

ELECTORAL SYSTEMS AND FORMS OF ABSTENTION

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Chapter 1

General Introduction

The present thesis lies at the intersection of economics and political science. Using methods, well developed in economics, I try to understand how electoral institutions shape individuals' behavior in different political contexts. The three chapters are closely related, and focus from three different perspectives on the decision of voters to abstain or to participate in an election.

Chapter 2 has a normative flavor, and is motivated by my personal frustration towards the dangerous combination of apathetic citizens and mediocre politicians prevailing in contemporary politics. Focusing on the interplay between low quality parties and citizens' apathy, I try to break the vicious cycle that links the two, by proposing two electoral rules that increase turnout in PR elections, and at the same time give incentives to parties to be of better quality.

First, I propose an electoral rule where the number of candidates elected depends on the level of participation. Second, I propose the introduction of a participation quorum that has to be met in order for the election to be valid. The common feature and innovation of these rules is that turnout affects the electoral outcome, and as a consequence these rules incentivize parties to care about the level of turnout. I show that both rules, while they increase turnout they imply lower profits for parties. My results explain why parties target to increase turnout through a certain type of measures that do not necessarily improve the quality of the vote. Moreover, I also explain the evolution of the use of the participation quorum in certain countries.

In reality, a participation quorum is not often present neither in parliamentary nor in presidential elections. On the contrary, in many small scale meetings, for example, meetings of shareholders, members of societies, and clubs, the number of attendees must exceed an exogenously given participation requirement in order for a decision to be taken. Otherwise, the meeting can not take place, and it has to be postponed for the future.

Chapter 3 is coauthored with Sabine Flamand, and tries to understand the effect of such a participation requirement on individuals' behavior and the decision outcome. To this end, we model a setup of repeated meetings, where a small group of individuals has to take a decision. We show that the decision is delayed when the quorum requirement is high and members are not harmed by postponing the decision. Surprisingly, the presence of the quorum may decrease the number of attendees taking the decision, while we show that in order to avoid policy distortions, the required number of participants must be even.

Although presented as the last chapter of this thesis, Chapter 4 is the one I completed first. While browsing the political science literature in order to motivate a previous version of Chapter 2, I realized that several questions that seemed natural to me, had not been answered. Apart from abstaining, voters that are not willing to support any of the candidates in most parliamentary elections, are given the choice to participate in the election and cast a blank or a null vote. A blank vote is a disapproval vote of all competing candidates, while a null vote is a vote cast erroneously or deliberately in a way not conforming with the legal voting procedure.

Political scientists were treating blank and null votes in an identical way. My attempt in this chapter is to study these two protest actions on a separate basis, in order to understand, why in some elections blank votes are many more than null votes and vice versa. After constructing a database considering the percentages of blank and null votes separately, I show that the amount of blank and null votes cast in an election are not affected by the same factors. Null votes convey dissatisfaction towards the electoral and democratic institutions, while blank votes convey dissatisfaction towards the parties. More important, my results go against one of the prevailing criticisms of compulsory voting. The latter has no significant effect on the amount of uninformative votes since it has no significant effect on the

amount of null votes. On the contrary, it increases only the amount of blank votes, which by definition disclose information, and in particular voters' disapproval of all competing parties.

Although abstention is one of the most studied issues both by political scientists and economists, I hope that the current thesis extends our knowledge, by giving insight into some of abstention's unexplored but widely observed aspects. Moreover, I hope that this thesis not only succeeds in analyzing from a positive perspective different forms of abstention as political actions, but also by taking a normative stance, and in particular by designing a set of modifications in existing electoral systems. If one believes that the quality of democracy is determined by the performance of its principal actors, then by giving incentives to parties to be of better quality, and at the same time involving more citizens in the electoral procedure, the suggestions of this thesis can eventually be considered as a way to establish a better functioning of contemporaneous democracy.

Chapter 2

Engineering Electoral Systems to Increase Turnout

2.1 Introduction

Voice and exit are often alternative ways of exerting influence, but with regard to voting the exit option spells no influence; only voice can have an effect , Lijphart (1997)

Lijphart refers to abstention as the exit option of voters in elections. Indeed in parliamentary elections voters abstaining do not affect the composition of the parliament. In this paper we introduce two electoral rules under which the exit option (i.e. abstention) influences the outcome of the election. Both electoral rules can be introduced as a solution to low turnout levels.

A well established empirical fact is that turnout is decreasing in most elections during the last decades and more specifically since the mid-eighties (see Figure 1 in the appendix). A natural question that may arise is why it is of interest going against this trend. In general, high levels of participation in democratic elections are desirable in order to guarantee the legitimacy of the election, of the elected government, of the representative institutions and of the democratic political system as a whole. This can be reflected in president Barroso s speech in 2009 right before the election for the EU parliament: *“A low turnout in the elections for the European Parliament next week would undermine the credibility of the European Union in general and of the parliament in particular”*. In this

particular election the turnout rate was the lowest in the history of the EU (see Figure 2 in the appendix) and as a result the vice-president of the EU commission claimed that “...low turnout was a “bad result”... there is need for a radical shake-up of how future European election campaigns are conducted by EU states, to try to boost voter turnout”.

The most extreme among all alternatives in order to increase turnout is the introduction of compulsory voting¹. As expected, it is shown both empirically (Jackman, 1987; Blais and Dobrzynska, 1998; Franklin, 1999, 2004) and experimentally (Hirczy, 1994) that compulsory voting is associated with higher levels of turnout. In his presidential speech of the American Political Science Association, Lijphart endorsed compulsory voting and claimed that “it is the only institutional mechanism that can assure high turnout virtually by itself”.

While the effectiveness of compulsory voting in increasing turnout can not be challenged, there exist several arguments against it. The typical normative argument is that in the effort to enhance democracy the tool is highly undemocratic since it restricts citizens right not to vote. Furthermore, although compulsory voting increases turnout, it does not improve on the quality of the vote since it increases the numbers of uninformed voters who are forced to participate.

As a solution to low turnout under a PR system we suggest two electoral rules that may affect the effort exerted by parties: a varying size parliament and the introduction of a participation quorum.

In the case of a varying size parliament (VSP) the number of candidates elected depends on the level of participation². It is actually determined after the election takes place and abstention is translated into empty seats in the parliament. The difference between a fixed size parliament (FSP) and a VSP is the seat allocation

¹As an alternative to compulsory voting, rather than punishing voters for not voting one could reward them for voting. This form of incentives could be either a flat-rate payment to every voter or the right to participate in a lottery that assigns a prize to a lucky participant. While a flat rate payment has not been implemented, the case of a lottery has been practiced in the 2005 parliamentary election in Bulgaria and in 1995 in municipality elections in Norway. Gerardi et al. (2009) compare theoretically and in laboratory experiments the case of compulsory voting versus incentives through a lottery. While both increase turnout, lotteries are more effective in terms of information aggregation.

²The idea of a VSP is similar to a seat allocation method based on a fixed quota. For example, in the elections of 1919 in Weimar Republic, parties had to obtain 60.000 votes for one seat in the parliament(Colomer, 2004).

after the election takes place. In the case of a FSP the number of candidates elected is fixed and does not depend on turnout while under a VSP the number of candidates elected depends on participation.

Alternatively, rather than varying the number of candidates elected, one could think of varying the length of the term that elected candidates stay in office. In this case the length of the term is endogenously determined and depends on the level of participation. If for example the normal term is four years and only half of the population participates then the term will be reduced to two years.

A last important interpretation of the VSP with direct policy implications is associated with parties public funding. A VSP can be thought as a funding system where parties are funded per vote and hence the total transfers to parties depends on the level of participation. On the contrary, the FSP can be interpreted as a funding system where the total amount of transfers to parties is fixed and the funds are allocated to each party based on each party's representation in the parliament.

The second electoral rule we analyze in order to increase turnout, is the introduction of a participation quorum. If the participation quorum is not met then candidates do not acquire any seats in the parliament thus they obtain zero benefit. In such a case a new election takes place. We assume that there are no fixed costs for organizing an election.

The common feature of both suggested electoral rules (i.e. a switch to a VSP and the introduction of a participation quorum) is that the level of abstention affects the electoral outcome and the composition of the parliament. The advantage of the two suggested alternatives over compulsory voting is that they do not oblige voters to participate³. They rather incentivize parties to care about the level of participation.

In order to study the effect of the two above alternatives we adapt a group-based model of turnout (Snyder, 1989; Shachar and Nalebuff, 1999; Herrera and Mattozzi, 2009). In this type of models parties exert some costly effort that is beneficial for voters. The effort that parties exert may be interpreted in several ways. The first interpretation is that effort is directly related to the mobilization

³Under compulsory voting a voter may still abstain. We say that voters are obliged to vote in order to avoid a possible sanction.

actions taken by parties. Gerber and Green (2000) through a random experiment show evidence of the relationship between mobilization and the decision to vote. Citizens that are informed by parties on a personal basis, vote with higher probability. In general, they conclude that time consuming and costly mobilization effort such as canvassing has a higher effect turnout than less costly effort such as telephone calls and electronic mails. Similarly, effort can be interpreted as the social pressure put by parties in order to mobilize voters. As Shachar and Nalebuff (1999) claim, through social pressure parties can mobilize voters by increasing the cost of non-voting. Indeed, Gerber et al. (2008) offer experimental evidence and show that individuals receiving social pressure participate in the election with higher probability.

Another interpretation of effort is the quality of each party as in Carrillo and Castanheira (2008). Hiring experts or the opportunity cost of parties of not being corrupt (Polo, 1998) are examples of why parties quality is costly. When parties are of better quality, voters obtain a higher benefit and are willing to pay the cost of participating. No matter which interpretation we give to effort, the important element for our analysis is that effort is costly for parties while it is desirable for voters.

Each voter has a personal cost of voting that is private information. We assume that each individual may vote for his favorite party or abstain, therefore there are no swing voters. Voters are expressive and support their preferred party if the effort exerted by the party is high enough to compensate the cost of participating.

In our model there are two exogenously given parties and the society is divided into two pools of potential voters for each party. One of the parties may have a larger support than the other and thus may enjoy an advantage. Parties profits depend on the the absolute number of seats won. The cost that parties have to bare depends on the effort they exert in order to convince their supporters to participate.

We begin our analysis with the case FSP which is our benchmark case. We show that despite a possible advantage of one party in terms of its initial support, there exists a unique symmetric equilibrium. On the contrary, under a VSP the equilibrium effort level is asymmetric in case of different initial support for each party. The reason why under a FSP we obtain only symmetric effort level is that

under a FSP the game is constant sum where parties share the same amount of seats in the parliament independently of the turnout level, while under a VSP this is not the case since the number of seats available is increasing in turnout.

More important, when comparing turnout under a FSP and a VSP, we show that a VSP implies higher turnout only when parties obtain high benefits for being represented in parliament. Moreover, when the introduction of a VSP implies higher turnout then parties obtain lower profits.

With respect to the second electoral rule, we show that there exists a range of participation quorums that boost turnout. If the exogenously given quorum falls in that range then in equilibrium it is always binding. We show that there exists a unique symmetric equilibrium, where both parties exert the same amount of effort. If so, parties obtain lower profits when competing in an election with a participation quorum requirement than when competing in an election with no quorum requirement. Despite the symmetric setup in terms of support, we show that there exist multiple asymmetric equilibria, where one party exerts higher effort than the other. Even in this case, the total effort exerted by parties is such that the participation quorum in equilibrium is binding.

The paper is organized as follows. In section two we compare our work to the existing literature and discuss our contribution to it. In section three we present the model. In section four we characterize and compare equilibria for both the FSP and the VSP. In section five we introduce a participation quorum and again we characterize and compare equilibria under both a FSP and a VSP. In section six we offer a spatial interpretation of the voters' decision rule and in section seven we conclude.

2.2 Related Literature

The characterization of the optimal level of turnout and the effects of compulsory voting had received very little attention by economists till recently. The first contribution focusing on compulsory voting is by Borgers (2004). He analyzes a costly voting model and he shows that in a symmetric setup, under majority voting, voluntary participation Pareto dominates compulsory voting. This is be-

cause voting can be thought of as a negative externality problem. Any additional vote makes it less likely that any other individual's vote is pivotal. Hence, in equilibrium participation in elections is already too high and compulsory voting moves turnout the wrong direction.

On the contrary, Krasa and Polborn (2009); Ghosal and Lockwood (2009) show that increasing turnout may be efficient. Under different assumptions they show that one's vote may create a positive externality on top of the negative pivot externality identified by Borgers (2004). When the positive externality compensates the negative externality then in equilibrium turnout is low and hence increasing turnout would be beneficial. This is the case when there exists a sufficiently large asymmetry in terms of the support of each candidate (Krasa and Polborn, 2009) or when voting takes place according to signals since one's vote may improve the quality of the decision for all voters (Ghosal and Lockwood, 2009).

Our group-turnout model (Snyder, 1989; Shachar and Nalebuff, 1999; Herrera and Mattozzi, 2009) differs from the above since our voters' decision to participate depends on a benefit associated with parties' effort rather than the probability of being pivotal. Regarding group-turnout models and although our research question is quite different from the one of Shachar and Nalebuff (1999), the authors offer interesting interpretations of why effort exerted by some voters (the so called leaders) may mobilize the rest of the voters (the so called followers). For example, effort exerted by leaders may decrease the direct cost of voting and the cost of information acquisition. We provide a spatial interpretation of the voter's decision and we motivate effort as the quality of the party.

A similar group-based model is used by Herrera and Mattozzi (2009) to study the effect of a participation quorum in referenda. In contrast to our setup, they show that the participation quorum in case of referenda may result into lower participation levels. This is the case when the party supporting the status quo has incentives to stay passive rather than mobilizing its supporters to vote against the change. By not mobilizing its voters it wins the referendum by not satisfying the participation quorum. This is not the case in our model, since both parties have incentives to mobilize the supporters in order to avoid a new election.

2.3 The Model

There are two exogenously given parties $j = L, R$. The society is divided into two pools of potential supporters of each party. Without loss of generality, we assume that fraction $\alpha \in [0, 1]$ supports party L , with the remaining $(1 - \alpha)$ denoting the support of party R . Each voter has a personal cost of voting $c \in [0, 1]$ that is private information. We assume that parties' beliefs on the value of c are represented by an i.i.d. uniform distribution on $[0, 1]$.

Parties decide simultaneously the amount of effort to exert in order to persuade their supporters to vote for them. We assume that voters receive a benefit from voting their preferred party that is strictly concave in parties' efforts. More specifically, if party j exerts effort ϵ_j , the benefit of voters supporting party j is captured by the function $\rho(\epsilon_j)$. Function $\rho(\epsilon_j) : \mathbb{R}_+ \rightarrow [0, 1]$, is continuous for $\epsilon_j \geq 0$, strictly increasing, strictly concave and takes value $\rho(0) = 0$. The voting rule followed by the individuals is as follows: For a given level of effort made by party j , a voter that supports party j and has a voting cost equal to c votes for party j if and only if $\rho(\epsilon_j) \geq c$.

From the parties' point of view and the uniformly distributed cost of voting, the probability that an individual participates is $Pr(\rho(\epsilon_j) \geq c) = \rho(\epsilon_j)$, which implies that the expected vote share for each party is equal to its initial support multiplied by the probability that an individual participates (i.e. $v_L = \alpha\rho(\epsilon_L)$ and $v_R = (1 - \alpha)\rho(\epsilon_R)$).

Abstention is given by: $v_A = 1 - v_L - v_R = 1 - \alpha\rho(\epsilon_L) - (1 - \alpha)\rho(\epsilon_R)$. Notice that abstention is always decreasing in both ϵ_L and ϵ_R .

Parties' profits are equal to the benefits parties obtain for holding seats in parliament minus the cost parties have to pay in order to exert some effort. Parties' profit function is defined as:

$$\pi_j = bs_j - \epsilon_j$$

where $b > 0$ is the payoff to a party if it were to obtain all the seats in the parliament and s_j is the percentage of seats the party obtains⁴. For simplicity, we

⁴The assumption of seat-maximizing parties is necessary in order to compare a VSP and a FSP system. The difference between the FSP and the VSP results from the seat allocation

assume that for both parties the marginal cost of effort is equal to one⁵.

2.4 Varying Size Parliament

We begin our analysis with our benchmark case, the FSP system. Party L exerting effort ϵ_L obtains vote share $v_L = \alpha\rho(\epsilon_L)$ and after the seat allocation its seat share is given by $s_L^F = \frac{v_L}{v_L+v_R} = \frac{\alpha\rho(\epsilon_L)}{\alpha\rho(\epsilon_L)+(1-\alpha)\rho(\epsilon_R)}$ which is the vote share of party L divided by the turnout level. In the same way party R obtains seat share $s_R^F = \frac{(1-\alpha)\rho(\epsilon_R)}{\alpha\rho(\epsilon_L)+(1-\alpha)\rho(\epsilon_R)}$ which is the vote share of party R divided by the turnout level.

Proposition 1. *Under a FSP there exists a unique equilibrium $\epsilon_L^F(b, \alpha) = \epsilon_R^F(b, \alpha) = \epsilon^F(b, \alpha)$, where $\epsilon^F(b, \alpha)$ is the unique solution of $\frac{\rho(\epsilon^F)}{\rho(\epsilon^F)} = \frac{1}{b\alpha(1-\alpha)}$*

All Proofs can be found in the Appendix

According to the above proposition in case of a FSP and even in cases that parties have different support (i.e. in case that $\alpha = \frac{1}{2}$), both parties exert the same level of effort $\epsilon^F(b, \alpha)$. The equilibrium effort level depends on both the benefit parties obtain and on the (a)symmetry of the society.

The symmetric equilibrium in terms of effort level despite a possible asymmetry in terms of each party's initial support, is a consequence of the fact that parties compete in a constant-sum game. The constant-sum game implies that what one party wins in terms of seat share by increasing its effort level is exactly what the other party loses in terms of seat share. Given that by assumption parties have the same marginal cost (in this case equal to one) and because of the constant-sum nature of the game, parties stop investing in effort at the same level (i.e. $\epsilon^F(b, \alpha)$).

and the absolute number of seats that each party obtains after the election. In order for the comparison between the two systems to make sense it has to be the case that parties care for the absolute number of seats rather than only the probability of winning. If parties care only for the majority the solution of the maximization problem would be the same under both a FSP and a VSP.

⁵This assumption can be relaxed and does not affect our analysis. In general if both parties have the same support and one party were able to increase its effort at a lower cost than the other party then it would exert higher effort than the other party.

Proposition 2. *Under a FSP, $\epsilon^F(b, \alpha)$ is strictly increasing in b and strictly decreasing in α .*

Effort level exerted by each party is higher in more symmetric societies. Given that in our setup, more symmetric societies implies a closer election, our result is in line with a well established empirical fact, turnout is higher in closer races. Moreover, effort level and turnout are increasing in the benefit parties obtain. When the benefit is high, parties have more incentives to mobilize their supporters to participate. As a result of this higher effort, turnout is higher.

Claim 1. *Under a FSP in equilibrium $\pi_L^F(b, \alpha) = \pi_R^F(b, \alpha)$, with the equality holding for $\alpha = 0.5$*

When one of the parties has an advantage in terms of support then it makes higher profits. This is because in equilibrium both parties exert the same effort, which implies that while they bare the same cost, the same amount of effort translates into more votes and hence seats in the parliament for the advantaged party.

Now we move to the case of the varying size parliament (VSP). The difference with the FSP is the seat allocation. In case of the VSP we have $s_L^V = v_L = \alpha\rho(\epsilon_L)$ and $s_R^V = v_R = (1 - \alpha)\rho(\epsilon_R)$. Since the vote share of each party is translated in the seat share of each party in the parliament there is no reason to divide by the participation level as in the case of a FSP. In case of a VSP the seat share of each party does not depend on the vote share of the other party.

Proposition 3. *Under a VSP there exists a unique equilibrium $(\epsilon_L^V(b, \alpha), \epsilon_R^V(b, \alpha))$, where $\epsilon_L^V(b, \alpha)$ is the unique solution of $\rho(\epsilon_L^V) = \frac{1}{b\alpha}$ and $\epsilon_R^V(b, \alpha)$ is the unique solution of $\rho(\epsilon_R^V) = \frac{1}{b(1-\alpha)}$.*

In the case of a VSP parties do not exert the same level of effort when they have different support. This is different than in the case of a FSP and this is a consequence of the fact that now the game is not a constant-sum one any longer since under a VSP the number of seats in the parliament is not fixed. Although both parties have the same marginal cost, the marginal benefit of their effort level is different (higher for the case of the advantaged party) and hence the solution of their maximization problem is different.

Proposition 4. *Under a VSP $\epsilon_L^V(b, \alpha) = \epsilon_R^V(b, \alpha)$ with the equality holding for $\alpha = 0.5$. Both $\epsilon_L^V(b, \alpha)$ and $\epsilon_R^V(b, \alpha)$ are strictly increasing in b . $\epsilon_L^V(b, \alpha)$ is strictly increasing in α , while $\epsilon_R^V(b, \alpha)$ is strictly decreasing in α .*

In contrast to the FSP, the advantaged party always exerts higher effort when $\alpha = 0.5$. As in the case of a FSP both parties exert higher effort when they obtain higher benefit for being in parliament. This is because they have more incentives to mobilize their supporters given the higher benefit. When α increases then party L support increases while party R support (i.e. $1 - \alpha$) decreases. Hence, when α increases then party L increases its effort and party R decreases its effort. This implies that when the society is more asymmetric, the difference between the effort level exerted by the two parties is larger.

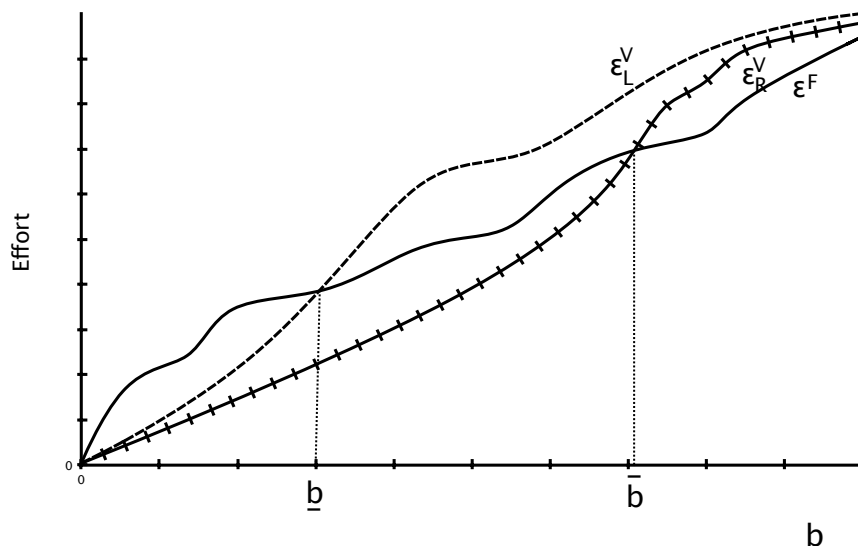
Claim 2. *Under VSP in equilibrium $\pi_L^V = \pi_R^V$ with the equality holding for $\alpha = 0.5$.*

If one party has an advantage then it makes higher profits. This time the intuition is slightly different than in the case of a FSP. In contrast to a FSP the advantaged party exerts higher effort than its opponent. This implies that it bears higher costs than the disadvantaged party. At the same time the advantaged party obtains more seats in the parliament than the disadvantaged both because of its higher effort and its higher support. Eventually, despite the higher cost that the advantaged party has to pay, because of the concavity of the voters' benefit function the advantaged party's higher effort implies much higher seat share than the disadvantaged party and hence higher profits for the advantaged party.

2.4.1 Comparing VSP and FSP

Having analyzed separately the two electoral systems and having proved the uniqueness of equilibria in both cases, the following proposition compares the two systems. The results of this proposition are illustrated in figure 1.

Proposition 5. *For a given α , $\epsilon_L^V(b) > \epsilon^F(b)$ if and only if $b > \underline{b}$. Moreover $\epsilon_R^V(b) > \epsilon^F(b)$ if and only if $b > \bar{b}$.*

Figure 1


From the above proposition we conclude that which electoral system implies higher effort depends on the level of b . If $b > \bar{b}$ then by introducing a VSP effort increases for both parties. In our model a VSP would imply higher effort by both parties if turnout under a FSP is larger than α . In the other extreme case, if $b < \underline{b}$ then a VSP implies lower effort by both parties. This is the case when turnout is lower than $1 - \alpha$. If $\underline{b} < b < \bar{b}$ (or turnout is between $1 - \alpha$ and α), then a VSP implies higher effort by the advantaged party and lower effort by the one with lower support.

Regarding turnout, if $b > \bar{b}$ both ϵ_L^V and ϵ_R^V are higher than ϵ^F and this implies that turnout under a VSP is higher than turnout under a FSP. On the other hand, if $b < \underline{b}$ both ϵ_L^V and ϵ_R^V are smaller than ϵ^F . This implies that if $b < \underline{b}$ then the introduction of a VSP would imply even lower turnout than in the case of a FSP. If $\underline{b} < b < \bar{b}$ then $\epsilon_L^V > \epsilon^F > \epsilon_R^V$. The combination of higher effort by the advantaged party and lower effort by the disadvantaged one under a VSP compared to the case of a FSP does not allow us to infer the total effect of the introduction of a VSP on turnout. Concluding, a VSP can always be used as a tool to increase turnout if under a FSP turnout is higher than α . In general, a VSP implies higher turnout if parties obtain high benefit for obtaining seats in the parliament.

The formal definition of \underline{b} and \bar{b} is the following.

Definition 1. *Given α , let \underline{b} be the unique b such that $\rho(\epsilon^F(b)) = 1 - \alpha$ and \bar{b} be the unique b such that $\rho(\epsilon^F(b)) = \alpha$.*

The above defined thresholds for b are related with turnout. As we have shown under a FSP turnout is increasing in b . The lower threshold \underline{b} is the unique value of b such that in equilibrium under a FSP we have participation equal to $1 - \alpha$. In the same way, \bar{b} defines the benefit such that participation is equal to α . If $\alpha = \frac{1}{2}$ this is the only case when $\underline{b} = \bar{b} = b^*$. If $\alpha > \frac{1}{2}$ and since $\rho(\epsilon_j)$ is strictly increasing in ϵ_j , then $\underline{b} < \bar{b}$.

2.5 Introducing a participation quorum $q > 0$

So far we have studied the effect of introducing the VSP in terms of participation and effort. In this section we analyze the effect of introducing a participation quorum in both the FSP and VSP systems. For tractability, in the participation quorum cases we assume that each party is supported by the same amount of potential voters (i.e. $\alpha = 0.5$). In case that the participation quorum is not satisfied the election is not valid and candidates obtain zero profits. We assume that parties are myopic and when deciding how much effort to exert do not form expectations for future elections.

On the other hand voters follow as before a simple expressive voting rule not taking in consideration possible future costs of an invalid election⁶. This myopic type of voters should be best understood as individuals obtaining a reward for supporting their favorite party rather than for affecting the electoral outcome. Voters in this sense obtain a reward for the action of voting independently of the outcome and hence are ready to support their favorite party and pay the cost of participating whenever they are satisfied with the effort the latter exerts.

A supporter of a party in our model may be comparable with a supporter of a football team deciding whether to attend Sunday's football game. The supporter

⁶If voters were to take into account the possible cost of a repeated election this would imply a much more complicated strategic participation calculation by each voter that may be a path for future research.

does not necessarily care for the final outcome of the game. He actually enjoys seeing the players of his favorite football team being of high quality and putting a lot of effort in the field. If the players offer a nice spectacle, then the supporter after taking in consideration the entrance fee, obtains positive utility independent of the final score.

Despite the fact that the each party has the same support and everything is very symmetric it may be the case that the equilibrium is asymmetric. First we concentrate on the unique symmetric equilibrium under each system and then we generalize and characterize all possible equilibria.

2.5.1 Symmetric Equilibria

In the following proposition we characterize the unique symmetric equilibrium under a FSP.

Proposition 6. *Under a FSP, for all b and q there exists a unique symmetric pure strategy Nash equilibrium characterized as follows:*

1. $\epsilon_L^{F,q}(b, q) = \epsilon_R^{F,q}(b, q) = \epsilon^F(b)$ if $q < \underline{q}^F$
2. $\epsilon_L^{F,q}(b, q) = \epsilon_R^{F,q}(b, q) = \epsilon^{F,q}(q)$ if $\underline{q}^F \leq q < \bar{q}^F$
3. $\epsilon_L^{F,q}(b, q) = \epsilon_R^{F,q}(b, q) = 0$ if $q \geq \bar{q}^F$

Regarding the symmetric equilibrium the above proposition suggests that introducing a participation quorum that is lower than the no quorum participation level (i.e. $q < \underline{q}^F$) there is no effect on equilibrium effort and parties follow the same strategy as in the non-quorum equilibrium. In case that the participation quorum introduced is not satisfied by the non-quorum equilibrium level of turnout (i.e. $\underline{q}^F \leq q$), parties exert higher effort than in the no-quorum case and participation is higher such that the participation quorum in equilibrium is binding and each of the parties fulfills half of it. An effective participation quorum (i.e. $q \leq \rho(\epsilon)$) always results in higher levels of effort and hence participation unless the participation quorum is too high (i.e. $q \geq \bar{q}^F$). If the later is the case parties do not exert any effort since this would imply negative profits.

The formal definition of the lower and upper threshold of the participation quorum is as follows.

Definition 2. Let $\underline{q}^F(b)$ be the unique solution of $\underline{q}^F = \rho(\epsilon^F(b))$ and $\bar{q}^F(b)$ the unique solution of $\bar{q}^F = \rho(\frac{b}{2})$. Moreover, let $\epsilon^{Fq}(q)$ be the unique solution of $\rho(\epsilon^{Fq}(q)) = q$.

The lower threshold of the participation quorum $\underline{q}^F(b)$ is defined as the equilibrium level of participation in the no-participation quorum election. The upper threshold $\bar{q}^F(b)$ is defined as the level of participation when each party exerts effort equal to the benefit of obtaining half of the seats in the parliament and zero profits. Finally, ϵ^{Fq} is the effort level required by each party in order to fulfill half of the participation quorum.

Claim 3. The set $[\underline{q}^F(b), \bar{q}^F(b)]$ is not empty.

This result tells us that in existing FSP systems, if we are interested in increasing effort and participation this can always be done by introducing a participation quorum. The participation quorum has to be chosen accordingly so that it is not too high. Hence, in contrast with the discussion of VSP versus FSP now we obtain that a participation quorum can always be used as an instrument to increase participation.

Claim 4. Both $\underline{q}^F(b)$ and $\bar{q}^F(b)$ are strictly increasing in b .

Regarding the upper threshold, the fact that it is increasing in the benefit implies that the potential effect of a participation quorum in terms of participation is higher in elections that parties obtain high benefit. As one would expect, the lower threshold is increasing in the benefit as well, given that it is defined as the no quorum participation level.

As before, moving to the case of a VSP, we present the results of a participation quorum in a VSP election and then we provide the formal definitions of the participation quorum thresholds.

Proposition 7. Under a VSP, for all b and q there exists a unique symmetric equilibrium characterized as follows:

1. $\epsilon_L^{Vq}(b, q) = \epsilon_R^{Vq}(b, q) = \epsilon^V(b)$ if $q < \underline{q}^V$
2. $\epsilon_L^{Vq}(b, q) = \epsilon_R^{Vq}(b, q) = \epsilon^{Vq}(q)$ if $\underline{q}^V \leq q < \bar{q}^V$
3. $\epsilon_L^{Vq}(b, q) = \epsilon_R^{Vq}(b, q) = 0$ if $q \geq \bar{q}^V$

The intuition is very similar to the case of a FSP. If the quorum is too low then there is no effect on the equilibrium effort. On the other extreme if the quorum is too high then parties exert zero effort since they are not able to fulfill it without making negative profits. Finally, if the quorum is appropriately chosen, then in equilibrium it is binding and parties exert higher effort than in the no quorum case. The formal definition of the thresholds is as follows.

Definition 3. Let $\underline{q}^V(b)$ be the unique solution of $\underline{q}^V = \rho(\epsilon^V(b))$ and $\bar{q}^V(b)$ the unique solution of $\bar{q}^V = \rho(\epsilon^{Vmax})$, where ϵ^{Vmax} is the unique solution of $\frac{1}{2}b\rho(\epsilon^{Vmax}) - \epsilon^{Vmax} = 0$. Moreover, let $\epsilon^{Vq}(q)$ be the unique solution of $\rho(\epsilon^{Vq}(q)) = q$.

Again, the lower threshold is defined as the no quorum VSP participation level. The upper threshold is the case that each party exerts effort equal to ϵ^{Vmax} and makes zero profits. Finally, $\epsilon^{Vq}(q)$ is the effort level required by each party competing in a VSP in order to fulfill half of the participation quorum.

Claim 5. Both $\underline{q}^V(b)$ and $\bar{q}^V(b)$ are strictly increasing in b .

The intuition of this claim is again very similar as in the case of a FSP. The lower threshold is increasing in b , since so is turnout in the no-quorum equilibrium. The upper threshold is defined as the turnout level when both parties exert the maximum effort possible and obtain zero profits. If b is higher then parties have more resources to invest in effort and hence ϵ^{Vmax} is higher. Given that ρ function is strictly increasing in ϵ and $\bar{q}^V = \rho(\epsilon^{Vmax})$, \bar{q}^V is strictly increasing in b as well.

Comparing FSP and VSP

As we have seen so far the participation quorum equilibria structure is the same both in case of a FSP and a VSP. In both cases if the participation quorum is lower than the no quorum equilibrium participation level then the equilibrium

is the same as in the case of no participation quorum. If the participation quorum is higher than the no quorum equilibrium participation level, then parties increase their effort level such that in equilibrium the participation quorum is binding. An effective participation quorum is never slack in equilibrium. In this way independently of b the designer can actually decide and impose the participation level in equilibrium. The only problem arises in case that the participation quorum is higher than the upper threshold. This is because if parties try to fulfill such a participation quorum this implies negative profits and hence parties choose not to exert any effort at all.

In both cases of a VSP or a FSP the upper threshold is increasing in b . This implies that the designer can actually impose higher quorums, which make parties exert higher effort and hence higher turnout, in elections that parties obtain high benefits.

If the participation quorum is higher than the lower threshold and lower than the upper threshold (i.e. $q \in [\underline{q}, \bar{q})$) then the quorum is always binding both under a FSP and a VSP. Hence participation is the same under both a FSP and a VSP and equal to the participation quorum. What actually varies between a VSP and a FSP are the values of the above mentioned thresholds. The comparison of the thresholds is the following.

Claim 6. *It holds that $\underline{q}^V(b) > \underline{q}^F(b)$ if and only if $b > b^*$. Moreover, $\bar{q}^F > \bar{q}^V$ for all values of b .*

The above claim compares the lower and upper thresholds of the set of efficient participation quorums. The upper threshold independent of b is always higher in case of a FSP. This implies that there exist $q \in (\bar{q}^V, \bar{q}^F)$ such that participation is zero under a VSP while it is equal to q in case of a FSP. Hence the designer can always impose higher quorums and as a result higher turnout in the case of a FSP than in the case of a VSP.

On the other hand, when comparing the lower thresholds, given that they are defined as the no quorum equilibrium turnout level, which one is higher depends on the level of b . When the society is equally split, remember that b^* is the unique value of b , such that turnout under a VSP and under a FSP is equal. In case that b is higher than b^* then in the absence of a participation quorum, participation is

higher in case of a VSP. This implies that the lower threshold for the VSP case is higher than the lower threshold for a FSP. The opposite holds if $b < b^*$ while the lower thresholds are equal only in the case that $b = b^*$.

2.5.2 Characterizing All Possible Equilibria

Despite the fact that each party has the same amount of initial support it may be the case that the equilibrium is asymmetric. This describes a situation when one of the parties exerts higher effort than the other and given the opponent's effort, none of the parties has incentives to deviate. Without loss of generality, for the asymmetric equilibria we characterize the case when $\epsilon_L > \epsilon_R$. This is just for convenience and one could think of exactly the opposite case.

Proposition 8. *Under a FSP, for all b and all $q \in (\underline{q}^F, \bar{q}^F)$, (ϵ_L, ϵ_R) is an equilibrium if and only if:*

1. $\epsilon_L > \epsilon_R$
2. $v_L(\epsilon_L) + v_R(\epsilon_R) = q$
3. $\epsilon_L < \frac{b\rho(\epsilon_L)}{2q}$
4. $\rho(\epsilon_R) > \frac{4q^2}{b\rho(\epsilon_L)}$

The above proposition provides the necessary and sufficient conditions for existence of both symmetric and asymmetric equilibria. Notice that the symmetric equilibrium as described in proposition 6 satisfies all four conditions (and obviously the first with equality). Regarding the asymmetric equilibria, having assumed without loss of generality that party L is the one that exerts higher effort (i.e. $\epsilon_L > \epsilon_R$), the remaining conditions can be interpreted in the following way:

The second condition implies that in equilibrium the participation quorum has to be binding (i.e. $v_L(\epsilon_L) + v_R(\epsilon_R) = q$). As we show in the appendix the participation quorum is never over-fulfilled given that one of the parties has incentives to deviate.

From the third condition, the party exerting higher effort must make positive profits (i.e. $\epsilon_L < \frac{b\rho(\epsilon_L)}{2q}$). Because of the concavity of $\rho(\epsilon_j)$ the latter guarantees

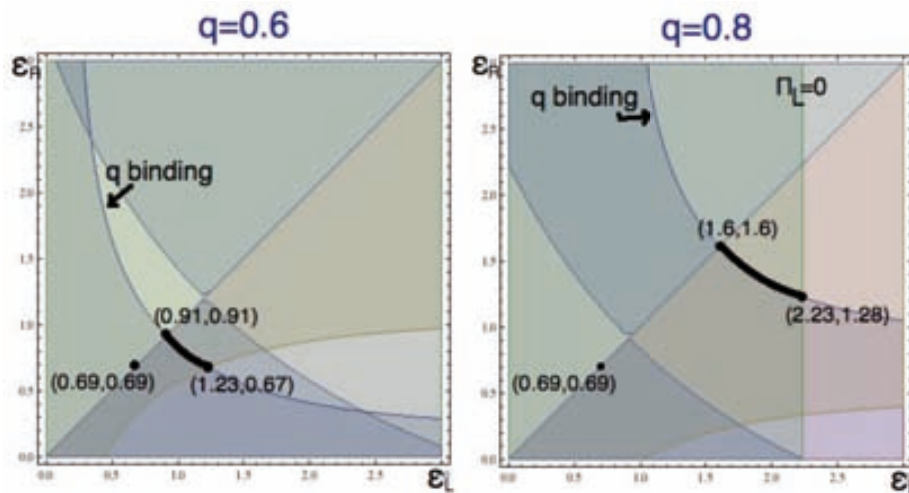
that the party exerting lower effort makes positive profits as well, and hence no party has incentives to deviate to zero effort.

Finally, the party exerting lower effort has no incentives to deviate by increasing its' effort level (i.e. $\rho'(\epsilon_R) \leq \frac{4q^2}{b\rho(\epsilon_L)}$). This is the case when given ϵ_L , at the necessary effort level ϵ_R such that the participation quorum is binding, party's R profits are decreasing in ϵ_R .

We illustrate the multiplicity of equilibria with an example.

Example - FSP & Quorum

Let $b = 4$ and $\rho(\epsilon_j) = 1 - e^{-\epsilon_j}$. The no participation quorum equilibrium as described in proposition 1 is unique and for this example implies $\epsilon_L = \epsilon_R = 0.69$ and turnout 50%. The lower threshold \underline{q}^F according to definition 2 is equal to the no quorum participation level in equilibrium, hence $\underline{q}^F = 0.5$. The upper threshold level of participation quorum is defined as the unique solution of $\bar{q}^F = \rho(\frac{b}{2})$ which for our example (i.e. $b = 4$) implies $\bar{q}^F = \rho(\frac{4}{2})$. Substituting with the explicit form of the benefit function we obtain $\bar{q}^F = 1 - e^{-2}$ and hence $\bar{q}^F = 0.86$. The set of effective participation quorums that implies higher participation in this example is $[0.5, 0.86)$. In the following graphs we analyze the case of a 60 and 80 percent participation quorum.



On the left graph we analyze the case of a 60% participation quorum. The horizontal axis represents the level of effort chosen by party L while the vertical

axis represents the level of effort chosen by party R . The point $(0.69, 0.69)$ is the unique no quorum equilibrium. The downward slope curve depicts all the combinations of ϵ_R and ϵ_L such that the participation quorum is binding. The unique symmetric 60% participation quorum equilibrium is $(0.91, 0.91)$. As expected under the presence of a participation quorum parties have to increase their effort in order to fulfill it.

The bold part of the binding quorum curve depicts all the possible asymmetric equilibria that satisfy the three conditions of proposition 8 and $\epsilon_L > \epsilon_R$. The point $(1.23, 0.67)$ is the most asymmetric among all asymmetric equilibria. This is because if $\epsilon_L > 1.23$ and the quorum is binding then $\frac{d\pi_R}{d\epsilon_R} > 0$ (i.e. $\rho(\epsilon_R) > \frac{4q^2}{b\rho(\epsilon_L)}$). Hence, party R has incentives to deviate.

The case of a 80% participation quorum is depicted on the right graph. The unique symmetric 80% participation quorum equilibrium is $(1.6, 1.6)$. As expected parties have to increase their effort in order to fulfill the 80% participation quorum more than in the case of the 60% quorum. Among all the asymmetric equilibria the most asymmetric is the pair $(2.23, 1.28)$. In contrast to the 60% participation quorum, now the reason why ϵ_L can not be higher is that if $\epsilon_L > 2.23$ and the quorum binding then $\pi_L < 0$ (i.e. $\epsilon_L > \frac{b\rho(\epsilon_L)}{2q}$).

In case of symmetric equilibria parties' profits are decreasing in the level of the participation quorum. Regarding asymmetric equilibria, in all cases of the 60% participation quorum, the party exerting higher effort (i.e. party L) makes higher profits. The relationship though, is exactly the opposite in the asymmetric equilibria of the 80% participation quorum. In this case the party exerting lower effort (i.e. party R) makes higher profits. When the participation quorum is high, in case of asymmetric equilibria there seems to be a free riding issue, while this is not the case when the participation quorum is low.

Under a VSP there may exist multiple equilibria as described in the following proposition.

Proposition 9. *Under a VSP, for all b and all $q \in (\underline{q}^V, \bar{q}^V)$, (ϵ_L, ϵ_R) is an equilibrium if and only if:*

1. $v_L(\epsilon_L) + v_R(\epsilon_R) = q$

$$2. \quad \epsilon_L < \frac{b\rho(\epsilon_L)}{2}$$

$$3. \quad \epsilon_L = \epsilon_R = \epsilon^V(b)$$

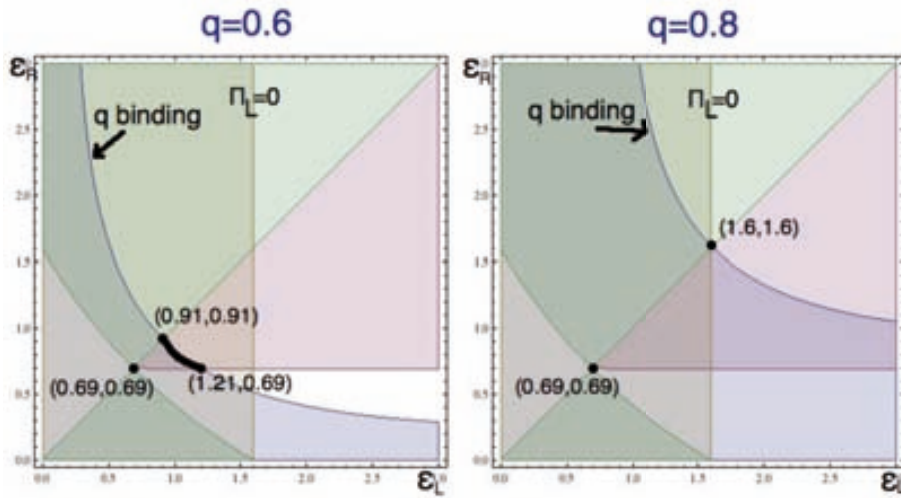
According to the above proposition, in any equilibrium the quorum is binding (i.e. $v_L(\epsilon_L) + v_R(\epsilon_R) = q$). Second, the party performing lower effort has to make positive profits (i.e. $\epsilon_L < \frac{b\rho(\epsilon_L)}{2}$). Third, the lower party's effort is equal or higher than the no quorum VSP equilibrium effort level ϵ^V (i.e. $\epsilon_R = \epsilon^V(b)$). Notice that the symmetric equilibrium as described in proposition 7 satisfies all the three conditions.

Claim 7. *In case of a VSP in all asymmetric equilibria party R makes higher profits than party L (free riding).*

The party exerting lower effort free rides and makes higher profits. This is not always the case in the asymmetric equilibria under a FSP where which party makes higher profits actually depends on the level of the quorum. To illustrate the multiplicity of equilibria we again proceed with an example.

Example - VSP & Quorum

As in the example of a FSP we assume that $b = 4$ and $\rho(\epsilon_j) = 1 - e^{-\epsilon_j}$. The VSP no participation quorum equilibrium as described in the above proposition is unique and implies $\epsilon_L = \epsilon_R = 0.69$ and turnout 50%. The lower threshold \underline{q}^F is equal to the participation, hence $\underline{q}^V = 0.5$. In order to find the upper threshold we first calculate the maximum effort level that each party may exert. ϵ^{Vmax} is the unique solution of $\frac{b}{2}\rho(\epsilon^{Vmax}) - \epsilon^{Vmax} = 0$ and by substituting b and the explicit form of function $\rho(\epsilon_j)$ we obtain $\frac{4}{2}(1 - e^{-\epsilon^{Vmax}}) - \epsilon^{Vmax} = 0$. Solving for ϵ^{Vmax} we obtain that $\epsilon^{Vmax} = 1.6$. By definition 3 we obtain that $\bar{q}^V = \rho(\epsilon^{Vmax}) = 0.8$. The set of effective participation quorums that implies higher participation in this example is $[0.5 \ 0.8)$. In the following graphs we analyze the case of a 60 and 80 percent participation quorum.



On the left graph we analyze the case of a 60% participation quorum. The point $(0.69, 0.69)$ is the unique no quorum equilibrium. The downward curve depicts all the combinations of ϵ_R and ϵ_L such that the participation quorum is binding. The unique symmetric 60% participation quorum equilibrium is $(0.91, 0.91)$. As expected, under the presence of a participation quorum, parties increase their effort in order to fulfill it.

The bold part of the binding quorum curve depicts all the possible asymmetric equilibria that satisfy the three conditions of proposition 9 and $\epsilon_L > \epsilon_R$. The point $(1.21, 0.69)$ is the most “asymmetric” among all asymmetric equilibria since for every point that $\epsilon_L > 1.21$ and the quorum binding it is the case that party R has incentives to deviate to $\epsilon_R = 0.69$.

The case of an 80% participation quorum is depicted on the right graph. The unique symmetric 80% participation quorum equilibrium is $(1.6, 1.6)$. As expected parties have to increase their effort even more in order to fulfill the 80% participation quorum. In this case there are no asymmetric equilibria since the participation quorum is equal to the upper threshold and both parties make zero profits. As in the case of a FSP in case of the symmetric equilibria parties’ profits are decreasing in the participation quorum level. The difference to the FSP system is that in all asymmetric equilibria party R free rides and makes higher profits than party L .

2.6 Discussion of the Voters' Decision

In group turnout models (Shachar and Nalebuff, 1999; Herrera and Mattozzi, 2009) each voter reacts to his favorite party's effort and once the exerted effort is high enough to compensate the voter's personal cost of voting, then the voter rewards the party by voting for it. In general in group turnout models the ideology of both parties and voters is not considered in detail. In this section we provide an interpretation of why effort may mobilize voters in a spatial setup.

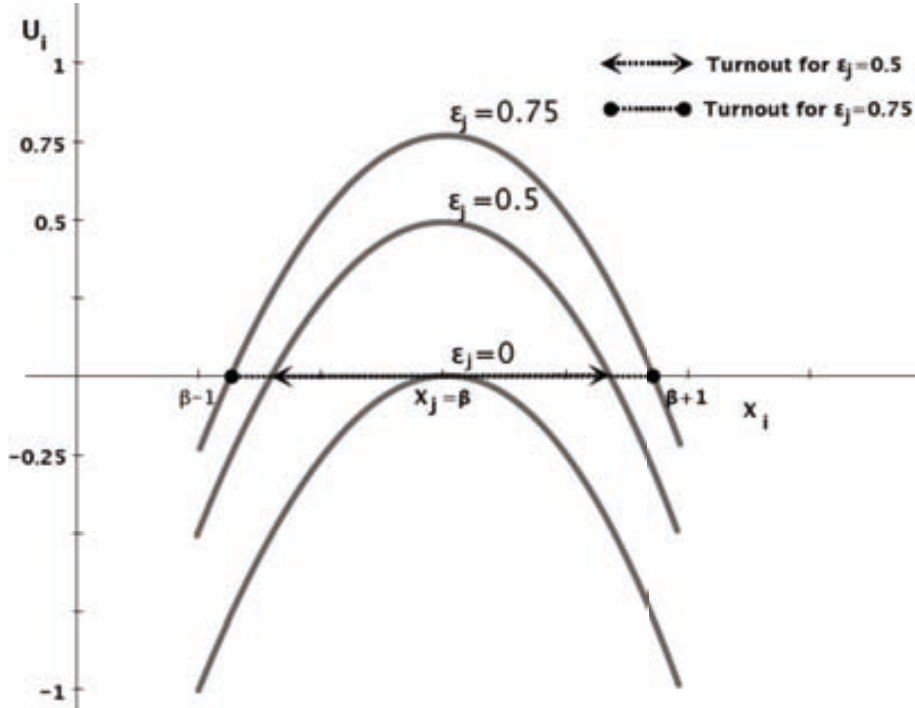
Let the society be split into two groups of voters each of them supporting one of the two parties. For each group, voters' ideal points are uniformly distributed on any closed interval of length two. Supporters of each party decide whether to support their favorite party or to abstain. Each party's announced platform is the mean of the distribution of the ideal points of its supporters.

Formally, let x_i denote voter i 's ideal point and be uniformly distributed on $[\beta - 1, \beta + 1]$ while $x_j = \beta$ denotes party j 's announced platform. Let $\epsilon_j \in [0, 1]$ be the effort exerted by party j . Voter i supporting party j obtains utility $U_i = \epsilon_j - (x_i - x_j)^2$ if he participates in the election and votes for his favorite party j and zero if he decides to abstain.

The above utility specification implies that voter i participates in the election and votes for his favorite party if and only if $\bar{\epsilon}_j \geq (x_i - x_j)$. Remember that according to the group-turnout model a voter participates in the election if and only if $\rho(\epsilon_j) \geq c_i$. By defining $c = x_i - x_j$ and function $\rho(\epsilon_j) = \bar{\epsilon}_j$ the voter's decision of the spatial model is equivalent to the voter's decision in the group-turnout model. Notice that c and $\rho(\epsilon_j)$ satisfy all the assumptions of our model.

As in a group-turnout model, by defining $\rho(\epsilon_j) = \bar{\epsilon}_j$, the benefit of each voter is associated with the effort exerted by his favorite party. By defining that $c = x_i - x_j$, in this spatial setup the personal cost of each voter is interpreted as the cost of supporting a party further from one's own ideal point.

Figure 2: *A spatial Interpretation of Group-Turnout Models*



As we see from figure two, in the spatial setup, as in the group-turnout model, turnout is increasing in effort. Actually, in the spatial setup, effort is the way to attract voters that are not so close to the platform of their favorite party. This is because, voters that have policy preferences different from the platform of their favorite party are willing to participate in the election if their favorite party's effort is high enough, such that it compensates the cost of supporting a policy further from their own ideal point.

2.7 Conclusion

In this paper we study the effect of different instruments that increase turnout in PR elections. We find that the introduction of a VSP system has a positive effect on turnout as long as parties obtain high benefits for being in office, while the establishment of certain participation quorums can always boost turnout.

In terms of welfare and in line with our model, if the utility of a voter is defined as the benefit a voter obtains as a result of the effort exerted by his favorite

party minus the personal cost of voting, then an increase in turnout through the introduction of a VSP or a participation quorum increases voters welfare. This is a consequence of the higher levels of turnout as a result of the higher effort exerted by parties in equilibrium in an expressive voting model.

We find that our mechanisms dominate compulsory voting in terms of voters welfare. If we were to introduce compulsory voting in our model, in equilibrium parties would exert zero effort since they would not have incentives to exert any effort in order to mobilize their voters to participate. This is because in our model we do not allow a none of the above option and when voters are forced to participate they would support their favorite party even when the latter does not exert any effort at all. Hence, compulsory voting would imply negative total welfare for voters, since all voters would obtain negative utility given that parties would not exert any effort at all while all voters would have to pay the cost of participating or the cost of a penalty in case they would decide to abstain.

The advantage of the electoral rules we propose over compulsory voting as a solution to low turnout is that they increase the effort put by parties in order to convince voters to participate. In other words, and in contrast to compulsory voting, our mechanisms target one of the underlying reasons of low turnout (i.e. low effort) rather than only the result itself (i.e. participation level).

A common feature of the two mechanisms we propose is that when appropriately used they increase turnout and at the same time they decrease parties welfare. This result may explain why parties often target to increase turnout through other alternatives such as organizing simultaneous elections, facilitating voters registration, switching the election to a weekend day or allowing absentee voting⁷. All these alternatives are costless for parties. They indeed increase turnout by reducing the cost of voting. At the same time and in contrast to our mechanisms they do not decrease parties benefits. Hence, even when parties recognize the need for higher turnout, given that they have the power over institutional changes, probably they will keep aiming to increase turnout through such alternatives.

Our results may as well explain the evolution of a participation quorum sys-

⁷See Wattenberg (1998); Blais et al. (2003); Geys (2006) for empirical evidence.

tem in several post-Soviet states⁸. As we have shown a participation quorum is associated with higher levels of turnout. This exactly was the reason why the participation quorum had been established in these countries right after the fall of Communism. The target of the electoral designers was to guarantee the legitimacy of the new form of government through high levels of turnout. Moreover, we have shown that parties' profits are decreasing in the level of the participation quorum. This may explain why with the passage of time and the strengthening of political parties, the participation quorum has been abolished or even modified to a lower level.

In well established democracies turnout levels are often decreasing. One of the common justifications for this, is that voters are tired of the existing political parties, elections and politics in general. According to our results parties and voters have interests in conflict. Hence, voters' apathy may not stem from voters' fatigue per se, rather than the fact that parties' profits are higher when citizens remain uninvolved.

⁸While a fifty percent quorum is still present in the cases of Moldova and Hungary, in Serbia and Ukraine the quorum has been abolished. Ukraine did so for the 1998 elections after the experience of repeated by-elections failing to reach the required turnout in 1994. Serbia abolished the quorum in 2004 since it had not been met and no president had been elected during two years and three elections. In Macedonia in 2009 the president decided to decrease the minimum level of participation from a 50% to a 40% probably to prevent the possibility of a new election.

2.8 Appendix

Proof of Proposition 1

Parties maximize profits. The optimization problem of party L is:

Max. $bs_L - \epsilon_L$ and by substituting s_L :

$$\text{Max } b \frac{\alpha \rho(\epsilon_L)}{\alpha \rho(\epsilon_L) + (1-\alpha) \rho(\epsilon_R)} - \epsilon_L$$

Taking F.O.C. with respect to effort for party L we obtain that it has to hold:

$$\frac{\alpha(1-\alpha)\rho(\epsilon_R)\rho(\epsilon_L)}{[\alpha\rho(\epsilon_L) + (1-\alpha)\rho(\epsilon_R)]^2} = \frac{1}{b} \quad (2.1)$$

Solving the same problem for party R we get:

$$\frac{\alpha(1-\alpha)\rho(\epsilon_L)\rho(\epsilon_R)}{[\alpha\rho(\epsilon_L) + (1-\alpha)\rho(\epsilon_R)]^2} = \frac{1}{b} \quad (2.2)$$

From equations (1) and (2) in equilibrium it has to hold that:

$$\frac{\rho(\epsilon_R)\rho(\epsilon_L)}{\rho(\epsilon_L)\rho(\epsilon_R)} = 1$$

which implies that in equilibrium:

$$\frac{\rho(\epsilon_L)}{\rho(\epsilon_L)} = \frac{\rho(\epsilon_R)}{\rho(\epsilon_R)} \quad (2.3)$$

Notice that $\frac{\rho(\epsilon_j)}{\rho(\epsilon_j)}$ is strictly decreasing in ϵ_j for all $\epsilon_j > 0$. This is because of the strict monotonicity and strict concavity of function $\rho(\epsilon_j)$. More specifically we have:

$$\frac{d \frac{\rho(\epsilon_j)}{\rho(\epsilon_j)}}{d\epsilon_j} = \frac{\rho'(\epsilon_j)\rho(\epsilon_j) - \rho(\epsilon_j)\rho'(\epsilon_j)}{[\rho(\epsilon_j)]^2} < 0$$

Given that $\frac{\rho(\epsilon_j)}{\rho(\epsilon_j)}$ is strictly decreasing in ϵ_j and equation (3) in equilibrium it hold that $\epsilon_R^F = \epsilon_L^F = \epsilon^F$. By substituting $\epsilon_R^F = \epsilon_L^F = \epsilon^F$ in equation (1) or (2) we obtain that both parties exert the same amount of effort that satisfies the

equilibrium condition:

$$\frac{\rho(\epsilon^F)}{\rho(\epsilon^F)} = \frac{1}{b\alpha(1-\alpha)} \quad (2.4)$$

In order to guarantee that ϵ^F is the equilibrium effort level it has to be the case that both parties make positive profits. Party L makes profits: $\pi_L^F = b \frac{\alpha\rho(\epsilon^F)}{\alpha\rho(\epsilon^F)+(1-\alpha)\rho(\epsilon^F)} - \epsilon^F = b\alpha - \epsilon^F$. Party R makes profits $\pi_R^F = b(1-\alpha) - \epsilon^F$. Given that $\alpha \in [0, 0.5, 1)$ it holds that $\pi_L^F \geq \pi_R^F$. In order to guarantee that both parties make positive profits it is enough to show that $\pi_R^F > 0$. From the profit function $\pi_R^F > 0$ is true if $\epsilon^F < b(1-\alpha)$. Because of the concavity of $\rho(\epsilon_j)$ it holds that $\epsilon^F < \frac{\rho(\epsilon^F)}{\rho(\epsilon^F)}$ and from (4) this implies that $\epsilon^F < \alpha(1-\alpha)b$. Given that $\alpha < 1$, the latter implies that $\pi_R^F > 0$ and hence in equilibrium both parties make positive profits for whatever values of α and b . \square

Proof of Proposition 2

From the equilibrium condition (4) if b increases then $\frac{\rho(\epsilon^F)}{\rho(\epsilon^F)}$ decreases. Given that $\frac{\rho(\epsilon^F)}{\rho(\epsilon^F)}$ is strictly decreasing in ϵ_j , we conclude that ϵ^F is strictly increasing in b .

If α increases then $\frac{1}{\alpha(1-\alpha)b}$ increases. Again from (4), this implies that $\frac{\rho(\epsilon^F)}{\rho(\epsilon^F)}$ increases and hence ϵ^F is strictly decreasing in α .

\square

Proof of Claim 1

Party L makes profits: $\pi_L^F = b \frac{\alpha\rho(\epsilon^F)}{\alpha\rho(\epsilon^F)+(1-\alpha)\rho(\epsilon^F)} - \epsilon^F = b\alpha - \epsilon^F$.

Party R makes profits $\pi_R^F = b(1-\alpha) - \epsilon^F$.

Given that $\alpha \in [0, 0.5, 1)$ it holds that $\pi_L^F \geq \pi_R^F$ with the equality holding for $\alpha = 0.5$.

\square

Proof of Proposition 3

Party L solves: $Max. b\alpha\rho(\epsilon_L) - \epsilon_L$. Taking F.O.C. with respect to ϵ_L we obtain that the solution has to satisfy:

$$\rho(\epsilon_L^V) = \frac{1}{b\alpha} \quad (2.5)$$

Notice that since $\rho(\epsilon_j)$ is strictly concave then the solution of the maximization is unique.

Performing the same maximization for party R we obtain that:

$$\rho(\epsilon_R^V) = \frac{1}{b(1-\alpha)} \quad (2.6)$$

In order to guarantee that ϵ_L^V and ϵ_R^V are part of an equilibrium both parties have to make positive profits. Because of the strict concavity of $\rho(\epsilon_j)$ it holds that $\epsilon_L^V < \frac{\rho(\epsilon_L^V)}{\rho(\epsilon_R^V)}$ and from equation (5), $\epsilon_L^V < b\alpha\rho(\epsilon_L^V)$. Given that $\pi_L^V = b\alpha\rho(\epsilon_L^V) - \epsilon_L^V$ we conclude that $\pi_L^V > 0$. In the same way for party R, $\epsilon_R^V < \frac{\rho(\epsilon_R^V)}{\rho(\epsilon_L^V)}$ which implies that $\epsilon_R^V < b(1-\alpha)\rho(\epsilon_R^V)$ and hence $\pi_R^V > 0$.

If $\alpha = 0.5$ then $\epsilon_L^V = \epsilon_R^V = \epsilon^V$. If $\alpha > 0.5$ then $\alpha > 1 - \alpha$ which implies that $\frac{1}{b\alpha} < \frac{1}{b(1-\alpha)}$ which implies that $\rho(\epsilon_L^V) < \rho(\epsilon_R^V)$ and because of the strict concavity of $\rho(\epsilon_j)$ this implies that $\epsilon_L^V > \epsilon_R^V$. \square

Proof of Proposition 4

From equilibrium condition (5) we know that $\rho(\epsilon_L^V) = \frac{1}{b\alpha}$ and from (6) that $\rho(\epsilon_R^V) = \frac{1}{b(1-\alpha)}$.

If b increases then both $\rho(\epsilon_L^V)$ and $\rho(\epsilon_R^V)$ decrease. Because of the strict concavity of $\rho(\epsilon_j)$ this implies that both ϵ_L^V and ϵ_R^V are strictly increasing in b .

If α increases then from (5) $\rho(\epsilon_L^V)$ decreases and from (6) $\rho(\epsilon_R^V)$ increases. Because of the strict concavity of $\rho(\epsilon_j)$ this implies that ϵ_L^V is strictly increasing in α while ϵ_R^V is strictly decreasing in α . \square

Proof of Claim 2

Profits are given by the following expressions: $\pi_L^V = b\alpha\rho(\epsilon_L^V) - \epsilon_L^V$ and $\pi_R^V = b(1-\alpha)\rho(\epsilon_R^V) - \epsilon_R^V$.

If $\alpha = 0.5$ then from equilibrium conditions (5) and (6) it holds that $\epsilon_L^V = \epsilon_R^V = \epsilon^V$ and hence $\pi_L^V = \pi_R^V$.

If $\alpha > 0.5$ then from proposition 3 it holds that $\epsilon_L^V > \epsilon_R^V$. Because of the strict concavity of $\rho(\epsilon_j)$ it holds that $\rho(\epsilon_L^V) < \frac{\rho(\epsilon_L^V) - \rho(\epsilon_R^V)}{\epsilon_L^V - \epsilon_R^V}$ which is equivalent to:

$$\epsilon_L^V - \epsilon_R^V < \frac{\rho(\epsilon_L^V) - \rho(\epsilon_R^V)}{\rho(\epsilon_L^V)} \quad (2.7)$$

In order to be true that $\pi_L^V > \pi_R^V$ it must hold that $b\alpha\rho(\epsilon_L^V) - \epsilon_L^V > b(1-\alpha)\rho(\epsilon_R^V) - \epsilon_R^V$

which is equivalent to:

$$\epsilon_L^V - \epsilon_R^V < b\alpha\rho(\epsilon_L^V) - b(1 - \alpha)\rho(\epsilon_R^V) \quad (2.8)$$

Given that inequality (7) always holds in order to show that $\pi_L^V > \pi_R^V$ it is enough to show that the RHS of inequality (7) is smaller than the RHS of equation (8). Hence we have to show that $\frac{\rho(\epsilon_L^V) - \rho(\epsilon_R^V)}{\rho(\epsilon_L^V)} < b\alpha\rho(\epsilon_L^V) - b(1 - \alpha)\rho(\epsilon_R^V)$. By substituting $\rho(\epsilon_L^V) = \frac{1}{b\alpha}$ we obtain that it has to hold $b\alpha[\rho(\epsilon_L^V) - \rho(\epsilon_R^V)] < b\alpha\rho(\epsilon_L^V) - b(1 - \alpha)\rho(\epsilon_R^V)$ or $b(1 - \alpha)\rho(\epsilon_R^V) < b\alpha\rho(\epsilon_R^V)$ which is equivalent to $1 - \alpha < \alpha$ and the latter is true given that $\alpha > 0.5$. Hence, if $\alpha < 0.5$ then $\pi_L^V > \pi_R^V$. \square

Proof of Proposition 5

From equilibrium conditions 4 and 5 we get that for party L it holds that $\rho(\epsilon_L^V) = \frac{1}{b\alpha}$ and $\frac{\rho(\epsilon^F)}{\rho(\epsilon^F)} = \frac{1}{b\alpha(1-\alpha)}$. Combining the two we obtain that $\frac{\rho(\epsilon_L^V)}{1-\alpha} = \frac{\rho(\epsilon^F)}{\rho(\epsilon^F)}$ which is equivalent to:

$$\frac{\rho(\epsilon_L^V)}{\rho(\epsilon^F)} = \frac{1-\alpha}{\rho(\epsilon^F)} \quad (2.9)$$

From condition (9) and the strict concavity of $\rho(\epsilon_j)$ we obtain that:

- $\epsilon_L^V = \epsilon^F$ if and only if $\rho(\epsilon^F) = 1 - \alpha$. The latter holds if and only if $b = \underline{b}$.
- $\epsilon_L^V > \epsilon^F$ if and only if $\frac{\rho(\epsilon_L^V)}{\rho(\epsilon^F)} < 1$ which is true if and only if $\frac{1-\alpha}{\rho(\epsilon^F)} < 1$ thus if and only if $\rho(\epsilon^F) > 1 - \alpha$. The latter is true if and only if $b > \underline{b}$.
- In the same way, $\epsilon_L^V < \epsilon^F$ if and only if $b < \underline{b}$.

Performing the same approach for party R , we obtain that $\epsilon_R^V = \epsilon^F$ if and only if $b = \bar{b}$, $\epsilon_R^V > \epsilon^F$ if and only if $b > \bar{b}$ and $\epsilon_R^V < \epsilon^F$ if and only if $b < \bar{b}$. \square

Proof of Proposition 6

We analyze all possible symmetric pure strategy Nash equilibria where both parties exert the same level of effort $\epsilon_L = \epsilon_R$. Those can belong in the following categories:

1. $v_L + v_R > q$ (i.e. the quorum is slack)
2. $v_L + v_R = q$ (i.e. the quorum is binding)

3. $v_L + v_R < q$ (i.e. the quorum is not fulfilled)

In general our approach is the following. For each pair of levels of effort (ϵ_L, ϵ_R) we have to guarantee that there are no incentives for any of the parties to deviate. Hence, if effort level ϵ_j is part of a Nash equilibrium it has to be the solution of the (constrained) maximization problem of party j given the effort of the opponent ϵ_{-j} . Moreover, we have to guarantee that each effort level being part of a Nash equilibrium implies positive profits given that the deviation $\epsilon_j = 0$ implying zero profits is always available.

1. Participation quorum slack.

Let $\epsilon_L = \epsilon_R = \epsilon$ be such that $v_L + v_R > q$.

Given $\epsilon_R = \epsilon$ the maximization problem of party L is:

Max. $b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon)} - \epsilon_L$ subject to $v_L + v_R > q$ and by substituting the vote shares in the constraint this is equivalent to:

Max. $b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon)} - \epsilon_L$ subject to $\rho(\epsilon_L) + \rho(\epsilon) = 2q$

The Lagrangian of this problem is:

$$L = b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon)} - \epsilon_L + \lambda [\rho(\epsilon_L) + \rho(\epsilon) - 2q]$$

Taking F.O.C. with respect to ϵ_L we obtain:

$$L_{\epsilon_L} = b \frac{\rho'(\epsilon_L)\rho(\epsilon)}{[\rho(\epsilon_L) + \rho(\epsilon)]^2} - 1 + \lambda \rho'(\epsilon_L)$$

If $\epsilon_L = \epsilon$ is part of the equilibrium it must satisfy the F.O.C. of party L . Given that the participation quorum is slack, this means that the associated multiplier is zero (i.e. $\lambda = 0$). Hence If $\epsilon_L = \epsilon$ is part of the equilibrium it must be true that:

$b \frac{\rho'(\epsilon)\rho(\epsilon)}{[\rho(\epsilon) + \rho(\epsilon)]^2} - 1 = 0$ which implies that $\frac{\rho'(\epsilon)}{\rho(\epsilon)} = \frac{4}{b}$. Notice that the unique solution of $\frac{\rho'(\epsilon)}{\rho(\epsilon)} = \frac{4}{b}$ is $\epsilon = \epsilon^F$ which is unique FSP no quorum equilibrium effort level for $\alpha = 0.5$.

In order the constraint to be slack it must be that that the level of participation is higher than the quorum. This is the case when $\rho(\epsilon^F) > q$ which is equivalent to $\underline{q}^F > q$.

In order to guarantee positive profits it must hold that $\pi_L = \frac{b}{2} - \epsilon^F > 0$. This is true if $\epsilon^F < \frac{b}{2}$.

Because of the strict concavity of ρ it holds that $\rho(\epsilon^F) < \frac{\rho(\epsilon^F)}{\epsilon^F}$ which is equivalent to $\epsilon^F < \frac{\rho(\epsilon^F)}{\rho'(\epsilon^F)}$ thus $\epsilon^F < \frac{b}{4}$. Hence, $\epsilon^F < \frac{b}{2}$ and this implies that party L makes positive profits.

The same applies for party R .

Hence, $\epsilon_L = \epsilon_R = \epsilon^F$ such that $\frac{\rho'(\epsilon^F)}{\rho(\epsilon^F)} = \frac{4}{b}$ is an equilibrium if $q < \underline{q}^F$

2. Participation Quorum Binding

Let $\epsilon_L = \epsilon_R = \epsilon^{F,q}$ be such that $v_L + v_R = q$.

Given effort $\epsilon_R = \epsilon^{F,q}$ the maximization problem of party L is:

$$\text{Max. } b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon^{F,q})} - \epsilon_L \quad \text{subject to } v_L + v_R = q$$

Given that with effort $\epsilon_R = \epsilon^{F,q}$ party R fulfills half of the quorum (i.e. $v_R = \frac{1}{2}\rho(\epsilon^{F,q}) = \frac{1}{2}q$), the problem can be written as:

$$\text{Max. } b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon^{F,q})} - \epsilon_L \quad \text{subject to } \rho(\epsilon_L) = q$$

The Lagrangian of this problem is:

$$L = b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon^{F,q})} - \epsilon_L + \lambda[\rho(\epsilon_L) - q]$$

Taking F.O.C. with respect to effort we obtain:

$$L_{\epsilon_L} = b \frac{\rho'(\epsilon_L)\rho(\epsilon^{F,q})}{[\rho(\epsilon_L) + \rho(\epsilon^{F,q})]^2} - 1 + \lambda\rho'(\epsilon_L)$$

If $\epsilon_L = \epsilon^{F,q} > 0$ is the best response it must hold that $b \frac{\rho'(\epsilon^{F,q})\rho(\epsilon^{F,q})}{[2\rho(\epsilon^{F,q})]^2} - 1 + \lambda\rho'(\epsilon^{F,q}) = 0$. Solving for the Lagrange multiplier we obtain that $\lambda = \frac{1}{\rho'(\epsilon^{F,q})} - \frac{b}{4\rho(\epsilon^{F,q})}$. Given that the constraint is binding it has to be that $\lambda = 0$ which is true if and only if $\rho(\epsilon^{F,q}) = \frac{4\rho(\epsilon^{F,q})}{b}$ which is true if $\frac{\rho'(\epsilon^{F,q})}{\rho(\epsilon^{F,q})} = \frac{\rho'(\epsilon^F)}{\rho(\epsilon^F)}$ or $\epsilon^{F,q} = \epsilon^F$ i.e. $\rho(\epsilon^{F,q}) = \rho(\epsilon^F)$ and hence $\lambda = 0$ if $q = \underline{q}^F$.

Last we have to guarantee that profits are positive which holds if $\epsilon^{F,q} < \frac{b}{2}$ i.e. $q < \bar{q}^F$. By symmetry the same applies for party R .

3. Participation Quorum not Fulfilled.

- $\epsilon_L = \epsilon_R = \epsilon > 0$

Given that the participation quorum is not satisfied and parties exert positive effort, parties make negative profits $\pi_j = -\epsilon < 0$. Hence (ϵ, ϵ) can not be an equilibrium given that party j has incentives to deviate to zero effort and zero profits respectively.

- $\epsilon_L = \epsilon_R = 0$

Given $\epsilon_{-j} = 0$ for whatever positive effort party j obtains all the seats in the parliament if the participation quorum is fulfilled. The constrained maximization problem of party j is: *Max.* $b - \epsilon_j$ subject to $\rho(\epsilon_j) = 2q$. The solution of this problem is ϵ_j^q such that $\rho(\epsilon_j^q) = 2q$. Notice that ϵ_j^q is the effort level required by party j in order to fulfill the quorum by itself. Finally, in order party j to deviate to effort ϵ_j^q it has to be that it makes positive profits. This is the true if $\epsilon_j^q < b$. If the latter inequality does not hold then $\epsilon_j = 0$. Hence, $\epsilon_L = \epsilon_R = 0$ is an equilibrium if $\epsilon_j^q < b$.

Summing up the cases regarding symmetric equilibria we have obtained:

1. $\epsilon_L = \epsilon_R = \epsilon^F$ such that $\frac{\rho'(\epsilon^F)}{\rho(\epsilon^F)} = \frac{4}{b}$ is an equilibrium if $q < \underline{q}^F$
2. $\epsilon_L = \epsilon_R = \epsilon^{F,q}$ such that $\rho(\epsilon^{F,q}) = q$ is an equilibrium if $\underline{q}^F < q < \bar{q}^F$
3. $\epsilon_L = \epsilon_R = 0$ is an equilibrium if $\epsilon_j^q > b$ where ϵ_j^q is the unique solution of $\rho(\epsilon_j^q) = 2q$

Hence if $q < \underline{q}^F$ then we fall in case 1. If $\underline{q}^F < q < \bar{q}^F$ then we fall in case 2.

Finally if $q > \bar{q}^F$ then $\epsilon_j^q > b$ which is the condition for case 3.

$\epsilon_j^q > b$ is true since $q > \bar{q}^F$ implies that $\rho(\epsilon_j^{F,q}) < \rho(\frac{b}{2})$ which implies that

$$\epsilon_j^{F,q} < \frac{b}{2} \quad (2.10)$$

Moreover it holds that $\epsilon_j^q > \epsilon_j^{F,q}$ which implies that $\frac{\rho(\epsilon_j^q)}{\epsilon_j^q} < \frac{\rho(\epsilon_j^{F,q})}{\epsilon_j^{F,q}}$ i.e. $\frac{2q}{\epsilon_j^q} < \frac{q}{\epsilon_j^{F,q}}$ which is equivalent to:

$$\epsilon_j^{F,q} < \frac{\epsilon_j^q}{2} \quad (2.11)$$

Combining the inequalities (10) and (11) we have shown that if $q < \bar{q}^F$ then $\epsilon_j^q > b$ and hence if $q < \bar{q}^F$ then from case 3 the equilibrium is $\epsilon_L = \epsilon_R = 0$. \square

Proof of Claim 3

As we have shown in the proof of proposition 1, for $\alpha = \frac{1}{2}$ it holds that $\epsilon^F(b) < \frac{b}{4}$. Given that function ρ is strictly increasing this implies that the set $[\rho(\epsilon^F(b)), \rho(\frac{b}{2})]$ is not-empty. \square

Proof of Claim 4

As we have shown in proposition 2 participation in a FSP with no participation quorum is increasing in the benefit parties obtain. Given that the lower threshold is defined as the no quorum participation level, then $\underline{q}^F(b)$ is increasing in the benefit as well.

The upper threshold is defined as $\bar{q}^F = \rho(\frac{b}{2})$. Given that the benefit function ρ is strictly increasing in b then $\bar{q}^F(b)$ is strictly increasing in b . \square

Proof of Proposition 7

We analyze all possible pure strategy symmetric Nash equilibria where $\epsilon_L = \epsilon_R$. Those can belong in the following categories:

1. $v_L + v_R > q$ (i.e. the quorum is slack)
2. $v_L + v_R = q$ (i.e. the quorum is binding)
3. $v_L + v_R < q$ (i.e. the quorum is not fulfilled)

1. Participation quorum is slack

Let $\epsilon_L = \epsilon_R = \epsilon$ be such that $v_L + v_R > q$.

Given effort $\epsilon_R = \epsilon$ the maximization problem of party L is:

$$Max. \frac{1}{2}b\rho(\epsilon_L) - \epsilon_L \quad \text{subject to } \rho(\epsilon_L) + \rho(\epsilon) = 2q$$

The Lagrangian of this problem is:

$$L = \frac{1}{2}b\rho(\epsilon_L) - \epsilon_L + \lambda[\rho(\epsilon_L) + \rho(\epsilon) - 2q]$$

Taking F.O.C. with respect to effort we obtain:

$$L_{\epsilon_L} = \frac{1}{2}b\rho'(\epsilon_L) - 1 + \lambda\rho'(\epsilon_L)$$

Given the participation quorum is slack the associated Lagrange multiplier takes value zero (i.e $\lambda = 0$). Hence, if $\epsilon_L = \epsilon$ is part of the equilibrium it must satisfy:

$$\rho(\epsilon_L) = \frac{2}{b} \text{ which is the no quorum equilibrium condition and hence } \epsilon_L = \epsilon^V.$$

In order the constraint to be slack it must be that $\rho(\epsilon^V) > q$ which is equivalent to $\underline{q}^V > q$.

In order to guarantee positive profits it must be that $\epsilon^V < \frac{1}{2}b\rho(\epsilon^V)$. Because of the strict concavity of ρ it holds that $\rho(\epsilon^V) < \frac{\rho(\epsilon^V)}{\epsilon^V}$ which implies that $\epsilon^V < \frac{\rho(\epsilon^V)}{\rho'(\epsilon^V)}$ and by substituting $\rho(\epsilon^V) = \frac{2}{b}$ we obtain that it holds that $\epsilon^V < \frac{1}{2}b\rho(\epsilon^V)$. Hence, profits are positive.

The same applies for party R .

2. Participation quorum is binding:

Let $\epsilon_L = \epsilon_R = \epsilon^{Vq}$ be such that $v_L + v_R = q$

Given effort $\epsilon_R = \epsilon^{Vq}$ the Lagrangian of this problem is:

$$L = \frac{1}{2}b\rho(\epsilon_L) - \epsilon_L + \lambda[\rho(\epsilon_L) + \rho(\epsilon^{Vq}) - 2q]$$

Taking F.O.C. with respect to effort we obtain:

$$L_{\epsilon_L} = \frac{1}{2}b\rho'(\epsilon_L) - 1 + \lambda\rho'(\epsilon_L)$$

If $\epsilon_L = \epsilon^{Vq} > 0$ is the solution it must hold that $\frac{1}{2}b\rho'(\epsilon^{Vq}) - 1 + \lambda\rho'(\epsilon^{Vq}) = 0$. Given that the constraint is binding it has to be that $\lambda > 0$ which is true if $\rho(\epsilon^{Vq}) < \frac{2}{b}$ and by substituting $\frac{2}{b} = \rho(\epsilon^V)$ we obtain $\rho(\epsilon^{Vq}) < \rho(\epsilon^V)$ which is true if $\epsilon^{Vq} > \epsilon^V$ i.e. $\rho(\epsilon^{Vq}) > \rho(\epsilon^V)$ or $q > \underline{q}^V$.

Finally it has to be that profits are positive which holds if $\epsilon^{Vq} < \frac{1}{2}b\rho(\epsilon^{Vq})$ which is true if $q < \bar{q}^V$. The same applies for party R .

3. Participation Quorum not Fulfilled

Let $\epsilon_L = \epsilon_R = \epsilon > 0$ be such that $v_L + v_R < 0$.

Party j makes profits $\pi_j = -\epsilon < 0$. Hence $\epsilon_L = \epsilon_R = \epsilon > 0$ can not be an equilibrium given that party j has incentives to deviate to $\epsilon_j = 0$.

Let $\epsilon_L = \epsilon_R = 0$ be such that $v_L + v_R < 0$.

This can be an equilibrium if no party can fulfill the quorum by itself and make positive profits.

Let ϵ_j^q be such that $\rho(\epsilon_j^q) = 2q$. Notice that ϵ_j^q is the effort level required by party j in order to fulfill the quorum by itself. In order to guarantee that profits are positive it must hold that $\epsilon_j^q < \frac{1}{2}b\rho(\epsilon_j^q)$. If the later inequality does not hold then $\epsilon_j = 0$. Hence, $\epsilon_L = \epsilon_R = 0$ is an equilibrium if $\epsilon_j^q < \frac{1}{2}b\rho(\epsilon_j^q)$ i.e. $\epsilon_j^q < \epsilon^{Vmax}$ i.e. $\rho(\epsilon_j^q) < \rho(\epsilon^{Vmax})$ i.e. $q < \bar{q}^V$.

Summing up the cases regarding pure strategy symmetric Nash equilibria under a VSP we have obtained:

1. $\epsilon_L = \epsilon_R = \epsilon^V$ such that $\rho(\epsilon^V) = \frac{2}{b}$ is an equilibrium if $q < \underline{q}^V$.
2. $\epsilon_L = \epsilon_R = \epsilon^{Vq}$ such that $\rho(\epsilon^{Vq}) = q$ is an equilibrium if $\underline{q}^V < q < \bar{q}^V$.
3. $\epsilon_L = \epsilon_R = 0$ is an equilibrium if $q > \bar{q}^V$.

□

Proof of Claim 5

As we have shown participation in a VSP with no participation quorum is strictly increasing in the benefit parties obtain. Given that the lower threshold is defined as the no quorum participation level, then $\underline{q}^V(b)$ is strictly increasing in the benefit as well.

The upper threshold is defined as $\bar{q}^V = \rho(\epsilon^{Vmax})$, where ϵ^{Vmax} is the unique solution of $F = \frac{1}{2}b\rho(\epsilon^{Vmax}) - \epsilon^{Vmax} = 0$. From the implicit function theorem we know that

$$\frac{d\epsilon^{Vmax}}{db} = -\frac{\frac{dF}{db}}{\frac{dF}{d\epsilon^{Vmax}}} = -\frac{\frac{\rho(\epsilon^{Vmax})}{2}}{\frac{b}{2}\rho(\epsilon^{Vmax}) - 1} \quad (2.12)$$

Given that $\epsilon^{Vmax} > \epsilon^V$ then it holds that $\frac{b}{2}\rho(\epsilon^{Vmax}) - 1 < 0$ and hence $\frac{d\epsilon^{Vmax}}{db} > 0$. Moreover given that $\bar{q}^V = \rho(\epsilon^{Vmax})$ and $\rho(\epsilon_j)$ is strictly increasing in ϵ_j we conclude that \bar{q}^V is strictly increasing in b . □

Proof of Claim 6

- The lower thresholds are defined as equilibrium turnout in the no quorum case. As we have shown in Proposition 5 for the case of $\alpha = \frac{1}{2}$ there exists

a unique b^* such that turnout under a FSP and a VSP is the same. Hence if $b = b^*$ then $\underline{q}^V(b) = \underline{q}^F(b)$. If $b > b^*$ then turnout under a VSP is higher and hence $\underline{q}^V(b) > \underline{q}^F(b)$. If $b < b^*$ then turnout under a VSP is lower and hence $\underline{q}^V(b) < \underline{q}^F(b)$.

- By definition $\bar{q}^F > \bar{q}^V$ if $\rho(\frac{b}{2}) > \rho(\epsilon^{Vmax})$ which is true if $\frac{b}{2} > \epsilon^{Vmax}$ and given that $\epsilon^{Vmax} = \frac{1}{2}b\rho(\epsilon^{Vmax})$ the latter inequality is equivalent to $\frac{b}{2} > \frac{b}{2}\rho(\epsilon^{Vmax})$. The latter holds since $\rho(\epsilon^{Vmax}) < 1$ for all finite b and hence $\bar{q}^F > \bar{q}^V$.

□

Proof of Proposition 8

We analyze all possible asymmetric Nash equilibria where without loss of generality $\epsilon_L > \epsilon_R$. Those can belong in the following categories:

1. $v_L + v_R > q$ (i.e. the quorum is slack)
2. $v_L + v_R = q$ (i.e. the quorum is binding)
3. $v_L + v_R < q$ (i.e. the quorum is not fulfilled)

1. Participation Quorum Slack

Let $\epsilon_L > \epsilon_R$ be such that $v_L + v_R > q$.

Given effort ϵ_R the maximization problem of party L is:

$$\text{Max. } b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon_R)} - \epsilon_L \quad \text{subject to } \rho(\epsilon_L) + \rho(\epsilon_R) = 2q$$

The Lagrangian of this problem is:

$$L = b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon_R)} - \epsilon_L + \lambda_L [\rho(\epsilon_L) + \rho(\epsilon_R) - 2q]$$

Taking F.O.C. with respect to effort we obtain:

$$L_{\epsilon_L} = b \frac{\rho'(\epsilon_L)\rho(\epsilon_R)}{[\rho(\epsilon_L) + \rho(\epsilon_R)]^2} - 1 + \lambda_L \rho'(\epsilon_L)$$

Given that the constraint is slack then it holds that $\lambda_L = 0$. Then it has to hold that: $b \frac{\rho'(\epsilon_L)\rho(\epsilon_R)}{[\rho(\epsilon_L) + \rho(\epsilon_R)]^2} - 1 = 0$.

In the same way for party R it has to hold that: $b \frac{\rho'(\epsilon_R)\rho(\epsilon_L)}{[\rho(\epsilon_L) + \rho(\epsilon_R)]^2} - 1 = 0$.

Combining the above two it has to be true that $\frac{\rho'(\epsilon_L)}{\rho(\epsilon_L)} = \frac{\rho'(\epsilon_R)}{\rho(\epsilon_R)}$ which as we have shown is true only for $\epsilon_L = \epsilon_R$. Hence, contradiction. $\epsilon_L > \epsilon_R$ such that $v_L + v_R > q$ can not be an equilibrium.

2. Participation Quorum Binding

Let $\epsilon_L > \epsilon_R$ be such that $v_L + v_R = q$.

Given effort ϵ_R the maximization problem of party L is:

$$\text{Max. } b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon_R)} - \epsilon_L \quad \text{subject to } \rho(\epsilon_L) + \rho(\epsilon_R) = 2q$$

The Lagrangian of this problem is:

$$L = b \frac{\rho(\epsilon_L)}{\rho(\epsilon_L) + \rho(\epsilon_R)} - \epsilon_L + \lambda_L [\rho(\epsilon_L) + \rho(\epsilon_R) - 2q]$$

Taking F.O.C. with respect to effort we obtain:

$$L_{\epsilon_L} = b \frac{\rho'(\epsilon_L)\rho(\epsilon_R)}{[\rho(\epsilon_L) + \rho(\epsilon_R)]^2} - 1 + \lambda_L \rho'(\epsilon_L)$$

In order the participation quorum to be binding it must be the case that $q = \rho(\epsilon^F)$ i.e. $q = \underline{q}^F$.

Given the participation quorum is binding it must be that $\lambda_L = 0$. This is true if and only if $\rho'(\epsilon_L) = \frac{4q^2}{b\rho(\epsilon_R)}$.

Performing the same maximization problem for party R we obtain that $\lambda_R = 0$ if and only if $\rho'(\epsilon_R) = \frac{4q^2}{b\rho(\epsilon_L)}$.

Notice that since $\epsilon_L > \epsilon_R$ it holds that $\rho'(\epsilon_L) < \rho'(\epsilon_R)$ and $\frac{4q^2}{b\rho(\epsilon_L)} < \frac{4q^2}{b\rho(\epsilon_R)}$. Hence $\rho'(\epsilon_R) = \frac{4q^2}{b\rho(\epsilon_L)}$ is sufficient to guarantee that both λ_L and λ_R are positive.

In order to guarantee positive profits for both parties it must hold that $\epsilon_L < \frac{b\rho(\epsilon_L)}{2q}$ and $\epsilon_R < \frac{b\rho(\epsilon_R)}{2q}$. Notice that since $\epsilon_L > \epsilon_R$ it holds that $\frac{\rho(\epsilon_L)}{\epsilon_L} < \frac{\rho(\epsilon_R)}{\epsilon_R}$. Hence, $\epsilon_L < \frac{b\rho(\epsilon_L)}{2q}$ is sufficient to guarantee that both parties make positive profits.

3. Participation Quorum not Fulfilled

Let $\epsilon_L > \epsilon_R = 0$ be such that $v_L + v_R < 0$.

Party L makes negative profits $\pi_L = -\epsilon_L < 0$. Hence it can not be an equilibrium given that it has incentives to deviate to $\epsilon = 0$

□

Proof of Proposition 9

We analyze all possible pure strategy asymmetric Nash equilibria where without loss of generality $\epsilon_L > \epsilon_R$. Those can belong in the following categories:

1. $v_L + v_R > q$ (i.e. the quorum is slack)
2. $v_L + v_R = q$ (i.e. the quorum is binding)
3. $v_L + v_R < q$ (i.e. the quorum is not fulfilled)

1. Participation Quorum Slack

Let $\epsilon_L > \epsilon_R$ be such that $v_L + v_R > q$.

Given effort ϵ_R the maximization problem of party L is:

$$\text{Max. } \frac{1}{2}b\rho(\epsilon_L) - \epsilon_L \quad \text{subject to } \rho(\epsilon_L) + \rho(\epsilon_R) = 2q$$

The Lagrangian of this problem is:

$$L = \frac{1}{2}b\rho(\epsilon_L) - \epsilon_L + \lambda[\rho(\epsilon_L) + \rho(\epsilon_R) - 2q]$$

Taking F.O.C. with respect to effort we obtain:

$$L_{\epsilon_L} = \frac{1}{2}b\rho'(\epsilon_L) - 1 + \lambda\rho'(\epsilon_L)$$

Given the participation quorum is slack the associated Lagrange multiplier takes value zero (i.e $\lambda = 0$). As before for $\lambda = 0$ we would get that for party L it should hold that: $\rho'(\epsilon_L) = \frac{2}{b}$.

Performing the same maximization problem for party R we obtain that it should hold that $\rho'(\epsilon_R) = \frac{2}{b}$.

Combining the two it has to be true that $\rho'(\epsilon_L) = \rho'(\epsilon_R)$ which because of the strict concavity of $\rho(\epsilon_j)$ is true if and only if $\epsilon_L = \epsilon_R$.

Hence, $\epsilon_L > \epsilon_R$ such that $v_L + v_R > q$ can not be an equilibrium.

2. Participation Quorum Binding

Let $\epsilon_L > \epsilon_R$ be such that $v_L + v_R = q$

Given effort ϵ_R the Lagrangian of the maximization problem of party L is:

$$L = \frac{1}{2}b\rho(\epsilon_L) - \epsilon_L + \lambda[\rho(\epsilon_L) + \rho(\epsilon_R) - 2q]$$

Taking F.O.C. with respect to effort we obtain:

$$L_{\epsilon_L} = \frac{1}{2}b\rho'(\epsilon_L) - 1 + \lambda\rho'(\epsilon_L)$$

If ϵ_L is part of an equilibrium it has to hold that $\frac{1}{2}b\rho'(\epsilon_L) - 1 + \lambda\rho'(\epsilon_L) = 0$. Given that the participation quorum is binding it must be that $\lambda = 0$ which is true if and only if $\rho'(\epsilon_L) = \frac{2}{b}$ which is equivalent to $\rho(\epsilon_L) = \rho(\epsilon^V)$ i.e. $\epsilon_L = \epsilon^V$.

Performing the the same maximization problem for party R we obtain that it has to hold that $\epsilon_R = \epsilon^V$.

In order to guarantee positive profits for party L it has to be that $\epsilon_L < \frac{b\rho(\epsilon_L)}{2}$ i.e. $\frac{\rho(\epsilon_L)}{\epsilon_L} > \frac{2}{b}$. For party R it has to hold that $\epsilon_R < \frac{b\rho(\epsilon_R)}{2}$ i.e. $\frac{\rho(\epsilon_R)}{\epsilon_R} > \frac{2}{b}$.

Given that $\epsilon_L > \epsilon_R$ and because of the strict concavity of the ρ function it holds that $\frac{\rho(\epsilon_R)}{\epsilon_R} > \frac{\rho(\epsilon_L)}{\epsilon_L}$. Hence in order to guarantee positive profits for both parties it is sufficient that $\epsilon_L < \frac{b\rho(\epsilon_L)}{2}$.

3. Participation Quorum not Fulfilled

Let $\epsilon_L > \epsilon_R = 0$ be such that $v_L + v_R < 0$.

Party L makes negative profits $\pi_L = -\epsilon_L < 0$. Hence it can not be an equilibrium given that it has incentives to deviate to $\epsilon = 0$

□

Proof of Claim 7

For the profit function under a VSP it holds that:

$$\frac{\pi^V}{\epsilon_j} = \frac{1}{2}b\rho'(\epsilon_j) - 1 \quad (2.13)$$

and

$$\frac{2\pi^V}{\epsilon_j^2} = \frac{1}{2}b\rho''(\epsilon_j) < 0 \quad (2.14)$$

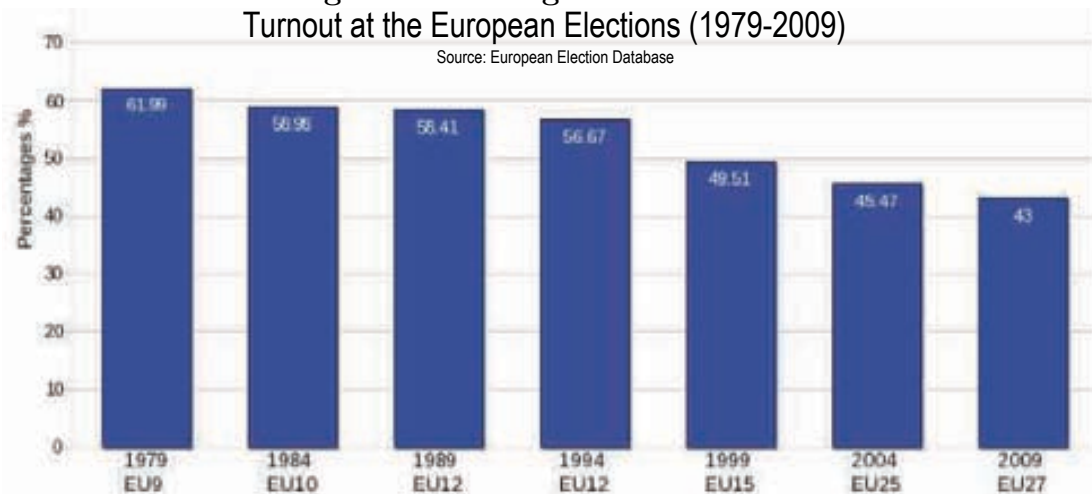
Hence, profits under a VSP are strictly concave in effort. Given that in asymmetric equilibria it holds that $\epsilon_L > \epsilon_R = \epsilon^V$ where ϵ^V is the effort that maximizes profits

(i.e. $\frac{1}{2}b\rho(\epsilon^V) - 1 = 0$) and given that the profit function is strictly concave it holds that $\pi_L^V < \pi_R^V$. \square

Table 2.1: Turnout Rates in OECD countries

Country	2000's	1950's	% Change	Electoral System
Australia	83.1	82.56	0.66%	Majoritarian & CV
Austria	74.1	89.34	-17.06%	Proportional & CV (until 1982/2004)
Belgium	85.98	88.02	-2.32%	Proportional & CV
Canada	55.48	69.56	-20.24%	Majoritarian
Denmark	82.96	78.05	6.30%	Proportional
Finland	69.07	76.31	-9.49%	Proportional
France	50.89	71.29	-28.63%	Majoritarian
Germany	72.73	84.07	-13.49%	Mixed
Iceland	86.89	91.16	-4.69%	Proportional
Ireland	67.94	73.37	-7.41%	Proportional
Italy	82.06	92.73	-11.50%	Proportional
Japan	61.57	73.66	-16.41%	Mixed
Luxembourg	56.5	61.57	-8.24%	Proportional & CV
Netherlands	77.28	87.9	-12.08%	Proportional & CV (until 1970)
New Zealand	76.52	90.58	-15.52%	Mixed
Norway	76.54	78.25	-2.18%	Proportional
Sweden	79.32	76.74	3.36%	Proportional
Switzerland	38.53	60.62	-36.44%	Proportional
United Kingdom	57.94	79.04	-26.70%	Majoritarian
United States	43.92	48.99	-10.34%	Majoritarian
				Source: IDEA

Figure 2: Average Turnout Rates
Turnout at the European Elections (1979-2009)



Bibliography

- BLAIS, A. AND A. DOBRZYNSKA (1998): Turnout in electoral democracies, *European Journal of Political Research*, 33, 239–262.
- BLAIS, A., L. MASSICOTTE, AND A. DOBRZYNSKA (2003): Why is turnout higher in some countries than in others, *Ottawa: Elections Canada*.
- BÖRGERS, T. (2004): Costly voting, *American Economic Review*, 94, 57–66.
- CARRILLO, J. AND M. CASTANHEIRA (2008): Information and Strategic Political Polarization, *Economic Journal*, 118, 845–874.
- COLOMER, J. M. (2004): *Handbook of electoral system choice*, Palgrave Macmillan.
- FRANKLIN, M. (1999): Electoral engineering and cross-national turnout differences: what role for compulsory voting? *British Journal of Political Science*.
- (2004): *Voter turnout and the dynamics of electoral competition in established democracies since 1945*, Cambridge Univ Pr.
- GERARDI, D., M. MCCONNELL, J. ROMERO, AND L. YARIV (2009): Get Out the (Costly) Vote: Institutional Design for Greater Participation, *Carlo Alberto Notebooks*.
- GERBER, A. AND D. GREEN (2000): The effects of canvassing, telephone calls, and direct mail on voter turnout: A field experiment, *American Political Science Review*, 94, 653–663.
- GERBER, A., D. GREEN, AND C. LARIMER (2008): Social pressure and voter turnout: Evidence from a large-scale field experiment, *American Political Science Review*, 102, 33–48.
- GEYS, B. (2006): Explaining voter turnout: A review of aggregate-level research, *Electoral Studies*, 25, 637–663.

BIBLIOGRAPHY

- GHOSAL, S. AND B. LOCKWOOD (2009): Costly voting when both information and preferences differ: is turnout too high or too low? *Social Choice and Welfare*.
- HERRERA, H. AND A. MATTOZZI (2009): Quorum and turnout in referenda, *Journal of the European Economic Association*.
- HIRCZY, W. (1994): The impact of mandatory voting laws on turnout: A quasi-experimental approach* 1, *Electoral Studies*.
- JACKMAN, R. (1987): Political institutions and voter turnout in the industrial democracies, *The American Political Science Review*, 405–423.
- KRASA, S. AND M. K. POLBORN (2009): Is mandatory voting better than voluntary voting? *Games and Economic Behavior*, 66, 275–291.
- LIJPHART, A. (1997): Unequal participation: Democracy's unresolved dilemma, *American Political Science Review*.
- POLO, M. (1998): Electoral competition and political rents, *IGIER e Bocconi University*.
- SHACHAR, R. AND B. NALEBUFF (1999): Follow the leader: Theory and evidence on political participation, *American Economic Review*, 89, 525–547.
- SNYDER, J. (1989): Election goals and the allocation of campaign resources, *Econometrica: Journal of the Econometric Society*.
- WATTENBERG, M. (1998): *The decline of American political parties, 1952-1996*, Harvard Univ Pr.

Chapter 3

Participation Quorums in Costly Meetings¹

3.1 Introduction

In any kind of decision making processes where a participation quorum requirement applies, the decision is valid only if participation is higher than a given level. Examples are various and include large elections such as referenda and presidential elections, but even more frequently smaller scale decision making meetings such as meetings of faculty members, shareholders, neighbors, or members of many societies, and clubs².

Although economists have extensively studied alternative voting rules, the use of a participation quorum has received very little attention. Recently, different authors (Maniquet and Morelli, 2010; Herrera and Mattozzi, 2009; Aguiar-Conraria and Magalhaes, 2010a,b; Houy, 2009) have studied the effect of a participation quorum in referenda. In this case, when not enough voters participate, the referendum is considered to be invalid and the outcome is a reversal to the status quo. On the contrary, in decision making meetings, in case the quorum is not fulfilled, the meeting has to be repeated since a decision must be taken (i.e. in contrast to the case of referenda there is no reversal to the status quo). The importance of analyzing such a situation lies in the fact that, although such a participation re-

¹This chapter is coauthored with Sabine Flamand

²The voting procedure in shareholders' meetings varies among most countries and is determined by each country's corporate law (Dornseifer, 2005).

quirement is widely applied, as we show, it often has negative welfare implications, including policy distortions.

We build on the model of costly meetings by Osborne et al. (2000)³. In the general setup, which may apply to any meeting and which we refer to as the No-Quorum Game, a group of individuals has to decide on a policy through a decision making meeting. Each member of the group independently decides whether to participate, at a cost, in the meeting, whose outcome is assumed to be a compromise between the participants' favorite policies. In deciding whether to attend or not, each member compares the cost of attending with the impact of her presence on the policy decision.

Similar to Bulkley et al. (2001), we characterize the Nash equilibrium of the one-shot *attendance* game by using a constructive approach as follows: We assume a sequential exit process that starts with the full set of attendees, and at each step of which the individual with the largest benefit from not attending the meeting is the one who exits. The process finishes when no further attendee is willing to exit. We fully characterize this exit process, and we argue that it ends at a unique Nash equilibrium belonging to the set of equilibria of the original Osborne et al. (2000) one-shot attendance game. Through this exit process, we are able to characterize the unique Nash equilibrium of any such attendance game, which consists in a set of attendees and the corresponding policy decision.

More importantly, in order to extend the attendance game to decision making meetings for which a participation quorum applies (which we refer to as the *quorum* game), we introduce two additional notions: the participation quorum itself, and how much members are harmed by the fact that a decision is delayed in case the meeting has to be postponed. Our ultimate goal is to analyze the effect of such a participation quorum on the equilibrium set of attendees, the policy decision and on total welfare.

As in Osborne et al. (2000), the equilibrium of the No-Quorum game that is reached through the exit process is such that moderates tend to abstain, while extremists tend to attend. Introducing a participation quorum in this setup leads to several insights. First, there is no situation in which some individuals show up

³For other applications of Osborne et al. (2000), see, for example, Aragonés and Sánchez-Pagés (2009)

in the first meeting and the quorum is not met. That is, either the participation quorum is fulfilled and the decision is taken in the first meeting, or everyone abstains in the first meeting and the decision is taken in the second one.

Second, the introduction of a quorum might either increase or decrease the number of attendees in equilibrium, depending on whether the second meeting is ever reached. More specifically, if the quorum requirement is higher than the number of members who would attend if there was no quorum, the decision is taken in the first meeting when individuals are harmed enough by delaying the decision. In that case, the quorum is binding, and its introduction yields to an increase in the equilibrium number of attendees with respect to the no-quorum case. Conversely, if the second meeting is ever reached, and provided that individuals value strictly less a decision that has been postponed, the introduction of a quorum has the opposite effect of lowering the number of attendees in equilibrium.

Third, given that we focus on symmetric individuals favorite policies, introducing a quorum has in most cases no effect on the chosen policy, no matter whether the second meeting is reached or not. In terms of welfare, it turns out that introducing a quorum, while having no direct policy effects, always yields to a loss of aggregate welfare, even in the cases for which less individuals attend the meeting as compared to the no-quorum equilibrium.

However, there does exist a situation in which the introduction of a quorum might yield to a policy distortion, this situation being that the quorum is odd and strictly higher than the number of attendees would there be no quorum. Intuitively, such a situation might, in some cases, allow an individual to free ride on the attendance of another committee member. As a result, the equilibrium policy ends up being biased. This result clearly has negative implications from a welfare perspective. Indeed, not only does the introduction of a quorum force more individuals to attend the meeting, which, given that the policy remains unaffected, is always a pure welfare loss, but it might also yield to policy distortions when it is odd, which is even worse in terms of aggregate welfare.

The paper is structured as follows: In Section 2, we present the model and we describe formally the attendance and the quorum game. In Section 3 we present the results of our analysis and in Section 4 we conclude. All proofs can be found in the Appendix.

3.2 The Model

3.2.1 The Setup

The policy space is continuous, one-dimensional, and represented by the interval $[0, 1]$. There is a finite group of $N \geq 2$ individuals who must collectively choose a policy, that is, a point $x \in [0, 1]$. We denote individual i 's favorite policy/position by $x_i \in [0, 1]$. We assume that individuals' preferences are single-peaked and uniformly distributed in the $[0, 1]$ interval, so that the distance between the ideal policy of any two individuals is $d = 1/(N - 1)$.

Each individual cares about the remoteness (but not the direction) of the collectively chosen policy x from his favorite policy x_i . Specifically, let individual i 's valuation of the distance between policy x and his ideal point x_i be $V_i(|x_i - x|)$, where $V : [0, 1] \rightarrow [0, 1]$ is a continuous function. We assume that with respect to the distance between x and x_i , V is strictly increasing, and strictly convex. Furthermore, notice that it is symmetric with respect to x_i , that is, the valuation of a policy x depends only on the distance between x and x_i and not on the direction in which x differs from x_i .

Each individual chooses whether or not to attend a meeting, at which a policy is to be selected. Hence, the available actions of individual i are either to attend the meeting or to abstain. Every individual who attends a meeting bears a cost $c > 0$. The final utility of individual i is then given by

$$U_i = 1 - V_i(|x_i - x|) - c\alpha_i$$

where $\alpha_i = 1$ if i attends the meeting (and so pays a cost c) and $\alpha_i = 0$ if i abstains. Therefore, $U(\cdot)$ is strictly decreasing and strictly concave with respect to the distance between x and x_i .

The incentive of every individual to attend a meeting lies in the fact that he can affect the chosen policy to some extent. For simplicity, we assume that the chosen policy x is the median of the favorite policies of the members who decide to attend⁴. Furthermore, given the symmetric distribution of individuals

⁴In case the number of attendees is even the median is defined by $\frac{x_{\frac{N}{2}} + x_{\frac{N}{2}+1}}{2}$

preferences in $[0, 1]$, we assume that the default policy (i.e. the one chosen when everyone abstains) is given by $x = \frac{1}{2}$. This is the most natural assumption, since any other choice for the default would mean introducing an arbitrary bias in favor of some individuals.

3.2.2 The Exit Process

The attendance game of Osborne et al. (2000) is a one-shot game in which all individuals simultaneously decide whether to abstain or to attend a (costly) meeting. In this setup, there may exist multiple pure strategy Nash equilibria, where in general, moderates tend to abstain, while extremists tend to pay the cost and attend⁵.

Given that the ultimate goal of our paper is to study the effect of an exogenously given (quorum) voting rule on the decision process, the multiplicity of equilibria may be problematic, as it may not allow us to draw sharp comparisons among different quorum rules regarding the decision outcome. In order for the model of Osborne et al. (2000) to serve our purposes, we are interested in an equilibrium refinement such that the Nash equilibrium of the attendance game is unique.

Similar to Bulkley et al. (2001), we assume that the attendance decision of each member is determined by a sequential exit process. At each step of the process, given any set of attendees, we assume that the attendee to exit is the individual with the highest potential benefit from doing so. Our exit process begins with the full set of individuals attending, and continues successively until no attendee has an incentive to exit.

Formally, let A be any set of attendees with decision outcome M . For some $i \in A$, let $A' = A - i$ with decision outcome M' . Let the benefit of exit of attendee i be given by the following function:

$$b_i(A) = U_i^{Exit} - U_i^{Attend} = V_i(x_i - M') - V_i(x_i - M) + c$$

Let E be the set of attendees with a positive benefit of exiting (i.e. $E = \{i : b_i(A) > 0\}$).

⁵For the issue of multiplicity of equilibria, see Dhillon and Lockwood (2002)

A and $b_i(A) > 0$). The exit process is defined as follows:

The process starts with the full set of individuals attending (i.e. $A = \{1, 2, \dots, N\}$) and proceeds in the following way:

For any committee A :

1. If $E = \emptyset$ then the process terminates;
2. If $E \neq \emptyset$ then attendee j exits, where j is defined by: $b_j(A)$ be the maximal element of $\{b_i(A) \mid i \in E\}$;
3. If there are more than one maximal elements in E , then the attendee who exits is drawn randomly among the ones having the maximal $b_i(A)$.

Intuitively, as in Osborne et al. (2000), when an attendee decides to exit, he does not have to pay the cost of attending (i.e. c), while he suffers some disutility in terms of the decision outcome since as a result of exiting, the decision moves further from his ideal policy (i.e. $V_i(x_i - M) - V_i(x_i - M^*) < 0$). Notice that at each step of the exit process, several attendees may have a positive benefit from exiting. Having assumed that at each step the attendee to exit is the one who has the highest benefit from doing so, and given that all individuals have to pay the same cost in case of attending, the individual that actually exits is the attendee with the smallest disutility from doing so in terms of the policy outcome (i.e. the attendee with the lowest $V_i(x_i - M) - V_i(x_i - M^*)$).

One can think of the above situation as following: Suppose that the committee members meet in a room where the meeting has to take place. Once in the room, the individuals have to decide whether to leave the room or to stay and pay the (opportunity) cost of attending the meeting, and thus influence to some extent the decision taken. Given such a situation, and for a given cost, we assume that the first individual to leave the room is the one for whom the benefit of leaving is the highest. After the first individual has left, the next member to exit (if any) is the one with the highest benefit from leaving given the remaining set of people in the room, and so on.

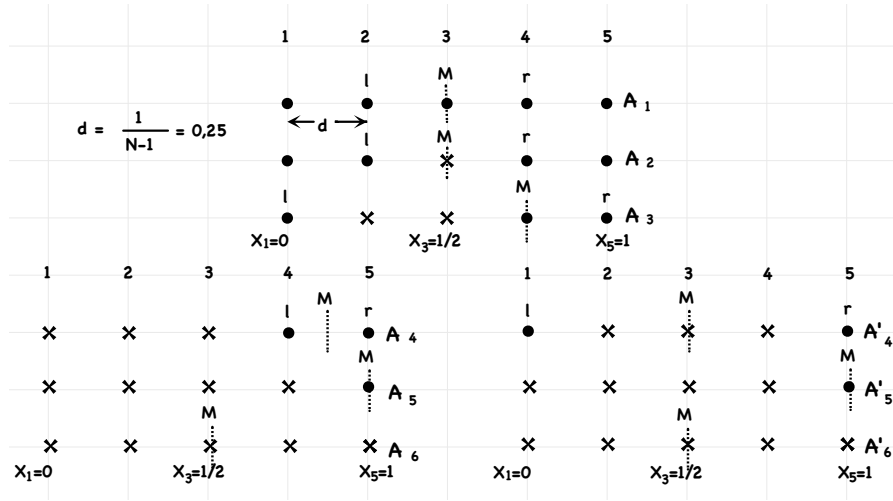


Figure 3.1: An Example of a 5 members committee. We denote by x a member who abstains, and with a circle a member who attends.

Example 1. In order to illustrate how a decision is reached given the exit process defined above, suppose $N = 5$, so that we start with a (full) set of 5 attendees as in situation A_1 . Let M denote the chosen policy for any given set of attendees, and let M' denote the new chosen policy resulting from the exit of some attendee from the committee. Then, for any given set of attendees, let m denote the attendee located at M (when the number of attendees is odd), and let l and r denote the first attendees on the left and on the right of M respectively.

Consider situation A_1 in the figure. In that situation, we have that $M = \frac{1}{2}$, which coincides with the location of $m = 3$, while $l = 2$ and $r = 4$. If m exits the committee, the policy remains at $\frac{1}{2}$, that is, $M = M'$, so that m is willing to exit whenever $V_m(0) > V_m(0) - c$, or, equivalently, $c > 0$. If any $i = m$ exits, he would suffer a strictly positive loss in terms of distance (i.e. $V_i(x_i - M) - V_i(x_i - M') < 0$ for all $i = m$). Hence, m is the first attendee to leave (i.e. $b_3(A)$ is the max element of E), so that we go to situation A_2 .

At A_2 , observe that both l and r are willing to leave if and only if $V(\frac{1}{4}) > V(\frac{1}{2}) - c$, or, equivalently, $c > V(\frac{1}{2}) - V(\frac{1}{4})$. If this inequality is not satisfied, the exit process stops here, and we are at an equilibrium (with the corresponding policy being $M = \frac{1}{2}$). Suppose now, on the contrary, that the inequality is satisfied

so that l and r want to leave. Notice then that at this stage, both attendees 1 and 5 might also want to exit. However, their disutility in terms of distance from doing so is given by $V(\frac{3}{4}) - V(\frac{1}{2}) > V(\frac{1}{2}) - V(\frac{1}{4})$ by the strict convexity of $V(\cdot)$. Therefore, the disutility from leaving is strictly higher for l and r than it is for 1 and 5, so that either one of the formers leaves first. Suppose, WLOG, that l exits, so that we go to A_3 .

At A_3 , it is clear that $m = 4$ would exit before $r = 5$. Indeed, m wants to exit if and only if $c > V(\frac{1}{4})$, which is true since we have assumed at stage A_2 that $c > V(\frac{1}{2}) - V(\frac{1}{4})$ and $V(\cdot)$ is strictly convex. Hence, m wants to leave. Then, observe that the potential disutility of r from exiting is given by $V(\frac{5}{8}) - V(\frac{1}{4}) > V(\frac{1}{4})$, so that m exits before r . Now, attendee l wants to exit if and only if $c > V(\frac{7}{8}) - V(\frac{3}{4})$. As we cannot compare the potential disutility from exiting between m and l , either of the two might leave first.

Suppose that $V(\cdot)$ is such that m exits before l , so that we are at stage A_4 . Again, observe that both $l = 1$ and $r = 5$ have the same incentives to leave the committee, that is, they want to exit if and only if $c > V(1) - V(\frac{1}{2})$. If this inequality is not satisfied, the exit process stops here, and we are at an equilibrium. Indeed, neither l nor r want to exit, and none of the abstainers want to attend: it is direct that neither 4 nor 2 (by symmetry) want to attend, as they just left. Then, it is also direct that abstainer 3 has no incentive to attend, as there would be no effect on the policy (which is already located at his ideal point) while he would have to bear the cost of attending. Hence, as no individual wants to deviate, we are at an equilibrium (with the corresponding policy being $M = \frac{1}{2}$). Suppose now, on the contrary, that $c > V(1) - V(\frac{1}{2})$, and suppose, WLOG, that l leaves, so that we reach A_5 .

At A_5 , only attendee m is left, and the chosen policy M is at his ideal point. As $c > V(\frac{1}{2})$, m leaves, and we reach A_6 , at which no attendee is left. As no abstainer wants to attend, we are at an equilibrium. Indeed, it is direct that neither 5 nor 1 (by symmetry) want to attend, as they just left. Then, it follows directly that neither 2 nor 4 want to attend either, as their potential benefit from doing so is

strictly lower than it is for 1 and 5. Finally, as M is already located at 3's ideal point, he has no incentive to attend either. Therefore, we are at an equilibrium, and the chosen policy is the default (i.e. $M = \frac{1}{2}$).

Suppose now that at A_3 , $V(\cdot)$ is such that l exits before m , so that we go to A_4 . The policy M is now located at $\frac{7}{8}$, and both l and r want to exit if and only if $c > V(\frac{1}{4}) - V(\frac{1}{8})$. As we have assumed that $c > V(\frac{7}{8}) - V(\frac{3}{4})$ at the previous stage, it follows directly that $c > V(\frac{1}{4}) - V(\frac{1}{8})$, so that both l and r want to leave. Suppose, WLOG⁶ that l leaves, so that we go to stage A_5 , which is identical to stage A_5 . From there on, we just saw that we end up at an equilibrium at which no attendee is left, and at which the chosen policy is the default (i.e. $M = \frac{1}{2}$).

3.2.3 The Quorum Game

In order to study the effect of a participation quorum, we introduce two additional ingredients in the attendance game: the quorum itself, and how much individuals discount the fact that the decision may be delayed in case the meeting has to be postponed.

Regarding the participation quorum, we start the analysis with the simplest case. In particular, we assume that in the first meeting, the number of attendees must be at least $Q \in (0, N]$ in order for a policy to be chosen. In case the attendees of the first meeting do not fulfill the quorum, the meeting has to be repeated. We assume that in the second meeting no participation quorum applies.

Individual i 's valuation of the policy chosen (in terms of final utility) in the second meeting is given by $\beta(1 - V_i(x_i - x))$, where $\beta \in [0, 1]$ denotes the loss in terms of utility that individuals suffer as a result of the decision being delayed. In situations where the final decision is the same no matter if delayed or not, a low β captures the idea that individuals are in a hurry to take the decision the soonest possible. On the contrary, individuals with a high β do not really care about postponing the decision to the following meeting, while individuals with

⁶This is WLOG because we are at the 5 individuals' example. Otherwise, it wouldn't be general to assume that l leaves first at this stage. In any case, this is of little importance, as it turns out that no matter what is the number of individuals, the structure of the equilibrium that is reached by applying the exit process is always the same (see section 3).

$\beta = 1$ are indifferent whether the decision is taken in the first or in the second meeting.

- **Stage 1 (1st Meeting):** Following the exit process defined above, individuals decide whether to attend the meeting or not. If the number of attendees is larger than Q , then a policy x is decided according to the compromise function (here the median). Each individual gets final utility $U_{1i} = 1 - V_i(x_i - x) - c\alpha_i$. If the quorum is not met, the game goes to stage two.
- **Stage 2 (2nd Meeting):** Following the exit process defined above, individuals decide whether to attend the meeting or not. No matter what is the number of participants this time, a policy x is to be chosen according to the same compromise function. Each individual gets final utility $U_{2i} = \beta(1 - V_i(x_i - x)) - c\alpha_i$.

3.3 Results

3.3.1 The Benchmark: The No-Quorum Game

In this section, we present the results for a one-shot attendance game for which no quorum applies. Naturally, these results are similar to the ones described in Osborne et al. (2000). To be more precise, our results characterize a unique equilibrium that belongs to the set of equilibria of the original work by Osborne et al. (2000). The novelty of our results lies in the fact that the equilibrium we obtain is unique, thanks to the equilibrium refinement through the defined exit process we based on Bulkeley et al. (2001).

Given the exit process we have defined previously, the following lemmas apply for the equilibrium:

Lemma 1. *The equilibrium number of attendees is even.*

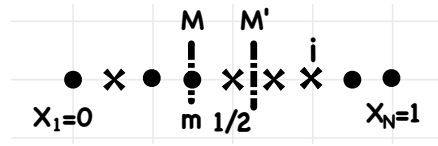


Figure 3.2: The equilibrium number of attendees cannot be odd.

Lemma 1 is intuitive. In any situation where the number of attendees is odd (see Figure 2), the policy is located at attendee m 's ideal point. Such a situation cannot be an equilibrium, since the non-attendee i who mirrors m with respect to M is willing to attend if m does so (M being the new policy would m leave). Indeed, observe that the policy moves by the same distance Δ would m leave or i attend (i.e. it goes to M). However, given that Δ occurs at a further distance from i than it does from m , and given that $V(\cdot)$ is strictly convex, i always wants to attend provided that m attends.

Lemma 2. *The equilibrium is such that there are no gaps between two given attendees on each side of $\frac{1}{2}$.*

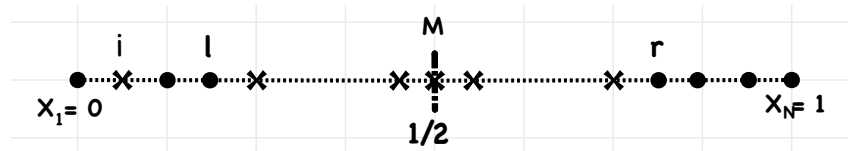


Figure 3.3: No gaps in equilibrium.

Lemma 2 is a consequence of the exit process. Starting from the full set of attendees N , and as $c > 0$, the first attendee to leave is m . Then, as the process keeps going, whenever the number of attendees is even, it is either l or r who leaves first. In order to see this, observe that if any attendee on the left of l leaves, the effect on the policy is exactly the same as if l leaves (i.e. it will coincide with x_r), while, being further from M , the disutility of doing so is strictly higher than the one of l by the strict convexity of $V(\cdot)$. As the same holds for any attendee on the right of r , the first one to leave will be either l or r (and thus, individual i in the figure cannot have left before l).

By the same reasoning, whenever the number of attendees is odd, m always leaves before all the attendees between himself and the extreme of the policy line

(i.e. 0 or 1), while the same holds for the first attendee on the other side on $1/2$ (i.e. l or r). Therefore, it is always either m , or r , or l leaving first, and there can be no gaps between any two attendees on each side of $1/2$ in equilibrium.

Corollary 1. *If there are attendees on both sides of $1/2$, the individuals located at 0 and 1 attend.*

At any stage of the exit process such that there are attendees on both sides of $1/2$, the attendee located at 0 (respectively 1) has strictly higher disutility from leaving than l (respectively r). Hence, it follows directly that any such stage, both the individuals at 0 and 1 attend.

Lemma 3. *The equilibrium is balanced. That is, the number of attendees on each side of $1/2$ is the same.*

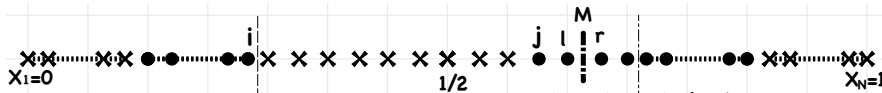


Figure 3.4: The equilibrium number of attendees can not be unbalanced.

Given that at an equilibrium, the number of attendees is even (Lemma 1), and that there can be no gaps between any two attendees on each side of $1/2$ (Lemma 2), then a situation in which the number of attendees is unbalanced is such that the number of attendees between 0 and $1/2$ is strictly smaller (or higher) than the number of attendees between $1/2$ and 1. Such a situation, as depicted in Figure 4, cannot be an equilibrium since if l and r are attending, any given abstainer is also willing to attend provided that $V(\cdot)$ is strictly convex.

We are now ready to characterize the (unique) equilibrium of the attendance game given the exit process:

Proposition 1. *For any $c > 0$, let t be the unique solution of $c = V(2t) - V(t)$. If $t \geq 1/2$, then for all $N \geq 2$, $A^* = 0$. If $t < 1/2$, then, for all $N \geq 2$, there exists a unique equilibrium at which any individual i with $x_i \in [0, 1/2 - t) \cup (1/2 + t, 1]$ attends. The equilibrium number of attendees is given by $A^* = 2(k + 1)$, where $k \in \mathbb{N}$ is the maximum natural number such that $k < (1/2 - t)(N - 1)$.*

The characterization, and, more importantly, the uniqueness of the equilibrium rely on the above lemmas and the associated corollaries.

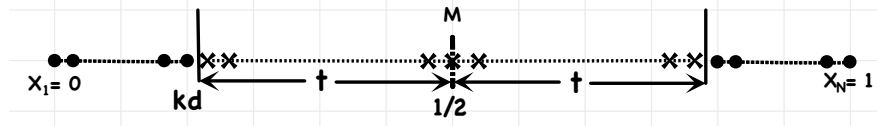


Figure 3.5: The Unique No-Quorum Equilibrium.

The threshold t represents the distance from $\frac{1}{2}$ such that an individual is indifferent between abstaining and attending. Observe that t does not depend on N . That is, no matter what is the number of individuals, the attendance condition is always the same, as for any given c , what matters for individual decisions is the *absolute* distance from the policy, which does not depend on N . On the contrary, t is increasing in c , as a higher participation cost obviously pushes the attendance threshold towards the extremes, meaning that the exit process continues further⁷.

If $t \geq \frac{1}{2}$, it means that c is so high that the abstention interval includes the whole policy line, so that no one attends (i.e. $A^* = 0$)⁸. If, on the contrary, $t < \frac{1}{2}$, it means that the policy line will contain both an abstention interval and two attendance regions on each side of $\frac{1}{2}$. Given the exit process, the equilibrium is characterized by an equal number of consecutive attendees on each side of $\frac{1}{2}$, from the extremists (i.e. the attendees located at 0 and 1) to the last individuals for whom it is worth attending given c (i.e. attendees l and r). Consider the individuals located on the left of $\frac{1}{2}$. The exit process continues until reaching the attendee located at kd , who is the first attendee for whom the benefit of leaving is negative (since $kd < \frac{1}{2} - t$). Therefore, the equilibrium number of attendees on the left of $\frac{1}{2}$ is given by $k + 1$, and thus $A^* = 2(k + 1)$.

Corollary 2. *The equilibrium number of attendees is zero if and only if $c > V(1) - V(\frac{1}{2})$.*

⁷see Proposition 2 and its proof.

⁸we assume that whenever an individual is indifferent between attending and abstaining, he abstains.

From the above corollary, there exists a threshold value of c (which corresponds to $t = 1/2$) such that no individual wants to attend the meeting. As it was the case for t , observe that the threshold value of c is constant. In particular, it does not depend on the number of individuals N .

Corollary 3. *The equilibrium policy x^* is always $\frac{1}{2}$.*

Given the symmetric structure of the equilibrium, it turns out that the equilibrium policy is always located at the middle of the policy line (i.e. at $\frac{1}{2}$) no matter what is the number of attendees.

Finally, Proposition 2 below gives some comparative statics results regarding the participation cost c and the number of individuals N on the equilibrium number of attendees.

Proposition 2. *The equilibrium number of attendees is decreasing in the attendance cost. Furthermore, both the number of attendees and abstainers is nondecreasing in the number of individuals.*

The comparative statics results in the above proposition are intuitive. For given N , an increase in the attendance cost c implies that the exit process continues further. That is, the most moderate attendees from the original set of attendees might not find it worthwhile any longer to attend, given that the cost of participation has increased. Therefore, the equilibrium number of attendees decreases. Moreover, given that an increase in the number of individuals N does not alter the indifference threshold on the policy line between attending and abstaining, it follows that such an increase directly translates into a higher (or at least equal) equilibrium number of both attendees and abstainers.

3.3.2 The Quorum Game

Having characterized the unique equilibrium of the No-Quorum Game, we can apply the results to the analysis of the Quorum Game. Remember that the Quorum Game consists of two meetings. In the second meeting there is no quorum requirement, while in the first one, a participation quorum $Q \in (0, N]$ has to be fulfilled in order for a decision to be taken.

Notation 1. We use subindexes 1 and 2 to refer to the first and second round of the quorum game, while no subindex refers to the one-shot no-quorum game.

We begin the analysis of the quorum game with the second meeting, if this were to be required.

Lemma 4 (Second Meeting). *For any $c > 0$, let t_2 be the unique solution of $c = \beta[V(2t_2) - V(t_2)]$. If $t_2 \geq \frac{1}{2}$, then for all $N \geq 2$, $A_2^* = 0$. If $t_2 < \frac{1}{2}$, then, for all $N \geq 2$, there exists a unique equilibrium at which any individual i with $x_i \in [0, \frac{1}{2} - t_2) \cup (\frac{1}{2} + t_2, 1]$ attends. The equilibrium number of attendees is given by $A_2^* = 2(k_2 + 1)$, where $k_2 \in \mathbb{N}$ is the maximum natural number such that $k_2 < (\frac{1}{2} - t_2)(N - 1)$.*

Lemma 4 has exactly the same flavor as Proposition 1. The equilibrium has a very similar structure, the chosen policy at the second stage is $x_2^* = \frac{1}{2}$, and the only (but important) difference is that the number of attendees may decrease.

Proposition 3. $A_2^* \leq A^*$

The number of attendees in the second meeting, if affected, is lower than that of the no-quorum game. Remember that in the second meeting, each individual discounts the decision taken (i.e. $\beta \in [0, 1]$). More specifically, the effect of one's attendance in terms of final utility is discounted, and hence some moderates that would have incentives to attend the one-shot no-quorum meeting may not participate in the second one. This is the case if their incentives to attend in terms of their (small) influence on the postponed decision do not compensate the cost of attending.

In order to characterize the equilibrium of the quorum game, the following definitions will be helpful. Thereafter we present the main contribution of our analysis in Proposition 4.

Definition 1. *Let the threshold values of the discount factor be defined as follows:*

$$\beta_1 = 1 - \frac{c}{1-V(kd-\frac{1}{2})} \quad \beta_2 = \frac{c}{V(1-2kd)-V(kd-\frac{1}{2})} \quad \beta_3 = 1 - \frac{c}{1-V((\frac{Q}{2}-1)d-\frac{1}{2})}$$

$$\beta_4 = \frac{1-c}{1-V((\frac{Q-1}{2})d-\frac{1}{2})} \quad \beta_5 = \frac{1-V(1-(\frac{Q-1}{2})d)}{1-V(\frac{1}{2})}$$

Proposition 4. *For any $c > 0$, $N \geq 2$ and $Q \in (0, N]$ there exists a unique equilibrium that is characterized as follows:*

Table 3.1: Equilibrium of the Quorum Game

Q	$Q < A^*$	$Q = A^*$			$Q > A^*$			
					Q even		Q odd	
β	$\beta \in [0, 1]$	$\beta \in (\beta_1, \beta_2]$	otherwise	$\beta < \beta_3$	otherwise	$\beta < \min\{\beta_4, \beta_5\}$	otherwise	
A_1^*	A^*	0	Q	Q	0	Q	0	
A_2^*		$2(k_2 + 1)$			$2(k_2 + 1)$		$2(k_2 + 1)$	
x^*	1/2	1/2	1/2	1/2	1/2	$(\frac{Q-1}{2})d$	1/2	

According to the above proposition, the effect of the quorum on the equilibrium (i.e. A_1^* , A_2^* and x^*) depends on the level of the quorum itself (i.e. Q), and on how much individuals discount a delayed decision (i.e. β). To be more precise, the only case for which the equilibrium outcome is independent of β (and the decision always taken in the first meeting), is when the quorum is lower than the no-quorum attendance rate (i.e. $Q < A^*$). On the contrary, when $Q \geq A^*$, how much individuals discount a delayed decision is crucial.

1. $Q < A^*$: If the quorum is smaller than the no-quorum attendance rate, then it has no effect on the equilibrium outcome. This is a consequence of the fact that the quorum does not alter the exit decision of any of the individuals during the first meeting and hence the policy decision remains unaffected.

2. $Q = A^*$: When the quorum is equal to the number of attendees in the no-quorum game, we know that A^* and hence the quorum is even. Moreover, we know that individual l located at kd is the first attendee who decided not to exit the no-quorum game (see Figure 6)⁹.

⁹WLOG we analyze the decision of individual l , while the same holds for individual r .

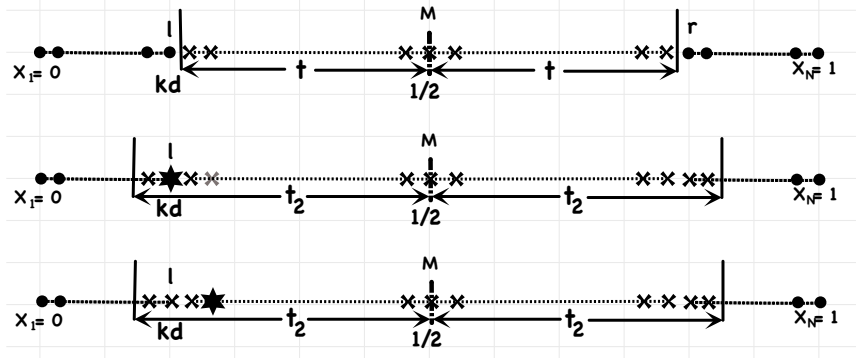


Figure 3.6: The meeting at the top is the one of the No-Quorum Game. At the middle the case of $Q = A^*$, and at the bottom the case of $Q > A^*$. The individual with a star is the pivotal individual (located at kd if $Q = A^*$, at $(\frac{Q}{2} - 1)d$ if $Q > A^*$ and even, and at $(\frac{Q-1}{2})d$ if $Q > A^*$ and odd).

Starting analyzing the exit process of the quorum game with the full set of attendees, the decision of all attendees between l and r is the same as in the no-quorum game, meaning that they exit. Notice though that because of the presence of the quorum, attendee l is now pivotal on whether the quorum is going to be met or not. Hence, his cost-benefit exit calculation is altered compared to the no-quorum case.

By exiting, he actually postpones the decision to the next meeting, while he knows that the policy will remain the same (that is, $x_2^* = 1/2$). The cost of such a decision stems from the fact that the decision is delayed, while its benefit lies in the fact that under certain conditions, he will not have to pay the cost of attending the second meeting.

When will the exit decision be worthwhile for the pivotal individual? Or, said in other words, when is the cost of exiting lower than the benefit? On the one hand, he must not discount the future “too much” (i.e. $\beta > \beta_1$), so that he does not care to delay the decision (i.e. low cost of exiting). On the other hand, however, he has to discount the future “enough” (i.e. $\beta < \beta_2$), so that he has no incentives to attend the second meeting (i.e. high benefit of exiting). Notice that if the pivotal individual exits (who is also the individual with the highest incentives to attend), then all the remaining attendees exit as well so as to avoid paying the cost of attending a meeting that is not fulfilling the quorum. Therefore,

if $\beta < (\beta_1, \beta_2)$, then $A_1^* = 0$, $A_2^* = 2(k_2 + 1)$ and $x^* = 1/2$.

If the above restriction on the discount factor does not hold (i.e. $\beta > (\beta_1, \beta_2)$), for example because the pivotal individual discounts the future a lot, then the quorum equilibrium is identical to the no-quorum one and the quorum is binding in the first round.

3. $Q > A^*$: If the quorum is larger than the number of attendees in the no-quorum game, its effect on the equilibrium varies depending on whether the quorum is even or odd.

(a) Q Even: The intuition in this case is similar to the one where $Q = A^*$. However, an important difference is that when $Q > A^*$, the pivotal individual is now located at $(\frac{Q}{2} - 1)d$, and hence is more moderate than the pivotal individual at $Q = A^*$ ¹⁰.

During the exit process of the quorum game, the pivotal attendee is the first individual whose exit decision may be altered because of the presence of the quorum. By exiting, he actually postpones the decision to the next meeting, being sure that the policy will remain the same (that is $x_2^* = 1/2$). Moreover, and contrary to the $Q = A^*$ case, he definitely has no incentives to attend the second meeting, given that he is an abstainer in the no-quorum game.

The benefit of attending stems from the fact that the decision is not delayed, while it implies an extra attendance cost. Hence, the pivotal individual is willing to pay the cost, attend and make the quorum binding in the first meeting if he discounts a lot a delayed decision (i.e. $\beta < \beta_3$). On the contrary, if $\beta > \beta_3$, the pivotal individual has no incentives to pay the cost, the quorum is not met and as before no one attends the first meeting (i.e. $A_1^* = 0$, $A_2^* = 2(k_2 + 1)$ and $x^* = 1/2$).

(b) Q Odd: The intuition in this case is similar but with a very important difference with respect to the case of an even quorum. The pivotal individual is located at $(\frac{Q-1}{2})d$, who is again more moderate than the pivotal individual at $Q = A^*$ ¹¹.

Through the exit process of the quorum game, the pivotal attendee is the first individual whose exit decision may be altered because of the presence of

¹⁰WLOG we analyze the individual located at $(\frac{Q}{2} - 1)d$. Because of symmetry, the same analysis holds for his mirror with respect to $1/2$.

¹¹For the same reasoning as Footnote 7, WLOG we analyze the individual located at $(\frac{Q-1}{2})d$.

the quorum. By exiting, he actually postpones the decision to the next meeting, being sure that the policy will remain the same (that is $x_2^* = 1/2$). As before, he definitely has no incentives to attend the second meeting, given that he is an abstainer in the no-quorum game.

The benefit of attending has now an additional component compared to the case of an even quorum. In addition to not being delayed, the elected policy moves to his ideal point (since the pivotal attendee is the new median). As before, the cost is what the individual has to pay in order to attend the meeting. Hence, the pivotal individual is willing to pay the cost, attend and make the quorum binding in the first meeting if he discounts a lot a delayed decision (i.e. $\beta < \beta_4$). On the contrary, if $\beta > \beta_4$, the pivotal individual has no incentives to pay the cost, the quorum is not met and as before no one attends the first meeting (i.e. $A_1^* = 0$, $A_2^* = 2(k_2 + 1)$ and $x^* = 1/2$).

Notice though that the attendance of the pivotal member implies a policy distortion. Indeed, the policy now moves to $x^* = \frac{Q-1}{2}d - \frac{1}{2}$, and given that the exit process is altered, it deserves a closer look. In order to guarantee that the exit process terminates at an equilibrium, we have to make sure that no further attendees have incentives to exit, while no further abstainers want to attend.

The process actually terminates if the attendee with the highest benefit of exiting is not willing to do so. Notice that the policy is biased towards the left (i.e. $x^* = \frac{Q-1}{2}d - \frac{1}{2}$). Moreover, any attendee that exits guarantees a delayed policy outcome of $1/2$. The individual with the highest incentives to do so is then the extremist located the furthest from $x^* = \frac{Q-1}{2}d$, that is, the individual located at 1. In order for this extremist not to exit, it has to be the case that he is harmed a lot by postponing the decision to the second meeting (i.e. $\beta < \beta_5$).

The individual with the highest benefit of entering is the mirror (with respect to $1/2$) of the individual located at x^* . Observe that this individual is never attending a no-quorum meeting, precisely because it is not worthwhile for him to pay the cost so as to push the policy from x^* to $1/2$. Therefore, he has no incentives to attend here either. Notice that in fact, this individual free rides on the attendance of his 'mirror' who participates so as to avoid postponing the decision.

To sum up, our results regarding the Quorum Game can be summarized as following:

- The quorum is met in the first round when:
 1. The quorum is lower than the no-quorum attendance rate.
 2. The quorum is higher than the no-quorum attendance rate and members discount a lot a delayed decision.
- If the second meeting takes place, the number of attendees is smaller or equal than in the no-quorum case.
- When the quorum is larger or equal than the no-quorum attendance rate, then either no member attends the first meeting or the quorum is binding. Notice that there exists no equilibrium in which a subset of members decide to attend the first meeting and the quorum is not met¹².
- The quorum has a different effect when it is even or odd. An odd quorum may create policy distortions.

Regarding the effect of the quorum on total welfare, we conclude that the introduction of quorum requirement is welfare decreasing. Whenever the quorum has no effect on the equilibrium, that is, when the decision is taken in the first meeting and both the number of attendees and the equilibrium policy remain the same, total welfare is clearly not affected.

Whenever the quorum is binding, but the equilibrium policy unchanged, total welfare is strictly lower than under the no-quorum equilibrium. Indeed, as the presence of the quorum forces some moderates to attend (and thus bear the corresponding cost) in order for the meeting not to be postponed, total welfare is strictly lower following the introduction of the quorum. If, in addition to that, the equilibrium policy gets distorted (i.e. $x^* = \frac{1}{2}$), the effect of the quorum is even worse from a welfare perspective.

¹²A possible way to obtain such a result would be to introduce an exogenous probability that each individual may be prevented from attending the meeting.

Finally, although not formally proven, through simulations we conclude that when the decision is postponed to the second meeting, and even though less individuals attend, total welfare is lowered as a result of the quorum being present. That is, the fact that less individuals pay the cost of attending does not compensate the aggregate loss of welfare in terms of policy valuation.

3.4 Conclusion

We believe that this piece of research is a first step towards the understanding of a widely used but not formally analyzed voting rule. Clearly, our results have important policy implications, and we think that policy makers deciding on the use of a participation quorum in small decision making meetings should take our results into consideration.

In particular, in meetings where the preferences of individuals are symmetrically distributed, introducing a participation quorum is a bad idea since it creates welfare losses. As explained, these losses may be the result of the decision being delayed, or the consequence of the fact that individuals who are better off abstaining are now forced to attend in order to fulfill the quorum.

One standard argument in favor of the use of a participation quorum is that it allows to protect individuals from a policy decision being taken by a small minority of the members. However, it turns out that the policy is in general not affected as it coincides with the one that would have been taken had there been no quorum. This is so because in a world where the costly committee meetings are ruled by the citizen-candidate spirit of Osborne et al. (2000), the individuals who are really afraid of a unwanted decision have incentives to attend anyway. In order to protect those individuals, there is then no need for a participation quorum since their participation is in line with their own interest, and hence this argument does not apply.

A final argument in favor of a participation quorum is that it guarantees the legitimacy of the decision taken. Think for instance of a meeting of the shareholders of an important enterprise deciding on the enterprise's strategy. Suppose that all shareholders have a very high opportunity cost of attending the meeting,

so that the equilibrium number of attendees under no quorum is very small. In such cases, one can think of a participation quorum as a tool to protect the legitimacy of the decision, since a decision taken by very few shareholders might have a rather negative impact on the enterprise's image. According to our results, if the shareholders are in a hurry, introducing a quorum then has the effect of increasing the number of shareholders who take part in the decision. In such situations, our analysis has policy recommendations as well. We showed that the use of a participation quorum always lowers welfare. In any case, however, if there is a quorum to be introduced, it should require an even number of individuals so as to avoid policy distortions (and thus minimize welfare losses).

As the present paper constitutes a first attempt to analyze the use of a participation quorum in meetings, many paths for future research remain open. Among those, a very important feature which we wish to capture is the case in which a positive number of members show up at the first meeting, but the quorum is not fulfilled. A way to allow for such a possibility could be the introduction of an exogenous probability that each member is prevented from attending the meeting. Intuitively, this would affect the number of attendees in each meeting and, more importantly, it would give rise to situations in which some individuals end up paying the cost of attending (even though the meeting has to be postponed), because they believe the quorum will be met.

So far our analysis has been focused on the case of uniformly distributed individuals' favorite positions. Although our results would still be valid for other symmetric distributions, we believe it is important to extend our analysis to asymmetric setups. Indeed, one of the main arguments favoring the use of a participation quorum is the fact that one wants to prevent the decision from being taken by a (non-representative) minority. However, in symmetric situations such as the one we have been analyzing here, there is no way a minority could possibly exploit a majority.

3.5 Appendix

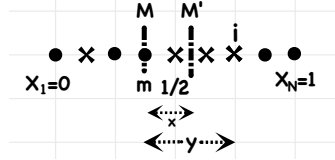


Figure 3.7: Proof of Lemma 1.

Proof of Lemma 1. Suppose the number of attendees is odd and individual m , by leaving, does not change the location of the policy M . Then he is willing to leave if and only if $V(0) > V(0) - c$ or, equivalently, $c > 0$. Therefore, m is always leaving.

Suppose the number of attendees is odd and individual m , by leaving, changes the location of the policy. Suppose, furthermore, that c is such that m is not willing to leave, and let the corresponding policy given this set of attendees be M . If this is the case, there is always a non-attendee i who is willing to attend, and whose location is such that by doing so, the policy moves to the same location as it does if m leaves. Let the distance between individual m and the new policy M in case he leaves be x . Furthermore, let the distance between the non-attendee i mentioned above and the policy M be y , where $y = 2x$. If m is not willing to leave, it means that $-V(x) < -V(0) - c$, or, equivalently, $c < V(x)$. Now, notice that i is willing to attend if and only if $-V(x) - c > -V(y)$, or, equivalently, $c < V(y) - V(x)$. As $y = 2x$ and $V(\cdot)$ is strictly convex, it follows that $V(x) < -V(x) + V(y)$, or, equivalently, $V(2x) > 2V(x)$. Therefore, i wants to attend. □

Proof of Lemma 2. Given the exit process, we know that at any stage, the first attendee to leave (if any) is the one with the lowest potential disutility from leaving. Starting from the full set of attendees N , where N is odd, we know that as $c > 0$, the first attendee to leave is m . Let l and r be the first attendees on the left and on the right of M respectively¹³. The potential disutility from leaving is

¹³Observe that assuming that N is odd is without loss of generality, since assuming instead that N is even simply means that we start from here on.

the same for both of them, and is given by $V(2d) - V(d)$. Consider now the next attendees on the left of l and on the right of r respectively. Their disutility from leaving is identical, and is given by $V(3d) - V(2d)$, which is strictly higher than $V(2d) - V(d)$ given the strict convexity of $V(\cdot)$. Obviously, this will also be true for any pair of attendees who are located even further from M . Therefore, the first one to leave is either l or r . From there on, if c is high enough so that the exit process keeps going,

1. At any stage for which the number of attendees is odd (case A_3 in Example 1), either m , or one (and only one) of the attendees l or r is the first one to leave. Given the exit process, a situation in which the number of attendees is odd is such that m is next to r (l), while, as some attendees have already left, there are gaps between m and l (r). Suppose, WLOG, that m is next to r (as in case A_3) and let kd be the distance between m and l (so that $k = 3$ in the example). We aim at showing that the first one to leave is either m or l (that is, either the median attendee or the furthest one next to him). If m leaves, his disutility from doing so is given by $V(\frac{(k-1)d}{2})$. If r leaves, he will suffer a disutility of $V(\frac{kd}{2} + d) - V(d)$. As $V(\frac{(k-1)d}{2}) = V(\frac{(k+1)d}{2} - d) < V(\frac{(k+1)d}{2}) - V(d)$, it follows that m always leaves before r . Obviously, this is also true for any attendee on the right of r . Indeed, if any such attendee leaves, the effect on the policy will be exactly the same as if r leaves, while, being strictly further, the disutility from leaving in terms of distance will be strictly higher than the one of r by the strict convexity of $V(\cdot)$. Now, if l leaves, his disutility from doing so is given by $V((k + \frac{1}{2})d) - V(kd)$. We do not know whether it is m or l who has the lowest disutility from leaving. However, by the same reasoning as the one just described above, we know that any attendee on the left of l will suffer a strictly higher disutility from leaving than l , so that l always leaves first. Therefore, either m or l leaves first.
2. At any stage for which the number of attendees is even (cases A_4 and A_4 in Example 1), so that the policy is $M = \frac{(x_l + x_r)}{2}$, either l or r is the first one to leave. Let kd be the distance between i and M , $i = l$ or r (so that $k = \frac{1}{2}$ in situation A_4 and $k = 2$ in situation A_4). The disutility from

leaving for individual $i = l, r$ is given by $V(2kd) - V(kd)$. Consider now the next attendees on the left of l and on the right of r respectively. Their disutility from leaving is strictly higher than $V(2kd) - V(kd)$ given the strict convexity of $V(\cdot)$. Furthermore, this is also true for any pair of attendees who are located even further from M .

□

Proof of Corollary 1. Consider any stage of the exit process such that there are still attendees on both sides of $1/2$ (i.e. we rule out situations such as A_4 in Example 1). From Lemma 3, we know that, no matter whether the number of attendees is even or odd, the first one to leave is always either m , or l , or r . Therefore, it follows directly that at any such stage (i.e. such that there are attendees on both sides of $1/2$), the attendee located at 0 (respectively 1) has strictly higher disutility from leaving than l (respectively r). Hence, at any such stage, both the individuals at 0 and 1 attend. □

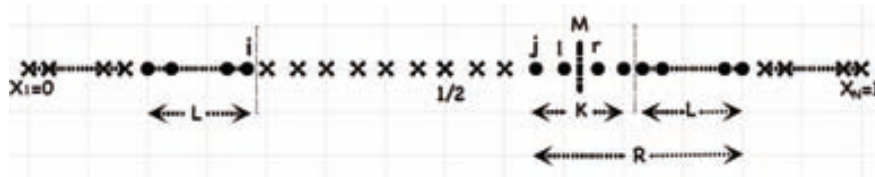


Figure 3.8: Proof of Lemma 3.

Proof of Lemma 3. Let $L \geq 0$ be the number of attendees on the left of $1/2$. Similarly, let R be the number of attendees on the right of $1/2$, and assume, WLOG, that $R \geq L$, and so $R = L + K$, where $K > 0$ and is even by Lemma 1. From Lemma 2, it follows that the L and R attendees respectively on the left and on the right of $1/2$ are consecutive (i.e. there are no gaps between them). Let M be the chosen policy given this set of attendees, which is located at the median of the K attendees. Observe that if the attendees l and r are not willing to leave, it means that c is small enough so that it is worth attending to prevent M from moving by $\frac{1}{2}d$, that is, it means that $c < V(d) - V(\frac{1}{2}d)$. Let i and j be the first attendees on the left and on the right of $1/2$ respectively, and consider any non-attendee between i and j . If any such non-attendee were to come back, the policy

would move to x_l , and he would do so if and only if $c < V((k + \frac{1}{2})d) - V(kd)$, where $kd > d$ is the distance between any non-attende and l . It turns out that if r attends, any abstainer between i and j wants to attend as well, that is, $V((k + \frac{1}{2})d) - V(kd) > V(d) - V(\frac{1}{2}d)$ for all $k > 1$ by the strict convexity of $V(\cdot)$. Therefore, the equilibrium is such that $K = 0$ (i.e. balanced). \square

Proof of Proposition 1. By lemmas 1 to 3 and the associated corollaries, we know that the equilibrium reached by means of the exit process is such that there is an equal number of attendees on each side of $\frac{1}{2}$ without gaps between any two of them, and such that the individuals located at the extremes of the policy line attend. Let t be the unique solution of $c = V(2t) - V(t)$ and consider a situation such as A_1 in Figure 9. By definition of t (and by the lemmas and associated corollaries), any such situation, that is, any situation with an equal number of consecutive attendees starting from the extremes on both sides of $\frac{1}{2}$, and such that only individuals with $x_i \in [0, \frac{1}{2} - t) \cup (\frac{1}{2} + t, 1]$ attend, is an equilibrium. Indeed, notice that the individuals with the highest potential benefit from leaving are l and r . However, as by definition of t , $b_l(A) = b_r(A) < 0$, they both attend. Furthermore, as by definition of t , $b_i(A) > 0$ for any i with $x_i \in [\frac{1}{2} - t, \frac{1}{2} + t]$, none of such individuals is willing to attend. Therefore, the situation depicted in the figure is an equilibrium.

In order to derive the equilibrium number of attendees A^* , observe that the exit process stops at the individual located at kd (i.e. attendee l), who is the first attendee (on the left side of $\frac{1}{2}$) for whom $b_i(A) < 0$. Therefore, the equilibrium number of attendees on the left of $\frac{1}{2}$ is given by $k + 1$, and thus $A^* = 2(k + 1)$.

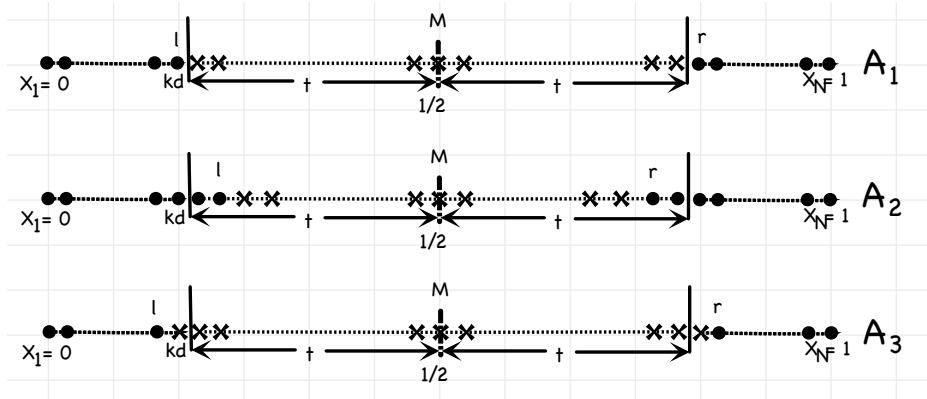


Figure 3.9: Proof of Proposition 1: Uniqueness

It now remains to be shown that the equilibrium is unique. Consider a situation such as A_2 in the figure. That is, a situation that satisfies the equilibrium requirements as described in the lemmas, but such that there is one (or more) pair(s) of consecutive attendees in the interval $[\frac{1}{2} - t, \frac{1}{2} + t]$ (or, equivalently, such that $x_l, x_r \in [\frac{1}{2} - t, \frac{1}{2} + t]$). As, by definition of t , $b_i(A) = 0$ for any i with $x_i \in [\frac{1}{2} - t, \frac{1}{2} + t]$, it follows that, for any t , any such situation cannot be an equilibrium.

Finally, consider a situation such as A_3 in the figure. That is, a situation that satisfies the equilibrium requirements as described in the lemmas, but such that there is one (or more) pair(s) of consecutive abstainers in the interval $[0, \frac{1}{2} - t) \cup (\frac{1}{2} + t, 1]$. As, by definition of t , $b_i(A) < 0$ for any i with $x_i \in [0, \frac{1}{2} - t) \cup (\frac{1}{2} + t, 1]$, it follows that, for any t , any such situation cannot be an equilibrium.

□

Proof of Corollary 2. According to the exit process, if everybody leaves, the last attendee to exit is either the one located at 0 or the one located at 1. This individual is indifferent to leave if and only if $c = V(1) - V(\frac{1}{2})$, or, equivalently, $t = \frac{1}{2}$. Obviously, if $c > V(1) - V(\frac{1}{2})$, or, equivalently, $t > \frac{1}{2}$, this last attendee wants to exit, which means that no one attends.

□

Proof of Proposition 2. The equilibrium value of t is implicitly defined by $\epsilon(t) =$

$c - V(2t) + V(t) = 0$. We have that

$$\frac{\epsilon}{t} = -2V'(2t) + V'(t) < 0$$

$$\frac{\epsilon}{c} = 1 > 0$$

and thus, by the implicit function theorem: $\frac{t}{c} = -\frac{\frac{\partial \phi}{\partial c}}{\frac{\partial \phi}{\partial t}} > 0$. Therefore, as the threshold for attendance is moving towards the extremes of the policy line as c increases, it follows directly that the equilibrium number of attendees decreases.

Given that t does not depend on the number of individuals N , it is also direct that the *absolute* number of both attendees and abstainers is increasing in N . Indeed, for a given $c > 0$ and given the (fixed) distance t , increasing the (uniformly distributed) number of individuals obviously means that there will be more (or at least an equal number of) individuals both between 0 and t (respectively t and 1) and in the abstention interval. \square

Proof of Proposition 3. As t is the unique solution of $c = V(2t) - V(t)$ and t_2 is the unique solution of $c = \beta [V(2t_2) - V(t_2)]$, we have that

$$\beta = \frac{V(2t) - V(t)}{V(2t_2) - V(t_2)}$$

As $\beta < 1$, it follows that $t_2 < t$, and thus $k_2 < k$ and $A_2^* < A^*$. \square

Proof of Proposition 4. 1. Suppose $Q < A^*$ and consider attendees l and r of the no-quorum game for given N and c . Thus, $b_l(A^*) = b_r(A^*) < 0$, the set E is empty and there are A^* attendees in equilibrium. Let now introduce a quorum $Q < A^*$. All moderates located between l and r of the no-quorum game exit, as before. Then, $b_l(A_1^*) = b_r(A_1^*) < 0$ and the set E is empty, so that the exit process stops at l and r . Therefore, $A_1^* = A^*$ and $x_1^* = x^* = \frac{1}{2}$. Said in words, the attendance decision of any individual is the same as in the no-quorum case (in particular, l is not pivotal regarding the quorum requirement), and hence the equilibrium is unaffected.

2. Suppose $Q = A^*$. All moderates located between l and r of the no-quorum game exit, as before. Then, WLOG, given that l is now pivotal regarding Q , he exits if and only if

$$1 - V\left(kd - \frac{1}{2}\right) - c < \beta \left[1 - V\left(kd - \frac{1}{2}\right)\right] - \alpha_l c$$

where $\alpha_l = 1$ if and only if

$$\beta > \frac{c}{V(1 - 2kd) - V(kd - \frac{1}{2})} = \beta_2$$

and $\alpha_l = 0$ otherwise. Suppose that $\beta < \beta_2$, so that l would not attend in the second round (otherwise, the second round is never reached as l has no incentive to exit in the first round). Then, he exits in the first round if and only if

$$\beta > 1 - \frac{c}{1 - V(kd - \frac{1}{2})} = \beta_1$$

Therefore, l exits in the first round if and only if $\beta_1 < \beta < \beta_2$. Suppose this is the case. Notice that l is the abstainer with the highest potential benefit from attending so as to fulfill Q , and thus no other abstainer has an incentive to attend either. Given that l exits, all other attendees exit as well since the quorum is not met and $c > 0$. Hence, $A_1^* = 0$ and we go to stage 2. From there on, the characterization of the equilibrium and its uniqueness follow the same reasoning as in Proposition 1 (with t_2 being the unique solution of $c = \beta [V(2t_2) - V(t_2)]$ as the new attendance threshold and $x_2^* = \frac{1}{2}$).

If $\beta < \beta_1$, l has no incentive to exit in the first round. The exit process stops here, and as no individual has an incentive to deviate, we are at an equilibrium, and it follows that $A_1^* = A^*$ and $x_1^* = x^* = \frac{1}{2}$. Similarly, if $\beta > \beta_2$, l would attend in the second round, so that he has no incentive to exit in the first round, and the same conclusion applies.

3. Suppose $Q > A^*$ is **even**. Then, during the exit process, WLOG, the attendee i located at $(\frac{Q}{2} - 1)d > kd$ is pivotal regarding Q , so that he exits

if and only if

$$\beta > 1 - \frac{c}{1 - V\left(\left(\frac{Q}{2} - 1\right)d - \frac{1}{2}\right)} = \beta_3$$

Suppose this is the case, so that i exits. Notice that i is the abstainer with the highest potential benefit from attending so as to fulfill Q , and thus no other abstainer has an incentive to attend either. Given that i exits, all other attendees exit as well since the quorum is not met and $c > 0$. Hence, $A_1^* = 0$ and we go to stage 2. From there on, the characterization of the equilibrium and its uniqueness follow the same reasoning as in Proposition 1.

Suppose now that $\beta < \beta_3$, so that i attends. Given that i is the attendee with the highest potential benefit from exiting, and $b_i(A) < 0$, the exit process stops here, and since no individual has an incentive to deviate, we are at an equilibrium. Thus, the equilibrium number of attendees is given by $A_1^* = Q$ and the equilibrium policy is $x_1^* = \frac{1}{2}$.

4. Suppose $Q > A^*$ is **odd**. Then, during the exit process, WLOG, the attendee i located at $\left(\frac{Q-1}{2}\right)d > kd$ is pivotal regarding Q , so that he exits if and only if

$$\beta > \frac{1 - c}{1 - V\left(\left(\frac{Q-1}{2}\right)d - \frac{1}{2}\right)} = \beta_4$$

Suppose this is the case, so that i exits. Notice that i is the abstainer with the highest potential benefit from attending so as to fulfill Q , and thus no other abstainer has an incentive to attend either. Given that i exits, all other attendees exit as well since the quorum is not met and $c > 0$. Hence, $A_1^* = 0$ and we go to stage 2. From there on, the characterization of the equilibrium and its uniqueness follow the same reasoning as in Proposition 1.

Suppose now that $\beta < \beta_4$, so that i attends. Notice that at this stage, the attendee located at 1 has the highest potential from exiting. Indeed, he is the furthest from the policy $M = \left(\frac{Q-1}{2}\right)d$, would it be implemented, and he is pivotal with respect to Q , so that it could be beneficial for him to exit

in order for the decision to be postponed and the policy to be closer to his ideal point. Formally, 1 exits if and only if

$$\beta > \frac{1 - V(1 - (\frac{Q-1}{2})d)}{1 - V(\frac{1}{2})} = \beta_5$$

Suppose that $\beta < \min \beta_4 \beta_5$, so that both i and 1 attend. Given that 1 is the attendee with the highest potential benefit from exiting, and $b_1(A) < 0$, the exit process stops here, and since no individual has an incentive to deviate, we are at an equilibrium. Thus, the equilibrium number of attendees is given by $A_1^* = Q$ and the equilibrium policy is $x_1^* = (\frac{Q-1}{2})d$.

□

Bibliography

- AGUIAR-CONRARIA, L. AND P. MAGALHÃES (2010a): How quorum rules distort referendum outcomes: evidence from a pivotal voter model, *European Journal of Political Economy*.
- (2010b): Referendum design, quorum rules and turnout, *Public Choice*, 144, 63–81.
- ARAGONES, E. AND S. SÁNCHEZ-PAGÉS (2009): A theory of participatory democracy based on the real case of Porto Alegre, *European Economic Review*, 53, 56–72.
- BULKLEY, G., G. MYLES, AND B. PEARSON (2001): On the membership of decision-making committees, *Public Choice*, 106, 1–22.
- DHILLON, A. AND B. LOCKWOOD (2002): Multiple equilibria in the citizen-candidate model of representative democracy: a comment, *Journal of Public Economic Theory*, 4, 171–184.
- DORNSEIFER, F. (2005): *Corporate business forms in Europe: a compendium of public and private limited companies in Europe*, Sellier European Law Pub.
- HERRERA, H. AND A. MATTOZZI (2009): Quorum and turnout in referenda, *Journal of the European Economic Association*.
- HOUY, N. (2009): A characterization of majority voting rules with quorums, *Theory and decision*, 67, 295–301.
- MANIQUET, F. AND M. MORELLI (2010): Approval Quorums Dominate Participation Quorums, *Working Paper, Series EUI ECO, 2010/13*.
- OSBORNE, M., J. ROSENTHAL, AND M. TURNER (2000): Meetings with costly participation, *American Economic Review*, 90, 927–943.

Chapter 4

Institutions, Society or Protest? Understanding Blank and Null Voting

4.1 Introduction

When Pierre Dupont leaves his home on those Sundays when he has the possibility of spending a few civic moments in his *bureau de vote*, he not at all so infrequently returns after having marked his ballot *va te faire foutre*, *enculé*, *con*, or some other libertarian expression. These ballots, along with those that are completely blank and those that contain errors in voting, form the category of blank and null ballots. Apart from those ballots cast in error, the blanks and null votes represent a definite refusal to choose among the various candidates in the election. (Rosenthal and Sen, 1973)

The above libertarian expressions are not a characteristic of the French electorate and can be found translated on ballots around the world. Pierre Dupont, Juan Perez or any voter who participates in the election but is not willing to support any of the competing candidates has to choose among two actions: vote blank or vote null.

A blank vote is a vote for none of the competing candidates (i.e. a so called None Of The Above vote), while a null vote is a vote that has been on pur-

pose or by mistake cast in a way not conforming with the legal voting procedure. Examples of null ballots can include libertarian expressions, votes for more candidates than allowed by the electoral law, usage of numbers rather than symbols next to the candidates ordering etc.

The common element of the above two is that they do not systematically affect the electoral outcome. Null votes are never considered to be valid and are always treated in exactly the same way as abstention¹. The case of blank votes is slightly more complicated since the way they are treated depends on the electoral law. In some countries, blank votes are not considered to be valid votes and hence are equivalent to abstention and null votes. In other countries, blank votes are considered to be valid votes, and as a result they may indirectly but not in a systematic way affect the electoral outcome in two ways.

The first way blank votes may affect the electoral outcome is when parties have to reach an exogenously given threshold of the vote share in order to obtain representation in the parliament. Since blank ballots are considered to be valid, the absolute number of votes required by parties (i.e. the threshold) in order to enter the parliament is increasing in the number of blank ballots. As a result, blank votes may harm small parties in their effort to obtain their first seat in the parliament.

The second way that blank votes may affect the constitution of the parliament is when the seat allocation is based on a largest remainder method (for example the Hare quota or the Droop quota method). In such methods, the seat allocation is determined by a quota and the so called remainder, both of which depend on the number of the total valid votes. Since blank votes are considered valid, they may affect both which party obtains the first seats (i.e. the ones allocated according to the quota), and which party obtains the last seats in the parliament (i.e. the seats distributed according to the larger remainders). However, in this case, it is random whether blank votes harm the small or the big parties.

The fact that blank and null votes do not systematically affect the electoral outcome is a first good reason why blank and null voting have received very little attention by scholars. Furthermore, and from a practical point of view, according

¹Throughout the paper, we refer to abstention as the decision of an eligible voter not to participate.

to Power and Garand (2007) *“from a macro-political and comparative perspective, blank and null voting is notoriously difficult to interpret. Do high rates of blank and null voting portend citizen dissatisfaction with the political system? Do blank and null votes signify deficiencies of the electoral system or ballot structure? What about voter error, perhaps due to socioeconomic characteristics of the electorate?”*

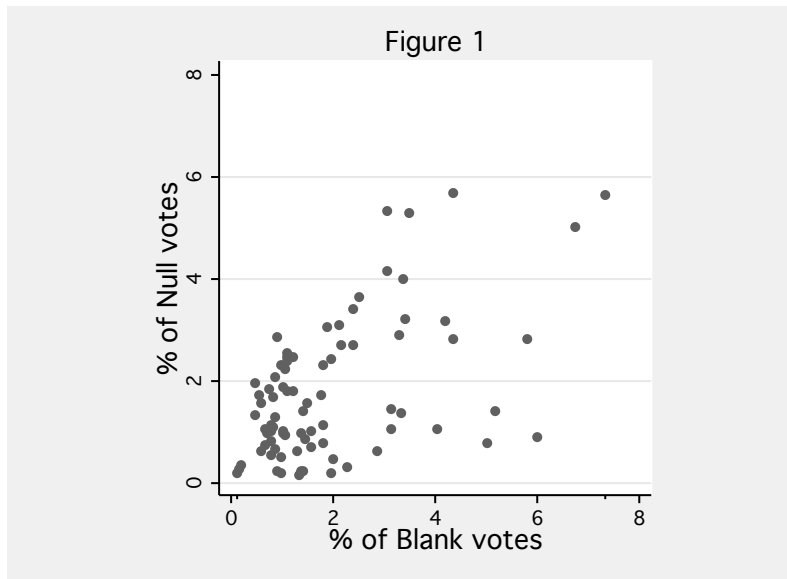
Recent empirical research (Power and Garand, 2007; Ugglå, 2008) has shed some light on the above questions. Existing approaches though answered the above questions analyzing invalid votes as a whole, where invalid votes are defined as the summation of blank and null votes. This is where our work differs from the existing literature by considering for the first time the analysis of blank and null votes in a separate basis.

A common assumption in the literature is that when voters are not interested in supporting any one of the candidates, they are indifferent between casting a blank or a null vote. A good justification of the this assumption, and as explained above, is that both blank and null votes do not affect the electoral outcome systematically. On top of that, this assumption allows an empirical analysis of a very broad sample, since many countries do not provide separate statistics for the percentages of blank and null votes. On the other hand, a drawback of this assumption, is that existing research does not explain why in some elections the percentages of blank votes are much higher than the percentages of null votes and vice versa.

For our purposes, we constructed a database considering blank and null votes as two different variables. The database includes 80 legislative lower-house elections that took place in 14 countries, during the period 1980-2007². Observing the percentages of blank and null votes in Figure 1, we claim that although there exists a positive correlation between the two, the way blank and null voting has been approached deserves a closer look³. This is because in many elections the percentage of blank votes is much higher than the percentage of null votes and vice versa. Thus, we aim to study empirically what are the different factors that may affect blank but not null voting and vice versa.

²Our initial target was a much broader sample (the countries used in Ugglå (2008)). Unfortunately, searching on-line and contacting the electoral authorities we realized that most countries do not offer the statistics separately. For the time being, our sample can not be further extended.

³For a better graphic representation we depict all observations that are lower than 8%.



Having in mind that our main goal is to disentangle the two dimensions of invalid voting, we agree with Stiefbold (1965) that there exist two separate categories of invalid voters. *“The first category is the “apathetic” invalid voters, who remain indifferent to the political system. The second category is the “highly politicized” invalid voters, who know exactly how they would vote if they could find the party corresponding to their ideas, but who, not finding that party, deliberately invalidate their ballots”.*

Our main hypothesis is that the *apathetic* invalid is expressed by a null vote, while the *highly politicized* invalid is expressed by a blank vote. Being more precise, we hypothesize that null votes cast deliberately, are cast by voters that are indifferent to the political system. On the contrary, we hypothesize that blank votes are cast by voters that are unwilling to support any of the candidates. The reason why we hypothesize that a null vote is *against* the political system, is because a null ballot is not conforming with the legal voting procedure. On the contrary, a blank vote can not be *against* the political system, since a blank vote is in line with the voting procedure. According to the electoral law though, a blank ballot is *against* the parties. When available to voters, a blank vote is a valid vote not offering support to any of the competing candidates.

In order to test the above hypothesis, our empirical approach builds on Powell (1982), who studies the impact of institutional, socioeconomic and political factors

on turnout. In order to have a complete picture of voters' decision not to vote for any of the candidates, parallel to the decision to cast a blank or a null vote we include in our analysis the decision to abstain.

Institutional Approach: This approach considers that invalid voting and abstention is a function of the institutional design. A typical example of this category is compulsory voting. In countries where participation is obligatory, abstention is lower and voters who would possibly abstain either because of alienation or indifference are obliged to participate and in that case cast an invalid ballot. Other examples are district magnitude, electoral system, electoral disproportionality, complexity of filling the ballot etc.

Socioeconomic Approach: This approach aims to capture how invalid ballots and abstention are affected by the socioeconomic characteristics of the country or the regions studied. Examples of variables that have been used are wealth of the country, development, urbanization, education and literacy, number of immigrants, social inequality etc.

Political (Protest) Approach: This approach captures how invalid ballots and the level of abstention are affected by the characteristic of the electoral competition and the political situation at the moment of the election. Variables belonging in this approach are the closeness of the election, the development and economic performance of the country during the pre-election period, any changes in levels of democratization, etc.

Empirically, we show that blank and null votes are not affected by the same factors. In particular the presence of an open list has a significant effect only on null votes while the number of strikes affects only the percentages of blank votes. The same holds for compulsory voting since it increases the level of blank votes while it has no significant effect on the level of null votes. Only two of our variables, the disproportionality of the electoral system and repression of political rights affect both variables in the same way.

In order to motivate theoretically our main hypothesis and understand what are the common and different features of abstaining, casting a blank or a null vote we build a theoretical model where voters perceive a consumption benefit of voting (*a la* Riker and Ordeshook (1968)) for casting a blank or null ballot. We provide conditions under which an individual may vote for a candidate, abstain, cast a

blank or a null ballot. We actually show that the decisions to abstain, vote blank or vote null are associated with voters that are either indifferent among parties, alienated by parties or both simultaneously.

The paper is organized as follows. In section two we present the existing literature and discuss our contribution to it. In section three we motivate theoretically why a voter may cast a blank or a null vote. In section four we perform our empirical analysis and in section five we conclude. In the Appendix we provide descriptive statistics and sources of our data.

4.2 Contribution to the Literature

Invalid voting literature is mainly empirical and consists of two groups of papers. Earlier approaches studying intra-country variation of invalid voting (Rosenthal and Sen, 1973; McAllister and Makkai, 1993; Power and Roberts, 1995; Zulfikarpasic, 2001) and more recent ones studying cross-country variation of invalid voting (Power and Garand, 2007; Uggla, 2008).

Regarding single country analyses one of the first contributions is by McAllister and Makkai (1993) for the case of Australia. Their results support the social structural hypothesis and in particular, invalid votes are explained by the number of immigrants, since the latter have poor language skills and higher probability to cast a null ballot by mistake. In a similar analysis for the case of Brazil by Power and Roberts (1995), the authors show that a model incorporating political, socioeconomic, and institutional factors is more powerful than a model relying on any one of these alone⁴. In the case of provincial elections in Ontario, Galatas (2008) shows that both demographic characteristics and party competition provide an explanation for casting blank ballots in a way consistent with expectations of rational choice theory.

Regarding cross-country evidence, Power and Garand (2007) focus on Latin American countries and show that variables of all the three approaches shape the rates of invalid voting. Variables included in the analysis with a significant effect are urbanization, income inequality, compulsory voting, electoral disproportion-

⁴The institutional approach for intra-country approaches refers to the characteristics of the electoral system that vary within the country (e.g. district magnitude).

ality, political violence and the level and direction of democratic change. Ugglå (2008) considers an extended panel, and parallel to invalid voting he analyzes voting for parties that do not reach the threshold required for representation in the parliament. Ugglå concludes that *“although at first glance, invalid voting appears irrational and politically inconsequential, it may in fact reflect political logic”*.

Our empirical approach falls in the category of papers providing cross-country evidence, and the main contribution in the invalid literature, is that we study for the first time blank and null votes on a separate basis.

Rosenthal and Sen (1973) are the only authors providing a theoretical model of voters participation decision. As in our setup, they use a spatial model in order to explain blank votes either because of alienation or because of indifference but they do not aim to approach blank and a null ballots on a separate basis.

4.3 Why Voters Cast a Blank or a Null Vote?

According to the well-known paradox of voting (Downs, 1957) voters should not be willing to pay the cost of voting and participate in large elections, since the probability of affecting the outcome is tiny. In their seminal work, Riker and Ordeshook (1968) (R&O) assume that voters obtain a consumption benefit when participating in large elections, and hence offer an explanation of why voters may do so. R&O actually interpret the consumption benefit of voting as *“the satisfaction from compliance with the ethic of voting, the satisfaction from affirming allegiance to the political system, the satisfaction of affirming one’s efficacy in the political system, the satisfaction of deciding, going to the polls, etc”*. When the benefit is high enough such that it compensates the cost of voting, it turns out that the decision of a voter to participate is rational.

In a similar spirit we adapt R&O’s approach to explain why a voter decides to cast a blank or a null ballot. Departing from the standard approach we allow the consumption benefit of voting to vary not only among voters but also for each voter to vary among the available choices. In particular, each voter may perceive a different benefit for casting a null vote, a blank vote or a vote for one of the candidates. A voter casts a blank or a null vote, when he obtains a high

consumption benefit of voting, and performing his calculus wants to participate but is not attracted by any of the competing candidates.

We assume a parliamentary election with two parties $j = L, R$. Each party is characterized by its ideal policy $x_j \in [0, 1]$ and a valence characteristic $\theta_j \in [0, 1]$.

For each voter i , the available strategies are $s \in \{j, B, N, A\}$ and for each voter each strategy is associated with a consumption benefit of voting (d_i^s). Each voter is characterized by her ideal point (x_i) and her personal cost of voting (c_i).

Voters evaluate both policy (x_j) and non-policy characteristics (θ_j) of each party. The evaluation of party j by voter i is given by: $u_i^j = \theta_j - x_j - x_i$.

The total reward that a voter obtains when following strategy s depends on the consumption benefit of voting associated with the strategy s (i.e. d_i^s), on the probability of determining the winner of the election (we refer to this as the instrumental dimension of a voter's decision), and also on supporting a "good" party in terms of policy and non-policy characteristics (we refer to this as the expressive dimension of a voter's decision). We assume that voter i puts weight $\alpha_i \in [0, 1]$ in the instrumental dimension (i.e. $(u_i^j - u_i^{-j})$ for all j), and weight $(1 - \alpha_i)$ in the expressive dimension (i.e. u_i^j for all j). Thus, $\alpha_i = 0$ describes a voter deciding in a purely expressive way, while $\alpha_i = 1$ describes a voter deciding in a purely instrumental way.

The following equation represents the above described reward of individual i from following strategy s :

$$R_i(s) = \begin{cases} \alpha_i p_i (u_i^L - u_i^R) + (1 - \alpha_i) u_i^L + d_i - c_i & \text{if } s = L \\ \alpha_i p_i (u_i^R - u_i^L) + (1 - \alpha_i) u_i^R + d_i - c_i & \text{if } s = R \\ d_i^B - c_i & \text{if } s = B \\ d_i^N - c_i & \text{if } s = N \\ 0 & \text{if } s = A \end{cases}$$

where p_i is the probability that the voter will break or create a tie between the two parties. Having in mind the above specification we can perform the calculus of individual i regarding the decision to abstain, vote blank or vote null.

In order for a voter to abstain the following three conditions have to hold simultaneously:

1. $\alpha_i p_i (u_i^j - u_i^{-j}) + (1 - \alpha_i) u_i^j + d_i^j - c_i < 0$ for both j : The probability of affecting the outcome (p_i), the corresponding utility difference if one party

wins versus the other ($u_i^j - u_i^{-j}$), the evaluation of the policy and non-policy characteristics of the parties (u_i^j), and the benefit of voting (d_i) do not compensate the cost of participating in the election (c_i).

2. $d_i^B - c_i < 0$: The benefit from voting blank is not high enough to compensate the cost of participating in the election.
3. $d_i^N - c_i < 0$: The benefit from voting null is not high enough to compensate the cost of participating in the election.

Given the first condition, abstention is increasing in the cost of voting (c_i), decreasing in the benefit of voting (d_i^j), decreasing in the probability of affecting the outcome (p_i), decreasing in the difference of policy and quality characteristics of the two parties ($u_i^j - u_i^{-j}$) and decreasing in the evaluation of parties by voters (i.e. u_i^j). Finally, given the second and third conditions abstention is decreasing in the consumption benefit voters obtain for voting blank or null (i.e. d_i^B and d_i^N).

In the literature, the two most common justifications of voters' decision to abstain are that voters are alienated from parties or indifferent among them. Notice that the predictions of our model are compatible with both explanations. The possible indifference of a voter is captured by the ($u_i^j - u_i^{-j}$) term. When this difference is small it implies that the voter is relatively indifferent among the parties and hence the probability to abstain is higher. The alienation of a voter is captured by low evaluations for all parties (i.e. low u_i^j for all j). Since the probability to abstain is decreasing in u_i^j , according to our model alienated voters, decide to abstain with higher probability.

Regarding blank voting, the conditions such that a voter casts a blank ballot are the following:

1. $\alpha_i p_i (u_i^j - u_i^{-j}) + (1 - \alpha_i) u_i^j + d_i < d_i^B$ for both j
2. $d_i^B - c_i > 0$
3. $d_i^B > d_i^N$

Similarly, regarding null voting, the conditions such that a voter casts a blank ballot are the following:

1. $\alpha_i p_i (u_i^j - u_i^{-j}) + (1 - \alpha_i) u_i^j + d_i < d_i^N$ for both j
2. $d_i^N - c_i > 0$
3. $d_i^N > d_i^B$

The first condition is very similar both for the decision to cast a blank and a null vote. Both blank and null votes are decreasing in the probability of affecting the outcome (p_i), decreasing in the difference between the evaluations of the two parties (i.e. $u_i^j - u_i^{-j}$ for all j) and decreasing in the evaluation of parties by voters (i.e. u_i^j for all j). As in the case of abstention, blank and null votes are associated both with alienated and indifferent voters.

Given the second and third conditions both blank and null votes are decreasing in c_i . Blank votes are increasing in d_i^B and decreasing in d_i^N while null votes are increasing in d_i^N and decreasing in d_i^B .

Notice that the conditions for a voter to cast a blank or a null ballot are very similar. In both cases voters are not willing to support any of the parties, and what determines whether an individual casts a blank or a null ballot is whether $d_i^B > d_i^N$ or $d_i^N > d_i^B$. Hence, the relevant question that we try to answer by our empirical analysis is the following: When is it the case that $d_i^B > d_i^N$ or vice versa? If we succeed in providing an answer to this question then we may have an answer to questions such as: *Do high rates of blank and null voting portend citizen dissatisfaction with the political system? Do blank and null votes signify deficiencies of the electoral system or ballot structure? Do high rates of blank and null voting portend citizen dissatisfaction with the competing parties?*

By definition and according to the electoral law, a blank ballot is a vote for none of the competing candidates and in line with the voting procedure. On the contrary, a null ballot is a vote not conforming with the rules of the voting procedure. If a blank and a null vote represent different strategies and indeed $d_i^B = d_i^N$ then our hypothesis is as follows: Voter i perceives benefit d_i^N that is higher than d_i^B when he is not willing to approve the voting procedure and the democratic institutions. Reinterpreting R&O we hypothesize that a high d_i^N is associated with *the satisfaction from non-compliance with the ethic of voting, the satisfaction from not affirming allegiance to the political system, the satisfaction of*

not affirming one's efficacy in the political system, the dissatisfaction of deciding, going to the polls, etc.

4.4 Empirical Evidence

4.4.1 Data and The Model

Our database consists of 80 lower house parliamentary elections in 14 countries, for the period 1980-2007⁵. The sample is relatively small but can not be further extended since the rest of the countries analyzed in the cross-country literature (Power and Garand, 2007; Ugglá, 2008) do not provide separate statistics for blank and null votes. Detailed descriptive statistics of the dependent variables and sources of our data can be found in the Appendix.

We follow the empirical literature by estimating a model using variables of the three defined approaches. We suggest that the four dependent variables (i.e. invalid, blank, null votes and abstention) are a function of institutional, socioeconomic and political variables. The main question we try to answer, is whether there exist a set of independent variables that may affect blank but not null votes and vice versa. As we have explained in the theory the two decisions are distinct and in order for a voter to choose among the two he must obtain a higher consumption benefit of voting one versus the other. Estimating the model, we will be able to see whether different factors influence each decision or not. The suggested model is the following:

$$y = \beta_0 + \beta_1 \text{Compulsory Voting} + \beta_2 \text{Disproportionality} + \beta_3 \text{Open List} + \beta_4 \text{Real GDP} + \beta_5 \text{Democracy Level} + \beta_6 \text{Closeness} + \beta_7 \text{Change in Democracy} + \beta_8 \text{Strikes} + \beta_9 \text{Growth}$$

where y in the four regressions of interest is the percentage of invalid, blank, null votes and abstention. Following Powell's classification, the institutional approach is captured by the three first independent variables (compulsory voting, disproportionality and open list), the socioeconomic approach is captured by the

⁵The countries that provide separate statistics for blank and null votes are: *Argentina, Bolivia, Chile, Costa Rica, Honduras, Iceland, Italy, Netherlands, Paraguay, Peru, Portugal, Spain, Switzerland, Uruguay*

following two (real GDP and democracy level), while the political protest approach is captured by the last four (closeness of the election, changes in levels of democracy, number of strikes and growth rate).

Through the institutional variables we aim to answer how institutions affect blank and null voting. Through the socioeconomic variables we focus on characteristics of the society, while through the political protest variables we study how the characteristics of the election and parties' actions shape blank and null voting.

Institutional Approach

The variables belonging in this approach are meant to answer in what ways institutions affect blank and null voting. This approach includes all those variables that are not related with parties' actions and the characteristics of the society but they will rather give us an answer on how voters react to the characteristics of the institutions. Out of the set of all institutional variables we include in our analysis the following three: compulsory voting, disproportionality and the presence of an open list. We believe that those three variables can in general offer a broad description of the institutions and in particular the electoral law.

Compulsory Voting: The variable is provided by the International Institute for Democracy and Electoral Assistance (IDEA) and measures not only presence but also enforcement of compulsory voting. This variable takes value zero in case of non-compulsory voting and value three for the case of compulsory voting with strict enforcement. In our sample in five of the countries voting is voluntarily while in the remaining nine some kind of enforcement applies.

Given that compulsory voting brings to the urns among all voters some that under non-compulsory voting would have abstained there is evidence that compulsory voting has a positive effect on invalid votes (Power and Garand, 2007). We expect that compulsory voting has a positive effect on the percentage of blank votes since voters not willing to vote for any of the candidates either from alienation (i.e. low u_i^j for all j) or from indifference (i.e. low $u_i^j - u_i^{-j}$ for all j) can not any longer abstain.

We expect that compulsory voting has a positive effect on null ballots if voters who are not satisfied by the electoral procedure (i.e. $d_i^N > d_i^B$) and in particular with the obligation to vote are forced to participate.

Regarding abstention, we expect that the stricter is the enforcement for a voter when he does not participate, the lower should be the level of abstention. This pattern is the most obvious and prevailing among empirical studies (Fornos et al., 2004; Blais and Dobrzynska, 1998; Franklin, 1999; Jackman, 1987). Hence, we expect that compulsory voting has a negative effect on the level of abstention.

Disproportionality: In order to measure the disproportionality of the electoral system we construct a disproportionality index following Lijphart and Aitkin (1994). The index measures the distortion between seat allocation and vote share obtained for the first two parties in the election. High values of the index are associated with disproportional electoral systems. In general, disproportional electoral systems tend to harm smaller parties.

We expect a more disproportional system to be correlated with more blank votes. Given the small number of parties competing, voters that perceive a high consumption benefit of voting and are willing to participate in the election, may not be able to find any attractive alternatives among the competing candidates and hence decide to cast a blank ballot.

As in the case of compulsory voting, if voters want to show their dissatisfaction towards the electoral procedure (i.e. $d_i^N > d_i^B$) and in particular the disproportionality of the electoral system then we expect a positive effect of this variable on null votes.

We expect that disproportionality has a positive effect on abstention. This is the case because more disproportional systems are associated with less parties competing in the election and hence less available alternatives for voters (Jackman, 1987; Franklin, 1999; Blais and Dobrzynska, 1998).

Open List: Is a dummy variable taking value one for electoral systems that allow voters to choose the ranking of candidates, approve a subset of them. etc. In general it allows voters to manipulate the ballot and voting becomes more complicated than casting a closed ballot. Clearly in countries using an open list the expected effect should be a higher percentage of null votes because of the higher probability of committing an error.

Regarding blank votes and abstention, this variable should have a negative on both. This is because in case of an open list voters are offered more alternatives among which they have to choose. Moreover, voters are involved in a more active

way in the electoral procedure than when they have to vote for a predetermined list of candidates. The latter may imply higher benefits of voting and hence higher participation in the case of an open list than in the case of a closed list .

Socioeconomic Approach

The two variables of this approach are included in our analysis in order to explain how the characteristics of the economy (real GDP) and the society (level of democracy) may affect blank and null votes. These two variables describe the economic and political situation of the country not as a result of parties actions.

Real GDP: Income per capita is the first socioeconomic variable. According to the empirical evidence turnout is higher in more economically developed countries (Blais and Dobrzynska, 1998; Norris, 2002; Fornos et al., 2004; Powell, 1982). We expect real GDP to have a negative effect on abstention if high levels of GDP are associated with more active citizens (and hence on average higher b_i^j).

Moreover, we expect that higher GDP per capita is associated with less null ballots cast by mistake. This is because on average, in rich countries, voters competency is higher than in poor countries (Power and Garand, 2007).

Level of Democracy: As defined by the Freedom House this variable takes value one for high levels of democracy and value seven for autocratic regimes.

We expect more null votes in more autocratic countries. The reasoning is that in more autocratic regimes voters not voting for any of the candidates are probably willing to show their dissatisfaction towards the autocratic regime and institutions through a null ballot rather than showing dissatisfaction towards the parties through a blank ballot (since $d_i^N > d_i^B$). If so, the level of democracy should not affect blank voting.

Regarding abstention, we expect that in more democratic regimes, voters participate with higher probability. This is because we expect that the consumption benefit of voting (b_i^j) in case of democratic regimes is higher than in autocratic regimes.

Political Protest Approach

The variables of this approach are included in our analysis in order to capture the effect of the characteristics of the election itself (closeness of the election) and the effect of parties actions (changes in level of democracy, number of strikes

and growth rate) on blank and null votes. The closeness of the election offers a proxy of the probability that a single vote may be decisive for the outcome of the election, while the other three variables are included in order to capture the (dis)satisfaction of voters with parties' actions.

Closeness: This variable measures the competitiveness of the election calculated as the difference between the first and second party in the election. This variable is associated with the p_i term of our theoretical motivation. The closer is the election, the higher is the actual probability of affecting the outcome of the election. If voters have rational expectations then the closeness of the election can be used as a proxy for the p_i term⁶. According to the predictions of our model, the lower is p_i the higher is the probability that a voter abstains, casts a null or blank ballot. Hence, we expect a positive effect of this variable on all independent variables.

Change in Democracy: This variable measures the change of political rights. Positive values are associated with a repression of political rights that during the period of the last government. Power and Garand (2007) actually show that when parties repress voters' rights then there is a significant effect on the level of invalid votes. According to our hypothesis, if a repression of political rights is associated with dissatisfaction towards the parties (i.e. low u_i^j for all j) then this variable should have a positive effect on the level of blank votes. On the other hand, if voters show their dissatisfaction towards then (un)democratic institutions (i.e. $d_i^N > d_i^B$) then we expect a positive effect on the percentages of null votes.

Strikes: This variable measures the number of strikes during the last two years. It is used in our model as a proxy of disapproval towards the parties (i.e. low u_i^j)⁷. We use this variable in a similar way that Power and Garand (2007) show that political violence has a positive effect on invalid ballots.

Since this variable captures dissatisfaction towards the political parties, we

⁶Notice that we proxy the ex-ante probability of being pivotal by the actual electoral outcome. This is a common feature of the empirical literature. We actually assume that voters have a presentiment of the outcome of the election (e.g. through opinion polls) and form expectations accordingly.

⁷We are aware of the fact that the number of strikes may be associated only with dissatisfaction towards rightist governments. Unfortunately, this is the only variable that we could find available for all elections and years of our database that can capture the notion of satisfaction.

expect a positive effect of the number of strikes on the level of abstention and blank votes. If our hypothesis is correct and null votes are used to show dissatisfaction towards the democratic institutions then we should not expect a significant effect of this variable on the percentage of null votes.

Growth: This variable is computed as the annual change in per capita GDP. We use this variable as a proxy of economic performance as in the approach by Power and Garand (2007). The expected effect of this variable is the same as of the number of strikes. We expect that high rates of growth are associated with high evaluations of parties by voters (i.e. high u_i^j). Hence, we expect that this variable has a negative effect on the level of blank votes and abstention. It will have no significant effect on the level of null votes, given that according to our hypothesis the latter are used as disapproval of the democratic institutions.

4.4.2 Empirical Results

A natural question is why the model we estimate combines variables of all the three approaches rather than studying the effect of each approach on each variable of interest in a separate regression for each approach. We have performed this task, and concluded that a model combining the three approaches, is associated with much higher goodness of fit (in terms of adjusted R^2) rather than a model focusing on the effect of each approach on each own, both for null and blank votes.

Most important, we observed joint significance of all the independent variables, in each of the three approaches when analyzed separately, for the case of blank votes. Hence, all the three approaches have predictive power on blank votes. This is not true though in the case of null votes. Only institutional variables had significant predictive power when studied separately. This result, gives us a first hint that blank and null votes are not affected by the same factors.

Table 1 summarizes the results of our empirical analysis. In the first column the dependent variable is invalid votes (i.e. the summation of blank and null votes). In the second and third columns, the dependent variables are blank and null votes respectively, while in the fourth column the dependent variable is abstention.

As we can see from the first column four variables have a significant effect on the percentage of invalid votes. Among the institutional variables, disproportion-

Table 4.1: Invalid, Blank, Null Votes and Abstention

Variable	Invalid	Blank	Null	Abstention
Compulsory	0.332 1.469	0.820 0.397	-0.487 1.242	-11.032** 3.220
Disproportionality	0.665* 0.280	0.335 0.163	0.331 0.160	0.626 0.618
Open List	7.619 3.835	1.398 1.037	6.221* 2.845	11.841 8.850
Real GDP per capita	-4.813 4.433	-1.067 1.720	-3.746 3.154	-16.153* 6.612
Level of Democracy	2.757 1.784	0.501 0.499	2.255 1.589	5.728* 1.949
Closeness	0.183 0.104	-0.023 0.021	0.205 0.111	-0.114 0.173
Change in Democracy	2.940 1.423	0.862 0.445	2.078 1.072	0.060 1.547
Strikes	0.736* 0.333	0.395** 0.109	0.341 0.259	-0.442 0.744
Growth	0.043 0.101	-0.094 0.085	0.137 0.125	0.012 0.256
Intercept	12.896 16.636	3.361 6.658	9.535 12.630	88.811** 25.252
N	80	80	80	80
R ²	0.688	0.677	0.611	0.374
R ² <i>adj</i>	0.648	0.635	0.561	0.294
F _(9 13)	5.479	26.528	9.904	24.059
Prob > F	0.003	0.000	0.000	0.000

Note: Our model is estimated using Robust OLS clustering for countries because of the presence of heteroskedasticity. Standard errors appear in parentheses.

Significance levels : : 10% * : 5% ** : 1%

ality and an open list have a positive significant effect (at 5% and 10% significance level). The sign is the one expected and is line with the approach by Power and Garand (2007). The socioeconomic variables have no significant effect as in the case of Ugglå (2008). Two of the variables of the political protest approach have a significant effect and is the expected one. More specifically, policies repressing political rights and higher number of strikes are associated with higher levels of invalid votes (significant at 10 and 5% respectively).

Regarding blank votes, four variables have a significant effect and in all cases the sign is the expected one. Out of the institutional variables, compulsory voting has a positive effect (at 10% significance level) and so does the disproportionality index. As expected, compulsory voting brings to the urns many voters that obtain low consumption benefit of voting, are not willing to support any party and under voluntarily voting would have abstained. These voters under compulsory voting, once obliged to participate, decide to cast a blank vote. Moreover, disproportional electoral systems are associated with more blank votes. A possible justification may be that the more disproportional the electoral system is, the less are the competing parties. As a consequence, in disproportional electoral systems, voters that obtain a high consumption benefit of voting, are not able to find a party that expresses their ideology, and hence decide to participate and cast a blank vote.

Out of the political approach, switches to more autocratic regimes and the number of strikes have a positive effect (at 10 and 1% respectively) on blank votes. In both cases the intuition is the same: dissatisfaction of voters towards political parties is associated with higher levels of blank votes.

Regarding null votes, four variables have a significant effect and all signs are the expected ones. Out of the institutional variables, disproportionality and an open list have a positive effect (significant at 10 and 5% level respectively). Regarding the political variables, the percentage of null votes is higher in less competitive elections (significant at 10% level) while switches to a more autocratic regime are associated with more null votes as well (significant at 10% level).

Interpreting the results regarding null votes, an open list is associated with more null votes since the probability that a voter make an error is higher when he is allowed to manipulate the ballot. In line with our main hypothesis, switches

to a more autocratic regime and disproportional electoral systems are associated with more null votes as a result of voters dissatisfaction towards the democratic and electoral institutions (i.e. $d_i^N > d_i^B$).

In all three regressions (invalid, null, blank) we observe very good fit of our model to the data (in terms of R^2 and adjusted R^2). In particular, 68% of the variance in blank votes, and 61% of the variation of null votes is explained by the independent variables of our model. In both cases the independent variables are jointly significant (i.e. $prob < 0.000$).

Interpreting our results from a comparative perspective, we observe that when disentangling the two dimensions of invalid voting different variables have a significant effect on blank and null voting. Notice that while compulsory voting has a significant effect on invalid votes, the effect comes only from the positive effect on blank votes, since it has no significant effect on the level of null votes. In the same way, only blank votes are increasing in the number of strikes.

The only variable that has a significant effect on null but not on blank votes is the presence of an open-list. This is because an open list captures the null votes cast by mistake. Contrary to our expectations, an open list does not reduce blank votes. Our prior that voters have the possibility to choose among more alternatives, and hence cast less blank votes is not confirmed. Finally, two variables, disproportionality and changes in levels of democracy, have the same (significant) effect on both blank and null voting.

The most important interpretation of the above results is that information is lost when the blank and null votes are assumed as the same political action and one considers only the case of invalid votes. According to our results, under compulsory voting, when voters are obliged to participate they do not show their dissatisfaction towards the obligation to vote by casting null ballots, they rather show their dissatisfaction towards parties through blank voting. In the same spirit when voters are not satisfied by parties policies (as proxied by the number of strikes), voters show their dissatisfaction only towards the parties through blank voting. This is not the case though when it comes to repression of democratic freedom, since after a switch towards a more autocratic regime voters show their dissatisfaction both by voting blank and null.

Our model is not as successful in explaining the variation of abstention, as it

is for the case of blank and null voting⁸. Nevertheless, we believe that the results of this regression are important. This part of our analysis provides a robustness check of the analysis of blank and null voting. Despite the restricted sample we study, the results obtained regarding abstention are in line with the much developed empirical literature. We believe that this gives more credit to the main results of our analysis, and the conclusion that blank and null ballots are not affected by the same factors.

In particular, abstention is lower in richer and more democratic countries (at 5% sign. level). A straightforward explanation is that voters in rich and highly democratic countries are more politically active, than voters in poor and autocratic countries. In line with one of the best established empirical regularities, compulsory voting has a negative effect on abstention (at 1% significance level). Under compulsory voting, voters participate with higher probability in order to avoid any sanctions.

4.5 Conclusion

In this paper we extend the restricted empirical evidence of the vision of invalid voting as a political protest action. Moreover, we make a step forward, by showing that different factors affect the decisions to cast a blank or a null ballot, and it turns out that information is lost when invalid voting is considered as the summation of null and blank votes. The kind of questions we aimed to answer were: *Do high rates of blank and null voting portend citizen dissatisfaction with the political system? Do blank and null votes signify deficiencies of the electoral system or ballot structure? Do high rates of blank and null voting portend citizen dissatisfaction with the competing parties?* After studying blank and null votes on a separate basis we conclude that null votes are definitely associated with the ballot structure. Dissatisfaction with the competing parties is associated with blank votes, while dissatisfaction towards the electoral system and democratic institutions is associated for different reasons, both with blank and null votes.

An important policy implication of our analysis and a lesson for electoral

⁸Only 37% of the variation of abstention is explained by the independent variables.

system designers stems from the following result: Compulsory voting has a positive significant effect on the level of blank votes, but has no significant effect on the level of null votes.

This results goes against an important criticism of compulsory voting. Opponents of it, claim that among other issues, compulsory voting is a bad idea, since it increases the amount of votes that convey no information. If compulsory voting was associated with more null votes, and given that null votes can always be attributed to votes cast erroneously, this criticism would have been fair. On the contrary, it turns out that compulsory voting is associated only with more blank votes, which are always cast deliberately, can not be misjudged, and actually convey a doubtless message of voters' disapproval of all competing candidates. Consequently, compulsory voting does not increase the amount of votes containing no information. It rather brings to the urns alienated and indifferent voters, who are not willing to support any of the competing parties, and once not allowed to abstain, express their disapproval by casting blank votes.

4.6 Appendix

Descriptive Statistics on Blank, Null ballots and Abstention, by country 1980-2007

Source: Electoral Authorities of each country & IPU database

Country	Variable	# Obs.	Mean	Std. Dev.	Min.	Max.
Argentina	Blank	10	3.96	2.17	1.31	8.2
	Null	10	2.1	3.81	0.47	12.9
	Abstention	10	18.39	3.57	14.39	26.3
Bolivia	Blank	6	4.15	1.77	2.15	7.34
	Null	6	3.95	1.4	2.69	5.67
	Abstention	6	24.09	5.78	15.49	28.64
Chile	Blank	4	3.54	0.48	3.07	4.2
	Null	4	8.35	3.91	5.3	13.5
	Abstention	4	11.8	2.12	8.7	13.4
Costa Rica	Blank	7	1.06	0.1	0.88	1.21
	Null	7	2.25	0.25	1.81	2.52
	Abstention	7	24.66	7.09	18.18	34.8
Honduras	Blank	5	2.55	1.88	1.23	5.79
	Null	5	2.42	0.89	1.42	3.64
	Abstention	5	26.93	8.01	15.96	35
Iceland	Blank	7	1.47	0.5	0.92	2.29
	Null	7	0.21	0.04	0.16	0.29
	Abstention	7	13.03	2.31	9.9	16.38
Italy	Blank	6	2.64	1.13	1.12	4.21
	Null	6	3.08	0.75	1.8	4.15
	Abstention	6	14.98	2.85	11.17	18.62
Netherlands	Blank	3	0.16	0.05	0.11	0.2
	Null	3	0.27	0.08	0.19	0.34
	Abstention	3	15.41	3.19	12.97	19.02
Paraguay	Blank	3	1.45	0.4	1.03	1.82
	Null	3	1.9	0.37	1.55	2.29
	Abstention	3	28.71	8.36	19.46	35.71
Peru	Blank	6	9.49	3.62	5.89	13.77
	Null	6	16.54	12.29	5.01	40.51
	Abstention	6	22.6	9.5	11.3	36.6
Portugal	Blank	6	1.04	0.4	0.74	1.8
	Null	6	1.19	0.33	0.95	1.83
	Abstention	6	33.53	6.13	22.21	38.91
Spain	Blank	7	0.96	0.45	0.47	1.58
	Null	7	1	0.56	0.5	1.95
	Abstention	7	25.94	4.36	20.03	31.29
Switzerland	Blank	6	0.72	0.14	0.47	0.86
	Null	6	0.97	0.23	0.65	1.33
	Abstention	6	54.35	2.66	51.1	57.8
Uruguay	Blank	4	2.22	1.18	1.02	3.34
	Null	4	1.09	0.18	0.96	1.35
	Abstention	4	9.66	1.44	8.3	11.33
All Countries	Blank	80	2.66	2.7	0.11	13.77
	Null	80	3.2	5.45	0.16	40.51
	Abstention	80	23.49	12	8.3	57.8

Bibliography

- BLAIS, A. AND A. DOBRZYNSKA (1998): Turnout in electoral democracies, *European Journal of Political Research*, 33, 239–262.
- DOWNES, A. (1957): An economic theory of political action in a democracy, *The Journal of Political Economy*, 65, 135–150.
- FORNOS, C., T. POWER, AND J. GARAND (2004): Explaining voter turnout in Latin America, 1980 to 2000, *Comparative Political Studies*, 37, 909.
- FRANKLIN, M. (1999): Electoral engineering and cross-national turnout differences: what role for compulsory voting? *British Journal of Political Science*.
- GALATAS, S. (2008): None of the Above? Casting Blank Ballots in Ontario Provincial Elections, *Politics & Policy*, 36, 448–473.
- JACKMAN, R. (1987): Political institutions and voter turnout in the industrial democracies, *The American Political Science Review*, 405–423.
- LIJPHART, A. AND D. AITKIN (1994): *Electoral systems and party systems: A study of twenty-seven democracies, 1945-1990*, Oxford University Press, USA.
- MCALLISTER, I. AND T. MAKKAI (1993): Institutions, society or protest? Explaining invalid votes in Australian elections, *Electoral Studies*, 12, 23–40.
- NORRIS, P. (2002): *Democratic phoenix: Reinventing political activism*, Cambridge Univ Pr.
- POWELL, G. (1982): *Contemporary democracies: Participation, stability, and violence*, Harvard Univ Pr.
- POWER, T. AND J. GARAND (2007): Determinants of invalid voting in Latin America, *Electoral Studies*, 26, 432–444.
- POWER, T. AND J. ROBERTS (1995): Compulsory voting, invalid ballots, and abstention in Brazil, *Political Research Quarterly*, 48, 795.

BIBLIOGRAPHY

- RIKER, W. AND P. ORDESHOOK (1968): A Theory of the Calculus of Voting, *The American Political Science Review*, 62, 25–42.
- ROSENTHAL, H. AND S. SEN (1973): Electoral participation in the French fifth republic, *The American Political Science Review*, 67, 29–54.
- STIEFBOLD, R. (1965): The significance of void ballots in West German elections, *The American Political Science Review*, 59, 391–407.
- UGGLA, F. (2008): Incompetence, Alienation, or Calculation?: Explaining Levels of Invalid Ballots and Extra-Parliamentary Votes, *Comparative Political Studies*, 41, 1141–1164.
- ZULFIKARPASIC, A. (2001): Le vote blanc: abstention civique ou expression politique? *Revue française de science politique*, 51, 247–268.