# *Models of Reasoning*

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# Agradecimientos

*"La vida no es la que uno vivió, sino la que recuerda y cómo la recuerda para contarla".* Con esta frase empieza Márquez su biografia. A lo largo de la vida conoces a muchas personas; de algunos no te acuerdas, de otros te acuerdas de vez en cuando, pero eres consciente de que todos ellos han marcado tu vida.

Cuando llegué al departamento eran las 10 menos cuarto de la mañana del 14 de septiembre de 1998. Al lado de una puerta ponia *Secretary IDEA* y me acerqué (debo confesar que estaba bastante perdida), entró una mujer con una sonrisa tranquilizadora, que me invitó sentarme, para explicarme un montón de cosas que, al oirlas de ella, parecían fáciles. Con el paso del tiempo aquel despacho se iba a convertir en mi confesionario y Mercè en alguien con quién siempre puedes contar.

Mi primera clase en el programa se anuló, porque el profe que tenía que darla acababa de tener un hijo. Lo vi como una buena señal, pero nunca pensé que este profe que me iba enseñar Estadística y que se llama Xavier Vilà, se iba convertir en mi director de tesis. Que Kaniska iba a empezar a llamarle *mi papa* y así se va quedar sin que él lo sepa. Al niño le pusieron Javier. Se lo puso su hermana Paula y a los dos los quiero muchísimo. He crecido mucho gracias a ellos.

Recuerdo el examen de Micro y la cara de comprensión de David. Sigo sin entender porqué le tiene tanto miedo la gente si es tan buena persona. No sé muy bien cuando nos hicimos amigas Ariadna y yo, quizás fue cuando me llamó por telefono para felicitarme para mi cumple. No importan ni la fecha y ni la hora, sino la amistad.

Juan Enrique consiquió que odiara las matemáticas durante un tiempo, pero

lo que me enseñó quedó para siempre. No sé si sabe, que gracias a él, me di cuenta de que tengo muy buena intuición matemática. Sí que aprendí que tengo que demostrar las cosas que parecen obvias. Para Ouayl las matemáticas eran su amor y hablaba de los conjuntos con mucho cariño. Ya sé que un conjunto cerrado se puede sentir cuando lo aprietas con las manos y uno abierto no.

Mi segundo año en IDEA fue duro. Me operaron y las dos personas que habián decidido acompañarme Ariadna y Ana lo pasaron peor que yo. Aquella experiencia me enseño muchas cosas, sobre todo que tengo amigos aquí, aunque algunos de ellos, como Kaniska, me quieren tanto que se preocupan demasiado.

El verano del 2000 fue decisivo para mi. Fui a San Sebastián y conocí a mucha gente. Algunos de ellos serían mis amigos, como Marco, y otros me harian ver mi trabajo desde otro punto de vista, como Gilboa.

El tercer año en IDEA fue cuando más trabajé y cuando mejor me lo pasé. Hacía cosas que me gustaban. Tenía con quien apostar una cena "a que voy a demostrar que éste grafo tiene diagonal $n-1$" y ganarla, aunque Joydeep nunca entendió qué magia hacía yo.

Todo esto son hechos, algunos fuera de contexto, pero ahora recuerdo las cenas del cumpleaños de Inés (que en teoría nunca eran para celebrar su cumpleaños) y todas las críticas después de mis charlas. Y todas las Pascuas de los búlgaros, donde nos lo pasábamos tan bien con Nasko, Rosi y Natalia, pintando huevos y preparándolo todo.

Hay muchos tipos de amigos, pero algunos son amigos del alma y no hace falta decirles mucho, ya lo saben todo, y Dora ya sabe que esto va para ella.

Mis padres y mi tía han soportado algo que quizás poca gente entienda cómo de difícil ha sido para ellos tenerme lejos tanto tiempo.

Toca acabar con los recuerdos y seguir adelante.

Muchííííííííííííííííííííííííssimas gracias a todos vosotros, no os olvidaré.

Mi Dani se pondrá muy contento cuando acabe la tesis y luego le tendré que dar las gracias por haber insistido.

Sólo antes de acabar tengo que decir:

Muchas gracias Bebo, por haberme elegido como tu madre.

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# Contents

# Chapter 1

# Introduction

It is not easy to write an introduction to something, which has been worked on during the last years. Many things have changed since I began to write this thesis.

People say that problems one studies are rooted somewhere in the personal life of the the author, but those roots are never cited inside the papers. In a certain way all problems with which this thesis deals appeared somehow natural in my life.

When I decided that there might be something interesting in economics, it was because by chance I read the papers of Gary Becker [9] and Akelof and Dickens [2]. Those papers where quite old but they were focusing on questions in economics, which I have never seen from this perspective during my undergraduate studies. I found them interesting and decided that I will work on something similar.

What made them interesting to me - a difficult question. Probably the perspective towards human behavior. I had some questions like; why, when I saw for the first time Prisoner's Dilemma and I was asked what I will do I said - cooperate. Or why, when I was asking my students in class - what they will do, many of them were willing to cooperate. Why the theory was not giving the same answer? What was missing there?

Constructing a model, which goes deeper in human reasoning and tries to explain for example why people keep going to a coffee-shop offering horrible coffee or why a student during an experiment, never though of adapting to the feedbacked he was receiving and this caused him loses, became a challenge.

Many people told me that I better give up with those issues. That there are open questions there, but if people do not manage, they should be very difficult. This made me look for a tool, which will allow me to go deeper inside the black box of human behavior. Simulation techniques where one of the options, which allowed me to look at the behavior of the agents from outside and go back to the code and see, which was the cause for this behavior. Computational economics models are usually quite simple constructions, which allow for complex behavior.

This thesis focuses on three problems.

The first one is the importance of the way the information is exchanged in the context of Repeated Prisoner's Dilemma game. In Chapter 2 we build a simulation model imitating the structure of human reasoning in order to study how people face a Repeated Prisoner's Dilemma game. The results are ranged starting from individual learning in which case the worst result -defection- is obtained, passing through a partial imitation, where individuals could end up in cooperation or defection, and reaching the other extreme of social learning, where mutual cooperation can be obtained. The influence of some particular strategies on the attainment of cooperation is also considered. Those differences in the results of the three scenarios we have constructed suggest that one should be very careful when deciding which one to choose.

Chapter 3 is a joint work with Xavier Vilà and studies the process of coalition formation when players are unsure about the true benefit of belonging to a given coalition. Under such strong incomplete information scenario, we use a Case-Based Decision Theory approach to study the underlying dynamic process. We show that such process can be modeled as a non-stationary Markov process. Our main result shows that any rest point of such dynamics can be approached by a sequence of similar "perturbed" dynamics in which players learn all the information about the value of each possible coalition

In Chapter 4 we study the dynamics of an experience good market using a two-sided adaptation Agent Based Computational Economics (ACE) model. The main focus of the analysis is the influence of consumers' habits on market structure. Our results show that given characteristics of consumers' behavior might sustain the diversity in the market both in terms of quality and firms' size. We observe that the more adaptive one side of the market is, the more the market reflects its interests.

# Chapter 2

# One Dilemma - Different Points of View

## 2.1 Introduction

Probably the most difficult problems are those that seem simple, and the most studied those faced by many people. A problem belonging to both groups is the so called Prisoner's Dilemma. The fable, which made it famous, is a story about two suspects in a crime who are put in separated cells and told the following rules: if both of them confess, each will be sentenced to three years in prison, if only one of them confesses, he will be free and the other one will be send to jail for four months, if neither of them confesses they will be send to prison only for one month. What makes the problem interesting is the conflict between the personal interest to defect (D) and confess, compared to the collective interest of cooperating (C) with the opponent. If the game is played only once then both suspects have no common interest and they will defect. Solving the repeated problem becomes a challenge. On the one hand, the solution of the classical Game Theory to a Finitely Repeated Prisoner's Dilemma is defection in all periods, but on the other, experiments reveal a significant level of cooperation. This contradiction raises the question of whether there exist some conditions, which promote cooperation.

In particular this work focuses on the learning process, and analyzes how different models of learning, can influence the final outcome of a Repeated Prisoner's Dilemma game.

We build an Agent Based Computational Economics (ACE) model of the repeated game in which, player's learning is modeled using an explicit evolutionary process - genetic algorithms. The pioneer in simulating Repeated Prisoner's Dilemma (RPD) was Axelrod [7], who applied genetic algorithms to evolve RPD strategies against a constant environment. His work was continued by many economists and computer scientists' papers. Among them is the paper of Miller [25], who studies the evolution of cooperation, starting from random population of strategies, which evolve in a changing environment. The authors mentioned above model learning at population level, i.e. each player learns from the best individuals in the population, assumption that is difficult to justify in the context of PD. This model of learning implicitly means that a player imitates the other players strategies, even from players he has never played with, which is equivalent to removing the assumption of no communication and changing the essence of the dilemma. With this scenario cooperation is easily obtained.

Another possibility is to construct a scenario of individual learning, where each individual learns only from his own experience and this becomes a reason for mutual defection.

The distinction between individual and social learning was studied in the work of Vriend [33], where he analyzed Cournot oligopoly and found that with social learning the results converge to the Walrasian output level, whereas with individual learning they converge to the Cournot - Nash outcome.

Our results move in the same directions as those of Vriend, i.e. social learning leads to socially optimal outcome, and individual to the egoistic one.

Having constructed these models we can notice that both assumptions are too extreme. People do not only learn from the others, neither do they learn only from proper experience. A scenario, which combines both assumptions, is more realistic. Here arises the problem of how to construct it. If we use the classical way of creating new strategies, half of the experiments end up defecting and the other half cooperating. But again we face a similar problem - how can a player imitate a strategy that he does not know. This problem can be avoided by using a different procedure, where each player is trying to understand and imitate to same extent (without copying the strategy) the behavior of his opponent (Vilà [32]). This reduces the number of cooperative outcomes and confirms that the procedure of creating new strategies is not irrelevant to the outcome, in other words the way the information is exchanged is important.

Under this scenario the elements that determine the outcome are the type of strategies the players have at the beginning and how fast is the process of evolution. If the evolution process is fast it is more likely that players end up defecting, but if it is slow they will experiment with more strategies and could obtain cooperation.

To analyze the importance of players' strategies we introduce players who always play a given strategy. The effects they induce are different. A player who always either defects or cooperates will induce defection, but one who plays a variation of Tit-For-Tat (TFT) (cooperates when the opponent is observed to cooperate and defects when the opponent has defected) makes his opponent cooperate.

The rest of the chapter is organized as follows. Next Section provides some technical details. Section 2.3 presents the model. Section 2.4 is dedicated to the results and the last Section 2.5 summarizes the results and concludes.

## 2.2 Finite Automata and Adaptation

This section is dedicated to explaining some technical details, which are necessary for constructing the model.

### 2.2.1 Finite Automata

Finite Automata model a system which responds to discrete inputs and outputs. In our context, the automata we are using for representing player's strategies is a Moore machine and has the following structure. The machine consists of 8 internal states and each internal state has 3 elements:

**1.** Action to be taken if the machine enters in this state;

**2.** Transition state if the opponent is observed to cooperate and

**3.** Transition state if the opponent is observed to defect.

There is one state, which has a different structure from the one explained above. It is called initial state and defines from which state the machine will start. The machine follows a simple algorithm.

**1.** Find out, which is the initial state and go there.

**2.** Take the action specified at this state;

**3.** Observe the action the opponent has taken

**a)** if the opponent has cooperated, check which is the transition state specified for this case and go to step 2;

**b)** if the opponent has defected, check which is the transition state specified for this case and go to step 2;

Repeat $r$ times, where $r$ is the number of rounds.

Each element of the machine is codified as binary decimal. Once codified the machine looks like this:

$$***|* *** ***|* *** ***|...|* *** ***|* *** ***|$$

where the stars replace 0's and 1's. The first three stars define the initial state. The first star of one internal state defines the action to be taken at that state. It is 0 if the machine cooperates and 1 if it defects. The first three stars define the transition state if the opponent has been observed to cooperated and the last three stars if he has defected.
If a state has this form:

$$|0\ 001\ 000|$$

it implies that if the machine enters this state it will cooperate and if the opponent cooperates it will go to state 1, if not to state 0.
Using this method a huge variety of machines can be created. How complex is a strategy depends on the number of states the machine enters. Notice that the fact that the machine has 8 states does not imply that the machine uses 8 states. It might very well be that there are states, where it never enters.

### 2.2.2 Adapation and Genethic Algorithms

Genetic algorithms are search algorithms developed by Holland (1975).The main goal of his research, was to abstract the adaptive process of the natural systems and design an artificial system, which follows it. The mechanism of the algorithm resembles the natural process of evolution and has the same elements:

1. Reproduction;

2. Selection - evaluation of the current existing forms according to their fitness to the environment;

3. Variation - generates new forms via:

    a) crossover

    b) mutation

These type of algorithms can be applied to solving problems in different domains and have been mainly used for solving problems in domains with enormous search space and objective function with non linearities, discontinuity or high dimensions. Their main advantages in our case are their robustness and the lack of any rationality assumptions.

## 2.3 The Model

The purpose of our work is to distinguish between three models of learning - social, individual and imitation. We start by building a basic framework which can be easily adapted to one of the three scenarios.

Two individuals are about to play a RPD. When facing the problem for the first time, each of them is assumed to have $K$ randomly generated strategies.[1]

Let $S_i^k$ be the $k$ strategy of individual $i$ ($k = 1,..K$, $i = 1, 2$). For the purposes of our model we need an abstraction of the process by which the player implements this strategy. This role will be played by a machine called finite automata.

Having constructed the sets of strategies for both players they are ready to play. Each player chooses one of his strategies to play $r$ times PD. When the repeated game is over the strategy $S_i^k$ receives payoff $\Pi_i^k$ being the sum of the payoffs in each single game. We assume that the next time players meet they will experiment with another strategy.

Once the players have tried all their strategies, they analyze the results obtained. A strategy that has relatively high payoff will be kept in the memory as a good one, and used in the future, and the strategies that performed bad will be replaced with a combination of the existing strategies. A strategy that performed well in one trail is not necessarily good, it just appeared to be good against a given strategy of the opponent. For example, a strategy that always defects will be a very good choice if played against a strategy that always cooperates, the first one will have an average payoff of five and the second of zero. But the first strategy may be a bad choice against TFT.

The process explained above is evolutionary and will be modeled using Genetic Algorithms. In our context the population will be each player's strategies and the environment they play in - his opponent's strategies.

Using this framework and changing only the process of formation of the new strategies we are able to construct the three scenarios.

[1] A strategy is a plan how to behave in all possible circumstances.

The first one resembles the model of Miller [25] and almost coincides with the classical framework in this problem. In Miller's model, each agent has only one strategy and the strategies of all agents evolve together i.e. there is only one population evolving. We use the same structure but our agents have more strategies. The strategies of both players evolve together, forming one population in which good strategies will be kept and bad replaced with a combination of the current strategies. This can be the case when players talk and discuss different plans. The result Miller obtained in the case of perfect information is that at the end all individuals have the same strategy and their behavior converges to the cooperative result.

Since the assumptions of the social learning scenario contradict with the essence of the dilemma, another possibility is to construct the other extreme - individual learning, where each player learns only from his own experience. Players stay at different rooms and they do not try to interpret each other's behavior, but just adapt. Technically this is a two population model based on Vilà [32], where a repeated discrete principal-agent game is analyzed. The difference between this model and the social learning is that now each player can use only his own strategies. The process of learning is internal and is based only on proper experience.

The two scenarios, as described above are good benchmark models, but too extreme. A better option is the imitation scenario, which combines the assumptions of the previous two. Players are not allowed to communicate, but they do make an attempt, to understand each other. It starts like the individual learning scenario until the moment when each player has his strategies with the same structure. After this point each of them imitates the strategy of his opponent. If we use the classical method for creating new strategies (single cut crossover[2]) we are facing the same problem as in the social learning. In order to avoid it we use a different procedure for generating new strategies i.e. different type of crossover due to Vilà [32]. Our player can not observe the strategies of his opponent, but he observes the history of the repeated game, which includes both players actions. Let $h_i$ be the history of player's $i$ actions. Then assume that player 1 asks himself - what would I do if I were him and somebody played against me in the way I have played against him. Since he does not know the strategy of his opponent he can only imagine playing with his strategy, from his opponent's position a history of random length and comparing his action after this history with the real action of the opponent. If they do not coincide player one replaces his action with his partner's action.

All the variations of the original framework are summarized in the following table:

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*The structure of the model:*

1. Generate K strategies for individual i, i=1,2;

2. Randomly match each strategy of one individual with one from the other;

3. Repeat until all strategies are played once. Each strategy receives payoff $\Pi_i^k$ i=1,2. k=1,..K;

4. Form new strategies

4.1 Social Learning (as in Miller [25])

4.2.Individual Learning

a) include the best K/2 strategies of each individual in his new group

[2]With single cut crossover you combine two strategies by cutting them in two parts and interchanging the second parts.

of strategies;

b) select two strategies of an individual i to be parents. The probability of strategy $S_i^k$ of being selected is:

$$P(S_i^k) = \frac{\Pi_i^k}{\sum_k \Pi_i^k} \tag{2.1}$$

c) create K/2 new strategies for each individual using the crossover and then apply the mutation;

4.3.Imitation - steps a, b and c remain the same until both individuals have their strategies with the same structure. After that step b is changed with

b′) one of the strategies selected belongs to the individual himself and the other is the best strategy of his opponent.

5. Repeat steps 1÷4 R times, where R is the number of repetitions.

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## 2.4 The Results

### 2.4.1 Parameter Values

The simulations were performed under the following conditions:

| | |
|---|---|
| Number of individuals | 2 |
| Number of strategies of each individual | 4 |
| Number of rounds | 300 |
| Number of repetitions | 10 000 |
| Probability of mutation | 0.001 |
| Overlapping generations | 1/2 |
| Crossover type | partial imitation or single cut |
| Length of bit string | 60 |

Payoff structure

| | C | D |
|---|---|---|
| C | 3 , 3 | 0 , 5 |
| D | 5 , 0 | 1 , 1 |

(2.2)

The results are robust to changes in most of the parameters (number of strategies, length of bit string and payoff structure), but the choice of some of them requires some discussion. One of them is the number of strategies. Usually the size of the population chosen in similar models is higher, but having in mind that in our context the population consists of strategies, it is difficult to assume that people have 50 or 100 strategies. We have run simulations with 50 or 100 strategies, but the results obtained were not different. The number of rounds and the number of repetitions, were chosen to guarantee convergence. The probability of mutation determines, among other things, how adaptive the players are. The higher is the probability of mutation, the easier it will be for one player to adapt to the changes of

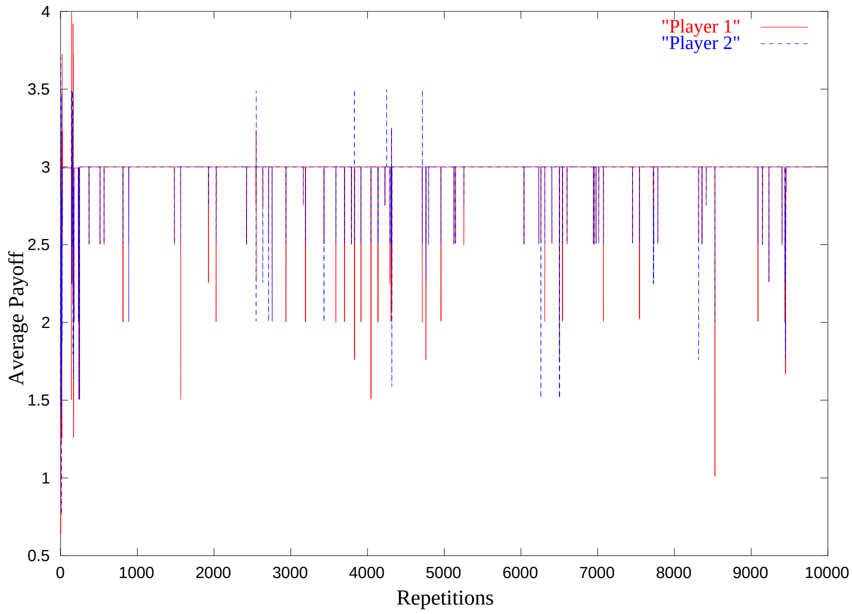


Figure 2.1: In the case of *Social Learning* players cooperate and receive average payoff of three.

the behavior of the other. But if it becomes too high they will modify their behavior too often. The value chosen is standard.

### 2.4.2 Results

Our result under the social learning scenario coincides with the one obtained in the literature i.e. cooperation. The outcome is independent of the type of crossover used. A typical evolution of the average payoffs of both players is depicted in Figure 2.1.

The same stability of the result is obtained also in the individual learning case but the outcome, as Figure 2.2 shows, is defection.

One possible explanation is that, when the evolution is common, the strategies that the players will have in the next period come out of the same

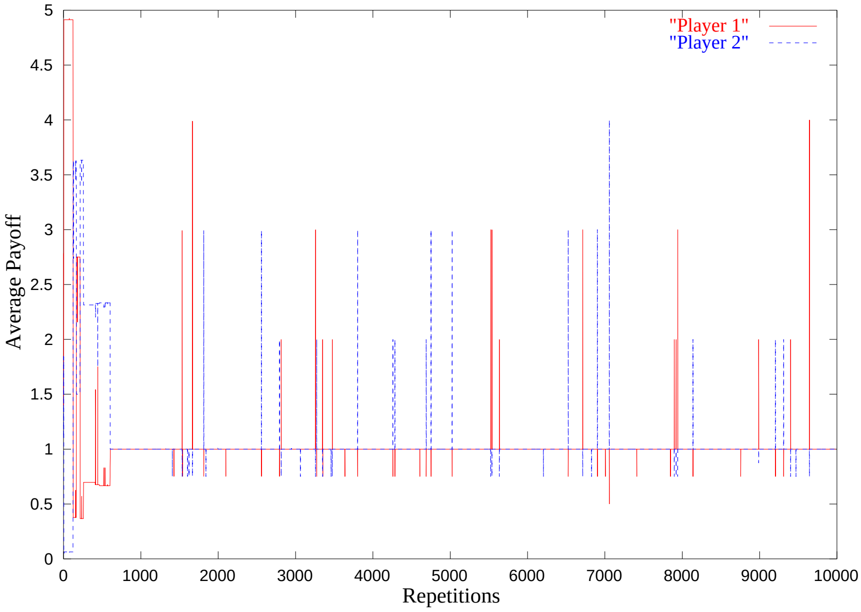


Figure 2.2: In the scenario of *Individual Learning* the only possible result is defection, which gives both players an average payoff of one.

process. Therefore it is very likely that they will be similar for both players and they will move together. Hence they have to choose between cooperate or not, and the first outcome is clearly better.

The imitation scenario has two possible results depending on the type of crossover assumed. If we use the traditional crossover (single cut) players can end up defecting or cooperating depending on how fast is the learning process and what type of strategy do they have before the imitation begins. This difference can be seen in the two graphs at Figure 2.3. Intuitively if players learn slowly the search field becomes bigger and the possibility of cooperation increases. But at the same time even if they learn fast but at least one of them uses a strategy that induces cooperation this is enough to make cooperation the only possible outcome.

In order to check the influence of different strategies we introduce a player who always plays the same strategy. We have chosen three types of players - player who cooperates, a player who defects and a player who plays TFT. If the first two are introduced they induce defection, but the third one is able to "teach" his opponent to cooperate. One possible explanation for the experiments, which converge to defection is that it is very difficult for a strategy like TFT to appear in a random environment.

If instead of adopting the classical crossover we choose the one that does not violate the assumptions of no communication the only possible result as depicted in Figure 4 is defection.

## 2.5 Concluding remarks

The results of the three scenarios we have constructed suggest that one should be very careful when deciding which one to choose.

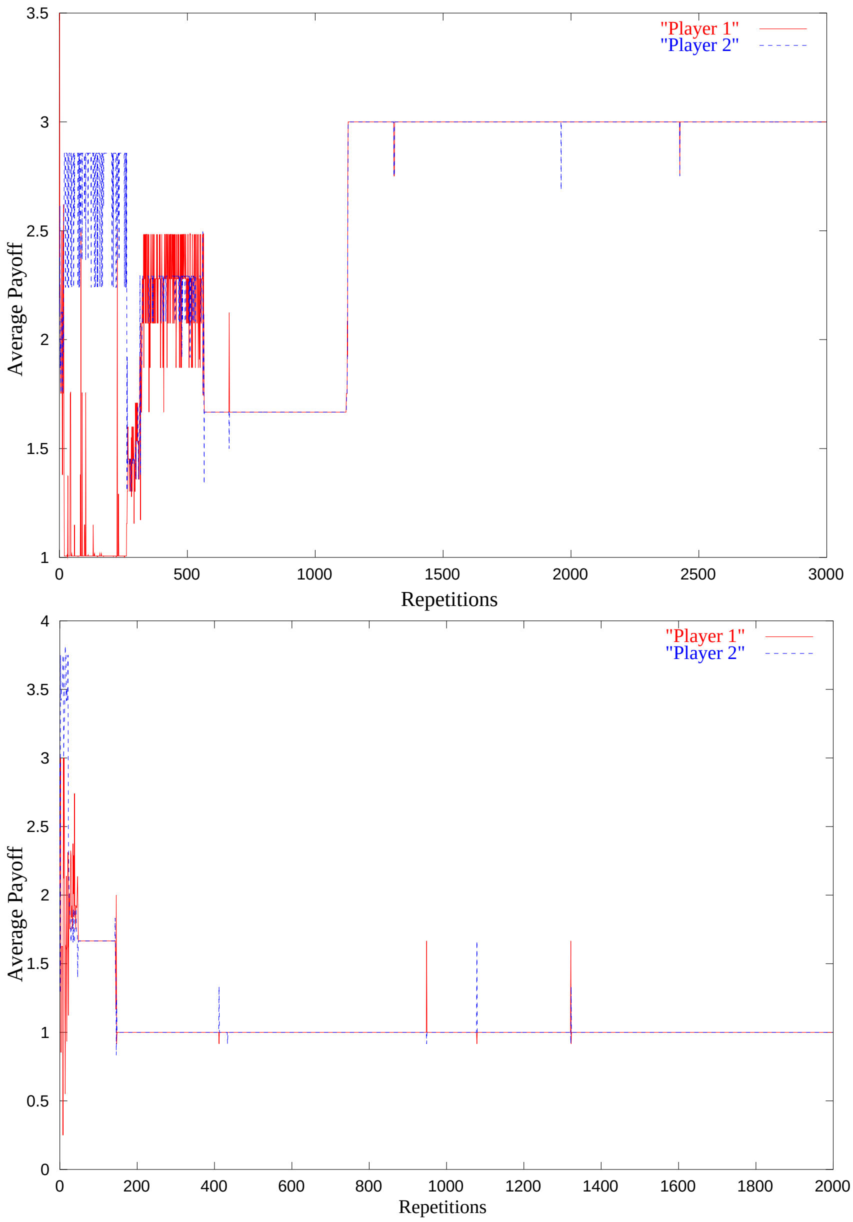


Figure 2.3: *Imitation scenario* The left graph depicts a case in which the learning process is slow and the result is cooperation. Exactly the opposite can be seen at the right graph, where the players end up defecting

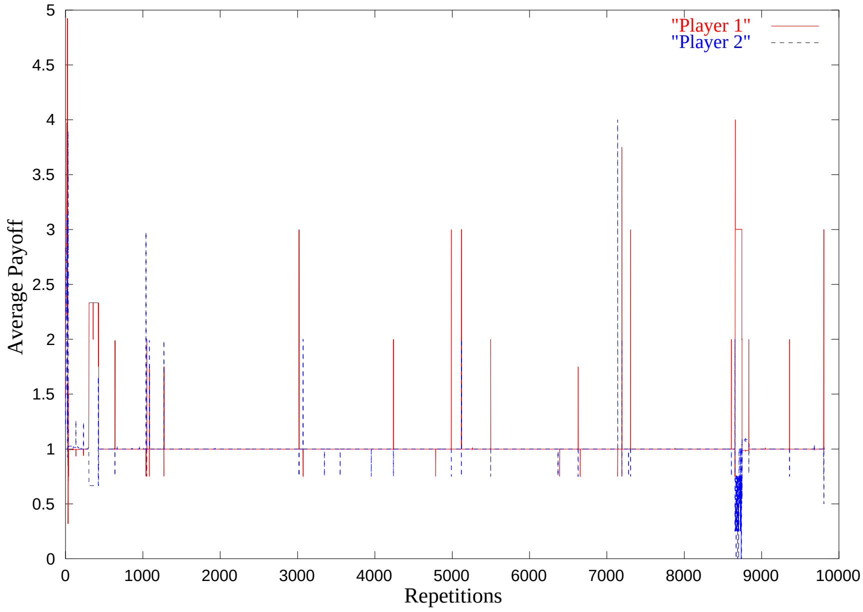


Figure 2.4: *Imitation scenario with partial imitation crossover*. The result is defection.

Special attention should be given to the assumption of no communication, which we found to be very influential. Intuitively if people cannot talk this lowers the confidence and decreases the probability of cooperation. Going back to the initial structure of the dilemma, if the two suspects can discuss, they will be able to find plans or strategies, which guarantee that neither will confess. Making the learning process internal, complicates it and independently on the possible attempts of reasoning about the other person's behavior, the cooperation fails. The imitation of the opponent's behavior leads to cooperation only if one of the agents plays TFT.

Summarizing, we have found two conditions, that promote cooperation. On the one hand, that is the possibility of communication and on the other the influence of a TFT player.

# Chapter 3

# Case-Based Coalition Formation

(joint with Xavier Vilà)

## 3.1 Introduction

Coalition Formation has traditionally been studied within the field of *Cooperative Game Theory*. Recently, though, researchers have focused their attention on other issues that are important for coalition formation and that had not been taken into account. Examples are considerations of credibility, strategic behavior on the side of individual players, farsighted strategies, and so on. In this sense, *non-Cooperative Game Theory* has started to play an active role in the study of coalition formation.

In this paper we approach the analysis of coalition formation in a situation in which individual players have *incomplete information* regarding the effective value that belonging to a specific group of people reports to them. For instance, firms involved in a merging project do not know in reality how their profits will be affected until the very moment in which the merging actually takes place. Also, countries discussing trading agreements do not actually know how their wealth or welfare might improve until a treaty is implemented. To our knowledge, this is a new approach to the topic and we think that it deserves some attention.

To start with, we focus our attention on *hedonic games*[1]. To deal with the fact that players do not know how valuable is for them to belong to a given coalition until the very moment such coalition is formed, we use a derivative of *Case-Based Theory* ([19], [20], and[21] ) by Gilboa and Schmeidler suited to our framework. Loosely speaking, Case-Based Theory assumes that when a player faces a new problem, his decision will be based on similar problems that he has solved successfully in the past.

Our framework is a very *simple* dynamic coalition formation model. Starting with a given situation (configuration of coalitions), any player decides at any moment whether to remain in the group he currently belongs to, or joint one of the other existing coalitions. If some of these *"other coalitions"* is a group to which the player has never belonged to, he will evaluate the group by comparing it to "similar" groups he has belonged to at some point and, hence, knows the *true* value they had for him.

In some sense, this model is close to the analysis by Konishi and Ray [17], with two main differences. First, they allow for *coalition moves* (i.e., not only individual players can move at some point, but also existing coalitions may merge or split) whereas in our model only individual moves are allowed, which emphasizes our non-cooperative approach. Second in their model players have perfect information about the value that each possible coalition has for them.

The rest of the paper is organized as follows. Section 2 introduces the

[1]Games in which each player only cares about people that belong to his group.

model. Section 3 provides an example that illustrates how difficult might be to reach an "optimal" coalition structure if players are *imperfectly informed* and how our approach overcomes these difficulties.

## 3.2 The Model

### 3.2.1 Coalition structures

Let $N = \{1, 2, \dots n\}$ be the set of players. A *coalition* is a set of one or more players. That is, there are as many as $2^n - 1$ non-empty possible *coalitions*. Any player $i \in N$ can choose from as many as $2^{n-1}$ possible coalitions. A *coalition structure* is a partition of $N$ into *coalitions*. Let $\mathcal{S}$ denote the set of all possible coalition structures. The cardinality of such set is given by the Bell number, which does not have a closed form but can be computed recursively in the following way: if $n$ is the number of players, then the number of possible *coalition* structures (excluding empty coalitions) is given by $b(n)$ as follows:

$$b(n) = \sum_{k=0}^{n} \begin{pmatrix} n-k \\ n \end{pmatrix} b(n-k-1), \text{ where } b(0) = 1$$

Formally, an element of $\mathcal{S}$ will be represented by a $n \times n$ matrix $S$. In order for such matrices to make sense as coalition structures, the following conditions must be satisfied

1. $s_{ij} \in \{0, 1\} \, \forall i, j \in N$
2. $s_{ii} = 1 \, \forall i \in N$
3. $s_{ij} = s_{ji} \, \forall i, j \in N$

4. $[s_{ij} = 1, s_{jk} = 1] \Rightarrow s_{ik} = 1$

The interpretation of such matrices as *coalition structures* is quite simple. For any coalition structure $S \in \mathcal{S}$, the entry $s_{ij}$ tells whether player $i$ and player $j$ belong to the same coalition. In this sense, $s_{ij} = 1$ means that players $i$ and $j$ do belong to the same coalition, whereas $s_{ij} = 0$ stands for the opposite. With this interpretation, conditions 2, 3 and 4 above are hence necessary so that the matrix $S$ makes sense as a coalition structure.

**Example** Let $N = \{1, 2, 3, 4, 5\}$. In this case there are 31 different non-empty coalitions and, hence, 52 possible coalition structures in $\mathcal{S}$. One such coalition structure could be represented by the following matrix as described above

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

This matrix represents, thus, the coalition structure best viewed as

$$\{\{1, 3\}, \{2, 4\}, \{5\}\}$$

Although the key point in our approach is that players do not know beforehand[2] how good (or bad) is a particular coalitions structure $S \in \mathcal{S}$, we

[2]As explained in the introductory section, we explicitly assume that players do not know what is the "value" of a given coalition structure until the moment that particular structure is realized.

assume that a certain true and objective[3] value function does exist. Formally this is a function

$$v : N \times \mathcal{S} \to \Re \tag{3.1}$$

That is, the function $v$ tells for each player $i \in N$ and for any coalition structure $S \in \mathcal{S}$ what is the "value" (utility, payoff, etc.) that this particular structure $S$ has for player $i$. This function could be understood as *profits* if players are firms seeking mergers.

For the following definition, let $S_i$ denote the $i-$th row of matrix $S$ (that is, the specification of what players belong to the same coalitions as player $i$)

**Definition 1** The function $v$ is said to be *hedonic* if

$$S_i = S'_i \Rightarrow v(i, S) = v(i, S') \ \forall i \in N \, \text{and} \, \forall S, S' \in \mathcal{S}$$

In our analysis, players will move from one coalition to another following a dynamic process that will be specified. Such process will be driven by player actions or *moves.* In this sense, a *move* is an action taken by a player that results in him entering (or leaving) a coalition. The following will be assumed regarding *moves* for player $i \in N$

1. Only *individual moves*[4] will be allowed: at any time only one player gets to move. That is, the dynamics can not go from the structure $\{\{1, 2\}, \{3, 4\}\}$ to $\{\{1, 2, 3, 4\}\}$ as that would require the coordinated (or agreed) move of at least 2 players.

2. Only *feasible moves* will be allowed.

[3] As opposed to subjective

[4] As opposed to *coalition moves* that allow coalitions larger than 2 merge.

Formally, the set of *feasible moves* for player $i \in N$ given the current structure $S \in \mathcal{S}$ is denoted by $M_i(S)$ and its elements are $n \times n$ matrices $M$ (whose entries are denoted by $m_{ij}$) satisfying

1. $m_{ij} \in \{0, 1, -1\}$

2. $m_{ii} = 0$

3. $s_{ij} + m_{ij} \in \{0, 1\}$

4. $m_{ij} = m_{ji}$

5. $m_{ik} = m_{ij} s_{jk} \, \forall i, j \neq i$

6. $m_{jk} = 0 \, \forall j, k \neq i \, j.k \in N$

Notice that $|M_i(S)| = \text{Rank}(S){+}1$, if player $i$ is not alone and $|M_i(S)| = \text{Rank}(S)$ otherwise.

The interpretation of this matrices as "moves" by players is quite simple. The matrix entries $m_{ij}$ just say what is the move of player $i$ with respect to player $j$. In this sense

1. $m_{ij} = 0$ means that the move of player $i$ does not change his "relation" with player $j$. That is, if players $i$ and $j$ are currently in the same coalition, they will remain in the same coalition after the move by player $i$ has been carried out.

2. $m_{ij} = 1$ means that the move of player $i$ includes joining player $j$. That is, if players $i$ and $j$ are not in the same coalition, they will be in the same coalition after the move by player $i$ has been carried out. Notice that condition 5 above ensures that if player $j$ is also with player $k$, then player $i$ will also be with player $k$ after his move has been implemented.

3. $m_{ij} = -1$ means that the move of player $i$ includes departing from player $j$. That is, if players $i$ and $j$ are in the same coalition, they will not be in the same coalition after the move by player $i$ has been carried out. Notice that condition 5 above ensures that if player $j$ is also with player $k$, then player $i$ will also depart from player $k$ after his move has been implemented.

Notice that if $S \in \mathcal{S}$ is the current coalition structure and player $i \in N$ chooses move $M \in M_i(S)$, then the structure resulting from this move, $S'$, can easily be found by

$$S' = S + M$$

In the example above, where the existing coalition structure is $\{\{1,3\},\{2,4\},\{5\}\}$ represented by the matrix

$$S = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

player 1 has 4 moves available, namely

1. $M^1$ = Leave player 3 alone and do not join any of the other existing coalitions;

2. $M^2$ = Leave player 3 alone and join the coalition composed of players 2 and 4;

3. $M^3$ = Leave player 3 alone and form a coalition with player 5;

4. $M^4$ = Remain with player 3 (do nothing).

If player 1 decides to leave player 3 alone and joint the coalition composed of players 2 and 4, its move ($M^2$) is represented by the matrix

$$M^2 = \begin{pmatrix} 0 & 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence, the coalition structure resulting from this move will be

$$S' = S + M^2 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

that corresponds to the structure $\{\{3\}, \{1, 2, 4\}, \{5\}\}$

**Definition 2** The Coalition Structure $S'$ is said to be *1-viable* from $S$ if $\exists i \in N$ and $\exists M \in M_i(S)$ such that $S' = S + M$. This is denoted

$$S \xrightarrow{1} S'$$

**Definition 3** The Coalition Structure $S'$ is said to be *t-viable* from $S$ if there are $t-1$ structure $S^1, S^2, \ldots S^{t-1}$ such that

$$S \xrightarrow{1} S^1 \xrightarrow{1} S^2 \xrightarrow{1} \cdots \xrightarrow{1} S^{t-1} \xrightarrow{1} S'$$

this is denoted

$$S \xrightarrow{t} S'$$

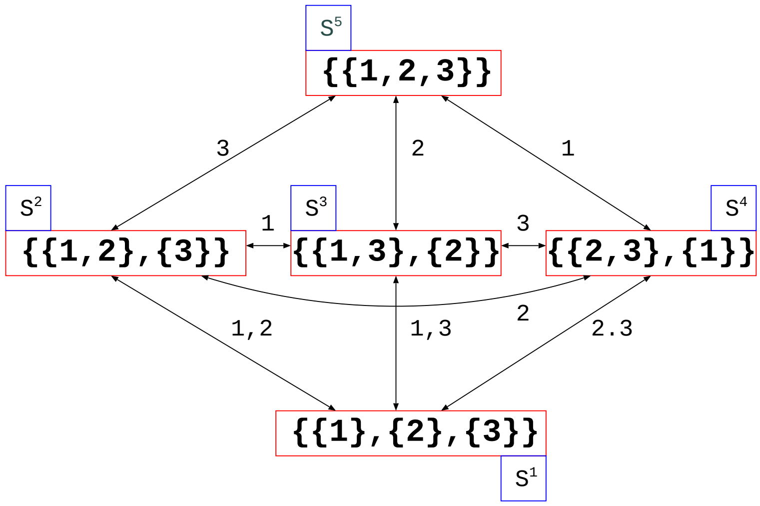


Figure 3.1: Transition graph with 3 players

The paths between coalition structures representing *1-viability* among them can be seen as a directed graph. This graphs rapidly become very involved when the number of players is high[5]. Figure 3.1 illustrates the case for 3 players

The results that follow refer to some features of such graphs

**Proposition 1** For any $S, S' \in \mathcal{S}$

**(i)** $S \xrightarrow{t} S'$ for some $t$

**(ii)** $t \leq n-1$

**Proof:** Denote with $G$ the graph, which vertices represent the possible coalition structures and edges the individual moves needed to go from one coalition structure to another.

[5]In fact, drawing the corresponding graph when the number of players is larger than 4 is already a challenge.

**Proof(i)** The proof is straightforward. If we take any coalition structure(a vertex of the graph $G$) by separating a player at each step we can reach the vertex, where each player is alone. Since any to vertices are connected with $S(0)$, there exists a path which connects them, hence the graph is connected.

**Proof(ii)** Our objective is to show that the length of the diagonal (the longest short path) of our graph is $n-1$.

Using the Breadth First Search Algorithm it is easy to show that there are $n-1$ moves between the coalition structure where all players are alone $S(0)$ and the one where all are together $S^n$. This is done by labeling the vertex of $S(0)$ with 0 and then repeatedly labeling all the adjacent vertices, which are not labeled, with the following number. Then using the Back-tracking Algorithm we find that the shortest path is n-1 and at each step starting from $S^n$ one player will be separated from the existing coalition and will not join any other. What is left to prove is that there exist no other pair of vertices whose distance exceeds $n-1$. Take any two coalition structures $S^k$ and $S^l$ ($S^k \neq S^l$ otherwise the distance is 0) where $k$ and $l$ stand for the labels they have in the previous algorithm. Notice the following, the label they have gives us the distance between this point and $S(0)$ and also the distance to $S^n$.
$d(S^k, S(0)) = k$ and $d(S^k, S^n) = (n-1) - k$
Following the properties of the distance we have the following inequalities:
$d(S^k, S^l) \leq d(S^k, S(0)) + d(S^k, S(0))$
$d(S^k, S^l) \leq d(S^k, S^n) + d(S^k, S^n)$
Rewriting we have:

$d(S^k, S^l) \leq k + l$ and

$d(S^k, S^l) \leq (n-1) - k + (n-1) - l.$

Summing up these two inequalities we get

$d(S^k, S^l) \leq n - 1.$ QED

**Proposition 2** For any $S, S' \in \mathcal{S}$

$$S \xrightarrow{t} S' \Longleftrightarrow S' \xrightarrow{t} S$$

**Proof:** Clearly if $S$ is connected to $S'$ via a path that has $t$ edges, $S'$ and $S$ are also connected via the reverse path.

### 3.2.2 Dynamics

The dynamics in our model comes from a very simple sequence of decisions by the players.

At time $t = 1$, players are initially arranged in coalitions, hence forming an initial coalition structure. Players are imperfectly informed on how valuable is for them to belong to a given coalition. Hence, at time $t = 1$, each player only knows how valuable is to belong to the current coalition. We also assume that players know how valuable is to be alone. The coalition structure in which all players are alone (no coalitions with more than one player exists) is represented by the *identity matrix* $I$. Therefore, $I \in \mathcal{S}$ represents the coalition structure were all players are alone. For notations consistency we often write $S(0) = I$.

Players have a memory that keeps track of all the coalitions they have known at some point. At time $t = 1$ the memory of each player will contain only $S(0)$ and the current structure. At any other point in time $t > 1$, the memory of a player (note that the memory is the same for all players) will contain the $k$ coalition structure that he knows. We denote the memory by $H(t)$ and we represented in a matrix $h(t)$. This matrix has on the main diagonal 1's for those states, which belong to the memory and 0's everywhere else.

At any point $t$ in time, one player in $N$ must decide whether to remain in the same coalition he belongs to or move to another coalition that is feasible given the current structure. What player in $N$ gets to choose his move at time $t$ is decided at random, having all players equal probability to be the one that chooses, namely $\frac{1}{n}$. As indicated before, the feasible moves available for player $i \in N$ at time $t$ are those in the set $M_i(S(t))$. When evaluating all the moves $m \in M_i(S(t))$, player $i$ will assign a value to all possible resulting coalition structures $S(t) + m$, $\forall m \in M_i(S(t))$ using the following criterion:

**(i)** Whenever his evaluating process involves considering a coalition structure the player knows (because such coalition structure did exist at some point in the past) he will attach to it the real value it has. If we multiply the matrix $h(t)$ by the vector $V_i$, which attaches to each coalition structure the value it has for player $i$, we obtain a vector which contains the values of the known coalitions and 0 at the places of the unknown.

**(ii)** From the above, when evaluating the move $m \in M_i(S(t))$, player $i$ will take into consideration whether the resulting coalition structure, $S(t) + m$ is in his memory (he knows the coalition he would belong to since $S(t) + m \in H(t)$) or, on the contrary, he never belonged to such coalition. In the former case, player $i$ will assign a value to $S(t) + m$ its true value. In the later case, player $i$ will assign a *utility value* to move $m$ at time $t$, $u_i(t, m)$, according to Case-based decision with respect to how similar is the resulting coalition structure $S(t) + m$ to all the coalition structures he has in the memory. The following expression combines the known and unknown coalition structure:

$$h(t) * V_i + (I - h(t)) * C * h(t) * V_i = u_i(t) \tag{3.2}$$

The first part of the expression attaches values to the coalition structures belonging to the memory and the second to the rest of the states.

$\mathcal{C}$ is a $\mathcal{S} \times \mathcal{S}$ matrix, which entries are the similarities between two coalition structures. The *similarity function* $s : \mathcal{S} \times \mathcal{S} \longrightarrow [0,1]$ is only axiomatically specified in the Case-Based Decision Theory, which allows the use of different type of functions. The functions described below are two examples of similarity function, which fit our framework.

**Definition 4** Given 2 different coalition structures $S, S' \in \mathcal{S}$, the *similarity* between them can be defined as

**(i)**

$$s(S, S') = 1 - \frac{d(S, S')}{n - 1}$$

where $d(S, S')$ is the *distance* between $S$ and $S'$, which corresponds to the smallest number of players that allow moving from $S$ to $S'$.

or

**(ii)**

$$s(S, S') = 1 - \frac{\sum_{k=1}^{n} (S_i - S'_i)^2}{(n - 1)}$$

Where $S_i$ and $S'_i$ are the $i$-th rows respectively of the matrix $S$ and $S'$.

The intuition behind these functions is different. For the first one two coalition structures will be viewed as less similar if the move from one of them to the other depends on many players. Note that this function has the same values for different players. The second function views $S$ and $S'$ as similar for player $i$ if he has some other players that are (are not) with him in both coalition structures.

The vector $u_i(t)$ allows the player to compare all possible coalition structures, but since most of them will not be reachable in one move it is sufficient to compare only the feasible moves. This gives us the following expression

$$F_i(S(t)) * u_i(t) = U_i(t) \tag{3.3}$$

Where the matrix $F_i(S(t))$ has 1's on the main diagonal if this move belongs to $M_i(S(t))$ and 0's everywhere else. Observe that since $u_i(t)$ is a function of $h(t)$, $U_i(t)$ is also a function of $h(t)$.

There are two ways to proceed at this point. If the player is taking always the optimal move this gives as a dynamics, which is significantly different from the one in which he assigns positive probability to moves which are not optimal.

In the first case we construct an indicator function, which fills in a matrix for each player by giving to each state the value of one to the optimal move, or $\frac{1}{l}$ if there are $l$ moves with equal value, and 0's elsewhere. The matrix of the transition probabilities is simply:

$$P(t) = \sum_{i=1}^{n} P_i(t) \tag{3.4}$$

In the second case the procedure is a bit different. To calculate the probabilities, with which player $i$ chooses each one of his moves we start by calculating:

$$\sigma_i(S(t)) = l * U_i(t) \tag{3.5}$$

where $l$ is a $1 \times \mathcal{S}$ vector of ones.

$$\sigma_i(S(t)) * (b(S(t)) * U_i(t)) = P_{iS(t)}(t) \tag{3.6}$$

where $b(S(t))$ is a $\mathcal{S} \times 1$ vector which has one at the current state and 0's elsewhere. Calculating those probabilities for all states and players gives us the transition probabilities at time $t$.

$$P(t) = \frac{1}{n} * \sum_{i=1}^{n} \sum_{s=1}^{\mathcal{S}} P_{is}(t) \tag{3.7}$$

Denote by $\phi(P)$ the mapping $\phi : \mathcal{P} \longrightarrow \mathcal{P}$ which maps $P(t)$ into $P(t+1)$.

With this we have completed the description of the model. Before we proceed to discuss the results notice that the process depends on the memory $h(t)$. All the equations above can be rewritten as functions of $h(t)$, which implies that if because of some reason players stop discovering new states i.e $h(t)$ remains constant this will be a fixed point for $\phi$.

## 3.3 Results

Our objective is to analyze the behavior of the dynamic system we have just described. Notice that how the system will behave will depend on many factors, in particular on the way players take their decisions (if they always choose the optimal move or make mistakes, or are willing to experiment), on the objective value function, and on the similarity function.

We begin by analyzing the dynamics of the system where players always choose the optimal move, what we call the unperturbed scenario. Observe first that in this case we need initial conditions to start the process. This is true in particular for graphs with small number of players. In the case of three players, no player will start to move if he knows only the values of two coalition structures. No matter which similarity function is chosen and which is the value function, we simply have a convex combination of two values, which at most will be equal to them if they are equal. In this case players need to know at least three different coalitions, which have similarity different from 0 with the coalition structure they are considering to move to. A general condition to start the dynamics and to guarantee afterwards that the players will continue to learn and will not get stuck into a local maxima will require in each moment the existence of an unknown move whose estimated value is superior to the value of the current coalition structure and all neighboring known coalition structures. This is a very strong condition to be fulfilled. In the most general case (without any restriction on similarity function or value function) this condition is obviously not satisfied. The result of this is that even if players start to move the process can stop fast. Observe that in the mapping $\phi$ any state in which the probability of staying there is 1 is a fixed point. If the process enters in such a state it will never leave, causing by this that players do not learn anymore and ignore the possible existence of a better coalition structure for all of them.

Another possibility is to allow players to make mistakes or tremble, or since everything is unknown decide to experiment with something new, or believe that the information they have is not sufficient to predict the behavior of a group after it has been formed (relax the myopic assumption) etc. This is what we call the perturbed scenario, in which moves that are not optimal are given positive probabilities $\varepsilon$. We then have the following result:

**Proposition 3** Under the perturbed scenario, and independently on the initial conditions, the mapping $\phi$ has a unique fixed point in which all states are discovered $h(t) = I$.

**Proof:** Proving that $h(t) = I$ is a fixed point is straightforward. Once they have discovered everything $h(t) = I$, $h(t)$ will remain constant and hence the same is true for $P(t)$.

There are two ways to prove the uniqueness part. One is just remembering that we have started by assuming that players know the coalition structure where there are alone and one more coalition structure. Assuming that there exist a $h'(t) \neq h(t)$, implies there is at least one state which is unknown and will remain unknown, hence the probability that it will be reached is 0. If this is true than the estimated value of moving to this state for all players should be 0. Since $V_i > 0$ this can be the case only if this state is not similar with any other, but all states have a positive similarity with $S(0)$, except the coalition structure where all players are together, but this state on the other hand has positive similarity with all the other states. We have reached a contradiction. The second approach is by induction. Assume that there exists another fixed point $h'(t) \neq h(t)$ and also that in there is only one coalition structure which have not been discovered yet. We will prove that this is a contradiction. Assume that this is the state $S'$, which makes the difference between the two histories. Since we have proved that the graph is connected and all the other states are known, this state has at least one neighbor $S''$, which is known, even if the move from $S''$ to $S'$ is not optimal it is feasible, and it has positive probability and we have reached a contradiction. Assume now that there are $k$ states which are unknown at the fixed point with a memory $h'(t)$. Again by connectivity of the graph, there exists a least one state which is a neighbor of a known state, follows it has a positive probability to be reached and the unknown states will remain $k - 1$. By induction all states will be reached and the fixed point is unique.

Notice also that this point is an attractor i.e independently on the initial conditions it will be reached.

The following results (Propositions 4 and 5) show that there exists a close relationship between both the perturbed and the unperturbed scenario as the size of the trembles vanishes. These results rely on the *Model of mutations* by Bergin and Lipman [10]. In such model, $\mathcal{P}$ denotes the set of Markov Matrices on $\mathcal{S}$ and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ a vector of mutation rates (trembles in our case). The following definition is due to Bergin and Lipman

**Definition 5** A model of mutations for $P$ is a continuous function $M : [0,1]^{b(n)} \to \mathcal{P}$ such that (a) $M(0) = P$, (b) $M(\varepsilon)$ is irreducible for all $\varepsilon >> 0$ and (c) the elements of the $i$th row of $M(\varepsilon)$ depend only on $\varepsilon_i$.

Notice that the perturbed scenario in our model is a special case of the $M$ function in this definition. Indeed, given an unperturbed Markov matrix in $\mathcal{P}$, the trembles map it onto another Markov matrix. We assume that $\varepsilon_i = \varepsilon$ for all feasible non-optimal moves, i.e, all non-optimal moves have the same probability of being followed.

Our model clearly satisfies the three conditions in the definition of Model of mutations above. Given a matrix $P$, condition (a) is trivially satisfied, and condition (c) is also satisfied by the way the "trembles" enter the Markov matrices $P$. Finally, condition (b) is satisfied because of the following proposition

**Proposition 4** Under the perturbed scenario, letting $\varepsilon \to 0$ induces a sequence of perturbed transition matrices $P_\varepsilon \to P$, each having a unique invariant distribution $\mu_\varepsilon$.

**Proof:** The proof is straightforward because of Proposition 3. Indeed, if "trembles" are taken into account, the $\phi$ mapping has a unique fixed point in which all states are discovered. This implies that the whole state space $S$ is minimally absorbing (no subset of it is absorbing) and hence $P_\varepsilon$ has a unique invariant distribution because of the standard properties of Markov processes.

Prior to our last result, the following definition and theorem from the Model of mutations must be introduced,where $\mathcal{I}(P)$ denotes the set of invariant distributions of $P \in \mathcal{P}$.

**Definition 6** A probability distribution $\mu$ is achievable with mutation model $M$ if there exists a sequence of strictly positive mutation rate vectors $\varepsilon^n \to 0$ such that $\mu^n \to \mu$ where $\{\mu^n\} = \mathcal{I}(M(\varepsilon^n))$. Let $\mathcal{A}(M)$ denote the set of achievable distributions with mutation model $M$.

**Theorem (Bergin and Lipman[10])** If $M$ is any mutation model for $P$, then $\mathcal{A}(M) = \mathcal{I}(P)$.

Hence, this theorem states that any invariant distribution of $P$ can be achieved (as the result of a limiting process) by a sequence of invariant distributions of "perturbed" (mutated) matrices. We can then easily prove the following result.

**Proposition 5** For any invariant distribution $\mu$ of the unperturbed transition matrix $P$, there exists a sequence $\varepsilon^n \to 0$ of perturbations such that $\mu^n \to \mu$, where $\mu^n$is the unique invariant distribution of the perturbed matrix $P_{\varepsilon^n}$

**Proof** The proof follows directly from the previous Theorem. Indeed, $\forall \mu \in \mathcal{I}(P)$ *any* mutation model achieves $\mu$. Since our perturbed model is a mutation model we have that there exists a sequence of strictly positive $\varepsilon^n \to 0$ such that $\mu^n \to \mu$, where $\mu^n = \mathcal{I}(P_{\varepsilon^m})$

This result provides a different, and perhaps more intuitive, explanation for the behavior that the dynamics described in the previous section might have. Indeed, as discussed above, it could easily be the case that with a small number of players (i.e. small number of states in the process) and a short history the system would no move (or stop relatively soon). This would happen because a case-based decision maker with little information (short memory) will never change to a different state unless there are states that are know to be "good" around it. Proposition 5 says that this behavior of the system can be approximated (as a limiting process) by a "similar" system in which players can make mistakes or experiment (trembles) and, hence, discover the true value of all the states. In other words, any rest point of our original dynamics (where players do not know the value of all coalitions) can be approximated by a sequence of rest point of similar dynamics in which players now the true values of all possible coalitions.

## 3.4 Concluding Remarks

We have studied a process of coalition formation in which players, when deciding whether to move to a different coalition or remain as at present, do not have all the information needed to determine the true value of each possible coalition. Assuming that in such cases the players follow a decision procedure based on Case-Based Decision Theory, we show that the underlying dynamics can be modeled as a non-stationary Markov process. Although such dynamics are usually difficult to study, we provide natural conditions under which our model fits the *Model of mutations* by Bergin and Lipman [10]. The results obtained show that the original dynamics (with unknown values for all possible coalitions) can be approximated by similar dynamics in which all possible coalitions are known.

# Chapter 4

# On the Influence of Consumers' Habits on Market Structure

## 4.1 Introduction

When we talk about market we usually have in mind a real or hypothetical place where buyers and sellers interchange (trade) a given good. At the same time, however,the market as a social institution is a complex system, whose dynamics is determined by the interactions of its components. In this context a new approach to market analysis is offered by Bowles [28], who addresses the question of how markets and other economic institutions influence the evolution of tastes and personalities, i.e how the society shapes the development of its members. We approach the problem from the opposite perspective, rising the question of how individuals' tastes, values and habits shape the market. We are aware of the fact that the influences are mutual, but we believe that having the two perspectives will improve our understanding of the market as a complex system. Defining a market is not an easy task itself, but it is out of the scope of this work. We will assume that the market is well defined and the good which is sold is an "experience" good. According to Nelson [27] this is a good, whose quality cannot be discovered before purchase. Some examples are the food in a restaurant, the coffee in a coffee-shop, the bread in a bakery, or even the choice of a doctor. The main reason why we have chosen this type of good for our analysis is that it allows us to avoid one of the main assumptions behind the competitive paradigm - the perfect information. We assume that when the two sides of the market meet for the first time they know nothing about each other. Imagine a situation, when a new person comes to the neighborhood. She knows what coffee she would like to drink, but she has no idea which is the best coffee-shop in her new neighborhood. Firms are assumed to be small and they do not perform any analysis of the market, but observe how customers react to the changes in the quality offered.

The model constructed here is a two-sided adaptation model in which each side of the market learns who is on the other side and adapts according to its own experience. The learning rule used is learning by doing. Individual's current decision is based on her previous experience in a similar situation. The criterion according to which the result of a decision is judged to be satisfactory is called "aspiration level". The aspiration level results from individual's own experience and is influenced by the social groups she belongs to. The concept of satisficing, was first introduced by Simon [29] and rediscovered lately in a large number of papers in different areas of economics. One example, which we will use throughout this work, is the coffee offered in a coffee shop. In general the price of a coffee does not vary considerably and before tasting it one cannot judge its quality. Normalizing the price of the good to one allows us to focus on the evolution of the quality. The owner of each coffee shop experiments with the quality of the coffee until he finds the one, which gives him the highest profit. The problem of each consumer is to find a coffee shop she likes. She is assumed not to have any previous experience with these coffee shops but to have an aspiration about the quality of the coffee. Every consumer selects one coffee shop. If the quality of the coffee is inferior to her aspiration, on the next morning she will experiment with another one and her aspiration will adapt taking into account the quality she has observed.

The market, in a broad sense, consists of buyers, sellers and the good sold. We present the related literature using this classification. There are not many papers analyzing the influence of consumers' behavior on market structure. Among them is the work of Janssen and Jager [24], which focuses on the influence the socialization process has on market dynamics. Closer to our objective is the work of Deneckere at all. [14], which studies the influence loyal consumers have on the existence and the identity of a price leader. However, up to our knowledge, there is no work addressing the issue of how consumers' behavior influences the market structure, which is the question this work rises.

Concerning the side of the firms, the first one to propose the idea of creating a model in which firms do not maximize profits, but adapt their behavior was Alchian [3]. His idea, however, has not been developed for almost forty years, when the first technical models appeared. There are two branches in this literature. In the first one there are papers studying the influence of the learning mechanism on the resulting market structure. For instance, Arthur [11] has argued that learning by doing can be considered as a form of dynamic increasing returns to scale, which induces a monopoly on a given market. Based on his work Mookherjee and Ray [13] analyze the influence of learning by doing and increasing returns to scale on the collusive market structure and show that learning does not reduce the viability of market-sharing collusion between a given number of firms, whereas increasing returns to scale does. In the second branch, and this is the main focus in the literature, are the papers characterizing the behavior of the system at equilibrium. Papers which belong to this group are Vega-Redondo [31], which shows that in a symmetric market, where firms learn by imitation and experimentation only the competitive Walrasian equilibrium is observed. A similar result but for a Bertrand competition is obtained in Alós Ferrer, Ania and Schenk-Hoppé [4]. Alós-Ferrer [12] proves that adding memory to the model of Cournot competition might sustain as a long term outcome any of the quantities in the interval between the Cournot quantity and the Walras one. The question we are trying to address here, namely, how the learning process influences the market structure connects this work mainly with the first of the groups of papers mentioned above.

To our knowledge there are only few papers, which use models, where the two sides of the market are learning. The first one is the paper by Kirman and Vriend [22], which creates an Agent-Based Computational Economics (ACE) model trying to explain the price dispersion and high loyalty observed at the wholesale market in Marseille. In this co-evolutionary process, buyers learn to become loyal as sellers learn to offer higher utility to loyal buyers. In our basic framework neither buyers, nor sellers distinguish who is on the other side of the market. Endowing consumers with memory changes significantly the results, but in any case no firm offers a special treatment to loyal customers, since it does not distinguish them. Harrington and Chang [23] consider a setting in which firms randomly discover new ideas and implement favorable ones. At the same time consumers are searching among firms for the best offers. This dynamics is shown to induce an increasing returns mechanism, which results in one firm dominating the market in the long run. The main differences with our model are: first that our model allows any number of firms and consumers, and second individuals can change their tastes. Probably the most recent paper in this group is the work by Hehenkamp [8], whose model is very similar with the one just mentioned, but he adds sluggish consumers[1]. This behavior is shown to promote monopoly pricing.

Our results suggest that one of the most important characteristics of consumers' behavior is the speed of adaptation to the feedback received from the environment. If consumers adapt easily this allows many firms to stay in the market offering different qualities of the same product. If individuals' valuations of the good are high this converts the best firm into a monopoly. When the above scenario is repeated many times firms exit the market or offer high quality and earn 0 profits. Adding memory to this model makes the best firm a monopoly with significant market power and this firm decreases its quality until it reaches the one of the second best. After this point qualities remain stable. Free entry changes the sign of the previous results, because even though the best firm gains the market at early periods, the entrance of new firms creates tough competition for clients and leads to high qualities and 0 profits. When customers adapt easily they are satisfied even if they do not go to the best place and this is the reason why all coffee-shops have positive profits and no firm leaves the market.

We observe that the more adaptive is one side of the market if at the same time it is persistent, the more the final outcome of the dynamics favors

[1]Consumers are sluggish if they do not learn sellers' prices with some positive probability.

this side.

The rest of the chapter is organized as follows. Section 4.2 presents the basic framework. In Section 4.3 we build different scenarios, which allow us to distinguish among the effects each of the parameters of the model has and to analyze deeply the results. Section 4.4 is dedicated to a discussion of some of the interesting issues revealed by the results. And Section 4.5 gives directions for future research and concludes.

## 4.2 The Framework

Our objective in this section is to build an ACE model of a single good market. One of the main advantages of ACE models is that they allow a detailed evaluation of the sensitivity of the model to changes of parameters' values. Another one is that in some cases, like the one discussed here, a really simple model induces quite compicated dynamics, hard to study analytically. We begin by constructing a simple benchmark model, which allows us to study deeply the factors influencing the dynamics of the market. We assume that the market is well defined and the good sold is an experience good. In order to be able to focus only on quality we normalize the price of the good to 1. The good we use as an example is the coffee offered in a coffee-shop. The quality takes values in the interval $(0, 1)$ and is perfectly observable by the consumer, once she has visited the coffee-shop.

The benchmark model analyses the dynamics in the following scenario. There are $N$ consumers $(i = 1, ....N)$, who are new to this town and they know nothing about the coffee offered in this place. Nevertheless, we assume that each individual has drunk coffee before and she has an aspiration level - kind of criterion, which allows her to judge if she likes or not a given coffee-shop. We will denote with $a_t^i$, the aspiration of individual $i$ at time $t$. On the other side of the market there are $M$ coffee-shops ($j = 1, ....M$), and each of them has its own quality. The quality coffee shop $j$ offers at time $t$ will be denoted by $q_t^j$. When the game starts each individual is assigned randomly an aspiration level and each coffee-shop - a given quality. The first day each individual chooses at random one coffee-shop. If she likes the coffee, which will be the case if $a_0^i < q_0^j$, she will go to the same coffee shop next morning. If not she will try a different one, but her aspiration will change according to the following rule:

$$a_{t+1}^i = (1 - \alpha)a_t^i + \alpha(q_t^j + \epsilon_t), \tag{4.1}$$

where $\alpha$ is an exogenous parameter in the interval $[0, 1]$, which will appear to be very important in our analysis.

This rule allows the individual to adapt her aspiration according to the feedback received from the environment. Notice that high $\alpha$ means that this person puts more weight on the feedback from the environment than on her own aspiration and vice versa. In our model $\alpha$ is exogenous and all individuals are assumed to have the same $\alpha$. Endogenizing $\alpha$ is an interesting problem itself and will be left for further research.

The $\epsilon_t$ in the above equation accounts for the fact that the consumer is not able to observe perfectly well the quality of the coffee. This parameter is uniformly distributed in the interval $[-0.001, 0.001]$.

The owners of the coffee shops also adapt their behavior. The only parameter, which is under their control is the quality of the coffee. Initially each coffee is endowed with quality $q_0^j$. After $k$ weeks with this quality the profit obtained is given by the following profit function:

$$\pi_{t/t+k}^j(q_t^j, q_t^{-j}) = C_{t/t+k}^j(q_t^j, q_t^{-j}) * (1 - (q_t^j)^2), \tag{4.2}$$

where $C$ is the number of clients this coffee shop had for the period between $t$ and $t+k$. $C$ does not have explicit form, but it obviously depends on the quality this coffee shop has chosen, and on the qualities of the other firms in the market. Notice also that firms do not base their behavior directly on the choices made by competitors, but their profits depend on each others' qualities.

From the profit function we can notice that increasing quality can have opposite effects. On the one hand, better quality (all other things kept equal) increases the number of clients, who come and stay. But on the other hand, it is very costly.

Every $k$ weeks the owners of the coffee shops experiment with a small change in the quality. If the change was made in period $l$, then in period $l+k$ the owner of the coffee shop $j$ can compare his profits. If $\pi^j_{l-k/l} < \pi^j_{l/l+k}$ he will experiment with a new quality, otherwise he will prefer the quality chosen at period $l-k$.

The benchmark model explained above is at the same time simple in its specification and complex in its behavior than similar theoretical model. Its structure allows us to add more dimensions to the process and analyze the behavior of the system. The model can evolve in many directions and we have chosen here the most intuitive ones. We enrich the model by endowing consumers with memory and allowing free entry on the firms' side. Notice that in the benchmark model individual's experience was reflected in her aspiration level, so if she drinks a bad coffee, this will reduce her aspirations and vice versa, but she was unable to remember where exactly was this coffee-shop. Adding memory implies adding an association rule for each individual. Each coffee-shop is remembered with the coffee she drunk, when she went there for the last time.

Free entry condition allows firms, which are earning zero profits for $n$ periods, to leave the market and new firms to enter. Three facts are important here. The first one is that when a firm leaves, it is automatically forgotten by its clients, i.e individuals know that this place does not exist anymore. The second issue concerns the parameter $n$. This period cannot be too short because each firm can have a bad day, but it cannot be too long either. And the last fact is related with our assumption that firms do not perform any market analysis, so the new firms are assumed to choose their initial qualities randomly.

## 4.3 Results

If we are trying to analyze how a system works, then probably the best way to do it is to start by allowing only one of its main parts to move and observe what happens, then do the same with the others. Once the main parts are working we can allow some of the additional parts to move and observe the whole system. This is the logic we have followed when analyzing the behavior of our market and we believe that it is natural to present the results in the same way.

### 4.3.1 Individuals' adaptation

To begin with we allow only individuals to adapt their behavior. So each person starts by going to a given place and if she likes it she will continue to go there, if not she will change, but in both cases her aspiration will change according to the rule specified above. In the benchmark model, individuals do not have memory, but all their experience is reflected in the evolution of the aspiration level, which implies that many positive experiences will lead to high aspiration levels and vise versa. How important is the feedback received from the environment depends on the value the parameter $\alpha$ takes. High values of $\alpha$ imply that individuals put more weight on the feedback they get from the environment than on their own aspiration and vice versa. As a matter of fact this parameter appears to be really important and requires some intuition. What we observe in Figure 4.1 is a clear difference in the way aspirations evolve, when $\alpha$ is low on the left graph and when it is high on the right. In the first case the evolution is smooth and aspirations are growing until they reach the level of the best quality offered in this market. As a consequence all individuals shop in different places, but very soon they discover the best coffee-shop and everybody goes there. Remember that coffee-shops cannot adapt, so the best coffee-shop becomes a monopoly, no matter how high is the quality it is sufficient to be higher than the others. In the second case, when $\alpha$ is high every experience has a dramatic influence on individuals' aspiration. If after being in a very good place, she decides to experiment something else and has a bad luck, the aspiration level will decrease so sharply that if her next choice is a medium quality, she will like it and she will stay there. Notice that this could not happened in the previous case, because the change in the aspiration level will be almost insignificant and the search for the best place will continue. The vulnerability of the aspirations when $\alpha$ is high explains why some individuals visit a medium quality place even after they have visited a better one. These groups of people are clients of those firms and even if they do not earn the highest profit they still have positive profits. Note that this is not always true. Depending on the values of the qualities of the two best firms it could happened that the quality of the first one is so high that the profit is lower than that earned by the second best with lower quality and less customers.

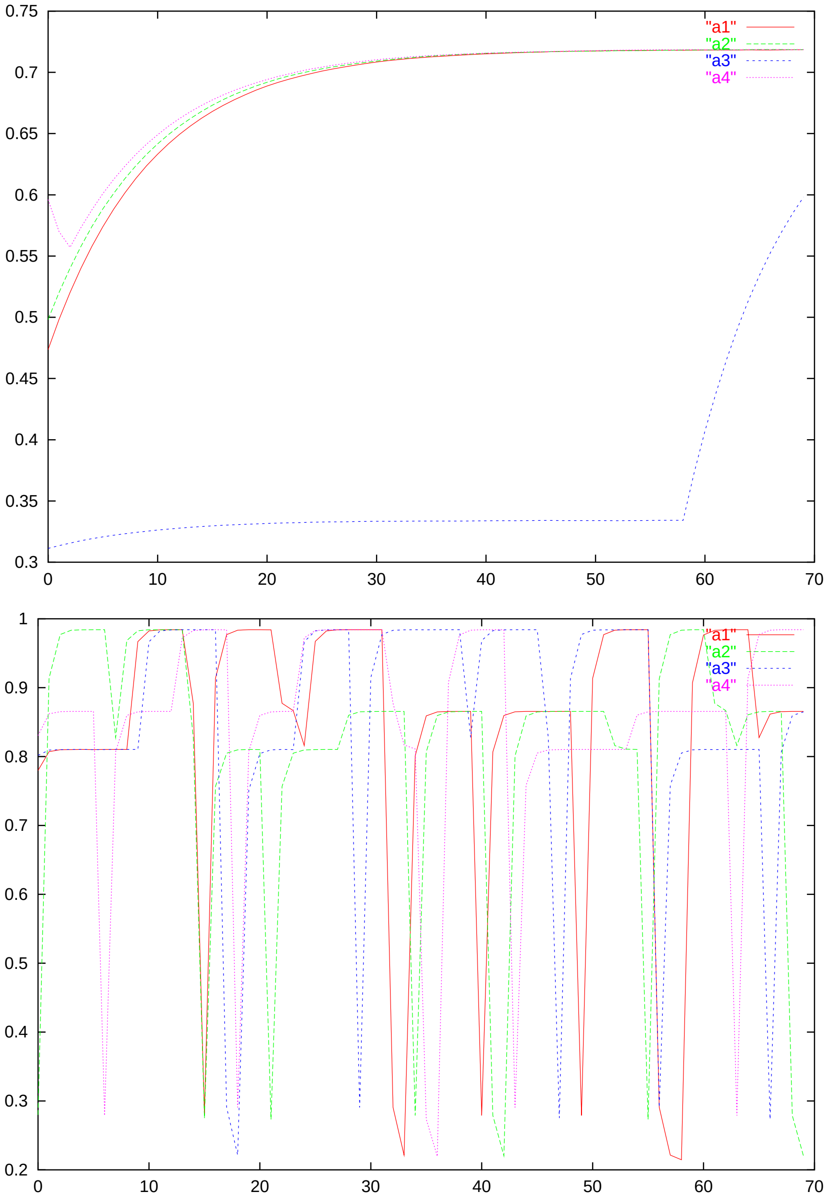


Figure 4.1: Individuals Adaptation

### 4.3.2 Firms' adaptation

When individuals' aspirations do not evolve and only firms adapt, coffee-shops have no incentives to increase quality, on the contrary they decrease it, which allows them to make huge profits. The intuition behind this result is more difficult to explain than in the previous case. The elements of the model which cause it are the fixed number of firms, the no entry - no exit condition and the fact that each individual drinks a coffee every morning. Firms realize fast that if all of them lower the quality almost all customers will be continously disappointed, but since they have a limited number of coffee shops and they have to drink coffee they will change the place every day. This guarantees clients for all firms independently on the quality offered and lowering the quality is a way to lower the cost and increase the profits. A straightforward question will be why a single firm does not increase quality. The problem is that if quality has already gone to 0, an increase to 0.1 will not induce almost any change in the number of customers, because the firm can get only those, whose aspiration is in the interval $[0, 0.1]$. They will not be many, but the cost will increase. If a firm in this market wants to make some customers loyal it needs to increase the quality a lot, which finally can be profitable, but a small increase is not, a firm facing loses will not continue.

### 4.3.3 Coevolution

If we allow both sides of the market to adapt the analysis becomes more involved. In the early period the evolution of the market structure appears to be depended on consumers' behavior and in particular on the values of $\alpha$. With low $\alpha$ the best firm emerges as a monopoly at early periods. The left graph in Figure 2 shows the evolution of profits in this case. It is easy to notice that the dynamics on the right side, with $\alpha$ high, are very different. All firms remain in the market and their profits are lower. The difference is due to both sides behavior. On the one hand consumers in the first case become very "loyal" to the best firm and this makes it difficult for the other firms to fight for clients. If a firm, different from the monopoly wants to attract customers who have visited the best coffee shop, this firm needs to increase the quality almost until reaching the one offered by the best firm. The evolution will be slow, because in general with this type of individuals' behavior it is very difficult to gain clients. On the other hand, in the second case with high $\alpha$, people are much more willing to change and it becomes easier for firms to gain clients by increasing quality. Since almost all firms do the same they share the market and profits are lower. A heterogeneous population of consumers leads to monopoly during the early periods, but not necessarily the highest quality firm. Increasing the number of periods shrinks the difference between the two cases. Firms which can do that increase their qualities to 1 and earn 0 profits. This result requires a note on the influence of initial conditions in this model. On the side of consumers we did not observe any dependence on initial conditions. On the firms' side, though, which firm survives or becomes a monopoly in early periods is mainly dependent on initial conditions. This observation confirms the results obtained by Arthur [11].

### 4.3.4 The influence of memory and free entry

Two elements can be added to this model. The first one is related with the assumption in the benchmark model that an individual does not remember, i.e. her experience influences her aspiration but she does not create a relation between a place and the coffee she had drunk there. Endowing our

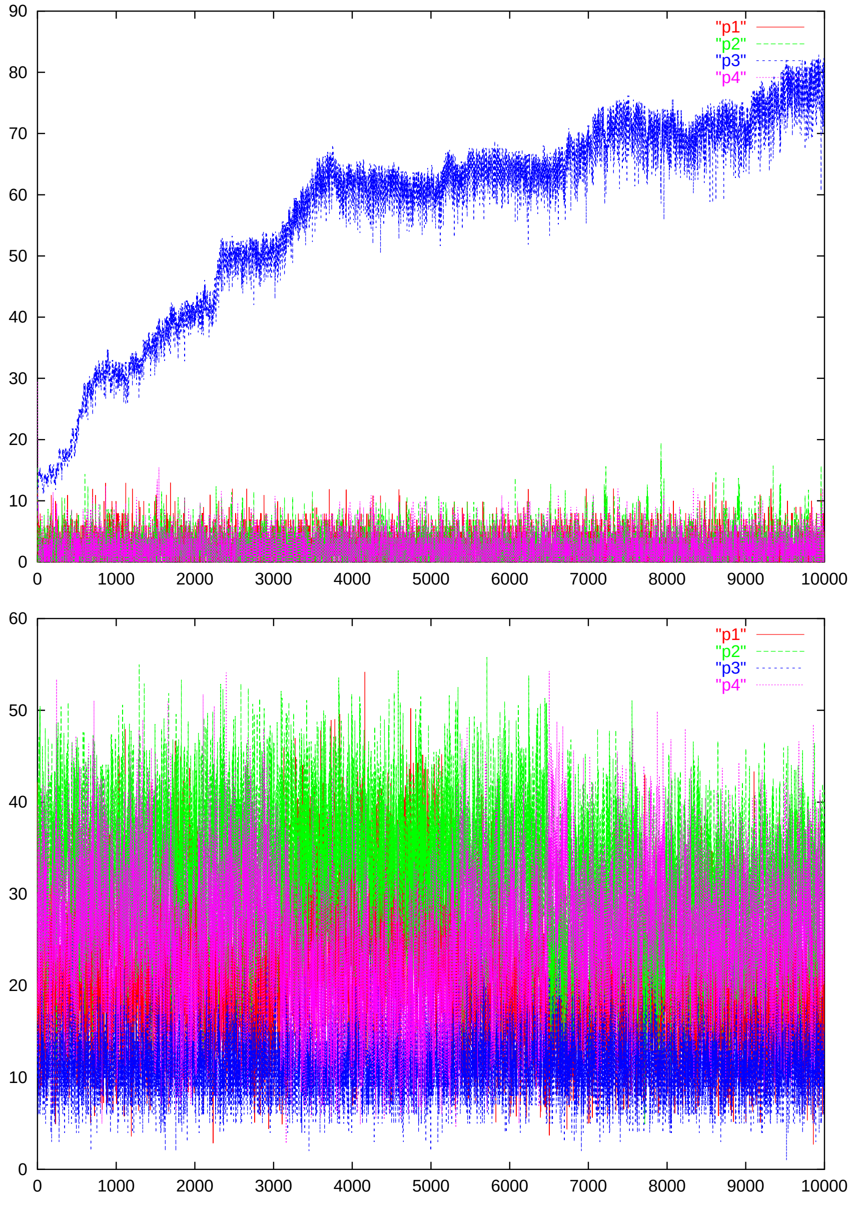


Figure 4.2: Short run profits

individuals with memory is not a difficult task, but their decision problem changes significantly. Having memory they become much more dependent on previous experience and if an agent goes by mistake in a low quality place and she remembers that there is another place, which she likes more, next time she will go there. If the best place she remembers does not satisfy her, she will continue experimenting. The results in this case are straightforward - the best firm emerges as a monopoly. It can afford to lower quality until it is almost equal to the second best quality in the market. From this moment on those two firms compete á la Bertrand. The qualities of those firms do not increase - they show a tendency of remaining stable or decrease. Which in this case goes against consumers' interests. Remembering the best coffee-shop as the best one does not give this firm any incentives to improve.

The second element is adding free entry. Allowing free entry has a dramatic influence on the dynamics of the market. The parameter $\alpha$ has the same influence, but together with the memory and free entry condition the behavior of the system is totally different. Our results show that when $\alpha$ is low (remember that in the first case this induces a monopoly in early periods) and individuals have memory, this makes the best firm monopoly in few steps, but all the other firms remain with 0 profits and leave the market. New firms enter in this market, and consumers have no experience with them, this reduces the effects memory has and creates a tough competition, inducing high qualities and 0 profits for all firms. However, when $\alpha$ is high the two effects do not overlap in the direction of creating a monopoly. The memory condition still promotes the existence of a monopoly, but the vulnerability of aspiration levels in this case increases the visits people make to other coffee-shops and they never reach zero profits, which implies that they never leave the market as well. In this case the market ends up with two firms competing á la Bertrand and other small firms making positive profits.

## 4.4 Discussion

From the analysis above we can notice that there are some characteristics of individuals' behavior or of the market, which influence the dynamics of the system. We focus here on few of those elements and provide some intuition about the outcomes obtained in the previous part.

The first one is the dependence on initial conditions. From individuals' behavior we did not observe any dependence on initial conditions, but this was not the case for firms' dynamics. Which firm will be the most visited one or become a monopoly mainly dependens on initial conditions. As well as having the best threshold between quality and clients, is in many cases a matter of good luck. The fact that one firm can become a market leader, because of initial conditions or random shocks is not a new idea and it was first noted by Alchian [3] and later developed in the work of Arthur [11]. Two things should be noted here. First, we obtain that monopoly is not the unique outcome when consumers' aspirations evolve as well and second, the paper of Mookherjee and Ray [13] shows that learning-by-doing does not promote collusions. This raises the question why in some cases, as those observed by Arthur, it does and in others it does not. We do not analyze here collusive behavior, but we believe that individuals' memory could provide one possible explanation to this contradiction. If agents remember the best coffee-shop as the best one, this induces a bias towards their past and obviously a tendency of creating the habit of going there.

The second element is the influence of the parameter $\alpha$. Why the speed of adaptation, or the willingness to adapt, should be so important? Notice that in our model there is no search cost and $\alpha$ cannot play the role of a cost, just because new experience can both decrease or increase the aspiration. Then what is the intuition behind $\alpha$? Consumers' Research show that individuals willingness to experiment with something new decreases with the age and brand loyalty increases. This result is closely related to the ability to adapt to new environments. Usually old people have more experience and stronger bias towards past. We can notice here two influences. On the one hand, $\alpha$ can be closely related with the individuals' valuation of the good in which case high $\alpha$ will imply that the quality of the coffee she drinks is really important for her and she is willing to search until she finds it and on the other hand it can be part of the characterization of the good. There are some goods like the salt for example, which is an experience good but most people have never though of its quality. A very interesting question for further research will be to study the evolution of $\alpha$, since it is closely related with endogenizing individuals' preferences.

The third issue is the influence of memory. Obviously, without the free entry condition the memory creates a monopoly and this firm begins to lower quality, until it reaches the one offered by the second best firm. On the one side this behavior does not allow individuals to visit coffee-shops offering inferior qualities, but on the other it provides incentives for the best firm to lower quality.

The forth issue concerns the importance of the term *monopoly.* By monopoly we mean that there is one firm dominating the market, but in most of the cases except those where the memory plays a strong role, this firm does not have any market power. In particular, in the benchmark model, when $\alpha$ is low and one firm emerges as a monopoly in early periods, because of the structure of consumers' preferences, this firm cannot lower the quality offered, because this will imply that most of its clients will leave. The existence of many firms in some cases can be seen as the persistence of different-quality goods at the same market, rather than quality competition.

All the scenarios constructed above, except the case with memory and high $\alpha$, end up with firms leaving the market or offering high qualities and earning zero profits in long run. Our interest, however, was much more in the dynamics during the early periods and the elements which influence them.

## 4.5 Concluding Remarks

We have studied in this work the dynamics of market behavior, treating markets as complex systems. What Bowles [28] has observed is that markets as social institutions shape individuals preferences and tastes. What we have shown is that individuals and firms also influence the behavior of the system. Our results show that the more adaptive one side of the market is, the more the market reflects its interests. Limitations, such as memory, make individuals stick to past feedbacks and work against their own interest. The cases in which one side was adapting and the other not, clearly show that the ability to adapt of one of the sides triggers the result in the direction of its interest. Nevertheless, this is not true if we increase this ability until the limit. The best combination requires strong internal values (individuals should know what they want) and at the same time match this desire with the feedback received from the environment.

We believe that further research should focus on combining the two directions, providing in this way a better insight on how markets work.

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