UNIVERSITAT AUTÓNOMA DE BARCELONA

DEPARTAMENT D'ECONOMIA I D'HISTÒRIA ECONÒMICA

PhD MEMORY

THEORY AND PRACTICE OF EQUILIBRIUM REAL EXCHANGE RATES.

LOOKING INTO THE EURO – AREA EMPIRICAL EVIDENCE

CARMEN MARÍN MARTÍNEZ

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TITLE: Theory and Practice of Equilibrium Real Exchange Rates. Looking into the Euro-Area Empirical Evidence

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To my mother

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Carmen Marín Martínez
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INTRODUCTION

GOALS OF THIS RESEARCH UNDER THE PERSPECTIVE OF SOME RELEVANT LITERATURE ON THE TOPIC

Most economists would agree that the real exchange rate is a determinant variable for a well working economy. Foreign investment, capital flows, or international trade, for instance, are deeply influenced by the shifts that a changing exchange rate brings about on goods and capital markets. Moreover, the quicker response of financial markets nowadays, largely encouraged by the economic phenomenon of globalisation, has even increased the traditional relevance of this variable. It is not strange so far that a big deal of different literature had come out to shed some light on exchange rate determination.

A main peculiarity of exchange rates is that they have been indistinctly understood as a monetary or a real phenomenon, since they can help to balance disequilibria in both financial and real markets. The usual reader knows well that, depending on the approach, the exchange rate is alternatively defined as the relative price between the national and foreign financial assets or, in real terms, as the relative price of domestic tradable goods either with respect to foreign tradable goods or with respect to non-tradables[1]. This double nature has given rise to the two prevalent approaches in the exchange rate literature, namely, the macroeconomic and the financial approach, which basically differ by the considered assumptions on the degree of market integration[2]. These assumptions have also constituted the basis for the different financial and macroeconomic approaches to have alternatively underlined the nominal or real aspects of the exchange rate behaviour.

Particularly, the financial approach is the one developed under the assumption of highly integrated good and capital markets, so in this case it has been standard practice to adopt the

[1] Even with respect to both of them.

[2] This argument can be also found, for instance, in Classen (1996).

long run anchor offered by the purchasing power parity (PPP) hypothesis. Rather on the contrary, the macroeconomic approach has repeatedly rejected any possibility that would approve PPP as a proper reference for equilibrium. That way, the result is that while the financial models have been mainly devoted to analyse the nominal exchange rate through the evolution of the asset and monetary markets, the macroeconomic approach has been concerned overall about the role played by the real fundamentals in the characterisation of the real exchange rate. This long run reference given by the PPP has induced to a great extent the different trajectories followed by the two mentioned approaches.

The PPP hypothesis is known to be one of the first attempts to explain the exchange rate behaviour. It dates indeed from the age of the Salmantine School in the XVI century, although it is not until the beginning of the XX century when, thanks to the contributions of Cassel in 1918 and 1922, it acquires a noteworthy relevance[3].

The original PPP version, known as the absolute version, is a simple generalisation of the law of the single price. This one asserts that, in a context of flexible prices and no trade restrictions, the price of relatively homogeneous goods will coincide in equilibrium[4]. In its turn, the PPP formulation simply extends this equilibrium condition to a set of goods and assumes that there will exist a long run convergence between relative (foreign and national) prices and nominal exchange rates[5]. Therefore, what the hypothesis (in its absolute version) is simply proposing is a real exchange rate understood as a relative price between the domestic and foreign (tradable) goods that, in the long run, will converge to unity.

Unfortunately, the data have not supported this strong equilibrium condition at all, so the absolute version was later on reformulated in relative terms. The PPP relative version accepts that different international peculiarities of the financial and good markets can prevent

[3] A mention to this particular can be also found in Bajo Rubio and Sosvilla Rivero (1993).

[4] International arbitrage is the economic mechanism that will guaranty the convergence process.

[5] An additional and restrictive assumption essential for PPP (although, by definition, not necessary for the law of the single price) is the necessity of a similar national and foreign weighting scheme for price index computation.

the real exchange rate from being exactly equal to unity although, in equilibrium, the real exchange rate will have to keep on satisfying the long run stability condition. In particular, the PPP proposal in relative terms is that, in the long run, the growth rate of the nominal exchange rate will have to compensate the differential between the national and foreign inflation rates. In some way, what the new version is assuming is that the differences among the economic systems can end up with different, but stationary, standards of life across countries.

There remain, however, a number of difficulties related to the restrictive assumption of stationarity. First, we should take into account that the PPP definition only considers the inclusion of traded goods. In the case of a price index composed of traded and non-traded goods, it can appear an important bias derived from differences in the relative sectorial productivity between countries[6]. In case that this deviation exists, it could easily introduce a non-stationary component. On the other hand, it also exists the unsolved question about how this long run equilibrium is in reality determined. Curiously, nothing about it is said in the theoretical model. Therefore, and despite its long historical tradition, a great deal of studies (trying with different periods and countries)[7] have concluded that, even in its relative version, the hypothesis is difficult to hold. Anyway, the PPP has been usually assumed more a long run anchor for the real exchange rate than a proper theory of exchange rate determination.

In addition to the PPP but regarding the short run, the financial approach has relied on the equilibrium condition proposed by the interest rate parity in its two alternative versions of uncovered and covered propositions[8]. The uncovered interest rate parity states that there will be short-term capital flows up to equalise the expected returns of a comparable short-term asset denominated in home and foreign currency respectively. In analytical terms, it implies

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[6] This is indeed the proposition of the Balassa-Samuelson hypothesis reviewed in detail further on in this introduction.

[7] Recent works about the topic are for instance that of Breuer (1994), Engle (2000) or Cheung and Lai (2000).

[8] The reader should notice at this point that both PPP and interest rate parity conform for the financial approach the basic long and short run equilibrium conditions of the good and capital markets respectively.

that the expected change in the nominal exchange rate (defined such that an increment means an appreciation) will be equal to the appropriate current short-term, foreign less national, interest rate differential[9]. Finally, the covered interest rate parity considers that the short-term interest rate differential will converge to the discounted forward premium quoted in exchange rate markets.

However, previous to any empirical implementation, the researcher will need a subjective specification of the particular mechanism used by the economic agents when they form their expectations. In this respect, the literature has overcome this drawback testing simultaneously the uncovered interest parity and the rational expectation hypothesis, as this one has been considered the most probable response of a usually assumed rational economic agent. Moreover, and also with relative frequency, it has been standard practice the combination of the rational expectation hypothesis along with the uncovered and covered interest rate parity conditions. That way, what the great majority of tests of this nature have ended up checking is if the regression value of the ratio between the spot variation and the forward premium could be assumed as settled around one.

In practice, the results have been very controversial and, in fact, the literature on the topic can be qualified as enormous. Unfortunately, the most usual finding has been that the uncovered interest rate parity is violated by the data. In view of these quite unfavourable results, some authors proposed a relaxation of the test simply introducing the possibility of a risk-premium that, in presence of uncertainty, would justify the continuous deviations showed by the empirical results. Nevertheless, most of the new estimations have ended up making use of this so-called forward premium anomaly as it could vary in time. So far, it is easy to guess that this sort of solution cannot be ever considered as such[10], as it was already defined in origin as a tautology.

Recent works of Meredith and Chinn (1998) or Baillie and Bollerslev (2000) have tried

[9] Note that this short-run hypothesis understands the nominal exchange rate as a relative price between national and foreign financial assets.

[10] In fact, quality research has never taken it seriously.

with other maturities different than the standard of monthly data and, fortunately, have found some favourable results. For the case of Europe, Ayuso and Restoy (1996) obtained that the standard uncovered interest rate parity defined between exchange rates -with the classical assumption of rational expectations- and interest rates was a reasonable approximation within the exchange rate mechanism. In any case, and as it was suggested with PPP, the uncovered or covered interest rate parity conditions cannot be considered at all as theories of exchange rate determination but, at most, as a short-run anchor for nominal exchange rates.

These two nominal anchors given by the PPP and the interest rate parity respectively have constituted the basis for the models developed under the so-called asset approach. This one conforms in fact one of the richest literatures of nominal exchange rate determination.

The asset approach was first developed in the early 70s and, unlike the flow theories in force at the moment, underlined the role of the capital account in a highly integrated world[11]. In particular, the approach proposed that the nominal exchange rate would be the effective variable to balance the asset demand and supply functions. Thereby, the nominal exchange rate has been naturally understood in this context as the relative price between domestic and foreign financial assets.

This asset approach literature throws into relief two alternative types of models, that is, the monetary and portfolio models. Both of them have been conceived in a world of free financial capital mobility where agents can immediately achieve the desired portfolio composition. In addition, the monetary approach introduces the assumption of perfect substitutability between national and foreign assets, so that the asset holders become completely indifferent between assets denominated in different currencies whenever expected returns coincide. According to this view, the national plus foreign money demand and supply is what will determine the current exchange rate evolution[12]. On the contrary, when the

[11] For an exhaustive analysis of the relationship between the flow (Keynesian) and the asset (monetarist) approach, see Ahtiala (1984).

[12] In this context it is assumed that there is only one alternative asset to money in which wealth can be invested. Remember that given the assumption of perfect substitution between the national and foreign assets, in order to reach equilibrium it will be enough to impose the equilibrium condition only on the monetary market.

assumption of perfect substitutability doesn't hold, we will be in front of the portfolio approach where the equilibrium condition of each different asset will need to be implemented in a separate way.

In its turn, the monetary approach has been presented in two different versions, the flexible and rigid price monetary models. Frenkel (1976), Mussa (1976) and Bilson (1978) are the authors that developed the easiest version of the flexible price monetary model. In short, what this model proposes is that -arranged in semi-logarithmic terms- the nominal exchange rate will be a function of the (national less foreign) differentials of the nominal money supply, gross domestic product and inflation rate. The analytical expression can be easily obtained taking into account the relationship between the national and foreign money market equilibrium conditions under the assumption of PPP plus the uncovered interest rate parity / rational expectation hypothesis.

On the other hand, the works of Dornbusch (1976) and Frankel (1979) have given rise to the rigid price monetary model. The common feature between this model and the flexible price version is the long-run equilibrium description. Alternatively, in the short run the predictions are completely different due to the assumed gradual response of good markets to the excess of demand and supply made by the rigid price monetary model. To say, if there was a shock causing a positive interest rate differential in the case of the flexible price model it would provoke a higher inflation rate differential and (according to PPP) a depreciation of the national currency. However, in the rigid price approach the positive interest rate differential would imply a short run inflow of capitals and, consequently, an appreciation. In fact, one of the most peculiar characteristics of the rigid price model is the prediction of a short run exchange rate overshooting in response to non-anticipated monetary policies.

Note that this overshooting effect depends exclusively on the assumptions taken about the different speed of adjustment of the asset and good markets; being the usual premise that, according to the empirical evidence, the good markets adjust slower than the monetary ones. The overshooting result then doesn’t seem to be something intrinsic to the asset market behaviour but, on the contrary, it is related to the particular assumptions of the model.

Particularly, the research on the topic has proven that the probability than an exchange rate overshooting occurs will depend positively on the degree of capital mobility, negatively on the trade balance response to the level of relative prices and negatively again on the semi-elasticity of the money demand with respect to the interest rate.

With regard to the empirical evidence available for the monetary model it is interesting that, despite the first favourable results achieved at the end of the 70s, a great deal of subsequent research had posed important doubts about its relevance to explain the recent evolution of exchange rates. Something similar occurs in the case of the portfolio balance approach, whose favourable evidence at the end of the 70s was not corroborated later on in the works of the middle 80s[13].

In this sense the most powerful argument comes from a paper of Meese and Rogoff (1983a), where they examined by means of the mean square errors of the corresponding prediction values, the out of sample predictive power of the monetary models in comparison to a simple random walk. Unfortunately, the conclusion was devastating for the monetary models, being the result that the random walk did not predict worse than any of the considered models. Moreover, the outcome was reinforced later on with the results of Meese and Rogoff (1983b). The conclusion of the moment was that the poor performance of the out of sample asset models did not seem to be due to any estimation difficulty, but to the own structure of the model.

Certainly, these bad empirical results gave rise to several alternative approaches; some of them involving an important degree of novelty, as it is the case of the consideration of non-linearities or the application of deterministic chaos to the exchange rate modelisation. Some others have come back to the line of the macroeconomic approach, with important improvements in the analysis of the effects that the intertemporal components have on exchange rate determination.

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[13] Complete surveys of this empirical evidence can be found in Frankel and Rose (1994), Flood and Rose (1995) and Taylor (1995).

Regarding the global assessment of the financial and macroeconomic approaches, the evidence shows that the existent research has faced this duality with quite unequal interest. In particular, while it is not difficult to encounter different literature devoted to review, compare or evaluate the attainments of the financial models -as it is for instance the case of the surveys of Frenkel and Mussa (1985), Obstfeld and Stockman (1985), Macdonald and Taylor (1992) or Taylor (1995)- it is not so easy to find many serious attempts that analyse in some depth the general performance of the macroeconomic models. Notwithstanding, and although quite recent in fact, the approach also includes some good examples like those of Clark, Bartolini, Bayoumi and Symansky (1994), Isard and Faruqee (1998)[14], Macdonald (1999) or Macdonald (2000). In any case, what is a fact nowadays is that this part of literature is experiencing a noteworthy growing process that, to a great extent, seems to be due to the problems encountered with the traditional exchange rate approaches.

Obstfeld and Rogoff (1996), for instance, have already mentioned in their compendium on International Economics, that the main drawback of the financial approach is that it lacks the basic micro-foundations needed for internal consistency, and that it fails to deal in a coherent way with the necessary dynamics of an open economy. In practical terms, perhaps the main failure of the old literature has been its inadequacy to give proper answers to some of the central questions of today policy makers and researchers; questions like the long run sustainability of some current account deficits or surpluses, or the consequences of an excessively enlarged government budget deficit. What seems evident is that this sort of questions can only be answered by a perspective that is able to integrate the different peculiarities of internal and external markets, so that something can be said about the long run real exchange rate.

This way, it is not difficult to understand the importance that the concept of exchange rate equilibrium has acquired nowadays, as well as the fact that any step forward to a better knowledge of the forces that underline the exchange rate misalignments is at the moment assumed as an undoubtedly worth success. In this respect, Macdonald and Stein already

[14] Both developed in the bosom of IMF.

stressed in their work of 1999 that economists are becoming now more conscious of the necessity of good implementable economic theories that can help them to better evaluate the consequences of specific policies on equilibrium. Just what a well-designed model of the macroeconomic approach would be able to offer.

The macroeconomic approach however is not an easy field of research. Far from constituting a unified theory of work, it gathers in reality a great deal of different literature that, depending on the context, has alternatively contributed to explain different aspects of markets behaviour. Notwithstanding, if the goal is to capture the dynamics of the system as better as possible, our bet is on the recent contributions of the so-called structural approach. In particular, many of these contributions have their origin in the work of the prestigious group of economists belonging to the International Monetary Fund or to the World Bank.

The interest of this structural approach is in reality two-fold. On one hand, it avoids the cyclical and speculative short run behaviour, largely proved transitory and unpredictable. And, on the other hand, the approach deliberately ignores the simplifying assumption of highly integrated markets to deal instead with a deeper analysis of the internal and external economic equilibrium conditions. The structural approach itself has followed two main tracks, the partial equilibrium specification and the general equilibrium one. The partial equilibrium approach, or macroeconomic balance approach, has focused exclusively in the current account trade balance component, avoiding completely the existent feedback between the current account and the real exchange rate; a feedback closely related to the international interest flow coming from the net foreign assets accumulation[15]. On the contrary, the general equilibrium approach is based on complete macroeconomic models that take care, in one way or another, of the long run sustainability condition and that are consequently conscious of the hysteresis phenomenon. In particular, to the extent of our knowledge we find this dynamic -or general- macroeconomic balance approach an adequate and coherent methodology to deal with the analysis of the long run real exchange rate. We declare in that way our intention of

[15] Note that only taking into account this mutual interaction, the persistence effect derived from the shocks influencing the equilibrium can be considered.

setting aside the nominal exchange rate analysis to centre instead on the real exchange rate determination.

The Meade model, the Mundell and Fleming model, the representative agent intertemporal model and the more contemporary FEER of Williamson, DEER of the IMF and Natrex of Stein and associates can be underlined as important theoretical contributions in the context of the macroeconomic approach. A particular case is the Balassa-Samuelson hypothesis, since strictly speaking it does not conform a complete macro model. Nevertheless, if something, it is clearly a theory closer to the macroeconomic than to the financial approach.

An early attempt to explain the systematic long run deviations from PPP is found in the Balassa-Samuelson hypothesis. This one, based on separate contributions of Balassa (1964) and Samuelson (1964), has been commonly understood by the literature on the topic as a simple supply-oriented model, but with the peculiarity of trying to explain the exchange rate PPP departures through the inclusion of non tradable goods. In this approach the starting point is the definition of a national aggregate price index from the single price indexes of the considered two key production sectors in an economy, that is, the tradable and non tradable one. Given that the PPP condition can only be satisfied in the tradable sector, in equilibrium the real exchange rate[16] will be equal to a generic constant multiplied by the ratio of the national price index of non-tradables with respect to tradables divided by the same ratio defined for the foreign variables.

Following the arguments of Balassa and Samuelson, a higher productivity level in the tradable sector will cause, in a world of free factor mobility, higher wages in both sectors. If productivity in nontradables does not experiment an increment of the same proportion, the result is that the relative price of nontradables with respect to tradables will end up growing in this economy. Given that developed countries are usually characterised by a higher increment

[16] Defined such that an increment means a real appreciation of the national currency.

in the relative productivity of the tradable sector, the more common expected result is to observe a natural tendency to the a appreciation over time.

In practise, many empirical works have made use of this proposition to conclude about the observed deterministic or stochastic long run tendencies in the real exchange rate behaviour. This is for instance the case of Asea and Mendoza (1994) or Micossi and Milesi-Ferretti (1994), which underline the role of productivity in the evolution of the relative price index of nontradables with respect to tradables. However, and despite its substantial empirical success, the Balassa-Samuelson model can only be considered a complementary view of other alternative approaches. In fact, many recent research has underlined the necessity of new theoretical explanations for that part of the real exchange rate evolution unexplained by the relative productivity hypothesis. In particular, works like that of Neary (1988), Bergstrand (1991) or De Gregorio et al (1994) have based their reasoning on the temporal evolution of the relative aggregate demand function[17] or, as it is the case of Lane and Milesi-Ferretti (2000), on the long run consequences of the transfer effect in the context of intertemporal equilibrium models.

A different type of modelisation, alternative to the previous one, belongs to the traditional flow approach, where the exchange rate moves up to equilibrate the international net demand flow of the national or foreign currency. This flow approach has gathered in fact the first developed macro models on exchange rate determination where it has been usual practise to assume that the national markets produce simply national single homogeneous goods. It introduces the simplification that the real exchange rate collapses to the international terms of trade.

A main peculiarity of these macro-models is that although they incorporated the

[17] This hypothesis suggests that, under the assumption of non-homotetic tastes, the relative price level will be higher in that countries with a higher income level. The reason is that nontradables can be considered as luxury goods while tradables use to be first-necessity goods. In that case an increment in the income level would increase more the demand of nontradables than the demand of tradables and, consequently, their corespondent price levels.

characteristic of free international good mobility since the first contributions in the 50's, it is not until the 60's that works like that of Fleming (1962) and Mundell (1963) introduced the international flow of capitals.

In particular, one of the oldest relevant flow models known up to the moment is the Meade model, developed in Meade (1951), which focused exclusively in the trade balance equilibrium. In this model the net demand of foreign currency comes mainly from the international exchange of goods, so it is characterised by a complete identification between the balance of payments as a whole and its trade balance component. In this type of models the Marshall-Lerner condition[18] is usually enough to assure the long run stability. Mainly, the reason why these theories appeared in the 50's is that this period was characterised by intense flows of international transactions almost exclusively constituted by exports and imports of goods, being the international flow of capitals considerable scarce.

Regarding the Mundell and Fleming model, it was at the beginning of sixties when the authors introduced the consideration of international capital flows[19]. In this model, the interest rate differential is the source of the international monetary flow, so it is a fundamental variable to determine the sign of the capital account and, consequently, of the real exchange rate. Under this approach the real exchange rate is incorporated into a macroeconomic model where it is determined together with the real interest rate and the level of gross domestic product.

Notwithstanding, there is a main critic to the Mundell and Fleming model that is related to the long-run consequences that the capital flow would have over the international net asset position. In this model if the interest rate differential is high enough, any stock of foreign assets, and consequently any exchange rate, can be possible in the long run. This is a main drawback that seemed difficult to solve in its own context, and that ended up causing the oblivion of the approach as a proper reference for real exchange rate determination.

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[18] This condition states that the addition of the price elasticities for the demand of imports and the demand of exports should be greater than unity.

[19] A good exposition of the Mundell and Fleming model is in Frenkel and Razin (1987).

Unlike the classic wisdom given by the Mundell and Fleming macro-model, the new research into the macroeconomic approach has underlined the intertemporal aspects of equilibrium[20]. As it is for instance mentioned in Obstfeld and Rogoff (1995), the main interest of these intertemporal models is that they can provide reliable grounds on the economic policy analysis of a standard open economy. In fact, a main factor influencing the boom of this kind of research has been the general concern about the current account evolution in developing countries that took place after the world oil-price shocks of the 70s. These events generated a generalised fear about the possibility that the external debt levels of developing countries could become unsustainable, with the subsequent problems for the developed economies that were acting as borrowers at the moment.

In all these models the external equilibrium condition has formally adopted the form of a current account intertemporal balance condition. In particular, the economic agents of this approach have all been considered as forward looking actors, so that the current account balance is viewed as an outcome of forward-looking dynamic savings and investment decisions. This way, it is provided an ideal conceptual framework for analysing the problem of the external long-run sustainability condition of the system. Moreover, it is also the appropriate context to undertake the concept of exchange rate equilibrium, not for nothing all the models under this approach have been used, in one way or another, to define particular concepts of exchange rate equilibrium.

A broad class of models in this approach have been constructed under the microfoundations of representative agents behaviour over an infinite horizon, assuming that the economic agents take investment and saving decisions simultaneously. We are referring to the representative agent intertemporal optimising models, shortened as RAIOM. A main problem of this approach, reviewed for instance in Stein and Paladino (1998) or Gandolfo (2002), is that it runs the risk of giving tautological explanations to exchange rate behaviour,

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[20] This intertemporal approach became common in the early 80s, although in fact it has its origin in the 70s with works like that of Hamada (1969) or Bruno (1970).

given that many of the crucial variables are subjectively measured. Indeed, the few attempts to apply the RAIOM in order to explain the current data have been quite unsuccessful.

On the contrary, an abundant alternative literature has avoided the idea of the representative agent, to consider instead that decisions on savings and investment are taken separately. Usually firms decide over investment, whereas consumers and government decide about savings. The outstanding models under this approach are the FEER model of Williamson, the DEER model of the group of research of IMF, and the NATREX model of Stein and associates. All of them can be understood as intertemporal macro-models worried about the long run real exchange rate of an open economy.

Williamson's first approach to his FEER equilibrium (initials of fundamental equilibrium exchange rate) is in Williamson (1985). In this approach the author, taking advantage of an initial work about the design of target zones, tries to calculate a sort of equilibrium exchange rate that could be useful to offer some consistency to the target zone proposal. Later on, being the profession as a whole more conscious of the importance of the equilibrium concept, Williamson (1994) presented a more mature version of his FEER concept in a volume specially devoted to the analysis of the exchange rate equilibrium.

The FEER, such as it is defined by Williamson, would be that exchange rate consistent with the medium run macroeconomic balance, meaning this one the simultaneous achievement of the internal and external balance. While the internal balance is defined by the level of activity consistent with the control of inflation, the external balance would be achieved when the country reaches a sustainable long run current account target. Certainly, this concept of sustainability contains an important normative element, as this current account target is completely a subjective element. In particular, and in words of the own Williamson, sustainability should not be understood merely as a steady-state outcome, but it is "any path that satisfies intertemporal budget constraints, and that can be followed indefinitely without surprises that would make agents wish that they had not acted as they did" (Williamson 1994: page 180).

Alternatively, a formal definition of the DEER (desired equilibrium exchange rate) is in Bayoumi et al (1994). Formally, the authors define the DEER as a “FEER-like” concept, so that their approach to the notion of equilibrium is in fact quite similar to that of Williamson. The difference, however, is that they emphasise more cautiously the normative element of the concept. In the case of a DEER model, in addition to the FEER alike current account objectives, there appear clear definitions of different target values for a real battery of alternative macroeconomic objectives.

A different approach to the concept of equilibrium is that posed by the Natrex (acronym of natural real exchange rate) model[21]. Stein defines the Natrex as the real exchange rate that would prevail if the cyclical and speculative factors were absent and the unemployment were at its natural rate. This is the case when desired social[22] savings minus social investment equal the current account. Therefore, in every Natrex model there is a formal attempt to explain the existent relationship between the real exchange rate and the set of fundamental variables explaining savings, investment and the current account levels.

A main peculiarity of the Natrex approach is that it implies a generalisation of the previous macroeconomic balance models, since it has been designed to explain the medium to long run movements in all, the real exchange rate, the current account and the net foreign liabilities based on a rigorous stock-flow interaction [23]. While in the medium run it is required that both the external and internal balance be satisfied, in the long-run the need would be that, in addition, the stock variables -net foreign debt and capital stock- reach their steady state levels.

The movements of the real exchange rate and the current account over time are indeed related to the fact that the capital stock and the net foreign debt move only slowly over time,

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[21] First developed by Stein (1990, 1994) and associates. See Stein, Allen and Associates (1995).

[22] We use the term social, according to the Stein proposal, to denote private plus public variables.

[23] For a detailed description of the model in contrast with other approaches see for instance Stein and Paladino (1998).

so that in the medium run they can be treated as exogenous variables. This medium term condition will necessarily evolve to its long-run equilibrium, where the capital stock and the foreign debt levels are evaluated at steady state levels.

In particular the Natrex approach is open to a wide variety of different model specifications, where savings, investment and the current account are endogenously determined. The most important exogenous factors within the Natrex concept are a thrift parameter measuring the preference for consumption and productivity. Moreover two important stability conditions are a negative relationship between the marginal productivity of capital and the capital stock and a positive relationship between savings and the net foreign debt level.

The standard response of the exchange rate to exogenous thrift and productivity shocks within the Natrex model as, for instance, suggested by Stein (1999) is the following: a negative shock to consumption preferences will clearly depreciate the currency in the medium run, but will lead to a real appreciation in the long run. The reason for the different sign effect of the medium and the long-run equilibria is due to the decumulation of foreign debt during the transition process. Eventually an improved net foreign asset position requires a stronger external value of the home currency and thus a weaker trade balance in order to counterbalance the current account. Alternatively a negative productivity shock will depreciate the equilibrium exchange rate in the medium-run, whereas the long-run effect can be ambiguous so it depends on the speed of decumulation of net foreign debt.

Formally, we find that the dynamic version of the macroeconomic approach in general and this Natrex approach in particular constitute from our point of view the adequate methodology to undertake our subsequent research. Under this approach then, we develop next three different research papers on the topic that, presented in different chapters, are going to conform the main body of this job. Moreover, they show the nice peculiarity that, even being all self-contained, each one can be also understood as a sort of step forward of the previous one. There exists then an interesting line of continuity throughout the whole work. To get a first flavour of it, a brief summary of the goals and contributions of every chapter is presented next.

To start with, chapter one analyses how the classical answer to the empirical relationship between the economic growth of a country and its long run real exchange rate can be enriched through the consideration of a more general, balanced growth, intertemporal equilibrium model. It is well known that, regarding this observed positive correlation between the per capita income level and the real exchange rate, the most popular theoretical proposal is the supply-sided Balassa-Samuelson hypothesis. This one concludes that it is the asymmetry in the relative growth rate of the sectorial productivity in tradables with respect to non-tradables among countries, which will mainly determine the international relative price level of a particular economy. Particularly, what the hypothesis predicts is simply that the higher the grade of economic development, the more appreciated the national currency of a country will be.

In our case, we consider a simple exogenous growth model where it is imposed the internal[24], external and intertemporal equilibrium conditions of a typical macroeconomic model; this last one through the inclusion of a balanced growth path for the foreign assets accumulation. The main result under this consideration is that the relationship defended by the Balassa-Samuelson hypothesis is no more so straightforward. In our particular approach, the mentioned bilateral relationship depends on a parameter measuring thriftiness in the economy. Therefore, the probability of ending up with a positive relationship between growth and real exchange rates –as the classical economic theory predicts– will be higher when the economy is able to maintain a minimum saving ratio. Moreover, given that our model considers a simple Keynesian consumption function, some explosive paths can be possible[25].

[24] Our supply condition, like in the case of the Balassa-Samuelson hypothesis, will joint the assumption of a perfectly elastic relative demand function.

[25] A quite preliminary approach to this research is in Marín (1999).

Chapter two gives a step forward and, following a general equilibrium macroeconomic approach, sets a closed micro-founded structural model to determine the long run real exchange rate of a developed economy. In particular, the analysis follows the structure of a Natrex model, whose main peculiarities within the structural approach are two-fold. On one hand, the own structure of the model presents a singular stock-flow interaction not known up to the moment. On the other hand, it introduces an exclusive distinction between the medium and long-run equilibrium that consists on the following: while in the medium-run only the external and internal equilibrium conditions are required, in the long-run it is also necessary that the net foreign debt plus the capital stock reach their steady state levels.

It is curious how many of the self-named structural models of exchange rate determination, included the known different versions of the Natrex approach, end up only partially deriving the equations that should compound the model. Moreover, it is also surprising how easy they forget implementing the particular restrictions that derive from the set of single equations, as well as it is common to observe that they avoid the problems related to the convergence and stability of the system. In that sense, the main contribution of this second chapter is the development of a solid theoretical framework that analyse in depth the basis of the real exchange rate and the details of the equilibrium dynamics after any shock influencing the steady state. In our case, the intertemporal factors derived from the stock-flow relationship will be particularly determinant.

The main results of the chapter can be summarised as follows. In first place, a complete well-integrated structural model for long-run real exchange rate determination is developed from first principles. Moreover, within the concrete dynamics of the model, it is found that some convergence restrictions will be necessary. On one hand, for the medium run convergence the sensitivity of the trade balance to changes in real exchange rate should be higher that the correspondent one to the investment decisions[26]. On the other hand, and regarding long-run convergence, it is also necessary both that there exists a negative

[26] The reader should know that in this approach investment is assumed to depend negatively on real exchange rates.

relationship between investment and capital stock accumulation and that the global saving of the economy (integrating public and private sectors) depends positively on net foreign debt accumulation. In addition, there are also interesting conclusions about the effects that certain shocks over the exogenous variables of the model have on real exchange rates.

Finally chapter three uses the previous theoretical model to check its performance in the particular case of the euro. Its contribution is two-fold. First of all, the Natrex model is estimated in its true structural form. So far the Natrex models had only been estimated in reduced forms or semi-reduced forms. Secondly, the model is applied to the effective euro exchange rate -period going from 1970 to 2000- using quarterly observations from the database of the ECB's area wide model (AWM). We thus contribute to the growing literature on the euro's fundamental value by using one of the more comprehensive databases for pre-Stage III euro area data available so far[27]. According to our structural model we can conclude about the main periods of over and undervaluation of the euro, being particularly interesting the significant undervaluation obtained at the end of the period under analysis.

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[27] See Detken and Marín (2003), and Detken, Dieppe, Henry, Marín and Smets (2002a and 2002b).

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CHAPTER 1

CURRENT ACCOUNT IMBALANCES, THRIFTINESS AND THE REAL EXCHANGE RATE IN A GROWING ECONOMY

1.1 Introduction

The purchasing power parity (PPP hereafter) assumes a constant level to which the real exchange rate would eventually converge in the long run. This is a classical reference in the literature on real exchange rates that however, and despite the attractiveness of its simplicity, has presented many problems in order to be empirically corroborated. In fact, the generalised result of a great deal of available research on the topic is precisely the non-stationarity of real exchange rates. This way, and against the PPP prevision, data seem to predict a systematic price level deviation among countries, which is in some degree independent of the economic integration between them.

An additional drawback regarding the PPP assumption is the simplification that it introduces on the price level behaviour of tradable and nontradable goods. This inconvenience is in fact related to the consideration of a long run stationary real exchange rate, since it necessarily involves that the tradable and nontradable sectorial price levels evolve similarly over time. However, given that nontradables are not subject in reality to the international competition, there is no reasonable argument that can justify this assumption of a similar sectorial evolution.

This kind of assumptions, extremely simplistic in many occasions, is what has led many authors in general and us in particular to the conviction that the real exchange rates do not have to be necessarily constant in the long run but, on the contrary, that there are real factors which affect and determine the real exchange rate behaviour[28].

There is also an interesting empirical regularity very frequently underlined by the international economic research and which highlights not only that the real exchange rate is a non-stationary variable, but that there is also a strong positive relationship between the real exchange rate and the per capita income level[29]. In this approach the two predominant competing theories underlying the role of the per capita income level are a supply-oriented approach, known as the Balassa-Samuelson hypothesis, and a demand-oriented one that, alternatively to the previous proposal, avoids the assumption of a perfectly elastic relative demand function. Empirical evidence on the Balassa-Samuelson model can be found, for instance, in Asea and Mendoza (1994) for a group of fourteen OECD countries and in Micossi and Milesi-Ferretti (1994) for the context of the EMS. Regarding the demand side approach the reader can refer to Neary (1988), Bergstrand (1991) or De Gregorio et al (1994).

In this chapter we go one step further and work on a third hypothesis, complementary in fact to the previous supply and demand-oriented ones, but that underlines instead the intertemporal restrictions that a process of economic growth necessarily imposes on the economy. This research is indeed related to the well-known transfer problem in international economics that, very recently, has been taken up again in order to highlight the influence that the net foreign asset position has on the real exchange rate determination.

As Obstfeld and Rogoff (1995) or Lane and Milesi-Ferretti (2000) underline, there is a persistence effect derived from the financing of the growth process that cannot be forgotten when analysing the foundations of the real exchange rate behaviour. It is well known that a period of high factor productivity is going to be characterised by capital inflows that, although being able to finance the new investment requirements, would also end up generating a stock

[28] In a small economy the terms of trade are commonly assumed an exogenous variable, so the real exchange rate results directly proportional to the ratio of relative prices between tradable and nontradable goods. Given that this will be a crucial assumption in the whole chapter, we will refer indifferently to the real exchange rate or to the ratio of relative prices.

[29] Evidence on this assertion can be found for instance in Kravis and Lipsey (1983, 1987, 1988).

of external liabilities that will need to be serviced in the near future. Countries in that case will have to run surpluses large enough so as to serve the net external payments, being a more depreciated national currency in this context finally required. Only the capital accumulation associated to the growth period could eventually reverse the process, giving the country the opportunity to end up as an asset holder.

Formally, the specific literature concerned with the intertemporal aspects of the exchange rate equilibrium has been mainly associated to the macroeconomic balance approach, which has insistently underlined the role of the intertemporal balance condition in the context of a growing economy. Within the current research in this area, and particularly in the context of developed economies, it has stood out the FEER approach of Williamson (1994), the DEER approach of the group of economists of the IMF[30] and the NATREX approach of Stein (1990, 1994) and Stein, Allen and associates (1995). For the case of developing economies, the works of Edwards (1988) and Eldabawi (1994) can be alternatively underlined. However, from the point of view of our research, the main problem of all this literature is that it has been designed following a pure empirical approach. None of them have devoted any special effort to formally analyse the determinants of the relationship between growth and real exchange rates. The theoretical analysis of this economic phenomenon, including the intertemporal aspects of equilibrium, is therefore the main contribution of this chapter to the existent literature on the topic.

Following this reasoning, the current chapter investigates how the classical theoretical approaches analysing the positive relationship between the income level and the real exchange rate can be enriched with the assumption that, in the long run, a balanced growth condition for the foreign asset accumulation has to be necessarily considered. With this objective, we develop a simple exogenous growth model where it is imposed the internal and external balance condition, plus the appropriate intertemporal asset restriction. The main conclusion of our particular small-scale model is that the result defended by the Balassa-

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[30] For this approach see for instance the works of Artis and Taylor (1993) or Bayoumi, Clark, Symansky and Taylor (1994).

Samuelson hypothesis is no more so straightforward, due to the existence of a thriftiness parameter that influences the bilateral relationship of real exchange rates and growth. In particular, our main result is that the probability of having a positive relationship between the two variables –as the classical economic theory predicts– will be higher when the economy is able to maintain a minimum saving ratio. In addition, as our model considers a simple Keynesian consumption function, some explosive paths can be possible.

The chapter is organised as follows. Next section goes briefly through the traditional demand and supply hypothesis to explain the classical relationship between the relative price index and income. Section three presents the main body of the work, being here where the intertemporal equilibrium model is fully developed. Formally, this equilibrium model stands out for the precise characterisation of the economic growth process, since only in this way the long run effects of the accumulation in the stock variables can be taken into account. Given that investment requirements are usually different from the volume of internal savings, the existence of transference of resources between complementary economies will be the most common result in an international context of openness. In particular, there are the conditions associated to this international loan that will determine the long run evolution of the real exchange rate. This is the reason why in this model the convergence process, as well as its stability, is considered a main element in the real exchange rate determination. Finally, section four summarises the main conclusions of the chapter.

1.2 Fundamental Determinants of the Relative Price Level

1.2.1 The Supply Approach

Under this supply-oriented approach we will simply underline the implications of the Balassa-Samuelson (BS) hypothesis[31]. Formally, the BS model[32] concludes that the greater

[31] The Heckscker-Ohlin model, although less known in the literature, is also a supply-sided approach that arises as an alternative to the Balassa-Samuelson hypothesis. This model bases its reasoning not in the role of the relative productivity of factors, but on the different factor intensity according to the degree of economic development. Bhagwati (1984) is a good reference to see an application of it.

[32] Developed from separated contributions of Balassa (1964) and Samuelson (1964).

the productivity of tradables in relation to nontradables, the more appreciated the real exchange rate will be in a particular economy. The importance of this hypothesis is that it can easily explain the stronger currencies observed in economies with higher income levels, given the consideration that these more developed countries present higher differentials in the level of relative productivity. Let us briefly revise the results of this proposition in analytical terms.

We consider the case of a small economy with two sectors, tradables and nontradables, each one with a Cobb-Douglas production function like the following,

$$Y_i = A_i K_i^{1-\alpha_i} L_i^{\alpha_i} \quad i = T, N \tag{1.1}$$

where T and N refer to the sector of tradables and nontradables respectively, Y is the level of production, K is the capital stock, L is the labour force and A_i is the global productivity factor in sector i.

In the case that the production factors are homogeneous, in equilibrium their costs will equalise across sectors. Therefore, assuming a competitive context and profit maximising firms, we can optimise with respect to K and L in order to obtain the equilibrium price levels of this economy. These appear in expressions (1.2) and (1.3) respectively,

$$P_T = \frac{1}{A_T} \frac{\omega^{\alpha_T} r^{1-\alpha_T}}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} \tag{1.2}$$

$$P_N = \frac{1}{A_N} \frac{\omega^{\alpha_N} r^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}} \tag{1.3}$$

being P_T the price level of tradables, P_N the price level of nontradables and ω and r the cost of labour and capital factors respectively.

Assuming that the tradable sector is subject to international competence, it will occur that $P_T = E \times P_T^*$ (being E the nominal exchange rate) so that P_T can be considered as given.

Hence, any relative improvement in the productivity of tradables will go to wages and, eventually, to the price of nontradables.

Specifically, if we consider that $P=P_N/P_T$, then P is defined as follows,

$$P=\frac{\omega^{\alpha_N} r^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}\frac{1}{A_N P_T} \qquad (1.4)$$

With P_T being an exogenous variable, changes in A_T will translate completely into wage changes. In that case we can find ω in (1.2) and substitute it in (1.4) to obtain P as a function of A_N, A_T and the corresponding exogenous variables,

$$P=\frac{A_T^{(\alpha_N/\alpha_T)}}{A_N}\left[\left(\frac{\alpha_T}{\alpha_N}\right)^{\alpha_N}\frac{(1-\alpha_T)^{(1-\alpha_T)(\alpha_N/\alpha_T)}}{(1-\alpha_N)^{1-\alpha_N}}\left(\frac{P_T}{r}\right)^{(\alpha_N-\alpha_T)/\alpha_T}\right] \qquad (1.5)$$

Finally, working with (1.5), we can easily derive the growth rate of the relative price index as a function of the respective productivity factor rates in both sectors, as it appears in (1.6),

$$\hat{P}=\frac{\alpha_N}{\alpha_T}\hat{A}_T-\hat{A}_N \qquad (1.6)$$

The conclusion under this approach is therefore that, in the context of small economy with given international price indexes and perfect factor mobility, a rise in the relative price ratio follows an improvement in the tradable good sector productivity.

1.2.2 The Demand Approach

The demand-oriented approach introduces the role of income in the analysis of real exchange rates through the different effect that it has on the income elasticity of the relative demand function. Under this approach the national currency will tend to appreciate whenever the income elasticity of nontradables is greater than the corresponding to tradables. Formally, this relationship between elasticities and the real exchange rate is carefully explained below.

In this approach it is assumed that agents incorporate a minimum subsistence level in their consumption of tradables, so we can get demand functions with income elasticities less than one for tradables and greater than one for nontradables respectively. In this case, a rise in income will lead to a greater increase in the wish to consume nontradables than tradables, what will cause a shift in the relative demand function to the right. However, the effect of this shift on demand will still depend on the supply side behaviour. Under the triple assumption of perfect competence in the good and factor markets, the fulfilment of the law of the single price and the existence of perfect capital mobility, a relative supply function put in the same terms that the demand function will be completely horizontal at the level determined in (1.5). In that case, the effect that the rise in income has on the relative demand function will simply lead to changes in the composition of production, but not in the relative price level. However, if we lose any of the previous assumptions, what is in fact the most common situation, the supply function will end up with a positive slope and a shift in relative demand will necessarily provoke a higher relative price index in a near future.

Note that this demand approach is in reality a complement of the previous supply-oriented one. Notwithstanding, for the case of an open economy, the external equilibrium condition should be also taken into account. The inflow and outflow of financial resources influences to a great extent the nominal exchange rate, so that the price level of tradables expressed in national currency is consequently affected and so is the supply function. This role of the external sector, constituting in fact our main contribution to the analysis of equilibrium, is thoroughly developed in the next section.

1.3 The Real Exchange Rate and the Income Growth Process: A Macroeconomic Equilibrium Approach

In this section, we propose a dynamic model where both the internal and external equilibrium conditions are going to characterise the long run of the national economy. The approach will particularly underline the effect that over the long run real exchange rate would have the intertemporal restriction derived from the external balance equilibrium. Formally, we will start identifying the peculiarities of this balance of payments equilibrium, as well as that of the national demand and supply conditions. Subsequently, the properties of the steady state will be carefully described.

1.3.1 The Balance of Payments Equilibrium

We know that the disposable income, determined by the national income plus the net transferences of the public and foreign sectors, is distributed between consumption and savings. However, in order to simplify we are going to consider both the absence of a public sector and that the net payments coming from the foreign sector will be merely composed of the net payment of interests from the rest of the world. This way, it is easy to obtain that the total production (Y) distributes between private consumption (C), private savings (S) and the service of the net foreign debt[33] (DS), such as it appears in expression (1.7),

$$Y = C + S + DS \tag{1.7}$$

Since the components of production are consumption (C), investment (I) and trade balance[34] (TB), making use of (1.7) the external equilibrium condition is determined as follows,

$$TB - DS + I - S = 0 \tag{1.8}$$

Alternatively, given the equality in (1.9),

[33] This amount must have the opposite sign in the case that the national economy would posses foreign assets instead of foreign debt.

[34] Remember that we have let aside the public sector.

$$TB - DS = CA = -KB = -\Delta D = S - I \tag{1.9}$$

where TB and KB are the trade and capital balances respectively, CA is the current account and D represents the stock of foreign debt, (1.8) can be in addition represented by the more informative expression (1.10),

$$TB + \underbrace{\left[\underbrace{I - S}_{\Delta D} - DS \right]}_{\text{Foreign Net Resources}} = 0 \tag{1.10}$$

Equation (1.10) says that, adopting the standard assumption of a negative relationship between the trade balance and the real exchange rate[35], the greater the inflow of foreign resources the stronger the currency of a country will be. Our aim is therefore to determine the conditions that in the steady state would characterise a net inflow of resources.

1.3.2 The Demand Equilibrium Condition

In the case of an open economy with absence of public sector, the aggregate demand function is simply determined by the addition of the consumption, investment and trade balance components, as expression (1.11) shows,

$$Y = C + I + TB \tag{1.11}$$

Regarding the trade balance, from (1.10) it can be specifically defined as in equation (1.12), where it appears as a function of the level of net foreign debt,

$$TB = (r - \hat{D})\, D \tag{1.12}$$

Including the result of (1.12) into equation (1.11), it is then obtained the expression

[35] Defined such that an increment means a real appreciation of the national currency.

(1.13) that represents an alternative to (1.11) in the definition of the aggregate demand function,

$$Y = C + I + (r - \hat{D})\ D \tag{1.13}$$

In per capita terms, equation (1.13) can be additionally simplified to expressions (1.14) or (1.15), depending on whether the relationship given by $\hat{D} = \hat{d} + n$ is finally taken into account,

$$y = c + i + (r - \hat{D})\ d \tag{1.14}$$

$$y = c + i + \left[(r - n) - \hat{d}\right] d \tag{1.15}$$

Lower case letters indicate per capita terms and n is the growth rate of the working population force. This one, without loss of generality, can be approached by the whole population.

Reformulating now expression (1.15) in growth rates and grouping conveniently, it is finally obtained the aggregate demand equilibrium condition given by (1.16),

$$0 = (\hat{c} - \hat{y})\ \frac{c}{y} + (\hat{i} - \hat{y})\ \frac{i}{y} + \left[(r - n) - \hat{d}\right](\hat{d} - \hat{y})\ \frac{d}{y} \tag{1.16}$$

Note that if the economy had foreign assets (named F in absolute terms and f in per capita terms) instead of foreign debt, it would follow that $f = -d$ and $\hat{f} = \hat{d}$. In that case, expression (1.16) would become the following,

$$0 = (\hat{c} - \hat{y})\ \frac{c}{y} + (\hat{i} - \hat{y})\ \frac{i}{y} + \left[\hat{f} - (r - n)\right](\hat{f} - \hat{y})\ \frac{f}{y} \tag{1.17}$$

Equations (1.16) and (1.17) will alternatively represent the demand side behaviour of the economy under analysis.

1.3.3 The Supply Equilibrium Condition

Regarding the supply equilibrium condition, we will start with the characterisation of the national production function that, in particular, is assumed as follows,

$$Y = A\,F(K,L) \qquad F'_K\,,\ F'_L > 0\ ;\ F''_{KK}\,,\ F''_{LL} \leq 0 \tag{1.18}$$

where K is the capital stock, L is the labour force and A a global productivity factor.

Assuming constant returns to scale[36], the production function is easily redefined as in (1.19) where variables are expressed in per capita terms,

$$y = A\,f\left(k\right) \tag{1.19}$$

Working in growth rates from (1.19), the equation (1.20) can be easily derived. This one relates the income growth rate with the rate of technological progress and the growth rate of the per capita stock of capital,

$$\hat{y} = \hat{A} + \gamma\,\hat{k}\ ;\ \ \gamma = \frac{f'_k}{f(k)}k \tag{1.20}$$

This equation (1.20) is then used to characterise the supply side behaviour of our growing economy.

[36] It is standard practice to assume constant returns to scale as this is commonly corroborated by the empirical analysis.

1.3.4 The Steady State

The steady state condition will satisfy the equations (1.10), (1.16) or (1.17), and (1.20). Next, we carefully analyse what these equilibrium conditions would imply over the long run levels of consumption, investment, trade balance, income and net foreign debt.

With regard to investment, we know that in economic terms the whole quantity of gross investment is devoted both to raise the capital stock and to pay its total amount of depreciation, that is to say,

$$I = \Delta K + \theta K \tag{1.21}$$

where θ is the depreciation rate.

Alternatively, in per capita terms equation (1.21) can be reformulated as follows,

$$i = \left(\hat{K} + \theta\right) k \tag{1.22}$$

From (1.22), and given that $\hat{K}$ and θ are stable in the steady state, the growth rates of both gross investment and capital stock in per capita terms should converge in equilibrium, i.e. $\hat{i} = \hat{k}$.

Regarding the demand side behaviour, we see that for the equation (1.16) to be fulfilled, it is a necessary equilibrium condition that $\hat{c} = \hat{i} = \hat{y} = \hat{d}$ or, alternatively, that $\hat{c} = \hat{i} = \hat{y} = \hat{f}$, so the condition $\hat{i} = \hat{k} = \hat{y}$ will be also satisfied. Once these relationships are included into (1.20), we can get the final expression (1.23) for the per capita income growth rate,

$$\hat{y} = \frac{1}{1-\gamma}\hat{A} \tag{1.23}$$

From (1.23) it is easily deduced that in equilibrium the per capita income will grow at a greater rate than the technological progress. The reason is that the technological factor is influencing income both directly and indirectly, this last one through its effect on the capital stock.

Also regarding equation (1.23), it is important to notice that for the result to be coherent, the parameter γ must be less than one, i.e. $f'_k < f(k)/k$. This would imply that in the steady state the marginal productivity of capital has to be lower than its mean productivity, which is always satisfied if the production function is a concave one.

Finally, we need still to conclude about the trade balance behaviour. In particular, from (1.12) it is easily deduced that, in equilibrium, the ratio TB/D is going to be stable over time, so in a steady state situation the trade balance growth rate should also equalise the growth rate of the stock of net foreign debt.

The next step should be then to characterise the convergence towards equilibrium. Our aim in this respect is to find out which are the circumstances that, in a steady state position, are going to determine the possible net inflow or outflow of foreign financial resources.

1.3.5 The Convergence to Equilibrium and its Fundamental Determinants

We know from the previous section that in equilibrium $\hat{c} = \hat{i} = \hat{k} = \hat{d}$ (or $\hat{f}$) $= \hat{tb} = \hat{y} = \eta \hat{A}$. However, without loss of generality we can assume that $\hat{A} = 0$. The steady state in that case would simplify to one where $\hat{C} = \hat{I} = \hat{K} = \hat{D}$ (or $\hat{F}$) $= \hat{TB} = \hat{Y} = n$, so in per capita terms these steady state growth rates will clearly approach to zero.

Let us start this analysis through the study of the investment and capital stock accumulation to continue next with the convergence and steady state properties of the rest of important variables in this model, as is the case of income, savings, foreign debt and real exchange rates.

1.3.5.1 Investment Decisions and the Capital Stock Dynamics

In a standard economy, the first stages of economic growth are usually characterised by a small accumulation of capital stock, so its marginal return becomes high enough as to promote a large investment rate. However, in the long run the arrival of diminishing returns in the production factor would cause that the capital stock growth rate decreases over time, until the steady state is finally reached and $F'_K(K,L) = r$. Let us analyse next the analytical implications of the process.

Expression (1.24) shows the particular determinant factors in the evolution of F'_K over time,

$$\Delta\left[F'_K(K,L)\right] = F''_{KK}\Delta K + F''_{KL}\Delta L = F''_{KK}K\left(\hat{K} + \frac{F''_{KL}}{F''_{KK}}\frac{L}{K}n\right) \tag{1.24}$$

However, using the assumption of constant returns to scale we can still simplify (1.24) into a more intuitive expression. Specifically, with constant returns to scale the following condition is necessarily satisfied,

$$F(K,L) = (F'_K)K + (F'_L)L \tag{1.25}$$

If we derive (1.25) with respect to K and reorganise[37], the following relationship can be easily obtained,

[37] Note that the labour force is by definition independent of the capital stock, so $dL/dK = 0$.

$$\frac{L}{K} = -\left(\frac{F''_{KK}}{F''_{KL}}\right) \tag{1.26}$$

Substituting next (1.26) into (1.25), the equation (1.27) is finally determined,

$$\Delta\left[F'_K\left(K,L\right)\right] = (\hat{K} - n)\ F''_{KK} K \tag{1.27}$$

This expression can be used to analyse the dynamic behaviour of the investment and capital stock variables.

With respect to the long run equilibrium, it is already known that an optimising agent is encouraged to invest whenever $F'_K > r$, so it should occur that $\Delta(F'_K) < 0$ over time. Following equation (1.27), we can see that the fact that F''_{KK} is negative is what guarantees this dynamic convergence to equilibrium. Additionally, since the steady state is reached when $\Delta(F'_K) = 0$, and given the condition $\hat{K} = (I/K) + \theta$, it should also occur in equilibrium that $\hat{I} = \hat{K} = n$; a result that is coherent with the previous conclusions about the steady state characterisation.

Coming back to expression (1.27), we can see that the circumstance of a decreasing marginal productivity of capital is obtained[38] whenever $\hat{K} > n$. The conclusion is therefore that in the case of a growing economy, the first stages of growth are usually characterised by a high capital stock growth rate along with a decreasing trend in the long run. Formally, this would imply that $\Delta(\hat{K})$ has to be negative for convergence to take place. In particular, we can use expression (1.28) to analyse what the previous requirement would imply on investment,

$$\Delta\hat{K} = \frac{\Delta^2 K}{K} - \hat{K}^2 = (\hat{I} - \hat{K})\ \hat{K} \tag{1.28}$$

[38] Remember that $F''_{KK} < 0$.

According to (1.28), for $\hat{K}$ to diminish in the process of convergence, investment must be such that $\hat{I} < \hat{K}$. Unfortunately, the knowledge of this condition does not allow us to specify if the per capita level of investment increases or decreases over time in its trajectory to the steady state position; both of these circumstances might happen in fact. We cannot forget at this point that this work does not enter in detail into the concrete dynamics of the investors' decisions[39].

In the same way, the stability of the process can be formally proved through the analysis of the capital stock dynamic equation. To start with, we can reformulate identity (1.21) in per capita terms as follows,

$$dk = i - (\theta + n)\,k \tag{1.29}$$

Or alternatively as in (1.30), abstracting from a particular functional form,

$$dk = J(k) \tag{1.30}$$

Note that convergence will take place whenever $J'_K < 0$, what would be satisfied if investment depends negatively on the capital stock. Obviously, this is a relationship guarantied by the assumption of F''_{KK} less than zero.

Let us analyse next the dynamic evolution of income and savings, two fundamental variables in the characterisation of a country's external equilibrium.

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[39] It could be possible, for instance, that in the first stages of growth the differential between F'_K and r provoked a rapid increase in investment, so that the latter increased in per capita terms. But, as the marginal productivity of capital and the interest rate got together, investment could slow down and even fall down in per capita terms. However, this is not the only way in which this dynamics can be characterised.

1.3.5.2 Income Level and the Saving Rate Behaviour

From the standard production function specified in (1.18), it is determined an useful expression that, in terms of growth rates, is relating the volume of national production with its corresponding productive factors, the capital stock and labour force,

$$\hat{Y} = \frac{F'_K K}{Y}\hat{K} + \frac{F'_L L}{Y} n \tag{1.31}$$

Moreover, since in the case of constant returns to scale equation (1.26) is fulfilled, we can take advantage of this particular property to additionally simplify expression (1.31). In that case, finding $(F'_L L/Y)$ in (1.26) and substituting it in (1.31), this one can be finally rewritten as follows,

$$\hat{Y} = \hat{K} - \left(1 - \frac{F'_K K}{Y}\right)(\hat{K} - n) \tag{1.32}$$

This expression (1.32) is very useful indeed to conclude about the dynamic behaviour of the income growth rate over time. In particular, we observe that with $0 < (F'_K K/Y) < 1$ the relationship between $\hat{Y}$ and $\hat{K}$ will depend on the accumulation rate of the capital stock, that is, if it overpasses or not the level of n. Obviously, in equilibrium it is satisfied that $\hat{K} = \hat{Y} = n$.

Regarding the saving rate, we can assume a simple case where savings are proportional to the disposable income, as it appears in (1.33),

$$S = sY_d = s(Y - rD) \tag{1.33}$$

This way, the evolution of savings will depend on the evolution of income and on the level of total foreign debt. Taking increments in (1.33) and dividing by S, we obtain expression (1.34) for the savings growth rate behaviour,

$$\hat{S} = \frac{\hat{Y} - (rD/Y)\hat{D}}{1 - (rD/Y)} = \hat{Y} + \frac{(rD/Y)\hat{D}}{1 - (rD/Y)}(\hat{Y} - \hat{D}) \tag{1.34}$$

In the first stages of growth, where D is sufficiently small, we find that $\hat{S} \cong \hat{Y} > n$. However, the empirical evidence shows that as foreign debt accumulates, the proportion of income that economies devote to the net payment of interests will be increasing over time. In terms of our model this would imply a per capita foreign debt growing faster than income, and a saving rate being lower that the income growth rate, i.e., $\hat{D} > \hat{Y} > \hat{S}$. In that case, only if the stock of foreign debt converges the economy will tend towards the steady state on its own. Most probably this steady state will be characterised by a situation where the national saving is insufficient to cover the investment needs of the economy.

However, we can consider instead another possibility; a situation where savings are large enough as for the stock of foreign debt to follow a decreasing process over time and the latter equilibrium conditions are eventually reversed. This dynamics opens the possibility for national savings to exceed investment necessities at a certain moment, so that the economy ends up accumulating foreign assets instead of debt. In that context, the steady state would have a considerable different characterisation.

Let us analyse next the specific conditions for the stock of foreign debt to converge.

1.3.5.3 Foreign Debt Convergence and the Possible Foreign Asset Accumulation

In order to analyse this process we start setting up in expression (1.35) the basic definition of the foreign debt growth rate,

$$\hat{D} = \frac{I - S}{D} \tag{1.35}$$

This one can be alternatively specified as in (1.36) in the case that the economy accumulates foreign debt, or as in (1.37) in the case that it accumulates foreign assets,

$$\hat{D} = rs + \frac{I - sY}{D} \tag{1.36}$$

$$\hat{F} = rs + \frac{sY - I}{F} \tag{1.37}$$

For the process to be stable, from (1.36) and (1.37) we need to check if $\Delta\hat{D}$ (or $\Delta\hat{F}$) tends to zero over time. An equation to characterise the $\Delta(\hat{D})$ dynamics is easily obtained from the original (1.35) formulation,

$$\Delta(\hat{D}) = \left[\frac{\hat{I} - \hat{S}(S/I)}{1-(S/I)} - \hat{D}\right]\hat{D} \tag{1.38}$$

Moreover, substituting the $\hat{S}$ variable by its definition in (1.34), we easily get the following result,

$$\Delta(\hat{D}) = \frac{(\hat{I} - \hat{D})(1 - rD/Y) - (S/I)(\hat{D} - \hat{Y})}{(1 - S/I)(1 - rD/Y)} \tag{1.39}$$

This equation (1.39) can be however additionally simplified by taking into account the savings definition given in (1.33). In particular, from this (1.33) expression, the following relationship will always occur,

$$\frac{rD}{Y} = 1 - \frac{S}{I}\,\frac{I}{sY} \tag{1.40}$$

Then, making use of equation (1.40), the previous (1.39) can be reformulated as follows,

$$\Delta(\hat{D}) = \frac{I(\hat{I} - \hat{D}) + sY(\hat{D} - \hat{Y})}{I - S} \tag{1.41}$$

As the aim is to analyse the convergence of the stock of foreign debt, it is determined next whether (1.41) would tend to zero over time. It is already known that as the diminishing returns of capital appear, the investment and income growth rates will progressively converge to n. In this respect, and in order to simplify, the conditions for convergence of the two dynamic variables of the problem, that is, the capital stock and the stock of foreign debt can be independently analysed. Provided then with the dynamic stability of investment, to examine (1.41) it is assumed that $\hat{I} = \hat{Y} = n$. In that case, (1.41) would reduce to the following expression,

$$\Delta(\hat{D}) = \frac{(\hat{D} - n)\ (sY - I)}{\Delta D} \tag{1.42}$$

which can be still simplified making use again of the savings definition. In particular, from (1.33) the next relationship can be also specified,

$$sY - I = (rs - \hat{D})\,D \tag{1.43}$$

So, taking into account the result in (1.43), equation (1.42) can be rewritten as follows,

$$\Delta(\hat{D}) = \frac{(\hat{D} - n)(rs - \hat{D})}{\hat{D}} \tag{1.44}$$

Or, alternatively, as in (1.45) in terms of foreign assets instead of foreign debt,

$$\Delta(\hat{F}) = \frac{(\hat{F} - n)(rs - \hat{F})}{\hat{F}} \tag{1.45}$$

From (1.44) and (1.45) it is clear that the convergence of the stock of foreign debt is not going to be so straightforward as in the case of investment but, on the contrary, it has to be carefully analysed. In particular, we will check that for the dynamics of the model, a key fact will be if the marginal savings variable s overpasses the level given by the ratio of n over r, i.e. n/r. Then, to conclude about this process we still need to look for some additional conditions that characterise the $\hat{D}$ dynamics[40].

Regarding the investment level we know that in equilibrium it will occur that $I=(n+\theta)K$. Hence, assuming the existence of constant returns to scale and that in the steady state $F'_K = r$, it would be true that[41] $I \leq (n/r)Y$. Moreover, taking into account the savings and foreign debt growth rate definitions given in expressions (1.33) and (1.35) respectively, we can finally get the following result,

$$\hat{D} - n \leq (n - rs)\left(\frac{Y - rD}{rD}\right) \qquad (1.46)$$

Since $(Y - rD) \geq 0$, the sign of the last expression will exclusively depend on the relationship between n and rs. Therefore, given the result in (1.46), for the foreign debt to converge the only possibility is that $n > rs$. That is, the model will present dynamic stability only in the case that $s < n/r$.

On the other hand, from (1.36) and (1.37) it is easily obtained that the corresponding steady state values for the stocks of foreign debt and foreign assets are respectively determined by the following expressions,

$$D^* = \frac{I^* - sY^*}{n - rs} \qquad (1.47)$$

[40] Always given that investment and income have already reached their particular equilibrium levels.

[41] Note that with constant returns to scale $Y = F'_K K + F'_L L$. If we consider that $F'_K = r$, then $Y = rK + F'_L L$, so we have that $K \leq (Y/r)$.

$$F^* = \frac{sY^* - I^*}{n - rs} \qquad (1.48)$$

where the star indicates the steady state position.

So, only if $s > (I/Y)^*$ the country will have the opportunity to end up as an asset holder. Let us simplify next the characteristics of this threshold condition.

In particular, we will assume a standard Cobb-Douglas production function that, in order to simplify, will not include technological progress. In this case, equation (1.18) can be specified in the following terms,

$$Y = K^{\alpha} L^{1-\alpha} \qquad (1.49)$$

Or, alternatively as in (1.50), with variables expressed in per capita terms,

$$y = k^{\alpha} \qquad (1.50)$$

Then, equation (1.47), again in per capita terms, can be reformulated as follows,

$$d^* = \frac{(n+\theta)(k^*)^{1-\alpha} - s}{n - rs} (k^*)^{\alpha} \qquad (1.51)$$

The reader should note that (1.51) could be still simplified by determining the steady state level of the capital stock. In particular, we already know that in steady state the capital stock should fulfil that its marginal productivity equalises the real interest rate, that is, $F'_K(K, L) = r$. Therefore, taking into account (1.49), this would imply the following result for the equilibrium capital stock if expressed in per capita terms,

$$k^* = \left(\frac{\alpha}{r}\right)^{1/1-\alpha} \tag{1.52}$$

Substituting then (1.52) into (1.51) we finally get the next steady state foreign debt level in (1.53),

$$d^* = \frac{\alpha\,(n+\theta) - r\,s}{n - rs}\left(\frac{\alpha^{\alpha}}{r}\right)^{1/1-\alpha} \tag{1.53}$$

Alternatively, without loss of generality we can avoid the capital stock depreciation rate, so that (1.53) would simplify to the next (1.54) equation,

$$d^* = \frac{\alpha\,n - r\,s}{n - rs}\left(\frac{\alpha^{\alpha}}{r}\right)^{1/1-\alpha} \tag{1.54}$$

In that case the condition saying that $s > I/Y$ would simply converge to the one where $s > \alpha\,(n/r)$. We have found therefore the main conclusions of the model.

- Corollary 1: *In this model of small economy with the assumptions of no public sector, no technological progress and a non-depreciating capital stock, it will exist convergence to a long run stock of foreign debt only if $s < n/r$.*

- Corollary 2: *Given the stability condition, the economy would end up in a net debtor position in the case that $s < \alpha(n/r)$.*

- Corollary 3: *Alternatively, if $\alpha(n/r) < s < n/r$ the country will have the opportunity to end up as an asset holder.*

Taking into account these results we can use expressions (1.36), (1.37) and (1.44) to represent the different alternative dynamics of the system.

The high factor productivity of the first stages of growth will be undoubtedly characterised by a high investment rate. This level of investment together with the low accumulated debt would lead the country to a situation where the foreign debt grows at an important rate. This one should normally decrease as investment reaches its steady state position. In this respect, if the economy satisfies that $s < \alpha(n/r)$, investment will reach its equilibrium at a higher value than sY, i.e. $I^* > sY^*$. At that level, from (1.36) it is obtained that $\hat{D} > rs$, so in equation (1.44) it can be easily checked the dynamical stability of the process. This is the case when the economy gets to its steady state with an international net debtor position. Nevertheless, it is also true that the larger the value of the marginal savings, the smaller the final accumulated per capita foreign debt will be and, therefore, more likely that the national currency ends up at a more appreciated level.

Eventually, in the case that s overpasses the threshold of $\alpha(n/r)$, the decrease in the foreign debt growth rate will continue over time until a situation where the economy changes from having a net debtor position to a net creditor one. Naturally, this transition process will be characterised by a high rhythm of asset accumulation until it successively converges to its steady state position. Again, the dynamic stability of the process can be checked in equations (1.37) and (1.46) respectively. From (1.37) it is clear that when investment reaches its steady state position the stock of foreign assets will be growing over rs, so the system will require a local stability condition that is easily derived from (1.46).

Thriftiness is therefore in this model a fundamental variable to characterise the growing process of the economy. Only if thriftiness is high enough, the economy will be able to move from a situation of net indebtedness, more likely in the first stages of growth, to a net creditor position. Of course, the real exchange rate will be directly influenced by this net foreign asset position.

This dynamic stability can be also proved through a rigorous analysis of the foreign debt dynamic equation. In particular expressions (1.55) and (1.56) give us the dynamic equilibrium of the stock of foreign debt in absolute levels and in per capita terms respectively,

$$d(D) = (I - S) \tag{1.55}$$

$$d(d) = i - (S/L) - n\,d \tag{1.56}$$

Substituting next the level of savings by its definition in (1.33), equation (1.56) can be also expressed as in (1.57),

$$d(d) = i - s\,y - (n - rs)\,d \tag{1.57}$$

Moreover, taking into account that both the investment and capital stock have reached their steady state levels, (1.57) will converge in equilibrium to equation (1.58),

$$d(d) = n\,k^* - s\,(k^*)^{\alpha} - (n - rs)\,d \tag{1.58}$$

And, if we substitute the steady state level of the per capita capital stock given in (1.52), the previous expression can be still reformulated as follows,

$$d(d) = \left(\frac{\alpha\, n}{r} - s\right)\left(\frac{\alpha}{r}\right)^{\alpha/1-\alpha} - \left(\frac{n}{r} - s\right) r\,d \tag{1.59}$$

From (1.59) it is clear that for the foreign debt -or net assets- stock to converge it is necessary that $s < (n/r)$, such as it was previously specified. Moreover, only in the case that $s > \alpha\,(n/r)$ the country will have the opportunity to end up as a foreign asset holder.

Next, charts from one to three show in graphical terms the alternative situations in which the economy would eventually converge to a net debtor position (Figure 1.1), to a net creditor position (Figure 1.2) or would follow a process of divergence (Figure 1.3).

Figure 1.1 Dynamics of Convergence in the Case of a Net Debtor Economy case $s<\alpha(n/r)$

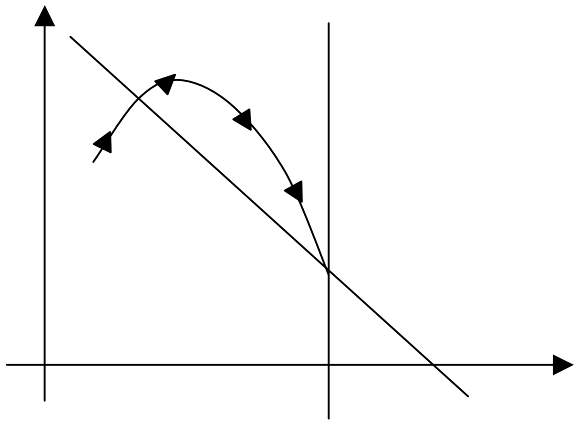


Figure 1.2 Dynamics of Convergence in the Case of a Net Creditor Economy case $\alpha(n/r)<s<(n/r)$

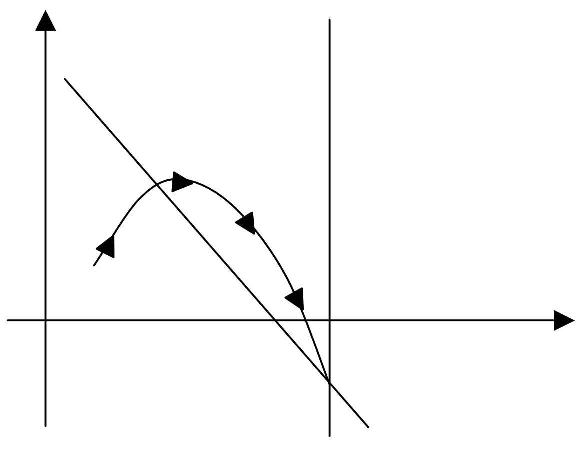
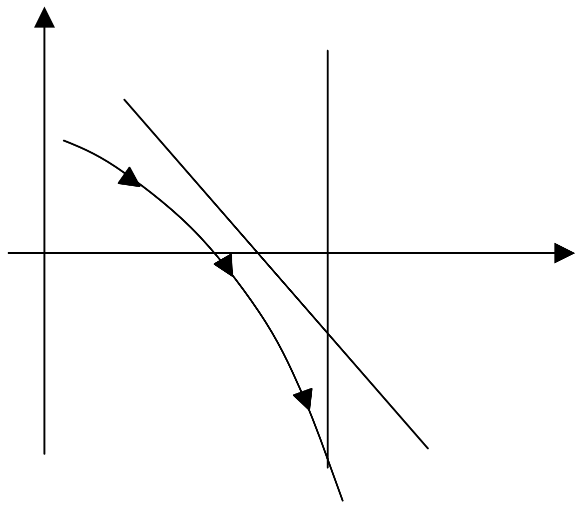


Figure 1.3 Dynamics of Divergence

s>(n/r)



1.3.5.4 The Real Exchange Rate and the Foreign Sector Path Towards Equilibrium

Finally, we will analyse in this section the implications of our previous results on the particular dynamics of the foreign sector.

The low level of income together with the high profitability of investment, commonly associated to the first stages of growth, is usually the cause by which the internal saving is not enough to cover the investment needs of an economy. In that case, it is usual that an international flow of resources from the surplus-producing countries to the loss-making ones appears, suffering these ones of increments in both the stock of foreign debt and the foreign interest payment. Notwithstanding, it is not habitual that the debt service of these first stages of growth overpasses the increment in foreign debt, so it should occur that the net transference of financial resources remains positive while the trade balance keeps on at negative values. To a great extent, the current account deficit is going to be due to the external capital inflow and not to the flow of interest payments.

From the current account and the trade balance definitions it is clear that their evolution will depend above all on the process of debt accumulation. Therefore, it is the steady state level of the foreign debt that will characterise the equilibrium of all, the current account, the trade balance and the real exchange rate variables. Let us analyse next this issue.

From (1.12) it is easy to obtain an expression for the trade balance growth rate,

$$\hat{TB} = \hat{D} - \frac{\Delta(\hat{D})}{r - \hat{D}} \tag{1.60}$$

Moreover, given the behaviour of debt provided in (1.42), the previous expressions can be also specified as follows,

$$\hat{TB} = \hat{D} - \frac{(\hat{D} - n)(rs - \hat{D})}{(r - \hat{D})\hat{D}} \tag{1.61}$$

Or, alternatively, as the symmetric equation (1.62),

$$\hat{TB} = \hat{F} - \frac{(\hat{F} - n)(rs - \hat{F})}{(r - \hat{F})\hat{F}} \tag{1.62}$$

From (1.61) and (1.62) it is clear that the trade balance would converge to a growth rate equal to n as the stock of foreign debt or foreign assets converges to its steady state level. The main differences will arise however in the absolute level of the trade balance position. In particular, the grater the accumulated stock of foreign debt, the greater the necessity of trade balance surplus for the economy to converge and, therefore, the more depreciated the real exchange rate will be in equilibrium. The idea underlying this result is that the trade balance should necessarily improve in order to finance the income outflow caused by the external indebtedness position.

Alternatively, in the case of asset accumulation, the process will lead the economy to a positive transference of resources that would eventually reverse the real exchange rate evolution.

In the first stages of repayment, the outflow of resources is going to be due not to the interest payment but to the external debt refund. In that case, and as foreign debt is repaid, the economy would accumulate assets until it ends up receiving positive financial flows. The process will necessarily result in a stronger national currency and a more deteriorated trade balance that, however, will keep on favouring the current account evolution.

1.3.5.5 The Real Exchange Rate, the Income Level and the Debt Cycle Theory

We will devote this section to summarise the main conclusions of our model on the theoretical relationship between income and real exchange rates over the different stages of growth. Curiously, our conclusions coincide broadly speaking with the idea of development proposed by the debt cycle theory. Specifically, we will highlight the three main characteristic periods in a common process of growth.

In a first stage, it will be usual that the volume of national savings cannot satisfy the high demand for investment. In that case, there will exist a large inflow of resources in the form of loans that will transitorily strengthen the national currency in real terms. Likewise, the economy would incur in important trade balance deficits accompanied in its turn by deficits on the current account.

In a second stage, the service of the net foreign debt will generate an international outflow of resources that at a certain moment would start to overpass the volume of resources coming from abroad. Sooner or later, this change provoked by the financing of the growth process will end up generating a weaker national currency. Naturally, the eventual result is a trade balance surplus, given the necessity of serving the debt, and a current account deficit.

Potentially, the economy will be able to enter in a new stage in which the total amount of debt is repaid and there is even an asset accumulation. In that case, there will be a net inflow of resources that, in the long run, would guarantee a more appreciated national currency and, indeed, a current account surplus. The result is therefore that, as long as the three stages of growth are covered, high income levels will be observed along with stronger national currencies. Only in some particular circumstances the economy is not going to acquire a creditor position, so it would be characterised as in the second stage of growth. Unfortunately, the economy will not be able to overcome in this case the initial weakness of its national currency.

Nevertheless, it is reasonable to think that although with low income levels consumption has a strong weight in the disposable income composition, as economies access to more resources a sensible evolution is that agents increase their thriftiness parameter[42]. Therefore, it will not be strange to observe stronger national currencies in economies with higher income levels. Exceptions to this rule will be those countries that have got into an excessive indebtedness position in their growing process.

1.4 Summary and Concluding Remarks

The empirical evidence shows that associated with the growing process of an economy, there is normally a process of loss of competitiveness against less developed countries. This evidence is what has contributed in fact to the development of the branch of literature that rejects PPP accomplishment and look for alternative explanations that highlight the role of real exchange rate fundamentals.

Specifically, some different theories have been proposed in order to explain the positive correlation between the real exchange rate and the per capita gross domestic product. These

[42] Note that this analysis does not consider agents thriftiness evolution over time.

theories concentrate on the case of an economy with two sectors, tradables and nontradables, and emphasise the role of GDP in the determination of the relative price level. The traditional theories under this approach are mainly supply-oriented models like the Balassa-Samuelson hypothesis or the Heckscher-Ohlin model. As opposed to these theories, demand-oriented approaches have been incorporated afterwards.

However, the main problem of these theoretical attempts is that they have not considered the financial restrictions imposed by the course of the economic growth. Thereby, for a comprehensive analysis of this economic problem under debate, it is also important to conclude about the intertemporal aspects that the financing of the growth process necessarily imposes. That is, in this context the equilibrium condition of the balance of payments should be also taken into account.

We highlight next the major conclusions of the chapter.

We can distinguish three differentiated stages in the growth process of a standard economy. In a first stage, it will occur that large inflows of external resources would transitorily appreciate the national currency. However, in a second stage, the appearance of the diminishing returns of capital will halt investment growth and, therefore, the need for external financing. In this case, a steady state can be reached where the economy held a net indebtedness position. Occasionally, in a third stage, if thriftiness is high enough, a new equilibrium can occur where the country held a net creditor position, and where there would be a net inflow of resources strengthening the national currency.

Nevertheless, if we take into account that as economies grow agents thriftiness can increase, it can be usual to observe, independently of the net indebtedness or creditor position, that the higher the income level of an economy, the more appreciated the real exchange rate will be. Exceptions to this rule will be those countries that have got into an excessive debt accumulation.

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CHAPTER 2

THE STRUCTURAL APPROACH OF A NATREX MODEL ON EQUILIBRIUM EXCHANGE RATES

2.1 Introduction

Certainly, economists have made significant efforts trying to report convincing explanations about the exchange rate behaviour. The results, however, have not been up to the dimension of the work and, as Meese and Rogoff (1983) underline, too frequently they have turned out a little bit disappointing. The new research on the topic has echoed this fact revealing the necessity of alternative results. This is the case of the quite recent contributions to the structural approach, which have clearly shown up their preference for the long-run analysis and for stronger theoretical structures. The interest of these new contributions is in reality two-fold. On one hand, they avoid the cyclical and speculative short run behaviour, proved to be transitory and rather unpredictable and, on the other hand, they deliberately oversight the simplifying assumption of highly integrated markets. Instead, the approach deals with the characteristics of the internal and external economic equilibrium, as well as with the stability requirements of the system.

To a large extent, the most popular approaches to exchange rate determination have ended up, in one way or another, bringing up single-equation systems where the nominal or real exchange rate has been simply related to a set of possible exogenous explanatory variables. From an economic point of view, this sort of proceeding can be justified by the useful, but important, simplifications that a reduced version imposes on the theoretical complexity of a model and on the intuitive interpretation of it. Probably, the best known examples of this nature are the PPP hypothesis, along with its subsequent theoretical developments[43], and those that, taking a step forward, have included some macroeconomic fundamental determinants. This is for instance the case of the vast research based on the Balassa-Samuelson supply hypothesis –originally developed from separated contributions of Balassa (1964) and Samuelson (1964)– or the demand-side fundamental approach developed in De Gregorio, Giovannini and Wolf (1994). Recent contributions on the PPP approach are those of Engel and Rogers (2001) and Rey and Varachaud (2002) for the case of the European economies; Strong and Sharma (2002) and Taylor (2002) for a group of industrialised countries and Anoruo, Braha and Ahmad (2002) for the case of developing economies. There are also recent notable contributions on the Balassa-Samuelson hypothesis like MacDonald and Ricci (2001), DeLoach (2001) or Canzoneri et al (2002).

Regarding exchange rate determination, it cannot be either ignored the extensive research done from a merely econometric point of view. This sort of literature has evolved from simple univariate time-series techniques, and its subsequent multivariate extensions, to the new multivariate techniques for non-stationary variables. These ones, on their own, conform a complete battery of work that goes from simple uniequational exchange rate cointegrating relationships to the more complicated SVAR (structural vector autoregressive) approach. Recent outstanding contributions using the cointegration technique are Karfakis and Phipps (1999) for the case of the American dollar against the Australian dollar and Detken et al (2002a, 2002b) for a composite index of the euro against its major trading partners. A good reference of the SVAR literature is the seminal work of Clarida and Gali (1994)[44] as well as that of MacDonald and Swagel (1998).

However, and although it would not be fair to refer to these theories as simply rough attempts to better understand the exchange rate behaviour, it is obvious to us that the great majority of them can be merely assumed as partial views of a more general global market equilibrium. The alternative can be found in the so-called structural approach, which follows in fact two main tracks, the partial equilibrium specification and the general equilibrium one.

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[43] That is, the asset monetary approach to exchange rate determination.

[44] Authors of the so-called CG model.

The partial equilibrium approach, or macroeconomic balance approach, has focused exclusively on the trade balance component of the current account, so that it has completely avoided the existent feedback between the capital account and the real exchange rate determination. This feedback, clearly induced by the payments for serving the accumulation of net foreign debt, generates a persistence effect that cannot be ignored. On the contrary, the general equilibrium approach is based on complete macroeconomic models that take care, in one way or another, of the problem of long-run sustainability, being consequently conscious of the hysteresis phenomenon. The most known applications of this general equilibrium framework are the Williamson's fundamental equilibrium real exchange rate (FEER), the IMF's desired equilibrium exchange rate (DEER) and the natural equilibrium real exchange rate (NATREX) of Stein. Representative works of these approaches are Williamson (1994) for the FEER approach, Bayoumi et al (1994) for the case of the DEER and Stein and associates (1995) for the NATREX approach.

This chapter aims to contribute to the theoretical literature on the structural macroeconomic approach analysing the real exchange rate from a dynamic general equilibrium perspective. In the chapter, we set a closed micro-founded structural model to determine the long run real exchange rate of a developed economy. In particular, the analysis will follow the structure of a Natrex model[45], whose main peculiarities within the structural approach are twofold. On one hand, the own structure of the model presents a singular stock-flow interaction not known up to the moment. On the other hand, it introduces an exclusive distinction between the medium and the long-run equilibrium that consists on the following: while in the medium-run only the external and internal equilibrium conditions will be required, in the long-run it will be also necessary that the net foreign debt plus the capital stock reach their steady state levels.

It is interesting how many of the self-named structural models of exchange rate determination, included the different versions of the Natrex approach, have ended up, in one way or another, only partially deriving the equations that constitute the theoretical models, so

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[45] Fist developed by Stein (1990, 1994).

the unfortunate result is that the approaches lack of well-integrated structures with relative frequency. Regarding the implementation of equilibriums, an additional objection is also the systematic omission of some important theoretical restrictions that are in fact determinant for the structural characterisation of the approach. Finally, and not less important, there is the fact of the continuous absence of a formal discussion about the convergence and stability of the systems under analysis. In that sense, the main contribution of this chapter is the development of a solid theoretical framework that analyses in depth the basis of the real exchange rate, as well as the details of the equilibrium dynamics after any shock influencing the steady state positions. In our case, the intertemporal factors derived from the stock-flow relationship will be particularly determinant.

The main results of the chapter are related next. In first place, a complete well-integrated structural model for long-run real exchange rate determination is developed from first principles. Moreover, we find that for convergence reasons, there are some restrictions that the model will necessarily need to satisfy. For the medium run convergence, the trade balance sensitivity to changes in the real exchange rate should be higher, in absolute values, than the correspondent one for investment decisions[46]. Regarding the long-run, it is also necessary both, that there exists a negative relationship between investment and capital stock accumulation and that the global saving of the economy (integrating public and private sectors) depends positively on the net foreign debt accumulation. In addition, there are also interesting conclusions about the results that certain shocks over the exogenous variables of the model have on real exchange rates.

The structure of the chapter is as follows. To start with, section two introduces the concept of equilibrium as well as the notion of stability used along the chapter, which is set up according to the Natrex approach. Next, in order to find well-defined investment, consumption and trade balance equations that describe the behaviour of our fully rational economic agents, in section three it is developed the correspondent micro-founded optimising programs. To continue, section four introduces the definitive characterisation of the equilibrium model both in a medium and long-run horizon, as well as it is introduced a detailed analysis of the

[46] Following Stein (1999), in this approach investment depends negatively on real exchange rates.

medium and long run convergence conditions of the system. Moreover, section four presents in addition interesting conclusions about the effect of some particular shocks over the fundamentals of equilibrium. Finally, section five concludes the chapter.

2.2 Equilibrium, Sustainability and the Natrex Approach

There is a wide consensus regarding what can be understood by long-run equilibrium of the real exchange rate. In this respect, Nurkse (1945) had already laid the foundations of the concept considering that it is the real exchange rate that is consistent with the dual objectives of internal and external balance of an economy, given the values of other variables that could influence the established equilibrium. However, although the definition is conceptually clear, in practical terms it is not so easy to find an agreement about how to deal with the implementation of these theoretical equilibriums.

In the particular case of Natrex, the approach introduces a distinction between the medium and long-run equilibrium depending on if the considered stock variables of the model, that is, the capital stock and the accumulated stock of foreign assets, have the time to converge. The Natrex, defined as a medium run equilibrium, is understood as the real exchange rate that prevails once speculative and cyclical factors are absent and the unemployment is at its natural rate. Therefore, in this medium term approach the internal equilibrium will be characterised by a non-inflationary employment rate, while for the external equilibrium it is simply required a balance between the international flow of capitals. This medium run analysis does not enter however into considerations about long-run sustainability conditions.

Alternatively, in the long-run the Natrex approach introduces the additional requirement that the capital stock and the stock of foreign debt should reach their steady state levels. For the external balance condition it implies not only that the current account will need to be financed by foreign capitals, but that the capital inflow should be sustainable over time.

Regarding the exogenous factors driving the equilibrium, the most important ones within the Natrex approach are a thrift parameter measuring the preference for consumption and a measure of productivity. Such as Stein suggested, in the context of the Natrex models the standard responses of exchange rates to exogenous shocks are the following: a negative shock to the consumption preferences will clearly depreciate the currency in the medium-run, although it will lead to a real appreciation in the long-run. However, a negative productivity shock will depreciate the equilibrium exchange rate in the medium-run, whereas the long-run effect can be ambiguous and will depend on the speed of decumulation of net foreign debt.

2.3 Behavioural Equations of the Model

In this model there are decisions on consumption, production, savings and investment, which will determine the equilibrium of the economy. To start with, the main assumptions of the model are carefully detailed next.

2.3.1 Assumptions

a) Consumption and investment decisions are decentralised. That is, the case of a representative agent who decides both variables simultaneously is specifically omitted. In this approach families will choose consumption and savings, while firms decide production and investment.

b) Savings are canalised to the national or international investment requirements through the financial system. That is, the national and foreign families can make savings profitable buying national or foreign bonds. Moreover, it is introduced here the assumption of perfect international financial capital mobility, what implies a domestic rate of return tied to the international one.

c) In this model the labour supply is assumed offered inelastically inside the country, although it is free to migrate between sectors. Regarding the capital stock, it will grow depending on investment decisions.

d) The role of the relative price between national and foreign goods is introduced through the differentiation between goods of tradable and non-tradable nature. Non-tradable goods will be produced and consumed inside the country, while tradables are freely dealt in international homogeneous markets. They will be therefore indistinctly produced and consumed both inside and outside the country, being their price exogenously determined in international markets[47].

e) Prices are considered in relative terms. In particular, the price of each good is defined in terms of a numerary that, in this case, is given by a good of tradable nature. The model is therefore characterised by the absence of a role for money, which is completely neutral in the analysis.

f) Finally, it is assumed that it does not exist any kind of international transactional costs or distorting taxes.

2.3.2 Investment Decisions

Let us assume common Cobb-Douglas production functions for the national production of tradable (T) and non-tradable (N) goods respectively,

$$Q_T = f(K_T, L_T) = A^{1-\alpha} K_T^{\alpha} L_T^{1-\alpha} \tag{2.1}$$

$$Q_N = f(K_N, L_N) = A^{1-\beta} K_N^{\beta} L_N^{1-\beta} \tag{2.2}$$

Being Q the real GDP, K the national capital stock -assumed exclusively composed of tradable goods-, L the total labour force and parameter A a labour augmenting productivity factor equal for the two sectors of the economy.

To determine investment we will start off with the definition in real terms of the sectorial

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[47] Note that in this particular case the terms of trade are assumed to be equal to one, so the real exchange rate can be approximated by the relative price of tradable to non-tradable goods. For an exhaustive analysis of this point see appendix 2.1.

profit equations, being all variables specified in terms of the numerary. These are expressions (2.3) and (2.4) respectively,

$$Pr_T = Q(K_T, L_T) - r K_T - \omega L_T \tag{2.3}$$

$$Pr_N = R\, Q(K_N, L_N) - r K_N - \omega L_N \tag{2.4}$$

where R is the real exchange rate approached by the price index ratio of non-tradable to tradable goods, and r and ω are the real interest rate and real wage respectively.

Moreover, the following existent relationships between the sectorial and global variables should be also taken into account,

$$K = K_T + K_N \tag{2.5}$$

$$L = L_T + L_N \tag{2.6}$$

In order to find the optimal levels of the labour force and capital stock, we should optimise equations (2.3) and (2.4) with respect to the two sectorial production factors. Conditions from (2.7) to (2.10) give us successively the obtained results,

$$\frac{\partial Pr_T}{\partial K_T} = \frac{\partial Q_T}{\partial K_T} - r = 0 \tag{2.7}$$

$$\frac{\partial Pr_N}{\partial K_N} = R\frac{\partial Q_N}{\partial K_N} - r = 0 \tag{2.8}$$

$$\frac{\partial Pr_T}{\partial L_T} = \frac{\partial Q_T}{\partial L_T} - \omega = 0 \tag{2.9}$$

$$\frac{\partial Pr_N}{\partial L_N} = R\frac{\partial Q_N}{\partial L_N} - \omega = 0 \tag{2.10}$$

Determining next the required derivatives from the correspondent production functions, the previous optimality conditions can give us final expressions for the labour and capital stock equilibrium levels,

$$K_T = A\,L_T\left(\frac{\alpha}{r}\right)^{1/1-\alpha} \tag{2.11}$$

$$K_N = A\,L_N\left(\frac{\beta R}{r}\right)^{1/1-\beta} \tag{2.12}$$

$$L_T = K_T\,A^{1-\alpha/\alpha}\left(\frac{1-\alpha}{\omega}\right)^{1/\alpha} \tag{2.13}$$

$$L_N = K_N\,A^{1-\beta/\beta}\left(\frac{1-\beta}{\omega/R}\right)^{1/\beta} \tag{2.14}$$

The optimal sectorial distribution can be also determined taking into account that the capital and labour remuneration, r and ω respectively, should equalise across sectors. For the capital stock, (2.11) and (2.12) will be the relevant expressions to work with, while (2.13) and (2.14) will be the ones for the case of the labour market. These subsequent equilibrium conditions are (2.15) and (2.16) respectively,

$$\frac{K_N^{\beta}}{L_N^{\beta}} = \frac{1}{A^{\alpha\text{-}\beta}}\frac{1}{R}\frac{\alpha}{\beta}\frac{L_T}{L_N}\frac{K_N}{K_T}\frac{K_T^{\alpha}}{L_T^{\alpha}} \tag{2.15}$$

$$\frac{K_T^{\alpha}}{L_T^{\alpha}} = A^{\alpha-\beta}R\,\frac{1-\beta}{1-\alpha}\frac{K_N^{\beta}}{L_N^{\beta}} \tag{2.16}$$

Moreover, when the capital stock has the time to redistribute between sectors, (2.15) and (2.16) can give us a unique equilibrium condition, shown next in (2.17),

$$\frac{K_T}{L_T} = \frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{K_N}{L_N} \tag{2.17}$$

At this point we can take advantage of equation (2.17) to determine expressions for the sectorial capital stocks as functions of the global one. They are easily obtained by the simple combination of the previous (2.17) equation with the (2.5) identity,

$$K_T = \frac{1}{1 + \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}} K \tag{2.18}$$

$$K_N = \frac{1}{1 + \frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}} K \tag{2.19}$$

Let us assume next that the engine moving the capital stock is the labour augmenting productivity growth rate. In that case, the first differences of identity (2.5) with respect to A will allow us to specify the first approach of the required investment function,

$$I = A\left(\frac{dK_T}{dA} + \frac{dK_N}{dA}\right)\hat{A} \tag{2.20}$$

Taking into account expressions (2.11) and (2.12) for K_T and K_N respectively, and given L as a fixed value, the correspondent derivatives necessary to solve equation (2.20) are then easily obtained,

$$\frac{dK_T}{dA} = \frac{K_T}{A} + \frac{K_T}{L_T}\frac{dL_T}{dA} \tag{2.21}$$

$$\frac{dK_N}{dA} = \frac{K_N}{A} + \frac{K_N}{L_N}\frac{dL_N}{dA} \tag{2.22}$$

Substituting next (2.21) and (2.22) into the correspondent (2.20) equation, it is finally obtained a concrete expression representing the required investment function,

$$I = A\left[\frac{K_T}{A} + \frac{K_N}{A} + \left(\frac{K_T}{L_T} - \frac{K_N}{L_N}\right)\frac{dL_T}{dA}\right]\hat{A} \tag{2.23}$$

Or alternatively,

$$I = A\,L_T\,\frac{K_T}{A\,L_T}\left[1 + \frac{K_N}{K_T} + \left(1 - \frac{K_N}{K_T}\frac{L_T}{L_N}\right)\frac{A}{L_T}\frac{dL_T}{dA}\right]\hat{A} \tag{2.24}$$

Moreover, making use again of the (2.11) and (2.17) expressions, the previous (2.24) equation can still simplify to the next one,

$$I = A\,L_T\left(\frac{\alpha}{r}\right)^{1/1-\alpha}\left[1 + \frac{K_N}{K_T} + \frac{\alpha - \beta}{\alpha\,(1-\beta)}\frac{A}{L_T}\frac{dL_T}{dA}\right]\hat{A} \tag{2.25}$$

In order to determine dL_T/dA we will start off with the derivative of equation (2.17) with respect to the A parameter,

$$\frac{dL_T}{dA} = \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{K_T}{K_N}L_N\left[\frac{1}{K_T}\left(\frac{dK_T}{dK} - \frac{K_T}{K_N}\frac{dK_N}{dK}\right)\frac{dK}{dA} + \frac{1}{L_N}\frac{dL_N}{dL_T}\frac{dL_T}{dA}\right] \tag{2.26}$$

Coming back to the use of the own (2.17) equation plus the (2.5) and (2.6) identities, (2.26) will subsequently simplify to equation (2.27),

$$\frac{dL_T}{dA} = \frac{\dfrac{L_T}{K_T}\left[\left(1 + \dfrac{K_T}{K_N}\right)\dfrac{dK_T}{dK} - \dfrac{K_T}{K_N}\right]\dfrac{dK}{dA}}{1 + \dfrac{L_T}{L_N}} \tag{2.27}$$

Finally, from the following (2.28) and (2.29) expressions, derived respectively from equations (2.17) and (2.18),

$$\frac{K_T}{K_N} = \frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N} \tag{2.28}$$

$$\frac{dK_T}{dK} = \frac{\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}{1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}} K \tag{2.29}$$

we can easily conclude the following,

$$\frac{dL_T}{dA} = 0 \tag{2.30}$$

And, consequently, the final expression for the investment function is given by equation (2.31),

$$I = A\,L_T \left(\frac{\alpha}{r}\right)^{1/1-\alpha} \left(1+\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}\right)\hat{A} \tag{2.31}$$

For simplicity reasons, the model will be presented with equations as ratios to GDP. Therefore, the next step should be to determine the explicative variables of production.

To start with, let us define the total real production Q (defined in terms of tradable good) as a function of the sectorial ones,

$$Q = Q_T + R\,Q_N \tag{2.32}$$

Given equation (2.15), which shows the optimal distribution of the capital stock between sectors, plus the (2.1) and (2.2) definitions of Q_T and Q_N respectively, the following relationship is easily obtained,

$$R\,Q_N = \frac{\alpha}{\beta}\frac{K_N}{K_T}Q_T \qquad (2.33)$$

Now, the previous production function (2.32) can be simplified to the next (2.34) one,

$$Q = \left(K + \frac{\alpha - \beta}{\beta}K_N\right)\frac{Q_T}{K_T} \qquad (2.34)$$

Moreover, taking into account the definition of K_N as it appears in (2.19), equation (2.34) can simplify again to the following,

$$Q = K\ \frac{\alpha}{\beta}\left(\frac{1+\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}{1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}\right)\frac{Q_T}{K_T} \qquad (2.35)$$

In the same way, the ratio of Q_T to K_T can be determined as a function of the global capital stock taking into account the production function (2.1) and expression (2.18) that connects K_T with the global capital stock,

$$\frac{Q_T}{K_T} = \left(\frac{A\,L_T}{K}\right)^{1-\alpha}\left(\frac{1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}{\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{L_T}{L_N}}\right)^{1-\alpha} \qquad (2.36)$$

Substituting then (2.36) into (2.35), this one easily collapses to the new production function (2.37),

$$Q = A^{1-\alpha}\,L_T^{1-\alpha}\,K^{\alpha}\,\frac{1+\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}}{\left(1+\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{L_N}{L_T}\right)^{\alpha}} \qquad (2.37)$$

From (2.37), rearranging and grouping conveniently, it is determined a final expression for production where the capital stock appears as a ratio to GDP,

$$Q = A\,L_T\,(K/Q)^{\alpha/1-\alpha}\left[\frac{1+\dfrac{1-\alpha}{1-\beta}\dfrac{L_N}{L_T}}{\left(1+\dfrac{\beta}{\alpha}\dfrac{1-\alpha}{1-\beta}\dfrac{L_N}{L_T}\right)^{\alpha}}\right]^{1/1-\alpha} \tag{2.38}$$

So we can now specify the investment function as a ratio to GDP as it appears in (2.39),

$$\frac{I}{Q} = \frac{\hat{A}}{(K/Q)^{\alpha/1-\alpha}}\left(\frac{\alpha}{r}\right)^{1/1-\alpha}\left(\frac{1+\dfrac{\beta}{\alpha}\dfrac{1-\alpha}{1-\beta}\dfrac{L_N}{L_T}}{1+\dfrac{1-\alpha}{1-\beta}\dfrac{L_N}{L_T}}\right)^{1/1-\alpha} \tag{2.39}$$

Finally, we should look for the explanatory variables of the ratio of L_N over L_T. Making use of definitions (2.13) and (2.14) for L_T and L_N respectively, the following relationship is easily obtained,

$$\frac{L_N}{L_T} = R^{1/\beta}\,\frac{(1-\beta)^{1/\beta}}{(1-\alpha)^{1/\alpha}}\left(\frac{A}{\omega}\right)^{\alpha-\beta/\alpha\beta}\frac{K_N}{K_T} \tag{2.40}$$

So, substituting this one into (2.39), equation (2.40) can be redefined as follows,

$$\frac{I}{Q} = \frac{\hat{A}}{(K/Q)^{\frac{\alpha}{1-\alpha}}}\,\frac{\beta}{\alpha}\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\left(\frac{\dfrac{\alpha}{\beta}+R^{1/\beta}\,\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}}{1+R^{1/\beta}\,\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}}\right)^{\frac{1}{1-\alpha}} \tag{2.41}$$

Given that the index A and real wages will grow at the same rate, and that after increments in the capital stock (every thing equal) the ratio K_N/K_T would keep constant, from equation (2.41) the main variables explaining the investment ratio are deduced. Summarising the most important factors we find that the ratio of the capital stock, the real interest rate and the real exchange rate are all variables influencing negatively the investment ratio, while improvements of the productivity index $(\hat{A})$ are alternatively boosting it. These results are summarised as follows,

$$\frac{I}{Q} = F_I\left(\hat{A}, \frac{K}{Q}, r, R\right) \tag{2.42}$$

Being $(F_I)'_{\hat{A}} > 0$, $(F_I)'_{K/Q} < 0$, $(F_I)'_r < 0$ and $(F_I)'_R < 0$. Appendix 2.2 proofs in detail the signs of the derivatives.

2.3.3 Consumption and Savings Decisions

Let us consider a representative family who decides between consumption and savings. Given that in this section we are interested in the national global levels of the two variables, in order to simplify we decide to undertake this particular problem working –at least temporarily– not in terms of tradable and non-tradable goods, but in aggregated terms.

In this case, it is assumed that the representative family choosing between consumption and savings is solving this dilemma by means of an intertemporal utility maximisation problem. The optimisation problem, without loss of generality, is simply specified over the horizon of two single periods. The periods, named present and future period, will be represented by the sub-indexes 1 and 2 respectively.

The families in this approach will not need to restrict their consumption to their current period incomes. That is to say, whenever the agents are able to predict future incomes and there are not liquidity constraints, they can change present for future consumption and vice versa. Moreover, the family can conclude its live with a predetermined quantity of assets inherited by the next generation. The only restriction imposed to this assumption is that the final quantity of accumulated assets should not evolve explosively over time.

Therefore, in order to induce convergence it is assumed that the decision to accumulate assets in the second period is going to be discouraged by the possession of some of them inherited from a previous generation. Moreover, we also find reasonable that this negative relationship is qualified by the families' preferences for consumption. In this respect, we are going to consider that a higher preference for consumption should cause a lower desire to accumulate assets at the end of the life, facilitating then an increment in the volume of consumption. From these assumptions the following relationship should occur,

$$\frac{a_2}{q_2} = f\left[\frac{a_0}{q_0}, \varphi\right] \quad f_1' < 0 \quad f_2' < 0 \tag{2.43}$$

The consideration that for the consumer the most important thing is not the possession of assets in absolute level but their proportion out of his total income is what motivates that a_0 an a_2 enter in (2.43) as ratios of the families' incomes. The parameter φ measures the preference for consumption.

To simplify the subsequent analysis, we will concretise the relationship (2.43) to a particular functional form. Being the linear specification the more comfortable one, (2.43) is finally defined as in (2.44),

$$\frac{a_2}{q_2} = -\alpha\left(\frac{a_0}{q_0}\right) - \varphi_1 \quad 0 < \alpha < 1 \tag{2.44}$$

To determine the intertemporal budget constraint we can start with the definition of savings taking into account that this variable has a dual perspective; from a financial point of view savings can be considered as the flow of accumulated assets, while from the point of view of consumption it can be defined as the differential between disposable income and the own quantity of consumption. These two aspects of savings have been reflected in expressions (2.45) and (2.46) respectively,

$$s_t = a_t - a_{t-1} \tag{2.45}$$

$$s_t = q_t + r_{t-1} a_{t-1} - c_t \tag{2.46}$$

Being *s* savings, *a* the total stock of accumulated assets, *q* the share of national production received by the family, *r* the real interest rate and *c* the level of consumption. Lower case means (with the exception of the real interest rate) that variables are defined for the sphere of activity of the family.

The total stock of assets *a* represents the family portfolio composed of both national bonds, named *b*, and foreign assets, named *f* or *d* alternatively. In particular, the name *f* is associated to the case when the family possesses rights over the foreign sector, while the name *d* is associated to the possession of foreign debt. The identity can be therefore indistinctly specified as in (2.47) or (2.48),

$$a_t = b_t + f_t \tag{2.47}$$

$$a_t = b_t - d_t \tag{2.48}$$

Working with expressions (2.45) and (2.46), the dynamic equation for the stock of total assets *a* is easily obtained and it remains as follows,

$$a_t = q_t - c_t + (1 + r_{t-1})\, a_{t-1} \tag{2.49}$$

Particularising (2.49) for the periods one and two, the following relationships are consequently obtained,

$$a_1 = q_1 - c_1 + (1 + r_0)\, a_0 \tag{2.50}$$

$$a_2 = q_2 - c_2 + (1+r_1)\, a_1 \tag{2.51}$$

These (2.50) and (2.51) equations are the base to determine the family's intertemporal budget constraint that appears in (2.52),

$$c_1 + \frac{c_2}{1+r_1} = q_1 + \frac{q_2}{1+r_1} + (1+r_0)\, a_0 - \frac{a_2}{1+r_1} \tag{2.52}$$

Regarding preferences, it is assumed a standard CES[48] utility function that depends positively on both the present and the future consumption as expression (2.53) shows,

$$U(c_1, c_2) = \left(c_1^{\gamma} + c_2^{\gamma}\right)^{1/\gamma} \tag{2.53}$$

The parameter γ is directly related to the intertemporal elasticity of substitution. In particular, appendix 2.3 displays the concrete relationship, shown in (2.54), connecting γ and the elasticity of substitution between the future and present consumption, named ε_{21}.

$$\varepsilon_{21} = \frac{1}{1-\gamma} \quad \gamma < 1 \quad \varepsilon'_{\gamma} > 0 \tag{2.54}$$

As (2.54) suggests, the parameter γ and ε_{21} move in the same direction. The intuition behind this elasticity of substitution is that in the case of a higher ε_{21} the consumer would remain indifferent renouncing to important quantities of present consumption in exchange of an additional unit of future consumption, and vice versa in the case of a lower ε_{21}.

We are now equipped with the appropriate elements to solve the consumer optimisation problem. The next step is therefore the resolution of the Lagrangian function given in equation (2.55),

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[48] CES is the acronym of constant elasticity of substitution.

$$\lambda(c_1,c_2)=c_1^{\gamma}+c_2^{\gamma}-\lambda\left[c_1+\frac{c_2}{1+r_1}-q_1-\frac{q_2}{1+r_1}-(1+r_0)\,a_0+\frac{a_2}{1+r_1}\right] \tag{2.55}$$

Equations (2.56), (2.57) and (2.58) are therefore the first order conditions of the problem,

$$\frac{\partial\lambda}{\partial c_1}=c_1^{\gamma-1}-\lambda=0 \tag{2.56}$$

$$\frac{\partial\lambda}{\partial c_2}=c_2^{\gamma-1}-\frac{\lambda}{1+r_1}=0 \tag{2.57}$$

$$\frac{\partial\lambda}{\partial\lambda}=c_1+\frac{c_2}{1+r_1}-q_1-\frac{q_2}{1+r_1}-(1+r_0)\,a_0+\frac{a_2}{1+r_1}=0 \tag{2.58}$$

From (2.56) and (2.57) it is obtained the equilibrium condition (2.59) that relates the present and future consumption,

$$\frac{c_2}{c_1}=(1+r_1)^{1/1-\gamma} \tag{2.59}$$

Equation (2.58) simply provides the intertemporal budget constraint previously detailed in (2.52).

Eventually, with the determination of (2.59) we are provided with the necessary equations to settle down the optimal consumption and savings decisions. The equations to consider in this problem are therefore (2.46), (2.59) and the budget constraint (2.52).

In particular, from (2.59) and (2.52), the expression (2.60) defining the present consumption is easily obtained. In the case of savings, it is (2.46) and the own (2.60) the ones used instead.

$$c_1 = \frac{(1+r_1)\left[q_1 + (1+r_0)\,a_0\right] + q_2 - a_2}{1 + r_1 + (1+r_1)^{1/1-\gamma}} \tag{2.60}$$

$$s_1 = q_1 + r_0\,a_0 - \frac{(1+r_1)\left[q_1 + (1+r_0)\,a_0\right] + q_2 - a_2}{1 + r_1 + (1+r_1)^{1/1-\gamma}} \tag{2.61}$$

However, given that we are interested in the national aggregated consumption and savings decisions, the obtained (2.60) and (2.61) individual equations will have to be extended to functions that represent the whole population. Expressions (2.62) and (2.63) show this aggregated versions, where we have also assumed equal interest rates between periods,

$$C_1 = \frac{(1+r)\left[Q_1 + (1+r)\,A_0\right] + Q_2 - A_2}{1 + r + (1+r)^{1/1-\gamma}} \tag{2.62}$$

$$S_1 = Q_1 + r\,A_0 - \frac{(1+r)\left[Q_1 + (1+r)\,A_0\right] + Q_2 - A_2}{1 + r + (1+r)^{1/1-\gamma}} \tag{2.63}$$

The capital letters means national levels and the total stock of assets A_2, from equations (2.44) and (2.48), is determined as follows,

$$A_2 = -Q_2\left[\alpha\left(\frac{A_0}{Q_0}\right) + \varphi_1\right] \tag{2.64}$$

$$A_0 = B_0 - D_0 \tag{2.65}$$

Given that in aggregated terms the variable B collapses to zero, (2.65) can be then reduced to the next simplified expression,

$$A_0 = -D_0 \tag{2.66}$$

Moreover, assuming that in the production function the capital stock enters with a lag, a production function for the level of Q_2 can be specified as follows,

$$Q_2 = F_Q(K_1) \tag{2.67}$$

Or, assuming constant returns to scale, it can be also settled as expression (2.68), where Q_2 is given as a ratio of the current level of production,

$$\frac{Q_2}{Q_1} = f_q\left(\frac{K_1}{Q_1}\right) \tag{2.68}$$

Both derivatives, $(F_Q)'_K$ and $(f_q)'_{K/Q}$, are considered greater than zero.

Now, making use of (2.64), (2.66) and (2.68), the consumption and saving equations (2.62) and (2.63) can be respectively reformulated as follows,

$$C_1 = \frac{Q_1 - Q_0\,(1+r)\,\frac{D_0}{Q_0} + \frac{Q_1}{1+r}\,f_q\left(1+\varphi_1 - \alpha\,\frac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{2.69}$$

$$S_1 = Q_1 - Q_0\,r\,\frac{D_0}{Q_0} - \frac{Q_1 - Q_0\,(1+r)\,\frac{D_0}{Q_0} + \frac{Q_1}{1+r}\,f_q\left(1+\varphi_1 - \alpha\,\frac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{2.70}$$

Or, alternatively, as in (2.71) and (2.72), if they are expressed as ratios to GDP,

$$\frac{C_1}{Q_1} = \frac{1-(Q_0/Q_1)\,(1+r)\,\frac{D_0}{Q_0} + \frac{f_q}{1+r}\left(1+\varphi_1 - \alpha\,\frac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{2.71}$$

$$\frac{S_1}{Q_1} = 1-(Q_0/Q_1)\ r\,\frac{D_0}{Q_0} - \frac{1-(Q_0/Q_1)(1+r)\dfrac{D_0}{Q_0} + \dfrac{f_q}{1+r}\left(1+\varphi_1 - \alpha\dfrac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \qquad (2.72)$$

From equations (2.71) and (2.72) we finally deduce the main variables explaining the saving and consumption ratios.

With regard to the consumption ratio, appendix 2.4 demonstrates that there exist unambiguous effects on behalf of all the main explanatory variables of consumption. Both, a higher capital stock as ratio to GDP and a higher preference for consumption will undoubtedly encourage the desire to consume while on the contrary, a higher interest rate or an increment in the amount of accumulated foreign debt are sure to provoke a continence of it.

Just the opposite reasoning can be deduced for the case of savings with the exception of both the real interest rate and the foreign debt variable. In this case, a change in the foreign debt ratio will affect in opposite directions the savings behaviour. The reason of it is that while higher foreign debt ratios will disincentive consumption, at the same time, the interest payment owed to the external sector will also decrease the level of disposable income. From a purely theoretical point of view our model cannot help us to forecast the sign relating the two variables. However, the empirical evidence seems to predict a positive relationship between foreign debt and saving, just the opposite to the case of consumption.

Moreover, appendix 2.5 shows how changes in the real interest rate are also ambiguous over the savings decisions. As it is standard in the literature, our approach to savings also predicts that a positive relationship between the real interest rates and savings is more probable in the case of the country being a net creditor. The probability decreases on the contrary when the country changes from being a net creditor to be a net debtor. Nevertheless, following the predictions of the empirical evidence the most probable result will be a positive relationship between the two variables.

All these results have been summarised in (2.73) and (2.74) respectively,

$$\frac{C}{Q} = F_C\left(\frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{2.73}$$

$$\frac{S}{Q} = F_S\left(\frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{2.74}$$

With $(F_C)'_{K/Q} > 0$, $(F_C)'_{D/Q} < 0$, $(F_C)'_r < 0$, $(F_C)'_\varphi > 0$ regarding the consumption ratio and $(F_S)'_{K/Q} < 0$, $(F_S)'_{D/Q} >^? 0$, $(F_S)'_r >^? 0$, $(F_S)'_\varphi < 0$ in the case of savings[49]. Appendix 2.4 and 2.5 proof in detail the signs of the derivatives.

2.3.4 Trade Balance and the Current Account

The trade balance is defined as the net income flow coming from the goods sold to the foreign sector (exports, X) once discounted the goods that the nationals acquire to the rest of the world (imports, M). Regarding the trade balance, the only relevant goods to be taken into account will be those that can be exchanged internationally. This is the reason why the trade balance is considered in analytical terms as the difference, shown in (2.75), between the national and foreign tradable goods exchanged with the rest of the world. In this section, it is again important the distinction between tradables and non-tradables left out in the analysis of consumption.

$$TB = X - M \tag{2.75}$$

In order to settle the trade balance determinants, we propose next the second step of an optimisation problem designed in two-stages. Provided the selected quantities of present and future consumption previously determined in the preceding section, we will consider here

[49] The question mark is introduced as a superscript of the inequality sign to remark the theoretical ambiguity of the result. If, for instance, following the empirical evidence the prospects of α being greater than β are high, then our terminological convention is to express that as $\alpha >^? \beta$.

a consumer that decides how to distribute his optimal aggregated consumption into goods of tradable and non-tradable nature.

Under this approach the following identity will be therefore true,

$$P_T\, C_T + P_N\, C_N = P\, C \tag{2.76}$$

where the sub-index T and N means tradable and non-tradable respectively, P is a price index and C is consumption. The price index P accompanying the global consumption C is considered as a composite index of the respective P_T and P_N ones. Particularly, it is defined as follows,

$$P = \theta P_T + (1-\theta) P_N \tag{2.77}$$

So we can make use of (2.77) to reformulate equation (2.76) as in (2.78), where it is expressed in terms of the tradable goods,

$$C_T + R\, C_N = \left[\theta + (1-\theta) R\right] C \tag{2.78}$$

In the same way, a standard intra-temporal utility function should be specified here. Following the usual practise, we assume next a simple Cobb-Douglas utility function that represents the consumer preferences on tradable and non-tradable goods,

$$U(C_T, C_N) = C_T^{\alpha}\, C_N^{\beta} \tag{2.79}$$

From (2.78) and (2.79), the optimisation problem will remain as follows,

$$\lambda(C_T, C_N) = C_T^{\alpha}\, C_N^{\beta} - \lambda\left[C_T + R\, C_N - \theta C - (1-\theta) R\, C\right] \tag{2.80}$$

Expressions (2.81) and (2.82) show the first order conditions,

$$\frac{\partial \lambda}{\partial C_T} = \frac{\alpha C_N^{\beta}}{C_T^{1-\alpha}} - \lambda = 0 \tag{2.81}$$

$$\frac{\partial \lambda}{\partial C_N} = \frac{\beta C_T^{\alpha}}{C_N^{1-\beta}} - \lambda R = 0 \tag{2.82}$$

From the combination of (2.81) and (2.82), it is obtained the equilibrium relationship in (2.83) between the C_T and C_N variables,

$$\frac{C_T}{C_N} = R\frac{\alpha}{\beta} \tag{2.83}$$

So, we can make use of (2.83) to solve finally equation (2.78) for the consumption of tradables,

$$C_T = \frac{\alpha}{\alpha+\beta}\left[\theta + (1-\theta)R\right]C \tag{2.84}$$

For simplifying reasons, we can assume that a fix proportion, for instance κ, of the desired consumption of tradables is from abroad, so that the volume of imports can be easily expressed as in (2.85),

$$M = \kappa\frac{\alpha}{\alpha+\beta}\left[\theta + (1-\theta)R\right]C \tag{2.85}$$

Or, alternatively, as in (2.86) in terms of GDP,

$$\frac{M}{Q} = \kappa\frac{\alpha}{\alpha+\beta}\left[\theta + (1-\theta)R\right]\frac{C}{Q} \tag{2.86}$$

Following equation (2.86), we can underline the real exchange rate and the consumption ratio as the main explanatory variables of the desired volume of imports. That is the following,

$$\frac{M}{Q} = F_M\left(R, \frac{C}{Q}\right) \tag{2.87}$$

where $(F_M)'_R > 0$ and $(F_M)'_{C/Q} > 0$.

In the same way, the reasoning developed for the analysis of imports can be extended to the case of exports, so that equation (2.88) would eventually represent the behaviour of exports in this economy,

$$\frac{X}{Q^*} = F_X\left(R, \frac{C^*}{Q^*}\right) \tag{2.88}$$

where $(F_X)'_R < 0$ and $(F_X)'_{C^*/Q^*} > 0$ in this case.

Provided now with equations (2.87) and (2.88), we can finally formulate the trade balance ratio as an unambiguous function of the real exchange rate, the national consumption ratio and the foreign consumption ratio. In the case of a stationary relationship between the national and foreign GDP's, the expression (2.89) will represent the desired trade balance equation,

$$\frac{TB}{Q} = F_{TB}\left(R, \frac{C^*}{Q^*}, \frac{C}{Q}\right) \tag{2.89}$$

where $(F_{TB})'_R < 0$, $(F_{TB})'_{C^*/Q^*} > 0$ and $(F_{TB})'_{C/Q} < 0$. See appendix 2.6 for details.

Regarding the current account behaviour, let us define this variable as follows,

$$\frac{CA}{Q} = \frac{TB}{Q} - \frac{DS}{Q} \tag{2.90}$$

Where CA is referred to the current account and DS to the total amount of debt service. The payments derived from the possession of foreign debt, the so-called DS, are assumed in this model to depend exclusively up on the accumulated stock of assets. In that case, a function specifying the current account behaviour can be therefore formulated as follows,

$$\frac{CA}{Q} = F_{TB}\left(R, \frac{C^*}{Q^*}, \frac{C}{Q}\right) - F_{DS}\left(\frac{D}{Q}\right) \tag{2.91}$$

where $(F_{DS})'_{D/Q} > 0$.

Moreover, taking into account expression (2.73), which explains the consumption ratio, equation (2.91) can be easily reformulated in terms of the predetermined and exogenous variables of the model,

$$CA = F_{CA}\left(R, \frac{C^*}{Q^*}, \frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{2.92}$$

where $(F_{CA})'_{R} < 0$, $(F_{CA})'_{C^*/Q^*} > 0$, $(F_{CA})'_{K/Q} < 0$, $(F_{CA})'_{D/Q} \overset{?}{<} 0$ $(F_{CA})'_{r} > 0$, $(F_{CA})'_{\varphi} < 0$.

The sign of all variables, with the exception of foreign debt, can be unambiguously deduced from the behavioural equations of the consumption and trade balance ratios to GDP. Regarding the relationship between foreign debt and current account, there is an ambiguous effect derived from the opposite influences of debt on consumption and on the service of the net foreign debt. Nevertheless, the more intuitive relationship between the two variables is a negative response of the current account to variations in foreign debt. The reason is that in an indebted economy, the introduction of risk premiums on behalf of the foreign sector will end up causing that the interest payment exceeds the improvement that the increment in debt has on the trade balance. Appendix 2.7 shows the details of this analysis.

2.4 The Model

In the previous section the main behavioural equations conforming the Natrex model, that is, the investment, consumption and trade balance equations, have been sufficiently founded. Provided then with the necessary aggregated macroeconomic functions, we develop in this section the complete set of equations that define both the medium and long run equilibrium that characterise the Natrex approach.

2.4.1 The Analytical Framework of Equilibrium

As section 2.2 details, the Natrex is understood as the real exchange rate that prevails once speculative and cyclical factors are absent and the unemployment is at its natural rate. This is the case when, in a non-inflationary context, there exists a balance between the international flow of capitals. Or, in analytical terms, when the desired social savings (S) minus social investment (I) equal the current account (CA) as expression (2.93), with variables given in ratios to GDP, particularly shows,

$$\frac{CA}{Q}=\frac{S}{Q}-\frac{I}{Q} \tag{2.93}$$

Alternatively, given the definitions of savings and current account in terms of the net payment of services, equation (2.93) can be also reformulated as follows,

$$\frac{TB}{Q}=1-\left(\frac{I}{Q}+\frac{C}{Q}\right) \tag{2.94}$$

Both, equations (2.93) and (2.94), alternatively represent the national account equilibrium condition of the economy under analysis.

In our model, we are going to consider that, given the decisions on investment and consumption, it is the trade balance -through the adjustments in the real exchange rate- which will accommodate to obtain the external equilibrium of the system. Under this assumption, the equation determining the real exchange rate is obtained from the trade balance equation (2.89), while the trade balance account can be determined by means of equation (2.94). The system in this approach would be then conformed by equations (2.42), (2.73) and (2.89), solved for the respective endogenous variables of the model, plus equation (2.94) describing the trade balance behaviour. In that case, expressions (2.42), (2.73) and (2.89) would remain as follows,

$$\frac{I}{Q} = \tilde{F}_I\,(\hat{A},\, r) + (F_I)'_{K/Q}\,\frac{K}{Q} + (F_I)'_R\, R \tag{2.95}$$

$$\frac{C}{Q} = (F_C)'_{K/Q}\,\frac{K}{Q} + (F_C)'_{D/Q}\,\frac{D}{Q} + \tilde{F}_C\left(r,\, \varphi\right) \tag{2.96}$$

$$R = \frac{1}{(F_{TB})'_R}\left[\frac{TB}{Q} - (F_{TB})'_{C/Q}\,\frac{C}{Q} - \tilde{F}_{TB}\left(\frac{C^*}{Q^*}\right)\right] \tag{2.97}$$

However, to define the complete model it is also necessary to include the uncovered interest rate parity condition, the Fisher equation and the dynamic equations for the capital stock and the stock of foreign debt. These are expressions from (2.98) to (2.101) respectively.

$$R - R^{eq}_{LR} = r - r^* \tag{2.98}$$

$$i - i^* = (\pi - \pi^*) + (r - r^*) \tag{2.99}$$

$$K = (1-\delta)\,K_{-1} + I \tag{2.100}$$

$$D - D_{-1} = I - S \tag{2.101}$$

Where the superscript *eq* means equilibrium level and the subscript LR means long run, π is the inflation rate and δ the depreciation rate of the capital stock.

The model is finally characterised by equations from (2.94) to (2.101).

2.4.2 Characterisation of the Medium Run Equilibrium

Provided the behavioural equations of the model, the real exchange rate derived from (2.93) or (2.94) is in fact a medium run market clearing condition given that, as the capital stock and the net foreign debt move only slowly over time, in the short run they can be treated as exogenous. Then, in the medium run equilibrium, conditions (2.94), (2.95), (2.96) and (2.97) will apply, so that equations from (2.102) to (2.105) will be the ones describing the equilibrium,

$$\left(\frac{I}{Q}\right)^{MR} = \tilde{F}_I(\hat{A}, r) + (F_I)'_{K/Q}\frac{K}{Q} + (F_I)'_R R^{MR} \qquad (2.102)$$

$$\left(\frac{C}{Q}\right)^{MR} = (F_C)'_{K/Q}\frac{K}{Q} + (F_C)'_{D/Q}\frac{D}{Q} + \tilde{F}_C(r, \varphi) \qquad (2.103)$$

$$\left(\frac{TB}{Q}\right)^{MR} = 1 - \left[\left(\frac{I}{Q}\right)^{MR} + \left(\frac{C}{Q}\right)^{MR}\right] \qquad (2.104)$$

$$R^{MR} = \frac{1}{(F_{TB})'_R}\left[\left(\frac{TB}{Q}\right)^{MR} - (F_{TB})'_{C/Q}\left(\frac{C}{Q}\right)^{MR} - \tilde{F}_{TB}\left(\frac{C^*}{Q^*}\right)\right] \qquad (2.105)$$

In this case, $\hat{A}$, K/Q, r, $\hat{Q}$, D/Q, φ, C*/Q* are the exogenous variables of the system, while the investment, consumption and trade balance ratios, together with the real exchange rate, are the endogenous ones to be solved in the context of the model.

This medium run equilibrium can be also specified in graphical terms taking into account the market clearing condition (2.94). From this equation, the medium run equilibrium can be understood as the level that would balance both, the trade balance economic function on one hand and, on the other, a new function defined by the difference between the unity and the sum of the investment and consumption ratios in terms of GDP. In that case, the national account identity (2.94) can be therefore reformulated as follows,

$$F_{TB}\left(R, \frac{C*}{Q*}, \frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) = F_{IC}\left(R, \hat{A}, \frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{2.106}$$

with $(F_{TB})'_R < 0,\ (F_{TB})'_{C*/Q*} > 0,\ (F_{TB})'_{K/Q} < 0,\ (F_{TB})'_{D/Q} > 0,\ (F_{TB})'_r > 0,\ (F_{TB})'_\varphi < 0$ and $(F_{IC})'_R > 0,\ (F_{IC})'_{\hat{A}} < 0,\ (F_{IC})'_{K/Q} >^{?} 0,\ (F_{IC})'_{D/Q} > 0,\ (F_{IC})'_r > 0,\ (F_{IC})'_\varphi < 0$. The only ambiguous relationship in this analysis is that of the derivative $(F_{IC})'_{K/Q}$. The reason is that the accumulation of capital stock influences oppositely investment and consumption decisions. While an increment in the stock of capital disincentives investment; it conversely boosts the level of consumption. Regarding this relationship, the empirical evidence seems to support a higher sensitivity of response in investment than in consumption to changes in the capital stock, so that the most probable result can be $(F_{IC})'_{K/Q}$ being higher than zero.

From (2.106) it is obtained figure 2.1, that shows the medium run equilibrium of the real exchange rate.

Moreover, from the resolution of the system formed by equations from (2.102) to (2.105) it is also possible to detail the exogenous fundamentals of this medium run equilibrium,

$$R^{MR} = \frac{\tilde{F}_R\left(\hat{A}, r, \varphi, \frac{C*}{Q*}\right) + \alpha^{MR}_{K/Q}\frac{K}{Q} + \alpha^{MR}_{D/Q}\frac{D}{Q}}{-\left[(F_{TB})'_R + (F_I)'_R\right]} \tag{2.107}$$

where $\tilde{F}_R$, $\alpha^{MR}_{K/Q}$ and $\alpha^{MR}_{D/Q}$ are defined as follows,

$$\tilde{F}_R\left(\hat{A}, r, \varphi, \frac{C*}{Q*}\right) = \tilde{F}_I(\hat{A}, r) + \left[1 + (F_{TB})'_{C/Q}\right]\tilde{F}_C(r, \varphi) + \tilde{F}_{TB}\left(\frac{C*}{Q*}\right) - 1 \tag{2.108}$$

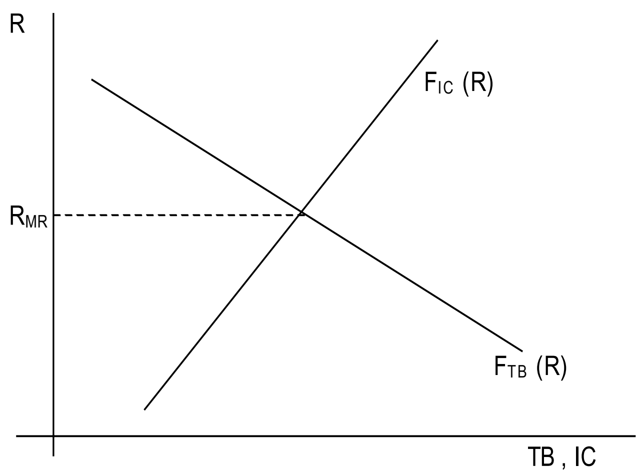
$$\alpha^{MR}_{K/Q} = (F_I)'_{K/Q} + (F_C)'_{K/Q}\left[1 + (F_{TB})'_{C/Q}\right] \qquad (2.109)$$

$$\alpha^{MR}_{D/Q} = (F_C)'_{D/Q}\left[1 + (F_{TB})'_{C/Q}\right] \qquad (2.110)$$

with $(\tilde{F}_R)'_{\hat{A}} > 0$, $(\tilde{F}_R)'_r < 0$, $(\tilde{F}_R)'_\varphi > 0$,$(\tilde{F}_R)'_{C*/Q*} > 0$; $\alpha^{MR}_{K/Q} \overset{?}{<} 0$ and $\alpha^{MR}_{D/Q} < 0$.

Figure 2.1

The Real Exchange Rate Medium Run Equilibrium



Summarising then these results, we can specify a simplified functional relationship between the real exchange rate and its medium run determinants as follows,

$$R^{MR} = F_{R_{MR}}\left(\hat{A}, r, \varphi, \frac{C*}{Q*}, \frac{K}{Q}, \frac{D}{Q}\right) \qquad (2.111)$$

with $(F_{R_{MR}})'_{\hat{A}} > 0$, $(F_{R_{MR}})'_r < 0$, $(F_{R_{MR}})'_\varphi > 0$, $(F_{R_{MR}})'_{C*/Q*} > 0$, $(F_{R_{MR}})'_{K/Q} \overset{?}{<} 0$ and $(F_{R_{MR}})'_{D/Q} < 0$.

2.4.3 Stock Variables and The Long Run Equilibrium

As the stock variables of the model move to their steady state positions, the medium run equilibrium will evolve to equilibrium situations of a longer horizon. The Natrex equilibrium is in fact a sequence of medium run equilibriums evolving to a long run reference.

In the long run, the steady state conditions for the capital stock and the net foreign asset position need also to be implemented. To determine those conditions, we can rewrite the dynamic equations (2.100) and (2.101) as ratios to GDP,

$$\frac{K}{Q}=\frac{1-\delta}{1+\hat{Q}}\left(\frac{K}{Q}\right)_{-1}+\frac{I}{Q} \tag{2.112}$$

$$\frac{D}{Q}=\frac{1}{1+\hat{Q}}\left(\frac{D}{Q}\right)_{-1}+\frac{I}{Q}-\frac{S}{Q} \tag{2.113}$$

Given the dynamic equations (2.112) and (2.113), the steady state conditions (2.114) and (2.115) can be easily obtained,

$$\frac{K}{Q}=\gamma\frac{I}{Q} \tag{2.114}$$

$$\frac{D}{Q}=\rho\left(\frac{I}{Q}-\frac{S}{Q}\right) \tag{2.115}$$

where γ and ρ are defined as follows,

$$\gamma=\frac{1+\hat{Q}}{\delta+\hat{Q}} \tag{2.116}$$

$$\rho=\frac{1+\hat{Q}}{\hat{Q}} \tag{2.117}$$

Regarding the determination of the real exchange rate long run equilibrium, besides the medium run condition (2.107), it should be also taken into account the steady state equations (2.114) and (2.115), plus the corresponding behavioural equations defining the investment and saving decisions. Specifying these ones solved for the endogenous variables of the model, the useful equation for investment would be (2.95), while for savings is (2.74) but reformulated in the following terms,

$$\frac{S}{Q} = \tilde{F}_S\left(r,\ \varphi\right) + (F_S)'_{K/Q}\frac{K}{Q} + (F_S)'_{D/Q}\frac{D}{Q} \tag{2.118}$$

Then, equations from (2.119) to (2.123) will describe in this case the desired long run equilibrium,

$$R^{LR} = \frac{\tilde{F}_R\left(\hat{A},\ r^*, \varphi, \dfrac{C^*}{Q^*}\right) + \alpha^{MR}_{K/Q}\left(\dfrac{K}{Q}\right)^S + \alpha^{MR}_{D/Q}\left(\dfrac{D}{Q}\right)^S}{-\left[(F_{TB})'_R + (F_I)'_R\right]} \tag{2.119}$$

$$\left(\frac{K}{Q}\right)^S = \gamma\left(\frac{I}{Q}\right)^{LR} \tag{2.120}$$

$$\left(\frac{D}{Q}\right)^S = \rho\left[\left(\frac{I}{Q}\right)^{LR} - \left(\frac{S}{Q}\right)^{LR}\right] \tag{2.121}$$

$$\left(\frac{I}{Q}\right)^{LR} = \tilde{F}_I\,(\hat{A},\ r^*) + (F_I)'_{K/Q}\left(\frac{K}{Q}\right)^S + (F_I)'_R\, R^{LR} \tag{2.122}$$

$$\left(\frac{S}{Q}\right)^{LR} = \tilde{F}_S\left(r^*,\ \varphi\right) + (F_S)'_{K/Q}\left(\frac{K}{Q}\right)^S + (F_S)'_{D/Q}\left(\frac{D}{Q}\right)^S \tag{2.123}$$

Or, alternatively, equations from (2.124) to (2.126) if it is solved for the investment and saving ratios,

$$R^{LR} = \frac{\tilde{F}_R\left(\hat{A}, r, \varphi, \frac{C^*}{Q^*}\right) + \alpha^{MR}_{K/Q}\left(\frac{K}{Q}\right)^S + \alpha^{MR}_{D/Q}\left(\frac{D}{Q}\right)^S}{-\left[(F_{TB})'_R + (F_I)'_R\right]} \tag{2.124}$$

$$\left(\frac{K}{Q}\right)^S = \frac{\tilde{F}_I(\hat{A}, r^*)}{\theta_1} + \frac{(F_I)'_R}{\theta_1} R^{LR} \tag{2.125}$$

$$\left(\frac{D}{Q}\right)^S = \frac{\theta_3 \tilde{F}_I(\hat{A}, r^*) - \theta_1 \tilde{F}_S(r^*, \varphi)}{\theta_1 \theta_2} + \frac{\theta_3 (F_I)'_R}{\theta_1 \theta_2} R^{LR} \tag{2.126}$$

with θ_1, θ_2 and θ_3 defined as follows,

$$\theta_1 = 1/\gamma - (F_I)'_{K/Q} \tag{2.127}$$

$$\theta_2 = 1/\rho + (F_S)'_{D/Q} \tag{2.128}$$

$$\theta_3 = 1/\gamma - (F_S)'_{K/Q} \tag{2.129}$$

being the three parameters positive values.

Finally, solving the system formed by equations from (2.124) to (2.126), we can obtain the real exchange rate long run equilibrium as a definitive function of its exogenous variables,

$$R^{LR} = \frac{\theta_1 \theta_2 \tilde{F}_R\left(\hat{A}, r, \varphi, \frac{C^*}{Q^*}\right) + \left[\theta_2 \alpha^{MR}_{K/Q} + \theta_3 \alpha^{MR}_{D/Q}\right] \tilde{F}_I(\hat{A}, r^*) - \theta_1 \alpha^{MR}_{D/Q} \tilde{F}_S(r^*, \varphi)}{\theta_1 \theta_2 (-F_{TB})'_R + \left[\theta_1 \theta_2 + \theta_2 \alpha^{MR}_{K/Q} + \theta_3 \alpha^{MR}_{D/Q}\right](-F_I)'_R} \tag{2.130}$$

2.4.4 Equilibrium and Stability Conditions

Regarding the medium run stability condition we should take into account the following dynamic equation obtained from the external equilibrium condition (2.94),

$$(F_{TB})'_R\,(dR) = -(F_I)'_R\,(dR_{-1}) \tag{2.131}$$

This equation is justified by the fact that the real exchange rate is the variable that accommodates the trade balance to the levels determined by the F_{IC} function.

From (2.131), the dynamic equation can be also specified as follows,

$$dR = -\frac{(F_I)'_R}{(F_{TB})'_R}\, dR_{-1} \tag{2.132}$$

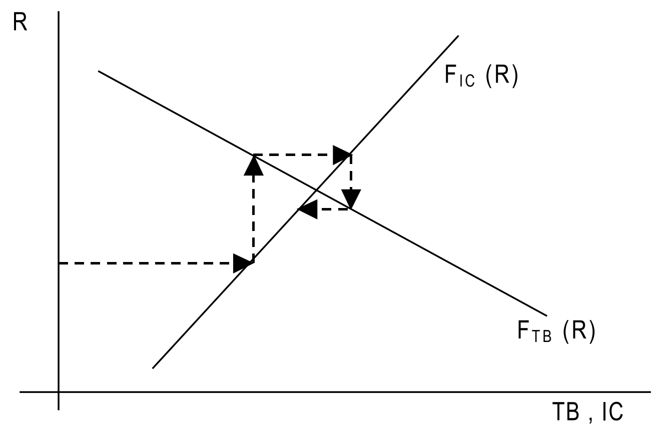
Equation (2.132) suggests an oscillatory process of convergence to equilibrium that will be stable only if the sensitivity of investment to the real exchange rate is in absolute values smaller than the corresponding one of the trade balance function. That is to say,

$$\left|(F_I)'_R\right| < \left|(F_{TB})'_R\right| \tag{2.133}$$

The intuition of this medium run condition can be easily observed in graphical terms as follows,

Figure 2.2

The Dynamics of the Medium Run Convergence



Regarding the long run equilibrium, let us start specifying the following dynamic equations for the stock variables of the system, which have been obtained from (2.112) and (2.113) respectively,

$$d\left(\frac{K}{Q}\right) = -\frac{\delta + \hat{Q}}{1+\hat{Q}}\frac{K}{Q} + F_I\left(\hat{A}, \frac{K}{Q}, r, R\right) \tag{2.134}$$

$$d\left(\frac{D}{Q}\right) = -\frac{\hat{Q}}{1+\hat{Q}}\left(\frac{D}{Q}\right) + F_I\left(\hat{A}, \frac{K}{Q}, r, R\right) - F_S\left(\frac{K}{Q}, \frac{D}{Q}, r, \varphi\right) \tag{2.135}$$

Making use of equations (2.95) and (2.118), the previous (2.134) and (2.135) can be solved by its corresponding endogenous variables,

$$d\left(\frac{K}{Q}\right) = \left[-\frac{\delta+\hat{Q}}{1+\hat{Q}} + (F_I)'_{K/Q}\right]\frac{K}{Q} + (F_I)'_R\, R + \tilde{F}_I\,(\hat{A}, r) \tag{2.136}$$

$$d\left(\frac{D}{Q}\right) = -\left[\frac{\hat{Q}}{1+\hat{Q}} + (F_S)'_{D/Q}\right]\frac{D}{Q} + F_I\left(\hat{A}, \frac{K}{Q}, r, R\right) - (F_S)'_{K/Q}\frac{K}{Q} - \tilde{F}_S(r, \varphi) \tag{2.137}$$

From (2.136) and (2.137) we can check that the fact of $(F_I)'_{K/Q}$ being negative and $(F_S)'_{D/Q}$ being positive are both sufficient conditions for the convergence of the stock variables in the long run equilibrium.

2.4.5 The Real Exchange Rate and its Fundamental Determinants

Finally, we analyse in this section the basic Natrex predictions about the medium and long run consequences of a shock over the productivity factor and the preferences for consumption.

Regarding the medium run effects we can obtain straightforward conclusions from equation (2.107), where the medium run equilibrium has been previously obtained. Given the results of $(F_{R_{MR}})'_{\hat{A}} > 0$ and $(F_{R_{MR}})'_{\varphi} > 0$ it is easy to conclude that, according with the Natrex suggestion, in the medium run a positive shock on productivity will appreciate the currency, while a lower preference for consumption will depreciate it.

However in the long run, it is an interesting conclusion of this research our result of an ambiguous effect after a shock affecting both, preferences for consumption and productivity. This contrasts strongly with Stein's prediction of a long run depreciating effect after a positive shock over the preference for consumption.

From (2.130) we obtain the derivatives of R^{LR} with respect to φ and $\hat{A}$,

$$\frac{\partial R^{LR}}{\partial \varphi} = \frac{\theta_1 \left[\theta_2 (\tilde{F}_R)'_{\varphi} - \alpha^{MR}_{D/Q} (\tilde{F}_S)'_{\varphi}\right]}{\theta_1 \theta_2 (-F_{TB})'_R + \left[\theta_2 \left[\theta_1 + \alpha^{MR}_{K/Q}\right] + \theta_3 \alpha^{MR}_{D/Q}\right] (-F_I)'_R} \tag{2.138}$$

$$\frac{\partial R^{LR}}{\partial \hat{A}} = \frac{\theta_1 \theta_2 (\tilde{F}_R)'_{\hat{A}} + \left[\theta_2 \alpha^{MR}_{K/Q} + \theta_3 \alpha^{MR}_{D/Q}\right] (\tilde{F}_I)'_{\hat{A}}}{\theta_1 \theta_2 (-F_{TB})'_R + \left[\theta_2 \left[\theta_1 + \alpha^{MR}_{K/Q}\right] + \theta_3 \alpha^{MR}_{D/Q}\right] (-F_I)'_R} \tag{2.139}$$

Given θ_1, θ_2, θ_3, defined in expressions from (2.127) to (2.129), $(\tilde{F}_R)'_{\varphi}$ and $(\tilde{F}_R)'_{\hat{A}}$, obtained from equation (2.108), and the conditions saying that $(\tilde{F}_S)'_{\varphi} = -(\tilde{F}_C)'_{\varphi}$, $(\tilde{F}_S)'_{K/Q} = -(\tilde{F}_C)'_{K/Q}$ and $(\tilde{F}_S)'_{D/Q} + (\tilde{F}_C)'_{D/Q} = -(\tilde{F}_{DS})'_{D/Q}$, the previous (2.138) and (2.139) equations can be reformulated as follows,

$$\frac{\partial R^{LR}}{\partial \varphi} = \frac{\alpha \theta_1 (\tilde{F}_C)'_{\varphi} \left[1 + (F_{TB})'_{C/Q}\right]}{\theta_1 \theta_2 (-F_{TB})'_R + \left[\alpha \left[\frac{1}{\gamma} + (F_C)'_{K/Q} \left[1 + (F_{TB})'_{C/Q}\right]\right] + \frac{(F_{TB})'_{C/Q} (F_C)'_{D/Q}}{\gamma}\right] (-F_I)'_R} \tag{2.140}$$

$$\frac{\partial R^{LR}}{\partial \hat{A}} = \frac{\left[\alpha\left[\frac{1}{\gamma}+(F_C)'_{K/Q}\left[1+(F_{TB})'_{C/Q}\right]\right]+\frac{(F_{TB})'_{C/Q}(F_C)'_{D/Q}}{\gamma}\right](\tilde{F}_I)'_{\hat{A}}}{\theta_1\,\theta_2\,(-F_{TB})'_R+\left[\alpha\left[\frac{1}{\gamma}+(F_C)'_{K/Q}\left[1+(F_{TB})'_{C/Q}\right]\right]+\frac{(F_{TB})'_{C/Q}(F_C)'_{D/Q}}{\gamma}\right](-F_I)'_R} \tag{2.141}$$

where

$$\alpha = 1/\rho-(F_{DS})'_{D/Q} \tag{2.142}$$

From (2.140) it is easy to conclude that after a positive shock on the preference for consumption, only in the case of α being negative, there exists an opportunity for the real exchange rate to depreciate in the long run, as it is the prediction of the Natrex approach. Rather on the contrary, with a positive α it is sure that both a medium and long run real appreciation will follow a decrease in thriftiness.

Alternatively, when a positive shock on productivity occurs, the evidence of $\alpha > 0$ will also assure the result of a long run appreciation in the real exchange rate. Nevertheless, from the parameters of the model it is deduced that, in contrast with the previous result, in the particular case of an increment in productivity, even with the alpha parameter being lower than zero, it still exists an opportunity for the real exchange rate to appreciate.

From these results, it is clear that some standard Natrex predictions about shocks on fundamentals are clearly broken down by the conclusions of this research.

2.5 Conclusions

This chapter aims to contribute to the theoretical literature on the structural macroeconomic approach analysing the real exchange rate from a dynamic general equilibrium perspective. In particular, the research has followed the philosophy of the Natrex approach due to its, at least from our point of view, interesting and accurate dynamic notion of the concept of equilibrium. The main peculiarities of a Natrex model within the context of the structural approach are related to its singular stock-flow interaction, and to the considered distinction between the medium and long-run equilibrium, differentiated both by the possibility that the stock variables have reached their steady state positions.

However, despite the theoretical spirit of Natrex, presented in origin as a set of solidly based models, the reality is that the approach has ended up circumscribed to the empirical estimation of reduced models of equilibrium. The unfortunate result is that the approach lacks of well-integrated structures, not to mention that it has completely omitted some important theoretical restrictions that are in fact determinant for the structural characterisation of the model. Moreover, and not less important, there is the fact of the absence of a formal discussion about the convergence and stability of the systems under analysis. In that sense, the main contribution of this chapter is the development of a solid theoretical framework that analyses in depth the basis of the real exchange rate, as well as the details of the equilibrium dynamics after any shock influencing steady state positions.

The main results of the chapter are the following. In first place, a complete well-integrated structural model not known up to the moment for the long-run real exchange rate determination is developed from first principles. Moreover, within the concrete dynamics of the model we find that, for convergence reasons, there are some restrictions that the model will necessarily need to satisfy. For the medium run convergence, the sensitivity of the trade balance to changes in the real exchange rate should be higher in absolute values than the correspondent ones in investment decisions. Regarding the long-run, it is also necessary both that there exists a negative relationship between investment and capital stock accumulation and that the global savings of the economy (integrating public and private sectors) depends positively on net foreign debt accumulation.

In addition, there are also interesting conclusions about the results that certain shocks over the exogenous variables of the model have on real exchange rates. Being more specific, the

Natrex approach predicts that in the medium run a real appreciation will follow both a positive shock on the preference for consumption and on productivity. However, the prediction changes dramatically in the long run. Following the Natrex suggestion, we expect a long run real depreciation in the case of a higher preference for consumption and an ambiguous result, although most probably a real appreciation, in the case of a better productivity factor. Our research, alternatively, breaks down these predictions and prognosticates that in the particular case of a shock on the thriftiness parameter, the mentioned long run real depreciation can be only assured when a particular restriction over the estimated parameters of the model occurs.

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APPENDIX

Appendix 2.1: The Real Exchange Rate Definition

Consider an economy where the produced and consumed goods are of tradable (T) and non-tradable (N) nature. Without loss of generality, tradables can be assumed homogeneous across countries, so that their price is therefore determined in international markets while, on the contrary, non-tradables will have to be dealt -produced and consumed- inside the country. In this case the terms of trade will collapse to one[50].

Under the previous assumption, the national and foreign price indexes –P and P* respectively– can be simply defined as follows,

$$P = P_N^{\alpha} P_T^{1-\alpha} \tag{A2.1}$$

$$P^{*} = \left(P_N^{*}\right)^{\beta} \left(P_T^{*}\right)^{1-\beta} \tag{A2.2}$$

The star indicates foreign sector.

Given next the definition in (A2.3) for the real exchange rate (R), where an increment means a real appreciation, price indexes (A2.1) and (A2.2) can be used to obtain expression (A2.4),

$$R = \frac{EP}{P^{*}} \tag{A2.3}$$

$$R = \left(\frac{P_N}{P_T}\right)^{\alpha} \left(\frac{P_T^{*}}{P_N^{*}}\right)^{\beta} \frac{EP_T}{P_T^{*}} \tag{A2.4}$$

---

[50] This simplifying assumption has not to be a problem in the case of an empirical estimation, given that R can be introduced as a global factor.

where E is the nominal exchange rate.

Taking into account the following (A2.5) and (A2.6) definitions referred to the relative price of non-tradables with respect to tradables for the national and foreign economies respectively,

$$R_n = \frac{P_N}{P_T} \tag{A2.5}$$

$$R_n^* = \frac{P_N^*}{P_T^*} \tag{A2.6}$$

expression (A2.4) can simplify to the next one,

$$R = \frac{R_n^{\alpha}}{\left(R_n^*\right)^{\beta}} \frac{E P_T}{P_T^*} \tag{A2.7}$$

Moreover, if the law of the single price is also into operation, the following (A2.8) condition should satisfy,

$$P_T^* = E P_T \tag{A2.8}$$

In that case, the real exchange rate can be finally reduced to expression (A2.9), where R^*_n is assumed an exogenous variable for the national country.

$$R = \frac{R_n^{\alpha}}{\left(R_n^*\right)^{\beta}} \tag{A2.9}$$

We then see a real exchange rate defined as a fixed proportion, δ, of R_n.

$$R = \delta R_n^{\alpha} \tag{A2.10}$$

where δ is defined as follows,

$$\delta = \frac{T^{\alpha+\beta-1}}{\left(R_n^*\right)^{\beta}} \tag{A2.11}$$

Appendix 2.2: Response of the Investment Ratio to its Explanatory Variables

Given the investment ratio obtained in (2.41) and reproduced here in (A2.12), the partial derivatives of this endogenous variable with respect to its explanatory variables are determined next,

$$\frac{I}{Q} = \frac{\hat{A}}{(K/Q)^{\frac{\alpha}{1-\alpha}}} \frac{\beta}{\alpha} \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \left(\frac{\frac{\alpha}{\beta} + R^{1/\beta} \frac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}} \left(\frac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}} \frac{K_N}{K_T}}{1 + R^{1/\beta} \frac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}} \left(\frac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}} \frac{K_N}{K_T}} \right)^{\frac{1}{1-\alpha}} \tag{A2.12}$$

Derivative with respect to the productivity index $\hat{A}$,

$$(F_I)'_{\hat{A}} = \frac{I/Q}{\hat{A}} > 0 \tag{A2.13}$$

Derivative with respect to the capital stock ratio K/Q,

$$(F_I)'_{K/Q} = -\frac{\alpha}{1-\alpha} \frac{I}{K} < 0 \tag{A2.14}$$

Derivative with respect to the real interest rate r,

$$(F_I)'_r = -\frac{1}{1-\alpha}\frac{1}{r}\frac{I}{Q} < 0 \qquad \text{(A2.15)}$$

Derivative with respect to the real exchange rate R,

$$(F_I)'_R = \frac{-(\alpha-\beta)\dfrac{R^{1-\beta/\beta}}{\beta^2}\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}\dfrac{I}{Q}}{\left(\dfrac{\alpha}{\beta}+R^{1/\beta}\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}\right)\left(1+R^{1/\beta}\dfrac{(1-\beta)^{1-\beta/\beta}}{(1-\alpha)^{1-\alpha/\alpha}}\left(\dfrac{A}{\omega}\right)^{\frac{\alpha-\beta}{\alpha\beta}}\dfrac{K_N}{K_T}\right)} \qquad \text{(A2.16)}$$

Expression (A2.16) would be negative whenever α is assumed greater than β, which is in fact commonly corroborated by the empirical evidence given that it implies the realistic assumption that the tradable sector is intensive in capital while the non-tradable is in labour.

Appendix 2.3: Elasticity of Substitution of a CES

In appendix 2.3 we are going to determine the elasticity of substitution of the following utility function,

$$U(c_1, c_2) = c_1^{\gamma} + c_2^{\gamma} \qquad \text{(A2.17)}$$

Let us start specifying in expression (A2.18) the concept of elasticity of substitution for the case of a generic utility function,

$$\varepsilon_{12} = \frac{\partial (c_2 / c_1)}{\partial MRS}\frac{MRS}{(c_2 / c_1)} \qquad \text{(A2.18)}$$

where MRS is the marginal rate of substitution, which in its turn is defined as follows,

$$MRS = -\frac{\partial c_2}{\partial c_1} = -\frac{\partial U/\partial c_1}{\partial U/\partial c_2} \tag{A2.19}$$

The marginal rate of substitution for the utility function (A2.17) is in this particular case the following,

$$MRS = -(1+r_1)\left(\frac{c_2}{c_1}\right)^{1-\gamma} \tag{A2.20}$$

Finally, making use of expression (A2.20), it is easily deduced the desired elasticity of substitution we are looking for,

$$\varepsilon_{12} = \frac{1}{1-\gamma} \tag{A2.21}$$

Appendix 2.4: Response of the Consumption Ratio to its Explanatory Variables

Given the consumption ratio obtained in (2.71) and written again here in equation (A2.22), in this appendix 2.4 it will be determined the partial derivatives of this consumption ratio with respect to each explanatory variable.

$$\frac{C_1}{Q_1} = \frac{1-(Q_0/Q_1)(1+r)\frac{D_0}{Q_0} + \frac{f_q}{1+r}\left(1+\varphi_1 - \alpha\frac{D_0}{Q_0}\right)}{1+(1+r)^{\gamma/1-\gamma}} \tag{A2.22}$$

Derivative with respect to the capital ratio K/Q,

$$(F_C)'_{K/Q} = \frac{(f_q)'_{K/Q}\left(1+\varphi_1-\alpha\,\dfrac{D_0}{Q_0}\right)}{1+r+(1+r)^{1/1-\gamma}} > 0 \qquad \text{(A2.23)}$$

Derivative with respect to the ratio of the foreign debt D/Q,

$$(F_C)'_{D/Q} = -\frac{(Q_0/Q_1)(1+r)+\dfrac{\alpha\, f_q}{1+r}}{1+(1+r)^{\gamma/1-\gamma}} < 0 \qquad \text{(A2.24)}$$

Derivative with respect to the real interest rate r,

$$(F_C)'_r = -\frac{(Q_0/Q_1)(1+r)+\dfrac{f_q}{1+r}\left(1+\varphi_1-\alpha\,\dfrac{D_0}{Q_0}\right)+\dfrac{\gamma}{1-\gamma}\dfrac{C_1}{Q_1}(1+r)^{\gamma/1-\gamma}}{1+r+(1+r)^{1/1-\gamma}} < 0 \qquad \text{(A2.25)}$$

Finally, we obtain the derivative with respect to the parameter φ_1 measuring the preference for consumption,

$$(F_C)'_\varphi = \frac{f_q}{1+r_1+(1+r_1)^{1/1-\gamma}} > 0 \qquad \text{(A2.26)}$$

Appendix 2.5: Response of the Saving Ratio to its Explanatory Variables

In order to determine the partial derivatives of the saving ratio, obtained in (2.72), with respect to its explanatory variables, this (2.72) equation can be rewritten as follows,

$$\frac{S_1}{Q_1} = 1-(Q_0/Q_1)\,r\,\frac{D_0}{Q_0}-\frac{C_1}{Q_1} \qquad \text{(A2.27)}$$

From (A2.27) it is clear that in this case the derivatives with respect to K/Q and φ_1 will have the opposite sign that in the case of the consumption ratio. That is the following,

$$(F_S)'_{\upsilon} = -(F_C)'_{\upsilon} \qquad \upsilon \neq \frac{D_0}{Q_0}, r \tag{A2.28}$$

However in the case of r and D_0/Q_0, it is necessary to perform separate derivatives,

$$(F_S)'_r = -(Q_0/Q_1)\frac{D_0}{Q_0} - (F_C)'_r \tag{A2.29}$$

$$(F_S)'_{D/Q} = -(Q_0/Q_1)r - (F_C)'_{D/Q} \tag{A2.30}$$

Being $(F_C)'_r$ and $(F_C)'_{D/Q}$ both lower than zero, the signs of (A2.29) and (A2.30) will result necessarily ambiguous.

Appendix 2.6: Response of the Trade Balance Ratio to its Explanatory Variables

In order to determined the partial derivatives of the trade balance ratio let us show here the obtained (2.86) equation for the imports ratio,

$$\frac{M}{Q} = \kappa\frac{\alpha}{\alpha+\beta}\left[\theta + (1-\theta)R\right]\frac{C}{Q} \tag{A2.31}$$

In the same way, the exports ratio can be formulated as a symmetric function of (A2.31),

$$\frac{X}{Q^*} = \kappa^*\frac{\alpha^*}{\alpha^*+\beta^*}\left(\theta^* + \frac{1-\theta^*}{R}\right)\frac{C^*}{Q^*} \tag{A2.32}$$

So that the trade balance equation will remain as follows,

$$\frac{TB}{Q}=\kappa^*\frac{\alpha^*(Q^*/Q)}{\alpha^*+\beta^*}\left(\theta^*+\frac{1-\theta^*}{R}\right)\frac{C^*}{Q^*}-\kappa\frac{\alpha}{\alpha+\beta}\left[\theta+(1-\theta)R\right]\frac{C}{Q} \qquad (A2.33)$$

Next the first derivatives of (A2.33) with respect to its main explanatory variables are obtained.

Derivative with respect to the real exchange rate R,

$$(F_{TB})'_R=-\left[\kappa^*\frac{\alpha^*(Q^*/Q)}{\alpha^*+\beta^*}\frac{C^*}{Q^*}\frac{1-\theta^*}{R^2}+\kappa\frac{\alpha(1-\theta)}{\alpha+\beta}\frac{C}{Q}\right]<0 \qquad (A2.34)$$

Derivative with respect to the foreign consumption ratio C*/Q*,

$$(F_{TB})'_{C^*/Q^*}=\kappa^*\frac{\alpha^*(Q^*/Q)}{\alpha^*+\beta^*}\left(\theta^*+\frac{1-\theta^*}{R}\right)>0 \qquad (A2.35)$$

Derivative with respect to the national consumption ratio C/Q,

$$(F_{TB})'_{C/Q}=-\kappa\frac{\alpha}{\alpha+\beta}\left[\theta+(1-\theta)R\right]<0 \qquad (A2.36)$$

Appendix 2.7: Response of the Current Account Ratio to its Explanatory Variables

To determine the partial derivatives of the current account ratio with respect to its explanatory variables, let us write here equations (2.91) and (2.73) again,

$$\frac{CA}{Q} = F_{TB}\left(R, \frac{C^*}{Q^*}, \frac{C}{Q}\right) - F_{DS}\left(\frac{D}{Q}\right) \qquad \text{(A2.37)}$$

$$\frac{C}{Q} = F_C\left(\hat{Q}, \frac{D}{Q}, r, \varphi\right) \qquad \text{(A2.38)}$$

From (A2.37) and (A2.38) it is clear that in this case the derivatives with respect to R, C*/Q*, $\hat{Q}$, r and φ will have the same sign that in the case of the trade balance ratio, that is to say,

$$(F_{CA})'_{\upsilon} = (F_{TB})'_{\upsilon} \frac{\partial}{\partial \upsilon}\left(\frac{TB}{Q}\right) \quad \upsilon \neq \frac{D}{Q} \qquad \text{(A2.39)}$$

However, in the case of D/Q, it is necessary to perform a separate analysis,

$$(F_{CA})'_{D/Q} = (F_{TB})'_{C/Q}\,(F_C)'_{D/Q} - (F_{DS})'_{D/Q} \qquad \text{(A2.40)}$$

Taking into account that the consumer distributes his disposable income (Q_D)[51] between consumption and savings, the following relationship will necessarily occur,

$$(F_C)'_{D/Q} = -\left[(F_{DS})'_{D/Q} + (F_S)'_{D/Q}\right] \qquad \text{(A2.41)}$$

So, substituting expression (A2.41) into the previous (A2.40), it is obtained the following,

$$(F_{CA})'_{D/Q} = -\left[(F_{TB})'_{C/Q}\,(F_S)'_{D/Q} + (F_{DS})'_{D/Q}\left[1 + (F_{TB})'_{C/Q}\right]\right] \qquad \text{(A2.42)}$$

Or renaming $(F_{TB})'_{C/Q}$ as -α in order to specify the variable in positive terms, (A2.42)

[51] Being this determined as follows, $Q_D = Q - DS$

can be reformulated as follows,

$$(F_{CA})'_{D/Q} = \alpha\,(F_S)'_{D/Q} - (1-\alpha)\,(F_{DS})'_{D/Q} \qquad \text{(A2.43)}$$

The necessary condition for $(F_{CA})'_{D/Q}$ to be negative is shown in equation (A2.44),

$$(F_{DS})'_{D/Q} > \frac{\alpha}{1-\alpha}(F_S)'_{D/Q} \qquad \text{(A2.44)}$$

As it was discussed in the main body of the chapter the condition for the negative relationship between the current account and the stock of foreign debt is that the interest payment exceeds the improvements that a higher debt causes in the volume of national savings.

CHAPTER 3

THE EURO EQUILIBRIUM LEVEL SINCE THE 70'S. A STRUCTURAL ESTIMATION BASED ON THE NATREX APPROACH[52]

3.1 Introduction

In spite of the abundant research on empirical exchange rate determination, economists have not yet been able to obtain conclusive results about the topic. Meese and Rogoff, in their seminal paper of 1983, have already specified this quite unsatisfying result which, unfortunately, does not seem to have changed dramatically since then. Nevertheless, this fact is not particularly surprising if we take into account that in the short run the exchange rate is in reality an asset price whose fluctuations are driven by potentially volatile expectations. Alternatively, more promising results have been obtained by the research based on the long-run exchange rate fundamental determinants. The peculiarity of this long-run analysis is that, by definition, it has completely avoided all the cyclical and speculative flows to evaluate instead only the forces that would eventually attract the real exchange rate to its equilibrium level.

The aim of this chapter is therefore to work on the basis of a rigorous long-run real exchange rate determination model. In particular, we deal with the theory and empirical implementation of an equilibrium concept based on a standard Natrex (acronym of natural real equilibrium exchange rate) model[53]. With regard to this approach, a key fact has been that

[52] This chapter follows to a great extent the work of Detken and Marín (2003). The authors make also use of a similar methodology in Detken et al (2002a) and Detken et al (2002b).

[53] The Natrex approach was first developed by Stein (1990, 1994).

although the structural estimation is what would exactly fit the spirit of a Natrex model, probably due to the problems encountered in the long-term convergence of both the capital stock and the foreign debt level, the model has been usually estimated in reduced form. The works of Crouhy-Veyrac and Saint Marc (1995), Lim and Stein (1995), Stein (1997) and Stein (1999), for instance, have all developed estimations of Natrex simply derived from single reduced equations.

There exist however two attempts to estimate a structural Natrex model, those of Gandolfo and Felettigh (1998) and Verrue and Colpaert (1999). Unfortunately, neither of them have been able to solve directly the problem of the structural estimation of a Natrex model, but they have circumvented it in different ways. Verrue and Colpaert eliminate the exchange rate from their system of behavioural equations and add a reduced form exchange rate equation; the result is that the Natrex is not finally derived from the structural equations. On the other hand, Gandolfo and Felettigh have explicitly ignored the long-run capital and foreign debt accumulation equations, as well as the structure of the investment and consumption functions. Our contribution is therefore an estimation of the model in its true structural form, with a special effort to derive the long-run convergence conditions of both the net foreign debt and the capital stock.

The model is applied to the effective exchange rate of a synthetic euro simulated from the 70's. The reason why we centre on the euro is two-fold. First of all, because not much work has yet been done on the euro exchange rate and second because the euro's depreciation since the beginning of Stage III of Monetary Union fuels economists curiosity to appraise whether this has been an equilibrium or disequilibrium phenomenon. The answer, according to the Natrex approach, is that this is likely to be a disequilibrium phenomenon. Other empirical work on the euro exchange rate has been conducted by van Aarle, Boss and Hlouskova (2000) and Chinn (2000) with monetary exchange rate models. Clostermann and Schnatz (2000) use a BEER (behavioural equilibrium real exchange rate) model, while Alberola, Cervero, Lopez and Ubide (1999) and Hansen and Roeger (2000) estimate a PEER (permanent equilibrium exchange rate) model. Barisone, Driver and Wren-Lewis (2000) estimate a FEER (fundamental equilibrium exchange rate) model while Detken, Dieppe, Henry, Marín and Smets (2002a, 2002b) simulate effective equilibrium exchange rates with a suite of four different models imposing an increasing degree of structure on the data. All of these studies find significant undervaluations of the euro against the dollar ranging between 10% and 40% by the end of 1999 or mid 2000[54]. The variety of results in these studies shows that the concept as such, as well as the empirical implementation of them through the different approaches to the equilibrium exchange rate, are miles apart from what one might call a consensus.

We employ the database of the ECB's area wide model described in Fagan, Henry and Mestre (2001). Our synthetic effective euro exchange rate is defined with respect to the currencies of the four largest trading partners of the euro area, i.e. the UK, the US, Switzerland and Japan.

The peculiarity of the Natrex approach with respect to other macroeconomic exchange rate models (e.g. the macroeconomic balance model) is that it distinguishes between the medium and long run horizons, through the consideration of a slow adjustment in two important stock variables, the capital stock and net foreign assets. Usually, these stock variables converge very slowly to their steady state paths, so that in reality and due to exogenous shocks, the economy will almost never reach its long-run equilibrium. Still, the definition of long-run equilibrium employed in the Natrex framework is in our opinion conceptually the cleanest way to deal with exchange rate equilibrium. We will both report the medium and the long-run equilibria resulting from the estimations.

The structure of the chapter is as follows. Next section summarises the main attributes of the dynamic structural model presented in chapter two, which is used for the empirical implementation of the effective euro equilibrium exchange rate. Section three presents the estimation procedure and the empirical results. The estimated equilibrium traces to a great extent the real depreciation occurred during the mid seventies and eighties, and the real

[54] Alberola et al. refer to end 1998. Detken et al. do not take the US dollar as the counterpart but use instead the effective exchange rate vis a vis the four most important trading partners (UK 34%, US 33.5%, CH 20% and JP 12.5%).

appreciation that took place after that moment and until the stabilisation happened in the second half of eighties and first half of nineties. It appears, however, an important discordance between the estimated and current real exchange rate in the last years of the analysis going from mid nineties until the end of 2000. The simulation of the model allows us to conclude that a main explicative factor of this discordance is the evolution of productivity. Finally, section four summarises and concludes the chapter.

3.2 A Dynamic Model for the Euro Exchange Rate

3.2.1 Stylised Facts

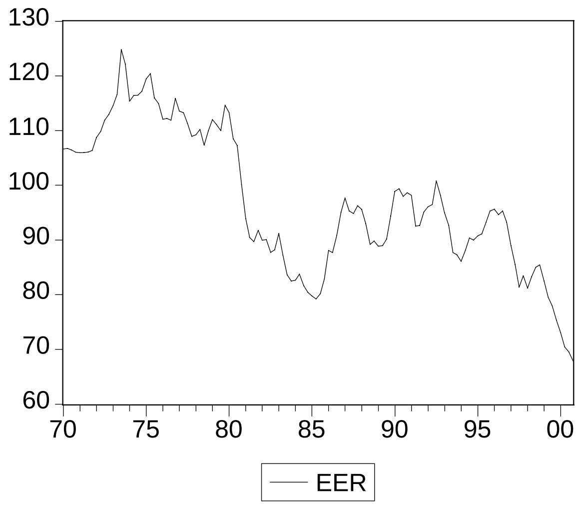
The preliminary data analysis in this section shows that the main stylised facts of Natrex can be reproduced with our data, so that our estimation is perfectly consistent with the approach. Some of these preliminary facts are the non-stationarity of the real exchange rate as well as of two of its main fundamental determinants, that is, a measure of the national consumption preferences and of productivity. Besides, it is also shown the stationarity properties of the national to foreign long run real interest rate differential.

In particular, graph 3.1 shows the evolution of the effective euro real exchange rate (EER), while graph 3.2 depicts the behaviour of the national productivity and consumption ratio respectively. Specifically, PROD means productivity and is defined as the growth rate of a specific labour-augmenting parameter of the production function[55]. Alternatively, CGDP is the ratio of total consumption to GDP and tries to give an idea of the national preference for consumption. Table 1 in appendix 3.1 formally proofs the non-stationarity of these series by means of augmented Dickey-Fuller unit root tests.

[55] This productivity parameter has been measured as the Solow residual of a Cobb-Douglas production function with constant returns to scale. The reader is referred to the empirical estimation of the model for details.

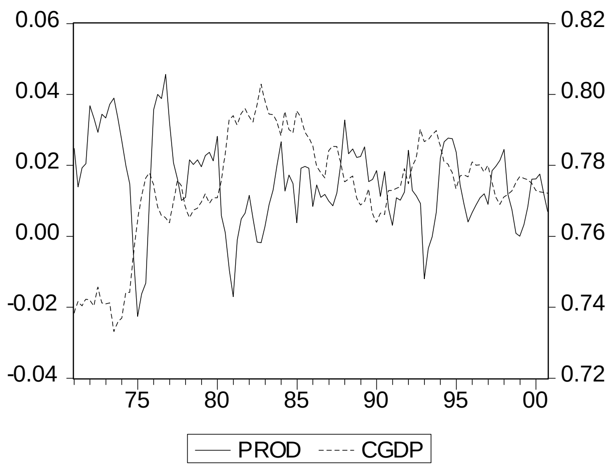
Graph 3.1

Real Exchange Rate Evolution



Graph 3.2

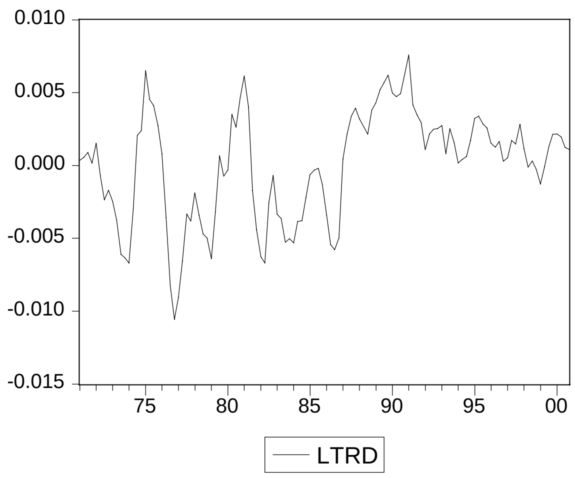
National Productivity and Consumption Ratio



Finally, graph 3.3 shows the difference between the national and foreign long-run real interest rates (LTRD). Also appendix 1 displays the augmented Dickey-Fuller test statistic of the variable, which suggests that it is clearly stationary.

Graph 3.3

Real Interest Rate Differential



Equipped with these facts we are ready to develop and estimate the Natrex model.

3.2.2 The Structural Model

In this section, we carefully summarise the peculiarities of the model that conforms the basis of the subsequent empirical estimation. Both the theoretical characterisation of the approach as well as its medium and long run stability conditions have been exhaustively analysed in the previous chapter. In this section then, it only remains to fix the main useful results important for its implementation.

In the model, decisions on consumption, production, savings and investment will determine the equilibrium of the economy. Given some previous assumptions, the basic structure of the model described below can be inferred from first principles.

The investment equation is derived from the decisions of profit maximising firms, so that the usual exogenous variables explaining the investment behaviour will apply in our model. Following however the most recent published versions of Natrex, our investment function includes moreover the non-standard explanatory variable given by the real exchange rate. In its formulation we have made use of the fact that in a multi-sectorial economy producing tradables and non-tradables the relative price between the two different goods will affect the value of the marginal productivity of capital and, consequently, the investment decisions. Without loss of generality, this relative price has been approximated by the real exchange rate. This insight is what motivates that in our empirical equation investment can thus be described by a function that depends positively on the productivity factor and negatively on the capital stock, the real interest rate and the real exchange rate.

Regarding consumption decisions, from the optimal control problem we know that social consumption will depend on both the capital stock and the net foreign debt (which together serve as a proxy for the total net wealth), as well as on the real interest rate (derived from the intertemporal decision problem) and a parameter measuring thrift. Given that capital positively contributes to the total net wealth while foreign debt subtracts from it, it should then occur that a higher capital stock boosts consumption while a higher foreign debt reduce it. The latter will also contribute to the long-run stability condition emphasised by the Natrex approach[56]. According to the first order condition of the optimal control problem, an increase in the real interest rate will depress consumption and the same will occur in the case of the thrift parameter.

Given the nature of savings, determined as the portion of disposable income that is not

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[56] As it has been previously suggested, the definitive long run steady state condition proposed by Natrex is on the relationship between savings and the stock of accumulated foreign debt.

devoted to consumption, all the explanatory variables of consumption will be also operative for explaining the saving decisions. Intuitively, their relationship with savings will consequently move savings in the opposite direction than consumption. The difference between the two behavioural equations arises in this case from the variables determining the service of the net foreign debt. That is the reason why the net effects of both the real interest rate and the ratio of foreign debt have ambiguous effects on savings while they are unambiguous on consumption. The empirical evidence helps us however to conclude about the sign of the derivatives, being the most probable result a positive relationship between savings and both the real interest rate and the ratio of the net foreign debt. Regarding this conclusion, the desire to save as a response to an increment in foreign debt is in this model an important restriction imposed by the theory in order to obtain the foreign debt long run convergence condition.

Finally, in relation to the trade balance component, this is considered to depend on the real exchange rate and on measures of the domestic and foreign demand levels. In our case, these two latter ones are going to be proxied by the national and foreign consumption to GDP ratios respectively. Foreign consumption will improve the domestic trade balance while domestic consumption as well as a real exchange rate appreciation will both worsen it.

In this approach, where investment responds to the real exchange rate, the medium run stability condition requires that the sensibility of response with respect to the real exchange rate is higher in the trade balance than in the investment function. This requirement, fortunately supported by the data, together with the previous restriction over the relationship between savings and foreign debt conform in this model the basic stability conditions of the system.

The model is completed with the national account identity, the real uncovered interest rate parity, the Fisher equation, and the dynamic stock-flow equations for the capital stock and the net foreign debt levels. Moreover, we know from the previous chapter that an alternative to the national account identity is the expression saying that in equilibrium the current account should equalise the differential of the desired national savings minus the investment level.

For the current account, as in the case of savings, it is the component describing the interest payment to the rest of the world which introduces an ambiguity in the relationship between the current account balance and the ratio of foreign debt. Again, it is the economic intuition and the empirical evidence the elements used to conclude. From the standard exposition of Natrex, it should occur that an increase in the net foreign debt level worsens the current account. In any case, and regarding the characterisation of equilibrium, provided that the effect of the debt ratio on savings will always be stronger that the effect on the current account[57], the fact that the empirical evidence do not support this last relationship it is not necessarily an inconvenience for convergence.

Given this brief characterisation of the model, we develop next an intuitive description of equilibrium by means of the example of a negative shock on the preferences for consumption or on the productivity growth rate.

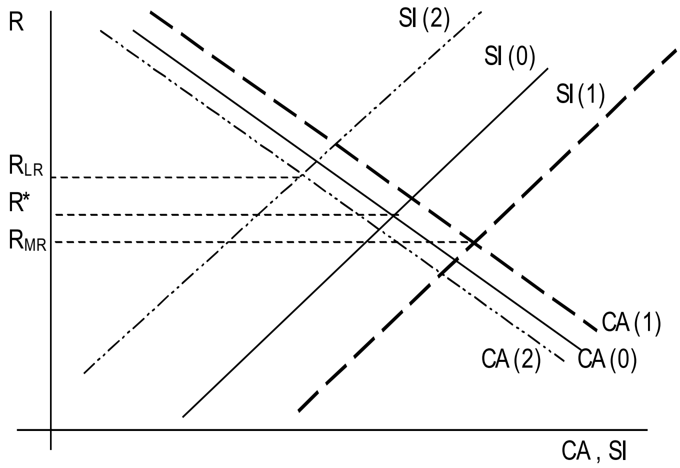
3.2.3 The Characterisation of Equilibrium

Figure 3.1 shows the graphical solution for the medium and long run equilibria after a permanent positive shock to savings minus investment. This one could be the consequence of both a lower preference for consumption or a negative shock to the exogenous productivity growth factor. In this analysis, the national account identity has been understood as the equation where the current account equalise the difference of savings minus investment. This is the reason why the current account (CA) and savings minus investment (SI) schedules have been separately analysed. For the current account curve, the negative slope is due to a real appreciation deteriorating the trade balance for given levels of the capital stock and foreign debt variables, while for the SI curve the slope is positive because a stronger currency crowds out investment.

[57] This is due to a marginal propensity to import lower than unity.

Figure 3.1

The Effect of a Positive Shock on Saving minus Investment



What happens in figure 3.1 after a permanent positive shock to savings minus investment is the following, less consumption or investment will require a depreciating currency that spurs the trade balance in order to achieve goods market equilibrium. This is the reason why the SI curve shifts downwards. However, with regard to the current account, and exclusively in the case that the shock is over the thriftiness parameter, this one will require an appreciating currency to compensate for the potential improvement in the trade balance that derives from the lower amount of consumption. The result is that the CA curve shifts upwards, although the absolute size of the shift will be smaller than the corresponding one for the SI schedule. At the old exchange rate R* the economy experiences an excess of desired savings over investment, so that the resulting capital outflow ends up depreciating the exchange rate and improving the current account. Therefore, in both cases the medium-run result is a real depreciation as depicted by R_{MR} and the intersection of the SI(1) and CA(1)[58] curves.

In the long run, the steady-state conditions for the capital stock and the net foreign asset position need also to be implemented. The transition from the medium to the long run can be explained as follows.

In the case of a negative shock in the preference for consumption, the improvement in both the current account and investment reduces the foreign debt and boots the capital stock ratios respectively. The positive wealth effect associated to both of these movements will lead to consumption slowly recovering from the initial drop. This together with an appreciation of the exchange rate with respect to the previous medium-run equilibrium is what stabilises the system, as the recovering of consumption scales back both the trade balance surplus and the current account up to their steady state levels. The reasoning just described features an overshooting of the real effective exchange rate. The overshooting is not due to rigid prices as in Dornbusch (1976) but to the fact that the accumulation of wealth through current account surpluses and investment in the capital stock takes time (like in Gärtner (1993, Ch. 7)). However, the overshooting result is not a necessary outcome of the Natrex model. Depending on the parameter estimates a further depreciation or even an appreciation with respect to the original equilibrium R* is possible. From our estimated parameters it is obtained the suggested overshooting effect.

Moreover, in the case of productivity shocks implying both a falling debt and capital stock ratio, it is obvious that the long-run effects sketched out by shifts in SI(2) and CA(2) in figure 3.1 become even more ambiguous. However, the appreciation result is in fact the standard outcome suggested by Stein (1994) and again supported by our data.

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58 The only difference between the thrift shock and the shock to productivity growth is that the latter does not shift the CA curve in period 1. Also, whether it will shift right or left in period 2 depends on the parameter estimates.

3.3 Empirical Estimation

3.3.1 The Data and Methodology

The empirical analysis makes use of the area wide model dataset developed by members of the Econometric Modeling Division in the European Central Bank[59]. The dataset available to us is quarterly with series starting in 1970Q1, where possible, and spans up to 2000Q4. It comprises aggregated data of individual country series for the EU11.

In the estimation we proceed as follows. We estimate separate vector error correction (VEC) models for the behavioural equations of the model. The existence of unit roots in the economic time series is tested with the Dickey-Fuller and augmented Dickey-Fuller unit root tests. In particular, we look for one co-integrating vector for each behavioural equation checked in this case with the Johansen cointegration test. The coefficients used to simulate the model are derived from the co-integrating vector of the respective VEC model. Whenever possible we try to find weak exogeneity for the right hand side variables of the main behavioural equations, what would be revealed by the insignificance of the respective error correction terms in the VEC specifications. Finally, with respect to the estimation period we begin as early as possible for each equation and cover until 1999Q4 for the estimation period. The last year is set aside simply to avoid the statistical distortions proper to the last data of an economic series.

3.3.2 The Estimation of Productivity

We start this empirical section estimating the productivity factor, named A. In particular, this has been approached by the Solow residual of a specific labour augmenting production function. Usually, for convergence reasons the best performance is achieved with a Cobb-Douglas with constant returns to scale[60]. In this case, the growth rate of the per capita GDP

[59] See Fagan, Henry and Mestre (2001).

[60] The assumption of constant returns to scale is also fortunately supported by empirical evidence.

(q) can be expressed as in (3.1), where variables are in real terms and lower case means per capita values. Parameters α and (1-α) depicts respectively the intensity of capital (k) and labour in the production function. Symbol ^ stands for the growth rate.

$$\hat{q} = (1-\alpha)\,\hat{A} + \alpha\,\hat{k} \tag{3.1}$$

For estimation reasons equation (3.1) is specified as in (3.2), where the unknown value of productivity is relegated to the residual term. Variable ε is therefore the residual that represents the growth rate[61] of A times the parameter measuring the intensity of labour in the production function.

$$\hat{q} = c + \alpha\,\hat{k} + \varepsilon \tag{3.2}$$

Appendix 3.2 shows the results of the unit root test performed for the per capita production and capital stock growth rates, named QRPCGR and KRPCGR respectively. Both have resulted being non-stationary variables with one unit root. Given these non-stationarity properties, equation (3.2) has to be necessarily estimated in first differences. In this particular case we follow the Hendry methodology in order to obtain a long run specification for equation (3.2). Appendix 3.2 shows the details of the estimation.

From the process, the estimated alpha parameter equals approximately 0.25, a similar level in fact to the one provided in the estimations for the United States. Some authors however have been sceptical with the assumption of QRPCGR and KRPCGR being I(1), so the estimation is also performed with variables in levels. Fortunately the new estimated alpha is very close to the previous one, what gives some consistency to the results. The details of this alternative procedure are also in appendix 3.2.

Once alpha is known, the A growth rate (named AGR hereafter) can be obtained from the assumed Cobb-Douglas production function. Equation (3.3) shows how it is determined,

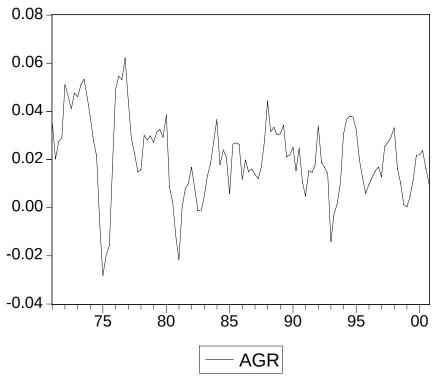
[61] Once discounted its constant mean.

while graph 3.4 give us its temporal evolution.

$$AGR = \frac{1}{1-\alpha}(QRPCGR - \alpha * KRPCGR) \tag{3.3}$$

Graph 3.4

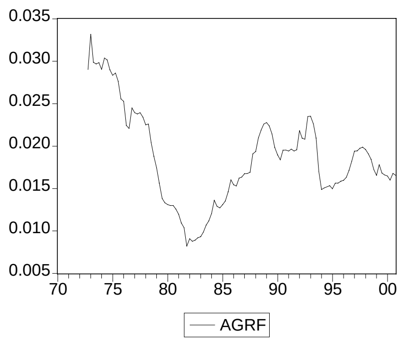
Productivity Growth Rate



Appendix 3.2 gives also details on the determination of a filtered AGR series (named AGRF). The reason of it is the necessity of avoiding the pure cyclical elements when estimating the medium and long run equilibriums. Graph 3.5 depicts this AGRF series.

Graph 3.5

Filtered Productivity Growth Rate



Once productivity is estimated, we are ready to deal with the main body of the structural equations.

3.3.3 The National Behavioural Equations

In this approach the behavioural equations are considered in linear form. In particular, expressions going from (3.4) to (3.6) show the relationships estimated in the exercise,

$$\frac{I}{Q} = \alpha_1 + \alpha_2 \hat{A} - \alpha_3 \frac{K}{Qr} - \alpha_4 r^L - \alpha_5 R \tag{3.4}$$

$$\frac{C}{Q} = \alpha_6 + \alpha_7 \frac{K}{Qr} - \alpha_8 \frac{D}{Q} - \alpha_9 r^S + \alpha_{10} i^L \tag{3.5}$$

$$\frac{TB}{Q} = \alpha_{11} - \alpha_{12} R - \alpha_{13} \frac{C}{Q} + \alpha_{14} \frac{C^*}{Q^*} \tag{3.6}$$

Where Q is GDP, I investment, C consumption and TB trade balance; all assumed in nominal terms. The only exception to these quotients of nominal variables is the capital stock ratio, given that in this case the more accurate way is to express it in real terms. The lower case letter *r* accompanying variable Q indicates therefore real terms. Regarding the exogenous and predetermined variables, $\hat{A}$ is the productivity factor growth rate, K the capital stock, r the real interest rate, R the real exchange rate , D the net foreign debt and *i* the nominal interest rate. Superscript L remarks long run and S short run interest rates. The star indicates foreign variables.

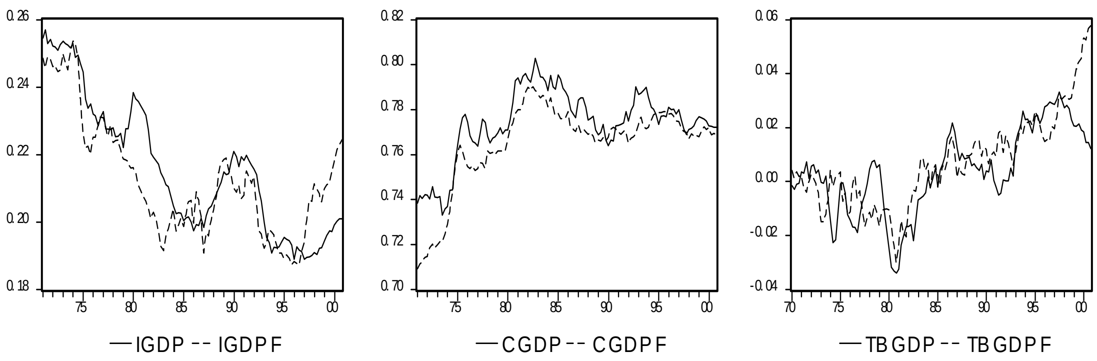
Given the procyclicality of the long-term nominal interest rate, we have chosen it to proxy for the exogenous parameter measuring consumption preferences. Consequently we expect α_{10} to be positive.

Appendix 3.3 shows the results about the unit root properties of the variables involved into the three behavioural equations as well as the Johansen cointegration tests and the VEC results. All variables considered are integrated of order 1 and in all the estimated equations we find one co-integrating vector of the respective VEC model. All the coefficients have the right sign and are significant at the 5% level. Weak exogeneity for the right hand side variables of equations (3.4), (3.5) and (3.6) (as revealed by the insignificance of the respective error correction terms in the VEC specifications) can be attested for all the explanatory variables in the three VEC estimations with the exceptions of the real exchange rate and the ratio of capital to GDP in the investment equation. The error correction terms for the respective endogenous variables under scrutiny are all significant at the 5% level in the three VEC models.

Graph 3.6 shows the estimated investment, consumption and trade balance series. The added F at the end of the word means forecasted.

Graph 3.6

Estimated National Behavioral Equations



Equipped with this equations we are ready to determine the medium run equilibrium where the two stock variables of the problem, that is, the capital stock and the net foreign debt are assumed exogenous. Moreover, given that this temporal horizon is not considered long enough as for the real interest rates to converge to international levels, in this medium term the current short and long run real interest rates are also used.

3.3.4 The Medium Run Equilibrium

For the medium run equilibrium the behavioural equations (3.4) to (3.6) plus the national account identity (3.7) will apply. Expression (3.7) differs from a standard theoretical formulation because in the dataset there is a distinction between gross investment (I) in capital stock and the variation of other stocks (named SCN).

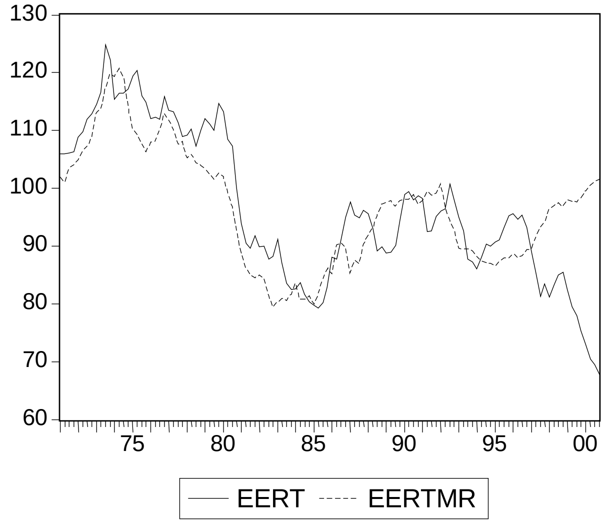
$$\frac{C}{Q}+\frac{I}{Q}+\frac{TB}{Q}=1-\frac{SCN}{Q} \tag{3.7}$$

To obtain this medium term equilibrium we also adopt some extra assumptions about the exogenous variables as it is carefully specified in appendix 3.4. In particular, as the Natrex should be evaluated at full-employment, i.e. internal equilibrium, we have tried to approximate this by using Hodrick/Prescott trends for business cycle sensitive variables. An special treatment has been dispensed to the SCN/Q variable given the high volatility induced by its residual nature. Due to the sensibility of the filter to the temporal horizon of a series, and in particular of this series, previous to the filtration process we have decided to expand this variable by means of an OLS estimation where this can be explained by its lags. The process is applied to the first differences due to the non-stationarity of the series. The details of this expansion are in appendix 3.4.

Graphs 3.7 and 3.8 depict the medium-run equilibrium.

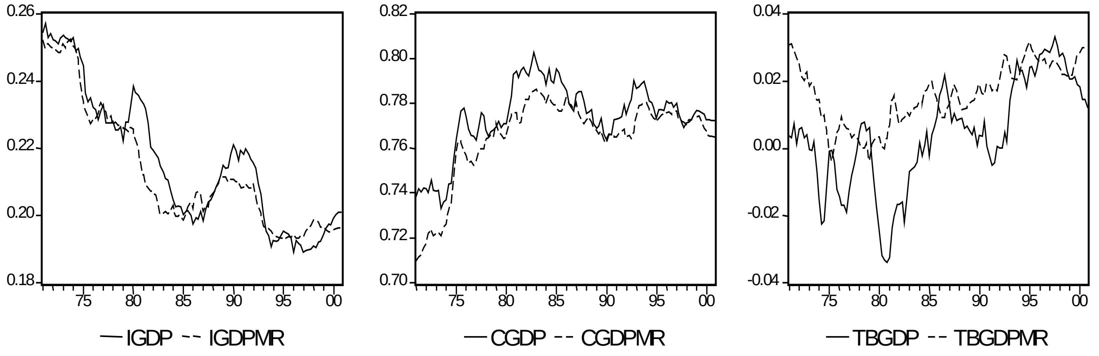
Graph 3.7

Real Exchange Rate Medium Run Equilibrium



Graph 3.8

National Behavioral Equations Medium Run Equilibrium



Given that some authors have pointed out their scepticism about the non-stationarity assumption of many real exchange rate determinants, alternative simple OLS estimations have been also performed. In this case, and in order to avoid the excessive variability of the quarterly data, for the estimation exercise the quarterly frequency has been reduced into annual frequency. The fact that the estimated real exchange rate follows a similar pattern that the resultant of the VEC estimations provide again robustness to the analysis. Appendix 3.5 shows the details of this alternative procedure.

With regard to the empirical results the first observation is that the medium-run Natrex equilibrium tracks the depreciation of the real effective euro in the first half of the 80s. During all the seventies up to the mid 80s the consumption ratio to GDP has been increasing at the same time that the ratio of investment to GDP has been decreasing. This is suggestive of a lower tendency to savings and, overall, of negative productivity shocks in the period[62]. The simulation of the model confirm in fact that productivity accounts for a considerable proportion of the real exchange rate evolution. Then, the depreciation in the early eighties can be considered an equilibrium reaction, as was the subsequent appreciation in the second half of the decade. From the second half of 80s onwards, and given an evolution in consumption considerably stable, the medium run equilibrium seems to follow the cycle pointed out by investment. It is particularly curious the wide gap opened between the current real exchange rate and its level of equilibrium since the beginning of 1997.

Nevertheless, from the simulation of the model it is interesting that a great proportion of the real exchange rate evolution is in fact explained by the behaviour of the stock variables, considered exogenous in this analysis. Therefore, for a more founded explanation of the exchange rate behaviour we should enter into long run considerations.

3.3.5 The Foreign Sector and the Real Interest Rate Parity Condition

In the long run both the capital stock and the net foreign debt must evolve to its steady

[62] In fact, the productivity growth rate turned negative in 74-75, 80-81 and 82.

state levels. Moreover, the long-term real interest rates should also converge internationally. To introduce this aspect of equilibrium it is required that our analysis of equilibrium changes to endogenise the real interest rate, so that the foreign sector is therefore incorporated.

In this case we should include an international equilibrium condition as the following,

$$\frac{S}{Q}-\frac{I}{Q}-\frac{SCN}{Q}=-\left(\frac{SI^*}{Q^*}-\frac{I^*}{Q^*}-\frac{SCN^*}{Q^*}\right) \tag{3.8}$$

where S represents savings, I investment, SCN variation of other stocks and the star means foreign sector. All the data, with the exception of SI*, can be derived from the available dataset. The foreign savings ratio, SI*/Q*, is alternatively inferred as a residual component of equation (3.8) due to the empirical difficulty of finding a proper measure of it.

Following the same reasoning that in the medium term equilibrium, we specify structural equations for the foreign savings and investment ratios to GDP. In this case, and in order to simplify, for the investment equation we are going to consider an aggregated I* + SCN* variable, so that both can be explained by the same variables. In particular, the following are the equations estimated in the exercise.

$$\frac{I^*}{Q^*}+\frac{SCN^*}{Q^*}=\alpha_{15}+\alpha_{16}\,\hat{Q}_r^*-\alpha_{17}\,(r^L)^* \tag{3.9}$$

$$\frac{SI^*}{Q^*}=\alpha_{18}+\alpha_{19}(r^S)^*+\alpha_{20}\frac{NFN}{Q}+\alpha \tag{3.10}$$

given the assumption that NFN*/Q* = -(NFN/Q).

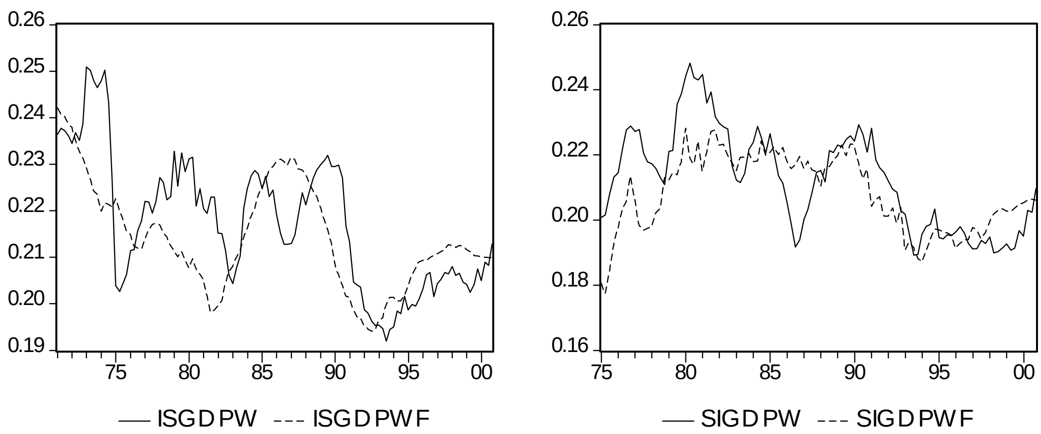
The growth rate of the foreign real GDP, $\hat{Q}_r^*$, approaches productivity in this case due to the lack of data that could allow a different estimation. Variable NFN/Q represents the net income of foreign factors as a ratio to GDP, and can be calculated as the difference between the current account and the trade balance. The basic stability condition adopted by the Natrex models saying that the accumulated stock of foreign debt should stimulate savings is the reason for the negative relationship between the income from foreign factors and the quantity of savings.

Appendix 3.6 shows the results about the unit root properties of the variables involved into this behavioural equations. Moreover, there is also information about the Johansen cointegration tests and the VEC estimations. All variables considered are integrated of order 1 and in all the estimated equations we find one co-integrating vector of the respective VEC model. All the coefficients have the right sign and are significant at the 5% level, the same that the error correction terms of the correspondent endogenous variables. In this case, weak exogeneity for the right hand side variables of equations is neither satisfied for productivity in the investment equation nor for the real interest rate in the saving equation.

Graph 3.9 shows the estimated foreign investment and saving equations. The added F at the end of the word means forecasted.

Graph 3.9

Foreign Behavioral Equations



Given the characterisation of the foreign sector we can start solving the long-run equilibrium by the first step of treating the real interest rate as an endogenous variable. In this case, equations (3.8), (3.9) and (3.10), which represent the international equilibrium, plus equation (3.11), which stands for the real interest rate parity condition, should be added to the medium run equilibrium,

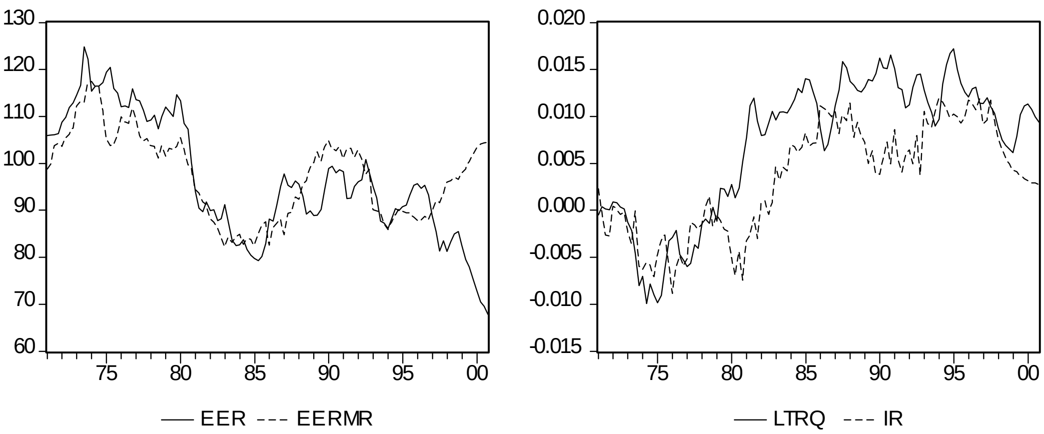
$$r - r^* = 0 \tag{3.11}$$

To determine a single homogeneous interest rate, previous to the analysis of equilibrium, we develop simple cointegrating relationships where both the national and foreign long-run real interest rates are considered explained by its corresponding short-term real interest rates. Appendix 3.7 details these results.

Graph 3.10 shows the new estimated real exchange rate and real interest rate.

Graph 3.10

Medium Run Equilibrium with Convergence in Real Interest Rates



From Graph 3.10 we can see that although in equilibrium the real interest rate is consistently lower than the national current level, the new path of equilibrium for the real exchange rate is in fact quite similar to the previous one without interest rate convergence.

3.3.6 The Long Run Equilibrium

To determine next the long run equilibrium, it should be also considered that the stock variables of the model have reached their respective steady state positions.

Equations (3.12) and (3.13) specify respectively the dynamic equations of the capital stock and the net foreign debt expressed as ratios to GDP,

$$\frac{K}{Qr} = \frac{1-\delta^q}{1+g^q}\left(\frac{K}{Qr}\right)_{-1} + \frac{P_Q}{P_I}\frac{I}{Q} \qquad (3.12)$$

$$\frac{D}{Q} = \frac{1}{1+q^q}\left(\frac{D}{Q}\right)_{-1} + \frac{I-S}{Q} \qquad (3.13)$$

Both have been adapted to the particular characteristics of our dataset. Therefore, variables g^q and q^q are respectively the quarterly real and nominal GDP growth rates, and in equation (3.13) it is introduced the ratio of GDP to investment deflators (P_Q and P_I respectively) as I/Q has been defined in nominal terms. The parameter δ^q is the quarterly depreciation rate of the capital stock variable.

From (3.12) and (3.13) it is easily obtained the following steady state equations ready to implement in the empirical estimation of the long run equilibrium,

$$\frac{K}{Qr} = \frac{1+g^q}{0.01+g^q}\frac{P_Q}{P_I}\frac{I}{Q} \qquad (3.14)$$

$$\frac{F}{Q}=\frac{1+q^{q}}{q^{q}}\left(\frac{I}{Q}+\frac{C}{Q}-\frac{NFN}{Q}-1\right) \tag{3.15}$$

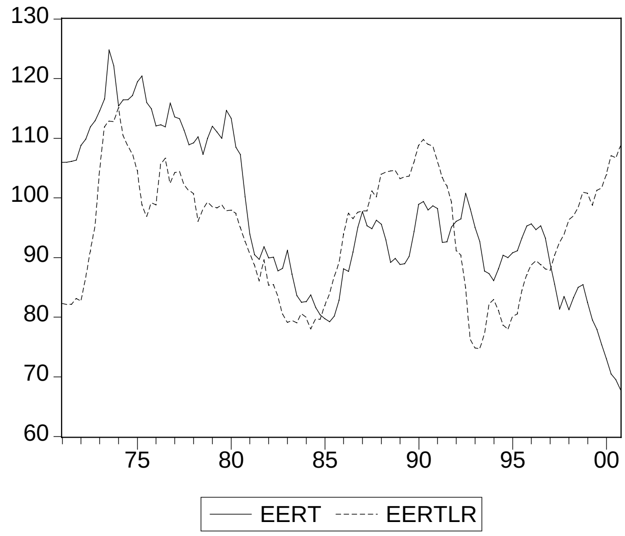
The quarterly depreciation rate of the capital stock is equal to 0.01. Savings has been substituted as a function of consumption and the net income from foreign factors (NFN).

Moreover, given the close relationship between the evolution of the NFN variable and the accumulation of foreign assets, for the long run equilibrium it should be also determined an equilibrium relationship between the two variables. Appendix 3.8 accounts for the details of this estimation.

To solve the long-run equilibrium we consider conditions (3.14) and (3.15) along with the previous medium run equilibrium with convergence in interest rates, the relationship between the NFN variable as a ratio to GDP, and the ratio of the net foreign debt. Graphs 3.11, 3.12 and 3.13 present these long-run equilibrium solutions.

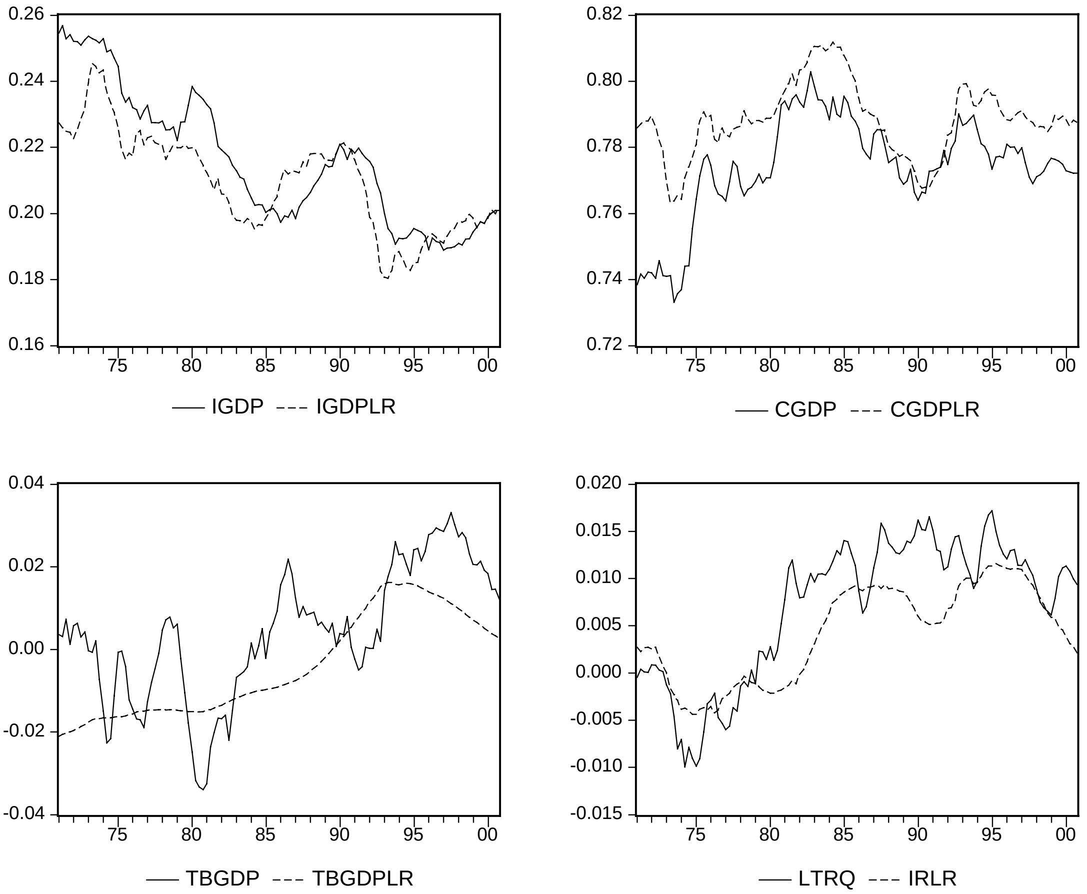
Graph 3.11

Real Exchange Rate Long Run Equilibrium



Graph 3.12

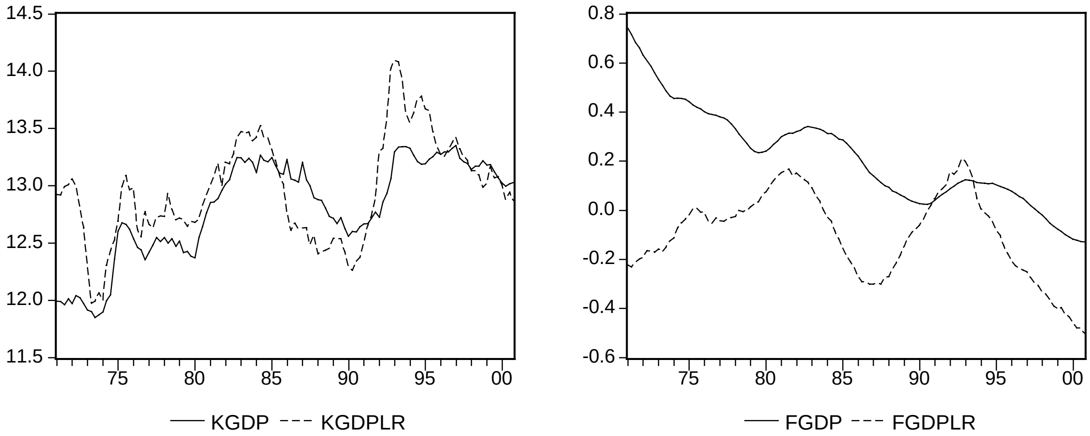
Real Interest Rate and National Behavioral Equations Long Run Equilibrium



Compared to the long-run equilibrium the effective euro rate appears overvalued all through the seventies and early eighties and undervalued from 1983 until 1991/92. The long-run depreciation of the equilibrium rate at the beginning of the 90s had been caused by the lower desired investment, which in general would tend to depreciate the currency both in the medium and in the long run. The most striking feature of the graphs is the opening gap between the equilibrium and the actual exchange rate since 1996/97.

Graph 3.13

Estimated Long Run National Stock Variables



At the end of the period our Natrex estimates suggest that the euro had been considerable undervalued. However, it is fair to say that although this tendency has been ratified by the opinion of many experts, and in the last days by the own market, the results of the estimation seem slightly excessive. From the observation of the data what has caused this evolution is mainly the results over the estimated long run foreign debt accumulation. Probably this suggested accumulation of foreign assets is due to the combination of a slight increment both in desired investment and savings.

It remains to be investigated whether the increasing undervaluation that at the end of the period takes place for both the medium and long-run equilibria is due to an out-of-sample breakdown of the model. First estimates show that the model seems to be robust concerning the use of additional data, and that also other equilibrium estimates tend to confirm the recent undervaluation of the euro.

3.4 Conclusions

We implement a dynamic structural model in the spirit of the Natrex approach. The Natrex models are based on a rigorous stock-flow interaction in a macroeconomic growth model.

In particular, we estimate a small-size dynamic model comprising behavioural equations for trade, consumption and investment both for the national and foreign economy. The main exogenous factors driving the equilibrium real exchange rate are productivity growth and consumption preferences.

The theoretical model is applied to determine the equilibrium effective exchange rate of a synthetic euro for both the medium-run, i.e. without adjustment of stock variables (net foreign debt and capital), and the long run after stock variables fully adjust to their steady state levels. From a theoretical point of view, we show that the transition from the medium to the long run is not strictly determined by the broad range of model specifications that could be summarized under the Natrex approach.

Overall the medium as well as the long run simulated equilibrium exchange rates track the broad pattern of actual exchange rate developments reasonably well. A notable exception is the gap opening since 1996/97, where the equilibrium rates would have predicted an appreciation of the euro but the opposite happened. Our model does not explicitly incorporate expectations, but nevertheless this result suggests a way to rationalize the depreciation of the real effective euro exchange rate with possible market expectations of a lasting productivity growth gap towards the rest of the world.

It remains to be seen if the increasing undervaluation since 1996/97 for the medium and long-run equilibria is robust or is due to an out-of-sample breakdown of the model. In principle, other equilibrium estimates in the literature confirm the finding of a recent euro undervaluation although the extent and the assumed source of it vary widely.

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APPENDIX

Appendix 3.1: Some Stylised Facts

Table 1

Augmented Dickey-Fuller Unit Root Test

$$\Delta x_t = \alpha + \beta t + \nu\, x_{t-1} + \text{lags}\,(\Delta x_t)$$

Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)*	ADF Statistic**	(α, β, lags)	ADF Statistic	
EER	(0, 0, 1)	-1.11	(0, 0, 4)	-5.18	I(1) without Drift
EERGR	(0, 0, 12)	-1.10	(0, 0, 11)	-5.64	I(1) without Drift
PROD	(0, 0, 29)	-1.15	(0, 0, 23)	-3.39	I(1) without Drift
CGDP	(0, 0, 13)	0.55	(0, 0, 12)	-2.46	I(1) without Drift
LTRD	(0, 0, 4)	-3.52	___	___	I(0)

(*) We test whether the constant α and the linear trend β are significant; the number of lags are chosen in order to obtain residuals that are white noise.

(**) The critical values depend on the number of observations and the inclusion of constant and linear trend in the ADF equation. All these considerations have been taken into account when performing the tests. In particular, our critical values are for 100 observations and at 5% significance level.

Appendix 3.2: Estimation of Productivity

Table 2

Augmented Dickey-Fuller Unit Root Test

$$\Delta x_t = \alpha + \beta t + \nu\, x_{t-1} + \text{lags}\,(\Delta x_t)$$

Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)	ADF Statistic	(α, β, lags)	ADF Statistic	
QRPCGR	(0, 0, 20)	-1.70	(0, 0, 7)	-6.49	I(1) without Drift
KRPCGR	(0, 0, 4)	-1.76	(0, 0, 3)	-5.87	I(1) without Drift

Table 3

Hendry Estimation

Variables(*)	Coefficient	t-Statistic	Prob.
QRPCGRD			
C(**)	-0.000476	-0.933425	0.3534
KRPCGRD	0.584235	2.590118	0.0114
KRPCGRD(-1)	-0.977509	-4.058995	0.0001
KRPCGRD(-4)	0.765871	3.415688	0.0010
KRPCGRD(-12)	0.487283	2.404919	0.0185
QRPCGRD(-4)	-0.779299	-8.400480	0.0000
QRPCGRD(-5)	-0.264737	-3.600369	0.0005
QRPCGRD(-8)	-0.539156	-5.317084	0.0000
QRPCGRD(-12)	-0.582714	-5.381360	0.0000
QRPCGRD(-13)	0.177425	2.545117	0.0128
QRPCGRD(-14)	0.230435	3.348376	0.0012
QRPCGRD(-16)	-0.292589	-2.793726	0.0065
QRPCGRD(-20)	-0.366729	-3.811783	0.0003
QRPCGRD(-21)	0.209353	3.104350	0.0026
QRPCGRD(-24)	-0.196152	-2.336321	0.0220
R-squared	0.632401		

(*) Letter D added at the end of the word means first differences

(**) The estimation keeps the constant value foreseeing the possibility that the growth rate of A is non-stationary. Once analysed the stationarity of the series the existence of one unit root can only be accepted in the limit, so it can be the reason why the constant in the Hendry equation has a t-statistic suggesting rejection.

From the previous estimation we can obtain the following long run relationship,

QRPCGRD = -0.000139743419493 + 0.252596492002*KRPCGRD

Or, alternatively, by means of an OLS estimation,

Table 4

OLS Estimation

Variables(*)	Coefficient	t-Statistic	Prob.
QRPCGR			
C	0.015636	6.388179	0.0000
KRPCGR	0.251132	2.921731	0.0042
Durbin-Watson Stat	0.416582	R-squared	0.067463

Exploiting the steady state conditions of the production function we can obtain a filtered A growth rate more founded than the ad-hoc Holdrick and Prescott (HP) filter. From the Cobb-Douglas function the following relationship is true,

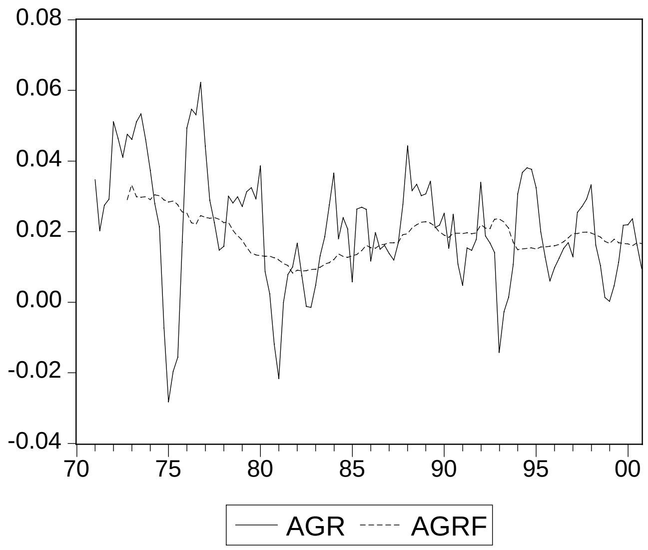
$$\hat{Q} = (1-\alpha)\,\hat{A} + \alpha\,\hat{K} + (1-\alpha)\,\hat{L} \tag{A3.1}$$

So, taking into account that in the steady state the capital stock growth rate has to equalise the production growth rate and that both labour and production should converge to their potential levels, equation (A3.2) can be easily reformulated in the following terms,

$$\hat{Q}_p = \hat{A} + \hat{L}_p \tag{A3.2}$$

where sub-index p indicates potential level.

From (A3.2) the filtered AGR (named AGRF) can be easily obtained.



Appendix 3.3: National Behavioural Equations

Next table shows the unit root tests of the following variables: I/Q (IGDP), $\hat{A}$ (AGR), K/Q (KGDP), r^L (LTRQ), R (EER), C/Q (CGDP), D/Q (DGDP), r^S (STRQ), i^L (LTNQ), TB/Q (TBGDP), C*/Q* (CGDPW)

Table 5

Augmented Dickey-Fuller Unit Root Test

$$\Delta x_t = \alpha + \beta t + \nu x_{t-1} + \text{lags}(\Delta x_t)$$

Investment Equation

Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)	ADF Statistic	(α, β, lags)	ADF Statistic	
IGDP	(0, 0, 6)	-1.09	(0, 0, 5)	-3.40	I(1) without Drift
AGR	(0, 0, 29)	-1.15	(0, 0, 23)	3.39	I(1) without Drift
KGDP	(0, 0, 21)	0.78	(0, 0, 1)	-5.47	I(1) without Drift
LTRQ	(0, 0, 4)	-0.30	(0, 0, 3)	-7.78	I(1) without Drift
EER	(0, 0, 1)	-1.11	(0, 0, 4)	-5.18	I(1) without Drift

Consumption Equation

Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)	ADF Statistic	(α, β, lags)	ADF Statistic	
CGDP	(0, 0, 13)	0.55	(0, 0, 12)	-2.46	I(1) without Drift
DGDP	(0, 0, 1)	-1.77	(0, 0, 1)	-3.17	I(1) without Drift
STRQ	(0, 0, 14)	-0.87	(0, 0, 2)	-6.82	I(1) without Drift
LTNQ	(0, 0, 7)	-0.57	(0, 0, 3)	-5.29	I(1) without Drift

Trade Balance Equation

Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)	ADF Statistic	(α, β, lags)	ADF Statistic	
TBGDP	(0, 0, 14)	-1.46	(0, 0, 4)	-8.61	I(1) without Drift
CGDPW	(0, 0, 8)	1.38	(0, 0, 6)	-4.81	I(1) without Drift

Table 6

Johansen Cointegration Test

	I/Q Equation	C/Q Equation	TB/Q Equation
VEC Specification*	(C, 3 to 4)	(C,1 to 4)	(0, 1 to 9)
Estimation Period	1972.2 – 1999.4	1972.2 – 1999.4	1972.3 – 1999.4
Cointegrating Equation	No trend in Data Intercept in CE	No trend in Data Intercept in CE	No trend in Data No Intercept CE
LR Test	1 Coint Eq at 1%	1 Coint Eq at 5%	1 Coint Eq at 5%

(*) C = Constant in VAR; integers are lags included

Table 7

National Variables Cointegrating Equations

Investment Cointegrating Equation
(t-statistics in parentheses)

IGDP	AGR	KGDP	LTRQ	EER	C
1.000000	-0.661894 (-1.97578)	0.037834 (4.25312)	1.111844 (2.38704)	0.000843 (2.23831)	-0.773942 (-6.01414)
Error Correction Term	D(IGDP)	D(AGR)	D(KGDP)	D(LTRQ)	D(EERT)
	-0.070642 (-3.78739)	-0.107775 (-1.61292)	2.080060 (3.22909)	0.007170 (0.60891)	-57.53664 (-2.58594)

Consumption Cointegrating Equation
(t-statistics in parentheses)

CGDP	KGDP	FGDP	STRQ	LTNQ	C
1.000000	-0.028394 (-5.24903)	0.056582 (2.26096)	0.899611 (2.39593)	-2.025427 (-4.23314)	-0.372659 (-5.40795)
Error Correction Term	D(CGDP)	D(KGDP)	D(FGDP)	D(STRQ)	D(LTNQ)
	-0.120625 (-2.39095)	1.349460 (1.26099)	-0.072923 (-1.17012)	0.034553 (1.48659)	0.018718 (1.33879)

Trade Balance Cointegrating Equation
(t-statistics in parentheses)

TBGDP	EER	CGDP	CGDPW	C
1.000000	0.000979	0.931097	-1.046889	0.000000
	(8.41900)	(9.96046)	(-10.8810)	

Error	D(TBGDP)	D(EER)	D(CGDP)	D(CGDPW)
Correction	-0.302943	-68.22343	-0.018364	-0.056770
Term	(-3.74176)	(-1.31681)	(-0.21580)	(-0.87809)

Appendix 3.4: Medium Run Equilibrium

Table 8

Assumptions for Solving the Medium-term Equilibrium

Imposed equations	(3.4) (3.5) (3.6) (3.7)
Endogenous Variables	I/Q; C/Q; TB/Q; R
Exogenous Variables	SCN/Q; $\hat{A}$; K/Qr; r^L; D/Q; r^S; i^L; C*/Q*
HP Filtered Series(*)	SCN/Q; $\hat{A}$; i^L; C*/Q*

(*) Previous to the filtration process the SCN/Q series has been extended by means of an ARIMA model.

Next, the results of an ARIMA adjustment to the first differences of the SCN/Q (SCNGDP) variable are shown. The reason of working in first differences is the non-stationarity of the series.

Table 9

Augmented Dickey-Fuller Unit Root Test

$$\Delta x_t = \alpha + \beta t + \nu\, x_{t-1} + lags\,(\Delta x_t)$$

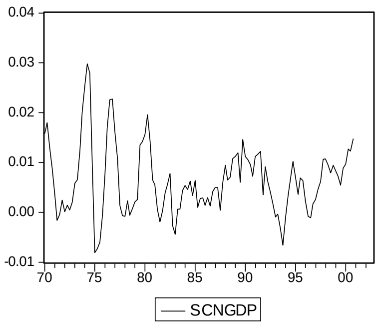
Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)	ADF Statistic	(α, β, lags)	ADF Statistic	
SCNGDP	(0, 0, 34)	-0.32	(0, 0, 9)	-5.99	I(1) without Drift

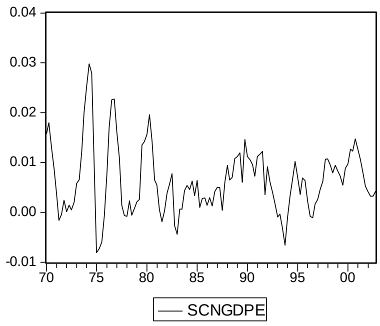
Table 10

OLS Estimation

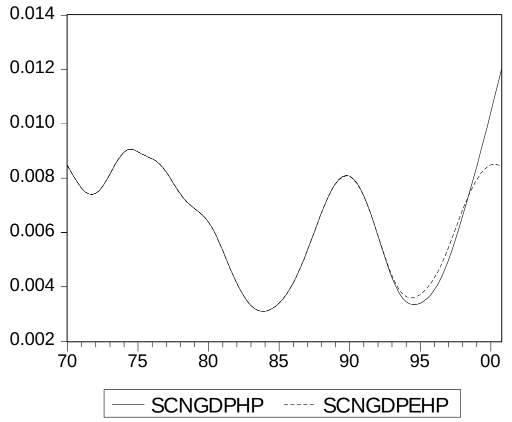
Variables	Coefficient	t-Statistic	Prob.
SCNGDPD			
AR(1)	0.469727	4.968954	0.0000
AR(3)	0.672233	17.20133	0.0000
AR(4)	-0.722654	-8.545320	0.0000
MA(1)	-0.403056	-3.109839	0.0024
MA(2)	-0.145777	-5.979035	0.0000
MA(3)	-0.912891	-36.12927	0.0000
MA(4)	0.483612	3.932838	0.0001
R-squared	0.365218		

The following is the expanded series (SCNGDPE) related to the non-expanded (SCNGDP):





So we can see the difference of applying the HP filter to both series in the period going until 2000.Q4.



Appendix 3.5: Estimation with Annual Data

In this appendix it is summarised the characteristics of the medium run equilibrium performed with annual data.

Table 11

Productivity Estimation

Variables	Coefficient	t-Statistic	Prob.
QRPCGR			
C	0.013150	3.181544	0.0036
KRPCGR	0.347945	2.353641	0.0258
Durbin-Watson Stat	2.006963		
R-squared	0.165167		

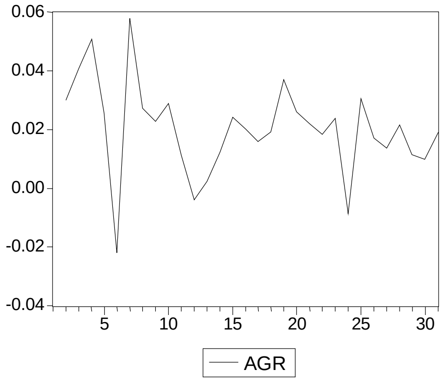
That is,

QRPCGR = 0.01315047481 + 0.3479447908*KRPCGR

Then,

AGR = (1/(1-alpha))*(QRPCGR - alpha*KRPCGR)

where alpha = 0.3479447908



Let us determine next a proper filter for AGR taking into account the characteristics of the production function in the steady state.

In the steady state, the following conditions are satisfied,

QRGR = YETGR

KRGR = QRGR = YETGR

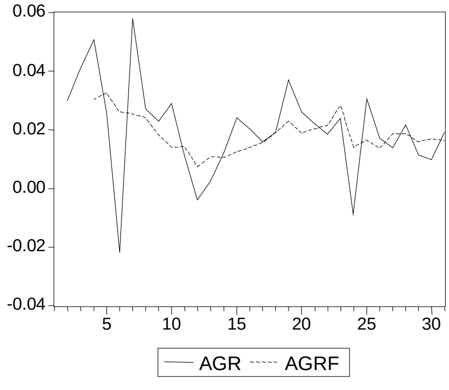
LNGR = LFNGR

So, given the following production function,

QRGR = (1-alpha)*AGR + alpha*KRGR + (1-alpha)*LNGR

in the steady state, the productivity growth rate is determined as follows,

AGRF = YETGR – LFNGR



Given the computation of productivity, these are the estimation results of the national behavioural equations.

Table 12

The Behavioural Equations Estimation

Investment Equation

Variables	Coefficient	t-Statistic	Prob.
IGDP			
C	0.826596	12.01552	0.0000
AGR	0.194942	3.334432	0.0028
KGDP(-1)	-0.182716	-10.13729	0.0000
LTR(-1)	-0.094957	-2.025147	0.0541
EER	-0.000236	-1.552897	0.1335
Durbin-Watson Stat	0.948946	R-squared	0.645686

Consumption Equation

Variables	Coefficient	t-Statistic	Prob.
CGDP			
C	0.420668	11.29056	0.0000
KGDP	0.100446	8.592694	0.0000
DGDP(-1)	-0.118596	-4.182296	0.0003
STR(-1)	-0.199468	-4.559253	0.0001
LTN(-1)	0.435248	9.882231	0.0000
Durbin-Watson Stat	1.457363	R-squared	0.939240

Trade Balance Equation

Variables	Coefficient	t-Statistic	Prob.
TBGDP			
EER	-0.000733	-5.625103	0.0000
CGDP	-0.645395	-4.860770	0.0000
CGDPW	0.733038	5.587078	0.0000
Durbin-Watson Stat	1.525254	R-squared	0.685639

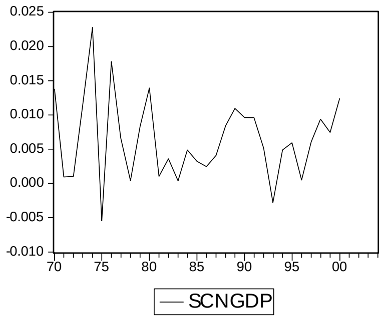
Finally, previous to determining the medium run equilibrium, it is still necessary to specify the extended and filtered SCNGDP series.

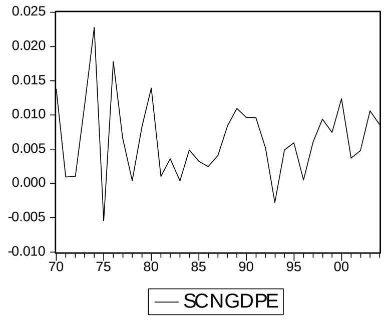
Table 13

SCNGDP Expansion

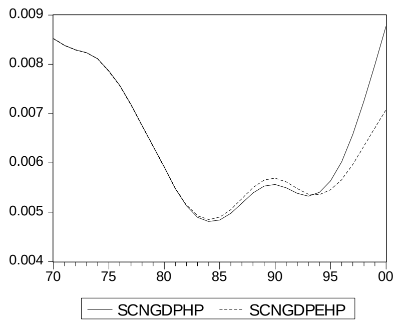
Variables	Coefficient	t-Statistic	Prob.
SCNGDP			
C	0.006543	5.014911	0.0000
AR(1)	0.381539	2.070517	0.0498
AR(2)	-0.493080	-2.656939	0.0141
MA(1)	-0.591637	-9.981078	0.0000
MA(2)	0.800227	11.12838	0.0000
MA(3)	0.216656	3.005766	0.0063
Durbin-Watson Stat	2.153080	R-squared	0.308895

So these are the non-extended (SCNGDP) versus the extended (SCNGDPE) series.

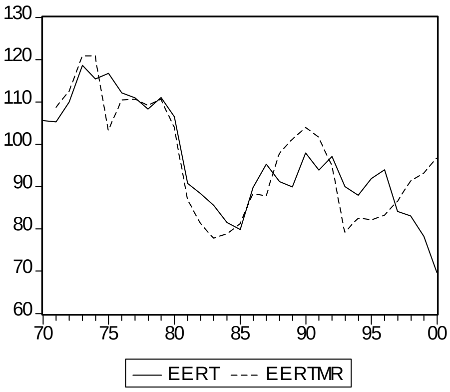


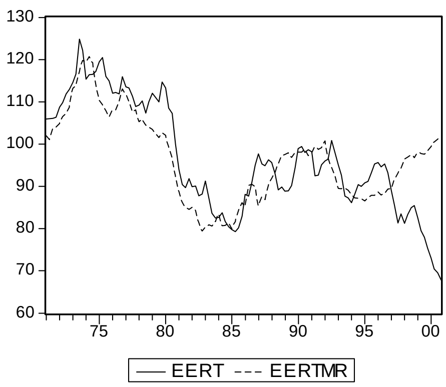


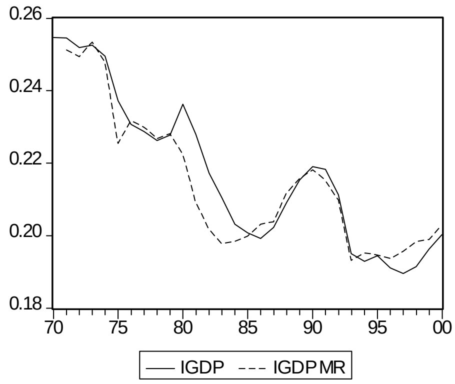
The difference after applying the HP filter is therefore evident.

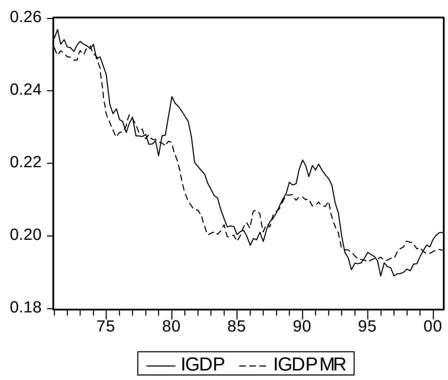


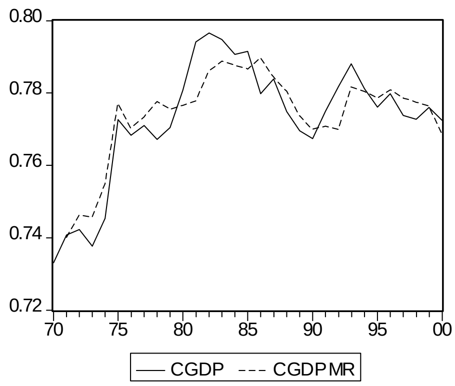
It is finally determined the medium run equilibrium under the same assumptions that in the case of quarterly data. These are the graphs comparing the estimated series of annual frequency versus the previously determined for quarterly frequency.

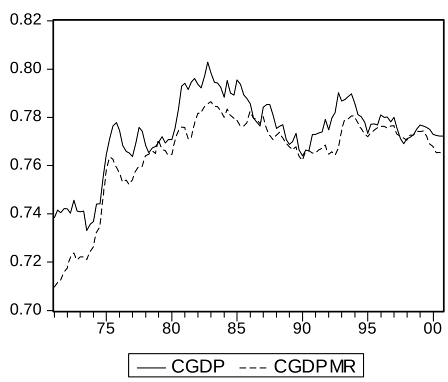


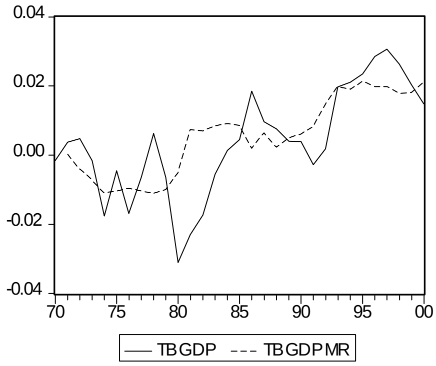


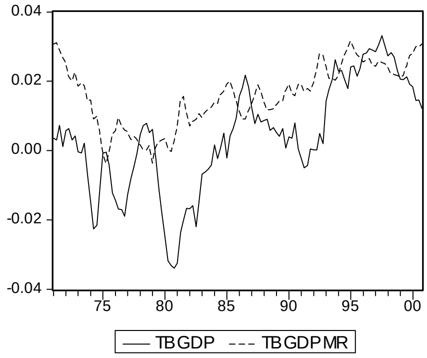












Appendix 3.6: Characterisation of the Foreign Sector

Next table shows the unit root tests of the following variables: I*/Q*+ SCN*/Q* (ISGDPW), $\hat{Q}_r^*$ (YERWGR), r^{L*} (LTRQW), SI*/Q* (SIGDP), r^{S*} (STRQW), NF*/Q* (NFGDPW)

Table 14

Augmented Dickey-Fuller Unit Root Test

$$\Delta x_t = \alpha + \beta t + \nu x_{t-1} + \text{lags}(\Delta x_t)$$

Foreign Investment Equation

Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)	ADF Statistic	(α, β, lags)	ADF Statistic	
ISGDPW	(0, 0, 23)	-0.49	(0, 0, 24)	-2.5	I(1) without Drift
YERWGR	(0, 0, 12)	-0.69	(0, 0, 15)	-5.15	I(1) without Drift
LTRQW	(0, 0, 5)	-0.67	(0, 0, 14)	-3.86	I(1) without Drift

Foreign Saving Equation

Variable	One Unit Root		Two Unit Roots		
	(α, β, lags)	ADF Statistic	(α, β, lags)	ADF Statistic	
SIGDPW	(0, 0, 1)	-0.53	(0, 0, 5)	-5.32	I(1) without Drift
STRQW	(0, 0, 23)	-0.94	(0, 0, 22)	-2.20	I(1) without Drift
NFGDPW	(0, 0, 8)	-1.05	(0, 0, 6)	-4.55	I(1) without Drift

Table 15

Johansen Cointegration Test

	IS*/Q* Equation	SI*/Q* Equation
VEC Specification*	(C, 1 to 3)	(C,2 to 5)
Estimation Period	1972.1 – 1999.4	1976.3 – 1999.4
Cointegrating Equation	No trend in Data Intercept in CE	No trend in Data Intercept in CE
LR Test	1 Coint Eq at 1%	1 Coint Eq at 5%

(*) C = Constant in VAR; integers are lags included

Table 16

Foreign Variables Cointegrating Equations

Foreign Investment Cointegrating Equation
(t-statistics in parentheses)

ISGDPW	YERWGR	LTRQ	C
1.000000	-1.811482 (-6.26316)	0.618724 (2.12917)	-0.170073 (-18.5368)
Error Correction Term	D(ISGDPW)	D(YERWGR)	D(LTRQW)
	-0.119970 (-5.83112)	0.189998 (3.99169)	0.013696 (1.29127)

Foreign Saving Cointegrating Equation
(t-statistics in parentheses)

SIGDP	STRQW	NFGDPW	C
1.000000	-1.331292 (-2.66526)	0.896886 (4.13691)	-0.202014 (-39.1486)
Error Correction Term	D(CGDP)	D(KGDP)	D(FGDP)
	-0.097338 (-2.29150)	0.074469 (3.94356)	-0.001567 (-0.04865)

Appendix 3.7: Long-Run versus Short-Run Real Interest Rates Relationship

Table 17

Johansen Cointegration Test

	National Equation	Foreign Equation
VEC Specification*	(C, 1 to 13)	(C, 1 to 5 and 7)
Estimation Period	1974.3 – 1999.4	1977.1 – 1999.4
Cointegrating Equation	No trend in Data Intercept in CE	No trend in Data Intercept in CE
LR Test	1 Coint Eq at 5%	1 Coint Eq at 1%

(*) C = Constant in VAR; integers are lags included

Table 18

National Interest Rate Equation
(t-statistics in parentheses)

National Equation			Foreign Equation		
LTRQ	STRQ	C	LTRQW	STRQW	C
1.000000	-0.930219 (-13.1383)	-0.002318 (-3.31860)	1.000000	-0.723060 (-6.30520)	-0.003751 (-3.46707)
Error Correction Term	D(LTRQ)	D(STRQ)	Error Correction Term	D(LTRQW)	D(STRQW)
	-0.164138 (-2.16551)	0.044655 (0.50590)		-0.189203 (-2.86440)	-0.019487 (-0.21121)

Appendix 3.8: NFGDP Determination

Next, we will try to estimate an equation for the endogenous NFGDP variable. We assume that it will depend basically on the following two variables:

- Positively on the foreign net transfers (TRGDP).
- Negatively on the net foreign debt accumulation (FGDP).

$$NFGDPF = TRGDP + C(1)*FGDP; \qquad C(1)<0$$

But, given that we cannot access to the TRGDP variable, this will be determined as a residual of the following equation,

$$NFGDP = C(1)*FGDP + Epsilon$$

Given the non-stationarity of FGDP and NFGDP, the estimation is performed in first differences. In particular, as in the case of productivity, we apply the Hendry methodology.

Table 19

NFGDP Equation

Variables	Coefficient	t-Statistic	Prob.
NFGDPD			
NFGDPD(-1)	-0.410584	-4.577982	0.0000
NFGDPD(-2)	-0.213535	-2.535824	0.0127
NFGDPD(-6)	0.142839	1.852923	0.0667
NFGDPD(-8)	0.198111	2.629746	0.0098
FGDPD	-0.124114	-2.047575	0.0431
FGDPD(-1)	0.209653	2.839189	0.0054
FGDPD(-3)	-0.177791	-3.896177	0.0002
FGDPD(-9)	0.058730	2.488478	0.0144
R-squared	0.308895		

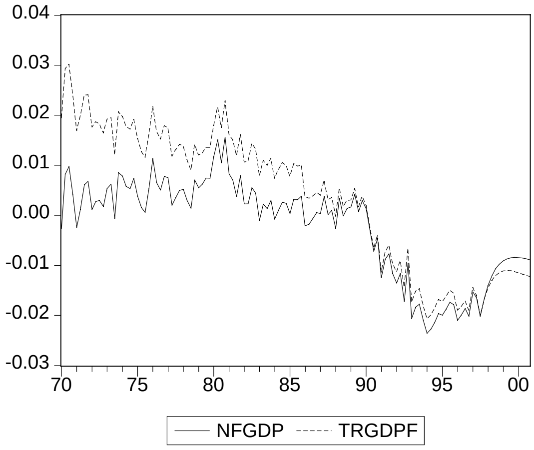
(*) D at the end of the word means first differentials

So this is the long run estimated equation,

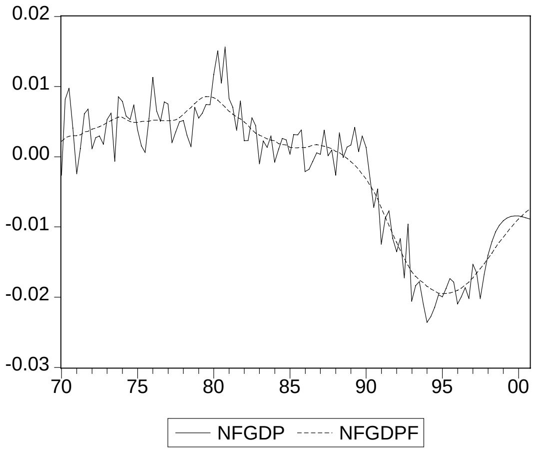
NFGDPD = -0.0261250047889*FGDPD

And TRGDP is determined as follows,

TRGDPF = NFGDP + 0.0261250047889*FGDP



Finally, this is the estimated NFGDP variable



FINAL CONSIDERATIONS

The main goal of this PhD memory is the analysis of the real exchange rate behaviour in a medium and long run horizon. With this objective we have developed three different research papers on the topic that, presented in different chapters, shed new light on the literature about real exchange rate determination.

In particular, chapter one analyses how the classical explanation to the empirical relationship between the economic growth of a country and its long run real exchange rate can be enriched through the consideration of a more general balanced growth intertemporal equilibrium model. In this chapter, we consider a simple exogenous growth model where it is imposed the internal, external and intertemporal equilibrium conditions of a typical macroeconomic model; this last one through the inclusion of a balanced growth path for the foreign assets accumulation. The main result under this consideration is that the relationship defended by the Balassa-Samuelson hypothesis is no more so straightforward. In our particular approach, the mentioned bilateral relationship depends on a parameter measuring thriftiness in the economy. Therefore, the probability to end up with a positive relationship between growth and real exchange rates –as the classical economic theory predicts– will be higher when the economy is able to maintain a minimum saving ratio. Moreover, given that our model considers a simple Keynesian consumption function, some explosive paths can be possible.

Chapter two gives a step forward and, following a general equilibrium macroeconomic approach, sets a closed micro-founded structural model to determine the long run real exchange rate of a developed economy. In particular, the analysis follows the structure of a Natrex model. The main contribution of this second chapter is the development of a solid theoretical framework that analyse in depth the basis of the real exchange rate and the details of the equilibrium dynamics after any shock influencing the steady state. In our case, the intertemporal factors derived from the stock-flow relationship will be particularly determinant. The main results of the chapter can be summarised as follows. In first place, a complete well-integrated structural model for long-run real exchange rate determination is developed from first principles. Moreover, within the concrete dynamics of the model, it is found that some convergence restrictions will be necessary. On the one hand, for the medium run convergence the sensitivity of the trade balance to changes in the real exchange rate should be higher than the correspondent one to the investment decisions. On the other hand, and regarding long-run convergence, it is also necessary both that there exists a negative relationship between investment and capital stock accumulation and that the global saving of the economy depends positively on the net foreign debt accumulation. In addition, there are also interesting conclusions about the effects that certain shocks over the exogenous variables of the model have on real exchange rates in a medium and long-run horizons.

Finally, chapter three uses the previous theoretical model to check its performance in the particular case of the euro. Its contribution is two-fold. First of all, the Natrex model is estimated in its true structural form. So far the Natrex models had only been estimated in reduced forms or semi-reduced forms. Secondly, the model is applied to the effective euro exchange rate -period going from 1970 to 2000- using quarterly observations from the database of the ECB's area wide model (AWM). We thus contribute to the growing literature on the euro's fundamental value by using one of the more comprehensive databases for pre-Stage III euro area data available so far. According to our structural model we can conclude about the main periods of over and undervaluation of the euro, being particularly interesting the significant undervaluation obtained at the end of the period under analysis.

Notwithstanding, the research developed in this memory should not be understood as a closed work. Rather on the contrary, each particular chapter opens different alternatives to interesting future research.

In particular, a main extension of chapter one is the empirical corroboration of its theoretical predictions by means of an appropriate panel of developing and developed economies. Regarding chapter two, we find interesting a deeper analysis of the medium and long run effects that other exogenous variables different than thriftiness and productivity will have over the medium and long run real exchange rate behaviour. An additional interesting possibility is a detailed analysis of the critical condition that after a shock on productivity would explain the long run trajectory of the real exchange rate equilibrium. Finally, chapter tree can be enriched in many directions. First of all, it should be taken into account that the set of tests applied at the different stages of the empirical analysis has always followed the most basic design. We also understand that a main step forward is to set aside the linear estimating approach to undertake the possibility of non linearities in the cointegrating relationships of the model.