Inventory Control in Supply Chains: An Internal Model Control Approach

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Ph.D. Thesis

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CERTIFY:

That the thesis entitled **Inventory control in supply chains: An internal model control approach** by Carlos Andrés García Salcedo, presented in partial fulfillment of the requirements for the degree of Doctor Engineer, has been developed and written under their supervision.

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Contents

Chapter 1

Introduction

The control theory has contributed to the improvement of numerous industrial processes. Its main purpose is the design of control schemes to keep the manipulated variables close to the set-points in spite of disturbances and changes in the plant. Thus, the control mechanism is essential for improving product quality and energy efficiency, which yields better (sustainable) economy.

Although the control theory has been essentially applied to chemical and industrial process, it may also be applied to other disciplines where the control of any variable is necessary. In this way, the management of inventory and information flow in supply chain systems by using control theory approaches is one of several examples and a current object of research in the last decades. A supply chain is a serial system where each element is coupled with the next one by orders and delivery signals. Moreover, the supply chain contains random delays (Lead times) and Integrators (Inventory levels) which make the inventory control a difficult task.

The aim of this introduction is to guide the reader through the contributions made during this thesis focused on improving the inventory control by using the control theory.

The main objective of this research is to propose a control scheme allowing to solve the principal problems appearing in the inventory control of the supply chain (uncertainty in the lead time, the bullwhip effect and the inventory drift). This objective is divided in two particular objectives:

1. On the one hand, the counteraction of effects caused by the existing

delay between the time at which an order is placed on the immediate supplier and the moment at which the petition is satisfied.

2. On the other hand, the design of inventory policies allowing a trade-off between the disturbance (demand) rejection and the inventory target tracking.

Since the nature of the internal model control scheme (IMC) allows to compensate the delay effects, the controllers included in the scheme can be designed for inventory target tracking and disturbance rejection disregarding the delay effects. Thus, the IMC scheme allows to tackle the objectives (1) and (2) under the same paradigm of control but in a decoupled way which is an advantage with respect to other control approaches: PID, space states, Smith predictor... Therefore, the IMC scheme is presented in this thesis as a novel inventory replenishment policy for the entire supply chain. Nevertheless, the IMC requires the perfect knowledge of the lead time to compensate its effects. This situation is not viable when the delay changes during the process which is a common situation in supply chains. Hence, first of all, the research is focused on the identification of the lead time.

As a result of this part of the research, the paper titled **Inventory control for the supply chain: An adaptive control approach based on the identification of the Lead-time** has been published in **Omega, The International Journal of Management Science, 2012**. This paper contains the formulation of an identification algorithm that allows to estimate the actual delay values of the entire supply chain at each time interval . Then, the estimated delay values are used to adapt the delay block in the control scheme. Moreover, this paper includes the theoretical proofs of the convergence to the actual delays and the stability of the system. The paper ends with the simulation results of the delay identification and inventory control of the entire supply chain. The simulation shows that the scheme identifies both constant and variable delays at the time. This publication represents the finalization of the particular objective (1).

Since the lead time uncertainty can be solved by the previously proposed identification scheme, the objective (2) relies on the design of the IMC scheme controllers. The aim of the IMC controllers is to keep the inventory levels of the supply chain close to the inventory target mitigating the aggresivity in the orders (bullwhip effect). Each echelon of the supply chain may operate under several working modes during the time:

- Infinite Supply and High stock, (ISHS)
- Infinite Supply and Low Stock, (ISLS)
- Limited Supply, (LS)

When the lead time of manufacturing is too long and there exist uncertainty in the demand level, many companies choose to use a safety stock strategy to assure the customer demand satisfaction. Under this strategy the supply chain is always in the (ISHS) working mode. When the stock is insufficient, the supply chain may work under any of (ISHS), (ISLS) or (LS) working modes. Therefore, we start proposing the design of the control scheme for supply chain under safety stock strategy (ISHS). After that, the control scheme is designed for a supply chain working without safety stock (i.e. under any working mode (ISHS), (ISLS) or (LS)). As a result of these control scheme designs two papers have been developed:

- The paper titled **Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches** has been published in the **European Journal of Operational Research, 2013**. This paper addresses the design of an inventory management policy for supply chains working under safety stock strategy (ISHS). The design is based on the IMC guidelines where a trade-off between the bullwhip effect avoidance and inventory tracking is carried out, which is the second objective of the research. Moreover, this paper includes the simulation results of the inventory control under two control strategies: centralized and decentralized control.
- The paper titled **A switched control strategy for inventory control of the supply chain** is an extension of the previous research and is submitted to a journal for its consideration. In this work a more complex model of the system which adds back-orders accumulation and switching characteristics between the different working modes (ISHS), (ISLS) or (LS) is taken into account. Therefore, we propose a switched inventory control system for a serial multivariable supply chain under decentralized control strategy (independent switched control to each echelon). This consists in a bank of controllers designed for each possible operation case of each echelon and a switching logic that selects the best controller

at each instant time. The two articles represent the finalization of the objective (2).

Since these papers contain the main contributions of this research, the thesis is submitted as a compendium of papers. The thesis has the following structure:

- Chapter 2 presents an introduction to the inventory control in supply chain. Thus, a description of the supply chain, the mathematical model, the definition of the inventory replenishment policies and the undesirable effects that may appear when an inventory replenishment policy is implemented in the supply chain are described in this chapter. Moreover, a summary of the contributions of this thesis is also included. Finally, the chapter ends with the conclusions and future research.

- Chapter 3, Section 3.1 presents the content of the paper **Inventory control for the supply chain: An adaptive control approach based on the identification of the Lead-time** where an adaptive control approach in the entire supply chain based on the identification of the Lead-time to solve its uncertainty is proposed.

- Chapter 3, Section 3.2 presents the content of the paper **Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches** where the design of an internal model control scheme in decentralized and centralized control strategies is developed in the entire supply chain. This design is based on a model of the supply chain where each echelon has infinite stock.

- Appendix A.1 presents the content of the paper **A switched control strategy for inventory control of the supply chain** where the design of the internal model control structure for a model of the supply chain where stock limitations and back orders accumulation are considered, providing a better description of the actual supply chain dynamics. Due to the stock limitations of the distributors, the supply chain turns out to be a switched system since its behaviour changes according the stock available of the distributors. This fact motivates us to propose a switched control system for a serial supply chain under a decentralized control strategy.

- Finally, references end the thesis.

Chapter 2

Inventory control problem

The management of supply chain systems has acquired great importance in the last decades in order to obtain competitive advantage for firms in the industrial sector that participate in the global markets. A common Supply Chain (SC) includes the necessary entities to provide goods to the customer from production centers. Thus, the main elements composing a general supply chain between the factory (F) and customer (C) are: warehouse (W), distributing center (D) and retailer (R) (Dejonckheere *et al.*, 2003),(Dejonckheere *et al.*, 2004), as is shown in Fig.2.1.

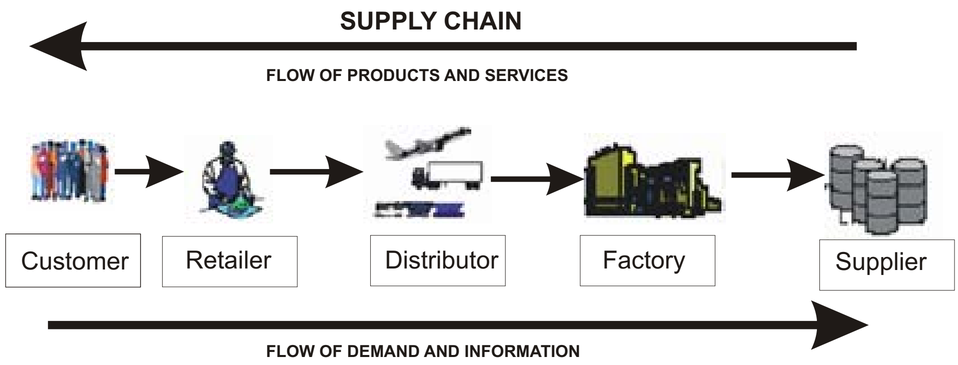

Figure 2.1: The main entities of a supply chain

There are many participants and processes as well as randomness in the information flow of a supply chain. Therefore, the coordination of the supply chain operation becomes a key point to optimize the use of its resources and compete on a global scale. There are many aspects to research in this complex network. One of these is the improvement of inventory management policies. The main objective of inventory management is to keep the inventory level of each element of the supply chain stable enough so as to satisfy the requirements of the customers by ordering products from its immediate supplier of the supply chain (Lin *et al.*, 2004). In this way, several works (Dejonckheere *et al.*, 2003), (Dejonckheere *et al.*, 2004), (Amini and Li, 2011; Aggelogiannaki and Sarimveis, 2007) have proposed control theory tools (transfer functions, frequency response...) to develop novel inventory control policies. In order to perform an inventory control policy, a model for a general supply chain must be developed. Therefore, a representative model of the main dynamics of the inventory level and information flow of the supply chain is shown in section 2.1.

2.1 Dynamic modelling of inventory of the supply chain

In this section the mathematical model of the main dynamics of the inventory level and information flow in an echelon is stated. In this model, enough stock is assumed in each echelon of the supply chain (ISHS working case) which is a common situation in several supply chains (Dejonckheere *et al.*, 2003), (Dejonckheere *et al.*, 2004). The model is amenable to develop inventory replenishment policies in a decentralized way. The model for the entire supply chain which is appropriate to develop inventory replenishment policies in a centralized way is presented in Chapter 3, Section 3.2. Moreover, a model of the inventory level and information flow where a possible insufficiency in the stock is taken into account is stated in Appendix A.1, which can be more representative for supply chains that works under low stocks. The model of an echelon is described below.

For the sake of simplicity, it is assumed a period base of time $T_m = 1$ which can be one day, one week or one month according to the dynamics of the supply chain. In this model there are N logistic echelons between the factory and the customer. The customer is considered the base while the factory is on the top of the supply chain. Thus, $j = 1, 2...N$ (where N is a finite integer) denotes each one of the intermediate logistic echelons of the supply chain, while $j = 1$ represents the retailer, $j = N + 1$ represents the factory. According to this notation, $(j + 1)$ represents an immediate supplier and $(j - 1)$ represents an immediate customer of the j^{th} echelon. A summarized list of variables is shown below:

- $\beta_{a,b}(t)$ denotes the amount of goods delivered by each logistic node a to the node b.
- $y_j(t)$ is the inventory level of the j^{th} echelon at any discrete time instant $t = nT_m$ where n is a natural number.
- $o_{j,j+1}(t)$ represents the order placed by the j^{th} echelon to its immediate supplier $j + 1$.
- $d_j(t)$ is the demand perceived by the j^{th} echelon from external customers.

Thus, the inventory balance in each echelon is given by the difference between the goods received from the immediate supplier and the goods delivered to the immediate customer as follows:

$$y_j(t) = y_j(t-1) + \beta_{j+1,j}(t) - \beta_{j,j-1}(t), \quad j = 1, 2, ..., N \tag{2.1}$$

where $j-1 = 0$ represents the final customer. A lead time $L_j \in \mathbb{N}$ is considered between the time when an order is placed by node j^{th} and the time when the goods are received from the immediate supplier (Amini and Li, 2011; Aggelogiannaki and Sarimveis, 2007; Dejonckheere *et al.*, 2003). It is also assumed that each node has enough existences to satisfy the demand of its immediate customer. In this way, the amount of goods ordered to an immediate supplier at time t will arrive at time $t + L_j$ i.e. $\beta_{j+1,j}(t) = o_{j,j+1}(t - L_j)$. Therefore, the sequence of events in the supply chain is the following:

1. At each discrete time t, the echelon j^{th} receives the goods ordered L_j periods ago.
2. The demand $d_j(t)$ is observed and satisfied immediately i.e $\beta_{j,j-1}(t) = d_j(t)$ (i.e there is no backlogged orders).
3. The new inventory level of each echelon $y_j(t)$, is observed.

4. Finally, an order $o_{j,j+1}(t)$ is placed on the $(j+1)^{th}$ level (upstream) according to the values of the inventory levels, $y_j(t)$.

Thus, the Eq. (2.1) relating the inventory balance with the demand $d_j(t)$ and order $o_{j,j+1}(t)$ at node j becomes under the previous series of events:

$$y_j(t) = y_j(t-1) + o_{j,j+1}(t - L_j) - d_j(t), \quad j = 1, 2, ..., N \tag{2.2}$$

Eq. (2.2) is a difference equation which can be solved directly in the time domain or by using transformation techniques. In particular, the z-transform is the most extended transformation because it transforms Eq. (2.2) into an algebraic equation. Then, applying the time shifting property of the z-transform, $Z\{x[t-k]\} = z^{-k}Z\{x[t]\} = z^{-k}X(z)$ (Ogata, 1996) to Eq. (2.2), where k is a finite integer, we obtain:

$$y_j(z) = y_j(z)z^{-1} + o_{j,j+1}(z)z^{-L_j} - d_j(z), \quad j = 1, 2, ..., N \tag{2.3}$$

Now, isolating $y_j(z)$ from (2.3) we get:

$$y_j(z) = \overbrace{\left[\frac{1}{1-z^{-1}}\right] z^{-L_j}}^{p_j(z)} o_{j,j+1}(z) - \overbrace{\left[\frac{1}{1-z^{-1}}\right]}^{p^m(z)} d_j(z) \quad j = 1, 2, ..., N \tag{2.4}$$

which relates the z-transform of the inventory level, $y_j(z)$, with the order and the demand only. For IMC design, $p_j(z)$ must be factored into a minimum-phase portion:

$$p^m(z) = \frac{1}{1-z^{-1}} \tag{2.5}$$

and a portion $p_j^a(z)$ that includes the delays of the system (Morari and Zafiriou, 1989):

$$p_j^a(z) = z^{-L_j} \tag{2.6}$$

The model for an echelon presented in Eq. (2.4) and shown in Fig. 2.2 is amenable to implement some inventory replenishment policies. In this way (Dejonckheere *et al.*, 2003) evaluates the bullwhip effect for the order-up-to policies with exponential smoothing forecast. (Disney and Towill, 2003) presents a trade-off between the inventory target tracking and the bullwhip effect under the order-up-to policies with exponential smoothing forecast.

(Dejonckheere *et al.*, 2004) studies the effects of the information enrichment on the bullwhip effect in supply chains. (Rivera and Pew, 2005) evaluates the PID controllers as inventory replenishment policy. (Schwartz *et al.*, 2006) and (Schwartz *et al.*, 2006) introduce the Internal Model Control and Predictive Control as inventory replenishment policies for a single echelon of the supply chain. Therefore, this model and its generalizations stated in Chapter 3, Section 3.2 and A.1 have been used in this work to design the inventory replenishment policies. The inventory replenishment policies are described

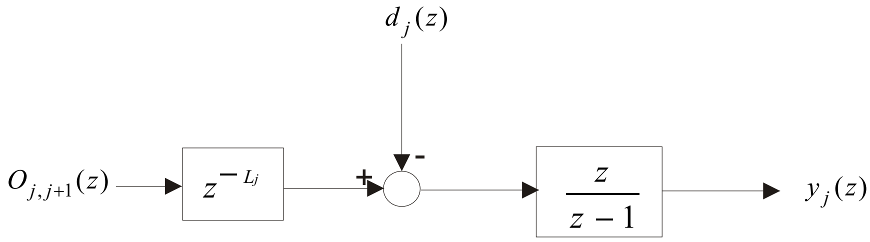

Figure 2.2: Block diagram of a single echelon according to Eq. 2.4.

below.

2.2 Inventory replenishment policies

An inventory replenishment policy is the strategy of each echelon to give orders to the immediate supplier in order to have enough existences to supply its customers. Many undesirable effects may appear when an inventory replenishment policy is implemented in the supply chain described above:

- Instability: The instability is the main problem since signals describing the inventory and orders may diverge as time goes on. (Hoberg *et al.*, 2007) applies linear control theory to study the effect of several inventory policies on order and inventory variability (using z-transform techniques) and their conditions for stability are examined by the Jury criteria. The work shows that an inadequate tuning of the inventory policy can produce instability in the supply chain. (Saada and Kadirkamanathan, 2006) also evaluates the effectiveness of various decision policies including he PID controller to provide stability to the supply chain.

- Bullwhip effect: Another inconvenience is that the variability in the ordering patterns often increases as we move upwards in the chain, from the customer to the factory. This phenomenon is broadly known as the *bullwhip effect* and is represented in Fig. 2.3. Some current studies, (Dejonckheere *et al.*, 2003),(Dejonckheere *et al.*, 2004),(Disney and Towill, 2003),(Hoberg *et al.*, 2007), have analyzed the effect of the replenishment policies focused on the bullwhip effect estimation and suppression. Moreover, (Lin *et al.*, 2004) presents Control Engineering based approaches, including proportional-integral (PI) controllers and cascade control as inventory replenishment policies, being the design of this controller also focused on the mitigation of the bullwhip effect. (Balan *et al.*, 2009) applies fuzzy logic theory control on inventory error and error changes associated with forecast demand among the nodes of a supply chain in order to allow smooth information flow in the chain.

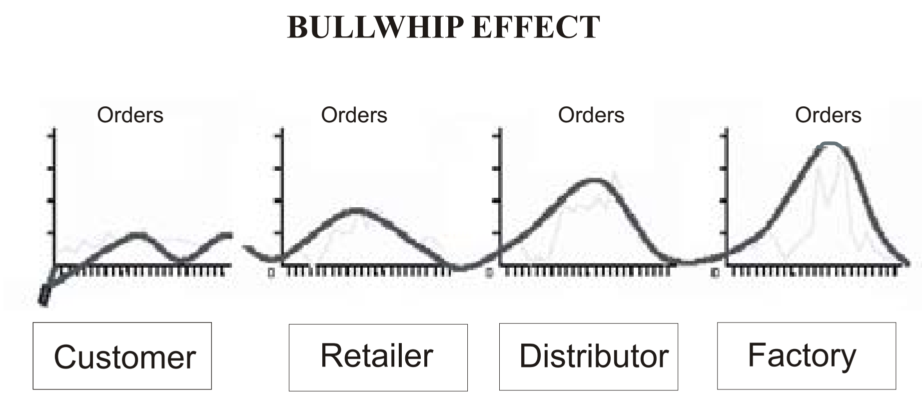

Figure 2.3: Bullwhip effect

- Inventory drift: Besides stability issues and bullwhip, the response of the net stock signal is an additional important subject of investigation. A major problem is inventory deficit existence (i.e. the difference between inventory target and the actual inventory level), usually referred to as inventory drift. (Tang, 2011) focuses on base-stock inventory models with and without expected demand and provides a computationally efficient method to set optimal inventory targets for finished products under capacitated postponement. (Silver and Bischak, 2011) considers a periodic review order-up-to-level (or basestock) inventory control system under normal distributed demand. (Aggelogiannaki and Sarimveis, 2007) presents a lead-time identification method to solve its uncertainty which is a cause of inventory driff.

One of the main causes of all these phenomena is attributed to the lead time, specially when it is not properly known. Therefore, counteracting the lead time effect is crucial so as to improve the supply chain management. There are two approaches towards the control of systems with external delays. The first one is the robust control approach that consists in designing a controller based on a nominal value for the delay and considering a degree of robustness of the controller to any mismatch in it. In this way, (Schwartz *et al.*, 2006; Schwartz and Rivera, 2010) introduce the Internal Model Control, as a robust control approach, and predictive control as novel decision replenishment policies.

A second control approach that allows us to tackle systems with delay without considering any approximation is the use of delay compensation schemes. The Smith Predictor (SP) and its generalizations such as those based on the Internal Model Control (IMC) are the most extended configurations.

We advocate on a control approach based on an IMC delay compensation scheme to tackle the inventory control problem for the multivariable supply chain. One of the main advantages of the use of an IMC-type structure is that, on the one hand, its structure can be formulated to compensate the lead time effects and, on the other hand, this scheme has different controllers that allow to tackle the nominal stability, the relation (Inventory level vs Inventory target) and the relation (Inventory level vs Demand) separately. Therefore, this control approach allows to tackle the inventory control problems (1),(2) formulated in Chapter 1 under the same frame but in a decoupled way. The structure of this control scheme to inventory management is presented in Section 2.3. The structure of the IMC control scheme under the decentralized and centralized control strategy is presented in Chapter 3, Section 3.2.

2.3 Internal model control as inventory replenishment policy

The controller structure proposed by (Tan *et al.*, 2003) is shown in Fig. 2.4 where $p^m(z)$ is the rational part of the model defined in Eq.(2.5). This part is related to the process of integration of material carried out in each echelon. Therefore, no uncertainty is considered in this part of the model. Also, z^{-L_j} denotes the transfer function of the real lead time. However, there would be an imperfect knowledge of the delay which implies that an estimated delay $\hat{L}_j$, rather than the actual delay is used for control purposes in the transfer function of the modeled lead time, $z^{-\hat{L}_j}$.

The proposed structure has three controllers for each echelon: $q_j^s(z)$, $q_j^t(z)$, and $q_j^{ld}(z)$, each having a distinctive use and influence on the overall closed-loop response:

- $q_j^s(z)$ is used to stabilize $p(z)$, the original (unstable) plant, ignoring the time-delay.
- $q_j^t(z)$ is an IMC controller for the stabilized model.
- $q_j^{ld}(z)$ is designed mainly to achieve the internal stability and load disturbance rejection (rejection to the orders placed from the immediate customer $O_{j-1,j}(z)$ in this case). Note that the orders $O_{j-1,j}(z)$ are considered as a perturbation since a decentralized control strategy is used instead of a centralized control where all orders vector $\mathbf{O}$(z) will be designed simultaneously (Chapter 3 Section 3.2).

In the IMC scheme, when the model of lead time is exact ($z^{-L_j} = z^{-\hat{L}_j}$) and there is no disturbance signal (in the supply chain case, when there is no demand signal $O_{j-1,j}(z) = 0$) , then the outputs $I_j(z)$ and $\hat{I}_j(z)$ are the same and the feedback signal $h_j(z)$ shown in Fig. 2.6 is zero. Thus, in this control system the lead time becomes external when there is no disturbance and no plant/model delay mismatch: the scheme compensates the delay. In order to point out this property, the equation of inventory balance for a single echelon j under this scheme is obtained and represented by Eq.(2.7):

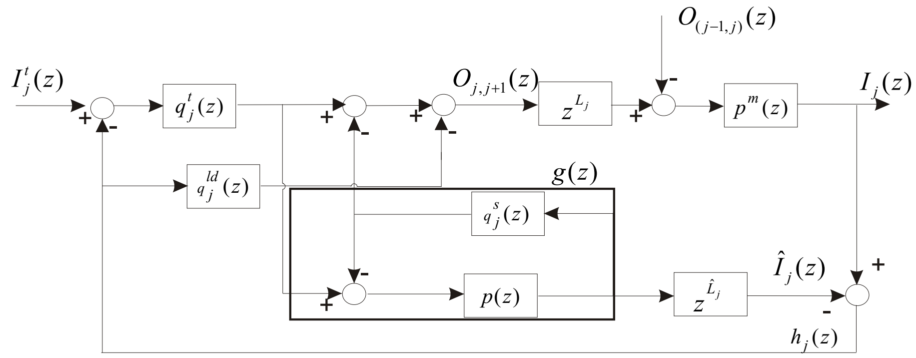

Figure 2.4: IMC control scheme

$$
\begin{aligned}
I_j(z) \quad &= \left\{ \frac{q_j^t(z)\left(\frac{p(z)}{1+q_j^s(z)p_j(z)}\right)z^{-L_j}(1+q_j^{ld}(z)p(z)z^{-\hat{L}_j})}{q_j^t(z)\left(\frac{p(z)}{1+q_j^s(z)p_j(z)}\right)(z^{-L_j}-z^{-\hat{L}_j})+(1+q_j^{ld}(z)p(z)z^{-\hat{L}_j})} \right\} I_j^t(z) \\
&- \left\{ \frac{\left[1-q_j^t(z)\left(\frac{p(z)}{1+q_j^s(z)p_j(z)}\right)z^{-\hat{L}_j}\right]p(z)}{1+q_j^{ld}(z)p(z)z^{-L_j}+q_j^t(z)\left(\frac{p(z)}{1+q_j^s(z)p_j(z)}\right)\left(z^{-L_j}-z^{-\hat{L}_j}\right)} \right\} O_{j-1,j}(z)
\end{aligned}
\qquad (2.7)
$$

Now, let us obtain the transfer function of the internal closed loop shown into the square in Fig. 2.6 and appearing in as a block in Eq. (2.7):

$$
g(z) = \frac{p(z)}{1 + q_j^s(z)p_j(z)} \qquad (2.8)
$$

From Eq.(2.8) it follows that q_j^s should be selected to stabilize $p(z)$. Then, replacing this expression in the equation (2.7):

$$
\begin{aligned}
I_j(z) &= \left\{ \frac{q_j^t(z)g(z)z^{-L_j}[1+q_j^{ld}(z)P(z)z^{-\hat{L}_j}]}{\underbrace{q_j^t(z)g(z)[z^{-L_j} - z^{-\hat{L}_j}]}_{x} + [1+q_j^{ld}(z)P(z)z^{-\hat{L}_j}]} \right\} I_j^t(z) \\
&- \left\{ \frac{[1-q_j^t(z)g(z)z^{-\hat{L}_j}]p(z)}{1+q_j^{ld}(z)p(z)z^{-L_j} + \underbrace{q_j^t(z)g(z)[z^{-L_j} - z^{-\hat{L}_j}]}_{y}} \right\} O_{j-1,j}(z)
\end{aligned} \tag{2.9}
$$

It can be seen in Eq.(2.9), that, if the lead time model is known $z^{-L_j} = z^{-\hat{L}_j}$, then the expressions x and y are zero. Thus, we get:

$$
I_j(z) = T_j(z)I_j^t(z) - S_j(z)O_{j-1,j}(z) \tag{2.10}
$$

where

$$
T_j(z) = q_j^t(z)g(z)z^{-L_j} \tag{2.11}
$$

and

$$
S_j(z) = [1 - q_j^t(z)g(z)z^{-\hat{L}_j}]\frac{p(z)}{1 + q_j^{ld}(z)p(z)z^{-L_j}} \tag{2.12}
$$

From (2.11) it follows that the lead time has disappeared from the denominator of the relation from target $I_j^t(z)$ to $I_j(z)$. Therefore, if there is no disturbance signal (The demand of the immediate customer $O_{j-1,j}(z) = 0$ in the supply chain case) the lead time has been decoupled of the rational part from the system. Thus, the tuning effort of the controller $q_j^t(z)$ can be reduced considerably because it depends only of the rational part of the system. Besides, it can be seen that $q_j^{ld}(z)$ only appears in the relation from $O_{j-1,j}(z)$ to $I_j(z)$. Thus, the problem of disturbance rejection can be tackled separately by tuning $q_j^{ld}(z)$.

However, delay compensation schemes have a drawback: the system's delay has to be known beforehand to perform its perfect compensation. This situation is not feasible when the delay is not known beforehand or the delay changes during the process which is a common situation in supply chains. An alternative to overcome this problem is to include a lead time identification method in the supply chain operation.

The inclusion of online identification methods to estimate the actual lead times in the supply chain is a novel topic of research. Therefore, it is difficult to find works that deal with lead time identification in entire supply chains since most of works normally deal with a single echelon. For instance, in ((Aggelogiannaki and Sarimveis, 2007)) a recursive prediction error method (RPEM) is proposed to identify the lead time online in a unique echelon (SISO system), based on historical data that includes order rate and received final products. Then one parameter of an Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS) is adjusted according the identified lead time.

Therefore, a delay identification method is proposed for the complete supply chain being able to identify the delays among the different echelons describing the supply chain. The identified values of the delays are then used to adjust the delay compensation in a IMC based decentralized compensation scheme. The approach is inspired in what are called Pattern Search Algorithms (Sriver *et al.*, 2009.; Herrera *et al.*, 2011), whose application in control is really novel. Besides the formulation of the complete control scheme, in the present work, theoretical proofs to guarantee that the algorithm identifies the actual lead times of the supply chain and closed-loop stability are provided, which is not common in works aimed at inventory control in the supply chain (Aggelogiannaki and Sarimveis, 2007). Indeed, these proofs are conceptually easier than the presented previous works (Bernstein and Rad., 2007), (Mirkin and Gutman., 2009), (S. S. Ge, 2004), (Zhang and Ge., 2010) and (Chen *et al.*, 2009) aimed at delay identification methods. Simulation results show that the algorithm identifies the lead time either constant or variable in the time. The identification scheme is described below.

2.4 Estimation of the Lead-time

The proposed identification scheme shown in Fig. 2.5 is based in a multi-model scheme. It is made up of a battery of different models operating in parallel (Ibeas *et al.*, 2008*a*; Ibeas *et al.*, 2008*b*; Herrera *et al.*, 2011). Each model includes the same rational component $p^m(z)$ but a different delay value $p_L^l(z)$.

A supervisory algorithm compares the mismatch between the actual system and each candidate models and determines, at each time interval, the one that best describes the behaviour of the real system, providing an estimation of the lead time. An additional block selects the best model for control purposes.

The formulation of the inventory control scheme, identification method, theoretical results and simulation results are condensed in the article titled **Inventory control for the supply chain: An adaptive control approach based on the identification of the Lead-time** and is presented in Chapter 3, Section 3.1. Therefore, the first objective of the research is carried out.

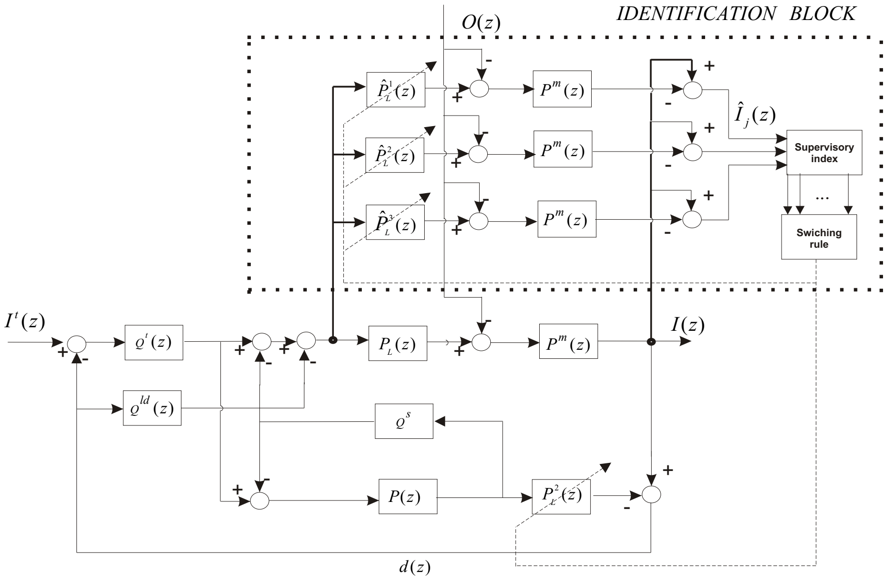

Figure 2.5: The block diagram of identification and control for the complete supply chain

Once the lead time uncertainty is overcome, the next objective is the design of an inventory control strategy to perform a trade-off between the optimal performance to inventory target tracking and the aggressivity in the orders (bullwhip effect) avoidance in the entire supply chain. Since the lead times can be identified by the delay identification method, the knowledge of the lead time is assumed in the controllers design.

There are two ways to perform an IMC based inventory control strategy for the entire supply chain: by using decentralized control, where an independent controller is applied to each echelon of the supply chain and by using centralized control, where a single controller is applied to the entire supply chain. The decentralized control approach is suitable for supply chains whose elements belong to different companies and do not share each others' information. On the other hand, when all or most of the supply chain elements belong to the same company or share internal information the centralized control approach would be applied. Therefore, in this research the internal model control has been designed under the two strategies in order to evaluate its performance to inventory control and bullwhip effect mitigation. A summary of the layout and design are presented below.

2.5 Decentralized and centralized IMC control approaches

In this section the two-degrees-of-freedom feedback IMC design for a complete supply chain applying both decentralized and centralized control strategies is introduced. Analytical guidelines to tune the controllers for bullwhip effect avoidance in the entire supply chain under centralized and decentralized inventory control strategies are also provided, which are not considered in previous works (Schwartz *et al.*, 2006; Schwartz and Rivera, 2010). The Two-degrees-of-

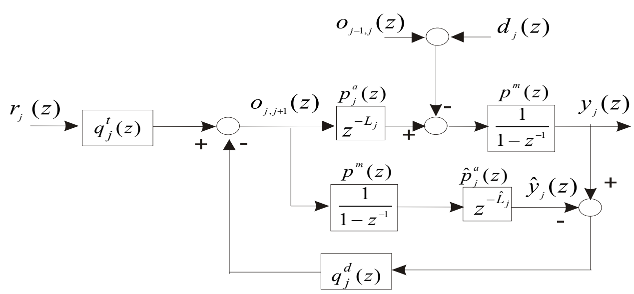

Figure 2.6: Two-degrees-of-freedom-feedback IMC scheme

freedom feedback IMC is shown in Fig. 2.6, where $r_j(z)$ denotes the inventory target for the control system of each echelon, $q_j^t(z)$ and $q_j^d(z)$ represent the two feedback controllers of the scheme, $p_j(z) = p^m(z)p_j^a(z)$ is the actual dynamics of the supply chain and $\hat{p}_j(z) = p^m(z)\hat{p}_j^a(z)$ represents the nominal model of the system. Each echelon may perceive demand from an external customer of the supply chain $d_j(z)$ and orders from the downstream echelon of the supply chain $o_{j-1,j}(z)$ as is shown in Fig. 2.6. Since in the decentralized control approach each echelon has no control on the downstream orders, $o_{j-1,j}(z)$ are added to $d_j(z)$ in a single disturbance input $v_j^m(z) = (o_{_{j-1,j}}(z) + d_j(z))$. Within this structure, the problems of inventory target tracking (Inventory target tracking) and disturbance rejection (demand rejection) can be tackled by separate controllers as it will pointed out.

When the model is exact, $p_j(z) = \hat{p}_j(z)$, (assuming the action of the lead time identification scheme), the lead time becomes external in closed-loop. Under these circumstances, the scheme compensates the delay and makes the control problem easier. The inventory level under the two-degrees-of-freedom feedback IMC scheme is given by:

$$\begin{aligned} y_j(z) \; &= q_j^t(z)p^m(z)p_j^a(z)r_j(z) \qquad\qquad j = 1, 2, ..., N \\ &- \Big(1 - p^m(z)p_j^a(z)q^d(z)\Big)\, p^d(z)v_j^m(z) \end{aligned} \tag{2.13}$$

where

- $q_j^t(z)$ is an IMC controller designed for inventory target tracking.
- $q_j^d(z)$ is designed mainly to achieve the internal stability and to satisfy the disturbance rejection objective (rejection to the demand perceived by each echelon $v_j^m(z)$).

Remark: In this scheme the controllers are directly designed for the unstable plant $p^m(z)$. Therefore, an internal loop to stabilize the plant is not necessary. Thereby, the expression of inventory level Eq. (2.13) is simplified respect to the presented in Eqs (2.10)-(2.12). This is a design advantage respect to the IMC scheme formulated in Section 2.3.

There are two ways to perform an IMC based inventory control strategy for the entire supply chain: by using the decentralized control, where a controller is applied to each echelon of the supply chain and the centralized control, where a single controller is applied to the entire supply chain. Hence, the IMC design under the two strategies is shown below. In both designs the bullwhip effect mitigation in the entire supply chain is taken into account.

2.5.1 IMC design under a decentralized control strategy

The IMC design for inventory control under a decentralized strategy is addressed to find the optimal controllers to inventory target tracking and disturbance rejection. After that, the optimal controllers are detuned to perform a trade-off between this optimal behaviour and the orders aggressively (bullwhip effect) mitigation. The procedure is presented as follows.

On the one hand, the H_2-optimal problem for inventory tracking is formulated in (Schwartz *et al.*, 2006), (Schwartz and Rivera, 2010) as:

$$min_{q_j^t(z)} \left\| \frac{y_j(z)}{r_j(z)} \right\|_2 = min_{q_j^t(z)} \|[1 - \underbrace{p^m(z) p_j^a(z)}_{p_j(z)} q_j^t(z)]\|_2 \tag{2.14}$$

The $q_j^t(z)$ obtained from Eq. (2.14) gives a optimal inventory tracking but, in consequence, the control action (orders) is aggressive which is not desirable for inventory managers. Therefore, the optimal controller is enhanced with a low pas filter to perform a trade-off between these behaviours. This procedure is shown in Chapter 3, Section 3.2.

On the other hand, the H_2-optimal problem for disturbance rejection is formulated in (Schwartz *et al.*, 2006), (Schwartz and Rivera, 2010) as:

$$min_{q_j^d(z)} \left\| \frac{y_j(z)}{v_j^m(z)} \right\|_2 = min_{q_j^d(z)} \|[1 - \underbrace{p^m(z) p_j^a(z)}_{p_j(z)} q_j^d(z)] p^m(z)\|_2 \tag{2.15}$$

- **Bullwhip effect formulation:**

  Besides the equation of inventory balance, the relation between the demand perceived by the echelon and the generated orders must be taken into account in the control design, since this relation determines the well known bullwhip effect constraint. The bullwhip effect can be characterized as an amplification of demand fluctuations ($v_j^m(z)$ in the decentralized control case) as one move upwards in the supply chain. This propagation of demand fluctuations is only possible when every node has sufficient stock. . If there are neither changes in the set point nor model mismatch, the relation between demand and orders to successive nodes under the two-degrees-of-freedom-feedback-IMC is given by:

and the ratio of orders to successive nodes can be expressed as:

$$|\gamma_j(z)| = \frac{|o_{j-1,j}(z)|}{|v_j^m(z)|} = |q_j^d(z)p_j^d(z)| = |q_j^d(e^{i\omega})p_j^d(e^{i\omega})| \quad \omega \in [0, 2\pi) \tag{2.16}$$

where i is the imaginary unity. (Lin *et al.*, 2004) have stated that the amplitude of demand fluctuations will not be amplified if

$$|\gamma_j(e^{j\omega})| \leq 1 \quad \forall\omega \in [0, 2\pi) \tag{2.17}$$

Notice that Eqs. (2.16) and (2.17) imply that a good bullwhip effect avoidance needs $|p_j(z)q_j^d(z)| << 1$ while Eq. (2.15) requires $|p_j(z)q_j^d(z)| \approx 1$ for step disturbance rejection in the inventory signal. Thus, the $q_j^d(z)$ controller must be designed taking into account two opposite objectives: step disturbance rejection in the inventory level and bullwhip effect avoidance. Therefore, an analytical detuning of the $\tilde{q}_j^d(z)$ optimal controller to obtain a trade-off between these two objectives is performed in the article titled **Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches** and is described in the Chapter 3, Section 3.2 of this thesis. This analytical detuning for IMC controllers for bullwhip effect is not explored in previous works (Schwartz *et al.*, 2006), (Schwartz and Rivera, 2010).

2.5.2 IMC design under a centralized control strategy

The centralized control design is based on the IMC scheme shown in Fig. 2.6 where the model formulated in Chapter 3, Section 3.2 for the entire supply chain, is taken into account. Thus, the vector of inventories, $\mathbf{Y}(z)$, is given by:

$$\mathbf{Y}(z) = \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^t(z)\mathbf{R}(z) - (\mathbf{I} - \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^d(z))\mathbf{P}^d(z)\mathbf{D}(z) \tag{2.18}$$

where the matrices $\mathbf{P}^A(z)$ and $\mathbf{P}^M(z)$ contain the delays and the rational part of the system respectively, $\mathbf{R}(z)$ represents the inventories target vector and $\mathbf{D}(z)$ represents the demands vector of the entire supply chain. In the centralized control strategy $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ can be designed for inventory target tracking and disturbance (demand) rejection respectively by using the IMC guidelines for multivariable (MIMO) systems.

- **Bullwhip effect formulation:**

  The bullwhip effect formulation for a single echelon can be generalized for a centralized control considering the transfer function matrix that relates the orders vector $\mathbf{O}(z)$ with the demand vector $\mathbf{D}(z)$. Then, considering no changes in the set point and no model mismatch, the relation between the set of demands and the set of orders is given by:

$$\mathbf{O}(z) = \mathbf{P}^d(z)\mathbf{Q}^d(z)\mathbf{D}(z) \tag{2.19}$$

  The generalization of the magnitude ratio of orders to successive nodes γ for a multivariable (MIMO) system under a centralized strategy can be expressed as:

$$|\Gamma(z)| = |\mathbf{P}^d(z)\mathbf{Q}^d(z)| = |\mathbf{P}^d(e^{i\omega})\mathbf{Q}^d(e^{i\omega})| \quad \omega \in [0, 2\pi) \tag{2.20}$$

  where the magnitude ratio is calculated component-wise. Thus, the demand signals perceived in the supply chain will not be amplified if:

$$|\Gamma_{ij}(e^{i\omega})| \leq 1 \quad i = 1, 2, ..., N \quad j = 1, 2, ..., N \quad \omega \in [0, 2\pi) \tag{2.21}$$

  In this case, the bullwhip effect implies that each demand signal represented by d_j introduced in the system is not amplified to subsequent suppliers represented by $o_{j,j+1}(z)$. Therefore, the bullwhip effect can be analyzed component-wise since each component of Γ contains the relation between each pair $d_j(z), o_{j,j+1}(z)$. Thus, in the centralized control approach multiple demand signals are taken into account i.e $d_j(z) \neq 0$ $j = 1, 2, ..., N$. After formulating the inventory control system in a centralized control way, the controller matrices $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ will be designed. The complete design is described in the Chapter 3, Section 3.2 of this thesis.

The design of the IMC under the decentralized and centralized control strategy addressed at the bullwhip effect mitigation is novel in the supply chain literature. This contribution is described in the paper titled **Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches**.

The strategies proposed in the previous works are based on linear models of the supply chain where suppliers stock limitations and back orders accumulation are not included. These strategies work for supply chains under high stocks which is common in practice). However, the policies based on these models can lead unrealistic behaviour (negative inventory) when are implemented in supply chains working under low stocks. This situation is undesirable for inventory managers. Therefore, in order to give a better mathematical description of the supply chain dynamics, (Lin *et al.*, 2004) developed a more complete supply chain model in the z-domain. This model takes into account stock limitations and back orders accumulation providing a more adequate representation of the supply chain dynamics. Due to the stock limitations of the distributors, the supply chain turns out to be a linear switched system corresponding to the stock limitations of the distributors (Wang *et al.*, 2009). The complete model is formulated in Appendix A.1.

Some strategies based on process control theory have been successfully applied using the model proposed by (Lin *et al.*, 2004). Among them, (Lin *et al.*, 2004) presents inventory replenishment policies based on Control Engineering, including proportional-integral (PI) controllers and cascade control. This approach applies a single controller through time whose design is focused on the mitigation of the bullwhip effect. (Balan *et al.*, 2009; Kristianto *et al.*, 2011) apply a fixed Fuzzy Logic Controller FLC through time. The tuning of this controller has been performed using adaptive neuro-fuzzy inference system (ANFIS). The design of this controller is also oriented to bullwhip effect mitigation. Nevertheless, since the supply chain changes its operation case through time, a control system that sets dynamically the appropriate controller for the current operation case would be more adequate.

Therefore, as an extension of the research we propose a switched inventory control system for a serial multivariable supply chain under decentralized control strategy (independent switched control to each echelon).

2.6 A switched control strategy for inventory control of the supply chain

This approach consists in a bank of controllers designed for each possible model case working in parallel at each echelon. (Schwartz and Rivera, 2010; Garcia *et al.*, 2012) introduces the application of Internal Model Control as a novel inventory replenishment policy in the supply chain, providing an important improvement in the performance of manufacturing systems. Therefore, in this work each controller for each echelon and operation mode is designed by following IMC guidelines taking into account the trade-off between the optimal inventory tracking and bullwhip effect mitigation. The system is enhanced with a switching logic that decides what controller is suitable at each instant time. A summary of the model is presented below.

2.6.1 Dynamic model of the supply chain

The system under study is a typical cascade production-distribution system consisting of two echelons: a retailer (R) and a distributor (D) between the factory (F) and final customer (C) as is shown in Fig. 2.7.

The model of a particular echelon presented by (Lin *et al.*, 2004) and shown in the box of Fig. 2.7 is adopted. In this way, let $I_j(t)$ denote the net stock inventory (the difference between on-hand inventory and backorders) of an echelon $j \in \{R, D\}$ of the chain at any discrete time $t = nT$ where T=1 is the base of time of the supply chain events (a day, a week or a month) and n is a nonnegative finite integer. The amount of goods to be delivered to downstream node $k \in \{C, R, D\}$ by the node j at the instant t is denoted by $Y_{j,k}(t)$. The demand received by node j from downstream node k is indicated by $U_{k,j}(t)$. In addition, a time delay of L_j time instants is assumed for all delivery of goods so that goods dispatched for a supplier at time t will arrive to the destination at time $t + L_j$. However, due to the need for examination and administrative processing, this new delivery is only available to the node j at $t + L_j + 1$ (Lin *et al.*, 2004). The supply chain model in the z-transform is shown below:

$$I_j(z) = \frac{1}{1 - z^{-1}}(z^{-L_j} Y_{i,j}(z) - Y_{j,k}(z)) \tag{2.22}$$

$$I_j^*(z) = \frac{1}{1 - z^{-1}}(Y_{i,j}(z) - Y_{j,k}(z)) \tag{2.23}$$

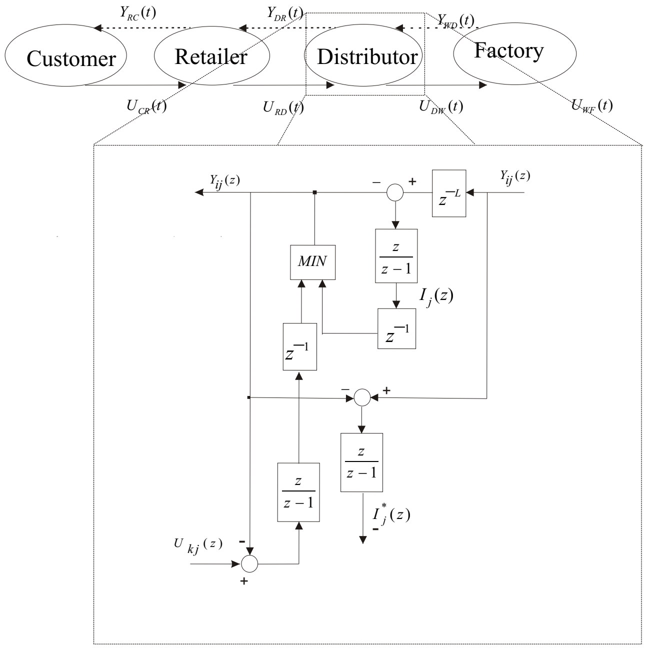

Figure 2.7: The block diagram of the supply chain under stock limitations and backorders.

$$O_j(z) = \frac{1}{1-z^{-1}}(U_{k,j}(z) - Y_{j,k}(z)) \tag{2.24}$$

$$Y_{j,k}(z) = \begin{cases} 0 & O_j(t-1) \leq 0 \\ z^{-1}O_j(z) & 0 \leq O_j(t-1) \leq I_j(t-1) \\ z^{-1}I_j(z) & 0 \leq I_j(t-1) \leq O_j(t-1) \end{cases} \tag{2.25}$$

When the supply chain is in operation, the stock conditions (enough or not enough) of the current echelon j can change at a determinate instant t, as is modelled in Eq. (2.25). Hence, the supply chain is a natural switched system where the general dynamic model given by Eqs. (2.22)-(2.25) can become in particular, one of the following models through the time (Lin *et al.*, 2004):

- Infinite Supply and High Stock, (ISHS).

- Infinite Supply and Low Stock, (ISLS).
- limited supply, (LS).

The particular models associated to the different cases described by Eq. (2.25) are specified in sections (2.6.2)-(2.6.3).

2.6.2 Infinite Supply and High Stock (ISHS)

In this case, the upstream supplier has sufficient inventory so that the demand of node j is always satisfied: i.e. $Y_{i,j}(z) = z^{-1}U_{j,i}(z)$. Furthermore, it is considered that the set point of node j is sufficiently high so that there will always be sufficient inventory to satisfy all customer demands, i.e. $Y_{j,k}(z) = z^{-1}O_j(z) = z^{-1}U_{k,j}$ which corresponds to the second case of Eq. (2.25). Under these conditions, the Eq. (2.23) for inventory position $I_j^*(t)$ is:

$$I_j^*(z) = \frac{1}{1-z^{-1}}z^{-1}U_{j,i}(z) - \frac{1}{1-z^{-1}}z^{-1}U_{k,j}(z) \tag{2.26}$$

Thereby, for the infinite supply and high stock case, the inventory position $I_j^*(t)$ is related with the orders $U_{j,i}(z)$ and the demand $U_{k,j}(z)$ by the following transfer function:

$$P_j^{(ISHS)}(z) = \frac{1}{1-z^{-1}}z^{-1} \tag{2.27}$$

For IMC design, Eq. (2.27) must be factored into a minimum-phase portion:

$$P_j^{(ISHS)m}(z) = \frac{1}{1-z^{-1}} \tag{2.28}$$

and a portion that includes the delays of the original transfer function:

$$P_j^{(ISHS)a}(z) = z^{-1} \tag{2.29}$$

2.6.3 Infinite Supply and Low Stock (ISLS)

If an upstream supplier has sufficient inventory so that the demand of node j is always satisfied, i.e. $Y_{i,j}(z) = z^{-1}U_{j,i}(z)$, but the set point of node j is low so that there will always be insufficient inventory to satisfy all customer demands, i.e. $Y_{jk}(z) = z^{-1}I_j(z)$ which corresponds to the third case of Eq. (2.25), then, the equation for inventory position $I_j^*(t)$ is determined by:

$$I_j^*(z) = \frac{z^{L_j} + z^{L_j-1} + ... + z + 1}{z^{L_j+1}} U_{j,i}(z) \tag{2.30}$$

In this case, the transfer function that relates the orders $U_{j,i}(z)$ with the inventory position is given by

$$P_j^{(ISLS)}(z) = \frac{z^{L_j} + z^{L_j-1} + ... + z + 1}{z^{L_j+1}} \tag{2.31}$$

For IMC design purposes, the minimum-phase portion of Eq. (2.31) is:

$$P_j^{(ISLS)m}(z) = \frac{z^{L_j} + z^{L_j-1} + ... + z + 1}{z^{L_j}} \tag{2.32}$$

and the portion that includes the delay of Eq. (2.31) is:

$$P_j^{(ISLS)a}(z) = z^{-1} \tag{2.33}$$

Limited Supply (LS)

In this case, the upstream supplier i does not have sufficient inventory to supply the node j which corresponds to the first case of Eq. (2.25). Therefore, the goods perceived for the echelon j in an instant t is equal to the existences of the immediate supplier i at the last time $t-1$ i.e $Y_{ij}(t) = z^{-1}I_i(t-1)$. In this case, we found the following transfer function:

$$I_j^*(z) = \begin{cases} \frac{1}{(z-1)}(I_i(z) - U_{j,k}(z)) & I_j(t) \geq U_{k,j}(t-1) \\ \frac{z^{L_j+1}-1}{z^{L_j+1}(z-1)} I_i(z) & Otherwise \end{cases} \tag{2.34}$$

In Eq. (2.34), the inventory position of node j depends neither on the set point nor on its ordering policy. This result is intuitive: If the supplier is low in stock no matter how node j orders, the inventory position is limited by the stock available in the supplier i.

The operation case of the supply chain depends on the relation $MIN(O_j(t), I_j(t))$, which determines if the j^{th} echelon have enough existences to supply the immediate customer k and the relation $MIN(O_i(t), I_i(t))$ which determines if the immediate supplier i have enough existences to supply the actual echelon. Thereby, the operation cases change as these relations change through time. Therefore, a system that sets automatically the adequate inventory control policy may be more suitable than a fixed control policy.

The objective is to design a switched control system for each j^{th} echelon (Decentralized strategy) that select automatically at each instant t the adequate controller according the current dynamics of the supply chain Eqs.(2.27), (2.31) or (2.34). Switching between controllers is carried out by measuring the signals $O_j(t)$, $I_j(t)$, $O_i(t)$ and $I_i(t)$). The controllers design for each echelon is based on internal model control guidelines. The complete system is described in Appendix A.1.

2.6.4 Switched control scheme for an echelon

A switched system is a hybrid system composed of a family of continuous-time or discrete-time subsystems and a rule orchestrating the switching between them (Ibeas *et al.*, 2004). Thus, the inventory control scheme proposed in this paper for each node, j, includes a bank of control policies $C_j(z)$ designed adequately for each supply chain operation case.

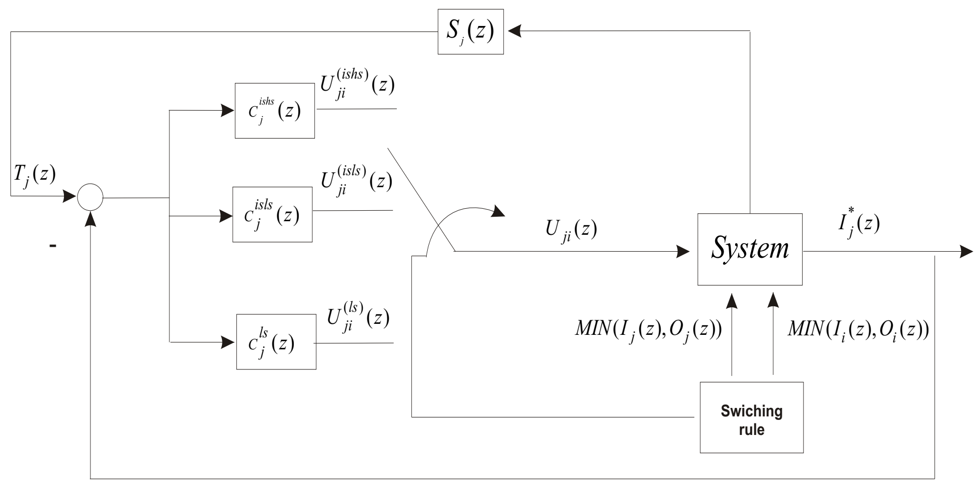

Figure 2.8: Switched control scheme for the supply chain system

In this work, a multi-degrees-of-freedom internal model control (IMC) structure (Morari and Zafiriou, 1989; Schwartz *et al.*, 2006; Schwartz and Rivera, 2010) in discrete-time described in Section 2.5 is adopted to design the $C_j^{(ISHS)}(z)$, $C_j^{(ISLS)}(z)$ and $C_j^{(LS)}(z)$ control policy for each node, j. Each policy is designed according to each supply chain operation case by using the corresponding models $P_j^{(ISHS)}(z)$, $P_j^{(ISLS)}(z)$ and $P_j^{(LS)}(z)$. The adequate controller at each instant time is selected according the following switching logic.

2.6.5 Switching logic

The switching logic evaluates if there is sufficient inventory in the node j, i.e $(MIN(O_j(t), I_j(t)))$ and immediate supplier $j+1$, i.e $(MIN(O_i(t), I_i(t)))$ and selects the respective controller designed for each case.
That is: if the node j and immediate supplier have sufficient inventory (Infinite Supply and High Stock case) then the switching logic chooses the output of the controller $C_j^{ishs}(z)$ i.e

$$if \quad I_{j+1}(t) > O_i(t) \wedge I_j(t) > O_j(t) \quad Then \quad C_j^{ishs}(z) \ . \tag{2.35}$$

If immediate supplier have sufficient inventory but the node j inventory is low (Infinite Supply and Low Stock case) then the switching logic chooses the output of the controller $C_j^{isls}(z)$ i.e

$$if \quad I_j(t) > O_j(t) \wedge I_j(t) < O_j(t) \quad Then \quad C_j^{isls}(z) \ . \tag{2.36}$$

If the immediate supplier have no sufficient existences to supply the node j (Limited Supply case) then the switching logic chooses the output of the controller $C_j^{ls}(z)$ i.e

$$\begin{array}{ll} if & I_i(t) < O_i(t) \wedge I_j(t) > O_j(t) \\ or & I_j(t) < O_j(t) \wedge I_j(t) < O_j(t) \\ Then & C_j^{ls}(z) \end{array} \tag{2.37}$$

The design of the controllers $C_j^{ishs}(z)$, $C_j^{isls}(z)$ and $C_j^{ls}(z)$ is carried out by using the Two-degrees-of-freedom feedback IMC scheme stated in Section 2.5. The complete design simulations an conclusions of the Switched Control Scheme are presented in Appendix A.1 which is the paper titled **A switched control strategy for inventory control of the supply chain** submitted to a journal for its consideration.

2.7 Results

On the one hand, theoretical proofs to guarantee that the algorithm identifies the actual lead times of the supply chain and closed-loop stability are provided in this research, which is not common in works aimed at inventory control in the supply chain. Indeed, these proofs are conceptually easier than for other identification methods presented in the previous works (Bernstein and Rad., 2007), (Mirkin and Gutman., 2009), (S. S. Ge, 2004), (Zhang and Ge., 2010) and (Chen *et al.*, 2009). Moreover, the simulations performed show that the inventory control system based in the Internal Model Control scheme enhanced with the delay identification algorithm is robust to changes in the lead time, and at the same time is able to present an acceptable performance to a stochastic customer demand. In these simulations the internal model control is implemented as a decentralized control strategy. The theoretical , quantitative and qualitative results are published in the paper titled **Inventory control for the supply chain: An adaptive control approach based on the identification of the Lead-time, Omega, The International Journal of Management Science** and are provided in Chapter 3, Section 3.1 of this thesis.

On the other hand, the article titled **Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches** shows the tuning of the internal model control to perform a trade-off between the inventory target tracking and bullwhip effect avoidance. The tuning is performed in the two cases: Centralized and decentralized control strategy. The proposed scheme allows us to tackle the two problems of inventory target tracking and demand rejection with two controllers separately. That is an advantage to other replenishment inventory policies based on control theory proposed in the literature (Hoberg *et al.*, 2007), (Jaksic and Rusjan, 2008), (Balan *et al.*, 2009),(Dejonckheere *et al.*, 2003). However, optimal tuning of these controllers produces aggressive orders that are unacceptable for factory managers. Then, two trade-off must be taken into account in the design of these controllers for inventory management in the supply chain, (Inventory target tracking vs aggressive orders mitigation for the $\mathbf{Q}^t(z)$ design) and (Demand rejection vs bullwhip effect mitigation for the $\mathbf{Q}^d(z)$ design).

In the decentralized control approach based on multi-degree-of-freedom controller, the tuning to perform the trade-offs (Inventory target tracking vs aggressive orders mitigation for the $\mathbf{Q}^t(z)$ design) and (Demand rejection vs bullwhip effect mitigation for the $\mathbf{Q}^d(z)$ design) works in the first echelon but is deteriorated in the subsequents echelons. Since the centralized control approach the controller for the entire supply chain is designed at once, the trade-offs hold for the entire supply chain showing the advantage of the centralization of the controller for bullwhip effect mitigation and inventory tracking in the entire supply chain.

A simulation of the inventory control under the centralized IMC enhanced with the identification scheme for a supply chain working in safety stock mode (ISHS) is performed to show that the overall system works. In this simulation the actual lead times of the three echelons are $L_1 = 2$, $L_2 = 3$, $L_3 = 4$ and the set point is 100 units. Fig. 2.9 shows that the actual lead times are identified after a time of residence $Tres = 10$. Moreover, Fig. 2.10 shows that the system is stable and presents a well inventory tracking.

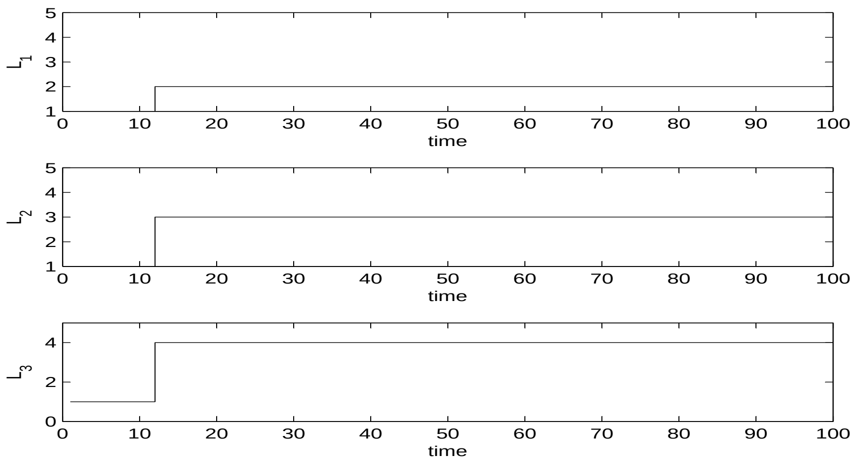

Figure 2.9: Lead times identified of the supply chain

Finally, the paper titled **A switched control strategy for inventory control of the supply chain** (Appendix A.1) develops a comparison between the performance of the PI controller proposed by (Lin *et al.*, 2004) and the switched control approach proposed in this work with respect to inventory

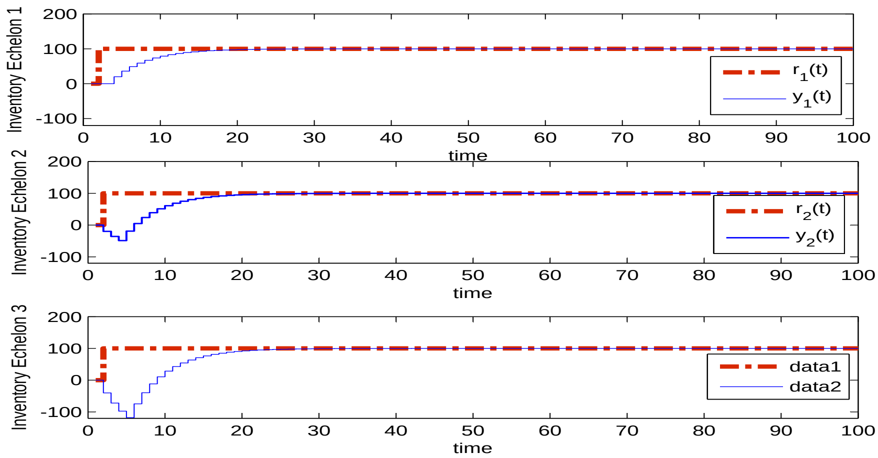

Figure 2.10: Inventory levels of the supply chain

tracking, bullwhip effect mitigation and customer satisfaction. The comparisons are performed by simulations of one and three echelons under PI and switched control. Moreover, in order to provide more quantitative results in all simulations, the basic performance indices of the supply chain Integral absolute error (IAE), Back-order (BO) and Bullwhip effect (BW) described in (Lin *et al.*, 2004; Balan *et al.*, 2009) are calculated. The qualitative and quantitative results shown in the Appendix A.1 evidence that the switched control strategy proposed in this work improves the inventory control in supply chains under multiple operation modes (ISHS, ISLS or LS).

2.8 Conclusions and future work

This thesis deals with the inventory control problem in the entire supply chain. The system has dynamics as the Lead time, that make the control problem a difficult task. Therefore, the problem was addressed with the Internal Model Control approach which compensates the lead time effects and presents a well performance to set point tracking and disturbance rejection. However, the uncertainty of the lead time does not allow to take advantage of the control scheme. Therefore, one of the contributions of this thesis was to overcome this problem by the lead time identification algorithm. This algorithm allows to obtain the real lead time and use this actual value in the control scheme. In this way, the advantages of the Internal model control scheme can be exploited in the Inventory control problem. Once this problem was solved, the research was focused in the evaluation of the Internal model control scheme to inventory target tracking disturbance rejection and bullwhip effect avoidance assuming the knowledge of the lead time. Hence, the internal model control was designed in the two ways: Centralized and decentralized strategies for the entire supply chain. The research has showed that the internal model designed under the centralized strategy allows a best trade-off between (the inventory tracking and disturbance rejection) with the bullwhip effect avoidance.

Finally, a non-linear and general model of the supply chain was taken into account for the inventory control problem. In this case, a single controller does not work very well. Therefore, the internal model control for the different cases of the supply chain was designed. Thus, the implementation of a switch system that selects the appropriate controller in each instant time was another important contribution in this research.

The Internal Model Control enhanced with the lead time identification algorithm and the switching scheme allows to solve the inventory control problem with acceptable results respect to inventory target tracking disturbance rejection and bullwhip effect avoidance in the entire supply chain. However, there are several contributions to improve the inventory control in the supply chain.

On the one hand, the design and simulation results presented in the articles titled "**Inventory control for the supply chain: An adaptive control approach based on the identification of the Lead-time**," "**Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches**" and the final results of this thesis (Section 2.7) shows that the IMC control scheme enhanced with a lead time identification scheme presents an acceptable results respect to inventory control in the entire supply chains under safety stock strategy (ISHS). Therefore, the approach can be extended to supply chains with more complex dynamics.

For instance, a branched supply chain is the most common configuration in the real life where the information sharing and the cooperation among entities play an important role in the customer satisfaction. These interactions represent an increase in the complexity of the dynamics of the chain. Under this situation an IMC control scheme under the centralized control strategy may be adequate to the inventory control in the entire supply chain mitigating the bullwhip effect which is not explored in the current literature of the supply chain.

Similarly, the design and simulation results presented in the article titled **A switched control strategy for inventory control of the supply chain** (Appendix A.1) evidence that the switched control system designed for the entire supply chain under multiple working cases (ISHS, ISLS or LS), is stable in closed-loop. Thus, there are many possibilities of future works using this approach. Among them, a theoretical study of the stability of the system must be performed by using the switched system theory and discrete system control theory (Wang *et al.*, 2009). This study will show the conditions for which the system will exhibit stable, periodic, and divergent behavior. Moreover, various complications can be introduced and analyzed using this basic model. For instance, it can be assumed the existence of different suppliers with different inventory levels and different transportation delays, and investigate the optimal order allocations. One may also impose constraints that total inventory is limited by storage space and devise an order strategy accordingly.

Up to now, the operation cost of the supply chain (inventory, transportation, distributing and manufacturing) are not taken into account in the control scheme. The minimization of the operation cost has an important role to maximize the profitability of the supply chain. Therefore, the IMC control scheme enhanced with a function that balance the supply chain cost (inventory, transportation, distributing and manufacturing) and selects the appropriate inventory targets in all points of the supply chain can be a future line of research. There are multiple optimization tools between the control theory to develop this research.

Chapter 3

Publications

This chapter present the set of journal publications.

- The paper titled **Inventory control for the supply chain: An adaptive control approach based on the identification of the Lead-time** which has been published in **Omega, The International Journal of Management Science, 2012**.
- The paper titled **Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches** which has been published in the **European Journal of Operational Research, 2013**.

3.1 Inventory control for the supply chain: An adaptive control approach based on the identification of the Lead-time

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abstract

In this paper, an Internal Model Control (IMC) scheme is incorporated in production inventory control systems in a complete supply chain. This control scheme presents a good target inventory tracking under the perfect knowledge of the system. Furthermore, the inventory tracking and load disturbance rejection control problems can be tackled separately. However, the closed-loop performance of the IMC scheme may be degraded due to a mismatch bewteen the modelled and actual delay or to the fact that delays may be time-varying. Thus, the IMC control scheme is enhanced in this work with a novel method for the online identification of lead times based on a multimodel scheme. In this way, all benefits of the IMC scheme can be exploited. A detailed discussion of the proposed production inventory system is provided including a stability and performance analysis as well as the identification capabilities of the algorithm. Several simulation examples illustrate the efficiency of the approach.

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Inventory control for the supply chain: An adaptive control approach based on the identification of the lead-time

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In this paper, an Internal Model Control (IMC) scheme is incorporated in production inventory control systems in a complete supply chain. This control scheme presents a good target inventory tracking under the perfect knowledge of the system. Furthermore, the inventory tracking and load disturbance rejection control problems can be tackled separately. However, the closed-loop performance of the IMC scheme may be degraded due to a mismatch between the modelled and actual delay or to the fact that delays may be time-varying. Thus, the IMC control scheme is enhanced in this work with a novel method for the online identification of lead times based on a multimodel scheme. In this way, all benefits of the IMC scheme can be exploited. A detailed discussion of the proposed production inventory system is provided including a stability and performance analysis as well as the identification capabilities of the algorithm. Several simulation examples illustrate the efficiency of the approach.

1. Introduction

A common supply chain (SC) includes the necessary entities to provide goods to the customer from production centers. Thus, the main elements composing a general supply chain between the factory (F) and customer (C) are: warehouse (W), distributing center (D) and retailer (R) [1,2]. There are many participants and processes as well as randomness in the information flow of a supply chain. Therefore, the coordination of the supply chain operation becomes a key point to optimise the use of its resources and compete on a global scale. There are many aspects to research in this complex network, one of these is the improvement of inventory management policies. The main objective of inventory management is to keep the inventory level of each element of the supply chain stable enough so as to satisfy the requirements of the customers by ordering products from its immediate supplier of the supply chain [3]. In this way, the supply chain is modeled as a serial process where each element gives orders to its immediate supplier in order to have enough goods to supply the orders of the immediate customer of the chain. The entire supply chain is a serial process since one element is strictly related to its immediate downstream and upstream elements. This kind of processes are commonly described as multi-variable (Multiple Inputs – Multiple Outputs) systems and they are represented by a matrix with a block diagonal structure [4]. Once an order is placed on the immediate supplier there is a time to the moment that the petition is satisfied. This is known as replenishment lead time and it consists of an ordering time-delay and a production or distribution time delay [5]. Many undesirable effects may appear when an inventory replenishment policy is implemented in the supply chain described above. Among these, the instability is the main problem since signals describing the inventory and orders may diverge as time goes on. Hoberg et al. [6] applies linear control theory to study the effect of several inventory policies on order and inventory variability (using z-transform techniques) and their conditions for stability are examined by the Jury criteria.

Another inconvenience is that the variability in the ordering patterns often increases as we move upwards in the chain, from the customer to the factory and the suppliers. This phenomenon is broadly known as the *bullwhip effect*. Some current studies [1,2,6,7], have analyzed the effect of the replenishment policies focused on the bullwhip effect estimation and suppression. Moreover, Lin et al. [3] presents Control Engineering based approaches, including proportional-integral (PI) controllers and cascade control as inventory replenishment policies, being the design of this controller also focused on the mitigation of the bullwhip effect. Balan et al. [8] applies fuzzy logic theory control on inventory error and error changes associated with forecast demand among the nodes of a supply chain in order to allow smooth information flow in the chain.

Besides stability issues and bullwhip, the response of the net stock signal is an additional important subject of investigation. Tang [9] focuses on base-stock inventory models with and without expected demand and provides a computationally efficient

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method to set optimal inventory targets for finished products under capacitated postponement. Silver and Bischak [10] considers a periodic review order-up-to-level (orbasestock) inventory control system under normal distributed demand. A major problem is inventory deficit existence (i.e. the difference between inventory target and the actual inventory level), usually referred to as inventory drift.

One of the main causes of all these phenomena is attributed to the lead time, specially when it is not properly known. Therefore, counteracting the lead time effect is crucial so as to improve supply chain management. There are two approaches towards the control of systems with external delays. The first one is the robust control approach that consists in designing a controller based on a nominal value for the delay and considering a grade of robustness of the controller to any mismatch in it. In this way, Aggelogiannaki and Sarimveis [11], Schwartz et al. [12], Schwartz and Rivera [13] introduce the Internal Model Control which is a robust control approach, and predictive control as novel decision replenishment policies. These works only consider an approximation of the delay to set up the control problem resulting in a good control of the inventory level avoiding the bullwhip effect. However, the changes in the delay through the time which is a typical situation in the supply chains are not considered in this approach.

A second control approach that allows us to tackle systems with delay without considering any approximation is the use of delay compensation schemes. The Smith Predictor (SP) and its generalizations such as those based on the Internal Model Control (IMC) are the more extended configurations [14]. These topologies were proposed in the 50 s and they have been extended to include robustness issues or the possibility of dealing with unstable processes [15] ever since. In this work we advocate on a decentralized control approach based on an IMC delay compensation scheme for the MIMO supply chain. One of the main advantages of the use of an IMC type structure is that in this scheme there are three controllers that allow to tackle the nominal stability, the relation (Inventory level vs Inventory target) and the relation (Inventory level vs Demand) separately. The use of the delay compensation scheme facilitates the controller design because it allows to disregard the delay when designing two of these controllers, which is an advantage in the design process.

However, delay compensation schemes have a drawback: the system's delay has to be known beforehand to perform its perfect compensation. This situation is not viable when the delay changes during the process which is a common situation in supply chains. An alternative to overcome this problem is to include a lead time identification method in the supply chain operation.

It is difficult to find works that deal with lead time identification in entire supply chains since most of works normally deal with a single echelon. In Aggelogiannaki and Sarimveis [11] a recursive prediction error method (RPEM) is proposed to identify the lead time online in a unique echelon (SISO system), based on historical data that includes order rate and received final products. Then one parameter of an Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS) is adjusted according the identified lead time. On the other hand, some researchers have developed algorithms from a control theoretic perspective for online identification and adaptive control of delayed MIMO systems, (see, for instance [16–18]). In Bernstein and Rad [16], a time-delay neural networks (NNs) model is employed to perform simultaneous system identification and time-delay estimation. The proposed network, for which stability is proved using Lyapunov theory, is an extended version of the delay-free dynamical NN. The adaptive controller, Mirkin and Gutman [17] presents a simple model reference adaptive control (MRAC) scheme which is also robust when dealing with external disturbance with unknown bound. A suitable Lyapunov–Krasovskii type functional with "virtual" gain is used to design the adaptation algorithm and prove stability. In Chen et al. [18], a novel adaptive neural controller based on a NN online approximation model is proposed. Its main contribution consists of the construction of a quadratic-type Lyapunov–Krasovskii functional which results in a number of online-adapted parameters independent from the number of nodes of the neural network which reduces complexity. Other significant results on this control issue have been reported in Ge [19], Zhang and Ge [20] and Zhang and Ge [20]. Overall, an important disadvantage of the aforementioned works is the large number of involved parameters and their theoretical complexity. Moreover, none of them have ever been applied to the supply chain control problem. In fact, there is no supply chain application of these delay identification works to lead time estimation. In this work, a delay identification algorithm is proposed for the complete supply chain being able to identify the delays among the different echelons describing the supply chain. The identified values of the delays are then used to adjust the delay compensation in a IMC based decentralized compensation scheme. The proposed identification scheme is based in a multi-model scheme, it made up of a battery of different models operating in parallel [21–23]. Each model includes the same rational component but a different delay value. A supervisory algorithm compares the mismatch between the actual system and each candidate models and it determines, for each time interval, the one that best describes the behaviour of the real system, providing an estimation of the lead time. An additional block selects the best model for control purposes. The approach is inspired in what are called Pattern Search Algorithms [24], whose application in control is really novel. Besides the formulation of the complete control scheme. In the present work, theoretical proofs to guarantee that the algorithm identifies the real lead times of the supply chain and closed-loop stability are provided, which is not common in works aimed at inventory control in the supply chain. Indeed, these proofs are conceptually easier than those presented in the mentioned works. The rest of the paper is formulated as follows: Section 2 presents the complete supply chain model using z-transform. As a result, a discrete multiple input multiple output (MIMO) system is obtained. Section 3 presents the formulation of the Internal model control as a delay compensation scheme to inventory control in supply chains. After that, Section 4 presents the adopted intelligent multimodel identification scheme. Section 5 presents theoretical results that ensure its convergence to the real lead time and the closed-loop stability, being the proofs of lemmas and theorems in the Appendix. Section 6 presents the simulation results. The paper ends with the concluding remarks in Section 7.

2. Supply chain model

Let us consider a general supply chain model used in Dejonckheere et al. [1], Hoberg [6]. For the sake of simplicity, assume a period base of time $T_m = 1$ which can be one day, one week or one month according to the dynamics of the supply chain. In this model there are three logistic echelons, warehouse (W), distributing center (D) and retailer (R) between the factory (F) and customer (C) as is shown in Fig. 1. The customer is considered to be the base of supply chain while the Factory is on the top of the supply chain. Thus, denote by $j = 0,1,2,3,4$ each of the logistic echelons of the supply chain. Thereby, for this specific supply chain $j = 0$ represent the Customer (C) and $j = 4$ represents the Factory (F). According to this notation, $(j+1)$ represent an immediate supplier and $(j-1)$ represent an immediate customer. The scheme of the supply chain in consideration is shown in Fig. 1.

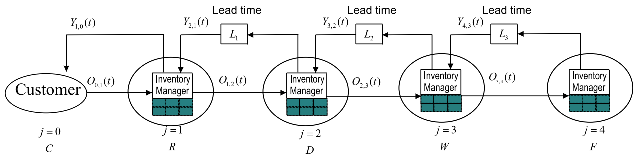

Fig. 1. The block diagram of supply chain.

Then, let $I_j(t)$ denote the inventory level of each logistic node j at any discrete time instant $t=nT_m$ where n is a natural. Let also denote by $O_{a,b}(z)$ the amount of orders placed to b from a. Thereby, the amount of orders placed by a participant j to an immediate supplier $j+1$ is denoted by $O_{j,j+1}(t)$. Finally, the amount of goods delivered by each logistic node a to the node b is denoted by $Y_{a,b}(t)$. Thereby, and the amount of goods delivered by each logistic node j to the immediate customer $j-1$ is denoted by $Y_{j,j-1}(t)$ Thus, the inventory balance in each echelon is given by the difference between the goods received from the immediate supplier and the goods delivered to the immediate customer as follows:

$$I_j(t)=I_j(t-1)+Y_{j+1,j}(t)-Y_{j,j-1}(t). \quad (1)$$

The following sequence of events in each echelon [1] is assumed:

i. At each discrete time t, the echelon jth receives goods.
ii. The demand $O_{j-1,j}(t)$ is observed and satisfied immediately, i.e. $Y_{j,j-1}(t)=O_{j-1,j}(t)$ (if not backlogged).
iii. The new inventory level, $I_j(t)$, is measured.
iv. Finally, the jth echelon places an order $O_{j,j+1}(t)$ on the superior level (upstream) depending on the value of the inventory level, $I_j(t)$.

There is a lead time $L_j \in \mathbb{N}$ between the time an order is placed by node jth and when it is received and is assumed that each node has enough existences to satisfy the demand of its inferior level. In this way, the amount of goods ordered to an immediate supplier at time t will arrive at time $t+L_j$, i.e. $Y_{j+1,j}(t)=O_j(t-L_j)$. Thus, the expression that relates the inventory balance with the demand $O_{j-1,j}(t)$ and order $O_{j,j+1}(t)$ at node j Eq. (1) becomes now:

$$I_j(t)=I_j(t-1)+O_{j,j+1}(t-L_j)-O_{j-1,j}(t). \quad (2)$$

Eq. (2) is a difference equation which can be solved directly in the time domain or by using transformation techniques. In particular, the z-transform is the most extended one because it transforms Eq. (2) into an algebraic equation [25]. Then, applying the time shifting property $Z\{x[t-k]\}=z^{-k}Z\{x[t]\}=z^{-k}X(z)$ where k is a finite integer to Eq. (2) it becomes:

$$I_j(z)=I_j(z)z^{-1}+O_{j,j+1}(z)z^{-L_j}-O_{j-1,j}(z). \quad (3)$$

Now, isolating $I_j(z)$ from (3) we get:

$$I_j(z)-I_j(z)z^{-1}=O_{j,j+1}(z)z^{-L_j}-O_{j-1,j}(z),$$

$$I_j(z)=\frac{O_{j,j+1}(z)z^{-L_j}}{1-z^{-1}}-\frac{O_{j-1,j}(z)}{1-z^{-1}}. \quad (4)$$

In this model:

1. $I_j(z)$ is the inventory level which is the variable to be controlled.
2. $O_{j,j+1}(z)$ represents the order placed to its immediate supplier and is the control action.
3. The demand perceived $O_{j-1,j}(z)$ is considered as a disturbance since this input depends on the decisions of the immediate customers. Since in a decentralized control strategy, each echelon takes decisions based on the global demand value, all customer demands can be lumped into an aggregate demand considering all requests [3]. Therefore, the inventory dynamics does not really depend on how many customers the node has.

Note that Eq. (4) represents an unstable relation (through an integrator) from $O_{j,j+1}(z)$ and $O_{j-1,j}(z)$ to $I_j(z)$ which is one of the problems appearing in Supply chain management as is commented in the Introduction. In the rest of the paper we will refer to this relation as

$$p(z)=\frac{1}{1-z^{-1}}. \quad (5)$$

Moreover, the relation from $O_{j,j+1}(z)$ to $I_j(z)$ involves a delay z^{-L_j} which makes the control problem more difficult. The model for an echelon in Eq. (4) is thus amenable to implement some controllers that exist in literature as is showed in last works [1,2,6,7], or an equivalent model in continuous time presented in Schwartz and Rivera [13]. However, these works only consider the control of an isolated echelon while in the present work the model is extended to the complete supply chain.

The generation of a model for the complete supply chain is carried out considering that the demand for a particular echelon is equal to the order generated by the downstream as shown

$$I_1(z)=\frac{z^{-L_1}}{1-z^{-1}}O_{1,2}(z)-\frac{1}{1-z^{-1}}O_{0,1}(z),$$

$$I_2(z)=\frac{z^{-L_2}}{1-z^{-1}}O_{2,3}(z)-\frac{1}{1-z^{-1}}O_{1,2}(z),$$

$$\vdots \quad \vdots$$

$$I_j(z)=\frac{z^{-L_j}}{1-z^{-1}}O_{j,j+1}(z)-\frac{1}{1-z^{-1}}O_{j-1,j}(z) \quad (6)$$

for $j=1,2,3,4$, the model expressed in Eq. (6) is a system of equations that can be represented in a matrix form. In this paper, the model contains three echelons between Customer and Factory and the complete supply chain is modeled by the matrix

$$\mathbf{I}(z)=\begin{pmatrix} I_2(z) \\ I_3(z) \\ I_4(z) \end{pmatrix}=\mathbf{P}(z)\mathbf{O}(z)=\mathbf{P}(z)\begin{pmatrix} O_{0,1}(z) \\ O_{1,2}(z) \\ O_{2,3}(z) \\ O_{3,4}(z) \end{pmatrix}, \quad (7)$$

where

$$\mathbf{P}(z)=\begin{pmatrix} -p(z) & p(z)z^{-L_1} & 0 & 0 \\ 0 & -p(z) & p(z)z^{-L_2} & 0 \\ 0 & 0 & -p(z) & p(z)z^{-L_3} \end{pmatrix} \quad (8)$$

is the matrix transfer function of the complete supply chain. The modeling scheme for the entire supply chain taken into account is illustrated in Fig. 2. The matrix expressed in Eq. (8) can be decomposed into a delay-free factor $\mathbf{P}_{Lf}(z)$ and a pure delay (Lead-time) term $\mathbf{P}_L(z)$ using the Schur product (or component-wise product) [26], in the form:

$$\mathbf{P}(z)=\mathbf{P}_{Lf}(z)\bullet\mathbf{P}_L(z), \quad (9)$$

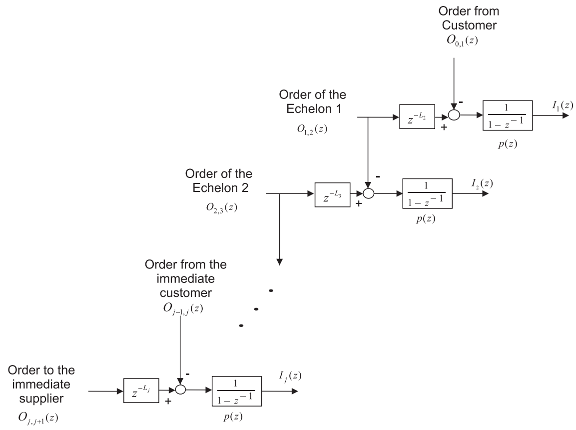

Fig. 2. The block diagram of the complete supply chain.

where

$$\mathbf{P}_{Lf}(z) = p(z)\begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \quad (10)$$

and

$$\mathbf{P}_{L}(z) = \begin{pmatrix} 1 & z^{-L_1} & 0 & 0 \\ 0 & 1 & z^{-L_2} & 0 \\ 0 & 0 & 1 & z^{-L_3} \end{pmatrix}. \quad (11)$$

The complete system is hence modeled by a multivariable system characterized by multiple delays contained in Eq. (11).

The objective of this work is to design a decentralized controller for the system Eqs. (10) and (11) under imperfect knowledge of delays given by Eq. (11). To achieve this objective, the different delays appearing in Eq. (11) are to be identified which is a topic that has not been analyzed previously in the supply chain literature.

In order to perform the identification of the delays the following assumptions are made.

Assumption 1. The rational part of the system, i.e. Eq. (10), is known.

Assumption 2. The delay between each pair output/input lies in a known compact interval. That is, if it is considered a matrix **H** which contains the nominal delays of the system:

$$\mathbf{H} = \begin{pmatrix} 0 & L_1 & 0 & 0 \\ 0 & 0 & L_2 & 0 \\ 0 & 0 & 0 & L_3 \end{pmatrix}, \quad (12)$$

there exist two known matrices $\underline{\mathbf{H}},\overline{\mathbf{H}}$, such that $\underline{\mathbf{H}} \le \mathbf{H} \le \overline{\mathbf{H}}$. □

Notice that both Assumptions are feasible in practice for the inventory control problem in the supply chain.

As it is mentioned in the Introduction, the previous work Aggelogiannaki and Sarimveis [11] has basically proposed the identification of a unique isolated echelon. The above model seems simple. Nevertheless, it captures the basic dynamics features of a supply chain system. In this model the vector $\mathbf{I}(z)$ represents the set of inventories which are the controlled variables, while $\mathbf{O}(z)$ represent the vector of the set of orders $O_{j,j+1}(z)$ of all the logistic echelons and the customer demand $O_{0,1}(z)$ of the supply chain.

Remark 1. Note that model (9)–(11) can be extended for an arbitrary number of supply chain participants, including the Factory and the customer making $j=0,1,2,\ldots N$ where N is a finite integer. Thus the general model for the complete supply chain is given by

$$\mathbf{P} = \mathbf{P}_{Lf}(z)\bullet\mathbf{P}_{L}(z), \quad (13)$$

with

$$\mathbf{P}_{Lf}(z) = p(z)\begin{pmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix}_{(N-1)\times(N)} \quad (14)$$

and

$$\mathbf{P}_{L}(z) = \begin{pmatrix} 1 & z^{-L_1} & 0 & \cdots & 0 \\ 0 & 1 & z^{-L_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & z^{-L_N} \end{pmatrix}_{(N-1)\times(N)}. \quad □ \quad (15)$$

Once the model for a supply chain is developed, the inventory control scheme and controller design are formulated.

Based on (4) it is possible to derive an IMC structure as a decision policy that manipulates orders $O_{j,j+1}(z)$ to maintain inventory level at a designated inventory target $I_j^t(z)$. The following section describes how this is possible.

3. Inventory control scheme for a supply chain

Once the model for a supply chain is developed, the inventory control scheme and controller design are formulated in this section. Initially, a decentralized control strategy is designed for each echelon. In future works, a centralized control for the complete supply chain will be researched in order to perform a comparative analysis (centralized control vs decentralized control) in both cases with the identification of the delay.

The proposed Internal Model Control scheme for the inventory control is described below.

3.1. Internal model control structure

In this section the internal model control (IMC) is introduced as a delay compensation scheme applied to each echelon of a supply chain. Since decentralized control is applied, we initially present the internal model control scheme for an echelon. Then the control scheme is generalized for the entire supply chain.

The controller structure proposed by Tan et al. [27] is shown in Fig. 3 where $p(z)$ is the rational part of the model defined in Eq. (5). This part is related to the process of integration of material carried out in each echelon. Therefore, no uncertainty is considered in this part of the model. Also, z^{-L_j} denotes the transfer function of the real lead time. However, there would be an imperfect knowledge of the delay which implies that an estimated delay $\hat{L}_j$, rather than the actual delay is used for control purposes is the transfer function of the modeled lead time.

The proposed structure has three controllers for each echelon, $q_j^s(z)$, $q_j^t(z)$, and $q_j^{ld}(z)$, each having a distinctive use and influence on the overall closed loop response:

1. $q_j^s(z)$ is used to stabilize $p(z)$, the original (unstable) plant, ignoring the time-delay.
2. $q_j^t(z)$ is an IMC controller for the stabilized model.
3. $q_j^{ld}(z)$ is designed mainly to achieve the internal stability and load disturbance rejection (rejection to the orders placed from the immediate customer $O_{j-1,j}(z)$ in this case). Note that the orders $O_{j-1,j}(z)$ are considered as a perturbation since a decentralized control strategy is used instead of a centralized control where all orders vector $\mathbf{O}(z)$ will be designed simultaneously.

In the IMC scheme, when the model of lead time is exact ($z^{-L_j}=z^{-\hat{L}_j}$) and there is no disturbance signal (in the supply chain case, when there is no demand signal $O_{j-1,j}(z)=0$), then the outputs $I_j(z)$ and $\hat{I}_j(z)$ are the same and the feedback signal $d_j(z)$ shown in Fig. 3 is zero. Thus, in this control system the lead time becomes external when there is no disturbance and no plant/model delay mismatch: the scheme compensates the delay. In order to point out this property, the equation of inventory balance for a single echelon j under this scheme is obtained and represented by

$$I_j(z)=\left\{\frac{q_j^t(z)\left(\dfrac{p(z)}{1+q_j^s(z)p_j(z)}\right)z^{-L_j}(1+q_j^{ld}(z)p(z)z^{-\hat{L}_j})}{q_j^t(z)\left(\dfrac{p(z)}{1+q_j^s(z)p_j(z)}\right)(z^{-L_j}-z^{-\hat{L}_j})+(1+q_j^{ld}(z)p(z)z^{-\hat{L}_j})}\right\}I_j^t(z)$$
$$-\left\{\frac{\left[1-q_j^t(z)\left(\dfrac{p(z)}{1+q_j^s(z)p_j(z)}\right)z^{-\hat{L}_j}\right]p(z)}{1+q_j^{ld}(z)p(z)z^{-L_j}+q_j^t(z)\left(\dfrac{p(z)}{1+q_j^s(z)p_j(z)}\right)(z^{-L_j}-z^{-\hat{L}_j})}\right\}O_{j-1,j}(z). \quad (16)$$

Now, let us obtain the transfer function of the internal closed loop shown into the square in Fig. 3 and appearing in as a block in Eq. (16):

$$g(z)=\frac{p(z)}{1+q_j^s(z)p_j(z)}. \quad (17)$$

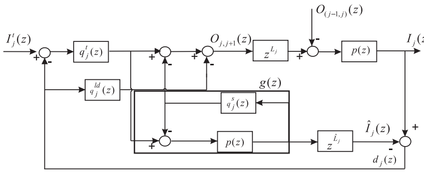

Fig. 3. The block diagram of supply chain.

From Eq. (17) it follows that q_j^s should be selected to stabilize $p(z)$. Then, replacing this expression in the Eq. (16):

$$I_j(z)=\left\{\frac{q_j^t(z)g(z)z^{-L_j}[1+q_j^{ld}(z)p(z)z^{-\hat{L}_j}]}{\underbrace{q_j^t(z)g(z)[z^{-L_j}-z^{-\hat{L}_j}]}_{x}+[1+q_j^{ld}(z)p(z)z^{-\hat{L}_j}]}\right\}I_j^t(z)$$
$$-\left\{\frac{[1-q_j^t(z)g(z)z^{-\hat{L}_j}]p(z)}{1+q_j^{ld}(z)p(z)z^{-L_j}+\underbrace{q_j^t(z)g(z)[z^{-L_j}-z^{-\hat{L}_j}]}_{y}}\right\}O_{j-1,j}(z). \quad (18)$$

It can be seen in Eq. (18), that, if the lead time model is known $z^{-L_j}=z^{-\hat{L}_j}$, then the expressions x and y are zero. Thus, we get

$$I_j(z)=T_jI_j^t(z)-S_jO_{j-1,j}(z), \quad (19)$$

where

$$T_j(z)=q_j^t(z)g(z)z^{-L_j} \quad (20)$$

and

$$S_j(z)=[1-q_j^t(z)g(z)z^{-\hat{L}_j}]\frac{p(z)}{1+q_j^{ld}(z)p(z)z^{-L_j}}. \quad (21)$$

From (20) it follows that the lead time has disappeared from the denominator of the relation from target $I_j^t(z)$ to $I_j(z)$. Therefore, if there is no disturbance signal (the demand of the immediate customer $O_{j-1,j}(z)=0$ in the supply chain case) the lead time has been decoupled of the rational part from the system. Thus, the tuning effort of the controller $q_j^t(z)$ can be reduced considerably because it depends only of the rational part of the system. Besides, it can be seen that $q_j^{ld}(z)$ only appears in the relation from $O_{j-1,j}(z)$ to $I_j(z)$. Thus, the problem of disturbance rejection can be tackled separately by tuning $q_j^{ld}(z)$.

Once the IMC scheme is formulated for a particular echelon (16)–(21) we can reproduce it for each echelon that belong to the supply chain. The general control system resulting for the complete supply chain is formulated as a diagonal matrix corresponding to the decentralized control case given by

$$\mathbf{I}(z)=Diag[T_1,\ldots,T_{(N-1)}]\mathbf{I}^t(z)-Diag[S_1,\ldots,S_{N-1}]\mathbf{O}(z), \quad (22)$$

where $\mathbf{I}^t(z)=[I_1^t,\ldots,I_{(N-1)}^t]^T$ denotes the vector of z-transform of the target inventories. At this point, assuming that the lead time is known, the rules for tuning the controllers are described below.

3.1.1. Controllers design

Initially, the procedure consists on stabilize $p(z)$. For simplicity, a proportional controller $q_j^s(z)=q_j^s\in\mathbb{R}$ is a common choice in many works for this purpose ([27]). Then, replacing $q_j^s(z)=q_j^s$ in Eq. (17) yields:

$$g(z)=\frac{z}{z(1+q_j^s)-1}, \quad (23)$$

which is a biproper transfer function with a characteristic equation given by

$$z(1+q_j^s)-1=0. \quad (24)$$

A sampled-data system is stable if all the roots of the characteristic equation lie within the unit open circle:

$$z(1+q_j^s)-1=0 \Rightarrow |z| = \left|\frac{1}{1+q_j^s}\right| < 1 \Rightarrow |1+q_j^s| > 1. \quad (25)$$

The condition expressed in Eq. (25) can be fulfilled by selecting $q_j^s > 0$.

Once the model is stabilized, the q_j^t controller is designed in the following two steps:

i. Design for nominal optimal performance: here, the IMC controller $q_j^t(z)$ is designed for inventory target tracking solving the following H_2-optimal problem formulated as

$$\min_{q^t(z)} \|g(z)q^t(z)\|_2. \quad (26)$$

Since $g(z)$ is stable, inversely stable and biproper, then its inverse $q_j^t(z)=g^{-1}(z)$ is an acceptable solution for this problem, Morari and Zafiriou [28].

ii. For robust stability: $q_j^t(z)$ is augmented with a low-pass filter $f(z)$ which is selected to detune the nominal performance but satisfying a grade of robustness to stability to some uncertainty in the parameters of the system or the delay. It is carried out with the adjustable real parameter λ. The structure of this filter is given by

$$f(z)=\frac{(1-\lambda)z}{z-\lambda}. \quad (27)$$

In the supply chain model, the rational part is usually completely known but the lead time may not be known with accuracy [11]. In fact, the Lead-time is normally fuzzy or stochastic in nature [29]. Hence, we focus on the identification on the delay.

In this paper, an online algorithm to identify the lead time is proposed to overcome the problem of delay imperfect knowledge. In this way, the delay identification is able to achieve perfect delay compensation and this problem is separated from the controller design procedure. Thereby, since the problem of the delay knowledge is overcome, and the rational part is known, the detuning of the optimal controller by the filter $f(z)$ can be done for bullwhip effect mitigation. The adjusting of these controllers for bullwhip effect mitigation in the entire supply chain is subject of future research.

The presented approach to identify the lead time is based on a multi-model scheme introduced in [21,22] for a SISO system. Basically, the scheme is composed of a battery of different models operating in parallel. Each model possesses the same rational component but a different time-varying value for the delay. A supervisory algorithm compares the mismatch between the actual system and each candidate model and selects, at each time instant, the one that best describes the behavior of the real system. Then, the set of models is updated providing an estimation of the delays of the system. An additional switching block selects the best model for control purposes, resulting an adaptive controller. The structure of the identification algorithm is described in the next section.

4. Proposed identification scheme

The basic structure of the proposed scheme is depicted in Fig. 4. Here, $\hat{\mathbf{P}}_L^{(l)}(z)$ with $l=1,2,\ldots,n_e$ denotes the matrix of transfer function associated with the different delay models where n_e represents the number of candidate models taken into account.

As it can be seen, the scheme is composed of three elements: a set of time-varying candidate models associated with different values for the delays, a figure of merit (performance or supervisory index) which evaluates the potential behavior of each model and a switching rule which monitors periodically this index and decides which model is the best to be used for control purposes. The switching mechanism is intended to reduce the possible mismatch between the nominal and the actual output of the system. In the following subsections the different elements of the presented architecture are considered in detail.

4.1. Set of nominal models

The proposed architecture is composed of a set of candidate models running in parallel, each one associated with a different

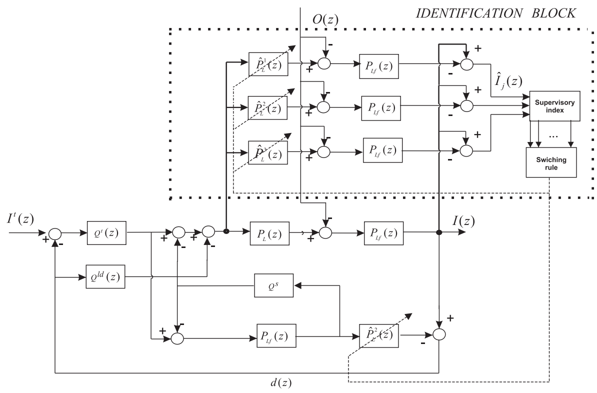

Fig. 4. The block diagram of identification and control for the complete supply chain.

delay matrix. These models are time-varying and automatically adjusted by the system. Therefore, the designer is exempted from having to define the exact values for the models. In order to describe the structure of the set of models, let us denote $M(t)$ as the set of models and its evolution through the time. It is considered a matrix $\hat{\mathbf{H}}(t)$ which contains the current nominal delays of $\hat{\mathbf{P}}_L(z)$ used for control purposes (see, Eq. (12)). It can be seen from Eq. (12), that for the supply chain case the delay appear in a superior diagonal. Then, the matrix $\hat{\mathbf{H}}$ has the respective delay components in a superior diagonal as is illustrated in Table 1.

Let $\Delta\hat{\mathbf{H}}^{inf}(t) \geq 0$, $\Delta\hat{\mathbf{H}}^{sup}(t) \geq 0$ and Γ be the upper and lower variations matrix, and the reduction factors matrix, respectively, also illustrated in (1).

Using the above defined matrices, the following ordered sets of nominal models are introduced:

$$M^-(t) = \{\hat{\mathbf{H}}(t) - \Delta\hat{\mathbf{H}}^{inf}(t)E_{i,i+1}\}, \quad i = 1,2\ldots N, \tag{28}$$

$$M^+(t) = \{\hat{\mathbf{H}}(z) + \Delta\hat{\mathbf{H}}^{sup}(t)E_{i,i+1}\}, \quad i = 1,2\ldots N, \tag{29}$$

$$M(t) = M^-(t) \cup \{\hat{\mathbf{H}}(t)\} \cup M^+(t), \tag{30}$$

where $E_{i,i+1}$ denotes the elements of the canonical basis of $\mathbb{R}^{(N-1)\times N}$ (matrices having a one in the $(i,i+1)$th entry and zeros elsewhere, Sriver et al. [24]).

From Eqs. (28)–(30) the time-dependent set of models $M(t)$ is formed by adding $\Delta\hat{\mathbf{H}}^{sup}(t)$ and subtracting $\Delta\hat{\mathbf{H}}^{inf}(t)$ to the nominal delay matrix $\hat{\mathbf{H}}(t)$ used for control purposes at time t.

4.2. Figure of merit

The second element of the proposed scheme is a figure of merit aimed at evaluating the behavior of each model in the set M. The suggested figure of merit is

$$J^{(l)}(\mathbf{H}) = \sum_{t-Tres}^{t} (I(\tau) - \hat{I}_{(i)}(\tau))^T (I(\tau) - \hat{I}_{(i)}(\tau)), \tag{31}$$

with $l = 1,2,\ldots,n_e$ and $I(\tau)$ is the vector output of the plant at the instant $t = \tau$ while $\hat{I}(\tau)$ denotes the (vector) output of each different model. The summation (31) takes place in the time interval in which all the models act simultaneously without being modified. T_{res} is the so-called residence time and defines the window where the different models are compared. Note that the figure of merit Eq. (31) is positive, i.e. $J^{(l)} \geq 0$, $\forall l = 1,2,\ldots,n_e$. The residence time is a positive number large enough be able to distinguish between the different models in parallel but, as shown in Section 5.1, its concrete value does not modify the identification properties of the algorithm. In Ibeas et al. [30], the reader can find alternative figures of merit that can be generalized to the multivariable case straightforwardly.

Table 1
The nominal delays $\hat{H}$, the upper $\hat{H}^{sup}$ and lower $\hat{H}^{inf}$ variations matrices and the reduction factors matrix Γ.

$\hat{\mathbf{H}}$	$\Delta\hat{\mathbf{H}}^{(sup)}$
$\begin{pmatrix} 0 & \hat{L}_1 & 0 & \cdots & 0 \\ 0 & \ddots & \hat{L}_2 & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 & \hat{L}_N \end{pmatrix}_{(N-1)\times(N)}$	$\begin{pmatrix} 0 & \Delta L_1^{(sup)} & 0 & \cdots & 0 \\ 0 & 0 & \Delta L_2^{(sup)} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \Delta L_N^{(sup)} \end{pmatrix}_{(N-1)\times(N)}$
Γ	$\Delta\hat{\mathbf{H}}^{(inf)}$
$\begin{pmatrix} 0 & \gamma_1 & 0 & 0 & 0 \\ 0 & \ddots & \gamma_2 & 0 & 0 \\ \vdots & \ddots & \ddots & 0 & 0 \\ 0 & \cdots & 0 & 0 & \gamma_N \end{pmatrix}_{(N-1)\times(N)}$	$\begin{pmatrix} 0 & \Delta L_1^{(inf)} & 0 & \cdots & 0 \\ 0 & 0 & \Delta L_2^{(inf)} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \Delta L_N^{(inf)} \end{pmatrix}_{(N-1)\times(N)}$

Defining the error between the plant output and the output of the lth model as

$$\mathbf{e}^{(l)}(t) \doteq \mathbf{I}(t) - \hat{\mathbf{I}}^{(l)}(t). \tag{32}$$

The z-transform of the error Eq. (32) is given by

$$\mathbf{E}(z)^{(l)} = \mathbf{P}_{Lf}(z)\bullet\mathbf{P}_L(z) - \mathbf{P}_{Lf}(z)\bullet\hat{\mathbf{P}}_L^{(l)}(z) = \mathbf{P}_{Lf}(z)\bullet(\mathbf{P}_L(z) - \hat{\mathbf{P}}_L^{(l)}(z)). \tag{33}$$

It is readily seen from Eq. (33) that the error Eq. (32) is zero when $\mathbf{P}_L(z) - \hat{\mathbf{P}}_L^{(l)}(z) = 0$, that is, when each modeled delay $\hat{L}_j^{(l)} \in \hat{\mathbf{P}}_L^{(l)}(z)$ is equal to each real delay $L_j \in \mathbf{P}_L(z)$. Thus, the identification algorithm acts as an optimization algorithm seeking the minimum of Eq. (31). This fact motivates the algorithm in the following subsection.

4.3. Switching logic

The switching logic monitors the value of the figure of merit at times instants multiples of T_{res} and selects the nominal delay which is the best estimation of the real one.

The initial nominal model is selected by the designer with the initialization of $\hat{\mathbf{H}}(0)$ and $\hat{H}^{sup/inf}(0)$. From this moment onwards, the switching logic can be expressed formally as the Algorithm 1.

Initially, the algorithm takes a set of data from the outputs $\mathbf{I}(t)$ and the inputs $\mathbf{O}(t)$ of the system. The length of this window depends on the T_{res}. Then, the **New_ models_ calculation** function is called until $\Delta\hat{H}^{sup} > 1 \,\&\, \Delta\hat{H}^{inf} > 1$ which implies that the Algorithm 1 has converged to the actual lead-times. In this function, the comparisons between the different models are carried out in groups of three consisting of the nominal, plus the additive and subtractive disturbances in the direction of $E_{i,i+1}$ (line 2 of the **New_models_calculation** function). Then, the output of the three candidate models are evaluated (lines from 4 to 8). These models (which only differ in the $(i,i+1)$th component of the matrix delay) are compared to each other using the performance index (31), used in line 10. As a result, the following outcomes are possible. If $Mc=1$ means that the model with the subtractive disturbance has been selected while $Mc=2$ stands for the previous nominal one and $Mc=3$ for the model with the additive disturbance. In this way the element associated with the lowest value of the figure of merit is obtained for each delay component, while the vector ν is the vector sum necessary to convert the nominal model into the best model of $M(t)$.

At the end, the vector ν results into the optimum direction change vector, allowing the update of the current nominal model so as to make it the best possible one. This vector is added to the nominal model to obtain the corresponding new nominal model from which a battery of models that will operate in parallel during the next interval of residence is generated again through Eqs. (28)–(30). To rebuilt the set $M(t)$, the search patterns $\hat{H}^{sup}_{i,i+1}(t), \hat{H}^{inf}_{i,i+1}(t)$ are adjusted (lines 12 to 22 of the function **New_models_calculation**), to ensure the convergence to the true delay taking a reduction factor that depends on the new nominal model value and reduction matrix Γ. This guarantees that all the search patterns converge to the same value and hence the scheme tends to a time invariant system with the real delays. After that, the function returns to the Algorithm 1 for a new period of identification.

Each element of the lead times matrix is estimated at each integer multiple of the residence time (line 9 of Algorithm 1).

Algorithm 1. MIMO identification.

$\hat{H}$: Matrix of delays.
$\Delta\hat{H}^{sup}$: Matrix upper-bound variation.
$\Delta\hat{H}^{inf}$: Matrix lower-bound Variation.
$\underline{H},\overline{H}$ Intervals of uncertainty of delays.
$\Gamma=1$: Matrix reduction factors for $\Delta\hat{H}$.
$T_{res}>0$: Residence time.
$v\leftarrow 0$
$M(0)=M^{-}(0)\cup\hat{H}(0)M^{+}(0)$
for $t>0$, At multiples of the residence time **do**.
  if $t=nT_{res},\ m\in\mathbb{N}$ **then**
    $\mathbf{Y}=[\mathbf{I}(t-1),\mathbf{I}(t-1),\ldots,\mathbf{I}(t-Tres)]$
    $\mathbf{X}=[\mathbf{U}(t-1),\mathbf{U}(t-1),\ldots,\mathbf{U}(t-Tres)]$
    $\mathbf{Z}=[\mathbf{O}(t-1),\mathbf{O}(t-1),\ldots,\mathbf{O}(t-Tres)]$
while $\Delta\hat{H}^{sup}>1\&\Delta\hat{\mathrm{H}}^{inf}>1$
New_ models_calculation
end while
end for

New_models_calculation
for $1\leq j\leq N$ **do**
  $M_c(t)\leftarrow\{\hat{H}_{ij}(t)-\Delta\hat{H}_{ij}^{inf}(t),\hat{H}_{j,j+1}(t),\hat{H}_{j,j+1}(t)+\Delta\hat{L}^{sup}(t)\}$
end for
for $0<i<3,0<h<T_{res}$
  $I_j^{(1)}(t)=I^{(1)}(t-1)+\mathbf{X}(t-\hat{H}_{j,j+1}^{sup})-\mathbf{Z}(t)$
  $I_j^{(2)}(t)=I^{(2)}(t-1)+\mathbf{X}(t-\hat{H}_{j,j+1})\mathbf{Z}(t)$
  $I_j^{(3)}(t)=I^{(3)}(t-1)+\mathbf{X}(t-\hat{H}_{j,j+1}^{inf})-\mathbf{Z}(t)$
end for
  for $1\leq j\leq N$ **do**
    $pos\leftarrow argmin_{1\leq k\leq 3}J_{ij}^{(k)}$
    $v\leftarrow v+(-2+pos)E_{ij}$
    if Mc=1 **then**
      $\Delta\hat{H}_{i,(i+1)}^{inf}(t)\leftarrow(2\Delta\hat{H}_{i(i+1)}^{inf}(t)+\gamma_{j(j+1)})$
      $\Delta\hat{H}_{j,(j+1)}^{sup}(t)\leftarrow(\Delta\hat{H}_{i(i+1)}^{sup}(t)+\gamma_{i(i+1)})$
    end if
    if Mc=2 **then**
      $\Delta\hat{H}_{i,(i+1)}^{inf}(t)\leftarrow(\Delta\hat{H}_{i(i+1)}^{inf}(t)+\gamma_{i(i+1)})$
      $\Delta\hat{H}_{i(i+1)}^{sup}(t)\leftarrow(\Delta\hat{H}_{i(i+1)}^{inf}(t)+\gamma_{i(i+1)})$
    end if
    if Mc=3 **then**
      $\Delta\hat{H}_{j,(j+1)}^{inf}(t)\leftarrow\Delta\hat{H}_{i(i+1)}^{inf}(t)+\gamma_{j,(j+1)}$
      $\Delta\hat{H}_{i(i+1)}^{sup}(t)\leftarrow(2\Delta\hat{H}_{i(i+1)}^{sup}(t)+\gamma_{i(i+1)})$
    end if
  end for
$\hat{H}(t)=proy_{\underline{H}_{i(i+1)},\overline{H}_{i(i+1)}}(\hat{H}(t)+v)$
$M(t)=M^{-}(t)\cup\{\hat{H}(z)\}\cup M^{+}(t)$
Return to Algorithm 1

5. Theoretical results

Once the adaptive control approach based on the identification of the lead-time for inventory control in the supply chain is formulated, we will develop the theoretical evaluation of the system. It consist in prove that the delay identification scheme is able to find the real lead times on the entire supply chain. After that, the stability of the closed loop system, provided that Assumptions 1 and 2 hold, is verified.

5.1. Results on convergence of the identification scheme

This section contains the theoretical basis to guarantee that the Algorithm 1 identifies the real lead-time matrix. In this way the convergence theorem can be formulated.

In the first step, the conditions to guarantee that Eq. (31) has a unique global minimum for $\hat{\mathbf{H}}=\mathbf{H}$ are stated. Having established that the real delay is the only global minimum of the figure of merit, the second step is to prove that the proposed Algorithm 1 is able to asymptotically find the global minimum of $J(\hat{H})$. The following Assumption 3 is used in the subsequent Lemma 5.1.

Assumption 3. The signal $O_{j-1,j}(t)$ is aperiodic in the interval $(t-Tres,t)$, i.e. $O_{j-1,j}(t-\mu)\neq O(t)$ where $\mu\in(0,Tres)$. □

Since the orders in the supply chain can be stochastic in nature, this assumption is feasible.

Lemma 5.1. *The global minimum of Eq.* (31) *is given by* $J=0$, *and this happens when the lead times of all the models are equal to the real ones, i.e.* $\hat{H}=H$, *provided that Assumption* 3 *holds and the initial estimated inventories are equal to the actual ones.*

The proof of this lemma is provided in Appendix A.1. Note that the initial condition of the estimated model and the real one being the same is a feasible condition since the inventory level is known at $t=0$ for all the supply chain echelons.

Once it has been established that the real delay is the unique global minimum of the figure of merit, the second step is prove that the Algorithm 1 is able to asymptotically find this global minimum of $J(\hat{H})$. In this way, the following theorem is formulated:

Theorem 5.2. *Consider the delayed supply chain system given by* (10) *and* (11) *satisfying Assumptions* 1, 2 *and* 3. *Thus, the multi-model based Algorithm*1 *can identify the real delays when the reduction matrix* $\Gamma=1$.

The proof of this theorem is developed in Appendix A.2. Although in the proof it is necessary that $\Gamma=1$, several simulations have shown that a greater value of this is able in practice to identify the lead times.

5.2. Stability properties

In this section we will prove that the scheme depicted in Fig. 4 is stable in closed-loop. From Eq. (18), it can be seen that when the lead time is perfectly compensated, i.e. $z^{-L}-z^{-\hat{L}}=0$ the stability of the system is determined by $g(z),q^{t}(z)$ which are stable. Therefore, the stability of the closed-loop system depends on the perfect lead time compensation. Since we advocate for a decentralized control the stability criteria can be extended to the general MIMO system. Thus, the following stability theorem can be formulated.

Theorem 5.3. *The control system depicted in Fig.* 4 *obtained from Eqs.* (7), (28)–(30) *through Algorithm* 1 *is asymptotically stable provided that Assumptions*1 *and* 2 *hold, and* $\gamma=1$.

The proof of this theorem is shown in Appendix A.3. The following section shows the usefulness of the presented scheme through simulation examples.

6. Simulation results

The basic supply chain composed by three echelons which was represented in Eqs. (10) and (11) is simulated in order to show the feasibility and usefulness of the proposed schemes. Two situations of lead time behavior will be taken into account. First, let us consider one change in the lead time for the deliberation of goods in each echelon under a deterministic demand. In the second example, it is considered that the real lead times values L_j change in several time instants in order to demonstrate that the Algorithm 1 converges to the true lead time quickly, so that the control scheme is adapted on time and thereby oscillations in the system are minimal. In this second example a stochastic customer demand is introduced to provide reality to the simulations.

6.1. Evaluation of the system under changes in the lead time and deterministic demand

In this case, the performance of the scheme when the actual lead times change online (which is a very common situation in many production inventory control systems) [11] is evaluated. It is consider a deterministic step change of customer demand from $t=10$ and onwards. The actual lead times are $L_1=5$, $L_2=5$, $L_3=10$. Then, a sudden step change of lead times is introduced at time instance $t=100$, setting the lead times value to $L_1=10$, $L_2=10$, $L_3=5$, as is illustrated in Fig. 5. The algorithm starts from time $t=1$ and the changes in the lead times are identified at $t=100+T_{res}$. In our simulation we chose the residence time $T_{res}=10$ which is the time window where the models are evaluated. Fig. 5 shows that the real lead time is identified with accuracy despite changes happening online. Respect the inventory control scheme behaviour, some schemes analyzed in the literature [11] present an inventory deficit as soon as the lead time is modified, in particular the Adaptive Automatic Pipeline, Inventory and Order Based Production Control System (AAPIOBPCS) is proposed by Aggelogiannaki and Sarimveis [11]. That work detects the change and updates the controller parameter in order to correct the inventory deficit. However, once produced the change in the lead time, this system presents oscillations over a long time (Approximately 100 periods of time) in the order and inventory signals. It can be seen in Fig. 6a that there are no deficit in the inventories due to changes in the lead time because of the each lead time L_j of the system is identified timely and the lead times used for control purposes $\hat{L}_j$ are adapted in the IMC scheme. Fig. 6(b) shows that changes in the lead time are not cause of distortion in the orders $O_{j,j+1}(t)$.

Thus, the presented approach offers a better behaviour of the system than previous.

6.2. Several changes in the lead times under a stochastic demand

This case corresponds to the situation where there is a strong case of variability in the lead times on the system. For the sake of simplicity, in this simulation at t=0 the modeled lead times coincide with the real lead times, i.e. $\hat{L}_1=L_1=4$, $\hat{L}_2=L_2=6$, $\hat{L}_3=L_3=3$. Then, multiples changes in the lead times in different periods of time are contemplated. That is, at time $t=40$ the real lead times change to $L_1=5$, $L_2=4$, $L_3=5$, at time $t=140$ the real lead times change to $L_1=7$, $L_2=3$, $L_3=8$, at time $t=240$ the real lead times change to $L_1=4$, $L_2=4$, $L_3=6$, and finally at time $t=340$ the real lead times change to $L_1=7$, $L_2=3$, $L_3=8$, at time $t=240$ the real lead times change to $L_1=3$, $L_2=5$, $L_3=5$. It is better illustrated in Fig. 7. A stochastic variability in the customer demand $O_{0,1}(z)$ is also applied to the system from time instance $t=10$ and onwards, in order to provide reality to the simulation. Customer demand is formulated as a normal function, with a average equal to 20 and a variance equal to 4, i.e. $\in N(20,4)$. Again all real lead times involved in the supply chain are identified with accuracy as is shown in Figs. 7 and 8 proves that the system is robust to changes in the lead time, and at the same time is able to present a well performance to a stochastic customer demand. There are some fluctuations while there is a mismatch in the lead time but once the lead time is identified the scheme correct timely these fluctuations. Fig. 9 evidence that the faster tracking of the lead time allows that its changes do not affect the behavior of the orders in the complete supply chain.

The presented approach presents an improvement in the inventory control in comparison to previous works.

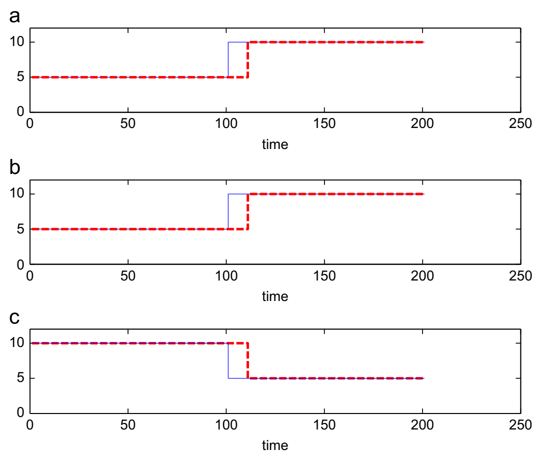

Fig. 5. (a) Solid line: real lead time L_1. Dash line: identified lead time $\hat{L}_1$. (b) Solid line: real lead time L_2. Dash line: identified lead time $\hat{L}_2$. (c) Solid line: real lead time L_3. Dash line: identified lead time $\hat{L}_3$.

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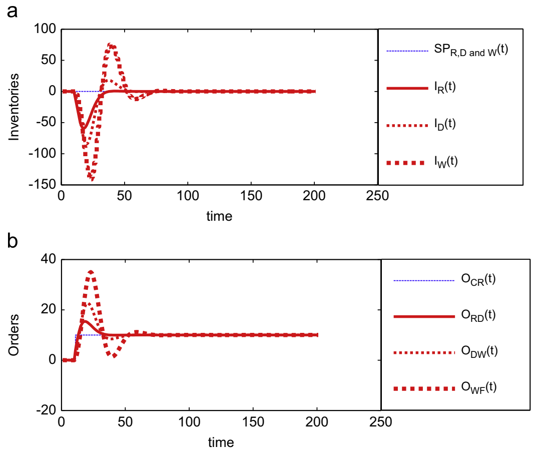

Fig. 6. Dynamic simulation results in all echelons with the proposed scheme under a constant customer demand. (a) The supply chain inventories. (b) Orders generated in the supply chain.

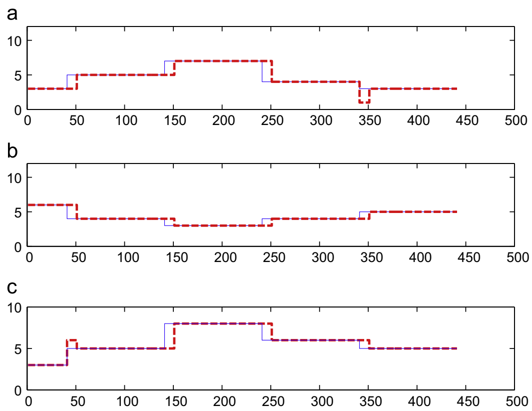

Fig. 7. (a) solid line: real lead time L_1. Dash line: identified lead time $\hat{L}_1$. (b) Solid line: real lead time L_2. Dash line: identified lead time $\hat{L}_2$. (c) Solid line: real lead time L_3. Dash line: identified lead time $\hat{L}_3$.

7. Conclusions and future research

In this paper, a decentralized inventory control approach based on an IMC delay compensation scheme for the MIMO supply chain is formulated. In this approach, each echelon takes decisions based on global values of the demand. Therefore, all customers demand are lumped into a single aggregate demand signal. In the IMC delay compensation scheme, the system's delay has to be known beforehand to perform its perfect compensation. However, this situation is not viable for the supply chain problem since delay may be unknown or even time-varying. A solution to this problem is to perform an identification of the delay using subsequently the estimated value to generate the control law. Thus, a delay identification algorithm is proposed in this work for the complete supply chain being able of identifying the delays between the different echelons describing the supply chain. The identified values of the delays are then used to adjust the delay compensation in a IMC based decentralized compensation scheme. The set of models is updated through time following a Pattern Search based algorithm which is a novel application in control and supply chain systems. The pattern search algorithm is implemented for practical purposes using a multimodel scheme. The convergence properties of the algorithm to the actual delay are proved and simulation examples have shown that the

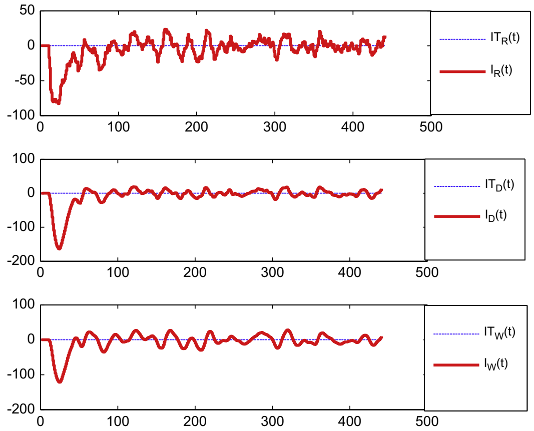

Fig. 8. Dynamic simulation results of inventories in all echelons with the proposed scheme under a constant customer demand.

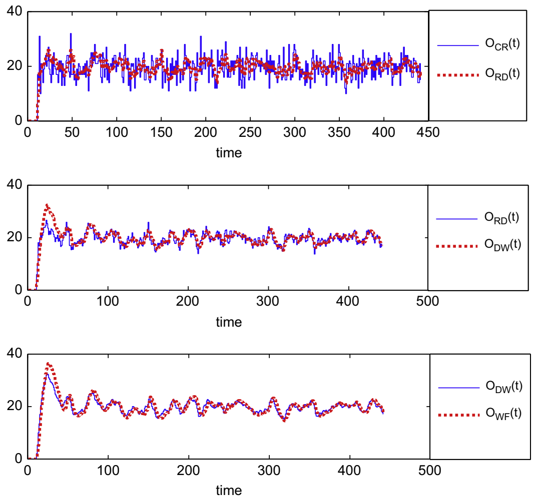

Fig. 9. Dynamic simulation results of orders in all echelons with the proposed scheme under a constant customer demand.

algorithm identifies the complete set of lead times of the supply chain timely improving the performance of the inventory control. The IMC delay compensation control structure complemented with the delay identification scheme has been applied in a serial supply chain with good results. Therefore, future works can be focused on the application of this scheme with a centralized control strategy where each demand signal is considered separately. Moreover, [31,32] have considered more complex supply chain models where multiple delays and cooperation between echelons are also considered. The mathematical model for this kind of systems is a general matrix instead of the block-diagonal one considered in this work. Hence, the delay mismatch becomes a harder problem in this situation. Therefore, the application of IMC strategies augmented with the lead time identification algorithm would be a promising way in order to improve the overall response of the system. Furthermore, the problem can be complicated assuming the lead times being sluggishly variable in the time.

Acknowledgements

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Appendix A. Proofs of lemmas and theorems contained in the paper

In this section we develop the theoretical proofs of the Lead time identification method and the stability of the closed-loop system.

A.1. Proof of Lemma 5.1

The figure of merit is evaluated on a window $(t-Tres,t)$ for a general model given by $\mathbf{H}$:

$$J(\hat{\mathbf{H}}) = \sum_{t-Tres}^{t} \mathbf{e}^T(\tau)\mathbf{e}(\tau), \tag{A.1}$$

where

$$\mathbf{e}^T(t)\mathbf{e}(t) = ((I_1(t)-\hat{I}_1)\ (I_j(t)-I_j(t))\ \cdots\ (I_N(t)-I_N(t))) \begin{pmatrix} (I_1(t)-I_1(t)) \\ (I_j(t)-I_j(t)) \\ \vdots \\ (I_N(t)-I_N(t)) \end{pmatrix}, \tag{A.2}$$

$$\mathbf{e}^T(t)\mathbf{e}(t) = (I_1(t)-\hat{I}_1)^2(I_j(t)-I_j(t))^2,\ldots,(I_N(t)-I_N(t))^2.$$

Then,

$$J^{(l)} = \sum_{t-Tres}^{t} (I_1(t)-\hat{I}_1(t))^2 + (I_j(t)-\hat{I}_j(t))^2 + \cdots + (I_N(t)-\hat{I}_N(t))^2 \geq 0, \tag{A.3}$$

where N denotes the number of entities involved in the supply chain in a general frame. Since Eq. (A.3) is a sum of squares function, $J=0$ is the unique possible minimum of this equation and it can only be identically zero if each one of the addends of Eq. (A.3) is zero, i.e. $(I_j(t)-\hat{I}_j(t))=0\ \forall j$ which implies that

$$I_j(t) = \hat{I}_j(t). \tag{A.4}$$

Then, according to Eq. (2), the real inventory at any discrete time t is given by

$$I_j(t) = I_j(t-1) + O_{j,j+1}(t-L_j) - O_{j-1,j}(t) \tag{A.5}$$

and its respective model of inventory at time t is

$$\hat{I}_j(t) = \hat{I}_j(t-1) + O_{j,j+1}(t-\hat{L}_j) - O_{j-1,j}(t). \tag{A.6}$$

Thus, Eq. (A.4) becomes

$$\hat{I}_j(t-1) + O_j(t-L_j) = \hat{I}_j(t-1) + O_j(t-\hat{L}_j). \tag{A.7}$$

Thus, if the initial condition of the model coincides with the initial condition of the real plant, $\hat{I}_j(0) = I_j(0)$, then the condition, $I_j(t)-\hat{I}_j(t)=0$ is fulfilled $\forall t$ if:

$$O_{j,j+1}(\tau-L_j) = O_{j,j+1}(\tau-\hat{L}_j) \quad \forall \tau \in [t-tres,t] \tag{A.8}$$

Hence, if Assumption 3 holds, then the condition Eq. (A.8) is fulfilled if:

$$L_j = \hat{L}_j, \quad j=1,2,\ldots,N \tag{A.9}$$

and the Lemma 5.1 is proved. □

A.2. Proof of Theorem 5.2

In order to face the proof of Theorem 5.2 we make use of induction. Since, the Algorithm 1 works echelon by echelon and each echelon contains a unique delay, it is suffices to prove that the system is able to identify the delay in one echelon and therefore, it is able to identify the delay in all of them.

Hence, it will be proved that if $L^* = L_j$ (we will omit the sub-index j from here onwards) being the unique global minimum of $J(L)$, contained in a search space given by: $C_r = \{L | \hat{L}_r - \Delta L_r^{inf} < \hat{L} < L_r + \Delta L_r^{sup}\}$, then, there exist an interval C_{r+1} generated by the Algorithm 1 which also contains the global minimum L^*. This interval can be generated from one of these posibilities:

$$C_{r+1} = \left\{ \begin{array}{l} C_{r+1}^1 = \{\hat{L} | \hat{L}_r - (2\Delta L_r^{inf} + \gamma) < \hat{L} < L_r + (\Delta L_r^{sup} + \gamma)\} \\ C_{r+1}^2 = \{\hat{L} | \hat{L}_r - (\Delta L_r^{inf} + \gamma) < \hat{L} < L_r + (\Delta L_r^{sup} + \gamma)\} \\ C_{r+1}^3 = \{\hat{L} | \hat{L}_r - (\Delta L_r^{inf} + \gamma) < \hat{L} < L_r + (2\Delta L_r^{sup} + \gamma)\} \end{array} \right\} \tag{A.10}$$

depending on the argument of the minimum value of the objective function Eq. (31). Hence if $\arg\min J = \hat{L}_1 - \Delta L_1^{inf}$ then $C_{r+1} = C_{r+1}^1$, if $\arg\min J = \hat{L}_1$ then $C_{r+1} = C_{r+1}^2$, if $\arg\min J = \hat{L}_1 - \Delta L_1^{sup}$ then $C_{r+1} = C_{r+1}^3$, where $C_{r+1} \subset C_r$ in all cases. We will proceed by induction:

1. Initially, we will prove that it holds for $r=1$, namely if L^* belongs to the first interval C_1 also belongs to the next interval C_2 chosen by the algorithm provided that $gamma=1$. From Assumption 2

$$L^* \in C_1. \tag{A.11}$$

Then, the following interval C_2 is chosen by the algorithm in function of the minimum value of the figure of merit (31). In this case, it can take any of these three values:

$$\arg\min J = \left\{ \begin{array}{l} \hat{L}_1 - \Delta L_1^{inf} \\ \hat{L}_1 \\ \hat{L}_1 - \Delta L_1^{sup} \end{array} \right\}. \tag{A.12}$$

It will be proved by contradiction that L^* is still included in C_2. Namely, since $C_2 \subset C_1$ if the minimum L^* does not belong to the complemented interval formed by $N_1 = C_2 - C_1$ then the minimum belongs to C_2. Therefore,

- Assume that $\arg\min J = \hat{L}_1$. Hence,

$$J(\hat{L}_1) < J(\hat{L}_1 - \Delta\hat{L}_1) \wedge J(\hat{L}_1) < J(\hat{L}_1 + \Delta\hat{L}_1). \tag{A.13}$$

Since Assumption 3 holds and thus, Lemma 5.1 holds and the global minimum is unique. If we find the complementary interval $N1 = C_1 - C_2 =$, which is formed by the union of N_a and N_b given by

$$N_a^1 = \{\hat{L}_1 - (\Delta L_1^{inf})\}$$

$$N_b^1 = \{\hat{L}_1 + (\Delta L_1^{sup})\} \tag{A.14}$$

provided that $\gamma = 1$ (element of the matrix Γ).
We can see that both N_a^1 and N_b^1 belong to C_1. Then, we want to prove that the global minimum does not belong to N_a^1 or N_b^1 and hence it is not lost by the algorithm. Suppose that $L^* \in N_a^1$. Then, $J(L_1) > J(L_1 + \Delta L_1^{inf})$ which is a contradiction of the initial (A.13). Now, suppose that $L^* \in N_b^1$. If we use $\gamma = 1$ then, $J(L_1) > J(L_1 + \Delta L_1^{sup})$ which is a contradiction of the initial (A.13).
- Suppose now that $\arg\min J = \hat{L}_1 - \Delta L_1^{inf}$. Hence,

$$J(\hat{L}_1 - \Delta\hat{L}_1) < J(\hat{L}_1) \wedge J(\hat{L}_1 - \Delta\hat{L}_1) < J(\hat{L}_1 + \Delta\hat{L}_1) \tag{A.15}$$

then the following two complementary sets are generated:

$$N_a^2 = \{\hat{L}_1 - (\Delta L_1^{inf} + \gamma)\},$$

$$N_b^2 = \{\hat{L}_1 + (\Delta L_1^{sup})\}. \quad (A.16)$$

The complementary $N_a^2 \notin C_1$ therefore $L^* \notin N_a^2$ the complementary $N_b^2 \in C_1$ and $N_b^2 = N_b^1$ we would reach the same conclusion as before and $L^* \notin N_a^2 \cup N_b^2$.

- Suppose now that $\arg\min J = \hat{L}_1 - \Delta L_1^{sup}$. Then, similar arguments as before can be used, showing that $L^* \notin N_a^3 \cup N_3^2$ and therefore, the minimum is not lost during the evolution of the algorithm. This confirms that the result holds for $r=1$.

2. Continuing with the proof, we assume that the result holds for the case $r=k$ and it will be proved that it is also true for the case $r=k+1$. From the induction step,

$$L^* \in C_{k+1}. \quad (A.17)$$

The algorithm proposes that the following interval C_{k+2} is chosen from Eq. (A.10) in function of the minimum value of the figure of merit (31). In this case, it can take on three values:

$$\arg\min J = \begin{Bmatrix} \hat{L}_{k+1} - \Delta L_{k+1}^{inf} \\ \hat{L}_{k+1} \\ \hat{L}_{k+1} - \Delta L_{k+1}^{sup} \end{Bmatrix}. \quad (A.18)$$

The result is again proved by contradiction.

(a) Assume that $\arg\min J = \hat{L}_{k+1}$. Hence,

$$J(\hat{L}_{k+1}) < J(\hat{L}_{k+1} - \Delta\hat{L}_{k+1}) \wedge J(\hat{L}_{k+1}) < J(\hat{L}_{k+1} + \Delta\hat{L}_{k+1}). \quad (A.19)$$

Since Assumption 3 holds and thus, Lemma 5.1 holds and the global minimum is unique.
If we find the complement $N^{k+1} = C_{k+1} - C_{k+2} =$, which is formed by the union of the intervals

$$N_a^{k+1} = \{\hat{L}_{k+1} - (\Delta L_{k+1}^{inf})\}$$

$$N_b^{k+1} = \{\hat{L}_{k+1} + (\Delta L_{k+1}^{sup})\} \quad (A.20)$$

provided that $\gamma = 1$
We can see that both N_a^1 and N_b^1 belong to C_1. Then, we want to prove that the global minimum does not belong to N_a^{k+1} or N_b^{k+1} and hence, it is not lost by the algorithm. Suppose that $L^* \in N_a^{k+1}$. Then, $J(L_{k+1}) > J(L_{k+1} + \Delta L_{k+1}^{inf})$ which is a contradiction of the initial (A.19). Now, suppose that $L^* \in N_b^{k+1}$. If we use $\gamma = 1$ then, $J(L_{k+1}) > J(L_{k+1} + \Delta L_{k+1}^{sup})$ which is a contradiction of the initial assumption Eq. (A.19).

(b) For the second and third cases, corresponding to *arg* $\min J = \hat{L}_{k+1} - \Delta L_{k+1}^{inf}$ and $J = \hat{L}_{k+1} + \Delta L_{k+1}^{inf}$ similar arguments as before can be used, and are therefore omitted.

We have completed the induction phase and, consequently, proved that $L^* \in C_r$ for all $r \in \mathbb{N}$. Now, denote with l_r the length of the interval $C = [a,b], l_r = b-a$. Since $C_{r-1} \subset C_r$ it can be deduced that $l_{r-1} < l_r$ then $\lim_{r\to\infty} l_r = 0$. In fact, $l_r = 0$ in a finite number of steps which implies that C_r converges to a single point. Since $L^* \in C_r \forall r \in \mathbb{N}$ then it converges to a single point, the lead time is identified.
Therefore, Theorem 5.2 is proved. □

A.3. Proof of Theorem 5.3

The proof is made by contradiction. Suppose that the output $I(t)$ diverges. If the output diverges, the input signal $O(t)$ also diverges satisfying Assumption 3, since diverging signals cannot satisfy the above periodicity condition. Theorem 5.2 guarantees that the nominal delay converges to the actual delay, and hence $\lim_{t\to\infty} \hat{\mathbf{H}} = \mathbf{H}$. Hence, there exists a sequence $\{t_n\}_{n=0}^{\infty}$ such that $|\mathbf{H} - \hat{\mathbf{H}}(t_n)| > |\mathbf{H} - \hat{\mathbf{H}}(t_{n-1})|$ for all $n \geq 1$. Thus, there exists a finite time $t_{DiacriticalGrave;n}$ such that $\mathbf{H} - \hat{\mathbf{H}}(t_n) < \varepsilon$ for all $t_n \geq t_{DiacriticalGrave;n}$ for an arbitrary positive value of the ε, being eventually zero. Thus, the algorithm reaches in finite time the actual delay where the control scheme is stable. Consequently, the output cannot be unbounded which is a contradiction and thus, the output is bounded. Accordingly, all the signals in the loop are bounded and the Theorem 5.3 is proved. □

Thus, we have proved that the closed-loop is stable using a conceptually simple proof.

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3.2 Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized internal model control approaches

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abstract

In this paper, a two-degrees-of-freedom Internal Model Control structure is incorporated in production inventory control for a supply chain system. This scheme presents an intuitive and simple parametrization of controllers, where inventory target tracking and disturbance (demand) rejection in the inventory level problems are treated separately. Moreover, considering that the lead times are known, this scheme presents a perfect compensation of the delay making the stabilization problem easier to handle. This control structure is formulated for a serial supply chain in two ways (by using a centralized and a decentralized control approach). The behaviour of these inventory control strategies is analyzed in the entire supply chain. Analytical tuning rules for bullwhip effect avoidance are developed for both strategies. The results of controller evaluations demonstrate that centralized control approach enhances the behaviour with respect to the inventory target tracking, demand rejection and bullwhip effect in the supply chain systems.

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Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized Internal Model Control approaches

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A B S T R A C T

In this paper, a two-degrees-of-freedom Internal Model Control structure is incorporated in production inventory control for a supply chain system. This scheme presents an intuitive and simple parametrization of controllers, where inventory target tracking and disturbance (demand) rejection in the inventory level problems are treated separately. Moreover, considering that the lead times are known, this scheme presents a perfect compensation of the delay making the stabilization problem easier to handle. This control structure is formulated for a serial supply chain in two ways (by using a centralized and a decentralized control approach). The behavior of these inventory control strategies is analyzed in the entire supply chain. Analytical tuning rules for bullwhip effect avoidance are developed for both strategies. The results of controller evaluations demonstrate that centralized control approach enhances the behavior with respect to the inventory target tracking, demand rejection and bullwhip effect in the supply chain systems.

1. Introduction

A common Supply Chain (SC) includes the necessary entities to provide the customer with goods from production centers. There are a large number of participants, processes and randomness in the information flow of a supply chain. Therefore, the coordination of the supply chain becomes a key point in order to optimize the use of its resources and compete on a global scale. There are many aspects to look at in this complex network. One of these focuses on the improvement of inventory management policies. The aim of inventory management is to maintain the inventory level of each element of the supply chain in order to satisfy the demands of its customers. It is carried out ordering products from its immediate supplier of the supply chain. Thus, the supply chain is modelled as a serial process where each element orders goods to its immediate supplier. In this way, each echelon may obtain enough goods to supply the orders of its immediate customer of the chain.

Once an order is placed on the immediate supplier, there is a time to satisfy it; this is known as the replenishment lead time and consists of a time period ordering delay and a time period of physical production or distribution delay. Each participant of the supply chain stores the goods received from its immediate supplier which implies integrative dynamics. Moreover, since the entire supply chain works as a serial process whose elements are only related to its immediate downstream and upstream elements, this kind of processes can also be described as a Multiple Inputs–Multiple Outputs (MIMOs) system represented by a matrix with a block-diagonal structure.

Lead times and integrative dynamics make the inventory control task harder. Therefore, they play an important role in the design of the inventory replenishment strategies. Moreover, the MIMO nature of the supply chain system increases the complexity of the inventories control task and helps many undesirable effects appear when an inadequate inventory control policy is applied. Among these ones, instability represents the main problem, which implies that signals describing the inventory and orders can diverge as time goes on Hoberg et al. (2007).

Another inconvenient associated to an inadequate inventory policy is that the variability in the ordering patterns often increases as we move up into the chain, from the customer towards the suppliers and factory. This phenomenon is known as the *bullwhip effect*. Zhang and Burke (2011) investigate compound causes of the bullwhip effect by considering an inventory system with multiple price-sensitive demand streams.

Besides the stability and bullwhip effect issues, another major problem is the possible existence of an inventory deficit (difference between inventory target and actual inventory level), usually called inventory drift (Aggelogiannaki and Sarimveis, 2008).

In order to overcome these problems regarding production inventory and supply chain inventory management, replenishment policies based on process control theory have been successfully applied. Among them, Hoberg et al. (2007) apply linear control theory

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to study the effect of various inventory policies on order and inventory variability while their conditions for stability are examined by the Jury criteria. Dejonckheere et al. (2003), Disney and Towill (2003), and Hoberg et al. (2007), have analyzed the effect of the replenishment policies focused on the bullwhip effect estimation and suppression. Moreover, Lin et al. (2004) present approaches based on Control Engineering, including proportional–integral (PI) controllers and cascade control as inventory replenishment policies. Balan et al. (2009), Kristianto et al. (2011), and Deshpande et al. (2011) apply a soft computing approach (fuzzy logic based control) to generate inventory replenishment policies. The design of these controllers is also focused on the mitigation of the bullwhip effect. All these approaches present an acceptable inventory control performance with smooth information flow when is implemented in a single echelon. Nevertheless, the analysis of the inventory control performance of these inventory control policies on the entire supply chain is not taken into account.

Schwartz and Rivera (2010) introduce the two degrees of freedom feedback and three degrees of freedom feedback–feedfordward Internal Model Control as well as the model predictive control as a novel inventory replenishment policy in the supply chain. The simulation results of these schemes are compared with smoothing replenishment rule presented by Dejonckheere et al. (2003) showing an important improvement in the performance of manufacturing systems with a long lead time and significant uncertainty. However, analytical tuning to guarantee bullwhip effect avoidance is omitted. Moreover, this work is oriented to inventory control of a single echelon instead of a complete supply chain composed by multiple echelons.

Since the supply chain is naturally described as a multivariable system, the generalization of the inventory control strategies to the entire supply chain can yield insights to improve the inventory control in an overall scale. In this way, Yanfeng and Xiaopeng (2010) have analyzed the bullwhip effect in supply chain networks operated with linear and time-invariant inventory management policies. Perea-López et al. (2003) and Schwartz et al. (2006) propose the predictive control as replenishment inventory policy. These works show that inventory control in the entire supply chain is a current subject of research. Henceforth, in this paper we focus on the development of an inventory control strategy for the complete supply chain. The inventory control will be based on Internal Model Control (Schwartz and Rivera, 2010; Morari and Zafiriou, 1989). An advantage of the multi-degrees-of-freedom IMC topologies is that their performance to set point tracking (i.e. meeting an inventory target), measured disturbance rejection (i.e. meeting forecasted demand), unmeasured disturbance rejection (i.e. satisfying unforecasted demand) is improved by using three independent controllers avoiding trade-off between these problems. Moreover, the design guidelines for a single echelon can be extended to the design of the general controller of the MIMO system. Nevertheless, the guidelines to tune these controllers for the MIMO case so as to avoid the bullwhip effect have not been explored yet. Therefore, we advocate to design a multiple degrees-of-freedom IMC scheme (Schwartz and Rivera, 2010) for the entire supply chain (MIMO system) where the bullwhip effect is taken into account. Generally, since the demand is considered completely stochastic, a feedforward degree of freedom based in the forecast of the demand does not contribute an improvement in the behavior of the system respect to the feedback configuration (Schwartz and Rivera, 2010).

Therefore, in this work a two-degrees-of-freedom-feedback IMC scheme to tackle the set point tracking (i.e. meeting an inventory target) and unmeasured disturbance rejection (i.e., satisfying unforecasted demand) is performed for the entire supply chain.

There are two ways to perform an IMC based inventory control strategy for the entire supply chain: by using decentralized control, where an independent controller is applied to each echelon of the supply chain and by using centralized control, where a single controller is applied to the entire supply chain. The decentralized control approach is suitable for supply chains where its elements belong to different companies and do not share each others' information. On the other hand, when all or most of the supply chain elements belong to the same company or share internal information the centralized control approach would be applied.

In this work the two-degrees-of-freedom feedback IMC design for a complete supply chain is performed by applying both decentralized and centralized control strategies. Analytical guidelines to tune the controllers for bullwhip effect avoidance in the entire supply chain under centralized and decentralized inventory control strategies are also provided, which are not considered in previous works. Moreover, a comparison between the performance of both approaches is included and discussed.

The rest of the paper is formulated as follows: Section 2 presents the complete supply chain model using z-transform. As a result, a discrete Multiple Input–Multiple Output (MIMO) system is obtained. Section 3 presents the formulation of the Internal Model Control as a delay compensation scheme to control the inventory level in a single echelon of supply chains. After that, the generalization of the IMC scheme for an entire supply chain by using decentralized control strategy is presented. Section 4 presents the generalization of the IMC design for the entire supply chain with centralized control. The paper ends with the discussion and concluding remarks in Section 6.

2. Supply chain model

The model for a general supply chain is developed in this section. For the sake of simplicity, it is assumed a period base of time $T_m = 1$ which can be 1 day, 1 week or 1 month according to the dynamics of the supply chain. In this model there are N logistic echelons between the factory and the customer. The customer is considered the base while the factory is on the top of the supply chain. Thus, $j = 1,2,\ldots,N$ (where N is a finite integer) denotes each one of the intermediate logistic echelons of the supply chain, while $j = 1$ represents the retailer, $j = N + 1$ represents the factory. According to this notation, $(j + 1)$ represents an immediate supplier and $(j - 1)$ represents an immediate customer of the jth echelon. A summarized list of variables is shown below:

- $\beta_{a,b}(t)$ denotes the amount of goods delivered by each logistic node a to the node b.
- $y_j(t)$ is the inventory level of the jth echelon at any discrete time instant $t = nT_m$ where n is a natural number.
- $o_{j,j+1}(t)$ represents the order placed by the jth echelon to its immediate supplier $j,j + 1$.
- $o_{j-1,j}(t)$ represents the order perceived by the jth echelon from its immediate downstream echelon $j - 1$, where $j - 1 > 0$.
- $d_j(t)$ is the demand perceived by the jth echelon from external customers.

Thus, the inventory balance in each echelon is given by the difference between the goods received from the immediate supplier and the goods delivered to the immediate customer as follows:

$$y_j(t) = y_j(t-1) + \beta_{j+1,j}(t) - \beta_{j,j-1}(t), \quad j = 1,2,\ldots,N \tag{1}$$

A lead time $L_j \in \mathbb{N}$ is considered between the time when an order is placed by node jth and the time when the goods are received from the immediate supplier (Amini and Li, 2011; Aggelogiannaki and Sarimveis, 2008; Dejonckheere et al., 2003). It is also assumed that each node has enough existences to satisfy the demand of its immediate customer. In this way, the amount of goods ordered to an immediate supplier at time t will arrive at time $t + L_j$ i.e. $\beta_{j+1,j}(t) = o_{j,j+1}(t - L_j)$. Therefore, the sequence of events in the supply chain is the following:

i. At each discrete time t, the echelon jth receives the goods ordered L_j periods ago.
ii. The demand $d_j(t)$ is observed and satisfied immediately i.e. $\beta_{j,j-1}(t) = d_j(t)$ (i.e. there is no backlogged orders).
iii. The new inventory level of each echelon $y_j(t)$, is observed.
iv. Finally, an order $o_{j,j+1}(t)$ is placed on the $(j+1)$th level (upstream) according to the values of the inventory levels, $y_j(t)$. The order-up-to-level replenishment policy based on the Two-degrees-of-freedom feedback IMC scheme is stated in Section 3.

Thus, the Eq. (1) that relates the inventory balance with the demand $d_j(t)$ and order $o_{j,j+1}(t)$ at node j becomes now:

$$y_j(t) = y_j(t-1) + o_{j,j+1}(t - L_j) - d_j(t), \quad j = 1, 2, \ldots, N \tag{2}$$

Eq. (2) is a difference equation which can be solved directly in the time domain or by using transformation techniques. In particular, the z-transform is the most extended one among transformations because it transforms Eq. (2) into an algebraic equation. Then, applying the time shifting property of the z-transform, $Z\{x[t-k]\} = z^{-k}Z\{x[t]\} = z^{-k}X(z)$ to Eq. (2), where k is a finite integer, Eq. (2) becomes:

$$y_j(z) = y_j(z)z^{-1} + o_{j,j+1}(z)z^{-L_j} - d_j(z), \quad j = 1, 2, \ldots, N \tag{3}$$

Now, isolating $y_j(z)$ from (3) we get:

$$y_j(z) = \overbrace{\left[\frac{1}{1-z^{-1}}\right] z^{-L_j}}^{p_j(z)} o_{j,j+1}(z) - \overbrace{\left[\frac{1}{1-z^{-1}}\right]}^{p^m(z)} d_j(z) \quad j = 1, 2, \ldots, N \tag{4}$$

which relates the z-transform of the inventory level, $y_j(z)$, with the order and the demand only. For IMC design, $p_j(z)$ must be factored into a minimum-phase portion:

$$p^m(z) = \frac{1}{1-z^{-1}} \tag{5}$$

and a portion $p_j^a(z)$ that includes the delays of the system (Morari and Zafiriou, 1989):

$$p_j^a(z) = z^{-L_j} \tag{6}$$

The model for an echelon presented in Eq. (4) is amenable to implement some controllers that exist in literature as is shown in the last works (Hoberg et al., 2007; Dejonckheere et al., 2003; Disney and Towill, 2003). Moreover, an equivalent model to Eq. (4), but in continuous time, is presented in Schwartz and Rivera (2010). However, these works only consider the inventory control of one echelon while in the present work the model is extended to the complete supply chain.

A model for the complete supply chain can be obtained considering that an order $o_{j-1,j}(z)$ generated by a downstream echelon $j-1$ is perceived and supplied by the immediate supplier j. In this way, the multivariable model described by Eq. (7) is obtained:

$$\begin{aligned}
y_1(z) &= \tfrac{z^{-L_1}}{1-z^{-1}} o_{1,2}(z) - \tfrac{1}{1-z^{-1}} d_1(z) \\
y_2(z) &= \tfrac{z^{-L_2}}{1-z^{-1}} o_{2,3}(z) - \tfrac{1}{1-z^{-1}} o_{1,2}(z) - \tfrac{1}{1-z^{-1}} d_2(z) \\
\vdots \quad & \quad \vdots \qquad \forall j = 1, 2, \ldots, N \\
y_j(z) &= \tfrac{z^{-L_j}}{1-z^{-1}} o_{j,j+1}(z) - \tfrac{1}{1-z^{-1}} o_{j-1,j}(z) - \tfrac{1}{1-z^{-1}} d_j(z)
\end{aligned} \tag{7}$$

Remark 1. In the decentralized control strategy all echelons take independent decisions and there is no information sharing. Therefore, each echelon must consider this input as a disturbance. Since in the centralized control strategy a single controller generates all orders of the supply chain, this input becomes a control action.

The model expressed in Eq. (7) is a linear system of equations that can be represented in a matrix form. Let the vector $\mathbf{Y}(z) = [y_1(z), y_2(z), \ldots, y_N(z)]^T$ represent the set of inventories, which are the controlled variables, and $\mathbf{O}(z) = [o_{1,2}(z), o_{2,3}(z), \ldots, o_{N,N+1}(z)]^T$ represent the vector of orders, which are the manipulated variables of the supply chain. Finally, the unknown demand signals perceived by each echelon are represented by the vector $\mathbf{D}(z) = [d_1(z), d_2(z), \ldots, d_N(z)]^T$.

Thus, the complete supply chain is modelled by the matrix Eq. (8):

$$\mathbf{Y}(z) = \mathbf{P}(z)\mathbf{O}(z) - \mathbf{P}^d(z)\mathbf{D}(z) \tag{8}$$

where the transfer function matrix that relates the set of inventories $\mathbf{Y}(z)$ with the orders vector $\mathbf{O}(z)$ is given by:

$$\mathbf{P}(z) = \begin{pmatrix} p^m(z)p_1^a(z) & 0 & 0 & \cdots & 0 \\ -p^m(z) & p^m(z)p_2^a(z) & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -p^m(z) & p^m(z)p_N^a(z) \end{pmatrix} \tag{9}$$

and the transfer function matrix that relates the set of inventories $\mathbf{Y}(z)$ with the set of demands $\mathbf{D}(z)$ is represented by:

$$\mathbf{P}^d(z) = p^m(z)\mathbf{I} \tag{10}$$

where $\mathbf{I}$ is the identity matrix.

For IMC design, $\mathbf{P}(z)$ must be factored into a portion $\mathbf{P}^A(z)$ that includes the delays of the system (Morari and Zafiriou, 1989):

$$\mathbf{P}^A(z) = \begin{pmatrix} p_1^a(z) & 0 & 0 & \cdots & 0 \\ p_2^a(z)-1 & p_2^a(z) & 0 & \ddots & \vdots \\ p_3^a(z)-1 & p_3^a(z)-1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ p_N^a(z)-1 & \cdots & p_N^a(z)-1 & p_N^a(z)-1 & p_N^a(z) \end{pmatrix} \tag{11}$$

and a minimum-phase portion given by:

$$\mathbf{P}^M(z) = p^m(z) \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix} \tag{12}$$

such that

$$\mathbf{P}(z) = \mathbf{P}^A(z)\mathbf{P}^M(z) \tag{13}$$

For control purposes, the following assumption is made in the rest of the paper:

Assumption 1. The rational part of the system, i.e. Eq. (12), and all the delays between each pair output/input, i.e. Eq. (11) are known. □

Since the rational part of the system describes the balance of material carried out in each echelon and the lead times can be determined by (on-line) identification algorithms (Aggelogiannaki and Sarimveis, 2008; Garcia et al., 2012), the Assumption 1 is feasible in practice for the inventory control problem in the supply chain.

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The objective of this work is to design a decentralized IMC control using the model described by Eq. (4), and a centralized IMC control using the MIMO model represented by Eqs. (8)–(10) for a complete supply chain in order to compare the two approaches.

There are two goals for this control system: The inventory target tracking and the demand rejection (i.e., the changes in the demand should not affect the inventory tracking). Moreover, the control system must satisfy these objectives avoiding aggressiveness in the orders (bullwhip effect).

Therefore, in the following section a two degrees-of-freedom Internal Model Control (IMC) structure (Schwartz et al., 2006; Schwartz and Rivera, 2010) for inventory control in a single echelon of the supply chain is formulated in discrete-time. After that, the design is extended to the complete supply chain. Moreover, novel guidelines for the controllers design and bullwhip effect formulation for an entire supply chain not taken into account in previous works are presented.

3. Decentralized control based in the two-degrees-of-freedom feedback IMC

The Two-degrees-of-freedom feedback IMC is shown in Fig. 1, where $r_j(z)$ denotes the inventory target for the control system of each echelon, $q_j^t(z)$ and $q_j^d(z)$ represent the two feedback controllers of the scheme, $p_j(z) = p^m(z)p_j^a(z)$ is the actual dynamics of the supply chain and $\hat{p}_j(z) = p^m(z)\hat{p}_j^a(z)$ represents the nominal model of the system. Each echelon may perceive demand from an external customer of the supply chain $d_j(z)$ and orders from the downstream echelon of the supply chain $o_{j-1,j}(z)$ as is shown in Fig. 1. Since in the decentralized control approach each echelon has no control on the downstream orders, $o_{j-1,j}(z)$ are added to $d_j(z)$ in a single disturbance input $\upsilon_j^m(z) = (o_{j-1,j}(z) + d_j(z))$. Within this structure, the problems of inventory target tracking (Inventory target tracking) and disturbance rejection (demand rejection) can be tackled by separate controllers as it will pointed out.

When the model is exact, $p_j(z) = \hat{p}_j(z)$, (i.e. under Assumption 1), the lead time becomes external in closed-loop. Under these circumstances, the scheme compensates the delay and makes the control problem easier. In order to point out this property, the equation of inventory balance for a single echelon j under this scheme is obtained and represented by Eq. (14):

$$y_j(z) = \left\{\frac{p^m(z)p_j^a(z)q_j^t(z)}{1 + p^m(z)\left[p_j^a(z) - \hat{p}_j^a(z)\right]q_j^d(z)}\right\}r_j(z) - \left\{1 - \frac{p^m(z)p_j^a(z)q_j^d(z)}{1 + p^m(z)\left[p_j^a(z) - \hat{p}_j^a(z)\right]q_j^d(z)}\right\}p^d(z)\upsilon_j^m(z)$$
$$j = 1, 2, \ldots, N \quad (14)$$

It can be seen in Eq. (14), that if the lead time model is known $p_j^a(z) = \hat{p}_j^a(z)$ (i.e. under Assumption 1), we get:

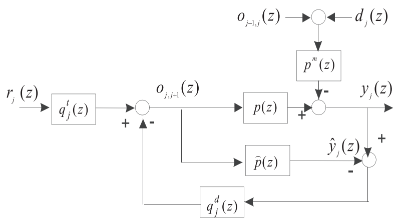

Fig. 1. Two-degrees-of-freedom-feedback IMC scheme.

$$y_j(z) = q_j^t(z)p^m(z)p_j^a(z)r_j(z) - \left(1 - p^m(z)p_j^a(z)q^d(z)\right)p^d(z)\upsilon_j^m(z)$$
$$j = 1, 2, \ldots, N \quad (15)$$

Eq. (15) shows that the lead-time has disappeared from the denominator of Eq. (14), converting the time-delay into external in closed-loop.

The main objective of the control system is to avoid the error between the inventory target and the inventory level, i.e. $r_j(z) - y_j(z) = 0$. Therefore, this error for the two degree of freedom IMC control system when $p_j^a(z) = \hat{p}_j^a(z)$ is calculated as follows:

$$e_j(z) = r_j(z) - y_j(z) = [1 - p^m(z)p_j^a(z)q_j^t(z)]r_j(z) - \left[1 - p^m(z)p_j^a(z)q_j^d(z)\right]p^d(z)\upsilon_j^m(z) \quad j = 1, 2, \ldots, N \quad (16)$$

Thereby, Eq. (16) shows that the $q_j^t(z)$ controller only affects the transfer function that relates the inventory target $r_j(z)$ with the error $e_j(z)$ (the first term of Eq. (16)). Then, the design of this controller is oriented to minimize this term (inventory target tracking). Similarly, $q_j^d(z)$ only affects the transfer function that relates the disturbance signal $\upsilon_j^m(z)$ with the inventory level (the second term of Eq. (16)). Therefore, these two controllers can be designed separately since both of them have a distinctive use and influence on the overall closed loop response:

- $q_j^t(z)$ is an IMC controller designed for inventory target tracking.
- $q_j^d(z)$ is designed mainly to achieve the internal stability and to satisfy the disturbance rejection objective (rejection to the demand perceived by each echelon $\upsilon_j^m(z)$).

3.1. Bullwhip effect formulation (effect of the disturbance on the order signal)

Besides the equation of inventory balance, the relation between the demand perceived by the echelon and the generated orders must be taken into account in the control design, since this relation determines the well known bullwhip effect constraint. The bullwhip effect can be characterized as an amplification of demand fluctuations ($\upsilon_j^m(z)$ in the decentralized control case) as one move upwards in the supply chain. This propagation of demand fluctuations is only possible when every node has sufficient stock. If there are neither changes in the set point nor model mismatch, the relation between demand and orders to successive nodes under the two-degrees-of-freedom-feedback-IMC is given by:

$$o_{j,j+1}(z) = q_j^d(z)p^d(z)\upsilon_j^m(z) \quad (17)$$

and the ratio of orders to successive nodes can be expressed as:

$$|\gamma_j(z)| = \frac{|o_{j-1,j}(z)|}{\left|\upsilon_j^m(z)\right|} = \left|q_j^d(z)p_j^d(z)\right| = \left|q_j^d(e^{i\omega})p_j^d(e^{i\omega})\right| \quad \omega \in [0, 2\pi) \quad (18)$$

where i is the imaginary unity. Lin et al. (2004) have stated that the amplitude of demand fluctuations will not be amplified if

$$|\gamma_j(e^{i\omega})| \leqslant 1 \quad \forall \omega \in [0, 2\pi) \quad (19)$$

Notice from Eq. (18) that if there are neither changes in the inventory target nor model mismatch, the q_j^d controller must be designed such that the bullwhip effect restriction Eq. (19) is satisfied. The IMC controllers design is shown in Section 3.2.

3.2. Controllers design

Once the control system requirements are stated, the Two-degrees-of-freedom-feedback IMC design is oriented to satisfy them.

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The IMC design for these controllers is comprised of the following two procedures.

3.2.1. Inventory target tracking design

For this control purpose $q^t(z)$ is designed for H_2-optimal set point tracking where the control policy is determined such that the sum of the square error

$$\|e\|_2^2 = \sum_{t=0}^{\infty} e^2(t) \tag{20}$$

is minimized.

The H_2-optimal problem for inventory tracking is formulated in Schwartz et al. (2006) and Schwartz and Rivera (2010) as:

$$min_{q_j^t(z)}\left\|\frac{y_j(z)}{r_j(z)}\right\|_2 min_{q_j^t(z)}\|[1-\underbrace{p^m(z)p_j^a(z)}_{p_j(z)}q_j^t(z)]\|_2 \tag{21}$$

which is the first term of Eq. (16).

The IMC solution for this problem is given by Schwartz et al. (2006) and Schwartz and Rivera (2010):

$$\tilde{q}_j^t(z) = z(p^m(z)r_j(z))^{-1}\{z^{-1}(p_j^a(z))^{-1}r_j(z)\}_* \tag{22}$$

where the $\{\cdot\}_*$ operator denotes that after a partial fraction expansion of the operand $\{\cdot\}$, all the terms involving the poles of $(p_j^a(z))^{-1}$ are omitted. Assuming a step change in the inventory target $r_j(z)$, the optimal controller obtained by applying this procedure is Schwartz et al. (2006) and Schwartz and Rivera (2010):

$$\tilde{q}_j^t(z) = (p^m)^{-1} \tag{23}$$

This controller provides a well inventory target tracking but as a result, orders are aggressive, which is unacceptable for factory managers. Therefore, the optimal controller is augmented with a low-pass filter in order to detune this optimal performance of the controller by a parameter $\lambda^t \in [0,1)$. In counteraction, this filter avoids the aggressive orders. Since step changes in the inventory target are considered, the optimal controller $\tilde{q}_j^t(z)$ for inventory target tracking at each echelon, j, is augmented with a type-1 filter (Morari and Zafiriou, 1989) defined as:

$$f_j^t(z) = \frac{(1-\lambda_j^t)z}{z-\lambda_j^t} \tag{24}$$

The final controller is given by:

$$q_j^t(z) = \tilde{q}_j^t(z)f_j^t(z) \tag{25}$$

3.2.2. Controller design to step disturbance rejection in the inventory signal

In this control problem, the $q_j^d(z)$ controller is designed specifically to provide a fast response of inventory level to step demand changes (abrupt changes in the demand). As a result of the integrative nature of the disturbance model $p^m(z)$ (Eq. (4)), a step change in demand becomes a Type-2(ramp) disturbance. Therefore the design procedure relies on solving the H_2-optimal control given by

$$min_{q_j^d(z)}\left\|\frac{y_j(z)}{v_j^m(z)}\right\|_2 = min_{q_j^d(z)}\|[1-\underbrace{p^m(z)p_j^a(z)}_{p_j(z)}q_j^d(z)]p^m(z)\|_2 \tag{26}$$

which is related with the second term of Eq. (16) where $v_j^m(z)$ is consider as an step signal. For disturbance rejection, the optimal controller is generally calculated as (Morari and Zafiriou, 1989):

$$\tilde{q}_j^d(z) = z(p^m(z)v_j^m(z))^{-1}\{z^{-1}(p_j^a(z))^{-1}(z)v_j^m(z)\}_* \tag{27}$$

The optimal IMC controller obtained for a ramp disturbance rejection is:

$$\tilde{q}_j^d(z) = (p^m(z))^{-1}\frac{(L_j+1)z-L_j}{z} \tag{28}$$

Notice that Eqs. (18) and (19) imply that a good bullwhip effect avoidance needs $|p_j(z)q_j^d(z)| \ll 1$ while Eq. (26) requires $|p_j(z)q_j^d(z)| \approx 1$ for step disturbance rejection in the inventory signal. Thus, the $q_j^d(z)$ controller must be designed taking into account two opposite objectives: step disturbance rejection in the inventory level and bullwhip effect avoidance. Therefore, an analytical detuning of the $\tilde{q}_j^d(z)$ optimal controller to obtain a trade-off between these two objectives is performed below. This analytical detuning for IMC controllers for bullwhip effect is not explored in previous works (Schwartz et al., 2006; Schwartz and Rivera, 2010).

- *Detuning of $q_j^d(z)$ for bullwhip effect avoidance*

In the IMC formulation for control systems, $q_j^d(z)$ is augmented with low-pass filter $f_j^d(z)$ to detune the nominal performance in order to satisfy a grade of stability robustness to uncertainty in the plant. However, in the supply chain case, provided that Assumption 1 holds, these filters will be used instead to counteract the bullwhip effect. Thereby, the nominal performance to an step change in the demand is deteriorated but, in exchange, high component frequencies of the demand are rejected in order to satisfy a grade of bullwhip effect avoidance i.e. to satisfy Eq. (19). In the disturbance rejection case, a generalized Type-2 filter is used to guarantee no asymptotic offset for both step and ramp disturbances. Moreover, two type-2 filters connected in series will be used generating a filter of order 4:

$$f_j^d(z) = \frac{((\alpha_j^1 z - \alpha_j^2)(1-\lambda_j^d)z)^2}{(z-\lambda_j^d)^4} \tag{29}$$

Thus, the final controller is given by:

$$q_j^d(z) = \tilde{q}_j^d(z)f_j^d(z) = (p^m(z))^{-1}\frac{((L_j+1)z-L_j)\left(\left(\alpha_j^1 z-\alpha_j^2\right)\left(1-\lambda_j^d\right)z\right)^2}{z\left(z-\lambda_j^d\right)^4} \tag{30}$$

Thereby, the bullwhip restriction for the two-degrees-of-freedom-feedback IMC scheme is given by:

$$|\gamma_j(z)| = \left|\frac{((L_j+1)z-L_j)\left(\left(\alpha_j^1 z-\alpha_j^2\right)\left(1-\lambda_j^d\right)z\right)^2}{z\left(z-\lambda_j^d\right)^4}\right| \leqslant 1 \tag{31}$$

Notice that, the bullwhip effect depends on the lead time L_j and the $q_j^d(z)$ parameters (λ_j^d, α_j^1 and α_j^2). Since the λ_j^d parameter modifies the bandwidth, this is selected so as to satisfy the bullwhip effect condition for a determined L_j value while the parameters α_j^1 and α_j^2 are adjusted to guarantee internal stability for this λ_j^d value. The $q_j^d(z)$ controller must be tuned such that the system has a fast response to low frequency demand changes. Thereby, the inventory level can be maintained. On the other hand, this controller must limit the ratio of orders less than 1 at high frequency to guarantee bullwhip effect mitigation.

In this way, Lin et al. (2004) have suggested to consider the following two factors on the magnitude ratio $|\gamma_j(z)|$.

1. Bandwidth: the frequency at which the magnitude ratio (Eq. (31)) is reduced to below 0.7. A wide bandwidth indicates a faster response but poorer bullwhip mitigation. Note that we are

dealing with a discrete-time system. Therefore, the highest frequency is at $\omega = \pi/T_m = \pi$ since $T_m = 1$. Thus, we can define a term γ_j^π as the magnitude ratio given by Eq. (31) at $\omega = \pi$ i.e. $\gamma_j^\pi = \gamma_j(\omega = \pi)$. Since a higher γ_j^π implies a wider bandwidth and a faster response, it results in more severe bullwhip.

2. Resonance peak (σ_j): the highest value of the amplitude ratio (Eq. (31)). A higher resonance peak indicates a fast response to low frequency demand changes (Disturbance rejection) but the closed-loop response may be more oscillatory. Suitable setting of σ_j ranges from 1.5 to 2.0.

The disadvantage of the controllers (PI and cascade PI) proposed in (Lin et al., 2004) is that there is no direct correspondence between the parameters of the controllers and the bandwidth of the magnitude ratio $|\gamma_j(z)|$. Therefore, that work performed an empirical tuning for bullwhip effect based on trial and error. In the Two-degrees-of-freedom feedback IMC scheme the bandwidth can be manipulated directly by using of the λ_j^d parameter of the $q_j^d(z)$ controller. Therefore, the application of this analytical tuning criterion for bullwhip effect avoidance is simplified. The application of this tuning criterion for the design of the IMC controllers is also novel in supply chain systems.

Fig. 2 shows a tuning example for an echelon with $L = 3$. In this figure, the magnitude ratio $|\gamma_j(z)|$ for several values of λ_j^d is plotted. It can be seen that for λ_j^d values close to 1 the system present strong mitigation of high frequency but low resonance peak σ_j. That means a mitigation of the bullwhip effect but a sluggish response to low frequency demand changes. On the other hand, for λ_j^d values close to 0 the system present poor mitigation of high frequency (severe bullwhip effect) but faster response to low frequency demand changes or step changes (disturbance rejection).

Therefore, the following approximate tuning criterion suggested by (Lin et al., 2004) to find a trade-off between fast inventory tracking and bullwhip effect mitigation can be used:

Choose a controller setting with $\gamma_j^\pi < 1$ and σ_j in the range 1.5–2. There are several λ_j^d solutions based on this criteria as is shown in Fig. 2 in solid lines. Therefore, in this work, to perform the simulations we chose a λ_j^d such that $\gamma_j^\pi < 1$ and σ_j be close to 1.8. Fig. 3 extends this criteria for delays between 1 and 10 periods of time.

Since $q_j^d(z)$ must satisfy asymptotically tracking and internal stability, the filter has to be designed in such a way that all these requirements hold. Hence, once the λ_j^d parameter is selected for the above commented trade-off, the α_j^1 and α_j^1 parameters must be adjusted so as to make the filter satisfy inventory tracking and internal stability. For this system with a pole of multiplicity 1 at $z = 1$, the filter has to satisfy the following conditions at $z = 1$ (Morari and Zafiriou, 1989):

$$f_j^d(z) = 1, \quad \frac{df_j^d(z)}{dz} = 0 \tag{32}$$

Solving this system, we get a mathematical relation which relates the α_j^1, α_j^2 parameters with λ_j^d as:

$$\alpha_j^2 = 2\lambda_j^d, \quad \alpha_j^1 = 1 + \lambda_j^d \tag{33}$$

After formulating the two degrees of freedom IMC scheme for a particular jth echelon, the control system is now generalized to a entire supply chain in a decentralized control way.

A decentralized control means that a controller is designed for each echelon of the supply chain. The resulting diagonal controller matrix for inventory target tracking is given by

$$\mathbf{Q}^t(z) = diag\left(q_1^t(z), \quad q_2^t(z), \ldots, q_N^t(z)\right) \tag{34}$$

In the same way, the resulting diagonal controller matrix for disturbance rejection is given by Eq. (35). When the decentralized control strategy is used, the orders $o_{j-1,j}(z)$ and $d_j(z)$ are considered as the perturbation for the echelon j since each echelon is autonomous to take decisions and these informations are not shared with the rest of entities.

$$\mathbf{Q}^d(z) = diag\left(q_1^d(z), \quad q_2^d(z), \ldots, q_N^d(z)\right) \tag{35}$$

Each controller $q_j^t(z)$ and $q_j^d(z)$ is designed independently for each echelon using the guidelines formulated in subSection 3.2.

An alternative to the decentralized control strategy is a full centralized control approach where all information of the supply chain is taken into account. In this approach, the entire orders vector $O(z)$ is designed simultaneously. This formulation is developed in Section 4.

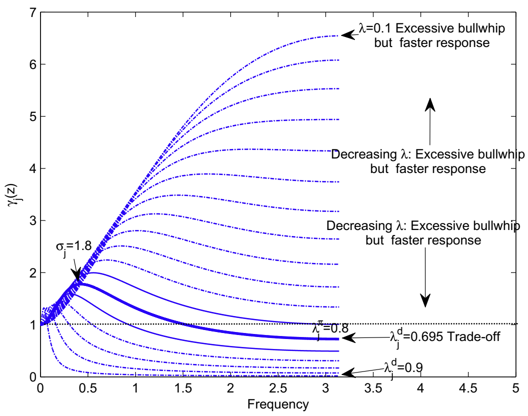

Fig. 2. Frequency response of $|\gamma_j(z)|$ with $L_j = 3$ for various λ_j^d values.

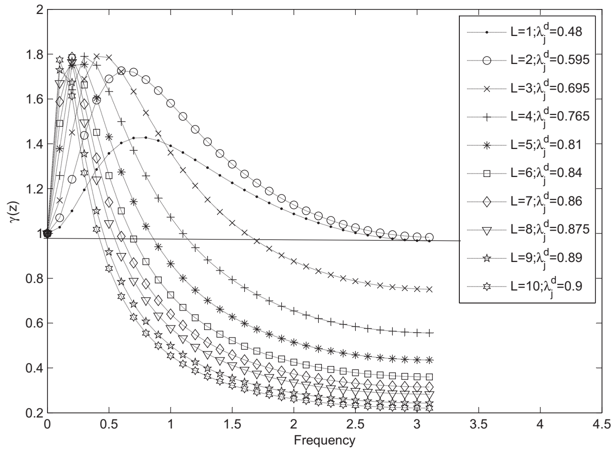

Fig. 3. Frequency response of $|\gamma_j(z)|$ for various L_j values.

4. Centralized control strategy

The centralized control design is based on the IMC scheme shown in Fig. 1 where the model formulated for the entire supply chain, Eqs. (11) and (10), is taken into account. Thus, the vector of inventories, $\mathbf{Y}(z)$, is given by:

$$\mathbf{Y}(z) = \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^t(z)\mathbf{R}(z) - (\mathbf{I} - \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^d(z))\mathbf{P}^d(z)\mathbf{D}(z) \quad (36)$$

In the centralized control strategy $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ can be designed for inventory target tracking and disturbance (demand) rejection respectively by using the IMC guidelines for multivariable (MIMO) systems.

4.1. Bullwhip effect formulation

The bullwhip effect formulation for a single echelon can be generalized for a centralized control considering the transfer function matrix that relates the orders vector $\mathbf{O}(z)$ with the demand vector $\mathbf{D}(z)$. Then, considering no changes in the set point and no model mismatch, the relation between the set of demands and the set of orders is given by:

$$\mathbf{O}(z) = \mathbf{P}^d(z)\mathbf{Q}^d(z)\mathbf{D}(z) \quad (37)$$

The generalization of the magnitude ratio of orders to successive nodes γ for a multivariable (MIMO) system under a centralized strategy can be expressed as:

$$|\Gamma(z)| = |\mathbf{P}^d(z)\mathbf{Q}^d(z)| = |\mathbf{P}^d(e^{i\omega})\mathbf{Q}^d(e^{i\omega})| \quad \omega \in [0, 2\pi) \quad (38)$$

where the magnitude ratio is calculated component-wise. Thus, the demand signals perceived in the supply chain will not be amplified if:

$$|\Gamma_{ij}(e^{i\omega})| \leqslant 1 \quad i = 1, 2, \ldots, N \quad j = 1, 2, \ldots, N \ \omega \in [0, 2\pi) \quad (39)$$

In this case, the bullwhip effect implies that each demand signal represented by d_j introduced in the system is not amplified to subsequents suppliers represented by $o_{j,j+1}(z)$. Therefore, the bullwhip effect can be analyzed component-wise since each component of Γ contains the relation between each pair $d_j(z), o_{j,j+1}(z)$. Thus, in the centralized control approach multiples demand signals are taken into account i.e. $d_j(z) \neq 0$ $j = 1, 2, \ldots, N$. After formulating the inventory control system in a centralized control way, the controller matrices $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ will be designed in subSection 4.2.

4.2. Controllers design

Notice from Eq. (36) that $\mathbf{Q}^t(z)$ only affects the relation between $\mathbf{Y}(z)$ and $\mathbf{R}(z)$ as well as $\mathbf{Q}^d(z)$ only affects the relation between $\mathbf{Y}(z)$ and $\mathbf{D}(z)$. Therefore these controllers can be designed separately. The design procedure is presented below:

4.2.1. Inventory target tracking design

For inventory target tracking $\mathbf{Q}^t(z)$ is designed to solve the H_2-optimal MIMO problem given by

$$min_{Q^t(z)} \|[\mathbf{I} - \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^t(z)]\mathbf{R}(z)\|_2 \quad (40)$$

where the vector $\mathbf{R}(z)$ contains the set of inventory targets $r_j(z)$ for the entire supply chain. Assuming a step change in the inventory target, the optimal IMC controller is:

$$\widetilde{\mathbf{Q}}^t(z) = (p^m(z))^{-1} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & 1 & 1 \end{pmatrix} \quad (41)$$

The optimal controller $\widetilde{\mathbf{Q}}^t(z)$ for inventory target tracking must be enhanced with a low-pass filters bank in order to degrade the fast response of this controller to changes in the inventory targets obtaining less aggressive orders. Thus, $\widetilde{\mathbf{Q}}^t(z)$ is augmented with a low-pass filter bank given by:

$$\mathbf{F}^t(z) = Diag[f_1(z), \quad f_2(z), \ldots, f_N(z)] \quad (42)$$

where each one type 1 filter $f_j(z)$ appearing in Eq. (42) is defined by Eq. (24). Thus, the final controller is:

$$Q^t(z) = \widetilde{Q}^t(z)\mathbf{F}^t \quad (43)$$

4.2.2. Design for disturbance rejection

The two-degrees-of-freedom-IMC scheme allows us to specify the system response to demand changes by using the $\mathbf{Q}^d(z)$ controller. As a result of the integrative nature of the inventory process, a step change in demand becomes a Type-2(ramp) disturbance. Therefore the design procedure relies on solve the H_2-optimal control given by

$$min_{\mathbf{Q}^d(z)} \|[1 - \mathbf{P}^M(z)\mathbf{P}^A(z)\mathbf{Q}^d(z)]\mathbf{P}^d(z)\mathbf{D}(z)\|_2 \tag{44}$$

The IMC controller that solve this problem for a ramp disturbance is:

$$\widetilde{\mathbf{Q}}^d(z) = \frac{1}{p^m}\begin{pmatrix} \frac{(1+L_1)z-(L_1)}{z} & 0 & \cdots & 0 \\ \frac{(1+L_1+L_2)z-(L_1+L_2)}{z} & \frac{(1+L_2)z-(L_2)}{z} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \frac{\left(1+\sum_1^N L_j\right)z-\left(\sum_1^N L_j\right)}{z} & \cdots & \frac{(1+L_{N-1}+L_N)z-(L_{N-1}+L_N)}{z} & \frac{(1+L_N)z-(L_N)}{z} \end{pmatrix} \tag{45}$$

Since the optimal controller for disturbance rejection yields strong variability in the orders, the bullwhip effect is also a restriction to be taken into account in the $Q^d(z)$ controller design. *Therefore, novel guidelines to detune $\widetilde{Q}^d(z)$ controller to satisfy bullwhip effect constraint are shown below.*

- *Detuning of $\widetilde{\mathbf{Q}}_j^d(z)$ for bullwhip effect avoidance*

Since each demand signal is related with each order, the detuning must be done component-wise with a low-pass filter. Each one of the low-pass filters appearing in Eq. (46) is defined by Eq. (29). Thus, the filter matrix is given by:

$$\mathbf{F}^d(z) = \begin{pmatrix} f_{1,1}^d(z) & 0 & \cdots & 0 \\ f_{2,1}^d(z) & f_{2,2}^d(z) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ f_{N,1}^d(z) & \cdots & f_{N,N-1}^d(z) & f_{N,N}^d(z) \end{pmatrix} \tag{46}$$

Therefore, the final controller can be obtained using the Schur product (or component-wise product), in the form:

$$\mathbf{Q}^d(z) = \widetilde{\mathbf{Q}}^d(z) \cdot \mathbf{F}^d \tag{47}$$

The tuning criterion applied in a single echelon in Section 3.2 is also considered in the centralized control, in this case component-wise. Thus, we chose λ_{ij}^d values such that each $\Gamma_{ij}(\omega = \pi) < 1$ and σ_{ij} close to 1.8 which determines the bandwidth and resonance peak of each component of the magnitude ratio of orders respectively.

Fig. 4 presents an example of the tuning of the λ_{ij}^d for the complete supply chain according to this tuning criterion. Notice that each component of Fig. 4 represents the magnitude ratio of an order $o_{j,j+1}(z)$, $\forall j = 1, 2, \ldots, N$ respect each demand signal $d_j(z)$, $\forall j = 1, 2, \ldots, N$. Moreover, all components can be designed at once. In this case, the actual delays are also considered known and $\widehat{L}_1 = L_1 = 3,\ \widehat{L}_2 = L_2 = 3,\ \widehat{L}_3 = L_3 = 3$.

Section 5 evaluates the basic supply chain with three echelons ($N = 3$), under a decentralized and centralized control ways in order to show the behavior respect to the inventory target tracking, demand rejection and bullwhip effect avoidance.

5. Controller evaluation

Since the Two-degrees-of-freedom feedback IMC scheme decouples the inventory tracking from the demand rejection and bullwhip effect avoidance, the simulations are performed in two different subsections. SubSection 5.1 is oriented to evaluate the inventory target tracking and subSection 5.2 evaluates the rejection to demand and bullwhip effect mitigation. In this last situation, the behavior of the decentralized control approach is performed assuming no changes in the inventory tracking.

5.1. Evaluation of the inventory target tracking

In this case of study, the performance of the two-degrees-of-freedom-feedback-IMC to inventory tracking, under the decentralized and centralized control strategies is evaluated for a step change in the setpoints (Inventory targets). In the simulations, no customer demand is considered and the actual lead times knowledge is assumed, i.e. $\widehat{L}_1 = L_1 = 3,\ \widehat{L}_2 = L_2 = 3,\ \widehat{L}_3 = L_3 = 3$. It is also considered an initial inventory of 100 unities in each echelon and suddenly a deterministic step change of 100 unities in each inventory target from $t = 20$ and onwards is introduced.

For the decentralized and centralized control strategies the λ_j^t parameter can take values in the interval [0,1), where 0 implies that the optimal controllers behavior are not deteriorated while 1 correspond to the worst degradation case of this optimal controllers behavior. In this simulation, the inventory tracking performance in a supply chain composed by three echelons under both strategies is evaluated for $\lambda_j^t = 0.2, 0.5$ and 0.8. In general, it can be seen in Figs. 5 and 6 that for values of λ_j^t close to zero, with both

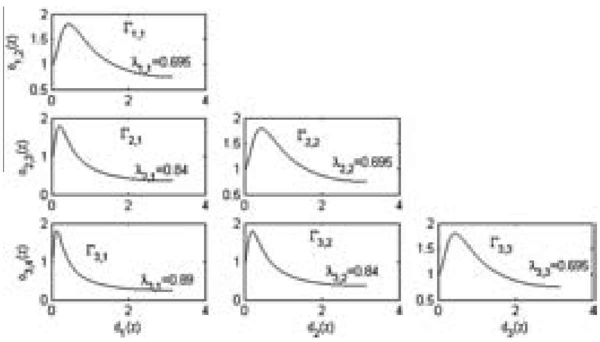

Fig. 4. Tuning of λ_{ij}^d for the entire supply chain.

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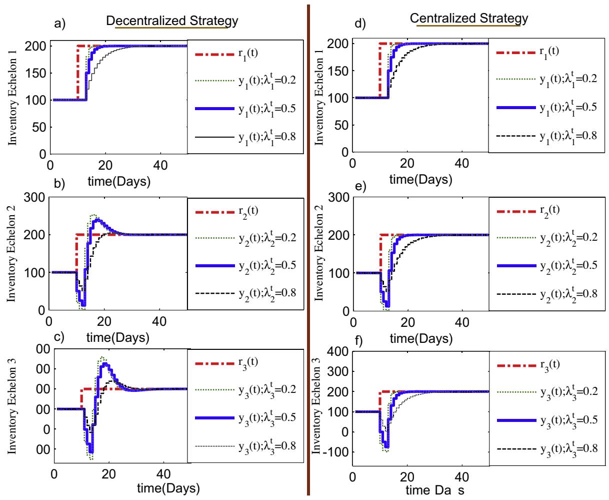

Fig. 5. Inventory responses to step changes in the inventory targets.

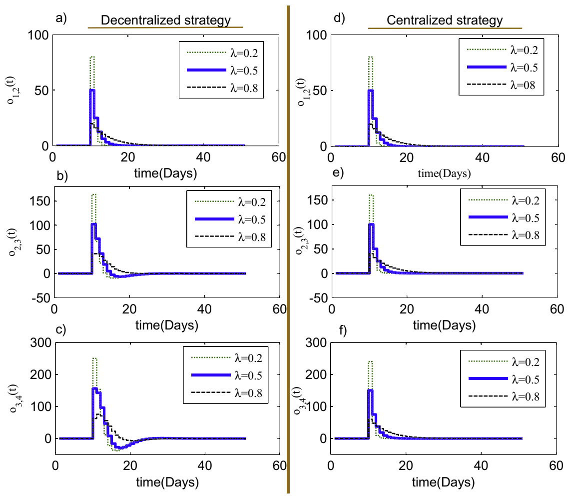

Fig. 6. Orders of the echelons in response to changes in the inventory target. (a) Orders of echelon 1. (b) Orders of echelon 2. (c) orders of echelon 3.

strategies, the first echelon presents a fast response to the changes in the setpoint but aggressive orders are generated, which is unacceptable for factory managers. For λ_j^t values close to 1 the system presents a slow response to the changes in the setpoint but the orders are smoothed. Therefore, a trade-off between these two behaviors is possible if the λ_j^t parameter is moved from zero to one.

In the second and third echelons, although the same $\lambda_j^t = 0.5, 0.5$ and 0.8 values are applied, the behavior is deteriorated significantly in the decentralized control strategy. That is, considerable overshoot in the inventory response to a setpoint change is presented as is shown in Fig. 5b and c. This is a result of the application of decentralized control strategy since the echelon controlled has no information of the dynamics of the chain. On the other hand, by modifying λ_j^t parameters of the controller $Q^t(z)$ from 0 to 1, the trade-off between a faster response of inventory to a step change in the inventory target and a smoothing of orders can be done for all supply chain. Fig. 5d–f shows that, the centralized control approach does not exhibit overshoot for any λ^t values when it is subjected to inventory target changes in the entire supply chain. Orders changes in response to inventory target changes can be less abrupt, consequently decreasing inventory holding costs, smoothing factory operations, and improving profitability.

In order to provide a quantitative evaluation of strategies performance to inventory tracking, the Integral Absolute Error (IAE) which is the cumulative difference between the controlled variable (inventory level) and its set point value (inventory target) is introduced below:

$$\text{IAE} = \sum_{t=0}^{\infty}(r_j(t) - y_j(t)) \tag{48}$$

Values of IAE close to zero means improvement in the performance to inventory tracking. In this way, Table 1, contains the values of IAE for the decentralized and centralized control strategies with the $\lambda_j^t = 0.2, 0.5$ and 0.8 values. This table shows that the centralized control obtains lower values of IAE when a supply chain with more than 1 echelon is considered. These results confirm that the centralized control strategy improves the performance to inventory tracking in the entire supply chain.

Table 1
IAE for the decentralized and centralized control strategies.

Strategy	Decentralized			Centralized		
	$\lambda_j^t = 0.2$	$\lambda_j^t = 0.5$	$\lambda_j^t = 0.8$	$\lambda_j^t = 0.2$	$\lambda_j^t = 0.5$	$\lambda_j^t = 0.8$
Echelon 1	325	400	700	325	400	700
Echelon 2	1127	1164	1201	625	700	1000
Echelon 3	2136	2154	2034	925	1000	1300

5.2. Evaluation of disturbance (demand) rejection and bullwhip effect avoidance

As it is stated in Sections 3 and 4 the design of the controllers for disturbance rejection ($q_j^d(z)$ and $Q^d(z)$) are restricted by the bullwhip effect condition. In this work the IMC scheme uses the λ_j^d parameter to degrade the optimal disturbance rejection obtain a trade-off between the disturbance rejection and bullwhip effect mitigation objectives. In order to show the systems performance to disturbance rejection, a step change in the demand is applied from time instant $t = 20$ and onwards. On the other hand, for bullwhip effect evaluation, a stochastic variability in the customer demand $d_j(z)$ is applied to the systems from time instant $t = 60$ and onwards. The customer demand is formulated as a normal function, with a average equal to 20 and a variance equal to 1, i.e. $\in N(20, 1)$.

On the one hand, for the decentralized control strategy the $\lambda_j^d = 0.695$ value is used according to the bullwhip tuning proposed in Section 3.2. Figs. 7 and 8 show the system behavior to the disturbance input. Again, the analytical tuning works for the first echelon, as is shown in Figs. 7 and 8a since the bullwhip effect is mitigated and an acceptable inventory response to step change in the demand is obtained. For the subsequent echelons 2 and 3 the performance to disturbance rejection and bullwhip effect avoidance are deteriorated. Fig. 7b and c shows that an overshot in the inventory level appears for the second echelon and onwards. The demand fluctuations in the echelons 2 and 3 are amplified (bullwhip effect) as is shown in Fig. 8b and c. Therefore, the application of the detuning for bullwhip effect mitigation in each particular echelon proposed in Section 3 is inefficient to mitigate the bullwhip effect in the entire supply chain with a considerable number of echelons. This behavior is result of the application of decentralized control strategy. An alternative to solve this problem

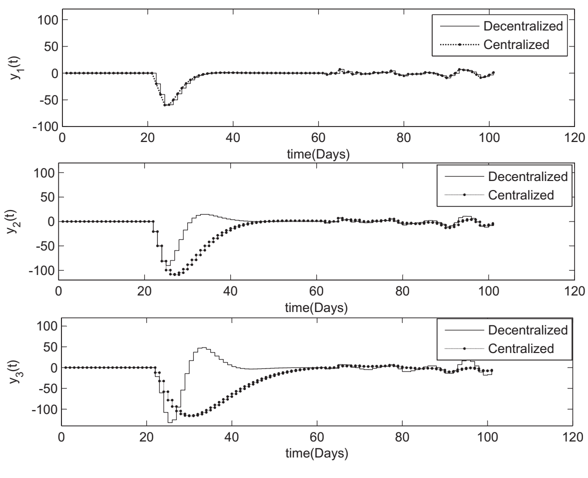

Fig. 7. Inventory behavior to changes in the demand assuming no changes in the inventory target.

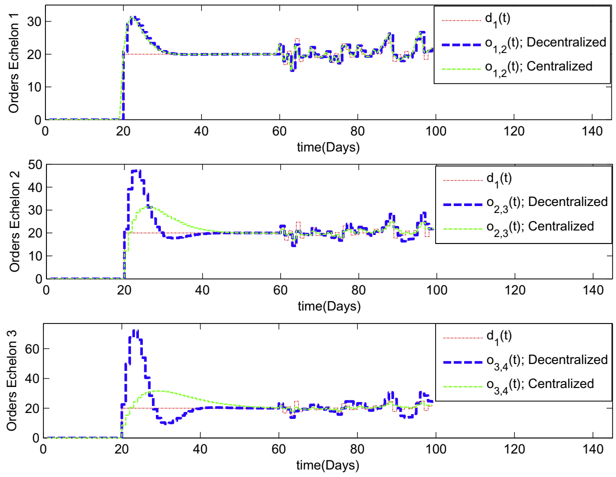

Fig. 8. Orders responses to changes in the demand assuming no changes in the inventory target.

Table 2
Ratio of the variance of the order rate to the variance of the demand rate (BW).

Strategy	Decentralized $\lambda_1^d = \lambda_2^d = \lambda_3^d = 0.695$	Centralized $\lambda_{1,1}^d = 0.695;\ \lambda_{2,1}^d = 0.84;\ \lambda_{3,1}^d = 0.89$
$\frac{Var(o_{1,2}(t))}{Var(d_1(t))}$	1.07	1.07
$\frac{Var(o_{2,3}(t))}{Var(d_1(t))}$	1.62	0.46
$\frac{Var(o_{3,4}(t))}{Var(d_1(t))}$	3.22	0.32

is to apply a significant detuning echelon by echelon mitigating progressively the bullwhip effect. In this case, an analytical trade-off is no possible. Since in this paper the main point is to compare the decentralized and centralized strategies, this analysis is proposed for future works.

On the other hand, Fig. 7 also shows that the centralized controller $Q^d(z)$ with the λ_{ij}^d values given in Fig. 4 provides a fast recuperation of inventory without overshoot in the first echelon. The second and third echelons present sluggish recuperation of inventory, that is due to the detuning for bullwhip mitigation. Fig. 8 shows that with the centralized control design, the overshots in the orders in response to a step change in the demand is slower than for the decentralized strategy. Moreover, this strategy presents a better bullwhip effect mitigation echelon by echelon in comparison with the decentralized strategy. Therefore, a trade-off between a fast response to the demand and bullwhip effect avoidance in the entire supply chain is available by the centralized control strategy.

There are several measures of the bullwhip effect proposed in the literature (Dejonckheere et al., 2003). The most common measure is the ratio of the variance of the order rate to the variance of the demand rate, i.e. $BW = \frac{Var(Orders(t))}{Var(Demand(t))}$, where BW values less than or equal to one means a total mitigation of bullwhip effect. Therefore, in Table 2, BW is calculated for the entire supply chain under the decentralized and centralized control strategies. The results confirm that the decentralized control strategy is effective in the first echelon but deteriorates dramatically echelon by echelon while in the centralized control strategy, the control of bullwhip effect is more effective echelon by echelon.

6. Discussion and conclusions

The proposed scheme allows us to tackle the two problems of inventory target tracking and demand rejection with two controllers separately. That is an advantage to other replenishment inventory policies based on control theory proposed in the literature (Hoberg et al., 2007; Jaksic and Rusjan, 2008; Balan et al., 2009; Dejonckheere et al., 2003). However, optimal tuning of these controllers produce aggressive orders that are unacceptable for factory managers. Then, two trade-off must be taken into account in the design of these controllers for inventory management in the supply chain, (Inventory target tracking vs. aggressive orders mitigation for the $\mathbf{Q}^t(z)$ design) and (Demand rejection vs. bullwhip effect mitigation for the $\mathbf{Q}^d(z)$ design). Previous works have analyzed these issues in a particular echelon. In this paper, these two control issues have been analyzed in the entire supply chain, under the decentralized and centralized control approaches. Since the interest of this work is to perform a comparative analysis between the decentralized and centralized control strategies in closed loop, forecasting demand which are feedforward schemes used to adjust the inventory target are excluded of the analysis. The analysis of appropriate schemes to adjust dynamically the inventory target are proposed for future works.

6.1. Inventory target tracking vs. aggressive orders mitigation for the $\mathbf{Q}^t(z)$ design

In the decentralized control approach based on multi-degree-of-freedom controller, the performance to inventory target tracking is optimal in the first echelon. That is, there is no overshoot in the inventory level when is subjected to inventory target changes. However, for the rest of the echelons, there are significant overshoots in the inventory response to an inventory target change which lead to a large spike in factory orders that is unacceptable for factory managers.

Since in the centralized control approach the controller for the entire supply chain is designed at once, the controller performance to inventory target tracking holds for all echelons. Therefore, the detuning of optimal controller to smooth the aggressive orders must be stronger for the decentralized control approach than for a centralized control approach.

6.2. Demand rejection vs. bullwhip effect mitigation for the $\mathbf{Q}^d(z)$ design

Respect to the trade-off between the demand rejection and the bullwhip effect mitigation in the $\mathbf{Q}^d(z)$ design, the tuning proposed by (Lin et al., 2004) for a single echelon is generalized in this work for the entire supply chain. Again, in the decentralized control approach this tuning works well in the first echelon. However, in the second echelon and onwards, the performance to bullwhip effect mitigation under this tuning is deteriorated. Since, in the centralized control approach the tuning for bullwhip effect mitigation is applied in each component of the Γ matrix, the tuning for bullwhip effect mitigation works successfully in the entire supply chain.

As is mentioned in the introduction, a decentralized control approach is more intuitive and easier to implement. Moreover, it is more suitable for supply chains where its elements belong to different companies and do not share information. Nevertheless, its performance to inventory target tracking and demand rejection and bullwhip effect mitigation of this controller can be deteriorated significantly in supply chains with several echelons and interactions.

On the other hand, when all or most of the supply chain elements belong to the same company or share internal information the centralized control approach is the more suitable. The IMC control scheme simplifies the design controllers for the multivariable system allowing to generalize the design guidelines for an echelon of the supply chain to multiple echelons in a decentralized and centralized control ways. Simulations shown in Section 5 evidence the improvement in the performance to inventory target tracking disturbance rejection and bullwhip effect mitigation provided by a centralized control implementation with information sharing. Therefore, the information sharing and the centralization of the inventory control are recommended in all cases where it is possible.

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Appendix A

Submitted paper

This Appendix presents the paper titled **A switched control strategy for inventory control of the supply chain** which is submited to a control journal for its consideration.

A.1 A switched control strategy for inventory control of the supply chain

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abstract

In this paper the inventory management is tackled by using control theory. For this, the complete model of the supply chain proposed by (Lin *et al.*, 2004) is taken into account. This general model considers stock limitations and back orders accumulation providing a better description of the actual supply chain dynamics. Due to the stock limitations of the distributors, the supply chain turns out to be a switched system since its behaviour changes according the stock available of the distributors. This fact motivates us to propose a switched control system for a serial supply chain under a decentralized control strategy (independent switched control to each echelon). This control system selects automatically the adequate control policy through time according the current dynamics of the supply chain. Moreover, the control policies are designed by Internal Model Control (IMC) techniques. The results of controller evaluations demonstrate that the proposed switched control approach enhances the behaviour with respect to the inventory target tracking, back-order reduction and bullwhip effect mitigation in the supply chain systems in comparison to previous approaches.

A switched control strategy for inventory control of the supply chain

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Abstract

In this paper the inventory management is tackled by using control theory. For this, the complete model of the supply chain proposed by (Lin et al., 2004) is taken into account. This general model considers stock limitations and back orders accumulation providing a better description of the actual supply chain dynamics. Due to the stock limitations of the distributors, the supply chain turns out to be a switched system since its behaviour changes according the stock available of the distributors. This fact motivates us to propose a switched control system for a serial supply chain under a decentralized control strategy (independent switched control to each echelon). This control system selects automatically the adequate control policy through time according the current dynamics of the supply chain. Moreover, the control policies are designed by Internal Model Control (IMC) techniques. The results of controller evaluations demonstrate that the proposed switched control approach enhances the behaviour with respect to the inventory target tracking, back-order reduction and bullwhip effect mitigation in the supply chain systems in comparison to previous approaches.

Keywords: Supply chain, inventory control, switching control.

1. Introduction

The improvement of inventory management in supply chain systems contributes to increased revenues, lower costs and greater customer satisfaction. In this way, development of inventory management policies has gained attention on the Engineering and logistic chains researchers. The main objective of an inventory policy is to obtain a desired inventory level avoiding severe order and inventory variability. Some of these researchers, have been inspired in control theory (transfer functions, frequency response...) to analyze and propose inventory replenishment policies. Thus, (Dejonckheere et al., 2003) measure the variance amplification of orders within order-up-to policies from a control engineering perspective. Also, (Dejonckheere et al., 2004) examine the beneficial impact of information sharing in the inventory control of multi-echelon supply

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chains. Moreover, (Disney and Towill, 2003) present a general solution to the bullwhip problem based on a sound mathematical background using the DE-APIOBPCS as inventory control policy. (Hoberg et al., 2007) apply linear control theory to study the effect of various inventory policies on order and inventory variability, which determine the inventory control performance. Furthermore, (Schwartz and Rivera, 2010) introduce the two degrees of freedom feedback and three degrees of freedom feedback-feedfordward Internal Model Control as well as the model predictive control as a novel inventory replenishment policy in the supply chain, providing an important improvement in the performance of manufacturing systems with long lead time and significant uncertainty.

However, these works are based on linear models of the supply chain where suppliers stock limitations and back orders accumulation are not included. The policies based on these models can lead unrealistic behaviour (negative inventory) when are implemented. This situation is undesirable for inventory managers. Therefore, in order to give a better mathematical description of the supply chain dynamics, (Lin et al., 2004) developed a more complete supply chain model in the z-domain. This model takes into account stock limitations and back orders accumulation providing a more adequate representation of the supply chain dynamics. Due to the stock limitations of the distributors, the supply chain turns out to be a linear switched system corresponding to the stock limitations of the distributors (Wang et al., 2009).

The general model of the system contains back-orders accumulation and switching characteristics in addition to lead-times and integrative dynamics already contained in linear models. This fact makes the control problem harder. Moreover, the multi-variable nature of the supply chain system increases the complexity of the inventory control and helps many undesirable effects appear when an inadequate inventory control policy is applied. Among these, instability represents the main problem, which implies that signals describing the inventory and orders can diverge as time goes on (Hoberg et al., 2007). Another inconvenient is that the variability in the ordering patterns often increases as we move up into the chain, from the customer or factory towards the suppliers. This phenomenon is known as the *bullwhip effect*.(Trapero et al., 2012; Dejonckheere et al., 2003, 2004; Disney and Towill, 2003).

In order to overcome these problems regarding production and supply chain inventory management, decisions based on process control theory have been successfully applied using the model proposed by (Lin et al., 2004). Among them, (Lin et al., 2004) present inventory replenishment policies based on Control Engineering, including proportional-integral (PI) controllers and cascade control. This approach applies a single controller through time whose design is focused on the mitigation of the bullwhip effect. (Balan et al., 2009; Kristianto et al., 2012) apply a fixed Fuzzy Logic Controller FLC through time. The tuning of this controller has been performed using adaptive neuro-fuzzy inference system (ANFIS). The design of this controller is also oriented to bullwhip effect mitigation. Nevertheless, since the supply chain changes its operation case through time, a control system that sets dynamically the appropriate controller for the current operation case is necessary.

Therefore, in this work we propose a switched inventory control system for a serial multivariable supply chain under decentralized control strategy (independent switched control to each echelon). This consists in a bank of controllers designed for each possible model case working in parallel at each echelon. (Schwartz and Rivera, 2010; Garcia et al., 2012) introduce the application of Internal Model Control as a novel inventory replenishment policy in the supply chain, providing an important improvement in the performance of manufacturing systems. Therefore, in this work each controller is designed by following IMC guidelines taking into account the trade-off between the optimal inventory tracking and bullwhip effect mitigation. The system is enhanced with a switching logic that decides what controller is suitable at each instant time.

Since the supply chain is described as a multi-variable system, the application of inventory control strategies to the entire supply chain can yield insights to improve the inventory control in an overall scale. Thus, a cascade production-distribution system consisting of two echelons as a multi-variable case is considered and simulated in this work as example, showing the main effects of the supply chain system under the proposed control approach . Moreover, a quantitative analysis of the control system performance (Integral Absolute Error (IAE), Back-order (BO) and Bullwhip effect (BW)) is performed and compared with the current literature. It is important to notice that the presented approach is applicable to any serial MIMO supply chain while the case of two echelons is considered only to present the theory and develop an illustrative example and simulation results.

The rest of the paper is formulated as follows: Section 2 presents the complete supply chain model using z-transform. As a result, a discrete Multiple-Input-Multiple-Output (MIMO) system is obtained. Section 3 presents the switched control scheme for a supply chain system. After that, section 4 presents the inventory control results in single and multi echelon scenery. The paper ends with the concluding remarks in Section 5.

2. Dynamic model of the supply chain

The system under study is a typical cascade production-distribution system consisting of two echelons: a retailer (R) and a distributor (D) between the factory (F) and final customer (C) as is shown in Fig. 1.

The model of a particular echelon presented by (Lin et al., 2004) and shown in the box of Fig. 1 is adopted. In this way, let $I_j(t)$ denote the net stock inventory (the difference between on-hand inventory and backorders) of an echelon $j \in \{R, D\}$ of the chain at any discrete time $t = nT$ where T=1 is the base of time of the supply chain events (a day, a week or a month) and n is a nonnegative finite integer. The amount of goods to be delivered to downstream node $k \in \{C, R, D\}$ by the node j at the instant t is denoted by $Y_{j,k}(t)$. The demand received by node j from downstream node k is indicated by $U_{k,j}(t)$. In addition, a time delay of L_j time instants is assumed for all delivery of goods so that goods dispatched for a supplier at time t will arrive to the destination at time $t + L_j$. However, due to the need for examination and administrative processing, this new delivery is only available to the node j at $t + L_j + 1$. The equation for inventory balance at each node, $I_j(t)$, is given by (Lin et al., 2004):

$$I_j(t) = I_j(t-1) + Y_{i,j}(t - L_j) - Y_{j,k}(t) \quad \begin{array}{l}(i, j) \in \{(D, R), (F, D)\} \\ (j, k) \in \{(R, C), (D, R)\}\end{array} \tag{1}$$

Due to the delay in delivery, an auxiliary inventory position $I^*_j(t)$, $j \in \{D, R\}$ is defined to better monitor the change in the inventory:

$$I^*_j(t) = I^*_j(t-1) + Y_{i,j}(t) - Y_{j,k}(t) \quad \begin{array}{l}(i, j) \in \{(D, R), (F, D)\} \\ (j, k) \in \{(R, C), (D, R)\}\end{array} \tag{2}$$

The orders placed in the node j by the immediate customer k are denoted by $U_{k,j}(t)$. The immediate customer can order as much as it wants, with no guarantee that the order can be fully fulfilled. It is assumed that ordering information is communicated instantly. However, an order at time t will only be processed at time $t + 1$, due to administrative delay. Therefore, the standing

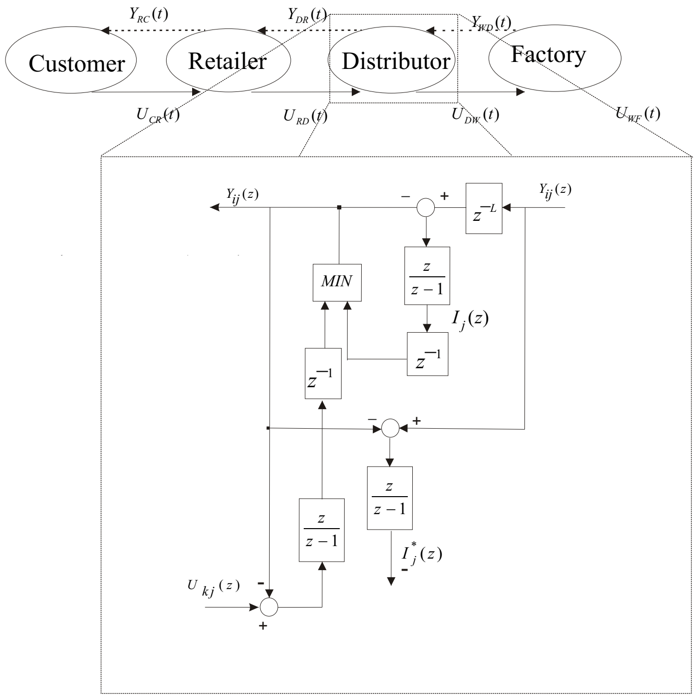

Figure 1: The block diagram of the supply chain

order for node j, $O_j(t)$, $j \in \{D, R\}$, is defined as the amount of order to be processed at time $t + 1$. Moreover, the model assumes that an order can be accumulated to the next time step if it has not been previously fully fulfilled, since each customer has only one supplier in our simple supply chain. Therefore, the standing order for node j at time t is the sum of the order placed plus any unfulfilled order at time t:

$$O_j(t) = U_{k,j}(t) + O_j(t-1) - Y_{j,k}(t) \quad (3)$$

Regarding the actual delivery goods, if there is enough inventory in the immediate supplier to satisfy the standing order at $t - 1$, all the orders will be delivered. Otherwise, all existences in the immediate supplier are delivered. Similarly, if the downstream node already has too much inventory, the supplier will just stop delivery; return of goods is not taken into consideration.

Therefore, the actual goods delivery to node is defined as:

$$Y_{j,k}(t) = \begin{cases} 0 & O_j(t-1) \le 0 \\ O_j(t-1) & 0 \le O_j(t-1) \le I_j(t-1) \\ I_j(t-1) & 0 \le I_j(t-1) \le O_j(t-1) \end{cases} \quad (j,k) \in \{(R,C),(D,R)\} \tag{4}$$

The customer satisfaction of node j is represented by a back-order defined as the difference between the total standing order at $t-1$, $O_j(t-1)$ and the amount of goods actually delivered at t $Y_{jk}(t)$:

$$BO_j(t) = O_j(t-1) - Y_{j,k}(t) \tag{5}$$

The larger the $BO_j(t)$, the poorer is the customer satisfaction.
The difference equations (1)-(5) described above can be solved directly in the time domain or by using transformation techniques. In particular, the z-transform is the most extended transformation because it converts the difference equations into algebraic ones (Ogata, 1996). Then, applying the time shifting property $Z\{x[t-n]\} = z^{-n}Z\{x[t]\} = z^{-n}X(z)$ where n is a finite integer, on Eqs. (1)-(5) we get the supply chain model in the z-transform as is shown below and represented in Fig. 1:

$$I_j(z) = \frac{1}{1-z^{-1}}(z^{-L_j}Y_{i,j}(z) - Y_{j,k}(z)) \tag{6}$$

$$I_j^*(z) = \frac{1}{1-z^{-1}}(Y_{i,j}(z) - Y_{j,k}(z)) \tag{7}$$

$$O_j(z) = \frac{1}{1-z^{-1}}(U_{k,j}(z) - Y_{j,k}(z)) \tag{8}$$

$$Y_{j,k}(z) = \begin{cases} 0 & O_j(t-1) \le 0 \\ z^{-1}O_j(z) & 0 \le O_j(t-1) \le I_j(t-1) \\ z^{-1}I_j(z) & 0 \le I_j(t-1) \le O_j(t-1) \end{cases} \tag{9}$$

When the supply chain is in operation, the stock conditions (enough or not enough) of the current echelon j can change at a determinate instant t, as is modelled in Eq. (9). Hence, the supply chain is a natural switched system where the general dynamic model given by Eqs. (6)-(9) can become in particular, one of the following models through the time (Lin et al., 2004) Infinite Supply and High Stock (ISHS), Infinite Supply and Low Stock (ISLS) or limited supply LS. The particular models associated to the different cases described by Eq. (9) are specified in sections (2.1-2.3).

2.1. Infinite Supply and High Stock (ISHS)

In this case, the upstream supplier has sufficient inventory so that the demand of node j is always satisfied: i.e. $Y_{i,j}(z) = z^{-1}U_{j,i}(z)$. Furthermore, it is considered that the set point of node j is sufficiently high so that there will always be sufficient inventory to satisfy all customer demands, i.e. $Y_{j,k}(z) = z^{-1}O_j(z) = z^{-1}U_{k,j}$ which corresponds to the second case of Eq. (9). Under these conditions, the Eq. (10) for inventory position $I_j^*(t)$ is:

$$I_j^*(z) = \frac{1}{1-z^{-1}}z^{-1}U_{j,i}(z) - \frac{1}{1-z^{-1}}z^{-1}U_{k,j}(z) \tag{10}$$

Thereby, for the infinite supply and high stock case, the inventory position $I_j^*(t)$ is related with the orders $U_{j,i}(z)$ and the demand $U_{k,j}(z)$ by the following transfer function:

$$P_j^{(ISHS)}(z) = \frac{1}{1 - z^{-1}} z^{-1} \tag{11}$$

For IMC design, Eq. (11) must be factored into a minimum-phase portion:

$$P_j^{(ISHS)m}(z) = \frac{1}{1 - z^{-1}} \tag{12}$$

and a portion that includes the delays of the original transfer function:

$$P_j^{(ISHS)a}(z) = z^{-1} \tag{13}$$

2.2. Infinite Supply and Low Stock (ISLS)

If an upstream supplier has sufficient inventory so that the demand of node j is always satisfied, i.e. $Y_{i,j}(z) = z^{-1}U_{j,i}(z)$, but the set point of node j is low so that there will always be insufficient inventory to satisfy all customer demands, i.e. $Y_{jk}(z) = z^{-1}I_j(z)$ which corresponds to the third case of Eq. (9), then, the equation for inventory position $I_j^*(t)$ is determined by (see Appendix A):

$$I_j^*(z) = \frac{z^{L_j} + z^{L_j-1} + ... + z + 1}{z^{L_j+1}} U_{j,i}(z) \tag{14}$$

In this case, the transfer function that relates the orders $U_{j,i}(z)$ with the inventory position is given by

$$P_j^{(ISLS)}(z) = \frac{z^{L_j} + z^{L_j-1} + ... + z + 1}{z^{L+1}} \tag{15}$$

For IMC design purposes, the minimum-phase portion of Eq. (15) is:

$$P_j^{(ISLS)m}(z) = \frac{z^{L_j} + z^{L_j-1} + ... + z + 1}{z^{L}} \tag{16}$$

and the portion that includes the delay of Eq. (15) is:

$$P_j^{(ISLS)a}(z) = z^{-1} \tag{17}$$

2.3. Limited Supply (LS)

In this case, the upstream supplier i does not have sufficient inventory to supply the node j which corresponds to the first case of Eq. (9). Therefore, the goods perceived for the echelon j in an instant t is equal to the existences of the immediate supplier i at the last time $t-1$ i.e $Y_{ij}(t) = z^{-1}I_i(t-1)$. In this case, we found the following transfer function (see Appendix B):

$$I_j^*(z) = \begin{cases} \frac{1}{(z-1)}(I_i(z) - U_{j,k}(z)) & I_j(t) \geq U_{k,j}(t-1) \\ \frac{z^{L_j+1}-1}{z^{L_j+1}(z-1)} I_i(z) & Otherwise \end{cases} \tag{18}$$

In Eq. (18), the inventory position of node j depends neither on the set point nor on its ordering policy. This result is intuitive: If the supplier is low in stock no matter how node j orders, the inventory position is limited by the stock available in the supplier i.

The operation case of the supply chain depends on the relation $MIN(O_j(t), I_j(t))$, which determines if the j^{th} echelon have enough existences to supply the immediate customer k and the relation $MIN(O_i(t), I_i(t))$ which determines if the immediate supplier i have enough existences to supply the actual echelon. Thereby, the operation cases change as these relations change through time. Therefore, a system that sets automatically the adequate inventory control policy may be more suitable than a fixed control policy.

The objective of this work is to design a switched control system for each j^{th} echelon (Decentralized strategy) that select automatically at each instant t the adequate controller according the current dynamics of the supply chain Eqs.(11), (15) or (18). Switching between controllers is carried out by measuring the signals $O_j(t)$, $I_j(t)$, $O_i(t)$ and $I_i(t)$). The controllers design for each echelon is based on internal model control guidelines. The complete system is described in Section 3.

3. Switched control approach for an echelon

A switched system is a hybrid system composed of a family of continuous-time or discrete-time subsystems and a rule orchestrating the switching between them. Thus, the inventory control scheme proposed in this paper for each node, j, includes a bank of control policies $C_j(z)$ designed adequately for each supply chain operation case described in Section 2. The control system is complemented with an inventory target calculator $S_j(z)$ to set the inventory position target $T_j(z)$ according to the demand perceived at each instant t (Lin et al., 2004; Dejonckheere et al., 2004). In addition, the system is enhanced with a switching logic block monitoring $MIN(O_j(t), I_j(t))$ and $MIN(O_i(t), I_i(t))$ which determine the change from one operation case to other. Then, the switching logic block selects the adequate controller through time. The complete system is illustrated in Fig. 2. The controllers design, the inventory target calculator $S_j(z)$ and the switching rules are explained below.

In this work, a multi-degrees-of-freedom internal model control (IMC) structure (Morari and Zafiriou, 1989; Schwartz et al., 2006; Schwartz and Rivera, 2010) in discrete-time is adopted to design the $C_j^{(IS\,HS)}(z)$, $C_j^{(IS\,LS)}(z)$ and $C_j^{(LS)}(z)$ control policy for each node, j. Each policy is designed according to each supply chain operation case by using the corresponding models $P_j^{(IS\,HS)}(z)$, $P_j^{(IS\,LS)}(z)$ and $P_j^{(LS)}(z)$ presented in section 2. The general guidelines for the multi-degrees-of-freedom internal model control (IMC) design are presented in Subsection 3.1 and the resulting controllers for each particular case are shown in Subsections 3.1.1, 3.1.2 and 3.1.3.

3.1. Two-degrees-of-freedom-feedback IMC scheme for an echelon

In this case we consider the Two-degrees-of-freedom feedback IMC as is shown in Fig. 3, where $T_j(z)$ denotes the inventory target for the control system of each echelon. $Q_j^t(z)$ and $Q_j^d(z)$ represent the two feedback controllers of the scheme and $\hat{P}(z)$ represents the nominal model of the system, all of them belonging to the control policy $C_j(z)$ as is shown in Fig. 3. Within this structure, the problems of inventory target tracking (Inventory target tracking) and disturbance rejection (demand rejection) can be tackled by separate controllers.

The main objective of the system is to avoid the error between the inventory target and the inventory level i.e. $T_j(z)$ -$I_j^*(z)$=0. Therefore, this error for the two degree of freedom IMC control system when $P_j(z) = \hat{P}_j(z)$ is calculated as follows:

$$\begin{array}{l} E_j(z) = T_j(z) - I_j^*(z) = [1 - PQ_j^t(z)]T_j(z) \quad j = R, D \\ -[1 - P(z)_j(z)Q_j^d(z)]P(z)U_{kj} \end{array} \tag{19}$$

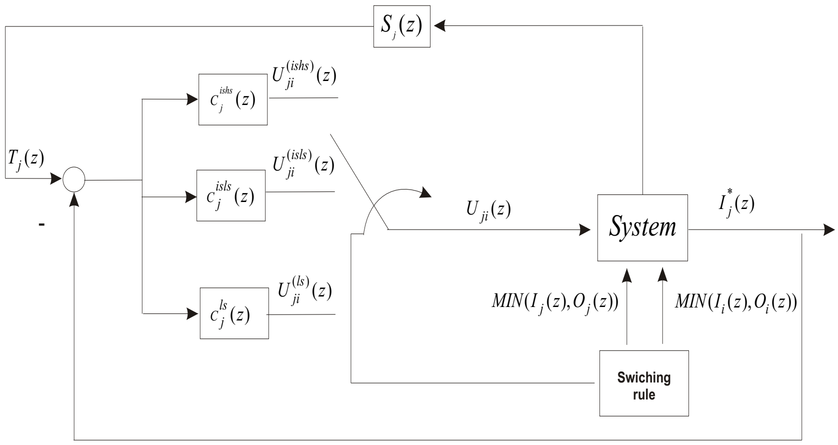

Figure 2: Switched control scheme for the supply chain system

Thereby, Eq. (19) shows that the $Q_j^t(z)$ controller only affects the transfer function that relates the inventory target $T_j(z)$ with the error $E_j(z)$ (the first term of Eq. (19)). Then, the design of this controller is oriented to minimize this term (inventory target tracking). Similarly, $Q_j^d(z)$ only affects the transfer function that relates the demand signal $U_k j(z)$ with the inventory level $E_j(z)$ (the second term of Eq. (19)). Therefore, these two controllers can be designed separately where both of them have a distinctive use and influence on the overall closed-loop response:

- $Q_j^t(z)$ is an IMC controller for inventory target tracking.
- $Q_j^d(z)$ is designed mainly to achieve the internal stability and to satisfy the disturbance rejection objective (rejection to the demand perceived by each echelon $U_k j(z)$).

The IMC design procedure for $Q_j^t(z)$ and $Q_j^d(z)$ controllers is comprised by the following two steps:

1. **Design for nominal optimal performance.** For this control purpose, $Q_j^t(z)$ and $Q_j^d(z)$ are designed for H_2-optimal set point tracking where the control policy is determined such that the sum of the square error

$$||E||_2^2 = \sum_{t=0}^{\infty} E^2(t) \tag{20}$$

is minimized.

In the ligh of Eq. (19), in the two-degrees-of-freedom-feedback IMC scheme, the minimization of 20 results in two distinct H_2-optimal problems that must be solved to obtain $Q_j^t(z)$ and $Q_j^d(z)$. On the one hand, the optimal inventory tracking problem is formulated as:

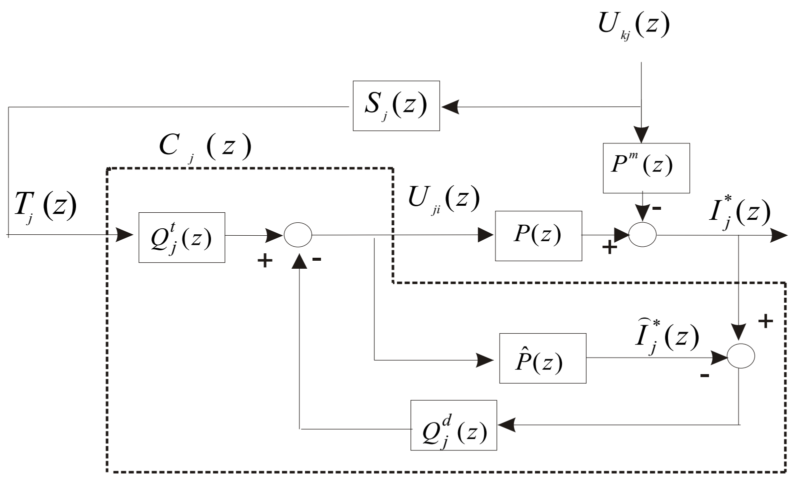

Figure 3: Two-degrees-of-freedom-feedback IMC scheme

$$min_{Q_j^t(z)} \|[1 - P_j(z)Q_j^t(z)]T(z)\|_2 \quad (21)$$

The IMC controller solution for this control problem is generally defined as follows (Morari and Zafiriou (1989); Schwartz et al. (2006); Schwartz and Rivera (2010)):

$$\tilde{Q}_j^t(z) = z(P_j^m(z)T_j(z))^{-1}\{z^{-1}(P_j^a(z))^{-1}T_j(z)\}_* \quad (22)$$

where the $\{.\}_*$ operator denotes that after a partial fraction expansion of the operand (.), all terms involving the poles of $P^a(z))^{-1}$ are omitted.

On the other hand, the optimal disturbance rejection is formulated as:

$$min_{Q_j^d(z)} \|[1 - Q_j^d(z)]P_j(z)U_{kj}(z)\|_2 \quad (23)$$

As a result of the integrating nature of the inventory process Eq. (10), a step change in demand becomes a Type-2(ramp) disturbance. This optimal controller is generally calculated by:

$$\tilde{Q}_j^d(z) = z(P_j^m(z)U_{kj}(z))^{-1}\{z^{-1}(P_j^a(z))^{-1}(z)U_{kj}(z)\}_* \quad (24)$$

2. **Detuning of $Q_j^t(z)$ and $Q_j^d(z)$ for bullwhip effect avoidance**. The resulting controllers $Q_j^t(z)$ and $Q_j^d(z)$ provide a good inventory target tracking and disturbance rejection. However, these controllers lead aggressive orders when the set point change abruptly. Moreover, if there are fluctuations in the orders from immediate customer, these fluctuations may be amplified. This behaviour is widely known as bullwhip effect. These situation are unacceptable for inventory managers. Therefore, the optimal controllers are augmented with

low-pass filters which allow us to manipulate their bandwidth. Then, in the supply chain case, these filters will be used to reduce the bandwidth of the system and avoid bullwhip. Thereby, the nominal performance to inventory target tracking and disturbance rejection is deteriorated but, in exchange, high component frequencies of the demand are rejected in order to satisfy a grade of bullwhip effect avoidance.

The relation between the orders from the immediate customer and the orders delivered to the immediate supplier determines the bullwhip effect constraint. In this way, the bullwhip effect under the two-degrees-of-freedom-feedback-IMC shown in Fig. 3 is given by:

$$U_{j,i}(z) \quad = [(L_j + 2)P_j Q_j^t(z) + Q_j^d(z)P^m(z)]U_{kj} \tag{25}$$

and the ratio of orders to successive nodes can be expressed as:

$$|\gamma_j(z)| = \frac{|U_{ji}(z)|}{|U_{k,j}(z)|} = |(L_j + 2)P_j(z)Q_j^t(z) + Q_j^d(z)P^m(z)| = |(L_j + 2)P_j(e^{i\omega})Q_j^t(e^{i\omega}) + Q_j^d(e^{i\omega})P^m(e^{i\omega})| \tag{26}$$

where i is the imaginary unity and $\omega \in [0, 2\pi)$. (Lin et al., 2004) stated that the amplitude of demand fluctuations will not be amplified if

$$|\gamma_j(z)| \leq 1 \quad \forall \omega \in [0, 2\pi) \tag{27}$$

Notice that Eqs. (26) and (27) imply that a good bullwhip effect avoidance needs $|(L_j + 2)P_j(z)Q_j^d(z) + P_j(z)Q_j^d(z)| << 1$ while Eqs. (21) and (23) require $|P_j(z)Q_j^t(z)| \approx 1$ and $|P_j(z)Q_j^d(z)| \approx 1$ for optimal inventory tracking. Thus, the $Q_j^t(z)$ and $Q_j^d(z)$ controllers must be designed taking into account two opposite objectives: inventory target tracking and bullwhip effect avoidance. Therefore, an analytical detuning of the $\tilde{Q}_j^d(z)$ optimal controller to obtain a trade-off between these objectives is performed by adding low pass filters to the optimal controllers.

Since step changes in the inventory target are considered, the optimal controller $\tilde{Q}_j^t(z)$ for inventory target tracking at each echelon, j, is augmented with a type 1 filter (Morari and Zafiriou, 1989) defined as :

$$f_j^t(z) = \frac{(1 - \lambda_j^t)z}{z - \lambda_j^t} \tag{28}$$

The final controller $Q_j^t(z)$ is given by:

$$Q_j^t(z) = \tilde{Q}_j^t(z)f_j^t(z) \tag{29}$$

In the disturbance rejection case, $\tilde{Q}_j^d(z)$ is augmented with a generalized Type-2 filter $f_j^d(z)$ in order to guarantee no offset for both asymptotically step and ramp disturbances.

$$f^d(z) = \frac{(\alpha_1 z - \alpha_2)(1 - \lambda^d)z}{(z - \lambda^d)^2} \tag{30}$$

The final controller $Q_j^d(z)$ is given by:

$$Q_j^d(z) = \tilde{Q}_j^d(z)f_j^t(z) \tag{31}$$

Remark: The propagation of demand fluctuations is only possible when every node has sufficient stock (Lin et al., 2004). Hence, the bullwhip effect restriction is only considered in the Infinite Supply and High Stock (ISHS) case.

Since the model of each node can become in particular, $P_j^{(ISHS)}(z)$, $P_j^{(ISLS)}(z)$, $P_j^{(LS)}(z)$) through time, the IMC guidelines stated in this section are applied to each one, resulting three different control strategies for each j^{th} echelon depending on the particular case: IMC controllers for Infinite Supply and High Stock case (ISHS), IMC controllers for Infinite Supply and Low Stock case (ISLS) and IMC controllers for Limited Supply case (LS). The resulting IMC controllers for each particular case are shown in the following subsections.

3.1.1. Resulting controllers for infinite supply and high stock case (ISHS)

The final controllers obtained by applying the procedure formulated in Section 3.1 to the model given by Eq. (11) are given by:

$$Q_j^{(ISHS)t}(z) = \frac{z-1}{z}\frac{(1-\lambda_j^t)z}{z-\lambda_j^t} \tag{32}$$

and

$$Q_j^{(ISHS)d}(z) = \frac{2z^2-3z+1}{z^2}\frac{(\alpha_1 z-\alpha_2)(1-\lambda^d)z}{(z-\lambda^d)^2} \tag{33}$$

Thus, replacing Eq. (32) and Eq. (33) in Eq. (26), the bullwhip restriction for the two-degrees-of-freedom-feedback IMC scheme is given by:

$$|\gamma_j(z)| = \left|\frac{(L_j+2)(1-\lambda^t)z}{z-\lambda^t} + \frac{(2z-1)(\alpha_j^1 z-\alpha_j^2)(1-\lambda^d)z}{z(z-\lambda^d)^2}\right| \leq 1 \tag{34}$$

Notice that the bullwhip effect depends on the lead time L_j and $Q^t(z)$ and $Q^d(z)$ parameters (λ_j^t, λ_j^d, α_j^1 and α_j^2). Since λ_j^t and λ_j^d modify the bandwidth of each controller, these are selected so as to satisfy the bullwhip effect condition for a determined L_j value while the parameters α_j^1 and α_j^2 are adjusted to guarantee internal stability for the so-designed λ_j^d value.

The $Q_j^t(z)$ and $Q_j^d(z)$ controllers must be tuned so that the system has a fast response to low frequency demand changes. Thereby, the inventory level can be maintained. On the other hand, this controller must limit the ratio of orders less than 1 at high frequency to guarantee bullwhip effect mitigation.

In this way, (Lin et al., 2004) have suggested to consider the following two factors on the magnitude ratio $|\gamma_j(z)|$ in order to select the values of the controller parameters.

1. Bandwidth: the frequency at which the magnitude ratio (Eq. 34) is reduced to below 0.7 times the value at low frequency.
   Note that we are dealing with a discrete-time system. Therefore, the highest frequency is at $\omega = \pi/T_m = \pi$ since $T_m = 1$. Thus, we can define a term γ_j^π as the magnitude ratio given by Eq. (34) at $\omega = \pi$ i.e $\gamma_j^\pi = \gamma_j(\omega = \pi)$.
   Notice that a higher γ_j^π implies a wider bandwidth and a faster response, it results in more severe bullwhip.

2. Resonance peak (σ_j): the highest value of the amplitude ratio (Eq. 34). A higher resonance peak indicates a fast response to low frequency demand changes (Disturbance rejection) but the closed-loop response may be more oscillatory. Suitable setting of σ_j ranges from 1.5 to 2.0.

The disadvantage of the controllers (PI and cascade PI) proposed in (Lin et al., 2004) is that there is no direct correspondence between the parameters of the controllers and the bandwidth of the magnitude ratio $|\gamma_j(z)|$. Therefore, that work performed an empirical tuning for bullwhip effect based on trial and error. In the Two-degrees-of-freedom feedback IMC scheme the bandwidth of the systems can be manipulated directly by using of the λ_j^t and λ_j^d parameters of the $Q_j^d(z)$ and $Q_j^d(z)$ controllers respectively. Therefore, the application of this analytical tuning criterion for bullwhip effect avoidance is simplified. The application of this tuning criterion for the design of the IMC controllers is also novel in supply chain systems.

Fig. 4 shows the tuning for an echelon with an arbitrary lead time $L = 3$ for illustrative purpose. In this figure, the magnitude ratio $|\gamma_j(z)|$ for several values of λ_j^t and λ_j^d is plotted. For simplicity, we have chosen $\lambda_j^t = \lambda_j^d$. It can be seen that for $\lambda_j^t = \lambda_j^d$ values close to 1 the system present strong mitigation of high frequency but low resonance peak σ_j. That means a mitigation of the bullwhip effect but a sluggish response to low frequency demand changes. On the other hand, for $\lambda_j^t = \lambda_j^d$ values close to 0 the system present poor mitigation of high frequency (severe bullwhip effect) but faster response to low frequency demand changes or step changes (disturbance rejection).

Therefore, the following approximate tuning criterion suggested by (Lin et al., 2004) to find a trade-off between fast inventory tracking and bullwhip effect mitigation can be used:
Choose a controller setting with $\gamma_j^\pi < 1$ and σ_j in the range 1.5 to 2. There are several $\lambda_j^t = \lambda_j^d$ solutions based on this criteria as is shown in Fig. 4 in solid lines. Therefore, in this work, to perform the simulations we chose $\lambda_j^t = \lambda_j^d$ such that $\gamma_j^\pi < 1$ and σ_j be close to 1.8. Fig. 5 extends this criteria for delays between 1 and 10 periods of time.

Since $Q_j^d(z)$ must satisfy asymptotic inventory tracking and internal stability, the filter has to be designed in such a way that all these requirements hold. Hence, once the λ_j^d parameter is selected for bullwhip effect avoidance, the α_j^1 and α_j^1 parameters must be adjusted so as to make the filter satisfy inventory tracking and internal stability. For this system with a pole of multiplicity 1 at z=1, the filter has to satisfy the following conditions at $z = 1$ (Morari and Zafiriou, 1989):

$$f_j^d(z) = 1, \quad \frac{df_j^d(z)}{dz} = 0 \tag{35}$$

Solving this system, we get a mathematical relation which relates the α_j^1, α_j^2 parameters with λ_j^d as:

$$\alpha_j^2 = 2\lambda_j^d, \quad \alpha_j^1 = 1 + \lambda_j^d \tag{36}$$

3.1.2. Resulting controllers for infinite supply and low stock case (ISLS)

The final controllers obtained by applying the procedure described in Section 3.1 to the model according to Eq. (11) are shown as follows:

$$\tilde{Q}^{(ISLS)t}(z) = \frac{z^{L_j}}{z^{L_j} + z^{L_j-1} + ... + z + 1} \tag{37}$$

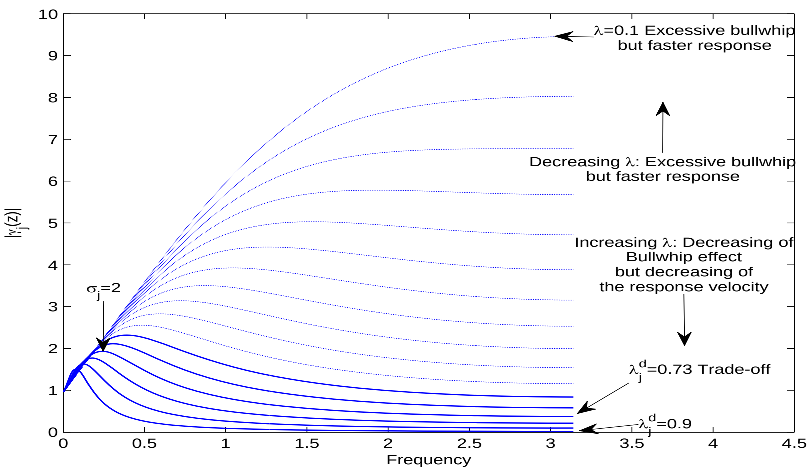

Figure 4: The magnitude ratio $|\gamma(z)|$ vs. frequency in a j echelon with $L_j = 3$ for various λ_j^d values.

Since the $\tilde{Q}^t(z)$ controller may contain unstable poles, a modification is necessary to apply it to the system. We can define now a new controller $\bar{Q}^t(z)$ as:

$$\bar{Q}^{(IS\,LS)t}(z) = \tilde{Q}^t(z)\hat{Q}^t(z) \tag{38}$$

where $\tilde{Q}_j^{(IS\,LS)t}(z)$ is obtained throw Eq.(22) and $\hat{Q}_j^t(z)$ cancels all undesirable poles of $\tilde{Q}_j^t(z)$ and substitutes them with poles at the origin. Let κ_h, $h = 1, ...\rho$ be the unstable poles of $\tilde{Q}_j^t(z)$ Then, we get (Morari and Zafiriou, 1989):

$$\hat{Q}^{(IS\,LS)t}(z) = z^{-\rho} \prod_{h=1}^{\rho} \frac{z - \kappa_h}{1 - \kappa_h} \tag{39}$$

Notice that, the optimal controller given by Eq.(40) and Eq.(39) depends on the lead time L_j. Therefore, in this paper se have choosen an arbitrary Lead time to perform a design example. That is:

$$\tilde{Q}^{(IS\,LS)t}(z) = \frac{z^3}{z^3 + z^2 + z + 1} \tag{40}$$

and

$$\hat{Q}^{(IS\,LS)t}(z) = z^{-3} \frac{z^3 + z^2 + z + 1}{4} \tag{41}$$

thus, the optimal controller is given by Eq. (38):

$$\tilde{Q}^{(IS\,LS)t}(z) = \frac{1}{4} \tag{42}$$

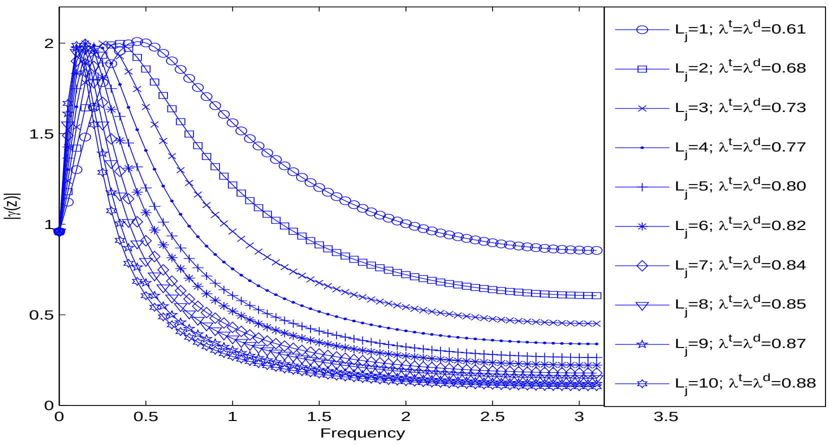

Figure 5: The magnitude ratio $|\gamma(z)|$ vs. frequency in a single echelon for various L_j values.

Finally, enhancing the optimal controller with the low pass filter $f(z)$ we get:

$$\hat{Q}_j^{(ISLS)t}(z) = 0.25\frac{(1-\lambda_j^t)z}{z-\lambda_j^t} \tag{43}$$

Since, in the limited supply case, the inventory position is not depending on the disturbance U_{kj}, the $Q^d(z)$ is not necessary. In order to avoid aggressivity in the orders with the switching of controllers, $\lambda_j^{(ISLS)t} = \lambda_j^{(ISHS)t} = \lambda_j^{(ISHS)d}$ so that $Q_j^{(ISLS)t}(z)$ be as fast as $Q_j^{(ISHS)t}(z)$.

3.1.3. Resulting controllers for Limited Supply (LS)

In this case, the inventory position of node j depends neither on the set point nor on the controller gain of the ordering policy of node j. Therefore, the system makes no control actions and the perceived demand is comunicated to the immediate supplier, i.e $U_{ji} = U_{kj}$.
After formulating the controllers design, the block to set an appropriate inventory target, $S_j(z)$, is stated now.

3.2. Inventory target calculator $S_j(z)$

This block is intended to set an adequate inventory target $T(t)$ at each instant t based on the measured demand signal $U_{kj}(t)$. Usually this block contains a factor of amplification $L_j + 2$ (Lin et al., 2004), and a low pas filter $g(z)$ to smooth the demand variations:

$$T_j(z) = \underbrace{(L_j+2)g(z)}_{S_j(z)} U_{kj}(z) \tag{44}$$

Since, the IMC controllers contain a low pass filter that can mitigate the high frequencies in the orders (bullwhip Effect), the g(z) filter is avoided in this scheme thus, in our case $T_j(z) = (L_j + 2)U_{kj}(z)$. Once the controllers design and the set point rule are stated, the switching rule to select between the presented controllers is introduced now.

3.3. Switching logic

The switching logic evaluates if there is sufficient inventory in the node *j*, i.e ($MIN(O_j(t), I_j(t))$) and immediate supplier $j + 1$, i.e ($MIN(O_i(t), I_i(t))$) and selects the respective controller designed for each case.
That is: if the node *j* and immediate supplier have sufficient inventory (Infinite Supply and High Stock case) then the switching logic chooses the output of the controller $C_j^{ishs}(z)$ i.e

$$if \quad I_{j+1}(t) > O_i(t) \wedge I_j(t) > O_j(t) \quad Then \quad C_j^{ishs}(z) \ . \tag{45}$$

If immediate supplier have sufficient inventory but the node *j* inventory is low (Infinite Supply and Low Stock case) then the switching logic chooses the output of the controller $C_j^{isls}(z)$ i.e

$$if \quad I_j(t) > O_j(t) \wedge I_j(t) < O_j(t) \quad Then \quad C_j^{isls}(z) \ . \tag{46}$$

If the immediate supplier have no sufficient existences to supply the node *j* (Limited Supply case) then the switching logic chooses the output of the controller $C_j^{ls}(z)$ i.e

$$if \quad I_i(t) < O_i(t) \wedge I_j(t) > O_j(t) \quad or \quad I_j(t) < O_j(t) \wedge I_j(t) < O_j(t) \quad Then \quad C_j^{ls}(z) \ . \tag{47}$$

thus, the complete structure of the proposed switched inventory management policy is defined. The following section shows the usefulness of the presented scheme through simulation examples.

4. Simulation results

This section starts comparing the performance of the PI controller proposed by (Lin et al., 2004) and the switched control approach proposed in this work with respect to inventory tracking, bullwhip effect mitigation and customer satisfaction. The comparisons are performed by simulations of an echelon under PI and switched control in Subsection (4.1). Moreover, in order to provide more quantitative results in all simulations, the basic performance indices of the supply chain Integral absolute error (IAE), Back-order (BO) and Bullwhip effect (BW) described in (Lin et al., 2004; Balan et al., 2009) are calculated:

$$IAE = \sum_{t=1}^{\infty} |R_j(t) - I_j^*(t)| dt \tag{48}$$

$$BO = \sum_{t=1}^{\infty} (Y_{j,k}(t) - O_j(t)) \tag{49}$$

$$BW = \sum_{t=1}^{\infty} |(U_{j,i}(t)/U_{kj}(t)) - 1| \tag{50}$$

After that, the simulation are extended to the multiechelon scenery in subsection (4.2) in order show the inventory control behaviour under a more realistic situation, which is novel.

4.1. A single echelon scenery

In this section, we simulate an echelon with the PI control policy proposed and designed by Lin et al. (2004) as is shown in Fig. 6 . In this policy, $C_j(z)$ is the proportional and integral controller which is widely known in the control literature. Thus, the controller for each echelon has the following structure:

$$C_j(z) = \tilde{k}_j \left(1 + \frac{1}{\tau_j} \frac{z}{z-1} \right) \tag{51}$$

The filter $g(z)$ proposed by the authors to forecast the demand is given by:

$$g_j(z) = \frac{\alpha}{z + \alpha - 1} \tag{52}$$

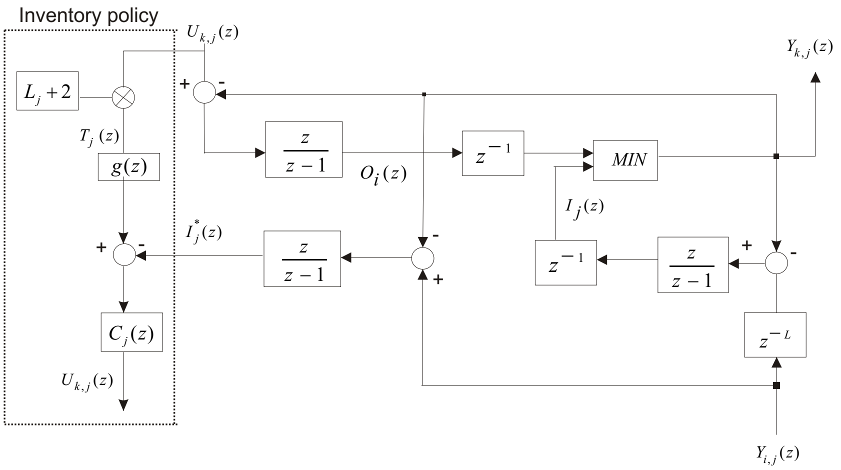

Figure 6: The block diagram of node j of a supply chain with demand forecasting and a PI controller.

The parameters of controller $C_j(z)$ and the parameter of the filter $g_j(z)$ are tuned for bullwhip effect avoidance in (Lin et al., 2004). That is, $\tilde{k}_j = 0.67$, $\tau_j = 3.3$and $\alpha = 0.1$. In this simulation, the customer demand is stochastic i.e $U_{CR} \in N(20, 4)$ at $t = 1$. Suddenly, the system perceives a change in the demand to $U_{CR} \in N(40, 4)$ at $t = 20$ and finally the demand changes to $U_{CR} \in N(30, 4)$ at $t = 60$ and onwards. Fig. (7) shows that, the filter usually used in the forecasted demand causes a sluggish set point. On the other hand, since the controller $C_j(z)$ is tuned to ISHS, this causes oscillations in the orders when the echelon change from ISHS to ISLS operation case. Thus, this policy presents slowly recuperation of the inventory. Therefore, it will lead to accumulation of a large amount of backorder and low customer satisfaction.
Fig. 8 illustrates that the presented approach avoids the filter in the forecasted demand. Therefore, the set point is faster than the presented in the PI policies. Since, this approach contains

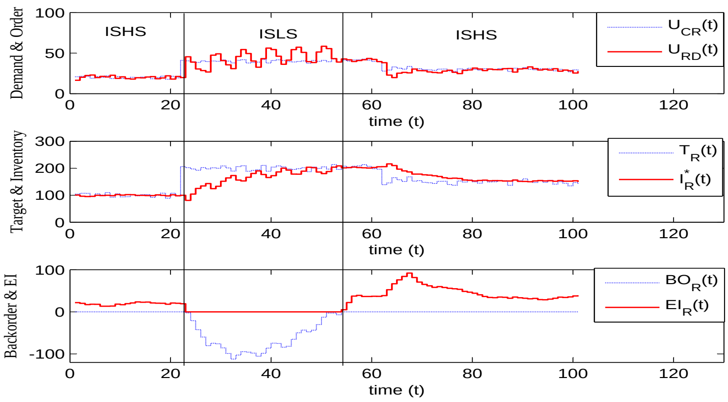

Figure 7: Dynamic simulation results of a supply chain unit with demand forecasting and a PI controller with $\tilde{k}_j = 0.67$ and $\tau_j = 3.3$.

Table 1: Performance indices of the control schemes

	PI controller	Switched system
IAE	1959	1837
BO	1956	883
BW	12.74	9.9

a switching logic to select the appropriate controller, once the operation case changes, the controller changes too. Therefore, there are no oscillations in the orders (control action). Moreover, the controllers are tuned appropriately to an acceptable set point tracking. Thus, the backorders are reduced and the bullwhip effect is mitigated. Table 1provides more quantitative results than Figs. 7 and 8. This table shows that for the switched control approach, the back-order and IAE index are lower than for the PI control strategies. Moreover, the bullwhip effect is reduced by the switched control respect to PI strategies.

4.2. Multi-echelon scenery

Once the inventory control system is evaluated for an echelon, the simulation for multi-echelons is performed which is not explored in previous works. In this case, we simulate two echelons (R,D). The simulation is performed with the same parameters and conditions presented in section 4.1 for each echelon. For the sake of simplicity the stock of the factory of the supply chain is considered infinite. Figs. 9 and 10 show that although the PI control policy presents an acceptable behavior for a single echelon, for several echelons this policy is unsuccessful.

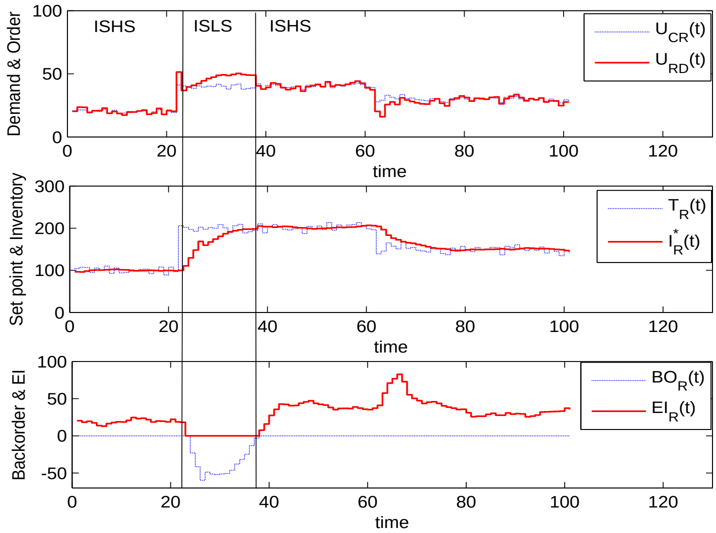

Figure 8: Dynamic simulation results of a supply chain unit with the switching control scheme

Fig. 11 shows the behaviour of the system under the switched control system. It can be seen that the performance to meet the set point is deteriorated in the echelon base (R). This is due to the fact that a decentralized control policy is implemented and that the replenishment depends on the existences in the immediate supplier. However, the inventories track the set point and the bullwhip effect is mitigated in the two echelons. Moreover, Table 2 shows the performance index of the retailer and distributor working at once. Since, there are not simulations of two echelons in previous works, the table values have illustrative purposes for future works.

5. Conclusions

The entire supply chain is a complex system where the operation modes changes with time. Therefore, in this work a switched inventory control system where the appropriate policy is set according to the operation mode is developed. The proposed approach includes a bank of control policies designed adequately for each supply chain operation case. The control system is complemented with an inventory target calculator to set the inventory position target according to the demand perceived through time. In addition, the system is enhanced with a switching logic block monitoring $MIN(O_j(t), I_j(t))$ and $MIN(O_i(t), I_i(t))$ which determine the change from one operation case to other. Then, the switching logic block selects the adequate controller through time. Simulation results in an echelon show that the switched control strategy is more successful

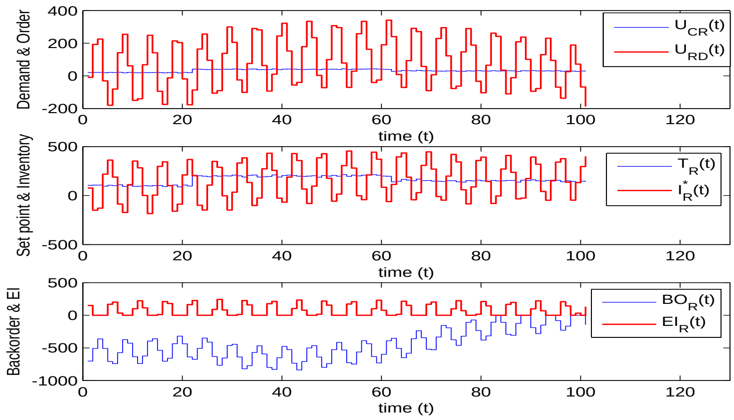

Figure 9: Dynamic simulation results of the Retailer with the PI control scheme

Table 2: Performance index of the PI control policy and the two-degrees-of-freedom-feedback IMC scheme

	PI controller		Switched system	
	Retailer	Distributor	Retailer	Distributor
IAE	16710	16724	2447	1878
BO	44780	213620	2554	863
BW	456	360	9.76	16.63

than a policy where the inventory control policy is fixed. Simulations developed in multi-echelon scenery show that the control strategy holds the efficacy in a general supply chain.

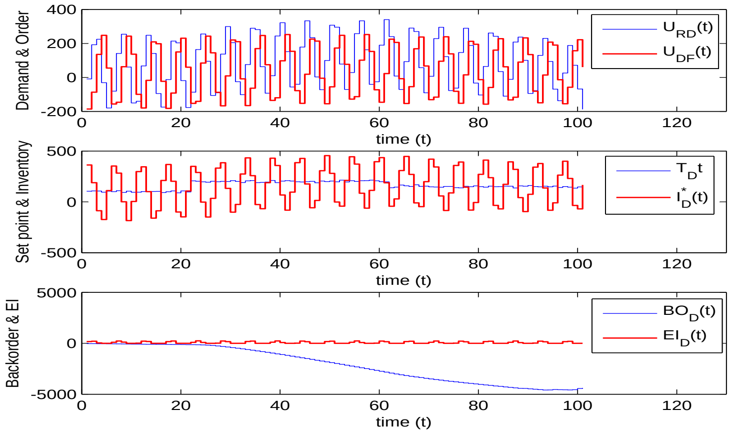

Figure 10: Dynamic simulation results of the Distributor with the PI control scheme

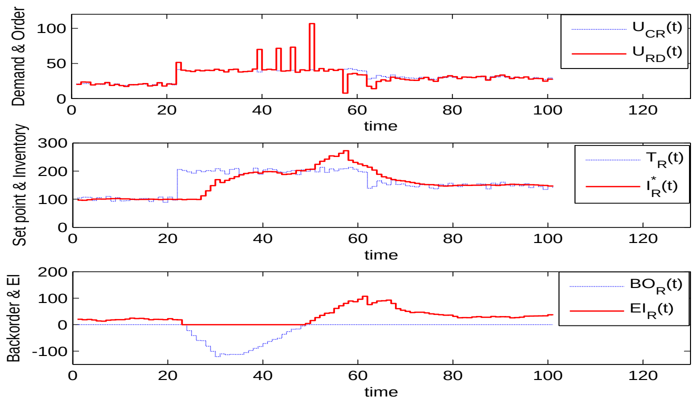

Figure 11: Dynamic simulation results of the Retailer with the switching control scheme

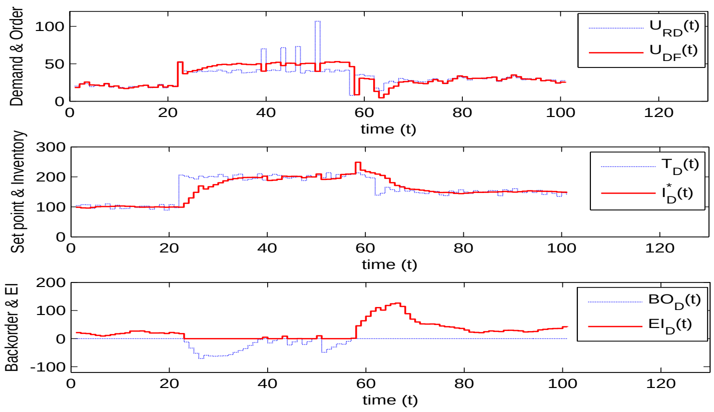

Figure 12: Dynamic simulation results of the Distributor with the switching control scheme

Appendix A. Derivation of the transfer function at Infinite Supply and Low Stock

If node j keeps a low stock so that there is always less inventory than the amount ordered by the customer, delivery is limited by the inventory:

$$Y_{j,k}(z) = z^{-1} I_j(z) \tag{A.1}$$

Therefore, substituting Eq. (6) into Eq. (A.1), we get

$$\begin{aligned} Y_{j,k}(z) &= \tfrac{z^{-(L_j+1)}}{1-z^{-1}} Y_{i,j}(z) - \tfrac{z^{-1}}{1-z^{-1}} Y_{j,k}(z) \\ &= z^{-(L_j+1)} Y_{i,j}(z) \end{aligned} \tag{A.2}$$

If we assume that the supplier has unlimited supply, delivery to node j will be according to what has been ordered:

$$Y_{i,j}(z) = z^{-1} O_i(z) = z^{-1} U_{j,i}(z) \tag{A.3}$$

Replacing Eq. (A.3) into (A.2) we get

$$Y_{j,k}(z) = z^{-(L_j+1)} U_{j,i}(z) \tag{A.4}$$

Using Eqs. (A.3) and (A.4) on the Eq. 10, inventory position for infinite supply and low stock is given by:

$$I_j^*(z) = \frac{z^{L_j+1} - 1}{z^{L_j+1}(z-1)} U_{j,i}(z) \tag{A.5}$$

Factoring the numerator of Eq. (A.5) the inventory position becomes:

$$I_j^*(z) = \frac{z^{L_j} + z^{L_j}+, ..., +z + 1}{z^{(L_j+1)}} U_{j,i}(z) \tag{A.6}$$

which is the equation given in Eq. (15).

Appendix B. Derivation of the transfer function at Limited Supply

If the supplier node i has insufficient inventory, then its delivery is limited by its stock level instead of the demand of node j:

$$Y_{i,j}(z) = z^{-1} I_i(z) \tag{B.1}$$

The inventory position becomes

$$I_j^*(z) = \frac{1}{z-1}(I_i(z) - zY_{j,k}(z)) \tag{B.2}$$

When node j keeps a high stock, then as given in Eq. (A.3):

$$I_j^*(z) = \frac{1}{z-1}(I_i(z) - U_{k,j}(z)) \tag{B.3}$$

When node j keeps a low stock:

$$\begin{aligned} I_j^*(z) &= \tfrac{1}{z-1}(I_i(z) - I_j(z)) \\ &= \tfrac{1}{z-1}\left(I_i(z) - I_j^*(z) - \tfrac{z(z^{-L_j}-1)}{z-1} Y_{i,j}(z)\right) \end{aligned} \tag{B.4}$$

Substituting Eq. (B.1) into Eq. (B.4) and rearrangint it:

$$\begin{aligned}\left(1+\frac{1}{z-1}\right)I_j^*(z) &= \frac{1}{z-1}\left(I_i(z)-\frac{z(z^{-L_j}-1)}{z-1}Y_{i,j}(z)\right)\\ &= \frac{z^{L_j+1}-1}{(z-1)^2 z^{L_j}}I_i(z)\end{aligned} \tag{B.5}$$

Further simplification gives:

$$I_j^*(z) = \frac{z^{L_j+1}-1}{(z-1)z^{L_j+1}}I_i(z) \tag{B.6}$$

Eqs. (B.5) and (B.6) are the equations given in Eq. (18) describing the Limited Supply case.

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