Horizontal Mergers and Equilibria Comparison in Oligopoly

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Contents

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Chapter 1

Introduction

The last two decades of twentieth century brought massive changes to the world economy. The leap in information technologies broadened to a great deal the organizational possibilities of firms, the deregulation and privatisation of many state managed sectors in developed markets generated new horizons for the firms, globalization and merger waves contributed to creating larger ever firms. In consequence, nowadays, anti-trust policies and the economic analysis that they are drawn from are crucial for the economy. Oligopoly theory to which the material eloborated in this thesis contributes, has become a field that receives much attention.

This thesis consists of four pieces of independent work. In broad terms two general themes that it addresses can be classified as horizontal mergers and comparison of equilibrium market parameters in Cournot and Bertrand competition. The analyses are theoretical, and they employ non-cooperative game theory tools. Supermodular game theory is used in one of the articles partly to get more general results and partly to make the formal mathematical derivations concise.

The concept of equilibrium used in the analyses is Nash equilibrium. Under this notion, firms in an oligopoly realize that their actions alter market outcomes and thus they behave strategically. At equilibrium, each firm is doing what is in his best interest given what other firms are doing. The strategic variable chosen by firms determines the mode of competition. Two most widely used competition modes are the Cournot one where firms choose quantities, and the Bertrand one where firms choose prices. Whether quantity or price is the appropriate choice variable has been much debated. In practice, firms seem to make both types of decisions. The relevant oligopoly model for a particular industry depends on the structural features existent in that industry. Depending on these, price or the quantity may emerge as the dominant variable. It is important to acknowledge that the theoretical economic models presented here contain certain hypothesis of industry behavior. If the goal is to draw conclusions or policy implications for a given industry, then one should either tailor the theoretical models according to market specifics or, when this is not possible, try to go for the setting that provides the correct intuition about the functioning of the market in question.

In oligopolistic markets, a merger may harm competition by eliminating competitive concern for some sellers who consequently would be able to increase their prices. The evaluation of a specific merger by an anti-trust agency depends on the relevant policy goals. In many occasions, these are related to consumer welfare or efficiency, however, other targets such as protection of small or medium sized enterprise, or achievement of single-market integration may be pursued by agencies. A merger raises concern due to two main categories of effects that it may lead to. The unilateral effects are the ones that result when firms compete before and after the merger according to some non-cooperative behavior. While, coordinated effects of a merger are related to the idea that it might be easier for firms to cooperate in the postmerger environment. Therefore, a merger might change the industry behavior from "competition" to "cooperation".

The first part of this thesis deals with mergers. In chapters two and three the unilateral effects of mergers under different market scenarios are considered. The approach is taken mainly from the firms perspective and most of the results presented concern merger profitability. Many theoretical models raise doubts about mergers being a fruitful business practice. The models presented here point out to some settings where mergers are benefical for firms.

Chapter two deals with mergers in a homogenous goods industry with supply function competition. Firms choose functions that determine the quantity that they are willing to supply for each possible equilibrium price. The canonical models of Cournot and Bertrand competition are the extreme cases of this broader competition mode: in the former a firm commits to supply the same quantity for any price, and in the latter a firm commits to supply any quantity at a single price. In terms of supply functions they mean horizontal and vertical schedules in the (price, quantity) space. Supply function competition, in effect, identifies if prices or quantities tend to be the dominant variable in a specific industry. Steep marginal costs make the outcome of the supply function competition resemble that of Cournot competition and flat marginal costs lead to outcomes that are closer to that of Bertrand competition.

Under quadratic cost functions for firms, it is shown that any merger leads to a reduction in total output of the industry. Contrary to what would happen in Cournot competition where only merging firms reduce supply and the remaining firms expand after a merger, in the present model all firms reduce their supply. The fact that non-participating firms decrease their supply in the postmerger equilibrium guarantees the profitability of mergers. The main driver of this result is the strategic complementarity of the supply function slopes. Strategic complementarity means that a firm finds it in his interest to acommodate the action of its rival. Here, it implies that a firm finds it more profitable to increase (decrease) his supply when its rival increases (decreases) his own. Thus, when merging firms decrease their supply after a merger the rivals also reduce theirs. This further benefits the merging parties. The reverse happens in quantity competition which is characterized by strategic substitutability. There, a firm finds it more profitable to increase his output when its rival decreases his. This being so, the merging firms can not internalize full effect of their quantity reduction. Their rivals increase output, and the market price does not increase as much, harming the profitability of the merger.

Apart from reducing outputs, a merger affects the market shares of the firms as well. In the quadratic cost setting this raises the possibility that mergers contribute to welfare by decreasing total costs. If firms have asymmetric cost functions, at equilibrium, the total output is not produced efficiently. In other words the marginal costs of firms are not equalized at equilibrium. A merger may increase the share of low marginal cost firms at equilibrium and thereby decrease the cost of total industry production.[1] If a merger increases sufficiently the symmetry of the cost structure in the industry it leads to more efficient production and contributes to welfare. Obviously if a merger reduces the symmetry in an industry it results in a welfare decrease.

Chapter three deals with mergers in a differentiated product setting under uncertainty. In many markets, firms face some sort of uncertainty. Fast pace of technological advance in hi-tech industries gives rise to dubious industry conditions, as often it is unpredictable which of the alternative production processes will turn out to be more efficient, or which product line will be the survivor in a rapidly changing environment. The speed and the extent of information diffusion has made consumers more responsive to shocks that occur elsewhere. Often the varying conditions in a market are observed by all firms active in that industry. For example, when SARS epidemic breaks out, all airlines know that travel to East Asia will fall. It is not necessary for an airline to be operating a route to/from there to sense the variation in demand conditions. Other times the variation is only privately observable. For intance, improvements or failures in a new production process are only observed by the firms that switch to the new technology.

In this work, uncertainty is modeled to be of the first type. Risk-neutral firms evaluate a merger decision when they expect an idiosyncratic shock to the market

[1] This is different than the output rationalization that will take place inside the merged firm. It will occur even if the merging firms have the same cost structure, if this structure is less efficient than the rivals' production technologies.

demand. However, they know that, once uncertainty is resolved all firms will learn the market conditions correctly. The analysis has two parts, first to compare the incentives to merge under uncertainty with the incentives in a benchmark deterministic market, and second to observe the preference of a firm over its possible partners when idiosyncratic random parameters are correlated. Under Bertrand competition, uncertainty always increases the incentives to merge. Under Cournot competition, it increases the incentives when they exist in the deterministic market, however, if a merger leads to a loss in the deterministic market, uncertainty may increase or decrease these losses. In price competition, a firm prefers a partner that has positively correlated random term with its own, while in quantity competition, it values more a partner that has negatively correlated random term with its own. The difference between the strategic natures of these competition games leads to this divergence in preferences.

The second part of the thesis provides some comparisons between Bertrand and Cournot equilibria. It points out, under particular settings, what would be the difference if an industry were characterized by price or quantity competition. Conventional wisdom in economics suggest that prices are lower in Bertrand competition than in Cournot competition. A particularly clear example of this phenomenon is the homogenous good common constant marginal cost setting where price is equal to marginal cost in price competition, but the equilibrium mark-up is positive under quantity competition. Although the intuition is valid in differentiated product setting as well, there are cases when it does not apply. Two particular instances of this type, dynamic competition and mixed differentiated product setting are reconsidered here.

Chapter four, a joint work with Ioana Chioveanu, is devoted to the analysis of research and development (R&D) investments in a duopoly model. Innovative activity has become quite important for firms in a wide range of industries. The origins of most modern microeconomic analyses related to innovation and technological change can be traced back to the ideas of Joseph A. Schumpeter. He described the process of creative destruction and the dynamics of innovation as the main drivers of competitive process:

"What we have got to accept is that innovation undertaken by large and dominant firms has come to be the most powerful engine of economic progress. In this respect perfect competition is not only impossible but inferior, and has no title to being set up as a model of ideal efficiency."

The fact that innovation is more likely in monopolistic rather than competitive markets, places it as a central theme in oligopoly theory. Economists classify innovation into two broad categories. Product innovation aspires to creating new goods or to improving the quality of existing products. Process innovation aims at increasing production efficiency and is typically modeled as an investment in cost reduction. The present work concentrates on process R&D, though, in part, it conveys a message that some of its results go beyond this setting. There are two main trends in modeling R&D competition. In tournament models, firms compete in a patent race, and R&D investment by a firm increases its probability of winning the race. The race is for a given innovation and competitors do not have control over innovation level. The winner of the race receives some degree of market power due to the innovation, which, in many cases, is represented as a fixed monetary prize. The alternative path is to model innovation as deterministic. In this case, the level of innovation is chosen by the firm through R&D investment. Thus, post innovation market results are endogenous to the model. Chapter four, which compares the dynamic equilibria of two competition modes when innovation is possible prior to market competition, deals with deterministic innovation.

A differentiated duopoly market with substitute goods is considered. Only one firm can reduce marginal cost of production before product market competition takes place. The demand originates from a representative consumer, R&D technology has decreasing returns to scale and innovation outcome spills imperfectly to the rival. Comparisons between the equilibrium results when firms compete in prices at the market stage and when they compete in quantities constitute the principal results of the paper. Previous literature has shown that, typically, R&D investment is higher under Cournot competition. This is mainly due to the difference in the strategic effects. A price-setting firm realizes that it will decrease its price after a cost decrease and this will induce price cuts by its rivals, which are detrimental to the profitability of the firm. A quantity-setter infers that a cost-reduction will lead him to expand output and this will induce output reductions by its rivals, which are beneficial to the firm. The ordinary result in terms of market quantities is that, although under Cournot competition firms reach to lower cost levels through higher investment, output levels remain higher under Bertrand competition. In this work, it is shown that, with high substitutability and low innovation costs: a) R&D investment can be higher under Bertrand competition, if spillovers are low, and b) output, consumer surplus and total welfare can be larger under Cournot, if spillovers are high. A new result is that, with process innovation, both consumers and producers can be better off under quantity competition. An extension concludes that the above novel ranking of the innovation level is robust to the consideration of product instead of process R&D. All the results can be encountered under both types of innovation as our remaining results for process innovation are present in the product innovation literature.

The last chapter is about a differentiated product setting where both complement and substitute goods are present on the market. Previous research in this setting, has found examples where Bertrand prices are higher for some products than the Cournot ones. This paper proposes a notion of "symmetry" for demand in such a market is developed. Whenever a mixed products market fulfills this condition prices of all products are lower under Bertrand competition.

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Chapter 2

Mergers with supply functions

(Another version of this chapter is forthcoming in the Journal of Industrial Economics under the same title.)

2.1 Introduction

Intuitively, it seems that a merger should not be harmful to the merging parties as their premerger actions can always be replicated by the new firm. However, the literature on homogenous product markets suggests that, in many cases, this intuition does not hold at equilibrium. In this paper, I analyze the effects of mergers in homogenous good industries *where firms compete by submitting supply functions.* I find that mergers always raise the joint profits of the participants.

Firms choose supply functions that relate their quantity to the market price. This generalizes the traditional models of price or quantity competition which exogenously impose vertical and horizontal supply schedules.[1] More importantly, supply function competition identifies when each of these special cases is more likely to arise. Klemperer and Meyer [1989] demonstrate that, with steep marginal costs, supply function competition resembles the Cournot model while, with flatter marginal costs,

[1] A vertical schedule in the (price, quantity) space corresponds to providing any quantity at a certain price.

it is closer to the Bertrand model.[2] The flexibility, obtained from choosing a relation between its quantity and the price rather than a single variable, is valuable for a firm that faces changing conditions. Thus, it is plausible that the first liberalized wholesale electricity market, the British Pool, was designed to let the generators submit supply schedules to adopt to changing demand conditions during the day.[3] The airlines, who also operate in a volatile market are known to use electronic reservation programs which offer different numbers of seats at different price levels on each flight. Arguably, choosing a particular program can be thought of as choosing a supply function.

In their paper which initiated the analysis of horizontal mergers, Salant, Switzer and Reynolds [1983] use a constant average cost Cournot model to conclude that mergers are generally unprofitable in homogenous good industries.[4] Under constant average costs the merged firm does not differ from the others. Perry and Porter [1985] consider Cournot competition and a cost function that depends on the assets, and thereby the size of a firm. They find that profitable mergers are possible when the merged firm is allowed to be twice as "large" as each partner in a two-firm merger. However, cases in which mergers lead to losses still exist. McAfee and Williams [1992] allow for different initial sizes for Cournot oligopolists, but use the particular case of an asset dependent cost function to characterize the welfare enhancing profitable mergers. Their analysis reveals that the profitability of the merger is only guaranteed if the total output share of the merging firms is large enough. By contrast, all mergers are profitable when firms compete by supply schedules under the same cost and demand structure.

I show that the linear equilibrium of the supply function game is the equilibrium

[2] A model considered by Vives [1986], where firms choose optimal scale first and then a competitive stage follows, produces qualitatively similar equilibrium results with respect to the flexibility of the production technology.

[3] Today, such liberalized electricity markets are in operation in many countries, although with differing rules. A few examples of the literature concentrating on the initial British case are Green and Newbery [1992], Green [1996, 1999].

[4] In their setting a merger is not beneficial unless it includes 80% of the firms in the industry.

of a game in which firms choose the slope of their linear supply functions. In the restricted game, the best responses are increasing, that is the slopes chosen are strategic complements. If a firm decreases its supply by choosing a lower slope its rivals do the same. Thus, when the merged entity reduces supply it benefits from the reaction of its rivals. This makes any merger profitable. On the contrary, the quantities are strategic substitutes in a Cournot game. If one firm decreases its supply its rivals react by expanding their output. Thus, a merged entity can not internalize all the benefits of its own output cut. The possibility of losses from a merger in Cournot competition originates from this fact.

In Section 2.2, the cost and demand structure is defined and competition in supply schedules is introduced. In Section 2.3, the equilibrium in linear supply functions is formulated. The equilibrium effects of mergers are analyzed in Section 2.4. Finally, Section 2.5 concludes.

2.2 The Model

There are n firms competing in an industry with a fixed total capital stock K^T. Following McAfee and Williams, I assume that industry demand is linear with $D(p) = \theta - bp$, and firm $i's$ cost of producing a quantity q_i is given by $C_i(q_i) = q_i^2/2K_i$ where K_i denotes its capital stock. Marginal cost, given by $c_i(q_i) = q_i/K_i$, is linear and decreases in the capital stock. An industry is symmetric when all firms have the same capital stock, and therefore the same costs.

A strategy for firm i is a supply function $S_i(p)$. The solution concept is Nash equilibrium. If there exists a unique market clearing price p^* such that $D(p^*) = \Sigma_i S_i(p^*)$, then each firm produces the output given by its supply function at p^* and receives the corresponding profits. Following Klemperer and Meyer [1989], I assume that, if there is no such price, or there is more than one solution, no production takes place and each firm gets zero profits.

In general, the model possesses multiple equilibria. Given the other firms' supply functions, a firm faces a certain residual demand and has a unique profit maximizing price-quantity pair. It can achieve this outcome with infinitely many different supply schedules, since it has a lot of freedom in constructing the supply schedule at prices that are not realized in equilibrium. However, as shown by Klemperer and Meyer, uncertain demand can reduce the multiplicity. The residual demand of the firm is no longer fixed, and as a result, the firm has different profit maximizing price-quantity pairs for each realization of the demand.[5] Thus, an optimal supply function for the firm should implement the profit maximizing pair for each possible realization of demand. As the support of the uncertainty increases, more and more functions cease to be ex-post optimal, causing the equilibrium set to shrink. In particular, when the support of the uncertainty is unbounded, Klemperer and Meyer have shown that, for a symmetric industry, the equilibrium is unique and the equilibrium supply function are linear. Laussel [1992] extended this result to asymmetric duopolies. It is worth noting that these results do not require any assumptions on the distribution of uncertainty. As long as the support is unbounded, even if the distribution degenerates into a mass point, the unique equilibrium consists of linear supply functions. This result motivates my restriction on the strategy space of each firm to linear supply functions.[6] In what follows, I will refer to an equilibrium in the space of linear supply functions as a linear equilibrium.

2.3 Equilibrium in Linear Supply Functions

In this section I establish that the linear equilibrium is unique and characterize its properties. The following lemma will prove useful in establishing uniqueness.

[5] A similar logic applies to the case of the firms in some liberalized electricity markets as they are allowed to submit only one supply schedule for a day although demand fluctuates during a day. Even if firms are allowed to submit different schedules for different hours they might abstain from variations to abuse their market power, in fear of the regulator.

[6] Klemperer and Meyer [1989].

Lemma 1. *Consider a game in which A_i is a compact interval of the non-negative reals and the best replies are continuous increasing functions of the type $\Psi_i(a_{-i})$ for all i where $a_{-i} = \Sigma_{j \neq i} a_j$. If $\Psi_i(a_{-i})/a_{-i}$ is strictly decreasing in a_{-i} for all $a_{-i} \neq 0$, then the equilibrium is unique provided that $\Psi_i(0) > 0$.*

Proof. Provided in the appendix. □

Proposition 1. *There exists a unique linear equilibrium in which $S_i(p) = \beta_i^* p$, and where $(\beta_1^*, .., \beta_n^*)$ solves the n-equation system given by*

$$\beta_i^* = \frac{K_i(b + \Sigma_{j \neq i}\beta_j^*)}{K_i + b + \Sigma_{j \neq i}\beta_j^*}.$$

Proof. For any firm i, if all other firms use linear supply functions, that is $S_j(p) = \beta_j p$ for all $j \neq i$, then the residual demand that i faces at any p is given by

$$\tilde{D}_i(p, \theta) = \theta - (b + \Sigma_{j \neq i}\beta_j)p.$$

Firm i will maximize its profits by choosing a point on its residual demand

$$\max_p [\theta - (b + \Sigma_{j \neq i}\beta_j)p]p - \frac{1}{2K_i}[\theta - (b + \Sigma_{j \neq i}\beta_j)p]^2.$$

Differentiating, the optimal price solves

$$\theta - (b + \Sigma_{j \neq i}\beta_j)p - (b + \Sigma_{j \neq i}\beta_j)p + \tfrac{1}{K_i}(b + \Sigma_{j \neq i}\beta_j)[\theta - (b + \Sigma_{j \neq i}\beta_j)p] = 0.$$

The second order conditions are met when $\Sigma_{j \neq i}\beta_j \geq 0$. The firm should choose its supply such that $S_i(p)$ is equal to its residual demand at the optimal price. It follows that

$$S_i(p)[K_i + b + \Sigma_{j \neq i}\beta_j] = K_i(b + \Sigma_{j \neq i}\beta_j)p,$$

and, solving for supply,

$$S_i(p) = \frac{K_i(b + \Sigma_{j \neq i}\beta_j)}{K_i + b + \Sigma_{j \neq i}\beta_j}p.$$

There is a one-to-one correspondence between the set of equilibria in the game where firms are restricted to submitting linear supply functions and the set of equilibria of the game in which each firm i chooses β_i from the set $[0, \bar{K}]$ where $\bar{K}$ is the capital stock of the largest firm, and the payoffs are defined as above with $S_i(p) = \beta_i p$.[7] The game is log-supermodular, that is, the payoffs satisfy $\partial (\log \pi_i) / \partial\beta_i \partial\beta_j \geq 0$ for $i \neq j$ and the slopes chosen by firms are strategic complements. Moreover, the strategy spaces are compact intervals. The set of equilibria is non-empty by supermodular theory.[8]

Let $\beta_{-i} = \Sigma_{j \neq i} \beta_j$. Then the best-response of firm i is given by

$$\Psi_i \left(\beta_{-i}\right) = \frac{K_i(b + \beta_{-i})}{K_i + b + \beta_{-i}}$$

and $\Psi_i \left(\beta_{-i}\right) / \beta_{-i}$ is strictly decreasing in β_{-i}. By Lemma 1, the equilibrium is unique and

$$\beta_i^* = \frac{K_i(b + \Sigma_{j \neq i} \beta_j^*)}{K_i + b + \Sigma_{j \neq i} \beta_j^*}$$

for $i \neq j, i = 1, .., n$. □

The unique linear equilibrium has several intuitively appealing properties. In particular, firms with the same stock of capital supply the same amount of output, and a firm with a larger capital stock supplies more output than a firm with a smaller capital stock.

Corollary 1. *At the unique linear equilibrium:*
(a) $K_i = K_j$ implies $\beta_i^* = \beta_j^*$ for any two firms i and j.
(b) $K_l > K_s$ implies $\beta_l^* > \beta_s^*$ for any two firms l and s.

Proof. In the restricted game $\Psi_i \left(\beta_{-i}\right) = \Psi \left(K_i, \beta_{-i}\right)$ for all i with $\Psi_K (\cdot, \cdot) > 0$ for all values. The result follows directly from Lemma 2 in the appendix. □

[7] The payoff to firm i in this game is: $\pi_i(\beta) = \left(\frac{1}{\beta_i} - \frac{1}{2K_i}\right)\left(\frac{\theta\beta_i}{b + \beta_i + \sum_{j \neq i} \beta_j}\right)^2$.
[8] A detailed discussion on this issue is in Vives [1999, section 2.2].

When K^T is distributed asymmetrically, the system of equations defining the equilibrium does not permit a closed-form solution except for the case of a duopoly. The duopoly solution is provided in the appendix. In the case of an oligopoly, let $B = b + \beta^T$ where $\beta^T = \Sigma_{i=1}^n \beta_i^*$. Then the system can be rewritten as

$$(\beta_i^*)^2 - (2K_i + B)\,\beta_i^* + K_i B = 0.$$

Its proper root satisfying $\beta_i^* \leq K_i$ is given by

$$\beta_i^* = \frac{1}{2}\left[2K_i + B - \sqrt{4K_i^2 + B^2}\right].$$

Summing over all firms, B is defined implicitly by

$$B = b + K^T + \frac{1}{2}\left[nB - \Sigma_{i=1}^n \sqrt{4K_i^2 + B^2}\right].$$

Letting $k = K^T/n$, the solution in the symmetric case is

$$\beta_i^* = \frac{1}{2\,(n-1)}\left[k(n-2) - b + \sqrt{\left[k(n-2) - b\right]^2 + 4(n-1)kb}\right].$$

It is interesting to compare the firm's behavior in the above equilibrium to the competitive benchmark. If firm i behaves competitively, its supply function is the inverse of its marginal cost function, that is, $S_i(p) = K_i p$. By contrast, in the Nash equilibrium, the slope of firm i's supply schedule, β_i^*, is equal to $\delta_i K_i$ where

$$\delta_i = \frac{(b + \Sigma_{j \neq i} \beta_j^*)}{K_i + b + \Sigma_{j \neq i} \beta_j^*} < 1.$$

Thus, in submitting its supply schedule, firm i in effect claims to have a fraction, δ_i, of its true capacity. I will refer to δ_i as firm i's abatement factor. Firms with more capital have lower abatement factors than firms with less capital since the former have relatively more to gain from the withdrawal of capacity.[9]

The market equilibrium yields $p = \theta / \left(b + \beta^T\right)$ and $Q = \theta\beta^T / \left(b + \beta^T\right)$. Since total quantity increases and price falls in β^T, higher values of β^T are better for consumers. The value of β^T depends on the distribution of capital. Note that β^T / K^T

[9] Corollary 1 implies that $K_i > K_j \Rightarrow \delta_i < \delta_j$.

is a measure of industry competitiveness as it is the ratio of aggregate supply to the supply that would obtain if firms behaved competitively. It follows that the competitiveness of the industry is just the capital weighted sum of the firms' abatement factors. Since higher capital firms have more weight and lower abatement factors, the symmetric industry is the most competitive industry.

Proposition 2. *For any n-firm industry with a total capital stock K^T, the slope of the equilibrium aggregate supply, β^T, is maximized when the industry is symmetric.*

Proof. The best responses in the restricted game satisfy the condition that

$$\frac{1}{2}\left[\Psi\left(K_i, \beta_{-i}\right) + \Psi\left(K_j, \beta_{-j}\right)\right] < \Psi\left(\frac{1}{2}\left[K_i + K_j\right], \frac{1}{2}\left[\beta_{-i} + \beta_{-j}\right]\right)$$

whenever $K_i < K_j$ and $\beta_{-i} > \beta_{-j}$. Lemma 3 in the appendix gives the result. □

Welfare is given by

$$W = \frac{\theta^2}{2b}\left[1 - \left(\frac{b}{b+\beta^T}\right)^2\right] - \frac{\theta^2}{2}\sum_{i=1}^{n}\frac{1}{K_i}\left(\frac{\beta_i^*}{b+\beta^T}\right)^2.$$

The symmetric capital distribution yields the lowest prices and highest output so it is the best distribution for consumers. Furthermore, output is produced efficiently only in symmetric industries. The efficient production plan requires that each firm i produce a share K_i/K^T of the total output. This can only be obtained at equilibrium when the abatement factors are the same for all firms, which in its turn requires K^T to be distributed equally among the n firms. Thus, in a symmetric industry, competition is toughest and production is efficient which leads to the following result.

Corollary 2. *For any n-firm industry with a total capital level K^T, total welfare is maximized when the industry is symmetric.*

A similar result is obtained by McAfee and Williams. However, the Cournot model with constant marginal costs gives the opposite result. Salant and Schafer

[1999] demonstrate that welfare is higher in asymmetric industries holding constant the sum of marginal costs. In such a case, the level of competition is invariant to a reallocation of the marginal costs between firms, since the total output is only dependent on their sum. However, the asymmetric distribution can lead to more efficient production as lower cost firms produce higher shares of the fixed output.

Consider next symmetric industries with differing number of firms but where the total capital K^T is fixed. The cost of a given output level is the same, regardless of the number of firms. Therefore, welfare comparisons are determined solely by the amount of competition, and naturally, competition amongst equals increases with their number.

Corollary 3. *For any two symmetric industries with the same amount of total capital K^T, welfare is higher in the industry with higher number of firms.*

2.4 Mergers

In the quantity-setting version of the model, McAfee and Williams show that, the merged entity produces less than the total pre-merger equilibrium output of its constituents (although higher than the larger one), and any outsider produces more after the merger. More precisely, the merged firms contract while the outsiders expand, although total output declines. Thus, the merger always results in a lower total output and a higher price. The latter result is true for a more general class of cost structures than the current one. Farrell and Shapiro [1990] show that when firms are playing Cournot, the price can decrease after a merger only if there are "synergies" created by the merger - the synergies being defined as the efficiencies resulting from a merger that are above the ones which could be created by reallocation of production amongst the participants.

Under the linear equilibrium in supply functions, the post-merger behavior of any outsider reflects the behavior of the merging firms. This is easy to see as the best response function is the same for a non-participating firm o before and after the merger, and $\Psi_o\left(\beta_{-o}\right)$ is strictly increasing in β_{-o}. Thus, non-participating firms lower their output if the merging firms decrease their output. This response represents a major difference from the Cournot model, and it has important consequences for mergers.

Let β^* (γ^*) denote the vector of pre-merger (post-merger) equilibrium supply slopes for the rest of this section.

Proposition 3. *Let I be the set of firms that merge and m be the resulting entity. Then, $\gamma_m^* < \Sigma_{i\in I}\beta_i^*$. Moreover, the abatement factor for any firm that is outside of the merger falls with the merger.*

Proof. Provided in the appendix. □

It follows from Proposition 3 that any merger increases price and decreases total output. Therefore mergers are harmful to consumers.

The merging firms are considered to have an incentive to merge if their post-merger profit is higher than the sum of their pre-merger profits.

Proposition 4. *Regardless of the capital distribution and the number of firms in the industry, any merger is profitable both for insiders and outsiders.*

Proof. It is straightforward that

$$\max_{\beta_m}\pi_m\left(K_m,\beta_m,\beta_{-m}^*\right)\geq\Sigma_{i\in I}\max_{\beta_i}\pi_i\left(K_i,\beta_i,\beta_{-i}^*\right)=\Sigma_{i\in I}\pi_i\left(\beta^*\right).$$

Since $\pi_m\left(K_m,\beta_m,\beta_{-m}\right)$ strictly decreases in β_{-m} and $\gamma_{-m}^* < \beta_{-m}^*$, the envelope theorem implies that

$$\pi_m\left(\gamma^*\right)=\max_{\beta_m}\pi_m\left(K_m,\beta_m,\gamma_{-m}^*\right)>\max_{\beta_m}\pi_m\left(K_m,\beta_m,\beta_{-m}^*\right).$$

Then, the merger is profitable for the insiders. Similarly, $\gamma_{-o}^* < \beta_{-o}^*$ results in

$$\pi_o\left(\gamma^*\right)=\max_{\beta_o}\pi_o\left(K_o,\beta_o,\gamma_{-o}^*\right)>\max_{\beta_o}\pi_o\left(K_o,\beta_o,\beta_{-o}^*\right)=\pi_o\left(\beta^*\right),$$

so the merger benefits the non-participating firms as well. □

The existence of incentives for any merger at any industry composition, differs from the result of this model under Cournot competition. In that case, outsiders free-ride as they expand and benefit from the higher prices due to the contraction of the merging parties. This creates an externality on the merging firms, decreasing the profitability of their merger to the extent of making it unprofitable sometimes.[10]

Although mergers are always profitable, the profitability of the merger depends upon the marginal cost functions. Below I use simulations to illustrate how the profitability of a merger varies with the slope of the marginal cost curve in a symmetric industry. The simulations assume that $\theta = 100$, $b = 1$, and that each firm owns the same amount of capital prior to the merger. The latter assumption implies that there are no efficiency gains from merging. The following table summarizes the results:

Table 1: Incentives to merge

K	*price scale*	*initial profits*	*change in profits*	Δ
.01	.9808	94.30	0.01	.01%
.1	.8538	609.85	3.81	.62%
1	.4692	938.36	129.4	13.8%
10	.1087	265.13	297.66	112%
100	.0130	32.47	171.63	529%
1000	.0013	3.32	63.89	1922%

price scale: The position of the market price given by the linear equilibrium in supply functions in the initial symmetric industry, on the $[0, 1]$ interval where 0 is the competitive price[11] and 1 is the Cournot price for the same market.

Δ: The percentage change in the total profits of two firms which merge.

[10] For example, McAfee and Williams find that there are no incentives to merge for any two firms which share less than 22% of the market in a triopoly.

[11] A discussion on why competitive equilibrium price is singled out as the relevant Bertrand equilibrium can be found in Klemperer and Meyer [1989] on page 1259.

The marginal cost curve of a firm becomes flatter as K increases. The second column of Table 1 demonstrates that prices approach Bertrand prices as marginal cost curves become flatter. The total pre-merger profits of two firms are presented in the third column and the change in profits from the merger are listed in the fourth column. The fifth column measures the increase relative to the pre-merger level of profits. Mergers are always profitable, but the increase in profits is largest when $K = 10$. However, as the initial level of profits is very low in the triopolies with higher capital stock, the relative incentives (Δ) continue increasing throughout. Then, if merging is not costly, the firms can be considered to get more eager to merge as the industry moves farther from the Cournot-like outcomes. The results belonging to models with extreme cases of competition can be seen as limits of this phenomenon in the intermediate range.

In the current model, apart from leading to an output decrease, a merger also alters the equilibrium market shares of the firms. In an asymmetric industry equilibrium $\delta_i > \delta_j$ whenever $K_i < K_j$, hence, the marginal costs are not equated and the production in the industry is carried out inefficiently. Through redistributing production, a merger can increase production efficiency by creating a more "symmetric" industry.[12] Furthermore, the competition reducing effect of such a merger will be less severe as competition amongst similars is tougher for a given number of firms. These suggest that a merger is more likely to favor welfare if it increases the "symmetry" in the industry. If the gains in production efficiency are higher than the loss in welfare due to weaker competition, the merger will increase welfare. However, when the initial situation is a symmetric industry, the merger creates asymmetry, and both effects work in the same direction leading to a detoriation in welfare. It follows directly from Corollary 2 and Corollary 3 that *any merger from a symmetric industry*

[12] The issue at stake is the efficiency of the overall production in an industry. Due to efficient operation of plants a merger will always weakly increase the production efficiency of the merged identity. However, in general, this does not mean that at the post-merger equilibrium the total production is carried out in a more efficient manner, because a merger might increase the share of the lower capital firms in the industry production.

results in a welfare loss.

A second numerical exercise proves useful to demonstrate the role of "symmetry" in the welfare implications of a merger in this market. This example involves an industry which has $n-2$ large firms that have equal capital stock and two small firms each equipped with half of the capital of a large firm. A merger of the two small firms results in a symmetric industry. Obviously, for higher n the initial structure is less asymmetric. Then as n gets lower the gain in symmetry created by the merger is higher. This simulation is made for an industry with $\theta = 100$, $b = 1$, and $K^T = 10$. The maximum amount of welfare is calculated to be $4545, 45$. Table 2 presents the results.

Table 2: Symmetry and welfare

n	*initial welfare*	*change in welfare*	Λ
9	4544.6	.092	.011 %
8	4544.2	.144	.017 %
7	4543.6	.200	.024 %
6	4542.5	.267	.032 %
5	4540.0	.101	.012 %
4	4533.6	-2.958	-.358 %
3	4514.5	-58.181	-7.040 %

Λ: The change in welfare as a percentage of the industry's revenues with competitive pricing.

All the mergers in this simulation are creating a symmetric industry, so their redistribution effect is positive. The sole counteracting effect working on welfare is from the descent in competition due fewer firms after merger. This negative effect gets stronger as n decreases. Initially, the rate of welfare change is positive and grows as n decreases until $n = 6$. From there onwards, it starts to decline and eventually, when there are $n^* = 4$ or less firms, the merger reduces welfare. This trend suggests that the welfare augmenting effect is getting larger as one moves down the table. Then, as the increase in symmetry created by the merger becomes larger, the positive effect is becoming stronger. The competition loss effect is also getting stronger as n decreases, and after a certain point on, dominates the total effect. Nevertheless, a positive relation between the increase in symmetry created by a merger and its welfare augmentation is evident. The gains in welfare are quite small as the industry is already producing a high amount of welfare relative to the competitive level. In fact, for the upper rows, the original situation being quite competitive makes easier for a merger to contribute to welfare as the competition loss is smaller. Further calculations show that n^* is decreasing in the capital stock of the industry. This confirms the previous affirmation since higher K^T is conducive to more competitive-like results for the industry.

2.5 Conclusions

I have analyzed the effects of a horizontal merger in a market where firms compete in supply functions. Any merger raises price and lowers aggregate output. However, unlike the Cournot model, the fall in output is not coming only from the merging firms. Non-merging firms also supply less, since their response to a decrease in the merging firms' aggregate supply function is to decrease their own supply schedules. As a result, mergers are always profitable. The possibility of a welfare increase depends crucially on gains in productive efficiency. It can happen if the merger makes the industry "more symmetric". Any merger from a symmetric industry results in a welfare loss.

This result should not be interpreted as predicting a monopoly. The model ignores the organizational cost of mergers and institutional concerns that are present in the real world. For example, potential mergers in the European aviation sector have been hampered by the fear that the merged entity would not be entitled to all the rights of the merging parties. Furthermore, some mergers proposed by firms' executives are not realized due to coordination problems and informational issues.

2.6 Appendix

Remark 1. *In a duopoly the equilibrium is:*

$\beta_i^* = \frac{1}{2}\left[-(b+\Gamma) + \sqrt{(b+\Gamma)^2 + 4K_i(b+\Gamma)}\right]$, $\beta_j^* = \beta_i^* + \Gamma$

where $\Gamma = \dfrac{b(K_j - K_i)}{K_i + K_j + b}$.

Proof. The two equations defining the equilibrium are: $\beta_j^* = \frac{K_j(b+\beta_i^*)}{K_j+b+\beta_i^*}$ and $\beta_i^* = \frac{K_i(b+\beta_j^*)}{K_i+b+\beta_j^*}$. Rearranging, $K_j\left(\beta_j^* - \beta_i^*\right) + b\left(\beta_j^* - K_j\right) + \beta_j^*\beta_i^* = 0$, and $-K_i\left(\beta_j^* - \beta_i^*\right) + b\left(\beta_i^* - K_i\right) + \beta_j^*\beta_i^* = 0$. Subtracting the second from the first:
$(K_j + K_i + b)\left(\beta_j^* - \beta_i^*\right) = b\left(K_j - K_i\right)$, so $\left(\beta_j^* - \beta_i^*\right) = \frac{b(K_j-K_i)}{(K_j+K_i+b)}$. Let $\Gamma = \beta_j^* - \beta_i^*$, then the equilibrium equation for i is $(\beta_i^*)^2 + \beta_i^*(b+\Gamma) - K_i(b+\Gamma) = 0$. Solving this yields $\beta_i^* = \frac{-(b+\Gamma)+\sqrt{(b+\Gamma)^2+4K_i(b+\Gamma)}}{2}$. □

Proof of Lemma 1. Take a' and a'' from the set of Nash equilibria such that $\Sigma a_i' \geq \Sigma a_i''$. From $\Psi_i(0) > 0$ and Ψ_i being increasing it is immediate that $a_i' \neq 0 \neq a_i''$ for all i. I claim that $a_{-i}' \geq a_{-i}''$ for all i. Assume $a_{-i}' < a_{-i}''$ for some i, then $a_i' = \Psi_i\left(a_{-i}'\right) \leq \Psi_i\left(a_{-i}''\right) = a_i''$ since $\Psi_i(a_{-i})$ increasing in a_{-i}. This results in $\Sigma a_i' = a_{-i}' + a_i' < a_{-i}'' + a_{-i}'' = \Sigma a_i''$ which contradicts $\Sigma a_i' \geq \Sigma a_i''$. Therefore $a_{-i}' \geq a_{-i}''$ and $a_i' \geq a_i''$for all i. Take j such that $a_j'/a_j'' \geq a_i'/a_i''$ for all i. It follows that $a_j' a_{-j}'' \geq a_j'' a_{-j}'$. Thus $\Psi_j\left(a_{-j}'\right)/a_{-j}' \geq \Psi_j\left(a_{-j}''\right)/a_{-j}''$, and it should be that $a_{-j}' \leq a_{-j}''$. But, then $a_j' = \Psi_j\left(a_{-j}'\right) \leq \Psi_j\left(a_{-j}''\right) = a_j''$ and $\Sigma a_i' \leq \Sigma a_i''$. Thus $a_{-i}' \leq a_{-i}''$ and $a_i' \leq a_i''$ for all i and I conclude that $a_i' = a_i''$. □

Lemma 2. *Consider a game as in Lemma 1 in which* $\Psi_i(a_{-i}) = \Psi(\alpha_i, a_{-i})$ *for all* i *where* α_i *is an exogenous parameter for each player and* $\Psi_\alpha(\cdot,\cdot) > 0$ *for all values. Then* $\alpha_i = \alpha_j \Rightarrow a_i^* = a_j^*$ *and* $\alpha_i > \alpha_j \Rightarrow a_i^* > a_j^*$ *at the Nash equilibrium.*

Proof. For the first part assume there exists a Nash equilibrium with $\alpha_i = \alpha_j$ and $a_i^* \neq a_j^*$. Let $a_i^* > a_j^*$, it implies $a_{-i}^* < a_{-j}^*$. This gives $a_i^* = \Psi_i\left(a_{-i}^*\right) = \Psi_j\left(a_{-i}^*\right) \leq$

$\Psi_j\left(a_{-j}^*\right) = a_j^*$ as $\Psi_j(\cdot)$ is increasing, which contradicts $a_i^* > a_j^*$. Thus, it must be that $a_i^* = a_j^*$. For the second part, assume there exists a Nash equilibrium with $\alpha_i > \alpha_j$ and $a_i^* \leq a_j^*$. Then $a_{-i}^* \geq a_{-j}^*$ and it follows that $a_j^* = \Psi_j\left(a_{-j}^*\right) < \Psi_i\left(a_{-j}^*\right) \leq \Psi_i\left(a_{-i}^*\right) = a_i^*$, but this contradicts $a_i^* \leq a_j^*$, so I conclude that $a_i^* > a_j^*$. □

In what follows I will make use of Thm. 2.10 on page 53 in Vives [1999]. Let $A_i^+ = \{a \in A : a_i \geq \overline{\Psi}_i(\widehat{a}_{-i})\}$, $A_i^- = \{a \in A : a_i \leq \underline{\Psi}_i(\widehat{a}_{-i})\}$,[13] and $A^+ = \cap_{i \in N} A_i^+$, $A^- = \cap_{i \in N} A_i^-$, where $\overline{\Psi}_i$ and $\underline{\Psi}_i$ are, respectively, the smallest and the largest best response function of player i.

Theorem 1. *(Vives) Let G be a supermodular game with continuous payoffs. If the players always select the largest (or the smallest) best response, then a Cournot tatônnement starting at any a^0 in A^+ (A^-) converges monotonically downward (upward) to an equilibrium point of the game.*

Lemma 3. *Let G be a supermodular game with continuous payoffs, satisfying the conditions of lemmas 1 and 2. If the best responses are such that $\frac{1}{2}\left[\Psi(\alpha_i, a_{-i}) + \Psi(\alpha_j, a_{-j})\right] < \Psi\left(\frac{1}{2}(\alpha_i + \alpha_j), \frac{1}{2}(a_{-i} + a_{-j})\right)$ whenever $\alpha_i > \alpha_j$ and $a_{-i} < a_{-j}$, then for any fixed level of $\overline{\alpha} = \Sigma\alpha_i$, the sum of the equilibrium actions is highest when $\overline{\alpha}$ is shared equally.*

Proof. Take any asymmetric distribution of α and denote its equilibrium by a^*. Let L and H be, respectively, the players that have the minimum and the maximum elements in the vector α. Let α' be such that $\alpha'_L = \alpha'_H = \frac{1}{2}(\alpha_L + \alpha_H)$ and $\alpha'_j = \alpha_j$ for any other player j and denote the Nash equilibrium of the game under this distribution by a'. I claim that $\Sigma a'_i > \Sigma a_i^*$. Consider the action profile a'' for which $a''_L = a''_H = \frac{1}{2}(a_L^* + a_H^*)$ and $a''_j = a_j^*$ for any other player j. For a'' it is true that $a''_{-L} = a''_{-H} = \frac{1}{2}\left(a_{-L}^* + a_{-H}^*\right)$ and $a''_{-j} = a_{-j}^*$ for any other player j. Since $\alpha_L < \alpha_H$ and $a_{-L}^* > a_{-H}^*$ the condition on best responses apply: $\Psi\left(\alpha'_L, a''_{-L}\right) = \Psi\left(\alpha'_H, a''_{-H}\right) >$

[13] Where $\widehat{a}_{-i}$ is a vector of size $n-1$ denoting the actions of players other than i.

$\frac{1}{2}\left[\Psi\left(\alpha_L, a^*_{-L}\right) + \Psi\left(\alpha_H, a^*_{-H}\right)\right] = a''_L = a''_H$. Then $a''_L \in A^-_L$ and $a''_H \in A^-_H$ for the game with α'. For any other player j, $\Psi\left(\alpha'_j, a''_{-j}\right) = \Psi\left(\alpha_j, a^*_{-j}\right) = a^*_j = a''_j$, so $a''_j \in A^-_j$ for the game with α'. Thus, by the above theorem the Cournot tatônnement starting at $a^0 = a''$ will converge monotonically upward to the Nash equilibrium of the game. Then, at Nash equilibrium a' I have $\Sigma a'_i \geq \Sigma\Psi\left(\alpha_i, a''_{-i}\right) > \Sigma a''_i = \Sigma a^*_i$. For any asymmetric α vector, α' exists, therefore I conclude that the sum of the equilibrium actions is maximum when $\overline{\alpha}$ is shared equally amongst players. □

Proof of Proposition 3. Take the action profile γ' for the postmerger situation where for the merged entity $\gamma'_m = \Sigma_{i\in I}\beta^*_i$ and $\gamma'_o = \beta^*_o$ for any firm o outsider to the merger, then $\Sigma\gamma'_i = \Sigma\beta^*_i$. Also $\gamma'_{-o} = \beta^*_{-o}$ for any o and it follows that $\Psi_o\left(\gamma'_{-o}\right) = \beta^*_o = \gamma'_o$. It is easy to verify that $\Psi\left(K_m, \gamma'_{-m}\right) \leq \Sigma_{i\in I}\Psi\left(K_i, \gamma'_{-m}\right)$ where $K_m = \Sigma_{i\in I}K_i$. Moreover, for any $i \in I$, I have $\beta^*_{-i} > \gamma'_{-m}$, so $\beta^*_i > \Psi\left(K_i, \gamma'_{-m}\right)$. Then, it is true that $\Psi_m\left(\gamma'_{-m}\right) < \Sigma_{i\in I}\beta^*_i = \gamma'_m$. Thus, for any firm l in the postmerger situation $\Psi_l\left(\gamma'_{-l}\right) \leq \gamma'_l$ (with a strict inequality for m) and by Theorem 1 the Cournot tatônnement starting at $\gamma^0 = \gamma'$ will converge monotonically downwards to the Nash equilibrium γ^*. I conclude $\gamma^*_m \leq \Psi_m\left(\gamma'_{-m}\right) < \Sigma_{i\in I}\beta^*_i$ and $\Sigma\gamma^*_i < \Sigma\gamma'_i = \Sigma\beta^*_i$. Then, for any outsider $\gamma^*_o < \beta_o$ since $\gamma^*_o \geq \beta^*_o$ would contradict $\Sigma\gamma^*_i < \Sigma\beta^*_i$. Thus, the abatement factor for any non-participating firm falls after a merger. □

2.7 References

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Chapter 3

Mergers under uncertainty

3.1 Introduction

Many merger decisions are taken by firms that anticipate near term shocks to the industry in which they operate. Firms may face some sort of uncertainty about the effects of these shocks on the future industry conditions. In this case, they evaluate a partnership differently in the presence of uncertainty. Therefore, analysis of mergers under uncertainty is important for our comprehension of this frequent business practice. The existing literature on mergers concentrates on uncertain environments with private information. Thus, those models do not differentiate between informational rationale for mergers and effects of uncertain environments on merger incentives. This paper intends to contribute to the issue by focusing solely on the effects of uncertainty on merger decisions.

In an industry facing fluctuations of underlying conditions, a shock may affect the market participants in different ways. Sometimes, the effects of a shock are only privately observable. For example, in the electricity generation industry, a change in wind conditions would most probably pass unnoticed by nuclear power generators while they would be observed with much attention by the wind power generators. Other times, the effects of a shock may be observed by all firms in the industry, even though they vary amongst firms. For instance, consider the tourism industry in the Meditarranean bowl. The effects of a war in the Persian Gulf are quite different for the firms that concentrate their operations on the western coast and the ones that are more active on the eastern one. However, all parties observe the outbreak of a war once it happens, and all are aware of its consequences.

In this paper, I try to assess the effects of uncertainty of the latter type on merger decisions of risk-neutral firms. More precisely, I consider two main questions when the uncertainty in the industry is such that all firms will possess perfect information after its resolution.[1] Does uncertain environments increase attractiveness of mergers? How do firms' preferences over possible merging partners depend on the way uncertainty affects them? I look for the answers under two standard competition models, the Bertrand one and the Cournot one.

In the analysis of mergers in deterministic markets, Salant et al. [1983] find that under homogenous product Cournot competition with constant marginal costs, profitable mergers are uncommon, while Deneckere and Davidson [1985] establish that under differentiated product Bertrand competition all mergers increase profits. Perry and Porter [1985], by using quadratic costs obtain a hybrid result for the homogenous product Cournot case where existence of incentives for merger depends on market parameters.

The research in uncertain markets concentrates on the case where information is private. Gal-Or [1985] uses a differentiated product model in which the uncertainty is on a common parameter affecting all firms symmetrically while firms have private information about it.[2] She demonstrates that a merger might impose an informational disadvantage to the partners at the Cournot equilibrium while it rewards an informational advantage at the Bertrand equilibrium. Hence, concludes that this type of uncertainty may reduce incentives when competition is in quantities while it

[1] In other words, each firm will learns the values of all variable parameters upon the realization of the uncertainty.

[2] Each firm receives equally precise imperfect information about a common parameter.

reinforces them when prices are the strategic variables.

A recent study by Banal-Estañol [2002] considers mergers under a Cournot model which can be seen as a quadratic cost homogenous product or constant marginal cost differentiated product set-up, where the uncertainty is on a firm specific variable. He shows that private information may turn a previously unprofitable merger in a certain market into a desirable one in the relevant uncertain market. Also, he establishes that under uncertainty firms always have more incentives to merge when the information is private compared to when it is public.

Here, I find that with publicly observed parameters, uncertainty makes mergers more attractive when competing à la Bertrand while it has an ambigous effect when competition is of the Cournot type. In quantity-setting, uncertainty augments the incentives if they already exist in the deterministic market, but it may increase or decrease the losses when a merger creates losses in the deterministic market. On the other hand, in both competition modes, firms have strict preferences over their partners: Under price competition each firm prefers to merge with the rival whose random shock is most positively correlated with its own, while with quantity competition the best partner is the one that possesses the most negatively correlated stochastic term with its own. This difference originates from the fact that prices are strategic complements while the quantities are strategic substitutes.

In Section 3.2 the model is presented. Premerger and postmerger Bertrand equilibria are derived in Section 3.3, and incentives to merge are analyzed in Section 3.4. Sections 3.5 and 3.6 repeat the previous analyses for Cournot competition. Finally, Section 3.7 concludes.

3.2 The model

There are $n \geq 3$ firms competing in a differentiated goods market. Each firm produces a single product and all products are substitutes. I consider the merger decisions of firms before the resolution of an uncertainty in demand, in two different scenarios: when firms compete in prices (Bertrand model), and when firms compete in quantities (Cournot model). I work with linear demand in both cases. This demand can be seen as originating from the maximization problem of a representative consumer with a utility linear in money (or in a numéraire representing the rest of the economy) and quadratic in the products of the industry under consideration.[3]

In the price-setting version, the demand for each segment of the market is given by

$$D_i(p) = a - \theta_i - bp_i + c\sum_{k\neq i} p_k \qquad i = 1,..,n;\ b,c > 0;\ b > (n-1)\,c$$

where θ_i is an idiosyncratic random parameter representing uncertainty in demand. The assumption that $b > (n-1)\,c$ means that the own effect on demand dominates the aggregate of the cross effects and guarantees the existence of a unique equilibrium in the price-setting game.

In the quantity-setting version, the inverse demand for each firm is given by

$$P_i(q) = a - \theta_i - bq_i - d\sum_{k\neq i} q_k \qquad i = 1,..,n;\ b,d > 0;\ b > d$$

where θ_i is an idiosyncratic random parameter representing uncertainty in inverse demand. The assumption $b > d$ means that the own effect on price is stronger than each of the cross effects and is enough for this game to have a unique equilibrium since the inverse demand of each firm can be written as a function of its quantity and the sum of its rivals' quantities.[4]

A firm faces a lower demand (inverse demand) when it receives a positive shock. I assume that the vector of idiosyncratic shocks is distributed such that $E(\theta_i) = \overline{\theta}$ and $var(\theta_i) = \sigma^2$ for all firms.[5] I denote $cov(\theta_i, \theta_j)$ by σ_{ij} and allow for these covariance

[3] A detailed analysis of the microfoundations for linear demands can be found in Vives (1999) pp.144-147.

[4] For a concise discussion on the uniqueness of equilibrium for both type of games, see Vives (1999) pp. 150.

[5] The support and distribution of θ is assumed to be such that all firms produce positive quantities and make positive profits at equilibrium for any resolution of the uncertainty.

parameters to differ amongst pairs of firms. I introduce these correlation parameters to observe a firm's preferences over its rivals in choosing a partner. A deterministic market where $\theta_i = \overline{\theta}$ constitutes a benchmark against which I compare the results of the markets with uncertainy.

I assume constant and equal marginal costs across firms. Since the level of constant marginal cost leads to quantitative but not qualitative differences, I normalize them to 0. In effect, for Cournot competiton, zero marginal cost and additive demand uncertainty set-up is equivalent to one in which the random parameter θ_i determines Firm i's marginal cost and the demand is deterministic.[6] For merger analysis constant marginal cost assumption represents the worst case scenario since two parties which merge do not increase their efficiency and have no gains on the cost side. By shutting off any effects coming from the cost side, this production technology allows to isolate the effect of uncertainty on mergers.

I only consider mergers that consist of two firms coming together. Let these two firms be 1 and 2. I compare the expected joint profits of these two firms before and after a a merger involving them to see if uncertainty increases or decreases the incentives to merge. An alternative would be to let any number of firms to merge at once. Then the effect of uncertainty on the incentives to merge could be followed by how the number of partners necessary to create a profitable merger changes when uncertainty is introduced. However, the generality gained by allowing multi-firm mergers does not justify the computational complexity with correlated parameters. Moreover, in practice most mergers include only two participants at once, although any of these participants itself might have resulted from a previous merger.[7]

The timing is as follows. First, two firms decide whether to merge or not. After the decision concerning the merger, the uncertainty is resolved and all firms learn the

[6] It is straightforward to see that the expressions for the profits of a firm are equivalent in the two cases.

[7] Recall that I deal with an initial structure with single product firms only, while existence of past mergers might create multiproduct firms competing in the market.

complete θ vector. There is no private information in the model as it would provide an additional advantage to the merging parties altering the effects of pure uncertainty. At last stage, firms make their pricing or quantity decisions, and afterwards receive the corresponding profits according to the market. Therefore, the initial merger decisions of the firms is based on their expected profits in the presence and absence of the merger. This particular timing is more appropriate when the industry receives shocks periodically over time so that there is always a shock that is expected. The assumption that all firms have perfect information about the resolution of uncertainty may seem to undermine this set-up's value for one time shocks. It might seem natural for firms to wait until things settle down and then take the merger decision rather than basing it on expectations. However, sometimes in practice there are strategic concerns that make it impossible for firms to wait. A prominent example is the 175 billion dollar takeover of Mannesmann by Vodafone which took place before the auctioning of British 3G mobile spectrum licences[8] and a few years before the actual implementation of this new technology. Notice that for the mobile phone operating industry, the UK auction process was an uncertain element whose resolution was known publicly once it was concluded.

3.3 Derivation of the Bertrand equilibria

3.3.1 No merger case

In this case after the resolution of uncertainty each firm faces the following maximization problem

$$\max \pi_i = \left[a - \theta_i - bp_i + c \sum_{k \neq i} p_k \right] p_i. \tag{3.1}$$

[8] In the fall of 1999, Vodafone announced its intentions for merger on November 16 and made its hostile bid for acquisation on November 19 with March 27, 2000 being the last day for Mannesmann shareholders to accept; while the auction ran from March 6 to April 27 of 2000.

Solving for all firms simultaneously I obtain the unique Nash equilibrium of the pricing game: Firm 1 prices as

$$p_1 = \frac{a - \theta_1}{[2b - (n-1)c]} + \frac{cn(\theta_1 - \theta^m)}{(2b + c)[2b - (n-1)c]},$$

where θ^m is the arithmetic mean of the θ vector after the uncertainty is resolved. The first term gives the price of the firm if there was a unique uncertainty parameter common to all firms, while the second is a term that corrects for having private parameters. This correction is positive for the firms whose draw is higher than the mean and negative for the ones with lower than mean draws. As the marginal gains from pricing higher decreases in θ_1 the Firm 1's price decreases in it. The expected price of Firm 1 ($\overline{p}_1$) is equal to its price in the benchmark deterministic market where each firm has $\theta_i = \overline{\theta}$. Substituting the first order condition of the maximization problem of the firm in its demand function one obtains that the amount sold by the firm at the market equilibrium is equal to $q_1 = bp_1$ and therefore its profits are equal to $\pi_1 = bp_1^2$. Then, its expected profits in the absence of a merger is given by

$$E(\pi_1) = b\left[\overline{p_1}^2 + var(p_1)\right] = b\left[\overline{p_1}^2 + C^2 V_1\right],$$

where $C = \dfrac{1}{(2b + c)[2b - (n-1)c]}$ and[9]

$$\begin{aligned} V_1 &= \left([2b - (n-2)c]^2 + (n-1)c^2\right)\sigma^2 + 2c[2b - (n-2)c]\sum\nolimits_{h\neq 1}\sigma_{1h} \\ &\quad + 2c^2\left(\sum_{k<l}\sigma_{kl} - \sum\nolimits_{h\neq 1}\sigma_{1h}\right). \end{aligned}$$

The firm's profits in the deterministic market would be $b\overline{p}^2$. Thus, under uncertainty expected profits increase in the variance of the firm's price. When a firm is charging the higher prices in its distribution it gains more compared to what it is loosing when it is charging the lower ones. This is a consequence of the scale effect that the additive

[9] $\sum\limits_{k<l}\sigma_{kl}$ denominates the sum of all possible correlation parameters amongst n firms which would be written formally as $\sum_{k=1}^{k=l-1}\sum_{l=2}^{l=n}\sigma_{kl}$.

uncertainty parameter creates.[10] It is straightforward to see from the above formula that a firm benefits, on expected terms, if its rivals' demand shocks are positively correlated with its own. The strategic complementarity of the prices is the key factor that leads to this effect. A firm chooses a high price when it receives a low shock, and it sells more at this price if its rivals also price high, that is, when rivals receive low shocks as well.

3.3.2 Postmerger case

After a merger the non-participating firms face the same maximization problem as in the premerger case, they each choose a price for their product facing (3.1). While the merged entity $1+2$ should decide upon the prices of its two products and it faces the following problem

$$\max \pi_{1+2}^{M} = \left[a - \theta_1 - bp_1^M + c \sum_{k \neq 1} p_k^M \right] p_1^M + \left[a - \theta_2 - bp_2^M + c \sum_{k \neq 2} p_k^M \right] p_2^M .$$

Solving for this and the outsider firms' problems simultaneously reveals that the merged entity prices its two products as

$$\begin{aligned} p_1^M &= \frac{(2b+c)(a-\theta_1) + nc(\theta_1 - \theta^m)}{2\left(b\left[2b-(n-1)c\right]-c^2\right)} + \frac{c\left[2b-(n-2)c\right](\theta_1-\theta_2)}{4(b+c)\left(b\left[2b-(n-1)c\right]-c^2\right)}, \\ p_2^M &= \frac{(2b+c)(a-\theta_2) + nc(\theta_2 - \theta^m)}{2\left(b\left[2b-(n-1)c\right]-c^2\right)} + \frac{c\left[2b-(n-2)c\right](\theta_2-\theta_1)}{4(b+c)\left(b\left[2b-(n-1)c\right]-c^2\right)}, \end{aligned}$$

and any outsider firm k chooses

$$\begin{aligned} p_k^M &= \frac{(a-\theta_k)}{\left[2b-(n-1)c\right]} + \frac{nc(\theta_k-\theta^m)}{(2b+c)\left[2b-(n-1)c\right]} \\ &\quad + \frac{(2b+c)(2a-\theta_1-\theta_2) + nc(\theta_1+\theta_2-2\theta^m)}{2(2b+c)\left[2b-(n-1)c\right]\left(b\left[2b-(n-1)c\right]-c^2\right)} c^2 . \end{aligned}$$

[10] For intuition, consider a monopolist facing an expected shock equal to $\overline{\theta}$. Its expected profits increase in the variance of the random parameter. It sells more than average at a higher than average price (when it receives a low shock), while it sells less than average at a lower than average price (when it receives a high shock).

The third term in the last formula is always positive under the assumptions made for the distribution of the uncertainty.[11] Thus, independently of how the uncertainty is resolved any outsider prices higher after the merger. As prices are strategic complements, the prices chosen by the merged entity for its products are higher than the prices its constituents would choose in the absence of a merger. This is easy to verify for the sum of the two prices, as the second terms cancel out. However it is true for individual prices too, since for the firm with a negative second term, the increase in the first term due to merger dominates the total effect on price.

Similar to the no merger case, any firm's equilibrium price decreases in its random parameter, and its expected price is equal to $\overline{p}_i^M$, its price in the equivalent deterministic market with merger, where each firm has $\theta_i = \overline{\theta}$. A similar substitution as in the previous case results in the market equilibrium quantities $q_1^M = bp_1^M - cp_2^M$ and $q_2^M = bp_2^M - cp_1^M$ for the merged entity which enables its total profits to be expressed as $\pi_{1+2}^M = b\left[\left(p_1^M\right)^2 + \left(p_2^M\right)^2\right] - 2cp_1^M p_2^M$. Then its total expected profits are

$$\begin{aligned} E\left(\pi_{1+2}^M\right) &= b\left[var\left(p_1^M\right) + \left(\overline{p_1^M}\right)^2 + var\left(p_2^M\right) + \left(\overline{p_2^M}\right)^2\right] \\ &\quad - 2c\left[cov\left(p_1^M, p_2^M\right) + \overline{p_1^M p_2^M}\right] \\ &= D^2\left[(b-c)\left(V_1+V_2\right) + (b+c)\, Z_{12} + 2\left[2b-(n-1)\,c\right]^2 c\left(\sigma^2 - \sigma_{12}\right)\right] \\ &\quad + b\left(\overline{p_1^M}\right)^2 + b\left(\overline{p_2^M}\right)^2 - 2c\overline{p_1^M p_2^M}, \end{aligned}$$

where $D = \dfrac{1}{2\left(b\left[2b-(n-1)\,c\right] - c^2\right)}$, and

$$Z = -c\left[\frac{(2b+c)\left[2b-(n-1)\,c\right] - c^2}{(b+c)^2}\right]\left[2b-(n-2)\,c\right]\left(\sigma^2 - \sigma_{12}\right).$$

In the equivalent deterministic market its total postmerger profit is $\pi_{1+2}^M = b\left[\left(\overline{p_1^M}\right)^2 + \left(\overline{p_2^M}\right)^2\right] - 2c\overline{p_1^M p_2^M}$. Total expected profits of the merged entity increases in the variance of its prices while it decreases in the covariance between its prices.

[11] This term is equal to the sum of the prices of firms 1 and 2 if no merger occurred multiplied by a positive constant.

3.4 Incentives to merge under Bertrand competition

The incentives to merge are given by the increase in the total profits of two merging firms. In the equivalent deterministic market it is easy to observe that the postmerger total profits are larger than the no merger profits, the difference is given by

$$\Delta^B = \pi_{1+2}^M - (\pi_1 + \pi_2) = 2b\left(\left(\overline{p_1^M}\right)^2 + \left(\overline{p_2^M}\right)^2 - \overline{p_1}^2\overline{p_2}^2\right) - 2c\overline{p_1^M p_2^M}.$$

The fact that there are incentives to merge in the price setting game is a consequence of the upward sloping best-response functions. As outsiders also increase their prices, the insiders gain from the merger.

In the market with uncertainty the difference between expected profits in the presence and absence of the merger is

$$\begin{aligned} E\left(\pi_{1+2}^M\right) - E\left(\pi_1 + \pi_2\right) &= \Delta^B + \left[(b-c)\,D^2 - bC^2\right](V_1 + V_2) + (b+c)\,D^2 Z_{12} \\ &\quad + D^2 2\left[2b - (n-1)\,c\right]^2 c\left(\sigma^2 - \sigma_{12}\right). \end{aligned}$$

The change in incentives due to the introduction of uncertainty, is

$$\begin{aligned} I_U^B &\equiv E\left(\pi_{1+2}^M\right) - E\left(\pi_1 + \pi_2\right) - \Delta^B \\ &= \left[(b-c)\,D^2 - bC^2\right](V_1 + V_2) + (b+c)\,D^2 Z_{12} \\ &\quad + D^2 2\left[2b - (n-1)\,c\right]^2 c\left(\sigma^2 - \sigma_{12}\right). \end{aligned}$$

Proposition 5. *With Bertrand competition, if there is no correlation between shocks of the firms $I_U^B > 0$, i.e. the incentives to merge are always higher under uncertainty than in the equivalent deterministic market.*

Proof. When $\sigma_{kl} = 0$ for all pairs $k \neq l$, substitution gives

$$\begin{aligned} \frac{I_U^B}{D^2\sigma^2} &= 2cG\left(\left[2b - (n-2)\,c\right]^2 + (n-1)\,c^2\right) \\ &\quad - \left[4bc^2 + \frac{2b+c}{(b+c)}\left[2b - (n-2)\,c\right]^2 c\right], \end{aligned}$$

$$\text{where } G = b\frac{\left[(4b+c)\left[2b-(n-1)c\right]-2c^2\right]\left[2b-(n-3)c\right]}{(2b+c)^2\left[2b-(n-1)c\right]^2}.$$

$G > 1$ when $n \geq 3$ and $b > (n-1)c$,[12] so:

$$\frac{I_U^B}{D^2\sigma^2} > \left(2G - 1 - \frac{b}{(b+c)}\right)\left[2b-(n-2)c\right]^2 c - 2\left[2b-(n-1)c\right]c^2.$$

Under the assumptions, the term on the right is positive as well as the denominator on the left, therefore $I_U^B > 0$. □

Due to the strategic complementarity of the prices, any firm, the merged entity and any outsider to the merger, is pricing less agressively after the merger(i.e. higher prices are chosen after a merger for any given parameter profile). Therefore, a given variance of idiosyncratic terms results in a higher variance of the prices in the post-merger world. Since expected profits increase in the variance of prices, σ^2 contributes more to profits after a merger leading to incentives to merge increase with uncertainty in the model.

Lemma 4. *Under Bertrand competition the incentives of two firms to merge increase in the covariance of their random parameters.*

Proof. The coefficient of σ_{12} in I_U^B is

$$\begin{aligned}\gamma \equiv\ & 2b\left(D^2 - C^2\right)2c\left[2b-(n-2)c\right] \\ & -D^2\left[\frac{c^2}{(b+c)}\left[2b-(n-2)c\right]^2 + 2c^2\left[2b-(n-3)c\right]\right].\end{aligned}$$

By algebraic manipulations the following equality is obtained

$$\begin{aligned}\frac{\gamma}{D^2c^2\left[2b-(n-3)c\right]} =\ & \frac{4b(4b-c)\left[2b-(n-1)c\right]-8bc^2}{(2b+c)^2\left[2b-(n-1)c\right]^2}c \\ & +\frac{(2b+c)\left(b^2-2bc-c^2\right)+b^2\left[4b-4(n-3)c+(2b-c)\right]}{(b+c)(2b+c)^2\left[2b-(n-3)c\right]}c \\ & +\frac{(n-3)}{(b+c)}c.\end{aligned}$$

[12] This is formally shown in the appendix.

Under the given assumptions of the model both the sum on the RHS and the denominator on the LHS are positive, therefore the coefficient of σ_{12} in I_U^B is positive. □

A positive σ_{12} means that the merged firm receives shocks that move together, making his price pair fluctuate more compared to the case of a negative correlation where the price pair would be formed of a high and a low price. This is because the strategic effect amplifies the price movement. As expected profits increase in the fluctuations, the contribution of σ_{12} to expected profits is stronger in the postmerger case. Thus, a firm with positive correlation helps more to the expected profits as a partner than as a rival in the no merger case.

Proposition 6. *Under Bertrand competition, all else equal, amongst its possible partners a firm prefers to merge with the one whose demand shock has highest positive correlation with its own.*

Proof. This result follows from the above lemma by reinterpretation of the parameters. Suppose that Firm 1 is considering to merge with Firm 2 or Firm 3. Then γ is actually the coefficient of $\sigma_{13} - \sigma_{12}$ in the difference $W_{13} - W_{12}$ where W_{12} is the increase in the total expected profits of 1 and 2 when they merge. As γ is shown to be positive, all else equal, a firm prefers to merge with the firm who has the highest correlated parameter with its own amongst its posssible partners. □

3.5 Derivation of the Cournot equilibria

3.5.1 No merger case

In this case, after the resolution of uncertainty each firm faces the following maximization problem

$$\max\ \pi_i = \left[a - \theta_i - bq_i - d \sum_{k \neq i} q_k \right] q_i. \tag{3.2}$$

The maximization problem in this case as given in (3.2) is exactly the one in (3.1) where c is replaced by $-d$.[13] Therefore, one can obtain the equilibrium under Cournot directly from the equilibrium under Bertrand just by substitution. When there is no merger Firm 1 chooses its quantity as

$$q_1 = \frac{a - \theta_1}{[2b + (n-1)\, d]} + \frac{dn\,(\theta^m - \theta_1)}{(2b - d)\,[2b + (n-1)\, d]},$$

where θ^m is the arithmetic mean of the θ vector after the uncertainty is resolved. The first term gives the quantity of the firm if there was a unique uncertainty parameter common to all firms, as before the second is a term that corrects for having private parameters. However, this time correction is negative for the firms whose draw is higher than the mean and positive for the ones with lower than mean draws. As the marginal gains from producing more decreases in θ_1 Firm 1's quantity decreases in it. The expected quantity of the firm ($\overline{q}_1$) is equal to its quantity in the equivalent deterministic market where each firm has $\theta_i = \overline{\theta}$. Proper substitution reveals that the price at the market equilibrium is equal to $p_1 = bq_1$ and therefore its profits are equal to $\pi_1 = bq_1^2$. Then, the expected profits of the firm in the absence of a merger is given by

$$E\,(\pi_1) = b\left[\overline{q}^2 + var\,(q_1)\right] = b\left[\overline{q}^2 + A^2 V_1'\right],$$

where $A = \dfrac{1}{(2b - d)\,[2b + (n-1)\, d]}$ and

$$\begin{aligned} V_1' &= \left([2b + (n-2)\, d]^2 + (n-1)\, d^2\right)\sigma^2 - 2d\,[2b + (n-2)\, d] \sum\nolimits_{h \neq 1} \sigma_{1h} \\ &\quad + 2d^2 \left(\sum_{k<l} \sigma_{kl} - \sum\nolimits_{h \neq 1} \sigma_{1h}\right). \end{aligned}$$

The firm's profits in the equivalent deterministic market would be $b\overline{q}^2$. Thus, under uncertainty expected profits are higher and they increase in the variance of the firm's quantity. When a firm is producing the higher quantities in its distribution, it gains

[13] This type of duality was pointed out first by Sonnenschein (1968) in a nondifferentiated framework, the application here is similar to Vives (1999) chapter 6.

more compared to what it is losing when it is produces the lower ones. This is because in the occasions where quantity is chosen relatively high(low) the price received is also higher(lower) than average. A firm benefits on expected terms from the existence of a firm amongst its rivals with a demand shock negatively correlated with its own. It is straightforward to see from the above formula that a firm benefits, on expected terms, if its rivals' demand shocks are negatively correlated with its own. The strategic substitutabilty of the quantities is the key factor that leads to this effect. A firm chooses a high quantity when it receives a low shock, and it receives a higher price if its rivals choose low quantities, that is, when rivals receive high shocks.

3.5.2 Postmerger case:

The proper substitutions to the Bertrand case yield that the merged entity chooses the quantities of its products as

$$
\begin{aligned}
q_1^M &= \frac{(2b-d)(a-\theta_1)+nd(\theta^m-\theta_1)}{2\left(b\left[2b+(n-1)d\right]-d^2\right)}+\frac{d\left[2b+(n-2)d\right](\theta_2-\theta_1)}{4(b-d)\left(b\left[2b+(n-1)d\right]-d^2\right)},\\
q_2^M &= \frac{(2b-d)(a-\theta_2)+nd(\theta^m-\theta_2)}{2\left(b\left[2b+(n-1)d\right]-d^2\right)}+\frac{d\left[2b+(n-2)d\right](\theta_1-\theta_2)}{4(b-d)\left(b\left[2b+(n-1)d\right]-d^2\right)},
\end{aligned}
$$

and any outsider firm k chooses

$$
\begin{aligned}
q_k^M &= \frac{(a-\theta_k)}{\left[2b+(n-1)d\right]}+\frac{nd(\theta^m-\theta_k)}{(2b-d)\left[2b+(n-1)d\right]}\\
&\quad+\frac{(2b-d)(2a-\theta_1-\theta_2)+nd(2\theta^m-\theta_1-\theta_2)}{2(2b-d)\left[2b+(n-1)d\right]\left(b\left[2b+(n-1)d\right]-d^2\right)}d^2.
\end{aligned}
$$

The third term in q_k^M is always positive under the assumption for the distribution of the uncertainty.[14] Thus regardless of how the shocks realize, any outsider expands production after the merger. As quantities are strategic substitutes in this game the merged entity chooses to produce less of each of its products than what its constituent would have chosen in the absence of the merger. The second terms cancel out, and each first term is smaller than the one in the no merger case, so the total quantity

[14] This term is equal to the sum of the quantities of 1 and 2 if no merger occurred multiplied by a positive constant.

produced by the constituents decrease. However it can be verified that their quantities decrease individually too, as even for the firm with a positive second term the decrease in the first term due to merger dominates the total effect on quantity.

As in the no merger case, any firm's quantity decreases in its random shock, and its expected quantity is equal to $\overline{q_i^M}$, its quantity in the equivalent deterministic market with merger where each firm has $\theta_i = \overline{\theta}$. A similar substitution as in the previous case results in the market equilibrium prices $p_1^M = bq_1^M + dq_2^M$ and $p_2^M = bq_2^M + dq_1^M$ for the merged entity which allows to express its total profits as $\pi_{1+2}^M = b\left[\left(q_1^M\right)^2 + \left(q_2^M\right)^2\right] + 2dq_1^M q_2^M$. Then its total expected profits are

$$\begin{aligned} E\left(\pi_{1+2}^M\right) &= b\left[var\left(q_1^M\right) + \left(\overline{q_1^M}\right)^2 + var\left(q_2^M\right) + \left(\overline{q_2^M}\right)^2\right] \\ &\quad +2d\left[cov\left(q_1^M, q_2^M\right) + \overline{q_1^M q_2^M}\right] \\ &= B^2\left[(b+d)\left(V_1' + V_2'\right) + (b-d)\,Z_{12}' - 2\left[2b + (n-1)\,d\right]^2 d\left(\sigma^2 - \sigma_{12}\right)\right] \\ &\quad +b\left(\overline{q_1^M}\right)^2 + b\left(\overline{q_2^M}\right)^2 + 2d\overline{q_1^M q_2^M} \end{aligned}$$

where $B = \dfrac{1}{2\left(b\left[2b + (n-1)\,d\right] - d^2\right)}$, and

$$\text{and } Z_{12}' = d\left[\frac{(2b-d)\left[2b + (n-1)\,d\right] - d^2}{(b-d)^2}\right]\left[2b + (n-2)\,d\right]\left(\sigma^2 - \sigma_{12}\right).$$

In the equivalent deterministic market its total postmerger profit is $\pi_{1+2}^M = b\left[\left(\overline{q_1^M}\right)^2 + \left(\overline{q_2^M}\right)^2\right] + 2d\overline{q_1^M q_2^M}$. Total expected profits of the merged entity increase in the variance of its quantities and in the covariance between its quantities.

3.6 Incentives to merge under Cournot

In the equivalent certain market it is ambiguous if postmerger or premerger profits are larger. Their difference is given by

$$\Delta^C = \pi_{1+2}^M - (\pi_1 + \pi_2) = 2b\left(\left(\overline{q_1^M}\right)^2 + \left(\overline{q_2^M}\right)^2 - \overline{q_1}^2\overline{q_2}^2\right) + 2d\overline{q_1^M q_2^M}.$$

The fact that sometimes it is disadvantageous to merge in the quantity setting game is a result of the downward sloping best-response functions. The insiders can not internalize full effect of their quantity cut as outsiders free ride and increase their quantities.

In the stochastic market the difference between expected profits in the presence and absence of the merger is

$$\begin{aligned} E\left(\pi_{1+2}^{M}\right) - E\left(\pi_1 + \pi_2\right) &= \Delta^{C} + \left[(b+d)\, B^2 - bA^2\right]\left(V_1' + V_2'\right) + (b-d)\, B^2 Z_{12}' \\ &\quad - B^2 2\left[2b + (n-1)\, d\right]^2 d\left(\sigma^2 - \sigma_{12}\right). \end{aligned}$$

The change in incentives due to the introduction of uncertainty, is

$$\begin{aligned} I_U^C &\equiv E\left(\pi_{1+2}^{M}\right) - E\left(\pi_1 + \pi_2\right) - \Delta^{C} \\ &= \left[(b+d)\, B^2 - bA^2\right]\left(V_1' + V_2'\right) + (b-d)\, B^2 Z_{12}' \\ &\quad - B^2 2\left[2b + (n-1)\, d\right]^2 d\left(\sigma^2 - \sigma_{12}\right) \end{aligned}$$

Even if all the correlation terms are zero it is not possible to say if uncertainty adds to the attractiveness of a merger or decreases it in general. In this respect, even knowing the sign of merger profitability in the deterministic market is not always sufficient to determine the sign of the change in the incentives due to uncertainty. Consider for example when d tends to b. Then, the market is arbitrarily close to a homogenous market where, in the deterministic case, a merger between two firms decrease their profits when $n \geq 3$. However, under uncertainty, the positive term(the second one) in the absence of correlation increases without bound as d tends to b (since Z' has $b - d$ to square in the denominator) suggesting that there must be a d^* above which the effect of uncertainty is always positive. Thus, it is not possible to decide on the effect of uncertainty on merger profitability even if one knows that the merger is detrimental to the total profits of two firms in the deterministic environment. Nevertheless, if a merger is profitable in the deterministic case, then uncertainty makes it more attractive.

Proposition 7. *With Cournot competition, if there is no correlation between shocks of the firms, and the merger is profitable in the deterministic setting then $I_U^C > 0$, i.e. the uncertainty contributes to incentives to merge.*

Proof. When $\sigma_{kl} = 0$ for all pairs $k \neq l$, substitution gives

$$\frac{I_U^C}{B^2\sigma^2} = \frac{1}{B^2\sigma^2}\left[(b+d)B^2 - bA^2\right]\left([2b+(n-2)d]^2 + (n-1)d^2\right) - \left[4bd^2 - \frac{2b-d}{(b-d)}[2b+(n-2)d]^2 d\right],$$

If the merger is profitable in the deterministic market then the first term is strictly positive. Moreover, $\frac{2b-d}{(b-d)}[2b+(n-2)d]^2 d > 4bd^2$ when $b > d$. Thus, under the assumptions, the term on the right is positive as well as the denominator on the left, therefore $I_U^C > 0$. □

The quantities are strategic substitutes and, thus, although merged firm behaves less agressively after a merger, the outsiders to the merger are more agressive in the postmerger world(i.e. for any given parameter profile, merged entity chooses lower quantities while the outsiders choose higher quantities after a merger). These two opposing effects are at play both when the deterministic market is considered and when the uncertain market is considered. However, they have different implications on merger profitability. The output reduction of the merged entity is beneficial while expansion of the outsiders is detrimental to merger profitability in a deterministic market. On the other hand, less agressive postmerger behavior of the merged entity means that a given variance level σ^2 has a weaker direct effect on the variance of its quantities, while the more agressive postmerger behavior of the outsiders means that a given variance level σ^2 has a stronger indirect effect on the variance of merged entity's quantities(both compared to the premerger case). Since, under uncertainty, the expected profits depend positively on the variance of quantities, I_U^C can be positive even when Δ^C is negative.

Lemma 5. *Under Cournot competition the incentives of two firms to merge decrease in the covariance of their random parameters.*

Proof. The coefficient of σ_{12} in I_U^C is

$$\begin{aligned}\delta &\equiv -2b\left(B^2 - A^2\right) 2d\left[2b - (n-2)\,c\right] \\ &\quad -B^2\left[\frac{d^2}{(b-d)}\left[2b + (n-2)\,d\right]^2 + 2d^2\left[2b + (n-3)\,d\right]\right]\end{aligned}$$

and $B^2 - A^2 = -\frac{\left[(4b-d)\left[2b+(n-1)\,d\right] - 2d^2\right] d\left[2b+(n-3)\,d\right]}{(2b-d)^2\left[2b+(n-1)\,d\right]^2} B^2$. The following equality can be obtained through algebraic manipulation:

$$\begin{aligned}\frac{\delta}{B^2 d^2\left[2b+(n-3)\,d\right]} &= -\frac{4b\,(4b+d)\left[2b+(n-1)\,d\right] - 8bd^2}{(2b-d)^2\left[2b+(n-1)\,d\right]^2} d \\ &\quad -\frac{(2b-d)\left(b+2bd-d^2\right)^2 + b^2\left[4b+4\,(n-3)\,d+(2b+d)\right]}{(b-d)\,(2b-d)^2\left[2b+(n-3)\,d\right]} d \\ &\quad -\frac{(n-3)}{(b-d)} d.\end{aligned}$$

Under the given assumptions of the model the expression on the right is negative and the denominator on the left is positive therefore the coefficient of σ_{12} in I_U^C is negative. □

A negative σ_{12} means that the merged firm receives shocks that move opposite and its pair of quantities fluctuate more since they are strategic substitutes. Expected profits increase in these fluctuations, so the contribution of σ_{12} to expected profits is stronger in the postmerger case. In consequence, a firm with a negative correlation helps more to the expected profits as a partner than as a rival in the no merger case.

Proposition 8. *Under Cournot competition, all else equal, amongst its possible partners a firm prefers to merge with the one whose demand shock has highest negative correlation with its own.*

Proof. This result is directly obtained from the above lemma by reinterpretation of the parameters. Suppose that Firm 1 is considering to merge with firm 2 or firm 3. Then δ is actually the coefficient of $\sigma_{12} - \sigma_{13}$ in the difference $W_{12} - W_{13}$ where W_{12} is the increase in the total expected profits of 1 and 2 when they merge. As δ is shown to be negative, the rise in total expected profits are highest when a firm merges with the firm who has the most negatively correlated shock with its own. □

The fact that a firm's preferences over its partners are completely reversed in the Cournot case as to that of the Bertrand is closely related to the fact that prices are strategic complements in the Bertrand game and quantities are strategic substitutes in the Cournot game when *(gross)* substitute products are considered. Naturally, all this would be reversed if products that are *(gross)* complements would be considered, as quantity competition in this case is the dual of price competition in the previous case.

3.7 Conclusions

I have analyzed the effects of uncertainty on the merger decisions of firms when the resolution of the uncertainty is publicly observed although it affects firm specific variables. In the case of Bertrand competition, the uncertainty has been found to amplify the private effects of a merger which are already positive in deterministic markets. However, the result is not so clear cut when competition is of the Cournot type in which case the effects in the deterministic market are, in general, ambiguous as well. If there are incentives to merge in the deterministic case then uncertainty increases them. The fundamental difference from the private information with idosyncratic random terms case is that there are no informational gains from the merger. There, a merged firm knows more parameters than the others and benefits from this.

The preferences of the firms over possible merger partners are reversed in passing from one type of competition to the other. Amongst price setters, partners whose shocks are positively correlated are more valuable, while quantity setters would rather merge with partners who have negatively correlated shocks. This is due to the different strategic nature of the two variables.

3.8 The Appendix

Lemma 6. $G-1$ *is positive under the assumptions of the model.*

Proof. I rewrite $G-1$ as

$$\begin{aligned} G-1 &= \frac{2bc}{(2b+c)^2\,[2b-(n-1)\,c]^2}\left\{(b-c)\,[2b-(n-1)\,c]-2c^2\right\} \\ &\quad +\frac{c\,(3b+c)\,(n-1)\,c}{(2b+c)^2\,[2b-(n-1)\,c]} \end{aligned}$$

I have $(b-c)\,[2b-(n-1)\,c]-2c^2>0$ when $b>(n-1)\,c$ thus I conclude that $G-1>0$. □

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Chapter 4

Innovation in an asymmetric setting: Comparing Cournot and Bertrand equilibria

(This chapter is joint work with Ioana Chioveanu.)

4.1 Introduction

The present note compares the outcomes and the dynamic efficiency of Cournot and Bertrand equilibria in a differentiated duopoly where only one firm can invest in cost reduction. *We show that output and consumer surplus can be larger under quantity competition, and that Bertrand firms may invest more in R&D than Cournot ones. These results differ from the existing ones in the process innovation literature.*

Singh and Vives [1984] show that when a duopoly interacts only in the product market Bertrand equilibrium results in larger output, consumer surplus and welfare, and lower prices than Cournot equilibrium. Vives [1985] shows that in a differentiated products oligopoly prices are lower under Bertrand competition. These results support the view that price competition is more efficient than quantity competition when a static market is considered. However, in a dynamic setting, where firms make some strategic choices before market competition, the situation might be different.

A number of more recent contributions compare Bertrand and Cournot competition modes in differentiated duopolies, when strategic investments in research and development (for process or product innovation) precede the market game. Qiu [1997] considers a symmetric duopoly and allows R&D outcomes to spill over. He shows that Cournot firms invest more in innovation than Bertrand firms. He also demonstrates that while quantity and consumer surplus are still larger under price competition, total welfare may be larger under quantity competition if the spillovers are large and the substitutability is high. *In this paper, we find well defined examples, in the symmetric setting, outside Qiu's parameter restrictions where Cournot quantities and consumer surplus are larger than Bertrand ones.*

Bester and Petrakis [1993] use an asymmetric setting where only one firm can pay a fixed amount to achieve a discrete cost reduction. They show that the incentives to invest in process innovation can be larger under price competition if the goods are close substitutes. Their analysis does not allow for spillovers and, as they work with global methods, does not provide market outcome or efficiency comparisons.

Symeonidis [2003] complements Qiu's analysis working in a symmetric setting with quality improvement instead of cost reduction investment. In his setting, R&D outcome directly enters consumer's utility unlike cost reduction that has only an indirect effect through the quantities. Qiu's results on innovation levels and total welfare are still valid with this new type of R&D. Symeonidis shows that it is possible to have larger quantities and consumer surplus under Cournot competition if spillovers are high and goods are close substitutes. *As he points out, product R&D boosts demand and helps this result. However, the result can be obtained under process R&D, as well.*

In a tournament model, Delbono and Denicolo [1990] consider a homogenous good oligopoly where firms first engage in an R&D race for a cost-reducing patent and then compete in the market in prices or in quantities. They find that, with linear demand, the R&D investment is larger when the market competition takes place in prices. However, as they show, this investment may be too excessive compared to the socially optimal level leading welfare to be lower than the quantity competition case.

In a setting similar to that of Qiu we allow only one firm to invest in cost reduction and show that:

a. Innovation maybe larger in Bertrand than in Cournot competition if goods are close substitutes, spillovers are low and efficiency of cost reduction is high;

b. Quantities of both firms are larger in Cournot than in Bertrand competition if goods are close substitutes, and spillovers and efficiency of cost reduction are high;

c. Consumer surplus and total welfare might be higher under quantity competition than under price competition if goods are close substitutes, and spillovers and efficiency of cost reduction are high.

Our first result (a), confirms the findings of Bester and Petrakis when innovation is chosen optimally, and extends them for low, but positive spillovers. In Bertrand competition, only the level of output has a positive effect on innovation. Spillovers, strategic complementarity of the prices, and the cost of R&D negatively affect innovation. Asymmetry in the R&D abilities and low differentiation favor the output effect, while low spillovers and R&D cost decrease the negative effects, so that under these conditions Bertrand firms may innovate more than Cournot ones.

A new result is the ranking of quantities (b). Unlike the previous papers dealing with process R&D, we report that the Cournot quantities may exceed the Bertrand ones. This happens in a region where Cournot firms innovate more than Bertrand ones and where the spillovers are high. Interestingly, this result does not depend on the asymmetry of the model or on the nature of innovation (process vs. product). In the asymmetric setting, in addition, it is possible to have the quantity of the non-innovator larger under quantity competition in cases where the output of the innovator is larger under price competition.

The dynamic efficiency comparison (c) shows that, consumer surplus is higher in Cournot than in Bertrand. This result is driven by the quantity ranking. Total welfare can be higher under quantity competition, like in the symmetric case. In fact, we point out that both consumers and producers can be better off under quantity competition.

Comparison of Bertrand and Cournot equilibria can be interpreted as an analysis of the effects of increased competition on innovation and dynamic efficiency. Our analysis reveals that with a high level of product substitutability and efficient R&D technology, both the innovation level and dynamic efficiency can be ranked in any order across different types of competition, depending on the level of spillovers.

Section 4.2 introduces our linear-quadratic model with asymmetric process innovation; Section 4.3 presents the market outcomes and the efficiency measures under Cournot and Bertrand competition. The comparisons between different competition modes follow in Section 4.4. Some final conclusions are contained in Section 4.5, and all proofs missing from the text are relegated to an appendix.

4.2 The Model

Consider a differentiated duopoly producing substitute goods. In the first stage, one of the firms can invest in marginal cost reduction. The outcome of the innovation is deterministic and it may spillover to the rival. In the second stage, firms compete in the product market. We consider two alternative competition modes, allowing firms to choose quantities and, respectively, prices. The timing of the game can be justified by the fact that R&D investment is a long term decision related to the production technology, while firms can change their output level or prices faster.

Following Singh and Vives [1984] and Qiu [1997] we work in a partial equilibrium setting and assume that the utility function of the representative consumer is given by:

$$U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + 2\gamma q_1 q_2 + q_2^2\right).$$

Then, q_i is the quantity of product i, and $\gamma \in (0, 1)$ is a measure of product substitutability: Product differentiation decreases with γ.

The inverse demand function is given by:

$$p_i = \alpha - q_i - \gamma q_j \qquad i, j = 1, 2,$$

and the direct demand is given by:

$$q_i = \frac{1}{1-\gamma^2}\left[\alpha(1-\gamma) - p_i + \gamma p_j\right] \qquad i, j = 1, 2.$$

Prior the R&D investment in the first stage, the duopolists share the same production technology, having equal constant marginal cost, $c < \alpha$. The innovation capabilities are asymmetric: Only one firm can invest an amount $V(x) = \frac{vx^2}{2}$ to achieve a cost reduction of x. The parameter v is inversely related to the efficiency of the R&D activity. Notice that the innovation technology exhibits decreasing returns to scale. This is necessary for concavity of the first stage profits. The innovation outcome spills over to the rival at a rate $\rho \in [0, 1]$. Thus, at the end of the first stage the innovator has a marginal cost $c - x$, and the rival has a cost $c - \rho x$.

We solve by backward induction for the Subgame Perfect Equilibrium of the two stage game. First, we consider quantity competition in the second stage and, then, price competition. We compare the market outcomes (innovation, quantities, prices) and the dynamic efficiency (consumer surplus, profits, total welfare) of the two competition modes.

4.3 Cournot and Bertrand equilibria

Consider, first, **quantity competition in the second stage**. Firms choose an output level to maximize their profits.

$$\pi_i = q_i (p_i - c_i) = q_i (\alpha - q_i - \gamma q_j - c_i) \qquad i, j = 1, 2, \text{ and } i \neq j.$$

The Cournot-Nash equilibrium, and the corresponding profits and prices are given by:

$$q_i^C = \frac{(2-\gamma)\alpha + \gamma c_j - 2c_i}{4-\gamma^2}, \; \pi_i^C = \left[\frac{(2-\gamma)\alpha + \gamma c_j - 2c_i}{4-\gamma^2}\right]^2$$

$$\text{and } p_i^C = \frac{(2-\gamma)\alpha + \gamma c_j - (\gamma^2 - 2) c_i}{4-\gamma^2} \qquad i, j = 1, 2, \text{ and } i \neq j.$$

Let Firm 1 be the innovator. In the first stage Firm 1 chooses a cost reduction level, x, to maximize its overall profit, $\Pi_1^C = \left[\frac{(2-\gamma)(\alpha-c)+(2-\gamma\rho)x}{4-\gamma^2}\right]^2 - \frac{vx^2}{2}$. The equilibrium R&D level is:

$$x^C = \frac{2(\alpha-c)(2-\gamma)(2-\gamma\rho)}{v(4-\gamma^2)^2 - 2(2-\gamma\rho)^2}. \tag{4.1}$$

The second order condition for an interior maximum requires:

$$v(4-\gamma^2)^2 - 2(2-\gamma\rho)^2 > 0.$$

The equilibrium quantities and prices are given by:

$$q_1^C = \frac{v(\alpha-c)(4-\gamma^2)(2-\gamma)}{v(4-\gamma^2)^2 - 2(2-\gamma\rho)^2} > 0 \text{ and} \tag{4.2}$$

$$q_2^C = \frac{v(4-\gamma^2)(2-\gamma)(\alpha-c) - 2(2-\gamma\rho)(\alpha-c)(1-\rho)}{v(4-\gamma^2)^2 - 2(2-\gamma\rho)^2}.$$

Lemma 7. *In the reduced form game,*

$$v > \frac{\alpha}{c}\frac{2(2-\gamma\rho)}{(2+\gamma)(4-\gamma^2)} + \frac{2(2-\gamma\rho)\gamma(1-\rho)}{(4-\gamma^2)^2}$$

is necessary and sufficient for positive post-innovation costs, and is sufficient for the second order condition of the maximization problem, while

$$v > \frac{2(2-\gamma\rho)(1-\rho)}{(4-\gamma^2)(2-\gamma)}$$

is necessary and sufficient for $q_2^C > 0$.

Equilibrium consumer surplus and total welfare are given by:

$$CS^C = \frac{\left(q_1^C\right)^2 + 2\gamma q_1^C q_2^C + \left(q_2^C\right)^2}{2},$$
$$W^C = \frac{3\left(q_1^C\right)^2 + 2\gamma q_1^C q_2^C + 3\left(q_2^C\right)^2}{2} - \frac{v\left(x^C\right)^2}{2}.$$

Finally, consider **price competition in the second stage**. Firms choose a price to maximize their profits.

$$\pi_i = q_i\left(p_i - c_i\right) = \frac{\left(p_i - c_i\right)}{1-\gamma^2}\left[\alpha\left(1-\gamma\right) - p_i + \gamma p_j\right] \qquad i,j = 1,2 \text{ and } i \neq j.$$

The Bertrand-Nash equilibrium, and the related equilibrium profits and quantities are:

$$p_i^B = \frac{\left(1-\gamma\right)\left(2+\gamma\right)\alpha + \gamma c_j + 2c_i}{4-\gamma^2},$$
$$\pi_i^B = \frac{1}{1-\gamma^2}\left[\frac{\left(1-\gamma\right)\left(2+\gamma\right)\alpha + \gamma c_j - \left(2-\gamma^2\right)c_i}{4-\gamma^2}\right]^2,$$
$$q_i^B = \frac{\left(1-\gamma\right)\left(2+\gamma\right)\alpha + \gamma c_j - \left(2-\gamma^2\right)c_i}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)} \qquad i,j = 1,2 \text{ and } i \neq j.$$

In the first stage, Firm 1, the innovator, chooses an R&D level, x, to maximize its overall profit, $\Pi_1^B = \frac{1}{1-\gamma^2}\left[\frac{(1-\gamma)(2+\gamma)\alpha+\gamma c_j-\left(2-\gamma^2\right)c_i}{4-\gamma^2}\right]^2 - \frac{vx^2}{2}$. The equilibrium innovation is:

$$x^B = \frac{2\left(\alpha-c\right)\left(\gamma+2\right)\left(1-\gamma\right)\left(2-\gamma^2-\gamma\rho\right)}{v\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2 - 2\left(2-\gamma^2-\gamma\rho\right)^2}. \tag{4.3}$$

The second order condition of the maximization problem requires:

$$v\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2 - 2\left(2-\gamma^2-\gamma\rho\right)^2 > 0.$$

The equilibrium quantities and prices are:

$$q_1^B = \frac{v\left(\alpha-c\right)\left(4-\gamma^2\right)\left(2+\gamma\right)\left(1-\gamma\right)}{v\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2 - 2\left(2-\gamma\rho-\gamma^2\right)^2} > 0 \text{ and} \tag{4.4}$$
$$q_2^B = \frac{v\left(\alpha-c\right)\left(4-\gamma^2\right)\left(2+\gamma\right)\left(1-\gamma\right) - 2\left(\alpha-c\right)\left(2-\gamma\rho-\gamma^2\right)\left(1-\rho\right)}{v\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2 - 2\left(2-\gamma\rho-\gamma^2\right)^2}.$$

Lemma 8. *In the reduced form game,*

$$v > \frac{\alpha}{c}\frac{2(\gamma+2)(1-\gamma)\left(2-\gamma^2-\gamma\rho\right)}{(1-\gamma^2)(4-\gamma^2)^2} + \frac{2\gamma(1-\rho)\left(2-\gamma^2-\gamma\rho\right)}{(1-\gamma^2)(4-\gamma^2)^2}$$

is necessary and sufficient for positive post-innovation costs, and is sufficient for the second order condition of the maximization problem, while

$$v > \frac{2\left(2-\gamma\rho-\gamma^2\right)(1-\rho)}{(4-\gamma^2)(2+\gamma)(1-\gamma)}$$

is necessary and sufficient for $q_2^B > 0$.

Notice that these conditions require the efficiency of R&D to be quite low when the goods are very close substitutes and spillovers are not very strong: If the costs of innovation are not high enough, Firm 2 would be pushed out of the market.

In equilibrium, consumer surplus, profits and welfare are:

$$CS^B = \frac{\left(q_1^B\right)^2 + 2\gamma q_1^B q_2^B + \left(q_2^B\right)^2}{2},$$
$$W^B = \frac{\left(3-2\gamma^2\right)\left(q_1^B\right)^2 + 2\gamma q_1^B q_2^B + \left(3-2\gamma^2\right)\left(q_2^B\right)^2}{2} - \frac{v\left(x^B\right)^2}{2}.$$

Finally, using Lemma 7 and 8 we can write the necessary condition for the equilibrium innovation to be well defined under both types of product market competition.

Assumption 1:

$$v > 2\max[A, B, C] \tag{A1}$$

where

$$A = \frac{\alpha}{c}\frac{(2-\gamma\rho)}{(2+\gamma)(4-\gamma^2)} + \frac{(2-\gamma\rho)\gamma(1-\rho)}{(4-\gamma^2)^2},$$
$$B = \frac{\left(2-\gamma\rho-\gamma^2\right)(1-\rho)}{(4-\gamma^2)(2+\gamma)(1-\gamma)},$$
$$C = \frac{\alpha}{c}\frac{(2+\gamma)(1-\gamma)\left(2-\gamma^2-\gamma\rho\right)}{(1-\gamma^2)(4-\gamma^2)^2} + \frac{\gamma(1-\rho)\left(2-\gamma^2-\gamma\rho\right)}{(1-\gamma^2)(4-\gamma^2)^2}.$$

4.4 Comparisons

4.4.1 Innovation comparison

We start the comparison of the outcomes under quantity and price competition with the R&D levels given by (4.1) and (4.3) .

Proposition 9. *Suppose A1 holds, then given* γ*:*

$$a)\ x^C > x^B \text{ if } \gamma < 1 - c/\alpha,$$
$$b) \text{ If } \gamma > 1 - c/\alpha \text{ then there exists } v^*(\gamma) \text{ such that:}$$

i) $x^C > x^B$ *for all* $v > v^*(\gamma)$*, and*
ii) for any $v < v^*(\gamma)$ *there exists* $\rho^*(\gamma) \in [0,1]$ *with* $x^C < x^B$ *for* $\rho < \rho^*$*,and* $x^C > x^B$ *for* $\rho > \rho^*$*.*
Furthermore, $v^*(\gamma)$ *and* $\rho^*(\gamma)$ *increase with* γ*.*

This result is consistent with the findings of Bester and Petrakis [1993]. In their linear-quadratic model, one of the duopolists in a differentiated market can buy a fixed level cost-reduction, and the rival firm does not benefit from any spillovers. They show that the incentives to innovate -the gain in profit due to the decrease in cost- might be larger in Bertrand than in Cournot if differentiation is low.[1] Then, it is intuitive that when the innovating firm has the option to choose cost-reduction optimally, it might invest more in R&D under Bertrand competition. Our findings confirm this intuition and extend the result to the case of low but strictly positive spillovers. When the products are close substitutes, and the efficiency of R&D activity is high enough, a threshold spillover, ρ^*, can be defined such that for levels below this, Bertrand firms innovate more. This threshold value increases with the substitutability.

[1] This happens in a region where a social planner does not produce both varieties. They also show that the incentives to innovate are socially excessive under market competition if the products are sufficiently close-substitutes and the fixed cost-reduction is low. On the contrary, market competitors underinvest in process R&D if the efficiency gain is large or the products are sufficiently differentiated. In the absence of spillovers, these results should also hold in our setting.

This model deals with constant marginal cost reduction, therefore a firm with a larger output has more incentives to innovate. This market size effect is positive regardless of the competition mode. The incentives to innovate are supported by the strategic effects in Cournot competition as the quantities are strategic substitutes. After a cost-reduction the innovator expands his output and this makes the rival contract his output increasing innovator's profits. Under Bertrand competition, where the prices are strategic complements, the strategic effect on innovation incentives is negative. A cost-reduction makes the innovator lower his price, inducing a price cut by the rival which decreases innovator's profits. The difference in the strategic effect is strong enough, so that symmetric firms invest more in R&D in Cournot competition. In our asymmetric setting, when spillovers are low, the cost reduction favors the innovator, whose market share is larger than in the symmetric setting. This makes the positive market size effect be stronger when only one firm innovates, and explains why Bertrand firms may invest more than Cournot ones.[2] However, this result depends on low differentiation, low spillovers and efficiency of the R&D activity. Both the spillovers and the costs of innovation have a negative effect on the levels of R&D, so they counteract the positive market size effect. If spillovers are high the market share advantage of the innovator gets smaller, and the results are similar to the symmetric setting. When the goods are close substitutes firms compete more and output is higher. Then, the market size effect helps Bertrand firms innovate more than the Cournot ones when spillovers are low. This still holds in the limit when the goods become perfect substitutes. An instance with perfect substitutes where Bertrand firms invest more in R&D is the tournament model considered by Delbono and Denicolo [1990].[3]

[2] Cournot R&D investment is larger in the asymmetric setting compared to the symmetric one if $\gamma - 2\rho > 0$ and Bertrand R&D investment is larger in the asymmetric setting if $\gamma - 2\rho + \gamma^2\rho > 0$. Also $\gamma - 2\rho > 0$ is necessary for $x^B > x^C$ in the asymmetric game.

[3] Notice that when firms engage in an R&D race for a cost-reducing patent the resulting marginal costs are asymmetric and the winner can make profits in the market game even if the products are homogenous.

4.4.2 Quantity comparisons

Consider, first, the quantities of the innovator.

Proposition 10. *Suppose A1 holds, then given* γ*:*

$$a)\ q_1^B > q_1^C \text{ if } \frac{\alpha}{c} > 3 \text{ or } \gamma < \gamma^* = \frac{2\left(\frac{\alpha}{c} - 1\right)}{\frac{\alpha}{c} + 1},$$

$$b)\ \text{If } \frac{\alpha}{c} > 3 \text{ and } \gamma > \gamma^* \text{ then there exists } v^{**}(\gamma) \text{ such that:}$$

i) $q_1^B > q_1^C$ *for all* $v > v^{**}(\gamma)$*, and*
ii) for any $v < v^{**}(\gamma)$ *there exists* $\rho^{**}(\gamma) \in [0,1]$ *with* $q_1^B > q_1^C$ *for* $\rho < \rho^{**}$*, and* $q_1^B < q_1^C$ *for* $\rho > \rho^{**}$*.*

Several conditions are necessary for the innovator to produce more under Cournot competition:

- Marginal cost before innovation, c, has to be high enough relative to total market demand;
- Product differentiation should be sufficiently low;
- R&D costs should not be too high;
- Spillovers have to be strong.

In the absence of innovation Cournot firms produce less than Bertrand firms. In order for the dynamic effects to overturn this ordering, the marginal cost reduction under quantity competition should be sufficiently high compared to the reduction under price competition. Hence, initial marginal cost has to be large enough for Cournot firms to achieve a significant cost advantage over Bertrand ones. Similarly, for the innovation under Cournot to be significant, the R&D technology should be efficient, and the products should be close substitutes. Low differentiation leads to stronger competition, and makes cost reductions more valuable. Unlike the former determinants, the spillovers have a negative effect on innovation. However, this negative effect is more detrimental in the case of price competition. For instance, in the extreme case of almost homogenous products and perfect spillovers, cost reduction is worthless for the innovator in the Bertrand market while it is still valuable in the Cournot one.

Next, we examine the quantities of Firm 2, prices and consumer surplus when Firm 1 produces more under Cournot competition.

Proposition 11. *If $q_1^C > q_1^B$ then $q_2^C > q_2^B$, and, consequently, $p_1^C < p_1^B$, $p_2^C < p_2^B$ and $CS^C > CS^B$.*

In a static model, for any given cost difference between duopolists, the quantity difference between low cost firm and high cost one is lower in Cournot competition. This is due to the low cost firm's less competitive behavior in quantity competition. In both types of competition, this quantity difference increases in cost difference at a decreasing rate with the rate being slower in Cournot. The innovator produces more in quantity competition when innovation is significantly larger for Cournot firms. However, the cost advantage gained is less significant since strong spillovers are necessary for this case. It turns out that at equilibrium innovation levels quantity difference is lower in quantity competition when $q_1^C > q_1^B$, and it follows that firm 2 is producing more as well. In fact, it is possible for this firm to be producing more in quantity competition even when the innovator produces more in price competition. For example, when $\alpha = 7$, $c = 3$, $\gamma = 0.9$, $\rho = 0.95$, $\upsilon = 0.63$ we have that $q_1^C = 2.338$, $q_1^B = 2.346$ together with $q_2^C = 2.213$, $q_1^B = 2.211$ at equilibrium.

When both quantities are larger under Cournot competition, it follows that prices are lower and consumer surplus is higher than in Bertrand competition. If only Firm 2 produces more under quantity competition, consumer surplus ordering depends on the amplitude of $q_1^B - q_1^C$ relative to the amplitude of $q_2^B - q_2^C$. In the previous numeric example, consumer surplus is larger under price competition, $CS^B = 9.8666$ $> CS^C = 9.8202$, and both prices are higher under Cournot competition, $p_1^B = 2.6636$

$< p_1^C = 2.6745$ and $p_2^B = 2.6771 < p_2^C = 2.6866$. However, considering $v = 0.625$ and the other parameters same as before, we obtain $q_1^C = 2.3468$, $q_1^B = 2.3504$ and $q_2^C = 2.2243$, $q_1^B = 2.2143$ at equilibrium. Consumer surplus is larger under quantity competition, $CS^B = 9.8978 < CS^C = 9.9255$, and both prices are higher under Bertrand competition, $p_1^B = 2.6567 > p_1^C = 2.6513$ and $p_2^B = 2.6703 > p_2^C = 2.6636$.

The fact that both quantities can be larger under Cournot competition is not driven by the asymmetry of the model. In a symmetric setting, Qiu [1997] reports that Bertrand quantities are always larger than Cournot ones whenever a necessary condition for the social planner's problem to have an interior solution holds.[4] Nevertheless, there are parameter ranges where both Cournot and Bertrand equilibria are well defined, and symmetric output is larger under quantity competition than under price competition.[5] In the symmetric setting, the conditions for well defined equilibria are more restrictive.[6] Symmetric post-innovation marginal costs approach zero faster because spillovers flow in both directions. With asymmetric R&D abilities, it is possible to have positive post innovation costs for more efficient innovation technology. Then, the resulting higher innovation levels allow the asymmetric Cournot quantities to be larger.[7]

[4]Under his condition optimal post-innovation costs are positive for *any* γ *and* ρ. This condition is sufficient, but not necessary for the market equilibria to be well defined.

[5]For instance, with $\alpha = 7$, $c = 3$, $\gamma = 0.95$, $v = 1.25$ and $\rho = 0.99$, the symmetric quantity is larger in Cournot, $q^C = 2.1495 > q^B = 2.1208$. In fact, for these parameters, in the asymmetric game, quantities are larger in Bertrand, $q_1^C = 1.6682 < q_1^B = 2.0396$ and $q_2^C = 1.6595 < q_2^B = 2.0284$.

[6]The positive post innovation costs constraint does not allow for relatively more efficient R&D technology. There is a range of low values for v, where the asymmetric equilibria are well defined, but the symmetric ones are not.

[7]This happens when spillovers and substitutability are high, so that Cournot firms innovate significantly more than Bertand ones. Notice that in the example in footnote 5, despite high spillovers and substitutability, asymmetric quantities are larger in Bertrand. This is due to the relatively high R&D cost that is needed for interior symmetric equilibria.

4.4.3 Welfare and profit comparisons

Singh and Vives [1984] show that Cournot duopolists make larger profits than Bertrand ones when they have the same profile of (possibly asymmetric) marginal costs. This means that, given a level of innovation, x, the profits of Firm 1 are higher under quantity competition, $\Pi_1^C(x) > \Pi_1^B(x)$. In our model, Firm 1 optimally chooses an innovation level, so that $\Pi_1^C\left(x^C\right) > \Pi_1^C\left(x^B\right)$ with x^C and x^B being the equilibrium R&D levels in Cournot and, respectively, in Bertrand. Then, it follows that $\Pi_1^C\left(x^C\right)$ $> \Pi_1^C\left(x^B\right) > \Pi_1^B\left(x^B\right)$, the equilibrium profits of the innovator are larger under Cournot competition than under Bertrand competition. The rival does not choose an optimal R&D level, it only benefits costlessly from spillovers whenever $\rho > 0$. The ranking of his profits depends on $sign\left(q_2^C - \sqrt{1-\gamma^2}q_2^B\right)$. From Proposition 3 it follows that whenever $q_1^C > q_1^B$ at equilibrium, Firm 2's profits are also larger under quantity competition than under price competition, $\Pi_2^C > \Pi_2^B$. These results together lead to $W^C = CS^C + \Sigma_i \Pi_i^C > W^B = CS^B + \Sigma_i \Pi_i^B$ and we have the following proposition.

Proposition 12. *Suppose A1 holds, then at equilibrium:*

$$i)\ \Pi_1^C > \Pi_1^B,$$
$$ii)\ if\ q_1^C > q_1^B\ then\ \Pi_2^C > \Pi_2^B\ and\ W^C > W^B.$$

Total welfare can be higher under Cournot even when both quantities are larger under price competition. This is due to the fact that, when innovation is higher in quantity competition, the benefits of larger cost-reduction may compensate the negative effect that lower output has on consumer surplus. This was observed by Qiu in his symmetric set-up. He showed that, in a dynamic model, quantity competition can produce more welfare even when quantities are larger in price competition. For our set-up, Proposition 12 already reports the possibility of Cournot competition being dynamically more efficient than the Bertrand one. However, the cases covered by this proposition do not conclude all situations where this occurs. For example, when $\alpha = 7$, $c = 3$, $\gamma = 0.9$, $\upsilon = 0.65$, $\rho = 0.95$ we have $q_1^C = 2.2852 < q_1^B = 2.33$ and $q_2^C = 2.1705 < q_2^B = 2.2002$, but $W^C = 17.294 > W^B = 11.516$.

4.5 Extensions and conclusions

R&D activity can focus on cost-reduction or, alternatively, on quality improvement. In Appendix B, we show that our ranking of the R&D levels extends to the case of product innovation. We consider a duopoly facing a linear quality-augmented demand following Symeonidis [2003]. In the first stage, only one firm can buy a fixed quality increase, and in the second stage, competition takes place in quantities or prices. We identify a parameter equivalence that proves that all the results of Bester and Petrakis generalize to the case of product R&D. As our model suggests, these results should continue to hold when the firm can optimally choose a product R&D level. That is, *in an asymmetric model of product innovation, it is possible to have larger R&D levels under Bertrand competition than under Cournot if products are not too differentiated*. This contrasts with the results of Symeonidis who shows that, in a symmetric model, innovation is always larger under quantity competition.

In a model where only one of the duopolists engage in cost reducing R&D we have shown that under price competition the innovating firm can be reducing costs *more or less* than under quantity competition depending on the level of product differentiation, the rate of spillovers and the R&D efficiency. Furthermore, we show that the duopoly can produce more of both products under Cournot competition leading to a higher surplus both for consumers and producers. Thus, a priori, both the ordering of innovation and the market quantities between the two competition modes are ambiguous, and the previously mentioned parameters play a crucial role in their determination.

4.6 Appendix A

Proof of Proposition 9.

$$sign\left[\frac{x^C - x^B}{2(\alpha - c)}\right] =$$
$$sign\left[v\left(4-\gamma^2\right)^2(1-\gamma)(1+\rho) - 2(1-\rho)\left(2-\gamma^2-\gamma\rho\right)(2-\gamma\rho)\right]$$

The condition for $x^B > x^C$ is

$$v < \frac{2(1-\rho)\left(2-\gamma^2-\gamma\rho\right)(2-\gamma\rho)}{\left(4-\gamma^2\right)^2(1-\gamma)(1+\rho)} \equiv D.$$

For this condition to hold under $A1$ we need the following signs to be positive:

$$sign\,(D-2A) = sign\,(D-2C) = sign\left[\frac{(1-\rho)(1-\gamma\rho)}{(1-\gamma)(1+\rho)} - \frac{\alpha}{c}\right],$$
$$sign\,(D-2B) = sign\,(\gamma - 2\rho)\,.$$

First notice that if $\gamma < 1 - c/\alpha$ then the sign of $D - 2A$ is negative for any $\rho \in [0,1]$ and, consequently $x^C > x^B$ whenever $A1$ holds. Second, when $\gamma > 1 - c/\alpha$ noticing that D is decreasing in ρ and letting $\rho = 0$ gives $v^*(\gamma) = \frac{4\left(2-\gamma^2\right)}{\left(4-\gamma^2\right)^2(1-\gamma)}$ with $x^C > x^B$ for any $v > v^*(\gamma)$. For any $v < v^*(\gamma)$ define $\rho^* = \min\left[\gamma/2, y, z\right]$ where y solves $v = \frac{2(1-y)\left(2-\gamma^2-\gamma y\right)(2-\gamma y)}{\left(4-\gamma^2\right)^2(1-\gamma)(1+y)}$ (there exists such $y \in [0,1]$ since $y = 0$ leads to $v < v^*(\gamma)$ and $y = 1$ leads to $v > 0$ and v is continuous) and

$$z = \frac{1}{2\gamma}\left[\left(1+\gamma+\tfrac{\alpha}{c}(1-\gamma)\right) - \sqrt{\left(1+\gamma+\tfrac{\alpha}{c}(1-\gamma)\right)^2 - 4\gamma\left(1-\tfrac{\alpha}{c}(1-\gamma)\right)}\right].$$

For any $\rho > \rho^*$ either $D - 2B$ or $D - 2C$ is negative or $v > D$, thus $x^C > x^B$ whenever $A1$ holds. If $A1$ holds and $\rho < \rho^*$ then we have $x^B > x^C$. □

Proof of Proposition 10.

$$sign\left(q_1^B - q_1^C\right) =$$
$$sign\left[v\left(4-\gamma^2\right)^2(1-\gamma) + 2(2-\gamma)\gamma^2 + 2(2-\gamma\rho)(2\gamma - 2 - \gamma\rho)\right]$$

The condition for $q_1^C > q_1^B$ is

$$v < \frac{2\left[(2-\gamma\rho)(2+\gamma\rho-2\gamma)-(2-\gamma)\gamma^2\right]}{\left(4-\gamma^2\right)^2(1-\gamma)} \equiv E$$

For this condition to be holding under $A1$ we need the following signs to be positive:

$$sign\,(E-2A) = sign\left[2-2\gamma+\rho\gamma-\gamma^2+\rho\gamma^2-\rho^2\gamma^2-\frac{\alpha}{c}(2-\gamma\rho)(1-\gamma)\right],$$

$$sign\,(E-2B) = sign\left[4\rho-2\gamma-\rho\gamma^2-2\rho^2\gamma+\rho\gamma^3\right],$$

$$sign\,(E-2C) =$$

$$sign\left[2-2\gamma+\rho\gamma-2\gamma^2+\gamma^3+\rho\gamma^2-\rho^2\gamma^2-\frac{\alpha}{c}(1-\gamma)\left(2-\gamma^2-\gamma\rho\right)\right].$$

We have that if $sign\,(E-2A)$ is positive $sign\,(E-2C)$ is positive as well.
First, notice that if $\frac{\alpha}{c} > 3$ then $sign\,(E-2A)$ is negative for any $\rho \in [0,1]$, and for any $\gamma \in (0,1)$ leading to $q_1^B > q_1^C$ under $A1$. Second if $\gamma < \frac{2\left(\frac{\alpha}{c}-1\right)}{\frac{\alpha}{c}+1} \equiv \gamma^*$ then $sign\,(E-2A)$ is negative for any $\rho \in [0,1]$, consequently $q_1^B > q_1^C$ under $A1$. For part b), noticing that E increases in ρ and letting $\rho = 1$ gives $v^{**}(\gamma) = \frac{2}{(4-\gamma^2)}$ with $q_1^B > q_1^C$ for any $v > v^{**}(\gamma)$. For any $v < v^{**}(\gamma)$ define $\rho^{**}(\gamma) = \max[f,g,h]$ where

$$f \text{ solves } 2-2\gamma+f\gamma-\gamma^2+f\gamma^2-f^2\gamma^2-\frac{\alpha}{c}(2-\gamma f)(1-\gamma) = 0,$$

$$g \text{ solves } 4g-2\gamma-g\gamma^2-2g^2\gamma+g\gamma^3 = 0,$$

$$h \text{ solves } v\left(4-\gamma^2\right)^2(1-\gamma)-2\left((2-\gamma h)(2+\gamma h-2\gamma)-(2-\gamma)\gamma^2\right) = 0.$$

For each equation LHS has different signs when 0 and 1 are substituted for the corresponding variable thus $f,g,h \in [0,1]$.
For any $\rho < \rho^{**}$ either $E-2A$ or $D-2B$ is negative or $v > E$, and $q_1^B > q_1^C$ whenever $A1$ holds. If $A1$ holds and $\rho > \rho^*$ then we have $q_1^C > q_1^B$. □

Proof of Proposition 11.

$$sign\left(q_2^B - q_2^C\right) = sign\,(F+G) \text{ where}$$

$F = v\gamma^2 \left(4-\gamma^2\right) \left[v \left(4-\gamma^2\right)^2 (1-\gamma) + 2(2-\gamma)\gamma^2 + 2(2-\gamma\rho)(2\gamma-2-\gamma\rho)\right]$ and $signF = sign\left(q_1^B - q_1^C\right)$,
$G = -2v\left(4-\gamma^2\right)^2 (1-\rho)\gamma^2(1-\rho\gamma) + 4\gamma^2(1-\rho)(2-\gamma\rho)\left(2-\gamma^2-\gamma\rho\right)$
Assume $q_1^C > q_1^B$ then

$$v\left(4-\gamma^2\right)^2(1-\gamma) + 2(2-\gamma)\gamma^2 + 2(2-\gamma\rho)(2\gamma-2-\gamma\rho) < 0. \tag{5}$$

We claim that G is strictly negative. Assume to the contrary that G is non-negative. Then

$$v\left(4-\gamma^2\right)^2(1-\rho\gamma) - 2(2-\gamma\rho)\left(2-\gamma^2-\gamma\rho\right) \leq 0. \tag{6}$$

Summing up inequalities (5) and (6) gives a contradiction:

$$v\left(4-\gamma^2\right)^2(2-\gamma-\rho\gamma) + 2(2-\gamma)\gamma^2 < 0.$$

Thus G is strictly negative whenever F is strictly negative. We conclude that $q_2^C > q_2^B$ if $q_1^C > q_1^B$. It is straightforward to check that if $q_2^C > q_2^B$ and $q_1^C > q_1^B$ then $p_2^C < p_2^B$, $p_1^C < p_1^B$ and $CS^C > CS^B$. □

4.7 Appendix B

Consider the linear-quadratic model of Bester and Petrakis [1993]. In the first stage, Firm 1 can buy a cost reduction of Δ by paying a fixed amount. In the second stage, firms compete in prices or quantities. Using the profits in the reduced form game, we can compute the innovation incentives of the firms.

$$I_C = \pi_1^C (c_1 - \Delta, c_2) - \pi_1^C (c_1, c_2) = \frac{4\beta^2\Delta [(2\beta - \gamma)\alpha - 2\beta c_1 + \gamma c_2 + \beta\Delta]}{(4\beta^2 - \gamma^2)^2},$$

$$I_B = \pi_1^B (c_1 - \Delta, c_2) - \pi_1^B (c_1, c_2) = \frac{(2\beta^2 - \gamma^2)\beta\Delta \left[2\alpha(\beta - \gamma)(2\beta + \gamma) - (2c_1 - \Delta)(2\beta^2 - \gamma^2) + 2\beta\gamma c_2\right]}{(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)^2}.$$

Bester and Petrakis show that for high values of the substitutability parameter (γ), it is possible to have $I^B > I^C$.

Consider now the quality augmented linear-quadratic model of Symeonidis [2003] with zero marginal cost, and a similar game. Firm 1 can buy a quality increase of Δ by paying a fixed amount. In the second stage, firms compete in prices or quantities. Using first stage profits, we can compute the innovation incentives of the firms.

$$I_C^* = \pi_1^C (u_1 + \Delta, u_2) - \pi_1^C (u_1, u_2) = \frac{16\Delta (4u_1 + 2\Delta - \sigma u_2)}{(16 - \sigma^2)^2},$$

$$I_B^* = \pi_1^B (u_1 + \Delta, u_2) - \pi_1^B (u_1, u_2) = \frac{2(8 - \sigma^2)\Delta \left[(8 - \sigma^2)(2u_1 + \Delta) - 4\sigma u_2\right]}{(4 - \sigma^2)(16 - \sigma^2)^2}.$$

Letting $c_i = 1 - u_i$, it can be shown that $I_C = I_C^*$ and $I_B = I_B^*$, for $\alpha = 1, \beta = 2, \gamma = \sigma$. Therefore, when substitutability is high, the incentive to invest in product quality may be higher under price competition. In fact, all results of Bester and Petrakis will continue to hold under product innovation, including comparisons with social incentives.

4.8 References

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Chapter 5

Comparing Bertrand and Cournot prices: A case of substitutes and complements

5.1 Introduction

It is an old conventional wisdom that Bertrand competition leads to lower prices than Cournot competition. Strongly supporting results have been obtained in certain types of differentiated products markets. This note, widens the scope of the belief to markets where substitutes and complements coexist. Under certain symmetry criteria, price competition results in lower prices for these mixed markets as well.

Vives [1985] confirms that Bertrand competition is more efficient when firms produce gross substitute products, or when they operate in a symmetric industry.[1] The first result is built on the fact that the price setting game is supermodular in the case of substitutes. The latter one relies on a contraction condition. This note relaxes the symmetry assumption to the extent of allowing coexistence of substitutes

[1] The symmetric industry can be complement type or the substitute type.

and complements, and dispenses with the contraction condition allowing multiple of equilibria. I provide conditions for the existence of equilibria leading to symmetric prices. When they are fulfilled, there is always a Bertrand equilibrium with a lower price for any Cournot equilibrium leading to a single price.

Okuguchi [1987] works with a model including complements and substitutes where the demand system satisfies a dominant diagonal condition. He finds that, at a Cournot equilibrium, firms would be willing to cut prices, if they were to choose prices.[2] Starting from this, he derives a comparison for the equilibrium prices in the two games. Although, his method relies on a mathematical theorem about non-negative solvability of linear equation systems, not on supermodular theory; in essence the result is driven by the assumption of supermodularity of the price game. When complement products exist in the model, this is a very strong supposition. Indeed, the fact, that the best response of a firm is increasing in the prices of some of its rivals and decreasing in the prices of others, is the main drawback for a straightforward comparison in the mixed case. Even though, starting from the Cournot equilibrium prices all firms decrease their prices in the initial stage of the price adjustment process, it is ambiguous where to the dynamics lead afterwards. Amir and Jin [2001] provide a triopoly example,[3] where one firm chooses a higher price in the price game while the others comply with the convention. In such situations where there is no unilateral ordering of equilibrium prices, the value of comparing them is questionable as the products are differentiated.

5.2 Demand system

Consider a market for n differentiated products where substitutes and complements coexist. I assume that the product set N has a partition formed of two sets, namely N_1 and N_2 such that within each set products are *gross substitutes* and across sets

[2] I rediscover this behavior under a milder assumption.
[3] There are two gross substitutes and a third product which is a gross complement to both.

they are *gross complements* according to the C^2 demand system h. Formally:

$$\partial h^i/\partial p_j \geq 0 \text{ if } i,j \in N_1 \text{ or } i,j \in N_2$$
$$\partial h^i/\partial p_j \leq 0 \text{ if } i \in N_1 \text{ and } j \in N_2, \text{ or } i \in N_2 \text{ and } j \in N_1.$$

I assume that the Jacobian of the demand, J_h is a negative definite matrix for any p such that $h^i(p) > 0$ for all i. Then, J_h is invertible in any rectangular region X in the price space which leads to positive demands for each firm, yielding the Jacobian of the inverse demand system f, denoted by J_f.

Let M be the matrix obtained by replacing the k^{th} column e^k of the identity matrix I_n by $-e^k$ for each $k \in N_2$. Then, the matrix $Z = M^{-1}J_hM$ is a negative definite matrix.[4] However, Z has all the offdiagonal elements nonnegative which implies that its inverse $Z^{-1} = M^{-1}J_fM$ has only nonpositive elements.[5] All this amounts to say that in such a product set, J_h being negative definite guarantees the preservation of substitute and complement relations with respect to the partition in the inverse demand system*:*

$$\partial f^{\,i}/\partial q_j \leq 0 \text{ if } i,j \in N_1 \text{ or } i,j \in N_2,$$
$$\partial f^{\,i}/\partial q_j \geq 0 \text{ if } i \in N_1 \text{ and } j \in N_2, \text{ or } i \in N_2 \text{ and } j \in N_1.$$[6]

Denote the elasticity of the demand by η_i and the elasticity of the inverse demand by ε_i, so $\eta_i = -(p_i/q_i)\left(\partial h^i/\partial p_i\right)$ and $\varepsilon_i = -(q_i/p_i)\left(\partial f^{\,i}/\partial q_i\right)$. Differentiating the identity $p_i = f^{\,i}\left(h^1(p), .., h^n(p)\right)$ with respect to p_i gives $1 = \left(\partial f^{\,i}/\partial q_i\right)\left(\partial h^i/\partial p_i\right) + \sum_{j\neq i}\left(\partial f^{\,i}/\partial q_j\right)\left(\partial h^j/\partial p_i\right)$. The previous analysis shows that each element of the second term is nonpositive, so $\left(\partial f^{\,i}/\partial q_i\right)\left(\partial h^i/\partial p_i\right) \geq 1$. Thus, as demonstrated in Vives (1999), for any price vector p and any product i we have $\varepsilon_i \geq 1/\eta_i$ (strict whenever a cross product is negative).

[4] By Lemma 20.3 on page 362 in Nikaido [1968].

[5] By Theorem 6.3 on page 95 in Nikaido [1968].

[6] Okuguchi (1987) uses a dominant diagonal assumption where $\left|\frac{\partial h^i}{\partial p_i}\right| > \sum_{j\neq i}\left|\frac{\partial h^i}{\partial pj}\right|$ for all i to establish the same.

5.3 Equilibrium price comparison

Assume that for each firm i, the cost function $c_i(q)$ is twice differentiable, increasing and convex. Then, at any interior Cournot equilibrium

$$\frac{p_i^C - c_i'\left(h^i\left(p^C\right)\right)}{p_i^C} = \varepsilon_i \text{ for each firm } i.$$

The optimal price adjustment requires

$$\frac{p_i - c_i'\left(h^i\left(p_i, p_{-i}\right)\right)}{p_i} = \frac{1}{\eta_i}.$$

As $\varepsilon_i\left(p^C\right) \geq 1/\eta_i\left(p^C\right)$ starting from a Cournot equilibrium, the best response of a firm i in the Bertrand game would satisfy $\phi_i\left(p_{-i}^C\right) \leq p_i^C$. Whenever the Bertrand game is supermodular this price cutting behavior at p^C can be used to show the existence of a Bertrand equilibrium with lower prices than any Cournot equilibrium.[7] However, for the current setting where substitutes and complements coexist, a supermodular price-setting game is a forced assumption. Nevertheless, it is possible to make a comparison when some symmetry is introduced.

Let $p_r \in R^{n-1}$ be a vector with all components equal to $p \in R$ and let $\psi^i\left(p_i, p\right) = h^i\left(p_i, p_r\right).$ In words, p_i is the firm's own price and p is a common price charged by its rivals in the derived demand ψ^i.

Definition 1. *The demand system h is two-dimensionally symmetric when $\psi^i\left(.\ , p\right) = \psi\left(.\ , p\right)$ for all i.*

Example 1. $h_A = 1 - p_A + \dfrac{p_B}{3} - \dfrac{p_C}{2}$, $h_B = 1 - p_B - \dfrac{p_C}{6}$,
$h_C = 1 - p_C - \dfrac{p_A + p_B}{12}$.

Example 1 presents a two-dimensionally symmetric demand system that is quite different from the usual symmetric demand. Nevertheless, when coupled with symmetric costs, this property guarantees a symmetric Bertrand equilibrium.

[7] Vives [1985].

Proposition 13. *If the cost structure is symmetric and the demand system is two-dimensionally symmetric with a price limit $\overline{p}$ at which no firm receives a strictly positive demand when it is the single price in the market, there exists a symmetric Bertrand equilibrium where all firms charge a price $p^B \leq p^C$ for any symmetric Cournot equilibrium price p^C.*

Proof. Notice that $\phi_i(p) = \phi_j(p)$ for all i, j whenever p is a symmetric price vector. Then, the restriction of the best response correspondence to symmetric price vectors can be represented by a continuous mapping ϕ_{sym} from $[0, \overline{p}]$ to itself. Take the lowest symmetric Cournot equilibrium price vector p^C, we know that $\phi_{sym}\left(p^C\right) \leq p^C$. Thus $\phi_{sym}\left(p^C\right)$ is a point below the diagonal in the square with side $[0, \overline{p}]$ in which the graph of ϕ_{sym} lies. Then ϕ_{sym} must have crossed the diagonal at least once below p^C, at a point p^B which is a symmetric Bertrand equilibrium price. *QED.* □

Notice that a two dimensionally symmetric demand system h does not guarantee a Cournot equilibrium leading to symmetric prices. Such equilibrium exists for sure, if additionally J_h is symmetric with equal diagonal elements and each of its rows is a permutation of another.[8] An example in the linear case is provided below.

Example 2. $h_A = 1 - \frac{5}{4}p_A + \frac{p_B}{2} - \frac{p_C}{3} - \frac{p_D}{4}$, $h_B = 1 - \frac{5}{4}p_B + \frac{p_A}{2} - \frac{p_D}{3} - \frac{p_C}{4}$, $h_C = 1 - \frac{5}{4}p_C + \frac{p_D}{2} - \frac{p_A}{3} - \frac{p_B}{4}$, $h_D = 1 - \frac{5}{4}p_D + \frac{p_C}{2} - \frac{p_B}{3} - \frac{p_A}{4}$.

If similar assumptions were put on the inverse demand system instead, the result would be: For any symmetric Cournot equilibrium, a Bertrand equilibrium leading to a higher quantity for each product exists.

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