Chapter 5

Towards Vessel Reconstruction from IVUS Data.

5.0.1 Methodology

In order to obtain experimental vessel wall morpho-geometric measures, an IVUS pullback of 2090 images corresponding to 45 mm long vessel segment from a pathological right coronary artery (See Fig. 4.2) was obtained. The sequence was acquired using a Boston Sci. equipment at 40 MHz at constant pullback speed of 0.5 mm/sec and segmented using the Neural Network procedure described in section 4.3.1. All geometric parameters have been obtained from an elliptical fitting to the vessel wall point found by the Neural Network made it using the procedure described by [52].

Model Assumption.

One of the nice advantages of the proposed approach consists of providing a way to obtain a 2.5D reconstruction of the vessel wall. The experimental analysis, has demonstrated that it is possible to separate the geometric vessel properties from heart dynamics contributions (See Chapter 4). We assume that the vessel wall shape \( \gamma = (x, y, z) \) at time \( t \) from catheter point of view, can be written as an elliptical approximation given by (See section 4.2):

\[
x(t) = a(t)\cos(\theta + \delta(t)) + cx(t), \quad y(t) = b(t)\sin(\theta + \delta(t)) + cy(t), \quad z(t) = cz(t); \quad (5.1)
\]

where \( 0 < \theta \leq 2\pi \), determines the angular position of the corresponding point on the ellipse, \((a(t), b(t))\) are the minor and major radii of the ellipse, given by (Eq. 5.2), \( \delta(t) \) is its orientation and \( C = (cx(t), cy(t), cz(t)) \) its center at time \( t \). In 2.5D IVUS reconstruction \( cz(t) \) is unknown, but in our model we can take it as the catheter position in longitudinal direction, \( cz = vt \), where \( v \) is the catheter velocity. Taking into account only the geometrical contributions, an approximated vessel geometry
description from catheter point of view is possible, when lumen center position major and minor ellipses radii and the ellipses orientation are known.

Parameters Evolution.

In order to reconstruct the vessel wall from catheter point of view the principal ellipses temporal evolution parameters will be used.

1. **Minor and Major Axis.** The functional dependence between ellipse eccentricity $\epsilon$ and ellipses axis (see Eq. (4.9)), can be used to model the temporal behavior of $a(t)$ and $b(t)$ (see Figures 5.1 and 5.2), as follows:

\[
a(t) = a_g(t) + a_d(t) \quad , \quad b(t) = b_g(t) + b_d(t)
\]

where $(a_g(t), b_g(t))$ and $(a_d(t), b_d(t))$ are the major and minor geometrical and dynamical ellipses axis temporal evolution. These contributions are given as a Fourier series:

\[
a_g(t) = \sum_{n=n_2}^{n_1} \left( A_n^a \cos(n\omega t) + B_n^a \sin(n\omega t) \right)
\]

\[
b_g(t) = \sum_{n=n_2}^{n_1} \left( C_n^b \cos(n\omega t) + D_n^b \sin(n\omega t) \right)
\]

\[
a_d(t) = \sum_{n=n_3}^{n_4} \left( A_n^a \cos(n\omega t) + B_n^a \sin(n\omega t) \right)
\]

\[
b_d(t) = \sum_{n=n_3}^{n_4} \left( C_n^b \cos(n\omega t) + D_n^b \sin(n\omega t) \right)
\]

2. **Ellipse Orientation** The ellipse orientation $\delta(t)$ shows a bimodal behavior such as illustrated in Fig 5.3. The temporal dependence can be written as follows:

\[
\delta(t) = \delta_g(t) + \delta_d(t)
\]

where $(\delta_g(t), \delta_d(t))$ are the geometric and dynamical dependence respectively. These contributions can be written as a Fourier series:

\[
\delta_g(t) = \sum_{n=n_2}^{n_1} \left( A_n^\delta \cos(n\omega t) + B_n^\delta \sin(n\omega t) \right)
\]

\[
\delta_d(t) = \sum_{n=n_3}^{n_4} \left( A_n^\delta \cos(n\omega t) + B_n^\delta \sin(n\omega t) \right)
\]
3. **Ellipses Center.** The coordinates ellipse centers $cx(t)$ and $cy(t)$ follow a periodic bimodal movement (see Figures 5.4 (a) and 5.5 (a)). Their spectral density shows that there is a bimodal behavior which comes from geometric and dynamical contributions (See Figures. 5.4 (b) and 5.5 (b)). Using these results we can write the ellipse centers temporal evolution as follows:

$$cx(t) = cx_g(t) + cx_d(t) \quad , \quad cy(t) = cy_g(t) + cy_d(t)$$  \hspace{1cm} (5.7)

where $(cx_g(t),cy_g(t))$ and $(cx_d(t),cy_d(t))$ are the geometrical and dynamical ellipse center temporal contributions. These contributions are given as a Fourier series:

$$cx_g(t) = \sum_{n=n_1}^{n=n_2} \left( A_n^c \cos(n\omega t) + B_n^c \sin(n\omega t) \right)$$  \hspace{1cm} (5.8)

$$cy_g(t) = \sum_{n=n_1}^{n=n_2} \left( C_n^c \cos(n\omega t) + D_n^c \sin(n\omega t) \right)$$  \hspace{1cm} (5.9)

$$cx_d(t) = \sum_{n=n_3}^{n=n_4} \left( A_n^d \cos(n\omega t) + B_n^d \sin(n\omega t) \right)$$

$$cy_d(t) = \sum_{n=n_3}^{n=n_4} \left( C_n^d \cos(n\omega t) + D_n^d \sin(n\omega t) \right)$$

---

**Figure 5.1:** Major $a(t)$ ellipses axis temporal evolution (a) and its corresponding power spectral density (b)
**Figure 5.2:** Minor $b(t)$ ellipses axis temporal evolution (a) and its corresponding power spectral density (b).

**Figure 5.3:** Ellipse orientation $\delta$ temporal evolution (a) and its corresponding power spectral density (b).
5.1. 2.5D Vessel Reconstruction.

The 2.5D vessel reconstruction is possible taking only the Fourier coefficient that corresponds to the geometrical contribution. Therefore, Eq. 5.10 can be rewritten as follows:

\[
\begin{align*}
    x(t) &= a_g(t)\cos(\theta + \delta_g(t)) + cx_g(t) \\
    y(t) &= b_g(t)\sin(\theta + \delta_g(t)) + cy_g(t) \\
    z(t) &= vt
\end{align*}
\]

where \(0 < \theta \leq 2\pi\) determines the angular position of the corresponding point on the ellipse, \((a_g(t), b_g(t), \delta_g, cx_g, cy_g)\) are the minor and major axis radii, orientation, ellipse centers at time \(t\) and \(v = 0.5 \text{ mm/s}\) the catheter velocity.

Figure 5.6 shows two views of a 2.5D vessel reconstruction of an IVUS pullback of 2090 images corresponding to 45 mm long vessel segment before rotation suppression and (Fig. 5.7) display two views of the same vessel when the dynamics suppression methodology has been applied.
Figure 5.5: Ellipse center $c_y(t)$ temporal evolution (a) and its corresponding spectral density (b)
Figure 5.6: Two views of 2.5D vessel wall reconstruction before dynamic suppression.
Figure 5.7: Two views of 2.5D vessel wall reconstruction after dynamic suppression.
Chapter 6

Conclusions and Future Lines

Although IVUS is continuously gaining its use in practice due to its multiple clinical advantages, the technical process of IVUS image generation, geometrical and dynamical aspect of the vessel wall evolution are not known by doctors and researchers developing IVUS image analysis. We developed in this thesis three complementary research studies: First one, we created a basic simulation model in order to generate 2D IVUS images. Second one, based on experimental results we introduced a new methodology to study the vessel wall appearance and its corresponding temporal evolution. Third, we introduced the main conceptual strategy that allows the 2.5D IVUS reconstruction.

1. **IVUS Images Simulation Model.** We discussed a basic physical model to generate synthetic 2D IVUS images. The model has different utilities: Firstly, expert can generate simulated IVUS images in order to observe different arterial structures of clinical interest and their grey level distribution in real images. Secondly, researchers and doctors can use our model to learn and to compare the influence of different physical parameters in the IVUS image formation, for example: the ultrasound frequency, the attenuation coefficient, the beam number influence, and the artifact generations. Third, this model can generate large database of synthetic data under different devices and acquisition parameters to be used to validate the robustness of image processing techniques. The IVUS image generation model provides a basic methodology that allows to observe the most important real image emulation aspects. This initial phase does not have the intention to compare pixel to pixel values generation, showing the coincidence with the real image, but looks for a global comparison method based on grey level difference distribution. The input model applies standard parameters that have been extracted from the literature. Hence this model is generic in terms that the model allows simulating different processes, parameters, and makes possible to compare to real data and to justify the generated data from the technical point of view.

The model is based on the interaction of the ultrasound waves with a discrete
scatterer distribution of the main arterial structures. The obtained results of the validation of our model illustrate a good approximation to the image formation process. The 2D IVUS images show a good correspondence between the arterial structures that generate the image structures and their grey level values. The simulations of the regions and tissue transitions of interest lumen and adventitia, have been achieved in a satisfactory degree. Interested readers are invited to check the generation model in (http://www.cvc.uab.es/~misael).

2. **Modelling Vessel Wall Dynamics.** We developed a geometric and kinematic model in order to study the evolution of coronary artery wall from catheter point of view. The model is based on the supposition that the evolution of the arterial wall, can be modelled assuming two principal contributions that come from different physical reasons. The first one, a systematic contribution caused by geometric intrinsic arterial properties and the second one, an oscillating contribution that comes from ventricle dynamics. These contributions govern in major degree the profiles appearance of arterial wall in longitudinal views. Using these assumption we generate the methodological strategy in order to estimate and suppress IVUS dynamical distortions. The vessel wall radial deformation on the IVUS images not only depends on the blood pressure variation, but also depends on the artifact produced by oscillating obliquity induced by the ventricle pulsatile dynamics. Any effort to obtain in situ vessel wall elastic properties using IVUS sequences should emphasize on the suppression of radial deformation that are blood pressure independent. We introduce a new conceptual formulation that permits to separate geometric contribution depending on intrinsical vessel wall micro-architecture from dynamical contribution that comes from heart movement. Our model only takes into account those transformations that maintain invariant the image dimensions, therefore the radial expansion and the catheter obliquity have not been treated, still these contributions predominate once suppressed the rotation.

3. **2.5D Vessel Reconstruction.** Being possible to separate the geometric contribution from dynamics contribution, we developed the basis to 2.5D vessel reconstruction only using IVUS data. This thesis gives an important advance in vessel wall dynamics estimation such as to introduce an alternative technique to estimate local heart dynamics. In this way, we provide a new possibility of studying robustly the vessel dynamics and establishing new diagnostic tools.
6.1 Future Lines

An important future line in this research can be oriented to IVUS Functional Image. The principal objective in this investigation way should be focused to find significative statistical correlation between static and dynamica morpho-geometric IVUS parameters and their corresponding cardiological functional dependence. In order to fulfill this general objective, the following research aspect can be consider as plausible:

1. Once separated the heart dynamics influence from geometric vessel wall contributions, we can characterize vessel wall mechanical properties evaluating the elastic constant $\sigma_k$ extracted from the relative radial deformation $\Delta R_k/R_k$ for each frame $k$. This technique can be a novel and direct method to obtain plaque and tissue characterization by patient in vivo.

2. Using IVUS technique by frequencies greater than 40 MHz it is possible to observe the fibers configuration in cross sectional images. An exhaustive study of IVUS images textural properties, could be used to estimate the fiber density, allowing to obtain intrinsical properties of vessel wall, that can give an important valuation on functional and local physiological state of an artery.

3. Our dynamical and geometric model can be used in two ways: 1.- Suppression of dynamical distortion in order to obtain 3D IVUS reconstruction. 2.- Once suppressed the dynamics effect, in vivo vessel wall elastic properties can be estimated. 3.- Parameters validation that can be used to estimate heart dynamics.

4. The IVUS rotation as a global phenomena can be used to evaluate the pumping heart percentage efficiency. An extensive study of the IVUS rotation images versus pumping heart efficiency obtained experimentally using a diastolic and systolic angiography heart views, should demonstrate a positive correlation between heart rotation and heart efficiency.
Appendix A

Eccentricity Definition

In order to describe the IVUS rotation effect respect to the catheter spatial position, we define from (Fig. A.1 (a)) the catheter eccentricity as: $\xi = |rc_1/r|$, where $rc_1 = (xc_1^2 + yc_1^2)^{1/2}$ is the spatial relative catheter position to the lumen center and $r$ is a region of interest (ROI) located on the vessel wall. Figure A.1 (b) shows the linear dependence of eccentricity $\xi$ versus lumen center spatial position, $rc_1$. The definition of catheter eccentricity $\xi$ can be used to achieve a better description of the heart dynamics influence and vessel geometry contributions to the longitudinal IVUS cuts’ shape appearance.
Appendix B

Kinematic Approach to IVUS Rotation Estimation.

In order to find the rotation profile of the sequence when $0 \leq \xi \leq 10\%$, we assume [27] that the vessel wall can be considered as a discrete linear elastic oscillating system [19]. Using polar coordinates to describe the vessel wall temporal evolution, it follows that its trajectory is given by: $(x, y) = (r(t)\cos(\theta(t)), r(t)\sin(\theta(t)))$, then an element of the vessel wall has the following total energy:

\[ E_i = T_i + U_i \] (B.1)

where\[ T_i = \frac{m_i v_i^2}{2} + \frac{m_i}{2} (\omega_i)^2 \quad U_i = \frac{k_i r_i^2}{2} \]

\[ v_i = \sqrt{v_x^2 + v_y^2} \quad r_i = \sqrt{x_i^2 + y_i^2} \quad w_i = \frac{\partial \theta_i}{\partial t} \]

$T_i$ and $U_i$, are the kinetic and elastic energy of the i-th discrete element of the vessel wall respectively, $m_i, v_i, \omega_i$ and $k_i$ are the mass, tangential velocity, angular velocity and elasticity constant of the i-th element of the vessel wall. The mass of one element can be estimated considering the minimal "voxel" volume sweeping by the ultrasound beam, being this $V_b \approx 6.4 \times 10^{-5} \ mm^3$ [51]. Using this fact, the element of mass is $m = \rho \cdot V_b \approx 1.09 \frac{grs}{cm^3} \cdot 6.4 \times 10^{-5} grs \approx 6.97 \times 10^{-5} kg$, where $\rho$ is the typical tissue density [48]. Within the above kinematic framework, it is sufficient to provide the temporal evolution of a single point on the vessel wall structure [42, 44], to measure the angular difference between two consecutive frames. Therefore, spatial location of this reference point was determined as the position $ij$ of the vessel wall that has a minimal total energy given by (Eq. B). The spatial location of this point is put into the image spatial coordinates which reaches the condition:

\[ (x_c, y_c) = \arg\min_{ij} f_n \sum_{k=1}^{f_n} E_{ij}^k \]

where $f_n = 25$ is the image number used to evaluate this condition and $ij$ are the row and columns of the average IVUS images.
KINEMATIC APPROACH TO IVUS ROTATION ESTIMATION.
Appendix C

Vessel and Ventricle Dynamic Interaction

The spatial-time evolution of an artery which is modelled for a generatrix curve \( G(s,t) \), is governed by the left ventricle (LV) evolution model specified in [40] (See Appendix C). The basic geometric model is given in a prolate sphere, whose parameters are shown in (Fig. C.1 (a)). A point \((\lambda, \eta, \phi)\) in prolate spheroidal coordinates has the following Cartesian coordinates:

\[
x = \delta \sinh \lambda \sin(\eta) \cos(\phi) , \quad y = \delta \sinh \lambda \sin(\eta) \sin(\phi) , \quad z = \delta \cosh \lambda \cos(\eta)
\]

where \((\lambda, \eta, \phi)\) are the radius, elevation and azimuthal angles respectively and \(\delta\) is the focal radius. The evolution spatial and temporal of the generatrix curve \( G(s,t) \) can be rewritten as a function of the curve \( g(u(\lambda, \phi, \eta), v(\lambda, \phi, \eta), w(\lambda, \phi, \eta)) \) on the

![Figure C.1: Prolate coordinates (a) used to represent the LV surface (b)](image-url)
The general matrix equation that transforms one point \( G \) on the surface of the ventricle is given as:

\[
G(s, \lambda, \phi, \eta, t) = gx(s(u(\lambda, \phi, \eta), t) + gy(s(v(\lambda, \phi, \eta), t) + gz(s(w(\lambda, \phi, \eta), t)
\]

(C.1)

The general matrix equation that transforms one point \( G(s, t) \) in \( G(s + \delta s, t + \delta t) \) on the surface of the ventricle is given as:

\[
G(s + \delta s, t + \delta t) = FaF6F5F4F3F2F1FoG(s, t)
\]

(C.2)

where \( a \) is the correctional parameter that transforms a prolate spheroidal shell into a more spherical shape in anticipation of the next transformation, \( \epsilon = \frac{3b_1V_w}{4\pi |FoG(s, t)|^{1/3}} \), \( V_w \) is the wall ventricle volume, in our model \( V_w = 1 \), \( b = ak_2z1 \) and \( r1 = F1FoG(s, t) = [x_1, y_1, z_1, 1]^t \)

\[
Fa = A4A3A2A1
\]

\[
A1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(k_8) & -\sin(k_8) & 0 \\ 0 & \sin(k_8) & \cos(k_8) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A2 = \begin{pmatrix} \cos(k_9) & 0 & \sin(k_9) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(k_9) & 0 & \cos(k_9) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\]

\[
A3 = \begin{pmatrix} \cos(k_{10}) & -\sin(k_{10}) & 0 & 0 \\ \sin(k_{10}) & \cos(k_{10}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\]

\[
A4 = \begin{pmatrix} 1 & 0 & 0 & k_{11} \\ 0 & 1 & 0 & k_{12} \\ 0 & 0 & 1 & k_{13} \\ 0 & 0 & 0 & 1 \end{pmatrix}
\]
The temporal and spatial evolution of the ventricle surface is given by the evolution of the $k_i$ parameters given in the model by Arts et al. [38]. Fig. C.2 show the temporal evolutions from $k_1$ to $k_{13}$ for two cardiac cycles.

<table>
<thead>
<tr>
<th>$k_i$</th>
<th>Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>Radially dependent compression</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Left ventricular torsion</td>
</tr>
<tr>
<td>$k_3$</td>
<td>Ellipticallization in long axis plane</td>
</tr>
<tr>
<td>$k_4$</td>
<td>Ellipticallization in short axis plane</td>
</tr>
<tr>
<td>$k_5$</td>
<td>Shear in x direction</td>
</tr>
<tr>
<td>$k_6$</td>
<td>Shear in y direction</td>
</tr>
<tr>
<td>$k_7$</td>
<td>Shear in z direction</td>
</tr>
<tr>
<td>$k_8$</td>
<td>Rotation about x-axis</td>
</tr>
<tr>
<td>$k_9$</td>
<td>Rotation about y-axis</td>
</tr>
<tr>
<td>$k_{10}$</td>
<td>Rotation about z-axis</td>
</tr>
<tr>
<td>$k_{11}$</td>
<td>Translation in x direction</td>
</tr>
<tr>
<td>$k_{12}$</td>
<td>Translation in y direction</td>
</tr>
<tr>
<td>$k_{13}$</td>
<td>Translation in z direction</td>
</tr>
</tbody>
</table>

**Figure C.2:** The temporal evolution of $k_i$ coefficients
VESSEL AND VENTRICLE DYNAMIC INTERACTION
Bibliography


[34] Jumbo G., Raimund E., Novel techniques of coronary artery imaging, in Beyond Angiography, Intra Vascular Ultrasound, state of the art, Vol. XX, Congress of the ESC Viena-Austria, University of Essen, Germany, 1998.


[37] Hiroshi Yamada., Strength of biological materials., edited by, F. Gay Nor Evans, Baltimore, 1970;


[40] Edo Waks., and etal, Cardiac Motion Simulator for Tagged MRI, Proceeding of MMBIA 1996, IEEE. John Hopkins University, Baltimore, MD 21218.


[61] Nissen SE. Application of intravascular ultrasound to characterize coronary artery disease and assess the progression or regression of atherosclerosis. Am J Cardiol 2002; 89(Suppl.):24B-31B.


Publications

- Empirical intravascular ultrasound simulation system. O Rodriguez Leor, J Mauri Ferrel, E Fernandez Nofrerias (1), E Tizon (2), A Tovar (1), V Valle Tudela1, M Rosales (2), P Radeva (2), Hospital Universitari Germans Trias i Pujol - Badalona - Spain, (1) HU Germans Trias i Pujol - Badalona - Spain, (2) Universitat Autonoma de Barcelona - Bellaterra - Spain, European Society of Cardiology Congress, Stockholm, Sweden, 3-7 sept 2005

- Misael Rosales, Petia Radeva, Debora Gil, Carlos Rodríguez and Oriol Rodríguez, Modelling of Image-Catheter Motion for 3-D IVUS. Medical Image Analysis, (Submitted 12/07/2005)


- Misael Rosales, Petia Radeva and Josepa Maury, Basic Model Of Simulation Of IVUS Data, (Part II: Model Validation). Computerized Medical Imaging and Graphics, (Submitted 08/08/2004)


- Modelo Físico para la Simulación de Imágenes de Ecografía Intracoronaria, Rodríguez O., Mauri J., Rosales Misael, Radeva Petia, Revista Española de Cardiología, 56(2), Congreso de las Enfermedades Cardiovasculares, Sevilla 10/2003

