INFORMATION AND CONTROL

SUPERVISION OF ADAPTIVE/ITERATIVE SCHEMES

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Submitted in partial fulfillment of the
requirements for the degree of
Doctor Engineer
at Autonomous University of Barcelona

June, 2007

Knowing ignorance is strength.
Ignoring knowledge is sickness.

Lao Tzu. Tao Te Ching.

Preface

The design of a controller is a procedure which requires acquiring and processing information in order to obtain a satisfactory design. For example, system identification requires experimental data, a model structure and an identification algorithm. Thus a nominal model plus uncertainty bounds can be identified (i.e. a model set) which is then used to design a controller.

It is widely recognized that if new information is added to the control design process, it is possible to improve the performance achieved by the designed controller. In fact that is the rationale behind adaptive control schemes. Adaptive control schemes are characterized by adding new information for control design purposes into the control design procedure with the aim of designing a better performing controller.

In this thesis we tackle the control design problem from the information point of view. In this way the problem of designing a controller is understood as the problem of managing the information flow required to design a proper controller (i.e. experimental data, system identification, fundamental limitations, control design, etc.).

The main contributions of this thesis are:

I) The control problem (i.e. the controller design problem) is posed in an information theoretic framework. Each element is endowed with an information measure on the basis of uncertainty information theory. The framework permits:

   i) to dissect all the available information sources for the control problem. An exhaustive analysis on the information sources is conducted which comprises not only well known elements such as experimental data but also not so well recognized elements such as a priori model information (e.g. model order) and a priori controller information (e.g. controller order). For example, apart from experimental data, it turns out that to increase the model order or the controller complexity can have a dramatic influence on the final designed controller, consequently they can also be regarded as information sources to be taken into account.

ii) Information relations among the elements of the control problem are established. In this way, for example, it is shown that an information increase on the model set (i.e. a model uncertainty reduction) comes necessarily by an information increase on the data set or on the a priori model set or both. However this requirement is necessary but no sufficient as the identification algorithm could not make proper use of the extra available information. The relationships are then useful to distinguish information sources and algorithm requirements in order to increase the information of certain elements belonging to the control problem.

The above information framework is derived by first defining the control problem from a holistic approach. The approach is holistic as firstly all the elements are considered and secondly the relations among these elements are established. In this way both the elements of the control problem and the relationships among the elements are established and related with existing control theory areas. Secondly distinct definitions of the information concept are reviewed on the basis of control theory. As a conclusion it can be seen that although the information concept plays a major role on the control theory, a complete information theoretic framework of control theory is still lacking. Finally distinct formalizations of the information concept are reviewed. The Uncertainty Information Based approach is taken in order to endow the elements of the holistic control problem with an information measure from which the already stated contributions follow.

II) On the basis of the information theoretic formalization of the control problem the following points are established:

i) The control design problem is considered as a problem of managing the information flow. The information flow management point of view permits to study and compare, under an unified framework, distinct control design methodologies. For example, in one shot designs (i.e. non adaptive) the data set is kept constant meanwhile in classical adaptive control new data are periodically added to the data set.

ii) A definition of adaptive control is given. The proposed definition tries to amalgamate the concepts present in existing adaptive control definitions. Thus a system is considered adaptive if new information is acquired and the control system is modified accordingly in order to achieve some goal (e.g. improve performance, maintain performance or minimize performance degradation).

iii) Iterative Control and Classical Adaptive Control are studied as special cases of the general adaptive control approach under the information theoretic framework of the control problem. For example, the only information source that is modified on classical adaptive control is the data set. On the contrary on iterative control schemes, apart of the data set, distinct model orders and controller orders are obtained at each iteration depending on the model identification and controller design approach taken. However the most important result of adaptive control and iterative control is the lack of information monotonicity. In fact although the mentioned adaptive control approaches incorporate new information, it is also true that already existing information is discarded. As a consequence the information monotonicity property is lost. The benefits of a monotonic information iterative control are shown by means of an example of classical iterative control scheme. In the example the open loop model discarded in the iterative control is used to design a better performing controller.

III) A new validation algorithm is proposed. The algorithm named *Control Oriented Frequency Dependent Model Validation* (COFDMV) is the frequency domain counterpart of a time domain whiteness test. The main contributions of the algorithm are:

i) The validation result is no longer a binary “validated/invalidated” answer but more informative. In fact a model can be validated for some frequency range but invalidated for other frequencies.

ii) The algorithm is control oriented in the sense that a model is not validated by itself in open loop but in closed loop. Actually what is validated is the performance of a model-controller pair .

iii) The validation procedure is suited for iterative control schemes in general as it provides the following features that help to manage the information flow of iterative schemes. First the algorithm is a guide to design the next experimental input. In fact if a better model is pursued around the frequency range where the former model was invalidated, the input should contain a high energy content around those frequencies. Secondly the algorithm helps to detect possible undermodelling problems. Finally the validation procedure gives a bound on the achievable controller bandwidth with the model at hand.

The main articles on international conferences the thesis has generated are:

- *P. Balaguer, R. Vilanova and R. Moreno.* ***"The Control Problem: A Framework for Holistic Design".*** *14th IEEE Mediterranean Conference on Control and Automation, 2006.*

  In this work the control problem is presented from a holistic point of view. Moreover the information flow nature of the control problem is introduced together with a proposal to manage the information flow.

- *P. Balaguer and R. Vilanova.* ***"Is Iterative Control Wasting Information?".*** *6th IEEE International Conference on Control and Automation, 2007.*

  The information properties of existing iterative control schemes are discussed and the problem of monotonicity is arisen. An example is provided in which it is shown that taking into account disregarded information from an iterative scheme improves control performance for a wider perturbation range.

- *P. Balaguer and R. Vilanova.* ***"Frequency Dependent Approach to Model Validation".*** *6th Asian Control Conference, 2006.*

  The paper presents de fundamentals of the frequency dependent model validation algorithm

- *P. Balaguer, R. Vilanova and R. Moreno.* ***"Control Oriented Frequency Dependent Model Validation".*** *International Control Conference UK, 2006.*

  The paper endows the frequency dependent model validation approach with control oriented issues. Residual generation structures are proposed to provide the Control Oriented Frequency Dependent Model Validation algorithm (COFDMV).

- *P. Balaguer and R. Vilanova.* ***"Quality Assessment of Models for Iterative/Adaptive Control".*** *45th IEEE Conference on Decision and Control, 2006.*

  The COFDMV algorithm is presented as a suited tool for model validation on iterative/adaptive control schemes.

The following journal articles have been submitted to:

- *P. Balaguer and R. Vilanova.* ***"Information Characterization of the Control Problem. Part I: The Framework".*** *International Journal of General Systems.* (submitted)
- *P. Balaguer and R. Vilanova.* ***"Information Characterization of the Control Problem. Part II: Analysis of Adaptive Control Schemes".*** *International Journal of General Systems.* (submitted)
- *P. Balaguer and R. Vilanova.* ***"Model Validation on Adaptive Control: A Frequency Dependent Approach".*** *International Journal of Control.* (submitted)

Agraïments

Este treball ha dut molt de sacrifici i és el resultat de moltes hores de faena, cabilacions, soletat, satisfaccions, frustracions, derrotes, i èxits. Indistintament del valor científic o acceptació que el treball puga tenir, cada paraula, cada frase, cada idea és resultat de molta faena i sacrifici. Es esta faena i sacrifici el que vull dedicar-vos i amb el qual vull donar-vos les gràcies.

No puc començar per altre lloc que per el principi. Vull agrair a ma mare i a mon pare tot el que han fet per mi en aquesta vida. El seu treball ha segut el meu descans, els seus patiments la meva tranquil·litat, i la meua alegria la seva. Ells m'han ensenyat les coses realment importants en la vida, les coses a partir de les quals tot lo demés ha eixit com una simple conseqüència .

Al meu iaio Vicent, per ensenyar-me el valor del coneiximent, la ciència, la paciència, la prudència i el gust per la faena ben feta.

Al meu iaio Pere, per ensenyar-me a alçar-me i tornar a pujar a la bicicleta quan estava en terra, a ser atrevit, valent, apretar les dents i seguir avant.

A la meua iaia Pilar amb qui tantes vespraes vaig passar jugant a cartes. Una pena que te'n anares tan pronte.

A la meua iaia Pascuala amb qui encara tinc la sort de poder passar algun ratet. Preparat que encara tenim que escriure les memòries de la guerra...

A la meua germana i a Felipe.

Al meu nebot, amb la il.lusió de tot allò que farà possible.

A un home que va lluitar per la llibertat,

Onofre Domenech Taurà

¡PRESENTE!

A tots aquells que van fer possible les festes del poble: les històries de Pascual “el chato”, l’alegria de Juan “el gordo”, els chistes de Pascualet “el carnisser”, l’esperit de xiquet de Pepe Luis “el pintor” (q.e.d), les converses en Julio, Pascual “el gelat”, Gerardo, Rosita, Pilar, Maria, Teresita, Encarnita, Enriqueta... I tots els xiquets que allí estavem, Maria Pilar, Silvia, Ana, Hector, Laura i Julio.

Als meus amics Julio i Mayca, Manuel i Isabel, Hector i Marian, Jose “el brother”, Javi, Pepe i Tere, Oscar i Marina, Dani i Bea i el “Tente”. Hem passat de jugar al “futbolin” a anar de boda. I el que vindrà...

A Carlos i a Alberto. Dos companys excepcionals de carrera. Dos grans enginyers.

A tots els companys del Departament de Telecomunicació i enginyeria de Sistemes de la Universitat Autònoma de Barcelona. En especial a Carles per la seua comprensió, anim i per ser un gran company de despatx, a Roman per el seu companyerisme, a Monica per aguantar els problemes informàtics, a Chantal per la seva “simpatia”, a Sònia i Laia, les meves companyes de viatje, a Dani (també conegut com D.G.) per la seva paciència davant l’adversitat, a Orlando per les seves cançons, a Asier per els seus comentaris, a Fausto per el seu orgull, a David per la seva simpatia, a Miguel per viure la vida i a Natasha per mostrar-me la dimensió espiritual de les coses. A Ramon per la seva disponibilitat, a Ignasi per el seu sentit de l’humor i a Romu. A Angels, Imma i Paqui per la seva ajuda a la secretaria. A Ernesto per la seva disposició. I encara que siguen “telecos”, a Gonzalo, a Gary i a Josep.

A tots vosatros, gràcies.

Contents

Chapter 1

Introduction

In this introductory chapter the problem to be considered throughout this thesis is introduced and motivated. Roughly speaking the problem to be tackled is the management of information for control design purposes. The problem is described and a brief analysis of the solution is introduced. The chapter finishes with an outline of the thesis.

1.1 Problem Statement

1.1.1 High Performance Control Systems

Control systems are present nearly in all areas of industry and research. The wide applicability of control is one of its major features. The fact that automatic control has become a widespread science is both an opportunity and a drawback. Its popularity makes automatic control to be full of application areas and intense research. Unfortunately automatic control is often hidden by the technology and not recognized by itself (Bastin and Gevers, 1997).

Notwithstanding, automatic control is a fundamental pilar of the current development state. The benefits of the current mass production system are mainly due to three factors, the development of assembly line production systems, the mass markets and the energy sources management. All this developments are deeply related with automatic control

- Assembly line production system: automatic control systems assure the repetibility of certain process variables (i.e. accuracy limits and tolerances), allowing assembly line production systems (i.e. mass production systems).
- Mass markets: the development of mass market was due not to only the mass production systems but to transportion evolution. Control systems are responsible for the current transportation development levels (e.g. aviation and shipping development levels).
- Energy management: the control systems applied to energy management have been the responsible of achieving good efficiency rates of energy production together with a broad distribution of energy for industrial use.

In order to improve the benefits of control systems a better control performance is pursued. Conceptually two are the advantages of improving performance:

- Productivity increase.
- New products development.

The first benefit is a quantitative improvement. The same result can be obtained in a more economic way (i.e. increasing outputs and/or decreasing inputs). On the other hand, the second advantage of a is qualitative nature. A different product can be obtained by designing a higher performing control systems (e.g. a chemical with a purity level of 99%).

Although the pursue of high performing control systems is desirable, the idea by itself can be misleading. Firstly, the highest performance achievable by any control system is limited by the plant to be controlled. In fact delays, sensors quality, actuators power and other hardware properties limit the achievable performance. Secondly any control design is a trade-off of competing factors, so high performance actually means a compromise of competing factors. These are the reasons that make control systems a nontrivial task and, even worse, a task with physical bounds that can not be crossed by any control design.

High performance control system is not only related with control design but it is a broader concept depending on:

- Plant Design.
- Controller Design.
- Maintenance.

Plant design is concerned with controller and actuator selection and allocation, plant size (e.g. civil engineering) etc. We assume that the plant to be controlled is given and we only focus on the Controller Design and Maintenance steps. Controller design comprises all the theoretical aspects involved on the model based control design (i.e. identification and control synthesis and analysis). Maintenance is concerned with an already existing control system. The procedure of maintenance in the present thesis is understood as a supervision system with two clear actions. First the system is monitored in order to detect changes (e.g. performance degradation, plant changes detection). Secondly actions are taken to achieve the goals based on the information at hand (e.g. controller redesign, plant reidentification).

Consequently in this thesis a High Performance Control System (HPCS) is defined as a system with High Performance Controller Design (HPCD) plus Maintenance considerations. Hence

$$HPCS = HPCD + Maintenance \tag{1.1}$$

The above definition relates HPCS with information through two terms, the HPCD and Maintenance. The main difference between both terms, as far as information is regarded, is that whereas the HPCD step is static (i.e. only information at hand is used) the Maintenance process is a dynamic one (i.e. new information is considered).

1.1.2 Information and High Performance Control Systems

In this thesis we focus on the problem of achieving High Performance Control Systems through the management of information. The idea is to manage the information flow trough the Maintenance term in order to increase the information available to the HPCD step, thus achieving HPCS. In this sense the Maintenance term is an information flow manager of the control system.

The idea of adding new information in the control design problem in order to improve performance is not new. It has been done since the 60's with the appearance of adaptive control (Harris and Billings, 1981) (Astrom and Wittenmark, 1989). However as stated in (Ioannou and Sun, 1996) "*the field of adaptive control may easily appear to an outsider as a collection of unrelated tricks and modifications*", being the most important reason that "*the lack of a conceptual framework for adaptive control has inhibited research in this area and made it difficult to compare alternative designs*" (Zames, 1998). It follows that although the idea of introducing new information in the process of designing a controller has been recognized since long time, the framework in which carry out this process is still lacking.

In fact, even the term information when applied to control theory context does not have a clear and unified definition. Not at least as in the case of communication theory (Shannon, 1948). Notwithstanding the information concept has been intimately joined with control theory in general through several areas, for example adaptive control, robust identification, robust control, fundamental limitations in control and networked control systems.

The deficiency on the role of information for control system design has been pointed out in several reports (Witsenhausen, 1971) (Lewis *et al.*, 1987) (Zames, 1998) (Touchette and Lloyd, 2004), For example in (Lewis *et al.*, 1987) it is stated: "*Adaptive control is a promising approach to achieve performance robustness. Its present setting is limited: it makes use of the most structured uncertainty in which the plant model has a known form, but unknown parameters*".

Iterative identification and control schemes (Bitmead, 1993) (Albertos and Sala, 2002) are more developed techniques for information management for control design. The objective is equal to the classical adaptive control (Astrom and Wittenmark, 1989), which is to improve the controller performance by means of adding new information on the control design step. However the information managed by iterative control approaches is much richer. For example in the windsurfer approach the following information related questions are pointed out (Lee *et al.*, 1995):

- *When can one redesign the controller and expand the bandwidth without re-identifying?*
- *When should one re-identify?*
- *What does one want to identify in the re-identification process?*
- *How can an identified model be verified against the desired purpose?*
- *Will re-identification always lead to improved closed-loop performance?*

The above considerations, which are not present on classical adaptive control schemes, can be casted in a more abstract information framework as follows:

- Is the model information at hand still valid?
- When should new information be added?
- What information is required?
- How can the information being validated?
- Is the lack of information limiting performance or there are any other causes?

These questions aim towards the problem of managing the information on the control design problem. Thus the problem is the management of the information flow in order to decide what information should be added, when it should be added and the mechanism and procedures necessary to accomplish the required tasks. Summing up, *the problem to be tackled in this thesis is how* *to manage the information flow of the control problem in order to improve the designed controller*. This problem is both a fundamental one and very general in nature. In this thesis we focus on two distinct aspects of the problem. The first one, that is more conceptual, tackles the problem of formalizing the framework in which study the information theoretic issues of the information flow for control design. The second aspect of the problem to be considered is of a more technical nature and deals with the development of a new model validation algorithm which provides guides to help in the information management on iterative identification and control schemes.

1.2 Introduction to the Solution

The first goal of the thesis is to define the conceptual framework which links the controller design problem with the information concept. The objective is to provide a framework in which the problem of managing the information of the control problem can be characterized, providing thus a baseline in which distinct approaches can be compared. The analysis of the framework also reveals necessary conditions among the elements in order to increase their information content.

Secondly, at the light of the proposed framework, two adaptive schemes, classical adaptive control and iterative control are compared. Their differences and similarities are established together with their advantages and disadvantages.

Finally, in a more technical context, a new validation procedure for iterative identification and control schemes is designed. The new algorithm provides a validation procedure that is more informative than classical methods which just provide a “validated/invalidated” result. The new validation algorithm provides new guides in order to manage the information of the control design problem.

1.2.1 A General Framework for Information and Control Design

In this part it is established a theoretical framework for the information management for control design purposes. First the control problem is formalized into its constitutive elements and relationships. It is shown how these elements and relationships can be related with existing control theory areas (e.g. model validation, control design, performance monitoring, etc).

Once the elements and relationships of the control problem are established, they are endowed with the information measure. The information measure is based on the set size. It then follows that it is possible to analyze their information relationships. For example it is possible to state necessary conditions over the data set (e.g. experimental data) and the a priori model set (e.g. model order) in order to increase the information of the model set (e.g. identified models family). The framework serves as a basis in which exhaustively enumerate all the possible information sources of the control design problem.

1.2.2 Information Supervision of Adaptive Control Schemes

Once the information theoretic framework is established, it is used to analyze how distinct adaptive schemes manage information. Thus classical adaptive control and iterative control (i.e. windsurfer approach) are dissected under the proposed framework. The similarities and differences clearly appear under the proposed framework. This analysis of the information management aims towards possible improvements on the way information is managed in the analyzed schemes. It can be seen that both schemes manage information in a distinct way. It can be concluded that iterative control takes into account more information management aspects of the control design problem than classical adaptive control. However it is showed that both, classical adaptive control and iterative control do not possess the information monotonicity property, that is, at each iteration step, information is lost.

1.2.3 Frequency Domain Model (In)Validation

A new model (in)validation algorithm is developed. The objective is to derive a model (in)validation procedure which is more informative than classical validation methods that just “validated/invalidate” a model. The algorithm is suited for iterative identification and control schemes. It is a frequency domain counterpart of a time domain whiteness test. In fact it is the frequency domain nature of the algorithm which provides a more insightful validation procedure. The algorithm allows to validated a model for certain frequency range, and invalidate the same model for other frequencies. This new available information is useful in several ways for managing the information flow.

It helps to define the experimental input for future experiments in order to obtain more informative data. It is useful also to select the model order to be fitted by the data and to decide the maximum allowable controller bandwidth with the model at hand. Thus the validation algorithm helps to manage the information flow of the control problem.

1.3 Thesis Outline

The thesis contents are mainly restricted to include only original discussions and contributions, so reviews are kept to a minimum. As a result this thesis is not utterly self contained but reviews and bibliographical references are provided for the sake of clarity. The thesis is divided in three parts, the first one is of a more conceptual nature meanwhile the second one is more technical. The last one includes the conclusions, open research areas and publications. The thesis outline together with a brief chapter content description is as follows:

Part I: Control and Information. On the first part the information theoretic framework is defined and the information relations of the control problem analyzed. Next, an analysis of existing adaptive control schemes (e.g. classical adaptive control and iterative control) are conducted on the basis of the cited information framework.

- **Chapter 2:** The chapter introduces the conceptualization of the control problem by presenting the constitutive elements and their relations. The elements and relations of the control problem are then dissected and related with existing control theory areas.
- **Chapter 3:** A review of the information concept applied to control theory is conducted. The objective is to study in how many distinct ways the information concept is applied to control theory. The definition of the information concept to be used in the formalization of the information theoretic framework for control design is introduced.
- **Chapter 4:** The information framework characterizing the control design problem is defined. The framework provides information relations among the elements in order to state the necessary requirements to increase the information of the elements. These relations are formalized by means of mathematical theorems.

Chapter 5: The task of managing the information flow for control design is associated with the concept of supervision. Adaptive control is reviewed from a conceptual point of view and a new definition developed. Finally classical adaptive control and iterative control are analyzed and compared under the information framework proposed.

Part II: Frequency Domain Model (In)Validation. The second part is completely devoted to the derivation of the frequency domain model (in)validation algorithm, its control oriented properties and application examples.

Chapter 6: The classical validation procedures are reviewed and discussed regarding the requirements of general adaptive schemes. The basis of the new validation algorithm, the Frequency Domain Model (In)Validation (FDMV) algorithm, are presented.

Chapter 7: In this chapter the control oriented requirements for the validation algorithm are presented. It is discussed how the FDMV algorithm is endowed with the control oriented property. The Control Oriented Frequency Dependent Model (In)Validation algorithm (COFDMV) is then developed.

Chapter 8: In this chapter application examples of the COFDMV algorithms are presented and discussed.

Part III: Epilogue. In this part the thesis conclusions are established.

Chapter 9: Finally the contributions of the thesis are summarized and the main conclusions together with future research areas are pointed out. A list of publications generated by the thesis can also be found.

Part I

Control and Information

Chapter 2

The Control Problem: A Holistic Approach

In this chapter we introduce a mathematical formalization of the problem of model based control design and maintenance. First we state the mathematical elements of the control problem. The elements, defined on the basis of set theory, are divided between a priori elements and a posteriori elements. The a priori elements are the elements defined without the necessity of real data. The a posteriori elements are the ones derived from real data either directly or indirectly.

Once the elements are defined, it is possible to study their relationships. The relationships are established formally and related with existing control topics. The analysis of the relationships shows that these relationships can be divided into two groups. The first gathers all the relationships responsible for transforming existing elements into new ones. The second group comprises all the relations that do not provide new elements but check consistency among the existing ones.

2.1 Introduction to the Control Problem

The objective of the control problem is to make a physical system behave in a desired manner. The control problem is then a control engineering problem. Control engineering can be divided into three points equally important in order to achieve a satisfactory controlled system. These points are controller design, controller implementation and controller maintenance. In what follows we focus on the control problem related with controller design and controller maintenance disregarding aspects concerning controller implementation (e.g. real-time software and hardware issues).

Controller design requires solving a variety of issues, ranging from system identification, model validation, control structure selection, assessing fundamental limitations in control and controller design among others. On the other hand, controller maintenance is related with performance monitoring, fault detection and isolation, system reidentification, controller redesign, etc. Moreover the areas of knowledge related with control systems are broad (e.g. mathematics, engineering, physics, etc). These facts make the control problem far for being trivial due to the broadness of questions involved and the technical knowledge required to solve the presented issues. Thus the control problem is a complex one.

Traditionally the complexity of the control problem has been overcome tackling the problem from a reductionist point of view (Gevers, 2006). The reductionist approach divides the main problem into more manageable subproblems which are solved separately. Thus the control problem is viewed from distinct points of view from the control community. For example the system identification community is concerned with algorithms for identifying "good" models or/and accurate model error bounds. On the contrary the control design community is concerned with the problem of controller design on the basis of models and "appropriate" specifications.

Notwithstanding the benefits of the reductionist approach, that has given expression to successful control designs, the reductionist approach suffers the following limitations:

- The subdivision introduces assumptions over the working elements which are usually accepted without discussion and/or can not be easily checked with the elements belonging to other subproblems (Balaguer *et al.*, 2006*b*). This fact can limit the achievable solutions.

- Recent advances and developments in identification for control (Gevers, 2002), robust identification (Chen and Gu, 2000) and iterative identification and control (Albertos and Sala, 2002) have arisen the necessity of tackling the identification and control design processes in an unified approach.

- Issues arising from the interrelation between subproblems belonging to the reductionist approach (e.g. fundamental limitations in control, monitoring performance issues) are difficult to arise and consider during the design.

- In order to design automatic supervisors, that is systems that automatically perform one or several functions of the control design step (e.g. identification, control design), high level issues must be considered. For example, it is necessary to know the reason that is limiting the control loop performance (e.g. fundamental limitations, bad controller design, bad model accuracy) in order to take the appropriate correcting actions (e.g. plant redesign, controller redesign, plant reidentification) (Balaguer *et al.*, 2006*b*).

It follows from the above difficulties that to formulate and answer higher level questions regarding the control design problem, a holistic point of view is necessary. This fact motivates to present a framework in which the control problem is presented from a holistic point of view.

In this chapter we analyze the model based control design procedure. The objective is to dissect both the elements and the relationships of the general model based control problem. Admittedly a model is not strictly necessary in order to design a proper controller. The literature shows approaches such as the Iterative Feedback Tuning (IFT) (Hjalmarsson *et al.*, 1998) in which controllers are designed without any model requirements. However the use of a model is more informative as for example, robust stability issues can be considered (Safonov and Tsao, 1997).

2.2 Elements of the Control Problem

We refer as the elements of the control problem the mathematical entities defined and used through the control design process. We group these mathematical entities on the following sets:

$\mathscr{A}$ A priori set. This is a set that comprises the tuple $\mathscr{A} = \{\mathscr{A}_{\mathscr{D}}, \mathscr{A}_{\mathscr{M}}, \mathscr{A}_{\mathscr{S}}, \mathscr{A}_{\mathscr{K}}\}$, where:

- $\mathscr{A}_{\mathscr{D}}$ A priori Data set.
- $\mathscr{A}_{\mathscr{M}}$ A priori Model set.
- $\mathscr{A}_{\mathscr{S}}$ A priori Specification set.
- $\mathscr{A}_{\mathscr{K}}$ A priori Controller set.

$\mathbb{D}$ Data set.

$\mathbb{G}$ Model set.

$\mathbb{S}$ Specification set.

$\mathbb{K}$ Controller set.

Figure 2.1: Elements of the Control Problem

The first difference among the sets introduced (see figure 2.1) is that whereas the set $\mathscr{A} = \{\mathscr{A}_{\mathscr{D}}, \mathscr{A}_{\mathscr{M}}, \mathscr{A}_{\mathscr{S}}, \mathscr{A}_{\mathscr{K}}\}$ refers to a priori elements (i.e. elements not based on any particular experiment result), the rest of the sets, that is, $\{\mathbb{D}, \mathbb{G}, \mathbb{S}, \mathbb{K}\}$ are the direct or indirect result of some experimental data, thus being defined as a posteriori elements[1]. Consequently the a priori set is the responsible of defining the whole mathematical structure in which the control problem is being tackled and possibly solved.

On the other hand, the a posteriori elements $\{\mathbb{D}, \mathbb{G}, \mathbb{S}, \mathbb{K}\}$ are tangible realizations of the mathematical environment described by the a priori set. The a posteriori elements are divided in four sets in which the data set is the origin of the rest of the sets, as in order to obtain any other a posteriori set, data is mandatory. The division of the sets follows the conceptual formulation of the control problem, that is, design a controller $\mathbb{K}$ for some given plant $\mathbb{G}$ that fulfils some requirements $\mathbb{S}$.

In the next paragraphs each set is described more thoroughly and we give some examples of the assumptions normally taken over these sets.

A Priori Set

This set gathers all the a priori elements (i.e. elements that are not based on data records). The a priori set is the responsible of defining the mathematical structure that will be assumed by the a posteriori elements of the control problem. In fact it is the a priori set which completely formalizes the control problem to be solved. Its importance is crucial, as the same problem, can be either solvable or unsolvable regarding the a priori set. For example the problem with the following elements: $\mathscr{A}_{\mathscr{S}}$=Zero Stationary Error, $\mathscr{A}_{\mathscr{M}}$=First Order LTI Plant, $\mathscr{A}_{\mathscr{K}}$=PD Controller, is not solvable, although changing the a priori set $\mathscr{A}_{\mathscr{K}}$ to be a PI Controller makes the solution achievable.

The a priori set is subdivided into four sets, $\mathscr{A}_{\mathscr{D}}, \mathscr{A}_{\mathscr{M}}, \mathscr{A}_{\mathscr{S}}$ and $\mathscr{A}_{\mathscr{K}}$, regarding the a priori considerations taken over the data , the model family, the specifications considered and the controller to be designed. These sets are:

$\mathscr{A}_{\mathscr{D}}$ A priori Data set. This set defines the mathematical characterization of the data. The related mathematical issues are defined from practical aspects such as:

- Sensors number and placement.

[1] A priori elements are the elements provided by the control engineer. On the other hand the a posteriori elements are elements obtained through the analysis and manipulation of collected data.

– Sensor noise level. The sensors noise level can be characterized either stochastically (e.g. gaussian noise) or deterministically (e.g. unknown but bounded).
– Sensor linearity characteristics.
– Sensor quantization level.

The set defines the structure of the data set. For example, if there is just one sensor, the data set is an array of reals (possible quantized) numbers. If two sensors are present, then the data set is formed by two data arrays. Moreover the set also defines all the possible outcomes of any experiment. This information is normally set at the plant design level and it is not changed as usually modifications implies hardware manipulation.

$\mathscr{A}_{\mathscr{M}}$ A priori Model set. This set defines the mathematical characteristics of the plant model considered. The plant model includes the nominal model and the uncertainty description of the model (if any). Among others, the set is mathematically defined on the basis of:

– Linear time invariant (LTI) models versus linear time variant (LTV) models.
– Linear versus Nonlinear models.
– Parametric models versus non parametric.
– Model structure (e.g. ARX, ARMA).
– Model order (e.g. polynomial degree).
– Model uncertainty description (e.g. parametric, multiplicative, etc.).

This set is seldom changed during the overall control design process if a reasonable description of the plant can be formalized within the mathematical assumptions taken.

$\mathscr{A}_{\mathscr{S}}$ A priori Specification set. This set defines the manner performance is assessed. The specifications are normally divided into stability specifications and performance specifications.

- Stability Specifications. The type of uncertainty against which the system is robust (e.g. gain margin uncertainty, multiplicative uncertainty, etc.) and the norm used to measure it (e.g. H_∞ norm, l_1 norm, gap metric, etc.).

- Performance Specifications. The cost functions which measures the performance (e.g. quadratic costs, etc).

$\mathscr{A}_{\mathscr{K}}$ A priori Controller set. This set establishes the mathematical definition of the controller to be designed. The main considerations are:

- Controller Architecture (e.g. 1 D.O.F. Vs 2 D.O.F., Smith Predictor, cascade configuration, internal model control, etc).
- Controller Order and Structure (e.g. PI, PID).

The importance of this set is that it defines, in mathematical terms, the search space of the controller. If the solution to the control problem does not lie inside the search space, it never will be found. On the other hand it can be the case that no solution exists hence the problem is unsolvable whatsoever search space is chosen.

Data Set

The data set $\mathbb{D}$ comprises all the measured variables from one or more experiments. The data can come from an experiment, either in open loop or closed loop, or from normal plant operation. It is clear that $\mathbb{D} \subset \mathscr{A}_{\mathscr{D}}$, namely, the result of any experiment (or experiments combination) must be a subset of the all possible experimental results given by the a priori data set. If the set $\mathbb{D}$ is empty (e.g. no experimental data) we are dealing only with a priori elements. All the a posteriori elements have their origin on this set.

Model Set

The model set $\mathbb{G}$ comprises the identified model family that are used to design a controller (a singleton if just a nominal model without error bounds is identified). The model family is a subset of the model set defined in the a priori information, thus $\mathbb{G} \subset \mathscr{A}_{\mathscr{M}}$. The set inclusion means that the model set is a particularization, for some given data, of the a priori model set. The model set is a posteriori set as data is mandatory to perform the identification. An example of the a priori model set is the set formed by first order plus time delay linear time invariant models (FOPTDLTI). On the other hand a posteriori model set is a FOPTDLTI with time constant $\tau = 5$, gain $k = 10$ and delay $d \in [0.5, 1]$.

Specification Set

The specification set $\mathbb{S}$ is the set containing the values and/or limits of the design specifications defined in the set $\mathscr{A}_{\mathscr{S}}$. For example this set contains all the specifications defined by a quadratic cost function which value is limited by the amount α imposed by the designer. Again $\mathbb{S} \subset \mathscr{A}_{\mathscr{S}}$, thus this set is a particularization for some performance level. An example of stability specification is the gain margin (e.g. a gain margin of 0.5).

Controller Set

The controller set $\mathbb{K}$ is the set of all controllers that accomplishes the specifications defined in the above set $\mathbb{S}$ for all the plants on the model family $\mathbb{G}$. These controllers are a subset of all the controllers defined in the a priori information (i.e. $\mathbb{K} \subset \mathscr{A}_{\mathscr{K}}$). If no controller can be found to accomplish the requirement then the controller set is empty, that is $\mathbb{K} = \varnothing$. The most common a priori controller set $\mathscr{A}_{\mathscr{K}}$ is the PID controller. The controller set is then defined by all the values of K_P, K_I and K_D which achieve the specifications.

2.3 Relationships Between Elements of the Control Problem

The elements of the control problem already introduced are necessary to solve the control problem but not sufficient. In fact some elements are the result of manipulations and combinations of other elements. In this section we state the relationships among the elements of the control problem in order to design a controller. First we identify each one of the element relations and define the relationship on the basis of function theory. Secondly it is shown how each one of these relationships can be related to areas of control theory (e.g. identification, control design, limitations in control, performance assessment, etc).

In figure 2.2 the elements are plotted together with their relationships. These relations are:

$\mathcal{I}$ Identification.

$\mathcal{C}$ Control Design.

$\mathcal{O}$ Consistency.

$\mathcal{V}$ Model Validation.

$\mathcal{L}$ Limitations in Control.

$\mathcal{M}$ Monitoring.

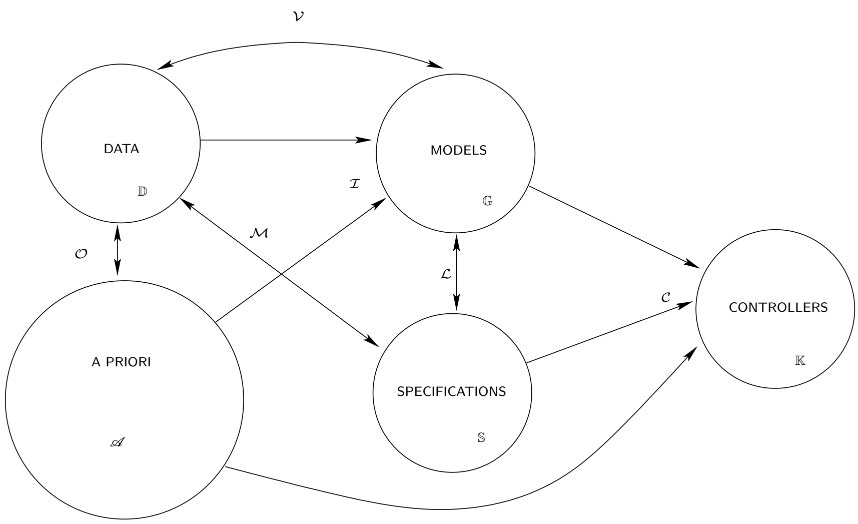

Figure 2.2: Relationships Among the Elements of the Control Problem

The five relations established can be divided into two groups regarding their functionality. On the one hand we have the relations which objective is to manipulate elements in order to derive new ones. To this group belongs the Identification, $\mathcal{I}$, and Control Design, $\mathcal{C}$, functions. Indeed the objective of identification is to manipulate data and the a priori model information in order to obtain a model set. Similarly, the objective of control design is to obtain a controller (or a family of controllers) derived from a model set $\mathbb{G}$, a specification set $\mathbb{S}$ and a priori controller set $\mathscr{A}_{\mathscr{K}}$.

On the other hand the relationships Consistency, $\mathcal{O}$, Model Validation, $\mathcal{V}$, Limitations in control, $\mathcal{L}$, and Monitoring, $\mathcal{M}$, are aimed towards validating the existing elements. The objective is to detect any possible mismatch between the elements. The consistency check, $\mathcal{O}$, is a validation of the collected data with the a priori data set. The Model Validation procedure, $\mathcal{V}$, is aimed at finding invalidation results between the model set identified and the experimental data collected. Similarly, the Monitoring relation $\mathcal{M}$ is a validation between the specification set and the actual achieved specifications calculated from collected data. Finally the Limitations in Control relation, $\mathcal{L}$, is a consistency check between the established designer specifications and the limitations imposed by the model set (e.g. right half plane zeroes and poles).

In the next paragraphs each relationship is established formally and described on the basis of well established control topics.

Identification

The aim of the identification is to derive a model (and possibly error bounds) for control design purposes. The identification is mathematically defined as an application from the set of possible models given by the a priori information $\mathscr{A}_{\mathscr{M}}$ and the set of data $\mathbb{D}$ to the set of possible models $\mathscr{A}_{\mathscr{M}}$.

Definition 2.3.1 *The identification function* $\mathcal{I} : \mathscr{A}_{\mathscr{M}} x\mathbb{D} \rightarrow \mathscr{A}_{\mathscr{M}}$ *is defined as*

$$\mathcal{I}(g, d) = \{g \in \mathscr{A}_{\mathscr{M}} | E(g) = d, \forall d \in \mathbb{D}\}$$

where $E(g) = d$ *is any operator relating the a priori information to the data. (e.g. convolution operator* $y = G * u$*).*

◊

The topic of system identification has been of major importance for control system design. The theory is well established and plenty of successful applications, however recently (last 15 years) fundamental issues regarding identification of models for controller design have modified the traditional point of view (Gevers, 2006). The angular stone has been to consider the final use of the model (e.g. prediction, control design) on the system identification process. Indeed, as it was pointed in (Skelton, 1989), small model-plant mismatches can lead to great different behaviors when both are operated in closed loop but two different systems can behave quite similarly under feedback. The reason is that under feedback some model errors are amplified whereas other model errors are attenuated. Additionally, the concept of identifying the “real” plant has proven to be bogus (Hjalmarsson, 2005) due to the following reasons:

- The order of real systems is infinite.
- Data collected from plant are always finite.
- Data are always corrupted by noise.

The above considerations have directed intensive research in order to identify “good” models for control design purposes. This research can be classified in two main points which are:

- Input experiment design. The fundamental importance of input experiment design for system identification has been pointed out in (Ljung, 1999) (Soderstrom and Stoica, 1989). In (Bitmead *et al.*, 1990) it is shown that under undermodelling conditions the identified plant depends on the input applied. Thus if a good model for control design purposes is pursued, the problem of input experiment design arises. This problem can be tackled by identifying the plant in a closed loop setting. The advantage is that the same control loop weights the energy input that is applied to the plant, thus providing more suited data for control design purposes.
- Identification criterion. Once data is collected and a model structure is chosen, it remains to fit the parameters of the model with the experimental data. Several possibilities arise:
    - Minimization of the discrepancy of a model and a plant. This is the approach taken by classical open loop identification (Ljung, 1999).
    - Minimization of the discrepancy of two controlled loops (Landau and Zito, 2006*a*).
    - Minimization of the model-plant discrepancy measured through a control design cost function. This is the approach taken in iterative identification and control design schemes (Albertos and Sala, 2002).

The input experiment design and the identification criterion entwine together arising the broader question of Open Loop Identification vs Closed Loop Identification. The results mentioned above aims towards closed loop identification, not to mention practical requirements such as identification of unstable plants. Adaptation of classical prediction error methods to be applied in closed loop settings can be found in (Landau and Zito, 2006*a*). The model error obtained is weighted by a function depending on the closed loop sensitivity function S (e.g. $S = (1+GH)^{-1}$), thus giving less error around the frequencies of interest in which the S function is “big”.

Another completely different area of system identification that has captured intensive research has been the identification of model error bounds, known also as robust identification or control oriented identification. The origin of this topic lies on the requirements of robust control design procedures of not only a nominal model but also a model error bound. The robust identification problem was posed originally in (Helmicki *et al.*, 1991) receiving considerably attention since then. Monographic publications are (Sanchez-Peña and Sznaier, 1998) and (Chen and Gu, 2000). The approach taken was a worst case deterministic approach, leading to controversial discussions between hard bounds and soft bounds. However after a decade of intensive research the real limitation of the procedures proposed was established. The robust identification methods where aimed towards identifying small uncertainty bounds, disregarding completely quality issues on the nominal model used to design the controller. This together with their inherent conservativeness due to the worst case approach has limited its popularization.

Control Design

The cornerstone of the control problem lies on the controller design step. In fact the problem is solved if a proper controller is designed and implemented. The control design is mathematically defined as an application from a model set $\mathbb{G}$, a priori assumptions of the controller $\mathscr{A}_{\mathscr{K}}$ and the specification set $\mathbb{S}$ to a controller $\mathscr{A}_{\mathscr{K}}$. It is expected that the designed controller performs accordingly with the specifications for the whole family of models $\mathbb{G}$.

Definition 2.3.2 *The control design function* $\mathcal{C} : \mathbb{G}x\mathbb{S}x\mathscr{A}_{\mathscr{K}} \rightarrow \mathscr{A}_{\mathscr{K}}$ *is defined as*

$$\mathcal{C}(g, s, k) = \{k \in \mathscr{A}_{\mathscr{K}} | F(g, k) = s, \forall g \in \mathbb{G}, \forall s \in \mathbb{S}\}$$

where $F(g, k) = s$ *states that the closed loop of* g *and* k *accomplishes the performance requirements.*

◊

Normally the knowledge on the plant is limited to certain levels of accuracy and thus uncertainty is present. This arises the problem of robustness.

A controller is robust if certain property (either stability or performance) is accomplished for all the members of certain family (Doyle *et al.*, 1992). This is the manner in which the robust control design approach tackles the problem of uncertainty. First the plant is modelled as a family of plants in which lies the real plant. Next, a robust controller is designed. As the resulting controller is robust, the real plant is satisfactorily controlled[2].

The above discussion allows us to classify model based control design approaches in two main areas regarding the model characteristics considered:

- Classical design approaches. It comprises a wide range of different techniques. However their common point is the lack of model uncertainty consideraions during the design step.

- Robust design approach. In the 80's a thorough investigation on robustness issues started. The objective was to explicitly take into account model uncertainty during the design step, which was found to be the main drawback of the classical approach. In the seminal papers (Zames, 1981) and (Doyle and Stein, 1981) the idea of robust control is suggested. The robust design approach is presented in (Zhou, 1998) and (Sanchez-Peña and Sznaier, 1998).

Consistency

Consistency checks if the a priori information agrees with the experimental data and vice versa. In fact the consistency check can serve either to disregard wrong a priori information when proper data is used or to disregard experimental procedures generating misleading data when truthful a priori information is present.

Definition 2.3.3 *We define the Consistency function* $\mathcal{O} : \mathscr{A} \rightarrow \mathscr{A}_{\mathscr{D}}$ *as the function that returns all the possible data generated by the a priori set.*

$\diamondsuit$

[2]It is necessary to mention that although the robust control design procedures produce robust controllers, this fact does not mean that any other methods of controller design produces non robust controllers. Indeed other control design methodologies can produce robust controllers too. The only difference is that the robustness requirement is not considered explicitly in the design whereas robust control design takes robustness issues in consideration.

Definition 2.3.4 *Given $\mathscr{A}, \mathbb{D}$ and $\mathcal{O}$, $\mathscr{A}$ and $\mathbb{D}$ are consistent if $\mathbb{D} \subset \mathcal{O}(\mathscr{A})$*

$\Diamond$

In words, the a priori set $\mathscr{A}$ and the a posteriori data set $\mathbb{D}$ are consistent if the data set is a subset of all the possible data generated accordingly with the a priori assumptions.

The consistency issue is not a topic by itself in the control theory literature. However it can be related with fault tolerant control and fault detection issues. Fault tolerant control (Isermann, 1997) deals with the design of systems tolerant to faults (i.e. non allowed deviation of a characteristic or parameter of a system), leading to a system that can cope with faults. To this end, fault detection (Blanke *et al.*, 2003) is a mandatory step. The fault detection step is a consistency check between current acquired data and a priori information. It should be noted however that fault detection includes changes on plant (e.g. time varying parameters). Thus plant variations are considered a type of fault. On the other hand the problem of plant variations can also be tackled from an identification point of view in the paragraph on model validation.

We expose the utility of consistency by means of a simple example.

Example 2.3.1 *The a priori information about a SISO plant states that we are dealing with a first order system. However experimental data shows an oscillatory behavior of the output. Hence inconsistency is present.*

$\triangle$

Limitations

The fact that the implementation of a controller can change the dynamic behavior of a system tends to mask the fundamental limitations of any physical system (e.g. system power is limited, delays are unavoidable, etc.). A control system can not be pushed further than its fundamental limitations (Seron *et al.*, 1997). Control designs based on specifications aiming at higher performances than those allowed by the physical limitations are a certain failure. Thus in order to avoid stating impossible designs, the compatibility between the designer proposed specifications and the plant allowable specifications must be checked.

Definition 2.3.5 *We define the Limitation function $\mathcal{L} : \mathbb{G} \to \mathscr{A}_{\mathscr{S}}$ as the function that returns the allowable performance values imposed by the models family $\mathbb{G}$.*

$\diamondsuit$

Definition 2.3.6 *A control design problem based on the models family $\mathbb{G}$ is feasible for some $\mathbb{S}$ if $\mathbb{S} \subset \mathcal{L}(\mathbb{G})$*

$\diamondsuit$

Thus the fundamental limitations requires that the set of desired specifications is a subset of the achievable specifications.

Limitations on control literature appeared at the same time as control theory (see for example (Bode, 1945) (Horowitz, 1963)). The limitations are due to plant structure (e.g. RHP poles and zeroes, delays) and actuator limitations (e.g. saturations, rate limitations, etc.). In (Skogestad and Postlethwaite, 1996) a good introduction to fundamental limitations is presented.

Model Validation

Once a model has been identified it remains to check its validity for the intended use. The term model validation is widely accepted although it is misleading. Admittedly, as stated in (Popper, 1958), the scientific method can only invalidate existing theories. Thus a model can only be invalidated never validated. This is so because further data could invalidate an existing validated model by former data sets. In what follows the terms model validation and model invalidation will be used indistinctly, however a model can only be invalidated.

There are certain identification algorithms that guarantees the identified plant is validated (e.g. interpolatory algorithms (Chen and Gu, 2000)). However this is not the general case, hence the necessity of the validation step.

Definition 2.3.7 *We define the Validation function $\mathcal{V} : \mathbb{G} \to \mathscr{A}_{\mathscr{D}}$ as the function that returns all the possible data generated by the models family $\mathbb{G}$.*

$\diamondsuit$

Definition 2.3.8 *A models family $\mathbb{G}$ is validated against certain data set $\mathbb{D}$ if $\mathbb{D} \subset \mathcal{V}(\mathbb{G})$*

◊

In words a model is validated if the data at hand is a subset of all the possible data that the model set can generate.

Model validation is highly related with the assumptions taken during the model identification step, thus it is discussed in the bibliography of model identification. See (Ljung, 1999) (Soderstrom and Stoica, 1989) for validation of classical models and (Chen and Gu, 2000) for validation of robust identified models.

Monitoring

The final objective of a control system is to achieve certain performance specifications. If they are fulfilled then the problem is solved and no further actions are needed. In order to assess performance a monitoring process over the data coming from the controlled loop is necessary.

Definition 2.3.9 *We define the Monitoring function* $\mathcal{M} : \mathbb{D} \rightarrow \mathscr{A}_{\mathscr{S}}$ *as the function that returns the performance index measured from the data set* $\mathbb{D}$.

◊

Definition 2.3.10 *A control system is under performance specifications* $\mathbb{S}$ *if* $\mathcal{M}(\mathbb{D}) \subset \mathbb{S}$

◊

If the specifications calculated from data are a subset of the desired specifications, the control system is performing as expected.

The monitoring process includes different issues. First the definition of the parameters in which the performance is based. In (Qin, 1998) (Harris *et al.*, 1999) a review of performance monitoring techniques is presented. They are based on the comparison of the current performance with the performance provided by a minimum variance controller. On the other hand, once a parameter for monitoring performance has been chosen, it is necessary to determine on the basis of the calculated parameter, if the system is under performance. This is not an easy task as normally we are dealing with variables defined on an stochastic framework. The use of hypothesis test and control charts have been presented as suited tools to manage the decision task. See for example (Balaguer and Vilanova, 2006*c*) (Balaguer and Vilanova, 2006*d*).

2.4 Summary

In this chapter an abstract study of the control problem from a holistic point of view has been conducted. The control problem has been divided between its elements and their relationships.

The elements have been divided in two subgroups regarding its dependence with data. On the one hand the a priori elements which are fixed by the control engineer and thus independent of future data. On the other hand data are mandatory in order to obtain the a posteriori elements. The objective of the a priori elements is to set the mathematical framework in which the problem is tackled and solved. On the contrary the a posteriori elements are the formalization inside the abstract framework for some data set which gives a tangible solution.

Once the elements are established, it is possible to define their relationships. The relationships among the elements are classified through existing control areas. Regarding the objective of the relationship it is possible to classify the relationships between the ones that transform current elements of the control problem into new ones and the ones which check consistency among these elements. It is worth nothing that any relation is an application requiring both, a priori elements and a posteriori elements. This implies the necessity of both elements in order to solve the problem.

As a result a framework is presented in which elements and relationships of the control problem which are normally hidden by the technicalities of the techniques used, come to the surface. This is useful in several ways:

- First the scheme provides a framework in which compare distinct control design approaches regarding the nature of the elements used, their allowed modifications and relationships taken.

- Secondly the scheme helps to envisage new ways to manage the elements and their relationships, helping in the design of new relations or adding new features to existing relations (e.g. iterative control).

- Finally the proposed scheme is of interest in control education in order to first establish the difference between the mathematical assumptions and the real data and their importance in order to solve the control problem. Secondly it presents the elements explicitly, stating the degrees of freedom in any design. The framework also presents, in an organized manner, the existing control topics in a context that helps its presentation, localization and understanding.

Chapter 3

Information and Control

The objective of this chapter is twofold, on the one hand to introduce the relationship between information and control and on the other hand to define the information theory approach used to define the information theoretic framework for control design introduced in the next chapter.

First it is presented how the information concept entwines several aspects of the control problem. In fact information is the basic concept on control theory which arises at signal and at system level. The distinct aspects of information on control theory are reviewed and finally the problem is focused on the interplay of information on the procedure of designing a controller

Secondly distinct formalizations of information are presented. Information is a term with different assumptions, definitions and meanings. We review the existing scientific approaches to the concept of information and present the Uncertainty Based Information theory as the one considered to formalize the problem of designing high performance controllers.

3.1 Information and Control Theory

The relation between information and control systems is an important and complex issue. Admittedly the goal of any feedback control system is to gain certainty on the system behavior under an uncertain environment by measuring some system variable. In (Newton *et al.*, 1957) one of the characteristics that a feedback system must have is:

"(1), the action of the system on the output is determined, in part, by the value of the output".

Additionally, regarding the justification of feedback, (Newton *et al.*, 1957) states:

"The three major reasons for employing feedback control are: (1) The process or actuator which supplies the output may have signal transmission characteristics that make open-loop operation very difficult. (2) With feedback the precision of control can be made to depend largely upon the equipment used to measure the output and to compare it with its ideal value. This fact may enable accurate control to be achieved in spite of inaccuracies and variable characteristics in the actuator or process. (3) The effect of disturbances on the output may be suppressed by employing feedback, thereby obviating the need to elaborate disturbance compensators that would be needed with open loop control."

On the above discussions we can relate the term information (or its absence) within different contexts. First feedback, in opposition to open loop schemes, requires extra information by measuring the output. Secondly, it is introduced the "signal transmission characteristics". This problem of characterizing the channel transmission characteristics was solved by the celebrated "A Mathematical Theory of Communication" (Shannon, 1948), which formed the basis on information theory during nearly forty years. Finally it appears the problem of lack of information in the systems itself "inaccuracies and variable characteristics in the actuator or process" or uncertainty due to unknown signals "effect of disturbances". Hence we can conclude that first, the term information is in the very basis of control systems and, secondly, it is a concept that appears through distinct aspects such as, channel transmission, feedback theory and uncertainty on signals and systems.

Despite the fundamental importance of information on control systems, a complete information-theoretic framework for control systems is still lacking (Zames, 1998) (Touchette and Lloyd, 2004).

Nonetheless information concepts have enriched control theory through several well established control theories as Adaptive Control (Astrom and Wittenmark, 1989), Robust Control (Doyle *et al.*, 1992), Fundamental Limitations in Control (Skogestad and Postlethwaite, 1996) and Networked Control Systems (Goodwin *et al.*, 2006). A brief description of the relation of each theory with information is presented.

- Adaptive Control. The main idea of adaptive control is to use the sensed output not only to calculate the control action but to modify the law responsible of calculating the control action. As a result information coming from the feedback is added to the controller itself, modifying its parameters. This is accomplished by the recursive identification of the plant parameters and modifying the controller parameters either directly or indirectly.

- Robust Control. The robust control paradigm was born as an answer to the problem of model uncertainty. In fact theoretical designs based on existing controllers failed when experimentally tested due to model error issues. The goal of robust control is to relate by means of some uncertainty measure, the plant uncertainty with stability and performance specifications. It is the first time that an information measure (e.g. a metric in H_∞, l_1, etc.) is used to establish a relation with either stability or performance margins.

- Fundamental Limitations in control. It is known that certain plant properties impose limitations on the achievable performance by any controller. Structural aspects such as time delays, input saturations, right half plane (RHP) poles and RHP zeros impose limitations on the bandwidth. For example a RHP pole requires a minimum amount of bandwidth in order to stabilize a system. On the other hand, RHP zeros and time delays impose a bound on the maximum bandwidth allowable. The plant is experimentally impossible to control if the bandwidth required for stabilization is greater than the allowable bandwidth of RHP zeros and delays. The fundamental limitations in control is a very important aspect that, if not taken into account properly, can lead to stating control problems with no feasible solution.

- Networked Control Systems. Traditionally control theory has assumed that controllers and plants communicate in an ideal manner. However with the technological developments of distributed control systems new issues relating communication and control has arisen. These issues are related with the effects of quantization and delays that exists in any real transmission channel. In fact they are related with information rates transmitted through the channels and the required information in order to achieve certain performance level.

The above information and control topics can be classified in two main groups. Firstly techniques that deal with information deficiencies at the plant level. Secondly topics that study transmission of information on the control problem. Adaptive control and robust control belong to the first group. In fact both are techniques to cope with uncertainty, although the solution they provide to the problem of lack of information is of different nature. On the one hand adaptive control tries to continuously capture information in order to reduce uncertainty. On the other hand, robust control tries to measure uncertainty and relate it with the pursued objective, in order to design a properly controller. Conversely fundamental limitations in control and networked control systems deal with information transmission at the signal level and how the limitations affects the overall performance.

On the rest of this thesis we focus on information concepts related with plant uncertainty, hence related to the adaptive and the robust control paradigm. Consequently the information-theoretic approach is aimed towards studying the information flow for control design purposes covering classical issues as model identification and controller design plus additional issues as information monitoring, fundamental limitations in control, model validation and consistency of a priori information. Our problem is how to *manage information sources* to design *high performance* controllers.

Summing up the concepts presented in this section are:

- Information plays a fundamental role in control theory.
- Information, as far as control theory is concerned, is a multiple concept. For example the information rate transmitted through the feedback channel, the lack of information (i.e. uncertainty) of a model, etc.
- We focus on information for designing high performing controllers.

- The main point that aims our study is that of adding new information and that the new information produces beneficial results.

3.2 What Is Information?

The concept of Information is a broad and elusive one. It is used for a wide variety of meanings and formalized within distinct theories. The objective of this section is first to review the existing scientific approaches to the concept of information and secondly to establish the meaning of the term information that is being used throughout this thesis.

The concept of information has been investigated under distinct conceptual frameworks as *Computational Complexity* (Kolmogorov, 1965) (Chaitin, 1987), *Uncertainty Based Information* (Klir and Wierman, 1999) (Klir, 2006), *Logic* (Devlin, 1991) and *Systems Organization* (Stonier, 1990). For each one of the proposed theories, information is defined in distinct terms. For example the computational complexity paradigm measures the information of an object by the length of the shortest possible program to define the object. On the other hand, uncertainty based information measures information as the capacity of reducing uncertainty whereas the systems organization approach to information views information as a property to organize a system.

In what follows we focus on *Uncertainty Based Information* as this is the conceptualization of information adopted in this thesis. Uncertainty Based Information theory considers uncertainty as a result of some information deficiency. Consequently certain amount of information can reduce uncertainty, thus implying that the amount of information gained can be measured by the reduction of uncertainty. Inside this framework uncertainty and information have an inverse relationship, thus

$$Information = Uncertainty^{-1} \tag{3.1}$$

The Uncertainty Based Information theory presented can be formalized under several mathematical frameworks. One of the main results of the Uncertainty Based Information Theory has been to establish the multidimensionality of the uncertainty concept. In fact, distinct types of uncertainty exists. For example, under classical set theory, uncertainty is related with the size (number of elements) of a set. On the other hand, the mathematical theory of communication, which is also an uncertainty based information theory, characterizes uncertainty on the basis of probability theory. Thus uncertainty can have a very different nature.

We formalize uncertainty on the basis of *Classical Set theory*. Thus uncertainty is related with the set's size. The bigger the set the more uncertain. This type of uncertainty is referred as nonspecificity (Klir and Wierman, 1999). Notwithstanding, in the information theoretic framework presented in the next chapter the information of some sets follows an inverse relation, that is the bigger the set the more informative the set. The rationale behind this choice will be clear in the next chapter, however consider that, as far as the a priori model set is considered, the bigger the set (e.g. the higher model order) the more informative the a priori model set (e.g. more representation capacity). The main idea is that information is measured through a set size.

The increase of information (reduction of uncertainty) is pursued for the benefit it can provide. In fact it is debatable that a more detailed model is more informative, for control design purposes, if it does not help to improve the designed performance controller. This arise the distinction between the terms *Information* and *Information Value*. As defined in (Sheridan, 1995), the term *Information* refers to the reduction of uncertainty whereas *Information Value* refers to what can be gained by the uncertainty reduction. Thus the concepts of Information and Relevant Information can be defined as:

$$Information = Reduction\ of\ Uncertainty \tag{3.2}$$

$$Relevant\ Information = Information\ that\ Improves\ Performance \tag{3.3}$$

We analyze the general control problem on the basis of existing information theory (i.e. Uncertainty Based Information theory) and arise the question of how information should be managed inside the uncertainty based framework in order to achieve high performance controllers.

The interplay of uncertainty and control systems is normally viewed as a negative unavoidable limitation. Indeed uncertainty is the result of some information deficiency. In control problems information deficiencies can arise due to:

- Incompleteness. The information at hand to solve the control problem is always incomplete. For instance, data are always finite, data contain some frequency components whereas other frequency ranges must be absent.

- Imprecision. Imprecision of information is another difficulty. For example data is normally corrupted by noise. As a result, even the information is complete (e.g. the whole frequencies of interest are captured), this information is imprecise.

- Contradiction. This lack of information is of special interest in the control problem as it is a warning of inconsistency (e.g. a model that is not validate). This can be a sign that the mathematical machinery is of limited capacity in order to capture the complexity of the problem.

These facts can limit the achievable performance of the control system[1].

Nonetheless uncertainty also plays a beneficial role on the control design problem. The positive aspect is due to complexity issues. Although it would be possible to obtain models which are a perfect representations of reality the mathematical framework required to manage these models would be too complex to be useful. It is possible to reduce the problem complexity by increasing the problem uncertainty (e.g. modelling a nonlinear plant as a liner system plus additive uncertainty). Thus it can be concluded that uncertainty is a two faced coin. Uncertainty can limit the achievable performance but, at the same time, uncertainty can transform complex problems into tractable ones.

To sum up the term information used throughout this thesis means information on the sense of Uncertainty-Information Theory, with the following properties:

- Classical Set theory description of uncertainty.

- $Information = Uncertainty^{-1}$

[1]It is not sure that the limits of performance of a control system are a consequence of plant uncertainty. It could be possible that the limitations on the control design follow their fundamental limitations. In that case it is arguable that a reduction of uncertainty could improve the performance. As an example imagine a first order plant with a delay of 10 minutes and a time constant of one second. Which is the benefit of reducing the time constant uncertainty by 1% on the overall performance?

- Information for control design purposes.

Chapter 4

Information Characterization of the Control Problem

In this chapter we endow the framework of the general control problem with the information measure by using the Uncertainty based Information theory. The approach gives insight on two important aspects for control design. First we determine exhaustively the information sources available to accomplish a successful control design. Secondly we study the relations between the information elements by deriving inclusion relations that must be fulfilled in order to assure an information increase. It is shown however that the inclusion relations are not sufficient and other aspects must be considered

4.1 Introduction

In Chapter 2, a framework to analyze the model based control problem has been present. The framework is divided in elements and relationships among these elements.

In the present chapter our aim is to endow the cited elements and relations with an information measure. To this end, the Uncertainty Information Theory is used to define the information capability of the elements and relations. The information definition hinges on the two main goals of the model based control problem. On the one hand to derive the smallest possible model set, as the smaller the model set the more informative the set. On the other hand, to design a controller. In this case the bigger the controller set the better. This fact is trivial in the case of the solvability of the control problem. In fact, if the problem is solvable the controller set has at lest one element: the designed controller. Conversely if the problem is not solvable the controller set is empty. Moreover, the "bigger" the controller set can imply benefits from a controller robustness point of view (i.e. robustness properties on the control loop due to controller uncertainty). For example imagine that the proportional controller that solves the control problem is the one with the gain value equal to K_0, and that any deviation for this value violates the specifications. From a practical point of view the problem is not solvable. On the other hand if the controller set is defined by a gain comprised between $[K_{min}, K_{max}]$, the controller is more likely to be solvable[1].

The benefits of characterizing the elements and relations with an information measure are:

- First we are able to enumerate all the possible information sources from which the control problem can beneficiate. We can classify the information sources in three groups regarding their nature. These are:
  - Data Set: Information coming from experimental data.
  - A Priori Set: Mathematical framework in which the control problem is posed.

[1] This discussion has nothing to do with the fact that the design algorithm returns just a single controller. We mean the set of all controllers that solve the control problem, regardless the ability to find any of them. Therefore this is related to the sensibility of the solution with respect to the problem posed.

   – Algorithms: Transformations of the elements (e.g. identification).

- Once an information measure is provided, it is possible to analyze the relations among the elements. Necessary conditions over the elements for information increase are stated. Moreover it is seen that to obtain also sufficiency conditions, the algorithms for identification and control design must possess certain properties.

4.2 Information and Identification

The goal of system identification could be understood, from a naive point of view, as identifying a mathematical model that represents the plant perfectly. This is however neither possible nor desirable. It is not possible due to fundamental limitations (e.g. signal to noise ratio, finite number of samples). It is not desirable because model complexity must be kept tractable to meet the level of current control design algorithms (e.g. LTI) when identifying real plants (e.g. non-lineal, time varying, infinite order).

Notwithstanding the identified model must capture the behavior of the real plant within some accuracy degree. This is accomplished by identifying not a nominal model but a family of models, that is a model set. For control design purposes, it is desirable that the model set be as fitted as possible. In the limit case the model set would consist in just one model, the real plant.

Following the Uncertainty Based Information theory we define the information relation between model sets as follows:

Definition 4.2.1 *A model set $\mathbb{G}_i$ is more informative than $\mathbb{G}_j$ if and only if (iff)*

$$\mathbb{G}_i \subset \mathbb{G}_j \quad \textit{and} \quad \mathbb{G}_i \neq \emptyset$$

◊

In words, the definition states that the lesser the elements of a model set the more informative the set. Additionally the set must consist of at least one element apart from the empty element (i.e. must be distinct than the empty set).

Recalling the definition of identification (i.e. $\mathcal{I} : \mathscr{A}_{\mathscr{M}} x \mathbb{D} \rightarrow \mathscr{A}_{\mathscr{M}}$), the increase of information on the model set can be pursued by modifying the following variables:

- The identification algorithm, $\mathcal{I}$.
- The data set, $\mathbb{D}$.
- The a priori model set, $\mathscr{A}_{\mathscr{M}}$.

The first variable, the identification algorithm, is a relationship of the elements of the control problem whereas the other two variables, the data set and the a priori model set, are elements of the control problem. In what follows, we analyze the mentioned variables and their properties in order to increase the information on the model set.

4.2.1 Identification Algorithm

For the sake of readability it is worth to remind that the result of the identification algorithm is a model set, so we can write the following equality relation, $\mathbb{G} = \mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}})$.

Definition 4.2.2 *For a given data set $\mathbb{D}$ and a priori model set $\mathscr{A}_{\mathscr{M}}$, the identification algorithm $\mathcal{I}_i$ is more informative than the algorithm $\mathcal{I}_j$ iff*

$$\mathcal{I}_i(\mathbb{D}, \mathscr{A}_{\mathscr{M}}) \subset \mathcal{I}_j(\mathbb{D}, \mathscr{A}_{\mathscr{M}})$$

and

$$\mathcal{I}_i(\mathbb{D}, \mathscr{A}_{\mathscr{M}}) \neq \emptyset$$

$\diamondsuit$

Definition 4.2.2 states that, for the same domain (i.e. elements $\mathbb{D}$ and $\mathscr{A}_{\mathscr{M}}$) if the model set resulting from algorithm $\mathcal{I}_i$ is smaller than the one resulting from algorithm $\mathcal{I}_j$, then the algorithm $\mathcal{I}_i$ is more informative than algorithm $\mathcal{I}_j$. It can be said that the algorithm that makes a better use of the same original information providing a smaller model set is more informative.

We however do not pursue longer in this thesis the possibility of gaining information through the identification algorithm. The main reason is that existing algorithms normally give the smallest possible set achievable with the current data and a priori model set assumptions, as they are formulated on the basis of some optimal problem, so no advantage can be foreseen by modifying the algorithm. Moreover the identification algorithm is very dependent on the a priori assumptions on the model set and data set. As a result the modification of the identification algorithm normally implies the modification of other elements (e.g. a priori model set, a priori data set, etc.) which can be not desirable.

4.2.2 Data Set

Definition 4.2.3 *For a given algorithm I and a priori model set $\mathscr{A}_{\mathscr{M}}$, the data set $\mathbb{D}_i$ is more informative than the data set $\mathbb{D}_j$ iff*

$$\mathcal{I}(\mathbb{D}_i, \mathscr{A}_{\mathscr{M}}) \subset \mathcal{I}(\mathbb{D}_j, \mathscr{A}_{\mathscr{M}})$$

and

$$\mathcal{I}(\mathbb{D}_i, \mathscr{A}_{\mathscr{M}}) \neq \emptyset$$

$\diamondsuit$

It should be noted that the measure of information content of a data set is performed through the identification algorithm and keeping the a priori model set constant. Under this framework, the more informative data are the data that provides the smallest model set.

A necessary condition for a data set $\mathbb{D}_i$ to be more informative than the data set $\mathbb{D}_j$, is that the data set $\mathbb{D}_j$ must be included in the data set $\mathbb{D}_i$, namely the more informative the data set, the "bigger" the data set (e.g. more samples, more experiments, etc).

Theorem 4.2.1 *If $\mathcal{I}(\mathbb{D}_i, \mathscr{A}_{\mathscr{M}}) \subset \mathcal{I}(\mathbb{D}_j, \mathscr{A}_{\mathscr{M}})$ then $\mathbb{D}_j \subset \mathbb{D}_i$*

$\square$

Proof: First of all assume that the data set $\mathbb{D}$ can be divided in distinct sets indexed by n (e.g. the index n can be the experiment number or the sample number). Given the data set $\mathbb{D}$ the smallest model set obtainable is given by the intersection of all the identifications performed with the given data set (e.g. the intersection of all the models identified with the experiments belonging to $\mathbb{D}$), that is $\mathbb{G} = \bigcap_{n=0}^{N} \mathcal{I}(\mathbb{D}_n, \mathscr{A}_{\mathscr{M}})$.

Given the data set $\mathbb{D}_j$, assume that the optimal model set obtained is $\mathbb{G}_{jo}$ which by the above discussion it turns out to be equal to $\bigcap_{n=0}^{N_i} \mathcal{I}(\mathbb{D}_n, \mathscr{A}_{\mathscr{M}})$.

If a new data set $\mathbb{D}_i$ provides a better model set $\mathbb{G}_{io}$ that is

$$\mathbb{G}_{io} \subset \mathbb{G}_{jo}$$

This implies that

$$\bigcap_{n=0}^{\mathbb{D}_i} \mathcal{I}(\mathbb{D}_n, \mathscr{A}_{\mathscr{M}}) \subset \bigcap_{n=0}^{\mathbb{D}_j} \mathcal{I}(\mathbb{D}_n, \mathscr{A}_{\mathscr{M}})$$

The inclusion operator means that the left hand side must content more set intersections in order to be smaller, then

$$\bigcap_{n=0}^{\mathbb{D}_j} \mathcal{I}(\mathbb{D}_n, \mathscr{A}_{\mathscr{M}}) \bigcap_{m=\mathbb{D}_j+1}^{\mathbb{D}_i} \mathcal{I}(\mathbb{D}_m, \mathscr{A}_{\mathscr{M}}) \subset \bigcap_{n=0}^{\mathbb{D}_j} \mathcal{I}(\mathbb{D}_n, \mathscr{A}_{\mathscr{M}})$$

As a result the set $\mathbb{D}_i$ contains elements that do not belong to the set $\mathbb{D}_j$ thus

$$\mathbb{D}_j \subset \mathbb{D}_i$$

■

Example 4.2.1 *In this example we consider the well known Least Squares Identification Method (LSM) which is a particular case of the Prediction Error Identification framework (Ljung, 1999). The method is based on finding the parameter vector $\hat{\theta}$ of the model $\hat{y}(t|\theta) = G(\theta)u(t)$ which minimizes the prediction error $\epsilon(t,\theta) = y(t) - \hat{y}(t|\theta)$. This is accomplished by minimizing the function*

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} \epsilon^2(t,\theta) \tag{4.1}$$

where N is the number of data samples. It can be proven that, asymptotically (no bias error), the parameters converge to the optimum limit θ^*.

We consider the parameter error variable given by $\hat{\theta}_N - \theta^*$, it turns out to be a normally distributed variable $N(0, \frac{1}{N}P)$. Thus the standard deviation of the error variable $\hat{\theta}_N - \theta^*$ is proportional to the $\frac{1}{\sqrt{N}}$, that is $\hat{\theta}_N - \theta^* \propto \frac{1}{\sqrt{N}}$. Thus it follows that if we have a data sets $\mathbb{D}_1$ with N_1 samples then the estimate $\hat{\theta}_{N_1} - \theta^* \propto \frac{1}{\sqrt{N_1}}$. If we want a smaller parameter variation, that is a smaller parameter set it follows that $\hat{\theta}_{N_2} - \theta^* \propto \frac{1}{\sqrt{N_2}}$, thus N_2 must be bigger than N_1.

However the condition of $\mathbb{D}_j \subset \mathbb{D}_i$ is not sufficient to guarantee the increase of information as it depends on the algorithm and data properties. This fact conducts us to the following theorem that is proven false by means of a counterexample.

Theorem 4.2.2 *The converse of theorem 4.2.1, that is*

$$\mathbb{D}_j \subset \mathbb{D}_i \longrightarrow \mathcal{I}(\mathbb{D}_i, \mathscr{A}_{\mathscr{M}}) \subset \mathcal{I}(\mathbb{D}_j, \mathscr{A}_{\mathscr{M}})$$

is, in general, not true.

Proof: The condition of data set increase is not sufficient for a model family reduction mainly for two reasons. First the algorithm must make use of the extra data. Secondly the extra data can contain data coming from a failure situation (e.g. plant variations, sensor and/or actuator failures, etc) or being not persistently exciting data, then although the new data set contains data that does not belong to the former data set, the data is not helpful in order to reduce the model set.

■

Example 4.2.2 *Assume that the model to be identified is a straight line (i.e. $y = m * x + b$). The data set consists of coordinate pais (i.e. (x_i, y_i), for $i = 1..N$). The proposed algorithm takes the first sample (x_1, y_1) and second sample (x_2, y_2) and calculates the parameters m and b by solving the linear equations system, that is:*

$$\begin{vmatrix} m \\ b \end{vmatrix} = \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix}^{-1} \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}$$

The identification is solvable whenever the number of data samples N is equal or bigger than 2. However although we take more samples, the new acquired information is not used by the algorithm. It follows that an increase on the data set does not imply an increase on model information. The limitation lies on the capability of the identification algorithm in making use of the extended data set. In fact the algorithm not even provides an error measure of the estimates.

Example 4.2.3 *Consider a static system which gain K_1 is to be identified. The samples are measured with a sensor affected by Gaussian noise N(0,σ^2). As a result the identified gain K_1 is also a Gaussian variable with distribution N($\bar{K}_1$,σ^2).*

Now assume that the system gain is not time invariant and that at some time instant "t_k" the system gain abruptly changes its value to K_2 and more data samples are acquired so the data set increases. However the data set is now composed by two distinct random variables, K_1 and K_2. If the gain is now estimated from this "bigger" set, K_{est} equals the sum of two random variables, K_1 and K_2, hence the distribution of K_{est} results to be N($\bar{K}_1 + \bar{K}_2$,$2\sigma^2$). As a result, not only the size of model set has increased by a factor of two (e.g. $2\sigma^2$) but also we can not guarantee that the real gain value belongs to the model set.

This example shows the necessity of using proper data for the identification procedure. That means to use data free of failures of any kind.

Summarizing, a necessary condition for increasing the model set information is that of increasing the data set. On the other hand an increase of the data set does not necessarily mean the increase of model set information. The reason is that the algorithm must be able to take advantage of the new incorporated data and that the new data must be generated under no failure situations.

4.2.3 A Priori Model Set

Definition 4.2.4 *For a given algorithm $\mathcal{I}$ and data set $\mathbb{D}$, the a priori model set $\mathscr{A}_{\mathscr{M}i}$ is more informative than the a priori model set $\mathscr{A}_{\mathscr{M}j}$ iff*

$$\mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}i}) \subset \mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}j})$$

and

$$\mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}i}) \neq \emptyset$$

$\diamondsuit$

Again, the measure of information content of the a priori model set is performed through the identification algorithm and keeping the other parameters constant. Under this framework, the more informative a priori model set is the one that provides the smallest model set.

A necessary condition for a priori model set $\mathscr{A}_{\mathscr{M}i}$ to be more informative than the a priori model set $\mathscr{A}_{\mathscr{M}j}$, is that the a priori model set $\mathscr{A}_{\mathscr{M}j}$ must be included in the a priori model set $\mathscr{A}_{\mathscr{M}i}$, namely the more informative the a priori model set, the "bigger" (in terms of elements) the a priori model set.

Theorem 4.2.3 *If* $\mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}i}) \subset \mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}j})$ *then* $\mathscr{A}_{\mathscr{M}j} \subset \mathscr{A}_{\mathscr{M}i}$

□

Proof: (by contradiction). Assume that $\mathscr{A}_{\mathscr{M}i} \subseteq \mathscr{A}_{\mathscr{M}j}$. By hypothesis we know that $\mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}i}) \subset \mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}j})$, which for short we can write as $\mathbb{G}_i \subset \mathbb{G}_j$.

If $\mathscr{A}_{\mathscr{M}i} = \mathscr{A}_{\mathscr{M}j}$ implies that $\mathbb{G}_i = \mathbb{G}_j$ which is a contradiction.

Consider now that $\mathscr{A}_{\mathscr{M}i} \subset \mathscr{A}_{\mathscr{M}j}$ and that the optimum a priori model set (i.e. the one that produces the smallest model set) $\mathscr{A}_{\mathscr{M}o}$ belongs to $\mathscr{A}_{\mathscr{M}i}$, that is $\mathscr{A}_{\mathscr{M}o} \in \mathscr{A}_{\mathscr{M}i}$. This however implies also that $\mathscr{A}_{\mathscr{M}o} \in \mathscr{A}_{\mathscr{M}j}$ which is a contradiction with the assumption of the model set $\mathbb{G}_i$ being optimal. Thus

$$\mathscr{A}_{\mathscr{M}j} \subset \mathscr{A}_{\mathscr{M}i}$$

■

However the condition $\mathscr{A}_{\mathscr{M}j} \subset \mathscr{A}_{\mathscr{M}i}$ is not sufficient to guarantee the increase of information as it again depends on the algorithm characteristics. This fact conducts us to the following theorem.

Theorem 4.2.4 *The converse of theorem 4.2.3, that is*

$$\mathscr{A}_{\mathscr{M}j} \subset \mathscr{A}_{\mathscr{M}i} \longrightarrow \mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}i}) \subset \mathcal{I}(\mathbb{D}, \mathscr{A}_{\mathscr{M}j})$$

is, in general, not true.

Proof: Think, for example in the trade-off of bias-variance error on the identification prediction error method (Ljung, 1999). If the model is overparameterized the model set increases due to variance errors.

■

In this section we have studied the influence of the a priori model set on the model set information. We can understand the a priori model set as a mathematical capacity of representation. A necessary condition for increasing the information of the model set is increasing the a priori model set, that is increasing the capacity of representation. On the other hand an increase of the a priori model set does not necessarily mean an increase of model set information as the problem of parsimony can limit the benefits of increasing the a priori model set if there is already enough degree of freedom to mathematically capture the plant.

4.3 Information and Control Design

The objective of the control design is to provide a controller or a family of controllers, defined by the controller set $\mathbb{K}$, that achieves the specifications level defined by the set $\mathbb{S}$ over the whole model set $\mathbb{G}$. The controller set $\mathbb{K}$ is normally a singleton (i.e. a unique controller) as it is implemented only one controller. However the controller set size is somehow related with controller robustness issues (that is tolerance to controller implementations imprecisions). Qualitatively, the more robust the controller (in the sense of controller uncertainty), the bigger the controller set. Thus the control problem is solvable whenever $\mathbb{K} \neq \emptyset$.

Definition 4.3.1 *A controller set* $\mathbb{K}_i$ *is more informative than* $\mathbb{K}_j$ *iff*

$$\mathbb{K}_j \subset \mathbb{K}_i$$

$\Diamond$

Definition1 4.3.1 introduces the informativeness concept of a controller set $\mathbb{K}$ by the set size. We pursue the biggest controller set as possible. First because this implies solvability of the problem and secondly, as discussed, due to robustness issues. We can conclude that for a given control problem, the greater the number of controllers that solve the problem the better.

Recalling the definition of control design (i.e. $\mathcal{C} : \mathbb{G}x\mathbb{S}x\mathscr{A}_{\mathscr{K}} \rightarrow \mathscr{A}_{\mathscr{K}}$), the information increase of the controller set can be pursued by modifying the following variables:

- The control design algorithm, $\mathcal{C}$.

- The model set, $\mathbb{G}$.
- The specifications set, $\mathbb{S}$.
- The a priori controller set, $\mathscr{A}_{\mathscr{K}}$.

The first variable, the identification algorithm, is a relationship of the elements of the control problem whereas the other three variables, the model set, the specification set and the a priori controller set are elements of the control problem. We analyze these variables and their properties in order to increase the size of the controller set.

4.3.1 Control Design Algorithm

The result of the control design algorithm is the controller set so we can write the following equality relation, $\mathbb{K} = \mathcal{C} : \mathbb{G} x \mathbb{S} x \mathscr{A}_{\mathscr{K}} \rightarrow \mathscr{A}_{\mathscr{K}}$.

Definition 4.3.2 *For a given model set $\mathbb{G}$, specifications set $\mathbb{S}$ and a priori controller set $\mathscr{A}_{\mathscr{K}}$, the control design algorithm $\mathcal{C}_i$ is more informative than the algorithm $\mathcal{C}_j$ iff*

$$\mathcal{C}_j(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}_i(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}})$$

$\diamondsuit$

Definition 4.3.2 states that, for the same domain (i.e. elements $\mathbb{G}$, $\mathbb{S}$ and $\mathscr{A}_{\mathscr{K}}$) if the controller set resulting from algorithm $\mathcal{C}_i$ is bigger than the one resulting from algorithm $\mathcal{C}_j$ then the algorithm $\mathcal{C}_i$ is more informative than $\mathcal{C}_j$. It can be said that the algorithm that make a better use of the same original information providing a bigger controller set is more informative.

We however do not pursue longer in this thesis the possibility of designing a bigger controller set through the design algorithm as normally design algorithms just return one controller (i.e. a singleton), thus Definition 4.3.2.

4.3.2 Model Set

Definition 4.3.3 *For a given design algorithm $\mathcal{C}$, a specifications set $\mathbb{S}$ and a priori controller set $\mathscr{A}_{\mathscr{K}}$, the model set $\mathbb{G}_i$ is more informative than the model set $\mathbb{G}_j$ iff*

$$\mathcal{C}(\mathbb{G}_j, \mathbb{S}, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}(\mathbb{G}_i, \mathbb{S}, \mathscr{A}_{\mathscr{K}})$$

◊

It should be noted that the measure of information content of a model set is performed through the control design algorithm and keeping the domain constant (i.e. $\mathcal{C}$, $\mathbb{S}$ and $\mathscr{A}_{\mathscr{K}}$). In words, the most informative model set is the one that provides the biggest controller set.

A necessary consequence for a model set $\mathbb{G}_i$ to be more informative than the model set $\mathbb{G}_j$, from a control design point of view, is that the model set $\mathbb{G}_i$ must be included in the model set $\mathbb{G}_j$, namely the more informative the model set, the "smaller" (in terms of elements) the model set, which agrees with Definition 4.2.1.

Theorem 4.3.1 *If* $\mathcal{C}(\mathbb{G}_j, \mathbb{S}, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}(\mathbb{G}_i, \mathbb{S}, \mathscr{A}_{\mathscr{K}})$ *then* $\mathbb{G}_i \subset \mathbb{G}_j$

□

Proof: (by contradiction) By hypothesis $\mathcal{C}(\mathbb{G}_j, \mathbb{S}, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}(\mathbb{G}_i, \mathbb{S}, \mathscr{A}_{\mathscr{K}})$, which for short let rewrite as $\mathbb{K}_j \subset \mathbb{K}_i$. The set $\mathbb{K}$ is the set of all controllers which achieve the specifications level $\mathbb{S}$ over all the model family $\mathbb{G}$. Assume that $\mathbb{G}_j \subseteq \mathbb{G}_i$.

If $\mathbb{G}_j = \mathbb{G}_i$ that implies $\mathbb{K}_j = \mathbb{K}_i$, a contradiction.

If $\mathbb{G}_j \subset \mathbb{G}_i$, all the controllers $\mathbb{K}_i$ achieve $\mathbb{S}$ not only over $\mathbb{G}_i$ but also $\mathbb{G}_j$. Then it follows that $\mathbb{K}_j \subset \mathbb{K}_i$ what is a contradiction, thus

$$\mathbb{G}_i \subset \mathbb{G}_j$$

■

However the condition of $\mathbb{G}_i \subset \mathbb{G}_j$ is not sufficient to guarantee the increase of controller set information as it depends on the algorithm characteristics and the relevancy of model set to the control design purposes. This fact conducts us to the following theorem:

Theorem 4.3.2 *The converse of theorem 4.3.1, that is*

$$\mathbb{G}_i \subset \mathbb{G}_j \longrightarrow \mathcal{C}(\mathbb{G}_j, \mathbb{S}, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}(\mathbb{G}_i, \mathbb{S}, \mathscr{A}_{\mathscr{K}})$$

is, in general, not true.

Proof: The decrease of models could be irrelevant from a control point of view (i.e. easy to control models, not limiting achievable performance).

■

As a conclusion a necessary property for an increase of the controller set is a decrease of the model set. However a model set reduction is not sufficient to guarantee the increase on the controller set.

4.3.3 Specification Set

We define the information content of the specifications set $\mathbb{S}$ in the same way that the information content of the model set $\mathbb{G}$.

Definition 4.3.4 *The specification set $\mathbb{S}_i$ is more informative than the specification set $\mathbb{S}_j$ iff*

$$\mathbb{S}_i \subset \mathbb{S}_j$$

◊

The rationale behind this definition is that, the smaller the specifications set the higher the performance level, as other lower performance levels are rejected and thus not included in the new specifications set. Normally this set is fixed by the designer. However, regarding the information content on the whole control problem, the specifications set could be managed as a new control variable. The information of the specification set could be understood as an information level required in order to solve the control problem.

The relations between controller set information and the specifications set information is given by the following two theorems.

Theorem 4.3.3 *If $\mathcal{C}(\mathbb{G}, \mathbb{S}_i, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}(\mathbb{G}, \mathbb{S}_j, \mathscr{A}_{\mathscr{K}})$ then $\mathbb{S}_i \subset \mathbb{S}_j$*

□

In words, if the controller set diminishes then the specifications level increase.

Proof: (by contradiction). Assume that $\mathcal{C}(\mathbb{G}, \mathbb{S}_i, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}(\mathbb{G}, \mathbb{S}_j, \mathscr{A}_{\mathscr{K}})$, for short, $\mathbb{K}_i \subset \mathbb{K}_j$. Assume also that $\mathbb{S}_j \subseteq \mathbb{S}_i$.

If $\mathbb{S}_j = \mathbb{S}_i$ then $\mathbb{K}_i = \mathbb{K}_j$, a contradiction.

If $\mathbb{S}_j \subset \mathbb{S}_i$ that would imply that $\mathbb{K}_j$ gives better performance that $\mathbb{K}_i$, which is a contradiction with the assumption that $\mathbb{K}_i \subset \mathbb{K}_j$, thus

$$\mathbb{S}_i \subset \mathbb{S}_j$$

■

However the condition of $\mathbb{S}_i \subset \mathbb{S}_j$ is not sufficient to guarantee the decrease of controller set information as it depends on the relevancy of model set for control design purposes. This fact conducts us to the following theorem.

Theorem 4.3.4 *The converse of theorem 4.3.3, that is*

$$\mathbb{S}_i \subset \mathbb{S}_j \longrightarrow \mathcal{C}(\mathbb{G}, \mathbb{S}_i, \mathscr{A}_{\mathscr{K}}) \subset \mathcal{C}(\mathbb{G}, \mathbb{S}_j, \mathscr{A}_{\mathscr{K}})$$

is, in general, not true.

Proof: The condition it is not sufficient as the increase of performance demanded could be provided by the whole existing controller set.

■

4.3.4 A Priori Controller Set

Definition 4.3.5 *For a given algorithm C, model set $\mathbb{G}$ and specifications set $\mathbb{S}$, the a priori controller set $\mathscr{A}_{\mathscr{K}i}$ is more informative than the a priori controller set $\mathscr{A}_{\mathscr{K}j}$ iff*

$$\mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}j}) \subset \mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}i})$$

.

◊

Measuring again the information content through the control design algorithm, the more informative a priori controller set is the one that provides the biggest controller set.

A necessary condition for a priori controller set $\mathscr{A}_{\mathscr{K}i}$ to be more informative than the a priori model set $\mathscr{A}_{\mathscr{M}j}$, is that the a priori model set $\mathscr{A}_{\mathscr{M}j}$ must be included in the a priori model set $\mathscr{A}_{\mathscr{M}i}$, namely the more informative the a priori controller set, the "bigger" (in terms of elements) the a priori controller set.

Theorem 4.3.5 *If* $\mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}i}) \subset \mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}j})$ *then* $\mathscr{A}_{\mathscr{K}i} \subset \mathscr{A}_{\mathscr{K}j}$

□

Proof: (by contradiction). Assume that $\mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}i}) \subset \mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}j})$, for short, $\mathbb{K}_i \subset \mathbb{K}_j$. Assume also that $\mathscr{A}_{\mathscr{K}j} \subseteq \mathscr{A}_{\mathscr{K}i}$.

If $\mathscr{A}_{\mathscr{K}j} = \mathscr{A}_{\mathscr{K}i}$ then $\mathbb{K}_i = \mathbb{K}_j$, a contradiction.

If $\mathscr{A}_{\mathscr{K}j} \subset \mathscr{A}_{\mathscr{K}i}$ that would imply that $\mathscr{A}_{\mathscr{K}i}$ has more representation capacity than $\mathscr{A}_{\mathscr{K}j}$, which is a contradiction with the assumption that $\mathbb{K}_i \subset \mathbb{K}_j$, thus

$$\mathscr{A}_{\mathscr{K}i} \subset \mathscr{A}_{\mathscr{K}j}$$

■

However the condition of $\mathscr{A}_{\mathscr{K}i} \subset \mathscr{A}_{\mathscr{K}j}$ is not sufficient to guarantee the increase of information as it again depends on the algorithm characteristics. This fact conducts us to the following theorem.

Theorem 4.3.6 *The converse of theorem 4.3.5, that is*

$$\mathscr{A}_{\mathscr{K}i} \subset \mathscr{A}_{\mathscr{K}j} \longrightarrow \mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}i}) \subset \mathcal{C}(\mathbb{G}, \mathbb{S}, \mathscr{A}_{\mathscr{K}j})$$

is, in general, not true.

Proof: The condition is no sufficient as the extra representation capacity and secondly the extra representation capacity could not be useful for finding a better controller set. For example a PID controller does not offer advantages over a PI if we are only interested in steady state error.

■

4.4 Summary

In this chapter we have seen the possible sources of information available to the control problem and the relations of these information elements in order to obtain better performing control systems.

We have seen that it is possible to reduce the models family by increasing the data set and the a priori model set. Of course the identification algorithm should make use of the extra information in order to be the goal achievable. Moreover the new data should be free of failures of any type.

Regarding the controller design, we have seen that the bigger the controller set that achieve the required specifications over a model set the better. It is possible to increase the controller set by diminishing the model set and by increasing the mathematical representation of the a priori controller set. The specifications set is a designer variable. However if the controller set diminishes the specification level increases.

On the contrary a sufficiency condition for uncertainty reduction must come from increasing the sets *and taking advantage of the new information* by means of algorithm capabilities. In table 4.4 we present a summary of the information sources of the control problem and the necessity conditions for information increase of the element sets.

Information Element	i more informative than j	Necessity
$\mathbb{G}$	$\mathbb{G}_i \subset \mathbb{G}_j$	$\mathbb{D}_j \subset \mathbb{D}_i$
		$\mathscr{A}_{\mathscr{M}j} \subset \mathscr{A}_{\mathscr{M}i}$
$\mathbb{K}$	$\mathbb{K}_j \subset \mathbb{K}_i$	$\mathbb{G}_i \subset \mathbb{G}_j$
		$\mathbb{S}_i \subset \mathbb{S}_j$
		$\mathscr{A}_{\mathscr{K}j} \subset \mathscr{A}_{\mathscr{K}i}$
$\mathbb{S}$	$\mathbb{S}_i \subset \mathbb{S}_j$	Design Parameter

Table 4.1: Summary table of Information elements and relations.

Chapter 5

Information Supervision of Adaptive Control Schemes

In this chapter the information framework presented in the previous chapter is used for analyzing existing adaptive control schemes. The objective is to study the information flow of adaptive schemes in order to compare and discuss problems and solutions of the approaches. The chapter covers the following sections:

- *First the term supervisor is defined as an entity that manages the information flow in order to accomplish some goal.*
- *The existing adaptive control definitions are reviewed from literature and a new definition that comprises all the particularities of the adaptive control term is established.*
- *Two adaptive control procedures, the "Model Reference Adaptive control" and the "Windsurfer approach" to iterative control, are analyzed regarding their information flow management. The comparison is accomplished under the information framework proposed in Chapter 4. Thus, it is possible to compare the approaches and to relate well know problems and possible improvements with the way information is managed.*

5.1 Information Flow Management: Supervisors

In Chapter 2, the control problem has been defined on the basis of its constitutive elements and relationships. Additionally the scheme has been endowed with an information measure that allows to establish and analyze information relations among the elements. However the conceived framework is a static one in the sense that no indications are given on how to process the information.

In fact the control design problem is a problem of managing the information flow. For example, first data is acquired and on the basis of some a priori model set, a nominal model is identified. Hence with the identified model a controller is designed and a solution proposed. The following considerations are in use:

- First we can identify an information flow. The first step is to acquire data and the controller is designed after a model is identified. There is a causality relation between information elements.
- The information relations considered are a designer choice. For example on the previous example there are no considerations about model validation, model uncertainty bounds, fundamental limitations, etc.
- The steps can be repeated and/or modified. Accordingly, even if the same actions are taken into account, the same process can be repeated several times (e.g. adaptive control) producing a distinct information flow management.
- Information can be stored for further use or just disregarded.

As a result of the above considerations, it is clear that the control design problem is a process in which the information elements stated in the framework of Chapter 4 must be managed in some way, resulting in an "information flow management".

In (Balaguer *et al.*, 2006*b*) the control problem is presented as a problem of managing the information flow. The information flow is divided in hierarchical blocks. The main contribution regarding the information flow management is that the information management should be watertight in the sense that information processed in one block should be validated before the information is delivered to other blocks. A more detailed discussion can be found in (Balaguer *et al.*, 2006*b*).

The issue of managing the information flow on the control problem has been tackled by the control community of distinct areas, such as adaptive control, fault tolerant control and intelligent control among others. The common concept gathered by the mentioned approaches is the concept of supervisor. The supervisor is the entity that manages the information flow in order to achieve certain goal. For example:

- "Fault Tolerant Control". The finality of fault tolerant control (Blanke *et al.*, 2003) is to design control systems that are able to cope with failures by maintaining the system functionality under abnormal conditions. This is accomplished by first detecting and isolating the failures. On the basis of this information the supervisor generates a diagnosis and correcting actions are taken (e.g. controller reconfiguration, change operation point, etc.). This is summarized in the following words from (Isermann, 1997), *"supervisory functions serve to indicate undesired or unpermitted process states, and to take appropriate actions in order to maintain the operation and to avoid damage or accidents."*

- "Adaptive Control". Adaptive control, that is discussed in detail in the next Section, has the objective of continuously adding new information, normally through new data, in order to improve the designed controller. The information flow on adaptive control schemes is controlled by a supervisor that, in general is quite simple performing the following actions (Balaguer *et al.*, 2006*b*): a new plant is identified constantly. As soon as the new plant is available, the controller is redesigned and applied to the closed loop. More developed adaptation schemes exists, for example iterative identification and control schemes, in which the supervisor takes into account further issues.

- "Probabilistic Robust Control". The objective of probabilistic robust control is to implement control systems with several controllers, each one having a distinct instability risk. This way high performance can be achieved with a high risk controller. However in order to minimize the instability risk a supervisor is designed in such a way that when a instability is detected, the supervisor changes the operating controller to one with a lower risk level. In (Horn *et al.*, 2005) a probabilistic robust control is proposed for the control of a helicopter rotorcraft.

- "Multimodels". In the multimodels approach, distinct models are evaluated at the same time in order to decide which one is the most suited representation of the plant. Then it is possible to select the most appropriated controller. A supervisor is then necessary to decide which model is the most suited and accordingly to select and implement the corresponding controller.

The supervisor concept has been presented by means of several application areas. It is possible to abstract the main features of a supervisory scheme. The supervisor is a set of rules that take actions (e.g. controller redesign, controller change, etc) on the basis of some index (e.g. performance index, model validation result, etc). Thus, the supervisor can be seen as the manager of the information flow. For example, the supervisor decides, after a model invalidation result, that a new model must be identified. Consequently in what follows we consider that the information flow management of the control problem is performed by a supervisor. Thus the properties of the information flow management are the properties imposed by the designed supervisor.

5.2 Adaptive Control

5.2.1 Definition of Adaptive Control

Adaptive control emerged in the 50's and since then it has been an important topic in control theory with intensive research. Notwithstanding the huge amount of research on adaptive control, a clear definition is still controversial. Indeed the definition of adaptive control was controversial even at the very beginning of the discipline (Harris and Billings, 1981).

"The lack of a conceptual framework for adaptive control has restrained research in this area and it has made difficult to compare alternative designs (Zames, 1998)". In fact a lot of effort has been made in order to avoid a "bag of tricks" type exposition of adaptive control (Ioannou and Sun, 1996).

In what follows we present a definition of adaptive control which abstracts the main features gathered on the existing adaptive control definitions. In order to accomplish the objective we start reviewing the distinct adaptive control definitions:

- (Margolis and Leondes, 1959): *An adaptive control system is defined as a feedback control system intelligent enough to adjust its characteristics in a changing environment so as to operate in an optimum manner according to some specified criterion.*

- (Bellman and Kabala, 1959): *An adaptive control is one that when faced with uncertain processes, its controller learns to improve its performance through the observation of the outputs of the process, hence as the process unfolds additional information becomes available and improved decisions become possible.*

- (Zadeh, 1963): *The system A is adaptive with respect to a family of time functions S_γ and a set of acceptable performance W if it performs acceptably well with every source in the family S_γ.*

- (Truxal, 1963): *An adaptive system is one defined from an adaptive point of view.*

- (Eveleigh, 1967): *An adaptive system is a system which is provided with a means of continuously monitoring its own performance in relation to a given figure of merit or optimal condition and a means of modifying its own parameters by a closed-loop action so as to approach this optimum.*

- (Gell-Mann, 1995): *A system is complex adaptive if it acquires information about its environment and the interactions between the environment and the system, identifies regularities and condense this regularities in models and interacts with the real world in base of the learned scheme.*

- (Zames, 1998): *A system is adaptive if its performance is better than the best possible based on a priori information.*

In table 5.1 the main features of the reviewed definitions are presented in an organized manner. Despite the variety of adaptive system definitions, it can be seen that whereas some definitions focus on the mechanism by which the adaptation is performed, others focus on the results provided by the adaptation. For example definition (Eveleigh, 1967) focuses on the mechanism (e.g. monitoring and parameters variation), definition (Zadeh, 1963) is based on performance issues and does not mention any mechanism[1].

As far as the mechanism of adaptive control is concerned the following actions are considered:

- Monitoring: Monitoring is the action of checking some variable of interest.
- Parameters change: It is the action of changing the parameters of the system (e.g. controller parameters).
- Supervisor: An intelligent system that decides to take some action.
- New info: New information of the environment that is made available to the system (e.g. a new measure).
- Learn: The act of modifying the system on the basis of some acquired information.

The supervisor should be understood as the system that take actions over the mechanisms above mentioned (e.g. change the controller parameters on the basis of some monitored variable). It is worth to remark the difference between the mechanisms named New info and Learn. New info refers to simply acquire information but the system is not modified. On the other hand the Learn procedure implies a change on the system (e.g. a new controller) on the basis of the new acquired information.

The definitions enumerated in table 5.1 are aimed to solve problems by acquiring and processing new information. However the nature of the problem to be solved is of very different kind:

- Performance Maintenance. The objective is to keep some optimum performance level in spite of a changing environment. Definitions (Eveleigh, 1967), (Margolis and Leondes, 1959) and (Zadeh, 1963) belong to this problem.
- Performance Improvement. Performance improvement is possible if new system information is made available to the control system. Definitions (Bellman and Kabala, 1959) and (Zames, 1998) refer to this problem.

[1] In fact definition (Zadeh, 1963) is the definition of robust performance defined on the robust control theory which is not considered a type of adaptive control but a distinct manner of tackling the problem of uncertainty (Landau, 1999).

	Mechanism					**Behavior**	
Definition	Monitoring	Parameters change	Supervisor	Add new Info.	Learn	Performance Maintenance	Performance Improvement
Eveleigh	x	x				x	
Margolis		x	x			x	
Bellman				x	x		x
Zadeh						x	
Zames				x			x
Gell-Mann				x	x		

Table 5.1: Features of Adaptive System Definitions.

The following definitions of the features regarding adaptation mechanism are in order:

- Monitoring: Monitoring is the action of checking some variable of interest.
- Parameters change: It is the action of changing the parameters of the system.
- Supervisor: An intelligent system that decides to take some action.
- New info: New information of the environment that is made available to the system (e.g. a new measure).
- Learn: The act of modifying the system on the basis of some acquired information.

In our conceptualization of adaptive control, we focus on the fact that in order to have an adaptive system the following actions are required:

- *Acquiring information.* The mechanisms related with the action of acquiring information comprises the Monitoring and Add new Info.
- *Modifying the own system* (e.g. parameters, structure, etc) on the basis of the new acquired information. This is accomplished by the mechanisms Parameters change and Learn.
- *The system modifications allow the system to perform better than without modifications.* This includes both, the performance maintenance over a changing environment and the performance improvement.

At this stage it is worth to state that *adaptive control does not have any to do with time varying systems or with nonlinear systems but with uncertainty.* In fact the above proposed scheme is a consequence of a lack of information as the first step of the schemes is to add new information. The lack of information is not caused only by an environment information deficiency (e.g. signal spectrum has no relevant content over some frequency range) but by a system representation fault (e.g. linear model for representing a nonlinear plant). In fact a linear model can capture accurately the nonlinear behavior only around some restricted neighborhood point. If the set point is changed the linear model is modified.

The second point of interest of adaptive control is the information supervision. In fact the above enumerated actions must be managed by means of some entity, the supervisor.

5.2.2 Supervision of Adaptive Control

The adaptive control actions mentioned above must be managed by a supervisor. The supervisor goal is to take the appropriate actions in order to improve system performance by acquiring new information by, for example, performing a new experiment. In this sense, as stated on the first section of the chapter, a supervisor is a logic that manages the control problem information flow.

The rationale behind the supervisor actions is that the acquired information helps to improve system performance. However it is not the general case that new information is useful. For example recall the concepts of Neutrality, Separability and Certainty Equivalence from stochastic control systems:

- Neutrality (Feldbaum, 1965). A stochastic control system is neutral if the rate of reduction of uncertainty about the states X is independent of the control u.

- Separability (Joseph and Tou, 1961). A stochastic control problem is separable if the only information which needs to be transmitted in order to calculate the control action is a point estimate of the current state, no information about accuracy being required.

- Certainty Equivalence (Simon, 1956). A separable problem is certainty-equivalent when the form of the control law is identical with that of the function specifying the optimal control law for an equivalent deterministic problem (i.e. a problem having no uncertainties about the state X).

The structural property of neutrality deternines that the information of the state is independent of the input. The main implication is that there is no reason to introduce any probing action (e.g. input experiment) in order to increase the rate of uncertainty reduction of the system states.

The Separability property indicates that the optimal control law does not depend on the uncertainty. Thus the parameters to be transmitted to the controller are just the state estimates, no its uncertainty.

The Certainty Equivalence property is a stronger property than separability. It refers to separable problems and states that the form of the optimal control law equals the control law of the equivalent deterministic problem (i.e. no uncertainty of the state). It follows that the control law can be designed without consider stochastic effects.

Sufficient conditions on systems for being neutral, separable and certain-equivalent can be found in (Harris and Billings, 1981). Linearity is an essential requirement in all the above conditions. Additionally the uncertainty considered is only stochastic (i.e. Gaussian noise). Notwithstanding supervisors that manage adaptive control schemes can indeed improve the system performance as:

- The problem of system identification (Ljung, 1999) is highly dependent on the experimental data used for data generation. Thus probing not only can improve the identified model but it is a necessity (e.g. persistently exciting signal).

- As stated in (Harris and Billings, 1981): *"It follows that adaptive control, which is an approach to non-linear problems, is concerned with problems which do not satisfy the known sufficient conditions for neutrality, separability or certainty equivalence and which are very unlikely to enjoy any of the three properties."*

It then follows that the supervision of adaptive systems can indeed improve the overall system performance. The idea is that if more information relevant for control purposes is added, then better controllers can be designed.

5.3 Information Flow Analysis of Adaptive Control Schemes

In this section two paradigms of adaptive control (following the adaptive control definition given in the preceding section) the classical adaptive control and iterative control are studied under the information framework presented in Chapter 4. The objective is to show the particularities of each one of the proposed schemes regarding its information flow management properties and how the analysis permits to detect limitations and improvements of the approaches.

5.3.1 Classical Adaptive Control

We refer to classical adaptive control as the adaptive control approach presented in (Astrom and Wittenmark, 1989). We focus our analysis to the model reference adaptive system (MRAS). The principal features of the scheme are:

- The performance of the system is specified by a model.

- The controller parameters are changed based on the error between the outputs of the system and the reference model.

In what follows, the classical adaptive control is analyzed on the basis of the information framework developed in Chapter 4. Thus the first step is to identify and define the elements of the control problem on the MRAC.

The data set $\mathbb{D}$ is composed by data acquired from closed loop operation with a variable controller in the loop. The data, together with certain model order and structure defined in the set $\mathscr{A}_{\mathscr{M}}$, is used to identify a single LTI model $\mathbb{G}$ as the certainty equivalence principle is assumed (i.e. no model undermodelling). The closed loop specifications $\mathbb{S}$ are defined by the reference model taken from the set of all possible reference models $\mathscr{A}_{\mathscr{S}}$. Finally the controller $\mathbb{K}$ with the structure defined in $\mathscr{A}_{\mathscr{K}}$ is designed by calculation of the controller parameters which depend on the output error. In table 5.2 a summary of the elements characterization is presented.

ELEMENT	**ASSUMPTIONS**
$\mathbb{D}$ (Data Set)	Closed loop data, variable controller.
$\mathscr{A}_{\mathscr{M}}$ (A Priori Model Set)	LTI Model.
$\mathbb{G}$ (Model Set)	Singleton. No uncertainty.
$\mathscr{A}_{\mathscr{S}}$ (A Priori Specification Set)	Model Reference.
$\mathbb{S}$ (Specification)	CL Reference Model
$\mathscr{A}_{\mathscr{K}}$ (A Priori Controller Set)	LTI Controllers.
$\mathbb{K}$ (Controller Set)	Parameters depend on error signal.

Table 5.2: MRAC Elements.

Once the elements of the MRAC are defined, we discuss the relationships considered among them. The relations considered are just the identification $\mathcal{I}$ and the control design algorithm $\mathcal{C}$. The rest of relationships are not considered. The identification algorithm is a recursive least square that whenever a new sample is acquired, a new model is identified. The control design is based on the calculation of the controller parameters which depend on the output error. In table 5.3 the MRAC relationships are summarized.

Finally, once the elements and the relationships are established it remains to discuss their supervision, that is how the elements and relations (i.e. information) are managed. The supervision of the elements and relations in the MRAC is simple. As stated in (Astrom and Wittenmark, 1989): "*In an adaptive system it is assumed that the regulator parameters are adjusted all the time*". This implies that whenever a new sample is acquired a new model

RELATIONSHIP	**CONSIDERED?**
$\mathcal{I}$ (Identification)	Recursive LSM
$\mathcal{C}$ (Controller Design)	Parameters depend on error signal
$\mathcal{O}$ (Consistency)	No
$\mathcal{V}$ (Model Validation)	No
$\mathcal{L}$ (Limitations)	No
$\mathcal{M}$ (Monitoring)	No

Table 5.3: MRAC Relationships.

is identified by the recursive algorithm. Thus a new output error can be calculated and the controller parameters modified accordingly. There are no other considerations taken.

The way that information is managed in the MRAC adaptive control in particular, and in the classical adaptive control in general, leads to the following problems:

- Stability Issues.
- Limited use of information sources.
- Limited management of information.

which are discussed in the next paragraphs.

Stability Issues

The stability problems of adaptive control have been recognized since the beginning of the field. However their causes were not recognized after some time. In (Anderson, 2005) a review on the instability causes is conducted:

- Bursting. Bursting is a phenomenon that shows transient instability. It is caused by deficient model identification due to the lack of persistently exciting signals. However due to unstable behavior signals become rich enough to identify a good model and the instability disappears.

- Process interaction. On adaptive systems there are distinct processes running in parallel. On the one hand the controlled plant and on the other hand the identification loop. If both processes have comparable time scales then interactions appear than can drive the system to instability.

It can be concluded from the adaptive control instability problems (Anderson, 2005) that the source is a misuse of information. For example a model is identified with data not informative enough (i.e. bursting problem). Another problem of information misuse is due to the interaction between system identification and the closed loop system. In this case the information coming from closed loop data is affected by the controller designed on the basis of the new data. As a result an interaction between both processes appears which can lead to instability. The framework we propose helps to tackle these questions relating them with information flow management.

Limited use of information sources

In adaptive schemes, the new information introduced in the controller design information flow is limited to new experimental data. Thus it would be beneficial to increase the information sources in order to achieve a more satisfactory controller design. For example in (Lewis *et al.*, 1987) it is stated: "*Adaptive control is a promising approach to achieve performance robustness. Its present setting is limited: it makes use of the most structured uncertainty in which the plant model has a known form, but unknown parameters*".

This leaves room to improve the information sources on adaptive control schemes. In (Balaguer *et al.*, 2006*b*) the following improvement is proposed: "*The only new information introduced in the information flow loop is through new experimental data. This leaves room to improve the a priori information by checking the consistency and validating new models identified with different models order and structure*".

The idea is to increase the a priori information content on the model set so the model can be improved not only through parameters reidentification but also by adapting the model order.

Limited management of information

In adaptive control schemes the only relations taken into account are the model identification step and the controller design step. The rest of relationships are just disregarded as can be seen from table 5.3. As a consequence the identified parameters are blindly replaced as soon as they are calculated. However due to the noise effect the parameters changes can be no significant. Thus although a new model has been identified, it could already belong to the models family identified in a former step, thus the following design step is meaningless as no new information is actually present.

In (Balaguer *et al.*, 2006*b*) it is proposed to identify not only a nominal model but also model error bounds. Thus it is possible to validate each new model in such a way that if the new model belongs to the already existing model set then no new information is present and the controller design and updating step is not performed. In this way the controller is not modified on the basis of non informative models.

5.3.2 Iterative Control

Iterative control is an adaptive control design scheme in the sense that i) new information is added to the control design procedure that ii) modifies the control system (i.e. controller) in such a way that iii) better performance level is achieved. Nonetheless the way these actions are supervised is different than in the classical adaptive control supervision introduced above.

The iterative control procedure is as follows:

1. An experiment in closed loop (i.e. controller K_{i-1} operating) is performed at step i and data set D_i is acquired.
2. On the basis of the new data set D_i a new model is identified G_i. It is assumed that the new model G_i is more accurate, for control design purposes, than the former model G_{i-1}.
3. On the basis of the new model G_i a new controller K_i is designed. The controller is applied to the loop and the procedure is repeated until some stoping condition is achieved, for example certain performance level.

Iterative control is based on two main concepts:

- If at each iteration a "better" model can be identified, a "better" controller can be designed (i.e. exploiting new information).

- The procedure is supervised by the control designer, with a constant controller in the loop, which prevents from unnoticed problems as in adaptive control (i.e. managing the information flow more safely).

Distinct iterative identification and control approaches have been proposed. In what follows we enumerate them and describe the identification and control design techniques taken. The iterative control design schemes are (Bitmead, 1993):

- *The Delft School* (Schrama, 1992): Coprime factor identification methods (Hansen *et al.*, 1989) together with H_∞ control design (Zhou, 1998) are used.

- *Windsurfer approach* (Albertos and Sala, 2002): Coprime factor identification methods (Hansen *et al.*, 1989) are joined with the Internal Model Control (IMC) design approach (Morari and Zafirou, 1990).

- *Zangstuff* (Zang *et al.*, 1995): Least Square Identification (Ljung, 1999) is used with LQG control (Bitmead *et al.*, 1990) providing a quadratic approach.

In order to perform the analysis of the iterative control approach on the basis of the information framework of Chapter 4 a specific scheme must be chosen, as distinct elements and relations are considered in each one of the presented schemes. As a result we focus on the Windsurfer approach (Albertos and Sala, 2002) and its extensions (Lee *et al.*, 1995).

Regarding the elements of the control problem, the data set $\mathbb{D}$ is composed by data acquired from closed loop operation with a constant controller on the loop. The data, together with certain model order and structure defined in $\mathscr{A}_{\mathscr{M}}$, is used to identify a single LTI model $\mathbb{G}$. It is worth to mention that the Hansen's identification method (Hansen *et al.*, 1989) can lead to a model order explosion as stated in page 154 (Albertos and Sala, 2002) thus a model reduction step can be necessary. The key point here is that if no model reduction is done, the model order, and thus the a priori model information $\mathscr{A}_{\mathscr{M}}$ is defined by the algorithm and the iterations performed.

The a priori specifications set $\mathscr{A}_{\mathscr{S}}$ is defined by the closed loop bandwidth λ. Then at each step i the specification set $\mathbb{S}$ is the value of the closed loop bandwidth λ_i.

Finally the controller $\mathbb{K}$ with the structure defined in $\mathscr{A}_{\mathscr{K}}$ is designed by the Internal Model Control (IMC) approach (Morari and Zafirou, 1990). In table 5.4 a summary of the elements characterization is presented.

ELEMENT	**ASSUMPTIONS**
$\mathbb{D}$ (Data Set)	Closed loop data, constant controller.
$\mathscr{A}_{\mathscr{M}}$ (A Priori Model Set)	LTI Model, variable order.
$\mathbb{G}$ (Model Set)	Singleton. No uncertainty.
$\mathscr{A}_{\mathscr{S}}$ (A Priori Specification Set)	Closed loop Bandwidth.
$\mathbb{S}$ (Specification)	λ_i
$\mathscr{A}_{\mathscr{K}}$ (A Priori Controller Set)	LTI Controllers.
$\mathbb{K}$ (Controller Set)	Parameters depend on model and λ_i.

Table 5.4: Windsurfer Iterative Control Elements.

Once the elements of the Windsurfer Iterative Control are defined, we discuss the relationships among them. The relations considered are the identification $\mathcal{I}$, the control design $\mathcal{C}$, the model validation $\mathcal{V}$, assessment of fundamental limitations $\mathcal{L}$ and performance monitoring $\mathcal{M}$.

The identification $\mathcal{I}$ is performed through a closed loop identification algorithm via the fractional representation (Hansen *et al.*, 1989). The novelty of the method is to restate the closed loop identification method as an open loop one, hence solving the well known difficulties of closed loop identification (Ljung, 1999).

The control design step $\mathcal{C}$ is performed by the Internal Model Control (IMC) approach (Morari and Zafirou, 1990). In this way first an H_2 optimal controller is designed on the basis of the model at hand. Next, the optimal controller is extended with a low pass filter in order to account for robustness and implementation issues. The low pass filter bandwidth is the value fixed by the specification of the closed loop bandwidth, that is λ_i.

Regarding the validation step $\mathcal{V}$, it is performed in two distinct ways. First a classical cross-correlation validation method is applied (Soderstrom and Stoica, 1989). Secondly the spectrum of the residual is compared with the spectrum of the noise. In fact if both spectrums are similar then the model is validated as there is no useful information on the residual spectrum. However if the spectrum of the residual is larger than the spectrum of the noise at some frequency band, then the model is invalidated. Moreover this indicates that the data is rich enough in order to attempt a new identification. To sum up the validation procedure is done by joining a classical validation procedure together with a visual comparison of the residual spectrum and the noise spectrum.

The fundamental limitations relation $\mathcal{L}$ is assessed through the knowledge of the unstable zeros of the existing model. In (Lee *et al.*, 1995) it is seen that it is not possible to improve the model if RHP zeroes exist. This imposes a bound on the achievable closed loop bandwidth.

Finally the Monitoring action $\mathcal{M}$ is performed at the end of each iteration (i.e. experiment). It is the result of the monitoring action together with the assessed fundamental limitations and control objectives what decides to stop the iterative procedure or to perform a new iteration. In table 5.5 the Windsurfer relations are summarized.

RELATIONSHIP	**CONSIDERED?**
$\mathcal{I}$ (Identification)	Hansen Method
$\mathcal{C}$ (Controller Design)	IMC
$\mathcal{O}$ (Consistency)	No
$\mathcal{V}$ (Model Validation)	Yes
$\mathcal{L}$ (Limitations)	Yes
$\mathcal{M}$ (Monitoring)	Yes

Table 5.5: Windsurfer Iterative Control Relationships.

However some problems of iterative control regarding the information management are:

- Necessity for Iterative Control (Boling and Makila, 1998).
- Information misuse (Balaguer and Vilanova, 2007).

Necessity for Iterative Control

In (Schrama, 1992) the iterative control is presented as a necessity for achieving high performance control. The technical procedure presented belongs to the above named "Delft School". The claimed necessity for iterative control is criticized in (Boling and Makila, 1998), where it is concluded the following:

"Thus this examples (it refers to (Schrama, 1992)) in the form presented in several conferences and published in the archival literature, can neither be used as an argument for the inadequacy of open-loop identification nor for the need of closed-loop iterations."

To our understanding, the key point lies on the problem statement that reads (Boling and Makila, 1998):

"It is assumed that the plant frequency response has been determined exactly from experimental data at a given set of frequencies Ω."

The assumption establishes that during the whole controller design process the data set is both constant and noiseless. Under this assumption, and from an information point of view, the issue of open-loop vs closed-loop identification is meaningless as the data set can not be further improved. Moreover assuming that the other information sources (e.g. the a priori model set, the identification algorithm, etc.) are equally informative, the obtainable result does not depend on iterative schemes. The fundamental issue is that if no new information is added at each iteration then the iterative solution is just a matter of choice as the same result could be obtained by suitable one-shot procedure.

However, without the assumption of perfect data knowledge, iterative schemes provide information benefits:

- The frequency information content can be different at each iteration, thus at each step new data information can be acquired.
- The selection of the information elements (e.g. model order) can be better chosen taking into account the information amount of the problem at each step.

Thus the potential of iterative identification and control schemes lies on the capacity of acquiring new information at each step and the feasibility of managing the current information.

Information misuse

We have seen how Iterative Control (i.e. Windsurfer approach) manages the information of the control problem on the basis of the information theoretic framework. It can be seen that iterative control is much richer than adaptive control regarding the elements and relationships considered. For example, the controller order of adaptive control is fixed meanwhile the controller order of an iterative scheme can be variable.

Notwithstanding the more developed scheme for information management of iterative control, it still remains to consider if the information management is optimal in some sense. We tackle the problem by analyzing how the information quantity of the elements of the control problem evolves during the iterative process.

We introduce the concept of information monotonicity on iterative identification and control schemes. Roughly speaking, an iterative identification and control scheme is monotonic if, at each iteration, the information content of the elements is greater or, at least, equal than the information content of the elements at former iterations.

In order to define the concept of information monotonicity we first recall the mathematical concept of monotonicity over sequences.

Definition 5.3.1 (Monotonic Sequence) *A sequence $\{a_n\}$ such that $a_{i+1} \geq a_i$ for every $i \geq 1$, is a monotonic increasing sequence.*

$\diamondsuit$

We assume that at each time step n we have certain amount of information of the control problem, say I_n. This amount of information is the information content of the elements of the control problem introduced in Chapter 4. Recalling that the information elements of the control problem are the sets $\{\mathscr{A}, \mathbb{D}, \mathbb{G}, \mathbb{S}, \mathbb{K}\}$ the amount of information of the control problem at step n is $I_n = \{\mathscr{A}_n, \mathbb{D}_n, \mathbb{G}_n, \mathbb{S}_n, \mathbb{K}_n\}$. The information content of the control problem is hence monotonic if at each time step i, the information content of the control problem at step $i+1$, that is I_{i+1}, is equal or "greater" than the information content of the control problem at time step i. It however remains to define the order relation between information elements. It is defined as follows:

Definition 5.3.2 (Information relation) *The control problem information element $I_{i+1} = \{\mathscr{A}_{i+1}, \mathbb{D}_{i+1}, \mathbb{G}_{i+1}, \mathbb{S}_{i+1}, \mathbb{K}_{i+1}\}$ is equally or more informative than the control problem information element $I_i = \{\mathscr{A}_i, \mathbb{D}_i, \mathbb{G}_i, \mathbb{S}_i, \mathbb{K}_i\}$, that is $I_{i+1} \succeq I_i$ iff the following set inclusion follows:*

$$\begin{aligned} \mathbb{D}_{i+1} &\supseteq \mathbb{D}_i \\ \mathscr{A}_{i+1} &\supseteq \mathscr{A}_i \\ \mathbb{G}_{i+1} &\subseteq \mathbb{G}_i \\ \mathbb{S}_{i+1} &\subseteq \mathbb{S}_i \\ \mathbb{K}_{i+1} &\supseteq \mathbb{K}_i \end{aligned}$$

$\diamondsuit$

Remark 5.3.1 *The inclusion set requirements introduced in definition 5.3.2 must be accomplished for all the elements. For example if the information element I_{i+1} increases the data set $\mathbb{D}_{i+1}$ but decrease the a priori information set $\mathscr{A}_{i+1}$ with respect to the information element I_i, the concept of information increase of the control problem is lost. However it is true that the data set has increased and that the a priori information set has decreased.*

$\diamondsuit$

Following the above discussion we introduce now the definition of information monotonicity of the control problem as follows:

Definition 5.3.3 (Monotonic Information) *The control problem information sequence $\{I_n\}$ such that $I_{i+1} \succeq I_i$ for every $i \geq 1$ is a monotonic control problem information sequence.*

$\diamondsuit$

Remark 5.3.2 *It follows from the above definitions that in order to have a monotonic increase of information of the control problem at each time step, the information content of all the elements of the control problem must increase or at least remain constant. In this way, classical one shot non adaptive design methods are monotonic as, although no new information is introduced in the design, no already taken information is lost.*

$\diamondsuit$

As a result of definition 5.3.3, the question of information monotonicity in iterative control schemes arises. In (Balaguer and Vilanova, 2007) it is shown that, in general, iterative identification and control schemes are not information monotonic. In fact, at each new iteration step, former data, models and controllers are disregarded and the whole control design process starts again on the basis of the a priori information and the new experimental data set. Thus former already taken information is just disregarded so the whole process does not manages information in a monotonic manner.

In what follows the example taken from (Balaguer and Vilanova, 2007) is presented. The example shows that it is possible to increase the performance (i.e. disturbance rejection) for a wider class of disturbances if former information (i.e. former model) produced by the iterative procedure is used to design the current controller. Moreover in the example it is envisaged a methodology to incorporate former information on the controller.

Example 5.3.1 Zang's Example

In (Zang et al.*, 1995) an iterative identification and control scheme is theoretically posed and a numerical example given. The setup of the example is as follows: the true plant is chosen to be of the form*

$$y_t = P(z)u_t + H(z)e_t \tag{5.1}$$

with

$$P(z) = \frac{z^{-1} - 1.2z^{-2} - 0.3z^{-3} + 0.156z^{-4} + 0.0845z^{-5}}{1 - 1.25z^{-1} + 0.4575z^{-2} + 0.0279z^{-3} - 0.0491z^{-4} + 0.0077z^{-5}} \tag{5.2–5.4}$$

$$H(z) = \frac{2}{1 + 0.6121z^{-1}} \tag{5.5}$$

$P(z)$ is a fifth-order stable non-minimum phase with a single delay. e_t is white noise of unit variance. The procedure commences identifying a model of the plant in open loop $\hat{P}_0$. The model order to be identified is assumed to be third order. The presented iterative control approach is then applied six times arriving at the final model $\hat{P}_6$ used to design the definitive controller C_6.

$$\hat{P}_6(z) = \frac{1.6066z^{-1} - 0.9120z^{-2} - 1.4404z^{-3}}{1 - 0.3139z^{-1} - 0.6349z^{-2} + 0.4839z^{-3}} \tag{5.6}$$

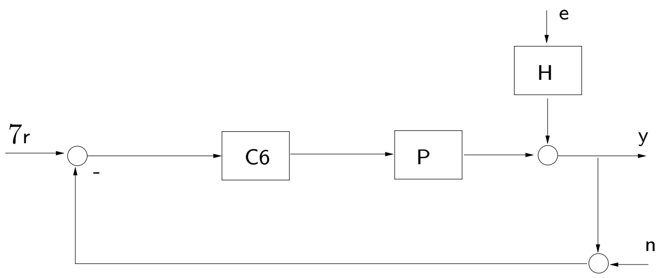

Figure 5.1: Zang's Example Control Structure

$$C_6(z) = \frac{0.038 + 0.1895z^{-1} - 0.0335z^{-2} \quad (5.7) \\ -0.1078z^{-3} - 0.0171z^{-4} - 0.0090z^{-5}}{10.7149z^{-1}0.4911z^{-2} - 0.4222z^{-3} \\ -0.0952z^{-4} - 0.0313z^{-5} + 0.0001z^{-6}} \quad (5.8),\ (5.9)$$

In figure 5.1 the control structure applied at iteration 6 is shown. The quadratic cost J is calculated as follows:

$$J = \frac{1}{N}\sum_{t=1}^{N}(y_y^2 + \lambda^2 u_t^2) \tag{5.10}$$

where the squared weight λ^2 is equal to 0.1. The quadratic cost value achieved by the last designed controller C_6 is shown in table 5.6 (upper left square).

Information Recovery

Now we show the procedure to incorporate former disregarded information of the iterative procedure. We begin with the information of the open loop model $\hat{P}_0$ and the model identified in the last operation $\hat{P}_6$. Note that we disregard the intermediate models. This is not a problem to convey the idea as the difference among the models $\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_6$ is just quantitative as all of them are aimed to capture accurately the same frequency range. The major difference is between the open loop model $\hat{P}_0$ and the closed loop one $\hat{P}_6$.

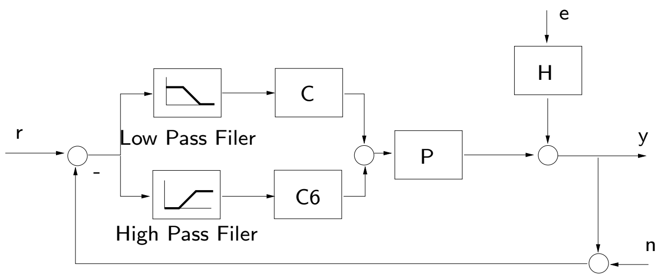

Figure 5.2: Augmented Controller and Zang's experimental conditions

For the open loop model $\hat{P}_0$ *we design a LQG controller* C *for the same quadratic cost as in 5.10. The designed controller* C *is:*

$$C(z) = \frac{-0.1749z^{-1} + 0.07712z^{-2} - 0.009421z^{-3}}{1 + 0.3821z^{-1} - 0.181z^{-2} - 0.07137z^{-3}} \tag{5.11}$$

The next step is to joint the existing controllers C *and* C_6*. This is done by augmenting each controller with a band-pass filter (e.g. Butterworth filter). In the present case, as we are only interested in two frequency bands (i.e. low frequency and high frequency), the controller* C *is multiplied by a low pass filter in order to allow the low frequencies to be processed by the controller* C*, whereas the controller* C_6 *(which is accurate for higher frequencies) is augmented with a high pass filter. Both filters are fourth order and have a cut frequency equal to 1 rad/seg. The final controller is formed by connecting in parallel the two original controllers extended with the filters. See figure 5.2.*

In order to compare the performance of the original controller C_6 *with the augmented version* C_e *two different experimental conditions have been prepared. First both controllers are simulated with the original perturbation proposed in (Zang* et al.*, 1995) and described in the first section of the example. In what follows we refer to this setup as Zang's Perturbation (see figure 5.2). The performance cost index is shown in table 5.6. As can be seen the cost difference between the controller* C_6 *and the controller* C_e *is relatively small. A second experimental setup (see figure 5.3) has been consider by adding to the Zang's Perturbation a step perturbation at the output. As shown in table 5.6 in this setting there is a clear cost reduction by the fact of applying the augmented controller* C_e*.*

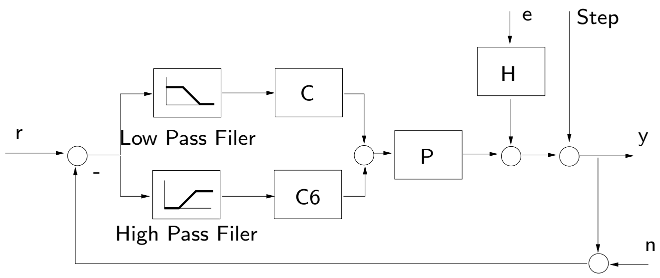

Figure 5.3: Augmented Controller, Zang's experimental conditions plus step disturbance

Table 5.6: Performance cost achieved by simulation

Controller	Zang's Perturbation	Zang's perturbation + Step
C_6	6.44	26.38
C_e	6.48	18.75

Deeper insight into the results presented can be gained by analyzing the Sensitivity functions plotted in figure 5.4. First the Sensitivity function of the C_6 *Zang's original controller (i.e.* S_{C6}*), secondly the Sensitivity function of the controller* C *designed on the basis of the open loop model (i.e.* S_C*) and finally the Sensitivity of the augmented controller (i.e.* S_{Ce}*). As can be seen at low frequencies the Sensitivity function of the augmented controller* S_{Ce} *is equal to the Sensitivity function of the controller* S_C *designed on the basis of an open loop model. It can be seen that gives better perturbation attenuation at low frequencies than the Sensitivity function of the original controller* S_{C6} *provided by Zang. On the other hand, for higher frequencies the Sensitivity function of the augmented controller* S_{Ce} *is equal to the Sensitivity function of the Zang's controller* S_{C6}*, thus performing well for the high frequency disturbance. Thus the controller* C_e *is able to capture the benefits of the two separate controllers* C *and* C_6*. However the resulting controller* C_e *is with no exemption subjected to the fundamental limitations on the Sensitivity function known as the waterbed effect or Bode's integral. In fact from figure 5.4 it can be seen that the Sensitivity function* S_{Ce} *has a low disturbance attenuation* *range around 0.25 rad/sec as a result of pushing down the Sensitivity function at low frequencies.*

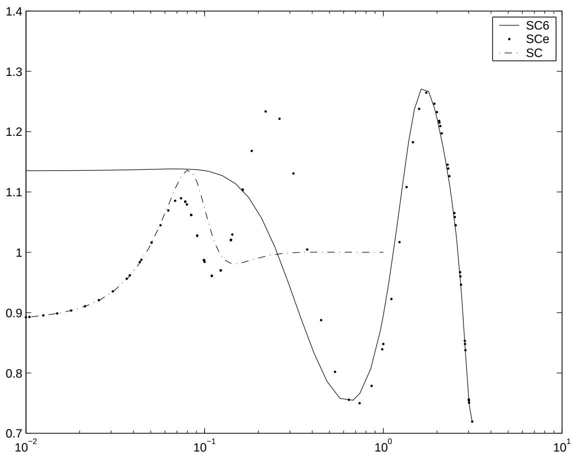

Figure 5.4: Comparison of Sensitivity Functions

Results Discussion

The example shows that it is possible to improve performance for a wider class of perturbation by taking advantage of already existing information, which is accurate for different frequency ranges. We acknowledge that the controllers comparison is not fair as a new perturbation not present in (Zang et al., *1995) has been added (i.e. the step disturbance at the output).*[2] *However both controllers design have the same model and data information for the controller design step.*

We have proven that taking into account former (already existing) information can help the design of controllers that perform better in front of a wider range of perturbations. Of course the new designed controller is under the rules of the fundamental limitations in control so the designed controller is a compromise. Notwithstanding it can be seen that the new augmented controller C_e *performs equally well as the original Zang's* C_6 *controller for the Zang's Perturbation (table 5.6).*

[2]however we consider that the new perturbation is far beyond of being artificial.

5.4 Conclusions

In this chapter the concept of supervisor is presented as an entity that manages the information flow in order to achieve certain objectives. Next the concept of adaptive control has been reviewed and a new definition which amalgamates former definitions is given. Thus an adaptive system is one that performs the following actions by means of a supervisor:

- "Acquiring information". The mechanisms related with the action of acquiring information comprises the Monitoring and Add new Info.
- "Modifying the own system (e.g. parameters, structure, etc) on the basis of the new acquired information". This are accomplished by the mechanisms Parameters change and Learn.
- "The system modifications allow the system to perform better than without modifications". This includes both, the performance maintenance over a changing environment and the performance improvement.

Once the concept of adaptive control is generally defined and related with the concept of information supervisor, the classical adaptive control and iterative control schemes are compared on the basis of the information framework introduced in Chapter 4. The result on the comparison shows that:

- Both classical adaptive control and iterative control are adaptive control schemes in the sense of the general definition of adaptive control introduced above.
- Iterative control schemes have a more developed way of supervising information than classical adaptive control. For example, the model order of iterative schemes is not necessarily fixed as occurs on classical adaptive control.

Furthermore the results of information analysis of classical adaptive control and iterative control are:

- Problems, limitations and possible improvements of classical adaptive control are related with information issues as follows:

- Stability issues of adaptive control can be explained through information deficiencies. For example the bursting phenomenon of classical adaptive control is related to a lack of information content if the data set (i.e. no persistently exciting signal).
- Classical adaptive control makes a limited use of the information sources. In fact new information is added only through the data set. However the a priori information set is not modified. The control design process is then limited to a fixed model order and a fixed controller order.
- The information supervisor actions of classical adaptive control are only the identification procedure and the control design procedure. Other actions such as model validation, assessment of fundamental limitations, etc. are not considered. In order to improve the information management of classical adaptive control it is proposed first to identify not only a nominal model but an error bound and secondly to validate each new identified model against the existing model set. Then if the new identified model belongs to the existing model set no actions are necessary as no new information has been gained through the new identified model.

• The relation of iterative control with information concepts results in:

- The necessity of iterative control schemes is discussed on the basis of controversial discussion on this issue arisen in the bibliography. The rationale behind iterative control schemes in particular and adaptive control schemes in general is the capacity of adding new information. If a control problem is posed with all the relevant information for control at hand, then adaptive control is meaningless. The power of adaptive control is to add possible useful information for control that was not know beforehand.
- The information management on iterative control is not monotonic. That means that although at each iteration new information is acquired, former information can be disregarded. In this sense it is shown by means of an example that if former information is used for control design, better performance levels can be achieved.

Part II

Frequency Domain Model (In)Validation

Chapter 6

Frequency Domain Model (In)Validation

A new (in)validation algorithm is developed in this chapter. The (in)validation procedure is designed to overcome the problems of classical model validation approaches and to provide required features for information management on iterative identification and control schemes

The main idea of the algorithm is to translate the residual from the time domain to the frequency domain and apply the frequency counterpart of the whiteness test. The output of the algorithm results to be more informative than the binary answers of "Validated/Invalidated" given by classical approaches. The algorithm provides, with a probability measure, frequency ranges for which the model is (in)validated.

6.1 Introduction

In order to have confidence in a model, it is necessary to validate it. Different model validation approaches exist. Their difference is based upon the assumptions on plant and models. Classical validation methods (based on classical model identification (Ljung, 1999) (Soderstrom and Stoica, 1989)) rely on statistical uncertainty assumptions due to stochastic noise only. On the other hand, control oriented identification methods (Chen and Gu, 2000) (i.e. H_∞ identification, stochastic embedding, set membership identification, etc.) lead to validation assumptions based on bounded noise and bounded model undermodelling.

However in all cases above mentioned the output of the validation process is a binary result either "validated" or "invalidated". This leads to the following problems (Balaguer and Vilanova, 2006*b*) (Balaguer and Vilanova, 2006*e*):

- Models are neither good nor bad. Normally models are good at capturing low frequency behaviour but their accuracy degrades at higher frequencies.
- This gives no insight into why the model is not useful for the intended use.
- No insight on the way the model should/could be improved.
- In iterative identification and control approaches, a low order model is fitted to capture the frequency range of interest for control. Hence undermodelling is always present, *which makes it difficult to apply traditional model validation schemes*, as stated in (Ljung, 1994).

These arguments question the suitability of classical validation approaches in general, and for Iterative Control schemes in particular. In fact the information flow management of the control problem elements requires more information than just a validated/invalidate test.

In order to solve the above problems, we present a new (in)validation[1]

[1] Although bibliography refers as model validation this is a misleading concept. In fact as discussed in (Popper, 1958) a model can not be validated but just invalidated, as future data could invalidate it. This is a fundamental limitation of the scientific method. However in what follows we use the terms model validation and model invalidation indistinctly keeping in mind that a model can just be invalidated.

procedure that (in)validates the model in the frequency domain. This permits us to ascertain at which frequency ranges the model is not invalidated. Thus the validation answer is no longer a binary result but frequency dependent.

6.2 Classical Model Validation

The problem of assessing the validity of an identified model has been traditionally linked with the problem of model order selection. The classical model validation literature (Soderstrom and Stoica, 1989) (Ljung, 1994) has approached the problem in two ways:

- Use of plots and common sense.
- Use of statistical tests on the residuals (i.e. the difference between the real output and the model output).

The first approach is basically based on the comparison of the data with the model output. If both are similar then the model can be considered a good one. However there are two unavoidable reasons that prevent the model output to fit data perfectly: the modelling errors and perturbations.

The second approach is to apply a hypothesis test over the residual. A hypothesis test is a statement about a random variable. This statement is expressed by means of two hypothesis H_0 and H_1. H_0 is called the null hypothesis and H_1 is the alternative hypothesis. In order to decide the validity of either the null hypothesis H_0 or the alternative hypothesis H_1, an estimation of a population parameter (e.g. mean or variance) is computed from a population sample and it is compared against the assumed population parameters. These population parameters are random variables too with certain mean and variance. If it is likely that the computed statistic is inside the population parameters distribution then H_0 is accepted, otherwise H_0 is rejected in favour of H_1. As a result, two errors are possible: to reject H_0 when it is true, that is a false alarm (type I error or α error), or to fail to reject H_0 when H_0 is false (type II error or β error).

The null hypothesis (i.e. H_0) taken on the model validation test is the hypothesis on the residual, which follows from the assumptions on the disturbance. The more common assumptions over the residual are (Soderstrom and Stoica, 1989):

h1 $\xi(t)$ is a zero mean white noise.

h2 $\xi(t)$ has a symmetric distribution.

h3 $\xi(t)$ is independent of past inputs $E\xi(t)u(\tau) = 0, t > \tau$.

h4 $\xi(t)$ is independent of all inputs $E\xi(t)u(\tau) = 0, \forall t, \tau$.

The above assumptions lead to check two main properties, the whiteness of the residuals (i.e. h1, h2) and the cross-correlation between residuals $\xi(t)$ and control actions $u(t)$ (i.e. h3, h4).

Hence classical validation tests can be classified as follows

- Whiteness Test.
  - Autocorrelation test. (h1)
  - Testing changes of sign. (h1 and h2)
- Independence between residuals and inputs.
  - Cross-correlation test of past inputs. (h3) or (h3 and h1)
  - Cross-correlation test of all inputs. (h4) or (h4 and h1)

The rationale of the tests is to detect causes of variation on the residual distinct than the ones assumed. For example if the residual is assumed to be white noise and the test shows that the whiteness statistical hypothesis is violated then we assume that there is a distinct cause causing the mismatch. The cause is a model error. The same example can be explained from a system identification perspective. On recursive least squares identification method, the residual can be interpreted as the direction through the parameters are modified. If the residual is white then there is no clear direction to follow (i.e. erratic movement). On the other hand, if the residual is not white there is the possibility of using the residuals to modify the parameters in a profitable way.

The result of the statistical tests above reviewed is a binary one. In fact the test either validates or invalidates the model. No further information is provided by the test. As a result, two important drawbacks are:

- There is no information on important model aspects such as (Balaguer and Vilanova, 2006*b*):

  - The reasons why the model is invalidate.
  - How to improve the model.
  - The model usefulness degree.

- In iterative identification and control schemes undermodelling is normally present (Balaguer and Vilanova, 2006*e*). In fact as stated in (Ljung, 1994): "For such a model (a model simpler that the one that minimizes the total error) typically the bias error is the dominating contribution to the total error. Consequently, *such models would normally be falsified during model validation.* These are then reduced complexity models".

Thus, as a conclusion, although the theory of classical validation methods is well developed and plenty of successful applications, it has limitations when a more informative validation procedure is required, as for example in iterative identification and control approaches.

6.3 Frequency Dependent Model (In)Validation

*The main objective of the new proposed algorithm is to validate a model on the frequency domain. To this end a time domain validation procedure based on testing the residual whiteness is modified to achieve the pursued objectives (Balaguer and Vilanova, 2006*a*).* The idea is as follows. It is assumed that if the residual is white noise the model is validated because the residual contains no further useful information that could be used to improve the model accuracy. Moreover it implies that there are no unmodelled dynamics as the uncertainty is due only to measurement noise. This test is usually performed in the time domain by testing the residual autocorrelation, the number of sign changes, etc. (Ljung, 1999).

The translation of the time domain whiteness test to the frequency domain is performed through two steps. Firstly we translate the time domain residual to the frequency domain by its Discrete Fourier Series. Secondly, the statistical properties of the spectrum of a white noise signal are calculated. The objective is to test if the spectrum calculated from the residual has properties of a white noise. As a result, one unique test in the time domain has been translated to N different tests in the frequency domain. We check if the k^{th} frequency component of the spectrum has the properties of a typical frequency component of a white noise. In case of an affirmative answer, we have no reason to believe that the model is invalidated on that frequency component. On the other hand, if there are certain frequency components which clearly do not behave accordingly to the statistical properties of white noise then it is likely that at this frequency range there is an important mismatch between the model and the plant. As a result the model is invalidated for that frequency range.

6.3.1 Whiteness Test on the Frequency Domain

In this section the statistical time domain properties of a white noise are translated to the frequency domain. This is accomplished by means of two theorems. The first one is an intermediate result that is used by the second theorem which describes the frequency domain distribution of the spectrum of a white noise.

Theorem 6.3.1 *Let $\xi(n)$ be a sequence of independent identically distributed (IID) samples of normal distribution $N(\mu_\xi, \sigma_\xi^2)$. If we express the Fourier coefficients by its real and imaginary part, that is $\xi_k = R_k + jI_k = \frac{1}{N}\sum_{n=0}^{N-1}\xi(n)e^{-j\Omega_0 kn}$, then the real part R_k is a random variable normally distributed ($R_k \in N(\mu_{R_k}, \sigma_{R_k}^2)$) with mean μ_{R_k} and variance $\sigma_{R_k}^2$ given by*

$$\begin{aligned} \mu_{R_k} &= \mu_\xi \frac{1}{N}\sum_{n=0}^{N-1}\cos(\Omega_0 kn) \\ \sigma_{R_k}^2 &= \sigma_\xi^2 \frac{1}{N^2}\sum_{n=0}^{N-1}\cos^2(\Omega_0 kn) \end{aligned} \tag{6.1}$$

Similarly the Imaginary part I_k is a random variable normally distributed $I_k \in N(\mu_{I_k}, \sigma_{I_k}^2)$ with mean μ_{I_k} and variance $\sigma_{I_k}^2$ given by

$$\begin{aligned} \mu_{I_k} &= \mu_\xi \frac{1}{N}\sum_{n=0}^{N-1}\sin(\Omega_0 kn) \\ \sigma_{I_k}^2 &= \sigma_\xi^2 \frac{1}{N^2}\sum_{n=0}^{N-1}\sin^2(\Omega_0 kn) \end{aligned} \tag{6.2}$$

□

Proof: The Discrete Fourier coefficients ξ_k of a discrete time signal $\xi(n)$ is given by

$$\xi_k = \frac{1}{N}\sum_{n=0}^{N-1}\xi(n)e^{-j\Omega_0 kn} \tag{6.3}$$

where $\Omega_0 = \frac{2\pi}{N}$ is the fundamental frequency in rad/sample. Decomposing the equation (6.3) into its real part and its imaginary part gives

$$\begin{aligned}
\xi_k &= \frac{1}{N}\sum_{n=0}^{N-1}\xi(n)e^{-j\Omega_0 kn} \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\xi(n)(\cos(\Omega_0 kn) - j\sin(\Omega_0 kn)) \\
&= \frac{1}{N}\sum_{n=0}^{N-1}\xi(n)\cos(\Omega_0 kn) - j\frac{1}{N}\sum_{n=0}^{N-1}\xi(n)\sin(\Omega_0 kn) \\
&= R_k - jI_k
\end{aligned}$$

where the second equality comes from Euler's identity. Hence the real and imaginary parts are linear combinations of normally distributed random variables

$$R_k = \frac{1}{N}\sum_{n=0}^{N-1}\xi(n)\cos(\Omega_0 kn) \tag{6.4}$$

$$I_k = \frac{1}{N}\sum_{n=0}^{N-1}\xi(n)\sin(\Omega_0 kn) \tag{6.5}$$

As a result, it follows that R_k and I_k are also normally distributed random variables. The distribution parameters for the stochastic variable R_k are calculated as follows (see, for example, (Box *et al.*, 1978), page 87)

$$\begin{aligned}\mu_{R_k} &= \mu_\xi \frac{1}{N}\sum_{n=0}^{N-1}\cos(\Omega_0 kn)\\ \sigma^2_{R_k} &= \sigma^2_{\xi_n}\frac{1}{N^2}\sum_{n=0}^{N-1}\cos^2(\Omega_0 kn)\end{aligned} \tag{6.6}$$

The same reasoning is applicable to the imaginary part I_k.

■

Remark 6.3.1 *μ_{R_k} is equal to zero for $k \in \{1, 2, ..., N-1\}$ and μ_{R_0} equals the mean value of the residual (i.e. $\mu_{R_0} = \mu_\xi$). μ_{I_k} is always equal to zero for $k \in \{0, 1, 2, ..., N-1\}$*

◊

Theorem 6.3.2 *The normalized squared gain M_k^2 defined as*

$$M_k^2 = \left(\frac{R_k - \mu_{R_k}}{\sigma_{R_k}}\right)^2 + \left(\frac{I_k - \mu_{I_k}}{\sigma_{I_k}}\right)^2 \tag{6.7}$$

has a χ^2 distribution of 2 degrees of freedom if R_k and I_k are independent.

□

Proof: By definition the sum of "" independent squared random normal variables $N(0,1)$ has a χ^2 distribution of r degrees of freedom. Due to the normalization of R_k and I_k, it follows that $\frac{R_k-\mu_{R_k}}{\sigma_{R_k}} \in N(0,1)$ and $\frac{I_k-\mu_{I_k}}{\sigma_{I_k}} \in N(0,1)$.

■

6.3.2 Procedure

The frequency domain model invalidation procedure is as follows

1. Calculate the residual as the difference of the real output and the model estimated output ($\xi(n) = y(n) - \hat{y}(n)$).

2. Calculate the Discrete Fourier coefficients of the residual (ξ_k)

3. Decompose each frequency component on its real part and imaginary part ($\xi_k = R_k + jI_k$).

4. Calculate the distribution parameters of the Real and Imaginary part of the residual spectrum (i.e. μ_{R_k}, μ_{I_k}, σ_{R_k}, σ_{I_k}).

5. Calculate the normalized magnitude spectrum as follows

$$M_k^2 = \left(\frac{R_k - \mu_{R_k}}{\sigma_{R_k}}\right)^2 + \left(\frac{I_k - \mu_{I_k}}{\sigma_{I_k}}\right)^2 \tag{6.8}$$

6. Perform an hypothesis test over the normalized magnitude spectrum calculated.

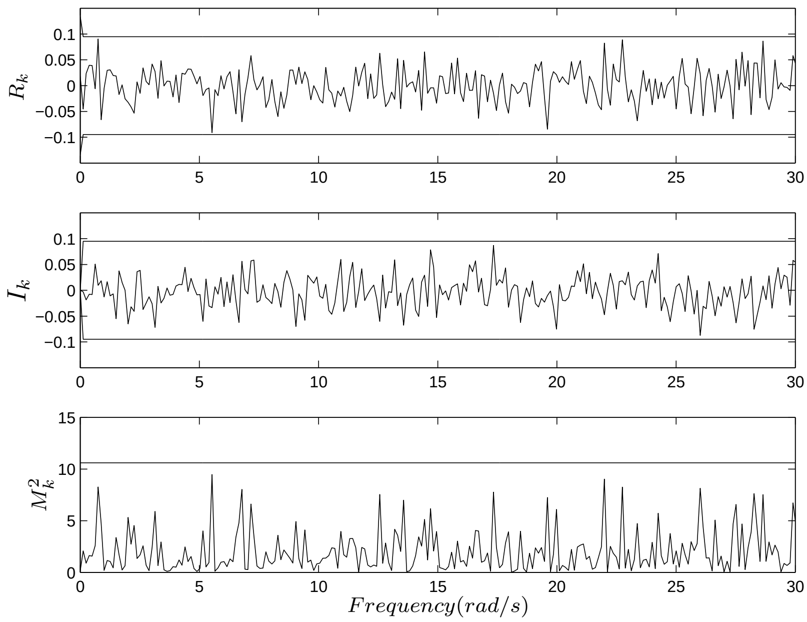

Figure 6.1: White noise example

The above steps are materialized in the following example. A realization of a normally distributed random variable of zero mean and unity variance is performed with 500 samples. The Discrete Fourier coefficients of the realization are calculated and decomposed into its real and imaginary parts (i.e. R_k, I_k). The values of R_k and I_k are shown in figure 6.1. They follow a normal variable distribution with parameters given by equations (6.1)-(6.2) (i.e. $R_k \in N(\mu_{R_k} = 0, \sigma^2_{R_k} = 0.001)$ and $I_k \in N(\mu_{I_k} = 0, \sigma^2_{I_k} = 0.001)$). On figure 6.1 the realizations of R_k and I_k are plotted together with the 3 sigma limits of their distribution (i.e. the 99.73% of the samples fall between the plotted limits). It can be seen that all the points fall inside this range.

Finally the normalized magnitude spectrum is calculated following equation (6.8). The magnitude spectrum can be seen in figure 6.1 together with the 99.5% confidence limit of the χ^2_2 distribution (i.e. the 99.5% of the samples fall between 0 and 10.6). All the magnitude frequency components remain below of the confidence limit so there are no reasons to invalidate the model. The whiteness test has passed.

Remark 6.3.2 *Two important variables of the residual $\xi(n)$ which have a fundamental influence on the validation test are the sample time T_s of the residual and the total number of samples N of the residual. The sample time T_s imposes the maximum frequency component that can be tested by the algorithm. By the Nyquist theorem the maximum frequency is $\Omega_{max} = \frac{\pi}{T_s}$ rad/sec. On the other hand the number of samples N of the residual provides the resolution of the validation test, as the frequency range is divided in multiples of the fundamental frequency given by $\Omega_0 = \frac{2\pi}{N}$. Thus the greater the N, the smaller the fundamental frequency and the higher the validation test resolution.*

◊

6.3.3 Hypothesis Test

The hypothesis test is the last step of the presented procedure, where the decision of validation/invalidation of certain frequency component is taken. An hypothesis test is a statement expressed by means of two hypothesis H_0 and H_1. H_0 is called the null hypothesis and H_1 is the alternative hypothesis. The hypothesis test to be applied in our procedure is:

$$\begin{aligned} H_0 &: M_k^2 \in \chi^2_2 \\ H_1 &: M_k^2 \notin \chi^2_2 \end{aligned} \tag{6.9}$$

The hypothesis H_0 states that the normalized modulus M_k^2 of the k^{th} frequency component is χ_2^2 distributed. On the other hand the hypothesis H_1 states that the normalized modulus M_k^2 of the k^{th} frequency component is not χ_2^2 distributed.

Remark 6.3.3 *The hypothesis test stated in (6.9) is applied to each frequency component, from 0 rad/sec up to the Nyquist frequency (i.e. π/Ts, where Ts is the sample time).*

$\diamondsuit$

In order to decide the validity of either the null hypothesis H_0 or the alternative hypothesis H_1, M_k^2 is computed. If it is “likely” that the value of M_k^2 lies inside the χ_2^2 distribution then H_0 is accepted, otherwise H_0 is rejected in favor of H_1. As a result, two errors are possible: to reject H_0 when it is true, that is a false alarm (type I error or α error), or to fail to reject H_0 when H_0 is false (type II error or β error).

The term “likely” introduced above is defined by the user by choosing the confidence limit. For example, if the confidence limit is chosen to be 10.6 then the 99.5% of the samples fall inside the limits. This confidence limit sets the type I error. Following the example presented in section 6.3.2 the type I error was of 0.5%, that is the 0.5% of the samples of a χ_2^2 should be greater than 10.6 in normal conditions. The type II error is more difficult to be calculated as it depends on the actual distribution followed by M_k^2.

The hypothesis test is then simply a check that any magnitude of the normalized spectrum is less than the test limit. If the value is greater than the test limit then the model results invalidate for that frequency.

6.4 Conclusions

In this chapter a new (in)validation procedure in the frequency domain has been presented. The main features of the algorithm are:

- The procedure permits to validate or invalidate models over certain frequency ranges.
- The procedure is the translation of a time domain residual whiteness test to a frequency dependent residual whiteness test.

- The validation/invalidation step is based on a hypothesis test applied to each frequency component. This determines if certain frequency components have an unusual content that discards the model validity for this frequency value. The acceptance/rejection decision of the frequency component validity comes with a probability measure.

The new validation procedure is aimed to overcome some well known limitations of classical validation procedures. The advantage of the new algorithm hinges on its frequency dependent information in order to decide the model validation. Thus a more informative answer is given that just a "validated/invalidated" result.

Chapter 7

Control Oriented (In)Validation

Once the frequency domain model validation procedure has been presented, in this chapter the control oriented properties of the algorithm are discussed. In order to obtain a meaningful model validation for control design purposes, the frequency domain algorithm is endowed with the control oriented property by means of generating the residuals through a suited structure.

Once the frequency domain model validation has been provided with the control oriented property the control oriented frequency dependent model validation (COFDMV) is analyzed regarding their properties for managing the information flow on iterative control schemes.

7.1 Introduction

Model validation theory is aimed towards checking the model usefulness for some intended use. Thus the model validation procedure should take into account the model use, for example control design or prediction purposes.

In fact, it is recognized in (Skelton, 1989) that arbitrary small model errors in open loop can lead to bad closed loop performance. On the other hand large open loop modelling errors do not necessarily lead to bad closed loop performance. As a result the model accuracy should be checked in such a way that the intended model use is taken into account in the model validation procedure.

An important aspect in the validation procedure to take into account the intended model use are the validation conditions. In fact validation from open loop data can provide a different result than validation with closed loop data. Furthermore it is completely different to validate an open loop model than to compare two closed loops, the one with the model and the real one. In (Gevers *et al.*, 1999) a closed loop validation procedure is presented aimed at validating a model for control design purposes. An open loop model is validated by comparing two closed loops, the real one and the closed loop with the open loop model. One of the main results is that "the same model may fail to be validated with open-loop data, while it is validated by data collected in closed loop". This result points out the importance of the information that is being validated. This is accomplished by means of setting the experimental conditions from which data are generated.

In conclusion a model is neither good nor bad by itself but regarding its intended use. In order to consider the model intended use in the validation procedure the conditions for data generation must be considered. In the following section we discuss how the new model validation procedure introduced in the preceding Chapter 6 is endowed with the control oriented property.

7.2 Control Oriented Validation

Once the fundamentals of the frequency dependent (in)validation algorithm are presented, it remains to assess the information that is being (in)validated in order to take into account control oriented aspects as discussed in the preceding section. In our case the model is pursued for control design purposes so the information that is being (in)validated should be relevant for control design purposes.

In the following subsections different structures are proposed in order to compute the residuals and it is shown that they have considerable importance on the actual information that is (in)validated (Balaguer *et al.*, 2006*a*). The residual statistical properties are reviewed as the residuals must be statistically white under perfect model matching in order to apply the proposed algorithm.

7.2.1 Open Loop Validation

A model is validated in open loop when no control loop is present in the validation procedure. The output of the model is compared with the output of the plant when both are affected by the same input which it is not generated by feedback. The structure used to validate the model is shown in figure 7.1.

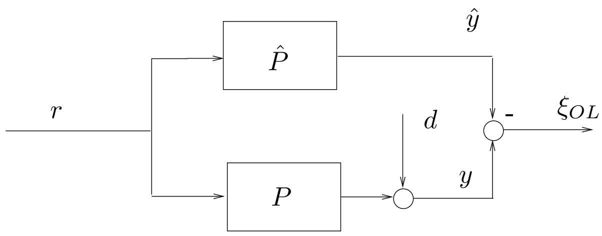

Figure 7.1: Open Loop Model Validation

The residual is given by the following expression

$$\xi_{OL} = d + (P - \hat{P})r \tag{7.1}$$

where P represents the plant, $\hat{P}$ is the model being validated, d is the process disturbance and r the input. As a result of r being produced in open loop, it follows that, in general, the signals r and d are not correlated.

The open loop residual ξ_{OL} given by equation 7.1 is analyzed under distinct model plant mismatch conditions. The residual ξ_{OL} is just the noise d if the model and the plant are equal (i.e. $\hat{P} = P$), thus no error modelling is present. As a result the residual has the same stochastic properties than the noise because $\xi_{OL} = d$. On the other hand if there exists a discrepancy between the model and the plant, the term $((P - \hat{P})r)$ appears in the residual. Now the stochastic properties are no longer equal to the disturbance d but affected by the term $(P - \hat{P})r$.

Finally it should be remarked that the model-plant error which will be detected when $\hat{P} \neq P$ (i.e. error model present) is highly dependent on the reference signal r. In fact in the trivial case that $r = 0$ no model error can be detected. The signal r should contain a high energy content around those frequencies which are interesting for validation in order to obtain a meaningful validation result. Nothing can be said about model errors around frequencies which have not been excited by r on the validation procedure. This fact remarks the importance on the input selection for model validation purposes.

7.2.2 Closed Loop Validation: Unstable Models

When a model for control design purposes is validated, the residuals (either ξ_{CLu} or ξ^u_{CLu}) are normally generated accordingly to figure 7.2 (Landau and Zito, 2006*b*). The residual ξ_{CLu} represents the error between two controlled loops instead that the error between a model and a plant. Thus the open loop model error is not assessed by the model-plant mismatch but by how the model error affects the controlled loop.

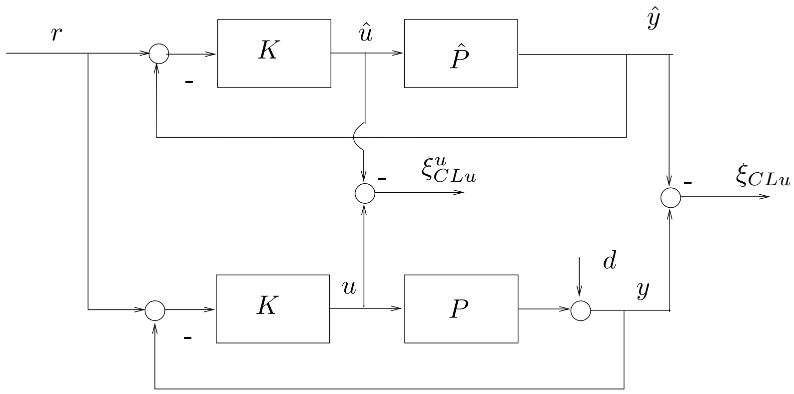

Figure 7.2: Closed Loop Model Validation for Unstable Plants

The residuals generated at the output ξ_{CLu} (at the input ξ^u_{CLu}) by means of the structure shown in figure 7.2 are:

$$\xi_{CLu} = Sd + KS\hat{S}(P - \hat{P})r \tag{7.2}$$
$$\xi^u_{CLu} = -KSd - KKS\hat{S}(P - \hat{P})r \tag{7.3}$$

The residual given by equation 7.2 (7.3) in case of perfect model (i.e. $\hat{P} = P$) is $\xi_{CLu} = Sd$ ($\xi^u_{CLu} = -KSd$). In this case the stochastic properties of the residual ξ_{CLu} (ξ^u_{CLu}) are no longer the ones of the disturbance d. For example if the disturbance d is assumed white, the residual ξ_{CLu} (ξ^u_{CLu}) is always autocorrelated as it is filtered by the sensitivity function S (KS) independently of the model-plant mismatch.

If there exists a model plant mismatch the new residual term $KS\hat{S}(P - \hat{P})r$ ($KKS\hat{S}(P - \hat{P})r$) arises. Again, the impact of the term in the residual depends on the reference r. However the model plant error is weighted by $KS\hat{S}$ ($KKS\hat{S}$) thus giving more importance to the frequencies where the Sensitivity function has a high value, that is around the crossover frequency. Thus the model errors are evaluated taking into account the closed loop.

The preceding discussion establishes two important facts:

- The residual generated in closed loop is suited for model validation for control design purposes. Even unstable models can be validated as far as the closed loop is stable.

- The residual generated is no longer equal to the disturbance d under perfect model plant fit. This fact prevents the residual to be used in a whiteness test of any kind.

As a result, although the control oriented nature of the residuals generated following diagram 7.2 is advantageous in front of the open loop counterpart, the auto-correlation of the produced residual ξ_{CLu} (ξ^u_{CLu}) prevents from applying any whiteness test. Accordingly the residuals thus generated are not suited to be used by the FDMV as the validation algorithm is based on a whiteness test in the frequency domain. In the next section a new structure is proposed in order to obtain control oriented residuals suited to be used by a whiteness test.

7.2.3 Closed Loop Validation: Stable Models

In this section a new structure for residual generation is introduced. The main benefit is that the residual generated is white under perfect mode plant match and it is control oriented. The proposed structure to validate stable models in closed loop is shown in figure 7.3.

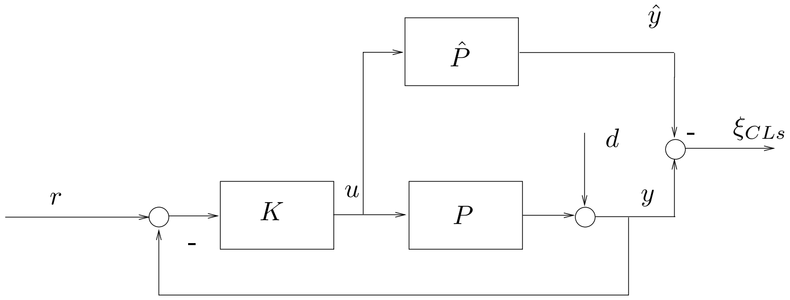

Figure 7.3: Closed Loop Model Validation for Stable Plants

The residual generated is

$$\xi_{CLs} = \frac{S}{\hat{S}} d + KS(P - \hat{P})r \tag{7.4}$$

If the model and the plant are equal (i.e. $\hat{P} = P$) then the real sensitivity function $S = (1 + GK)^{-1}$ and the estimated sensitivity function $\hat{S} = (1 + \hat{G}K)^{-1}$) are equal, so the first term of equation 7.4 yields the noise d. Moreover the second term, under the same perfect model-plant matching assumption, is zero. Hence in this case the residuals are again the noise d, having both the same stochastic properties.

If there exist a discrepancy between the model and the plant then the division between S and $\hat{S}$ is no longer unity but a transfer function resulting from the noise d filtered by $S/\hat{S}$ (i.e. autocorrelated). Additionally the second term of equation 7.4 gives a signal proportional to the model-plant error weighted by the control sensitivity function KS thus giving more emphasis on errors around the frequencies where the sensitivity function is high (i.e. control oriented).

Consequently the residuals generated accordingly the structure shown in 7.3 have the same properties than the disturbance in the case of perfect model plant matching and are control oriented. Thus if the disturbance is white it is possible to apply any whiteness test over the residuals (i.e. FDMV). The only drawback of the structure presented in 7.3 is that it is not feasible for unstable models, as the model $\hat{P}$ is operated in open loop.

7.3 Model Validation on Iterative/Adaptive Schemes

The objective assumed by classical model validation approaches is to ascertain the validity of the model. Consequently if a model is validated there are no reasons to doubt of the suitability of the data set $\mathbb{D}$, the a priori model information set $\mathscr{A}_{\mathscr{M}}$ and the identification algorithm $\mathcal{I}$. As discussed in Chapter 2, model validation does not produce any new element of the control problem. The only relationships which transform existing elements into new ones are the identification and the control design. The mission of the rest of the relationships (e.g. model validation) are to check the validity among the elements.

Notwithstanding the model validation procedure plays a more active role on adaptive schemes. The requirement of adding periodically new information arises new requirements on the validation procedure beyond that the "validated/invalidated" answer. The new requirements are:

- Is it possible to improve an existing model? Is the data informative enough to attempt a new identification?
- How can the model be improved? Is the model order/structure rich enough to capture the interesting features of the plant?
- How authoritative can be the controller designed on the basis of the new model? Which is the validity frequency range of the model?

The above requirements for iterative control can not be provided by the classical model validation approaches above introduced because:

- No indication on the possibility to improve an existing model. This problem is tackled in (Lee *et al.*, 1995) by the use of classical validation methods (i.e. crosscorrelation test) together with the visual comparison of two power spectrum.

- In iterative identification and control approaches a low order model is fitted to capture the frequency range of interest for control. Hence undermodelling is always present. This fact makes it difficult to apply traditional model validation schemes as the output of the validation procedure is a binary answer (i.e. validated/no validated) (Ljung, 1994).

- No indication on how to improve the model on the next iteration (i.e. model order selection and/or input experiment design).

- No indication on the model validity range for control design (i.e. controller bandwidth selection).

These limitations arise the need for a more informative validation procedure. In fact, the windsurfer approach to iterative control (see for example (Albertos and Sala, 2002)), is enhanced with some new features introduced in (Lee *et al.*, 1995). Regarding model validation, two complementary validation methods are applied, a time domain method and a frequency domain method. The time domain validation is a classical cross-correlation test between the residual and the filtered input. In fact no whiteness test could be applied to the residual as they are generated following the structure of figure 7.2. The frequency domain method for model validation is based on comparing two power spectrum, the noise spectrum and the residual spectrum. When the residual spectrum is larger than the noise spectrum for some frequency range, that means that the error cause is the undermodelling. Thus the model is invalidated for that frequency range. Moreover the spectrum comparison reveals frequency bands where high signal-to-noise ratio is present and thus the data is suited to attempt a new identification procedure, at least for some frequency band. The frequency domain information is obtained by visual comparison of both spectrums.

The Control Oriented Frequency Dependent Model Validation algorithm (COFDMV), that is the frequency domain validation procedure presented in the preceding Chaptertogether with the residual generated by the structure presented in figure 7.3, is of interest in iterative identification and control schemes in general. The benefits of the COFDMV for the iterative identification and control schemes hinge on the frequency domain information produced by the algorithm and its control oriented nature. The COFDMV has the following features:

- The validation algorithm permits to distinguish between the lack of information shown by imprecision due to noise and imprecision due to undermodelling. Moreover the undermodelling severity can be assessed in the frequency domain by the spectrum magnitude.
- The lack of information due to incompleteness can be also managed. In fact, if the algorithm does not invalidate a model, it can be used in order to design a higher bandwidth controller. However the new pair model controller can be invalidated in the next step. Then the algorithm indicates for what frequency range the current model is not valid and more information about the system behavior around those frequencies should be acquired.

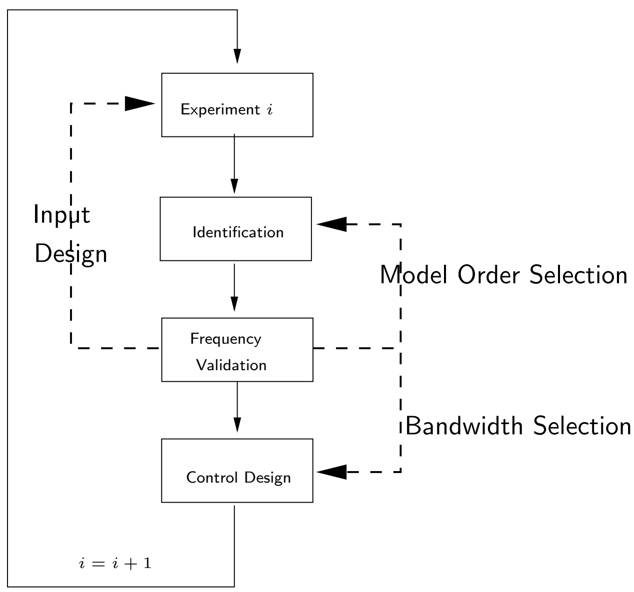

Figure 7.4: Guided actions by the COFDMV on iterative identification and control schemes

The above information analysis permits to take the actions presented schematically in figure 7.4. These are:

- Designing the input experiment for the next identification step. It is well known that the identified model quality is strongly dependent on the experimental input applied to generate the data (Ljung, 1999) (Bitmead *et al.*, 1990). The experiment should contain high energy components on the frequency range where the model is being invalidated if a new identification aimed at improving the model at that frequency range is pursued. Furthermore the COFDMV indicates for what frequency bands the data contains meaningful information.

- Detecting model undermodelling and/or choosing model order. A higher order model can be fitted when the current model is being invalidated. It can be done even inside the current iteration step without the need of performing a new experiment.

- Selecting the controller bandwidth on the controller design step. If the COFDMV algorithm does not invalidate a model then it is feasible to increase the model bandwidth. However the model could be invalidated for the new designed controller. On the other hand if a model is invalidated at some frequency then, it is not sensible to increase the controller bandwidth. If a higher bandwidth is desired further actions should be performed (e.g. a new model identification).

7.4 Conclusions

In this chapter we have pointed out the importance of the final intended use of the model when a validation is performed. The frequency dependent model validation algorithm proposed is control oriented because the residuals are generated in a suitable way. Three distinct structures for generating the residuals have been presented. In particular the structure plot on figure 7.3 generates residuals which are firstly control oriented and secondly suited to be tested by any whiteness test.

Finally the control oriented frequency dependent model validation algorithm has been discussed on the basis of the iterative control. First the frequency validation proposed on the windsurfer approach has been presented. Compared with the COFDMV we can extract that the main feature of the validation algorithm to be useful for iterative control hinges on the frequency domain nature of the information produced. However the main advantage of the COFDMV is that provides a qualitative assessment on the frequency validation/invalidation question by means of a probability measure. It follows that the benefits of the COFDMV for iterative control are the better management of information flow of the control problem by:

- Designing the experimental input in order to obtain a informative data set for some frequency band of interest (the band for what the model is not validated).
- Increasing the model order.
- Selecting the allowable controller bandwidth with the model at hand.

Chapter 8

Application Examples

The usefulness of the new frequency domain model validation approach is presented by means of three examples. The first one is a classical open loop model validation. It shows the test when a model is invalidated. Moreover the importance of the reference signal on model validation is stressed. Next the COFDMV algorithm is applied to existing iterative identification and control procedures. Thus at each step a new model is available, it is validated regarding the current loop controller.

8.1 Open Loop FDMV

The proposed validation procedure in the frequency domain is applied to a stable plant in open loop (see figure 8.1).

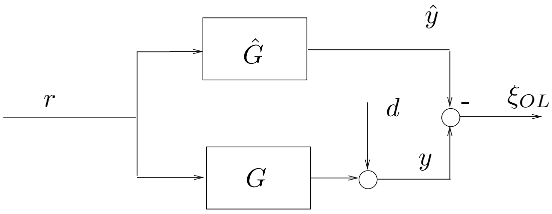

Figure 8.1: Open Loop Model Validation Structure

The real plant G and the model $\hat{G}$ chosen to approximate it are:

$$G = \frac{10}{(s+1)(s+10)}, \qquad \hat{G} = \frac{1}{(s+1)}$$

The Bode diagram comparing the real plant with the model is shown in figure 8.2.

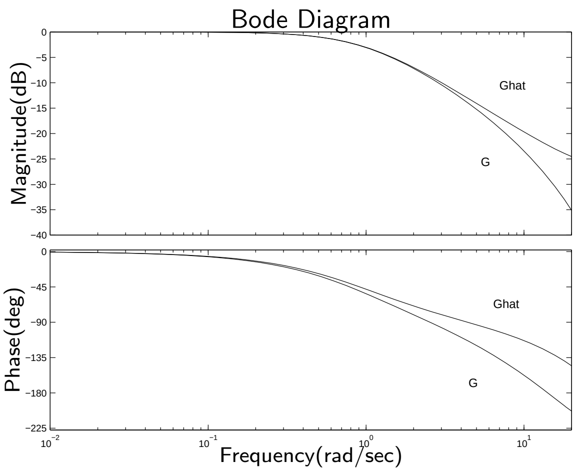

Figure 8.2: Comparison of Bode diagrams

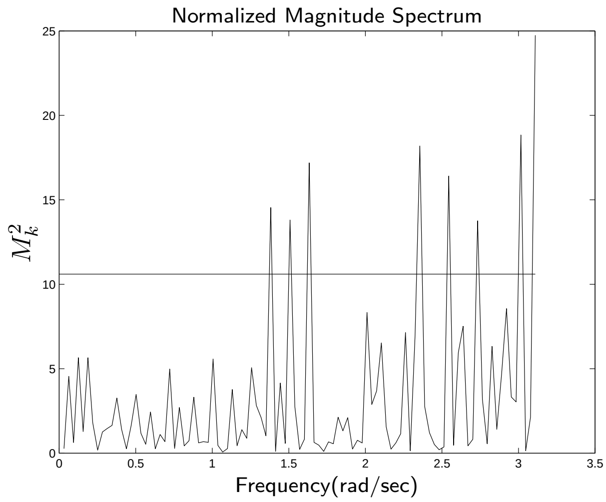

Figure 8.3: Normalized Magnitude Spectrum and Confidence Limits

The experimental setup is as follows, the reference input r is a train of sinusoids up to frequency 3 rad/sec, of amplitude $A = 5$. The perturbation d is assumed to be white noise with σ=1. After the experiment the FMVD algorithm is applied to the residuals ξ_{OL}, which are sampled at 1 second (i.e. $T_s = 1$ *sec*). The selected sample time gives a Nyquist frequency around 3 rad/sec. The validation procedure results can be seen in figure 8.3. The model $\hat{G}$ shows no invalidation signs up to 1.4 rad/sec. However for higher frequencies the hypothesis test fails to validate the model. As a conclusion we can state that, for the input applied, the model is correct for frequencies below to 1.4 rad/sec. It is worth to mention that although between 1.7 rad/sec and 2.3 rad/sec there are no spikes out of the confidence limit, a deeper examination reveals that several consecutive spikes are abnormally high to belong a χ_2^2 distribution. In order to detect this situations further probabilities should be checked (e.g. the probability that two consecutive points of a χ_2^2 distribution be higher than some given value, etc.).

Conclusions

As a conclusion, the model $\hat{G}$ can be accepted as a good approximation of the plant G up to frequency 1.4 rad/sec. For higher frequencies the mismatch between model and plant is present up to the input bandwidth (i.e. 3 rad/sec). It should be mention that the result is input dependent. For example, for a lower amplitude of the signal r no invalidation is detected. However the results obtained up to now can serve as a guideline to design new input signals with suitable frequency contents for new identification steps (e.g. high energy around the frequencies were a significant error exists, that is between 1.4 rad/sec and 3 rad/sec).

8.2 Iterative Control COFDMV

8.2.1 Example 1

The present example is the application of the proposed frequency domain model validation to an Iterative Control Design. As baseline we take the Iterative Control Design example presented in (Albertos and Sala, 2002) page 126, where a stable plant with high-frequency resonant modes is controlled by successive plant identification (e.g. step response) and the subsequent controller design (e.g. model matching and cancellation controller). We apply to the successive models and controllers given in (Albertos and Sala, 2002) our frequency domain model validation procedure. Furthermore we use the structure shown in figure 8.4 to generate the residuals in a control oriented way.

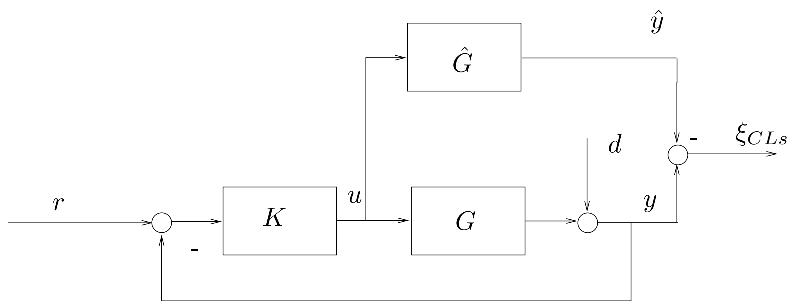

Figure 8.4: Closed Loop Model Validation for Stable Plants

The plant $G(s)$ to be controlled is:

$$G(s) = \frac{10^6(s+1000)}{(s^2+0.002s+1000^2)(s^2+0.1s+50^2)(s+0.1)(s+0.2)} \tag{8.1}$$

The experimental setup is as follows. First a model of the plant $\hat{G}$ is obtained by a step response identification. For this model successive controllers K are designed by imposing more stringent reference models M. When the closed loop step response is unsatisfactory, a new model is identified and the controller design steps are repeated. The measurement noise d is white noise with $\sigma = 10^{-2}$. The reference input r is a train of sinusoids up to frequency 200 rad/sec.

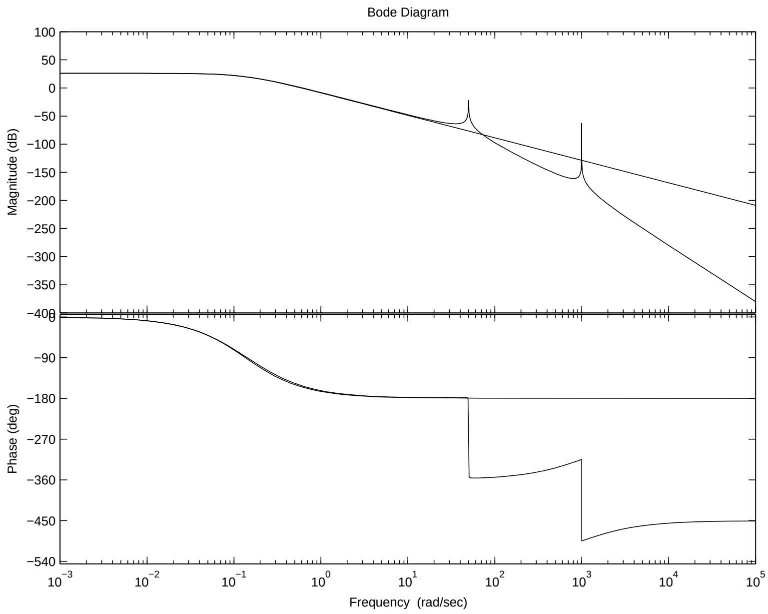

Figure 8.5: Bode Plot

First Iteration

The first identified model and the model reference used for controller design are:

$$\hat{G}_0 = \frac{20}{(1+7.4s)^2}, \qquad M_{01} = \frac{0.5^2}{(s+0.5)^2}$$

The bode plot of the real plant G and the first model $\hat{G}_0$ is shown in figure 8.5. The frequency domain validation is applied, giving a positive validation result, as can be seen in the first plot of figure 8.6.

Second Iteration

In the example the model $\hat{G}_0$ is kept as a valid one and the performance is pushed forward by a new, more stringent, reference model:

$$\hat{G}_0 = \frac{20}{(1+7.4s)^2}, \qquad M_{02} = \frac{3^2}{(s+3)^2}$$

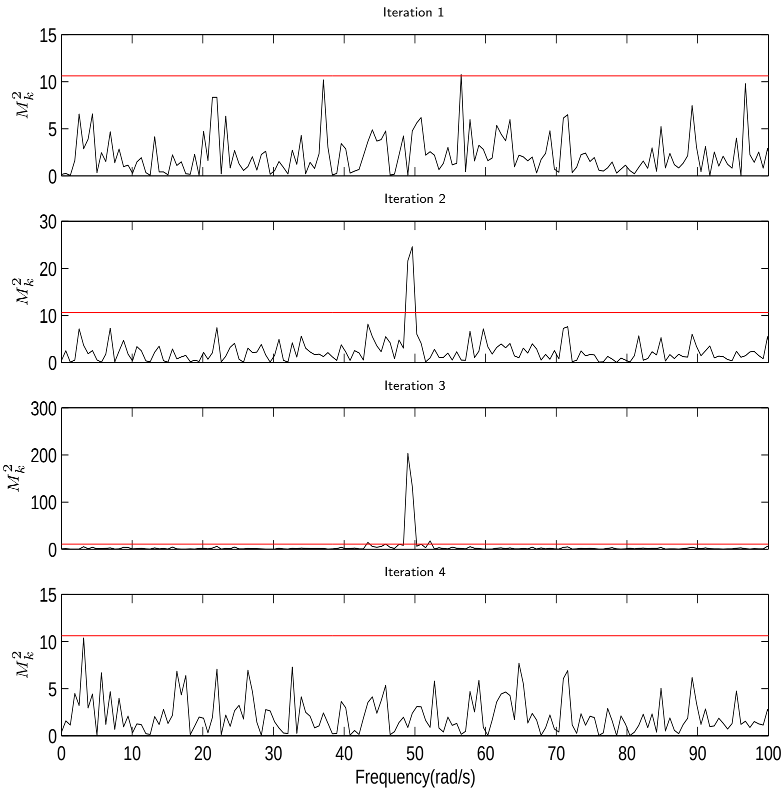

Figure 8.6: Normalized Magnitude Spectrum and Confidence Limits

The validation test invalidates the model for frequencies around 50 rad/sec (see plot 2 of figure 8.6). This is due to the non modelled resonance peak as can be seen in the bode diagram of figure 8.5.

Third Iteration

A new controller is designed after pushing even further the desired reference model:

$$\hat{G}_0 = \frac{20}{(1+7.4s)^2}, \qquad M_{03} = \frac{5^2}{(s+5)^2}$$

The invalidation of the model for frequencies around 50 rad/sec for this controller is evident (plot 3 of figure 8.6). The new controller yields an unstable closed loop.

Fourth Iteration

In (Albertos and Sala, 2002) a new model plant is identified due to the unacceptable closed loop behaviour for the controller designed with the reference model M_{03}. The new identified plant captures the first resonance peak of the plant.

$$\begin{aligned}\hat{G}_1 &= \hat{G}_0 \frac{0.01^2 + 50^2}{(s + 0.01 + 50i)(s + 0.01 - 50i)}, \\ M_{11} &= \frac{5^4}{(s+5)^4}\end{aligned}$$

The model validation result shows that now, the model is validated for all the frequency range covered by the input (plot 4 of figure 8.6).

Conclusions

In this example the proposed COFDMV algorithm has been presented to an already developed example of iterative identification and control example. Although the validation algorithm has not been used to modify any of the decisions taken on the example, it is clear from the validation plots of figure 8.6 that the algorithm detects the model frequency range that is not an accurate plant representation. At the first iteration, the pair controller-model is not invalidated. However for the second iteration it can be seen that the same model has problems in order to capture the plant behavior at around 50 rad/sec (i.e. the first resonant peak) due to the new more demanding controller designed. At this iteration no actions are taken although the model is clearly invalidated for the mentioned frequency range. On the third step the model is also invalidated around the frequency of 50 rad/sec but the normalized magnitude spectrum is one order of magnitude greater due to an even more demanding controller. Finally the model order is augmented with the resonant peak and this new model together with the last designed controller is not invalidated.

8.2.2 Example 2

This example deals also with the application of the COFDMV algorithm to iterative control. As a baseline we take the example of (Gunnarsson and Hjalmarsson, 1994). Now it is studied the performance of the COFDMV under distinct model order conditions. In order to accomplish the goal three independent iterative experiments are performed. First the whole iterative procedure is performed assuming a first order model of the plant. Secondly the same iterative procedure is repeated but now considering a second order model. Finally the iterative procedure is performed starting with a first order model but its order is increased when necessary. In the example it is shown not only the effect of the model undermodelling but the effect of a wrong model identification due to the use of feedback data (i.e. closed loop identification).

The plant to be controlled $G(s)$ is assumed to be third order:

$$G(s) = \frac{2}{(s+1)} \frac{229}{(s^2+30s+229)} \tag{8.2}$$

The control objective is to make the closed loop to behave as a desired reference model, thus the controller specification is set by the model reference bandwidth w_B. The controller design is performed through pole placement.

When a first order model of the plant is assumed, the reference model selected is:

$$G(s) = \frac{a}{(s+a)} \tag{8.3}$$

which imposes a bandwidth $w_B = a$. The designed controller is discrete and designed by pole placement, thus the closed loop discrete poles are placed at $z = e^{-T_s a}$. The sample time is $T_s = \; 0.04 sec$.

If a second order model is assumed then the reference model is given by:

$$G(s) = \frac{a^2}{(s+a)^2} \tag{8.4}$$

which has a bandwidth of $w_B = a\sqrt{\sqrt{2}-1}$. The rest of required poles are placed at $w_B = a$. As a result the discrete poles are placed at $z = e^{-T_s w_B}$.

The experimental layout is presented in figure 8.7. The reference r is white noise with variance $\sigma^2 = 1$. The reference r is filtered through a second order Butterworth filter with bandwidth equal to w_B (i.e. the closed loop bandwidth selected for the current iteration), which gives the signal r'. The disturbance v is assumed to be white noise with variance $\sigma^2 = 0.1$. The plant is G, the current identified model is $\hat{G}_i$ and the current controller is K_i. The simulation is performed in discrete time.

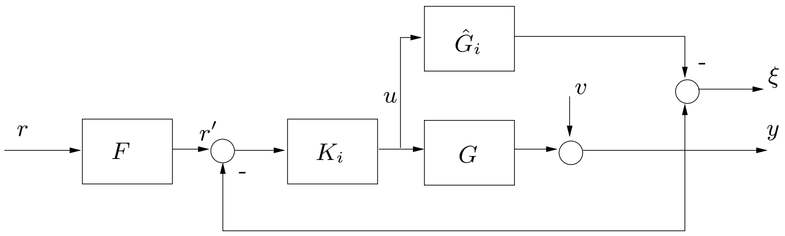

Figure 8.7: Experiment layout

The iterative procedure is as follows. At each iteration i, a new model $\hat{G}_i$ is identified using output error models. The new controller K_i is designed on the basis of the new identified model $\hat{G}_i$ and the chosen closed loop bandwidth is $w_{Bi} = w_{Bi-1} + 1$, that is the former closed loop bandwidth w_{Bi-1} increased in 1 rad/sec.

First order model

In the first experiment we assume a first order model of the plant. As a result the first order reference model is accordingly selected. The desired closed loop bandwidth starts with the value of $w_B = 1$ rad/sec and it is increased in 1 rad/sec at each new iteration. The validation results from the COFDMV are presented in figure 8.8.

The first pair model-controller, $\hat{G}_1(z)$-$K_1(z)$ is considered to be validated despite three frequency components cross the confidence limit. The rationale of this decision lies in two facts:

- The number of frequency components k is equal to 625. The false alarm (type I error) is equal to 0.5%. Thus it follows that from a realization of 625 samples following a χ^2_2 distribution it is normal that approximately 3 of the samples fall outside the limit.

- The invalidating spikes do not show a pattern or a frequency range in which the model is not accurate.

On the other hand, the second pair model-controller $\hat{G}_2(z)$-$K_2(z)$ clearly shows that a higher number of spikes, and of higher magnitude, are invalidating the model between 5 rad/sec and 10 rad/sec.

The third pair model-controller $\hat{G}_3(z)$-$K_3(z)$ shows the same behavior although the spikes magnitude doubles the former one. Moreover it appears a D.C. Fourier coefficient (i.e. a spike at frequency 0 rad/sec).

Finally the pair model-controller $\hat{G}_4(z)$-$K_4(z)$ is invalidated and the spikes size are increased one order of magnitude. Following with the same framework provides an unstable controller for higher bandwidths w_B.

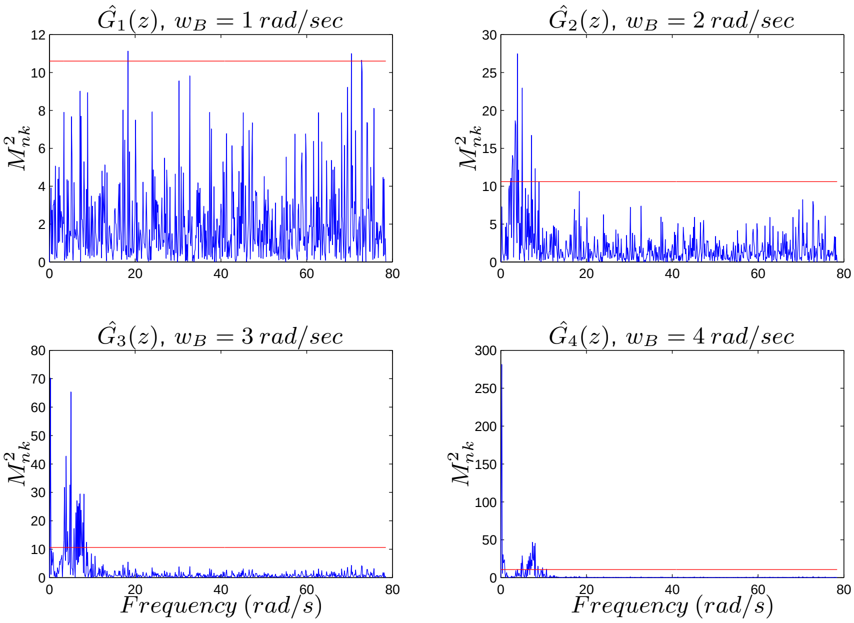

Figure 8.8: Validation first order model

Second order model

In the second experiment the assumed model order is second order. It is expected that the undermodelling problems are alleviated. Again, the desired closed loop bandwidth starts with the value of $w_B = 1$ rad/sec and it is increased in 1 rad/sec at each new iteration. The validation results from the COFDMV for this experiment are presented in figure 8.9.

The first pair model-controller, $\hat{G}_1(z)$-$K_1(z)$ is validated as the same discussion of the preceding section applies here.

Surprisingly, the second pair model-controller $\hat{G}_2(z)$-$K_2(z)$ not only is invalidated but also shows a worse invalidation picture that its first order counterpart (see figure 8.8). In fact the spikes are greater and the frequency range of invalidation affects not only low frequencies (i.e. 5 rad/sec) but medium frequencies (i.e. 18 rad/sec). The cause is the well known problem of model identification from closed loop data (Ljung, 1999).

The third pair model-controller $\hat{G}_3(z)$-$K_3(z)$ although shows invalidation signs around 5-10 rad/sec the spikes magnitude is not dramatic. The same result is obtained for the fourth pair model-controller $\hat{G}_4(z)$-$K_4(z)$ although the spikes magnitude and the invalidation frequency range is increased slightly. However in both cases the invalidation results for bandwidth $w_B = 3$ rad/sec and $w_B = 4$ rad/sec are much better than with a first order model.

Variable order model

Finally the same procedure is repeated starting with a first order model and increasing the order to a second order one when the first invalidation is achieved. In figure 8.10 the validation tests are shown. The first pair model-controller that results invalidated is the second one $\hat{G}_2(z)$-$K_2(z)$ (i.e. first order model plus desired bandwidth $w_B = 2\ rad/sec$). Then the third model to be identified is selected to be second order. As shown in 8.10 the third pair model-controller $\hat{G}_3(z)$-$K_3(z)$ can be considered validated. The third and fourth pairs model-controller behave similarly than the ones discussed in the preceding section where a second model order was chosen.

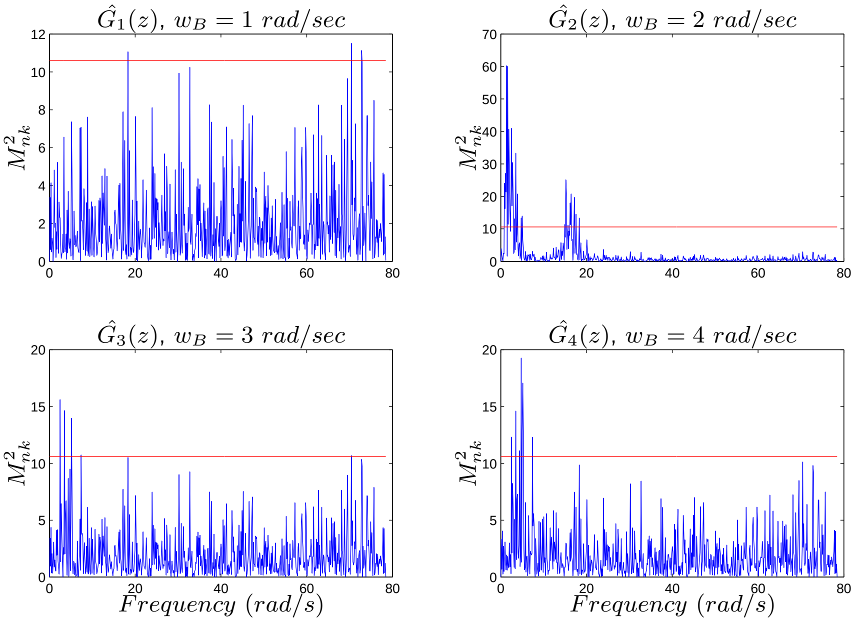

Figure 8.9: Validation second order model

Conclusions

In this example we have applied the COFDMV algorithm to an iterative identification and control schemes considering distinct undermodelling degrees. The COFDMV is a suitable tool to detect the undermodelling problems. Moreover it is possible to detect also “bad” models due to other problem sources such as lack of identification capacity due to closed loop data.

The example also points out the dependence of the necessary model order with the closed loop specifications, showing the fact that increasing the model order does not necessary implies and increase of the model accuracy. Then the algorithm helps to decide when to increase the model order.

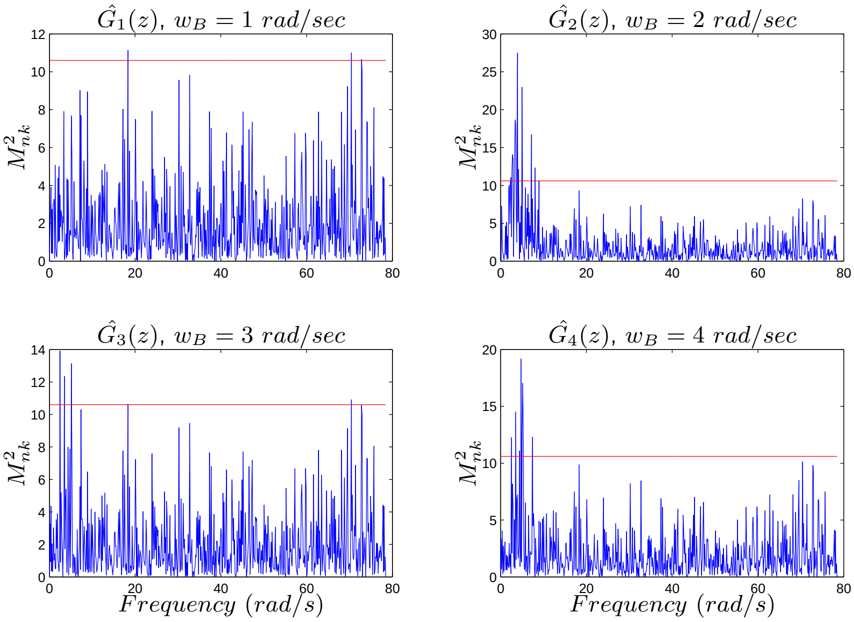

Figure 8.10: Validation variable order model. $\hat{G}_1(z)$ and $\hat{G}_2(z)$ first order. $\hat{G}_3(z)$ and $\hat{G}_4(z)$ second order

Part III

Epilogue

Chapter 9

Conclusions and Perspectives

9.1 Contributions and Conclusions

The thesis presents the following contributions:

- *The control problem has been formalized within the information theoretic framework.* The goal has been accomplished by:
  - Defining the control problem from a holistic point of view. That means that the problem has been defined taking in consideration all the possible constitutive elements and their relationships. As a result the elements have been dissected and their relationships related with existing control theory areas.
  - Reviewing the information approaches to control theory. It is concluded that although the information concept is a fundamental one on control theory, it is by no means a well defined concept.

  The formalization is based on very general assumptions. The advantage of this approach lies in the *generality of the results*. In fact the results are independent of the algorithm or the mathematical characterization of the problem (e.g. linear vs non-linear models). In order to derive more detailed relations, lower level mathematical assumptions are required. The proposed framework forms the conceptual basis and provides guidelines in which derive these schemes.

- On the basis of the information formalization of the control problem, *relations between the information content of the elements are established.* Thus theorems are proved that show necessary conditions for an information increase of certain element on the basis of the information variation of other elements. These conditions are fundamental ones affecting any algorithm and any mathematical formalization of the control problem.

- *The concept of adaptive control is reviewed and a new definition that amalgamates all the concepts is presented.* It serves to present the basis of adaptive control which is i) acquiring information, ii) modifying the system on the basis of the new information iii) the modification presents a beneficial result over the unchanged system. The definition is general and it does not rely on any technicalities.

- Two formalizations of *adaptive control, classical adaptive control and iterative control are studied under the proposed information theoretic framework.* It serves to compare their similarities and differences. Thus the information theoretic framework proposed is useful for comparing distinct adaptive schemes as it provides a baseline for comparison. It follows from the results that iterative control manages the information to be used in the control design procedure in a much richer way. In fact more questions and validation tests are performed before new information is acquired and used to modify the system. On the contrary classical adaptive control manages information in a more restricted way as no questions about the quality or necessity of information are arisen. However it is seen that *neither classical adaptive control nor iterative control manage the information in a monotonic way.* That means that although both schemes are able to incorporate new information to the problem, former information is discarded, so at each time step there is no guarantee that the information amount increases.

- *A new model validation algorithm COFDMV is developed.* The novelty of the algorithm is that it goes further than just a "validated/invalidated" result as the model validation is *frequency dependent*, thus a model is neither validated nor invalidated but the model frequency ranges validity. Moreover the algorithm helps on the management of information in the control problem by i) designing the experimental energy content in order to obtain a good model where the current one is invalidated, ii) helping in the selection of the model order, iii) deciding the expected controller bandwidth that a model can tolerate.

The algorithm, due to its control oriented nature, results to be suited for iterative control schemes in which i) during the whole process several models are identified and ii) the models always suffer from undermodelling.

9.2 Open Research Areas

- The study of the control design problem by means of information concepts it is by no means closed. The information approach can give answer to really important questions for control theoreticians and control practitioners. In order to answer questions such as i) when it is necessary to perform an new identification, ii) what are the benefits expected for the new identification, etc., it is necessary to apply the presented framework together with more lower level assumptions. Thus the general abstractly presented concepts can be posed in a more detailed way assuming more mathematical formalizations and assumptions. *It is expected that posing the presented abstract framework in a more specific basis will provide more detailed answers regarding the information flow management.*

- One of the results of the thesis has been to notice the lack of information monotonicity of adaptive control schemes in general. *It is interesting the problem of designing an adaptive control scheme which is monotonic.* The implications of monotonicity of information over the performance of the scheme should be addressed in order to ascertain the necessity of information monotonicity.

  In this thesis a methodology to obtain a monotonic information iterative identification and control procedure has been envisaged. The idea is to form the controller joining in parallel distinct controllers which have been designed from models which are good for distinct frequency ranges. In order to minimize the interaction among controllers, these are augmented with appropriate band-pass filters. The idea is that each particular controller feeds back only those frequencies for which the model that was used for its design is good. The effect of the filters on the overall controller is currently under investigation.

- The Control Oriented Frequency Dependent Control Algorithm is suited only for stable models due to the structure used to generate the residual. The extension of the COFDMV for unstable plant models would provide a greater applicability. Currently two possible ways of accomplishing this point are under investigation. The first approach tries to whiten a residual which has been generated by a white noise filtered by a system. The idea is to be able to transform the residual in order to be tractable by the current COFDMV algorithm. The second approach is more involved and tries to developed a new frequency dependent model validation algorithm by translating to the frequency domain a cross-correlation time domain model validation test. In this case the whiteness of the residual is no longer required.

9.3 Publications

The main articles on international conferences the thesis has generated are:

- *P. Balaguer, R. Vilanova and R. Moreno.* ***"The Control Problem: A Framework for Holistic Design".*** *14th IEEE Mediterranean Conference on Control and Automation, 2006.*

- *P. Balaguer and R. Vilanova.* ***"Is Iterative Control Wasting Information?".*** *6th IEEE International Conference on Control and Automation, 2007.*

- *P. Balaguer and R. Vilanova.* ***"Frequency Dependent Approach to Model Validation".*** *6th Asian Control Conference, 2006.*

- *P. Balaguer, R. Vilanova and R. Moreno.* ***"Control Oriented Frequency Dependent Model Validation".*** *International Control Conference UK, 2006.*

- *P. Balaguer and R. Vilanova.* ***"Quality Assessment of Models for Iterative/Adaptive Control".*** *45th IEEE Conference on Decision and Control, 2006.*

The thesis also has generated the following journal papers:

- *P. Balaguer and R. Vilanova.* ***"Information Characterization of the Control Problem. Part I: The Framework".*** *International Journal of General Systems, 2007.* (submitted)

- *P. Balaguer and R. Vilanova.* ***"Information Characterization of the Control Problem. Part II: Analysis of Adaptive Control Schemes".*** *International Journal of General Systems, 2007.* (submitted)

- *P. Balaguer and R. Vilanova.* ***"Model Validation on Adaptive Control: A Frequency Dependent Approach".*** *International Journal of Control, 2007.* (submitted)

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Balaguer, P. and R. Vilanova (2006*b*). Model validation on iterative identification and control schemes: A frequency domain approach. In: *7th Portuguese Conference on Automatic Control.*

Balaguer, P. and R. Vilanova (2006*c*). Monitoring plant variations with control charts. In: *The IEEE International Conference on Methods and Models in Automation and Robotics.*

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