S}_t^i) = U_t^U(\widehat{S}_t^i)$.

Note that young individuals belonging to the same parent's type, either educated or uneducated, differ in their critical level of education subsidies due to differences in education costs, h^i . Given the bequest, w^i , an individual with a higher education cost or

a lower ability requires a higher subsidy to invest in education, $\frac{\partial \hat{S}_t^i}{\partial h^i} > 0$. Alternatively, a higher inherited income, w^i , given individual's cost, h^i , decreases individual's critical level of education subsidies, $\frac{\partial \hat{S}_t^i}{\partial w^i} < 0$.

The indirect utility function of an individual i can be written as a function of education subsidies as follows:

$$U_t^i(S_t) = \begin{cases} U_t^U & \text{if } S_t < \hat{S}_t^i, \\ U_t^E & \text{if } S_t \geq \hat{S}_t^i, \end{cases} \quad (2.14)$$

where $U_t^U = \ln((1 - \theta_t(S_t))w^i) + \ln w^U$ is the utility obtained by a young individual i if he remains uneducated in period t and $U_t^E = \ln((1 - \theta_t(S_t))w^i + S_t - h^i) + \ln w^E$ is the indirect utility function of this individual if he acquires education in period t .

An individual i chooses the subsidy level that maximizes his utility:

$$\begin{aligned} \max_{S_t} U_t^i(S_t) \\ \text{s.t. } 0 \leq S_t \leq S_t^{\max}, \end{aligned} \quad (2.15)$$

where S_t^{\max} is the maximum subsidy given economic resources, i.e., $\theta_t(S_t^{\max}) = 1$.

Since utility is not a quasiconcave function of the level of the subsidy, each individual must compare his maximum utility if he does not acquire education, U_t^U with the maximum utility obtained if he invests in education, U_t^E , in order to find his most preferred subsidy level. Intuitively, if an individual i does not invest in education, his preferred subsidy level is zero, since in this case he does not benefit from the subsidy but he instead has to contribute to finance it paying taxes. The following proposition shows that preferred subsidies of individuals who acquire education only depend on their parental income.

Proposition 2.3. *If an individual i invests in education, his preferred subsidy is S_t^i and it only depends on his parent's income, w^i , $i = E, U$ and $S_t^U > S_t^E$ since $w^U < w^E$.*

Proof. If individual i acquires education in period t , he chooses the subsidy level, S_t^i , that

maximizes his indirect utility,

$$\begin{aligned} \max_{S_t} U_t^E \\ \text{s.t. } 0 \leq S_t \leq S_t^{\max}. \end{aligned}$$

The optimal subsidy of individual i , S_t^i is the subsidy that maximizes the difference between the subsidy received and the taxes payed, $S_t^i = \arg \max (S_t - \theta_t(S_t)w^i)$, and thus, it only depends on parental income, S_t^i . The interior optimal subsidy $0 < S_t^i < S_t^{\max}$ satisfies the following condition:

$$\frac{\partial U_t^E}{\partial S_t} = 0 \Leftrightarrow \frac{\partial \theta_t(S_t^i)}{\partial S_t} = \frac{1}{w^i}, \quad i = E, U. \quad (2.16)$$

Under (A2), $S_t^E > 0$ and from Proposition 2.2, we know that an increase in education subsidies raises the proportion of individuals who invest in education in period t , E_{t+1} and hence, the tax rate is an increasing and convex function of the level of the subsidy. This implies that the indirect utility, U_t^E is a strictly concave function of S_t and from (2.16), the subsidy preferred by individuals born of educated parents is strictly lower than the one preferred by individuals of uneducated parents, $S_t^E < S_t^U$ since $w^E > w^U$. ■

Intuitively, young individuals from poor families prefer a higher subsidy than individuals born of rich parents because their net gains from redistribution, i.e., the difference between the subsidy received and the taxes payed, are higher. Note that in contrast to a purely redistributive policy, interior subsidy levels now appear. This result is rooted in the fact that the size of the subsidy determines which individuals are going to receive the transfers. Individuals, when they decide their preferred subsidy levels, they take into account how the subsidy determines who are those individuals who invest in education. Thus, they may wish to reduce the subsidy in order to prevent others to invest in education and share the subsidy, extracting resources from them.

An individual i finds his most preferred education subsidy by means of comparing the utility obtained if he acquires education evaluated at the local maximum S_t^i , $U_t^E(S_t^i)$, with

the utility at $S_t = 0$ in case of remaining uneducated, $U_t^U(S_t = 0)$. Thus, an individual i prefers S_t^i to $S_t = 0$ if and only if the following inequality holds:

$$U_t^E(S_t^i) \geq U_t^U(S_t = 0). \quad (2.17)$$

From (2.17) it follows that an individual i prefers the subsidy level S_t^i to a zero subsidy if his education cost h^i is small enough

$$h^i \leq \tilde{h}_t^i(S_t^i), \quad (2.18)$$

where $\tilde{h}_t^i(S_t^i) = w^i \left(1 - \frac{w^U}{w^E}\right) + S_t^i - \theta_t(S_t^i)w^i \equiv \tilde{h}(w^E, w^U, E_t)$ is the critical value of the cost of education for individuals with a parent of type $i = E, U$, or alternatively, it is the education cost of the individual who is indifferent between S_t^i and $S_t = 0$ in each income group. Thus, a lower education cost or a higher ability raises the utility from investing in education and thus, the support for S_t^i against $S_t = 0$.

Since education subsidies are only transferred to those who invest in education, individuals' preferences over subsidies are non single-peaked for some individuals.¹⁷ Intuitively, single-peakedness fails to exist because at low levels of education subsidies, an individual is not willing to purchase education and thus, he prefers zero subsidies since he does not receive the subsidy but he has to pay the tax used to finance it. However, as the level of education subsidies increases, he is willing to become educated and in this case, he prefers a positive subsidy. We present different examples of individuals' preferences over education subsidies. In Figures 2.2 and 2.3, we represent the preferences of individuals whose preferred subsidy level is $S_t = 0$, while in Figures 2.4 and 2.5 we present two different cases of individuals whose preferred subsidy is S_t^i .¹⁸

¹⁷Non single-peakedness appears also when both public and private education coexist. This is because those individuals attending private schools must opt out of publicly provided education. Stiglitz (1974) was among the first to study this problem.

¹⁸In Figure 2, the utility function of individual i is represented by the bold curve.

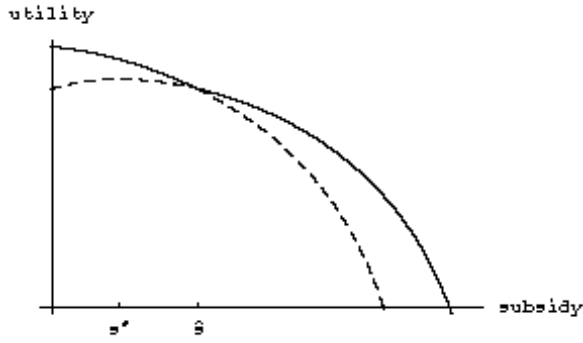


Figure 2.2: $U_t^i(S)$ if $\widehat{S}_t^i > S_t^i$.

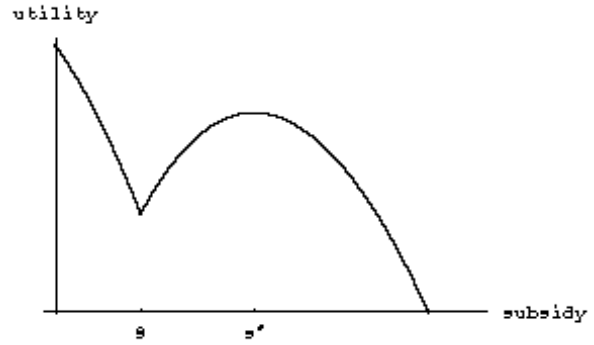


Figure 2.3: $U_t^i(S)$ if $\widehat{S}_t^i < S_t^i$.

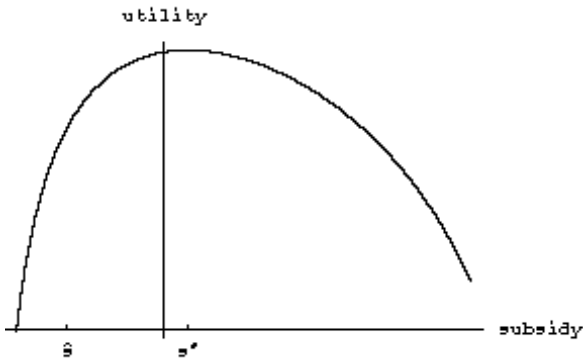


Figure 2.4: $U_t^i(S)$ if $\widehat{S}_t^i < 0$.

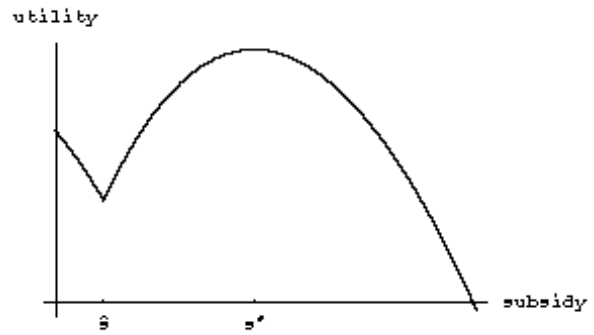


Figure 2.5: $U_t^i(S)$ if $\widehat{S}_t^i < S_t^i$.

2.3.2 Majority Voting Equilibrium

In this section we analyze the political equilibrium level of education subsidies. The equilibrium degree of subsidization of education is decided by majority voting. As we have

already showed in the previous section, individuals' preferences over education subsidies are non single-peaked.¹⁹ It is well known in the voting literature that a majority voting equilibrium may exist, even if preferences fail to be single-peaked. It is the case when preferences of individuals over the public policy satisfy a single-crossing property.²⁰ Intuitively, this property means that it is possible to order individuals by their characteristics according to their preferences for the public policy.²¹

In our model, the conflict of interests among individuals regarding the preferred level of subsidization of education has two dimensions. On the one hand, high-ability individuals have a lower opportunity cost of investing in education than low-ability individuals with the same income. Thus, preferred subsidies may differ across individuals belonging to the same type of family, either educated or uneducated, due to differences in education costs. It is possible to order individuals belonging to the same parent's type by their ability to determine their preferred subsidy, since individuals with an education cost sufficiently small, $h^i \leq \tilde{h}_t^i(S_t^i)$, $i = E, U$ prefer S_t^i to $S_t = 0$, whereas those with high education costs, $h^i > \tilde{h}_t^i(S_t^i)$ prefer a zero subsidy level.

On the other hand, high-income individuals prefer lower interior subsidies than low-income individuals, $S_t^E < S_t^U$, because their gains from redistribution are lower. However, it is not possible to determine how are the preferred education subsidies of two individuals with the same education cost h^i but different parental income. The most preferred subsidy of the low-income individual may be lower or higher than the most preferred subsidy of the rich individual, depending on their income and education costs. Therefore, it is not possible to order individuals by a single characteristic to determine their preferred subsidy level, which means that in our model individuals' preferences over education subsidies are

¹⁹This feature implies that a majority voting may not exist.

²⁰See Roberts (1977) and Gans and Smart (1996)).

²¹Non single-peakedness of individuals' preferences over tax rates to finance public education appears also when both public and private education coexist. This is because those individuals attending private schools must opt out of publicly provided education. Stiglitz (1974) was among the first to study this problem.

not single-crossing.

We proceed as follows: first, we provide the necessary and sufficient conditions for the existence of a majority voting equilibrium and then, we show that there always exists a majority voting equilibrium subsidy in the economy. Let define the political equilibrium level of education subsidies in the economy. This subsidy is the Condorcet winner of the voting process.

Definition *The Condorcet winner is the subsidy level S_t^c , $0 \leq S_t^c \leq S_t^{\max}$, that beats any other subsidy in pairwise comparison, i.e., for all $S \in [0, S_t^{\max}]$, the fraction of agents with $U_t^i(S_t^c) \geq U_t^i(S)$ is strictly greater than half the total number of individuals in the economy.*

In order to find the Condorcet winner of the political process, we define as p_t^i , the number of individuals of each parent's type, $i = E, U$, whose preferred subsidy level is $S_t^i = \arg \max U_t^E(w^i)$. These individuals are those who prefer to acquire education when the education subsidy is S_t^i to remain uneducated at $S_t = 0$ and they have an education cost small enough, $h^i \leq \tilde{h}_t^i(S_t^i)$. Accordingly, the number of individuals of each parent's type whose preferred subsidy is $S_t = 0$ are those with high education costs, $h^i > \tilde{h}_t^i(S_t^i)$. This group of individuals proportion is $p_t^0 = 1 - (p_t^E + p_t^U)$.

These three groups of individuals of size p_t^E , p_t^U and p_t^0 respectively, are the following:

- $p_t^E = E_t F(\tilde{h}_t^E(S_t^E))$.
- $p_t^U = (1 - E_t) F(\tilde{h}_t^U(S_t^U))$.
- $p_t^0 = E_t(1 - F(\tilde{h}_t^E(S_t^E))) + (1 - E_t)(1 - F(\tilde{h}_t^U(S_t^U)))$.

In the case in which one of these three groups consists of more than half the total number of individuals in the economy, i.e., $p_t^i > \frac{1}{2}$, $i = \{E, U, 0\}$, there exists a *trivial* political equilibrium subsidy, which is the subsidy preferred by this group. Intuitively, $S_t = 0$ and S_t^E are *trivial* majority voting equilibria respectively if either the proportion

of educated individuals, E_t , is low or sufficiently high. We may interpret E_t as the level of development of the economy. Thus, when the level of development of the economy is low, the proportion of individuals who acquire education is also low and a majority of individuals support $S_t = 0$, i.e., $p_t^0 > \frac{1}{2}$. Conversely, S_t^E is a trivial equilibrium when the level of development is high and the proportion of individuals born of educated parents are a majority, $E_t > \frac{1}{2}$. In such an economy, a majority of individuals born of educated parents acquire education and support S_t^E against $S_t = 0$ and thus, $p_t^E > \frac{1}{2}$.

The preferred subsidy of individuals born of uneducated parents who invest in education, S_t^U is a *trivial* political equilibrium, i.e., $p_t^U > \frac{1}{2}$, if uneducated individuals are a majority in the economy, $1 - E_t > \frac{1}{2}$ and the uneducated labor wage w^U is sufficiently high to allow a majority of poor individuals to acquire education, $F(\tilde{h}_t^U(S_t^U)) > \frac{1}{2}$.²²

We can write the requirements that the economy must satisfy in each of these cases as implicit conditions over the level of development of the economy E_t .²³ Thus, $S_t = 0$ is a trivial equilibrium if and only if $F(\tilde{h}_t^U(S_t^U)) < \frac{1}{2}$ and $E_t < \tilde{E}_t^0$, while S_t^E is an equilibrium if $E_t > \tilde{E}_t^E$. Finally, S_t^U only appears if $F(\tilde{h}_t^U(S_t^U)) > \frac{1}{2}$ and $E_t < \tilde{E}_t^U$.

To characterize the *non-trivial* political equilibrium we consider the case in which these groups are strictly smaller than half the total population in the economy, i.e., $p_t^i < \frac{1}{2}$ for all i . Thus, the sum of any two groups consists of more than half the total number of individuals. In order to find the political equilibrium level of education subsidies S_t^c , we first identify which are the candidates to be the Condorcet winner. We state a lemma that provides the necessary condition that a subsidy level must satisfy to be the Condorcet winner of the voting process.²⁴

Lemma 2.1. *If S_t^c is a majority voting equilibrium, then it must be a local maximum for*

²²This equilibrium is not relevant empirically since it would imply that a majority of children of poor families have access to university.

²³Note that the proportion of individuals who strictly prefer S_t^i , $i = E, U$ to $S_t = 0$, $F(\tilde{h}_t^i(S_t^i))$ only depends on period t through E_t .

²⁴A parallel result is obtained by Fernández and Rogerson (1995).

at least one group of individuals.

Proof. Assume that no group has a local maximum at S_t^c , this implies that $\frac{\partial U_t}{\partial S_t}(S_t^c) \neq 0$ for all individuals. Since S_t^c is the Condorcet winner, it is strictly preferred to any other alternative by more than half the total number of individuals in the economy. Then, for more than half of individuals S_t^c is strictly preferred to any other alternative arbitrarily closed and smaller than S_t^c , i.e., $U_t^i(S_t^c) > U_t^i(S_t^c - \varepsilon)$. Thus, the utility of more than half the population in the economy is upward sloping at S_t^c . Since S_t^c is not a local maximum, then necessarily any alternative bigger and arbitrarily closed to S_t^c will be preferred to S_t^c by a majority of individuals, i.e., $U_t^i(S_t^c + \varepsilon) > U_t^i(S_t^c)$, and then S_t^c cannot be the Condorcet winner. ■

This lemma establishes that the candidates to majority voting equilibrium are the local maxima for the groups defined above, i.e., $\{0, S_t^E, S_t^U\}$. Now we can prove that if a candidate beats the other two, it beats any other subsidy and therefore, it is a majority voting equilibrium. Thus, the following lemma provides the sufficient condition for a candidate to be the Condorcet winner of the voting process.

Lemma 2.2. *If one candidate to majority voting equilibrium, $\{0, S_t^E, S_t^U\}$, beats the other two candidates, it is the Condorcet winner of the majority voting process.*

Proof. See the Appendix. ■

The more intuitive case is the one in which S_t^E , which is the most preferred subsidy for individuals born of educated parents who acquire education, is strictly preferred to both $S_t = 0$ and S_t^U by a majority of individuals in the economy. The argument used in the proof is similar to the standard arguments in the median voter theorem. In this case, those individuals whose preferred subsidy is $S_t = 0$ and those who support S_t^E against $S_t = 0$, they are both better off at S_t^E than at any $S \in (S_t^E, S_t^{\max}]$ and they are more than half the total number of individuals in the economy, since $p_t^0 + p_t^E > \frac{1}{2}$. On the

other hand, those individuals who prefer S_t^U to $S_t = 0$ and S_t^E to $S_t = 0$ respectively, they also prefer S_t^E to any $S \in (0, S_t^E)$ and they are also a majority in the economy since $p_t^U + p_t^E > \frac{1}{2}$.

In the case of $S_t = 0$, it is easy to check that if $S_t = 0$ beats both S_t^E and S_t^U , then it beats any $S \in (0, S_t^E)$, since those who prefer $S_t = 0$ to S_t^E strictly prefer a zero subsidy to any $S \in (0, S_t^E)$ and the same argument holds for any $S \in (S_t^U, S_t^{\max}]$ because those who support $S_t = 0$ against S_t^U also support a zero subsidy against $S \in (S_t^U, S_t^{\max}]$. Thus, we turn to prove that $S_t = 0$ also beats any subsidy, $S \in [S_t^E, S_t^U]$. This result holds because the increase in the support for $S_t = 0$ against S , with respect to the support for $S_t = 0$ against S_t^E or S_t^U , is always higher than the increase in the support for S against $S_t = 0$.

Finally, it is not difficult to prove that if S_t^U beats S_t^E and $S_t = 0$, then it trivially beats any other education subsidy. This is because individuals born of educated parents, whether they acquire education or not, always prefer $S_t = 0$ to S_t^U . We can show that the net gains at S_t^U are negative for individuals born of educated parents, i.e., $S_t^U - \theta(S_t^U)w^E < 0$. Thus, if S_t^U beats $S_t = 0$ it is required that those individuals born of uneducated parents who strictly prefer S_t^U to $S_t = 0$ are more than half the total number of individuals in the economy, and thus, S_t^U is a *trivial* majority voting equilibrium.

In the following theorem we show that there always exists a local maximum that beats the other two. Therefore, we obtain the following result:

Theorem 2.1. *There always exists a majority voting equilibrium in the economy.*

Proof. See the Appendix. ■

In the proof of this theorem we show that there does not exist any voting cycle between the candidates and therefore, the existence of a majority voting equilibrium level of education subsidies is always guaranteed.

In Lemma 2.2 we have shown that the set of *non-trivial* majority candidates can be reduced to $\{0, S_t^E\}$ since both groups p_t^E and p_t^0 strictly prefer $S_t = 0$ to S_t^U and they

are a majority in the economy since $p_t^E + p_t^0 > \frac{1}{2}$. A zero subsidy is an equilibrium when individuals who strictly prefer $S_t = 0$ to S_t^E are more than a half. These individuals are those born of educated and uneducated parents who obtain more utility remaining uneducated at $S_t = 0$ than acquiring education at S_t^E , i.e., $U_t^U(S_t = 0) > U_t^E(S_t^E)$ and have high education costs, $h^i > \tilde{h}_t^i(S_t^E)$, $i = E, U$. Conversely, S_t^E is the equilibrium subsidy when the proportion of individuals of both income groups who prefer S_t^E to $S_t = 0$ are a majority in the economy.

Thus, $S_t = 0$ is an equilibrium if the level of development of the economy is low, $E_t < \hat{E}_t$ and hence, $E_t F(\tilde{h}_t^E(S_t^E)) + (1 - E_t) F(\tilde{h}_t^U(S_t^E)) < \frac{1}{2}$ and S_t^E is non-trivial political equilibrium if the level of development of the economy is sufficiently high, $E_t > \hat{E}_t$ and thus, $E_t F(\tilde{h}_t^E(S_t^E)) + (1 - E_t) F(\tilde{h}_t^U(S_t^E)) > \frac{1}{2}$. Intuitively, these type of political equilibria appear at intermediate levels of development compared to trivial political equilibria. Note that the conditions that the economy must fulfilled in terms of E_t are less restrictive than those for trivial political equilibria since $\tilde{E}_t^E > \hat{E}_t > \tilde{E}_t^0$ as represented in Figure 2.6.

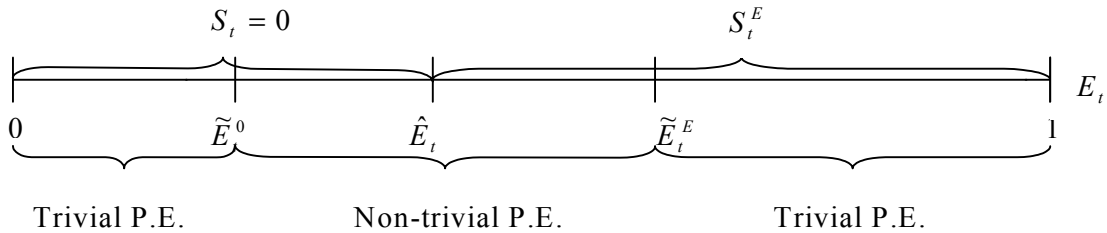


Figure 2.6: Political equilibrium subsidies as a function of economic development

2.3.3 Equilibrium Subsidies and Inequality

The size of subsidization of education not only depends on the proportion of educated individuals in the economy, E_t but also on the level of income inequality. In our model,

we may measure inequality by the wage gap, $w^E - w^U$.²⁵ Inequality has two effects of opposite sign on the political support of education subsidies. On the one hand, a higher wage gap affects positively the incentives of individuals to invest in education and a higher number of individuals acquiring education raises the support of education subsidies. On the other hand, the conflict of interests between rich and poor individuals regarding the level of subsidization of education is also higher. Intuitively, a higher inequality reduces the gains from redistribution for individuals born of educated parents while it raises the gains for individuals of uneducated parents. Thus, a higher wage gap decreases the preferred subsidy of individuals born of educated parents S_t^E relative to the preferred subsidy of uneducated individuals S_t^U , which makes more difficult the support of subsidies in economies with intermediate levels of development.²⁶

The impact of the level of development of the economy on the size of education subsidies crucially depends on income inequality. The critical number of educated individuals required for the support of education subsidies, \hat{E}_t changes with the wage gap. If we compare economies with the same level of development, E_t , we could expect that those with a lower wage gap would require a higher number of educated individuals than those with a higher wage gap for the political support of subsidies, since incentives to acquire education are higher in the latter economies. This result is illustrated in Table 2.1.²⁷ In this table, we present the equilibrium subsidies in economies with different wage premia. We observe that the critical level of development required for the support of education subsidies, \hat{E}_t decreases with inequality. On the other hand, the size of the subsidy is not monotone in the level of income inequality, being higher at intermediate levels of inequality.²⁸

²⁵Note that the Gini index is monotone with the wage gap in our model.

²⁶Note that in economies with high levels of development, educated individuals supporting positive levels of subsidization are a majority and the trivial political equilibrium is S_t^E . Thus, a higher wage gap also results in a lower subsidy in these economies.

²⁷The example underlying Tables 2.1 and 2.2 is based on the following parameters: $\bar{h} = 5$, $\underline{h} = 1$ and $w^U = 3.5$. The wage gap in the example of Table 2.2 is $w^E - w^U = 4.5$

Table 2.1
Equilibrium subsidies in economies with the same number of
educated individuals ($E_t = 0.35$) and different wage premia

Wage gap $w^E - w^U$	Critical level \hat{E}_t	Income Inequality (Gini index)	Equilibrium Subsidy S_t^c	Tax Rate θ_t
3.5	0.4782	0.16852	0	0
4.0	0.3752	0.18571	0	0
4.5	0.3131	0.20172	0.51236	0.05572
4.75	0.2909	0.20932	0.40311	0.04383
5.0	0.2685	0.21667	0.29395	0.03196

In Table 2.2 we compute the equilibrium subsidies in economies with the same wage gap, $w^E - w^U$, and different number of educated individuals, E_t . We observe that income inequality is an inverse U-shaped function of the level of development of the economy. Inequality raises at low stages of development and then decreases. Therefore, it is higher at intermediate levels of development. This result is consistent with the Kuznets curve.²⁹ Table 2.2 also shows that a minimum level of development is required for the political support of education subsidies. Economies with a very low proportion of educated individuals are characterized by zero subsidies and as the level of development raises, the political support for education subsidies increases.

²⁸The Gini index measures inequality in a scale of zero to one, with zero being complete equality and one complete inequality. The formula for the Gini index in our model is $\frac{E_t(1-E_t)(w^E - w^U)}{\bar{w}_t}$.

²⁹See Kuznets (1955).

Table 2.2
Equilibrium subsidies in economies with
different levels of development

Number of Educated individuals: E_t	Income per Capita \bar{w}_t	Income Inequality (Gini index)	Equilibrium Subsidy: S_t^c	Tax Rate θ_t
0.10	3.95	0.10253	0	0
0.25	4.62	0.18243	0	0
0.35	5.07	0.20172	0.51236	0.05572
0.50	5.75	0.19565	0.51878	0.05787
0.70	6.65	0.14211	0.52911	0.06053

2.4 The Evolution of the Economy

In this section we study the evolution of the economy when education subsidies are endogenously determined by majority voting. We first analyze the dynamic behavior of intergenerational mobility and education subsidies and afterwards, we characterize the steady state levels of educated individuals and subsidies. Consistent with the empirical evidence of a monotone increase in education over time, we limit the convergence to a steady state to be from below, i.e, we assume that the initial number of educated individuals, E_0 is smaller than the steady state level.

2.4.1 Dynamics

The economy evolves when the proportion of individuals who acquire education raises over time as a result of net upward mobility of individuals born of uneducated parents who invest in education. The proportion of individuals who invest in education in period t , E_{t+1} is determined implicitly by the proportion of educated individuals in the economy

E_t and the political equilibrium level of education subsidies S_t^c :

$$E_{t+1} = E_t F(\widehat{h}_t^E(S_t^c)) + (1 - E_t) F(\widehat{h}_t^U(S_t^c)),$$

$$\text{where } S_t^c = \begin{cases} 0 & \text{if } E_t \leq \widehat{E}_t, \\ S_t^E & \text{if } E_t > \widehat{E}_t, \end{cases} \quad (2.19)$$

where $\widehat{h}_t^E \equiv \widehat{h}^E(E_t, E_{t+1}, S_t^c(E_t, E_{t+1}))$, $\widehat{h}_t^U \equiv \widehat{h}^U(E_t, E_{t+1}, S_t^c(E_t, E_{t+1}))$ and \widehat{E}_t is the critical proportion of educated individuals required for the support of education subsidies. Thus, E_{t+1} is derived from (2.19) as an implicit function of E_t :

$$E_{t+1} = \varphi(E_t). \quad (2.20)$$

The following proposition states that E_{t+1} is uniquely determined by E_t when education subsidies are endogenously determined by majority voting.

Proposition 2.4. *In the range $0 < E_t < 1$, there exists a unique $E_{t+1} \in (0, 1]$, which is the solution to (2.19) for any E_t .*

Proof. See the Appendix. ■

To understand which is the role of endogenous education subsidies in the evolution of the economy, consider an economy without education subsidies. The evolution of this economy depends on the wages of educated and uneducated individuals in comparison with education costs. If inequality is such that w^E is sufficiently high to allow even the least talented child of educated workers to find optimal to acquire education, $\widehat{h}_t^E(0) \geq \bar{h}$, and w^U is so low that even the most talented child of uneducated dynasties does not invest in education, $\widehat{h}_t^U(0) \leq \underline{h}$, there is no mobility and the economy is in a *poverty trap*.

However, if the wage gap is very low, $w^E - w^U < \underline{h}$, nobody is willing to invest in education, there is no mobility and $E_{t+1} = 0$. Thus, intergenerational mobility exists at intermediate levels of wage inequality.

The number of individuals who acquire education raises over time when there exists net upward mobility, i.e. $(1 - E_t) F(\widehat{h}_t^U(0)) > E_t(1 - F(\widehat{h}_t^E(0)))$.

The evolution of the economy depends not only on the wage gap, $w^E - w^U$, but also on the initial level of development, E_0 , when the level of subsidization of education is determined by majority voting. If the initial level of development is low, $E_0 \leq \widehat{E}_0$, the political equilibrium level of education subsidies is $S_0 = 0$.³⁰ In this economy, the proportion of individuals raises over time if there exists net upward mobility and the wage gap is sufficiently high:

$$w^E - w^U > \underline{h} \frac{w^E}{w^U}. \quad (2.21)$$

If (2.21) holds, some individuals born of uneducated parents acquire education since $\widehat{h}_t^U(0) > \underline{h}$. The economy may reach a level of development in some period \widehat{t} , such that $E_{\widehat{t}} > \widehat{E}_{\widehat{t}}$, and the proportion of educated individuals in the economy is sufficiently high to support $S_{\widehat{t}}^E$ as a majority voting equilibrium in the economy. Notice that $S_{\widehat{t}}^E$ is also the political equilibrium subsidy in the following periods, since the proportion of educated individuals grows over time, i.e., $S_t^c = S_{\widehat{t}}^E, \forall t \geq \widehat{t}$. Once the economy reaches this level of development, it evolves according to a new law of motion and converges to a steady state level of educated individuals equilibrium which is strictly higher than the one achieved without education subsidies. Education subsidies raise mobility because they allow more individuals born to uneducated parents to acquire education.

Figure 2.7 shows the evolution of an economy with endogenous education subsidies. The bold curve corresponds to the law of motion of the economy with subsidies and the dotted line represents the growth path of this economy without education subsidies. We observe that the economy has a switch of regime once it reaches a level of development, \widehat{E}_t high enough to support S_t^E as a political equilibrium. The steady state level of educated

³⁰If the level of development is low, another possible equilibrium is S_0^U , which is the preferred subsidy of individuals born of uneducated parents who acquire education. In this case, the economy converges immediately to a higher growth path with $S_t^E, \forall t > 0$, as a political equilibrium.

individuals is higher with education subsidies: $E^*(S) > E^*(0)$.

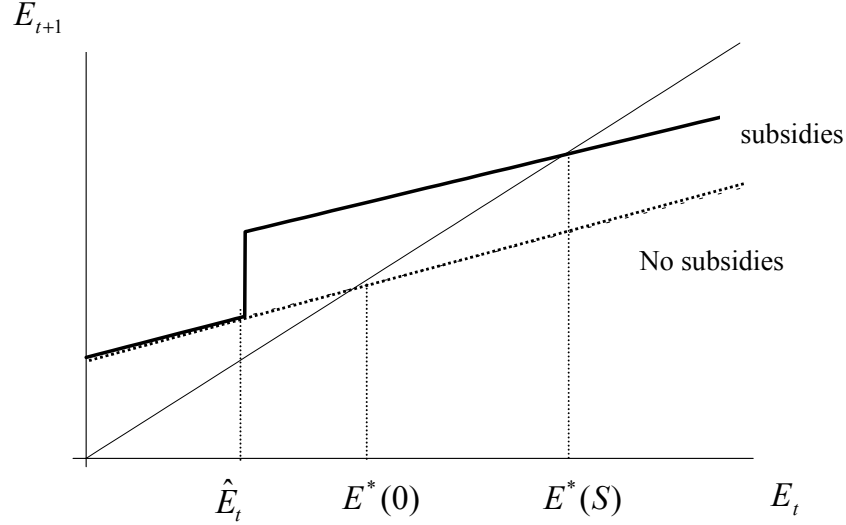


Figure 2.7: Evolution of the economy with endogenous education subsidies

Now we turn to study the evolution of education subsidies in the growth process. In each period, economic conditions represented by the state variable E_t , determine simultaneously the equilibrium level of education subsidies in this period, S_t^c and the proportion of individuals purchasing education in next period, E_{t+1} . If $E_t > \hat{E}_t$, the equilibrium subsidy is S_t^E , which is the preferred subsidy of individuals of educated parents who acquire education. The following proposition states that the evolution of subsidies crucially depends on the level of wage inequality of the economy.

Proposition 2.5. *The transitional dynamics of S_t^E is the following:*

$$\frac{\partial S_t^E}{\partial E_t} \geq 0, \text{ if } w^E - w^U \leq \bar{h} - \underline{h} \frac{w^U}{w^E} \text{ and } \frac{\partial S_t^E}{\partial E_t} < 0 \text{ if } w^E - w^U > \bar{h} - \underline{h} \frac{w^U}{w^E}.$$

Proof. See the Appendix. ■

Intuitively, an increase in E_t has two effects of opposite sign on preferred subsidies of individuals born of educated parents who invest in education, S_t^E . On the one hand, an increase in E_t raises total income in the economy, \bar{w}_t , and hence, the total amount of resources to redistribute, which has a positive effect on S_t^E . On the other hand, it raises the proportion of individuals who acquire education in the next period, E_{t+1} , which has a negative effect on S_t^E . The above proposition shows that the net effect is positive if the wage gap is not too high. In this case, individuals born of educated parents can extract resources from those who remain uneducated.

2.4.2 Steady State Analysis

The initial conditions of the economy, which are the level of development, E_0 and the wage gap, $w^E - w^U$, crucially determine the evolution of the economy and the steady state levels of educated individuals and education subsidies. Economies with different initial number of educated individuals and different levels of wage inequality choose different levels of education subsidies and they exhibit different patterns of mobility.

The economy converges to a steady state level of educated individuals, E^* which is the solution to (2.19) when the proportion of individuals who invest in education is time invariant, i.e., $E_{t+1} = E_t = E^*$. There exists mobility between education classes if the proportion of individuals of both types of parents who invest in education in steady state is strictly positive and smaller than one, i.e., $0 < F(\hat{h}^E(S^*)) < 1$ and $0 < F(\hat{h}^U(S^*)) < 1$. In this case, the steady state level of educated individuals is $0 < E^* < 1$ and it is defined implicitly as follows:

$$E^* = E^* F(\hat{h}^E(S^*)) + (1 - E^*) F(\hat{h}^U(S^*)),$$

$$S^* = \begin{cases} 0 & \text{if } E^* \leq \hat{E}(E^*), \\ S^E(E^*) & \text{if } E^* > \hat{E}(E^*). \end{cases} \quad (2.22)$$

Depending on initial conditions, the economy may also converge to a steady state in which there does not exist mobility between education classes. For instance, in rich

economies in which there exists only upward mobility, all individuals born of educated parents acquire education and some individuals of uneducated families also do. This economy converges to a steady state in which all individuals in the economy invest in education, $E^* = 1$ and the equilibrium subsidy is $S^* = S^E(E^*) = 0$. Since in steady state all individuals invest in education, there are no education subsidies in equilibrium due to gains from redistribution are not possible.

Conversely, in poor economies there exists only downward mobility. Thus, only some individuals born of educated parents acquire education and the economy converges to a steady state in which no individual invest in education, $E^* = 0$ and equilibrium subsidies are also zero.

Finally, an economy in a *poverty trap* is characterized by no mobility between education classes since children's educational attainment is perfectly correlated to parental education. In this economy, the steady state level of educated individuals is the initial number of educated individuals, $E^* = E_0$ and thus, education subsidies are either $S^* = 0$ if $E_0 \leq \frac{1}{2}$ or $S^E(E_0)$ if $E_0 > \frac{1}{2}$. Note that in the case in which educated individuals are a majority, their preferred subsidy is positive if they can extract resources from uneducated individuals. Thus, $S^E(E_0)$ is the maximum subsidy given available resources, $S^{\max}(E_0)$ if the number of educated individuals is sufficiently low, $E_0 < \frac{\bar{w}_0}{w^E}$ and $S^E(E_0) = 0$ if $E_0 \geq \frac{\bar{w}_0}{w^E}$. Note that a higher inequality makes more difficult the support of positive subsidies in economies without mobility since the ratio $\frac{\bar{w}_0}{w^E}$ decreases with the wage gap between educated and uneducated labor, $w^E - w^U$.

2.5 Concluding Remarks

This chapter tries to shed some light on the complex relationship between income inequality, intergenerational mobility and endogenous redistributive policies. We analyze the political economy of education subsidies in a dynamic model of development through intergenerational mobility. We also investigate the role of such subsidies in the develop-

ment process and as an engine to generate mobility in economies stuck in poverty traps. Our results point out the crucial role of initial economic conditions in shaping equilibrium education policies. We find that a minimum level of development is required for the subsidization of education. However, the impact of income per capita on the size of the subsidies depends crucially on the degree of income inequality of the economy. It is the interaction of both variables the one determining equilibrium level of redistribution in this model. A similar result is obtained by Perotti (1993) who shows that the effect of initial income inequality on redistribution depends on total output.

We also obtain that the magnitude of subsidization of education is higher at intermediate levels of income inequality. The wage premium obtained by educated individuals may help explain this phenomenon. On the one hand, a high wage gap affects positively incentives to invest in human capital, and this, in turn has a positive impact on the magnitude of education subsidies. On the other hand, the conflict of interests on the equilibrium level of redistribution raises with inequality, which makes more difficult the political support of education subsidies. Benabou (2000) and Lee and Roemer (1998) also show that the relationship between income inequality and redistribution is not monotone, although in their models redistribution is lump-sum.

Finally, we find that education subsidies may have a positive effect on economic development. Education subsidies may increase the level of mobility in the economy and thus, they may be an endogenous mechanism to avoid countries to get stuck in poverty traps. However, our results also suggest that the education policy considered here is more likely to be implemented in more developed economies. Thus, other policies, such as transfers to low income individuals, could be more effective for economies at early stages of the development process.

2.6 Appendix

Proof of Lemma 2.2. We prove that if one local maximum, $\{0, S_t^E, S_t^U\}$ defeats the other two, it is strictly preferred to any subsidy level $S \in (0, S_t^{\max}]$ by a majority of individuals, and thus, it is the political equilibrium level of education subsidies. Recall that S_t^E is the subsidy that maximizes the indirect utility of acquiring education of individuals born of educated parents, while S_t^U is the subsidy that maximizes the utility of individuals born of uneducated parents who invest education .

First, we prove that if S_t^E beats S_t^U and $S_t = 0$, it also defeats any $S \in (0, S_t^{\max}]$. Intuitively, all individuals of educated individuals support S_t^E against S_t^U . Those who acquire education strictly prefer S_t^E to S_t^U since $U_t^E(S_t^E) > U_t^E(S_t^U)$ because $S_t^E = \arg \max U_t^E(w^E)$ and those who remain uneducated also prefer S_t^E to S_t^U because $U_t^U(S_t^E) > U_t^U(S_t^U)$, since $S_t^E < S_t^U$. Among individuals born of uneducated parents, some are going to support S_t^E against S_t^U . These individuals are those with $U_t^U(S_t^E) > U_t^U(S_t^U)$ and high education costs, $h^i > \bar{h}_t^U \equiv w^U \left(1 - \frac{w^U}{w^E}\right) + S_t^U - \theta(S_t^U)w^U + \theta(S_t^E)\frac{w^U}{w^E}$, where $\bar{h}_t^U > \tilde{h}_t^U(S_t^U)$. Thus, S_t^E is strictly preferred to S_t^U by a majority in the economy if the following condition holds:

$$E_t + (1 - E_t)(1 - F(\bar{h}_t^U)) > \frac{1}{2}. \quad (2.23)$$

Correspondingly, the support for S_t^E against $S_t = 0$ comes from individuals of both educated and uneducated parents with $U_t^E(S_t^E) \geq U_t^U(S_t = 0)$. Thus, a majority strictly prefer S_t^E to $S_t = 0$ if the following condition is satisfied:

$$E_t F(\tilde{h}_t^E(S_t^E)) + (1 - E_t)F(\tilde{h}_t^U(S_t^E)) > \frac{1}{2}. \quad (2.24)$$

Now we prove that if (2.23) and (2.24) hold, then S_t^E defeats any $S \in (0, S_t^{\max}]$. First we compute the support for S_t^E against $S \in (0, S_t^E)$. This support comes from individuals born of educated and uneducated parents with $U_t^E(S_t^E) \geq U_t^U(S)$. It is easy to show that the support for S_t^E against S is higher than the support for S_t^E against $S_t = 0$ since $U_t^U(S) < U_t^U(S_t = 0)$. Therefore, by (2.24) S_t^E defeats any $S \in (0, S_t^E)$.

Consider a subsidy $S \in (S_t^E, S_t^U)$. Intuitively, all individuals born of educated parents, whether they acquire education or not, they strictly prefer S_t^E to any $S \in (S_t^E, S_t^U)$. Individuals born of uneducated parents who support S_t^E against $S \in (S_t^E, S_t^U)$ are those with $U_t^U(S_t^E) > U_t^E(S)$ and education costs $h^i > w^U \left(1 - \frac{w^U}{w^E}\right) + S - \theta(S) w^U + \theta(S_t^E) \frac{w^U}{w^E}$. Note that the number of individuals of uneducated parents who support S_t^E against $S \in (S_t^E, S_t^U)$ are more than those who supported S_t^E against S_t^U since $\bar{h}_t^U > w^U \left(1 - \frac{w^U}{w^E}\right) + S - \theta(S) w^U + \theta(S_t^E) \frac{w^U}{w^E}$ or equivalently, $U_t^E(S) < U_t^U(S_t^U)$. Thus, from (2.23), S_t^E defeats any $S \in (S_t^E, S_t^U)$ and the same argument holds to prove that S_t^E beats $S \in (S_t^U, S_t^{\max}]$.

Consider now the case in which a zero subsidy is strictly preferred to both S_t^E and S_t^U by a majority. This is the case when the number of individuals who prefer $S_t = 0$ to S_t^i , $i = E, U$ is a majority in the economy. These individuals are those born of educated and uneducated parents whose utility if they do not invest in education and $S_t = 0$ is strictly higher than their utility of acquiring education when the subsidy is either S_t^E or S_t^U , i.e., $U_t^U(S_t = 0) > U_t^E(S_t^i)$, $i = E, U$ and are those with an education cost high enough. Thus, $S_t = 0$ is strictly preferred to S_t^E and S_t^U respectively if the following conditions hold:

$$E_t(1 - F(\tilde{h}_t^E(S_t^E))) + (1 - E_t)(1 - F(\tilde{h}_t^U(S_t^E))) > \frac{1}{2}, \quad (2.25)$$

$$E_t(1 - F(\tilde{h}_t^E(S_t^U))) + (1 - E_t)(1 - F(\tilde{h}_t^U(S_t^U))) > \frac{1}{2}. \quad (2.26)$$

Notice that it is immediate to check that if the above conditions hold, $S_t = 0$ beats any subsidy $S \in (0, S_t^E)$ and $S \in (S_t^U, S_t^{\max}]$. This is because those who vote for $S_t = 0$ against S_t^E strictly prefer $S_t = 0$ to $S \in (0, S_t^E)$ and they are a majority. Equivalently, those who prefer $S_t = 0$ to S_t^U strictly prefer a zero subsidy to $S \in (S_t^U, S_t^{\max}]$. Thus, we must prove that a zero subsidy also beats any $S \in [S_t^E, S_t^U]$.

Consider first the case in which $E_t \geq \frac{1}{2}$. If $S \geq S_t^E$, from (2.25), the increase in the support for a zero subsidy against S comes from individuals born of educated parents, since $\tilde{h}_t^E(S_t^E) > \tilde{h}_t^E(S)$. The proportion of individuals supporting zero now

raises in $E_t(F(\tilde{h}_t^E(S_t^E)) - F(\tilde{h}_t^E(S))) = E_t\left(\frac{S_t^E - S + w^E(\theta(S) - \theta(S_t^E))}{\bar{h} - \underline{h}}\right)$. However, some individuals born of educated parents who vote for $S_t = 0$ against S_t^E are now going to support S since $\tilde{h}_t^U(S) > \tilde{h}_t^U(S_t^E)$. Thus, the decrease in the support for zero is $-(1 - E_t)\left(\frac{S_t^E - S + w^U(\theta(S) - \theta(S_t^E))}{\bar{h} - \underline{h}}\right)$. Since $E_t \geq \frac{1}{2}$ and $w^E > w^U$, the support for $S_t = 0$ against S raises with respect to the support for $S_t = 0$ against S_t^E .

Now consider that $E_t \leq \frac{1}{2}$ and $S \leq S_t^U$. From (2.26), the support for $S_t = 0$ against S now comes from individuals of uneducated parents since $\tilde{h}_t^U(S_t^U) > \tilde{h}_t^U(S)$. The support for $S_t = 0$ increases in $(1 - E_t)\left(\frac{S_t^U - S - w^U(\theta(S_t^U) - \theta(S))}{\bar{h} - \underline{h}}\right)$. However, some individuals of educated parents who supported $S_t = 0$ against S_t^U now prefer S to a zero subsidy. The support for $S_t = 0$ decreases in $-E_t\left(\frac{S_t^U - S - w^E(\theta(S_t^U) - \theta(S))}{\bar{h} - \underline{h}}\right)$ since $\tilde{h}_t^E(S) > \tilde{h}_t^E(S_t^U)$. In this case the support for $S_t = 0$ against $S \in (0, S_t^{\max}]$ also raises compared to the support obtained by $S_t = 0$ against S_t^U , since $E_t \leq \frac{1}{2}$ and $w^E > w^U$. Therefore, if $S_t = 0$ beats S_t^E and S_t^U , then it beats any other subsidy level, $S \in (0, S_t^{\max}]$ and it is the majority voting equilibrium.

Finally, it is immediate to show that if S_t^U beats S_t^E and $S_t = 0$, it beats trivially any other subsidy level, $S \in (0, S_t^{\max}]$. Note that S_t^U beats S_t^E if the number of individuals of uneducated parents with $U_t^E(S_t^U) \geq U_t^E(S_t^E)$ are a majority in the economy, i.e., $(1 - E_t)F(\bar{h}_t^U) > \frac{1}{2}$ and thus, it must be necessarily the case that $(1 - E_t) > \frac{1}{2}$. If S_t^U beats $S_t = 0$, then a majority of individuals in the economy strictly prefer S_t^U to $S_t = 0$. We can prove that in this case, S_t^U is a trivial political equilibrium and therefore, it beats any $S \in (0, S_t^{\max}]$ in pairwise comparison.

First, we prove that $S_t^U - \theta(S_t^U)w^E < 0$:

S_t^U satisfies $\left.\frac{\partial\theta(S_t)}{\partial S_t}\right|_{S_t=S_t^U} \geq \frac{1}{w^U}$ since $S_t^U \leq S_t^{\max}$, where $\theta(S_t) = \frac{E_{t+1}S_t}{\bar{w}_t}$ and $\frac{\partial\theta(S_t)}{\partial S_t} = \frac{E_{t+1} + S_t \frac{\partial E_{t+1}}{\partial S_t}}{\bar{w}_t}$. Note that if $S_t^U - \theta(S_t^U)w^E \leq 0$, then S_t^U must be such that $E_{t+1}(S_t^U) \frac{w^E}{\bar{w}_t} \geq 1$ or equivalently, $E_{t+1}(S_t^U) \frac{w^E}{\bar{w}_t} \geq \frac{E_{t+1}(S_t^U)w^U + S_t^U \frac{\partial E_{t+1}}{\partial S_t} w^U}{\bar{w}_t} \geq 1$. Using (2.9) and (2.10),

$E_{t+1}(S_t^U)$ may be expressed as follows:

$$E_{t+1} = \frac{\bar{w}_t \left(1 - \frac{w^U}{w^E}\right) + S_t^U - \underline{h}}{\bar{h} - \underline{h} + S_t^U \left(1 - \frac{w^U}{w^E}\right)},$$

and thus, $S_t^U \frac{\partial E_{t+1}}{\partial S_t} \leq E_{t+1}(S_t^U)$ since $\bar{w}_t \left(1 - \frac{w^U}{w^E}\right) - \underline{h} \geq 0$ from (A2).

Therefore, $S_t^U - \theta(S_t^U)w^E < 0$ holds since $w^E \geq 2w$. This means that no individual born of educated parents prefer S_t^U to $S_t = 0$ and necessarily it is required that the number of individuals born of uneducated parents who support S_t^U against to $S_t = 0$ are more than half the total number of individuals in the economy, i.e., $(1 - E_t)F(\tilde{h}_t^U(S_t^U)) > \frac{1}{2}$, which implies that S_t^U is a trivial political equilibrium. ■

Proof of Theorem 2.1. In Lemma 2.2, we provide the sufficient condition for each candidate, $\{0, S_t^E, S_t^U\}$, to be a majority voting equilibrium. This lemma states that if a local maximum beats the other two, it is the Condorcet winner of the voting process. To show that there always exists a majority voting equilibrium, it is sufficient to prove that there are no majority voting cycles among the three candidates. We prove it by contradiction. Assume there does exist a cycle between the candidates. We have two possible cycles; either $S_t = 0 \succ S_t^E, S_t^E \succ S_t^U$ and $S_t^U \succ S_t = 0$ or $S_t^U \succ S_t^E, S_t^E \succ S_t = 0$ and $S_t = 0 \succ S_t^U$. Note that the first cycle cannot occur since by Lemma 2.2, if S_t^U is strictly preferred to $S_t = 0$ by a majority of individuals in the economy, it is then a trivial majority voting equilibrium. The second case only can occur if $1 - E_t > \frac{1}{2}$ since this condition is required for $S_t^U \succ S_t^E$ but note that in this case, we showed in Lemma 2.2 that if $1 - E_t > \frac{1}{2}$ and $S_t = 0$ beats S_t^U by majority, then $S_t = 0$ beats any subsidy $S \in [S_t^E, S_t^U]$ which contradicts $S_t^E \succ S_t = 0$, and thus this cycle does neither exist. ■

Proof of Proposition 2.4. Following the arguments used in Proposition 2.1, we first prove that $E_{t+1} > 0, \forall E_t \in (0, 1)$. Assume that $E_{t+1} = 0$, thus $S_t^c = 0$ and $\hat{h}_t^E > \underline{h}$ under (A1), which implies that the right-hand side of (2.19) is strictly positive which contradicts

$E_{t+1} = 0$. To prove existence of a unique $E_{t+1} \in (0, 1]$, we first consider the case in which $E_t \leq \widehat{E}_t$. In this case, the political equilibrium level of subsidies is $S_t^c = 0$ and from (2.7), it is trivial to show that E_{t+1} is uniquely determined by E_t .

Consider the case in which $E_t > \widehat{E}_t$. In this case, the political equilibrium level of education subsidies is S_t^E and it is determined by (2.16). We can show that in this case the right-hand side of (2.19), $E_t F(\widehat{h}_t^E(S_t^E)) + (1 - E_t) F(\widehat{h}_t^U(S_t^E))$, is decreasing in E_{t+1} .

Note that $\widehat{h}_t^i = \left(1 - \frac{E_{t+1} S_t^E}{\bar{w}_t}\right) w^i \left(1 - \frac{w^U}{w^E}\right) + S_t^E$, $i = E, U$ and thus the right-hand side of (2.19) may be written as follows:

$$\frac{\bar{w}_t \left(1 - \frac{w^U}{w^E}\right) + S_t^E \left(1 - E_{t+1} \left(1 - \frac{w^U}{w^E}\right)\right)}{\bar{h} - \underline{h}}. \quad (2.27)$$

It is thus sufficient to prove that $\frac{\partial S_t^E}{\partial E_{t+1}} < 0$ to show that (2.27) is decreasing in E_{t+1} .

From (2.16), we define the following function:

$$H(E_{t+1}, S_t^E) = \frac{\frac{\partial E_{t+1}}{\partial S_t^E} S_t^E + E_{t+1}}{\bar{w}_t} - \frac{1}{w^E} = 0. \quad (2.28)$$

We can compute $\frac{\partial S_t^E}{\partial E_{t+1}}$ applying the implicit function theorem. This theorem holds since H is a continuous function of E_{t+1} and S_t^E .

We obtain that $\frac{\partial S_t^E}{\partial E_{t+1}} = \frac{-\frac{\partial H}{\partial E_{t+1}}}{\frac{\partial H}{\partial S_t^E}} < 0$ since $\frac{\partial H}{\partial E_{t+1}} = \frac{1}{\bar{w}_t} \left(1 - \frac{S_t^E \left(1 - \frac{w^U}{w^E}\right)}{1 + S_t^E \left(1 - \frac{w^U}{w^E}\right)}\right) > 0$ from (2.19)

and (2.28) and $\frac{\partial H}{\partial S_t^E} = \frac{1 - E_{t+1} \left(1 - \frac{w^U}{w^E}\right)}{1 + S_t^E \left(1 - \frac{w^U}{w^E}\right)} \left(2 - \frac{1 - \frac{w^U}{w^E}}{1 + S_t^E \left(1 - \frac{w^U}{w^E}\right)}\right) > 0$ since $E_{t+1} \leq 1$ and $1 - \frac{w^U}{w^E} < 1$.

Therefore, existence of a unique $E_{t+1} \in (0, 1]$ holds also if $E_t > \widehat{E}_t$ since the right-hand side of (2.19) is decreasing in E_{t+1} , whereas the left-hand side is increasing with a slope of one in E_{t+1} . ■

Proof of Proposition 2.5. The preferred interior subsidy of individuals born of educated parents, S_t^E is determined by (2.16). Solving (2.19) for E_{t+1} and plugging into (2.10), we obtain S_t^E as a function of E_t :

$$S_t^E = \frac{(\bar{h} - \underline{h})(z^E - 1)}{\left(1 - \frac{w^U}{w^E}\right)},$$

where $z^E = \left(\frac{w^E(\bar{h}-\underline{h}-(1-\frac{w^U}{w^E})(\bar{w}_t(1-\frac{w^U}{w^E})-\underline{h}))}{(\bar{h}-\underline{h})(w^E-\bar{w}_t(1-\frac{w^U}{w^E}))} \right)^{\frac{1}{2}} > 1$ and $\bar{w}_t = E_t w^E + (1 - E_t) w^U$.

Thus, $\frac{\partial S_t^E}{\partial E_t} = -\frac{1}{2} \frac{\left((1-\frac{w^U}{w^E})(w^E-\underline{h})-(\bar{h}-\underline{h}) \right)}{z^E(w^E-\bar{w}_t(1-\frac{w^U}{w^E}))^2}$ is positive if $w^E - w^U \leq \bar{h} - \underline{h} \frac{w^U}{w^E}$ and negative otherwise. We also find that $\frac{\partial^2 S_t^E}{\partial^2 \bar{w}_t} < 0$. ■

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Chapter 3

Sorting into Public and Private Schools and Education Vouchers

3.1 Introduction

Primary and secondary education in most countries is, to a large extent, provided by the government. Public educational services are funded with taxes and provided at a very low price, although these services are available in the private sector. State intervention in education may be rationalized on the basis of its external benefits, but there exist also other reasons such as the protection of children against negligent parents and the need to ensure equal opportunity. Compulsory schooling is regarded as satisfying the first two arguments for state intervention, since the society as a whole benefits from a minimum degree of literacy of its citizens.

The argument based on the need to ensure equal opportunity deals with the distributional role of public education in allowing low income individuals to obtain an educational achievement at almost no cost, therefore increasing the prospects of upward mobility in children belonging to poor families. However, it is also argued that a public system offering a uniform -and frequently low- educational quality independently of individu-

als' specific needs can actually reduce mobility. The discontent with the primary and secondary educational system in the United States has centered the political debate in recent presidential elections on issues of school choice, including voucher programs.

Typical voucher proposals provide students attending private schools a tax-financed payment covering all or most of the tuition charged. Such payments can be made directly to parents or indirectly to the selected schools. The type of voucher in which the government subsidizes schools in proportion to enrollment has been adopted by countries such as Chile, Colombia, Sweden and the United Kingdom. In the United States, the cities of Milwaukee and Cleveland have introduced voucher systems.¹ Arguments in favor of education vouchers include the increase in educational choice for poor families and the promotion of competition among public and private schools to attract students. Opponents argue that vouchers channel resources to a small number of students attending private schools, while the majority of students at public schools are made worse-off.

This chapter analyzes the effect of education vouchers on the efficient sorting of students, who differ over ability and family income, into public and private schools. We also evaluate the welfare and distributional effects of different voucher policies. In our model, school quality is measured by per-student educational expenditures and the human capital of students depends both on their ability and on the quality of the school they attend. Both inputs are complements in the production of human capital. Thus, education vouchers may increase aggregate human capital by means of allowing high-ability individuals to acquire a higher level of educational quality in the private market.

If capital markets to finance private educational expenditures are perfect and ability is observable, the allocation of students to schools is efficient. In the absence of borrowing constraints and as long as the marginal return from investing in higher educational quality is greater for talented students, high-ability individuals will be willing to spend more in education than low-ability individuals. In this case, government intervention may be

¹See West (1997) for a survey of voucher systems in operation.

justified if it is desirable to require a minimum level of schooling or a longer period of education for those individuals that otherwise would choose too little educational quality.

Borrowing constraints, however, are an important impediment to achieve this efficient allocation, and parents' inability to borrow against the future income of their children makes parental income a relevant factor in the acquisition of education. The framework chosen for our analysis of education vouchers is an economy with borrowing constraints and a dual education provision system in which public schools coexist with private education alternatives. Individuals who prefer a higher educational quality than the one provided publicly, must opt out of public educational services.

We find that the introduction of education vouchers raises aggregate human capital by means of a more efficient sorting of students across schools. However, in the relevant empirical case, the implementation of vouchers is only possible if it entails welfare gains for the majority of students remaining at public schools. If public quality at public schools remains constant after the introduction of education vouchers, it will be approved by majority the voucher that minimizes taxes. We find that this voucher is positive if public school quality is sufficiently high. In such case, the introduction of a voucher program is welfare-enhancing.

We calibrate the model to existing empirical evidence for the US economy in order to illustrate the impact of different voucher policies on welfare, human capital accumulation and income inequality. Our benchmark economy without education vouchers is characterized by a majority of students attending public schools. Borrowing constraints crucially affect the educational expenditures of poor families who send their children to private schools. We find that the tax-minimizing voucher chosen by majority entails welfare gains but higher inequality compared to the mixed education system without vouchers. The impact of different voucher levels is also analyzed. We find that total welfare increases with the magnitude of the voucher, but the impact of the voucher size on income inequality is not monotone. Small vouchers entail low taxes and redistribution and thus,

higher inequality. As the voucher size increases, its redistributive impact is higher and income inequality decreases.

The welfare and distributional effects of uniform education vouchers have been analyzed by Ireland (1990), Epple and Romano (1996) and Hoyt and Lee (1998). In these papers, the only source of heterogeneity among individuals is income and they do not investigate the impact of vouchers on human capital accumulation. On the other hand, Barse, Glomm and Ravikumar (2000) use computational experiments to quantify the effect of means-tested vouchers on the level and distribution of educational expenditures, when both the funding and the allocation of education vouchers are determined by majority vote. Means-tested vouchers are given to individuals only if their income is sufficiently small.

The effects of education vouchers on the sorting of students across schools in the presence of peer-group effects have been investigated by Epple and Romano (1998) and Caucutt (2002). Epple and Romano (1998) analyze the impact of vouchers on competition between public and private schools and on the efficient sorting of students across schools. The demand for education in their model depends on both income of the parents and ability of the children. However, school quality only depends on the mean ability of the students attending this school. Their work differs crucially from other papers modelling public and private schooling, since they provide an active role for the private education sector. In their computational model, both public educational quality and education vouchers are determined exogenously. Their main findings are that education vouchers yield small welfare gains but they have an important distributional impact.

Caucutt (2002) develops a model in which educational achievement depends not only on student's ability and educational expenditure, but also on the school's peer group. She evaluates the distributional and welfare impacts of switching from a public system to a completely private system with education vouchers. She finds that low voucher levels are associated with welfare losses that decrease monotonically in the voucher size,

but the impact of the magnitude of the voucher on income inequality is, as in our case, non-monotone.

Our work is similar in objectives to these two papers, but it differs mainly in two features. First, our model does not incorporate peer group effects in the production of human capital or as a determinant of school quality. Instead, school quality is measured by per-student expenditures. School quality and child's own ability are the sole determinants of student's earnings. We have chosen this framework to focus the analysis on the role of vouchers in allowing poor families to invest an efficient amount in their children's education. The complementarities existing between student's own ability and school quality allow us to characterize the efficient allocation of students to schools. Secondly, we characterize the equilibrium level of vouchers chosen by majority for a given level of public school quality, whereas these papers abstract from political economy issues raised by the introduction of vouchers.

The organization of the chapter is as follows. In Section 3.2, we present a simple model in which public education provision coexists with a competitive private school system. In this framework, we characterize the equilibrium allocation of students across schools under perfect capital markets and under borrowing constraints. Section 3.3 investigates first the effects of education vouchers on the sorting of students across schools and secondly, the political economy of education vouchers and its welfare and distributional effects. In Section 3.4 we calibrate the model and present some illustrative results of the effects of education vouchers on students' sorting across schools, welfare and income distribution. In the last section, we present some concluding remarks.

3.2 The Model

The economy consists of a continuum of individuals, characterized by a predetermined level of income y^i , which takes two values, either high (H) or low (L). The proportion of individuals with low income is γ and the size of the population is normalized to unity.

Each individual is a parent of one student of ability a^i and child's ability is distributed uniformly over the interval $[\underline{a}, \bar{a}]$, with $\underline{a} \geq 0$. The child's learning ability, a^i is assumed to be perfectly observable by his parent and it is uncorrelated with his parent's income.

Parents are altruistic toward their children and they derive utility from their own consumption, c^i and from the human capital or educational achievement of their children, h^i .² The human capital a child acquires depends on the quality of the school he attends, q^i and on his ability, a^i . We assume that school quality and student's ability are complements in the production of human capital and human capital's technology is Cobb-Douglas:

$$h^i = k (q^i)^\alpha (a^i)^{1-\alpha}, \quad \alpha < 1, \quad k \geq 1, \quad (3.1)$$

where α is the elasticity of the child's human capital with respect to educational expenditures and $1 - \alpha$ is the elasticity of earnings with respect to student's learning ability.

School quality, q^i is determined by per-student educational expenditures.³ There are two types of schools: private and public. Private schools provide a continuum variety of educational qualities, $q^i \in [\underline{q}, \bar{q}]$ at the competitive price p . We assume that each private school uses a technology that transforms educational expenditures, pq^i into a unit of educational quality, q^i .

Public schools provide a uniform quality of education, Q , funded by proportional taxes over total income, t . Hence, the government budget constraint is the following:

$$t\bar{y} = QF^*, \quad (3.2)$$

where F^* is the number of students attending public schools and $\bar{y} = \gamma y^L + (1 - \gamma)y^H$ is total income in the economy.⁴

²Parents obtain utility from the human capital of their children and not from the utility their children receive.

³In the education literature school quality is frequently considered as depending also on peer-group effects (see de Bartolome (1990), Epple and Romano (1998)).

⁴Since all public schools have the same school quality, Q , we may consider the public sector as consisting of one (large) school.

Those students who attend private schools must opt out of publicly provided education and thus, the choice between public and private education is not convex. Parents who send their children to public schools cannot supplement the publicly provided quality with private services. Thus, parent i 's utility depends crucially on the educational system chosen,

$$\begin{aligned}
 U^i(c^i, h^i) &= c^i + h^i, \\
 \text{where } c^i &= \begin{cases} (1-t)y^i, & \text{if he chooses public education,} \\ (1-t)y^i - pq^i, & \text{if he acquires private education,} \end{cases} \\
 \text{and } h^i &= \begin{cases} k(Q)^\alpha (a^i)^{1-\alpha}, & \text{if public education is chosen,} \\ k(q^i)^\alpha (a^i)^{1-\alpha}, & \text{if private education is chosen.} \end{cases}
 \end{aligned} \tag{3.3}$$

The utility maximization problem of an individual i may be solved backwards in two stages. In the first stage, each parent chooses whether to send his child to public or private schools, given the price of private quality, p , the quality of public education, Q , and the tax rate, t . In the second stage, individuals who acquire educational quality in the private system decide their optimal level of private school quality, q^{i*} , given the price, p and the tax rate t .

In the next subsections we investigate the allocation of students to public and private schools. First, we analyze the sorting of students into public and private schools when capital markets are perfect and then we turn to study the distribution of students across schools in the presence of borrowing constraints.

3.2.1 Equilibrium under Perfect Capital Markets

In this section we characterize the efficient allocation of students to schools under perfect capital markets. For this purpose, we consider that parents have access to perfect capital markets to finance investments in their children's education. We assume that there exists an external capital market for loans operating at a constant and riskless interest rate, which is normalized to zero for simplicity.

We solve the problem of a parent i backwards. In the second stage, if a parent i sends his child to a private school he decides his optimal level of educational quality, q^{i*} given the tax rate, t as follows:

$$q^{i*} = \arg \max U^i (c^i, h^i). \quad (3.4)$$

Under perfect capital markets, we obtain that the optimal investment in education of parent i , q^{i*} depends only on the ability of his child, a^i :

$$q^{i*} = q^{iP} = a^i \left(\frac{k\alpha}{p} \right)^{\frac{1}{1-\alpha}}. \quad (3.5)$$

This result follows immediately from the assumption of perfect capital markets and the complementarities between educational investments and children's ability. Since parents can borrow any amount to finance their investments in education, those parents with talented children are willing to spend more in school quality than those with less talented children. Thus, under perfect capital markets we obtain that the allocation of students to schools in the private education market is efficient.

In the first stage, individuals decide whether to send their children to public or private schools, given the quality of public education, Q , the tax rate, t and the price of private education, p . A parent i will acquire education in the public system if the utility he derives from sending his child to the public school is higher or equal than his indirect utility if he chooses the private system:

$$U^{pb} \geq U^{pr*}, \quad (3.6)$$

where $U^{pb} = (1-t)y^i + k(Q)^\alpha (a^i)^{1-\alpha}$ is the indirect utility if he chooses the public system and $U^{pr*} = (1-t)y^i - pq^{iP} + k(q^{iP})^\alpha (a^i)^{1-\alpha}$ is the indirect utility if his child attends a private school.

From (3.6) it follows that given parental income, y^i , the ability of his child, a^i , and the tax rate to fund education, t , an individual i will choose the public system if public

educational quality, Q is sufficiently high. Let \widehat{Q}^{iP} denote the critical level of public school quality such that a parent i will send his child to a public school if and only if:

$$Q \geq \widehat{Q}^{iP} = q^{iP}(1 - \alpha)^{\frac{1}{\alpha}}, \tag{3.7}$$

where $q^{iP} = a^i \left(\frac{k\alpha}{p}\right)^{\frac{1}{1-\alpha}}$ is the optimal private educational quality of individual i under perfect capital markets.

Alternatively, we may express (3.6) in terms of student's ability. Let \widehat{a} denote the critical value of child's ability such that a parent i will choose the public system if and only if the ability of his child is smaller or equal than \widehat{a} :

$$a^i \leq \widehat{a} = \frac{Q}{\left(\frac{k\alpha}{p}\right)^{\frac{1}{1-\alpha}} (1 - \alpha)^{\frac{1}{\alpha}}}. \tag{3.8}$$

Note that $\frac{\partial \widehat{a}}{\partial Q} > 0$ and $\frac{\partial \widehat{a}}{\partial p} > 0$; a higher level of public educational quality, Q and a higher price of private school quality, p both increase the critical level of ability and so as the proportion of students who are willing to attend public schools.

Hence, if capital markets are perfect, the public provision of education provides low-ability students with a higher level of educational quality than the one they would acquire in the private market. In the absence of borrowing constraints, government intervention in education is not efficient and in the context of this model, it would be justified only on distributional grounds.

The allocation of students to schools is the following: those students with high ability, $a^i > \widehat{a}$ are perfectly sorted into private schools according to their ability while low-ability students, $a^i \leq \widehat{a}$ are pooled in public schools, obtaining the same educational quality, Q .

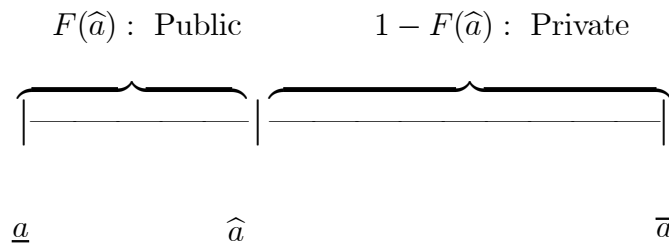


Figure 3.1: Sorting under Perfect Capital Markets

3.2.2 Equilibrium under Borrowing Constraints

We now turn to investigate the equilibrium distribution of students across schools under the assumption that parents cannot borrow against the future income of their children. In such an economy, a parent i who opt for private educational services chooses the level of private educational quality, q^i that maximizes his utility, given the tax rate, t

$$\begin{aligned} \max_{q^i} U^i(c^i, h^i) \\ \text{s.t. } c^i \geq 0, \end{aligned} \quad (3.9)$$

where $c^i = (1-t)y^i - pq^i$ is individual's consumption, which consists of individual's after-tax income $(1-t)y^i$ minus education purchases pq^i .

In the absence of capital markets, individuals' investments in education may be crucially affected by their income, since the inability of parents to borrow may result in inefficiently low investments in educational quality. Under borrowing constraints, the optimal investment in education of parent i , q^{i*} is the following:

$$q^{i*} = \begin{cases} q^{iP} = a^i \left(\frac{k\alpha}{p}\right)^{\frac{1}{1-\alpha}} & \text{if } a^i \leq \tilde{a}^i, \\ q^{iC} = \frac{(1-t)y^i}{p} & \text{if } a^i > \tilde{a}^i, \end{cases} \quad (3.10)$$

where $\tilde{a}^i \equiv \frac{(1-t)y^i}{p\left(\frac{k\alpha}{p}\right)^{\frac{1}{1-\alpha}}}$, $i = H, L$.

The presence of borrowing constraints affects the educational investments of poor families with talented children. Let denote \tilde{a}^i the critical level of ability such that a parent with a child of ability, $a^i > \tilde{a}^i$ invests inefficiently in educational quality. Note that this critical level is higher for high-income families, $\tilde{a}^H > \tilde{a}^L$ since $y^H > y^L$, which means that poor families are more likely to be constrained than rich families.⁵

⁵Note that those families who invest inefficiently in the school quality of their children devote all their after-tax income to investments in human capital, i.e., $c^i = 0$. Since parent's consumption and children's human capital are perfect substitutes in the model, some families find optimal to substitute entirely their own consumption for their children's human capital.

Hence, the equilibrium allocation of students to schools in the private education market is inefficient when capital markets are inexistent. High-ability students from constrained families, $a^i > \tilde{a}^i$, $i = H, L$ attend the same type of school while low-ability students are perfectly sorted into different schools according to their ability.

In the first stage of the utility maximization problem, a parent i chooses the public education system if and only if the utility he derives from sending his child to public schools is higher or equal than the utility he obtains if he chooses the private system,

$$U^{pb} \geq U^{pv*}, \quad (3.11)$$

where $U^{pb} = (1-t)y^i + kQ^\alpha (a^i)^{1-\alpha}$ is the indirect utility obtained by parent i if he sends his child to a public school and $U^{pv*} = (1-t)y^i - pq^{i*} + k(q^{i*})^\alpha (a^i)^{1-\alpha}$ is his indirect utility if his child attends a private school, where q^{i*} is the optimal expenditure on education of parent i under borrowing constraints and it is given by (3.10).

We can express (3.11) as an implicit function of per-student expenditures in public schools. Let \hat{Q}^{iC} denote the critical level of public quality such that a parent i sends his child to a public school if and only if:

$$Q \geq q^{i*} \left(1 - \frac{p}{k} \left(\frac{q^{i*}}{a^i} \right)^{1-\alpha} \right)^{\frac{1}{\alpha}} = \hat{Q}^{iC}, \quad (3.12)$$

where q^{i*} is given by (3.10). Using the implicit function theorem, we obtain that $\frac{\partial \hat{Q}^{iC}}{\partial a^i} > 0$ and $\frac{\partial \hat{Q}^{iC}}{\partial y^i} \geq 0$.⁶

Let define implicitly $\tilde{Q}^i = (1-\alpha)^{\frac{1}{\alpha}} \frac{(1-t)y^i}{p}$ as the critical level of public quality such that a parent with a child of ability \tilde{a}^i , $i = H, L$, sends his child to the public system if and only if public school quality is higher or equal than this critical level, $Q \geq \tilde{Q}^i$.⁷

⁶The presence of borrowing constraints reduces the critical level of public quality that a family with credit constraints requires to send his child to a public school, i.e., $\hat{Q}^{iC} < \hat{Q}^{iP}$. This is because constrained families invest an inefficiently low amount in the school quality of their children, i.e., $q^{iC} < q^{iP}$.

⁷The optimal investment in education of a parent whose child has an ability \tilde{a}^i is $q^{iP} = a^i \left(\frac{k\alpha}{p} \right)^{\frac{1}{1-\alpha}} = \frac{(1-t)y^i}{p}$. Therefore, although this parent spends entirely his after-tax income in his child's education, he invests an efficient amount in school quality.

Intuitively, if per-student expenditures at public schools are higher than this critical level, \tilde{Q}^i some families who are constrained in the private market start sending their children to public schools. Using the government budget constraint, given by (3.2), we obtain the following derivative signs:

$$\frac{\partial \tilde{Q}^i}{\partial \alpha} < 0, \quad \frac{\partial \tilde{Q}^i}{\partial p} < 0, \quad \frac{\partial \tilde{Q}^i}{\partial \bar{y}} > 0, \quad \frac{\partial \tilde{Q}^i}{\partial y^i} > 0. \quad (3.13)$$

The first derivative shows how a higher elasticity of human capital with respect to educational expenditures, α , increases the efficient level of private quality q^{i*} and hence, it makes easier that an individual with a child of ability \tilde{a}^i is credit constrained. In this case, the level of public quality required for this individual to send his child to the public school is lower. A higher price of private school quality, p also decreases the critical level of public quality, \tilde{Q}^i since the amount of quality that this individual may acquire in the private market is lower. In contrast, an increase in total income, \bar{y} and in individual's income, y^i raise the critical level of public quality since both increase disposable income of poor families.⁸

It also follows from (3.11) that a parent i sends his child to a public school if his child's ability is sufficiently low. Let \hat{a}^i be the critical level of ability such that a parent with income $i = H, L$ sends his child to public schools if and only if: $a^i \leq \hat{a}^i$. This critical ability is implicitly defined as follows:

$$\hat{a}^i = \begin{cases} \frac{Q}{\left(\frac{k\alpha}{p}\right)^{\frac{1}{1-\alpha}} (1-\alpha)^{\frac{1}{\alpha}}} & \text{if } Q \leq \tilde{Q}^i, i = H, L, \\ \left(\frac{(1-t)y^i}{k\left(\left(\frac{(1-t)y^i}{p}\right)^\alpha - Q^\alpha\right)}\right)^{\frac{1}{1-\alpha}} & \text{if } Q > \tilde{Q}^i, i = H, L. \end{cases} \quad (3.14)$$

An important implication of (3.14) is that the parent who is indifferent between sending his child to public or private schools may vary across income groups. Note that if some constrained families send their children to public schools, then $\hat{a}^L > \hat{a}^H$ since $y^L < y^H$,

⁸Note that a higher \bar{y} decreases the level of taxes required to fund a given level of public quality and thus raises individuals' after-tax income.

which means that the number of children of poor families at public schools is higher than the number of children of rich families.

Moreover, in the presence of borrowing constraints we obtain the following result:

Proposition 3.1. *Under borrowing constraints, the number of students attending public schools, F^* , is higher or equal than under perfect capital markets:*

$$F^* = \begin{cases} F(\hat{a}) & \text{if } Q \leq \tilde{Q}^L, \\ \gamma F(\hat{a}^L) + (1 - \gamma)F(\hat{a}) & \text{if } \tilde{Q}^L < Q \leq \tilde{Q}^H, \\ \gamma F(\hat{a}^L) + (1 - \gamma)F(\hat{a}^H) & \text{if } Q > \tilde{Q}^H. \end{cases}$$

This result holds since the presence of borrowing constraints reduces the utility that credit constrained families obtain at the private education system. Hence, for a sufficiently high level of public quality, a higher number of this type of families send their children to public schools compared to perfect capital markets.

3.3 Analysis of Education Vouchers

In this section we consider the introduction of education vouchers in an economy with borrowing constraints and a mixed education system. Education vouchers consist of a uniform monetary transfer of size v that the government makes to all students attending private schools.⁹ Students can only use the voucher to fund expenditures on education. However, parents who wish to spend more than this amount can supplement it out of their own funds. The voucher, as public educational quality, is financed by proportional taxation over income. Thus, the government balanced budget constraint after the introduction

⁹Other vouchers programs are the *means-tested* voucher in which only families with an income below some threshold receive a voucher of equal value and the *power-equalizing* voucher which gives families below some income a voucher that depends both on their income and the amount of funds they devote to education. See Fernández and Rogerson (2001).

of the voucher program is the following:

$$t\bar{y} = F_v^*Q + (1 - F_v^*)v, \quad (3.15)$$

where F_v^* is the proportion of individuals who remain at public schools after the introduction of education vouchers.

We first study the equilibrium allocation of students to schools in the presence of education vouchers. Then we turn to characterize the equilibrium voucher chosen by majority when public school quality does not change after the introduction of the voucher program. Finally, we investigate the welfare and distributional effects associated to different voucher policies.

3.3.1 Equilibrium with Education Vouchers

In the second stage of the utility maximization problem, each parent i chooses the amount of private school quality, q^{i*} that maximizes his utility:

$$\begin{aligned} & \max_{q^i} U^i(c^i, h^i) \\ & \text{s.t.} \begin{cases} c^i \geq 0, \\ pq^i \geq v, \end{cases} \end{aligned} \quad (3.16)$$

where $c^i = (1 - t)y^i + v - pq^i$ is parent's consumption, the constraint $c^i \geq 0$ means that parents cannot devote more than their disposable income to expenditures on education and the constraint $pq^i \geq v$ implies that the voucher v must be spent entirely on educational investments.

In the presence of education vouchers, the optimal investment in education of a parent i , q^{i*} is the following:

$$q^{i*} = \begin{cases} \frac{v}{p} & \text{if } a^i < a^*, \\ q^{iP} = a^i \left(\frac{k\alpha}{p} \right)^{\frac{1}{1-\alpha}} & \text{if } a^* \leq a^i \leq \tilde{a}^{iv}, \\ \frac{(1-t)y^i + v}{p} & \text{if } a^i > \tilde{a}^{iv}, \end{cases}$$

where $a^* = \frac{v}{p(\frac{k\alpha}{p})^{\frac{1}{1-\alpha}}}$ and $\tilde{a}^{iv} = \frac{(1-t)y^i+v}{p(\frac{k\alpha}{p})^{\frac{1}{1-\alpha}}}$, $i = H, L$.

Hence, education vouchers affect crucially optimal investments in education in the private market. It is interesting to note that the voucher induces *overinvestment* in educational expenditures in families with children of low ability, $a^i < a^*$.¹⁰ These families spend the voucher, v on quality investments, while their optimal level of educational quality without vouchers is lower. Individuals with students of ability, $a^* \leq a^i \leq \tilde{a}^{iv}$, $i = H, L$ purchase the same amount of educational quality as under perfect capital markets, i.e., $q^{i*} = q^{iP}$. Due to the presence of borrowing constraints, the proportion of families of this type is higher among the rich since $\tilde{a}^{Hv} > \tilde{a}^{Lv}$. Finally, those parents with children of ability $a^i > \tilde{a}^{iv}$ invest an inefficiently low amount of resources in education and the proportion of constrained families is higher among the poor.

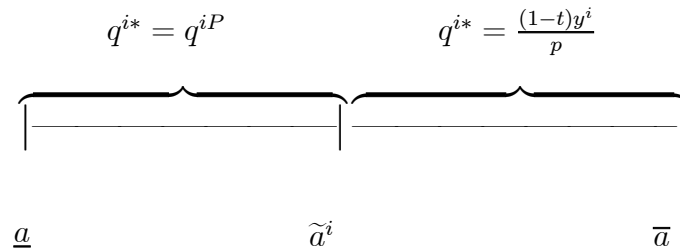


Figure 3.2: Private quality without education vouchers

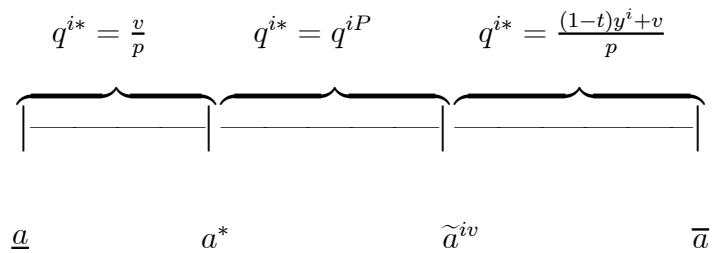


Figure 3.3: Private quality with education vouchers

¹⁰Notice that this *overinvestment* effect also appears when education is publicly provided.

In the first stage, given public educational quality, Q , the tax rate, t and the voucher level, v , a parent i will send his child to a public school in the presence of education vouchers if the following condition holds:

$$U^{pb} \geq U^{pr*}(v), \quad (3.17)$$

where $U^{pb} = (1-t)y^i + kQ^\alpha (a^i)^{1-\alpha}$ is the indirect utility if he sends his child to a public school and $U^{pr*}(v) = (1-t)y^i + v - pq^{i*} + k(q^{i*})^\alpha (a^i)^{1-\alpha}$ is the indirect utility if he chooses the private system.

We must impose some conditions over the magnitude of the voucher introduced. Note that if the government provides a quality of public education smaller than the market value of the voucher, i.e., $Q < \frac{v}{p}$ nobody will be willing to send their children to public schools since a higher level of quality is available at no cost at the private system. Thus, we consider that the voucher implemented is $v \in (0, Qp]$.

In the benchmark economy without vouchers, the allocation of students to schools is completely characterized by the level of public educational quality, Q . We can distinguish two possible cases, depending on whether constrained families send their children to public schools. We focus our study on the case in which only unconstrained families send their children to public schools and hence, borrowing constraints are not very binding. However, the presence of borrowing constraints crucially affects the expenditures on education of poor families with talented students in the private market. In this case, we can obtain unambiguous analytical results of the effects of vouchers on the sorting of students into public and private schools.

Note that in the case in which families with credit constraints send their children to public schools we cannot predict the impact of education vouchers on the number of individuals attending public schools. The introduction of a voucher decreases the number of students at public schools if the utility obtained at the private system by some families whose children attended public schools before the introduction of the voucher program is higher than their utility at the public system. The introduction of a voucher program has

two effects of opposite sign on the utility that constrained families obtain sending their children to the public system. On the hand one, the utility of these families at the public system raises compared to the utility at the public system because they receive a voucher from the government and on the other, the impact of vouchers on taxes decreases their utility at the private system compared to the public.

Hence, we assume hereinafter that per-student expenditures at public schools satisfy the following condition:

$$Q \leq \tilde{Q}^L. \quad (3.18)$$

Let denote $\hat{a}(0)$ the ability of the child whose parent is indifferent between public and private schools before education vouchers are introduced. The equilibrium number of students at public schools, before the introduction of the voucher plan, is $F_{v=0}^* = F(\hat{a}(0))$.

We can state the following proposition:

Proposition 3.2. *The introduction of education vouchers always decreases the equilibrium number of students attending public schools.*

Proof. The indifferent individual between public and private schools in the presence of education vouchers is an individual with a child of ability $\hat{a}(v)$ and he derives the same utility from sending his child either to public or private schools, i.e., $U^{pb} = U^{pr}(q^{i*})$:

$$(1-t)y^i + kQ^\alpha (\hat{a}(v))^{1-\alpha} = (1-t)y^i + v + k\hat{a}(v) \left(\frac{k\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha). \quad (3.19)$$

Using the implicit function theorem we find that a marginal increase in the voucher level marginally decreases the critical level of ability, $\hat{a}(v)$,

$$\frac{\partial \hat{a}(v)}{\partial v} = \frac{-1}{k(1-\alpha) \left[\left(\frac{k\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{Q}{\hat{a}(v)}\right)^\alpha \right]} < 0, \quad (3.20)$$

where $\left(\frac{k\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{Q}{\hat{a}(v)}\right)^\alpha \geq 0$ since $\hat{a}(v) \geq \frac{Q}{\left(\frac{k\alpha}{p}\right)^{\frac{1}{1-\alpha}}} = a^*$, $\forall v \in (0, \bar{v}]$. ■

The intuition for this result is the following: the introduction of education vouchers, given the level of educational quality, increases the utility derived from choosing private schools compared to the utility obtained in the public education system. Then, less parents will be willing to send their children to the public system after the introduction of the voucher plan. Note that when the indifferent parent is not constrained in the private market, he does not care about the effect of education vouchers on the level of taxes when he decides whether to send his child to public or private schools.

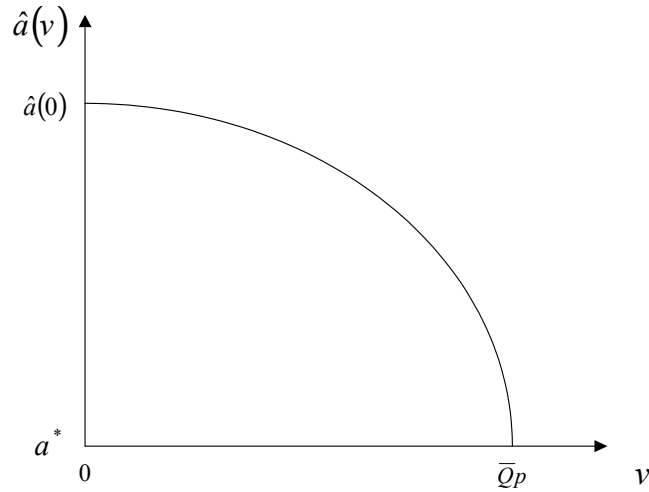


Figure 3.4: Critical ability as a function of the voucher

In Figure 3.4 we illustrate the negative effect that the size of the voucher has on the critical level of ability, $\hat{a}(v)$ and we observe that the critical ability is a decreasing and convex function of the voucher level.

3.3.2 Political Economy of Education Vouchers

In this subsection we characterize the voucher level chosen by majority considering that public school quality does not change after the introduction of the voucher program. We consider that the level of public school quality represents the status quo in the society. The government proposes the introduction of a voucher program. Such proposal will obtain

the political support required to be approved if it benefits a majority of individuals in the economy. We consider the relevant empirical case in which a majority of students remain at public schools after the introduction of the voucher program. We first analyze the preferred education vouchers of different types of families and then we characterize the equilibrium level chosen by majority voting. Finally, we investigate the welfare and distributional effects of different voucher policies.

Preferred Education Vouchers

After the introduction of the voucher, a parent i sends his child to public schools if and only if $U^{pb} \geq U^{pr*}(v)$. We may define $\widehat{v}^i(a^i)$ as the minimum voucher level such that a parent i with a child of ability a^i sends his child to a private school if and only if the voucher is sufficiently high, $v > \widehat{v}^i(a^i)$, while for voucher levels, $v \leq \widehat{v}^i(a^i)$, he chooses the public system.

Those students with high-ability, $a^i \geq \widehat{a}(0)$ who attended private schools before the introduction of the voucher, have a negative critical voucher level, $\widehat{v}^i \leq 0$ and thus, they continue attending private schools after the introduction of the voucher program. Conversely, students with a sufficiently low ability, $a^i \leq \frac{Q}{\left(\frac{k\alpha}{p}\right)^{\frac{1}{1-\alpha}}} = a^*$ remain at public schools independently of the size of the voucher, since their critical voucher is equal to the maximum voucher, $\widehat{v}^i = Qp = \bar{v}$. Finally, students of intermediate ability, $a^* < a^i < \widehat{a}(0)$ are those who have a positive critical voucher, $\widehat{v}^i(a^i)$, which is decreasing in their ability as represented in Figure 3.5.

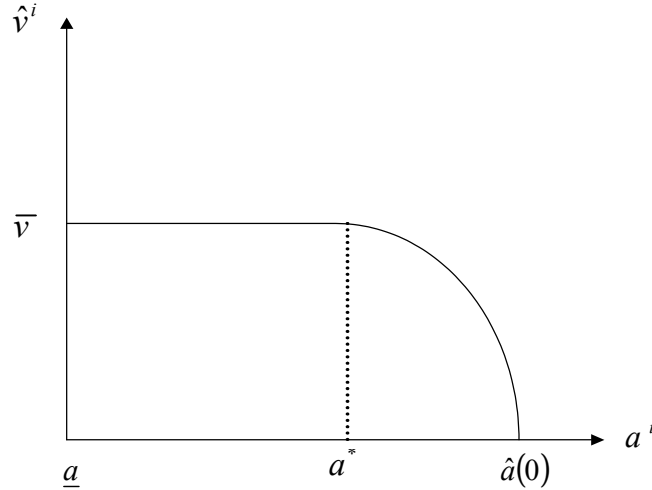


Figure 3.5: $\hat{a}(v)$ as a function of the voucher

To characterize parent's preferences over education vouchers, we can classify families in the following groups:

- Those families with students of low-ability $a^i \leq a^*$ who choose public schools independently of the level of education vouchers, $v \in (0, \bar{v}]$. These families prefer the voucher level that minimizes taxes, provided that school quality does not change after the introduction of education vouchers.
- Families with students of high-ability $a^i > \hat{a}(0)$ always send their kids to private schools, independently of the size of the voucher and thus, their most preferred voucher level is v^{i*} , $i = H, L$, which is the voucher that maximizes their disposable income, $(1 - t)y^i + v$.
- Finally, families with children of ability $a^* < a^i < \hat{a}(0)$ are those who have a positive critical level of vouchers, $\hat{v}^i > 0$, and thus, their decision of choosing public or private schools depends crucially on the voucher level. Hence, their utility function is the

following:

$$U^i(v) = \begin{cases} U^{pb} & \text{if } v \leq \hat{v}^i(a^i), \\ U^{pr*}(v) & \text{if } v > \hat{v}^i(a^i). \end{cases} \quad (3.21)$$

A parent i prefers the tax-minimizing voucher level, which is the voucher that maximizes his indirect utility if he chooses the public system, to the voucher v^i , which is the voucher level that maximizes his disposable income if he sends his child to the private system, if and only if $U^{pb}(v^*) \geq U^{pr}(v^i)$.

Equilibrium Education Vouchers

The fact that only those families who send their children to private schools receive the voucher makes preferences over vouchers to be non-single peaked for some families. We focus in the relevant empirical case, in which the number of students who remain at public schools after the introduction of vouchers are the majority in the economy. These families will vote for the voucher level that minimizes taxes. We now turn to investigate under which circumstances this voucher level is different from zero and hence, a voucher program may be implemented with the political support of a majority in the economy.

The level of taxes after the introduction of education vouchers is $t(v) = \frac{F(\hat{a}(v))Q + (1 - F(\hat{a}(v)))v}{\bar{y}}$, where $\hat{a}(v)$ is determined by (3.19). The effect of a marginal increase in the level of the voucher on the tax rate is the following:

$$\frac{\partial t(v)}{\partial v} = \frac{1}{\bar{y}} \left[\frac{\partial F(\hat{a}(v))}{\partial v} (Q - v) + 1 - F(\hat{a}(v)) \right], \quad (3.22)$$

where $F(\hat{a}(v)) = \frac{\hat{a}(v) - \underline{a}}{\bar{a} - \underline{a}}$, $\frac{\partial F(\hat{a}(v))}{\partial v} < 0$ and then, $\frac{\partial F(\hat{a}(v))}{\partial v} (Q - v) < 0$ if $Q - v > 0$.¹¹ Thus, an increase in the level of education vouchers marginally decreases taxes, $\frac{\partial t(v)}{\partial v} < 0$ if the following inequality holds:

$$\left| \frac{\partial F(\hat{a}(v))}{\partial v} (Q - v) \right| > 1 - F(\hat{a}(v)). \quad (3.23)$$

¹¹The introduction of education vouchers may result in a reduction of the tax rate only if per-student expenditures in public schools are strictly higher than per-student voucher expenditures in private schools.

From (3.23), it follows that an increase in the voucher level results in a decrease of taxes if the marginal reduction in the expenditures of the government due to the transfer of individuals from public to private schools in response to an increase in the voucher, $\left| \frac{\partial F(\hat{a}(v))}{\partial v} (Q - v) \right|$ is higher than the marginal increase in expenditures as a result of voucher payments to students in private schools, $1 - F(\hat{a}(v))$.

The existence of a positive tax-minimizing voucher, $v^* > 0$ crucially depends on per-student expenditures in public education, as the next proposition shows:

Proposition 3.3. *The interior tax-minimizing level of education vouchers is increasing in public educational quality.*

Proof. If there exists an interior tax-minimizing voucher level, v^{int} it satisfies the following condition:

$$\frac{1}{y} \left[\frac{\partial F(\hat{a}(v^{int}))}{\partial v} (Q - v^{int}) + 1 - F(\hat{a}(v^{int})) \right] = 0.$$

Using the implicit function theorem, we may compute $\frac{\partial v^{int}}{\partial Q}$:

$$\frac{\partial v^{int}}{\partial Q} = \frac{- \left(\frac{\partial \left(\frac{\partial \hat{a}(v)}{\partial v} \right)}{\partial Q} (Q - v^{int}) + \frac{\partial \hat{a}(v)}{\partial v} - \frac{\partial \hat{a}(v)}{\partial Q} \right)}{\frac{\partial^2 \hat{a}(v)}{\partial^2 v} (Q - v) - 2 \frac{\partial \hat{a}(v)}{\partial v}} > 0,$$

where $\frac{\partial \left(\frac{\partial \hat{a}(v)}{\partial v} \right)}{\partial Q} (Q - v^{int}) + \frac{\partial \hat{a}(v)}{\partial v} - \frac{\partial \hat{a}(v)}{\partial Q} < 0$ and $\frac{\partial^2 \hat{a}(v)}{\partial^2 v} (Q - v^{int}) - 2 \frac{\partial \hat{a}(v)}{\partial v} > 0$ as v^{int} is a minimum of $t(v)$. ■

We can examine under which conditions there exists a positive tax-minimizing voucher level in the economy. This is the case if the introduction of the voucher decreases the tax rate compared to the situation without education vouchers:

$$\frac{\partial t}{\partial v}(v = 0) < 0. \quad (3.24)$$

Condition (3.24) may be written in terms of a critical level of public educational quality, Q^* such that a voucher program is tax-decreasing if and only if public quality is

higher than this critical level:

$$Q > \bar{a} \left(\frac{k\alpha\theta}{\alpha k + \theta^{1-\alpha}} \right) = Q^*, \quad (3.25)$$

where $\theta = \left(\frac{k\alpha}{p} \right)^{\frac{1}{1-\alpha}} (1-\alpha)^{\frac{1}{\alpha}}$.

This result is illustrated in Figures 3.6 and 3.7.¹² In Figure 3.6 we observe that there exists a positive tax-minimizing level of education vouchers because the level of school quality is sufficiently high. However, we observe in Figure 3.7 that a low level of per-student expenditures at public schools makes impossible any gains from vouchers in terms of tax-reductions.

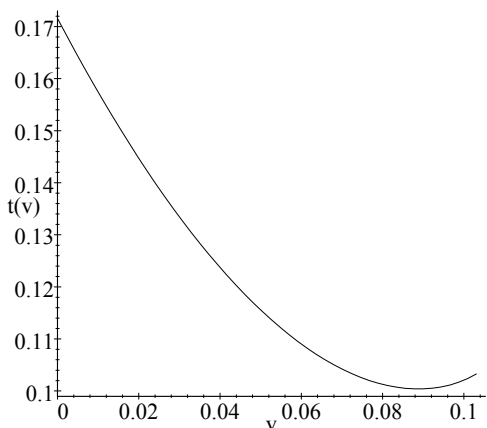


Figure 3.6: Tax rate if $\bar{Q} > Q^{int}$

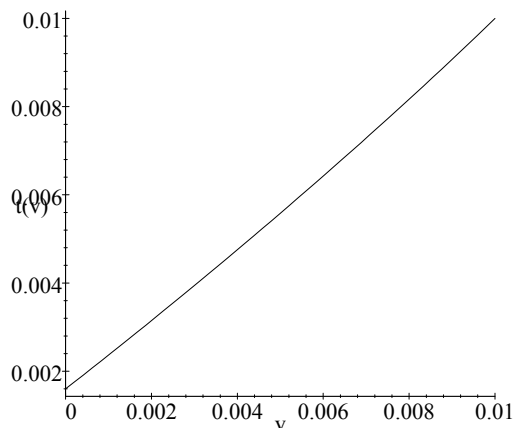


Figure 3.7: Tax rate if $\bar{Q} < Q^{int}$

The tax-minimizing voucher level, v^* , is a majority-voting equilibrium if the number of families who send their children to public schools when the voucher is v^* is the majority in the economy:

$$F(\hat{a}(v^*)) > 0.5. \quad (3.26)$$

Condition (3.26) is always satisfied if the proportion of students attending public schools are the majority, independently on the value of the voucher introduced, i.e.,

¹²These figures are computed for the following parameters: $y^H = 1.5$, $y^L = 0.5$, $\gamma = 0.5$, $\underline{a} = 0$, $\bar{a} = 1$.

$F(\hat{a}(v)) > 0.5, \forall v \in (0, \bar{v}]$. This is the case if the level of public school quality satisfies the following condition:

$$Q > 0.5 (\bar{a} + \underline{a}) \left(\frac{k\alpha}{p} \right)^{\frac{1}{1-\alpha}} = Q^m. \quad (3.27)$$

In economies characterized by (3.25) and (3.27), a majority of students remain at public schools after the introduction of the voucher program and the tax-minimizing voucher is different from zero. In these economies, a voucher program may be implemented since it will be approved by the majority of families whose children remain at public schools.

Welfare and Distributional Effects of Education Vouchers

The introduction of education vouchers entails important distributional and welfare effects and such effects depend crucially on the impact that a voucher program may have on taxes, since public quality remains fixed after the introduction of the voucher. In our model, total welfare is the sum of aggregate parents' consumption and children's accumulated human capital, so we may study the effect of the voucher on each of these magnitudes separately.

First, consider the case in which the level of the voucher introduced is such that it reduces taxes. This voucher level has positive welfare effects for all individuals and hence, it raises total welfare. In the previous subsection we have characterized under which conditions a tax-minimizing voucher may obtain the political support necessary to be implemented. Note that the effect of such voucher on aggregate consumption is ambiguous. On the one hand, a reduction in taxes raises the consumption of those families whose children remain in public schools after the introduction of the voucher. On the other hand, some families whose children attend private schools may be willing to reduce their consumption to increase expenditures in education after the introduction of the voucher since the voucher allows them to purchase more educational quality than before.

To evaluate the impact of a tax-reducing voucher on aggregate human capital, consider the aggregate human capital in the absence of education vouchers, $W_{v=0}^*$:

$$\begin{aligned} W_{v=0}^* &= \int_{\underline{a}}^{\hat{a}(0)} kQ^\alpha (a^i)^{1-\alpha} f(a) da \\ &+ \gamma \left(\int_{\hat{a}(0)}^{\tilde{a}^L} ka^i \left(\frac{k\alpha}{p} \right)^{\frac{\alpha}{1-\alpha}} f(a) da + \int_{\tilde{a}^L}^{\bar{a}} k (q^{LC})^\alpha (a^i)^{1-\alpha} f(a) da \right) \\ &+ (1-\gamma) \left(\int_{\hat{a}(0)}^{\tilde{a}^H} ka^i \left(\frac{k\alpha}{p} \right)^{\frac{\alpha}{1-\alpha}} da + \int_{\tilde{a}^H}^{\bar{a}} k (q^{HC})^\alpha (a^i)^{1-\alpha} f(a) da \right), \end{aligned}$$

where $f(a) = \frac{1}{\bar{a}-\underline{a}}$, since a^i is uniformly distributed over $[\underline{a}, \bar{a}]$, $q^{LC} = \frac{(1-t(0))y^L}{p}$ and $q^{HC} = \frac{(1-t(0))y^H}{p}$.

Aggregate human capital after the introduction of voucher program, $W_{v>0}^*$ is the following:

$$\begin{aligned} W_{v>0}^* &= \int_{\underline{a}}^{\hat{a}(v)} kQ^\alpha (a^i)^{1-\alpha} f(a) da \\ &+ \gamma \left(\int_{\hat{a}(v)}^{\tilde{a}^{Lv}} ka^i \left(\frac{\alpha}{p} \right)^{\frac{\alpha}{1-\alpha}} f(a) da + \int_{\tilde{a}^{Lv}}^{\bar{a}} k (q^{LCv})^\alpha (a^i)^{1-\alpha} f(a) da \right) \\ &+ (1-\gamma) \left(\int_{\hat{a}(v)}^{\tilde{a}^{Hv}} ka^i \left(\frac{\alpha}{p} \right)^{\frac{\alpha}{1-\alpha}} f(a) da + \int_{\tilde{a}^{Hv}}^{\bar{a}} k (q^{HCv})^\alpha (a^i)^{1-\alpha} f(a) da \right), \end{aligned}$$

where $q^{LvC} = \frac{(1-t(v))y^{L+v}}{p}$, $q^{HvC} = \frac{(1-t(v))y^{H+v}}{p}$, and $\tilde{a}^{iv} = \frac{(1-t(v))y^i+v}{p} > \tilde{a}^i = \frac{(1-t(0))y^i}{p}$, $i = H, L$, since $t(v) < t(0)$.

To compare $W_{v=0}^*$ and $W_{v>0}^*$, notice that the introduction of an education voucher that reduces taxes has two effects on aggregate human capital: first, the voucher induces a transfer of the more talented students from public to private schools. Since student's ability and school quality are complements in the production of human capital, this transfer increases aggregate human capital. The second effect of this voucher policy is that it makes borrowing constraints less tighten for those families, either rich or poor, whose children attended private schools before the introduction of the voucher. This is because disposable income raises with the voucher, i.e., $(1-t(v))y^i + v > (1-t(0))y^i$, $i = H, L$ since $t(v) < t(0)$.

Therefore, total welfare increases after the introduction of the voucher if the increase in aggregate human capital compensates the decrease in total consumption. Since individuals derive utility from the sum of their own consumption and the human capital of their children and a tax reduction makes all type of families strictly better-off, total welfare raises with a tax-decreasing voucher. Those who remain sending their children to public schools after the introduction of the voucher pay now lower taxes, while those who choose the private system not only pay lower taxes but also receive the voucher.

Now consider the case of a voucher policy that leads to an increase in taxes, i.e., $t(v) > t(0)$. If a majority of families remain sending their children to public schools after the introduction of the voucher, a tax-increasing voucher cannot obtain the political support required to be implemented. However, the effects of this type of voucher on total welfare are ambiguous. Note that all families sending their children to public schools are strictly worse-off after the introduction of the voucher. Their consumption is lower and their children acquire the same human capital than before, since public quality does not change with the voucher.

In order to evaluate the impact of the voucher on the welfare of families with students attending private schools, we can distinguish between rich and poor individuals. A high voucher level makes rich families with children attending the private system to be worse-off compared to the situation without vouchers. There may exist a voucher level, v^H such that $\forall v \geq v^H$, $[t(v) - t(0)] y^H \geq v$. However, the redistributive role of vouchers makes low-income families to be always better-off after the introduction of a voucher, independently of the voucher level and the effect of the voucher on taxes since

$$(t(v) - t(0))y^L < v, \forall v > 0. \quad (3.28)$$

where $(t(v) - t(0))y^L = \frac{(1 - F(\hat{a}(v))v - Q(F(\hat{a}(0) - \hat{a}(v)))y^L}{\bar{y}} < \frac{v}{\bar{y}}$ and then, $[t(v) - t(0)] y^L < v$, since $(1 - F(\hat{a}(v))v - Q(F(\hat{a}(0) - \hat{a}(v))) < v$ and $\frac{y^L}{\bar{y}} < 1$.

The effect of a tax-increasing voucher on total welfare is difficult to predict. On the one hand, it decreases the consumption of families whose children attend public schools,

and on the other hand, it generates a more efficient allocation of students to schools and higher human capital accumulation. Thus, the impact of this voucher on total welfare depends on the relative proportion of poor and rich individuals in the economy and on the distribution of children across public and private schools.

3.4 Calibration and Illustrative Results

3.4.1 Benchmark Economy

We calibrate the model to existing empirical evidence for the US. Our benchmark is an economy with borrowing constraints and without education vouchers. First, we start with the distribution of income in the economy. We proceed as Caucutt (2002), taking data from the 1992 census on total money earnings of full-time workers in the United States. The earnings of the poor are \$19,325 and the earnings of the rich are \$57,065. With units of earnings in thousand of dollars, $y^L = 19.325$ and $y^H = 57.065$. We assume that poor families are the 80 percent of the population and rich families 20 percent, and then, $\gamma = 0.8$.

The distribution of child's ability is uniform, $a^i \sim U [\underline{a}, \bar{a}]$. We assume that the learning ability of the least talented child is $\underline{a} = 1$ and the ability of the most talented is $\bar{a} = 6$. Since child's ability and parental income are uncorrelated, the distribution of children's ability is the same for rich and poor families.

The human capital production function is the following:

$$h^i = k (q^i)^\alpha (a^i)^{1-\alpha}.$$

We calibrate this function following the empirical literature on the determinants of educational achievement. The Cobb-Douglas functional form restricts the number of parameters to match to just one, α . We choose this parameter such that the elasticity of child's earnings with respect to child's ability is $1 - \alpha = 0.7$. This specification implies

that for the same level of expenditures in education, the child endowed with the highest ability, $\bar{a} = 6$ obtains three times and a half as much human capital as the child of the lowest ability, $\underline{a} = 1$. Henderson et al. (1978) report a parameter range for this elasticity from 0.5 to 0.7.

The elasticity of earnings with respect to educational expenditures is $\alpha = 0.3$. The empirical literature on the effects of school expenditures on educational outcomes estimate an elasticity between 0.1 and 0.2 (see Card and Krueger (1992) and Grogger (1996)). This value, however is obtained after controlling for family characteristics, such as income or education. Our choice of a slightly higher value for this elasticity is justified on the grounds of the correlation between father's income and son's income, which is found to be close to 0.4 (see Solon (1992)). Note that in our model intergenerational correlation in income arises only from the correlation between parental income and the quality of the school attended.

The parameter k is chosen so next period's human capital is approximately of the same magnitude as first period's human capital, $k = 6.900$. In the United States in 1998, per student expenditure in public schools in 1988 was \$4,222, and thus, public school quality is $Q = 4.222$ in thousand of dollars. The proportion of students at public schools is approximately 90 percent of the student population. We match the proportion of students at public schools in the US adjusting the price of educational quality at private schools which is found to be approximately equal to unity.

The allocation of students to schools in the benchmark economy is described in Table 3.1. Poor students at the private system are pooled into School 1 due to the presence of borrowing constraints. Poor families who send their children to private schools devote all their after-tax income to expenditures on education, $q_1^L = \frac{(1-t)}{p}y^L = 14.120$ (in thousand of dollars). On the other hand, rich students are perfectly sorted across a continuum of private schools according to their ability and the mean expenditures at these schools is 14.170.

Table 3.1
Schools: Benchmark Economy without Vouchers

	Public School	School 1	Rest of Schools (continuum)
Measure	0.90	0.08	0.02
Fraction of poor	0.72	0.08	0
Fraction of rich	0.13	0	0.02
Mean Expenditures	4.222	14.120	14.170

3.4.2 Introduction of Education Vouchers

Equilibrium Education Voucher

We find that in the benchmark economy a majority of students attend public schools after the introduction of the tax-minimizing voucher, and hence, this is the voucher level approved by majority. In our economy, the equilibrium voucher is $v^* = \$1,389$, given that per-student expenditures at public schools remain constant at the pre-voucher level, $Q = \$4,222$. The implementation of this voucher program decreases the tax rate from $t(0) = 0.1414$ to $t(v^*) = 0.1258$.

The allocation of students to schools under the equilibrium voucher is described in Table 3.2. The proportion of students at public schools decreases in 20 percent. We observe a perfect stratification of students across private schools according to their learning ability. This is because the introduction of the voucher policy allows poor families who send their children to private schools to invest an efficient amount in their education.

Table 3.2
 Schools: Equilibrium voucher of 1.390 (thousand of dollars)
 Per-Student Expenditures in Public Schools Constant

	Public School	Private Schools (continuum)
Measure	0.70	0.30
Fraction of poor	0.56	0.28
Fraction of rich	0.14	0.07
Mean Expenditures	4.222	13.496

Welfare and Distributional Effects of Different Voucher Policies

In this section we investigate the welfare and distributional effects of different voucher policies. In this exercise we try to shed some light on some issues for which theoretical analysis yields ambiguous effects, such as the impact of voucher on the distribution of income in the economy. First, we analyze the impact of different voucher levels, including the equilibrium voucher, on the mean and the distribution of human capital. We use the coefficient of variation as a measure of income inequality, which is the standard deviation of the distribution divided by the mean.

The results obtained are described in Table 3.3. We observe that the average human capital increases with the voucher size. A high level of vouchers raises the proportion of individuals at private schools and the allocation of students to schools becomes gradually more efficient. Note that as the size of the voucher increases, poor students at private schools are also less constrained, so poor families can gradually invest according to their children's ability.

Interestingly, the impact of the magnitude of the voucher on income inequality is not monotone. The explanation for this result is the following: at low voucher levels, the distributional impact of this policy is low, and only a small number of poor families make use of the voucher, and thus, income inequality raises. As the voucher increases, there

is more redistribution and thus, income inequality is lower. Thus, low voucher levels are associated to higher inequality than high voucher levels.

Table 3.3
Comparisons to the Benchmark Economy (voucher = 0),
percentage changes in parenthesis

Case	Mean (h)	CV(h)	Tax rate
Benchmark	26.858	0.4501	0.1414
Voucher = 1.000	29.218 (8.78)	0.4577 (1.68)	0.1263 (-10.68)
Voucher = 1.390	29.706 (10.60)	0.4604 (2.28)	0.1258 (-11.03)
Voucher = 2.000	30.429 (13.30)	0.4577 (1.68)	0.1271 (-10.11)
Voucher = 2.300	30.762 (14.54)	0.4549 (1.06)	0.1287 (-8.98)

Table 3.4
Welfare comparisons to the Benchmark Economy without Vouchers,
percentage changes in parenthesis

	Consumption	Human Capital	Total Welfare
Benchmark	21.399	26.858	48.257
Voucher = 1.000	19.585 (-8.47)	29.218 (8.78)	48.803 (1.19)
Voucher = 1.390	19.194 (-10.30)	29.706 (10.60)	48.900 (1.33)
Voucher = 2.000	18.607 (-13.05)	30.429 (13.30)	49.036 (1.61)
Voucher = 2.300	18.330 (-14.34)	30.762 (14.54)	49.092 (1.73)

The effects that different voucher policies may have on total welfare are explored in Table 3.4. Total welfare is the sum of aggregate parents' consumption and children's human capital. We observe that aggregate consumption decreases with the size of the

voucher while total human capital increases. The effect of an increase in the magnitude of the voucher on total welfare is positive. Moreover, welfare gains raise with the voucher size, ranging from 1.19 percent, when a voucher of \$1,000 is introduced, to 1.73 percent when the voucher is \$2,300.

Epple and Romano (1998) also obtain that welfare gains increase monotonically in the size of the voucher. In contrast, in Caucutt (2002) low voucher levels are associated with welfare losses that decrease with the size of the voucher and become welfare gains for sufficiently high levels of the voucher. However, her results are not comparable to ours since she focuses on the welfare gains of switching from a public school system to a completely private system with education vouchers. We instead investigate the efficient and distributional effects of vouchers on a mixed education regime with a public and a private sector.

3.5 Concluding Remarks

In this chapter we present a model in which families differ in over income and ability of their children and there are public and private schools. Private schools offer a variety of educational quality levels, while public schools provide a uniform quality, funded with taxes. We show that the introduction of education vouchers crucially affects the allocation of individuals to public and private schools and thus, the accumulation of human capital. A voucher program also entails important welfare and distributional effects.

If borrowing constraints are not very binding, the introduction of vouchers always decreases the proportion of students attending public schools independently of the impact of vouchers on taxes. This is because not only rich, but also poor families make use of the voucher to acquire educational quality in the private system. If a majority of the student population attend public schools, education vouchers will be supported politically only if they entail welfare gains for the majority of the population. Assuming that per-student expenditures at public schools do not change after the introduction of the voucher, it will

be approved the voucher that minimizes taxes. We find that the equilibrium voucher will be different from zero if public school quality is sufficiently high.

We calibrate the model to existing empirical evidence for the US. The benchmark economy is characterized by 90 percent of students attending public schools and an imperfect allocation of poor students across private schools due to borrowing constraints. The equilibrium voucher in this economy entails a 20 percent reduction of the public student body. The transfer of individuals from public to private schools increases aggregate human capital and welfare. On the other hand, this voucher policy leads to greater inequality. We also investigate the welfare and distributional impact of different voucher policies. We find that welfare gains are increasing in the voucher size. However, the impact of the magnitude of the voucher on income inequality is not monotone, being low voucher levels more unequal than high levels.

In order to concentrate our analysis on the impact of vouchers on the efficient allocation of students in the private sector, we abstract from several important issues. First, we consider that school quality is only determined by expenditures on education. School quality may also depend on the type of students who compose the school. For instance, if the mean ability of the student body affects school quality, a transfer of the more talented students from public to private schools as a consequence of the voucher could be detrimental for students who are left behind in public schools. Second, our work does not address the impact of vouchers on competition between public and private schools. One of the arguments in favor of education vouchers is that they raise competition between educational institutions to attract students, which may result in efficiency gains at both types of schools. To evaluate this issue in our model, it would be necessary to provide schools with an active role in the selection of students.

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Chapter 4

Competition between Public and Private Universities: Exams versus Prices

4.1 Introduction

Higher education institutions usually use exams as an instrument to select their students. The importance of this instrument as a mechanism to allocate students to schools varies depending on the country and institution. In many European countries, public universities set very low prices and use exams as the main guide to determine their admissions while private universities are usually of lower quality. In contrast, in the United States the majority of universities are private and non-profit institutions and both exams and prices play an important role as instruments to allocate students.¹

The main difference between prices and exams as allocation devices is that while fees make students to self-select according to willingness and ability to pay, exams are used by

¹See Whinston (1999) for a detailed discussion of the features of the higher education sector in the US.

schools to select students according to revealed ability to learn. If student's ability and school quality are complements in the production of education, which seems reasonable in the case of higher education, then willingness to pay is a good approximation for student's ability as long as the marginal return from investing in education of higher quality is larger for high-ability students. In this case, and in the absence of borrowing constraints, students make adequate choices regarding their enrolment to existing schools.

However, it is generally acknowledged that the existence of borrowing constraints prevents a pure price mechanism from attaining the optimal allocation of students to schools of different quality. The relative efficiency of each device then depends on the ability of exams to identify unobservable ability without wasting too much resources (the exam technology). Fernández (1998) and Fernández and Galí (1999) investigate the properties of markets and exams as alternative assignment mechanisms under borrowing constraints. They show that exams are more efficient allocation devices if the exam technology is *sufficiently powerful*. These papers consider a perfectly competitive market in which schools vary in their exogenous quality. Each school has a fixed capacity and charges a market clearing price to students who vary in their income and ability.

This perfect competition assumption is probably not a good approximation to the market for higher education, which does not clear in the usual sense.² Moreover, imperfect competition among higher education institutions provides exams and prices with an additional strategic role that has been ignored in the scarce literature on university behavior.³ One of the problems when dealing with the higher education market rests in determining which are the objectives of the decisive units. Since our work is intended to capture some of the distinctive features of the European higher education market, we focus our analysis on two types of institutions: public and private. The objective of the public university is to maximize public surplus, that is, the sum of earnings of students attending the public university minus the cost incurred to provide education, while the

²Most higher education systems allocate places to students by administrative rationing.

³Competition between universities has been studied by Del Rey (2000) and De Fraja and Iossa (2002).

private institution maximizes profits.

The concern for competition among public and private institutions is motivated by the increasing debate on privatization within the European higher education sector. The increasing budgetary restrictions it faces are rooted in a generalized fall in government spending as well as the slow-down of population growth. Moreover, public universities are increasingly seen as huge, bureaucratized institutions with very slow capacity of reaction and in fact very aloof from the real needs of society.⁴ In this context, not only public universities are raising their fees, but also private institutions are being welcome to enter the higher education sector.

In this chapter we aim at shedding some light on the role of universities' objectives in shaping optimal choices concerning prices and exams in the presence of borrowing constraints. We investigate on the one hand, the determinants of the observed different combinations of exams and prices across institutions and countries and on the other, the impact that competition between institutions may have on such choices and on total welfare.

We consider a simple model in which two institutions, one public and one private, provide educational quality in the same market. Universities may use exams, prices or both instruments to allocate students to schools.⁵ Students differ in their unobservable ability and in their income endowment and they choose whether to attend a university or remain uneducated in order to maximize their lifetime income. We consider that students' ability and educational quality are complements in the production of human capital, so the return from educational quality is higher for high-ability students.

We first establish a benchmark in which there is only one institution, either private or public, in the higher education market and there are perfect capital markets to finance educational investments. We use this benchmark to investigate the role of borrowing

⁴In France, the university system is referred to as the "Mammouth" (see Gary-Bobo and Trannoy (1998a) and (1998b)).

⁵In our model, exams do not involve any wasteful expenditure in contrast to Fernández (1998).

constraints on universities' optimal choices concerning prices and exams. In the presence of borrowing constraints, the private and profit-maximizing university only uses prices or tuition fees to determine admissions. In contrast, the public university uses exams and sets a zero price for its educational services. Exams are preferred to prices because they allocate students to the public institution according to their ability. The utility of the public institution is positively affected by the ability and the quantity of students enrolled in this institution. In the presence of borrowing constraints, prices limit admissions of high-ability and poor individuals who are unable to pay university's fees, which in turn, reduces public institution's utility. Interestingly, we find that both institutions provide the same educational quality without borrowing constraints under monopoly, although the private institution is more selective than the public. The presence of borrowing constraints only affects the type of instrument chosen by the public university to select its students, prices under perfect capital markets and exams under borrowing constraints, while the quality provided at the public institution is the same than under perfect capital markets. However, both the price and educational quality at the private institution are lower due to the presence of borrowing constraints.

Next, we model competition between a public and a private institution. Our main result is the following: the private institution provides a quality lower than the public university. This result is independent of the existence of borrowing constraints and it is driven by the different strategies followed by the public and the private institution when competing for students. In the presence of borrowing constraints, the choice of exams allows the public university to behave as a monopoly in the higher education market. Hence, the private university is attractive just for those students of lower ability who are not accepted at the public university. Competition between a public and a private university involves positive welfare gains compared to monopoly. Those students attending the public institution under monopoly are not affected by competition and the presence of a private university of lower quality allows new students to have access to

higher education. This, in turn increases total income in the economy.

This chapter is organized as follows: Section 4.2 presents a simple model of the higher education sector. In this section, we describe the characteristics and behavior of students and universities and the alternative instruments to allocate students to schools. In Section 4.3 we first analyze a benchmark economy without borrowing constraints, in which there is only one university, either public or private, in the higher education market. Later on, we introduce borrowing constraints and compare universities' optimal choices when they may use either prices, exams or both instruments to select their students. Section 4.4 models competition between a public and a private university with and without borrowing constraints. Section 4.5 concludes.

4.2 The Model

4.2.1 Individuals

The economy consists of a continuum of individuals of measure one. Each individual i is characterized by a different and unobservable ability, a_i and an initial income endowment, w_i , which are uniformly and independently distributed over the interval $[0, 1]$. Individuals derive utility from their total lifetime income, which consists of their initial income endowment, w_i and their earnings or accumulated human capital if they become educated, $a_i Q_j$, where Q_j is the educational quality provided by university j . We assume that individual's ability and educational quality are complements in the production of human capital.⁶

Hence, a student of ability a_i and income w_i who attends a university of quality Q_j , that charges a tuition fee of p_j , enjoys the following utility:

$$U_j^i = w_i - p_j + a_i Q_j. \quad (4.1)$$

Conversely, a student who does not attend university obtains a utility equal to his

⁶This assumption is crucial to characterize the efficient allocation of students to schools in our model.

initial endowment, i.e., $U_0^i = w^i$ since earnings of uneducated individuals are normalized to zero.

4.2.2 Universities

We consider universities that produce educational services of quality Q_j where $j = b, v$ stand respectively for *public* and *private*. Educational quality may be interpreted as the *prestige* of the higher education institution.⁷ Public and private universities differ in their objectives: while the public university maximizes public surplus (the difference between the sum of earnings of students attending the public university and the costs incurred to provide education), the private institution maximizes profits.⁸

University j incurs in per-student costs $C(Q_j)$, which have the following functional form:

$$C(Q_j) = \alpha Q_j^k, \quad k > 1, \quad \alpha > 0. \quad (4.2)$$

Therefore, university's costs are an increasing and convex function of educational quality, $C'(Q_j) > 0$, $C''(Q_j) > 0$.

4.2.3 Allocation Mechanisms

Exams

Universities may use an entry exam to select the best students among those who are willing to attend the university. In order to do so, they establish a minimum score such that those who obtain a score equal or higher are accepted at the university. We assume

⁷School quality is usually measured by per-student expenditures in the empirical and theoretical education literature. Some authors consider that school quality also depends positively on the mean ability of students attending the school (peer-group effects).

⁸Public surplus is the sum of the utility of students attending the public university and the utility of the university. Notice that tuition fees are merely a monetary transfer from students to the university and thus, it cancels out when these utilities are aggregated.

that the exam technology is able to perfectly reveal the student's ability, which means that those students who obtain a score higher or equal than the minimum score established by the university are those of ability $a_i \geq a_j^E$, $j = b, v$.

Prices

Students may be allocated to schools also by means of prices or fees. This mechanism selects students according to their willingness and ability to pay. Then, by choosing a price p_j , the university indirectly determines the type of students (characterized by their ability and income) who are willing and able to enrol the university, given quality Q_j .

When there is only one school in the market, students compare their utility with and without education. Let \hat{a}_j be the ability of the student who is indifferent between attending school j and remaining uneducated, i.e., $\hat{U}_0 = \hat{U}_j$:

$$\hat{a}_j = \frac{p_j}{Q_j}. \quad (4.3)$$

All students of ability $a_i \geq \hat{a}_j$ are willing to attend university j . Among those, only students with income $w_i \geq p_j$ are able to do so in the presence of borrowing constraints.

If there are two universities in the market, university h and university l , and $Q_h \geq Q_l$, students compare the utility obtained at both universities. Let \tilde{a}_h be the ability of the student who is indifferent between attending university h and university l , i.e., $\hat{U}_h = \hat{U}_l$:

$$\tilde{a}_h = \frac{p_h - p_l}{Q_h - Q_l}. \quad (4.4)$$

Students of high ability, $a_i \geq \tilde{a}_h$, are willing to attend university h , while students of lower ability, $\hat{a}_l \leq a_i < \tilde{a}_h$, prefer to attend university l .

4.3 The monopoly benchmark

In this section we investigate how universities' different objectives shape optimal choices regarding prices and exams in the case in which there is only one institution, either

public or private, in the higher education market. We also want to evaluate the impact of borrowing constraints on such decisions. For this purpose, our benchmark economy is an economy with perfect capital markets. In such an economy, students can borrow any amount in the capital market to finance their education investments at a constant tax rate, which is normalized to zero for simplicity.

First, we consider that educational quality is given and institutions may choose either exams or prices as allocation mechanisms. Next, we allow each institution to use both instruments as assignment devices and to decide its optimal level of educational quality. The timing of decisions is the following: in a first stage, each monopoly chooses the level of educational quality, Q_j . In a second stage, it decides the price, p_j for its educational services. Finally, the institution may use a selective exam to admit only the more talented students, $a_i \geq a_j^E$ among those who are willing to attend the university, $a_i \geq \hat{a}_j$. Afterwards, we introduce borrowing constraints in the model and compare the results with those obtained for the benchmark economy.

4.3.1 Perfect Capital Markets

Exams

In this section we consider that the price and the quality of the university are already given and each monopoly decides whether to run an exam or accept all applications. This assumption may be justified due to the existence of a strong regulation concerning prices or because we are in the short run (prices have been already set and universities try to raise their utility via exams). Under perfect capital markets, the public monopoly chooses the exam, a_b^E that maximizes public surplus, U_b^p where the superscript p stands for *perfect* capital markets,

$$U_b^p = \int_0^1 \int_{a_b}^1 (aQ_b - C(Q_b)) da dw. \quad (4.5)$$

Given public quality, Q_b and price, p_b , the limiting admission grade must be such that

$a_b^E \geq \hat{a}_b = \frac{p_b}{Q_b}$. This means that exams must select students among those who are willing to attend the university, $a_i \geq \hat{a}_b$, otherwise exams are useless. The optimal grade a_b^E is the following:

$$a_b^E = \begin{cases} \frac{C(Q_b)}{Q_b} & \text{if } p_b \leq C(Q_b), \\ \hat{a}_b & \text{if } p_b > C(Q_b). \end{cases} \quad (4.6)$$

Similarly, the private monopoly chooses the exam, a_v^E that maximizes profits, U_v^P

$$U_v^P = \int_0^1 \int_{a_v}^1 (p_v - C(Q_v)) da dw, \quad (4.7)$$

subject to $a_v^E \geq \hat{a}_v = \frac{p_v}{Q_v}$.

We observe that private university's utility is strictly decreasing in the ability of the last student admitted at the private university, a_v and thus, this institution establishes the minimum exam, $a_v^E = \hat{a}_v$, which means that the private institution does not use selective exams in the absence of borrowing constraints.

Prices

Assume now that admissions are only determined by prices. This may be the case if exams are explicitly forbidden or too expensive to implement. We consider that the quality the university provides is given. Those students of ability $a_i \geq \hat{a}_b = \frac{p_b}{Q_b}$ are willing to attend the public university. Under perfect capital markets, prices do not limit admissions and the public monopoly chooses p_b to maximize public surplus, given by (4.5), where $a_b = \hat{a}_b = \frac{p_b}{Q_b}$.

The optimal public price, p_b satisfies the following condition:

$$\frac{da_b}{dp_b} (-a_b Q_b + C(Q_b)) = 0. \quad (4.8)$$

Thus, in the absence of borrowing constraints the optimal public price is equal to the per-student cost, $p_b = C(Q_b)$.

From the comparison of (4.6) and (4.8), we observe that both the optimal public price and the optimal exam yield the same allocation of students to schools under perfect

capital markets since $a_b^E = \hat{a}_b = \frac{C(Q_b)}{Q_b}$. The explanation is the following: both instruments allocate students according to their ability when there are no borrowing constraints and thus, they are equally efficient as allocation devices.

The private monopoly chooses p_v to maximize (4.7), where $a_v = \hat{a}_v$. The optimal private price, p_v is determined by the following condition:

$$(1 - a_v) - \frac{da_v}{dp_v} (p_v - C(Q_v)) = 0. \quad (4.9)$$

Notice that, given educational quality, the private institution sets a higher price than the one set by the public university, $p_v > C(Q_v)$. This result can be explained by the different role of prices for both institutions; for the public university, prices are only an instrument to select students while for private university, prices are not only an allocation device but also a source of revenues. Moreover, the tuition fee at the private university must be strictly higher than the per-student cost for private profits to be strictly positive.

Prices and exams

Now we allow each monopoly to use both prices and exams as allocation devices and we also endogeneize the choice of educational quality by institutions. Each monopoly chooses first its optimal level of educational quality. In a second stage, it decides its optimal price and afterwards, the university decides whether to run an exam or accept all applications.

We first solve the problem of the public monopoly by backward induction. In the last stage, both the public quality, Q_b and the price, p_b are already determined. Thus, the public institution chooses exams according to (4.6), given Q_b and p_b . In the second stage and given public quality, Q_b the public institution sets the price, p_b that maximizes (4.5), where $a_b = \max \{a_b^E, \hat{a}_b\}$. The public price is chosen optimally according to (4.8) and thus, $a_b^E = \hat{a}_b = \frac{C(Q_b)}{Q_b}$, which means that the instrument chosen by the public institution to determine admissions is the price or alternatively, exams are not selective under perfect capital markets.

In the first stage, the public institution chooses educational quality, Q_b to maximize (4.5). Optimal public quality satisfies the following condition:

$$\frac{dC(Q_b)}{dQ_b} = \frac{1 + a_b}{2}. \quad (4.10)$$

where $a_b = \hat{a}_b = \frac{C(Q_b)}{Q_b}$.

From (4.10), it follows that the public institution chooses the level of educational quality so as to make the marginal cost of providing quality equal to the mean ability of students attending the public institution. Hence, optimal public quality depends positively on the mean ability of the student body.

We now turn to investigate private monopoly's optimal choices concerning educational quality, prices and exams. In the last stage, given Q_v and p_v , the private institution chooses not to use selective exams, $a_v^E = \hat{a}_v$. In the second stage, the private monopoly sets the price optimally according to (4.9). Using this condition, we may write admission standards at the private university as follows:

$$a_v = \hat{a}_v = \frac{1 + \frac{C(Q_v)}{Q_v}}{2}, \quad (4.11)$$

where $\hat{a}_v = \frac{p_v}{Q_v}$ is the ability of the last student admitted at the private institution.

In the first stage, the private university chooses educational quality Q_v according to this condition:

$$-\frac{dC(Q_v)}{dQ_v} (1 - a_v) + \frac{da_v}{dQ_v} (-p_v + C(Q_v)) = 0. \quad (4.12)$$

We may rewrite (4.12) as follows:

$$\frac{dC(Q_v)}{dQ_v} = \frac{a_v}{1 - a_v} \left(a_v - \frac{C(Q_v)}{Q_v} \right). \quad (4.13)$$

Substituting (4.11) into (4.13), we obtain that optimal educational quality at the private institution is chosen such that the marginal cost of providing education is equal to the ability of the last student admitted at this institution:

$$\frac{dC(Q_v)}{dQ_v} = a_v. \quad (4.14)$$

Finally, it is interesting to compare optimal choices of both monopolies in the absence of credit constraints, in order to investigate how their different objectives affect these choices. Proposition 4.1 summarizes the results obtained in this section:

Proposition 4.1. *Under perfect capital markets, if there is only one institution, either public or private, in the higher education market we obtain the following results:*

- (a) *Each type of monopoly uses only prices to guide their admission policies.*
- (b) *Both institutions provide the same educational quality under monopoly but the price is higher at the private university.*
- (c) *Welfare is higher under the public monopoly than under the private education regime.*

Proof. Result (a) has been already proved and (b) follows directly from the comparison of optimal conditions for the public institution, (4.8) and (4.10) and for the private institution, (4.11) and (4.14). Since educational quality is the same under both monopolies, we obtain that the private university is more selective than the public since $a_v = \frac{1}{2}(1 + a_b) \geq a_b$. Finally, (c) holds because public surplus (the difference between the sum of earnings of students attending the public university and the costs incurred to provide education) coincides with total surplus when the public institution is a monopoly in the higher education market and hence, welfare is maximized under the public monopoly.

■

4.3.2 Borrowing Constraints

We next turn to analyze optimal choices by different types of institutions in the presence of borrowing constraints. We assume that there is only one institution in the market and students cannot borrow at all to finance their education investments. In such economy, prices limit admissions since only those individuals with an income $w^i \geq p_j$ are able to attend university j . In this environment, exams and prices have different properties as allocation mechanisms; while exams allocate students according to their ability, prices

select students not only by their ability (willingness to pay), but also by their income (ability to pay).

Exams

We consider that universities choose whether to run an exam or not, given the price and educational quality. We start with the choice of exams by the public institution. This institution chooses the limiting admission grade, a_b^E that maximizes public surplus, U_b^c , where the superscript c stands for *credit constraints*

$$U_b^c = \int_{p_b}^1 \int_{a_b}^1 (aQ_b - C(Q_b)) da dw, \quad (4.15)$$

subject to $a_b^E \geq \hat{a}_b$.

Given public quality, Q_b and price, p_b the optimal exam at the public university, a_b^E , is the same as the one chosen under perfect capital markets:

$$a_b^E = \begin{cases} \frac{C(Q_b)}{Q_b} & \text{if } p_b \leq C(Q_b), \\ \hat{a}_b & \text{if } p_b > C(Q_b). \end{cases}$$

The private monopoly chooses a_v^E to maximize profits, U_v^c

$$U_v^c = \int_{p_v}^1 \int_{a_v}^1 (p_v - C(Q_v)) da dw, \quad (4.16)$$

subject $a_v^E \geq \hat{a}_v$.

The private institution does not use exams in the presence of borrowing constraints, since given its price, p_v and its quality, Q_v , the profits of the private institution are strictly decreasing in the ability of the last student admitted and thus, $a_v^E = \hat{a}_v$.

Thus, we observe that the optimal choice of exams is the same under borrowing constraints than under perfect capital markets for both types of institutions.

Prices

We assume now that universities use prices as allocation device and educational quality is given. The public monopoly chooses the price, p_b that maximizes (4.15), where $a_b =$

$\hat{a}_b = \frac{p_b}{Q_b}$. The optimal public price p_b satisfies the following condition:

$$-\left(\frac{1-a_b^2}{2}Q_b - C(Q_b)(1-a_b)\right) + \frac{da_b}{dp_b}(1-p_b)(-a_bQ_b + C(Q_b)) = 0. \quad (4.17)$$

The first term is negative since $\frac{(1-a_b)^2}{2}Q_b - C(Q_b)(1-a_b) > 0$ and it represents the effect that an increase in the price has on public university's demand. Assuming that there exists an interior solution, $-a_bQ_b + C(Q_b) > 0$ and thus, $p_b < C(Q_b)$. From the comparison of (4.8) and (4.17), it follows that the optimal price at the public institution is smaller under borrowing constraints than under perfect capital markets.

It is of interest to compare the allocation of students resulting from the use of prices compared to exams. We observe that admission standards are higher under exams than under prices, i.e., $a_b^E > \hat{a}_b$ since $p_b < C(Q_b)$. We have already shown that the optimal choice of exams by the public university is the same under borrowing constraints and perfect capital markets. However, the use of prices reduces the proportion of students who attend the university compared to perfect capital markets, since only those students with income $w_i \geq p_b$ are able to pay the university's fees. Thus, in the presence of borrowing constraints, the public university finds optimal to reduce the price and thus, it attracts students of lower ability.

The private institution chooses p_v to maximize (4.16), where $a_v = \hat{a}_v$. Given private quality, the optimal private price may be expressed as follows:

$$(1-a_v)(1-2p_v + C(Q_v)) - \frac{da_v}{dp_v}(1-p_v)(p_v - C(Q_v)) = 0. \quad (4.18)$$

From the comparison of (4.18) and (4.9), we observe that given educational quality, the optimal private price is lower under borrowing constraints than under perfect capital markets.

We also may write (4.18) in terms of the ability of the last student admitted at the private monopoly under borrowing constraints:

$$\hat{a}_v = \frac{(1-p_v)\left(1 + \frac{C(Q_v)}{Q_v}\right) - (p_v - C(Q_v))}{2(1-p_v) - (p_v - C(Q_v))}. \quad (4.19)$$

The above results show that the reaction of both universities to the existence of borrowing constraints is similar when they use only prices: both institutions find optimal to reduce its price compared to perfect capital markets. This is because the presence of borrowing constraints decreases the proportion of individuals who are able to pay the university's fees. Institutions react accordingly decreasing their prices and the lower price makes the university attractive to students of lower ability.

Prices and Exams

In this section we consider that each university optimally determines the level of educational quality and it also may choose either prices or exams or both instruments to allocate its students. The timing is the same as the one established under perfect capital markets and we solve the problem of each type of institution backwards.

In the last stage, public quality and price are already determined and the public institution chooses the optimal limiting admission grade as in the absence of borrowing constraints, according to (4.6). In the second stage, we find that the price has a negative effect on public university's utility since

$$-\left(\frac{1-a_b^2}{2}Q_b - C(Q_b)(1-a_b)\right) + \frac{da_b}{dp_b}(1-p_b)(-a_bQ_b + C(Q_b)) < 0, \quad (4.20)$$

where $a_b = \max\{a_b^E, \hat{a}_b\}$, $-\left(\frac{1-a_b^2}{2}Q_b - C(Q_b)(1-a_b)\right) < 0$ and $-a_bQ_b + C(Q_b) \leq 0$ from (4.6). Thus, public price is set at its minimum level under borrowing constraints, $p_b = 0$ and selection at the public university is guided only by means of exams, $a_b^E > 0$.

An intuition for this result is the following: under borrowing constraints, exams are more efficient in allocating students than prices. As the objective of the public monopoly is to maximize public surplus, exams are preferred to prices as an instrument to allocate students.

In the first stage, optimal public quality is chosen optimally as without borrowing constraints, such that the marginal cost of providing quality is equal to the mean ability of students attending the public university, $\frac{dC(Q_b)}{dQ_b} = \frac{1+a_b}{2}$, where $a_b = a_b^E = \frac{C(Q_b)}{Q_b}$.

Therefore, public quality and admission standards (the ability of the last student admitted at the public university) are the same under borrowing constraints than under perfect capital markets. However, the public university uses exams instead of prices as allocation device in the presence of borrowing constraints.

Now we solve the problem of the private monopoly. In the last stage, when price and quality are given, we have already shown that the private institution does not use exams, $a_v^E = \hat{a}_v$ and in a second stage, prices are optimally chosen according to (4.18). Finally, in the first stage, optimal private quality is determined by (4.13) as in the case without borrowing constraints. Plugging (4.19) into (4.13), we obtain the following expression:

$$\frac{dC(Q_v)}{dQ_v} = a_v \left(1 - \frac{p_v - C(Q_v)}{1 - p_v} \right). \quad (4.21)$$

We have already shown that, given educational quality, selectivity standards are lower under borrowing constraints than under perfect capital markets at the private university. According to (4.21), lower admission standards decrease the marginal cost of quality and therefore, by convexity of the cost function, education quality is lower under borrowing constraints than under capital markets.

We summarize below the results obtained in this subsection in the following proposition:

Proposition 4.2. *The presence of borrowing constraints affects optimal choices of public and private institutions compared to perfect capital markets as follows:*

- (a) *The public monopoly chooses exams instead of prices as allocation device but admission standards and educational quality at the public institution do not change.*
- (b) *The private monopoly reduces the price and the quality provided.*
- (c) *Welfare is higher under the public monopoly than under the private monopoly.*

Proof. Result (a) has been showed above and (b) holds since the quality provided at the private monopoly is lower under borrowing constraints. Using the implicit function theorem, we obtain from (4.18) that $\frac{dp_v}{dQ_v} > 0$ and from (4.21), we find that $\frac{da_v}{dQ_v} > 0$

and therefore, both the price and the admission standards at the private university are lower under borrowing constraints than under perfect capital markets. Result (c) follows immediately since the public university maximizes total welfare under monopoly. ■

4.4 Competition between a Public and a Private University

In this section we consider that there are two institutions, one public and one private, competing for students in the higher education market. We analyze the following game: in a first stage, universities simultaneously choose quality, then prices and in a third stage, once their demand is determined, they decide whether to run an exam or accept all their applications. We are interested in the Subgame Perfect Nash Equilibrium (henceforth equilibrium) of the game played by both institutions. We solve this game using backward induction. On the one hand, we are interested in analyzing optimal choices of exams, prices and quality by institutions under competition and on the other hand, in evaluating the welfare effects of competition compared to monopoly.

4.4.1 Equilibrium under Perfect Capital markets

Under perfect capital markets, prices only affect students' willingness to attend a university. The shape of demand and hence of the payoffs of the universities differ depending on students' relative preferences for both institutions, which, in turn, depend upon prices and qualities. We first consider the case in which public quality is higher than private quality, $Q_b \geq Q_v$. Hence, the private institution is active in the market only if it is less selective than the public, i.e., $a_b \geq a_v$, since under perfect capital markets, high-ability individuals outspend low-ability individuals to attend the high-quality university.

The objective function of the public university is the following:

$$U_b = \int_0^1 \int_{a_b}^1 (aQ_b - C(Q_b)) da dw, \quad (4.22)$$

where $a_b = \max \{a_b^E, \tilde{a}_b, \hat{a}_b\}$.

Private university's profits are the following:

$$U_v = \int_0^1 \int_{a_v}^{a_b} (p_v - C(Q_v)) da dw, \quad (4.23)$$

where $a_v = \max \{a_v^E, \hat{a}_v\}$.

We consider all possible combinations of prices, qualities and exams (market partitions) such that $Q_b \geq Q_v$ and $a_b \geq a_v$ and both institutions are active in the higher education market:

1. Public price is lower than private price, $p_b < p_v$ and thus, all individuals prefer the public university to the private institution. However, if the public university uses selective exams, such that $a_b^E \geq a_v$, some students who are not admitted in the public institution will attend the private university.
2. Public price is higher or equal than private price, $p_b \geq p_v$. We may distinguish two cases:
 - $\frac{p_b}{p_v} \geq \frac{Q_b}{Q_v}$, which implies that $\hat{a}_v \leq \hat{a}_b \leq \tilde{a}_b$. In this case, the public institution attracts the more talented students but some students of lower ability are willing to attend the private university.
 - $\frac{p_b}{p_v} < \frac{Q_b}{Q_v}$, satisfies $\tilde{a}_b < \hat{a}_b < \hat{a}_v$. As in the situation in which public prices are lower than private prices, all individuals prefer the public to the private university. Hence, only if the public institution uses selective exams such that $a_b^E \geq a_v$ some students may attend the private university.

Market partitions in the case in which the private university provides higher quality than the public institution, $Q_b < Q_v$ may be derived analogously, just permuting the subindexes b and v .

After identifying which are the market partitions with both institutions in the higher education market, we investigate the equilibrium under competition in the presence of perfect capital markets. Notice that the objective function of the public university is the same as under monopoly, when this institution provides a higher quality than the private university. In the presence of perfect capital markets, we have shown that the public monopoly does not use selective exams and it instead uses prices to allocate students. Thus, the only combination of prices and qualities such that both institutions compete in the higher education market is the one in which the ratio of prices is higher or equal than the ratio of qualities, $\frac{p_b}{p_v} \geq \frac{Q_b}{Q_v}$. Hence, we investigate the optimal choices of universities if $Q_b \geq Q_v$ and $\frac{p_b}{p_v} \geq \frac{Q_b}{Q_v}$:

In the last stage of the game, institutions choose simultaneously their optimal exams and prices and qualities are already determined. The public university chooses a_b^E to maximize (4.22) subject to $a_b^E \geq \tilde{a}_b$. The optimal exam at the public university is the following:

$$a_b^E = \begin{cases} \frac{C(Q_b)}{Q_b} & \text{if } p_b \leq \frac{C(Q_b)}{Q_b} (Q_b - Q_v) + p_v, \\ \tilde{a}_b & \text{if } p_b > \frac{C(Q_b)}{Q_b} (Q_b - Q_v) + p_v. \end{cases} \quad (4.24)$$

The private institution chooses the optimal limiting admission grade, a_v^E that maximizes (4.23) subject to $a_v^E \geq \hat{a}_v$. Since private university's utility is strictly decreasing in the ability of the last student admitted at the public university, this institution does not use selective exams, i.e., $a_v^E = \hat{a}_v$.

In the second stage, universities set simultaneously their optimal prices. The price chosen by the public university is the following:

$$p_b = \frac{C(Q_b)}{Q_b} (Q_b - Q_v) + p_v. \quad (4.25)$$

It follows from (4.24) that $a_b^E = \tilde{a}_b = \frac{C(Q_b)}{Q_b}$ and thus, the public university only uses prices as allocation device.

The private university chooses the price that maximizes (4.23), where $a_v = \hat{a}_v$. The

optimal private price is the following:

$$p_v = \begin{cases} \frac{\frac{C(Q_b)}{Q_b}Q_v + C(Q_v)}{2} & \text{if } p_b \leq \frac{C(Q_b)}{Q_b}(Q_b - Q_v) + p_v, \\ \frac{\frac{p_b}{Q_b}Q_v + C(Q_v)}{2} & \text{if } p_b > \frac{C(Q_b)}{Q_b}(Q_b - Q_v) + p_v. \end{cases} \quad (4.26)$$

From (4.26) and (4.25), we may write the ability of the last student admitted at the private institution, \hat{a}_v as follows:

$$\hat{a}_v = \frac{\tilde{a}_b + \frac{C(Q_v)}{Q_v}}{2}. \quad (4.27)$$

From the comparison of (4.11) and (4.27), we find that admission standards at the private institution are lower than under the private monopoly since $\tilde{a}_b < 1$. Given private quality, Q_v , the private institution reduces its price and thus, it attracts students of lower ability compared to monopoly, when it competes with an institution of higher quality.

In the first stage of the game, the public institution chooses Q_b , given the quality of its competitor, as under monopoly, i.e., $\frac{dC(Q_b)}{dQ_b} = \frac{1+\tilde{a}_b}{2}$, where $\tilde{a}_b = \frac{C(Q_b)}{Q_b}$.

The private university chooses private quality, taking as given the quality chosen by the public university, according to the following condition:

$$-\frac{dC(Q_v)}{dQ_v}(\tilde{a}_b - \hat{a}_v) + \frac{d\hat{a}_v}{dQ_v}(-p_v + C(Q_v)) = 0. \quad (4.28)$$

Plugging (4.27) into (4.28), we obtain that optimal private quality may be written as follows:

$$\frac{dC(Q_v)}{dQ_v} = \hat{a}_v. \quad (4.29)$$

We can state the following proposition:

Proposition 4.3. *In equilibrium, the public university provides a higher educational quality and it is more selective than the private university. This equilibrium may be characterized as follows:*

(a) *The public institution behaves as under monopoly: it uses prices as allocation device*

and provides the monopoly level of quality.

(b) The private institution serves the residual demand and provides a lower quality at a lower price than the public institution.

(c) Competition raises total welfare in the economy compared to monopoly.

Proof. We prove the existence of this equilibrium. First, we obtain that public quality is strictly higher than private quality, $Q_b > Q_v$ since both educational qualities are equal under monopoly and under competition the private institution provides lower quality than under monopoly, as it follows from (4.29). Notice that the marginal cost of providing private quality is increasing in \hat{a}_v , and we have already shown that the ability of the last student admitted at the private institution is lower under competition than under the private monopoly which means that private quality is also lower.

Next we turn to prove that the public university is more selective than the private, i.e., $\tilde{a}_b > \hat{a}_v$. According to (4.27), the public university is more selective than the private if $\tilde{a}_b > \frac{C(Q_v)}{Q_v}$, or equivalently, $\frac{C(Q_b)}{Q_b} > \frac{C(Q_v)}{Q_v}$. This condition holds if the marginal cost is higher than the average cost, which implies that the average cost is increasing in educational quality. This property is satisfied since $C(Q)$ is an increasing and convex function of the level of quality Q , as it follows from (4.2).

Results (a) and (b) have been already proved and (c) holds if welfare raises with competition compared to the public monopoly. This is the case if the presence of the private institution increases total income in the economy:

$$\int_{\tilde{a}_b}^{\hat{a}_v} (aQ_v - C(Q_v)) da > 0, \quad (4.30)$$

where $\int_{\tilde{a}_b}^{\hat{a}_v} (aQ_v - C(Q_v)) da$ is the surplus generated by the private university.

Condition (4.30) may be written as follows:

$$\frac{\tilde{a}_b + \hat{a}_v}{2} > \frac{C(Q_v)}{Q_v} \Leftrightarrow \frac{C(Q_b)}{Q_b} > \frac{C(Q_v)}{Q_v}. \quad (4.31)$$

Note that (4.31) holds since $Q_b > Q_v$. ■

We show in the following proposition that the equilibrium described above is the only possible equilibrium with both universities active in the higher education market.

Proposition 4.4. *There does not exist an equilibrium in which the private institution provides a higher quality than the public university.*

Proof. See Appendix A. ■

Intuitively, this result holds because in the first stage the public institution finds optimal, given private quality, to provide the monopoly level of quality. The best response of the private university in the next stage is to set a price so high that no student is willing to attend this university. This result may be explained by the strategy of using prices followed by the private university since this allows the public institution to affect admissions at the private institution just varying the level of public quality.

4.4.2 Equilibrium under Borrowing Constraints

Under borrowing constraints, prices affect not only the willingness to attend a university, given educational quality, but also the capacity of students to pay university's fees. For this reason, the number of market partitions such that both universities may coexist in the higher education market is higher than those existing under perfect capital markets. Thus, we refer the reader to Appendix B, where all the possible market partitions under borrowing constraints are specified.

We can simplify the analysis of competition under borrowing constraints restricting our attention to those cases in which public price is strictly lower than private price, $p_b < p_v$. We observe in all the possible cases specified in Appendix B that public price has a negative effect on public university's utility. Hence, in equilibrium the public university chooses a zero price and uses exams as allocation device.

The number of cases with a private and a public institution in the market are reduced to the following:

1. The case in which public quality is higher or equal than private quality, $Q_b \geq Q_v$ and the public institution is more selective than the private, $a_b \geq a_v$ (corresponding to the Case A.1 in Appendix B). Note that if public quality is higher than private quality, it cannot be the case that the private institution is more selective (since $p_b < p_v$ and $Q_b \geq Q_v$, all students strictly prefer the public to the private school).
2. There exists also the possibility that the private university provides higher quality than the public, $Q_b < Q_v$. In this case, we can distinguish two different situations:
 - The case in which the public institution is more selective than the private, $a_b \geq a_v$, (the public university, in spite of providing lower quality than the private, attracts students of higher ability because of its lower price), which corresponds to the Case B.2 in Appendix B.
 - Another possibility is that the private institution is more selective than the public, $a_b < a_v$ (this situation corresponds to the Case B.3 in Appendix B).

We first consider the case in which public quality is higher or equal than private quality, $Q_b \geq Q_v$ and the public university is more selective than the private, $a_b \geq a_v$:

In the last stage of the game, universities simultaneously choose exams. The public institution chooses a_b^E to maximize (4.15) subject to $a_b^E \geq \hat{a}_b$. Public exams are chosen as under monopoly, according to (4.6). Correspondingly, the private institution chooses a_v^E , subject to $a_v^E \geq \hat{a}_v$, to maximize private utility:

$$U_v = \int_{p_v}^1 \int_{a_v}^{a_b} (p_v - C(Q_v)) da dw. \quad (4.32)$$

The private university does not use exams, $a_v^E = \hat{a}_v$, since (4.32) is strictly decreasing in a_v .

In the second stage of the game, universities simultaneously choose their prices, given the price of the other institution. The public institution chooses $p_b = 0$, since public

utility, given by (4.15), is strictly decreasing in p_b . Thus, the public institution only uses exams and behaves as a monopoly in the higher education market.

Private prices are chosen to maximize (4.32) where $a_b = \hat{a}_v$. Prices at the private institution, p_v are chosen optimally according to this condition:

$$(1 - 2p_v)(a_b^E - \hat{a}_v) + C(Q_v)(a_b^E - \hat{a}_v) + \frac{\partial a_v}{\partial p_v}(1 - p_v)(-p_v + C(Q_v)) = 0, \quad (4.33)$$

We may rewrite (4.33) in terms of the last student admitted at the private university:

$$\hat{a}_v = \frac{(1 - p_v)\left(a_b^E + \frac{C(Q_v)}{Q_v}\right) - a_b^E(p_v - C(Q_v))}{2(1 - p_v) - (p_v - C(Q_v))}. \quad (4.34)$$

From the comparison of (4.34) and (4.19), we observe that given private quality, the ability of the last student admitted at the private university, \hat{a}_v is lower than under the private monopoly with borrowing constraints. Since $\hat{a}_v = \frac{p_v}{Q_v}$, this means that given Q_v , optimal private price is lower than under monopoly.

In a first stage, universities choose simultaneously their optimal quality, given the quality of its competitor. It is easy to see that the public university chooses quality as under monopoly, $\frac{dC(Q_b)}{dQ_b} = \frac{1+a_b^E}{2}$, while the private institution sets its quality optimally according to the following condition:

$$-\frac{dC(Q_v)}{dQ_v}(a_b^E - \hat{a}_v) + \frac{d\hat{a}_v}{dQ_v}(-p_v + C(Q_v)) = 0. \quad (4.35)$$

Equation (4.35) may be rewritten as follows:

$$\frac{dC(Q_v)}{dQ_v} = \hat{a}_v \left(1 - \frac{p_v - C(Q_v)}{1 - p_v}\right). \quad (4.36)$$

Note that the condition for optimal private quality is the same that determines quality in the private monopoly with borrowing constraints. Since \hat{a}_v is lower than under monopoly and marginal costs are increasing in quality and then, by convexity of the cost function, private quality is lower under competition than under the private monopoly.

The following proposition shows that $Q_b \geq Q_v$, $p_b < p_v$ and $a_b \geq a_v$ is an equilibrium:

Proposition 4.5. *In equilibrium under borrowing constraints, the public university provides higher quality and attracts higher-ability students than the private institution. This equilibrium may be described as follows:*

- (a) *The public institution behaves as a monopoly in the market: it uses exams and provides its monopoly level of quality.*
- (b) *The private university uses prices and provides a lower quality than the public institution.*
- (c) *Competition raises total welfare.*

Proof. We prove the existence of this equilibrium: first, we show that $Q_b > Q_v$. It follows from (4.19), (4.34), and (4.36) that, in the presence of borrowing constraints, the private university provides a lower quality than under monopoly when it competes with a public institution of higher quality. Since the public university provides the monopoly level of quality, $Q_b > Q_v$ holds. Secondly, we show that the public institution is more selective than the private, i.e., $a_b^E > \hat{a}_v$:

$$a_b^E > \frac{(1 - p_v) \left(a_b^E + \frac{C(Q_v)}{Q_v} \right) - a_b^E (p_v - C(Q_v))}{2(1 - p_v) - (p_v - C(Q_v))} \Leftrightarrow a_b^E > \frac{C(Q_v)}{Q_v}.$$

This condition holds with strict inequality since $Q_b > Q_v$ and therefore, $a_b^E = \frac{C(Q_b)}{Q_b} > \frac{C(Q_v)}{Q_v}$, provided that $\frac{C(Q)}{Q}$ are strictly increasing in Q .

Results (a) and (b) have been showed above and (c) holds if total income in the economy raises with the presence of the private university with respect to the public monopoly:

$$\int_{p_v}^1 \int_{a_b^E}^{\hat{a}_v} (aQ_v - C(Q_v)) da > 0.$$

This condition holds since $Q_b > Q_v$ and thus, $\frac{C(Q_b)}{Q_b} > \frac{C(Q_v)}{Q_v}$. ■

We can also show, as in the case of perfect capital markets, that this equilibrium is unique.

Proposition 4.6. *Under borrowing constraints, there does not exist an equilibrium in which the private university provides higher quality than the public.*

Proof. See Appendix A. ■

In the first part of the proof, we show that the private institution cannot provide a higher quality than the public, being less selective. In the second part, we show that, as in the case of perfect capital markets, there does not exist an equilibrium with the private institution as the high-quality and selective institution in the market. The intuition for this last result is the same as for the case of perfect capital markets: the public institution chooses the monopoly level of educational quality and this level is high enough to push the private institution out of the market.

4.5 Concluding Remarks

In this chapter we investigate the strategic role of prices and exams for public and private universities in the presence of borrowing constraints. First, we compare the optimal choices of a public and a private monopoly and we find fundamental differences in the behavior of both types of institutions. The private university does never use exams as allocation device and chooses instead prices. On the contrary, the public university selects its students by means of exams and sets a zero price for its educational services under borrowing constraints while it uses prices under perfect capital markets. This result may be explained by the fact that the private university maximizes profits while the public institution maximizes public surplus. In the presence of borrowing constraints, prices have a negative effect on the payoff of the public university since they limit the admission of talented and poor students.

Next, we model competition between a public and a private institution. After characterizing all possible configurations with both institutions in the market under borrowing constraints, we find that there exists a unique equilibrium under competition. In such

an equilibrium, the public university always provides a higher quality than the private institution. This result may be explained by the fact that the public university uses exams while the private institution uses prices to select students in the presence of borrowing constraints. Exams not only allow the public university to behave as a monopoly in the market but also prevents the private institution from providing a higher quality than the public university. We also find that competition between educational institutions raises welfare compared to monopoly.

Our paper fits quite well the features of the European higher education market. The majority of European countries are characterized by the presence of public universities that use primarily exams to allocate their students and set very low prices, while private universities are usually of lower quality. We show that public and private universities' objectives are crucial to understand the characteristics of this market. We leave for future research the analysis of private and non-profit higher education institutions, which use both prices and exams to allocate their students. Interestingly, these institutions share common features with both public and private, profit-maximizing, universities, which may help explain why they use both prices and exams as allocation mechanisms.

4.6 Appendix

Appendix A. Proofs of Propositions 4.4 and 4.6

Proof of Proposition 4.4. We prove that the case in which the high-quality and selective institution is the private university is not an equilibrium under perfect capital markets. For this purpose, we consider that $Q_b < Q_v$ and $a_b < a_v$.

The utility of the public university is the following:

$$U_b = \int_0^1 \int_{a_v}^{a_b} (aQ_b - C(Q_b)) da dw, \quad (4.37)$$

where $a_b = \max \{a_b^E, \hat{a}_b\}$ and private utility may be written as follows:

$$U_v = \int_0^1 \int_{a_v}^1 (aQ_v - C(Q_v)) da dw, \quad (4.38)$$

where $a_v = \max \{a_v^E, \tilde{a}_v, \hat{a}_v\}$.

In the last stage of the game, universities simultaneously choose their optimal exam. The public university chooses the exam, a_b^E that maximizes (4.37) subject to $a_b^E \geq \hat{a}_b$ and exams at the public institution are chosen optimally according to (4.6). The private university chooses $a_v^E = \max \{\tilde{a}_v, \hat{a}_v\}$ and hence, it does not use selective exams since the private pay-off is strictly decreasing in a_v .

In the second stage, universities simultaneously decide their optimal prices. We find that public utility is strictly decreasing in p_b :

$$\frac{da_b}{dp_b} (-a_b Q_b + C(Q_b)) + \frac{da_v}{dp_b} (a_v Q_b - C(Q_b)) < 0, \quad (4.39)$$

since $-a_b Q_b + C(Q_b) \leq 0$ according to (4.6), $\frac{da_b}{dp_b} \geq 0$, $\frac{da_v}{dp_b} \leq 0$ and $a_v Q_b - C(Q_b) > 0$ since $a_v > a_b$. Thus, optimal public price is zero and the public university only uses exams as allocation device.

The private institution chooses p_v as follows:

$$p_v = \begin{cases} \frac{Q_v + \frac{C(Q_v)}{Q_v - Q_b} Q_v}{2} & \text{if } p_b \leq p_v \frac{Q_b}{Q_v}, \\ \frac{Q_v + C(Q_v)}{2} & \text{if } p_b > p_v \frac{Q_b}{Q_v}. \end{cases} \quad (4.40)$$

From (4.40) and (4.39), admission standards at the private university may be expressed as follows:

$$\tilde{a}_v = \frac{1 + \frac{C(Q_v)}{Q_v - Q_b}}{2}, \quad (4.41)$$

where $\frac{d\tilde{a}_v}{dQ_b} > 0$. Thus, an increase in public educational quality, given private quality, increases selectivity at the private university.

In the first stage, the private institution chooses educational quality, given the quality chosen by the public competitor, according to (4.12), where $a_v = \tilde{a}_v$. Optimal private

quality satisfies the following condition:

$$\frac{dC(Q_v)}{dQ_v} = \tilde{a}_v. \quad (4.42)$$

Public university's utility may be written as follows:

$$U_b = \begin{cases} \int_{a_b^E}^{\tilde{a}_v} (aQ_b - C(Q_b)) da & \text{if } Q_b \leq Q_v - C(Q_v), \\ \int_{a_b^E}^1 (aQ_b - C(Q_b)) da & \text{if } Q_b \geq Q_v - C(Q_v), \end{cases} \quad (4.43)$$

where $a_b^E = \frac{C(Q_b)}{Q_b}$ and $\tilde{a}_v = \frac{1 + \frac{C(Q_v)}{Q_v - Q_b}}{2}$.

Note that if $Q_b < Q_v - C(Q_v)$, $\tilde{a}_v < 1$ from (4.41). This means that the private institution is active in the market if public quality is not too high compared to private quality. On the contrary, if public quality is sufficiently high, $Q_b \geq Q_v - C(Q_v)$, then $\tilde{a}_v = 1$ and thus, the public university is a monopoly in the market. We identify this threshold level of public quality as $\bar{Q}_b \equiv Q_v - C(Q_v)$.

Note first that public utility under monopoly is higher than under competition:

$$\int_{a_b^E}^1 (aQ_b - C(Q_b)) da \geq \int_{a_b^E}^{\tilde{a}_v} (aQ_b - C(Q_b)) da, \forall Q_b \in (0, \bar{Q}_b],$$

since $\tilde{a}_v \leq 1$.

We now turn to show that it is optimal for the public university, given private quality, to choose the monopoly level of quality and this level is such that the private institution is no longer active of the market. The argument of the proof is the following: if the minimum level of quality required to push the private institution out of the market (and become a monopoly), \bar{Q}_b is smaller or equal than optimal quality at monopoly, Q_b^m , we obtain that there does not exist an equilibrium with the private institution providing higher quality than the public. Given public quality, it is optimal for the public university to provide the monopoly educational quality, Q_b^m which is sufficiently high to send the private university out of the market.

We show that $Q_b^m \geq \bar{Q}_b$, for all possible levels of private quality, Q_v :

Optimal quality under monopoly, Q_b^m is determined optimally by the following condition:

$$\frac{dC(Q_b)}{dQ_b} = \frac{1 + \frac{C(Q_b)}{Q_b}}{2}, \quad (4.44)$$

where $C(Q_b) = \alpha Q_b^k$, $k > 1$, $\alpha > 0$. Therefore, substituting $C(Q_b)$ and $\frac{dC(Q_b)}{dQ_b}$, we solve (4.44) for Q_b^m :

$$Q_b^m = \left(\frac{1}{\alpha(2k-1)} \right)^{\frac{1}{k-1}}. \quad (4.45)$$

The minimum level of public quality required for the public university to become a monopoly is $\bar{Q}_b = Q_v - C(Q_v)$, and this level is maximum when private quality, Q_v satisfies, $C'(Q_v) = 1$. In this case, \bar{Q}_b is the following:

$$\bar{Q}_b = \left(\frac{1}{\alpha k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right). \quad (4.46)$$

We find that $Q_b^m \geq \bar{Q}_b$ if the following condition holds: $\left(\frac{2k-1}{k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right) \leq 1$. The above inequality holds since $\left(\frac{2k-1}{k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right)$ is strictly increasing in k and $\lim_{k \rightarrow \infty} \left(\frac{2k-1}{k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right) = 1$, and thus, $Q_b^m \geq \bar{Q}_b$. ■

Proof of Proposition 4.6. We prove that in equilibrium under competition and borrowing constraints, the private university does not provide a quality higher than the private university. We start showing that $Q_b < Q_v$, $p_b < p_v$ and $a_b \geq a_v$ (Case B.2. in Appendix B) is not an equilibrium. The pay-offs of the universities are the following:

$$\begin{aligned} U_b &= \int_{p_b}^{p_v} \int_{a_b}^1 (aQ_b - C(Q_b)) da dw, \\ U_v &= \int_{p_v}^1 \int_{a_v}^1 (p_v - C(Q_v)) da dw, \end{aligned} \quad (4.47)$$

where $a_b = \max \{a_b^E, \hat{a}_b\}$ and $a_v = \hat{a}_{pv}$, since $a_b \geq a_v$.

We solve the game backwards: in the last stage of the game, it is easy to see that the public institution is going to choose exams as under monopoly: $a_b^E = \frac{C(Q_b)}{Q_b}$, while

the private institution does not use exams. In the second stage of the game, the public institution finds optimal to set a public price equal to zero, while the private university chooses its price according to (4.18). In the first stage of the game, both universities choose educational quality simultaneously. The private university behaves as a monopoly and private quality is determined according to this condition:

$$\frac{dC(Q_v)}{dQ_v} = \frac{a_v}{1 - a_v} \left(a_v - \frac{C(Q_v)}{Q_v} \right).$$

Optimal quality satisfies $\frac{dC(Q_v)}{dQ_v} > 0$ if $a_v - \frac{C(Q_v)}{Q_v} > 0$. Thus, if $Q_v > Q_b$, by the properties of the cost function, $\frac{C(Q_v)}{Q_v} > \frac{C(Q_b)}{Q_b}$ and hence, $a_v > \frac{C(Q_b)}{Q_b}$ which contradicts $a_b \geq a_v$ and therefore, this equilibrium does not exist.

Now we show that the case in which $Q_b < Q_v$, $p_b < p_v$ and $a_v > a_b$ (Case B.3 in Appendix B) is not an equilibrium:

The payoff function of the public university is the following:

$$U_b = \int_{p_b}^{p_v} \int_{a_v}^1 (aQ_b - C(Q_b)) da dw + \int_{p_b}^1 \int_{a_b}^{a_v} (aQ_b - C(Q_b)) da dw, \quad (4.48)$$

where $a_b = \max\{a_b^E, \hat{a}_b\}$ and $a_v = \max\{a_v^E, \tilde{a}_v, \hat{a}_v\}$.

Private utility is the following:

$$U_v = \int_{p_v}^1 \int_{a_v}^1 (p_v - C(Q_v)) da dw. \quad (4.49)$$

The public institution chooses exams optimally as in monopoly, according to (4.6). The private institution does not use exams since private utility is strictly decreasing in a_v . Optimal public prices are zero since public utility is strictly decreasing in p_b :

$$\begin{aligned} & -\frac{(1 - a_v^2)}{2} Q_b - \frac{a_{pv}^2 - a_b}{2} Q_b + C(Q_b)(1 - a_b) \\ & + \frac{da_b}{dp_b} (1 - p_b) (-a_b Q_b + C(Q_b)) \\ & + \frac{da_v}{dp_b} (1 - p_v) (a_v Q_b - C(Q_b)) < 0, \end{aligned} \quad (4.50)$$

since $-\frac{(1 - a_{pv}^2)}{2} Q_b - \frac{a_{pv}^2 - a_b}{2} Q_b + C(Q_b)(1 - a_b) < 0$, $\frac{da_b}{dp_b} \geq 0$, $-a_b Q_b + C(Q_b) \leq 0$, $\frac{da_v}{dp_b} < 0$ and $(a_v Q_b - C(Q_b)) > 0$, since $a_v > a_b^E = \frac{C(Q_b)}{Q_b}$.

The optimal private price is determined by (4.18), where $a_v = \tilde{a}_v = \frac{p_v}{Q_v - Q_b}$. The ability of the last student admitted at the private university may be written as follows:

$$\hat{a}_v = \frac{(1 - p_v) \left(1 + \frac{C(Q_v)}{Q_v - Q_b}\right) - (p_v - C(Q_v))}{2(1 - p_v) - (p_v - C(Q_v))}.$$

Therefore, $\hat{a}_v \leq 1$ if $Q_b \leq Q_v - C(Q_v)$ as it was the case under perfect capital markets.

Educational quality at the private university is chosen according to the following condition:

$$\frac{dC(Q_v)}{dQ_v} = \tilde{a}_v \left(1 - \frac{p_v - C(Q_v)}{1 - p_v}\right).$$

Public university's utility may be written as follows:

$$U_b = \begin{cases} \int_{p_b}^{p_v} \int_{\tilde{a}_v}^1 (aQ_b - C(Q_b)) da dw & \text{if } Q_b < Q_v - C(Q_v), \\ + \int_{p_b}^1 \int_{a_b^E}^{\tilde{a}_v} (aQ_b - C(Q_b)) da dw & \\ \int_{p_b}^1 \int_{a_b^E}^{\tilde{a}_v} (aQ_b - C(Q_b)) da dw & \text{if } Q_b \geq Q_v - C(Q_v). \end{cases}$$

Following the same argument as in the proof of Proposition 4.4, we can show that the public university is going to choose the monopoly optimal level of educational quality and this quality is high enough to send the other institution out of the market, i.e., $Q_b^m \geq \bar{Q}_b \equiv Q_v - C(Q_v)$. Thus, this equilibrium does not exist. ■

Appendix B. Possible Market Partitions with Borrowing Constraints

In this appendix we consider all possible situations or market partitions with both institutions in the higher education market in the presence of borrowing constraints. First, we consider the case in which public quality is higher or equal than private quality, $Q_b \geq Q_v$ (Case A). In this case, if the public institution sets a lower price than the private university, $p_b < p_v$, then all students prefer to attend the public university, $\tilde{a}_b = \frac{p_b - p_v}{Q_b - Q_v} < 0$. Thus, the only possibility with both institutions in the market is the one in which the public university is more selective than the private by means of using exams, $a_b^E > a_v = \max\{\hat{a}_v, a_v^E\}$. Some of the students rejected from the public university are

willing and can afford to attend the private university. These are individuals of ability $a_i \geq a_v$ and initial endowment $w_i \geq p_v$. This allocation of students is represented in Figure 4.1, where the darker area represents enrolments at the public university.

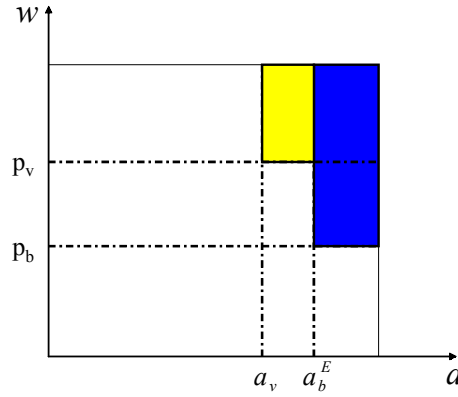


Figure 4.1

The pay-off of the public and the private university are respectively:

$$U_b = \int_{p_b}^1 \int_{a_b}^1 (aQ_b - C(Q_b)) da dw, \quad (4.51)$$

and

$$U_v = \int_{p_v}^1 \int_{a_v}^{a_b} (p_v - C(Q_v)) da dw, \quad (4.52)$$

where $a_b = a_b^E$ and $a_v = \max\{\hat{a}_v, a_v^E\}$.

Conversely, if the price of the public and high-quality university is higher or equal than the price of its private competitor, $p_b \geq p_v$, we must differentiate two cases:

- $\frac{p_b}{p_v} \geq \frac{Q_b}{Q_v}$, which implies that $\hat{a}_b \geq \hat{a}_v$ and also $\tilde{a}_b \geq \hat{a}_b$:

$$\frac{p_b - p_v}{Q_b - Q_v} \geq \frac{p_b}{Q_b} \Leftrightarrow \frac{p_b}{p_v} \geq \frac{Q_b}{Q_v}.$$

Therefore, we obtain the following ordering of ability thresholds, $\hat{a}_v \leq \hat{a}_b \leq \tilde{a}_b$, which corresponds to the following ordering of preferences:

- $a_i \in [0, \hat{a}_v) : 0 \succ v \succ b$
- $a_i \in (\hat{a}_v, \hat{a}_b) : v \succ 0 \succ b$.
- $a_i \in (\hat{a}_b, \tilde{a}_b) : v \succ b \succ 0$.
- $a_i \in (\tilde{a}_b, 1] : b \succ v \succ 0$.

The possible allocations of students to schools or market partitions in this case are represented in Figure 4.2:

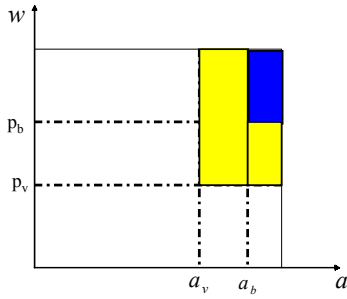


Figure 4.2a

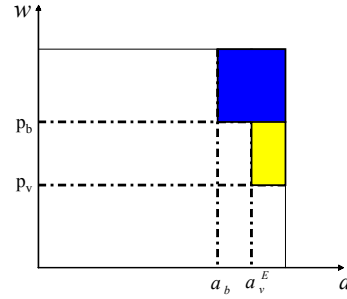


Figure 4.2b

In the Figure 4.2a, $a_b = \max \{ \tilde{a}_b, a_b^E \}$ since $a_b \geq a_v = \max \{ \hat{a}_v, a_v^E \}$, while in Figure 4.2b, $a_b = \max \{ \hat{a}_b, a_b^E \}$ because in this case the private university is more selective using exams, $a_v = a_v^E > a_b$ and students attending the public university are those who prefer to attend the public to remain uneducated.

$$- \frac{p_b}{p_v} < \frac{Q_b}{Q_v} \Leftrightarrow \hat{a}_b < \hat{a}_v \text{ and also } \tilde{a}_b < \hat{a}_b \text{ since}$$

$$\frac{p_b}{Q_b} > \frac{p_b - p_v}{Q_b - Q_v} \Leftrightarrow \frac{p_b}{p_v} < \frac{Q_b}{Q_v}.$$

The ordering of ability thresholds is thus, $\hat{a}_v > \hat{a}_b > \tilde{a}_b$ and the preference ordering is the following:

- $a_i \in [0, \tilde{a}_b) : 0 \succ v \succ b$.
- $a_i \in (\tilde{a}_b, \hat{a}_b) : 0 \succ b \succ v$.
- $a_i \in (\hat{a}_b, \hat{a}_v) : b \succ 0 \succ v$.
- $a_i \in (\hat{a}_v, 1] : b \succ v \succ 0$.

The possible allocation of students are represented in Figure 4.3, where, as before, the darker area represents enrolments at the public university:

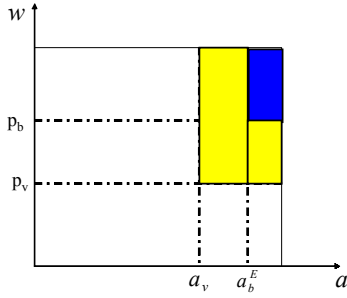


Figure 4.3a

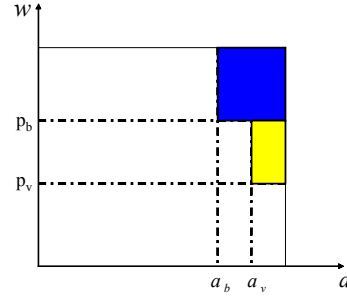


Figure 4.3b

In Figure 4.3a, the public university is required to use exams to be more selective than the private university, $a_b = a_b^E \geq a_v = \max \{\hat{a}_v, a_v^E\}$ and in Figure 4.3b, $a_b = \max \{\hat{a}_b, a_b^E\}$ and $a_v = \max \{\hat{a}_v, a_v^E\}$.

Summarizing, when both public price and quality are higher than price and quality at the private university, universities' enrolments can follow one of these patterns:

$$p_b \geq p_v \begin{cases} \frac{p_b}{p_v} \geq \frac{Q_b}{Q_v} \Rightarrow \hat{a}_v \leq \hat{a}_b \leq \tilde{a}_b \begin{cases} a_b \geq a_v \text{ (i)} \\ a_b < a_v^E \text{ (ii)} \end{cases} \\ \frac{p_b}{p_v} < \frac{Q_b}{Q_v} \Rightarrow \tilde{a}_b < \hat{a}_b < \hat{a}_{pv} \begin{cases} a_b^E \geq a_v \text{ (iii)} \\ a_b < a_v \text{ (iv)} \end{cases} \end{cases}$$

Note that the only difference between cases (ii) and (iv) is that, in (ii) the private institution must use exams in order to be more selective than the public, whereas in (iv)

it is more selective even if it does not use exams due to the configuration of relative prices and qualities. Since the shape of the payoff functions is the same in both cases, we group (ii) and (iv) in Case A.2, which hence studies the possibility that the private institution enrolls students of higher average quality when public quality is larger. The payoffs of the public and private university are respectively

$$U_b = \int_{p_b}^1 \int_{a_b}^1 (aQ_b - C(Q_b)) da dw, \quad (4.53)$$

and

$$U_v = \int_{p_v}^{p_b} \int_{a_v}^1 (p_v - C(Q_v)) da dw. \quad (4.54)$$

Similarly, the only difference between cases (i) and (iii) is that, in (iii) the public institution must necessarily use exams in order to be more selective than the private whereas in (i) it is more selective due to the relative levels of prices and qualities. We hence group cases (i) and (iii) in Case A.3, which analyzes the possibility that the public university is more selective when its prices are higher. The payoffs of the public and the private university are respectively

$$U_b = \int_{p_b}^1 \int_{a_b}^1 (aQ_b - C(Q_b)) da dw, \quad (4.55)$$

and

$$U_v = \int_{p_v}^{p_b} \int_{a_b}^1 (p_v - C(Q_v)) da dw + \int_{p_v}^1 \int_{a_v}^{a_b} (p_v - C(Q_v)) da dw, \quad (4.56)$$

where $a_b = \max \{a_b^E, \hat{a}_b, \tilde{a}_b\}$ and $a_v = \max \{a_v^E, \hat{a}_v\}$.

A similar procedure identifies all possible enrolment patterns when private quality is strictly higher than public quality, $Q_v > Q_b$ (case B). By permuting subindexes v (private) and b (public), we obtain the corresponding cases B.1, B.2 and B.3.

It can be shown that in the second stage of the game, the public university is going to set public prices optimally at zero.⁹ Note that this fact reduces the scope of our analysis

⁹This can be verified rapidly by observing that *public* price always has a negative effect on the payoff of the public university for all possible market partitions specified above, taking into account that exams have been chosen optimally in the following stage.

to the cases in which public prices are lower than private prices, $p_b < p_v$, which are Case A.1, when public quality is higher $Q_b \geq Q_v$, and Case B.2 and Case B.3 if private quality is higher than public, $Q_v > Q_b$.

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