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On Conflict and Power

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Contents

A mi familia, simplemente por estar ahi.
A Begotxu, mi *sine qua non*.

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"This is not an argument that economic analysis *should* be political.
It is an argument that it inevitably *is* political"
Randall Bartlett (1989), *Economics and Power.*

Part I

Paradoxes of Conflict and Power

"Ultimately, good Economics will also have to be good Anthropology and Sociology and Political Science and Psychology."

Jack Hirshleifer (1985), *The Expanding Domain of Economics.*

Economists neglect the analysis of the interplay between conflict and power. Conflict is not considered an economic activity. Yet selfish agents are eager to engage in it. On its side, the exercise of power is linked to that of coercion or imposition and that seems very far from the world of mutual consent where consumers and firms exchange goods and money[1]. Consequently, conflict and power are usually considered alien concepts to the domain of Economics. But, as Skaperdas (2002) argues, it is scientifically misguided to assume that rational agents interact in a well-defined "bubble of sainthood" and to relegate (economic) institutions to the role of contract enforcers.

Agents often devote resources to and engage in coercive and violent activities in order to attain their goals. This has an evident impact on the Economy since conflict activities are typically wasteful from a social point of view (by consuming resources otherwise used in production.) In addition, there is also a subtler channel conflict acts through: Individuals an groups seek for partial or universal agreements as a way to avoid the inefficiencies associated with conflict. But the possibility and stability of such arrangements demands that everybody receives at least what they could obtain by fighting.

Once we recognize the importance of conflict, the concept of power must as well enter in the realm of economic analysis. Although, this is not an easy task; power is a difficult concept to grasp[2]. However, little effort has been devoted to the study of the *sources* of power; research that focuses on how power influences the patterns of conflict fails to account for the effects of conflict in the determination of power relationships.

In this dissertation we propose a thesis on the interplay between conflict and power: If economic power is based on the access to information and resources, conflict is actually a mechanism to establish differences in power since it generates differences on the access to these assets.

In particular, we will be interested on three simple and natural questions:

[1] This idea is at the origin of the old tradition that considers any economic intervention as a distorsion In this line, Milton Friedman argued in "Capitalism and Freedom" (1962) that markets equal freedom since exchange is made in the absence of power or coercion.

[2] In one of the very few attempts to address the issue, Bardhan (1991) devotes a big effort to define power. However, none of the definitions proposed seems to satisfy him.

1.) Does conflict make "the rich get richer and the poor poorer"?

2.) Does conflict promote cooperation?

3.) Under which circumstances is peace attained?

In this introductory part we briefly and broadly present the answers we provide to these questions.

I.

Hirshleifer (1991) introduced what he called the *Paradox of power* (POP), a reformulation of the famous Olson-Tullock's *Group size paradox* of rent-seeking models: Poor or small combatants end up improving their position with respect to richer or larger ones. The poor specialize in conflict and rich in production, so to speak. Hirshleifer claimed that this paradox explains the established pattern of political redistribution of income from rich to poor and in favour of interest groups[3]. At some point, Hirshleifer, somewhat prophetically, goes as far a saying that

> "Nations with wealth-enhancing laws and institutions will not able to enjoy the fruits thereof unless, when challenged, they can put up a tough fight."[4]

It is true that many times the apparent balance of strengths between agents does not match the observed shares of income or other sources of power. However, we claim that explanations based on POP are misguided. First, the model it is based on is non-robust to very slight technical modifications (it has been more carefully modelled, and weakened, by Skaperdas (1992).) But, more importantly, it is also seriously flawed in two other aspects.

The POP rests heavily on the fact that contenders fight over a common pool of income. That is, the winner of the contest receives the output produced by all the agents. Whereas this formulation can be adequate to pre-modern or close to the "state of nature" periods, it seems much less valid in recent history. A simple repeated-interactions argument will be enough to show this: If I know that the fruits of my labor are to be grabbed by someone specialized in predative activities, next time I will probably try to defend myself or fight to obtain full control rights (maybe

[3] It is very curious to compare Hirshleifer's question of why *so much* income is redistributed with John Roemer's (1998) of why the poor do not expropriate the rich, in other words, why *so little* income is redistributed.

[4] Hirshelifer, 1994, p. 4.

by eliminating the appropriators.) In fact, History shows that struggles over objects or income evolved into struggle over rights. In short, what we want to point out, as we do in Chapter 2, is that the absence of property rights may open the door to the exercise of power, but more importantly, that property rights emerge from an exercise of power. Although apparently trivial, the difference between these two types of conflicts is similar to the distance between Pareto and Marx; the difference between the analysis of *the allocative distorsions due to conflict* and the analysis of *conflict as an allocation mechanism* itself.

And this is the second, and most important fallacy of Hirshleifer's argument: The separation between the distribution of resources, that eventually represents power, from the events that lead to such distribution. The process defining property, the ability to force such initial conditions, is also a form of power[5]. In this sense, Hirshleifer simply inherits Demsetz's notion of the emergence of property as the outcome of some sort of group consensus. According to him, property rights simply evolved into a Pareto superior situation. Such candid (yet ideological) view underlies in his following *Gedankenexperiment*: "Let us initially distribute private titles to land randomly among existing individuals [...] These owners will now negotiate among themselves to internalize any remaining externalities."[6] But property rights were seldom randomly distributed! The relevant issue is instead how did these rights actually evolve.

What Hirshleifer, Coase or Demsetz forgot is that trade and many other economic interactions occur in a framework of property rights, under a pattern of initial endowments, that themselves reflect power relationships. Exchange may be efficient and mutually beneficial. But the process that generated the initial conditions where exchange takes place are often the fruit of a (sometimes brutal) exercise of force. And losers are not compensated.

Surprisingly, as we will see in Chapter 2, conflict may also have a welfare enhancing effect when it serves as an allocation mechanism; the exclusion introduced by the restriction of property rights may Pareto dominate common ownership *ex-ante* whenever free access implied overexploitation[7]. But the *ex-post* distribution of income will be extremely unequal since a part of the population gets excluded.

The ethical trade-off inherent of conflicts over property rights gives raise to situations similar to *the fireman paradox*[8] scenario: In that paradox, a house is on fire and a fireman has enough time to open only one room and save the people inside. There are two rooms, one with five people and another with one person alone. Utilitarianism

[5] This is called "Event power" by Bartlett (1989).

[6] Demsetz, 1967, pp. 356-7.

[7] This obsevation introduces a deeper challenge to the standard economic approach to conflict since it would call for a *moral* condemn rather than for a purely efficiency-based one.

[8] That I owe to Joan Esteban.

would prescribe to save the five people since this maximizes social welfare. However, the resulting allocation of life ends up being very unfavorable to the left-over person as long as she is facing death. Similarly, the creation and allocation of property rights through conflict, "unlike exchange, can rarely benefit all participants."[9]

Summarizing, the definition of property rights through exclusion and conflict implies that some had the power to impose their will on others; in order to save some from the externalities due to common ownership we must exert on others the externality of being excluded from property.

But we cannot stop here. Once we have recognized that conflict is an allocation mechanism, a final criticism to the Coase-Demsetz Nirvana arises: If agents know that their final position in the economy is going to depend on the set of established rights, they will be eager to participate in the rights-definition process, based on conflict and the exercise of power, in order to affect the outcome of the subsequent trade. So can we be sure that there is no connection at all between the process of right determination and the process of exchange? Does the market truly eliminate coercive power? The old suspect that these questions have negative answers is summarized by Bartlett who pointed out that

> “Power may be absent from any particular exchange. It cannot be absent from a system of market-exchanges”[10]

In a very recent paper, Rubinstein and Piccione (2003) explore this direction and end up blurring the edges between standard competitive equilibrium and the equilibrium resulting from the "law of the strongest", what they call the "jungle equilibrium"; this allocation stems out from a power relation allowing stronger agents to appropriate everything from those weaker than them. If initial endowments correspond to this jungle equilibrium allocation, there exists a vector of prices that also supports it as a competitive equilibrium. Finally, and contrary to POP, some mild restrictions on preferences and consumption sets allow to set a one-to-one correspondence in equilibrium between strength and income. As a result, the standard arguments used to justify "free markets" get seriously undermined; as the authors point out, if the initial wealth is allocated unfairly or obtained by dishonest means (conflict as above) it is hard to find reasons to prefer markets over the jungle.

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[9] Hirshleifer, 1985, p. 64.
[10] Bartlett, 1989, p. 166.

II.

In Chapters 1 and 2 we deal with the role of groups in conflict. Agents join together in order to fight rather than engaging in "wars of all against all". This is what Coser (1956) called *the unifier* role of conflict. Coser distinguished between *realistic conflicts,* where groups use conflict as a mean to attain their goals, from *non-realisitic conflicts*, where the need for tensions release makes conflict an end itself. Sociological tradition tends thus to consider the formation of groups in these contexts as a way to alleviate social tensions, following Sigmund Freud who stated that

> "It is always possible to bind together a considerably number people in love, so long as there are other people left over to receive the manifestations of their aggressiveness."[11]

Here we will embrace a more economicist (and operative) interpretation of the role of group formation in the shadow of conflict. It is plausible to assume that there are increasing returns to size in conflicts and contests. Individuals would thus initially join in order to improve their position in case of conflict. These brand new groups typically reach compromises over the alternative they will implement in case of winning or about the share of the object they contest over. This process will eventually stops once the costs of sharing the spoils of victory outweight the benefits from size.

However, there is a powerful force pushing in favor of this clustering process, specially if losers get almost nothing. As noted by Tan and Wang (2000), the costs of conflict and of sharing the spoils with increasing numbers decrease as other groups form; if one is going to loose more likely it is less important what one would get in the unlikely event of winning. This effect generates a strong tendency towards bi-partisan conflicts. A tendency that would explain why when more than two agents (individuals or nations) fight, they form alliances instead of engaging in multi-sided conflicts.

But the exercise of considering group formation in conflicts has also an immediate consequence: The grand coalition can be thought off as a state of universal peace or universal agreement. It would be equivalent to a social contract binding all the agents in the society, who commit on not fighting each other, and specifying rights and shares. On the contrary, when fragmented structures form, a system of partial social contracts binding only members arises. Such contexts are characterized by a fierce *non-cooperative interaction among coalitions* and a more or less *cooperative behavior within coalitions*, depending on how tight such partial contracts are.

[11] Freud, 1929 [1986], p. 114.

III.

In the Chapters 3 and 4, we try to give answer to the third question posed above by moving to the analysis of the role of confrontation in negotiations. We assume first that some process of group formation has already determined *who* is going to fight over some control right or object. The strong tendency towards bi-partisan conflicts mentioned above gives foundations to a two-person bargaining framework. Second, we discretize the range of choices between conflict and peace and fix the strength of the parties (to size, for instance).

In the study of the attainment of peaceful agreements, there is an enduring puzzle, known in Economics as *the Hicks' paradox*: In view of mutually beneficial agreements, confrontation and disagreement are the rule rather than the exception. Incomplete information about some critical aspects of the negotiation, the opponent's strength for instance, has been systematically invoked as an explanation. However, this is not enough: Uncertainty is less likely to arise in situations where the parties' strengths are very unequal. But one can observe that clearly small and weak agents often fight much larger and powerful ones[12]. We will call this phenomenon *the Uneven contenders paradox.*

Here, we will challenge the common assumption underlying these puzzles, namely that power is solely reflected by military or material capabilities. We want to argue that information is also a source of power and that the interplay between the material and informational spheres of power can help us to solve the two proposed puzzles *at once.*

There is a very long standing tradition, dating back to Georg Simmel, the master of sociological paradoxes, that recognizes that non-economic power is not as easy to measure as its economic counterpart. The consequence, Simmel said, is that the most effective deterrent of conflict, the perfect revelation of relative strength, is only possible through conflict itself[13]. Given the well documented tendency of agents to make optimistic estimations of their own strength in conflict, Simmel's statement becomes tremendously frightening.

This conclusion is too simplistic since evidence also suggests that conflicts terminate mainly by mutual agreement rather than through the total collapse of one of the sides. All-out conflicts are not the only outcome of disagreement, as implicitly assumed so far. If that would be the case, we should observe little conflict between uneven contenders. Agents choose instead the scope of the conflict they will fight and many

[12] As we will see next, POP as an explanation of this phenomenon is as inadequate as thinking that the Vietcong's purpose was to march over Washington D.C.

[13] As we can see, power is difficult to define and difficult to measure. Maybe that is why some believe that power does not exist!

times engage in limited conflicts that do not imply the breakdown of negotiations. Skirmishes, strikes or family arguments fit this pattern. The role of these confrontations is to test (or reveal) the true relative strength of the rivals and, paradoxical as it may seem, open the door to agreement.

On the one hand this definitively sheds light on the puzzle of uneven contenders: If the strong side is too optimistic and consequently offers almost nothing to its opponent, the weaker side may have incentives to engage in a limited confrontation in order to make clear the true balance of power[14]. But on the other hand, the informational role of conflict may provide an answer to *Hicks' paradox*: If agents know they can affect their opponent's expectations about their comparative strength, they will be eager to fight even when no optimism precludes agreement. This suggest that conflict will be pervasive as long as its returns as an informational device outweight the returns of diplomacy.

Is there any peaceful way out of Simmel's tragic paradox? In Lewis Coser's book *Continuities in the Study of Social Conflict* (1957) there is a very revealing chapter entitled "The dysfunction of military secret" where the author notes that if parties believe that only conflict can transmit information, they will tend to maximize secrecy making confrontation even more likely. Coser advocated for a policy of cooperation where international actors embraced a strategy of revelation and communication in order to make trial-by-fighting a useless instrument. In particular, he proposed the creation of an international list of scientific and military personnel and of a professional body of inspectors and military observers and the improvement of communication channels among nations as ways to avoid war. Unfortunately, as Max Weber pointed out

> "Everywhere that the power interests of the domination structure towards *the outside* are at stake, whether it is an economic competitor of a private enterprise, or a foreign, potentially hostile polity, we find secrecy."[15]

At this point, we would like to present our last paradox, namely that the candidness of Coser's proposals is only surpassed by our desire of seeing them come true.

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[14] Maybe that was the true Vietcong's purpose.
[15] Weber, 1948, p. 233.

IV.

The remainder of this thesis is organized as follows:

Chapter 1 presents a generalized rent-seeking model in partial equilibrium. Our main goal is to analyze the stability of universal peace, the efficient situation. It turns out to be very resilient to possible deviations, specially if individuals behave cooperatively within coalitions. Universal peace is also the outcome of the sequential game of coalition formation introduced by Bloch (1996). Conflict situations can be sustained as stable outcomes only if players hold optimistic (and not necessarily rational) expectations about outsiders reactions to deviation.

In the Chapter 2 we move to a general equilibrium model where groups fight for the right to control a resource. Access to that resource is driven by an exclusion contest that is won by only one coalition. If the resource is exploited cooperatively and conflict technology is relatively better than the production technology, universal peace is not stable. If the resource is exploited non-cooperatively, it becomes a common property resource and in that case, conflict may be socially efficient because it alleviates overexploitation.

In Chapter 3 we explore a two-person bargaining model where one player has incomplete about the opponent strength. We consider the possibility that at each period parties can fight either a total conflict that ends the game or a "battle" that only causes delay but whose outcome conveys information about the true strengths in case of absolute conflict. Then, limited conflict may actually help parties to settle because it precludes optimism in the long run. This feature introduces a novelty with respect to previous bargaining models with incomplete information because here information transmission is hardly manipulable. The main result is that conflict opens the door to agreement if too optimistic expectations precluded it, but delays it when the informed party uses conflict to improve their bargaining positions.

Finally, in Chapter 4 we test empirically the implications of the bargaining model through a duration analysis. We focus on the duration dependence displayed by the hazard rate of real conflicts: If conflict is a learning-persuasion device, this dependence must be positive, that is, the more a conflict lasts the more likely it ends. We perform the analysis with data on colonial and imperial wars from 1816 to 1988: The results obtained give support to the informational hypothesis.

Part II

Conflict and Group Formation

Chapter 1

When does Universal Peace prevail?

(This chapter is jointly written with Francis Bloch and Raphaël Soubeyran.)

"VITO CORLEONE: I swore that I would never break the peace.
MICHAEL: But won't they take that as a sign of weakness?
VITO CORLEONE: It is a sign of weakness..."
The Godfather

1.1 Introduction

Why doesn't universal peace prevail? The world is riddled with conflicts: states fight over territories, firms over markets, individuals over honors and prizes, political parties and interest groups over policies. In each of these situations, agents are willing to waste valuable resources in order to compete while they could enter into an efficient peaceful agreement.

There is of course a distinguished literature in peace and conflict theory (and its natural extension in economics– the rent seeking theory pioneered by Tullock (1967)) whose objective is precisely to understand how conflicts emerge and can be resolved.[1] The focus of the theory of rent seeking has always been on the level of resources spent in contests. For example, in a recent article, Esteban and Ray (1999) analyze how the total amount of resources spent in contests depends on the distribution of a population with heterogeneous characteristics. But while the theory of contests has been extended in a number of directions, it is still almost silent on one important issue: why do agents form groups, or engage in contests when they could agree to a universal agreement?

Our objective in this Chapter is to shed light on this issue, by studying the incentives to secede from a universal agreement and to form groups in a general model of contest. More precisely, we consider the following set of questions. Given that the efficient structure is universal peace, where all agents form a single group to divide rents or choose policy, why do we observe conflict among agents or groups of agents? Which agents have an incentive to secede from the universal agreement? What conjectures should they form on the reaction of other agents to make the secession profitable? Alternatively, if agents are initially isolated, what is the process by which they end up forming a single, efficient group?

To answer these questions, we rely on the recent noncooperative models of coalition formation developed, among others, by Hart and Kurz (1983), Bloch (1996), Yi (1997) and Ray and Vohra (1999) . (See Bloch (1997) for a survey.) These models, which to the best of the knowledge have not yet systematically been applied to the study of conflicts, enable us to obtain sharp, general conclusions on the viability of universal agreements and the formation of groups. Another distinguishing feature of our approach is that we consider a general model of conflict, which admits as special cases the traditional rent-seeking model as well as models of policy conflicts, where interest groups located on a one-dimensional space lobby for the adoption of a policy. Our analysis sheds light on the common structure of conflict models as well as on the specific features of rent-seeking contests and policy conflicts.

[1] For an introduction to conflicts and collective action, see the classical book of Olson (1965) and the book by Sandler (1992)).)

Our analysis starts with a description of a general model of conflict, adapted from Esteban and Ray (1999). In this model, we explicitly allow for the formation of groups and the existence of external effects across groups. This general model encompasses as specific cases pure rent seeking (with a collective or private divisible good), as well as policy conflicts where the choice of a policy by the winning group induces external effects on all the agents. Our first results show that the efficient coalition structure is always the grand coalition, where no resources are wasted on conflict and agents divide rents or choose policy inside a single group. While this result is well known and obvious in the case of rent seeking, it is not immediately obtained in the case of policy conflict, and requires some qualification. We show that, as long as the utility loss is a convex function of the distance between an agent's ideal point and the policy chosen, universal agreement will always be the efficient coalition structure in the model of policy conflict.

Our study then focuses on the incentives to secede from the grand coalition. For the rent seeking contests and policy conflicts, we construct a valuation, expressing the utility of every player in every coalition structure. In the rent seeking model, this valuation can be explicitly computed while in the policy conflict, we can only construct the valuation for a small number of players. However, our analysis shows that, generally, the payoffs obtained by every player in a symmetric coalition structure is lower than the payoff obtained in the grand coalition. This result suggests that if agents want to secede, they only have an incentive to do so if the resulting coalition structure is asymmetric. In fact, we establish that a single player (in the rent seeking contest) or a single extremist (in the policy conflict) always has an incentive to secede when all other players form a single group. Hence, it appears that individual players have no incentive to secede when their secession results in a complete collapse of the universal agreement (a symmetric coalition structure where all groups are singletons), but are always willing to secede when their secession is not followed by any additional change. We formalize this observation, using the terminology introduced by Hart and Kurz (1983). In the γ model (where a secession is followed by the collapse of the group), the grand coalition is an equilibrium, whereas it is not an equilibrium in the δ model (where after a secession, members of a group remain together). In the rent seeking model, we are able to go one step further, and endogenize the reaction of other players to a secession. Considering the sequential model of coalition formation proposed in Bloch (1996) and Ray and Vohra (1999), we show that the grand coalition is indeed the unique equilibrium outcome of the process of coalition formation.

While the previous results were obtained under the assumption that every agent chooses noncooperatively the amount of resources spent in the conflict, we also consider a cooperative model where members of a group coordinate their investments in the contest. Our main finding is that incentives to secede are lower in the cooperative model, as seceding players face a higher level of conflict than in the noncooperative model. In fact, while an individual still has an incentive to secede in the cooperative rent seeking contest when all other agents remain together, in the policy conflict, an extremist no longer has an incentive to secede, once she knows that all other agents will choose their outlays cooperatively in the remaining group.

This chapter draws its inspiration from recent studies by Esteban and Ray (Esteban and Ray (1999), (2001a) and (2001b)). Esteban and Ray (1999) introduce the general model of conflict that we use. Their analysis focuses on the relation between distribution and the level of conflict, and shows that this relation is nonmonotonic and usually quite complex. We encounter the same complexity in our study, but focus our attention to a different problem: the endogenous formation of groups in models of conflicts. By simplifying their model in some dimensions (considering a specific contest technology and assuming that agents are uniformly distributed along the line in the policy conflict), we are able to obtain new results on the incentives to secede and form groups in models of conflicts, thereby progressing on a research agenda which is implicit in their analysis (Section 4.3.2 on group mergers in Esteban and Ray (1999), pp. 396-397.) Esteban and Ray (2001b) study explicitly the effect of changes in group sizes in a model of rent seeking with increasing marginal cost and prizes having both private and collective components. Again, they focus their attention on the global level of conflict, and do not discuss incentives to form groups or secede from the grand coalition.

In the rent-seeking literature, the issue of group and alliance formation has received some attention since the early 80's (See Tullock (1980), Katz, Nitzan and Rosenberg (1991), Nitzan (1991), and the survey by Sandler (1993).) The early literature treated groups and alliances as exogenous, and did not consider incentives to form groups in contests. Baik and Shogren (1995), Baik and Lee (1997) and Baik and Lee (2001) obtain partial results on group formation in rent seeking models with linear costs. They consider a three-stage model, where players form groups, decide on a sharing rule, and then choose noncooperatively the resources they spend on conflict. Baik and Shogren (1995) analyze a situation where a single group faces isolated players, Baik and Lee (1997) consider competition between two groups and Baik and Lee (2001) analyze a general model with an arbitrary number of groups. In all three models, it appears that the group formation model leads to the formation of groups containing approximately one half of the players. Our paper is closest to Baik and Lee (2001) because we consider the formation of arbitrary groups. Our analysis differs from theirs in two important respects: we consider very different models of group formation, where players can choose to exclude other players from the group (they only consider open membership games), and we analyze a variety of models of conflicts, whereas they focus on a pure rent seeking model with linear costs. A recent strand of the literature (Skaperdas (1998) and Tan and Wang (2000)) analyzes the formation of alliances in models with continuing conflict: once an alliance has won a contest, a new contest is played among members of the winning alliance. Tan and Wang (2000) consider a general model with asymmetric players, but suppose that the amount resources spent of conflict is exogenous. Skaperdas (1998) allows for an endogenous choice of fighting expenses, but limits his analysis to three players. The main distinction between these models and ours is that we only consider one conflict: once a group has won the contest, either it obtains the right to decide collectively on the policy, or it shares the prize between its members according to a fixed sharing rule.

Finally, our analysis of policy conflicts bears some resemblance to the study of country formation and secession in local public goods games. (Alesina and Spolaore (1997) and Le Breton and Weber (2000).) As in these models, we analyze incentives to form groups for agents located on a line and whose utility depends on the distance between their location and the location of the local public good (or policy). There are two important differences between local public goods economies and policy conflicts, which make the comparison between the two models difficult to interpret. First, in local public goods economies, it may be efficient to divide the population into different groups (when the cost of providing the public good is low with respect to the utility loss due to distances between the location of the agent and of the public good), whereas in the policy conflict the grand coalition is always efficient. Second, in local public goods economies, as agents do not benefit from public goods offered outside their jurisdiction, there are no externalities across groups, whereas in the policy conflict, an agent's utility depends on the entire coalition structure, as it determines both the location of the policies and the winning probabilities of the different groups.

The remainder of the paper is organized as follows. Section 2 describes the model and preliminary results on the equilibrium of the games of conflict. Section 3 focuses on rent seeking contests, and Section 4 discusses policy conflicts. Section 5 contains our conclusions and discussion of the limitations of the analysis and future research.

1.2 A Model of Conflicts and Contests

We borrow the model of conflicts and contests from Esteban and Ray (1999), and extend it to allow for the formation of groups of agents. This is a general model encompassing as special cases the pure rent seeking contest and conflict among lobbyists over the choice of social policies. We assume that there are $n+1$ players, indexed by $i = 0, 1, 2, ..., n$. The set of all players (with cardinality $n+1$) is denoted N. A coalition C_j is a nonempty subset of N, and a coalition structure $\pi = \{C_1, C_2, .., C_m\}$ is a partition of the set of players into coalitions. Once a group of players C_j is formed, its members spend effort (or invest resources) in order to make the group win the contest. We adopt the simple contest technology initially advocated by Tullock (1967), and axiomatized by Skaperdas (1996). The probability that group C_j wins is given by

$$p_j = \frac{\sum_{i \in C_j} r_i}{R},$$

where r_i denotes the resources spent by agent i, and $R = \sum_{i \in N} r_i$ the total amount of resources spent on conflict by all the agents. Resources are costly to acquire, and each agent faces an identical quadratic cost function,

$$c(r_i) = \frac{1}{2} r_i^2.$$

This specification of the cost function differs from the linear function usually assumed in the rent seeking literature, and is adapted from the general cost functions analyzed by Esteban and Ray (1999).[2] We depart from the usual linear specification because, with heterogeneous agents and groups, the cost function must satisfy $c'(0) = 0$ to guarantee the existence of an interior equilibrium.

Upon winning the contest, the group C_j either obtains a fixed prize (in the case of rent seeking contests) or the right to choose the policy implemented for all agents (in the case of policy conflicts). We denote by u_{ij} the utility obtained by agent i when group C_j wins the contest. With all these notations in mind, the utility of agent i can be written as

$$U_i = \sum_{j=1}^{m} p_j u_{ij} - c(r_i).$$

As in Esteban and Ray (1999), this formulation is general enough to cover the case of pure rent seeking contests (where agents only derive positive utility when their group wins the contest), and conflicts (or contests with externalities), where agents derive different utilities, when losing the contest, according to the identity of the winning group. However, as opposed to Esteban and Ray (1999), we do not suppose that all agents inside a group obtain the same utility level, (u_{ij} may be different from $u_{i'j}$ for two agents i and i' in group C_j), nor that agents systematically favor the group they belong to (u_{ij} may be smaller than $u_{ij'}$ for two disjoint coalitions C_j and $C_{j'}$ where $i \in C_j$). However, we will maintain Esteban and Ray (1999)'s assumption that the

[2] Esteban and Ray (1999) conduct their analysis for cost functions satisfying $c'(0) = 0, c' > 0, c'' \geq 0$ and $c''' \geq 0$. The quadratic cost is a special case of their general family of cost functions.

total utility obtained by group C_j is higher when the group wins than when any other group wins the contest, i.e.

$$\sum_{i\in C_j} u_{ij} > \sum_{i\in C_j} u_{ik} \text{ for all } k \neq j$$

We distinguish between two models of interaction between members of a group. In the *noncooperative* model, every agent chooses her contribution r_i individually. In the *cooperative* model, total contributions are chosen cooperatively (and denoted R_j for the coalition C_j). Hence, in the cooperative model, we can collapse the game into a game played by representatives of each group, where each representative has a utility function given by

$$U_j = \sum_{j=1}^{m} p_j \sum_{i\in C_j} u_{ij} - \sum_{i\in C_j} c(r_i).$$

We start our analysis by deriving, for any coalition structure π, the Nash equilibrium of the game of conflict and contest, where players choose (either noncooperatively or cooperatively) the level of resources they spend on conflict. It is easy to see that the cooperative conflict game is formally identical to the game considered by Esteban and Ray (1999). Hence, we refer to their Propositions 3.2 and 3.3 (Esteban and Ray (1999), p. 386) to state:

Proposition 1 *(Esteban and Ray (1999)). The cooperative game of conflict admits a unique equilibrium* $(R_1^*, R_2^*, ..., R_m^*)$*, characterized by the interior first order conditions:*

$$\frac{\sum_{k\neq j} R_k(\sum_{i\in C_j} u_{ij} - \sum_{i\in C_k} u_{ik})}{R^2} = \frac{R_i}{|C_i|}$$

One can use the same proof as in Esteban and Ray (1999) since our model is a special case of theirs with a quadratic cost of acquiring conflict resources.

Using Proposition 1, we derive the indirect utility function of each agent as

$$v_i = \sum_{j=1}^{m} \frac{R_j^*}{R^*} u_{ij} - \frac{1}{2} \frac{R_j^{*2}}{|C_j|^2}.$$

This indirect utility function assigns to each coalition structure π a vector of payoffs for all the agents. It enables players to evaluate the coalition structures they form, and has been labeled a "valuation" in the literature. (See Hart and Kurz (1983) for an early example and Bloch (1997) for a general discussion.) We denote this valuation by $v_i^C(\pi)$.

We now turn to the noncooperative game of conflict, which was not considered by Esteban and Ray (1999), but retains close similarities with the cooperative game. We obtain the first order condition:

$$\sum_{k\neq j} \frac{R_k}{R^2}(u_{ij} - u_{ik}) - r_i = 0. \tag{1.1}$$

Notice that condition (1.1) does not guarantee that individual contributions to the contest will always be positive. If in fact,

$$\sum_{k\neq j} R_k(u_{ij} - u_{ik}) < 0,$$

the agent will prefer to see her group lose, and will make *negative* contributions to the contest.[3]

Following the same lines as Esteban and Ray (1999), we can prove:

Proposition 2 *The noncooperative game of conflict admits a unique Nash equilibrium* $(r_1^*, r_2^*, ..., r_n^*)$ *characterized by the interior first order conditions:.*

$$\sum_{k\neq j} \frac{R_k}{R^2}(u_{ij} - u_{ik}) - r_i = 0.$$

Again, we define the valuation for each agent in the noncooperative model as the indirect utility function

$$v_i^N(\pi) = \sum_{j=1}^{m} \frac{R_j^*}{R^*} u_{ij} - r_i^*$$

1.3 Rent Seeking Contests

In this Section, we analyze a first model of contest, where agents fight over a fixed prize V. The literature on group rent seeking discusses various alternatives for the sharing of the prize among members of the winning group (see Nitzan (1991) and Baik and Shogren (1995)). Typically, one considers a sharing rule which is a weighted combination of equal sharing and sharing proportional to individual investments in the group. Equal sharing induces group members to free-ride on the contribution of other members, and results in lower investments in the contest ; proportional sharing, on the other hand, induces a "rat race" effect, and results in higher investments in the contest. While the role of various sharing rules and the endogenous determination of the optimal rule have been emphasized in the literature on group rent seeking, we focus in this paper on a different issue, and simply assume that the prize is equally

[3]Negative contributions have to be understood as investments undermining the probability of success of the group. An alternative model could be considered, where players make nonnegative contributions. The analysis and results would not be altered by placing a positivity constraint on investments.

shared among members of the winning group. Hence, the utility of an agent is given by

$$\begin{aligned} u_{ij} &= V/|C_j| \text{ if } i \in C_j, \\ u_{ij} &= 0 \text{ if } i \notin C_j. \end{aligned}$$

In this simple group rent seeking model, it is well known that the efficient coalition structure is the grand coalition. Formally, a coalition structure π is *efficient* (in the cooperative or noncooperative sense) if there exists no coalition structure π' such that $\sum_{i \in N} v_i(\pi') > \sum_{i \in N} v_i(\pi)$, where the valuation v is defined respectively in the cooperative or noncooperative model. We can state:

Lemma 3 *In the rent seeking contest, the efficient coalition structure is the grand coalition both in the noncooperative and cooperative models.*

The proof of this statement is obvious: In the grand coalition, no resources are dissipated and the sum of utilities is equal to the prize. Any model of conflict (cooperative or noncooperative) with at least two groups results in rent dissipation, and yields a smaller total payoff.

Our next result shows that the payoff received by any agent in a symmetric coalition structure is always lower than the payoff received in the grand coalition. Formally, a coalition structure is *symmetric* if and only if $|C_j| = |C_k|$ for all groups C_j and C_k in π.

Lemma 4 *In the rent seeking contest, both in the cooperative and noncooperative models, for any symmetric coalition structure* $\pi, v_i(\pi) < v_i(\{N\})\ \forall i \in N$.

The proof is extremely simple: In any regular coalition structure with m groups of $(n + 1/m)$ players, the expected utility of a player is:

$$\frac{V}{m(n+1/m)} - c(r_i) < \frac{V}{n+1}.$$

Hence, any player gets a smaller payoff in a regular coalition structure than in the grand coalition.

The intuition underlying Lemma 4 is easily grasped. In a symmetric coalition structure, all agents are symmetric, and obtain the same expected gain than in the grand coalition, but must also incur the cost of conflict. While this Lemma is very simple, it will prove helpful in the analysis of secession and group formation.

1.3.1 Valuations in rent seeking contests

We now derive explicitly the valuations in the noncooperative and cooperative models of rent seeking contests. In the noncooperative model, we are able to derive an explicit analytical formula for the valuation. The interior first order condition gives

$$\frac{V}{|C_j|}\frac{\sum_{k\neq j} R_k}{R^2} = r_i \ \forall i \in C_j$$

Summing over all members of group C_j,

$$V\frac{\sum_{k\neq j} R_k}{R^2} = R_j.$$

Notice that this last expression is symmetric for all groups. Hence, in equilibrium, every group will spend the same resources in the conflict, and the winning probability is identical across groups. Straightforward computations then show that the total level of conflict and individual expenses can be computed as:

$$\begin{aligned} R &= \sqrt{V(m-1)} \\ r_i &= \frac{\sqrt{V(m-1)}}{m|C_j|} \end{aligned}$$

The valuation is thus given by

$$v_i^N(\pi) = V\{\frac{1}{m|C_j|} - \frac{1}{2}\frac{m-1}{m^2|C_j|^2}\} \tag{1.2}$$

In the noncooperative model of rent seeking contest, the valuation thus takes a particularly easy form. It only depends on the total number of groups formed (m) and on the size of the group to which player i belongs ($|C_j|$). The valuation is independent of the size distribution of coalitions to which the player does not belong, and of the total number of agents in the society.[4] We use the analytical expression to compute the valuation for small numbers of players. (Tables only report the values for some of the partitions. The values for partitions which can be obtained by permutation of the players are not given here.)

Player/Coalition Structure	0	1	2
012	$V/3$	$V/3$	$V/3$
0|12	$3V/8$	$7V/32$	$7V/32$
0|1|2	$2V/9$	$2V/9$	$2V/9$

[4] This very simple expression is of course only obtained under very specific assumptions on the contest technology, and would not obtain for alternative specifications. It is however illustrative of the qualitative properties of the valuation in rent seeking contests. Notice that a similar simple expression can be found in a very different context – cartel formation in linear Cournot oligopolies studied in Bloch (1996) and Ray and Vohra (1999).

TABLE 1: VALUATIONS FOR THE NONCOOPERATIVE RENT SEEKING CONTEST (3 PLAYERS).

Player/Coalition Structure	0	1	2	3
0123	$V/4$	$V/4$	$V/4$	$V/4$
0|123	$3V/8$	$11V/72$	$11V/72$	$11V/72$
01|23	$7V/32$	$7V/32$	$7V/32$	$7V/32$
01|2|3	$5V/36$	$5V/36$	$2V/9$	$2V/9$
0|1|2|3	$5V/32$	$5V/32$	$5V/32$	$5V/32$

TABLE 2: VALUATIONS FOR THE NONCOOPERATIVE RENT SEEKING CONTEST (4 PLAYERS).

Tables 1 and 2 illustrate some important properties of the valuation. First of all, it appears that the payoff a players receives in the grand coalition is only dominated by the payoff she receives when she is an isolated player, facing a group of size $(n-1)$. Any other coalition structure results in lower payoffs for all the players. Furthermore, it appears that the formation of a group (or the merger between groups) always creates *positive spillovers* to the other players. (As can be seen from the analytical expression for the valuation, a decrease in the total number of groups m induces an increase in the payoff for any player not affected by the merger.) This positive externality is the source of a free-riding problem, which leads any player to prefer to let the other players form groups. This free-riding problem is highlighted by the fact that the only case where a player obtains a higher payoff than in the grand coalition is when it faces a group formed by all the other players.[5]

When players choose cooperatively their contributions, an analytical expression for the pure rent seeking contest cannot be obtained. Instead, we compute below the valuation for small numbers of players

Player/Coalition Structure	0	1	2
012	$V/3$	$V/3$	$V/3$
0|12	$0.29V$	$0.21V$	$0.21V$
0|1|2	$2V/9$	$2V/9$	$2V/9$

TABLE 3: VALUATIONS FOR THE COOPERATIVE RENT SEEKING CONTEST (3 PLAYERS)

[5]A similar free-riding problem appears in the study of cartel formation. The cartel game is also a game with positive spillovers. See Bloch (1997) and Yi (1997) for a general discussion of games with positive spillovers.

Player/Coalition Structure	0	1	2	3
0123	$V/4$	$V/4$	$V/4$	$V/4$
0|123	$V/4$	$0.14V$	$0.14V$	$0.14V$
01|23	$0.19V$	$0.19V$	$0.19V$	$0.19V$
01|2|3	$0.16V$	$0.16V$	$0.18V$	$0.18V$
0|1|2|3	$5V/32$	$5V/32$	$5V/32$	$5V/32$

TABLE 4: VALUATION FOR THE COOPERATIVE RENT SEEKING CONTEST (4 PLAYERS)

Tables 3 and 4 show that the valuation in the cooperative model displays the same qualitative properties as the valuation in the noncooperative model. In the cooperative model, the payoff received in the grand coalition dominates the payoff received in any other coalition structure.(The two payoffs are equal when one agent faces a group of three other agents). One can also check that, for a small number of players, the cooperative model displays positive spillovers. Finally, the payoffs are typically lower in the cooperative model than in the noncooperative model. This observation (which may seem counterintuitive at first glance) is due to the fact that the total level of conflict is higher in the cooperative model, as members of a group coordinate their choices of investments in contest, and do not face free-riding from other group members.

1.3.2 Secession in rent seeking contests

Given that the efficient coalition structure is the grand coalition, we now analyze under which conditions the grand coalition is immune to secession. Our analysis will be centered around *individual* deviations, and we ask: When does an individual agent have an incentive to leave the group and initiate a contest? The previous tables show that the answer to this question depends on the anticipated reaction of the other players to the initial secession. As a first step, we analyze individual incentives to secede, with an exogenous description of the reaction of other agents.

Borrowing from Hart and Kurz (1983), we define two possible reactions of the external players. In the γ model, the grand coalition dissolves, and all the players become singletons. In the δ model, after the secession of a player, all other players remain together in a complementary coalition.[6] We thus say that the grand coalition is $\gamma-$*immune to secession* by player i if $v_i(\{N\}) \geq v_i(\{\{0\},...\{n\}\}$. (As the valuations

[6]In Hart and Kurz (1983)'s original formulation, the γ and δ models were defined in terms of noncooperative games of coalition formation. In the γ model, a coalition is formed if all its members unanimously agree on the coalition ; in the δ model, a coalition is formed by all players who have announced the same coalition. A coalition structure is then γ (respectively δ) immune to secession if and only if it is a Nash equilibrium outcome of the γ (respectively δ) game of coalition formation.

obtained by a player in the grand coalition and in the coalition structure formed of singletons are identical in the cooperative and noncooperative models, we do not need to specify the model we use in the γ case). The grand coalition is $\delta-$*immune to secession* by player i in the noncooperative (respectively cooperative) models if $v_i(\{N\}) \geq v_i^N(\{\{i\}, N\backslash\{i\}\})$ (respectively $v_i(\{N\}) \geq v_i^C(\{\{i\}, N\backslash\{i\}\})$

Proposition 5 *In the rent seeking contest, the grand coalition is $\gamma-$immune to secession for all the players. The grand coalition is not $\delta-$immune to secession in the noncooperative model for $n \geq 2$ and it is not δ-immune to secession in the cooperative model for $n \geq 4$.*

Proposition 5 shows that the profitability of a secession depends on the anticipated reaction of the other players. If the other players react by breaking into singletons, the deviation is not profitable ; if, on the other hand, they react by staying into a single group, an individual deviation becomes profitable. Furthermore, payoffs obtained in a cooperative contest are lower than the payoffs obtained in a noncooperative contest, so that the incentive to secede is *lower* in the cooperative model.

1.3.3 Group formation in rent seeking contests

The analysis of the previous subsection relies on an exogenous specification of the behavior of players following a secession. We now turn to a group formation model where the reaction of players to a secession is endogenized. Bloch (1996) and Ray and Vohra (1999) propose a sequential model of coalition formation, where every player acts optimally, anticipating the behavior of subsequent players. This forward looking game of coalition formation is formalized as follows. At each period t, one player is chosen to make a proposal (a coalition to which it belongs), and all the prospective members of the coalition respond in turn to the proposal. If the proposal is accepted by all, the coalition is formed and another player is designated to make a proposal at $t+1$; if some of the players reject the proposal, the coalition is not formed, and the first player to reject the offer makes a counteroffer at period $t+1$. The identity of the different proposers and the order of response are given by an exogenous rule of order. There is no discounting in the game but all players receive a zero payoff in case of an infinite play. As the game is a sequential game of complete information and infinite horizon, we use as a solution concept *stationary perfect equilibria.*

When players are ex ante identical, it can be shown that the coalition structures generated by stationary perfect equilibria can also be obtained by analyzing the following simple finite game. The first player announces an integer k_1, corresponding to the size of the coalition she wants to see formed, player $k_1 + 1$ announces an integer k_2, etc.;, until the total number n of players is exhausted. An equilibrium of the finite game determines a sequence of integers adding up to n, which completely characterizes the coalition structure as all players are ex ante identical.

The characterization of the subgame perfect equilibrium outcome of the sequential game of group formation requires an explicit analytical expression for the valuation, and hence can only be done in the noncooperative rent seeking contest. We obtain

Proposition 6 *In the rent seeking contest, the grand coalition is the unique equilibrium coalition structure of the sequential game of coalition formation.*

1.4 Policy conflicts

The second model we consider is a model of *policy conflict* inspired by Esteban and Ray (1999). In this model, agents lobby for a policy and each agent receives utility from the policy chosen in the contest. We take the policy space to be the segment $[0, 1]$ and suppose that the $n + 1$ are equally spaced along the line. The location of agent i (which corresponds to the point i/n on the segment) represents her optimal policy. We suppose that agents have Euclidean preferences and suffer a loss from the choice of a policy different from their bliss point. The primitive utility of agent i is thus a decreasing function of the distance between the policy x and her ideal point i/n. More precisely, we describe the primitive utility of agent i as

$$u_i = V - f(|i/n - x|),$$

where V denotes a common payoff for all agents, and f is a strictly increasing and convex function of the distance between agent i and the implemented policy x, with $f(0) = 0$.[7]

We restrict our attention to the formation of *consecutive* groups of agents, i.e. groups which contain all the players in the interval $[i, k]$ whenever they contain the two agents i and k. If a group $C_j = [i, k]$ wins the contest, we suppose that the policy chosen is at the mid-point of the interval $[i, k]$. Whenever the group C_j contains an odd number of players, this point is the policy chosen by the median voter. If the group C_j contains an even number of players, this point can be understood as a random draw between the optimal policies of the two middle voters.[8] Furthermore, it is clear

[7] In some of the computations to follow, we will focus on *linear utilities*, and assume that the function f is the identity.

[8] We are of course aware of the fact that, with an even number of group members, the choice of this policy cannot be rationalized by a voting model. However, we have chosen to make this assumption in order to keep the model simple, and allow us to derive results independently of the fact that the number of agents is a group is odd or even.

that this policy choice is the one which maximizes the sum of payoffs of all the group members.

Hence, letting m_j denote the midpoint of group C_j, the utility of an agent i is given by

$$u_{ij} = V - f(|i/n - m_j|).$$

The policy conflict is thus a contest model with externalities: the payoff of a losing agent depends on the identity of the winning group. An added complexity of the model stems from the fact that agents are ex ante asymmetric. It thus appears that policy conflicts are much more complex to analyze than rent seeking contests. However, in spite of these complexities, we are able to obtain results which parallel the results obtained for the rent seeking model. In particular, we can show:

Proposition 7 *In the policy conflict, the efficient coalition structure is the grand coalition both in the cooperative and noncooperative models..*

Proposition 7 shows that the grand coalition is also the efficient structure in the policy conflict game. A careful reading of the proof shows that this result is independent of the contest technology, and only relies on the convexity of the distance function. Because the distance function is convex, the sum of utility losses incurred by the agents is minimized when the grand coalition is formed, and the policy 1/2 is chosen with certainty.

The next Proposition parallels Lemma 4 and shows that every player obtains a lower payoff in a symmetric coalition structure than in the grand coalition. Given that players are ex ante asymmetric, we define a symmetric coalition structure as a partition which is symmetric around the point 1/2. Formally, a coalition structure π is *symmetric* if, whenever two players i and j belong to the same coalition in π, players $n - i$ and $n - j$ also belong to the same coalition in π.

Proposition 8 *In the policy conflict, both in the cooperative and noncooperative models, for any symmetric coalition structure* $\pi, v_i(\pi) < v_i(\{N\})\ \forall i \in N$.

Proposition 8 again is independent of the contest technology and only relies on the convexity of the distance function. The proof of the Proposition exploits the fact that, in a symmetric coalition structure, the winning probabilities of two coalitions which are symmetric around 1/2 are equal. Hence, for any player i, the expected distance to the chosen policy point is equal to $|i/n - 1/2|$. However, since the distance function is convex, the total utility loss is necessarily at least as large as the loss incurred in the grand coalition where the policy 1/2 is chosen with certainty.

1.4.1 Valuations in policy conflicts

We now turn to a computation of the valuation for the policy conflict. Not surprisingly, we have been unable to obtain an analytical expression for the valuation, and derive below the valuations in the noncooperative policy conflict for 3 and 4 players and linear utilities. Again, we have omitted from the tables those coalition structures which can be obtained by a permutation of the agents.

Player/Coalition Structure	0	1	2
012	$V-0.5$	V	$V-0.5$
0|12	$V-0.49$	$V-0.37$	$V-0.74$
0|1|2	$V-0.59$	$V-0.41$	$V-0.59$

TABLE 5: VALUATION FOR THE NONCOOPERATIVE POLICY CONFLICT, (3 PLAYERS)

Player/Coalition Structure	0	1	2	3
0123	$V-0.5$	$V-0.167$	$V-0.167$	$V-0.5$
0|123	$V-0.47$	$V-0.33$	$V-0.32$	$V-0.65$
01|23	$V-0.55$	$V-0.34$	$V-0.34$	$V-0.55$
01|2|3	$V-0.61$	$V-0.38$	$V-0.36$	$V-0.51$
0|12|3	$V-0.58$	$V-0.41$	$V-0.41$	$V-0.58$
0|1|2|3	$V-0.57$	$V-0.40$	$V-0.40$	$V-0.57$

TABLE 6: VALUATION FOR THE NONCOOPERATIVE POLICY CONFLICT, (4 PLAYERS)

Tables 5 and 6 clearly demonstrate the complexity of the structure of the valuation in the policy conflict. It appears that, as in the case of the rent seeking contest, the only case where an agent obtains a higher payoff than in the grand coalition is when an extremist individual breaks away from the grand coalition while all other players remain together. Furthermore, notice that the spillovers due to the formation of a group are either positive or negative depending on the coalition structure. In the four player case, when players 0 and 1 have formed a group, they obtain a higher payoff when 2 and 3 merge than when 2 and 3 are independent agents. On the other hand, it turns out that player 0 obtains a higher payoff in the coalition structure 0|1|2|3 than in the coalition structure 0|12|3. There does not seem to be any regularity in the direction of externalities induced by mergers between groups of agents!

Player/Coalition Structure	0	1	2
012	$V-0.5$	V	$V-0.5$
0|12	$V-0.55$	$V-0.405$	$V-0.595$
0|1|2	$V-0.59$	$V-0.41$	$V-0.59$

TABLE 7: VALUATION FOR THE COOPERATIVE POLICY CONFLICT, (3 PLAYERS)

Player/Coalition Structure	0	1	2	3
0123	$V - 0.5$	$V - 0.167$	$V - 0.167$	$V - 0.5$
0|123	$V - 0.54$	$V - 0.38$	$V - 0.24$	$V - 0.57$
01|23	$V - 0.56$	$V - 0.39$	$V - 0.39$	$V - 0.56$
01|2|3	$V - 0.56$	$V - 0.40$	$V - 0.39$	$V - 0.58$
0|12|3	$V - 0.57$	$V - 0.40$	$V - 0.40$	$V - 0.57$
0|1|2|3	$V - 0.57$	$V - 0.40$	$V - 0.40$	$V - 0.57$

TABLE 8: VALUATION FOR THE COOPERATIVE POLICY CONFLICT (4 PLAYERS)

Tables 7 and 8 again illustrate the complexity of the valuation in the policy conflict, which does not seem to display any regularity. Notice that, as opposed to the noncooperative case, the grand coalition dominates all coalition structures: an extremist never benefits from breaking away. A comparison between Tables 6 and 8 shows that agents do not necessarily benefit from choosing their resources collectively. This is due to the fact that we do not allow transfers among agents in a group. Hence, even though agents collectively benefit from cooperating in their choices of investment, some agents may end up with a lower utility in the cooperative model.

1.4.2 Secession in policy conflicts

As in the rent seeking contests, we now investigate whether the grand coalition is immune to secession in the policy conflicts.

Proposition 9 *In the policy conflict, the grand coalition is $\gamma-$immune to secession for all the players. The grand coalition is not $\delta-$immune to secession by an extremist player in the noncooperative model with linear utilities for $n \geq 2$. However, the grand coalition is $\delta-$immune to secession by an extremist player in the cooperative model with linear utilities.*

Proposition 8 establishes a close parallel between incentives to secede in rent seeking contests and policy conflicts. An extremist agent has an incentive to secede in the noncooperative policy conflict only when she anticipates that all other agents remain in a single group (the δ model). If she believes that her secession will lead to a dissolution of the group, an extremist agent has no incentive to break away from the grand coalition. Interestingly, in the cooperative policy conflict, an extremist agent does not have an incentive to secede from the grand coalition, even when all other agents remain together. This result is due to the fact that, by cooperating inside a group, all other agents are able to increase the amount of resources spent on the contest, so that the payoff of a seceding extremist is always lower in the cooperative model than in the noncooperative model. Finally, note that we have been unable to characterize the incentives to secede by agents who are not at the extreme points of the segment. While we strongly believe that these agents have less incentive to secede than extremists, we have not been able to prove it formally.

1.5 Conclusion

This paper analyzes secession and group formation in a general model of contest inspired by Esteban and Ray (1999). This model encompasses as special cases rent seeking contests and policy conflicts, where agents lobby over the choice of a policy in a one-dimensional policy space. We show that in both models the grand coalition is the efficient coalition structure and that agents are always better off in the grand coalition than in a symmetric coalition structure. As a consequence, individual agents only have an incentive to secede if their secession results in an asymmetric structure. We show that individual agents (in the rent seeking contest) and extremists (in the policy conflict) only have an incentive to secede when they anticipate that their secession will not be followed by additional secessions. Furthermore, if group members choose cooperatively their investments in conflict, incentives to secede are lower. In the policy conflict, an extremist never has an incentive to secede when she faces a group of agents coordinating the amount they spend in the conflict.

We should stress that our analysis suffers from severe limitations. We have only considered *individual* incentives to secede, and do not consider joint secessions by groups of agents. This focus on individual deviations is motivated by the analysis of valuations with small numbers of players, where it appears that the most favorable cases for secessions are secessions by individual players (in the rent seeking contest) or individual extremists (in the policy conflict). However, a complete analysis of group secessions is still needed to analyze the stability of the grand coalition. We have also limited our analysis by forbidding transfers across group members. Allowing for transfers in a model with individual secessions can only bias the analysis in favor of the grand coalition, as the grand coalition could implement a transfer scheme to prevent deviations by individuals. In a model with group secession, the effect of transfers is less transparent, as transfers would simultaneously increase the set of feasible utility allocations in the grand coalition and in deviating groups. This is an issue that we plan to tackle in future research.

Finally, the main findings of our analysis leave us somewhat dissatisfied. We have found that the grand coalition is surprisingly resilient. In the rent seeking contest, it is the only outcome of a natural procedure of group formation. In the policy conflict, the grand coalition is immune to secession when group members coordinate their choice of investments. This suggests that the level of conflict, and the formation of groups and alliances that we observe in reality cannot be justified purely on strategic grounds. In order to explain conflict, we probably need to resort to other elements – group identity, ethnic belonging– which are not easily incorporated in an economic model.

Chapter 2

Rivalry, Exclusion and Coalitions

"The emergence of new property rights takes place in response to the desires of the interacting persons for adjustment to new benefit-cost possibilities."

Harold Demsetz (1967), *Towards a Theory of Property Rights.*

2.1 Introduction

Individuals often face situations in which to interact with many other agents provokes a decline in individual payments (the exploitation of natural resources or markets for instance.) In these cases, agents perceive the presence of others as potentially dangerous or harmful; They are *rivals in nature*. But what if they can anticipate this "tragic" result? Will not they be tempted to invest effort in non-economic means in order to avoid such ending? In these settings, diverting part of the productive endowments into *appropriative* activities aiming to reduce the number of rivals or competitors arises as a natural option.

Historically, fights for the control over or access to resources have been one of the main roots of conflict among individuals and states: During the English Enclosure on the 18th century, land, traditionally of common property, was privatized through the political initiative of the upper classes[1]; in 1998, the Project on Environmental Scarcities, State Capacity and Civil Violence of the University of Toronto concluded that resource scarcity has triggered predative behavior by elite groups in Indonesia, China and India (among other countries.) These groups aim to change property rights in order to obtain monopolistic access to the resources. The immediate consequence is the defensive reaction of excluded groups[2].

Rivalry leads to competition, but competition may lead to cooperation: Even if individuals are rivals in nature, it is not obvious that they will remain in the state of "the war of all against all". Sooner or later they realize that by joining with others individuals and agreeing on a peaceful arrangement, groups may face external hostilities in a much better position. A natural question is: Does this clustering process eventually lead to universal agreement or to social fragmentation?

This paper investigates the formation of groups or coalitions when individuals may engage in activities aiming to exclude others from a resource of common ownership. With that purpose we explore a general equilibrium model where, once the population is partitioned into a coalition structure, members of coalitions allocate their endowments into conflict effort (*effort* henceforth) and productive activities (*labor* henceforth). Agents are identical and budget constrained by their initial endowments. For each coalition, a *conflict technology* maps the profile of total coalitional efforts

[1] "Where enclosure involved significant redistribution of wealth it led to widespread rioting and even open rebellion" (North and Thomas, 1973).

[2] This was the case of the events in the Senegal River valley in 1989: Anticipating the construction of a dam that increased land values, the Moor elite in Mauritania rewrote legislation governing land ownership, effectively abrogating the rights the black Africans to continue their economic activities on that lands. After the subsequent explosion of violence in response, the black Mauritanians were stripped of their citizenship, expelled from the country and their properties seized. For more examples of recent conflicts over water or land supply see Homer-Dixon (1994).

to the probability of winning the *exclusion contest* that follows the group formation stage. According to these probabilities, Nature selects one coalition as the winner of the contest. Therefore, *agents use conflict to create effective property rights*[3]. Once control is granted, members of the winning coalition exploit the resource with the supplied labor. The sharing rule employed is a convex combination between equal sharing and proportional to labor contributions. This specification not only satisfies some desirable properties, but also encompasses as specific cases *joint production*, where agents can sign binding agreements over labor contributions, and *individual production*, where members of the winning coalition exploit the resource non-cooperatively and the 'tragedy of the commons' arises.

In this model the coalition formation process presents two particular features. First, coalitions face a trade-off when they decide to incorporate a new member: Given that output is shared among the members of the winning coalition, the more players join in a coalition the more likely is that it obtains control but the more diluted the control rights are. Second, group formation induces externalities in non-members: When two individuals merge they agree not to fight each other. Consequently, their exclusion effort changes. This affects the winning probabilities across coalitions, and thus payoffs.

We are interested on what coalition structures arise in this game and their impact on efficiency.

We first analyze the basic properties of the non-cooperative game played after coalitions have formed: For any partition of the set of players and any sharing rule considered, there exists a unique interior Nash Equilibrium. This result is very important because it allows us to associate a unique vector of individual payoffs to each possible coalition structure. We then explore comparative statics and show that low-elasticity production technologies and more egalitarian sharing rules lead to higher total levels of conflict.

Next, we focus on the coalition formation stage for the two polar cases described above. There is not a unique approach to this issue. We consider the following procedures. First, we simplify the existence of externalities by assuming a fixed pattern of behavior on outsiders. This approach defines game in characteristic form where the coalitional worth is independent of outsiders' movements. In the second approach we assume that coalitions play only best responses and form sequentially following the game proposed by Bloch (1996). In the game ensuing payoffs depend on the entire coalition structure. Unfortunately, general closed forms are not possible and results

[3]Following Grossman (2001), we say that an agent has an effective property right over an object means that this agent controls its allocation and distribution.

are thus limited. However, the qualitative features can be displayed by examples.

Next we analyze the joint production case. We show that sufficiently effective conflict technologies make coarser coalition structures induce higher levels of overall conflict. Under the assumption that players are committed to inflict as much harm as possible to possible deviators the grand coalition is stable. When only best responses are employed, a conflict among coalitions of the same size cannot improve upon universal agreement. The game shows a strong tendency towards (at most) bi-partisan conflicts, the incentives to open conflict depending upon the relation between the returns to scale of conflict and rivalry/congestion: Under constant returns, coalitions do not find profitable to open conflict if they believe that the complementary coalition will break up into singletons; but it may be profitable for small coalitions to break the grand coalition if they expect that their deviation will not be followed by any other. Increasing returns to effort reverse these results. In any case the size of the population turns out to be critical in the generation of deviations.

We continue addressing the case of individual production in which, once a coalition obtains control over the resource, its members exploit it non-cooperatively. In this case, the production stage takes the form of the 'tragedy of the commons', one of the most clear examples of economic rivalry. Recall that this phenomenon occurs when, due to negative externalities, a resource of common use becomes overexploited. For instance, when herdsmen put the individually optimal amount of cattle in the pasture they do not take into account that this decreases the available pasture for other herdsmen's cattle. Inefficiency increases as the number of individuals who exploit the resource grows.

Normative approaches to the problem of the commons are perhaps too naive if appropriative activities are available. In fact, we show that conflict may be *socially efficient.* It acts as a discipline device that deters players from devoting too much labor in the exploitation of the resource: Under these circumstances, the formation of the grand coalition is much more difficult than under joint production because coalitions may prefer to expel members and put up with higher hostilities in order to avoid overexploitation of the resource. Moreover, contrary to the joint production case, it is very likely that a conflict among coalitions of the same size Pareto dominates free access.

The paper is structured as follows: In the subsection below the present work is related with the literature. In Section 2 we give some basic notation and assumptions. In Section 3 we show the uniqueness of the Nash equilibrium for any coalition structure and do some comparative statics. Section 4 and 5 address coalition formation for the polar cases of joint and individual production respectively. In Section 6 we comment the related literature and in Section 7 we conclude and discuss open questions for further research.

2.2 The model

Consider a set $N = \{1, 2, ..., n\}$ of identical players. Each of them owns one unit of endowment that can be transformed into *effort* in the exclusion contest (effort henceforth) or in *labor*. We denote these investments by r_i and l_i respectively, subject to the constraint $r_i + l_i \leq 1$.

Players may form coalitions. A *coalition structure* π is a partition of N in a collection of disjoint coalitions $\{S_k\}_{k \in K}$. Let us denote by s_k the cardinality of S_k. We say that a coalition structure is *symmetric* when all coalitions in it are of the same size. Finally, the structure π is said to be a *coarsening* of π' if π can be obtained from π' by merging coalitions in π'.

Once a coalition structure π has formed an *exclusion contest* takes place: Denote by $\mathbf{r}(\pi) = (r^{S_1}, r^{S_2}, ..., r^{S_K})$, where $r^{S_k} = \sum_{i \in S_k} r_i$, the vector of coalitional efforts (we will denote individuals by subscripts and coalitions by superscripts). The result of the contest among coalitions is driven by the *conflict technology* that maps $\mathbf{r}(\pi)$ to a vector $\mathbf{p} = \{p^{S_k}\}_{k \in K}$ of coalitional winning probabilities (with probability p^{S_k} the coalition S_k attains the control of the resource and so on). We adopt a simple functional form where a generic element of π denoted, with some abuse of notation, by S accesses to the resource with probability

$$p^S(\mathbf{r}) = \frac{(r^S)^m}{(r^S)^m + r^{-S}}, \tag{2.1}$$

where $(r^S)^m$ is the *coalitional outlay*, $r^{-S} = \sum\limits_{S_k \in \pi \setminus \{S\}} (r^{S_k})^m$ is the sum of all coalitional outlays outside S, and m represents the returns to scale or *effectivity* of conflict effort. It is assumed that $m \geq 1$. Notice that S cares only about the supply of effort r^{-S} and not about the exact composition of π. However, the particular π we are considering makes a difference: The *total* of coalitional efforts may be the same for two different coalition structures but, for $m > 1$, they lead to different levels of *total* coalitional outlays; the limit case when $m = \infty$ the exclusion contest is formally equivalent to a first-price auction where the coalition with the highest coalitional effort wins the contest with probability 1.

Exploitation of the resource is carried through the production function $f(L)$, where $L = \sum_{i \in S} l_i$, satisfying $f(0) = 0$. This technology is continuous and concave in labor and satisfies that $f'(0) > n\omega$, where ω is the unit cost of labor, in order to ensure the existence of an interior solution to the production problem of all coalitions.

The elasticity of production with respect to labor

$$\varepsilon = \frac{f'(L)L}{f(L)},$$

is a useful proxy for scarcity; by concavity $\varepsilon \leq 1$. We establish a partial ordering: technology f is said to *dominate* technology g if and only if $\varepsilon_f > \varepsilon_g$ for any L.

Each member of the winning coalition receives a share

$$\alpha_i = \frac{\lambda}{s} + (1-\lambda)\frac{l_i}{L}, \tag{2.2}$$

of the output generated. The individual payoff in the production stage is $\alpha_i f(L) - \omega l_i$.

This proposed family of sharing rules presents some advantages: The parameter λ can be related with the *enforceability* of the contracts over labor that members of a coalition can sign. If $\lambda = 1$ we are in a case of joint or cooperative production in which players are pre-committed to share the final output equally. However, if $\lambda = 0$ production is totally individual or non-cooperative; sharing is proportional to labor contributions and members of S play the 'tragedy of the commons'[4]. Second, it is the only family that satisfies the axioms of Additivity and Non Advantageous Reallocation (NAR)[5]; the latter ensures that no sub-coalition in S can benefit from redistributing labor contributions among its members; The total dividend for any subgroup depends only upon its contribution and the total labor contribution[6].

2.3 The Exclusion game

In this Section we explore the game agents play once a particular coalition structure π has formed and analyze its properties.

The individual payoff for an individual $i \in S$ is

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}}[\alpha_i f(L) - \omega l_i]. \tag{2.3}$$

Players in N are identical: All of them are equally efficient when transforming their endowments in effort or labor and the constant marginal cost of labor ω is also the same for all players.

[4] This formulation of the tragedy of the commons can be found, among others, in Cornes and Sandler (1983). With it we try to focus only on the trade-off between exclusionary and productive activities rather than between labor and leisure.

[5] See Moulin (1987).

[6] This is equivalent to a decentralization property because in order to compute his payoff an agent does not need to know who contributed by how much. Other rules like $\alpha_i = \frac{l_i^\lambda}{\sum_{i \in N} l_i^\lambda}$ does not satisfy this property.

It is easy to see that at any optimal decision, individuals will employ their entire endowments in both activities. Consequently, $L = s - r^S$ and one can rewrite (2.3) as

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}}[\alpha_i f(s - r^S) - \omega(1 - r_i)]. \tag{2.4}$$

where α_i is now equal to $\frac{\lambda}{s} + (1-\lambda)\frac{1-r_i}{s-r^S}$. Then, the strategy space of all individuals is $R_i = [0, 1]$. They make their choice of r_i simultaneously and non-cooperatively.

Definition 10 *The **Exclusion game** $\Gamma = (N, \{X_i, u_i^S\}_{i\in S\in\pi}, f, \omega, \lambda, m)$ induced by the coalition structure π is defined by the payoff function in (2.4)*

The profile of individual choices yields both the vector $\mathbf{r}(\pi)$ of coalitional effort and individual payoff in the production stage. If π is the grand coalition, all players accede to the resource without contest.

Let us now define the best reply of an agent: Denote by $\mathbf{r}(\pi)\backslash r_i$ the strategy profile under the unilateral deviation of player i from the strategy profile $\mathbf{r}(\pi)$.

Definition 11 (Individual Best reply) *Given a coalition structure π, the set of individual best replies, denoted by $B_i^S(r_{-i})$, of agent $i \in S$ to the strategy profile $r_{-i} = \{r_j\}_{i\neq j}$, chosen by the rest of members of S (if any) and the outsiders is*

$$B_i^S(r_{-i}) = \left\{r_i \in [0,1] \;/\; u_i^S(\mathbf{r}(\pi)) \geq u_i^S(\mathbf{r}(\pi)\backslash r_i)\right\}.$$

In the Nash Equilibrium of Γ all players are playing their best response $r_i(r_{-i})$ to the strategy profile r_{-i}. More formally:

Definition 12 (Nash Equilibrium of the Exclusion game) *A profile of effort choices $(r_1, ..., r_n)$ is a Nash Equilibrium of the Exclusion game Γ induced by π if and only if $u_i^S(\mathbf{r}(\pi)) \geq u_i^S(\mathbf{r}(\pi)\backslash r_i)$ $\forall i \in N$.*

Proposition 13 *The Exclusion game Γ induced by any coalition structure π has a unique interior Nash Equilibrium. Moreover, it is symmetric, i.e. $r_i = r_j$ $\forall i, j \in S$, $\forall S \in \pi$, and it is given by the following system of equations*

$$\frac{m}{sr^S}\frac{r^{-S}}{(r^S)^m + r^{-S}}[f(s - r^S) - \omega(s - r^S)] = \tag{2.5}$$
$$(1-\lambda)\frac{s-1}{s}\frac{f(s - r^S)}{s - r^S} + \frac{1}{s}f'(s - r^S) - \omega \qquad \forall S \in \pi.$$

When we introduce the exclusion contest, the supply of effort becomes a "rat race": Every unit spent by outsiders in excluding me reduces my opportunity cost of investing one more unit in excluding others, i.e. best response functions are increasing in r^{-S}. On the other hand, the efforts of the members of a given coalition are strategic substitutes. The reason is that α_i is non increasing in r_i whereas the winning probability is a public good. So there are always incentives to free ride on other members' effort[7].

Our target now is to investigate the effect of different productive and conflict technologies, parametrized by ε and m respectively and the particular sharing rule employed (parametrized by λ) on the agents' optimal and equilibrium choices.

Proposition 14 *In the Exclusion game Γ the equilibrium level of total effort $\sum_{s_k \in \pi}(r^{S_k})^m$*

(i) is higher under g than under f provided that f dominates g;

(ii) is increasing in λ;

(ii) is increasing in m if

$$\sum_{k \in \pi \setminus S} (r^{S_k})^m (\ln \frac{r^{S_k}}{r^S}) \geq 0.$$

Conflict is linked to scarcity: In a world of constant returns to labor conflict makes less sense. As the opportunity cost of labor increases exclusion effort may be advantageous. In the same fashion, more egalitarian groups behave more aggressively because they can overcome free-riding in effort contributions. If the coalitions in π would differ in λ, these groups would have an advantage in the exclusion contest.

Unfortunately, the last part of the Proposition allow us to extract partial conclusions only: We can ensure that symmetric coalition structures induce higher conflict expenditures in equilibrium when the conflict technology improves.

For the rest of the paper we will consider two families of production functions:

1) Linear quadratic: with $f(L) = aL - bL^2$ that can be parametrized through $\theta = \frac{a}{b}$ as measure of linearity

2) Exponential: with $f(L) = L^\alpha$ where $\alpha \leq 1$ that satisfies constant elasticity of labor, i.e. $\varepsilon = \alpha$.

Example 1: Let us illustrate Proposition 5 with the following example. Take $N = \{a, b, c, d, e\}$. Initially, let $f(L) = \sqrt{L}$, $m = 1$, $\omega = 0.1$ and $\lambda = 0$. The coalition structure we assume that has previously form is $\pi = \{\{a, b, c\}, \{d, e\}\}$.

[7] Given the public nature of the winning probability we will say that agents free-ride on effort.

In Figure 1 we plot the effect of different sharing rules and conflict technologies. In the vertical axis we project the best response coalitional effort of $\{a, b, c\}$ and of $\{d, e\}$ in the horizontal one.

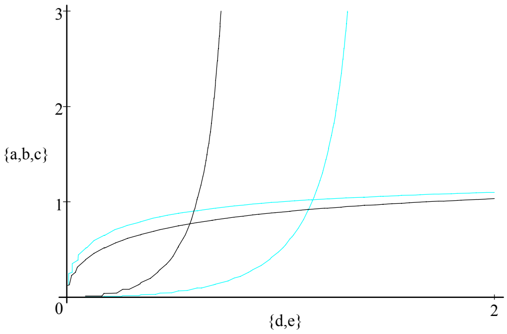


Fig. 1a: The effect of λ.

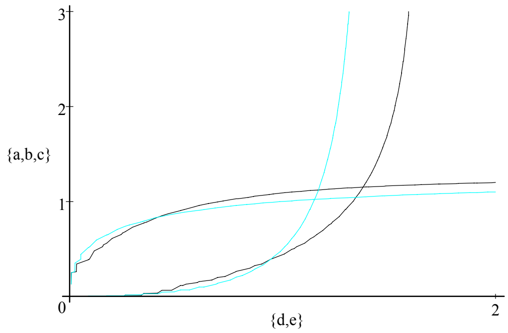


Fig 1b: The effect of m.

The lighter lines represent the baseline case. In panel 1a, the darker line depicts the case $\lambda = 0.5$. As stated above, players put less effort as λ decreases because free-riding. In panel 1b, the dark lines correspond to the case when $m = 2$. As shown in the last Proposition the effect of this change is ambiguous: For low values of outsiders' effort, best replies below the ones for the baseline. However, the equilibrium takes place at higher investments of effort.

2.4 Coalition formation under Joint production

When $\lambda = 1$, agents in the winning coalition share the output equally. This is equivalent to say that they can sign binding agreements on labor contributions. As a consequence the resource is never overexploited, that is the total labor supply always satisfies $f'(L) > \omega$ (this can be seen in the RHS of expression (2.5).) Hence, for simplicity and throughout this Section we will assume that $\omega = 0$: An Exclusion game with a production function $h(L) = f(L) + \omega L$ would yield the same equilibrium[8].

Then, the payoff function can be rewritten as

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}} \frac{1}{s} f(L). \tag{2.6}$$

Let us now state some basic properties of the coalition formation game in this case. First. we identify the sign of the spillovers that coalition formation generates: Games with positive externalities are those where mergers of coalitions produce positive effects

[8] The only relevant effect of doing this is that the Exclusion game has multiple payoff equivalent equilibria. A game where players are coalitions instead of individuals can be defined. It yields a unique Nash equilibrium profile $\mathbf{r}^*(\pi)$. Given that there is no personal cost of contributing labor, any sharing of the equilibrium coalitional effort among its members is also an equilibrium at individual level.

on non-members; with negative externalities, the effect is the opposite. It turns out that in our case this depends on the effectivity of conflict effort

Proposition 15 *The coalition formation game under joint production (i) is of negative externalities if $m \geq 2$ (ii) and of positive externalities under linear quadratic and exponential technologies if $m = 1$.*

Proposition 15 and the fact that coalitional effort is increasing in r^{-S} allow us to state the following Corollary

Corollary 16 *When $m \geq 2$ coarser coalition structures induce higher levels of total effort.*

Another basic property of the coalition formation game is the following

Proposition 17 *Under joint production and productive linear quadratic and exponential technologies, in any symmetric coalition structure π, $u_i^S < u_i^N$.*

It immediately implies that the 'war of all against all' should never be the outcome of a coalition formation process; universal agreement must be broken only in favor of an asymmetric coalition structure.

2.4.1 Characteristic function approach

The main difficulty of the non-orthogonal games of coalition formation, in contrast with standard characteristic form games, is that outsiders' actions affect coalitional payoffs. Static approaches simplify this issue by assuming a specific pattern of behavior for the rest of players that pins down only one coalitional payoff. In this context there are two alternatives: One can make assumptions over the particular *strategies* that outsiders will choose and/or about the *coalition structure* they will form.

The α and β characteristic functions

The α and β concepts, introduced by Aumann (1959), assume that players are committed to punish deviations as much as they can. It implies as well the formation of a particular coalition structure in order to achieve that goal. Notice that this behavior will not be rational most of the times.

For our purposes, it will be important to define an indirect payoff function. Denote by $r^S(r^{-S})$ the maximizer of expression (2.6).

Definition 18 *The **indirect payoff function** is*

$$u^*(r^{-S}) = u^S(r^S(r^{-S}), r^{-S}) = \underset{r^S}{Max}\ u^S(r^S, r^{-S}) \tag{2.7}$$

Now we can define the α and β characteristic functions:

Definition 19 *The α-characteristic function, v_α, in the Exclusion game is defined by:*

$$v_\alpha(S) = \underset{r^S}{Max}\ \underset{r^{-S}}{Min}\ u^S(r^S, r^{-S}) = \underset{r^S}{Max}\ u^S(r^S, \widehat{r}^{-S}) = u^S(r^S(\widehat{r}^{-S}), \widehat{r}^{-S}) = u^*(\widehat{r}^{-S}),$$

where $\widehat{r}^{-S}$ is the minimizer of the coalitional payoff.

This expression corresponds to the indirect payoff function (2.7) when the outsiders have chosen the action (and therefore a partition) that minimizes the coalitional payoff: It is the minimum payoff that coalition S can guarantee to itself.

The beta notion defines the payoff coalition cannot prevented from for any choice of outsiders:

Definition 20 *The β-characteristic function v_β in the Exclusion game is defined by:*

$$v_\beta(S) = \underset{r^{-S}}{Min}\ \underset{r^S}{Max}\ u^S(r^S, r^{-S}) = \underset{r^{-S}}{Min}\ u^S(r^S(r^{-S}), r^{-S}) = \underset{r^{-S}}{Min}\ u^*(r^{-S}).$$

Notice that both characteristics functions coincide if $\widehat{r}^{-S} = Min_{r^{-S}}\ u^*(r^{-S})$. Under joint production this holds.

Proposition 21 *Under joint production, the indirect payoff function is decreasing in r^{-S}. Therefore the α and β characteristic functions coincide, i.e. $v_\alpha(S) = v_\beta(S)$.*

The minimizer of the coalitional payoff is the same regardless of whether players react passively (after) or actively (before) to outsider's "best" punishment: We obtain the coincidence result also obtained for Common-Pool games (Meinhardt (1999)) and Cournot games (Zhao (1999).)

Now we ask: Can the grand coalition be blocked by some coalition $S \subset N$ under these assumptions about outsiders' behavior? Is there any room for cooperation?

Definition 22 *The α-core (β-core) is nonempty if there is no coalition $S \subset N$ such that $v_\alpha(S) > v_\alpha(N)$ ($v_\beta(S) > v_\beta(N)$.)*

Scarf (1971) showed that the α-core of a NTU game is non-empty if the strategy space for each player is compact and convex and payoff functions are all continuous and quasiconcave. This conditions are satisfied by our game . Then, Proposition 21 implies the next result.

Proposition 23 *Under joint production, the $\alpha-$core and $\beta-$core are nonempty. Moreover, they coincide.*

Given that Proposition 23 is important we briefly outline its proof: Simple inspection of (2.6) show us that the worst case scenario for S when they are waiting for the choice of their rivals occurs when r^{-S} attains its maximum. When $m \geq 1$ the coalition $N \backslash S$ must form and all its members must put their entire endowment; then $\widehat{r}^{-S} = (n-s)^m$. Then, the alpha characteristic function is just the best response to $(n-s)^m$.

We know by Proposition 21 that the indirect characteristic function is decreasing in r^{-S}. This ensures that for any coalition $u^*(r^{-S})$ attains its minimum when $r^{-S} = (n-s)^m$. So finally we have:

$$v_\alpha(S) = u^S(r^S((n-s)^m), (n-s)^m) = u^*((n-s)^m) = \underset{r^{-S}}{Min}\ u^*(r^{-S}) = v_\beta(S).$$

Hence, if individuals are committed to inflict as much harm as possible to potential deviators universal agreement prevails.

The γ and δ characteristic functions

The static approach to coalition formation allows for non optimal reactions because it is never optimal for them to invest the entire endowment in conflict. Agents in the complement coalition may not be able thus to commit to total warfare in case of deviation. Hence, the next step would be to exogenously impose a coalition structure but allow players to use best responses. Coalitions will still be associated with a single payoff.

The idea, first introduced by Hart and Kurtz (1983) is to model the coalition formation process as a normal form game where the strategy space of the players is the set $S_i = \{S \subseteq N \ / \ i \in S\}$. They define two possible games: In the γ game, a coalition forms if and only if all its members announced that coalition; in the δ game a coalition forms among those that announced the same coalition even though some of its prospective members announced something else.

Given that in the Joint production case the grand coalition is the efficient coalition structure, in the sense that the sum of individual payoffs is the maximum[9], a natural question is if the universal agreement is stable, that is, if it can be supported as a (Strong) Nash equilibrium of these games.

When analyzing the stability of the grand coalition the γ and δ concepts can be easily interpreted as expectations of players about the coalition structure that outsiders will form after an individual or group decides to open hostilities: In the γ case they believe that the deviation will trigger a chain reaction until all remaining players form

[9] Notice that the total output is maximized under the grand coalition: Any other coalition structure yields a convex combination among lower total productions.

singletons; in the δ case they believe that remaining players will stick together. This allow us to define again two characteristic functions.

Definition 24 *The grand coalition is $\gamma-$immune or γ stable ($\delta-$immune or δ stable) if there is no coalition $S \subset N$ such that $v_\gamma(S)$ $(v_\delta(S)) > \sum_{i\in S} u_i^N$.*

Proposition 25 *Under joint production, constant returns to scale of effort and an exponential production function*

(i) the grand coalition is γ stable;

(ii) the grand coalition is δ immune to deviations by coalitions with $s \geq \frac{n}{2}$ but it is not δ immune to deviations by smaller coalitions.

In particular for $\alpha = 0.1$ the grand coalition is not δ immune to the deviation by a single player when $n \geq 15$ and to the deviation of a two players coalition when $n \geq 31$.

When players hold "optimistic" expectations about the behavior of outsiders is hard to get the stability of the grand coalition. To have optimistic expectations means different things depending on m : When $m = 1$ the coalition formation presents, optimism happens under the δ concept and, as Proposition 25 shows, universal agreement breaks up. However, it is in the other way around when $m \geq 2$. In such case we show with the following example that the previous result get reversed

Example 3: Suppose that $n = 5$ and $\alpha = 0.1$. Then the γ and δ characteristic functions are

m	$v_\delta(1)$	$v_\delta(2)$	$v_\delta(3)$	$v_\delta(4)$	$v_\delta(5)$
1	0.160	0.183	0.185	0.193	0.234
2	0.080	0.147	0.210	0.228	0.234
3	0.039	0.105	0.218	**0.246**	0.234

Table 1: δ characteristic function when $n = 5$

m	$v_\gamma(1)$	$v_\gamma(2)$	$v_\gamma(3)$	$v_\gamma(4)$	$v_\gamma(5)$
1	0.160	0.172	0.182	0.193	0.234
2	0.142	0.233	**0.241**	0.228	0.234
3	0.138	**0.291**	**0.280**	**0.246**	0.234

Table 2: γ characteristic function when $n = 5$

Payoffs in bold are those higher than the payoff under the grand coalition. Notice first that $v_\delta(s) \leq v_\gamma(s)$ only when $m \geq 2$. In that case, incentives to deviate are in the hands of big coalitions: When $m = 2$ the grand coalition is not γ-immune and it is neither γ or δ-immune for $m = 3$. It seems that the stability of the grand coalition m depends critically on the relationship between m and α : Given a productive technology one only needs to have a sufficiently effective technology of conflict to attack N successfully.

2.4.2 Sequential coalition formation approach

We now assume that players rationally predict the coalition structure that outsiders will form after a deviation. There is no a unique approach to tackle this issue. Here, we will follow Bloch (1996), where coalitions form if and only if all members agree to do it *à la* Rubinstein: The first player in a pre-determined protocol makes a proposal for a coalition; the players in this proposed coalition decide sequentially to accept or not. The process stops when all members accept or one rejects. In the former case, the coalition finally forms; in the latter, the rejector must make another proposal. Bloch (1996) shows that this game yields the same stationary subgame perfect equilibrium coalition structure as the much simpler "Size Announcement game": First player proposes a coalition of size s_1 that immediately forms. Then the $(s_1 + 1)$-th player in the protocol proposes a coalition s_2 and so on, until the player set is exhausted. The game is solved through backward induction and has generally a unique subgame perfect equilibrium.

However, we face a new difficulty: Contrary to the existing literature on coalition formation games with externalities, the payoff function cannot be characterized uniquely by the number of coalitions in π. So we can only provide some partial results that show the importance of the relationship between the productive and conflict technology for the stability of the grand coalition.

Proposition 26 *Under joint production and exponential production functions, the Bloch's stable coalition structure*

- (i) *is the grand coalition under constant returns of labor* $(\alpha = 1)$.

- (ii) *[Tan and Wang (2000)] is not the grand coalition if labor is unproductive* $(\alpha = 0)$ *and under increasing returns of effort. In particular it is of the form* $\{s, n - s\}$ *when* $m \geq 2$ *and where* s *satisfies*

$$(\frac{n}{s} - 1)^{m-1}((m-1)\frac{n}{s} + 1) = 1. \tag{2.8}$$

- (iii) *is of the form* $\{s, n - s\}$ *with* $s \geq \min\{\frac{n}{2(1-\alpha)}, n\}$ *in the first-price auction-like case if the coarsest stable partition is selected.*

With constant returns of labor, conflict activities are wasteful from an individual point of view: The first player in the protocol announces the coalition that maximizes his probability of access because the cost of absorbing one rival is zero.

The total congestion scenario coincides with Tan and Wang (2000) for the case of n identical players: Given that the prize is unaffected by labor investments, players invest all their endowments in the exclusion contest and this allows to derive a closed form for u_i^S. Under increasing returns to scale of effort there is a strong tendency towards bi-partisan conflicts because the formation of an outside group reduces the cost of joining with others. The first coalition formed is less inclusive as m increases: It is of size $\frac{\sqrt{2}}{2}n$ (ignoring the integer problem) when $m = 2$, $\frac{2}{3}n$ when $m = 3$, and $0.54n$ when $m = 20$.

Finally, in the first-price-auction-like any coalition structure in which the first coalition is greater or equal than $\frac{n}{2}$ is Bloch's stable. In order to be consistent with the previous result, we take the classification of selecting the coarsest stable partition (because then the first coalition formed when $\alpha = 0$ is of size $\frac{n}{2}$.) In that case, the grand coalition is Bloch stable if and only if $\alpha \geq \frac{1}{2}$. Again, universal agreement can only be supported as a Bloch stable coalition structure if returns to scale of labor are not too decreasing with respect to the effectivity of effort.

Let us illustrate the points made so far with an example for a linear quadratic production function.

Example 4: Assume that $N = 4$. In order to obtain reader friendly figures we assume that players have 35 units of initial endowment. Let $f(l) = 20L - \frac{1}{8}L^2$. We allow m to be 1 or 2. Payoffs are displayed in the following tables.

	$m=1$				$m=2$			
π	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$
$a \mid b \mid c \mid d$	83	83	83	83	60	60	60	60
$ab \mid c \mid d$	144	144	85	85	151	151	50	50
$abc \mid d$	184	184	184	90	202	202	202	42
$ab \mid cd$	150	150	150	150	123	123	123	123
$abcd$	200	200	200	200	200	200	200	200

It is easy to see what the Bloch stable coalitions structures are: For player a (the first in the protocol) it is dominant to announce $\{N\}$ when $m = 1$ and to announce $\{3\}$ when $m = 2$. In the latter case a deviation is possible because increasing returns to effort make exclusion cheap for big coalitions. In that case, the possibility of conflict breaks up the efficiency of universal agreement.

2.5 Coalition formation and commons

A common good is an object that is owned by nobody or, equivalently, by everybody: a fishery, a pasture... In this Section we will be concerned with how the existence of an exclusion contest and the possibility of coalition formation affect the creation of effective property rights over common goods.

If a good is of common property, one would think that it cannot be owned by a few![10] However, as pointed out by Grossman (2001), there is a clear difference between *effective* and *formal* property rights: The former entail control, the latter are those stated by legal ownership and may not confer control rights by themselves. In fact, sufficiently strong control rights are the main step for the recognition of formal ones if they were previously undefined. For example, in the 1960s, oil and gas were found under the North Sea. Several countries contested for the exploitation rights. Although the United Nations' Law of the Sea claims that resources in the seabed are "the common heritage of all mankind", Britain and Norway finally obtained such rights because they were able to impose the "smallest distance to the coast" classification.

Let us briefly described the basic analysis of the 'tragedy of the commons': In the unique symmetric Nash Equilibrium of the game the total labor input l^F when s players has free access to the common

$$\frac{1}{s}f'(l^F) + \frac{s-1}{s}\frac{f(l^F)}{l^F} = \omega. \tag{2.9}$$

Efficiency would require that $f'(l^S) = \omega$. However the total labor input yields a weighted average between the efficiency level (achieved only when one agent entries) and the equalization to the average productivity, where the resource is overexploited. Moreover, the equilibrium payoff is decreasing in s because inefficiency becomes more severe as s grows; as $s \to \infty$ individual payoff approaches zero.

As we know the Exclusion game in the presence of a common pool resource corresponds to the case of $\lambda = 0$. In that case condition (2.5) becomes

$$\frac{m}{s-l^E}\frac{r^{-S}}{(r^S)^m + r^{-S}}\frac{1}{s}[f(l^E) - \omega l^E] = \frac{s-1}{s}\frac{f(l^E)}{l^E} + \frac{1}{s}f'(l^E) - \omega. \tag{2.10}$$

The RHS of this expression is precisely the difference between the terms in (2.9) that now is positive instead of zero. It implies that $l^F > l^E$. Hence *conflict acts as a discipline device* because it deters players from contributing too much labor. This result opens the door to the *social efficiency of conflict*, because exclusion activities (partially) alleviate the tragedy of the commons. However, it would be totally trivial and vacuous if the stable coalition structures, according to the concepts employed

[10] I thank Carmen Beviá for this point.

above, yielded always free access. Next, we analyze this issue by using linear quadratic production functions.

The first result is in sharp contrast with the Joint production case: Players in a symmetric coalition structure may be better off than in the grand coalition and hence conflict may Pareto dominate free access.

Proposition 27 *Suppose that the free access problem case has an interior solution under a linear quadratic technology. Then there exist a threshold $\widetilde{m}(n,k,\omega,\theta)$ such that a k-sided symmetric conflict Pareto dominates free access if and only if $m \leq \widetilde{m}(n,k,\omega,\theta)$.*

As expected the threshold $\widetilde{m}$ is increasing in θ : If conflict technology is too effective and players underexploit the common free access can be again dominated by rising θ and making production technology sufficiently concave.

The second observation is that the α and β characteristic functions may not coincide, as the following Lemma illustrates.

Lemma 28 *The indirect payoff function of a player $i \in S$ is strictly decreasing in r^{-S} if*

$$s \geq (m+1)\frac{1-\varepsilon}{1-(\omega(s-r^S)/f(s-r^S))} - m. \tag{2.11}$$

Why can we only state a sufficient condition? Notice first that if a coalition is underexploiting, $f'(L) > \omega$, condition (2.11) holds because the right hand side is negative. However this is not necessarily true when the coalition is overexploiting the resource. In such cases, in increment of the effort by outsiders reduce the total labor contribution of the coalition and payoff in the production stage increases. One can only ensure that $v_\alpha(s) = v_b(s)$ as long as coalition S cannot overexploit, i.e. $f'(s) > \omega$. As a consequence only when no coalition can overexploit the common[11].

Now we illustrate Proposition 27 and Lemma 28 by means of the following example where we also show that the α characteristic function is not convex contrary to what happens in common pool games (see Meinhardt (1999).)

Example 5: The initial data of this game are taken from Meinhardt (1999). Note that, by symmetry, they are equivalent to those used in Example 3. This allows to compare the three models, Meinhardt's and joint and individual production.

[11]This does not imply that the β core is empty. In fact, if $\widehat{r}^{-S} = (n-s)^m$ is the actual minimizer of u_i^*, the exclusion game satisfies all the conditions posed in Theorem 1 in Zhao (1999) for the non-emptiness of the β core. The real problem is that it is not possible in this framework to compare a corner solution with possible interior minimizers.

Let $n = 4$, players have 35 units of initial endowment, the unit cost of labor is 3 and let $f(L) = 23L - \frac{1}{8}L^2$. Again, we consider the cases of m equal to 1 and 2.

First, we compare the alpha (and beta, because they coincide for the individual production case too) characteristic functions. The characteristic form game of Meinhardt (1999) is convex, that is, individual contributions to coalitional worth are greater the bigger the coalition the player joins with. However, this does not hold for our exclusion games

	$v_\alpha(\{1\})$	$v_\alpha(\{2\})$	$v_\alpha(\{3\})$	$v_\alpha(\{4\})$
Meinhardt (1999)	95	253	488	800
Joint production ($m = 2$)	10.2	123.4	454.5	800
Individual production ($m = 2$)	10.2	106.5	380.3	512

where $v_\alpha(\{s\})$ is the value generated by a coalition of size s when outsiders behave in the α fashion. Note that the joint production case is intermediate between Meinhardt's and individual production. Anyway, values for the exclusion games are always below Meinhardt's ones. Furthermore, Lemma 28 does not apply: Suppose that $n > 4$. Then, the indirect payoff function for the member of coalition of four players attains a maximum when $r^{-S} \approx 0.8$.

Let us now assume that coalitions play best responses. We compute the partition function for Meinhardt (1999):

π	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$
$a \mid b \mid c \mid d$	128	128	128	128
$ab \mid c \mid d$	100	100	200	200
$abc \mid d$	118	118	118	335
$ab \mid cd$	177	177	177	177
$abcd$	200	200	200	200

Notice that this game is of positive externalities: When players merge they reduce their labor input because they internalize part of the social costs. Outsiders take advantage from it: They have now a bigger share of a higher overall production. Then, players are reluctant to form coalitions: They want others to do so. Knowing this, the best announcement for the first player in the protocol can do is the grand coalition.

Things change dramatically for the individual production case:

	$m = 1$				$m = 2$			
π	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$	$u_a(\pi)$	$u_b(\pi)$	$u_c(\pi)$	$u_d(\pi)$
$a \mid b \mid c \mid d$	83	83	83	83	61	61	61	61
$ab \mid c \mid d$	134	134	102	102	142	142	67	67
$abc \mid d$	152	152	152	156	171	171	171	94
$ab \mid cd$	177	177	177	177	158	158	158	158
$abcd$	128	128	128	128	128	128	128	128

Note first that, in contrast with the joint production case, this game is of positive externalities for both values of m. When $m = 1$, it is dominant for a to announce a two-player coalition because for c it will optimal to form $\{cd\}$. On the other side, when $m = 2$ it is dominant for a to form $\{abc\}$. The reason for this difference lies at the fact that when $m = 1$ the three players coalition is overexploiting and the winning probability does not decrease too much by expelling c although it joined d.

Observe that, as pointed out in Proposition 27, for both values of m the structure $\{ab \mid cd\}$ is the most efficient one and Pareto dominates free access. The consequence is that the latter is neither δ or γ immune. The same happens with $\{abc \mid d\}$ when $m = 1$. However, when $m = 2$, although players in $\{abc\}$ no longer overexploit the resource because their effort has increased, d is in a too weak position. So big coalitions "need" the presence of outsiders. Not too many, as in the latter case for $\{ab\}$, but not too few, as in the former for $\{abc\}$.

Finally let us compare the stable structures of the three models:

	$m = 1$	$m = 2$
Meinhardt (1999)	$abcd$	$abcd$
Joint exploitation	$abcd$	$abc \mid d$
Separate Exploitation	$ab \mid cd$	$abc \mid d$

This results suggest that even if agents can communicate, very effective conflict technologies make a difference. On the other side, by accepting the possibility of conflict in non-cooperative environments, the 'tragedy of the commons' is partially alleviated: The expected production is closer to the joint production of the resource, the best case scenario.

2.6 Related literature

This paper is related with three different strands of the economic literature: Economic models of conflict, coalition formation games with externalities and common property resources.

Economic models of conflict date back to Bush and Meyer (1974) and have received important contributions by Skaperdas (1992), Hirshleifer (1995) and Neary (1996)[12]. The basic idea underlying this literature is that if property rights are not properly defined individuals face a trade-off between undertaking productive and non-economic or appropriative activities. The main consequence is that the allocations resulting from economic interactions may not be exclusively those derived from productivity but also from relative performance in a conflict stage: Agents engage first in productive activities -labor is transformed into output- and output is redistributed by force in the second stage when players devote effort to appropriation. The probability of winning the conflict can thus be identified as the proportion of the total output allocated to each agent.

This canonical model however has been criticized because it can be interpreted only as a theory of the right of access to common property, but fails to account for the creation of private property rights[13]. Grossman and Kim (1995) and Muthoo (2002) deal with the enforcement of the right to enjoy the fruits of one's labor. Rather than over some aggregate, the contest is over individual productions. In any case, all these models render conflict activities as socially wasteful because resources are diverted away from productive uses. On the contrary, we show that when players fight for the right to exploit a common good, conflict activities may be socially efficient precisely because of that.

All the mentioned models ignore as well the issue of coalition formation[14]. Moreover, they share the unsatisfactory feature of focusing on struggles over objects rather than over *rights*: In the former case agents produce in the shadow of expropriation of the common output so they might be appropriate to discuss pre-modern conflicts. However, victory in present-day contests (as the cases above show) implies that winners control (or have effective property rights) some contested object that enables them to produce without further opposition[15]. Under this broader view the role of coalitions in conflicts is more sensible and clearer[16]. Then, the main trade-off is no longer between productive and unproductive activities but between a higher chance of success and diluted property rights.

[12] These models are closely related to rent-seeking models. In fact, they belong to the more general class of models of rivalry. We refer the reader to Neary (1997) for a nice exposition of these issues.

[13] See Grossman and Kim (1995), Neary (1996) and Muthoo (2002)

[14] The only exception is the recent paper by Noh (2002) who extends the canonical conflict model to the case of three heterogeneous players.

[15] To the best of our knowledge, the only model of coflict that makes this distinction is Skaperdas and Syropoulos (1998), where two agents fight for the right to access to some fixed factor that they can use in production in case of victory.

[16] On the models of enforcement of private property rights, the simple existence of more than two players may lead to inconsitencies: If an agent challenges two outsiders he may loose two times his individual production!

The issue of coalition formation in common-pool resources in the absence of conflict for control has been explored by Funaki and Yamato (1999) and Meinhardt (1999). If players can communicate, they can form groups in order to exploit the common. This (partially) solves the externality problem because members internalize the negative effects on other members. Although the grand coalition is the most desirable outcome, the presence of external effects may prevent its formation because all players prefer others to form coalitions: Funaki and Yamato (1999), in a partition function approach, show that the core of their game is non-empty if players have the most pessimistic expectations but not if they have the most optimistic ones. Meinhardt (1999) addresses the issue through a characteristic function approach that turns out to be convex and whose core coincides with the Von Neumann-Morgenstern's stable set.

Our model would be therefore complementary to these two because we all address, from a positive point of view, situations in which the tragedy of the commons may not be an irreversible outcome.

Third, the present work adds up to the existing literature on coalition formation games with externalities, surveyed in Yi (1999), that departs from traditional characteristic form games in that coalitional payoffs depend on outsiders actions. With the exception of Tan and Wang (2000) and Noh (2002), appropriative activities have been ignored as a source of externalities. In this context, we also try to provide foundations to conflict models by analyzing what coalition structures arise in our game.

Finally, our model is somehow related with some works in the field of sociobiology as an instance of the *competitive exclusion* principle[17] that states that two species cannot coexist indefinitely under a limited amount of resource. Anyway, we assume implicitly this principle rather than proving it.

2.7 Conclusion

We have presented an economic model of conflict where agents reduce the rivalry or congestion over some resource by excluding others. We have considered also the possibility that these agents may form coalitions in order to be more successful in that endeavour. Effective property rights are created through contests that determine what coalition gains access to the resource. One important feature is that coalitions face a trade off: As they incorporate more members they attain control more likely, but individual property rights within it dilute. Moreover, individual and coalitional payoffs depend on the entire coalition structure.

We show that the more concave the technology of production is the more likely a sufficiently effective conflict technology will break up universal agreement. Under in-

[17] See Carneiro (1970) and (1978).

creasing returns to scale of exclusion efforts, the formation of other groups reduces the costs of sharing property rights for the rest of agents, leading to bi-partisan coalition structures. This would support the standard two-player models of conflict. However, the main result under individual production is completely new in this literature: Conflict may be socially efficient because deters individuals from using too much labor in the exploitation of the resource. Moreover, it is relatively easy to generate coalition structures that Pareto dominate free access and to support them as the outcome of some well-know coalition formation games. Our results must be interpreted with caution: We do not advocate that property rights over commons should be allocated through conflict. Our contribution should be regarded from a purely positive perspective; as an alternative that would emerge when agents are not able to contract for welfare enhancing arrangements nor communicate.

Some other comments are in order. First, our game is one shot: Once a coalition has won the contest, its members agree not to fight again. Other models, like Tan and Wang (2000), consider the scenario of *continuing conflict* where conflict is assumed to be fought until a conflict-proof coalition (that is, one immune to the re-opening of conflict) prevails. We think that this is a very relevant question with a population of identical agents where only size matters: Why should a group of agents that fought mercilessly with others cooperate forever once conflict is solved?[18]. Continuing conflict seems to be better suited than cooperative bargaining or fixed sharing rules to overcome this problem for instance in rent-seeking setups where the value of the prize is fixed. However, it seems less valid in a setting like ours where the after-conflict stage is a production stage with its own structure.

One possible objection would be that our players are identical. It can be argued that conflicts many times arise because agents are different. Beyond the problem of tractability -Noh (2002) points out how complex is to consider just three heterogenous players- we can answer that assuming inequality of endowments or strengths may be self-explanatory of the presumable unequal allocations resting on conflict (or power relationships). We are mainly interested on exploring the validity of conflict as a *mechanism* that generates such inequality when agents use it to attains their goals.

Another possible extension of the model would be to relax the assumption that the losing players face "death". This may be a source of additional conflict investments. In Skaperdas and Syropoulos (1996) victory means that the winners trade with the losers in a dominant position. In these lines, it could be assumed that the winning coalition also gains the power to hire labor (or take it freely) from the losers. Then, if the payoff after exclusion is not zero or the winners care about the left over endowments of the losers exclusion races might be alleviated and conflict less fierce.

[18]We thank Serge Kolm for pointing out this question.

Part III

Bargaining and Conflict

Chapter 3

Theory

"War and peace are more than opposites. They have so much in common that neither can be understood without the other."

Geoffrey Blainey (1973), *The Causes of War.*

3.1 Introduction

The starting point of this paper is the following idea: In order to understand *how* parties reach an agreement one has to understand first how they disagree.

The economic approach to disagreement is strongly tailored by Nash's seminal contribution. In his description of the *bargaining problem* Nash (1950) embeds disagreement in the threat point; it means to be the outcome of a hypothetical non-cooperative game played after parties fail to agree how they share the surplus of cooperation. However, no information about that game or the forces that determine the location of such point is incorporated into the description of the problem.

This paper turns around two considerations challenging this position. First, disagreement is often followed by a conflict or confrontation whose outcome is driven by the *power relationship* or *relative strength* of the parties. Examples are the renegotiation of the terms of a contract between a soccer player and his club; the negotiations between two countries on the division of some piece of territory; between workers and management on wages; among social groups on the share of political power; or simply how a just married couple will share the chores. All these bargaining situations have the common feature of occurring *in the shadow of conflict:* If parties fail to agree they can resort to coercive methods. They can go to court, they can go to war or strike; they can divorce. In any case, these are probabilistic conflicts. Their outcomes depend on military strengths, the extent of the union membership, the quality of the lawyers... That is, they depend on *power*. Consequently, any sensible agreement will be conditioned by how the conflict triggered by disagreement is resolved.

But why should disagreement be only an outside option? The second observation we want to point out is that parties do choose the way in which they disagree; they actually choose the *scope* of the conflict they are going to fight[1]: India and Pakistan has not used nuclear weapons, they only engage in skirmishes; Pepsi and Coca-cola do not engage in worldwide price wars, but only national; family arguments do not necessarily imply divorce... To assume that conflicts are just "fights to the finish" neglects the fact that the main cause for the end of a confrontation is the parties' agreement on stopping hostilities rather than the total collapse of one side. In short, it prevents us to see that conflict is *part of the bargaining process*.

Suppose that parties can only engage in conflicts aiming to the complete defeat of the opponent and consider the following situation: Two agents have incomplete information on the strength of their opponent in case of conflict; and both parties are strong but believe that they are facing a weak rival. Then, the perceived threat-

[1] As Wagner (2000) pointed out, few strikes or wars would occur if their consequences were always the total defeat of one of the parties.

point will be out of the bargaining set and the result of the negotiation will be a total confrontation[2]. But what if parties can engage in a conflict of limited scope that does not entail the end of negotiations? Given that its outcome would also be determined power, it would convey information about the true relative strengths and thus it might open a range of possible agreements.

The aim of this paper is to explore the role of confrontation in negotiations by integrating these two considerations. With that purpose we explore a very simple bargaining model with one-sided incomplete information and one-sided offers. Inspired by the pioneer analysis of Clausewitz (1832) we consider two types of conflicts: *Absolute conflicts* (AC), equivalent to an outside option and that end the game when taken[3], and *Real conflicts,* that can be thought as battles and that allow the game to continue. Given that we assume that parties' winning probabilities in both types of conflicts are a function of their relative strength, Real conflicts become experiments that transmit information about it and about the expected outcome of the AC by extension. The latter is fought when one party losses completely the hope of reaching an agreement through ordinary methods; battles are fought before or during normal bargaining because they can help to improve positions by changing opponent's beliefs; therefore they do not impede the game to end in a settlement.

However, information is not only transmitted in the battlefield. Offers may also signal the type of the proponent, because the toughest players will never make certain offers. Unfortunately, the analysis when the uninformed party learns from these two sources is difficult and we leave it for further research. We thus assume that the uninformed player is boundly rational: She learns the information conveyed through confrontation but not that transmitted through ordinary bargaining.

Once battles have changed enough the expectations of the uninformed party to create a range of possible agreements, the informed agent has incentives to trigger further battles in order to gain even more advantage. Then the proponent's strategy collapses into an *optimal stopping* problem: He must decide at each period either to stop the game (by making an acceptable offer or triggering AC), or to keep gambling[4]. From this point of view our model introduces a novelty with respect to standard models of bargaining under incomplete information: The outcome of a confrontation is not subject to manipulation because it depends on the true relative strengths of the parties. The only strategic variable is the decision of invoking such conflict or not;

[2] Then our answer to the Hicks' paradox would be very extreme: Not only parties do not reach an agreement when a mutually beneficial one exists; moreover, one of them will bite the dust in the ensuing (wasteful) conflict.

[3] These are a very extreme conflicts because parties aim to render the opponent defenseless and impose their most preferred outcome without opposition

[4] "Of all the branches of human activity, war is the most like a gambling game" (Clausewitz (1832)).

bluffing the other party becomes a very difficult task.

We obtain the following results: Even though the game has an infinite horizon it ends in a finite number of periods. The reason being that the improvement in the expectations produced by an additional victory of the informed party is decreasing both in time and in the number of partial victories already obtained. Given that players are impatient, incentives to gamble decrease with time as well. Agreement is thus immediate if costs from confrontation -either from delay or from AC- are high enough.

A second result is that the more powerful the informed agent the more likely is the use of Real conflicts during the bargaining process: In our model, the informed party owns a *persuasion device;* and the ease to "sell" information to the other party is increasing in his strength. Hence, if he is not powerful enough, he prefers to make an acceptable offer immediately. Beyond this case, we show that the existence of a bargaining range is necessary but not sufficient for a settlement to be reached. Hence, the role of conflict as a source of information would help to explain why we observe confrontation in situations where agreement is possible.

We distinguish between two scenarios: *Advantage conflicts,* where mutually beneficial agreements exist at *any* state of the game, and *Unavoidable confrontations,* where excessive optimism precludes agreement at some states. In the former case we show that the informed party's equilibrium strategy can be characterized by a sequence of integers, one for each period before the (endogenous) finite horizon. If the number of victories obtained at some period t is greater than the corresponding integer in the sequence, the informed party makes an offer that is accepted by his opponent. Otherwise, he prefers to trigger another battle. This sequence is first non-decreasing and non-increasing afterwards. In Unavoidable confrontations, a non-decreasing sequence is added, such that AC occurs if the number of victories steps below it. We are able thus to generate an *ex-post* interpretation of conflict as an outside option: If Real conflicts cannot create a bargaining range early enough or if bad luck in the battlefield makes persuasion attempts fail, the informed party loses any hope of extracting surplus and conflict is total.

The rest of the paper is structured as follows: In Section 2 we present the basic elements of the model. In Section 3 we show how the bargaining game collapses into an optimal stopping problem and in Section 4 we solve it. In Section 5 we revise the related literature.

3.2 The model

Consider a game, denoted by $G[\delta, \theta]$, where two risk neutral players bargain over the division of a cake one euro worth. We will assign to P1 the male gender and the female gender to P2. They are impatient an discount future at a common factor $\delta \in (0, 1)$. Let us denote by $p \in [0, 1]$ the *relative strength* of player P1 in case of conflict. The p is perfectly observed by P1 but not by P2, who believes at $t = 0$ that p is uniformly distributed[5].

Players act in discrete time under an infinite horizon. In each period $t = 0, 1, 2, ...$ player P1 chooses an action in $\{AC, B, x\}$, where $x \in [0, 1]$ denotes an *offer* to P2; AC is the outside option of *Absolute conflict* (AC henceforth) that ends the game; and B means that a *battle* between the players is fought, making the game proceed to period $t + 1$.

P2 only moves if P1 makes an offer. In that case, her available actions are $\{$*Accept*, *Reject*$\}$. If P2 accepts, agreement is reached at that period and payoffs are $(\delta^t(1 - x), \delta^t x)$. Rejection triggers AC.

AC is a "fight to the finish", a confrontation where parties perfectly commit to try to defeat their opponent. Therefore, it necessarily ends the game. We model AC as a costly lottery, yielding payoffs that depend on the realization of p : With such probability P1 wins the conflict and P2 is defeated. This confrontation entails a fixed loss; the value of the cake reduces to $0 \leq \theta \leq 1$. Therefore, the payoffs from AC, conditional on p, are

$$d = (d_1, d_2) = (\theta p, \theta(1 - p)).$$

Finally, a battle is a conflict of limited scope that does not entail the end of the game: Nature simply announces a winner and the next period is reached. We will assume that the outcome of these battles is a function of the relative strength p too. But since p is not known by P2, this implies that *battles convey information* about p. For simplicity, we will assume that the battle winning probabilities are precisely p and $1 - p$ respectively.

Battles are thus Bernoulli trials. The belief updating due to battles can be easily computed: Posterior beliefs on p follow a Beta distribution with parameters $(k+1, t-k+1)$ (see De Groot (1970)). The probability density function of p at period t after k successes of P1 is thus

$$f(z, k, t) = \frac{z^k(1 - z)^{t-k}}{\int_0^1 u^k(1 - u)^{t-k} du} \tag{3.1}$$

[5]As we will see below, this assumption can be relaxed to any priors following a Beta distribution.

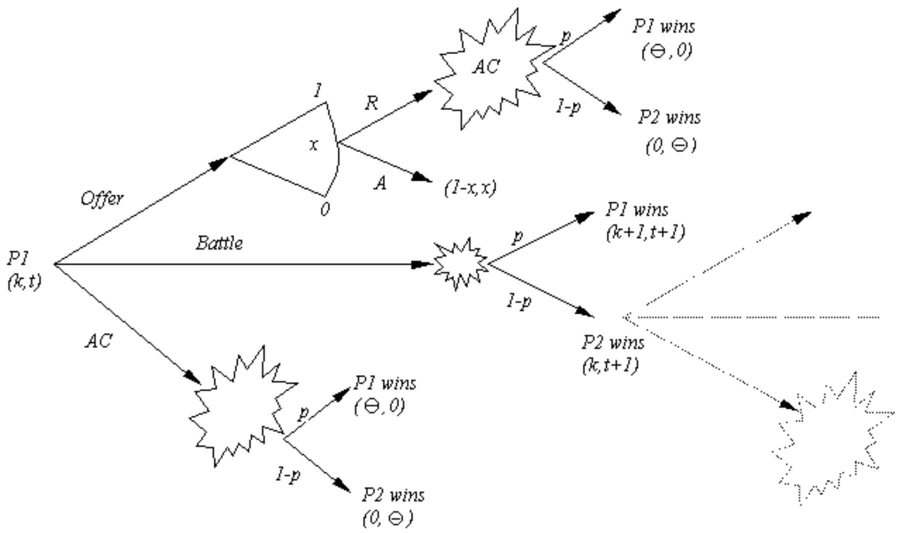


Figure 3.1: The game

and its expectation, $E(p \mid k,t)$, is equal to $\frac{k+1}{t+2}$. This allows us to characterize completely P2's beliefs at any information set in which no offer has been made yet through a vector $h_t = (k,t)$ where k is the number of battles won by P1. Thus the set $\Theta = \{(k,t) \in \mathbb{N}^2 \ / \ k \leq t\}$ describes the possible states in the battlefield. And P2's expected payoff from AC at any state $h_t \in \Theta$ is

$$E(d_2 \mid h_t) = \theta(1 - \frac{k+1}{t+2}).$$

P2 will not accept offers such that $x < E(d_2 \mid h_t)$ because she believes that she can do better by rejecting and triggering AC.

Hence, a strategy for P1 is simply a function σ_1 mapping the set of states into the set of actions $\{AC, B, x\}$; similarly, a strategy for P2 is a function σ_2 mapping states into $\{Accept,\ Reject\}^6$.

We will extensively focus on the case when P2 only learns the information transmitted in the battlefield. In that case, we will say that P2 is *unsophisticated.*

Definition 29 *A* ***Unsophisticated Equilibrium*** *(UE) of the game $G[\delta,\theta]$ is a pair of strategies (σ_1^*, σ_2^*) such that σ_1^* maximizes P1's continuation value of the game at each h_t and P2 accepts x if and only if $x \geq E(d_2 \mid h_t)$.*

[6] Note that we will restrict our analysis to strategies that do not depend on the concrete order of victories and defeats.

The reader will note that by assuming that every offer is final we avoid further signalling through rejectable P1's offers. We claim that this is assumed without loss of generality: If P2 had the option of rejecting the offer and trigger a new battle, all offers would be either uninformative or accepted in equilibrium since the information transmitted through offers would decrease P2's estimates of p so she would become more demanding. Therefore, P1 *cannot gain* from making that offer.

3.3 P1's strategy as an optimal stopping problem

In this Section we show that the P1's strategy against an unsophisticated opponent can be characterized as an optimal stopping problem.

We know that an equilibrium offer must leave P1 at least as well as under AC and give P2 at least her expected payoff in that situation. Hence, at each state (k, t) the set of relevant proposals for P1 is

$$\mathsf{X}_{k,t} = [\theta(1 - \frac{k+1}{t+2}), 1 - \theta p].$$

Under complete information, agreement would be achieved immediately: P1 would offer $\theta(1-p)$ and P2 would accept. However, under incomplete information, the interval $\mathsf{X}_{k,t}$ may be empty: If P2 is too optimistic about her probability of winning AC, the sum of the perceived disagreement payoffs may be greater than one and agreement is impossible.

Remark 30 *The sum of the "perceived" disagreement payoffs exceeds the surplus at history* (k, t) *if and only if*

$$\theta(1 - \frac{k+1}{t+2}) + \theta p > 1, \tag{3.2}$$

The key ingredient of our model is that the transition from period t to $t+1$ through a battle conveys information because its outcome is driven by the true relative strength of the players. Therefore conflict, in the form of battles, may actually *open the door to agreement* by making P2 less optimistic about her prospects in case of AC.

Let us denote by

$$S = \{h_t \in \Theta \;/\; \theta(1 - \frac{k+1}{t+2}) + \theta p \leq 1\}$$

the set of states where agreement is possible. Observe that a non-empty bargaining range at state h_t (i.e., $h_t \in S$) is not sufficient for agreement: P1 knows that he can make P2 even more pessimistic about the outcome of AC by winning one additional battle; he is endowed with a "persuasion device".

Thus P1's problem can be treated as an *optimal stopping* problem because he has to decide at each stage whether to stop or not and how (through agreement or AC). One can express P1's objective problem by means of the following value function

$$v(k,t) = \max\{r(k,t),\ \delta E(v \mid k,t)\}, \tag{3.3}$$

where

$$E(v \mid k,t) = p \cdot v(k+1,t+1) + (1-p) \cdot v(k,t+1),$$

is the expected continuation value of the game, and

$$r(k,t) = \max\{1 - \theta(1 - \frac{k+1}{t+2}),\ \theta p\},$$

is the *immediate reward function*; this is the best payoff P1 can get by stopping at (k,t): If $h_t \in S$, then $r(k,t) = 1 - E(d_2 \mid h_t)$[7]; if not P1 would trigger AC in case he decides to stop, so $r(k,t) = \theta p$.

The solution to P1's optimal stopping problem is typically a mapping from the space of states Θ to a decision of continuing or stopping.

We will say that P1 *prefers to stop the game* whenever

$$r(k,t) \geq \delta E(v \mid k,t). \tag{3.4}$$

Our first result is that in equilibrium P1 stops the game in finite time.

Proposition 31 (Game ends in finite time) *For each pair (δ,θ) there exists a period $\bar{t} < \infty$ such that in any UE the game ends no later than $\bar{t}$. Furthermore, $\bar{t} > 1$ only if $p < \frac{\delta}{\theta}$.*

A couple of remarks are in order: First that the battles become a relevant instrument for P1 only when the cost of delay they introduce is low relative to the loss from AC. Therefore the *endogenous finite horizon* comes *sooner the more impatient* the players are and the *higher is the conflict loss,* because the outside option becomes less attractive.

Second, the returns of conflict as an informational device are decreasing in time[8]. This, together with impatience, sets an upper bound to the number of persuasion

[7] Given that P2 knows that at each period she will never receive an offer greater than that, the continuation value of rejecting x is at most $\delta\frac{k+1}{t+2}\theta(1-\frac{k+2}{t+3}) + \delta(1-\frac{k+1}{t+2})\theta(1-\frac{k+1}{t+3}) = \delta\theta(1-\frac{k+1}{t+2})$; that is always dominated by the value of accepting x now.

[8] The increment in P2's expectation of p after a P1's victory is

$$\Delta E(p \mid k,t) = \frac{t+1-k}{(t+2)\,(t+3)},$$

It is easy to check that

$$\frac{\Delta E(p \mid k+1,t+1)}{\Delta E(p \mid k,t)} = \frac{t+2}{t+4}.$$

attempts. In addition, if the game is in the set of states where the bargaining range is empty, P1 may (at some point) lose the hope of extracting some surplus, and then he trigger AC.

Thus, the grid $H = \{h_t \in \Theta/\ t \leq \bar{t}\}$ covers all states of the game that are possible along any equilibrium path. Let us denote by

$$\Gamma = \{h_t \in H/\ v(k,t) = r(k,t)\},$$

the *Stopping region,* i.e. the set of states where P1 prefers to stop the game. It can be shown (see Dynkin and Yushkevich, (1969)) that it is optimal for P1 to stop at the first time the state of the game visits Γ. Therefore, to characterize the UE outcome it is sufficient to characterize Γ. The following condition will be useful for that characterization.

Weak concavity: *The immediate reward function satisfies weak concavity at state* (k,t) *if and only if*

$$r(k,t) \geq \delta[p \cdot r(k+1,t+1) + (1-p) \cdot r(k,t+1)],$$

Weak concavity is a state-dependent property that the immediate reward function satisfies whenever the current reward is greater than the discounted expected reward in the next period. It is immediate to see that this condition is *necessary* for P1 to prefer to stop. Hence, in order to characterize Γ we must investigate when it is also *sufficient.* This is the main goal of the next Section.

3.4 Conflicts against unsophisticated opponents

In this Section we will characterize the *stopping region* for the case where P2 is unsophisticated. As we will see, it consists on a sequence of integers separating those states of the game where it is optimal for P1 to stop from those where it is not. By construction, this sequence will characterize the UE of our game.

Two scenarios arise here: *Advantage conflicts*, where agreements are possible at any state of the game; and *Unavoidable confrontations*, where optimism may preclude agreement and confrontation becomes the unique mean to bargain.

3.4.1 Advantage conflicts

In Advantage conflicts, the conflict loss $1-\theta$ is so high that AC never occurs in equilibrium. But P1 may still find incentives to delay agreement by fighting battles in order to change P2's beliefs.

In order to characterize the Stopping region Γ we will make use of the property of weak concavity. Since

$$r(k,t) = 1 - \theta(1 - \frac{k+1}{t+2}) \qquad \forall h_t \in H,$$

weak concavity at state (k,t) reduces to

$$1 - \theta(1 - \frac{k+1}{t+2}) \geq \delta(1 - \theta(1 - \frac{k+1+p}{t+3})). \tag{3.5}$$

Expression (3.5) implicitly defines a boundary on the set H denoted by

$$k^p(t) = \left\{ k \in \mathbb{R} \quad / \quad 1 - \theta(1 - \frac{k+1}{t+2}) = \delta(1 - \theta(1 - \frac{k+1+p}{t+3}) \right\}. \tag{3.6}$$

If, at a given state, $k < k^p(t)$ then weak concavity fails and consequently P1 will surely fight an additional battle.

Note that if weak concavity holds for all $h_t \in H$, i.e. $k^p(t) < 0\ \forall t$, backwards induction from for $\bar{t}$ yields that P1 prefers to stop at any state of the game. That is, *agreement must be immediate.*

Proposition 32 (UE with Immediate agreement) *There exist some threshold* $\widetilde{p}(\delta, \theta)$ *such that in any UE agreement is immediate if the realization of* p *satisfies* $p \leq \widetilde{p}(\delta, \theta)$. *Moreover, there exists a threshold* $\widetilde{\theta}(\delta)$, *decreasing in* δ *such that, if* $\theta < \widetilde{\theta}(\delta)$, *in any UE agreement is immediate for any realization of* p.

When P1 is strong enough his "persuasion device" is very effective and the option of acquiring additional advantage is thus attractive. The threshold

$$\widetilde{p}(\delta, \theta) = \frac{1 - \delta + 2\sqrt{\theta\,(1-\delta)\,(1-\theta)}}{\theta\delta},$$

is decreasing in both δ and θ. Hence, as the delay cost and the loss due to AC increase, only stronger types of P1 find battles worthy as a way to improve their bargaining position. If these costs are high enough, i.e. $\widetilde{p}(\delta, \theta) > 1$, agreement is immediate *for any type* of P1.

Note that Propositions 31 and 32 together show that if p is too low or too high, battles become useless for P1 in Advantage conflicts; either because he prefers to agree immediately or to trigger AC.

For the remainder of the Section, we will focus on environments where agreement is not immediate for some P1's types.

Assumption A1 (AC not too costly): $\theta \geq \widetilde{\theta}(\delta) = (1-\delta)\frac{(\sqrt{2}-\sqrt{\delta})^2}{(2-\delta)^2}$.

Note that this threshold is decreasing in δ. Now we are ready to characterize the stopping region and therefore the UE, with the help of the boundary (3.6).

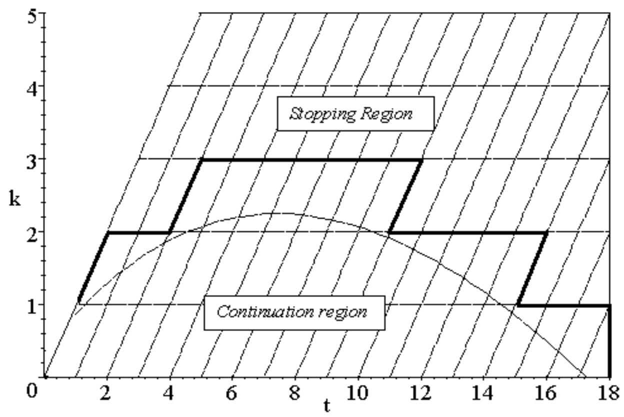
Proposition 33 (UE of Advantage conflicts) *Suppose that A1 holds. If* $p > \widetilde{p}(\delta, \theta)$ *there exists a sequence of integers* $\{k_o^p(t)\}_{t=1}^{\bar{t}}$*, that is first non decreasing and non increasing afterwards, such that the profile*

$$\sigma_1^* = \begin{cases} x^* = E(d_2 \mid h_t) & \text{if } k \geq k_o^p(t) \\ B & \text{otherwise} \end{cases}$$

$$\sigma_2^* = \begin{cases} \text{Accept} & \text{if } x \geq E(d_2 \mid h_t) \\ \text{Reject} & \text{otherwise} \end{cases}$$

constitutes the unique UE of $G^A[\delta, \theta]$.

Example 1: Let $(\theta, \delta, p) = (0.45, 0.97, 0.85)$. In this case, the type p falls above the threshold $\widetilde{p}(\delta, \theta) = 0.463$.



Stopping and Continuation regions for Example 1

The boundary $k_o^p(t)$ (the dashed curve in Figure 2) shows that the game ends in at most 18 periods since for $t > 18$, $k_o^p(t) < 0$. The thick black line characterizes the stopping region: P1 fights battles as long as the states remains below it.

Note that at $t = 0$, P1 is eager to fight. But, if the game lasts more than two periods (an event that occurs with probability 0.04), he ends up worse off than under immediate settlement even before correcting for the discounting.

3.4.2 Unavoidable confrontations

If P1 is sufficiently strong and the loss from AC is relatively low, P2 may be optimistic enough to make conflict unavoidable. Excessive optimism may be present from the very beginning (if $p > \frac{1-2\theta}{\theta}$ agreement is impossible at $t = 0$); or it may arise after a series of defeats on a conflict that P1 started as a simple Advantage conflict. In such cases, battles will be necessary for agreement. But if bad luck in the battlefield is pervasive, P1 resorts to AC.

In this Section we will thus make the following assumption.

Assumption A2 (Unavoidable confrontations): θ is such that at some states of the game agreement is impossible, i.e. $H \nsubseteq S$.

Denote by $G^U[\delta, \theta]$ the game satisfying A1-A2.

The main difference with the case of Advantage conflicts is thus the fact that for states $h_t \notin S$, the immediate reward function dictates P1 to stop by triggering AC, i.e. $r(k,t) = \theta p$. Therefore, when checking weak concavity one has to consider several cases depending on the values that $r(k,t)$ and $r(k,t+1)$ take[9]:

(*i*) State $(k,t) \in S$, but $(k,t+1) \notin S$. In that case, $r(k,t) = 1 - \theta(1 - \frac{k+1}{t+2})$ and $r(k,t+1) = \theta p$. Weak concavity induces a boundary in H, denoted by

$$k_1^p(t) = \left\{ k \in \mathbb{R} \quad / \quad 1 - \theta(1 - \frac{k+1}{t+2}) = \delta p[1 - \theta(1 - \frac{k+2}{t+3})] + \delta(1-p)\theta p \right\} \tag{3.7}$$

If $k \geq k_1^p(t)$, then $r(k,t)$ is weakly concave.

(*ii*) State $(k,t) \notin S$, but $(k+1,t+1) \in S$. In this case, $r(k,t) = \theta p$ and $r(k+1,t+1) = 1 - \theta(1 - \frac{k+2}{t+3})$. By the same token denote as

$$k_2^p(t) = \left\{ k \in \mathbb{R} \quad / \quad \theta p = \delta p[1 - \theta(1 - \frac{k+2}{t+3})] + \delta(1-p)\theta p \right\} \tag{3.8}$$

If $k \leq k_2^p(t)$, then $r(k,t)$ is weakly concave. Note that weak concavity implies here that P1 will stop the game by triggering AC.

Recall that the sequence $k_o^p(t)$ applies to the case when agreement is possible at (k,t) and $(k,t+1)$. H is partitioned in three regions; in order to characterize the Stopping region weak concavity must be checked through the corresponding boundary at each state .

The boundary $k_o^p(t)$ applies only to states satisfying

$$k \geq (1 + p - \frac{1}{\theta})(t+3) - 1,$$

[9] If $r(k,t) = r(k+1,t+1) = \theta p$. Then the optimal decision is always to stop at (k,t).

because $(k, t+1) \in S$. For k's below this threshold the boundary $k_1^p(t)$ starts being relevant. It is so until

$$k \leq (1 + p - \frac{1}{\theta})(t+2) - 1, \tag{3.9}$$

because at states satisfying this inequality the bargaining range is empty at (k, t) and P1 will trigger AC in case he prefers to stop (note that this is just the reformulation of expression (3.2)). For k's below this second threshold, $k_2^p(t)$ becomes the relevant boundary and if k fails to step above it P1 triggers AC.

The preceding analysis essentially describes P1's equilibrium behavior. The game is essentially an Advantage conflict as long as P1 obtains sufficiently many victories to keep the bargaining range non-empty. But if P1 is systematically defeated, and P2 becomes too optimistic in consequence, he may lose hope of extracting surplus before $\bar{t}$. So P1 will prefer to stop gambling and terminate the game in AC. This is formally stated in the following Proposition.

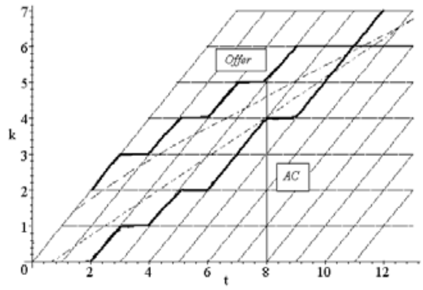
Proposition 34 (UE in Unavoidable confrontations) *Suppose that A1-A2 hold and consider an UE. Then there exists two sequences of integers* $\{\overline{k}^p(t)\}_{t=1}^{\bar{t}}$ *and* $\{\underline{k}^p(t)\}_{t=1}^{\bar{t}}$, *where the latter is non-decreasing, satisfying that* $\overline{k}^p(t) \geq \underline{k}^p(t)$ *for any* t, *such that the strategy profile*

$$\sigma_1^* = \begin{cases} x^* = E(d_2 \mid h_t) & \text{if} \quad k \geq \overline{k}^p(t) \\ AC & \text{if} \quad k < \underline{k}^p(t) \\ B & \text{otherwise} \end{cases} \tag{3.10}$$

and $\sigma_2^* = \{Accept\}$ *if and only if* $x \geq E(d_2 \mid h_t)$ *constitutes the unique UE of* $G^U[\delta, \theta]$.

A battle is a risky gamble that P1 may use to attain advantageous positions in bargaining. However, if these limited confrontations do not help to create a bargaining range soon enough, the delay they introduce provokes the total breakdown of negotiations and precipitates AC.

Example 2: Let $(\theta, \delta, p) = (0.75, 0.97, 0.85)$. The black thick lines in Figure 2 are the boundaries $\overline{k}^p(t)$ and $\underline{k}^p(t)$. In between, P1 finds worthy to trigger additional battles. Note that the game ends in at most eight periods.



Stopping and continuation regions for Example 2.

At $t = 0$ the bargaining range is empty and P1 tries to generate a non-empty range of possible agreements by fighting one battle. At state $(1,1)$ agreement is no longer impossible but there P1 finds incentives to improve his position, so he prefers to trigger an additional battle. On the contrary, if he is defeated in the first two battles, he knows that he will not do better for the rest of the game, so he takes the outside option. In fact, AC is the outcome of the game with probability 0.07.

3.5 Related literature

The present paper is related with two lines of research: The economic literature on bargaining under incomplete information and the literature on conflict, both in Economics and Political Science.

The term *bargaining in the shadow of power* was introduced by Powell (1996) as a mean to refer to those situations where agreements in negotiations must mirror the structure of disagreement. This idea has been explored in the last years from a wide variety of perspectives: In the spirit of Nash's bargaining problem Anbarci et al. (2002) analyze a model where the threat point is endogenously determined by agents' investments in arms; they modify favorably the location of the threat point but are wasteful. The authors compare several bargaining solutions and show that Equal Sacrifice is the one that induces the lower loss of efficiency. On their side, and following an axiomatic approach, Esteban and Sakovics (2002) introduce the concept of *disagreement function.* This function determines the threat point for each possible set of feasible outcomes, embedding thus the power relationship between the parties. They characterize a solution, the Agreement in the Shadow of Conflict (ASC) as the result of a sequence of partial agreements.

Some non-cooperative bargaining games with complete information also take into account the issue of power: Horowitz (1993) models land reform as a gradual process where landowners and peasants can accept or contest by arms the allocation proposed by the social planner: Land reform is thus optimal response of the landowning class to the threat of expropriation (a sort of AC since victor takes all loser's land). As expected in a complete information setting, a safe reform plan always exists but it reflects the balance of power. On the other hand, the model proposed by Fearon (1996) obtains similar results in a different setting: In his model two states bargaining over a piece of territory. Concessions accelerate because winning probabilities augment as the country obtains more land, contrary to Horowitz (1993) where they remain constant[10].

These four papers recognize that agreements will be conditioned by the structure of disagreement. But their common pitfall is that they fail to explain the actual occurrence of conflict. Hence, incomplete information arises as an appealing explanation for the systematic difficulty of parties to reach peaceful agreements[11].

In this context, the contributions by Myerson and Satterthwaite (1983) and Banks (1990) are crucial: Taking a mechanism design approach, i.e. focusing on incentive compatibility constraints, they were able to derive properties of the Bayesian equilibria of any bargaining game where conflict is an outside option. Banks' (1990) results, namely that the more powerful the informed agent the higher his equilibrium payoff and the more likely the war, have been paralleled by models in extensive form. In the same lines, Bester and Wärneryd (1999) show that under two-sided incomplete information and if the loss due to conflict is sufficiently low, it is impossible to design a *peaceful mechanism*, i.e. a mechanism that assigns zero probability to conflict.

Brito and Intriligator (1985) develop a model of conflict and war that is strongly related with the pioneer work of Sobel and Takahashi (1983): There is private information about the costs of going to war. When receiving offers, the informed party has incentives to misrepresent his type by trying to look tough. War is thus the result of a *separating strategy* taken by the uninformed party. On his side, Powell (1996) presents an alternating offer game where players can impose a settlement at a cost players have private information about. In this game, like in ours, when parties become too pessimistic they take the outside option of conflict.

All the models above depict conflict simply as a costly lottery: Either one party or the other wins and captures the surplus. Thus, invoking conflict is a game-ending move, an alternative to the bargaining process.

Clausewitz (1832) coined the terms of Absolute and Real war we heavily borrow. He was the first one in noticing the distinction between those wars that seem uniquely intended to the destruction of the enemy from those that are "simply a continuation

[10]Note that this model could be translated to the Esteban and Sakovics (2002) framework by constructing the adequate disagreement function.

[11]For a very exahustive survey of this issue see Ausubel et al. (2001). However, t does not need to be the unique explanation: Garfinkel and Skaperdas (2000) show that conflict can also occur under complete information.

of politics by other means."[12]

Georg Simmel reconsidered this provocative idea and his work "The Sociology of Conflict" (1904, p. 501) he pointed out the following paradox:

> "the most effective prerequisite for preventing struggle, the exact knowledge of the comparative strength of the two parties, is very often attainable only by the actual fighting out of the conflict".

Geoffrey Blainey (1973) pursued this idea and asserted that "war itself provides the stinging ice of reality", because it helps to solve the optimism arising from conflicting expectations about the outcome of war. Furthering this reasoning, Wittman (1979) noted that if conflict is a source of information, war would occur also if there is no optimism because parties can use confrontation to extract better terms from the opponent. However, all these contributions did not provide any foundation to explain *why* parties should go to conflict when they disagree on the perception of their relative strength[13].

Wagner (2000) developed the incomplete information approach underlying Simmel and Blainey's analysis: Absolute war is indeed an outside option. However, it is not the only way to solve a situation where a contradiction in the perceived relative strengths locates the threat point outside the bargaining set. Real war arises as a solution by providing Blainey's "stinging ice of reality". Wagner also presented a model (although he did not solve it) trying to formalize Wittman (1979) ideas[14]. But, in his analysis, the author is too biased towards the study of war. This makes him assume for instance that the option of AC is only available after Real conflicts occur. Our contention is that the fact that confrontation is part of the bargaining process is common to many other contexts under the shadow of conflict; by allowing parties to *choose* the type of confrontation they want to fight, we can gain insights on the rational foundations of conflict.

Among the several attempts to formalize Wagner's ideas the two most relevant and closest contributions are the papers by Filson and Werner (2002) and Slantchev (2002). Like ours, these models introduce one-sided incomplete information but model conflict as a succession of battles only. In the former the uninformed party is the one who makes offers. They are not final, so rejection transmits information along with

[12] However, he failed to articulate these two apparently opposite motivations. See Gallie (1978).

[13] Vey interestengly, this idea is also hidden in the economic literature: Schnell and Gramm (1982) study empirically how unions learn by striking: Lagged strike experience is shown to reduce the propensity to strike.

[14] This lack of formalization allows Wagner to make risky statements like "bargaining does not occur until states no longer have an incentive to reveal information by fighting" (p. 472). Although our model supports it, it is very far from being an obvious statement.

battles, which are resource consuming in case of losing. When her resources run out, a player can no longer fight. However, and in spite of considering only two types of informed players, the game become so complex that the authors only present the results for three-period games. In Slantchev (2002) parties use the alternating-offers protocol so the informed party screens her opponent and the uninformed signals her type through non-serious offers. Battles help to advance in a finite scale of positions. Conflict is over when a player controls all of them.

Apart from their complexity, fighting makes no sense in both models once beliefs have sufficiently converged to avoid optimism. Moreover, the *physical* meaning given to battles limit their role during the bargaining process as long as the total defeat they may induce is near. In our model and by abstracting from a particular interpretation, we are in addition able to explain the use of confrontation in situations where there is no optimism, but just uncertainty.

Finally, the informed party's optimal stopping problem is related to the literature on dynamic programming and optimal stopping problems. In particular, the game that we propose is similar to the discrete time version of the model of job matching proposed by Jovanovic (1979) and presented in Stokey and Lucas (1989): In that case an employer and a worker get matched but they do not know the quality of the match. They can learn about it by observing the output produced so far. When the expectation of the quality of the match falls below a given target, the match is terminated. One important difference with our paper is that our game can end in agreement.

Chapter 4

Empirical Evidence

> "The aggressor is always peace-loving; he would prefer to take over our country unopposed. To prevent his doing so one must be willing to make war and be prepared for it. In other words it is the weak who should always be armed."

Carl von Clausewitz (1832), *On war.*

4.1 Introduction

Power is very difficult to measure. However, measuring it is essential for reaching peaceful agreements: If agents hold incompatible (too optimistic) expectations about the balance of power no settlement can satisfy them at the same time. They prefer conflict. Paradoxically, the actual exercise of power through conflict is very often the only way to appraise it (Simmel (1904).)

The aim of this chapter is to test the implications of the bargaining model proposed in the preceding Chapter by means of a duration analysis approach. The main feature of that model was to approach the role of conflict in negotiations from the mentioned perspective: If parties can engage in limited confrontations, these violent trials become a channel to transmit information about strength. Since their outcome is not manipulable (no one is interested in losing), conflict becomes thus part of the bargaining process. By fighting, individuals, unions or nations can signal credibly their power; and take advantage, if they succeed, to change favorably the opponent's beliefs.

The informational role of conflict has necessary implications on how contenders behave and, by extension, on the onset, duration and termination of real-world conflicts. In an international relations framework Werner (1999) pointed out that if war is an informational device a new war should be less likely to erupt between countries that have already fought. Furthering this reasoning, one should conclude that the probability of a new conflict should be even lower if the previous war lasted long since protracted confrontations make estimates of true strengths sharper. This observation is our starting point. But we will focus instead on the intra-war implications of the analysis above: Whenever "learning by fighting" is relevant, conflicts should be more likely to end the more they last. They must exhibit an increasing hazard rate (positive duration dependence) in Duration analysis language.

We test the informational hypothesis by means of a sample of 82 Extra-systemic wars between 1817 and 1988. These wars were fought between a state and an entity that did not belong to the interstate system at that moment (a tribe, a colony, a protectorate). We focus on this kind of conflicts for two reasons: They are the best suited for testing our hypothesis because their asymmetric nature and the plausible presence of incomplete information. Moreover, this data-set has not been subject to Duration analysis so far[1].

Wars have been extensively subject to this sort of empirical analysis. Interstate wars have been approached from a wide variety of perspectives, as many as hypoth-

[1] The only exception is Ravlo et al. (2003) who use this dataset to test the Democratic peace hypothesis.

esis have been tested. In this aspect, the closest paper to ours is Bennett and Stam (1996) who where also mainly interested both on duration dependence and on the effect of several variables on duration. They concluded that interstate wars do not present duration dependence, i.e. they display a constant hazard rate. Hence, our hypothesis would not hold for such wars. They claim that residual duration dependence is the fruit of unexplained factors and, in consequence, that their model is well specified. However, these authors fail to account for the well-documented differences of behavior of democracies between autocracies (as they note in Bennett and Stam (1998)). Furthermore they do not justify the proportional (time invariant) effect of variables. Nevertheless, they are correct in observing that if the analysis uncovers duration dependence, researchers must argue that it is in fact due to the conjectured causes.

Here we pursue a double approach. First, we perform a preliminary non-parametric in order to uncover the possible particularities of our data: We find that the traditional classification into colonial (fought against a colony or dependency) and imperial wars (against independent entities) is not relevant from a duration analysis perspective. We follow instead a slightly modified version of the temporal classification proposed by Ravlo et al. (2003). We also obtain evidence of different patterns of behavior between those wars that ended in agreement and those which did not. According to the theoretical model, we should expect that the former type (called *Advantage conflicts* there) presents a stronger bargaining component and should display an increasing hazard rate. On the contrary, those that ended in the total victory of one of the sides (or at least part of them) were closer to pure military contests where incomplete information was absent or of minor relevance. The sharp differences revealed by the non-parametric analysis support this hypothesis.

Second, we pursue a discrete-time duration analysis. We incorporate several explanatory variables as proxies for the parameters of the theoretical model. In order to disentangle the source of duration dependence, we employ the evidence previously obtained and estimate a *competing risks* model where wars are classified according to the type of ending, agreement vs no agreement. The suspicion raised above is confirmed, supporting the informational hypothesis: Wars that ended in agreement have positive duration dependence whereas those that ended in total defeat display a constant hazard rate. Our contention is that the failure to distinguish these two cases contributed to the results displaying decreasing or constant pooled hazard rates in previous empirical analysis.

Other hypothesis are also supported: The more powerful the informed party, the shorter the war and the more likely they end in no agreement; moreover, the hazard also increases with the costs of fighting.

4.2 Hypothesis to test

In our theoretical model parties trigger conflict in disagreement, but they can choose the type of conflict they want to fight: An Absolute conflict (AC henceforth) that ends the game but destroys some part of the surplus, or a battle that makes the game proceed to the next period. The outcome of the latter is a function of the relative strength of the parties in AC that is known only by the party that makes the offers. Thus the informed player can change the beliefs of the uninformed by triggering a series of battles. Conflict becomes a double-edged information revelation process: It may help parties to settle because it reduces optimism, but it may introduce delay as well. Once agreement is possible, the informed party has incentives to keep fighting in order to attain even more advantage.

There are however two factors that limit the use of conflict as a mean to bargain: On the one hand, the returns of conflict as an informational device decrease over time. This is a natural property of Bayesian updating; one additional victory once a thousand battles have been fought induces a negligible change in beliefs. On the other hand, in the long run, the more the parties fight the sharper their estimates of the true balance of strengths (they converge in probability to the actual value), so the closest they are to a complete information scenario where agreement is immediate. These two reasons suggest that conflict is a self-limiting phenomenon and allow us to state the main hypothesis we want to test:

H1:The more a conflict lasts, the more likely it ends.

This will be the main focus of our empirical analysis. As we will see below, this hypothesis corresponds to the concept of an increasing hazard rate in Duration analysis language.

It is conceivable that some conflicts are pure contests, where incomplete information is irrelevant, with no bargaining component at all. Since the increasing hazard rates hypothesis does not apply for those, the analysis should be able to discriminate them.

The theoretical model distinguishes Advantage conflicts -where agreement is never impossible and aiming to improve the bargaining position- from Unavoidable confrontations -where excessive optimism due to unfortunate persuasion attempts empty the bargaining range. The main difference between these two scenarios is thus the possibility of reaching a negotiated settlement: The former can only end with an agreement. The latter may also end in AC if the informed party loses the hope of extracting better terms; and they are therefore more likely to end because the outside option becomes more attractive as time passes. Hence the second hypothesis we will test is

H2: Conflicts ending in agreement (Advantage conflicts) last longer.

As mentioned, one reason for the use of conflict (although not the only one) is that optimism of the uninformed party about the outcome of AC can preclude agreement: The perceived disagreement outcome may lie outside the set of feasible agreements. Therefore,

H3: The more optimistic the uninformed party, the longer the conflict.

Conflict is a mechanism of persuasion in the hands of the informed party. However, the "quality" of this mechanism determines the extent of its use. If the informed party is not sufficiently powerful he will prefer to reach an agreement immediately. Hence:

H4: The less powerful the informed party the shorter the conflict.

Finally, conflict introduces two types of costs: First, players are impatient and limited conflicts delay the game. On the other hand, absolute conflict destroys some portion of the surplus. Then, even if the informed party is very powerful, sufficiently high costs may cause immediate agreement. Therefore,

H5: The more impatient the players and the higher the costs, the shorter the conflict.

In the remainder of the paper we will develop an empirical analysis aiming to test hypothesis H1 to H5. In the next two Sections we present the methodology and the data we will employ. In Sections 5 and 6 we present the results.

4.3 Methodology

The duration of events can be seen as a random variable with its own probability distribution,

$$F(t) = \Pr(T < t),$$

specifying the probability that the random variable T takes a value lower than t. Symmetrically, the survivor function

$$S(t) = 1 - F(t) = \Pr(T > t),$$

is the probability that the duration will exceed t.

The main focus of our analysis is the hazard rate

$$h(t) = \frac{f(t)}{S(t)},$$

that can be thought as a conditional probability of event termination at duration t. The hazard rate allows us to study easily the issue of duration dependence: We say that an event exhibits positive duration dependence if the probability that an event will end shortly increases as its duration increases, i.e. $\frac{\partial h(t)}{\partial t} > 0$.

As mentioned, we will take both a parametric and a non-parametric approach: The non-parametric approach is very useful for a preliminary study of the data because it helps to uncover the presence of heterogeneity and also to correct this problem by suggesting ways to group explanatory variables.

The duration of a sample of size n can be ordered in increasing manner as $t_1 < t_2 < ... < t_n$. Define by m_j the number of completed observations at t_j and as n_j the number of events not completed before that date. Then, a natural estimator of the hazard rate is

$$\widehat{h}_j = \frac{m_j}{n_j},$$

the proportion of events "at risk" at t_j that actually ended at that date. The corresponding estimator of the survivor function is the Kaplan-Meier estimator

$$\widehat{S}_j = \prod_{i=1}^{j} (1 - \widehat{h}_j).$$

Once data is grouped, graphical analysis and several numerical test can be used to check if the survivor functions estimated for each group are significantly different.

The main object of interest for our parametric approach will be a *competing risks* model, useful to investigate multiple modes of termination. The key ingredient is the introduction of r risk-specific hazard rates

$$h_r(t) = \Pr[T = t, R = r \mid T > t].$$

Here we will consider two risks depending on whether contenders reached an agreement, coded as $r = 1$, or one of the sides was totally defeated, coded as $r = 0$. Assuming that risks are independent, the overall hazard becomes

$$h(t) = \sum_{r=0,1} h_r(t).$$

In this paper we will adopt a discrete-time duration analysis with months as the units of observation. Although wars are likely continuous-time processes, the data concerning them are collected at discrete time intervals. Since our time unit will be a month, ties among observations are likely to arise; and some continuous-time models have difficulties to deal with them. Another advantage of this approach is that, unlike continuous-time models, it imposes few distributional assumptions on the shape of the hazard rate. In particular we will assume that it follows the logistic specification

$$h_r(t) = \frac{\exp\{A_r\alpha_r + \beta_1 t + \beta_2 t^2 + ... + \beta_q t^q\}}{1 + \exp\{A_r\alpha_r + \beta_1 t + \beta_2 t^2 + ... + \beta_q t^q\}}, \tag{4.1}$$

where A_r is a vector of cause-specific covariates and α_r is the vector of associated coefficients. For simplicity we will use a data set with one observation per event, using variables measured at its beginning. The interpretation of the parameters in (4.1) is not as straightforward as in linear regression models: A positive (negative) sign implies that the variable increases (decreases) the hazard but its magnitude is better understood by e^{β_i}, that is, the *odd ratio* of variable X^i: The deviation from one can be interpreted as the increase or decrease in the likelihood of event termination caused by this variable. Duration dependence is captured by the polynomial of order q in t, also with cause-specific coefficients. Therefore, the model is quite flexible; for instance and contrary to other specifications (like Weibull) it does no restrict the hazard to be monotonic.

The competing risk model allows to distinguish the effect of the explanatory variables on each type of ending. In fact, it is more interesting to analyze the effect of the variables on the probability of exiting via a certain state conditioned on exiting at t that for termination in agreement is simply

$$P_1(t) = \frac{h_1(t)}{h_0(t) + h_1(t)}, \tag{4.2}$$

rather than on the unconditional probability (Thomas (1996).) The comparison of the effect of a change on a covariate X^i across cause specific- hazards is given by the derivative,

$$\frac{\partial P_1(t)}{\partial X^i} = \frac{h_1(t)h_0(t)(\alpha_1^i - \alpha_o^i)}{h(t)^2}. \tag{4.3}$$

That is, the conditional probability of a war ending in agreement increases with variable X^i if and only if the corresponding coefficient is greater than the one for the other no-agreement termination. We will be also interested on the effect of the selected variables on the expected waiting time until end via each state r.

4.4 The data

Population of cases and dependent variable

Our main sample consists of 82 Extra-systemic wars disputed between 1817 and 1988. We use the version 3.0 of the Correlates of War (COW) project database initiated by

Small and Singer (1982). Extra-systemic wars were those fought between a state belonging to the interstate system and an entity that did not qualify as so: The database distinguishes between Imperial wars, those fought against independent political entities, and Colonial wars, against colonies, dependencies or protectorates of the state.

A couples of remarks are in order: Our contention is that this data has not been analyzed so far because many dyadic variables cannot be measured. Given that the theoretical model is asymmetric, we do not need to rely so much on this conception of war as a bilateral phenomenon (that we do not question). Second, in this kind of wars, the concept of AC is not purely theoretical: Many of them ended with the annihilation of the non-sate entity.

The dependent variable is war duration, measured in months. We update the data of the COW database with information found in Clodfelter (1992), Dupuy and Dupuy (1993) and Goldstein (1992).

Independent variables and hypothesis

Agreement: This variable is going to be central in our analysis since it will determine the type of ending. It is a dummy taking the value 0 if the war did not end with an agreement and 1 otherwise. Our data come mainly from Clodfelter (1992), Dupuy and Dupuy (1993) and Goldstein (1992). We consider that a war did not end in agreement when the states stormed the capital of the opponent, the latter lost its autonomy or its population was annihilated; we consider as agreement even very unfavorable settlements for the losers, like the acceptance of a protectorate. We expect that wars that do not end in agreement tend to be pure military contests which last less therefore.

Average deaths: The costs of conflict in the theoretical models are two: The fixed AC loss and the discount factor. There is no clear indicator for the former. The variable proposed by Bueno de Mesquita and Siverson (1995), battle deaths per 10,000 population, seems to be endogenous to the model. Therefore we only try to proxy the discount factor through the monthly average of battle deaths for the non-state entity (the average for the state yields similar results). For this variable we use the data supplied by the COW database and Clodfelter (1992).

Democracy: Previous empirical analysis have shown that democracies and autocracies wage war differently: Whereas democracies are less likely to support long wars because the costs for their leaders increase in time (due to the existence of a public opinion and free press) they tend to fight and win more wars in the short run than autocracies (Bennett and Stam (1998)). Hence, one must control for these differences.

We employ the Polity III dataset, also obtained from Bennet and Stam (2000), to construct the widely used continuous index of democracy running from -10 to 10.

State's optimism: Following the proposal of Blainey (1973) we consider that "if a country had serious tensions it would seem to have less hope of waging a successful foreign war" (p. 81). We use several political variables proposed by Dassel and Reinhardt (1999) to measure this link between internal tensions and optimism:

- <u>General violence</u>: is a dummy variable coded 1 if a coup or civil war takes place or if the state initiates high levels of violence abroad, including interstate war.

- <u>Contested Institutions</u>: This measure proposed by Dassel and Reinhardt (1999) is a moving average from year $t-10$ to $t-1$ that takes into account those instances of "Major Abrupt Polity Changes" that are the outcome of struggle among domestic groups.

- <u>Security dilemma</u>: Is a also a moving average of the number of MID (Military Interstate Disputes) dyads were the state under consideration was a target but not the initiator.

The effects of these variables is far from unambiguous. Security or domestic problems will make expectations more pessimistic for states; but it is also true that if one party becomes weaker this might encourage the opponent to use confrontation in order to gain even more concessions.

Relative strength or balance of power: As mentioned above, many of the possible proxies for this concept cannot be dyadic due to the lack of statistical information. We consider several unilateral variables that can be conceptualized as dyadic measures if all the non-state entities are assumed to be of the same strength.

- <u>Capabilities</u>: We use the COW composite index as indicator of the state power. If multiple states fought one war we summed their indexes.

- <u>Population</u>: We measure the total states' population in hundreds of billions.

- <u>Military personnel</u>: Measures the state's total military personnel in millions of soldiers.

- <u>Military Quality</u>, that estimates the quality of the state's army by the military expenditure per unit of military personnel. Prices indexes and exchange rates were provided by Global Financial Data Inc.

The main components of these three variables were obtained from the version 2.25 of the EUGENE program (Bennett and Stam (2000).) Finally we consider the following dyadic variable

- <u>Ratio of battle deaths</u>: We divide the state's battle deaths by the total of battle deaths[2].

[2] When there were multiples states figthing, we created a weighted average for the last four variables

We expect that these four variables increase the hazard rate.

Type of war: Motivated by the non-parametric results below, we separate our observations among Colonial, Imperial and Decolonization wars.

Previous disputes: We count the average number of years with disputes between the considered state in the 25 years before each war. Our hypothesis is that the more the disputes, the shorter the war because part of the "learning" process is already done.

4.5 Preliminary analysis: Non-parametric approach

Non-parametric approaches are very useful both for suggesting a particular specification and for uncovering problems of the data to be corrected. In this Section we use the Kaplan-Meier (KM henceforth) estimates of the survivor function and show that these problems exist indeed.

Figure 1 displays the KM survivor function following the COW classification that distinguishes between colonial wars - fought between a state and one of its colonies- and imperial wars, against independent entities. It seems that there are clear differences in duration: Colonial wars tend to survive more.

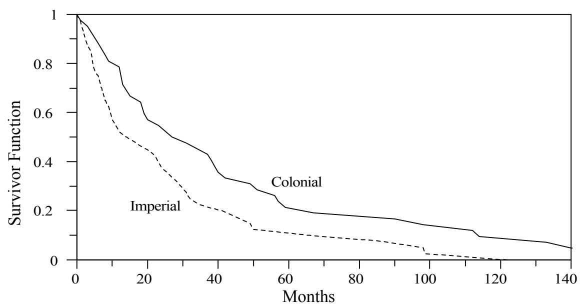


Figure 1: Survival functions according to COW classification

However, if one separates from the sample those wars occurred after 1918 (that we call Decolonization wars) but maintain the COW classification for the rest of observations, one can see that the two categories proposed are arbitrary from the point of view of duration analysis. This is displayed in Figure 2.

whereby a state's contribution is proportional to its capability index.

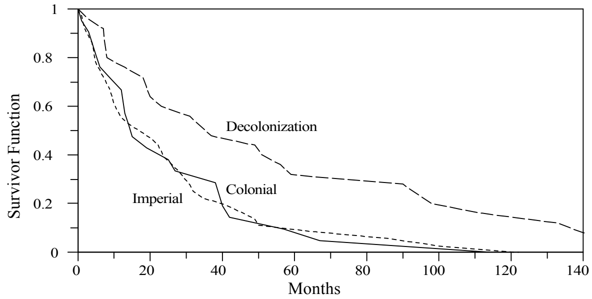


Figure 2: Survivor functions folowing COW classification except after 1918

Four different numerical tests (Savage, Breslow, Tarone-Ware and Prentice) cannot reject the null hypothesis of no differences between the survivor functions of Colonial and Imperial wars. Nevertheless, significant differences exist between each of these two survivor functions and the one of later wars. Therefore, we conclude that from the perspective of duration analysis, the qualitative classification is irrelevant.

We adopt a temporal classification instead. Following the analysis of this kind of conflicts in Ravlo et. al (2003), we propose three categories: We say that a war is Colonial if it is fought in the period 1816-1870, when wars were mainly wars of conquest. Imperial if it was fought in the period 1871-1918, when the colonized areas were expanded; and we say that a war is of Decolonization otherwise. Their survivor functions are displayed in Figure 3.

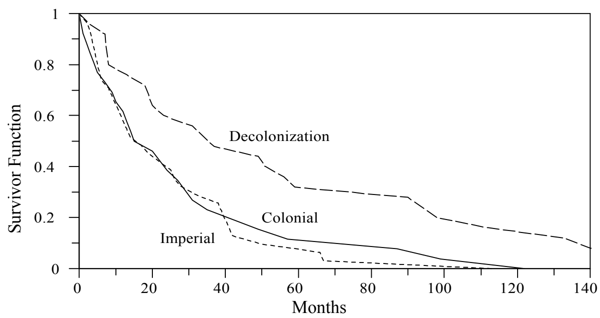


Figure 3: Survivor functions folowing temporal criterion.

Intuitively, grouping all observation may be dangerous: Conflicts in the sample are too heterogeneous to be treated equally, they are structurally different. In fact, as can be seen in the Table 1, the median duration is quite different among them.

Type of war	*Colonial*	*Imperial*	*Decolonization*
Mean duration (months)	28.88	25.16	58.84

Table 1: Mean duration by types of war.

Recent wars tend to last more. Why? One answer would be that Decolonization wars lasted longer because they had a stronger bargaining component. On the contrary, Colonial and Imperial wars were based mainly on "take it or leave it offers" since states had high chances of reaching total victory.

However this does not seem to be confirmed by the data. Table 2 classifies wars according to their type of ending and our temporal classification.

End \ Type of war	*Colonial*	*Imperial*	*Decolonization*	*Total*
Agreement	14	13	13	40
No agreement	12	18	12	42
Total	26	31	25	82

Table 2: Ends of wars by type.

The Chi-square test on this stable cannot reject the null hypothesis of independence between the type of wars and the termination mode. The reason for different duration patterns seems to be then that the balance of strengths between contenders become more even as time passed. Sharp differences do emerge regarding this latter dimension. as Table 3 shows

Type of ending	*No agreement*	*Agreement*
Mean duration (months)	25.1	48.7
Median duration (months)	17.5	31.5

Table 3: Mean and median duration according to type of ending.

As expected, conflicts that ended in total defeat of one of the sides tended to be shorter than those wars that ended in agreement. This pattern can be confirmed through the survivor functions of the wars according to how they ended. Those are displayed in Figure 4.

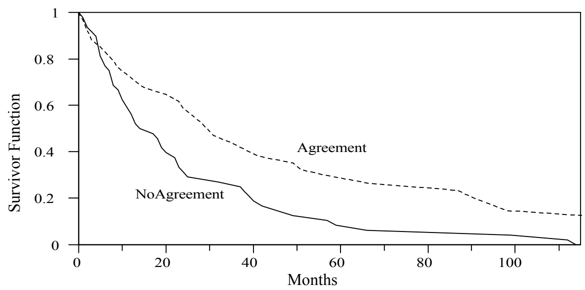


Figure 4: Survivor functions according to type of ending.

The preceding figure also shows that, apart from differences in duration, the survival patterns of wars are not very different across termination modes. But note that these functions were estimated by separating populations; in order to uncover possible differences on behavior one should take into account that all observations and treated as censored those observations with termination modes different from the one we are analyzing.

Figure 5 displays the cumulative hazard rate using this procedure. The cumulative hazard gives hints about duration dependence: If it is convex (concave) it means that the hazard is increasing at an increasing (decreasing) rate.

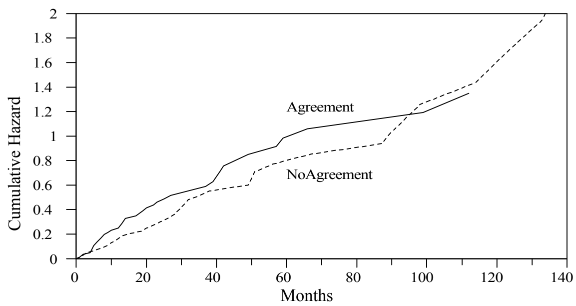


Figure 5: Cumulative hazard according to type of ending.

Wars that ended in no agreement seem to display a concave hazard rate. The opposite holds for wars that ended in agreement. This evidence suggests the adoption of a competing risks formulation in the ensuing parametric analysis.

Finally it is worth mentioning the effect of the state regime type. Table 4 shows the mean duration of wars according to regime type[3]

Regime type	*Democracy*	*Autocracy*
Mean duration (months)	31.66	47.88
Total	57	25

Table 4: Mean duration of wars by state regime type

Why were wars fought by democracies shorter? A Chi-square test in a table similar to Table 2 shows no relation between regime type and the termination mode. This evidence challenges the idea that democracies apply peaceful norms for conflict resolution. In fact, *most of the Extra-systemic wars in our sample were fought by democracies.* Differences in duration seem to support instead the hypothesis that democracies carefully select the wars they fight, those with low costs (Bennet and Stam (1998).)

[3] An observation is classified as a democracy if its score in our measure of democracy is positive.

4.6 Parametric approach: Results

In this Section we follow a parametric procedure and assume that the cause-specific hazard rate takes the functional form (4.1). As mentioned, we pursue this analysis for two reasons (*i*) several variables are expected to have different risk-specific effects and (*ii*) rather than claiming that all Extra-systemic wars in the world display an increasing (or decreasing) hazard rate we want to check that it is due to the reasons we are aiming to test.

Table 5 presents the result for the estimates of the model with the proposed variables[4].

Table 5

Logistic competing risks model results for Extra-systemic wars data.

	Termination states	
Variables	No agreement	Agreement
Constant	-4.395(0.660)***	-5.827(0.708)***
Average Death	0.0001(0.0001)	0.0002(0.0001)*
Democracy	0.071(0.042)*	0.031(0.038)
Military personnel	-0.0006(0.033)	0.050(0.025)**
Military quality	-0.058(0.037)	-0.026(0.029)
Population	0.137(0.061)**	-1.879(1.612)
Capability index	2.761(1.551)*	3.410(1.840)*
Deaths Ratio	-0.807(0.868)	1.670(0.782)**
General violence	-0.685(0.377)*	-0.916(0.417)**
Security dilemma	0.193(0.149)	0.290(0.181)*
Contested institutions	-7.127(5.921)	1.399(0.384)
Imperial war	0.732(0.393)*	0.876(0.435)**
Previous disputes	-0.120(1.549)	-1.741(1.683)
Time interaction	-0.002(0.006)	0.020(0.005)***
Log likelihood	-400.67	
$-2(L_{null} - L_{model})$	64.249	
N	82	

Note: Numbers in parentheses are standard errors. One asterisk indicates p<0.10, two indicate p<0.05 and three indicate p<0.01.

[4] The program TDA (Transition Data Analysis) by Blossfled and Rohwer (1995) was used to perform the estimation.

Duration dependence is captured by the time interactions coefficients. Our strategy was to estimate models with increasing degrees of the polynomial: There was a significant improvement when introducing a first-degree interaction. This was not the case when a quadratic specification was added so the estimated model is a polynomial of degree one.

Previous estimations also show that the temporal classification introduced in the previous section is only partially significative: Once Imperial wars are selected as the reference group, the null hypothesis of equal coefficients of the dummies for Colonial and Imperial wars cannot be rejected. Therefore, we only include the dummy for Imperial wars.

The most important result displayed in Table 5 is the different duration dependence of the two types of ending. As hypothesized, wars that terminated in agreement display an increasing hazard rate, they are more likely to end the more they last. On the contrary, those wars that did in the total collapse of one of the parties and that we conjecture to be mainly composed by pure military contests, display a flat hazard rate. This evidence strongly supports the informational hypothesis.

This can be emphasized by paying attention to the shape of the conditional probability of exiting via agreement given by expression (4.2). In Figure 6 we display such probability with continuous covariates evaluated at the sample means and the binary variables evaluated at zero.

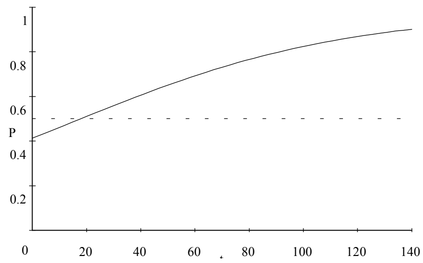


Figure 6: Conditional probability of ending via agreement.

As time passes, wars are more likely to end in agreement. The conditional hazard is greater than 0.5 after roughly 18 months; once those total conflicts finished, only the conflicts where confrontation is used as an instrument to reveal and signal the actual balance of power remain. The main issue in decolonization wars was independence or autonomy and therefore they were more about bargaining than about conquering. These wars reached long durations precisely because a long confrontation was needed for the state to realize what kind of conflict was being fought, an Advantage conflict rather a simple war of conquest.

With respect to the rest of hypothesis, the effects of the variables can be better understood by looking at the odds ratios of the significative variables. The difference from 1 yields the percentile change in the unconditional cause-specific hazard due to an increment in one unit of the covariate.

Cause-specific odds ratios for the significative variables are displayed in Table 6. The highest cause-specific coefficients for each variable are in bold. This will allow us to analyze how conditional hazards move.

Variables	*No agreement*	*Agreement*
Average deaths	-	**1.0002**
Democracy	**1.073**	-
Military personnel	-	**1.659**
Population	**1.146**	-
Deaths ratio	-	**5.311**
Capability index	15.811	**30.278**
General violence	**0.504**	0.400
Security dilemma	-	**1.336**
Imperial war	2.079	**2.401**

Table 6: Odds ratios for significative variables

The variable Average deaths, that we associate to the cost of confrontation, presents an odds ratio slightly greater than one; it implies that an increase in one hundred monthly average casualties for the non-state entity increased in a 2% the hazard. Moreover, since the coefficient is not significative for no-agreement terminations, it implies that *higher costs make more likely a negotiated settlement*, supporting our hypothesis H5.

Similarly, an increase in one point in the democratic score increases the hazard in a 7.3%. Moreover, this effect only applies to no-agreement termination. These results support the ones in Bennet and Stam (1998): Democracies select "easy" wars and therefore the ones they fight are shorter and are more likely to be won.

Mixed evidence results from the variables aimed to proxy the effect of the balance of power. It is true that, as expected, the four of them increase the hazard, but three increase the conditional probability of exiting via agreement. Only the population variable makes more likely total victory.

As expected, variables measuring the state's optimism also yield mixed predictions. It seems that the *source* of the stress for the state is of great importance: One additional interstate dispute of the state makes the hazard grow in a 33.6% and only affect the agreement hazard; security concerns make states more prone to settlement. On the other hand, if the state is in a situation of generalized violence including domestic strife the hazard rate decreases in about a 50%; moreover, if the war ends it is less likely to do so via agreement.

The cause-specific effects of the variables can be see more clearly by analyzing the changes they induce in the expect duration of wars until termination via agreement and total victory. We increase the continuous variables in one standard deviation and set the dummies equal to one. Note that variables have an impact on the expected waiting times for both termination modes.

	No agreement	*Agreement*
Baseline expected duration	17.44	22.87
Variables		
Average deaths	-2.4	-3.9
Democracy	-2.7	-4
Military personnel	-2.6	-4.2
Population	-2.7	-4
Deaths ratio	-2.9	-4.6
Capability index	-4.12	-6
General violence	+9.6	+16.2
Security dilemma	-3.8	-5.4
Imperial war	-8	-11

Table 7: Variables effects on expected duration until exit

Most of the variables have similar qualitative and quantitative effects. They reduce in 2.5 months the expected waiting time until end via total victory and in 4 months via agreement. Stronger but similar effects are obtained if the state becomes stronger (*Capability index)* or if it is involved in interstate disputes (expected duration decreases in 4 and 6 months respectively). It is striking that if the war is an Imperial one, the expected durations decrease and differences between the competing risks dilute. Strikingly, a situation of generalized not only *increases* substantially the expected duration of wars, that come close to their sample mean duration, but also widens the gap between types of completion: It takes more time for a Extra-systemic war to terminating in a negotiated settlement.

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Appendix A

Proofs of Chapter 1

Proof of Proposition 5. To prove existence, note that the first order condition (1.1) define a *unique* best response of player i for all vectors of contributions (r_{-i}). Furthermore, this best response is a continuous function of the contributions (r_{-i}). As $(u_{ij} - u_{ik})$ is finite, condition (1.1) guarantees that r_i is bounded above by some positive real number $\overline{R}$. Now consider the function $\Phi_i(r_{-i})$ defined over $[0, \overline{R}]$ by the first order conditions. Let $\Phi = \times_i \Phi_i$.The function Φ is a continuous map from a compact space into itself, and hence admits a fixed point by Brouwer's fixed point theorem. The fixed point of the function Φ is clearly a Nash equilibrium of the game of the noncooperative game of conflict.

To prove that the equilibrium is unique, suppose by contradiction that there exist two equilibria r and r'. Without loss of generality, suppose that $R' \leq R$. Pick the index j for which the ratio $\frac{p_k}{p'_k}$ is maximal. Consider the total contributions made by players in group C_j equilibrium r. A simple summation of the individual first order conditions gives:

$$\begin{aligned} R_j &= \frac{1}{R} \sum_{i \in C_j} \sum_{k \neq j} (u_{ij} - u_{ik}) p_k \\ &= \frac{1}{R} \sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p_k \end{aligned}$$

As $\sum_{i \in C_j} (u_{ij} - u_{ik}) > 0$, total contributions of group j are positive. Now comparing total contributions made in the two equilibria r and r' we obtain

$$\begin{aligned} \frac{R_j}{R'_j} &= \frac{R'}{R} \frac{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p_k}{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p'_k} \\ &= \frac{R'}{R} \frac{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p'_k (p_k / p'_k)}{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p'_k} \\ &< \frac{R'}{R} \frac{p_j}{p'_j} = (\frac{R'}{R})^2 \frac{R_j}{R'_j} \leq \frac{R_j}{R'_j}, \end{aligned}$$

yielding a contradiction. ■

Proof of Proposition 6. In the γ model, Lemma 4 immediately shows that the value of every player in a coalition structure formed of singletons is lower than in the grand coalition.

In the δ model, for the noncooperative case, a direct computation gives the value $v_i^N(\{\{i\}, N\backslash\{i\}\}) = V(\frac{1}{2} - \frac{1}{8}) = \frac{3V}{8} > \frac{V}{n+1}$ for $n \geq 2$.

In the cooperative case, a simple computation shows that

$$v_i^N(\{\{i\}, N\backslash\{i\}\}) = V\frac{2+\sqrt{n}}{2(1+\sqrt{n})^2} > \frac{V}{n+1} \text{ for } n \geq 4.$$

■

Proof. To prove the Proposition, we consider the finite game of announcement of coalition sizes, and compute by backward induction the unique subgame perfect equilibrium. The proof of the Proposition relies on the following Lemma.

Lemma 35 *Suppose that $K \geq 1$ coalitions have been formed and that there are j remaining players in the game, with $j \geq 2$. Then player $(n+1-j)$ optimally chooses to form a coalition of size 1 when she anticipates that all subsequent players form singletons.*

To prove the Lemma, we compute the payoff of player $n+1-j$ as a function of the size μ of the coalition she forms, anticipating that all subsequent $j-\mu$ players form singletons.

$$F(\mu) = \frac{1}{(K+j-\mu+1)\mu} - \frac{1}{2}\frac{K+j-\mu}{(K+j-\mu+1)^2\mu^2}$$

Let $a = K + j$ and define

$$G(\mu) = \frac{F(\mu)}{F(1)} = \frac{a^2\left[-2\mu^2 + \mu(2a+3) - a\right]}{(a-\mu+1)^2\mu^2\,(a+1)}$$

and

$$h(\mu) = (a-\mu+1)^2\mu^2\,(a+1) - a^2\left[-2\mu^2 + \mu(2a+3) - a\right].$$

We will show that $h(\mu) > 0$ for all $j \geq \mu > 1$, thus establishing that the optimal choice of player $n+1-j$ is to choose a coalition of size 1. We first note that $h(1) = 0$ and

$$h(j) = j[(j + \frac{1}{j} - 2)K^3 + j\,(j-1)\left(K^2 - 1\right)] > 0 \quad \text{as } K \geq 1 \text{ and } j \geq 2.$$

Next we compute

$$h'(\mu) = 2(a+1)\,(a+1-\mu)\,(a+1-2\mu)\,\mu - a^2\,[2a+3-4\mu]$$

and obtain

$$h'(1) = 2a(a-2) \geq 0 \text{ as } a \geq 2,$$

$$h'(j) = 2(K+1-j)[(j-1)K^2 + j^2K + j)] - (K+j).$$

Finally, we compute the second derivative

$$h''(\mu) = 2(a+1)[6\mu^2 - 6\mu(a+1) + (a+1)^2] + 4a^2$$

The second derivative h'' is a quadratic function, and the equation $h''(x) = 0$ admits two roots given by

$$\begin{aligned} x_1 &= \frac{a+1}{2} - \sqrt{\Delta}, x_2 = \frac{a+1}{2} + \sqrt{\Delta} \\ \text{with } \Delta &= 48\left[(a+1)^4 - 4a^2(a+1)\right] \end{aligned}$$

We conclude that the function h' is increasing over the interval $[-\infty, x_1]$, decreasing over the interval $[x_1, x_2]$ and increasing over the interval $[x_2, +\infty]$.

We now distinguish between two cases. If $h'(j) < 0$, as the function h' is continuous over $[1, j]$, and $h'(1) > 0 > h'(j)$, there exists a value x for which $h(x) = 0$.We show that this value is unique. Suppose by contradiction that $h'(x) = 0$ admits multiple roots over the interval $[1, j]$. As $h'(1) > 0$ and $h'(j) < 0$, there must exist at least three values $y_1 < y_2 < y_3$ with $h'(y_1) = h'(y_2) = h'(y_3) = 0$ and $h''(y_1) < 0, h''(y_2) > 0, h''(y_3) < 0$. However, our earlier study of the second derivative established that there exist no values satisfying these conditions. Hence, there exists a unique root $x^*$of the equation $h'(x) = 0$ in the interval $[1, j]$ and $h'(x) \geq 0$ for all $x \in [1, x^*], h'(x) \leq 0$ for all $x \in [x^*, j]$. Hence, the function h attains its minimum either at $\mu = 1$ or $\mu = j$ and as $h(j) > h(1) = 0, h(\mu) > 0$ for all $j \geq \mu > 1$.

If now $h'(j) > 0$, we necessarily have $j < K + 1$. Hence, $j < \frac{a+1}{2} < x_2$. In that case, we show that there is no value $x \in [1, j]$ for which $h'(x) = 0$. Suppose by contradiction that the function crosses the horizontal axis. Then there exists at least two values $y_1 < y_2 < x_2$ for which $h'(y_1) = h'(y_2) = 0$ and $h''(y_1) < 0, h''(y_2) > 0$. Our earlier study of the second derivative h'' shows that there exist no values satisfying those conditions. Hence $h'(\mu) > 0$ for all $\mu \in [1, j]$ and as $h(1) = 0, h(\mu) > 0$ for all $j \geq \mu > 1$, completing the proof of the Lemma.

We now use the preceding Lemma to finish the proof. We first claim that, in a subgame perfect equilibrium, after any coalition has been formed, all players choose to form singletons. The proof of this claim is obtained by induction on the number j of remaining players. If $j = 1$, the result is immediate. Suppose now that the induction hypothesis is true for all $t < j$. By the induction hypothesis, in equilibrium, all players following player $(n-j+1)$ form singletons. By the preceding Lemma, player $(n-j+1)$ optimally chooses to form a coalition of size 1.

Finally, consider the first player. In a subgame perfect equilibrium, she knows that players form singletons after she moved. Hence, she computes her expected profit as

$$\begin{aligned} F(\mu) &= \frac{1}{(n-\mu+2)\mu} - \frac{1}{2}\frac{n-\mu+1}{(n-\mu+2)^2\mu^2} \\ &= \frac{(n-\mu+1)(2\mu-1)+2\mu}{2(n-\mu+2)^2\mu^2}. \end{aligned}$$

To show that $F(\mu) < F(n+1)$ for all $\mu < n+1$, notice first that

$$n+1 \leq \mu(n-\mu+2),$$

as the left hand side of this inequality defines a concave function of μ, which is increasing until $\mu = \frac{n}{2}+1$, then decreasing and attains the values $n+1$ for $\mu = 1$ and $\mu = n+1$. We thus have:

$$\begin{aligned}\frac{(n-\mu+1)(2\mu-1)+2\mu}{2(n-\mu+2)^2\mu^2} &\leq \frac{(n-\mu+1)(2\mu-1)+2\mu}{2(n-\mu+2)\mu(n+1)} \\ &< \frac{2\mu(n-\mu+2)}{2(n-\mu+2)\mu(n+1)} = \frac{1}{n+1},\end{aligned}$$

establishing that the first player chooses to form the grand coalition. ■

Proof of Proposition 7. The proof of the proposition amounts to showing that the sum of utilities of all agents is higher in the grand coalition than in any efficient structure and does not distinguish between the cooperative and noncooperative cases. In fact, both in the cooperative and noncooperative cases, for any coalition structure π,

$$\begin{aligned}\sum_i v_i(\pi) &= nV - \sum_i \sum_j p_j f(|i/n - m_j|) - \sum_i c(r_i) \\ &\leq nV - \sum_i \sum_j p_j f(|i/n - m_j|).\end{aligned}$$

Now, reversing the order of summation,

$$\sum_i \sum_j p_j f(|i/n - m_j|) = \sum_j p_j \sum_i f(|i/n - m_j|)$$

We will show that for any median midpoint m_j,

$$\sum_i f(|i/n - m_j|) - \sum_i f(|i/n - 1/2|) \geq 0,$$

so the highest sum of utilities is obtained when the grand coalition is formed, the policy chosen is 1/2 and no resources are dissipated in the conflict.

The computation of the sum of utilities depends on the parity of the cardinal of the coalition C_j and the total number of players, $n+1$. A straightforward computation shows that

$$\begin{aligned}\sum_i f(|i/n - m_j|) &= \sum_{i \leq m_j} f(m_j - i/n) + \sum_{i \geq m_j} f(i/n - m_j) \\ &= \sum_{t=1}^{m_j} f(t/n) + \sum_{t=1}^{n-m_j} f(t/n) \text{ if } |C_j| \text{ is odd} \\ &= \sum_{t=0}^{m_j-1/2} f(\frac{2t+1}{2n}) + \sum_{t=0}^{n-1/2-m_j} f(\frac{2t+1}{2n}) \text{ if } |C_j| \text{ is even.}\end{aligned}$$

Similarly,

$$\begin{aligned}\sum_i f(|i/n - 1/2|) &= 2\sum_{t=1}^{n/2} f(t/n) \text{ if } n \text{ is even}\\ &= 2\sum_{t=0}^{(n-1)/2} f(\frac{2t+1}{2n}) \text{ if } n \text{ is odd.}\end{aligned}$$

Without loss of generality, we suppose that $m_j \leq 1/2$. If $|C_j|$ and $n+1$ are odd, we compute

$$\begin{aligned}\sum_i f(|i/n - m_j|) - \sum_i f(|i/n - 1/2|) &= 0 \text{ if } m_j = 1/2\\ &= \sum_{t=n/2+1}^{n-nm_j} f(t/n) - \sum_{t=nm_j+1}^{n/2} f(t/n) \ \geq 0\\ \text{if } m_j &< 1/2\end{aligned}$$

where the last inequality is obtained because f is increasing. If $|C_j|$ and $n+1$ are even, we obtain

$$\begin{aligned}\sum_i f(|i/n - m_j|) - \sum_i f(|i/n - 1/2|) &= 0 \text{ if } m_j = 1/2\\ &= \sum_{t=n/2+1/2}^{n-1/2-nm_j} f(\frac{2t+1}{2n}) - \sum_{t=nm_j-1/2}^{n/2-1/2} f(\frac{2t+1}{2n}) \geq 0\\ \text{if } m_j &< 1/2.\end{aligned}$$

Next suppose that $|C_j|$ is odd and $n+1$ is even. By convexity of the function f,

$$2f(\frac{2t+1}{2n}) \leq f(t/n) + f((t+1)/n).$$

Hence,

$$2\sum_{t=0}^{(n-1)/2} f(\frac{2t+1}{2n}) \leq f(0) + 2\sum_{t=1}^{(n-1)/2} f(t/n) + f((n+1)/2n).$$

and as $f(0) = 0$,

$$\sum_i f(|i - n/2|) \leq 2\sum_{t=1}^{(n-1)/2} f(t/n) + f((n+1)/2n)$$

As nm_j is an integer and $n/2$ is not, the condition $m_j \leq 1/2$ implies that $nm_j \leq$

$(n-1)/2$. Then,

$$\begin{aligned}\sum_i f(|i/n-m_j|)-\sum_i f(|i/n-1/2|) &\geq \sum_{t=1}^{nm_j} f(t/n)+\sum_{t=1}^{n-nm_j} f(t/n) \\ &\quad -2\sum_{t=1}^{(n-1)/2} f(t/n)-f((n+1)/2n) \\ &= 0 \text{ if } nm_j=(n-1)/2 \\ &= \sum_{t=(n+3)/2}^{n-nm_j} f(t/n)-\sum_{t=nm_j+1}^{(n-1)/2} f(t/n)\geq 0 \\ \text{if } nm_j &< (n-1)/2.\end{aligned}$$

Finally, suppose that $|C_j|$ is even and $n+1$ is odd. By convexity of the function f, for any $t \geq 1$

$$2f(t/n)\leq f(\frac{2t-1}{2n})+f(\frac{2t+1}{2n}).$$

Hence,

$$\begin{aligned}\sum_i f(|i/n-1/2|) &= 2\sum_{t=1}^{n/2} f(t/n)\leq f(0)+2\sum_{t=0}^{n/2-1} f(\frac{2t+1}{2n})+f((n+1)/2n) \\ &= 2\sum_{t=0}^{n/2-1} f(\frac{2t+1}{2n})+f((n+1)/2n).\end{aligned}$$

As $n/2$ is an integer and nm_j is not, the condition $m_j \leq 1/2$ implies $nm_j \leq (n-1)/2$. Hence,

$$\begin{aligned}\sum_i f(|i/n-m_j|)-\sum_i f(|i/n-1/2|) &\geq \sum_{t=0}^{nm_j-1/2} f(\frac{2t+1}{2n})+\sum_{t=0}^{n-1/2-nm_j} f(\frac{2t+1}{2n}) \\ &\quad -2\sum_{t=0}^{n/2-1} f(\frac{2t+1}{2n})-f((n+1)/2n) \\ &= 0 \text{ if } nm_j=(n-1)/2 \\ &= \sum_{t=n/2}^{n-1/2-nm_j} f(\frac{2t+1}{2n})-\sum_{t=nm_j-1/2}^{n/2-1} f(\frac{2t+1}{2n})\geq 0 \\ \text{if } nm_j &< (n-1)/2\end{aligned}$$

■

Proof of Proposition 8. For a symmetric coalition structure, consider the coalitions to the left of $1/2$, $C_1, ..., C_J$, and let $p_1, .., p_J$ denote the winning probabilities of the corresponding groups. We distinguish between two cases. (i) If C_J contains players to the right of 1/2, there are in total $2J-1$ coalitions in π, $2\sum_{j=1}^{J-1} p_j + p_J = 1$, and the coalition C_J is centered around 1/2 (ii) If C_J does not contain any player to the right of the 1/2, then there are $2J$ coalitions in π and $2\sum_{j=1}^{J} p_j = 1$. In both cases, we compute the payoff of any player $i \leq n/2$ in the coalition structure π. It turns out that the computation does not rely on a specification of the resources spent in rent seeking and hence is identical in the cooperative and noncooperative cases.

Case (i) $v_i(\pi) = V - \sum_{j=1}^{J-1} p_j(f(|i/n - m_j|) + f(1 - m_j - i/n)) - p_J f(1/2 - i/n) - c(r_i)$. Now, if $i/n \leq m_j$, by convexity of the function f,

$$f(m_j - i/n) + f(1 - m_j - i/n) \geq 2f(1/2 - i/n).$$

If $i/n \geq m_j$, by convexity of the function f,

$$f(i/n - m_j) + f(1 - m_j - i/n) \geq 2f(1/2 - m_j) \geq 2f(1/2 - i/n).$$

Hence,

$$v_i(\pi) \leq V - 2\sum_{j=1}^{J-1} p_j f(1/2 - i/n) - p_J f(1/2 - i/n) - c(r_i).$$

As $2\sum_{j=1}^{J-1} p_j + p_J = 1$,

$$v_i(\pi) \leq V - f(1/2 - i/n) - c(r_i) < V - f(1/2 - i/n) = v_i(\{N\}).$$

Case (ii). By a similar computation, we obtain:

$$\begin{aligned} v_i(\pi) &\leq V - 2\sum_{j=1}^{J} p_j f(1/2 - i/n) - c(r_i) = V - f(1/2 - i/n) - c(r_i) \\ &< V - f(1/2 - i/n) = v_i(\{N\}). \end{aligned}$$

■

Proof of Proposition 9. The fact that the grand coalition is γ immune to secession is a direct consequence of Proposition 8, as the coalition structure formed of singletons is symmetric.

To show that an extremist benefits from breaking away in the noncooperative model with linear utilities, we compute the equilibrium payoffs. Let C denote the coalition $\{1, .., n\}$. We denote by r_0 the equilibrium investment of agent 0 and by R_C the total equilibrium investments of group C. The distance between 0 and the midpoint of C is $\frac{n+1}{2n}$. Hence the first order condition for player 0 is:

$$\frac{n+1}{2n}\frac{R_C}{R^2} = r_0.$$

Now consider players in C. As long as $i \leq \frac{n+1}{4}$, player i prefers the policy choice of player 0 to the policy choice of the coalition C and contributes a negative amount:

$$r_i = \frac{r_0}{R^2}\frac{(4i - (n+1))}{2n}.$$

For $\frac{n+1}{4} \leq i \leq \frac{n+1}{2}$, player i contributes a positive amount:

$$r_i = \frac{(4i-(n+1))}{2n}\frac{r_0}{R^2}.$$

For players to the right of $\frac{n+1}{2}$, the difference in distances is

$$(\frac{n+1}{2n}) - \frac{i}{n} + \frac{i}{n} = \frac{n+1}{2n},$$

and the contribution is given by the first order condition

$$\frac{n+1}{2n}\frac{r_0}{R^2} = r_i.$$

Let r_C denote the solution to this last equation. Then

$$R_C = \sum_{i>0} r_i = r_c(Card\{i, i > \frac{n+1}{2}\} + \sum_{1 \leq i \leq \frac{n+1}{2}} \frac{4i-(n+1)}{n+1}).$$

We define

$$A(n) = Card\{i, i > \frac{n+1}{2}\} + \sum_{1 \leq i \leq \frac{n+1}{2}} \frac{4i-(n+1)}{n+1},$$

and the Nash equilibrium of the game of individual contributions can be obtained by solving the system of two equations:

$$\frac{r_c A(n)}{R^2}\frac{n+1}{2n} = r_0, \tag{A.1}$$
$$\frac{r_0}{R^2}\frac{n+1}{2n} = r_c. \tag{A.2}$$

Dividing the two equations, we obtain $r_0 = \sqrt{A(n)}\, r_c$, and equation A.1 yields:

$$r_0^2 = \frac{n+1}{2n}\frac{\sqrt{A(n)}}{(1+\sqrt{A(n)})^2}.$$

Hence,

$$\begin{aligned} U_0 &= V - \frac{\sqrt{A(n)}}{1+\sqrt{A(n)}}\frac{n+1}{2n} - \frac{n+1}{4n}\frac{\sqrt{A(n)}}{(1+\sqrt{A(n)})^2} \\ &= V - \frac{n+1}{2n}\frac{\sqrt{A(n)}(3+2\sqrt{A(n)})}{2(1+\sqrt{A(n)})^2}. \end{aligned}$$

To show that player 0 obtains a higher profit than in the grand coalition, it thus suffices to show

$$\frac{n+1}{2n}\frac{\sqrt{A(n)}(3+2\sqrt{A(n)})}{2(1+\sqrt{A(n)})^2} < \frac{1}{2}. \tag{A.3}$$

Inequality A.3 is equivalent to

$$-2A(n) + (n-3)\sqrt{A(n)} + 2n > 0.$$

As $A(n) < n$, this inequality is always satisfied for $n \geq 3$. A direct computation (Table 5) shows that the inequality is also satisfied for $n = 2$.

In the cooperative model, two cases must be considered according to the parity of the number of elements in the set $C = \{1, ..., n\}$. The first order condition for the extremist remains

$$\frac{R_C}{R^2}\frac{n+1}{2n} = r_0$$

If n is odd, the first order condition for the complement coalition is

$$\frac{r_0}{R^2}\frac{(n+1)^2}{4n} = \frac{R_C}{n}$$

and if n is even,

$$\frac{r_0}{R^2}\frac{n+2}{4} = \frac{R_C}{n}$$

In the latter case,

$$\begin{aligned} r_0 &= \frac{(2(n+1)(n+2))^{\frac{1}{4}}}{\sqrt{2(n+1)} + n\sqrt{(n+2)}}\sqrt{\frac{n+1}{2}} \\ R &= \frac{1}{2}(2(n+1)(n+2))^{\frac{1}{4}} \end{aligned}$$

and the individual payoff is

$$u_0^e = V - \frac{n+1}{4}\frac{3\sqrt{2(n+1)(n+2)} + 2n(n+2)}{(\sqrt{2(n+1)} + n\sqrt{(n+2)})^2}.$$

When n is odd, an analogous computation shows:

$$u_0^o = V - \frac{n+1}{4n}\frac{3\sqrt{2n(n+1)} + 2n(n+1)}{(\sqrt{2} + \sqrt{n(n+1)})^2}$$

It can be checked that

$$u_0^o < x_0 \Leftrightarrow \frac{n+1}{4n}\frac{3\sqrt{2n(n+1)} + 2n(n+1)}{(\sqrt{2} + \sqrt{n(n+1)})^2} > \frac{1}{2} \Leftrightarrow (3-n)\sqrt{(n+1)} + \sqrt{2n}(n-1) > 0$$

The latter expression is increasing in n and positive for $n = 1$. Hence it is always positive. In the even case

$$u_0^e < x_0 \Leftrightarrow \frac{n+1}{4}\frac{3\sqrt{2(n+1)(n+2)} + 2n(n+2)}{(\sqrt{2(n+1)} + n\sqrt{(n+2)})^2} > \frac{1}{2} \Leftrightarrow (3-n)\sqrt{(n+1)(n+2)} + \sqrt{2}(n^2-2) > 0 \tag{A.4}$$

Again the last term is increasing in n and positive for $n = 2$. We conclude that an extremist never has an incentive to break away from the grand coalition in the cooperative model. ■

Appendix B

Proofs of Chapter 2

Proof of Proposition 13. Let us denote $f(s-r^S)$ by f and so on. Moreover, let us denote $R=(sr)^m+r^{-S}$. Then, the first order condition for the maximization problem faced by $i \in S$ states

$$m(r^S)^{m-1}\frac{r^{-S}}{R^2}[\alpha_i f-\omega(1-r_i)]-\frac{(r^S)^m}{R}[(1-\lambda)\frac{s-1-r^{S\backslash i}}{s-r^S}\frac{f}{s-r^S}+\alpha_i f'-\omega]=0 \quad \text{(B.1)}$$

Now we show that all optimal decisions inside any coalition S must be the same across its members: Expression (B.1) can be rewritten as

$$m\frac{r^{-S}}{R}[\alpha_i f-\omega(1-r_i)]=r^S[(1-\lambda)\frac{1-r_i}{s-r^S}(f'-\frac{f}{s-r^S})+(1-\lambda)\frac{f}{s-r^S}-\omega] \quad \text{(B.2)}$$

Dividing (B.2) by the analogous expression a member j of S and rearranging yields

$$\frac{\alpha_i f-\omega(1-r_i)}{a_j f-\omega(1-r_j)}=1+\frac{(1-\lambda)\frac{r_j-r_i}{s-r^S}(f'-\frac{f}{s-r^S})}{(1-\lambda)\frac{1-r_j}{s-r^S}(f'-\frac{f}{s-r^S})+(1-\lambda)\frac{f}{s-r^S}-\omega} \quad \text{(B.3)}$$

Suppose that contrary to our conjecture $r_i > r_j$. Then the LHS of (B.3) must be smaller than one. Otherwise r_i would not be optimal for i because by decreasing his effort would get a higher share of a higher total output. However, the RHS of (B.3) is clearly greater than one because by concavity $f' < \frac{f}{s-r^S}$. Therefore, expression (B.3) only holds true when $r_i = r_j$.

Denote by r the individual level of effort inside S. Now we show that there is a unique value of $r > 0$ that satisfies (B.1). Let

$$g(r)=m\frac{r^{-S}}{R}[\frac{1}{s}f-\omega(1-r)]-r[(1-\lambda)(s-1)\frac{f}{s-sr}+f'-s\omega].$$

Nest we show that whenever $g(r) \leq 0$ then $g'(r) < 0$.

$$\begin{aligned} g'(r) &= -m\frac{r^{-S}}{R^2}[f-\omega(s-sr)] - m\frac{r^{-S}}{R}[f'-\omega] \\ &\quad -[(1-\lambda)(s-1)\frac{f}{s-sr} + f' - s\omega] \\ &\quad -sr[(1-\lambda)\frac{(s-1)}{s-sr}(\frac{f}{s-sr} - f') - f''] \\ &\leq m\frac{r^{-S}}{R}[\omega - f'] + sr(1-\lambda)\frac{(s-1)}{s-sr}(f' - \frac{f}{s-sr}) \\ &\leq (f' - \frac{f}{s-sr})[sr(1-\lambda)\frac{(s-1)}{s-sr} - m\frac{r^{-S}}{R}] \leq 0. \end{aligned}$$

where the last inequality follows from the fact that when $g(r) \leq 0$

$$\frac{m}{sr}\frac{r^{-S}}{R} \leq \frac{(1-\lambda)(s-1)\frac{f}{s-sr} + f' - s\omega}{f - \omega(s-sr)} < (1-\lambda)\frac{(s-1)}{s-sr}, \tag{B.4}$$

because $f' < \omega \leq (s(1-\lambda)+1)\omega$. Therefore, if $g(r)$ has a critical point or it is decreasing, it is concave. This result implies that there exist at most one r that makes $g(r) = 0$. Now we show that this r exists: $\lim_{r\to 0} g(r) = m[\frac{1}{s}f - \omega] > 0$[1] whereas by L'Hôpital rule $\lim_{r\to 1} g(r) = -[(s(1-\lambda)+\lambda)f'(0) - s\omega] < 0$ by assumption. ■

Proof of Proposition 14. Let us denote by γ the parameter of interest. By total differentiation

$$\frac{dr^S}{d\gamma} = \frac{\partial r^S}{\partial \gamma} + \frac{\partial r^S}{\partial r^{-S}}\frac{dr^{-S}}{d\gamma},$$

Define

$$H(r^S, r^{-S}) = m\frac{r^{-S}}{R}[\alpha_i f - \omega(1-r_i)] - r^S[(1-\lambda)\frac{s-1-r^{S\backslash i}}{s-r^S}\frac{f}{s-r^S} + \alpha_i f' - \omega] \tag{B.5}$$

One can repeat easily the procedure of the previous proposition and show that $\partial H(r^S, r^{-S})/\partial r_i < 0$ when $H(r^S, r^{-S}) = 0$. First we show that the best reply effort is increasing in r^{-S} : By the Implicit Function Theorem, the sign of derivative is simply given by

$$\frac{\partial H(r^S, r^{-S})}{\partial r^{-S}} = m\frac{(r^S)^m}{R^2}[\alpha_i f - \omega(1-r_i)] > 0.$$

Therefore, for any parameter γ the sign of $\frac{dr^S}{d\gamma}$ and thus the direction in which R moves when γ increases is totally described by the sign of $\partial H(r^S, r^{-S})/\partial\gamma$.

In the case of the elasticity. Condition (B.5) can be rewritten as

$$H(r^S, r^{-S}) = m\frac{r^{-S}}{R}(1-r_i)[\frac{s-r^S}{1-r_i}\frac{\alpha_i}{\varepsilon} - \frac{\omega}{f'}] - r^S[(1-\lambda)\frac{s-1-r^{S\backslash i}}{s-r^S}\varepsilon + \alpha_i - \frac{\omega}{f'}]$$

[1] If $\frac{1}{s}f < \omega$ then $g(r)$ is clearly bounded from below by a level $\underline{r}$ such that when $r \to \underline{r}$, $g(r) \to 0^+$.

and the sign of that derivative is clearly negative, so the effort is inversely related with the elasticity of output with respect to labor. Following the same procedure, the sign of the derivative of the best reply with respect to λ is given by the sign of

$$\frac{\partial H(r^S, r^{-S})}{\partial \lambda} = r^S \frac{s - 1 - r^{S\backslash i}}{s - r^S} \frac{f}{s - r^S} > 0.$$

so more egalitarian groups are more aggressive. Finally, in the case of m

$$\frac{\partial H(r^S, r^{-S})}{\partial m} > m \frac{(r^S)^m \sum_{S_k \in \pi \backslash S} (r^{S_k})^m [\ln r^{S_k} - \ln r^S]}{R^2} [\frac{1}{s} f - \omega(1 - r)].$$

Hence, the condition stated in the text is enough to obtain the desired result. For symmetric coalition structures $\ln \frac{r^{S_k}}{r^S} = 0$ for any $S_k \in \pi$ and the equilibrium level of effort is increasing in m for sure. ■

Proof of Proposition 15. It is shown in Proposition 12 (proved below) that the indirect payoff function in the joint production case is decreasing in r^{-S} then the coalition formation game is of positive (negative) externalities if given two coalition structures π and π' it happens that $r^{-S}(\pi') < (>)\ r^{-S}(\pi)$, where $\pi' \backslash \{S\}$ can be obtained by merging coalitions in $\pi \backslash \{S\}$.

The next step is to look to the convexity or concavity of the coalitional outlay with respect to size in order to discern how the overall level of hostility changes when coalition structures become coarser. Let us define $r = r^S/s$. Then $\partial r^S / \partial s = r + s \frac{\partial r}{\partial s}$. Monotone average coalitional outlay is sufficient to ensure that $(r^{S \cup T})^m \geq (r^S)^m + (r^T)^m$. One can rewrite the average coalitional outlay as a function of r, i.e. $\overline{r} = s^{m-1} r^m$, and its derivative:

$$\frac{\partial \overline{r}}{\partial s} = s^{m-2} r^m [(m - 1) + \frac{s}{r} \frac{\partial r}{\partial s}].$$

Let us first investigate the sign of $\frac{\partial r}{\partial s} = -\frac{\partial g(r)/\partial s}{\partial g(r)/\partial r}$ where

$$g(r) = m \frac{r^{-S}}{R} \frac{1}{s} f - r f'.$$

Then

$$\begin{aligned} \frac{\partial g(r)}{\partial s} &= -m^2 \frac{(sr)^{m-1} r^{-S}}{R^2} \frac{r}{s} f - m \frac{r^{-S}}{R} \frac{1}{s^2} f + m \frac{r^{-S}}{R} \frac{1 - r}{s} f' - (1 - r) r f'' \\ &= m \frac{r^{-S}}{R} \frac{1}{s^2} f [\frac{1 - p^S}{r} - (m + 1)] - (1 - r) r f'' \end{aligned}$$

when $g(r) = 0$. And

$$g'(r) = -m^2 \frac{r^{-S} (sr)^{m-1}}{R^2} f - m \frac{r^{-S}}{R} f' - f' + s r f''.$$

But when $g(r) = 0$

$$m \frac{r^{-S}}{(sr)^m + r^{-S}} f' = m^2 \frac{(r^{-S})^2}{[(sr)^m + r^{-S}]^2} \frac{1}{sr} f.$$

Then

$$\frac{\partial r}{\partial s}=\frac{m\frac{r^{-S}}{R}\frac{1}{s^2}f[\frac{1-p^S}{r}-(m+1)]-(1-r)rf''}{m\frac{(m+1)r^{-S}(sr)^{-1}}{R}f-srf''},$$

and it can be easily checked that $\frac{\partial r}{\partial s}>-\frac{r}{s}$. Then $\frac{\partial \bar{r}}{\partial s}>s^{m-2}r^m(m-2)$ and hence that is positive for any $m\geq 2$. On the other side, concavity of the coalitional outlay implies that it is sub-additive.

$$\frac{\partial (r^S)^m}{\partial^2 s}=m(r^S)^{m-1}[\frac{m-1}{r^S}(\frac{\partial r^S}{\partial s})^2+\frac{\partial^2 r^S}{\partial^2 s}].$$

For linear quadratic and exponential production functions second derivatives are:

$$\frac{\partial^2 r^S}{\partial^2 s}=\frac{2\frac{m^2(r^S)^2(r^{-S})^2}{R^2}[\theta(s-r^S)-(s-r^S)^2][\frac{mr^{-S}}{R}-(m+1)]}{[\frac{m(m+1)(r^S)^{-2}r^{-S}}{R}[\theta(s-r^S)-(s-r^S)^2]+2]^2}\leq 0,$$

$$\frac{\partial^2 r^S}{\partial^2 s}=\frac{2\frac{m(r^S)^{-2}r^{-S}}{R}[\frac{mr^{-S}}{R}-(m+1)]\alpha(1-\alpha)(s-r^S)^{2\alpha-3}}{[\frac{m(m+1)(r^S)^{-2}r^{-S}}{R}(s-r^S)^{\alpha}+\alpha(1-\alpha)(s-r^S)^{\alpha-2}]^2}\leq 0.$$

Then, $m\leq 1$ is sufficient for concavity in size. ■

Proof of Proposition 17. Under a symmetric coalition structure all coalitions exert the same effort so $p^S=\frac{1}{k}$, where k is the number of coalitions in π. Therefore, the FOC can be rewritten as

$$m\frac{k-1}{k}(s-r^S)=\alpha r^S,$$

Hence the equilibrium level of coalitional effort and the payoff for all individuals in N are

$$\begin{aligned} r^S &= \frac{n}{k}\frac{m(k-1)}{m(k-1)+\alpha k}, \\ u_i^S &= n^{\alpha-1}(\frac{1}{k}\frac{\alpha k}{m(k-1)+\alpha k})^{\alpha}, \end{aligned}$$

and it is easy to see that $u_i^S<n^{\alpha-1}=u_i^N$.

Now, under the linear quadratic production technologies we compute the best case scenario for a coalition and show that the resulting payoff is lower than the one received under the grand coalition:

By the results on comparative statics we know that in symmetric coalition structures r^S is increasing in m and therefore payoffs are decreasing. So let us fix $m=1$. Then the FOC states:

$$(k-1)(s-r^S-\theta(s-r^S)^2)-(kr^S-2\theta kr^S(s-r^S))=0,$$

and then the equilibrium level of effort as a function of the k and the parameters of the game is:

$$r^S(k)=\frac{2\theta n-k}{2k\theta}+\frac{k-2\theta n+\sqrt{(-4k\theta n-4k+4\theta^2n^2+1+4k^2)}}{2\theta\,(3k-1)}.$$

The next step is to obtain the $k \in [2, n]$ for which individual payoff is maximized. Given $p^S = \frac{1}{k}$, and $s = \frac{n}{k}$ this reduces to know when total level of labor is maximum. The derivative of $s - r^S$ w.r.t k is

$$\frac{\partial(s - r^S)}{\partial k} = -\frac{n}{k^2} - \frac{\partial r^S}{\partial k},$$

where

$$\frac{\partial r^S}{\partial k} = \frac{(6\theta n-1)\sqrt{\left(-4k\theta n-4k+4\theta^2n^2+1+4k^2\right)}+6k\theta n+2\theta n+2k-1-12\theta^2n^2}{2\sqrt{\left(-4k\theta n-4k+4\theta^2n^2+1+4k^2\right)}\theta(3k-1)^2}.$$

The latter expression only equals zero at $k = \frac{1}{3}$, so r^S is either increasing or decreasing in k. Now we evaluate the derivative at $k = 2$ in order to obtain its sign; It turns out that

$$\left.\frac{\partial r^S}{\partial k}\right|_{k=2} = \frac{(6\theta n-1)\sqrt{\left(-8\theta n+9+4\theta^2n^2\right)}+14\theta n+3-12\theta^2n^2}{50\sqrt{\left(-8\theta n+9+4\theta^2n^2\right)}\theta} > 0,$$

where the last inequality follows after some tedious algebra; the derivative is thus always positive and r^S is increasing in k. Finally $\frac{\partial(s-r^S)}{\partial k} < 0$, so the total level of labor is decreasing in k. Therefore, it remains to check that a symmetric two sided conflict is worse than universal peace

$$\begin{aligned} r^S(2) &= \frac{3\theta n - 3 + \sqrt{\left(-8\theta n + 9 + 4\theta^2 n^2\right)}}{10\theta}, \\ u_i^S &= \frac{4\theta n + 3 - 2\theta^2 n^2 + (\theta n - 1)\sqrt{-8\theta n + 9 + 4\theta^2 n^2}}{25 n\theta}. \end{aligned}$$

Next we compute the payoff under universal peace: For interior solution we need $2\theta n > 1$. Then

$$u_i^N = \frac{1}{4\theta n}.$$

If $2\theta n \leq 1$ we have a corner solution and

$$u_i^N = 1 - \theta n.$$

Now we show for each of these cases that this payoffs improve u_i^S:

(i) $2\theta n \leq 1$. The inequality $\frac{4\theta n+3-2\theta^2n^2+(\theta n-1)\sqrt{\left(-8\theta n+9+4\theta^2n^2\right)}}{25n\theta} < 1 - \theta n$ holds whenever θn is positive and smaller than 0.775, which is satisfied by assumption

(ii) $2\theta n > 1$. Simple algebra shows that the inequality $\frac{4\theta n+3-2\theta^2n^2+(\theta n-1)\sqrt{\left(-8\theta n+9+4\theta^2n^2\right)}}{25n\theta} < \frac{1}{4\theta n}$ holds for any value of θn.

Hence, $u_i^S < u_i^N$ in any symmetric coalition structure. ■

Proof of Proposition 21. As stated in the text, both characteristic functions coincide if and only if $\widehat{r}^{-S} = Min_{r^{-S}}\ u^*(r^{-S})$. Then, to prove that the indirect payoff function is decreasing in r^{-S} is a sufficient condition.

$$u^* = \frac{(r^S(r^{-S}))^m}{(r^S(r^{-S}))^m + r^{-S}}[f(s - r^S(r^{-S}))] \tag{B.6}$$

Let r' be the short hand notation of $\frac{\partial r(r^{-S})}{\partial r^{-S}}$. Then, by the envelope theorem

$$\frac{\partial u^*}{\partial r^{-S}} = -\frac{(sr)^m}{R^2}f + sr'[m\frac{(sr)^{m-1}r^{-S}}{R^2}f - \frac{(sr)^m}{R}f']$$
$$= -\frac{(sr)^m}{R^2}f < 0.$$

where the last equality follows from the fact that the terms in brackets is exactly the first order condition for the joint production problem. ■

Proof of Proposition 25. The first order conditions for the problem of a coalition of size s against $t(= n - s)$ individual players are

$$\begin{aligned} \frac{tr^1}{tr^1 + r^S}(s - r^S) &= \alpha r^S \\ \frac{(t-1)r^1 + r^S}{tr^1 + r^S}(1 - r^1) &= \alpha r^1 \end{aligned}$$

where r^1 is the effort exerted by each singleton. Now

$$\frac{tr^1}{r^S}(s - r^S) = \frac{(t-1)r^1 + r^S}{r^1}(1 - r^1),$$

yielding that

$$r^1 = r^S \frac{t - 1 - r^S + \sqrt{(t - 1 - r^S)^2 + 4(ts - r^S)}}{2\,(st - r^S)}.$$

Now we can rewrite the equilibrium winning probability for coalition S as a function of the equilibrium level of r^S :

$$p^S(r^S) = \frac{2\big(ts - r^S\big)}{2(ts - r^S) + t(t - 1 - r^S) + t\sqrt{(t - 1 - r^S)^2 + 4(ts - r^S)}}$$

Now we establish the bounds for this probability. When

$\frac{2s}{2s + t - 1 + \sqrt{(t-1)^2 + 4ts}}$ $r^S = 0$

$$p^S(0) = \frac{2s}{s + n - 1 + \sqrt{n - s - 1)^2 + 4(n - s)s}}.$$

whereas when $r^S = s$, $p^S(s) = \frac{s}{n}$. It is easy to check that $p^S(s) > p^S(0)$. Finally,

$$\frac{\partial p^S}{\partial r^S} = \frac{2t(ts - t + 1)(1 + \frac{\frac{2(r - ts)}{(ts - t + 1)} + t - r^S - 1}{\sqrt{(n-s)(n - s - 2 - 2r^S) + 1 - 2r^S + (r^S)^2 + 4st}})}{\left(2ts - 2r^S + t(t - 1 - r^S) + t\sqrt{t(t - 2 - 2r^S) + 1 - 2r^S + (r^S)^2 + 4st}\right)^2}.$$

Some algebra shows that this derivative does not equal zero in the interval $[0, s]$ and hence the winning probability in equilibrium will lie for sure in the interval $[p^S(0), p^S(s)]$. Therefore

$$u_i^S = p^S \frac{1}{s}(s - r^S)^\alpha \leq \frac{1}{n}(s - r^S)^\alpha < n^{\alpha-1} = u_i^n.$$

So the grand coalition is γ stable.

For δ stability we follow the same procedure. First order conditions are

$$\frac{r^T}{r^T + r^S}(s - r^S) = \alpha r^S, \tag{B.7}$$

$$\frac{r^S}{r^T + r^S}(t - r^T) = \alpha r^T. \tag{B.8}$$

where r^T and t are the coalitional effort and cardinality respectively of the complement coalition of S. Then

$$\frac{r^T}{r^S}(s - r^S) = \frac{r^S}{r^T}(t - r^T)$$

and

$$r^T = r^S \frac{-r^S + \sqrt{(r^S)^2 + 4t(s - r^S)}}{2(s - r^S)}..$$

Again the equilibrium winning probability as a function of r^S is

$$p^S(r^S) = \frac{2(s - r^S)}{2s - 3r^S + \sqrt{(r^S)^2 + 4t(s - r^S)}}$$

Evaluated at the extremes $p^S(0) = \frac{\sqrt{s}}{\sqrt{s}+\sqrt{n-s}}$ and $p^S(s) = \frac{s}{n}$. It turns out that $p^S(0) \lesseqgtr p^S(s)$ if and only if $s \gtreqless \frac{n}{2}$.

Again, the derivative

$$\frac{\partial p^S}{\partial r^S} = \frac{2}{\left(2s-3r^S+\sqrt{(r^S)^2+4t(s-r^S)}\right)^2}\left(s - \frac{2t(s-r^S)+r^S s}{\sqrt{(r^S)^2+4t(s-r^S)}}\right)$$

shows that $p^S(r^S)$ has no critical point in $(0, s)$. Therefore the equilibrium winning probability will lie in $[\frac{\sqrt{s}}{\sqrt{s}+\sqrt{n-s}}, \frac{s}{n}]$ if $s > \frac{n}{2}$ and in $[\frac{s}{n}, \frac{\sqrt{s}}{\sqrt{s}+\sqrt{n-s}}]$ otherwise. It immediately implies that no coalition greater or equal than half of the population will deviate. Now we show that this is not the case for small coalitions.

Manipulation of (B.7) shows that in equilibrium

$$\alpha\frac{\left(r^S\right)^2}{s-(1+\alpha)r^S} = r^T = \frac{\sqrt{(1+\alpha)^2(r^S)^2+4\alpha t r^S}-(1+\alpha)r^S}{2\alpha}.$$

Therefore we can be sure that in equilibrium $r^S < \frac{s}{1+\alpha}$. This, together with the fact that when $s < \frac{n}{2}$ $p^S(r^S)$ is decreasing in r^S implies that

$$u_i^S \geq p^S(\frac{s}{1+\alpha})(\frac{\alpha}{1+\alpha})^\alpha s^{\alpha-1} = \frac{2\alpha(\frac{\alpha}{1+\alpha})^\alpha s^{\alpha-1}}{2\alpha - 1 + \sqrt{1 + 4\alpha(1+\alpha)\frac{n-s}{s}}}$$

and it is easy to generate examples where the latter term is greater than $n^{\alpha-1}$, the individual payoff under the grand coalition. ■

Proof of Proposition 26. When $\alpha = 1$ the individual payoff is

$$u_i^S = \frac{(r^S)^m}{(r^S)^m + r^{-S}}(1 - \frac{r^S}{s}) < 1 = u_i^N.$$

So the first player in the protocol will announce $\{n\}$. When $\alpha = 0$ the game is exactly the one of Tan and Wang (2000) with n identical rivals. For the (lengthy) proof of the statement for $m \geq 2$ we refer the reader to their paper. We briefly describe where the condition stated comes from: Once one proves that at most two coalitions will form the first player in the protocol announces the coalition

$$s_1 = \arg\max_{1 \leq s_1 \leq n} \frac{s_1^{m-1}}{s_1^{m-1} + (n - s_1)^{m-1}}.$$

Condition (8) is the FOC of this problem. For the case of $m \in (1, 2)$ notice that in that case the game is of negative externalities. In that case we only need to find a size s such that $\frac{s^{m-1}}{s^{m-1}+(n-s)^{m-1}} > \frac{1}{n} = u_i^n$ because to announce s for the first player in the protocol dominates n. If the $s + 1$ player does not announce $n - s$ payoff is even higher.

Finally, as in a first-price-auction, when $m \to \infty$ in the Nash equilibrium given a π the all coalition except the biggest one will use their entire endowments in effort. The biggest one will invest $t + \varepsilon$ where t is the size of the second biggest coalition. Let us assume that in the case of ties the contest is won by the first coalition formed. Therefore, in the Bloch stable coalition structure s_1 must be the biggest coalition. However the player $s_1 + 1$-th player in th protocol is indifferent among all the announcements in $\{1, n - s_1\}$ because all yield a payoff of zero. In order to select only the coarsest stable coalition structure the player $s_1 + 1$ will announce $n - s_1$. Knowing this player 1 in the protocol will announce

$$s_1 = \arg\max_{s \in \{1,\ldots,n\}} \frac{(2s - n)^\alpha}{s} = \min\{\frac{1}{2}\frac{n}{1 - \alpha}, n\}.$$

and the payoff will be

$$u_i^{s_1} = \begin{cases} 2\alpha^\alpha(1 - \alpha)^{1-\alpha}n^{\alpha-1} & \text{if } \alpha < \frac{1}{2} \\ n^{\alpha-1} & \text{otherwise} \end{cases}$$

■

Proof of Proposition 27. In equilibrium the FOC states.

$$m(k - 1)((s - r^S)(1 - \omega) - \theta(s - r^S)^2) = kr^S(s(1 - \omega) - (s + 1)\theta(s - r^S)).$$

From here, some calculation yields the equilibrium level of effort

$$r^S = \frac{n}{2k} + \frac{(m - n - mk)k(1 - \omega) + \theta nm(k - 1)}{2\theta\,(mk + k - m + n)\,k}$$
$$\frac{\sqrt{(m - n - mk)^2k^2(1 - \omega)^2 + (m(k - n)(k - 1) - n(n + k))2\theta nk(1 - \omega) + \theta^2n^2(n + k)^2}}{2\theta\,(mk + k - m + n)\,k}$$

so one can get a (complicated) expression for u_i^S. If $\theta > \frac{1-\omega}{n+1}$ then the solution to the production problem of the grand coalition is interior and the individual payoff under free access is

$$u_i^N = \frac{(1-\omega)(1+n\omega)}{(n+1)^2\theta}.$$

Computations yield that

$$u_i^S(r^S) > u_i^N \Leftrightarrow m < \frac{(\theta n - k)(n+1) + kn(1-\omega)}{k(k-1)} \frac{n(2n\omega + \omega - n) + k(1+n\omega)}{(1+n\omega)(2n\omega + \omega - n)} = \widetilde{m}(n,k,\omega,\theta).$$

Moreover, given that $\widetilde{m}(n,k,\omega,\theta)$ is increasing in θ it is clear that one can derive a threshold $\widetilde{\theta}$ such that $m = \widetilde{m}$. Hence, the existence of an interior solution of the free access problem guarantees as well the Pareto dominance of conflict if $\frac{1-\omega}{n+1} \geq \widetilde{\theta}$. ■

Proof of Lemma 28. Let r' be the short hand notation of $\frac{\partial r}{\partial r^{-S}}$ and $r(r^{-S})$ the best response strategy of the member of S. Then indirect payoff function and its derivative with respect to r^{-S} are

$u_i^*(r^{-S}) = \frac{(sr(r^{-S}))^m}{(sr(r^{-S}))^m + r^{-S}}[\frac{1}{s}f(s - sr(r^{-S})) - \omega(1 - r(r^{-S}))]$.

$$\begin{aligned} \frac{\partial u_i^*(r^{-S})}{\partial r^{-S}} &= \frac{\partial u_i^*}{\partial r} r' + \frac{\partial u_i^*}{\partial r^{-S}} \\ &= sp^S[r'\frac{s-1}{1-r} - \frac{1}{R}][\frac{1}{s}f - \omega(1-r)] \end{aligned}$$

where:

$\frac{\partial r(r^{-S}))}{\partial r^{-S}} = \frac{\frac{m(sr)^{2m-1}}{R^2}[\frac{1}{s}f(s-sr)-\omega(1-r)]}{\frac{mr^{-S}(sr)^{m-1}}{R}[\frac{m+1}{r}-\frac{s-1}{1-r}][\frac{1}{s}f-\omega(1-r)]-f''-(\frac{(s-1)}{s-sr}+\frac{1}{s^2})(f'-\frac{f}{(s-r^S)})} > 0$. Then $\frac{\partial u_i^*}{\partial r^{-S}}$ has no clear sign because $r' > 0$. After making some computations, one can check that the sign of $\frac{\partial u_i^*}{\partial r^{-S}}$ is given by the sign of

$$\frac{-\frac{mr^{-S}(sr)^{m-1}}{R^2}[\frac{m+1}{r}-\frac{s-1}{1-r}\frac{1}{1-p^S}]][\frac{1}{s}f-\omega(1-r)]+f''+(\frac{(s-1)}{s-sr}+\frac{1}{s^2})(f'-\frac{f}{(s-r^S)})}{\frac{mr^{-S}(sr)^{m-1}}{R}[\frac{m+1}{r}-\frac{s-1}{1-r}][\frac{1}{s}f-\omega(1-r)]-f''-(\frac{(s-1)}{s-sr}+\frac{1}{s^2})(f'-\frac{f}{(s-r^S)})}$$

Unfortunately we should restrict to conditions that ensure the negative sign of this derivative. It is sufficient to show that

$\frac{m+1}{r} - \frac{s-1}{1-r}\frac{1}{1-p^S} > 0$

Finally, first order condition allows us to rewrite this condition in terms of production and conflict technology only as stated in the text. ■

Appendix C

Proofs of Chapter 3

Proof of Proposition 32. First, note that when $p > \frac{\delta}{\theta}$, the optimal stopping problem is trivial: Since $E(v \mid k, t) \leq 1$, P1 will decide to stop for sure when $r(k,t) > \delta$. Then $r(k,t) \geq \theta p > \delta$ and P1 would stop for sure at any state.

So take the case when $p \leq \frac{\delta}{\theta}$. We will find a period t such that if P1 prefers to stop at $(k+1, t+1)$ then he also does at (k,t). That is, t such that

$$v(k+1,t+1) = r(k+1,t+1) \Rightarrow v(k,t) = r(k,t) \tag{C.1}$$

We must consider two cases depending on the value that $r(k,t)$ takes:

- Case 1: Let us consider the case when $(k,t) \in S$. The necessary condition for (C.1) is
$$1 - \theta + \theta \frac{k+1}{t+2} \geq \delta(1 - \theta + \theta \frac{k+2}{t+3}). \tag{C.2}$$
And one can be sure that this hold for any $k \leq t$ when
$$\frac{(1-\theta)(1-\delta)}{\delta\theta} \geq \frac{t-k+1}{(t+3)(t+2)}.$$
Given that $(k,t) \in S$ this implies[1]
$$\frac{t-k+1}{(t+3)(t+2)} \leq \frac{1-\theta p}{\theta(t+3)}$$
Then, the condition (C.2) holds for sure if
$$t \geq \frac{1-\theta p}{1-p} \frac{\delta}{1-\delta} - 3.$$
Now define the period
$$\bar{t}_1 = \left\lceil \frac{1-\theta p}{1-p} \frac{\delta}{1-\delta} - 3 \right\rceil$$

[1] If all possible histories belong to S, one should replace this step and take $k \geq 0$.

where $\lceil \cdot \rceil$ denotes the upper integer operator. It only remains to find a period where P1 wants to stop in order to show that the game will end at t_1 since (C.1) holds for any state $h_t \in S$ from t_1 and on.

Given that $E(v \mid k, t) \leq 1$, P1 will stop for sure when $r(k,t) \geq \delta$. And observe that with a sufficient number of victories any possible state $(k, \bar{t}_1) \in S$ can be traced to a state $(k+m, \bar{t}_1+m)$ where $r(k+m, \bar{t}_1+m) \geq \delta$. Hence, for states $h_t \in S$ the game will end at most at $\bar{t}_1$.

- Case 2: For $h_t \notin S$ we follow the same procedure. If $(k+1, t+1) \notin S$ then P1 prefers to stop at (k,t), so only needs to consider the case when $(k+1, t+1) \in S$. Then the necessary condition for (C.1) is

$$\theta p \geq \delta(1 - \theta + \theta \frac{k+2}{t+3}).$$

It holds for any k when

$$\frac{1-\delta}{\delta} p \geq \frac{t-k+1}{(t+3)(t+2)}.$$

By using the fact that at $(k+1, t+1) \in S$ this is equivalent to condition

$$t \geq \frac{1-\theta p}{\theta p} \frac{\delta}{1-\delta} - 2.$$

Analogously, let

$$\bar{t}_2 = \left\lceil \frac{1-\theta p}{\theta p} \frac{\delta}{1-\delta} - 2 \right\rceil$$

denote the period such at states $h_t \notin S$ the game ends at most.

We can conclude that the game will end for sure no later than $\bar{t} = \max\{\bar{t}_1, \bar{t}_2\}$ when $p \leq \frac{\delta}{\theta}$. ■

Proof of Proposition 33. Let us treat t as a continuous variable. Then

$$k^p(t) = \frac{\delta\theta p(t+2) - (1-\theta)(1-\delta)(t+2)(t+3)}{\theta(t+2)(1-\delta) + \theta} - 1.$$

Then, the maximum is attained at

$$t^* = \frac{\sqrt{\delta(1-\theta+\theta p)}}{(1-\delta)\sqrt{1-\theta}} - \frac{3-2\delta}{1-\delta}.$$

One can check that this function is strictly concave:

$$\frac{\partial^2 k^p(t)}{\partial^2 t} = \frac{-2\delta(1-\delta)}{((t+2)(1-\delta)+1)^3}(1 + p - \frac{1}{\theta}) < 0.$$

A sufficient condition for agreement to be immediate is thus $k^p(t^*) < 0$. Tedious algebra shows that this holds true if and only if

$$p < \frac{1-\delta+2\sqrt{\theta(1-\delta)(1-\theta)}}{\theta\delta} = \widetilde{p}(\delta, \theta) \tag{C.3}$$

Finally, simple calculations yield that if

$$\theta < (1-\delta)\frac{\left(\sqrt{2}-\sqrt{\delta}\right)^2}{(2-\delta)^2} = \widetilde{\theta}(\delta) \Rightarrow \widetilde{p}(\delta,\theta) > 1. \tag{C.4}$$

■

Proof of Proposition 34. Let

$$j_t = \lceil k^p(t) \rceil .$$

Note that $k \geq j_t$ for all $t > t^*$ and $k \geq \lceil k^p(t^*) \rceil$ for any t are sufficient conditions for P1 to prefer to stop because at any posterior state weak concavity always holds. Then, from t^* and on weak concavity is sufficient and we can set $k_o^p(t) = j_t$.

Due to the lack of monotonicity of $k^p(t)$ it is possible that for some states h_t with $t < t^*$ and $k \geq j_t$, $r(k, t+1) < v(k, t+1)$. Weak concavity is thus no longer sufficient. However, this does not invalidate the existence of our separating sequence: Note that at $\lceil t^* \rceil$ continuation values are non-decreasing in k. By the same token fixed $t < t^*$, $E(v \mid k, t)$ is non-decreasing in k.

We know that given a $t < t^*$ there exists a $\widetilde{k} \leq \lceil k^p(t^*) \rceil$ such that P1 prefers to stop for any $k \geq \widetilde{k}$. At that states it holds that $r(\widetilde{k}, t) > \delta E_v(\widetilde{k}, t)$ but $r(\widetilde{k}-1, t) < \delta E_v(\widetilde{k}-1, t)$, implying that

$$r(\widetilde{k}, t) - r(\widetilde{k}-1, t) = \frac{\theta}{t+2} > \delta(E_v(\widetilde{k}, t) - E_v(\widetilde{k}-1, t)).$$

Hence, it cannot be the case that at t P1 does not prefer to stop if he obtains $k < \widetilde{k}$ victories but he does so if he obtains $k-1$. This implies existence of the separating sequence, that we keep denoting by $k_o^p(t)$. Its elements are most equal to $\lceil k^p(t^*) \rceil$ for $t < t^*$. It only remains to check that $k_o^p(t)$ is non decreasing before t^*. Suppose on the contrary that $k_o^p(t) > k_o^p(t+1)$. Then there exist at least one state (k,t) such that $k_o^p(t+1) \leq k < k_o^p(t)$. At this state, P1 prefers to fight one more battle, but he will stop for sure in the next period, implying that $k < k^p(t)$ necessarily, because weak concavity cannot hold. However, given that $k^p(t)$ is increasing in this range, $k < k^p(t+1)$ contradicting the fact that $k_o^p(t+1) \leq k$. ■

Proof of Proposition 35. Simple calculations show that $k_1^p(t)$ is also concave and attains a maximum at.

$$t^{**} = \frac{1}{1-\delta p}\left(\sqrt{\delta p(1+\theta\frac{1-\delta p}{1-\theta-\delta p(1-\theta p)})} - 1\right) - 2.$$

It crosses with $k_o^p(t)$ at the date

$$t_1 = \frac{1-\theta+\sqrt{(1-\theta)^2 - 4\delta\theta^2 p^2(1-\delta)}}{2(1-\delta)\theta p} - 3, \tag{C.5}$$

such that $k_o^p(t) \gtrless k_1^p(t)$ for $t \lessgtr t_1$. P1 becomes indifferent between triggering AC and making a definitive offer in case of losing the next battle when

$$k_o^p(t_1) = k_1^p(t_1) = (1 + p - \frac{1}{\theta})(t_1 + 3) - 1.$$

The same happens with $k_1^p(t)$ and $k_2^p(t)$. P1 is indifferent between triggering AC and making an offer at the current state when

$$k_1^p(t_2) = k_2^p(t_2) = (1 + p - \frac{1}{\theta})(t_2 + 2) - 1,$$

where

$$t_2 = \frac{\delta(1 - \theta p)}{\theta\,(1 - \delta)} - 3.$$

Again $k_1^p(t) \gtrless k_2^p(t)$ for $t \lessgtr t_2$. Note that the game ends at most at $\lceil t_2 + 1 \rceil$.

Hence, weak concavity offers a candidate to characterize the stopping-by-offer Stopping region.

$$\overline{j}(t) = \begin{cases} \lceil k_o^p(t) \rceil & \text{if} \quad t \le t_1 \\ \lceil k_1^p(t) \rceil & \text{if} \quad t \in (t_1, t_2) \\ \lceil k_2^p(t) \rceil & \quad \text{otherwise} \end{cases} \tag{C.6}$$

If k steps below $\overline{j}(t)$, we can be sure that P1 will not make an offer. Again the problem is that weak concavity is not sufficient to characterize Γ by the same reasons as above. Analogously, we can be sure that P1 will stop if $k > \overline{j}(t)$ for $t > t^{**}$ or if $k \ge \lceil \max\{k_o^p(t^*), k_1^p(t^{**})\} \rceil$; and that a sequence separating the stopping states exists, by following the same procedure as in Proposition 8.

In addition, we must also keep track of the possibility of AC. It generates a boundary, that is only valid when states $h_t \notin S$ do exist. This happens when

$$t \ge t_1' = \frac{1}{1 + p - \frac{1}{\theta}} - 2 \tag{C.7}$$

It is easy to check that for those states between t_1' and t_2 the boundary defined by taking equality in expression (3.9), separates those states where $r(k, t) = \theta p$. Hence, at any state lying in between, the bargaining range is empty but P1 still finds worthy fighting one more battle. Weak concavity yields again a candidate for the boundary that characterizes the stopping-by-AC region

$$\underline{j}(t) = \begin{cases} \lceil (1 + p - \frac{1}{\theta})(t + 2) - 1 \rceil & \text{if} \quad t \in [t_1', t_2]; \\ \lceil k_2^p(t) \rceil & \text{if} \quad t > t_2. \end{cases} \tag{C.8}$$

After t_2, all states in which agreement is not feasible lie below $k_2^p(t)$ and P1 will opt out because there is no possibility of continuation (P1 has stopped either by making an offer or by triggering AC) so Weak concavity characterizes completely Γ. However, it is not sufficient between t_1' and t_2: At state (k, t) a victory may lead the game to the continuation region and make $v(k + 1, t + 1) \neq r(k + 1, t + 1)$.

It is easy to check that such inferior sequence exists. Suppose on the contrary that for some state $h_t \notin S$, $v(k,t) = \theta p$ but that $v(k-1,t) > \theta p$. This cannot be the case because it would imply that

$$\delta E_v(k-1,t)) > r(k-1,t) = \theta p = r(k,t) > \delta E(v \mid k,t),$$

and we know, through backwards induction from $\bar{t}$, that the expected continuation value is non-decreasing in k for a fixed t. By the same token, one can show that this inferior sequence is non-decreasing as well: On the contrary, suppose that $\underline{k}^p(t) > \underline{k}^p(t+1)$ so at state $(\underline{k}^p(t), t)$, $\theta p > \delta E(v \mid k, t)$. This implies that at state $(\underline{k}^p(t), t+1)$, P1 prefers to fight one more battle. Hence $\theta p < \delta E_V(k, t+1)$ holds true . But it is impossible because $E(v \mid k,t) \geq E_V(k, t+1)$. ■