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Abstract

Essays on Intersectoral Dynamics

My thesis aims at analyzing intersectoral dynamics through the lens of political economy and industrial organization. Since I treat technology innovation and policy determination as the equilibrium product of political processes and structural changes in the economy, I shed light on the engines and hurdles of economic development.

First, I analyze the effect of technology innovation on the relative price of the productive factors. As a consequence of a technological change, the productivity of a factor may increase even when its supply increases. I analyze the determinants of this technological bias. I present a general equilibrium model, where a good is produced in the final sector using both a factor and a technology, and the technology is produced in the intermediate sector. I allow for different market structures in the intermediate sector, and I prove that both competition and a variable set of technology producers may affect the occurrence of the technological bias, since they affect the necessary nonconvexities in the equilibrium allocation.

Second, I try to explain why the misallocation of resources across different productive sectors tends to persist over time. I document that there is a link between the distribution of the public expenditure across sectors and the sectoral composition of an economy. I propose a general equilibrium model that interprets this stylized fact as a reduced form representation of two structural relations, namely, the dynamic effect of the public expenditure on the future distribution of value added and the influence of the distribution of vested interests across sectors on current public policy decisions. The model predicts that different initial sectoral compositions cause different future streams of public expenditures and therefore different paces of development.

Third, I construct a theoretical model that encompasses both firms' and sectors' network structure by considering a lower-dimension economic unit, that is, sector-specific establishments of multi-sectoral firms. The model suggests a reduced-form relation where aggregate production is a function of all the idiosyncratic shocks filtered by the network structure of the economy. I show that aggregate fluctuations depend on the geometry and magnitude of cross-effects across establishments, which is measured by the eigenvalues and eigenvectors of the network matrix. Moreover, the equilibrium levels and their dispersion depend on the Bonacich centrality of establishments within the network structure of the economy. Different network structures entail different aggregate volatilities due to the fact that the presence of direct relations averages out the idiosyncrasies across establishments.

Contents

Chapter 1

Introduction

The process of structural change is tightly connected to economic development. As productive factors shift from traditional sectors to modern economic activities, so do economic and social interests. The migration of these interests influences the intensity of technological innovation and the support of public policy, thought in the development literature to be the engines of economic development. Hence, technology and policy are not the product of chance or good will but rather the equilibrium outcome of economic and political processes. The analysis of economic development should therefore treat technology and policy as endogenous.

My thesis aims at explaining certain aspects of intersectoral dynamics from the perspectives of political economy and industrial organization. In the following three chapters I use general equilibrium models to explain certain empirical puzzles of structural change and development. In fact, general equilibrium models permit the endogenous determination of technology and policy in the development process.

In Chapter 2, I study the relation between technology and factor supply. Firms that produce factor-specific technologies tend to concentrate where those factors are abundant. An increase in the factor supply may then cause an increase in the supply of the factor-specific technology, boosting the factor's productivity. This would cause the equilibrium factor price to increase when the factor supply increases. This phenomenon is called technological bias and it is a credible explanation for the historical increase in wage inequality among different skill levels and sectors. I analyze how changes in the way firms interact among them can affect the occurrence of the technological bias. The intuition behind is that the equilibrium factor price increases when the factor supply increases only if the technology production is profitable enough. Once firms' profits are low enough, technological change is not strong enough to dominate the standard decrease in marginal productivity.

I present a two-sector model of general equilibrium, where a good is produced in the final sector using both a factor and a technology. Firms produce technology in the intermediate sector. We provide three cases of market structure in the intermediate sector, i.e., a single monopolistic producer, oligopolists that compete in quantities, and variety-specific monopolistic producers. I show that the technological bias is connected to the existence of nonconvexities in the equilibrium surface, and that the market structure of the intermediate sector affects their occurrence. Moreover, I characterize the case of a variable number of technology producers that depends on the factor supply, and analyze its effect on the existence of technological bias. I find that competition is incompatible with technological bias and that, if the set of technology producers depends on the factor supply, the equilibrium factor price may increase even when the technological production is not particularly profitable.

In Chapter 3, I analyze policy determination and its relation to the sectoral composition of an economy. I try to explain why suboptimal allocations of public resources tend to persist over time. The Development Accounting literature has already documented that the differences in the sectoral composition of an economy can account for a relevant part of the wide differences in TFP across countries. In a nutshell, depending on whether the sectors are complementary or not, sectoral diversification or sectoral specialization increase aggregate efficiency. Nevertheless, it is still unclear why such misallocations of value added and public resources do not disappear over time or are at least very resistant to change, even among developed and democratic countries.

I checked therefore the Eurostat data on national accounts detailed by productive branches and I looked at the distribution of value added across different productive sectors and at the distribution of current public expenditure across sectors for European countries, and I noticed that the concentration of these distributions, for example measured by the Gini coefficients of these distributions, evolve together over time. Countries that concentrate their value added over time from 2000 to 2008 also concentrate their public expenditure, while countries that diversify their value added also diversify their public expenditure. Hence, I constructed a model that interprets this correlation as a reduced form representation of two structural relations. On the one hand, the distribution of public expenditure drives the future distribution of value added. On the other hand, the distribution of value added mirrors the distribution of vested interests in the economy, so that public expenditure tends to mirror the value added distribution at each point in time.

By connecting public policy to the sectoral composition of an economy, I am able to shed light on the reasons why misallocations of resources persist over time. New public policies that might increase future aggregate efficiency are actually politically unfeasible because the sectors that would lose shares in the government's budget due to these changes oppose the policy reforms, that is, the individuals working in the losing sectors oppose the reforms. The existence of political blockages of growth-enhancing reforms is not new in the literature. In fact, all the research efforts on the effects of institutions on growth focuses on the conflicting interests between the ruling elites and the rest of the population. What is new in my work is that by allowing for a proper voting process I obtain that even the majority of the population might oppose growth-enhancing reforms, so that even in perfectly functioning democratic societies it is natural to think of delays in the development path due to persistent suboptimal allocations of resources.

In Chapter 4, I look at how the transmission of shocks through the network of intersectoral linkages can originate aggregate fluctuations. The main mechanism through which the sectoral composition of an economy influences aggregate efficiency is the complementarity or substitutability of sector-specific value added in the composition of aggregate production. If the sectors are complementary enough, then diversification would increase aggregate efficiency, and if the sectors are substitutable then it's specialization that yields higher returns. Sectoral complementarity is though a complicated object to analyze, and its connection to the moments of aggregate production is much more complex than just the long-run effect on the level of aggregate output. In fact, the complementarity among sectors depends on the input-output structure of the economy, which consists of a network of sectors supplying intermediate goods to each other. The literature has already evidenced how aggregate efficiency and aggregate volatility are tightly connected to the network structure of the economy. In particular, there are two streams of literature that explain how idiosyncratic shocks can transmit to the aggregate level and cause fluctuations of the GDP. Normally, we would expect the law of large number to apply and the idiosyncratic shocks to smooth out at the aggregate. On the one hand though, idiosyncratic shocks to firms can transmit to the aggregate level due to the size distribution of firms, that is, identical shocks to big and small firms have very different consequences to the aggregate output. On the other hand, idiosyncratic shocks to sectors transmit to the aggregate level because there are some sectors that work as hubs to the economy, that is, they supply their intermediate goods to almost all the sectors of the economy. So these sectors amplify the effect of idiosyncratic shocks making them have an effect on aggregate production.

I construct a theoretical framework that embraces the volatility coming from both sectors and firms. Since shocks to sectors and firms alone explain a certain percentage of aggregate volatility, including both sources into a unified framework can potentially explain a greater part of aggregate fluctuations. The theoretical tool to do that is to consider lower-dimensional units, that is, the establishments of each firms. I consider each firm as a collection of sector-specific establishments, and sector production as the sum of the production of establishments of different firms. This set-up yields a network structure of the economy that encompasses both the network of sectors and the network of firms. I obtain a linear reduced form function that connects the network centrality of each establishment to its contribution to the aggregate volatility. From this there are two empirical implications. On the one hand, my model would contribute to the literature on the great moderation and its undoing, since a relevant part of aggregate volatility that we cannot explain with neither a better policy nor with a different transmission process could actually be explained by a change in the network structure of the economy. On the other hand, the matrix representation of the equilibrium production as a function of the idiosyncratic shocks filtered by the network structure of the economy permits me to identify the establishments that contribute the most to aggregate volatility, that is, the most path-central units of the economy. This is possible by computing the eigenvalues of the network matrix. So, if we had to choose which establishment to stabilize in order to reduce the most the aggregate volatility, my model tells that we should target the most path-central establishment of the economy that results from taking into account the network structure of both sectors and firm conglomerates.

Chapter 2

Market Structure, Nonconvexities, and Equilibrium Bias of Technology

As a consequence of a technological change, the productivity of a factor may increase even when its supply increases. In this paper we analyze the determinants of this technological bias. We present a general equilibrium model, where a good is produced in the final sector using both a factor and a technology, and the technology is produced in the intermediate sector. We allow for different market structures in the intermediate sector, and we prove that both competition and a variable set of technology producers may affect the occurrence of the technological bias, since they affect the necessary nonconvexities in the equilibrium allocation.

Keywords: Technological Bias, Market Structure, Nonconvexities.

2.1 Introduction

In this paper, we look at the determinants of the so-called technological bias in a general equilibrium set-up. We relate the existence of technological bias to the presence of nonconvexities in the equilibrium allocation as in Acemoglu [3]. We prove that the market structure of the sector where the technology is produced affects the equilibrium nonconvexity and is therefore a salient feature of the technological bias.

When innovations alter the ratio of marginal products of the different factors of an economy, the technology is said to be biased. Hicks [12], Solow [16], and Acemoglu [2] are, among others, seminal works on the subject and boosted a wide empirical literature on the existence of technological bias, e.g., Goldin and Katz [11] and Autor, Katz, and Krueger [6]. If the marginal product of a factor increases when its supply increases, then, the technology is said to be biased or directed towards that factor. The rationale behind this is the endogeneity of the technological change. A factor may need a technology produced in a certain sector in order to be operative. Hence, if the supply of such a factor increases, then the factor-specific technology is more profitable. This stimulates the production of such a technology, which in turn increases the factor's productivity. In a situation of perfect competition this causes an increase in the factor price in equilibrium. See, e.g., Acemoglu [1] for the case of skill-biased technical change and its effects on wage inequalities.

We define the technological bias as an equilibrium phenomenon, where an increase in a factor's supply causes an increase in its equilibrium price. The existence of technological bias is tightly connected to the presence of nonconvexities at equilibrium, that is, non-standard features of a general equilibrium allocation such as saddle points, kinks, discontinuities, and so on. We present a general equilibrium model of a two-sector economy, a final good sector and a technological sector.[1] The final sector employs factors and intermediate goods in the production of a final good. The intermediate goods embody a certain level of homogeneous technology, which is supplied by the firms in the technological sector.[2] We distinguish between three market structures of the technological sector. In the Benchmark Case, the technological sector is occupied by a monopolist producer that embeds the technology in the intermediate good used in the final good production. In the Cournot Case, the technological sector is populated by a finite number of firms that produce the same variety of intermediate good and compete à la Cournot among them. In the Dixit-Stiglitz Case, a continuum of intermediate monopolistic

[1]For a similar set-up, see Acemoglu [3].

[2]Results would hold even if we allowed for firm-specific types of technology, as Acemoglu [3] shows.

firms supply different varieties of intermediate goods to the final sector.

We prove in all three set-ups that the nonconvexity of the equilibrium surface is necessary and sufficient to the existence of technological bias. We show also how such a nonconvexity depends not only on the shape of the production function but also on the set of intermediate good producers. In particular, first, we show that there exists technological bias only if the mark-ups in the technological sector are high enough. In other words, competition in the technological sector convexifies the equilibrium surface and rules out the possibility of technological bias. Second, we show that there may exist technological bias when the entry of new firms in the technological sector depends on the factor supply. In other words, an endogenous number of technology producers may entail a nonconvex equilibrium surface due to indirectly increasing returns of the factor supply.

We could list other market structures of the technological sector that would determine the occurrence of technological bias such as contestable markets, price competition with differentiated products, or price competition with capacity constraints.[3] Our results are based on a Schumpeterian approach where market power provides the necessary incentives to endogenous and possibly biased technological change.[4] We acknowledge nevertheless the alternative perspective where competition for survival in highly innovative environments is the main channel of technological change.[5] In fact, the two approaches refer to inherently different industry-specific dynamics, as pointed out in Aghion et al. [4].

The paper is organized as follows. In Section 2 we simplify the framework of Acemoglu [3] and we formulate the Benchmark Case. Sections 3 and 4 provide the Cournot and Dixit-Stiglitz Cases. Section 5 draws the final conclusions. The Appendix collects all the proofs.

2.2 The Benchmark Case - Economy M

We present a simplified version of Acemoglu [3, Economy M] as the benchmark of our analysis.[6] Consider a static economy, which we call Economy M, composed of two sectors, the final good sector and the intermediate good sector. There is

[3] See respectively Baumol, Panzar, and Willig [7], Salop [15], and Kreps and Scheinkman [13]. Epifani and Gancia [10] present a similar approach to ours, that is, a comparative statics exercise with different types of market structure.

[4] See, e.g., Romer [14] and Aghion and Howitt [5].

[5] See, e.g., Boldrin and Levine [8].

[6] The differences with respect to the original version are two. First, we consider a unique final good firm instead of a continuum of mass 1 of final good firms. Second, we consider a unique factor of production, since the focus of our analysis is on the factor's absolute price and not on its relative price. None of these simplifications are relevant for our results.

a unique final good, a unique intermediate good embodying a unique technology, and a unique factor of production.

The final good sector consists of a final good producer which operates under perfect competition. Since the final good is unique, the price of the final good can be normalized to one. The problem of the firm is

$$\max_{Z\in\mathbb{R}_+, q\in\mathbb{R}_+} \pi(Z, q \mid \theta, w, \chi) = \alpha^{-\alpha}(1-\alpha)^{-1}[G(Z,\theta)]^{\alpha} q^{1-\alpha} - wZ - \chi q, \tag{2.1}$$

where $Z \in \mathbb{R}_+$ is the amount used of the single factor in the economy, $\theta \in \mathbb{R}_+$ is the scalar parameterizing the general level of the unique non-rivalrous technology, and $q \in \mathbb{R}_+$ is the quantity of the unique intermediate good, the ownership of which guarantees access to the embedded technology θ. The real-valued function $G(\cdot,\cdot)$ is twice continuously differentiable in (Z,θ) and is called the productive kernel. This function is combined with the intermediate good's quantity q into the production function

$$Y = \alpha^{-\alpha}(1-\alpha)^{-1}[G(Z,\theta)]^{\alpha} q^{1-\alpha}, \tag{2.2}$$

so that the shares of the two subcomponents G and q are described by the constant $\alpha \in (0,1)$. The term $\alpha^{-\alpha}(1-\alpha)^{-1}$ is simply a useful normalization. The element w is the price of the factor Z and χ is the price of the intermediate good q, which are both taken as given by the firm since it operates under perfect competition. Also the level of technology θ is exogenous to the final good firm's problem.

The optimal quantity of the intermediate good must satisfy the first order condition (henceforth, FOC) of this problem, from which we derive a demand function for the intermediate good,

$$q = q(\chi \mid Z, \theta) \equiv \alpha^{-1} G(Z,\theta)\chi^{-\frac{1}{\alpha}}, \tag{2.3}$$

where the demanded quantity is expressed as a function of its price χ that is influenced by the factor's employment Z and the technological level θ.

In the intermediate good sector there is a monopolist that supplies the technology to the final good firm. The monopolist sells an intermediate good that embodies the produced technology. We can think of the intermediate good as a patent that allows access to the embedded technology. The demand for the intermediate good, (2.3), is anticipated in the intermediate firm's problem. The monopolist decides the optimal intermediate good price and the optimal level of technology. We can view technology as an innovative effort that the monopolistic technology provider may exert at a certain cost in order to maximize its own profits. The problem of the firm in the intermediate sector is then

$$\max_{\theta\in R_+, \chi\in R_+} \Pi\left(\theta, \chi \mid q(\chi \mid Z, \theta)\right) = \chi q - (1-\alpha)q - C(\theta), \tag{2.4}$$

where χq are the total revenues from the sale of the intermediate good, $(1-\alpha)$ is the unitary production cost of the intermediate good, and $C(\theta)$ is the cost of producing the technological level θ.[7] The real-valued function $C(\cdot)$ is twice-continuously differentiable. We assume that $C(\cdot)$ is increasing and convex in the level θ of technology. The intermediate good firm solves the problem (2.4) subject to the demand function (2.3) for the intermediate good.

The technology menu consists of just one type of technology. Once the latter is created the technology provider can produce as many units of the intermediate good as it prefers. Each and every unit of the intermediate guarantees to the purchaser, that is, the final good firm, access to whatever level of technology is embedded in the patent. Therefore, the incentive to purchase more than just one unit of the intermediate good is the role that this intermediate asset plays in the final good production function, (2.2), rather than the level of technology embedded in it.

The unique factor Z is supplied inelastically, so the market clearing condition for the factor is

$$Z \leq \bar{Z}, \tag{2.5}$$

where $\bar{Z} \in \mathbb{R}_+$ is the exogenous inelastic supply of the factor, and Z is the employment level, that is, the demand, chosen by the final good firm.

The optimization problems in the two sectors and the relevant market clearing condition permit us to define a concept of equilibrium for this economy.

Definition 1 (Equilibrium in Economy M.)**.** An equilibrium is a set of firm decisions $\{Z, q\}$, technology level θ, intermediate price χ, and factor price w such that $\{Z, q\}$ solve the problem (2.1) of the final good firm given prices $\{w, \chi\}$ and technology level θ, the technology level θ and the intermediate price χ solve the monopolist problem (2.4) subject to the demand (2.3) for the intermediate good, and the market clearing condition (2.5) holds.

We refer to a technology level θ at equilibrium as an equilibrium technology. The equilibrium allocations are given by the FOCs of the respective problems, under the sufficient conditions for optimality represented by the second order conditions (SOC) in each problem. We impose the following assumption, which restricts the shape of the productive kernel G in its first argument, Z.

Assumption 1 (Strict Concavity)**.** For all $\theta \in R_+$, the function $G(\cdot, \theta)$ is increasing and either strictly concave or exhibiting constant returns to scale.

The function G does not need to be jointly strictly concave in (Z, θ). Assumption 1 simply represents a sufficient condition for optimality in (2.1). The final

[7] This constant unitary cost is a convenient normalization of the marginal cost of production of the intermediate which does not entail implications for the quality of the results.

good firm chooses Z and the intermediate good firm chooses θ. Hence, the (Z, θ) plane is the collection of the control variables of the two agents of the economy. We are led therefore to the following solution for the equilibrium allocation.

Proposition 1 (Equilibrium-Equivalent Problem of Economy M)**.** *Suppose Assumption 1 holds. Then, a technology level θ is an equilibrium technology if and only if θ is a solution to*

$$\max_{\theta \in \mathbb{R}_+} F_M(Z, \theta)|_{Z=\bar{Z}} \equiv G(Z, \theta)|_{Z=\bar{Z}} - C(\theta). \tag{2.6}$$

Note that, after we substitute for the optimal quantity and price of the intermediate good in (2.1) and (2.4), these two optimization problems have separate control spaces, since in (2.1) the control variable is Z and in (2.4) the control variable is θ. This is a key point for the results, since the separation of the optimization problems permits the presence of nonconvexities in the overall (Z, θ) space, although not in Z or θ separately. This means that in the (Z, θ) plane there might exist saddle points, kinks, and other non-standard features of the equilibrium surface.

From now on we will call (2.6) the equilibrium-equivalent problem of Economy M, and its solution the equilibrium level of technology, $\theta(\bar{Z})$. The latter depends on the level of the exogenous and inelastic factor supply, $\bar{Z}$, that is, the state variable of the whole model. We will call the objective function of (2.6), F_M, the equilibrium function of Economy M.

We substitute the equilibrium quantity and price of the intermediate good, q^* and χ^*, inside the objective function of (2.1) and we obtain

$$\max_{Z \in \mathbb{R}_+} \frac{G(Z, \theta)}{1 - \alpha} - wZ.$$

The FOC with respect to the factor's employment yields

$$w^* = w(Z, \theta)\bigg|_{\substack{Z=\bar{Z} \\ \theta=\theta(\bar{Z})}} \equiv (1 - \alpha)^{-1} \frac{\partial G(Z, \theta)}{\partial Z}\bigg|_{\substack{Z=\bar{Z} \\ \theta=\theta(\bar{Z})}}, \tag{2.7}$$

that is, the competitive price of the factor equals its marginal productivity for each given level of technology. Since it is computed at the equilibrium point $(\bar{Z}, \theta(\bar{Z}))$, we call (2.7) the factor price in equilibrium.

Suppose that $w(Z, \theta)$ is differentiable in both its arguments at the equilibrium point $(\bar{Z}, \theta(\bar{Z}))$ and that $\partial\theta(Z)/\partial Z$ exists at $\bar{Z}$. Then, we define the technological bias in the following way.[8]

[8] Definition 2 corresponds to the strong absolute equilibrium bias in Acemoglu [3, Definition 7].

Definition 2 (Technological Bias)**.** There is technological bias at $\bar{Z}$ if

$$\left.\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \left.\frac{\partial w(Z,\theta)}{\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} + \left.\frac{\partial w(Z,\theta)}{\partial \theta}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} \left.\frac{\partial \theta(Z)}{\partial Z}\right|_{Z=\bar{Z}} > 0, \tag{2.8}$$

that is, if the total derivative of the equilibrium price obtained through the chain rule is locally strictly positive.

We call the first element of the total derivative the Hicksian component, and the second element the equilibrium bias component. Note that, since Assumption 1 holds, the Hicksian component is negative for all (Z,θ). Hence, a precondition for the existence of technological bias is that the equilibrium bias component is locally positive in $(\bar{Z},\theta(\bar{Z}))$. We want to focus on an equilibrium phenomenon that refers to a change in the equilibrium price in absolute terms and not relative to the wage of other possible factors, that is at least locally valid at the equilibrium point $(\bar{Z},\theta(\bar{Z}))$, and that consists of an equilibrium bias component strong enough to overtake the negative effect of the Hicksian component.

We can formulate a theorem that identifies a necessary and sufficient condition for the existence of technological bias in Economy M.

Theorem 1 (Technological Bias in Economy M)**.** *Consider Economy M. There is technological bias at $\bar{Z}$ if and only if the Hessian ∇^2_M of $F_M(Z,\theta)$ is not negative semi-definite at $(\bar{Z},\theta(\bar{Z}))$.*

This theorem is a reformulation of the result in [3, Theorem 4]. The failure of joint concavity of the equilibrium function and the conditions induced by the agents' maximization problems imply the presence of technological bias.[9] Since technology and factor demands are chosen by different agents, that is, the technology monopolist and the final good firm, the equilibrium allocation may not be a maximum on the entire space of both factor quantity and technology, that is, in the (Z,θ) plane. It may be instead a saddle point which results as a maximum of F_M only separately in the control spaces of the final good firm, that is, in the factor quantity Z, and of the monopolist, that is, in the technology level θ. The economy's equilibrium set-up originate the nonconvexity, not the productive technologies of the agents. Thus, if we change the market relations, we affect the nonconvexity and therefore the possibility of technological bias. The claim of the present work is that the market structure plays a key role in the formation of nonconvexities in equilibrium, and therefore in the existence of technological bias.

[9]Note that in order to have F_M jointly concave in (Z,θ) at the equilibrium point $(\bar{Z},\theta(\bar{Z}))$, its Hessian with respect to (Z,θ), ∇^2_M, should necessarily be negative semi-definite at this point, even though negative semi-definiteness is not a sufficient condition for local joint concavity. This means therefore that we adopt a loose concept of nonconvexity, albeit more versatile.

2.3 The Cournot Case - Economy Q

In this section we present a model that casts a link between the Benchmark Case and the perfect competition case in the technological sector. This is important in order to determine why technological bias is not compatible with high levels of competition.

Consider a two-sector economy, called Economy Q, where there is unique firm in the final good sector operating under perfect competition and N firms in the technological sector that compete in quantities à la Cournot. There exists a unique type of nonrival technology. Moreover, we assume that the intermediate goods where the technology is embedded are perfectly substitutable.

The final good firm's problem is

$$\max_{Z\in\mathbb{R}_+,Q\in\mathbb{R}_+} \pi(Z,Q \mid \theta,\chi) = \alpha^{-\alpha}(1-\alpha)^{-1}[G(Z,\theta)]^{\alpha}Q^{1-\alpha} - wZ - \chi Q, \tag{2.9}$$

where Q represents the quantity of intermediate goods employed by the final good firm. The intermediate good's price χ is the equilibrium result of a Cournot interaction in the intermediate sector. We apply to (2.9) the FOC with respect to Q and obtain the total demand for intermediates, which we express as an inverse demand function, that is,

$$\chi = \chi(Q \mid Z,\theta) \equiv \alpha^{-\alpha}[G(Z,\theta)]^{\alpha}Q^{-\alpha}, \tag{2.10}$$

where θ is again the technology to which the final good firm has access.

Let N be the number of technology producers, and let n indicate the generic producer, that is, $n = 1,...,N$. Each technology producer supplies its own quantity of intermediate good q_n and provides its own technology level θ_n. Since technology is unique and nonrival, the technology level to which the final good firm has access, θ, is the maximum among the levels provided by all the technological producers, that is,

$$\theta = \max\{\theta_n\}_{n=1}^{N}. \tag{2.11}$$

Each intermediate producer solves the same symmetric problem, that is,

$$\max_{\theta_n\in\mathbb{R}_+,q_n\in\mathbb{R}_+} \Pi_n(\theta_n,q_n \mid \chi(Q \mid Z,\theta)) = (\chi-(1-\alpha))q_n - C(\theta_n), \tag{2.12}$$

where q_n is the individual quantity, χ the intermediate price, and θ_n the firm-specific technology production. The control variables of the technology producers are different from (2.4), since now firms choose the optimal quantity and the price is determined by the market clearing. The factor market clears when (2.5) holds. The market clearing condition for the intermediate goods market is instead

$$Q \leq \sum_{n=1}^{N} q_n, \tag{2.13}$$

that is, total demand of intermediate goods is equal to total supply. We can formulate a definition of equilibrium in Economy Q that is similar to the definition of equilibrium in Economy M.

Definition 3 (Equilibrium in Economy Q)**.** An equilibrium in Economy Q is a set of firm decisions $\{Z, Q, q_1, ..., q_N\}$, technology levels $\{\theta_1, ..., \theta_N\}$ and θ, intermediate price χ, and factor price w such that θ solves (2.11), $\{Z, Q\}$ solve the problem (2.9) of the final good firm given prices $\{\chi, w\}$ and technology θ, $\{\theta_n, q_n\}$ solve the problem (2.12) of the n-th intermediate good firm for any $n \in \{1, ..., N\}$ given (2.10), and the market clearing conditions (2.5) and (2.13) hold.

On the one hand, since the demand for intermediate goods depends on the maximum among all the technology productions, each technology producer has the incentive to free ride on the technology production of the other providers. So the symmetric solution for which all firms produce the maximal technology is not a (Nash) equilibrium. On the other hand, the solution where all the firms produce a nil technology is not an equilibrium either, since each firm in this case would make zero profits while producing some technology and stimulating a nonnil demand for the intermediate goods would yield stricly positive profits, even if shared with the competitors. Hence, if one of the technology producers, with probability $1/N$, decides to produce a nonnil amount of technology, then everybody gains, although the actual producer pays all the costs of technology production. The other technology providers produce a nonnil level of technology as well, since otherwise the final good firm has no incentive to buy their intermediate goods. At the margin, the actual producer wants to produce the amount of technology that maximizes its profits, no matter how much profit the other free-riding competitors realize. Thus, it maximizes its how profit function regardless of the others. There exists N asymmetric pure strategy equilibria. All of them yield the same maximal technology, θ. The following proposition characterizes the equilibrium technology.

Proposition 2 (Equilibrium-Equivalent Problem of Economy Q)**.** *Suppose Assumption 1 holds. Then, a technology level θ is an equilibrium technology if and only if θ is a solution to*

$$\max_{\theta \in \mathbb{R}_+} F_Q(Z, \theta)|_{Z=\bar{Z}} \equiv \frac{\Omega(N, \alpha)}{N^2} G(Z, \theta)|_{Z=\bar{Z}} - C(\theta), \tag{2.14}$$

where

$$\Omega(N, \alpha) \equiv \left(\frac{N - \alpha}{N(1 - \alpha)} \right)^{\frac{1-\alpha}{\alpha}}. \tag{2.15}$$

The intuition is that (2.14) implies that the higher the number of intermediate good producers, N, the lower the individual share in aggregate demand and the higher the competition, and therefore the lower the incentive for the single firm to embed technology in the intermediate good. An increase in the number of firms reduces technology production. This has consequences on the existence of the technological bias. According to Definition 2, there exists technological bias only if the total derivative of the factor price is positive. The number of firms in the technological sector reduces the incentives for technology production and lowers therefore the equilibrium bias component of the total derivative. This means that, even though technology increases as the factor supply increases, it may not increase enough to compensate the negative Hicksian effect on the factor price.

We substitute for the equilibrium level of Q and χ in (2.1) and we obtain

$$\max_{Z\in\mathbb{R}_+} \frac{\Omega(N,\alpha)}{1-\alpha} G(Z,\theta) - wZ.$$

Hence, the factor price in equilibrium

$$w(Z,\theta)\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} \equiv \frac{\Omega(N,\alpha)}{1-\alpha}\frac{\partial G(Z,\theta)}{\partial Z}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}, \tag{2.16}$$

where $\Omega(N,\alpha)$ is defined in (2.15).

The following theorem states the equivalence between equilibrium nonconvexity and technological bias in this economy.

Theorem 2 (Technological Bias in Economy Q)**.** *Consider Economy Q. There is technological bias in Economy Q at $\bar{Z}$ if and only if the Hessian ∇^2_Q of $F_Q(Z,\theta)$ is not negative semi-definite at $(\bar{Z},\theta(\bar{Z}))$.*

The technological bias is incompatible with low individual incentives to technology production. The high number of firms affects the potential nonconvexity of the equilibrium, that is, it makes the equilibrium function's Hessian negative semi-definite.

Corollary 1 (Technological Bias and Number of Firms in Economy Q)**.** *Suppose Assumption 1 holds. If*

$$\frac{G(Z,\theta)}{C(\theta)}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} > \frac{\partial^2 G(Z,\theta)}{\partial\theta^2}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} - \frac{(\partial^2 G(Z,\theta)/\partial Z\partial\theta)^2}{(\partial^2 G(Z,\theta)/\partial\theta^2)C''(\theta)}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}, \tag{2.17}$$

then, there exists a number N of firms high enough such that the technology producer's profits are nonnegative and

$$\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} < 0.$$

The intuition behind (2.17) is the following. On the one hand, technology producers make nonnegative profits at equilibrium only if

$$\frac{N^2}{\Omega(N)} \leq \left.\frac{G(Z,\theta)}{C(\theta)}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}},$$

where $G(Z,\theta)/C(\theta)$ is a measure of the monopolistic mark-up. If the number of firms, N, is too high, the mark-up cannot be enough for all the participants to make positive profits. On the other hand, the total derivative of the factor price in equilibrium is negative when

$$\frac{N^2}{\Omega(N)} > \left.\frac{\partial^2 G(Z,\theta)}{\partial\theta^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} - \left.\frac{(\partial^2 G(Z,\theta)/\partial Z\partial\theta)^2}{(\partial^2 G(Z,\theta)/\partial\theta^2)C''(\theta)}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}},$$

that is, when the joint concavity of the equilibrium surface is strong enough. The element on the right hand side measures both the joint concavity of the production kernel, G, and the convexity of the cost function, C. First, the more concave the production with respect to the factor, the stronger the Hicksian effect, and therefore the more likely the total derivative to be negative. Second, the more concave the production with respect to the technology, the less the increase in demand due to technology production. Third, the higher the production cross-effects between the factor and the technology, the higher the interaction between factor supply and technology availability. And fourth, the more convex the cost function, the more costly are increases in technology. There is technological bias only if this element is low with respect to the number of firms. Hence, if the mark-up and the joint concavity of the equilibrium solution are high enough, there exists a number of firms such that the equilibrium bias is not enough to compensate the Hicksian effect. If the mark-up was too low, a few additional firms in the technological sector would imply already negative profits. If the joint concavity was moderate, the technological bias would disappear only with a high number of firms.

The Cournot case bridges the Benchmark Case with the presence of competition. In fact, when $N = 1$ we are back to Economy M. This fills a gap in Acemoglu [3], where only the dichotomic difference between perfect competition with no bias and monopoly with bias is analyzed. Economy Q permits to map the effect of competition on firms' profits and consequently on the technological bias.

2.4 The Dixit-Stiglitz Case - Economy DS

In this section the set of technology producers changes in response to changes in the supply of a factor. We present an oligopolistic model that is closer to

Acemoglu [3, Economy O] than Economy Q, in order to rule out the possibility of negative profits with high levels of competition. This is important in order to characterize the effects of a variable set of technology producers on the existence of technological bias.

Consider a two-sector economy, called Economy DS, where there is a unique firm in the final good sector operating under perfect competition and the intermediate sector is populated by a continuum of mass N of symmetric intermediate firms. Each firm is the monopolistic producer of a specific intermediate good, and the unique final good firm combines the entire range of intermediate goods in its production function according to a symmetric level of substitutability. All the firms produce the same type of technonogy.[10]

The final good firm uses a production function à la Dixit and Stiglitz.[11] Hence, its problem is

$$\max_{Z\in\mathbb{R}_+,\{q_j\}_{j\in[0,N]}} \alpha^{-\alpha}(1-\alpha)^{-1}[G(Z,\theta)]^{\alpha}\int_0^N q_j^{1-\alpha}dj - wZ - \int_0^N \chi_j q_j dj, \tag{2.18}$$

where j is the continuous type index, $[0,N]$ is the range of available types, q_j is the quantity produced by the j-th technology provider, χ_j is its price, θ is the type-independent level of technology, and $1-\alpha$ represents the substitutability of the intermediate goods in the final good production. The FOCs of (2.18) with respect to q_j yield a demand function for every intermediate type j that depends on the price χ_j of that intermediate, that is,

$$q_j = q(\chi_j \mid Z,\theta) \equiv \alpha^{-1}G(Z,\theta)\chi_j^{-\frac{1}{\alpha}}. \tag{2.19}$$

The monopolistic intermediate producer of type j maximizes its own profits subject to the intermediate good-specific demand expressed by the final good firm. We suppose symmetry among intermediate producers. Thus, the problem of the j-th monopolist is

$$\max_{\theta_n,\chi_j} \Pi_j(\theta_n,\chi_j \mid q(\chi_j \mid Z,\theta)) = (\chi_j-(1-\alpha))q_j - C(\theta_n), \tag{2.20}$$

subject to (2.19). The market clearing condition for the factor is given by (2.5). We define the equilibrium in Economy DS in a similar way to the definition of equilibrium in Economy M and Economy Q.

[10] The case for firm-specific technology types corresponds to [3, Economy O]. The only consequence is on the equilibrium-equivalent problem, while our main results would not change.

[11] See Dixit and Stiglitz [9]. For an application to a similar problem, see, e.g., Epifani and Gancia [10].

Definition 4 (Equilibrium in Economy DS)**.** An equilibrium in Economy DS is a set of firm decisions $\{Z, \{q_j\}_{j\in[0,N]}\}$, technology levels $\{\theta_j\}_{j\in[0,N]}$ and θ, factor price w, and intermediate prices $\{\chi_j\}_{j\in[0,N]}$ such that $\{Z, \{q_j\}_{j\in[0,N]}\}$ solve the problem (2.18) of the final good firm given prices $\{w, \{\chi_j\}_{j\in[0,N]}\}$ and technology θ, θ_j and q_j solve the problem (2.20) of the j-th monopolist subject to the demand (2.19) for every $j \in [0, N]$, (2.11) holds, and the market clearing condition (2.5) is satisfied.

The existence and uniqueness of the maximal produced technology is in line with Economy Q. If the technology type was firm-specific and not unique, the equilibrium technology would be the fixed point of a mass N of maximization problems.[12] This would not affect our results below.

Proposition 3 (Equilibrium-Equivalent Problem in Economy DS)**.** *Suppose Assumption 1 holds. Then, a technology level θ is an equilibrium technology if and only if θ is a solution to*

$$\max_{\theta\in\mathbb{R}_+} F_{DS}(Z,\theta)|_{Z=\bar{Z}} \equiv G(Z,\theta)|_{Z=\bar{Z}} - C(\theta). \tag{2.21}$$

The problem in (2.21) coincides with (2.6), since it is the result of a mass N of symmetric monopolistic problems. Proposition 3 is different from Proposition 2 because the number N of competitors does not enter the profits of the intermediate firms.

We substitute the equilibrium quantities and prices of the intermediate goods into (2.18) and obttain

$$\max_{Z} N(1-\alpha)^{-1}G(Z,\theta) - wZ.$$

We then impose the FOC and obtain the factor price in equilibrium, that is,

$$w(Z,\theta)\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} \equiv \frac{N}{1-\alpha}\frac{\partial G(Z,\theta)}{\partial Z}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}. \tag{2.22}$$

The mass of technological firms, N, enters the equilibrium price of the factor because there exists a love for variety of inputs in the production of the final good. This means that the higher the mass N, the higher the final production. While in Economy Q the number of firms enters also the profit function of the technology producer, (2.14), in Economy DS the mass of firms enters only the

[12] If the cross-derivative between technology types in the production kernel G was nil, then, the fixed point would correspond to a unique problem. See Acemoglu [3, Economy O] for more on this.

factor price. Note that the degree of input substitutability, parameterized by $1-\alpha$, is the inverse of the mark-up, $1/(1-\alpha)$. Thus, the more substitutable the intermediate goods, the lower the incentive to technology production, as we can see in (2.22).

Theorem 3 (Technological Bias in Economy DS)**.** *Consider Economy DS. There is technological bias at $\bar{Z}$ if and only if the Hessian ∇^2_{DS} of $F_{DS}(Z,\theta)$ is not negative semi-definite at $(\bar{Z},\theta(\bar{Z}))$.*

Since every technology producer is a monopolist in the production of the intermediate good, the equivalence between equilibrium nonconvexity and existence of technological bias is the same as in Economy M. Theorem 3 holds for every exogenous N.

We propose a new set-up of Economy DS where the mass of firms is an increasing function of the amount of the aggregate factor.

Assumption 2. The mass N of firms is an increasing function of the amount Z of the aggregate factor, that is,

$$N = N(Z), \tag{2.23}$$

where $N'(Z) > 0$ for every $Z \in \mathbb{R}_+$.

Since the profits of the technology producers do not depend on the level of competition, firm entry does not depend on any nonnegative-profits requirement. Hence, Assumption 2 is compatible with the rest of the set-up.

Remark. Note that the amount Z of the aggregate factor in (2.23) is not the control variable of the perfectly competitive final good firm. In fact, we can think of a mass 1 of symmetric final good firms under perfect competition, where each firm i chooses its firm specific employment Z_i and aggregate factor demand is $Z \equiv \int_0^1 Z_i di$, with the market clearing condition $\int_0^1 Z_i di \le \bar{Z}$. Hence, the single firm i does not control Z. We skip this notation for simplicity.

If Assumption 2 holds, then the total derivative of the equilibrium price of the factor has an additional element. This implies that we need to redefine the technological bias.

Definition 5 (Technological Bias with a Variable Number of Technology Producers)**.** There is technological bias at $\bar{Z}$ if

$$\begin{aligned}\left.\frac{\mathrm{d}w(Z,\theta,N)}{\mathrm{d}Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = &\left.\frac{\partial w(Z,\theta,N)}{\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} + \\ &+\left.\frac{\partial w(Z,\theta,N)}{\partial \theta}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} \left.\frac{\partial \theta(Z)}{\partial Z}\right|_{Z=\bar{Z}} + \\ &+\left.\frac{\partial w(Z,\theta,N)}{\partial N}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} \left.\frac{\partial N(Z)}{\partial Z}\right|_{Z=\bar{Z}} > 0,\end{aligned} \tag{2.24}$$

that is, if the total derivative of the equilibrium price obtained through the chain rule is locally strictly positive.

The third element of the total derivative is positive as long as Assumption 2 holds and the factor price is given by (2.22). Hence, although the Hicksian component was bigger than the equilibrium bias component, the third component could add enough bias so as to make the total derivative positive.

Theorem 4 (Technological Bias with a Variable Number of Technology Producers)**.** *Consider Economy DS. Suppose that Assumption 2 holds. Then, there is technological bias at* $\bar{Z}$ *if the Hessian* ∇^2_{DS} *of* $F_{DS}(Z, \theta)$ *is not negative semi-definite at* $(\bar{Z}, \theta(\bar{Z}))$*.*

The failure of equilibrium joint concavity in the (Z, θ) plane is only a sufficient condition for the existence of technological bias when Assumption 2 holds. There may exist the technological bias also if the third element of the total derivative of the factor price is high enough. We propose hereafter a characterization of Economy DS where there is technological bias although the production kernel is jointly concave.

Example 1. Consider a pharmaceutical firm that provides the market with its final good, that is, medicines. The production of such medicines and their quality depends on the number of people employed Z, their technical preparation and effort θ, and on the quantity of patents q used in the preparation of the final medicines Y. Since success in a high-technology sector depends crucially on the access to the research frontier and the exclusion of competitors from the latest research developments, pharmaceutical firms tend to internalize research activity rather than use a patents market. The firms employ researchers with contracts that both force the researchers to submit their research results exclusively to the firm and reward them according to the commercial potential of their findings. We can break down these rewards into two components. A fixed wage w and a rent χ that varies across employees within the firm depending on their research performance. Thus, the researcher works both for the production of the final good based on the existing technology and for the development of chemical formulas that could lead to new and commercially valuable products. She sells her work time in the final good firm for the wage w but she also sells her innovations within the firm's research department for the extra compensation χ. Her innovations are more valuable the higher her research effort θ. The effort is costly for the researcher, that is, the researcher faces a cost function $C(\theta)$ for her research effort. The researcher can either sell her innovations at the price χ to the pharmaceutical firm that hired her or destroy them. She cannot freely sell her innovations to other firms due to her employment contract. In this way, the number of workers Z that the pharmaceutical firm hires to produce medicines coincides with the number of researchers N that give their innovations to the firm for private profit, that is,

$$N = Z.$$

Suppose that the productive kernel G has a Cobb-Douglas form satisfying Assumption 1, e.g.,

$$G(Z,\theta) = Z^{\epsilon}\theta^{1-\epsilon},$$

with $\epsilon \in (0,1)$. Moreover, suppose that technology production costs are quadratic, that is,

$$C(\theta) = \theta^2/2.$$

Then, the factor price is

$$w(Z,\theta)\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N}{1-\alpha}\epsilon\bar{Z}^{\epsilon-1}\theta(\bar{Z})^{1-\epsilon}. \tag{2.25}$$

The equilibrium level of technology is the solution of (2.21), that is,

$$\theta(Z) = (1-\epsilon)^{\frac{1}{1+\epsilon}} Z^{\frac{\epsilon}{1+\epsilon}}.$$

Moreover, $N = Z$ and at equilibrium $Z = \bar{Z}$, so the total derivative is

$$\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\Big|_{\substack{N=Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{2\epsilon^2(1-\epsilon)^{\frac{1-\epsilon}{1+\epsilon}}}{(1-\alpha)(1+\epsilon)}\bar{Z}^{\frac{\epsilon-1}{1+\epsilon}} > 0.$$

On the one hand, there exists technological bias. On the other hand, the determinant of the Hessian of the equilibrium function F_{DS} at equilibrium is

$$\left|\nabla^2_{DS}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \epsilon\left(\frac{\bar{Z}}{1-\epsilon}\right)^{-\frac{2}{1+\epsilon}} > 0,$$

that is, the Hessian of F_{DS} is negative definite. Hence, there is technological bias even if the equilibrium is jointly concave in the (Z,θ) plane.

The increase in the factor supply has three effects. One, it decreases the marginal productivity of workers in the final good sector. Two, it induces a wider variety of intermediate goods, that is, a more sophisticated production technology. Thus, the potential productivity of the researchers increases, once we expand their number. And three, the agents, whose compensation scheme depends on their productivity, perceive this opportunity and put more effort into providing the innovations that may enhance their productivity, that is, they induce a technological change in the economy. Hence, the first negative effect is overruled by the two positive effects on the equilibrium wage.

2.5 Conclusion

In this paper we have investigated the determinants of the technological bias. We show that the technological bias exists when the equilibrium surface is nonconvex. We cast a link between the market structure of the technological sector and the existence of the nonconvexities in equilibrium. We compare the Benchmark Case of monopoly in the technological sector with two alternative settings. In the Cournot Case we show that technological bias is incompatible with high levels of competition. In the Dixit-Stiglitz Case we claim that if the set of active firms in the technological sector depends on the factor supply, the technological bias may occur even when the equilibrium surface is jointly concave in the factor employment and the technology. We provide an example that characterizes the claim.

An implication of our model is that, if we want to include a technological bias mechanism in, e.g., a growth model, we can simply control for the market structure of the technological sector or the entry decision of the technological producers. We do not need to impose assumptions on the aggregate production function. Moreover, in a structural regression, if the technological bias is an endogenous regressor, we need an instrument for it. Our model proposes the market structure of the technological sector as a theoretically consistent instrument. This suggests an empirical application of Acemoglu [3]'s results.

Our model fills a gap in Acemoglu [3]. Economy Q encompasses both the monopoly case of [3, Economy M] and the competition case. It shows how the equilibrium nonconvexity depends on the market structure, due to the effect of competition on the technology producers' incentives. Economy DS analyzes the case where the set of technological firms depends on the factor employment. This helps us understand how the size effect of an increase in the factor supply may influence the factor prize.

Our analysis of the existence of equilibrium bias in presence of alternative market structures is similar to Epifani and Gancia [10]. Nevertheless, they diversify the market structure of the final good sector while we focus on the technological sector. We consider similar market structures, that is, imperfect competition frameworks where the agents alternatively compete in quantities or provide firm-specific intermediate goods. Their conclusions about the relation between the existence of scale bias and the level of competition are the same as ours, although they deal with relative bias while we concentrate on absolute bias.

The last remark we ought to make is about welfare issues related to the effects of the technological bias. What we have analyzed here is not, for example, the evolution of skilled workers' conditions in the presence of skilled immigration. In fact, our price representation of absolute technological bias neglects the relative position of the increased factor with respect to others. The social condition of a factor may deteriorate even in the presence of absolute technological bias once we allow for an increase in the factor supply. What we would need for a welfare improvement of the increasing factors is factor-augmenting or at least factor-specific technologies. Furthermore, we can argue that redistributing the gains given by the increase in total factor productivity induced by the technological bias would yield Pareto-improving increases of factor supplies.

An important area of future research is an empirical test of the structural relations implied by the present work. For instance, in a setting of increasing factor supply, we could test whether increasing or decreasing factor prices are correlated with changes in the market structure of the technological sector.

2.6 Appendix: Proofs

Proof of Proposition 1. Since G is increasing in Z under Assumption 1 and (2.5) holds, the optimal level of employment for the factor is the border solution $Z^* = \bar{Z}$. Therefore, the FOC with respect to the quantity of intermediate good q is, according to (2.3),

$$q(\chi \mid Z, \theta)\big|_{Z=\bar{Z}} = \alpha^{-1} G(Z,\theta)|_{Z=\bar{Z}} \chi^{-\frac{1}{\alpha}}.$$

We substitute q into the monopolist problem (2.4), that is,

$$\max_{\theta \in R_+, \chi \in R_+} (\chi - (1-\alpha))\, \alpha^{-1} G(Z,\theta)\big|_{Z=\bar{Z}} \chi^{-\frac{1}{\alpha}} - C(\theta).$$

The FOC with respect to χ yields $\chi^* = 1$. We plug this into the demand function and obtain

$$q(\chi \mid Z, \theta)\Big|_{\substack{Z=\bar{Z} \\ \chi=1}} = \alpha^{-1} G(Z,\theta)\big|_{Z=\bar{Z}}.$$

We substitute the values for the intermediate good's equilibrium price and quantity into (2.4), that is,

$$\max_{\theta \in R_+} = G(\bar{Z},\theta) - C(\theta).$$

□

Proof of Theorem 1. The function $F_M(\cdot,\cdot)$ in (2.6) is twice continuously differentiable in (Z,θ), since both $G(\cdot,\cdot)$ and $C(\cdot)$ are twice continuously differentiable. Since Assumption 1 holds,

$$\frac{\partial F_M(Z,\theta)}{\partial Z} = \frac{\partial G(Z,\theta)}{\partial Z} \quad \text{and} \quad \frac{\partial^2 F_M(Z,\theta)}{\partial Z^2} = \frac{\partial^2 G(Z,\theta)}{\partial Z^2} < 0.$$

We impose the FOC and the SOC on (2.6) and obtain

$$\left.\frac{\partial F_M(Z,\theta)}{\partial\theta}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}=0 \quad\text{and}\quad \left.\frac{\partial^2 F_M(Z,\theta)}{\partial\theta^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}<0. \tag{2.26}$$

First, the function $\partial F_M(\cdot,\cdot)/\partial\theta$ is a mapping from $\mathbb{R}_+^2$ to $\mathbb{R}_+$. Second, it is locally C^1. Third, its value is zero at the equilibrium point $(\bar{Z},\theta(\bar{Z}))$. Fourth, its derivative with respect to the θ, $\partial^2 F_M(\cdot,\cdot)/\partial\theta^2$, is not nil at equilibrium because of the SOC. Hence, we satisfy all the hypotheses of the Implicit Function Theorem (IFT) and we can apply it to the FOC of (2.6), (2.26), that is,

$$\left.\frac{\partial\theta(Z)}{\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}=-\left.\frac{\dfrac{\partial\left(\partial F_M(Z,\theta)/\partial\theta\right)}{\partial Z}}{\dfrac{\partial\left(\partial F_M(Z,\theta)/\partial\theta\right)}{\partial\theta}}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}=-\left.\frac{\dfrac{\partial^2 F_M\left(Z,\theta\right)}{\partial Z\partial\theta}}{\dfrac{\partial^2 F_M\left(Z,\theta\right)}{\partial\theta^2}}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

However,

$$\frac{\partial w(Z,\theta)}{\partial Z}=(1-\alpha)^{-1}\frac{\partial^2 G(Z,\theta)}{\partial Z^2}=(1-\alpha)^{-1}\frac{\partial^2 F_M(Z,\theta)}{\partial Z^2},$$

and

$$\frac{\partial w(Z,\theta)}{\partial\theta}=(1-\alpha)^{-1}\frac{\partial^2 G(Z,\theta)}{\partial Z\partial\theta}=(1-\alpha)^{-1}\frac{\partial^2 F_M(Z,\theta)}{\partial Z\partial\theta}.$$

Hence, we can rewrite (2.8) as

$$\left.\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}=(1-\alpha)^{-1}\left.\left[\frac{\partial^2 F_M(Z,\theta)}{\partial Z^2}-\frac{\left(\partial^2 F_M(Z,\theta)/\partial Z\partial\theta\right)^2}{\partial^2 F_M(Z,\theta)/\partial\theta^2}\right]\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}>0.$$

The Hessian of F_M is

$$\nabla_M^2=\begin{pmatrix}\partial^2 F_M(Z,\theta)/\partial\theta^2 & \partial^2 F_M(Z,\theta)/\partial\theta\partial Z\\ \partial^2 F_M(Z,\theta)/\partial\theta\partial Z & \partial^2 F_M(Z,\theta)/\partial Z^2\end{pmatrix}.$$

Since (2.26) holds,, ∇_M^2 is not locally negative semi-definiteness if

$$\left.\frac{\partial^2 F_M(Z,\theta)}{\partial\theta^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\times\left.\frac{\partial^2 F_M(Z,\theta)}{\partial Z^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}<\left(\left.\frac{\partial^2 F_M(Z,\theta)}{\partial\theta\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\right)^2,$$

that is, if

$$\left.\frac{\partial^2 F_M(Z,\theta)}{\partial Z^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}>\left.\frac{\left(\partial^2 F_M\left(Z,\theta\right)/\partial\theta\partial Z\right)^2}{\partial^2 F_M\left(Z,\theta\right)/\partial\theta^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

Suppose that $\nabla_M(Z,\theta)$ is not locally negative semi-definite. Then, since $(1-\alpha)^{-1}>0$,

$$\left.\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}>0,$$

that is, there is technological bias.

Conversely, suppose that the Hessian of F_M is negative semi-definite at $(\bar{Z},\theta\bar{Z})$. Then,

$$\left.\frac{\partial^2 F_M(Z,\theta)}{\partial\theta^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\times\left.\frac{\partial^2 F_M(Z,\theta)}{\partial Z^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\geq\left(\left.\frac{\partial^2 F_M(Z,\theta)}{\partial\theta\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\right)^2.$$

But (2.26) holds, so

$$\left.\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\leq 0,$$

that is, there is no technological bias.

□

Proof of Proposition 2. Since Assumption 1 and (2.5) hold, $Z^*=\bar{Z}$. Then, from (2.13) and (2.10) we obtain that

$$\chi=\alpha^{-\alpha}\left[G(Z,\theta)|_{Z=\bar{Z}}\right]^{\alpha}\left(\sum_{n=1}^{N}q_n\right)^{-\alpha}.$$

Thus, the individual intermediate problem,

$$\max_{\theta_n,q_n}\left[\alpha^{-\alpha}\left[G(Z,\theta)|_{Z=\bar{Z}}\right]^{\alpha}\left(\sum_{n=1}^{N}q_n\right)^{-\alpha}-(1-\alpha)\right]q_n-C(\theta_n)$$

yields the following FOC with respect to q_n,

$$q_n(\chi\mid Q)=Q\frac{\chi-(1-\alpha)}{\alpha\chi}=q.$$

Hence, the optimal quantity of the intermediate good, q, is symmetric. The market clearing condition (2.13) becomes

$$Q=\sum_{n=1}^{N}q=Nq.$$

We substitute for this value in the FOC above and obtain the equilibrium price for the intermediate,

$$\chi^*=\frac{N(1-\alpha)}{N-\alpha}.$$

Thus, the optimal individual quantity of the intermediate is

$$q(\chi \mid Z,\theta)\Big|_{\substack{Z=\bar{Z}\\ \chi=\chi^*}} = \left(\frac{N-\alpha}{N(1-\alpha)}\right)^{\frac{1}{\alpha}} \frac{G(Z,\theta)|_{Z=\bar{Z}}}{\alpha N},$$

which implies that

$$Q(\chi \mid Z,\theta)\Big|_{\substack{Z=\bar{Z}\\ \chi=\chi^*}} = \left(\frac{N-\alpha}{N(1-\alpha)}\right)^{\frac{1}{\alpha}} \frac{G(Z,\theta)|_{Z=\bar{Z}}}{\alpha}$$

We plug these values in (2.12) to obtain

$$\max_{\theta_n} \left(\frac{N-\alpha}{N(1-\alpha)}\right)^{\frac{1-\alpha}{\alpha}} \frac{G(Z,\theta)|_{Z=\bar{Z}}}{N^2} - C(\theta_n).$$

Since there exist N asymmetric pure strategy equilibria (see discussion in Section 2), the maximum level of available technology, θ, is equivalent to the solution of the above problem for some n in $\{1,\cdots,N\}$.

□

Proof of Theorem 2. The function $F_Q(\cdot,\cdot)$ is twice continuously differentiable. Since the factor price in equilibrium is given by (2.16),

$$\frac{\partial w(Z,\theta)}{\partial Z}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{\Omega(N)}{1-\alpha}\frac{\partial^2 G(Z,\theta)}{\partial Z^2}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N^2}{1-\alpha}\frac{\partial^2 F_Q(Z,\theta)}{\partial Z^2}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}},$$

and

$$\frac{\partial w(Z,\theta)}{\partial \theta}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{\Omega(N)}{1-\alpha}\frac{\partial^2 G(Z,\theta)}{\partial Z\partial\theta}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N^2}{1-\alpha}\frac{\partial^2 F_Q(Z,\theta)}{\partial Z\partial\theta}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

We apply the IFT to the FOC of (2.14), which yields

$$\frac{\partial\theta(Z)}{\partial Z}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = -\frac{\partial^2 F_Q(Z,\theta)/\partial Z\partial\theta}{\partial^2 F_Q\left(Z,\theta\right)/\partial\theta^2}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

We substitute the relations above into the total derivative of (2.16) and obtain

$$\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N^2}{1-\alpha}\left[\frac{\partial^2 F_Q(Z,\theta)}{\partial Z^2} - \frac{\left(\partial^2 F_Q(Z,\theta)/\partial\theta\partial Z\right)^2}{\partial^2 F_Q\left(Z,\theta\right)/\partial\theta^2}\right]\Big|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

Hence, the failure of negative semi-definiteness of the Hessian of F_Q, ∇^2_Q, is a necessary and sufficient condition for the existence of technological bias. □

Proof of Corollary 1. According to (2.14), the profits of the actual technology producer at equilibrium are nonnegative if and only if

$$\left[\frac{\Omega(N,\alpha)}{N^2}G(Z,\theta)-C(\theta)\right]\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\geq 0,$$

for any N, that is,

$$\frac{N^2}{\Omega(N,\alpha)}\leq\frac{G(Z,\theta)}{C(\theta)}\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

We can rewrite the total derivative of the factor price as

$$\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}=\frac{\Omega(N)}{1-\alpha}\left[\frac{\partial^2 G(Z,\theta)}{\partial Z^2}+\frac{\partial^2 G(Z,\theta)}{\partial Z\partial\theta}\frac{\partial\theta(Z)}{\partial Z}\right]\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}},$$

that is,

$$\frac{\mathrm{d}w(Z,\theta)}{\mathrm{d}Z}\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}=\frac{\Omega(N)}{1-\alpha}\left[\frac{\partial^2 G(Z,\theta)}{\partial Z^2}-\frac{\left(\dfrac{\partial^2 G(Z,\theta)}{\partial Z\partial\theta}\right)^2}{\dfrac{\partial^2 G(Z,\theta)}{\partial\theta^2}-\dfrac{N^2}{\Omega(N)}C''(\theta)}\right]\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

Hence, the total derivative is negative if and only if

$$\frac{N^2}{\Omega(N,\alpha)}>\frac{\partial^2 G(Z,\theta)}{\partial\theta^2}\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}-\frac{(\partial^2 G(Z,\theta)/\partial Z\partial\theta)^2}{(\partial^2 G(Z,\theta)/\partial\theta^2)C''(\theta)}\Bigg|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

□

Proof of Proposition 3. From Assumption 1 and (2.5) we set $Z^*=\bar{Z}$. Hence, the demand for the j-th intermediate good is

$$q_j(\chi_j\mid Z,\theta)|_{Z=\bar{Z}}=\alpha^{-1}G(Z,\theta)|_{Z=\bar{Z}}\chi_j^{-\frac{1}{\alpha}},$$

for all $j\in[0,N]$. The j-th intermediate firm solves (2.20) subject to the demand above. The FOC with respect to the j-th intermediate good price, χ_j, yields the symmetric equilibrium price for the intermediate good $\chi_j^*=1$, for all $j\in[0,N]$. We substitute the intermediate price into the intermediate demand and obtain

$$q_j(\chi_j\mid Z,\theta)\Bigg|_{\substack{Z=\bar{Z}\\ \chi_j=1}}=\alpha^{-1}G(Z,\theta)|_{Z=\bar{Z}}=q,$$

for all $j\in[0,N]$. The problem (2.20) is then equivalent to

$$\max_{\theta_j}G(Z,\theta)|_{Z=\bar{Z}}-C(\theta_j),$$

for all $j \in [0, N]$. Since there exists a mass N of asymmetric pure strategy equilibria (see discussion in Section 2 and 3), the maximum level of available technology, θ, is equivalent to the solution of the above problem for some n in $[0, N]$.

□

Proof of Theorem 3. The function $F_{DS}(\cdot,\cdot)$ is twice continuously differentiable. Since the factor price in equilibrium is given by (2.22),

$$\left.\frac{\partial w(Z,\theta)}{\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N}{1-\alpha}\left.\frac{\partial^2 G(Z,\theta)}{\partial Z^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N}{1-\alpha}\left.\frac{\partial^2 F_{DS}(Z,\theta)}{\partial Z^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}},$$

and

$$\left.\frac{\partial w(Z,\theta)}{\partial \theta}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N}{1-\alpha}\left.\frac{\partial^2 G(Z,\theta)}{\partial Z\partial\theta}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N}{1-\alpha}\left.\frac{\partial^2 F_{DS}(Z,\theta)}{\partial Z\partial\theta}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

We apply the IFT to the FOC of (2.21), which yields

$$\left.\frac{\partial \theta(Z)}{\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = -\left.\frac{\partial^2 F_{DS}(Z,\theta)/\partial Z\partial\theta}{\partial^2 F_{DS}\left(Z,\theta\right)/\partial\theta^2}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

Thus, the total derivative of the equilibrium price is

$$\left.\frac{\mathrm{d} w(Z,\theta)}{\mathrm{d} Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} = \frac{N}{1-\alpha}\left.\left[\frac{\partial^2 F_{DS}(Z,\theta)}{\partial Z^2} - \frac{\left(\partial^2 F_{DS}(Z,\theta)/\partial\theta\partial Z\right)^2}{\partial^2 F_{DS}(Z,\theta)/\partial\theta^2}\right]\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}.$$

Hence, the failure of negative semi-definiteness of the Hessian of F_{DS}, ∇^2_{DS}, is a necessary and sufficient condition for the existence of technological bias.

□

Proof of Theorem 4. The factor price in equilibrium is given by (2.22). Since Assumption 2 holds,

$$\left.\frac{\partial w(Z,\theta,N)}{\partial N}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}}\left.\frac{\partial N(Z)}{\partial Z}\right|_{Z=\bar{Z}} > 0.$$

On the one hand, if the Hessian of F_Q, ∇^2_{DS}, is not negative semi-definite at $(\bar{Z}, \theta(\bar{Z}))$, then,

$$\left.\frac{\mathrm{d} w(Z,\theta,N)}{\mathrm{d} Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} > 0,$$

as in Theorem 3. On the other hand, if

$$\left.\frac{\partial w(Z,\theta,N)}{\partial N}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} \left.\frac{\partial N(Z)}{\partial Z}\right|_{Z=\bar{Z}} > \\ > \left.\frac{\partial w(Z,\theta,N)}{\partial Z}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} + \left.\frac{\partial w(Z,\theta,N)}{\partial \theta}\right|_{\substack{Z=\bar{Z}\\ \theta=\theta(\bar{Z})}} \left.\frac{\partial \theta(Z)}{\partial Z}\right|_{Z=\bar{Z}},$$

then there exists technological bias even if ∇^2_{DS} is negative semi-definite at $(\bar{Z}, \theta(\bar{Z}))$.

□

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Chapter 3

Public Expenditure Distribution, Voting, and Growth

In this paper we try to explain why the misallocation of resources across different productive sectors tends to persist over time. We document that there is a link between the distribution of the public expenditure across sectors and the sectoral composition of an economy. We propose a general equilibrium model that interprets this stylized fact as a reduced form representation of two structural relations, namely, the dynamic effect of the public expenditure on the future distribution of value added and the influence of the distribution of vested interests across sectors on current public policy decisions. The model predicts that different initial sectoral compositions cause different future streams of public expenditures and therefore different paces of development.

Keywords: Public Expenditure; Sectoral Composition; Vested Interests; Economic Growth.

3.1 Introduction

We present a general equilibrium growth model that helps us understand how the public expenditure distribution, the sectoral composition, and the productive efficiency of an economy are intertwined along a development path. The mechanism that connects these three elements is a voting process that drives public policy.

Given an economy's stock of physical and human capital, labor, and technology, the way in which these are allocated across sectors -and across firms or even across plants and individuals- determines the productive capacity of an economy. See Jones [27] for a recent analysis of the role of misallocations in explaining income differences across countries. Once we acknowledge the importance of the misallocation of resources, we might ask ourselves why countries with less-than-efficient allocations of resources do not shift to more efficient ones, or why different allocations exist in the first place. The literature on political economy and economic growth answers this questions interpreting the misallocation as the equilibrium outcome of a political process, where institutions determine the way in which resources are distributed and the distribution of resources itself influences the type of institutions an economy adopts. See Acemoglu, Johnson, and Robinson [1] for an extensive overview. For example, it might not be in the interests of the ruling elite to improve the allocation of resources, even though the aggregate efficiency of the economy as a whole might increase.

The differences in institutions may explain well the differences in efficiency between developing and developed countries.[1] Nevertheless, it is not clear why differences in allocative efficiency should exist between countries with the same institutional quality. For example, why do misallocations persist even among countries endowed with democratic systems? In this paper we claim that the link between public policy, sectoral composition, and aggregate efficiency helps explaining this question.

We employ a dynamic model with voting applied to economic growth.[2] Public decisions may influence the sectoral evolution of an economy through taxation and public investment.[3] At the same time, policies are usually tailored to or influenced by the productive structure of an economy.[4] Moreover, the sectoral composition of an economy explains a significant part of aggregate efficiency.[5] We combine in

[1]For example, Cuberes and Jerzmanowski [15] show how growth reversals are more likely in less democratic countries.

[2]See Krusell, Quadrini, and Ríos-Rull [30] for an overview of the related literature.

[3]Galor and Moav [22] for example model the transition to a different sectoral composition as driven by publicly provided education programs.

[4]For example, Galor, Moav, and Vollrath [23] show how the inequality in land ownership caused delays in the emergence of public schooling and therefore in the transition from agricultural to industrial economies.

[5]For an analysis of the role of structural transformation on aggregate efficiency, see for

a growth model the endogenous determination of both the policy and the sectoral composition of an economy. Policies influence the future sectoral composition, the current sectoral composition drives the policies, and both contribute to the evolution of aggregate efficiency over time.

On the one hand, the sectoral composition of the economy affects its aggregate efficiency. If the different sectoral inputs are substitutable enough in the production of the final consumption good, then specialization increases aggregate efficiency. If instead the sectoral inputs are complementary, then diversification increases aggregate efficiency.[6] In our framework, the misallocation of resources consists of the distance of an economy from its efficient level of diversification or specialization. Let us illustrate the effect of misallocation on aggregate efficiency by means of an example.

Example 2. Suppose an economy that produces a unique final good using human capital coming from two sectors, 1 and 2. The aggregation in this economy is given by

$$Y = H_1^{1/3} + H_2^{1/3},$$

where H_1 is the human capital of sector 1 and H_2 is the human capital of sector 2. Suppose furthermore that the human capital in sector 1 is equal to the individual efficiency endowment h_1 of the individuals in sector 1 multiplied by the population θ_1 that works in sector 1. The same holds in sector 2. In other words,

$$H_1 = \theta_1 h_1 \text{ and } H_2 = \theta_2 h_2,$$

where the total population, $\theta_1 + \theta_2$, is equal to 1 for simplicity. Moreover, suppose that at equilibrium the population distribution mirrors the distribution of individual efficiency, that is,

$$\frac{\theta_1}{\theta_2} = \frac{h_1}{h_2}.$$

Let us call x the fraction of efficiency endowment in sector 1, that is, $x \equiv h_1/(h_1 + h_2)$. If x is equal to $1/2$ then there is perfect diversification and if x tends to 0 or 1 there is perfect specialization in sector 1 or 2. Solving for the aggregation Y we obtain

$$Y = A(x)(h_1 + h_2)^{1/3},$$

where $A(x)$ represents the aggregate efficiency of the economy and is given by

$$A(x) = \left[x^{2/3} + (1-x)^{2/3}\right].$$

example Caselli [12], Chanda and Dalgaard [13], Córdoba and Ripoll [14], and Duarte and Restuccia [17].

[6] By diversification we mean a more even distribution of resources across the same set of sectors, not an expansion of the set. Similarly specialization refers to a more uneven distribution on the same set of sectors.

In this case, aggregate efficiency is maximal when the economy is perfectly diversified, that is, when $x = 1/2$. This is due to the fact that the human capital types are relatively complementary in the production of the final good. If instead the types of sector-specific human capital are more substitutable, for example if the aggregation is given by

$$Y = H_1^{2/3} + H_2^{2/3},$$

then $A(x) = \left[x^{4/3} + (1-x)^{4/3}\right]$ and aggregate efficiency increases with specialization.

On the other hand, redistributions of public expenditure can be blocked in the voting process because they affect the current interests of the individuals working in different sectors. Hence, there exists the possibility of political blockages of reforms, whose likelihood depend on the initial sectoral composition of an economy. A change towards sectoral specialization is more likely in economies that are already specialized and a change towards sectoral diversification is more likely in economies with an already diversified distribution. Let us provide another illustrative example.

Example 3. Suppose two economies, A and B, that have two sectors each, 1 and 2. Suppose furthermore that the population distribution across sectors mirrors the value added distribution. Economy A's value added is composed 60% by Sector 1 and 40% by Sector 2. The policy of its government mirrors these relative magnitudes and distributes public expenditure 60% to Sector 1 and 40% to Sector 2. In Economy B instead the shares are 70% for Sector 1 and 30% for Sector 2, and the public expenditure is distributed accordingly. Both governments propose the same new public expenditure distribution, that is, 80% to Sector 1 and 20% to Sector 2. In both economies, the population in Sector 2 opposes the proposal because it would lose shares within government's budget if the proposal was approved. The population in Sector 1 instead supports the proposal. While in Economy A up to 40% of the population opposes the proposal, in Economy B only 30% of the population does so. Hence, Economy B is more likely to approve the proposal. If instead the proposal is to distribute expenditure $50\% - 50\%$ between Sector 1 and 2, Economy A is more likely to approve the proposal.

The reduced form prediction of the model is that the output depends on the history of government proposals' successes and failures, whose probabilities depend on the sectoral composition of the economy in the initial period.[7] This generates delays in the development paths of economies with too diversified or

[7]Krusell and Ríos-Rull [31] cast a similar connection between the initial skill composition on an economy and the likelihood to adopt technological innovations.

too concentrated sectoral compositions.[8] Hence, any exogenous alterations of the initial sectoral composition can trigger different streams of reforms and therefore different development paths. If the economy is stuck to an inferior development path characterized by a persistent political blockage of possibly growth-enhancing proposals, any exogenous shock that modifies the sectoral composition may remove the political blockage and shift the economy to higher long run income levels. This may refer, first, to economic crises, where mass unemployment rises unevenly across sectors and reallocates the political opposition to reforms. Second, to trade liberalizations and technological innovations, where for example the introduction of new tradable goods changes the combination of vested interests into organized political blockages. Third, to institutional changes such as the decentralization of political, administrative, and fiscal authority, where the subnational public decision units may face less sectoral complexity in the allocation of public resources, and the possibility of different political majorities across regions may remove the political blockages that occur at the national level.

The paper is organized as follows. Section 2 presents some stylized facts behind the theory of the model. Section 3 presents the set-up and the equilibrium solution. Section 4 analyzes the dynamics of the development process and their relation to the public expenditure distribution. Section 5 characterizes the political dynamics and the determinants of political blockages and approvals. Section 6 discusses the results and Section 7 draws the final conclusions. Tables, figures, proofs, and numerical exercises are provided in the Appendix.

3.2 Stylized facts

In this section we provide an empirical intuition of the phenomena we deal with. The stylized facts reported here justify the qualitative properties of the structural relations we can extract from the model. We use data from Eurostat's government statistics (gov_a) and national accounts (nama_nace). In particular, we consider annual national data on general expenditure by function (COFOG), that decomposes total public expenditure into 10 categories, namely, general public services, defense, public order and safety, economic affairs, environmental protection, housing and community amenities, health, recreation, culture and religion, education, and social protection. Since in our model we consider public resources devoted to productive sectors, we focus on public expenditure in economic affairs, which

[8]Alesina and Drazen [2] and Fernandez and Rodrik [19] obtain similar status quo biases due to the nonneutrality in which the gains and losses from a reform are distributed within the society.

is itself decomposed into 9 sectors.[9] Eurostat provides data for 29 countries for different time periods, from the 1990's to 2008. Total government expenditure has different components, that is, compensation of employees, subsidies, property income, social benefits, other current transfers, investment grants, capital transfers, acquisitions less disposals of non-financial non-produced assets, intermediate consumption, final consumption expenditure, and gross capital formation. The distribution across sectors is similar for each component, which suggests that both productive and unproductive public expenditures are distributed similarly across sectors.

As a proxy for the sectoral composition, we consider annual accounts detailed breakdowns by branches (NACE), that decomposes gross value added into 7 sectors.[10] In this dataset there are 33 countries. We drop from both government expenditure and value added datasets the countries with excessive missing values, we intersect the two sets of countries, and eventually we are left with a balanced panel of 20 countries for 9 years, from 2000 to 2008. We provide the list of countries and some summary statistics on government expenditure in Table 3.1 in the Appendix. Table 3.1 contains the values of total general government expenditure in economic affairs as a percentage of GDP. This values are roughly stable through time and show quite a large cross-country variation. The average expenditure in the whole sample is 4.6% of GDP, with no significant changes through time, which corresponds on average to 10% of total public expenditure.

For each country, we order the sectors increasingly according to their share in overall economic affairs expenditure in the first available period, which is 2000 for most countries. We correct only for some isolated negative values in the government expenditure due to disposals of non-financial non-produced assets. We then compute each country's public expenditure and value added cumulative distribution function for each year. In the countries where the value added is more concentrated in 2008 than in 2000, also the government expenditure seems more concentrated, while in the countries where the value added is more dispersed,

[9]The sectors are general economic, commercial, and labour affairs; agriculture, forestry, fishing and hunting; fuel and energy; mining, manufacturing and construction; transport; communication; other industries; research and development; and a residual category, economic affairs n.e.c.

[10]We could choose finer decompositions such as 31 and 60 branches, but since the Gini coefficients we derive later are sensible to the decomposition level, we choose the closest level to the 9 categories from the government statistics. The sectors are agriculture, hunting, forestry and fishing; total industry excluding manufacturing and construction; manufacturing; construction; wholesale and retail trade, repair of motor vehicles, motorcycles and personal and household goods, hotels and restaurants, transport, storage and communication; financial intermediation, real estate, renting and business activities; public administration and defense, compulsory social security, education, health and social work, other community, social and personal service activities, private households with employed persons.

also the government expenditure is more dispersed. In particular, there seems to be a shift in the degree of first-order stochastic dominance from 2000 to 2008. Hence, using Gini coefficients suits the observed distributional dynamics, since an increase in the stochastic dominance corresponds unambiguously to an increase in the Gini coefficient.[11] We compute the Gini coefficient of all distributions, we normalize it to the 2000 level, and compare the evolution of value added and public expenditure for each country. For illustrative reasons, we report here only the cases of Germany in Figure 3.1 and of Italy in Figure 3.2.

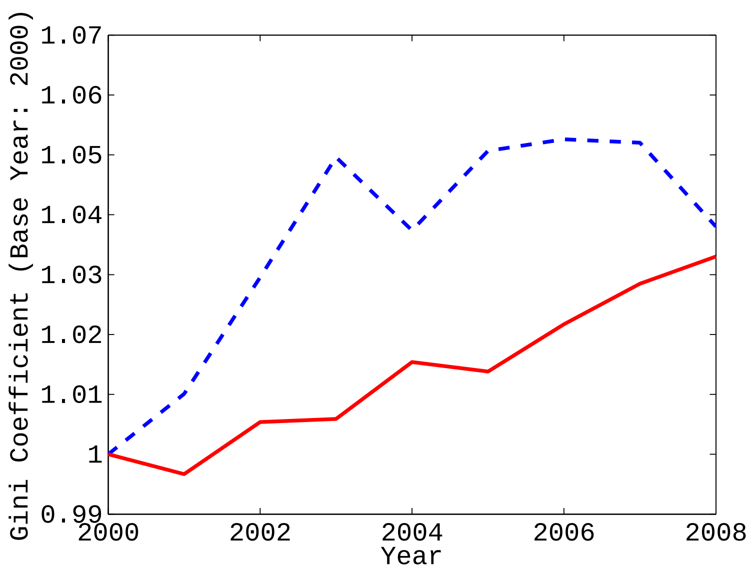

Figure 3.1: Germany's Value Added (dashed) and Government Expenditure (solid) Gini Coefficients through time.

From 2000 to 2008, Germany's expenditure and value added Gini coefficients increase by 3.3% and 3.8%. Italy's coefficients increase by 7.6% and 5.0%.[12] Germany's expenditure is more stable than Italy's, while value added seems to follow a similar pattern. The main message we draw from this exercise is that there seems to be a comovement between the value added and the public expenditure over time. In particular, in the countries where the value added gets more concentrated, also the public expenditure gets more concentrated. In the countries where instead the value added gets less concentrated, also the public expenditure gets less concentrated over time.

[11] We cannot analyze sector-specific pairwise correlations because the sectors in which Eurostat classifies value added and government expenditure do not coincide. This is why we are forced to employ a synthetic measure such as the Gini coefficient.

[12] The spike in 2006 for Italy is probably due to an electoral-year bias.

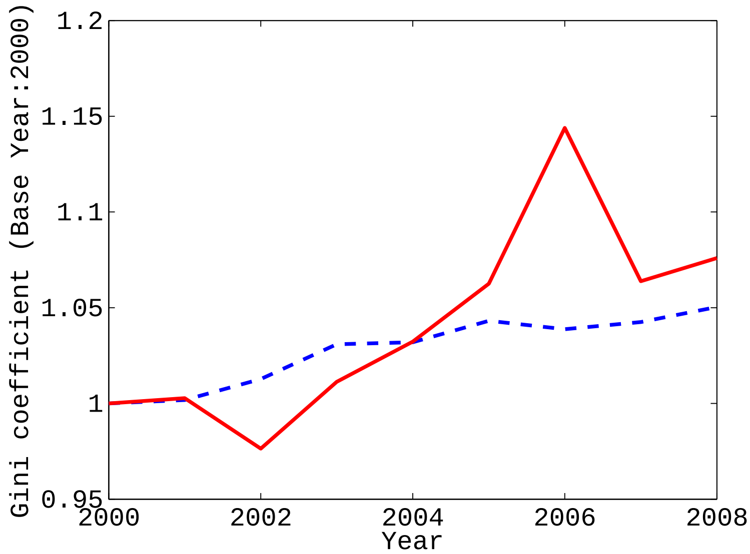

Figure 3.2: Italy's Value Added (dashed) and Government Expenditure (solid) Gini Coefficients through time.

In our model we claim that the joint evolution of value added and expenditure is driven by two mechanisms. First, if the government expenditure gets more concentrated, so does the value added in the following period. Second, the value added distribution partially describes the relative political influence of each sector within the economy. Hence, the government expenditure distribution in each period tends to mirror the value added distribution in the same period. The model therefore suggests two simultaneous and autonomous equations where the endogenous variables are the value added and government expenditure Gini coefficients, that is,

$$\begin{cases} \mathrm{VA}_t = f_t\left(\mathrm{GE}_{t-1}\right), \\ \mathrm{GE}_t = g_t\left(\mathrm{VA}_t\right), \end{cases} \tag{3.1}$$

where VA_t is the value added Gini coefficient in period t, GE_t is the government expenditure Gini coefficient in period t, and f_t and g_t are possibly non-linear equations that contain exogenous variables and error terms. Figure 3.3 represents the first equation of (3.1) and Figure 3.4 represents the second equation for Germany and Italy.

The lines in Figure 3.3 and in Figure 3.4 represent the predicted values of linear regressions. We choose two specific countries for the clarity of exposition. With respect to our sample of European Countries, Germany and Italy have a comparable level of development, geographical extension, and institutional frame-

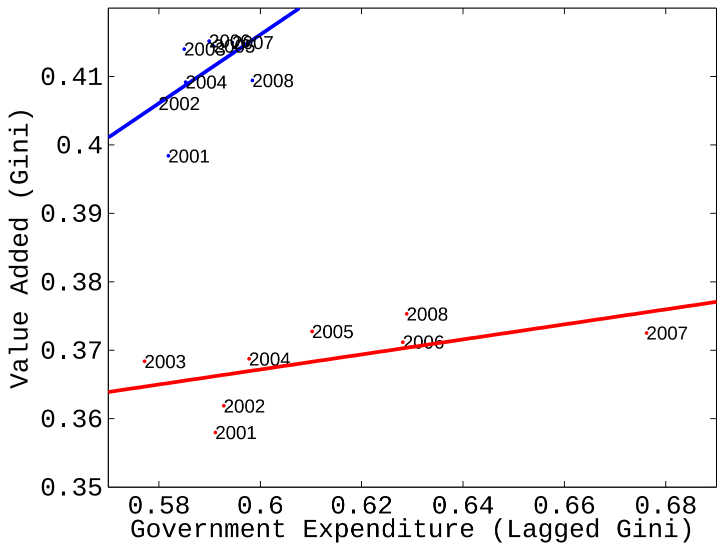

Figure 3.3: Value added as a function of lagged government expenditure for Germany (upper side) and Italy (lower side).

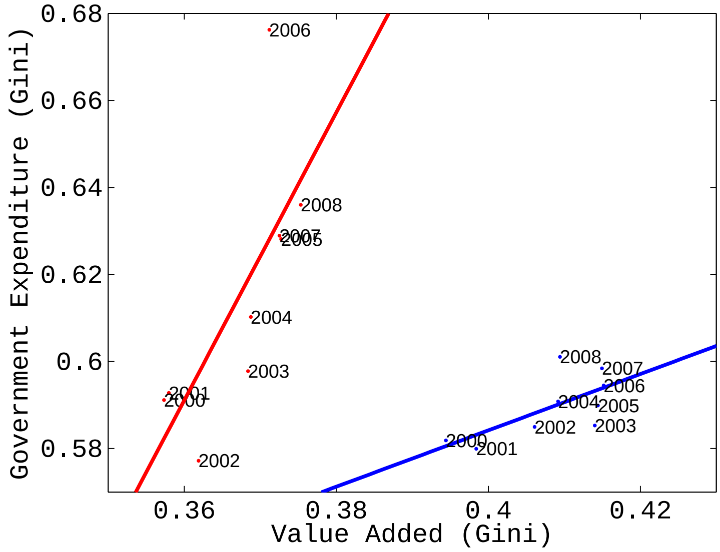

Figure 3.4: Government expenditure as a function of value added for Germany (on the right) and Italy (on the left).

work, in the sense that they were part of the European Union for the whole time period. In the case of Germany and Italy the relations of (3.1) seem to be positive.

The same relations for the whole pool of countries are reported in Figures 3.13 and 3.14 in the Appendix. There seems to be a high cross-country dispersion in levels but the relations seem to be similar across countries. The value added and government expenditure distributions tend to move in the same direction, either towards concentration or towards diversification. My model interprets the distinction between these two sets of countries as the difference in the complementarity among sectors. If the productive sectors of a country are complementary, for example if the sector-specific value added growth rates are correlated over time, then the country tends to diversify its production among different sectors, that is, the Gini coefficient of the country will decrease over time. If instead the productive sectors are substitutable, the Gini coefficient will increase.

Once we merge the two structural relations in (3.1), we obtain two laws of motions for both the value added and the public expenditure, that is,

$$\mathrm{VA}_t = f_t\left(g_t\left(\mathrm{VA}_{t-1}\right)\right),$$

and

$$\mathrm{GE}_t = g_t\left(f_t\left(\mathrm{GE}_{t-1}\right)\right),$$

which means that the value added and the public expenditure are autocorrelated. We represent these laws of motion for our pool of countries in Figure 3.5 and in Figure 3.6.[13] We assign a color to each country and we plot its observations through time with the same color.

There is a wide cross-country dispersion, which means that there must be country-specific characteristics driving the - levels of - Gini coefficients of both government expenditure and value added. In other words, the role of the exogenous variables in (3.1) is relevant for the levels of concentration. Some examples of relevant country-specific characteristics are the level of development, the geographical extension, and the institutional framework. First, more developed countries seem to be more sectorally diversified. Second, since industrial sectors tend to cluster geographically, the smaller the country, the more concentrated the value added distribution. Third, if a country commits to certain structural reforms, like the Eastern European countries that accessed the European Union in 2004, this affects their government expenditure distribution independently from their value added distribution.

[13]In Figure 3.5 we drop Bulgaria and Luxembourg as outliers. They would simply increase the scale of the graph.

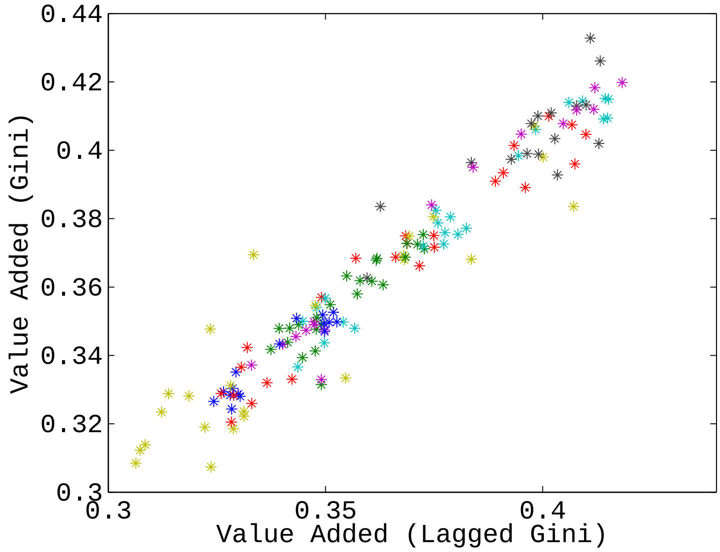

Figure 3.5: Law of motion for the value added distribution.

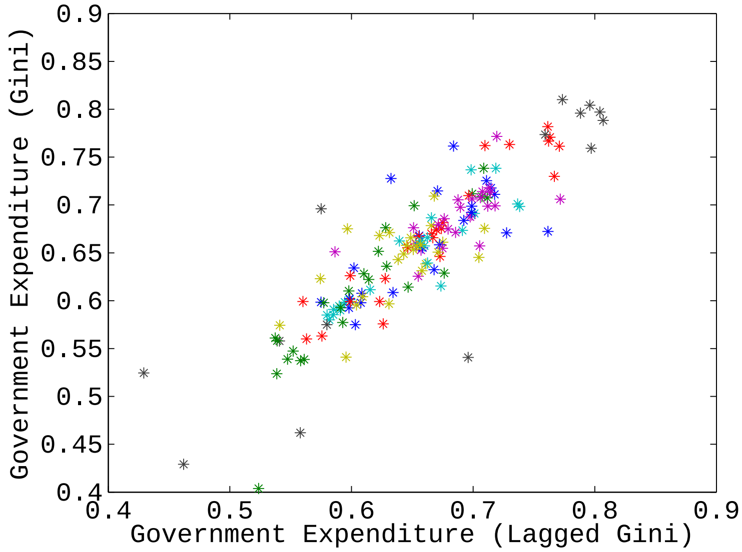

Figure 3.6: Law of motion for the government expenditure distribution.

3.3 The Model

Consider an economy where a firm produces a unique final good according to a Cobb-Douglas production function,

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \tag{3.2}$$

where $K_t \in \mathbb{R}_+$ is the stock of physical capital at time t and $H_t \in \mathbb{R}_+$ parameterizes the aggregate level of human capital. The shares of these two components are described by the constant $\alpha \in (0,1)$. The human capital at the aggregate level is a combination of a continuous variety of sectoral specializations, each of them representing a different type of sectoral expertise. Therefore aggregate human capital is an additively separable sum of a continuum of mass J of sector-specific levels of human capital, that is,

$$H_t \equiv \left[\int_0^J [H_t(j)]^{1-\alpha} \, dj \right]^{\frac{1}{1-\alpha}}, \tag{3.3}$$

where j indexes the generic sector in the interval $[0, J] \subset \mathbb{R}_+$, and $H_t(j) \in \mathbb{R}_+$ indicates the sector-specific human capital. The firm operates under perfect competition. Since there is a unique final good, we can normalize its price to 1. The maximization problem for the final good firm at time t is

$$\max_{K_t, \{H_t(j)\}_{j \in [0,J]}} K_t^\alpha \int_0^J [H_t(j)]^{1-\alpha} \, dj - \int_0^J w_t(j) H_t(j) dj - R_t K_t, \tag{3.4}$$

where $w_t(j) \in \mathbb{R}_+$ is the wage per efficiency unit in sector j and $R_t \in \mathbb{R}_+$ is the rate of return on capital. The first order condition (FOC) with respect to the physical capital is

$$R_t = \alpha K_t^{\alpha-1} H_t^{1-\alpha}, \tag{3.5}$$

while the FOC with respect to the j-th sector-specific human capital $H_t(j)$ is

$$w_t(j) = (1-\alpha) K_t^\alpha [H_t(j)]^{-\alpha}. \tag{3.6}$$

From the FOC we can recover the optimal shares of physical and human capital within the final output, i.e.,

$$R_t K_t = \alpha Y_t \quad \text{and} \quad \int_0^J w_t(j) H_t(j) dj = (1-\alpha) Y_t. \tag{3.7}$$

The accumulation of physical capital is driven by private investment under full depreciation, i.e., $i_t = K_{t+1}$.

There is a continuum of individuals of mass 1, each of them indexed by i. Each individual lives for two periods, t and $t+1$, and at the beginning of the second she has a single child, so that every period a constant cohort of individuals of mass 1 is born. Every individual selects the sector where to work in the first period, and works, votes, consumes and leaves a bequest in the second period. The timing in period t unfolds as follows:

1. the individual receives an after-tax bequest from her parent,

$$(1-\tau_t^A)b_t^i(j'),$$

where $b_t^i(j') \in \mathbb{R}_+$ is the bequest given the sector j' where the parent worked and τ_t^A is the tax rate on bequests,

2. she saves inelastically all her financial endowment and lends it to the firm,

3. she observes how total public investment $A_t = \int_0^J a_t(j)dj$ is allocated across sectors according to the distribution F_t,

4. she selects the sector j that guarantees her the highest wage income in period $t+1$. Once she chooses the sector, she receives from the government the right to access the sector-specific public investment $a_t(j)$.[14]

In period $t+1$, the individual realizes income.

1. She accumulates human capital $h_{t+1}(j)$ that depends on the public investment directed to the sector she chose in the previous period, i.e., $h_{t+1}(j) \equiv h(a_t(j))$.[15] We assume decreasing individual returns to scale on public investment, i.e., the function h is homogeneous of degree $\epsilon \in (0,1)$.

2. She supplies inelastically one unit of labor in the sector she has chosen and receives an interest on her savings. Hence, her realized income is the sum of wage and capital income, i.e., $w_{t+1}(j)h_{t+1}(j) + R_{t+1}(1-\tau_t^A)b_t^i(j')$.

The realized income is taxed by the government at rate τ_t^E. After being taxed, an individual working in sector j is supposed to receive a lump-sum transfer $\xi_{t+1}(j)$ from the government. Hence, she votes in the following way:

3. she observes a proposal presented by the government. This proposal consists of an alternative public expenditure distribution, $\bar{F}_{t+1}$,

4. she votes in favor of the proposal if and only if $\xi_{t+1}(j)|_{\bar{F}_{t+1}} \geq \xi_{t+1}(j)|_{F_t}$, i.e., if the transfer she would get is higher with the proposed distribution than with the old one. Otherwise, she votes against it.

[14] We can think of $a_t(j)$ as the public investment in sector j-specific research and development activities, or as the level of congestion-free public education guaranteed for the formation in sector j. For example, imagine that each individual that wants to work in sector j has to attend a sector-specific university. Then, $a_t(j)$ may represent public financing of that university.

[15] We rule out any congestion effect by considering public investment in a sector-specific pure public good. This simplification defines the distinction from the opposite case of complete congestion, where the equilibrium population distribution would be independent of the public expenditure distribution.

The individual final income is

$$I^i_{t+1}(j) \equiv (1-\tau^E_{t+1})\left[w_{t+1}(j)h_{t+1}(j) + R_{t+1}(1-\tau^A_t)b^i_t(j')\right] + \xi_{t+1}(j), \tag{3.8}$$

where for simplicity we avoid to report the dependence of an individual's income on the sector where her parent worked, j'. The preferences of individual i born at time t and working in sector j in $t+1$ are represented by a log-linear utility function,

$$u^i_t = (1-\beta)\ln c^i_{t+1} + \beta \ln b^i_{t+1},$$

where $\beta \in (0,1)$ indicates the preference share of bequests, $c^i_{t+1} \in \mathbb{R}_+$ is second period's consumption and $b^i_{t+1} \in \mathbb{R}_+$ is the transfer to the offspring due to a joy-of-giving bequest motive.[16] The optimal choice of how much to consume and how much to leave as a bequest for member i of generation t in period $t+1$ is then the solution to the following problem,

$$\begin{aligned} \max_{c^i_{t+1}, b^i_{t+1}} \; & u^i_t = (1-\beta)\ln c^i_{t+1} + \beta \ln b^i_{t+1} \\ \text{subject to} \; & c^i_{t+1} + b^i_{t+1} \leq I^i_{t+1}(j), \end{aligned} \tag{3.9}$$

where $I^i_{t+1}(j)$ is defined in (3.8). The first order conditions yield

$$c^i_{t+1}(j) = (1-\beta)I^i_{t+1}(j) \;\text{ and }\; b^i_{t+1}(j) = \beta I^i_{t+1}(j). \tag{3.10}$$

Note again that the optimal choices and total income should be indexed also by j', as the parental bequest $b_t(j')$ is a state variable of the problem. We neglect this element because it does not affect the aggregate dynamics of the model, although it does indeed drive the evolution of a dynasty's income through time and affect the cross-sectional income inequality.

The indirect utility function,

$$v^i_t(j) = \ln I^i_{t+1}(j) + (1-\beta)\ln(1-\beta) + \beta \ln \beta, \tag{3.11}$$

is a monotonically increasing function of final income, which depends positively on the lump-sum transfer devoted to sector j. This is the interest that drives the

[16] We choose a warm-glow-of-giving type of utility function merely for analytical simplicity. Allowing for an alternative bequest motive such as in Alonso-Carrera, Caballé, and Raurich [4], that is, an interest in the after-tax contribution to the future life-time income of the offspring, would not change qualitatively the optimal choice of bequest and consumption. See Michel, Thibault, and Vidal [34] for an overview of alternative mechanisms of intergenerational altruism.

individual's voting behavior, that is, voting yes to the proposal $\bar{F}_{t+1}$ if and only if $\xi_{t+1}(j)|_{\bar{F}_{t+1}} \geq \xi_{t+1}(j)|_{F_t}$.[17]

It is analytically difficult to solve models with sequential voting and growth since the voters' policy preferences should be in general formed on the prediction of the equilibrium effects of a change in the current policy on the future path of both the economic state variable and the policies. This difficulty originated several shortcuts in the literature. First, we could restrict the voting at time zero only, as in Bertola [10] and Alesina and Rodrik [3]. Second, we could assume an equilibrium law of motion for policies consistent with individual maximization and market clearing which is used by the agents to form preferences over alternative policies, as in Krusell and Ríos-Rull [31] and Krusell, Quadrini, and Ríos-Rull [30]. Third, we could focus on Markov-perfect equilibria á la Meltzer and Richard [33] under aggregation, for which the policies result as a function of the pay-off relevant state and nothing else, as in Krusell and Ríos-Rull [32] and Azzimonti, de Francisco, and Krusell [6] and [7]. Fourth, we could restrict the ability of the agents to predict the policy outcome, as in Boldrin [11], Cukierman and Meltzer [16], and Huffman [26]. Fifth, we could restrict the ability of agents to predict the economic outcome, or at least to make it not necessary for the median voter to be forward looking, as in Persson and Tabellini [37], Saint-Paul and Verdier [42] and [43], Glomm and Ravikumar [24], Perotti [36], Fernandez and Rogerson [20]. The latter stream of literature is closer to our approach, in the sense that the overlapping generations framework helps in cutting the ties to the future. In order to solve the problem of sequential voting without commitment on the government side, we make the following key assumptions. First, voters vote only once. Second, agents have a joy-of-giving bequest motive. Third, they vote only when they are old. Fourth, leisure does not enter the utility function. In this way, we obtain that voters do not care about the next vote, neither through the equilibrium effects on the stocks nor on the prices.

The government follows two balanced-budget constraints. It finances public investment A_t by taxing bequests, i.e.,

$$A_t = \tau_t^A \int_0^1 b_t^i(j')di, \tag{3.12}$$

[17]We stress the timing of the individuals because the results below are sensible to it. In particular, if we put the voting decision in the first period and the sector selection in the second, on the one hand, individuals would not have a vested interest in voting in favor of a sector, as ex-ante they would not belong to any. On the other hand, individuals' final income would depend on aggregate efficiency, so they would vote in favor of efficiency-increasing public expenditure distribution proposals. There would be no conflict of interests and efficiency dynamics would be entirely driven by government proposals.

and finances aggregate lump-sum transfers E_t by taxing realized income, i.e.,

$$E_t = \tau_t^E \int_0^1 \left[w_t(j)h_t(j) + R_t(1-\tau_{t-1}^A)b_{t-1}^i(j')\right] di. \tag{3.13}$$

We could consider different forms of taxation, but since the focus of our paper is on the public expenditure distribution we can neglect the distortions introduced by a bequest flat tax rate τ_t^A, which for our purposes are not qualitatively different from the distortions of a labor, income, or savings tax. This indifference is mainly due to the inelastic labor supply assumed above. We do not consider lump-sum taxation because we want to keep the trade-off between private investment in physical capital and public investment in human capital.[18] Moreover, the government imposes a tax rate τ_t^E on realized income and it redistributes the revenue E_t through lump-sum transfers. The proportional tax rate τ_t^E does not introduce any distortion in the dynamics of the economy because the imposition occurs when the individual has already realized her income. We can think of A_t as the investment in public education or infrastructures, and E_t as a system of transfers for redistributive purposes.

The public resources available at any period, A_t and E_t, are distributed across sectors according to the public expenditure distribution F_t.[19] Thus, each sector j receives the sector-specific public investment $a_t(j) \in [0, A_t]$ and the sector-specific transfer $e_t(j) \in [0, E_t]$ according to F_t. Note that by construction

$$\int_0^J a_t(j)dj = A_t$$

and

$$\int_0^J e_t(j)dj = E_t.$$

The functions a_t and e_t map each sector j in $[0, J]$ to levels of sector-specific public investment $a_t(j)$ and transfer $e_t(j)$. Let us define the share $\tilde{a}_t(j) \equiv a_t(j)/A_t$ and the share $\tilde{e}_t \equiv e_t(j)/E_t$. By construction, $\tilde{a}_t(j) = \tilde{e}_t(j)$ for every j in $[0, J]$, since the public expenditure distribution across sectors F_t affects all types of public expenditure. Therefore, from now on we refer to $\tilde{a}_t(j)$ bearing in mind that it coincides with the distribution of overall public expenditure across sectors. We can view $\tilde{a}_t$ as a density function over the support set $[0, J]$, since $\tilde{a}_t(j) \geq 0$ for every j and $\int_0^J \tilde{a}_t(j)dj = \int_0^J [a_t(j)/A_t]\, dj = 1$. We define $\mathscr{A}_t(j) \equiv \int_0^j \tilde{a}_t(s)ds$ as the

[18] See Galor and Moav [22] and Galor, Moav, and Vollrath [23] for the effects of this trade-off on the growth regimes of an economy.

[19] The restriction of a common distribution for both productive and unproductive public expenditure is justified by the fact that in the data analyzed above the distribution across sectors is similar between different components of public expenditure.

cumulative distribution function of public expenditure over the sectors' interval. Hence, F_t defines $\tilde{a}_t(j)$ and $\mathscr{A}_t(j)$ for every j in $[0, J]$.

The public expenditure distribution evolves over time. In every period $t+1$ the government formulates a proposal $\bar{F}_{t+1}$. If the individuals' voting outcome approves the proposal, then the government updates the distribution according to $F_{t+1} = \bar{F}_{t+1}$. Otherwise, the government ignores the proposal and sets $F_{t+1} = F_t$. The government formulates the proposal by making a random draw from the set of all possible public expenditure distributions which would increase next period's aggregate efficiency, that is,

$$\left\{ \bar{F}_{t+1} \text{ such that } Y_{t+2}|_{\bar{F}_{t+1}} \geq Y_{t+2}|_{F_t} \right\},$$

where $Y_{t+2}|_{\bar{F}_{t+1}}$ is the value of Y_{t+2} if $F_{t+1} = \bar{F}_{t+1}$ and $Y_{t+2}|_{F_t}$ is the value of Y_{t+2} if $F_{t+1} = F_t$. In our model, the government does not coincide with a central planner. It does not maximize any welfare function, it simply follows every period the constitutional rule described above. Its decisions are simply the result of the chance in the draws of $\bar{F}_{t+1}$ every period and individuals' voting outcomes. In a sense, it is not an agent of our economy, but rather an aspect of the model's technology.

The reason for the adoption of this set-up for the formulation of policies is that the policy space is multidimensional. The government has to choose a share $\tilde{a}_t(j)$ in $[0, 1]$ for each sector j in the continuum $[0, J]$, under the constraint that $\int_0^J \tilde{a}_t(j)dj = 1$. Hence, the median voter theorem yields no prediction, despite individual preferences are single-peaked over any proposal due to (3.11) and (3.8). As an alternative to the median voter theorem we could allow for probabilistic voting, where social groups with greater homogeneity of preferences would be more politically powerful than those whose preferences are dispersed because the equilibrium policy would depend on the magnitude and density of social groups rather than on the median position of voters.[20] Nevertheless, we do not adopt probabilistic voting because we want to obtain the possibility of political blockages of possibly growth-enhancing reforms and not simply the absence of politically unfeasible policy proposals. In this sense, our framework is reminiscent of the agenda-setting model of Romer and Rosenthal [40] and [41], since voters let the government control the agenda and respond as price takers to the government's supply offer, where in our case the supply consists of a given distribution of public expenditure. Voters can only choose between accepting the proposal of the "setter" or rejecting it in favor of an institutionalized "reversion" distribution, which in our case is the previous period's distribution.

[20] See, e.g., the applications to special-interest politics in Persson and Tabellini [38, Chapter 7] and Bellettini and Ottaviano [9] or to social security in Profeta [39] and Galasso and Profeta [21].

The connections between the different sides of this economy consist of, first, the savings-investment market equilibrium condition,

$$K_{t+1} = i_t = (1 - \tau_t^A) \int_0^1 b_t^i(j')di, \tag{3.14}$$

where the right hand side is the sum of all after-tax bequests saved by individuals in the first period of life. Second, we consider the aggregation rule for sector-specific human capital,

$$H_t(j) = \theta_t(j)h_t(j), \tag{3.15}$$

where $\theta_t(j) \in [0, 1]$ is the portion of total population working in sector j at time t. Since the mass of the population is 1, $\int_0^J \theta_t(j)dj = 1$. $\theta_t(j)$ represents therefore the population density in sector j at time t. Note that $H_t(j)$ parameterizes the aggregate demand for sector j-specific efficiency units expressed by the firm. In equilibrium this must be equal to the total supply, which is the individual supply $h_t(j)$ of efficiency units of those who chose sector j multiplied by the total number $\theta(j)$ of individuals in sector j. Third, the transfer received by sector j is equally distributed among the population working in sector j, i.e.,

$$e_t(j) = \xi_t(j)\theta_t(j), \tag{3.16}$$

for every j in $[0, J]$. Fourth, the workers select the sector according to the wage income that working in that sector guarantees. In equilibrium this implies that the wage income must be the same across sectors, that is to say,

$$w_t(j)h_t(j) = W_t, \tag{3.17}$$

for all $j \in [0, J]$. If this was not true, at the moment of choosing the sector any individual would have the incentive to choose the higher wage income sector, increasing the supply in that sector and lowering thereafter its wage per efficiency unit. Finally, we impose a political threshold for successful proposals, namely that proposals are approved if and only if the majority of the population expresses a vote in their favor.

Assumption 3 (Approval Threshold)**.** A proposal $\bar{F}_{t+1}$ is approved if the mass of individuals in favor is greater or equal than 1/2.

The existence of an approval threshold is a salient element of the results below, but the level of such a threshold is qualitatively irrelevant. We choose majority voting for simplicity and consistency with widespread concepts in the literature such as median voters and Condorcet winners.

We define and solve for the intertemporal equilibrium as follows.

Definition 6 (Intertemporal Equilibrium)**.** An intertemporal equilibrium is a set of firm decisions $\{K_t\}_{t\in[0,\infty)}$ and $\{H_t(j)\}_{t\in[0,\infty)}^{j\in[0,J]}$, individual decisions $\{c_t^i, b_t^i\}_{t\in[0,\infty)}^{i\in[0,1]}$, government instruments $\{\tau_t^A, \tau_t^E\}_{t\in[0,\infty)}$, public expenditure distributions $\{F_t\}_{t\in[0,\infty)}$ and prices $\{R_t\}_{t\in[0,\infty)}$ and $\{w_t(j)\}_{t\in[0,\infty)}^{j\in[0,J]}$, such that

a) $\{K_t, \{H_t(j)\}_{j\in[0,J]}\}$ is a solution to problem (3.4) given R_t and $\{w_t(j)\}_{j\in[0,J]}$ for every $t \in [0,\infty)$,

b) $\{c_{t+1}^i, b_{t+1}^i\}_{i\in[0,1]}$ is a solution to problem (3.9) given (3.8), for all $i \in [0,1]$ and all $t \in [0,\infty)$,

c) $\{A_t, E_t, \tau_t^A, \tau_t^E\}$ satisfies the balanced-budget constraints of the government (3.12) and (3.13) for all $t \in [0,\infty)$,

d) and the market clearing conditions (3.14), (3.15), (3.16), (3.17), and Assumption 3 are satisfied for all $t \in [0,\infty)$.

Suppose that we order the sectors increasingly according to their sector-specific public investment, $a_t(j)$. In what follows, we make an assumption in order to obtain an analytically tractable framework. This assumption though is not necessary for the derivation of the equilibrium solution.

Assumption 4. The function a_t is differentiable and strictly increasing on $(0, J)$ for every t.

The rationale behind this assumption is that sectors in our model are different between each other only in terms of the public expenditure directed to them. There is no other intrinsic technological endowment that characterizes each sector. Hence, the sector-specific public investment differs between any pair of sectors j and k, that is, $a_t(j) \neq a_t(k)$. Assumption 4 implies that the function a_t is continuous and by construction that $a_t' > 0$. Consequently, also $\tilde{a}_t$ is differentiable and strictly increasing. Thus, there exists an inverse function $\tilde{a}_t^{-1}$ such that, for any $j \in [0,J]$, $\tilde{a}_t^{-1}(\tilde{a}_t(j)) = j$. If we call $x \equiv \tilde{a}_t(j)$ and its distribution $F_t(x)$, the latter is the same as the distribution of j, i.e., $F_t(\tilde{a}_t(j)) = \mathscr{A}_t(j)$. We can combine the first order conditions of the different agents and the market clearing conditions in order to summarize the equilibrium solution in the following proposition.

Proposition 4 (The Equilibrium Solution)**.** *Given the tax rate τ_t^A, total output evolves over time depending on the public expenditure distribution F_t, i.e.,*

$$Y_{t+1} = \psi_t(Y_t|F_t) \equiv M(\tau_t^A)\phi(F_t)^\alpha Y_t^{\epsilon(1-\alpha)+\alpha}, \tag{3.18}$$

where

$$M(\tau_t^A) \equiv \left[1-\tau_t^A\right]^\alpha \left[\tau_t^A\right]^{\epsilon(1-\alpha)} \beta^{\epsilon(1-\alpha)+\alpha} h(1)^{1-\alpha}$$

and

$$\phi(F_t) \equiv \int_0^1 x^{\frac{\epsilon(1-\alpha)}{\alpha}-1} dF_t(x).$$

Moreover, the distribution of population across sectors at time $t+1$ mirrors the public expenditure distribution decided at time t, that is,

$$\theta_{t+1}(j) = \left(\frac{h_{t+1}(j)}{H_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = \frac{\tilde{a}_t(j)^{\frac{\epsilon(1-\alpha)}{\alpha}}}{\phi(F_t)}. \tag{3.19}$$

and the transfer that each individual working in sector j receives is a linear function of the share of that sector within the government budget, that is,

$$\xi_{t+1}(j) = \tilde{a}_{t+1}(j)\frac{\tau_{t+1}^E Y_{t+1}}{\theta_{t+1}(j)}. \tag{3.20}$$

See the Appendix for the proofs of all propositions.

3.4 The Development Process

We show how the public expenditure distribution affects aggregate human capital and therefore the law of motion of total output. The equilibrium law of motion, (3.18), expresses future output Y_{t+1} as a function of the public expenditure distribution F_t and current output level Y_t. Suppose we fix the distribution to its initial level, i.e., $F_t = F_0$ for every t. Furthermore, we also fix the exogenously given tax rate on bequests to its initial level, $\tau_t^A = \tau_0^A$. The law of motion ψ_t is in this case time invariant, that is, $Y_{t+1} = \psi(Y_t|F_0)$ for every t. In other words, we neglect for the moment that F_t is a product of the voting equilibrium in every period. Thus, there is only one endogenous state variable, namely the current level of output Y_t. Then we have that

$$Y_{t+1} = \psi(Y_t|F_0) = M(\tau_0^A)\phi(F_0)^\alpha Y_t^{\epsilon(1-\alpha)+\alpha},$$

where

$$M(\tau_0^A) \equiv \left[1-\tau_0^A\right]^\alpha \left[\tau_0^A\right]^{\epsilon(1-\alpha)} \beta^{\epsilon(1-\alpha)+\alpha} h(1)^{1-\alpha}$$

and

$$\phi(F_0) \equiv \int_0^1 x^{\frac{\epsilon(1-\alpha)}{\alpha}-1} dF_0(x).$$

In order to analyze how output evolves over time maintaining constant the other variables, we state the following proposition.

Proposition 5 (Development Path with Constant Distribution)**.** *Suppose that $F_t = F_0$ and $\tau_t^A = \tau_0^A$ for every t. Then the following applies:*

a) $\psi'(Y_t|F_0) > 0$ *for every* Y_t,

b) $\psi''(Y_t|F_0) < 0$ *for every* Y_t,

c) $\lim_{Y_t \to \infty} \psi'(Y_t|F_0) = 0$,

d) $\lim_{Y_t \to 0} \psi'(Y_t|F_0) = +\infty$,

e) $\psi(0|F_0) = 0$,

that is, the law of motion is strictly increasing, strictly concave, and respects the Inada conditions.

This proposition assures that the law of motion is well behaved and that the economy is able to follow a development process independently of the public expenditures distribution. In particular, let us define a steady state.

Definition 7 (Steady State with Constant Distribution)**.** Consider a fixed initial public expenditure distribution, F_0. A steady state Y_s is a level of income such that $\psi(Y_s|F_0) = Y_s$.

Proposition 5 implies that, if there exists a non-trivial steady state $Y_s > 0$, it is unique and identified by the public expenditure distribution, F_0. In order to obtain a closed form solution for it, we set

$$\psi(Y_s|F_0) = M(\tau_0^A)\phi(F_0)^\alpha Y_s^{\epsilon(1-\alpha)+\alpha} = Y_s,$$

which yields a unique solution greater than 0,

$$Y_s = Y_s(F_0) \equiv \left[M(\tau_0^A)\phi(F_0)^\alpha\right]^{\frac{1}{1-\epsilon(1-\alpha)-\alpha}} . \tag{3.21}$$

The steady state level of total output depends on F_0. This unique non-trivial steady state is also stable, because the condition under which $Y_{t+1} > Y_t$ is

$$M(\tau_0^A)\phi(F_0)^\alpha Y_t^{\epsilon(1-\alpha)+\alpha} > Y_t,$$

which corresponds to the condition $Y_t < Y_s$. Hence, given the law of motion with the properties defined in Proposition 5, there exists a unique non-trivial steady state which is stable and whose level depends on the public expenditure distribution. If the public expenditure distribution is constant, the economy follows a standard development path towards a unique stable steady state, as Figure 3.7 shows.

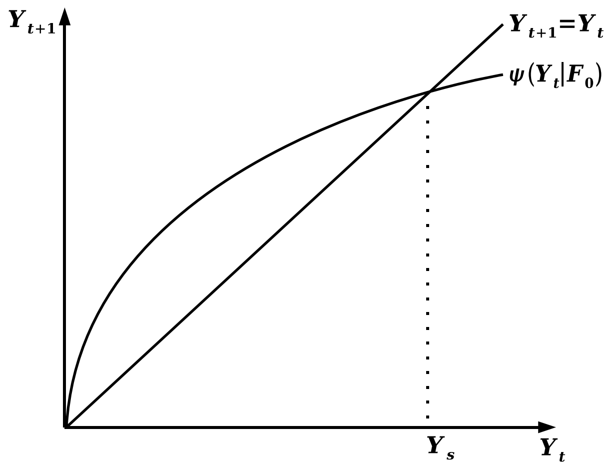

Figure 3.7: Development Path with a Constant Distribution

There exists a multiplicity of steady states, each one identified by a different public expenditure distribution. It is then crucial to understand the effect of changes in the public expenditure distribution on the development path. In order to do so, we have to choose how to sort different distributions and see if there is a relation between this sorting and different development paths. In what follows, we choose to classify distributions according to their degree of first-order stochastic dominance, as this delivers the most clear-cut results and it directly connects with the use of Gini coefficients in Section 2. It is possible to obtain also results for second-order stochastic dominance, but the necessary restrictions on the parameter space are even stronger. The element $\phi(F_t)$ in the law of motion for total output, which depends on the public expenditure distribution, can be expressed as

$$\phi(F_t) = \int_0^1 g(x) dF_t(x), \tag{3.22}$$

where $g(x) \equiv x^{\frac{\epsilon(1-\alpha)}{\alpha}-1}$. Let us now state the following assumptions.

Assumption 5. The function g is nondecreasing.

Assumption 6. The function g is strictly increasing and strictly concave.

Given that $g(x) \equiv x^{\frac{\epsilon(1-\alpha)}{\alpha}-1}$, the conditions in Assumptions 5 and 6 imply respectively that $\epsilon(1-\alpha) \geq \alpha$ and $\alpha < \epsilon(1-\alpha) < 2\alpha$. These are the restrictions on the parameter space that permit us to link the ordering of the public expenditure distributions by first-order stochastic dominance with different development paths. As a preliminary discussion about the plausibility of these assumptions, let us remember that the parameter α measures the share of physical capital in the production function. If we take the standard value of 0.3 for α, and therefore $(1-\alpha) = 0.7$, then g is nondecreasing for values of the homogeneity degree ϵ of the human capital accumulation function close to 1. This means that the returns of public investment on human capital must be not too decreasing for Assumptions 5 and 6 to hold. Moreover, Assumption 6 holds only if α is not excessively small. Let us remember that a distribution $\hat{F}_t$ first-order stochastically dominates (henceforth, dominates) the distribution F_t when $\hat{F}_t(x) \leq F_t(x)$ for every $x \in [0,1]$. Moreover, $\hat{F}_t$ strictly dominates F_t when there exist $a < b$ in $(0,1)$ such that $F_t'(x) < F_t(x)$ for every $x \in [a,b]$ and $F_t'(x) = F_t(x)$ for any other value of x. We call the shift from F_t to $\hat{F}_t$ an increase in dominance. Such an increase means intuitively that sectors with high portions of overall public expenditure increase their shares, and sectors with low portions decrease them. Note that this implies an increase in the mean and variance of the public expenditure distribution.

Proposition 6 (Efficiency with Dominance)**.** *If Assumption 5 holds and $\hat{F}_t$ dominates F_t, then, $\phi(\hat{F}_t) \geq \phi(F_t)$. If Assumption 6 holds and $\hat{F}_t$ strictly dominates F_t, then $\phi(\hat{F}_t) > \phi(F_t)$.*

This proposition suggests specialization as a channel of productive efficiency. The origin of the increase in aggregate productivity is the concentration of the population in the most efficient sectors. The population equilibrium distribution, (3.19), mirrors the public investment distribution across sectors. Hence, an increase in dominance causes a corresponding increase in the population distribution dominance. Since $\theta_{t+1}(j)$ is strictly increasing in $\tilde{a}_t(j)$, if the latter increases, then the former increases as well. Hence, population migrates from low-productive and less-populated sectors to high-productive and more-populated sectors. The concentration occurs because the public investment in a sector is a pure public good for the individuals that choose that sector. This creates the possibility of aggregate positive returns on concentration. An increase in dominance causes, on the one hand, an increase in individual productivity and in the number of workers in a few sectors and, on the other hand, a decrease in productivity and in the number of workers in all the other sectors. If the parameter restrictions implied by Assumptions 5 and 6 hold, the former effect overcome the latter for any degree of concentration, paving the way for persistent positive returns. Thus, the plausibility of the parameter restrictions implied by these assumptions is key to Proposition 6. The parameter α plays a specific role in the production function, namely in (3.2) and (3.3), since if α is equal to 0 the sectors are perfectly substitutable, while if α tends to 1 we approach perfect complementarity. Then, Assumptions 5 and 6 imply that aggregate improvement through sectoral specialization is possible only if the sectors are sufficiently substitutable for the production of the final good. Indeed, if the sectors are enough complementary to reverse Assumptions 5 and 6, Proposition 6 is also reversed and sectoral diversification leads to aggregate efficiency. Hence, Assumptions 5 and 6 help in defining a perspective of analysis. If they hold, specialization leads to aggregate efficiency. If they do not hold, diversification is the efficient policy. In the data we may find an unclear correlation between specialization and economic growth across countries. Our model interprets this ambiguous relation as the occurrence, across countries and over time, of Assumptions 5 and 6. The main message of Proposition 6 is that, although there are no aggregate increasing returns of the amount A_t of public investment, there may be a limited scope for efficiency gains from the direction of such an investment.

An increase in dominance causes an increase in future output and therefore a generalized increase in future wage income, because $W_{t+1} = (1-\alpha)Y_{t+1}$. Note though that this is not necessarily a Pareto-improvement. As (3.20) states, transfers decrease for individuals who work in sectors that lose shares in the overall public expenditure. This has also a repercussion on the bequest that these individuals leave to their offspring, and therefore on the disposable income of part of the future generation. Hence, an increase in dominance in period t increases overall income in period $t+1$, but it also prejudices the current income of some individuals and consequently of their offspring. In other words, it generates income redistributions through time.

Proposition 6 states that the function ϕ in the law of motion for output (3.18) depends negatively on F_t, namely that the more dominant the initial public expenditure distribution the higher the aggregate efficiency. This has consequences on the development path that an economy follows depending on the initial distribution, as the next proposition clarifies.

Proposition 7 (Initial Distribution and Development)**.** *Suppose Assumption 5 holds. If* $\hat{F}_0$ *dominates* F_0*, then*

a) $\psi(Y_t|\hat{F}_0) \geq \psi(Y_t|F_0)$,

b) $\psi'(Y_t|\hat{F}_0) \geq \psi'(Y_t|F_0)$,

c) $\psi''(Y_t|\hat{F}_0) \leq \psi''(Y_t|F_0)$,

for every Y_t*, that is, an initial distribution that shows a higher dominance degree generates a superior transition path. Moreover, a higher initial dominance leads to a higher steady state, i.e.,* $Y_s(\hat{F}_0) \geq Y_s(F_0)$*. Strict inequalities apply if Assumption 6 holds and* $\hat{F}_0$ *strictly dominates* F_0*.*

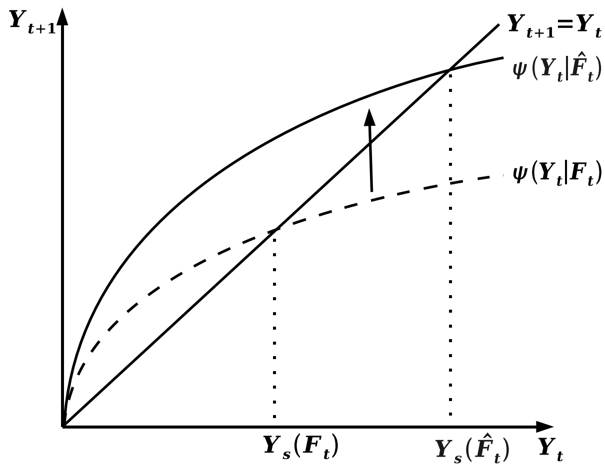

Figure 3.8: Dominance and Superior Development Paths

A direct implication of this proposition is that, if at a certain stage of the development process the dominance degree changes, the economy shifts to a development path that would have been unreachable with the original distribution. This effect is common to any level of income Y_t and to any initial distribution F_0, as we can see in Figure 3.8. Thus, the more regressive the redistributions across sectors along the development process are, the higher the development path is.

The effect of changes in the exogenous variables τ_t^A and β on the steady state and on the transition path are not the focus of this paper. Nevertheless, we can show that there exists an interior tax rate $\tau^* = \epsilon(1-\alpha)/(\alpha + \epsilon(1-\alpha)) \in (0,1)$ above which increases in the tax rate τ have negative effects on future output and below which the effects are positive. Therefore, there exists an optimal tax rate that assures the highest transition path and the highest steady state. The share of bequests within individuals' incomes, β, fastens the development path, though the returns of β on $M(\tau_t^A)$ are decreasing. Since in our model bequests act as intergenerational savings, this is consistent with the standard effect of the saving rate on growth.

3.5 Political Opposition and Blockages

Each period the government makes a random draw from the set of all possible distributions that would increase efficiency and proposes such a distribution to the population. According to Assumption 5 and Proposition 6, the set from which the government makes its draws is

$$\{\bar{F}_{t+1}|\bar{F}_{t+1}(x) \leq F_t(x) \text{ for all } x \in [0,1]\},$$

i.e., the set of the distributions which dominate F_t. If Assumption 5 did not hold, the set would be

$$\{\bar{F}_{t+1}|\bar{F}_{t+1}(x) \geq F_t(x) \text{ for all } x \in [0,1]\}.$$

We restrict ourselves to the case where Assumption 5 does hold, although all the following results hold under the alternative assumption with the adequate sign inversions. The population observes the proposal and decides whether to approve it. The proposal is approved if the majority of the population is in favor, as Assumption 3 states. In case the proposal $\bar{F}_{t+1}$ passes the voting test in period $t+1$, the government sets the public expenditure distribution to $F_{t+1} = \bar{F}_{t+1}$. The individual expresses her vote on the proposal in her second period of life, depending on whether $\xi_{t+1}(j)|_{\bar{F}_{t+1}} \geq \xi_{t+1}(j)|_{F_t}$. If this condition holds, the individual votes in favor of the proposal. Otherwise, she votes against it. This leads to the following proposition.

Proposition 8 (Opposition to Proposals). *An individual in sector j votes against a proposal if and only if $\tilde{a}_{t+1}(j)|_{\bar{F}_{t+1}} < \tilde{a}_{t+1}(j)|_{F_t} = \tilde{a}_t(j)$.*

In other words, if the share of the sector where the individual works would decrease with the proposal, she votes against it because the transfer that she would get in case the proposal was approved is lower than the transfer she would get if the public expenditure distribution remained the same. Given a sector j such that $\tilde{a}_{t+1}(j) \geq \tilde{a}_t(j)$ if $\bar{F}_{t+1}$ was approved, all the individuals who work in that sector in $t+1$ vote in favor of the proposal. Hence, the number of "yes" votes coming from sector j in period $t+1$ for the proposed proposal $\bar{F}_{t+1}$ is $\theta_{t+1}(j)$, which is a function of $\tilde{a}_t(j)$ and F_t according to (3.19). Let us consider the equilibrium population distribution as described by its cumulative distribution function, that is,

$$\Theta_{t+1}(j) \equiv \int_0^j \theta_{t+1}(s)ds = \frac{\int_0^{\tilde{a}_t(j)} g(x)dF_t(x)}{\int_0^1 g(x)dF_t(x)}. \tag{3.23}$$

The sectors are ordered increasingly according to their share in government's budget and $\bar{F}_{t+1}$ represents an increase in the dominance. Thus, if $\tilde{a}_{t+1}(j) < \tilde{a}_t(j)$, then for every sector k such that $\tilde{a}_t(k) \leq \tilde{a}_t(j)$, that is, for every sector k such that $k < j$, $\tilde{a}_{t+1}(k) < \tilde{a}_t(k)$ as well. An increase in dominance implies that, if a sector loses shares with a proposal, every sector that is currently entitled with a lower share loses shares as well. So, $\Theta_{t+1}(j)$ expresses the amount of population who would vote "no" to the proposal up to sector j. The following key concept helps to compute the total amount of population that opposes the proposal.

Definition 8 (Neutral Sector)**.** A neutral sector j^n is a sector such that $\tilde{a}_{t+1}(j^n) = \tilde{a}_t(j^n)$ if $F_{t+1} = \bar{F}_{t+1}$. In other words, a neutral sector is a sector whose share within government's budget would not change with the proposal $\bar{F}_{t+1}$.

Since the share in government's budget would remain the same under the proposal, the individuals that work in the neutral sector are indifferent between the proposal and the current distribution, in the sense that the transfer they receive is the same under both distributions. We state the following assumption to ensure the existence of at least one neutral sector.

Assumption 7. The function a_t satisfies $\lim_{j \to 0} a_t(j) = 0$ for every t.

We want to rule out the possibility of proposing redistributions of public investment from the single sector with the lowest share to the others. Redistributions in this way affect a strictly positive mass of sectors.

Proposition 9 (Existence of the Neutral Sector)**.** *Suppose Assumptions 4 and 7 hold. If a proposal $\bar{F}_{t+1}$ strictly dominates the initial distribution F_t, there exists a neutral sector, j^n.*

We can consider a particular case of growth-enhancing proposal, namely such that, if $F_{t+1} = \bar{F}_{t+1}$, $\mathscr{A}''_{t+1}(j) > \mathscr{A}''_t(j)$ for every j in $(0, J)$, where $\mathscr{A}_{t+1}$ is the public expenditure cumulative distribution function at time $t+1$. The intuition behind this case is that, given a sector j, the sectors that have slightly higher shares in government's budget than j's would increase the distance of their shares from j's. In other words, $\tilde{a}'_{t+1}(j) > \tilde{a}'_t(j)$. In this case, the neutral sector is unique.

Proposition 10 (Uniqueness of the Neutral Sector)**.** *Suppose Assumptions 4 and 7 hold. If a proposal $\bar{F}_{t+1}$ strictly dominates the initial distribution F_t and if $\mathscr{A}''_{t+1}(j) > \mathscr{A}''_t(j)$ for every j in $(0, J)$ whenever $F_{t+1} = \bar{F}_{t+1}$, then there exists a unique neutral sector, j^n. Moreover, for every $j < j^n$ we have $\tilde{a}(j)_{t+1} < \tilde{a}_t(j)$ and for every $j > j^n$ we have $\tilde{a}_{t+1}(j) > \tilde{a}_t(j)$.*

An individual in a sector $j < j^n$ expresses her vote against the proposal, because the share of her sector would decrease with the proposed distribution and this would have negative effects on her transfer. Instead, all the individuals that work in a sector $j > j^n$ vote in favor of the proposal. Hence, the amount of negative votes is $\Theta_{t+1}(j^n)$, and the amount of positive votes is $1 - \Theta_{t+1}(j^n)$. Since the threshold for approval of a proposal is given by Assumption 3, we can state in the following proposition.

Proposition 11 (Political Blockage and Approval)**.** *Suppose there exists a unique neutral sector. Then, if $1 - \Theta_{t+1}(j^n) > 1/2$, the government sets $F_{t+1} = \bar{F}_{t+1}$. If $\Theta_{t+1}(j^n) \geq 1/2$, there is a blockage and the government sets $F_{t+1} = F_t$.*

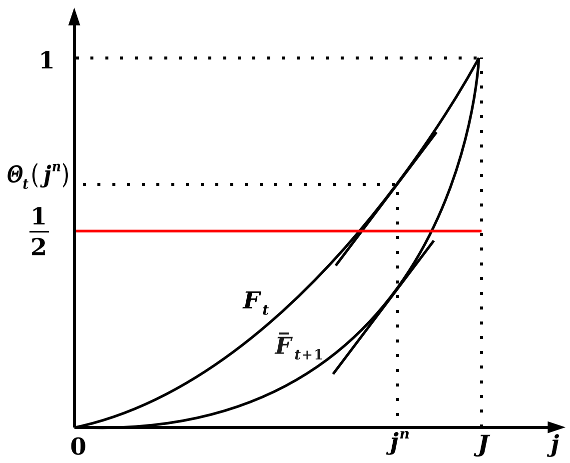

Figure 3.9: The Neutral Sector and Political Opposition to Proposals

According to this proposition and to Proposition 6, since $\bar{F}_{t+1}$ would exhibit a higher dominance than F_t, the consequence of a blockage in period $t+1$ is a lower dominance in $t+1$ and therefore a lower level of total output in $t+2$. If the government made the proposal only in period $t+1$ and not in every period, this would mean that the economy would be bound to an inferior development path from period $t+1$ onward, as Proposition 7 states.

We now turn to the probability of political blockage, that is, the probability that $\Theta_{t+1}(j^n) \geq 1/2$. This probability depends on two elements, the endogenous state F_t and the exogenous random component $\bar{F}_{t+1}$, which interacting with F_t generates the neutral sector j^n.

Claim (Likelihood of a Political Blockage)**.** *The probability of political blockage in period $t+1$ decreases with the dominance degree of F_t.*

There are four classes of results for this claim. First, let us consider a certain $\bar{F}_{t+1}$ and a fixed j^n. Suppose that an alternative initial distribution $\hat{F}_t$ generates the same j^n in combination with $\bar{F}_{t+1}$ and at the same time dominates F_t. Then, $\Theta_{t+1}(j^n)$ would be lower, which implies it would be more likely for $\Theta_{t+1}(j^n)$ to stand below the 1/2 blockage threshold. Hence, an increase in the initial dominance that maintains the same j^n increases the probability that the same proposal $\bar{F}_{t+1}$ is approved.

Second, let us formalize the concept of change degree.

Definition 9 (Change Degree)**.** The change degree δ associated to a given initial distribution F_t and a proposal $\bar{F}_{t+1}$ is such that

$$\delta \equiv \max_j \{\mathscr{A}_t(j) - \mathscr{A}_{t+1}(j)\},$$

that is, the change degree is the maximal distance between the initial $\mathscr{A}_t$ and $\mathscr{A}_{t+1}$ if $F_{t+1} = \bar{F}_{t+1}$.

By (3.23), Definition 8, and Proposition 10, we have that $\delta = \mathscr{A}_t(j^n) - \mathscr{A}_{t+1}(j^n) > 0$. The change degree is the cumulated distribution shift of the neutral sector between the initial and the proposed distribution. Then, every proposed change from an $\hat{F}_t$ to an $\hat{\bar{F}}_{t+1}$ that exhibits the same j^n as a change from F_t and $\bar{F}_{t+1}$, where $\hat{F}_t$ dominates F_t and $\hat{\bar{F}}_{t+1}$ dominates $\bar{F}_{t+1}$, is more likely to be approved than the change from F_t to $\bar{F}_{t+1}$. This is true for all δ's within the class of proposals that generate the same j^n. Hence, a higher initial dominance increases the possibility of approval of all proposals that generate the same j^n.[21]

Third, the subset of proposals that are approved for sure depends on the initial distribution. Let us define another important sector in the political equilibrium.

Definition 10 (Approval Sector)**.** An approval sector j^a is a sector such that $\Theta_t(j^a) = 1/2$. In other words, the approval sector measures the span of sectors whose population would be sufficient to approve a proposal.

If the neutral sector j^n is lower in ranking than the approval sector j^a, that is, if $j^n \leq j^a$, then the proposal generating such a neutral sector is approved, since in this case $\Theta_t(j^n) \leq \Theta_t(j^a) = 1/2$. Hence, the condition $j^n \leq j^a$ is sufficient for the approval. If we increase the dominance degree of the initial distribution the ranking of j^a increases, so the span of sectors that might result as a successful neutral sector of a proposal increases with the dominance degree of the initial distribution. If each sector had the same probability of resulting as the neutral sector generated by the randomly drawn proposal, then the probability of $j^n \leq j^a$ would increase with the dominance degree of the initial distribution.

Fourth, the claim that the probability of blockage at t decreases with the degree of dominance at t cannot be generalized explicitly outside stylized cases such as a fixed j^n or a uniform distribution of the event $j = j^n$ on $[0, J]$. We explore therefore the case of a variable j^n by means of a numerical exercise. Up to now we considered a continuous variety of sectors. This permitted us to obtain

[21] Our concept of change degree is reminiscent of the KolmogorovSmirnov test, a nonparametric test for the equality of continuous, one-dimensional probability distributions. In this sense, we are comparing couples of distributions that share the same value of the K-S statistic, which usually quantifies the distance between the empirical distribution function of a sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples. See Kolmogorov [28] and Smirnov [44].

a series of neat propositions. However, the qualitative results would still hold if we dropped the continuity assumption. We consider therefore a discrete number of sectors, namely $J = 1000$. Given an initial randomly assigned F_t, we consider a reallocation algorithm that increases the dominance according to a given j^n and a given proxy for the change degree $\hat{\delta}$. The algorithm consists of taking a portion $\hat{\delta}$, which we call proposed change degree, of the mass of public expenditure assigned to all the sectors that have a lower share than $\tilde{a}_t(j^n)$, and transfer it to the sectors that have a higher share than $\tilde{a}_t(j^n)$. In other words, we transfer an amount $\hat{\delta}\mathscr{A}_t(j^n)$ of public expenditure. The reallocation within each subset of the support set $\{1, ..., J\}$, namely within $\{1, ..., j^n - 1\}$ and within $\{j^n + 1, ..., J\}$, is made equally. In other words, the total transfer is divided by the number of sectors within each subset and equally distributed. We repeat this reallocation considering as neutral one every 10 sectors on the support set, that is, we consider 100 possible neutral sectors. For each neutral sector we consider 9 different change degrees, that is, $\hat{\delta}$ can take values $\{0.1, 0.2, ..., 0.9\}$. Thus, for a given initial F_t we obtain 100 possible proposals that could originate from it, as Figure 3.10 in the Appendix shows. Each graph corresponds to a different change degree, and for each $\hat{\delta}$ we represent the 100 possible proposals, each one corresponding to a different $j^n \in \{1, ..., J\}$. We have then a span of possible proposals for each $\hat{\delta}$, from which we can extract the mean and median values. The mean though seems to overestimate the likelihood of a blockage, while the median seems to underestimate it. In fact, if we trim the simulated data by eliminating the lowest and highest 10% of each span, that is, if we eliminate the proposals whose neutral sectors are at the boundaries of the support set, we obtain a result for the mean value that is intermediate between the pure mean and median cases. This means that the cumulative distribution functions computed for the extreme values of j^n are outliers in our sample, so they may bias the likelihood of a blockage. We therefore take into account the trimmed-down dataset, as Figure 3.11 shows. By increasing the change degree, the neutral sector shifts towards higher ranks of the sequence $\{1, ..., J\}$, up to the point where the proposals are likely to be blocked. We then compute an alternative initial distribution $\hat{F}_t$ that first-order stochastically dominates the previous one, that is, we simulate a higher initial dominance. This alternative initial distribution is obtained by reallocating the initial distribution around $\hat{j} = J/2$. The reallocation consists of the transfer of 10% of total public expenditures from all the sectors $j < \hat{j}$ to all the sectors $j > j^n$. The following results are consistent for values of $\hat{j}$ in a trimmed subset of sectors, that is, excluding the highest and lowest 10% of $\{1, ..., J\}$, and any change degree of the reallocation in $\{0.1, ..., 0.9\}$. We compute again all the possible proposals starting from the alternative initial distribution. We trim the data by eliminating the lowest and highest 10%, and we compute the mean cumulative distribution function of the proposals. To show how the likelihood of a blockage decreases as we increase the initial dominance, we compare the cumulative distribution function computed at j^n before and after the shift in the initial dominance, that is, $\Theta_t(j^n)$ and $\hat{\Theta}_t(j^n)$. We repeat this for every change degree. In Figure 3.12 we can see both how the blockage likelihood increases as the change degree increases, and how a higher initial dominance generates a lower likelihood for any change degree. This simulation exercise supports the claim that blockage probability in period $t+1$ decreases with F_t's dominance degree.

3.6 Discussion

An economy exhibits a certain sectoral composition F_t at a certain period t with a probability that depends on previous period's public expenditure distribution F_{t-1}. We can think of public expenditure F_t as distributed with mean $E_{F_{t-1}}\left[\bar{F}_t\right]$, that is, over the possible proposals of the government $\bar{F}_t$ whose probability of being chosen and approved depends on F_{t-1}. We can therefore think of the public expenditure distribution GE_t as distributed with mean $E_{VA_t}\left[\overline{GE}_t\right]$. Hence, we obtain one of the structural relations of (3.1), that is, $GE_t = g_t(VA_t)$. Moreover, we can compute the cumulative distribution function $\mathscr{H}_t$ of human capital in sector j in $[0, J]$,

$$\mathscr{H}_t(j) = \frac{\int_0^{\tilde{a}_t(j)} g(x)dF_{t-1}(x)}{\int_0^1 g(x)dF_{t-1}(x)},$$

which is our proxy for the sectoral composition VA_t of the economy. Hence, the value added distribution VA_t depends on previous period's distribution of public expenditure GE_{t-1}. This yields the second structural relations in (3.1), $VA_t = f_t(GE_{t-1})$.

The system of two structural relations yields a law of motion for the public expenditure distribution. This can be iterated backwards up to the initial distribution F_{-1}, which is the endowment of an economy together with $b_0^i > 0$ for some i. Given a certain infinite stream of per-period growth-enhancing proposals $\{\bar{F}_t\}_{t=0}^{\infty}$ under substitutability of sectors, the probability that an economy reaches a certain sectoral composition F, i.e., that an economy settles in a certain development path described by $Y_{t+1} = \psi(Y_t|F)$ and therefore by $Y_s = Y_s(F)$, is higher if F_{-1} shows a higher dominance degree. Since the time horizon is infinite, two economies with different initial distributions differ only in the timing in which they reach the development path described by $Y_{t+1} = \psi(Y_t|F)$. In case of only growth-enhancing distribution changes, there exists a unique steady state to which all economies with the same parameter values tend, which is the fixed point of $Y_{t+1} = \psi(Y_t|F_{\min})$, where $F_{\min}$ is the Dirac mass on a single sector. However, this upper bound for steady states would mean that $\tilde{a}_t(J) = 1$ and $\tilde{a}_t(j) = 0$ for every $j < J$. This is not compatible with Assumptions 4 and 7. Hence, the upper bound is not reachable under Assumptions 4 and 7. The model sheds light on the differences in timing in the transition rather than on the differences in steady state output levels. Note that the closer is the economy to the upper bound steady state level of income, the more negligible the gain from an approval, although the approval itself is more likely.

In the case where the government proposes not only growth-enhancing changes, but also growth-reducing ones, the economy would fluctuate among different development paths through time. It would alternate growth with recession depending both on whether the proposal in the previous period was growth-enhancing and on whether such a proposal was approved. If the economy at a certain time t is considerably diversified, then in the following periods the economy is likely to iterate a similar distribution. This is due to the fact that, on the one hand, the economy is unlikely to approve proposals towards more concentration and, on the other hand, the economy cannot diversify much more than what it has already done. The same occurs if the economy at time t is specialized. Hence, if the government proposes randomly both growth-enhancing and growth-reducing distributions, then the economy fluctuates across time but tends to degenerate in the long run to either perfect diversification or perfect concentration, being the two scenarios the product of both chance and the initial distribution.

Our model basically suggests that the level of total output at a certain point in time is a product of the starting value of output and of the history of political blockages and approvals of an economy, which is a function of the initial public expenditure distribution. Given the initial degree of sectoral diversification of an economy, the status quo is broken and the economy shifts to higher development paths only if the government manages to formulate a proposal that is both growth-enhancing through the public investment and capable of collecting the sufficient political support through the redistribution of transfers. Moreover, if an economy is stuck into a development path characterized by a persistent public expenditure distribution, any event that modifies the initial distribution of vested interests, such as an economic crisis, the discovery of a natural resource, or an institutional change, may remove the political blockage and let the economy shift to different development paths. For example, suppose we decentralize political, fiscal, and administrative authority from the national to the regional level. This creates subnational economies out of an overall national economy, where each of the regions would have its own tax system and public expenditure distribution. For simplification suppose that the population cannot migrate across regions. If the productive sectors were concentrated into geographic clusters due to agglomeration externalities and common local facilities, then the regional economies would appear more specialized than the national economy. Hence, the conflicts of interests within each region would be less intense, and therefore the likelihood of political blockages would be lower. Thus, the blockage that might have occurred at the national level does not occur at the regional level, and each region starts to follow its own development path. This blockage removal is due to the fact that the decentralization of political, fiscal, and administrative authority generates the possibility of different political majorities across regions, and therefore each regional government is more likely to formulate politically implementable proposals.[22]

3.7 Conclusion

The misallocation of resources between productive sectors explains a relevant part of TFP differences across countries. Since there are allocations that are efficient and others that are not, a natural question to ask is why such differences across countries tend to persist over time or to be at least very resistant to change. We provide an explanation for this persistence even in presence of a democratic voting process. The basic idea is that the sectoral composition of an economy mirrors the distribution of vested interests across sectors. Hence, the distribution of public resources tends to mimic the distribution of value added, as we document in the data. Drastic changes in the distribution of public resources that might be growth-enhancing are therefore politically unfeasible and the sectoral composition is slacker to change than what would maximize growth. This generates cross-country differences in the pace of the path towards the efficient allocation of resources, where the differences are due to the initial sectoral composition of an economy.

We presented a general equilibrium model of growth able to generate the expected structural relations. The production side is characterized by a variety of sectors, each of them contributing to the production of the final consumption good. The equilibrium solution leads to a law of motion for total output that depends on the distribution of public expenditure across sectors. Given a sufficient level of substitutability among sectors, an increase in the concentration of the public expenditure shifts the economy to a superior development path due to the migration of the population towards highly productive sectors. We make the public expenditure distribution an outcome of a voting process on new distributions proposed by the government. Due to a transfers scheme, individuals hold interests in the share within the government budget of the sector where they work.

[22] Here we are obviously neglecting all the possibly negative effects of decentralization such as tax competition, scale effects, and rent-seeking behavior of local monopolies.

Hence, they vote in favor of proposals that increase their sector's share. Proposals can be either approved or blocked, depending on whether the mass of population supporting the proposal is greater than the mass of population opposing it. Thus, the likelihood of a political blockage depends on the population distribution each period. This implies that the level of development of an economy is a product of the history of political blockages and approvals, whose likelihood depends on the initial sectoral composition of the economy.

The main way of modeling the interrelation between structural change and development in the literature presents the sectoral composition of an economy as a steady state distribution of economic activity across sectors which is ergodic to the initial composition. Differences in the steady state distributions across countries are then a product of intrinsic differences in either the preferences or the production technology.[23] Our model can be extended to both explanations, since it focuses on the differences in timing along the transition rather than on the differences in the steady states. The difference with respect to the present version would be that, instead of converging towards a degenerate steady state distribution, the model would converge towards the ergodic steady state distribution determined by either non-homothetic preferences or sector-biased technological change. Future work could also reformulate the model so as to allow for a non-degenerate ergodic public expenditure distribution to arise in the long run, where the long-run Gini index of the value added and public expenditure distributions would mirror the fundamental complementarity among the productive sectors of each economy.

In our model the comovement between the distribution across sectors of productive and unproductive components of the public expenditure is obtained through a restriction in the set-up of the model. We avoid to make it the equilibrium outcome of a political process only because we want to focus on the distribution of total public expenditure across sectors. Nevertheless, future research could for example interpret the aforementioned comovement as the product of a political economy mechanism involving probability voting with lobbies or demographic aspects of the population.

On the empirical side, the stylized facts presented in Section 2 should be supported by a more thorough econometric exercise on the relationship between sectoral diversification, development, and the public expenditure distribution. This includes for instance a simultaneous equations model of panel estimation or the

[23]For the first approach, see for example Echevarria [18], Kongsamut, Rebelo, and Xie [29], and Alonso-Carrera and Raurich [5], where the presence of non-homothetic preferences is key to the emergence of structural change. For the second approach, see Baumol [8] and Ngai and Pissarides [35], where the presence of sector-biased technological change generates the structural change. For a recent assessment of the relative quantitative importance of the two explanations, see Guilló, Papageorgiou, and Pérez-Sebastián [25].

analysis of different measures of sectoral diversification in terms of employment and sector-specific total factor productivity. Future work could also take into account a broader sample that would represent, on the one hand, a wider cross-country spectrum of development stages, and, on the other hand, longer time series where the phenomena we deal with would arise more clearly. However, any estimation exercise would suffer from the several dimensions of endogeneity in the variables. Moreover, the overlapping generations and the perfect depreciation of our model's set-up, along with other simplifications, are neither suitable for calibration exercises nor for empirical exercises. Hence, the reformulation of the results allowing for infinitely lived heterogeneous agents and partial depreciation of both physical and human capital would enrich quantitatively the dynamics of the model and permit to isolate empirically the relations between sectoral diversification, public expenditure distribution, and development stages.

3.8 Appendix: Proofs, Numerical Exercise, Figures, and Tables

Proof of Proposition 4. Consider the FOC of the final good firm, (3.6). The total share of human capital into final output is $1-\alpha$, so substituting for (3.15) we obtain that

$$\int_0^J w_{t+1}(j)\theta_{t+1}(j)h_{t+1}(j)dj = (1-\alpha)Y_{t+1}.$$

In equilibrium (3.17) must hold, so

$$W_{t+1}\int_0^J \theta_{t+1}(j)dj = (1-\alpha)Y_{t+1},$$

and, since $\int_0^J \theta_{t+1}(j)dj = 1$, we have

$$W_{t+1} = (1-\alpha)Y_{t+1}.$$

The aggregate level of income is equal to

$$\int_0^1 I_{t+1}^i(j)di = \int_0^1 \left[(1-\tau_{t+1}^E)\left[W_{t+1}+R_{t+1}(1-\tau_t^A)b_t^i(j')\right]+\xi_{t+1}(j)\right]di,$$

i.e.,

$$\int_0^1 I_{t+1}^i(j)di = (1-\tau_{t+1}^E)\int_0^1 \left[W_{t+1}+R_{t+1}(1-\tau_t^A)b_t^i(j')\right]di + \int_0^J \xi_{t+1}(j)\theta_{t+1}(j)dj.$$

Moreover, from (3.16) we know that

$$\xi_{t+1}(j) = \frac{e_{t+1}(j)}{\theta_{t+1}(j)},$$

so, by (3.13),

$$\int_0^J \xi_{t+1}(j)\theta_{t+1}(j)dj = \int_0^J e_{t+1}(j)dj = \tau_{t+1}^E \int_0^1 \left[W_{t+1} + R_{t+1}(1-\tau_t^A)b_t^i(j')\right] di.$$

Thus,

$$\int_0^1 I_{t+1}^i(j)di = \int_0^1 \left[W_{t+1} + R_{t+1}(1-\tau_t^A)b_t^i(j')\right] di,$$

which, under (3.14) and (3.7), yields

$$\int_0^1 I_{t+1}^i(j)di = Y_{t+1},$$

that is, total income is equal to total output. With this result, we deduce from (3.16) and (3.10) that

$$\int_0^1 b_t^i(j')di = \beta \int_0^1 I_t^i(j)di = \beta Y_t.$$

Hence, the level of physical capital in the next period is, according to (3.14),

$$K_{t+1} = (1-\tau_t^A) \int_0^1 b_t^i(j')di = (1-\tau_t^A)\beta Y_t. \tag{3.24}$$

If we substitute for (3.15) in (3.6), then

$$\theta_{t+1}(j)w_{t+1}(j)h_{t+1}(j) = w_{t+1}(j)H_{t+1}(j) = (1-\alpha)K_{t+1}^{\alpha}\left[H_{t+1}(j)\right]^{1-\alpha}.$$

Taking into account (3.17), we have that

$$\theta_{t+1}(j)W_{t+1} = (1-\alpha)K_{t+1}^{\alpha}\left[\theta_{t+1}(j)h_{t+1}(j)\right]^{1-\alpha},$$

i.e.,

$$\left[\theta_{t+1}(j)\right]^{\alpha} = \frac{(1-\alpha)K_{t+1}^{\alpha}}{W_{t+1}}\left[h_{t+1}(j)\right]^{1-\alpha}.$$

Substituting for (3.7) and (3.2), we can express $\theta_{t+1}(j)$ as

$$\theta_{t+1}(j) = \left[\frac{h_{t+1}(j)}{H_{t+1}}\right]^{\frac{1-\alpha}{\alpha}}.$$

If we sum up all the $\theta_t(j)$'s through all the j's, we obtain that

$$1 = \frac{\int_0^J [h_{t+1}(j)]^{\frac{1-\alpha}{\alpha}} dj}{H_{t+1}^{\frac{1-\alpha}{\alpha}}},$$

that is,

$$H_{t+1} = \left(\int_0^J [h_{t+1}(j)]^{\frac{1-\alpha}{\alpha}} dj \right)^{\frac{\alpha}{1-\alpha}}.$$

Hence, we can rewrite $\theta_{t+1}(j)$ as

$$\theta_{t+1}(j) = \frac{[h_{t+1}(j)]^{\frac{1-\alpha}{\alpha}}}{\int_0^J [h_{t+1}(j)]^{\frac{1-\alpha}{\alpha}} dj},$$

which, since $h_{t+1}(j) = h(a_t(j)) = a_t(j)^\epsilon h(1) = A_t^\epsilon \tilde{a}_t(j)^\epsilon h(1)$, is equivalent to

$$\theta_{t+1}(j) = \frac{\tilde{a}_t(j)^{\frac{\epsilon(1-\alpha)}{\alpha}}}{\int_0^J \tilde{a}_t(j)^{\frac{\epsilon(1-\alpha)}{\alpha}} dj}.$$

If we consider that $d\mathscr{A}_t(j) = \tilde{a}_t(j)dj$ and substituting for $x = \tilde{a}_t(j)$ and $F_t(\tilde{a}_t(j)) = \mathscr{A}_t(j)$, we obtain (3.19). Let us consider again H_{t+1}. If we substitute for $h_{t+1}(j) = A_t^\epsilon \tilde{a}_t(j)^\epsilon h(1)$, we obtain

$$H_{t+1} = (A_t)^\epsilon h(1) \left[\int_0^1 x^{\frac{\epsilon(1-\alpha)}{\alpha} - 1} dF_t(x) \right]^{\frac{\alpha}{1-\alpha}} = (A_t)^\epsilon h(1) \phi(F_t)^{\frac{\alpha}{1-\alpha}},$$

but by (3.12) $A_t = \tau_t^A \int_0^1 b_t^i(j')di = \tau_t^A \beta Y_t$, thus

$$H_{t+1} = \left[\tau_t^A \beta Y_t \right]^\epsilon h(1) \phi(F_t)^{\frac{\alpha}{1-\alpha}}. \tag{3.25}$$

If we substitute the equilibrium levels of physical and human capital, (3.24) and (3.25), inside next period's production function, we obtain

$$Y_{t+1} = K_{t+1}^\alpha H_{t+1}^{1-\alpha} = \left[\left[1 - \tau_t^A \right] \beta Y_t \right]^\alpha \left[\left[\tau_t^A \beta Y_t \right]^\epsilon h(1) \phi(F_t)^{\frac{\alpha}{1-\alpha}} \right]^{1-\alpha},$$

which leads to (3.18) after rearranging the components. From (3.16) we know that

$$\xi_{t+1}(j) = \frac{e_{t+1}(j)}{\theta_{t+1}(j)} = \frac{\tilde{e}_{t+1}(j)}{\theta_{t+1}(j)} E_{t+1},$$

which, by (3.19) and since $\tilde{a}_{t+1}(j) = \tilde{e}_{t+1}(j)$, yields (3.20). □

Proof of Proposition 5. Since $\alpha \in (0,1)$ and $\epsilon \in (0,1)$, $\epsilon(1-\alpha)+\alpha \in (0,1)$. On the one hand, the first order derivative of ψ is always strictly positive and tends to 0 if Y_t goes to infinity, and to infinity as Y_t approaches 0. On the other hand, the second order derivative is always negative. Moreover, $\psi(Y_t|F_0) \geq 0$ for every Y_t and $\psi(0|F_0) = 0$. □

Proof of Proposition 6. According to (3.22), $\phi(F_t) = \int_0^1 g_t(x)dF_t(x)$. If F_t' first-order stochastically dominates F_t, then for every nondecreasing function u defined on $[0,1]$, $\int_0^1 u(x)dF_t'(x) \geq \int_0^1 u(x)dF_t(x)$. In particular, this holds for $u = g$, as long as g is nondecreasing under Assumption 5. Hence, $\int_0^1 g_t(x)dF_t'(x) \geq \int_0^1 g_t(x)dF_t(x)$. We can interpret ϕ as decreasing in F_t, that is, every increase in dominance leads to a higher value of ϕ. According to (3.18), if $\phi(F_t') \geq \phi(F_t)$, then $\psi_t(Y_t|F_t') \geq \psi_t(Y_t|F_t)$, i.e., an increase in dominance causes a higher future output. If Assumption 6 holds and F_t' strictly dominates F_t, then $\int_0^1 g_t(x)dF_t'(x) > \int_0^1 g_t(x)dF_t(x)$ and therefore $\psi_t(Y_t|F_t') > \psi_t(Y_t|F_t)$. □

Proof of Proposition 7. Let us remember that

$$\psi(Y_t|F_0) \equiv M(\tau_0)\phi(F_0)^\alpha Y_t^{\epsilon(1-\alpha)+\alpha}.$$

Thus,

$$\psi'(Y_t|F_0) = (\epsilon(1-\alpha)+\alpha)M(\tau_0)\phi(F_0)^\alpha Y_t^{\epsilon(1-\alpha)+\alpha-1}$$

and

$$\psi''(Y_t|F_0) = (\epsilon(1-\alpha)+\alpha)(\epsilon(1-\alpha)+\alpha-1)M(\tau_0)\phi(F_0)^\alpha Y_t^{\epsilon(1-\alpha)+\alpha-2}.$$

Since Assumption 5 holds, then $\phi(\hat{F}_0) \geq \phi(F_0)$ if $\hat{F}_0$ dominates F_0. Hence, $\psi(Y_t|\hat{F}_0) \geq \psi(Y_t|F_0)$, $\psi'(Y_t|\hat{F}_0) \geq \psi'(Y_t|F_0)$, and $\psi''(Y_t|\hat{F}_0) \leq \psi''(Y_t|F_0)$, for every Y_t. Moreover, according to (3.21),

$$Y_s(F_0) \equiv \left[M(\tau_0^A)\phi(F_0)^\alpha\right]^{\frac{1}{1-\epsilon(1-\alpha)-\alpha}},$$

so $Y_s(\hat{F}_0) \geq Y_s(F_0)$. If Assumption 6 holds and $\hat{F}_0$ strictly dominates F_0, then $\phi(\hat{F}_0) > \phi(F_0)$ and strict inequalities apply to all results. □

Proof of Proposition 8. From (3.20) and (3.18),

$$\xi_{t+1}(j) = \tilde{a}_{t+1}(j)\frac{\tau_{t+1}^E M(\tau_t^A)\phi(F_t)^{\alpha+1}Y_t^{\epsilon(1-\alpha)+\alpha}}{\tilde{a}_t(j)^{\frac{\epsilon(1-\alpha)}{\alpha}}}.$$

Hence, individual transfers are proportional to $\tilde{a}_{t+1}(j)$, where the proportion is given only by period t's variables and the exogenous tax rate τ_{t+1}^E. Since $\bar{F}_{t+1}$ would only affect $\tilde{a}_{t+1}(j)$, the comparison between $\xi_{t+1}(j)|_{\bar{F}_{t+1}}$ and $\xi_{t+1}(j)|_{F_t}$ is the same as the comparison between $\tilde{a}_{t+1}(j)|_{\bar{F}_{t+1}}$ and $\tilde{a}_{t+1}(j)|_{F_t} = \tilde{a}_t(j)$. Thus, $\xi_{t+1}(j)|_{\bar{F}_{t+1}} < \xi_{t+1}(j)|_{F_t}$ if and only if $\tilde{a}_{t+1}(j)|_{\bar{F}_{t+1}} < \tilde{a}_t(j)$. □

Proof of Proposition 9. If Assumption 4 holds, $\mathscr{A}_t''(j) = \tilde{a}_t'(j) > 0$ for every j in $(0, J)$. Since $\bar{F}_{t+1}(x) < F_t(x)$ for every x in $[a, b] \subseteq [0, 1]$, there exist a' and b' in $(0, J)$ such that, if $F_{t+1} = \bar{F}_{t+1}$, $\mathscr{A}_{t+1}(j) < \mathscr{A}_t(j)$ for every j in $[a', b']$. But $\mathscr{A}_t''(j) > 0$ for every j in $(0, J)$. Hence, $\mathscr{A}_{t+1}(j) < \mathscr{A}_t(j)$ for every j in $(0, J)$. By Assumption 7, $\mathscr{A}_{t+1}(0) = \mathscr{A}_t(0)$, and $\mathscr{A}_{t+1}(J) = \mathscr{A}_t(J)$ by construction. Thus, by Rolle's theorem, there exists a sector j^n in $[0, J]$ such that the local derivatives of $\mathscr{A}_{t+1}(j)$ and $\mathscr{A}_t(j)$ are the same, that is, $\tilde{a}_{t+1}(j^n) = \mathscr{A}_{t+1}'(j^n) = \mathscr{A}_t'(j^n) = \tilde{a}_t(j^n)$. □

Proof of Proposition 10. According to Proposition 9, there exists at least one neutral sector. Let us define $\delta(j) \equiv \mathscr{A}_t(j) - \mathscr{A}_{t+1}(j)$. Since $F_{t+1} = \bar{F}_{t+1}$ and as we proved in Proposition 9 $\mathscr{A}_{t+1}(j) < \mathscr{A}_t(j)$ for every j in $(0, J)$, then $\delta(j) > 0$ for every j in $(0, J)$. Moreover, $\delta(0) = \delta(J) = 0$ by construction. The values of j in $[0, J]$ for which $\delta(j)$ is maximal are given by the first order condition $\delta'(j) = 0$, that is, $\mathscr{A}_t'(j) - \mathscr{A}_{t+1}'(j) = \tilde{a}_t(j) - \tilde{a}_{t+1}(j) = 0$. In other words, the $\delta(j)$ is maximal when $j = j^n$, since by definition $\tilde{a}_t(j^n) = \tilde{a}_{t+1}(j^n)$. If $\mathscr{A}_{t+1}''(j) > \mathscr{A}_t''(j)$ for every j in $(0, J)$, then $\delta''(j) < 0$ for every j in $(0, J)$. Hence, there exists a unique j^n in $(0, J)$ satisfying $\tilde{a}_{t+1}(j^n) = \tilde{a}_t(j^n)$. If $j < j^n$, then $\delta'(j) < 0$, that is, $\tilde{a}_{t+1}(j) < \tilde{a}_t(j)$. Otherwise if $j > j^n$, then $\tilde{a}_{t+1}(j) > \tilde{a}_t(j)$. □

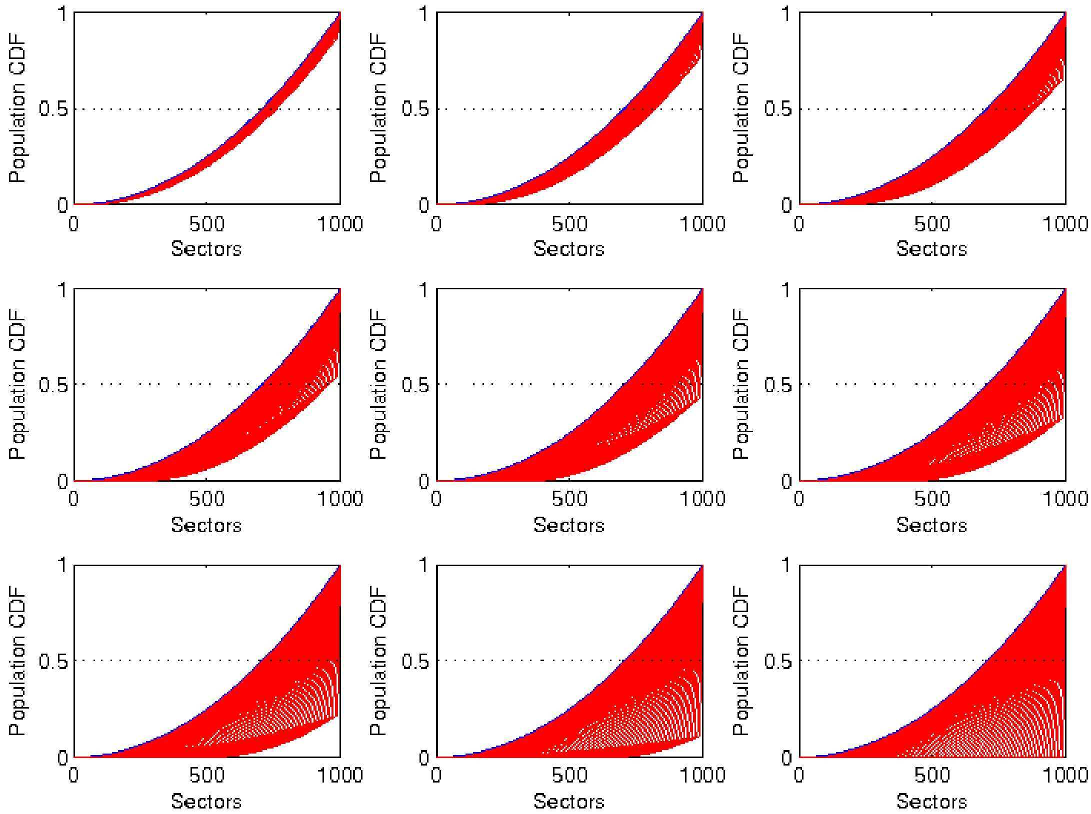

Figure 3.10: Proposals for different change degrees.

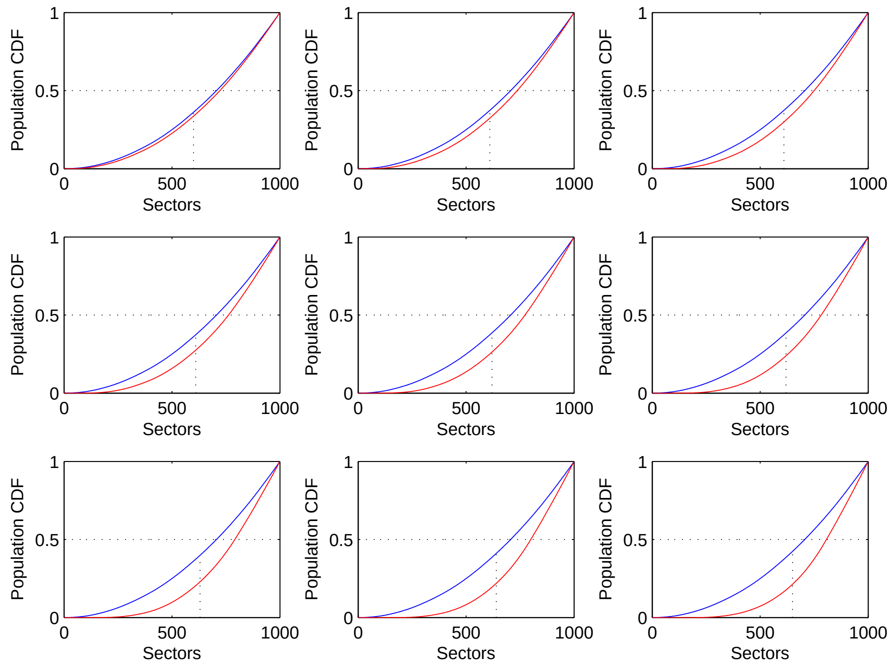

Figure 3.11: Proposals' mean values for different change degrees.

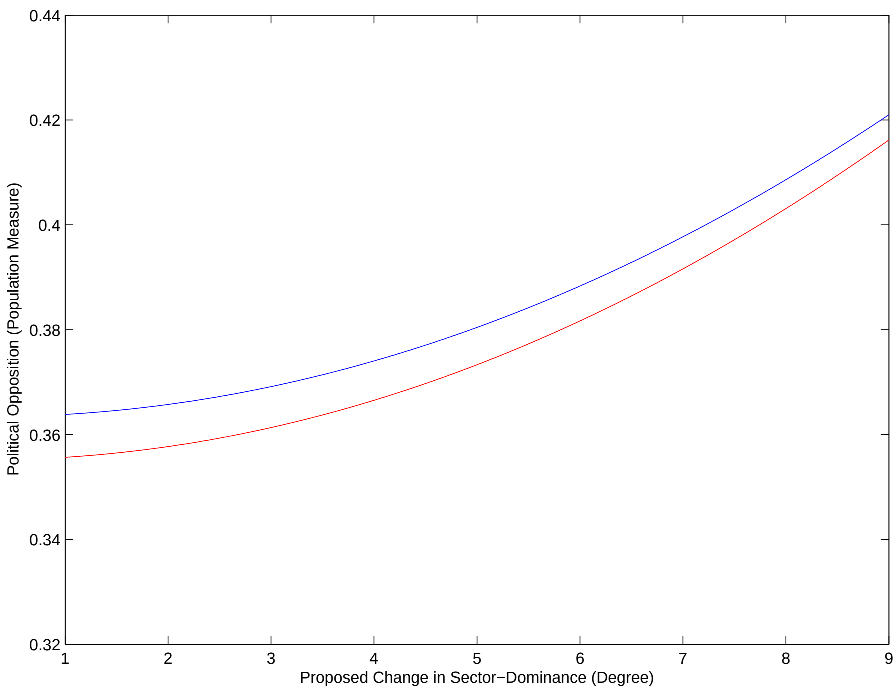

Figure 3.12: Blockage likelihood with lower (line above) and higher (line below) initial dominance.

GEO/TIME	2000	2001	2002	2003	2004	2005	2006	2007	2008	Mean
Bulgaria	3.5	4.2	3.7	5.5	5.0	4.8	4.6	5.0	4.9	4.6
Czech Rep.	7.3	9.3	8.9	8.3	7.5	6.9	7.1	6.9	7.2	7.7
Denmark	3.4	3.2	3.2	3.1	3.1	3.1	3.0	2.9	2.9	3.1
Germany	1.7	4.1	4.0	3.9	3.6	3.6	3.3	3.3	3.5	3.4
Estonia	3.8	3.7	3.7	3.5	3.9	4.0	4.7	4.6	4.9	4.1
Ireland	3.9	4.4	4.1	4.1	3.8	3.7	4.0	4.1	5.3	4.2
Greece	5.9	5.1	4.6	5.0	5.9	4.6	4.0	4.4	6.7	5.1
Spain	4.7	4.6	4.7	4.5	5.2	4.8	4.8	5.1	5.1	4.8
Italy	2.7	4.4	4.1	4.0	3.8	3.7	4.8	4.0	3.7	3.9
Cyprus	4.8	5.1	5.1	5.7	5.1	4.8	4.7	4.2	4.2	4.9
Lithuania	6.1	4.4	4.3	3.9	4.3	3.9	4.2	4.2	4.5	4.4
Luxembourg	4.1	2.8	5.0	4.7	4.8	4.4	4.7	4.0	4.2	4.3
Hungary	6.4	5.9	7.6	5.7	5.5	5.6	6.3	6.5	5.9	6.2
Malta	6.4	6.6	6.1	9.4	6.2	6.3	6.0	5.9	7.4	6.7
Austria	4.3	5.2	4.7	4.9	8.4	4.9	5.0	4.6	4.9	5.2
Portugal	5.3	5.6	4.8	4.6	4.4	4.4	3.8	3.8	4.5	4.6
Slovenia	5.2	4.3	4.4	4.8	4.1	3.9	4.1	4.1	4.7	4.4
Sweden	4.1	4.3	4.7	4.8	4.7	5.0	4.7	4.7	5.0	4.7
UK	1.9	2.5	2.6	2.9	2.6	2.9	2.9	2.9	4.8	2.9
Norway	4.9	4.7	4.8	4.3	4.0	3.9	3.6	3.7	3.6	4.2
Mean	4.5	4.7	4.6	4.7	4.7	4.5	4.6	4.5	4.9	4.6
StDev	1.4	1.4	1.4	1.5	1.3	0.9	1.0	1.0	1.1	1.1

Table 3.1: Total general government expenditure in economic affairs as percentage of GDP.

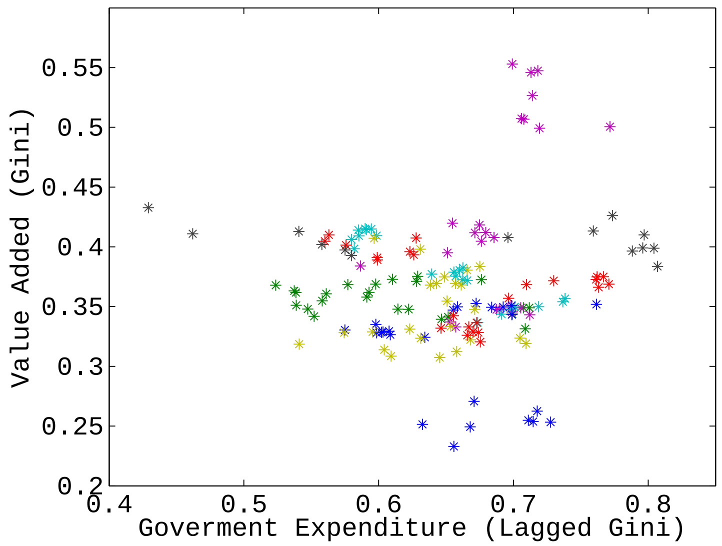

Figure 3.13: Value added Distribution as a function of previous period government expenditure distribution, cross-country evidence.

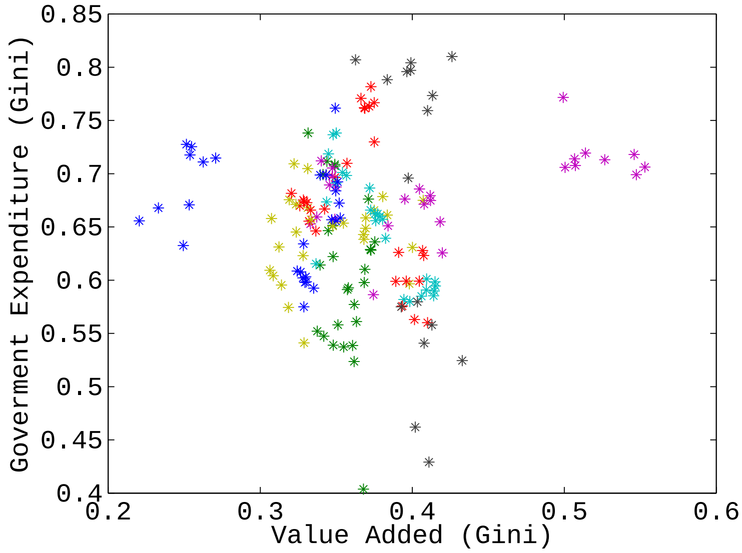

Figure 3.14: Government expenditure distribution as a function of value added distribution, cross-country evidence.

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Chapter 4

Do Aggregate Fluctuations Depend on the Network Structure of Firms and Sectors?

We construct a theoretical model that encompasses both firms' and sectors' network structure by considering a lower-dimension economic unit, that is, sector-specific establishments of multi-sectoral firms. The model suggests a reduced-form relation where aggregate production is a function of all the establishment-specific idiosyncratic shocks filtered by the network structure of the economy. We show that aggregate fluctuations depend on the geometry and magnitude of cross-effects across establishments, which is measured by the eigenvalues and eigenvectors of the network matrix. Moreover, the equilibrium levels and their dispersion depend on the Bonacich centrality of establishments within the network structure of the economy. Different network structures entail different aggregate volatilities due to the fact that the presence of direct relations averages out the idiosyncrasies across establishments.

Keywords: Granular hypothesis, Aggregate fluctuations, Networks, Conglomerates, Intersectoral linkages

4.1 Introduction

The standard diversification argument claims that independent sectoral shocks tend to average out as the level of disaggregation increases, as well as idiosyncratic shocks to firms average out as the number of firms increases.[1] In this perspective, the existence of aggregate fluctuations is only or at least mainly due to shocks that affect contemporaneously all the *grains* of the economy, such as fluctuations in intrinsically macroeconomic variables, e.g., the inflation rate, financial turmoils, or policy shocks.

A recent stream of literature tries to provide a microfoundation for the existence of aggregate shocks. We will refer to this stream as the granular hypothesis (GH) literature. The GH suggests two possible types of "grains" from which aggregate fluctuations might originate, that is, firms or sectors. For example, Gabaix [16] notices that the empirical size distribution of firms is fat-tailed. Hence, the baseline assumption of the diversification argument, that is, the finite variance of the distribution from which firms are drawn, is not supported empirically. For example, a power law for the size-distribution of firms allows for a relevant impact of idiosyncratic shocks on aggregate fluctuations. As a consequence, the variability in sales of the 100 top US firms can explain as much as 1/3 of aggregate variability. Other studies that look at firms to at least partially explain aggregate fluctuations are, among others, Jovanovic [24], Durlauf [15], Bak et al. [2], and Nirei [31]. In the case of sectoral shocks, the seminal paper of Long and Plosser [26] has been followed by several works, namely, Horvath [21] and [22], Conley and Dupor [9], Dupor [14], Shea [33], the same Bak et al. [2], Scheinkman and Woodford [32], Carvalho [7], Acemoglu, Ozdaglar, and Tahbaz-Salehi [1], and Carvalho and Gabaix [8]. In particular, Carvalho [7] applies the tools of network theory to the input-output tables, linking aggregate variability to the network structure of intersectoral trade. The presence of sectors that work as hubs to the economy make idiosyncratic shocks that would normally be irrelevant propagate to the aggregate level.

These two explanations may be overlapping. On the one hand, aggregate fluctuations can be originated through idiosyncratic shocks to sectors. On the other hand, the existence of big firms allows for the transmission of micro-level shocks to macro-level variables. Aggregate fluctuations are therefore either alternatively or jointly facilitated by idiosyncratic shocks to sectors and firms. In this paper we consider the two features, that is, sector- and firm-specific variability, as the two faces of the same phenomenon, which jointly contribute to the propagation of idiosyncratic shocks to the aggregate. The baseline intuition of our model is that big firms are not sector-specific. In other words, we can view firms as an

[1] See, e.g., Lucas [27] and, more recently, the irrelevance theorem of Dupor [14].

intersectoral network of sector-specific business units. From now on, we will refer to the networks of business units with the term "conglomerates," and we will call each atomistic business unit a "firm." Each firm produces a sector-specific commodity, and it can be part of a conglomerate. We make shocks originate at the firm level, so that conglomerate- or sector-wide fluctuations result as aggregations of multiple firm-specific shocks.

We present a static economy with multiple sectors. A continuum of households consumes a combination of good types according to their respective complementarity. Each good type is produced in a sector, where sector-specific firms compete à la Cournot among them. Each firm can have links with other firms in other sectors. If a link exists, then the two linked firms are part of the same conglomerate. The marginal cost of each firm depends on the production of the other firms linked to it, that is, on the production of the conglomerate. In network theory terminology, firms represent the vertices of a graph where we can note different components, that is, distinct path-connected subnetworks of firms. The network structure of conglomerates overlaps independently with the distribution of firms among sectors. We take the network structure as given and we explore the equilibrium output given the network.

We model peer effects among firms in a way similar to Ballester et al. [3], where the profit of each agent-firm is concave with respect to its own production and linear with respect to other firms' actions. This leads to a very tractable linear structure of the equilibrium solution and therefore permits a matrix representation. The basic result of this literature is that each agent's action is a function of its network centrality, a concept originally borrowed from the sociological literature. For example, see Bonacich [5]. The novelty of our model with respect to this stream of literature is the presence of multiple interrelated markets.[2] This permits the existence of intersectoral spillovers and transmission mechanisms that can be reduced to their peer-effects nature. A negative shock to firm i in sector s transmits not only as a positive shock to other competitor firms in the same sector but also as a negative shock to firms in other sectors both directly, if they are part of the same multisectoral conglomerate, and indirectly, through the complementarities on the demand side across commodities. Thus, the diffusion of idiosyncratic shocks from one firm to another depends on the existence of a transmission path that connects the two firms. This transmission path does not coincide with the network path of a conglomerate. It is instead a mix of network components and existence of markets for more or less related goods.

The multisectoral models à la Long and Plosser permit the transmission to

[2] İlkılıç [23] shares with our model the same logic and assumes profit maximization to be taken at the conglomerate level. This is game-theoretically more sophisticated but less tractable once we analyze aggregate volatility and we try to bring the model to the data.

the aggregate level of idiosyncratic sectoral shocks. The use of input-output tables permits to track the transmission mechanism of shocks across sectors. This explains partially the micro-foundation of aggregate fluctuations and intersectoral comovement as a product of sector-specific technological fluctuations in the style of RBC models. Nevertheless, it is not clear what a sector-specific shock is, in the sense that a sectoral shock is likely to be the aggregation of lower-dimensional "granular" shocks. Similarly, Gabaix [16] considers firm-specific shocks, even accounting for the industrial specialization of the core business of each firm. Nevertheless, it is not clear in this case either what a firm-specific shock is, being firm-specific production the complex aggregation of contract relations, internal organization, and so on. We partially overcome this ambiguities by considering establishment-specific shocks. Given the high level of disaggregation, we are more likely to characterize the idiosyncratic shocks as a product of chance, like mistakes in accounting, strikes, mismatches in the logistics procedures, or simply temporary bad luck in production. In our framework, sector- and firm-specific shocks result as aggregations at the sectoral and firm level of establishment-specific volatility. We would like to contribute to this literature by showing how the transmission of idiosyncratic shocks can account for a greater part of aggregate fluctuations once we take into account not only shocks to sectors or to firms separately, but considering the joint structure of intersectoral linkages and conglomerate relationships.

The paper is organized as follows. Section 2 maps the data on intersectoral linkages and conglomerates into graph-theoretic language. Section 3 presents the set-up for the model and derives the equilibrium solution. Section 4 expresses the equilibrium solution in terms of Bonacich centrality measures. Section 5 shows the effects of the network structure on aggregate volatility and performs some counterfactual exercises using both data from US Census Bureau and BEA and simulated random networks. Section 6 draws the final conclusions and suggests future lines of research. Proofs, figures, and tables are provided in the Appendix.

4.2 The Network Structure of the Economy

In this section we use Detailed Benchmark Input-Output data compiled by the Bureau of Economic Analysis to describe intersectoral linkages and the County Business Patterns by the US Census Bureau to describe some features of the conglomerate relations. We consider 2002 data.

Let $\mathscr{S} \equiv \{1, \cdots, S\}$ be the set of sectors ordered by the N.A.I.C.S. code at a certain digit level, where S is the total number of sectors. In the commodity-by-commodity direct requirements tables the typical (s, s') entry gives the input share evaluated at producers' prices of (row) commodity s as an intermediate input in the production of (column) commodity s'. In order to interpret the data, we assume that each commodity s is produced only in sector s. This is an approximation of what we can extract from the input-output tables, where each commodity can be produced by several sectors. Nevertheless, we can always assign for each commodity a typical sector that produces the great majority of the quantity of a specific commodity. This simplification does not entail qualitative problems and can be seen more as an abuse of notation. We use the requirements tables as a proxy for (aggregate) complementarity across sectors. If commodity s is used as an intermediate input in the production of commodity s' with a share $\tilde{\beta}_{ss'}$, then we say that commodity s is complementary to commodity s' and $\tilde{\beta}_{ss'}$ parameterizes the degree of complementarity. From a theoretical point of view, the complementarity between sectors can be technological or related to preferences, and can relate to both intermediate and final products. In this sense, the data in the direct requirements tables are an equilibrium product of the interaction between technological and consumption complementarities across sectors and at different levels of intermediation. From an aggregative perspective, if we treat the direct requirements as an exogenous endowment of the economy, then interpreting them as consumption or production complementarities does not yield different implications qualitatively and the problem becomes a matter of analytical tractability, as we see in the model below. According to this interpretation, the requirements tables tell us that, while commodity s is a complement of commodity s' with a certain degree $\tilde{\beta}_{ss'}$, commodity s' is a complement of commodity s with degree $\tilde{\beta}_{s's}$, where $\tilde{\beta}_{ss}$ is generally not equal to $\tilde{\beta}_{s's}$. Shocks to the production of a certain commodity can affect the production of other commodities either downstream, that is, from a sector that supplies the commodity to other sectors that demand the commodity, or upstream, that is, from a sector that demands the commodity to other sectors that supply the commodity. In order to analyze the transmission of shocks both downstream and upstream, we can construct a symmetric version of the input-output matrix that accounts for both downstream and upstream complementarity across sectors. Hence, we map the asymmetric input-output matrix into a symmetric complementarity matrix by adding the two corresponding entries for each pair of sectors (s, s') and normalizing them in the interval $[0, 1/(S-1)]$, that is,

$$\beta_{ss'} = \beta_{s's} = \frac{1}{S-1} \frac{\tilde{\beta}_{ss'} + \tilde{\beta}_{s's}}{\max\{\tilde{\beta}_{ss'} + \tilde{\beta}_{s's}\}},$$

for every $s \neq s'$ in $\mathscr{S}$. This transformation dismisses part the information contained in the input-output table but it is necessary for analytical tractability later on. Moreover, we ignore for the moment the complementarity of each commodity with respect to itself.

We use the elements of the set $\mathscr{S}$ of sectors as labels for the vertex set $V(\mathscr{S})$, that is, $V(\mathscr{S}) \equiv \{v_1, \cdots, v_S\}$.

Definition 11. The edge set of intersectoral linkages The edge set $E(\mathscr{S})$ of intersectoral linkages is a subset of $[V(\mathscr{S})]^2$ such that

$$E(\mathscr{S}) \equiv \{\{v_s, v_{s'}\} \in V(\mathscr{S})^2 \,| s \text{ is a complement of } s', \text{ with } s \neq s'\}.$$

In other words, $E(\mathscr{S})$ is defined by the link between elements of the set $V(\mathscr{S})$ of all nodes-sectors. Note that according to our definition of $E(\mathscr{S})$ a sector cannot be complementary to itself. The direct requirement of sector s of intermediate inputs from sector s itself does not enter the edge set, so that there are no self-links. The maximal consumption of commodity s, which is given by α_s/β_{ss}, captures this feature of the input-output tables. The link that defines $E(\mathscr{S})$ is undirected, that is, if $\{v_s, v_{s'}\}$ belongs to E, then also $\{v_{s'}, v_s\}$ belongs to E. We define also the weight function W as a real-valued function from $E(\mathscr{S})$ to $[0, 1/(S-1)]$ that assigns to each element $\{v_s, v_{s'}\}$ in $E(\mathscr{S})$ a weight equal to $W_S(\{v_s, v_{s'}\}) = \beta_{ss'}$.[3]

Definition 12 (The network of intersectoral linkages)**.** The network $\mathbf{g}(\mathscr{S}) \equiv (V(\mathscr{S}), E(\mathscr{S}), W_S)$ of intersectoral linkages is a network of vertex set $V(\mathscr{S})$, edge set $E(\mathscr{S})$, and weight function W_S, where every element of $E(\mathscr{S})$ is an undirected link between two distinct elements v_s and $v_{s'}$ of $V(\mathscr{S})$ with associated weight $W_S(\{v_s, v_{s'}\}) = \beta_{ss'}$.

For example, we represent graphically the network structure of intersectoral linkages in Figure 4.3.[4] The relative central position of the nodes reflects the weights associated to the links. Central sectors have strong links with the other sectors, that is, they are more complementary to the other sectors, while peripheral nodes have weak links. The central sectors for the US economy are "manufacturing" (node 5), "professional and business services" (node 10), and "educational services, health care, and social assistance" (node 11). The centrality of these sectors is clearer in Figure 4.4, where we dichotomize the network by constructing a new edge set where there is a link between two nodes only if the original link is greater than the threshold level of 0.01, and there is no link otherwise. The alternative weight function assigns the weight 1 to all links in the new edge set.

The adjacency matrix is a particular representation of weighted networks, which reproduces which pairs of nodes are linked together and with which weight.

[3]We could use the term vertex instead of the term node. Similarly, we could use the terms adjacency relation, edge, or arc instead of the term link, the term link strength instead of weight, and the term graph instead of network. The literature uses this terminology interchangeably.

[4]For the representations of graphs we use the software UCINET 6 by Borgatti, Everett, and Freeman, distributed by Analytic Technologies.

Definition 13 (The network matrix of intersectoral linkages)**.** The network matrix $\mathbf{B}$ of intersectoral linkages is the adjacency matrix of the network $\mathbf{g}(\mathscr{S})$. In other words, $\mathbf{B}$ is a real-valued symmetric $S \times S$ matrix with typical element $\mathbf{B}_{ss'} = \beta_{ss'} \in [0, 1/(S-1)]$ if $\{v_s, v'_s\}$ belong to $E(\mathscr{S})$ and $W_S(\{v_s, v_{s'}\}) = \beta_{ss'}$, and $\mathbf{B}_{ss'} = 0$ if $\{v_s, v'_s\}$ does not belong to $E(\mathscr{S})$.

We reproduce in Table 4.3 in the Appendix the matrix of intersectoral linkages for the case $S = 17$, derived from BEA's commodity-by-commodity direct requirements tables for 2002.

An *establishment* is a single physical location at which business is conducted or services are provided. Each establishment is classified on the basis of its major activity, that is, each establishment is characterized by a sector of specialization. An establishment is not necessarily identical with a company or an enterprise, which may consist of one establishment or more. It constitutes the very "grains" of organized economic activity, in the sense that no matter what conceptual framework we employ we can always express any corporation, division, subsidiary, company, and so on, as a differently organized combination of a certain number of establishments. For example, a conglomerate is a collection of corporations involving a parent company and one or more subsidiaries. Each subsidiary can have its own subsidiaries itself. Each company of the conglomerate, be it a parent company or a subsidiary, can be decomposed into one or more establishments, connected to one another by one or more legal, accounting, or economic ties. From now on, we call each establishment a *firm* and any collection of establishments a *conglomerate*.

Let $\mathscr{F}$ be the set of establishments/firms and F be the total number of firms. Each firm is sector-specific, so the set of sectors defines a partition P_S of the set of firms. Let $P_S \equiv \{\mathscr{F}_1, \cdots, \mathscr{F}_S\}$ be a set of subsets of $\mathscr{F}$ such that $\bigcup_{s\in\mathscr{S}} \mathscr{F}_s = \mathscr{F}$ and $\mathscr{F}_s \cap \mathscr{F}_{s'} = \emptyset$ if $\mathscr{F}_s$ and $\mathscr{F}_{s'}$ belong to P_S, for every $s \neq s'$ in $\mathscr{S}$. For every sector s in $\mathscr{S}$, we call n_s the cardinality of $\mathscr{F}_s$, that is, $n_s \equiv \#(\mathscr{F}_s)$. By construction, $\sum_{s\in\mathscr{S}} n_s = F$. We order the firms in $\mathscr{F}$ by their sector of activity, that is,

$$\mathscr{F} = \{1, \cdots, n_1, n_1 + 1, \cdots, n_1 + n_2, \cdots, F\}, \tag{4.1}$$

where n_s is the number of establishments in sector s, for all s in $\mathscr{S}$.[5]

We use the elements of the set $\mathscr{S}$ of firms as labels for the vertex set $V(\mathscr{F})$, that is, $V(\mathscr{F}) \equiv \{v_1, \cdots, v_F\}$.

Definition 14 (The edge set of linked firms)**.** The edge set $\tilde{E}(\mathscr{F})$ of linked firms

[5]According to the County Business Patterns of the US Census Bureau, there were around 5.5 million employer establishments in the US in 2002. The number of establishments and their dimension, measured in terms of average number of employees per establishment, are distributed unevenly across sectors and change over time.

is a subset of $[V(\mathscr{F})]^2$ such that

$$\tilde{E}(\mathscr{F}) \equiv \{\{v_i, v_j\} \in V(\mathscr{F})^2 \,|i\text{'s output depends on } j\text{'s output, with } i \neq j\}.$$

In other words, $\tilde{E}(\mathscr{F})$ is defined by the link between elements of the set $V(\mathscr{F})$ of all nodes-firms. Note that there are no self-links, that is, a firm's performance cannot depend on its own performance. The links that define $E(\mathscr{F})$ are undirected, that is, if $\{v_i, v_j\}$ belongs to E, then also $\{v_j, v_i\}$ belongs to E.

Definition 15 (The network of linked firms)**.** The network $\tilde{\mathbf{g}}(\mathscr{F}) \equiv (V(\mathscr{F}), \tilde{E}(\mathscr{F}))$ of linked firms is an undirected network of vertex set $V(\mathscr{F})$ and edge set $\tilde{E}(\mathscr{F})$, where every element of $\tilde{E}(\mathscr{F})$ is an undirected link between two distinct elements v_i and v_j of $V(\mathscr{F})$.

A path from i to j of length l is a sequence of $l \in \mathbb{N}$ links that indirectly connects node v_i to node v_j. For example, there may be no direct link between i and j, that is, $\{v_i, v_j\}$ may not belong to the edge set $\tilde{E}(\mathscr{F})$. Nevertheless, if there exists a node v_k such that both $\{v_i, v_k\}$ and $\{v_k, v_j\}$ belong to $\tilde{E}(\mathscr{F})$, then there is path of length 2 from i to j. A component $C(\tilde{E})$ of vertex set $V(\mathscr{F})$ and edge set $\tilde{E}(\mathscr{F})$ is a subset of the vertex set $V(\mathscr{F})$ such that for each pair of nodes v_i and v_j in the component C it is possible to find a path of some length l in $\tilde{E}(\mathscr{F})$ between v_i and v_j. There may be one or more components in the vertex set. The set of components constitutes a partition of the vertex set $V(\mathscr{F})$.

In the model we assume that a firm does not compete with firms within the same conglomerate. This is not necessarily true in practice but it is a necessary assumption for analytical tractability.[6] Hence, we construct a network that respects this limitation. Let $V(\mathscr{F}_s)$ be the vertex set of the firms that belong to sector s in $\mathscr{S}$. By construction, $v_i \in V(\mathscr{F}_s)$ if and only if $i \in \mathscr{F}_s$. Moreover, if $v_i \in V(\mathscr{F}_s)$ and $v_j \in V(\mathscr{F}_{s'})$ with $s \neq s'$, then necessarily $i \neq j$.

Definition 16 (The edge set of conglomerates)**.** The edge set $E(\mathscr{S})$ of conglomerates is a subset of $\tilde{E}(\mathscr{F})$ such that

$$\begin{aligned} E(\mathscr{F}) \equiv \{\{v_i, v_j\} \in V(\mathscr{F}_s) \times V(\mathscr{F}_{s'}) \,|&\text{if } v_i, v_j \in C(\tilde{E}), \\ &\text{then } \nexists v_k \in C(\tilde{E}) \setminus \{v_i, v_j\} \text{ such that } v_k \in \mathscr{F}_s \cup \mathscr{F}_{s'}\}. \end{aligned}$$

In other words, the edge set of conglomerates is a subset of the edge set where each path can only include nodes that belong to different sectors.

For every component $C(E)$ in $V(\mathscr{F})$ that we obtain from the edge set $E(\mathscr{F})$ and for every sector s in $\mathscr{S}$, we can find at most one node v_i in $C(E)$ such that firm i belongs to $\mathscr{F}_s$. This leads us to the definition of conglomerate.

[6] In reality a parent company and a subsidiary can be competitors in the same market and this may be an important source - or lack thereof - of comovement across firms.

Definition 17 (Conglomerate)**.** A conglomerate $C(E)$ is a component of the vertex set $V(\mathscr{F})$ and edge set $E(\mathscr{F})$.

The conglomerates are multisectoral components of $V(\mathscr{F})$. For each firm i whose node v_i belongs to the component $C(E)$, we say that firm i is part of conglomerate $C(E)$. A conglomerate cannot have more than one firm in each sector, and each sector can have at most one firm that belong to a certain conglomerate. We call $\mathscr{C}(E)$ the set of all components $C(E)$ of the vertex set $V(\mathscr{F})$ through the edge set $E(\mathscr{F})$. By construction, $\mathscr{C}(E)$ is a partition of the vertex set $V(\mathscr{F})$, and for every $C(E)$ in $\mathscr{C}(E)$ and every s in $\mathscr{S}$ we have that $\#(C(E) \cap V(\mathscr{F}_s)) \leq 1$. The cardinality of each conglomerate measures its sectoral diversification.

We define also the weight function W_F as a real-valued function from $E(\mathscr{F})$ to $[0, 1]$ that assigns to each element $\{v_i, v_j\}$ in $E(\mathscr{F})$ a weight, or link strength, equal to $W_F(\{v_i, v_j\}) = \epsilon$, where $\epsilon \in [0, 1]$.

Definition 18 (The network of conglomerates)**.** The network $\mathbf{g}(\mathscr{F}) \equiv (V(\mathscr{F}), E(\mathscr{F}), W_F)$ of conglomerates is an undirected network of vertex set $V(\mathscr{F})$, edge set $E(\mathscr{F})$, and weight function W_F, where every element of $E(\mathscr{F})$ is an undirected link between two distinct elements v_i and v_j of $V(\mathscr{F})$ with associated weight $W(\{v_i, v_j\}) = \epsilon \in [0, 1]$.

We present in Figure 4.5 a simulated random network of conglomerations with $F = 100$ and $S = 20$. Each node is a firm, and each group of linked nodes is a conglomerate. Each conglomerate is characterized by a different shape of its nodes. On the upper left side there is a group of one-firm conglomerates, while the rest of the firms are distributed among conglomerates of different sizes. Since each firm belongs to a different sector within the same component, the size of the components reflects the diversification of each conglomerate. A random network that satisfies the properties of Definition 18 tends generate a size distribution of conglomerates, as we can see in Figure 4.6. This resembles the empirical sectoral diversification of large conglomerates, which we reproduce in Figure 4.7.

We can define the adjacency matrix of the conglomerations, that reproduces which of nodes of the vertex set of firms $V(\mathscr{F})$ are linked according to $E(\mathscr{F})$ and with which weight.

Definition 19 (The network matrix of conglomerations)**.** The network matrix Γ of conglomerations is the adjacency matrix of the network $\mathbf{g}(\mathscr{F})$. In other words, Γ is a real-valued symmetric $F \times F$ matrix with typical element $\gamma_{ij} = \epsilon \in [0, 1]$ if $\{v_i, v_j\}$ belong to $E(\mathscr{F})$ and $W(\{v_i, v_j\}) = \epsilon$, and $\gamma_{ij} = 0$ if $\{v_i, v_j\}$ does not belong to $E(\mathscr{F})$.

Given the ordering of $\mathscr{F}$ in (4.1) and Definition 18, if the number F of firms is enough greater than the number S of sectors, then the matrix Γ of conglomerations is a rather sparse matrix, with several zero entries and just a few non-nil entries.

4.3 The Model

Consider a multisector economy. There is a continuum of mass 1 of identical households whose utility depends on the consumption c_s of different commodities. There are $S \in \mathbb{N}$ commodities, where $s \in \mathscr{S} = \{1, \cdots, S\}$. There are S productive sectors, each of them producing a different sector-specific commodity.[7] Each sector s is populated by n_s firms, and each firm i within sector s produces an undifferentiated quantity q_i of good s competing à la Cournot with the other firms within the same sector. Each firm may have a link with firms in other sectors, and the more productive the linked firms, the lower the marginal production cost.

On the preference side, each household owns a symmetric share of the profits realized on the production side, and employs these resources to consume different good types. We omit an index for the generic household, though we bear in mind that each control variable on the preference side should have it. The utility function of the household is linear-quadratic, that is,

$$U(c_1, \cdots, c_S) = \sum_{s \in \mathscr{S}} \alpha_s c_s - \frac{1}{2} \sum_{s \in \mathscr{S}} \beta_{ss} c_s^2 + \sum_{s \in \mathscr{S}} \sum_{s' \neq s} \beta_{ss'} c_s c_{s'},$$

with $\alpha_s \geq 0$, $\beta_{ss} > \beta_{ss'}$, and $\beta_{ss'} \in [0, 1/(S-1)]$ for all s and s' in $\mathscr{S}$.

The value of $\sum_{s=1}^{S} \alpha_s$ measures the absolute size of the economy, that is, α_s is high enough so that the utility function is strictly increasing and strictly concave separately with respect to the consumption of each commodity c_s. The intuition is the following: suppose that $\alpha_s = \sum_{s'=1}^{S} \alpha_{s'}$, that is, the household only consumes the good of type s. The marginal utility is always decreasing but positive only up to $c_s < \alpha_s/\beta_{ss}$. Hence, α_s represents the upper bound for the values of c_s such that the utility function shows the standard properties of strictly positive, strictly decreasing marginal utility. Below this level, consumption is compatible with the standard properties of utility functions. The parameter β_{ss} measures the concavity of the utility function separately with respect to each good, and $\beta_{ss'}$ parameterizes instead the degree of complementarity between different consumption goods. If $\beta_{ss'} = 0$, the goods are independent. If $0 < \beta_{ss'} \leq 1/(S-1)$, the goods are complements at different degrees. The upper bound on the possible values of $\beta_{ss'}$ avoids increasing returns in the model. For a similar version of this utility function, see Bloch [4]. In order to rule out the possibility of nil demand of any good type, we may assume that $\alpha_s > 0$ for every s. For simplicity, we also assume that $\beta_{ss'} = \beta_{s's}$, that is, the relation of complementarity between two good types is reciprocal.[8]

[7] We use the term commodity and sector interchangeably because we assume that only sector s produces commodity s.

[8] Since the model does not incorporate any input-use mechanism on the production side,

The household maximizes its utility subject to the budget constraint

$$\sum_{s=1}^{S} p_s c_s = \sum_{s=1}^{S} \Pi_s,$$

where p_s is the price of each good s and

$$\Pi_s \equiv \sum_{i \in \mathscr{F}_s} \pi_i^s \tag{4.2}$$

represents the share of the household in the profits realized by each firm i operating in sector s. The set $\mathscr{F}_s$ is the set of firms that operate in sector s and π_i^s is the profit of the i-th firm in $\mathscr{F}_s$. Since there is a continuum of households of mass 1 and shares are equal across households, each dividend coincides analytically with total profits. The maximization problem is

$$\begin{aligned} \max_{\{c_s\}_{s\in\mathscr{S}}} \sum_{s\in\mathscr{S}} \alpha_s c_s - \frac{1}{2}\sum_{s\in\mathscr{S}} \beta_{ss} c_s^2 + \sum_{s\in\mathscr{S}} \sum_{s'\neq s} \beta_{ss'} c_s c_{s'} \\ \text{subject to} \quad \sum_{s\in\mathscr{S}} p_s c_s = \sum_{s=1}^{S} \Pi_s. \end{aligned} \tag{4.3}$$

The first order condition (FOC) yields a linear inverse demand function for each commodity,

$$p_s = \alpha_s - \beta_{ss} c_s + \sum_{s'\neq s} \beta_{ss'} c_{s'}. \tag{4.4}$$

For a discussion of the parameter values for which we have positive prices and quantities, see Bloch [4].

On the production side, there are n_s firms in each sector s. They compete à la Cournot and share the sector-specific demand expressed by the households. The maximization problem for firm i in sector s is

$$\max_{q_i} \quad \pi_i^s \equiv p_s q_i - m_i q_i, \tag{4.5}$$

where m_i is the marginal cost of producing one unit of good s. We assume that the marginal cost is invariant across good types. Moreover, we suppose that the

this is a fair approximation of the aggregate technological relation between two sectors. An alternative modeling strategy is to consider a unique final good consumed by the households and produced using the S intermediate commodities through an aggregative equation that takes into account the technological complementarities among commodities. Since this would not change the structure of the aggregate equilibrium, we choose the simplest formulation.

marginal cost is linearly increasing in the firm i's own production and decreasing in the production of any other firm that has a link with firm i. Definition 18 implies that the maximum degree of any firm i is $S-1$, that is, the number of sectors other than its own. The marginal cost m_i of firm i is

$$m_i = \frac{\delta}{2} q_i - \sum_{\substack{j \in \mathscr{F} \\ j \neq i}} \gamma_{ij} q_j - \xi_i, \tag{4.6}$$

where $\delta > 0$ parameterizes the concavity of each firm's profits in own production. The idiosyncratic component ξ_i is an iid random variable with mean μ and finite variance σ^2. The element $-\sum_{\substack{j \in \mathscr{F} \\ j \neq i}} \gamma_{ij} q_j$ represents how the marginal cost of a firm decreases with the production of the firms that are linked to it. According to Definition 18, γ_{ij} can be either 0 if i does not share a link with j or ϵ if i does share a link with j. The conjecture behind lies on a similar way of thinking as Goyal and Joshi [17], where the marginal cost is linearly decreasing in the number of links that a firm has. We provide hereafter two examples in which the marginal cost of production is connected to the production of the linked firms. For a similar argument, see Bloch [4].

Example 4 (Common use of a facility.)**.** Firms in the same conglomerate have to pay a fixed amount ε for every produced unit in order to construct a common warehouse, independently of the good type produced. We can treat this portion as a proportional taxation that every firm in the conglomerate agrees to pay. The rationale for this commitment is that the cost of each produced unit decreases with the size of the common warehouse, a sort of a public good for the firms that are part of the conglomerate. Firm i in conglomerate κ_i has therefore a unitary cost that is composed of $\tilde{\delta} q_i$ for contracting its own workers, $-\tilde{\xi}_i$ for some idiosyncratic technological endowment, ε as the compulsory proportional contribution to the common warehouse, and $-\varepsilon \sum_{j \in \kappa_i} q_j$ as the unit cost of operating the warehouse which decreases with the size of the warehouse. Hence, the marginal cost is

$$m_i = \tilde{\delta} q_i - \tilde{\xi}_i + \varepsilon - \varepsilon \sum_{j \in \kappa_i} q_j = (\tilde{\delta} - \varepsilon) q_i - \sum_{\substack{j \in \kappa_i \\ j \neq i}} \varepsilon q_j + \varepsilon - \tilde{\xi}_i,$$

which has the same form as (4.6), once we impose $\delta \equiv 2(\tilde{\delta} - \varepsilon)$, $\tilde{\xi}_i$ distributed iid with mean ε and finite variance σ, and $\gamma_{ij} \equiv \mathbb{I}_i^j \varepsilon$, where $\mathbb{I}_i^j = 1$ if $j \in \kappa_i$ and $\mathbb{I}_i^j = 0$ otherwise.

Example 5 (Common R&D production.)**.** Every firm within a conglomerate produces a technology which is nonrival for the members of the conglomerate. As in

Goyal and Moraga [18] for example, we characterize production as an innovation effort. Technology produced in one sector by a firm of the conglomerate can be applied in another sector with an adaptation cost, $(1-\varepsilon)$, that we can treat as an iceberg cost on the original technology. In order to be sold in the market, that is, outside the borders of the conglomerate, the product must be patented and therefore the complementarity or the substitutability on the demand side depend on the legal rather than technological compatibility between one good and another. The costs of each firms are convex in the firm's own innovation effort and depend on the effort exerted by the other firms in the same conglomerate, whereas the marginal cost is equal across firms of the same conglomerate up to a scaling constant ξ_i. The resulting expression for m_i is the same as in (4.6).

Remember that Definition 18 implies that $\gamma_{ij} = 0$ if i and j produce the same good type s, that is, two firms that belong to the same conglomerate cannot compete with one another. Moreover, $\gamma_{ij} = 0$ also if there exists a path between i and j in $\mathbf{g}(\mathscr{F})$ such that a firm k on that path belongs to either the sector of firm i or the sector of firm j. The structure of the marginal costs we assume basically implies that the correlation of produced quantities between two firms that share a link should be higher than the correlation between two firms that do not share a link, which is at the origin of our concept of linkage between two firms. The share of each unit i within sector s depends on the conglomerate to which i belongs, and more specifically on the conglomerate's dimension and the dimension of the sectors in which the conglomerate has a firm.

There is a market clearing condition for each sector that guarantees that at equilibrium the demand for good s expressed on the preference side is equal to the supply provided by the production side, i.e.,

$$c_s \leq \sum_{i \in \mathscr{F}_s} q_i, \tag{4.7}$$

for every s in $\mathscr{S}$. This means that the households can consume only up to the total production provided by all the firms that operate in sector s. Another market clearing condition refers to the financial market, that is, all the profits realized by the firms are distributed among the households in equal shares, as (4.2) states.

Definition 20 (Equilibrium)**.** An equilibrium for the economy is a set of household decisions $\{c_s\}_{s \in \mathscr{S}}$, firms decisions $\{q_i\}_{i \in \mathscr{F}}$, dividends $\{\Pi_s\}_{s \in \mathscr{S}}$, profits $\{\pi_i\}_{i \in \mathscr{F}}$, and prices $\{p_s\}_{s \in \mathscr{S}}$ such that $\{c_s\}_{s \in \mathscr{S}}$ solves (4.3) given $\{p_s\}_{s \in \mathscr{S}}$ and $\{\Pi_s\}_{s \in \mathscr{S}}$, $\{q_i\}_{i \in \mathscr{F}}$ solves (4.5) given (4.4), and the market clearing conditions (4.7) and (4.2) hold for every $s \in \mathscr{S}$.

Let us consider the FOC of (4.5) and the market clearing condition (4.7). If we substitute for $c_{s'} = \sum_{j \in F_{s'}} q_j$ in (4.4), we obtain

$$p_s = \alpha_s - \beta_{ss} \sum_{j \in F_s} q_j + \sum_{s' \neq s} \beta_{ss'} \left(\sum_{j \in F_{s'}} q_j \right),$$

for every s in $\mathscr{S}$. We can plug this inverse demand function and the expression for the marginal cost (4.6) inside the i-th firm's problem (4.5). The resulting firm i's problem is

$$\max_{q_i} \quad \left[\alpha_s - \beta_{ss} \sum_{j \in F_s} q_j + \sum_{s' \neq s} \beta_{ss'} \left(\sum_{j \in F_{s'}} q_j \right) \right] q_i - \left[\frac{\delta}{2} q_i - \sum_{\substack{j \in \mathscr{F} \\ j \neq i}} \gamma_{ij} q_j - \xi_i \right] q_i,$$

whose FOC with respect to q_i yields

$$\alpha_s - \beta_{ss} q_i - \beta_{ss} \sum_{j \in F_s} q_j + \sum_{s' \neq s} \beta_{ss'} \left(\sum_{j \in F_{s'}} q_j \right) - \delta q_i + \sum_{\substack{j \in \mathscr{F} \\ j \neq i}} \gamma_{ij} q_j + \xi_i = 0.$$

Thus, the equilibrium solution satisfies

$$(\delta + \beta_{ss}) q_i + \beta_{ss} \sum_{j \in F_s} q_j - \sum_{s' \neq s} \beta_{ss'} \left(\sum_{j \in F_{s'}} q_j \right) - \sum_{\substack{j \in \mathscr{F} \\ j \neq i}} \gamma_{ij} q_j = \alpha_s + \xi_i, \tag{4.8}$$

for every s in $\mathscr{S}$ and every i in $\mathscr{F}_s$. The FOC (4.8) identifies three aspects that characterize a firm's optimal production choice. First, the width of the sector in which it operates, parameterized by α_s. Second, its idiosyncratic level of technology, identified by ξ_i, which affects the cost-efficiency of production. Third, the position of the firm within the network structure of production, that is, some centrality measure relative to the network of conglomerates Γ and its interaction with the sectoral composition of the economy $\mathbf{B}$. We assume for simplicity that the concavity of the utility function with respect to the consumption of each commodity alone is the same across good types. This does not affect relevantly the results.

Assumption 8. The concavity of the utility function with respect to each good's consumption is the same across all good types, that is, $\beta_{ss} = \beta$ for all s in $\mathscr{S}$.

We use Definition 13 and Definition 19 to express the equilibrium solution in matrix form. Let $\bar{\xi} \equiv [\xi_1, \cdots, \xi_N]'$ be the vector of length F of the idiosyncratic shocks to all firms grouped by sector. Similarly, let $\bar{q} \equiv [q_1, \cdots, q_N]'$ be the vector of optimal quantities. We define $\bar{\alpha} \equiv [\alpha_1, \cdots, \alpha_1, \cdots, \alpha_S, \cdots, \alpha_S]'$ as the vector of length F of the sectors' shares within the overall size of the economy, where each sector share α_s is repeated n_s times, where n_s is the number of firms within sector s. Let $\mathbf{I}_S$ be the $S \times S$ identity matrix and $\mathbf{U}_s$ be the $n_s \times n_s$ matrix of ones. If we compute $\mathbf{B} - \beta * \mathbf{I}_S$, we can construct the $F \times F$ block matrix $\bar{\mathbf{B}}$, where each block is the expansion of $\mathbf{B} - \beta * \mathbf{I}_S$ by $\mathbf{U}_s$. In order to represent the equilibrium in matrix form, we need a matrix that accounts for both the network of intersectoral linkages and the network of conglomerates.

Definition 21 (The interaction matrix of the economy)**.** The interaction matrix Θ of the economy is the real-valued symmetric $F \times F$ matrix given by $\Theta \equiv \bar{\mathbf{B}} + \Gamma$.

The matrix Θ summarizes both the network of intersectoral linkages and the network of conglomerates. The typical element θ_{ij} of Θ is either $\theta_{ij} = \beta_{ss'} + \gamma_{ij}$ if $s \neq s'$, or $\theta_{ij} = -\beta$ if $s = s'$, where firm i operates in sector s and firm j operates in sector s'. Let $\mathbf{I}_F$ be the $F \times F$ identity matrix. Hence, the necessary condition (4.8) for the equilibrium solution consists of the matrix equation

$$\Psi \bar{q} = \bar{\alpha} + \bar{\xi}, \tag{4.9}$$

where Ψ is

$$\Psi \equiv \left[(\delta + \beta) \mathbf{I}_F - \Theta \right]. \tag{4.10}$$

We call $\bar{q}^*$ the solution of the matrix equation (4.9).

Proposition 12. *The matrix equation (4.9) has a unique generic solution $\bar{q}^*$, and*

$$\bar{q}^* = \Psi^{-1} \left[\bar{\alpha} + \bar{\xi} \right], \tag{4.11}$$

where Ψ^{-1} is the inverse of Ψ.

The equilibrium production is a function of the relative dimensions of the sectors composing the economy and the idiosyncratic productivities of all the firms in the economy filtered by the network matrix of the economy. The existence and uniqueness of the equilibrium solution does not mean that $q_i^* \geq 0$ for every i in $\mathscr{F}$. This depends on the interaction between Ψ^{-1} and $\bar{\alpha} + \bar{\xi}$, which may contain negative elements.

4.4 Equilibrium and Bonacich Centrality

We need to characterize the equilibrium solution to obtain sufficient conditions for which $\bar{q}^* \geqq 0$. Let $\underline{\theta} \equiv \min\{\theta_{ij} | i \neq j\}$ and $\overline{\theta} \equiv \max\{\theta_{ij} | i \neq j\}$. Since $\gamma_{ij} \geq 0$

for every i and j in $\mathscr{F}$ and $\beta_{ss} > \beta_{ss'} \geq 0$ for every $s \neq s'$ in $\mathscr{S}$, then $\underline{\theta} = -\beta < 0$ and $\overline{\theta} \geq 0$. In order to obtain sufficient conditions for which $\bar{q}^* \geqq 0$ we need to decompose Ψ. Since $\Psi = [(\delta + \beta)\mathbf{I}_F - \Theta]$ according to (4.10), we have to reformulate Θ. First, we isolate the positive cross-effects of Θ and we normalize them by the highest cross-effect $\overline{\theta}$.

Definition 22 (The network matrix of the economy)**.** The network matrix $\mathbf{G}$ of the economy is the $F \times F$ real-valued matrix where the typical element is

$$g_{ij} \equiv \frac{\theta_{ij} + \beta}{\overline{\theta} + \beta}$$

for $i \neq j$ and $g_{ii} = 0$ otherwise.

Note that $\overline{\theta} + \beta > 0$ since $\beta > 0$. By construction, $g_{ij} \in [0, 1]$ for all i and j in $\mathscr{F}$.

We can decompose the matrix Ψ into three components of cross-effects between firms,

$$\Psi = -\left[-(\delta + \beta)\mathbf{I}_F - \beta\mathbf{U}_F + \left(\overline{\theta} + \beta\right)\mathbf{G}\right], \tag{4.12}$$

where $\mathbf{U}_F$ is the $F \times F$ matrix of ones. The component $-(\delta + \beta)\mathbf{I}_F$ reflects the concavity of firms' profits in own production, the component $-\beta\mathbf{U}_F$ mirrors the competition in homogeneous quantities within the same sector and uses this uniform substitutability as a benchmark value for all the firms, and the component $\left(\overline{\theta} + \beta\right)\mathbf{G}$ represents the complementarity of production decisions between firms within the same conglomerate, measured from the benchmark value of the competition à la Cournot.

Note that we can treat the network matrix of the economy as the adjacency matrix of a weighted network, which we call network of the economy, that accounts for both the network of intersectoral linkages and the network of conglomerates. In order to define the network of the economy, we need a vertex set, an edge set, and a weight function.

Definition 23. The edge set of the economy The edge set E of the economy is a subset of $[V(\mathscr{F})]^2$ such that

$$E \equiv \{\{v_i, v_j\} \in [V(\mathscr{F})]^2 \;|\{v_i, v_j\} \in E(\mathscr{F}) \text{ or } \{v_s, v_{s'}\} \in E(\mathscr{S})\},$$

where $i \in \mathscr{F}_s$ and $j \in \mathscr{F}_{s'}$.

The weight function W is a real-valued function from $V(\mathscr{F})^2$ to $[0, 1]$.

Definition 24 (The network of the economy)**.** The network $\mathbf{g} \equiv (V(\mathscr{F}), E, W)$ of the economy is an undirected network of vertex set $V(\mathscr{F})$ and edge set E, where every element of E is an undirected link between two distinct elements v_i and v_j of $V(\mathscr{F})$, with associated weight $W(\{v_i, v_j\}) = g_{ij} \in [0, 1]$.

As an example, we report in Figure 4.8 the representation of the network matrix $\mathbf{G}$ for the US economy, where we can see the blocks of intersectoral linkages at different complementarity levels and the conglomerate relations across the sectors.

Let $\lambda_{max}(\mathbf{G})$ be the highest eigenvalue of $\mathbf{G}$. This value is crucial for the characterization of the equilibrium solution.

Assumption 9. The network matrix $\mathbf{G}$ satisfies

$$\lambda_{max}(\mathbf{G}) < \frac{\delta + \beta}{\overline{\theta} + \beta}.$$

The intuition behind Assumption 9 is that the concavity of own production, measured by $\delta + \beta$, must be higher than the maximal complementarity between own production and other firms' choices, measured by $(\overline{\theta} + \beta)\lambda_{max}(\mathbf{G})$.

Proposition 13 (Debreu and Herstein, 1953)**.** $\Psi^{-1} \geqq 0$ *if and only if Assumption 9 holds.*

Let $\lambda_{min}(\mathbf{G})$ be the lowest eigenvalue of $\mathbf{G}$. Moreover, we call $\rho(\mathbf{G}) \equiv \max\left(|\lambda_{min}(\mathbf{G})|, |\lambda_{max}(\mathbf{G}\right.$ the spectral radius of $\mathbf{G}$. Since $\mathbf{G}$ is square and symmetric, $\lambda_{max}(\mathbf{G}) \geq 0$ and $\lambda_{min}(\mathbf{G}) \leq 0$. Moreover, $\lambda_{max}(\mathbf{G}) \geq -\lambda_{min}(\mathbf{G})$.[9] Hence, Assumption (9) implies that

$$\lambda_{min}(\mathbf{G}) > -\frac{\delta + \beta}{\overline{\theta} + \beta}, \tag{4.13}$$

and therefore the spectral radius $\rho(\Theta) \equiv \max\{|\lambda_{max}(\mathbf{G})|, |\lambda_{min}(\mathbf{G})|\} < (\delta + \beta)/(\overline{\theta} + \beta)$. The weaker condition in (4.13) is sufficient for $\Psi^{-1} \geqq 0$, and it is necessary if $\mathbf{G}$ describes a regular network, that is, a network without loops and multiple edges where each node has the same number of neighbors. See Bramoullé, Kranton, and D'Amours [6, Corollary 1].[10] As the highest eigenvalue represents the population-wide pattern and level of positive cross-effects, the lowest eigenvalue represents another important feature of the network. The lower it is, the higher the number of links that connect distinct sets of firms, and the greater the impact of each firm's production decision on other firms' production.

Note that

$$\mathbf{U}_F \bar{q}^* = q^* \bar{1}_F,$$

where $q^* \equiv \sum_{i \in \mathscr{F}} q_i$. Using (4.12), we can rewrite (4.9) as

$$\left[(\delta + \beta)\mathbf{I}_F - \left(\overline{\theta} + \beta\right) \mathbf{G}\right] \bar{q}^* = -\beta q^* \bar{1}_F + \bar{\alpha} + \bar{\xi},$$

[9] See Cvetković et al. [10, Theorem 0.13].

[10] In our framework, the network structure of the economy would be a regular network if all the conglomerates had the same number of firms distributed in the same collection of sectors and each firm within a conglomerate was connected to each other.

that is,

$$(\delta+\beta)\left[\mathbf{I}_F - \frac{\overline{\theta}+\beta}{\delta+\beta}\mathbf{G}\right]\bar{q}^* = -\beta q^* \bar{1}_F + \bar{\alpha} + \bar{\xi}.$$

If Assumption 9 holds, the matrix $\left[\mathbf{I}_F - (\overline{\theta}+\beta)/(\delta+\beta)\mathbf{G}\right]$ is invertible.[11] If it is invertible and $(\overline{\theta}+\beta)/(\delta+\beta)$ is small enough, then we can express $\left[\mathbf{I}_F - (\overline{\theta}+\beta)/(\delta+\beta)\mathbf{G}\right]$ by a Newman series, that is,

$$\left[\mathbf{I}_F - \frac{\overline{\theta}+\beta}{\delta+\beta}\mathbf{G}\right]^{-1} = \sum_{k=0}^{+\infty}\left(\frac{\overline{\theta}+\beta}{\delta+\beta}\right)^k \mathbf{G}^k,$$

where $\mathbf{G}^k$ as the k-th power of $\mathbf{G}$. We call $g_{ij}^{[k]}$ the typical element of $\mathbf{G}^k$, where k is a positive integer. The iterations of $\mathbf{G}$ for $k \in \{2, 3, \cdots\}$ keep track of the indirect connections in the network of both conglomerates and sectors. The entry $g_{ij}^{[k]}$ yields the number of paths of length k necessary to pass from i to j in the network $\mathbf{g}$ of Definition 24. Hence, the typical element of

$$\left(\frac{\overline{\theta}+\beta}{\delta+\beta}\right)^k \mathbf{G}^k$$

measures the number of paths number of paths of length k necessary to pass from any firm to any other firm in $\mathbf{g}$ weighted by $(\overline{\theta}+\beta)/(\delta+\beta)$. Thus, from (4.9) we derive

$$(\delta+\beta)\bar{q}^* = \sum_{k=0}^{+\infty}\left(\frac{\overline{\theta}+\beta}{\delta+\beta}\right)^k \mathbf{G}^k\left[-\beta q^* \bar{1}_F + \bar{\alpha} + \bar{\xi}\right].$$

Our framework of establishment-level production mixes the network of conglomerates with the network of intersectoral linkages. A change in one firm's production does not only affect the production of the firms that are connected to it through the conglomerate network, but also the production in the sector in which the firm operates. This increases or decreases the demand for other good types, depending on whether these other goods are complementary to the firm's good. Hence, the production in other sectors is affected as well. For example, let us take the first iteration of $\mathbf{G}$. If we want to see the direct effect of a shock from i to j, we can look at $g_{ij}^{[1]} = (\gamma_{ij} + \beta_{ss'} + \beta)/(\overline{\theta}+\beta)$, where i belongs to s and j belongs to s'. The conglomerate effect is measured by γ_{ij}, while the intersectoral relation through the demand side is measured by $\beta_{ss'}$. The rest of the elements simply rescale the effect with respect to the substitutability among homogeneous goods β. Even if the firms are not part of the same conglomerate, a shock to i has

[11] The Frobenius theory of nonnegative matrices implies that Assumption 9 holds if $(\overline{\theta}+\beta)/(\delta+\beta)$ is bounded above by the largest column sum of $\mathbf{G}$.

an effect on the demand for the good type produced by j. Let us now take the second iteration of $\mathbf{G}$, so as to see the indirect effect of a shock from i to j. By indirect effect we mean the effect of a shock to i on j through the impact on all the firms m that are directly connected both to i and to j. We can look at $g_{ij}^{[2]} = \sum_{m=1}^{F}(\gamma_{im} + \beta_{ss''})(\gamma_{mj} + \beta_{s''s'})$, where $i \in F_s$, $j \in F_{s'}$, and $m \in F_{s''}$, for every m and every s''. The element $g_{ij}^{[2]}$ sums up all the possible firms m's and sectors s'''s that could be the conduit of propagation of a shock from i to j. Higher powers of $\mathbf{G}$ yield weaker levels of intermediation, and the sum of $\mathbf{G}^k$ for every $k \geq 0$ accounts for the whole stream of intermediate degrees, that is, for any path length k from 0 to ∞. The decay factor $(\overline{\theta} + \beta)/(\delta + \beta)$ assures that there exists always a level of intermediation such that the impact is negligible.

Definition 25 (Weighted Bonacich Centrality)**.** Consider a network $\mathbf{g}$ with adjacency matrix $\mathbf{G}$, a scalar a such that $[\mathbf{I}_F - a\mathbf{G}]^{-1}$ is well defined and nonnegative, and a vector $\bar{x} \in \mathbb{R}^F$. The vector of *weighted Bonacich centralities* of parameter a in $\mathbf{g}$ and weights $\bar{x}$ is

$$\bar{b}(\mathbf{g}, a, \bar{x}) \equiv [\mathbf{I}_F - a\mathbf{G}]^{-1}\, \bar{x}.$$

The weighted Bonacich centrality of firm i is $b_i(\mathbf{g}, a, \bar{x}) = \sum_{j\in\mathscr{F}} x_j m_{ij}(\mathbf{g}, a)$, where x_j is the j-th element of the vector $\bar{x}$ of weights and $m_{ij}(\mathbf{g}, a)$ is the typical element of $[\mathbf{I}_F - a\mathbf{G}]^{-1}$. This centrality measure is the sum of all the weighted loops $x_i m_{ii}(\mathbf{g}, a, \bar{x})$ from firm i to itself and of all the other paths $\sum_{j\neq i} x_j m_{ij}(\mathbf{g}, a, \bar{x})$ from i to every other firm j. In our framework, we set $a = (\overline{\theta} + \beta)/(\delta + \beta)$ and we refer to $\bar{b}(\mathbf{g}, (\overline{\theta} + \beta)/(\delta + \beta), \bar{x})$ as $\bar{b}(\bar{x})$ for simplicity. We report in Figure 4.1 the unweighted Bonacich centrality for the 558 firms that mirrors the distribution across sectors of the 5.6 million establishments of the County Business Patterns. In other words, on the horizontal axis there are all the firms in $\mathscr{F}$, ordered by sector as in (4.1), while on the vertical axis there are the Bonacich centralities $b_i(\bar{1}_F)$ of each firm i in $\mathscr{F}$. The different segments of Bonacich centrality correspond to firms that belong to different sectors.

Proposition 14 (The Equilibrium Solution)**.** *Suppose Assumption 9 holds. Then, the unique interior equilibrium solution is*

$$\bar{q}^* = \frac{1}{\delta + \beta}\left[\bar{b}(\bar{\alpha}) + \bar{b}(\bar{\xi})\right] - \frac{\beta\left(b(\bar{\alpha}) + b(\bar{\xi})\right)}{(\delta + \beta)(\delta + \beta + \beta b(\bar{1}_F))}\bar{b}(\bar{1}_F). \tag{4.14}$$

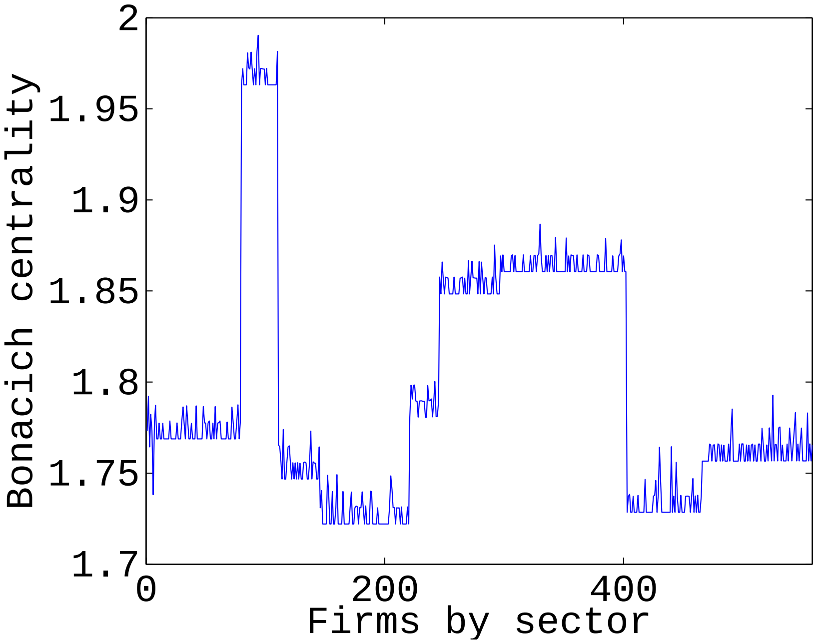

Figure 4.1: Unweighted Bonacich centrality for 558 firms in 14 sectors. The different segments corresponds to different sectors.

Note that if $\alpha_s = \tilde{\alpha}$ for all s in $\mathscr{S}$ and $\xi_i = 0$ for all i in $\mathscr{F}$, then $\bar{b}(\bar{\alpha}) = \alpha\bar{b}(\bar{1}_F)$, $\bar{b}(\bar{\xi}) = \bar{0}_F$, and the solution boils down to

$$\bar{q}^* = \frac{\tilde{\alpha}}{\delta + \beta + \beta b(\bar{1}_F)}\bar{b}(\bar{1}_F),$$

which is the equilibrium solution of Ballester et al. [3, Theorem 1]. Hence, our model can be seen as a generalization of that result to case of S sectors and idiosyncratic shocks. Our general result shares with the special case the dependence of each firm's production on the - in our case weighted - centrality degree of the firm. The weights are the dimensions of the sectors in which the firms operate and the idiosyncratic productivities. The source of heterogeneity for equilibrium production is the position of each firm with respect to the complex nexus of intersectoral relations, conglomerates among sectors, and the whole list of technological idiosyncrasies of all the firms in the economy. The production of each firm depends positively on how path-central that firm is in the network structure $\mathbf{g}$.

Let us call $\alpha_{min} \equiv \min_s\{\alpha_s\}$, $\xi_{min} \equiv \min_i\{\xi_i\}$, and $\xi_{max} \equiv \max_i\{\xi_i\}$. Note that since $m_{ij} > 0$ for every i and j in $\mathscr{F}$, $b_i(\bar{x}) > 0$ for every i in $\mathscr{F}$ and every vector $\bar{x}$ of strictly positive weights.

Proposition 15. *If the condition*

$$\alpha_{min} > \frac{\beta}{\delta+\beta} b(\bar{1}_F)(\xi_{max} - \xi_{min}) - \xi_{min}$$

holds, then $q_i^* > 0$ *for every* i *in* $\mathscr{F}$.

If the smallest sector of the economy is large enough with respect to the magnitude of the idiosyncratic shocks, then the production of any firm is always positive. If the fundamental uncertainty coming from the idiosyncratic shocks is too wide, that is, if the firm-specific productivities vary too much across firms, then there is the possibility that some firms do not produce positive quantities. Take the case where ξ_i follows a uniform distribution over the interval $[0, 1]$. Then, $\xi_{max} \leq 1$, $\xi_{min} \geq 0$, and $\xi_{max} - \xi_{min} \leq 1$. Hence, the condition of Proposition 15 holds if

$$\alpha_{min} > \frac{\beta}{\delta+\beta} b(\bar{1}_F).$$

Remark. The symmetry of **G** is not relevant for Proposition 14. In case of asymmetry of **G**, we can substitute Assumption 9 with the condition $(\overline{\theta}+\beta)\rho(\mathbf{G}) < \delta + \beta$, where $\rho(\mathbf{G})$ is the spectral radius of **G**, and the rest of the results follow.

Remark. We impose $n_s = n$ for all s in $\mathscr{S}$ without any loss of generality. Letting α_s to vary across sectors simply complicates the notation but does not affect the conditions behind Proposition 14.

4.5 Network structure and aggregate volatility

Now we will study how aggregate volatility, that is, the variance of the production choices vector, depends on the network structure of the economy. The standard diversification argument maintains that a series of idiosyncratic shocks to the *grains* of the economy, which in our case are the single firms, average out at the aggregate level due to the law of large numbers. In particular, we can apply a variation of the Central Limit Theorem (CLT) to argue that the variance of aggregate production, that is, σ_{GDP}, decays at rate $1/\sqrt{F}$ as the number of grains pertubed by iid shocks, F, increases.[12] In our model, we expect the variance of our aggregate production measure to decrease at a lower rate then $1/\sqrt{F}$ as F increases. This is due to the fact that equilibrium production choices are connected through the network structure of the economy. A similar argument was presented

[12] See for example Gabaix [16] for the aggregation of firms' production into a unique final good, and Dupor [14] for the aggregation of sector-specific productions through a standard dynamic multisector model à la Long and Plosser [26].

in Jovanovic [24], where the author easily provides examples where the endogenous data do not obey the law of large numbers, albeit the exogenous data do. For even earlier contributions, see Diamond [13] and Mortensen [29].

We can express $\bar{b}(\bar{x})$ as

$$\bar{b}(\bar{x}) = \left[\mathbf{I}_F - \frac{\overline{\theta}+\beta}{\delta+\beta}\mathbf{G}\right]^{-1}\bar{x} = \sum_{k=0}^{+\infty}\left(\frac{\overline{\theta}+\beta}{\delta+\beta}\right)^k \mathbf{G}^k\bar{x}.$$

Proposition 16 (Finite dimensional spectral theorem)**.** *Let G in $\mathbb{R}$ be symmetric. Then, G has F linearly independent real eigenvectors. Moreover, these eigenvectors can be chosen such that they are orthogonal to each other and have norm one.*

According to Theorem 16 and given that G is symmetric by construction, there exists a real orthonormal matrix $\mathbf{V}$ such that

$$\Lambda = \mathbf{V}^T\mathbf{G}\mathbf{V}$$

is a diagonal matrix with elements $\Lambda_{ii} = \lambda_i(\mathbf{G})$, where $\lambda_i(\mathbf{G})$ is the i-th eigenvalue of $\mathbf{G}$. Remember that a matrix $\mathbf{V}$ is orthonormal if its transpose is equal to its inverse, that is, $\mathbf{V}^{-1} = \mathbf{V}^T$.

Proposition 17. *Suppose Assumption 9 holds. Then,*

$$\frac{1}{\delta+\beta}\left[\mathbf{I}_F - \frac{\overline{\theta}+\beta}{\delta+\beta}\mathbf{G}\right]^{-1} = \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1},$$

where $\tilde{\Lambda}$ is a diagonal matrix whose generic diagonal element is

$$\tilde{\lambda}_i(\mathbf{G}) \equiv \frac{1}{\delta+\beta-(\overline{\theta}+\beta)\lambda_i(\mathbf{G})}, \tag{4.15}$$

and $\mathbf{V}$ is an orthonormal matrix of eigenvectors of $\mathbf{G}$.

Proposition 17 tells us that the equilibrium production levels depend on the eigenvalues and the eigenvectors of the network matrix $\mathbf{G}$, where $\tilde{\Lambda}$ contains the eigenvalues and $\mathbf{V}$ contains the eigenvectors. The eigenvalues measure the cross-effects of each firm, while the eigenvectors account for the particular position of each firm within the network. The more the positive cross-effects across firms, the higher the eigenvalues and therefore the higher the production. Let v_{im} be the row i-column m element of $\mathbf{V}$ and v_{mj}^{-1} be the row m-column j element of $\mathbf{V}^{-1}$.

In order to study the volatility of our economy, we can look at the variance-covariance matrix of the equilibrium production. Since we deal with a static model, we will use the concepts of variance and volatility interchangeably.

Definition 26. The variance-covariance matrix The variance-covariance matrix $\Sigma(\mathbf{G})$ is the $F \times F$ real-valued matrix given by

$$\Sigma(\mathbf{G}) \equiv E\left[(\bar{q}^* - E[\bar{q}^*])(\bar{q}^* - E[\bar{q}^*])^T\right],$$

where the expectation operator $E[\cdot]$ is defined over the probability distribution of $\bar{\xi}$.

The diagonal entries of variance-covariance matrix account for the volatility of each firm, while the off-diagonal entries account for the comovement between different firms. Since there exist links between firms, both in the network of intersectoral linkages and in the network of conglomerates, there exists covariance in equilibrium production across firms although the idiosyncratic shocks $\bar{\xi}$ are independently distributed. Hence, the variance-covariance matrix depends on the network structure of the economy.

Proposition 18. *The variance-covariance matrix of the equilibrium production can be decomposed into three components, that is,*

$$\Sigma(\mathbf{G}) = \sigma^2 [\mathbf{I}_F + \Sigma_U + \Sigma_G], \tag{4.16}$$

where

$$\Sigma_U \equiv D^2 F \left(\sum_{i\in\mathscr{F}} \sum_{m\in\mathscr{F}} \sum_{j\in\mathscr{F}} v_{im} \tilde{\lambda}_m(\mathbf{G}) v_{mj}^{-1} \right)^2 \sum_{i\in\mathscr{F}} \sum_{m\in\mathscr{F}} \sum_{j\in\mathscr{F}} \left(v_{im} \tilde{\lambda}_m(\mathbf{G}) v_{mj}^{-1} \right)^2 \mathbf{U}_F,$$

$$\Sigma_G \equiv D \mathbf{V} \tilde{\Lambda} \mathbf{V}^{-1} \mathscr{V} \mathbf{V} \tilde{\Lambda} \mathbf{V}^{-1},$$

$$D \equiv \frac{-\beta(\delta+\beta)}{\delta+\beta+\beta b(\bar{1}_F)},$$

and the typical element of $\mathscr{V}$ is

$$\mathscr{V}_{jk} = \sum_{i\in\mathscr{F}} \sum_{m\in\mathscr{F}} \left(v_{im} \tilde{\lambda}_m(\mathbf{G}) \right)^2 \left((v_{mj}^{-1})^2 + (v_{mk}^{-1})^2 \right).$$

We can also derive a measure of aggregate volatility that is comparable with the one used in Horvath [21], Dupor [14], and Carvalho [7]. This formulation is a simplified measure of the actual aggregate variance, whose properties are best described by the variance-covariance matrix $\Sigma(\mathbf{G})$.

Definition 27 (Aggregate volatility)**.** The aggregate volatility $\sigma_Y^2(\mathbf{G})$ is a scalar given by

$$\sigma_Y^2(\mathbf{G}) \equiv E\left[\left(\frac{1}{F}\sum_{i=1}^{F}(q_i - E[q_i])\right)^2\right].$$

The advantage of $\sigma_Y^2(\mathbf{G})$ is that it shuts down the covariance between firms and focuses on the aggregate variance. In fact, $\sigma_Y^2(\mathbf{G})$ corresponds to a transformation of the diagonal elements of $\Sigma(\mathbf{G})$. Consequently, it depends on the network structure of the economy.

Proposition 19. *Suppose Assumption 9 holds. Then, the aggregate volatility is a function of the eigenvalues of the network matrix, that is,*

$$\sigma_Y^2(\mathbf{G}) = \frac{\sigma^2}{F^2}\left(\frac{\delta+\beta}{\delta+\beta+\beta b(\bar{1}_F)}\right)^2 \sum_{i\in\mathscr{F}}\sum_{j\in\mathscr{F}}\sum_{m\in\mathscr{F}}\left(v_{im}\tilde{\lambda}_m(\mathbf{G})v_{mj}^{-1}\right)^2, \tag{4.17}$$

where v_{im} is the row i-column m element of $\mathbf{V}$, v_{mj} is the row m-column j element of $\mathbf{V}^{-1}$, and $\tilde{\lambda}_m(\mathbf{G})$ is defined in (4.15).

The elements v_{im} and v_{mj} account for the presence of a path from i to j that involves m as an intermediary node, so as the number of links grows their values will become non-nil. Hence, if the number of links grows at the same rate as F, then the aggregate volatility will decline at a rate lower than F, and if the link rate is even higher than F, then aggregate volatility may not decline at all. See Acemoglu, Ozdaglar, and Tahbaz-Salehi [1] for the limiting distributions of aggregate volatility in presence of cross-effects.

Equations (4.16) and (4.17) relate the aggregate variance and covariance to the absolute value of the network matrix' eigenvalues. Hence, the typical element of the variance-covariance matrix $\Sigma(\mathbf{G})$ increases if the absolute value of any eigenvalue of $\mathbf{G}$ increases, that is, if the high and positive eigenvalues increase and the low and negative eigenvalues decrease. In other words, the variance-covariance matrix shrinks if the most path-central elements of the economy lose some weight and the least path-central gain some weight.[13] The heterogeneity in the path-centrality degrees of firms is directly related to the heterogeneity of the equilibrium production levels and, since firms are path-connected, to the covariance of equilibrium quantities. The higher the difference in the path-centrality degrees the higher the dispersion of the production levels.

[13] See Bramoullé, Kranton, and D'Amours [6] for an interpretation of the extreme eigenvalues of a network matrix.

We run some simulations of our model to understand the relationship between the network structure $\mathbf{g}$ of the economy and aggregate volatility. We derive the matrix $\mathbf{B}$ of intersectoral linkages from the direct requirements tables of the BEA for 2002, as in Table 4.3. We assign to each sector s a certain number of firms n_s and a certain dimension α_s. We take the n_s's from the County Business Patterns of the US Census Bureau for 2002 and the α_s's from the Gross Ouput by Industry accounts of the BEA for 2002. We report the summary of the data in Table 4.2. We construct a matrix $\hat{\Gamma}$ of conglomerations using an algorithm that generates a random network that satisfies Definition 19. We use $\mathbf{B}$ and $\hat{\Gamma}$ to derive the network matrix $\hat{\mathbf{G}}$ of the economy as in Definition 22. Moreover, we consider $T = 100$ periods. For each period t, we make a different draw $\bar{\xi}_t$ of $\bar{\xi}$. For each draw, we derive $\bar{q}_t$ holding the rest of the parameters and the network structure $\hat{\mathbf{G}}$ constant. We define real GDP at time t as follows,

$$\text{GDP}_t \equiv \sum_{i \in \mathscr{F}} q_{it}. \tag{4.18}$$

Figure 4.2 represents how the real GDP fluctuates across time due to idiosyncratic shocks, given a fixed network matrix $\hat{\mathbf{G}}$ determining aggregate volatility $\sigma^2_Y(\hat{\mathbf{G}})$.

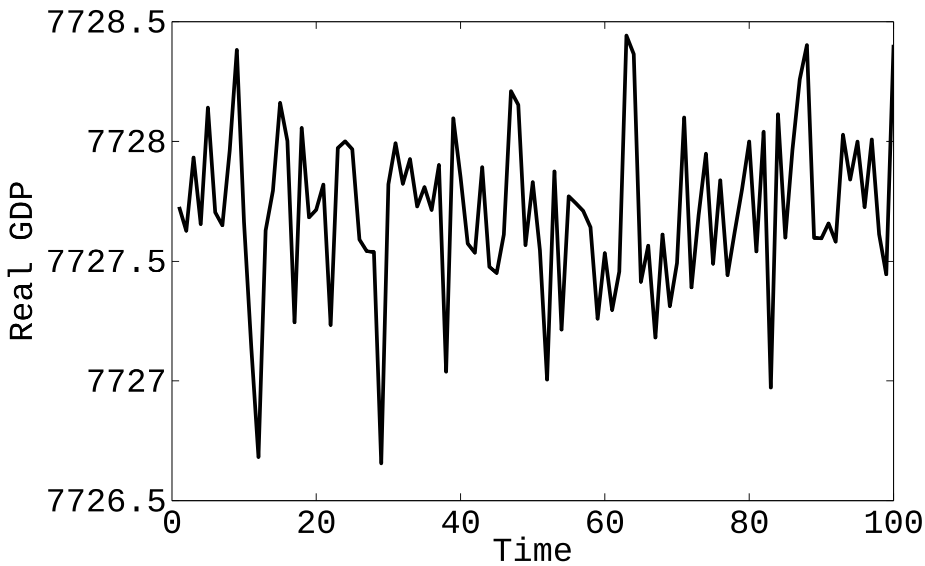

Figure 4.2: Aggregate fluctuations over time given a fixed network structure.

We make different counterfactual exercises to analyze the effect of the network structure on aggregate volatility. We keep the number of firms across sectors taken from the County Business Patterns constant across cases. In Case 1, we suppose that there are no idiosyncrasies across sectors and firms. The sectors have the same size $\alpha_s = \alpha/S$ for every s in $\mathscr{S}$ and the firms have the same productivity $\xi_i = 0$ for every i in $\mathscr{F}$. In Case 2, we introduce different sector shares that mirror the distribution of industry shares in the US economy. In Case 3, we introduce idiosyncratic productivities across firms that follow a normal distribution of mean 0 and standard deviation 1. In Case 4, we introduce intersectoral linkages that mirror the complementarity across sectors derived from the inpu-output tables in the US economy. In Case 5, we introduce conglomerate relations simulated through a random algorithm that respects Definition 18. In Case 6, we increase by 20% the number of links in the conglomerate network. Table 4.1 reports the results about different aggregate statistics, that is, aggregate volatility according to Definition 27, standard deviation of output levels (STD), and GDP as defined in (4.18).[14]

	$\sigma^2_Y(\mathbf{G}) * 10^8$	STD	GDP/F
Case 1	7.5339	0.0000	7.1386
Case 2	7.5339	0.0100	7.1377
Case 3	7.5339	5.8059	8.7806
Case 4	6.7717	5.9500	9.8161
Case 5	6.7437	5.9477	9.8481
Case 6	6.7381	5.9472	9.8542

Table 4.1: Numerical exercises. Sources: BEA. Year: 2002. Simulation algorithms available upon request.

Cases 1, 2, and 3 confirm the theory of Proposition 18 and Proposition 27, that is, aggregate volatility changes only if we change the network structure $\hat{\mathbf{G}}$. The aggregate volatility does not decrease because we introduce changes in $\hat{\mathbf{G}}$ only from Case 4. The standard deviation instead increases with the introduction of the idiosyncrasies in sector shares and firm-specific productivities of Cases 2 and 3. The inclusion of the network of intersectoral linkages in Case 4 decreases the volatility by around 10%. The reason is that the idiosyncrasies introduced by the different sector shares in Case 2 are smoothed out partially by the intersectoral linkages, which make the production in a sector have positive feedbacks into the production of another sector. The standard deviation increases in Case 4 simply because of the different number of firms across sectors. The existence of the conglomerate relations in Case 5 reduces even further aggregate volatility. The rationale behind is the same: the idiosyncratic shocks to the single firms are

[14]We include the standard deviation of outputs in order to make our results comparable with Acemoglu, Ozdaglar, Tahbaz-Salehi [1].

smoothed out across firms within the same conglomerate. The magnitude of the drop in volatility from Case 4 to Case 5 is smaller than from Case 3 to Case 4 because of the sparseness of the network matrix of conglomerate relations. For example, in the numerical exercise of Table 4.1, Case 5 introduces 171 links to the overall network matrix $\mathbf{g}$ out of the $558 * 557/2$ possible links. The standard deviation decreases as well from Case 4 to Case 5 for the same reason as the aggregate volatility. As a control, in Case 6 we introduce around 20% more links with respect to the benchmark of Case 5. The volatility and the standard deviation decrease accordingly. The new links are randomly assigned to the network of conglomerates in Case 5. Note that the aggregate product, measured by the average of our definition of GDP in (4.18), increases along the different cases. This is due to the positive spillovers on the production side across firms and on the demand side across sectors/commodities. In fact, the most relevant shifts in aggregate production are due to the introduction of the intersectoral linkages in Case 3 and of the conglomerate relations in Case 4. We can measure these increases by the increases in the (average) Bonacich centralities of the network.[15] These centrality measures crucially depend on the eigenvalues and eigenvectors of the network matrix, as Proposition 17 suggests.

4.6 Conclusion

We explain the transmission of idiosyncratic shocks to the aggregate level by considering intersectoral linkages and conglomerate relations across different firms. We express aggregate output as a reduced-form function of the idiosyncratic shocks filtered by the network structure of the economy. The aggregate volatility depends on the network structure of the economy. In particular, we can express the dispersion in production levels as a function of the eigenvalues and eigenvectors of the network matrix.

The model helps us to understand how the network structure influences aggregate volatility. We show that the more connected the economy, the more the law of large numbers smooths out the idiosyncrasies across sectors and firms. Our counterfactual exercises in Section 6 show that the introduction of intersectoral linkages and conglomerates decreases aggregate volatility. There are three extensions that future research might explore. First, it remains to understand which types of network structure favor aggregate volatility and which others temper it, other things equal. We could compare two extreme cases. On the one hand, a certain set of conglomerates with a certain size distribution consists of complete

[15] See Ballester et al. [3, Theorem 2] for an early characterization of the dependence of equilibrium outcome on the Bonacich centrality.

components of firms, that is, components where each node is connected to each other. On the other hand, the same conglomerates with the same size distribution consist of star-like components, with one central node and two or more peripheral nodes. The aggregate volatility might change if we pass from one case to the other. Moreover, we could simulate the transition between the two extreme cases by parameterizing the intermediate network structures with some key variables. Following Newman, Strogatz, and Watts [30], we could derive the average component size in a random network from the moments of the degree distribution. Hence, different average component sizes correspond, other things equal, to different levels of, say, the average degree. We could generate a family of random networks for each chosen average component size. For example, a high average component size would mean an economy characterized by big firms. With the same logic, we could derive families of network structures that would adhere to some stylized facts that we may want to examine, for example, the intersectoral diffusion of conglomerates, the concentration ratios within each sector, and so on. If we analyze the intermediate cases, the volatility might not change monotonically from one extreme to the other.

Second, future research could explore a framework of network formation using the payoff structure introduced in this paper. We could set up a two-stage game where the model presented so far would represent the second stage, being the first stage devoted to network formation. There would be a trade-off between the cost of forming a link and the benefits of belonging to a component. Firms would act strategically and decide with whom to share a link depending on the potential equilibrium outcomes in the second stage. Another possibility is dynamic network formation model as in König, Tessone, and Zenou [25], where the timing of the two stages is inverted. First, the agents realize their equilibrium production given previous period's network. Second, given the equilibrium result they choose which other agents to share a link with. Different network structures arise and it is possible to identify stationary network structures that follow the properties of nested split graphs.[16] These networks are also known as "interlink stars" in Goyal and Joshi [17] and Goyal et al. [19]. The main property of these networks is the core-periphery structure, which reminds the stylized structure of the conglomerates, that is, central parent companies that specialize the conglomerate into a core business and peripheral subsidiaries that diversify the production to smooth out sector-specific fluctuations.

Third, the model has also several policy implications. For instance, future work might analyze the application of our framework to model discretionary policy interventions. In times of crisis, there may be interventions aimed at stabilizing aggregate output. Each intervention entails a public cost, so a key question is which

[16] See, for example, Mahadev and Peled [28, Chapter 5].

economic agent we should stabilize first in order to decrease aggregate volatility the most with the least public cost. Suppose that a given set of firms is subject to idiosyncratic shocks such as demand shifts, strikes, productivity fluctuations, or simply bad luck. The law of large numbers reduces the scope of public intervention since the random idiosyncratic happenings in different directions would compensate each other if the set of firms is large enough. Nevertheless, this argument does not hold if there exist direct connections between firms, for example financial liability relations. Hence, idiosyncratic shocks can transmit to the aggregate level and can generate aggregate fluctuations in presence of a network structure of inter-firm relations. Discretionary policy in this sense plays a key role in stabilizing output: as much as - negative - idiosyncratic shocks transmit from one agent to the other up to the aggregate level, so does the stabilization policy of the public authority. If the public authority bails out a troubled firm, it stabilizes the performance of all the firms directly or indirectly connected to it by a path of inter-firm financial ties. My model suggests that the firm that should be stabilized in order to obtain the most substantial aggregate effect is the most path-central firm of the economy, considering both intersectoral linkages and conglomerate relations. Moreover, the reduced-form of the model suggests a practical way of identifying the key firm(s), that is, by deriving the eigenvalues of the network matrix, isolating the highest, and deducing which is the most path-central firm. This method has important analogies with the static Principal Component Analysis (PCA), where my model identifies the common components with the bundled up cross-effects among different agents. Future research could explore further the link between centrality measures and PCA in networks.

4.7 Appendix: Proofs, Figures, and Tables

Proof of Proposition 12. The set of parameters β, δ, and ε for which $\det(\Psi) = 0$ has Lebesgue measure zero in $\mathbb{R}^3$. Hence, the matrix of cross-effects Ψ is generally nonsingular. Hence, Ψ is generically nonsingular, so it is invertible and we call its inverse Ψ^{-1}. □

Proof of Proposition 13. See Theorem III* in Debreu and Herstein [12, p. 601]. □

Proof of Proposition 14. We can express $\bar{q}^*$ in terms of weighted Bonacich centralities, that is,

$$(\delta + \beta)\bar{q}^* = -\beta q^* \bar{b}(\bar{1}_F) + \bar{b}(\bar{\alpha}) + \bar{b}(\bar{\xi}). \tag{4.19}$$

The individual firm's production is

$$(\delta + \beta)q_i^* = -\beta q^* b_i(\bar{1}_F) + b_i(\bar{\alpha}) + b_i(\bar{\xi}),$$

which we can sum up over all i's in $\mathscr{F}$ and obtain

$$(\delta+\beta)q^* = (\delta+\beta)\sum_{i\in\mathscr{F}} q_i^* = -\beta q^* b(\bar{1}_F) + b(\bar{\alpha}) + b(\bar{\xi}),$$

where $b(\bar{x}) \equiv \sum_{i\in\mathscr{F}} b_i(\bar{x})$ for any $\bar{x}\in\mathbb{R}^F$. Thus,

$$q^* = \frac{b(\bar{\alpha}) + b(\bar{\xi})}{\delta+\beta+\beta b(\bar{1}_F)}, \tag{4.20}$$

which we substitute in (4.19). □

Proof of Proposition 15. If $\alpha_s = \alpha_{min}$ for every s in $\mathscr{S}$, then $\bar{b}(\bar{\alpha}) = \alpha_{min}\bar{b}(\bar{1}_F)$. Since $\alpha_{min} \equiv \min_s\{\alpha_s\} > 0$ by construction and $b_i(\bar{x}) > 0$ for every i in $\mathscr{F}$ and every vector $\bar{x}$ of strictly positive weights, we have that $\bar{b}(\bar{\alpha}) \geq \alpha_{min}\bar{b}(\bar{1}_F)$. Similarly, $\bar{b}(\bar{\xi}) \geq \xi_{min}\bar{b}(\bar{1}_F)$ and $\bar{b}(\bar{\xi}) \leq \xi_{max}\bar{b}(\bar{1}_F)$. Substituting these values in the equilibrium expression of Proposition 14 we obtain the condition for $\bar{q}^*$ being a vector of strictly positive entries of Proposition 15. □

Proof of Proposition 16. See, for example, Halmos [20, Chapter 79]. □

Proof of Proposition 17. We perform the eigendecomposition of $\mathbf{G}$ and obtain

$$(\delta+\beta)\bar{q}^* = \sum_{k=0}^{+\infty}\left(\frac{\overline{\theta}+\beta}{\delta+\beta}\right)^k (\mathbf{V}\Lambda\mathbf{V}^{-1})^k\left[-\beta q^*\bar{b}(\bar{1}_F) + \bar{\alpha} + \bar{\xi}\right],$$

that is,

$$(\delta+\beta)\bar{q}^* = \mathbf{V}\sum_{k=0}^{+\infty}\left(\frac{\overline{\theta}+\beta}{\delta+\beta}\right)^k \Lambda^k\mathbf{V}^{-1}\left[-\beta q^*\bar{b}(\bar{1}_F) + \bar{\alpha} + \bar{\xi}\right].$$

If Assumption 9 holds, then $(\delta+\beta)/(\overline{\theta}+\beta) > \lambda_{max}(\mathbf{G}) \geq \lambda_i(\mathbf{G})$ for every i in $\mathscr{F}$, that is,

$$\frac{\overline{\theta}+\beta}{\delta+\beta}\lambda_i(\mathbf{G}) < 1,$$

for every i in $\mathscr{F}$. Hence,

$$\sum_{k=0}^{+\infty}\left(\frac{\overline{\theta}+\beta}{\delta+\beta}\lambda_i(\mathbf{G})\right)^k = \frac{\delta+\beta}{\delta+\beta-(\overline{\theta}+\beta)\lambda_i(\mathbf{G})}$$

for every i in $\mathscr{F}$. □

Proof of Proposition 19. According to Proposition 17, if Assumption 9 holds we can express the equilibrium production as

$$\bar{q}^* = \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\left[-\beta q^*\bar{1}_F + \bar{\alpha} + \bar{\xi}\right].$$

According to Definition 25,

$$\bar{b}(\bar{x}) = \left[\mathbf{I}_F - \frac{\overline{\theta} + \beta}{\delta + \beta}\mathbf{G}\right]^{-1}\bar{x} = (\delta + \beta)\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{x},$$

and

$$b(\bar{x}) = (\delta + \beta)\bar{1}_F^T\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{x}. \tag{4.21}$$

Hence,

$$\bar{q}^* = -\frac{\beta}{\delta + \beta}q^*\bar{b}(\bar{1}_F) + \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\left[\bar{\alpha} + \bar{\xi}\right],$$

which given (4.20) is equivalent to

$$\bar{q}^* = -\frac{\beta}{\delta + \beta}\frac{b(\bar{\alpha}) + b(\bar{\xi})}{\delta + \beta + \beta b(\bar{1}_F)}\bar{b}(\bar{1}_F) + \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\left[\bar{\alpha} + \bar{\xi}\right],$$

which given (4.21) gives us

$$\bar{q}^* = -\frac{\beta\bar{1}_F^T\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\left[\bar{\alpha} + \bar{\xi}\right]}{\delta + \beta + \beta b(\bar{1}_F)}\bar{b}(\bar{1}_F) + \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\left[\bar{\alpha} + \bar{\xi}\right]. \tag{4.22}$$

Since there is a source of uncertainty in (4.22) given by the stochastic ξ, let us compute the expected value of $\bar{q}^*$,

$$E\left[\bar{q}^*\right] = -\frac{\beta\bar{1}_F^T\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{\alpha}}{\delta + \beta + \beta b(\bar{1}_F)}\bar{b}(\bar{1}_F) + \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{\alpha} \tag{4.23}$$

We look at the deviation from the expected production of firm i,

$$q_i^* - E\left[q_i^*\right] = -\frac{\beta\bar{1}_F^T\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{\xi}}{\delta + \beta + \beta b(\bar{1}_F)}b_i(\bar{1}_F) + \left[\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{\xi}\right]_i,$$

that is,

$$q_i^* - E\left[q_i^*\right] = -\frac{\beta\sum_{i\in\mathscr{F}}\sum_{m\in\mathscr{F}}\sum_{j\in\mathscr{F}}v_{im}\tilde{\lambda}_m v_{mj}^{-1}\xi_j}{\delta + \beta + \beta b(\bar{1}_F)}b_i(\bar{1}_F) + \sum_{m\in\mathscr{F}}\sum_{j\in\mathscr{F}}v_{im}\tilde{\lambda}_m v_{mj}^{-1}\xi_j,$$

where v_{im} is the row i-column m element of $\mathbf{V}$ and v_{mj}^{-1} is the row m-column j element of $\mathbf{V}^{-1}$. Thus,

$$\frac{1}{F}\sum_{i=1}^{F}\left(q_i^* - E\left[q_i^*\right]\right) = \frac{1}{F}\left(1 - \frac{\beta b(\bar{1}_F)}{\delta + \beta + \beta b(\bar{1}_F)}\right)\sum_{i\in\mathscr{F}}\sum_{m\in\mathscr{F}}\sum_{j\in\mathscr{F}} v_{im}\tilde{\lambda}_m v_{mj}^{-1}\xi_j,$$

from which we obtain

$$\sigma_Y^2 = \frac{1}{F^2}\left(\frac{\delta+\beta}{\delta + \beta + \beta b(\bar{1}_F)}\right)^2 E\left[\left(\sum_{i\in\mathscr{F}}\sum_{m\in\mathscr{F}}\sum_{j\in\mathscr{F}} v_{im}\tilde{\lambda}_m v_{mj}^{-1}\xi_j\right)^2\right].$$

The idiosyncratic shocks are independently and identically distributed across firms, so

$$E\left[\left(\sum_{i\in\mathscr{F}}\sum_{m\in\mathscr{F}}\sum_{j\in\mathscr{F}} v_{im}\tilde{\lambda}_m v_{mj}^{-1}\xi_j\right)^2\right] = \sum_{i\in\mathscr{F}}\sum_{m\in\mathscr{F}}\sum_{j\in\mathscr{F}}\left(v_{im}\tilde{\lambda}_m v_{mj}^{-1}\right)^2 E\left[\xi_j^2\right],$$

where $E\left[\xi_j^2\right] = \sigma^2$ for every j in $\mathscr{F}$.

□

Proof of Proposition 18. Given (4.23) and (4.22), we have that

$$\bar{q}^* - E\left[\bar{q}^*\right] = -\frac{\beta\bar{1}_F^T\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{\xi}}{\delta + \beta + \beta b(\bar{1}_F)}\bar{b}(\bar{1}_F) + \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{\xi}.$$

Moreover, given (4.21), we know that

$$\bar{b}(\bar{1}_F) = (\delta+\beta)\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{1}_F.$$

Hence, we obtain that

$$\bar{q}^* - E\left[\bar{q}^*\right] = \mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\left[\frac{-\beta(\delta+\beta)}{\delta + \beta + \beta b(\bar{1}_F)}\bar{1}_F^T\mathbf{V}\tilde{\Lambda}\mathbf{V}^{-1}\bar{\xi}\bar{1}_F + \bar{\xi}\right].$$

Since ξ_i is identically and independently distributed for every i in $\mathscr{F}$ with mean 0 and variance σ^2, we can decompose the variance-covariance matrix of equilibrium production into the three components $\mathbf{I}_F$, Σ_U, and Σ_G. □

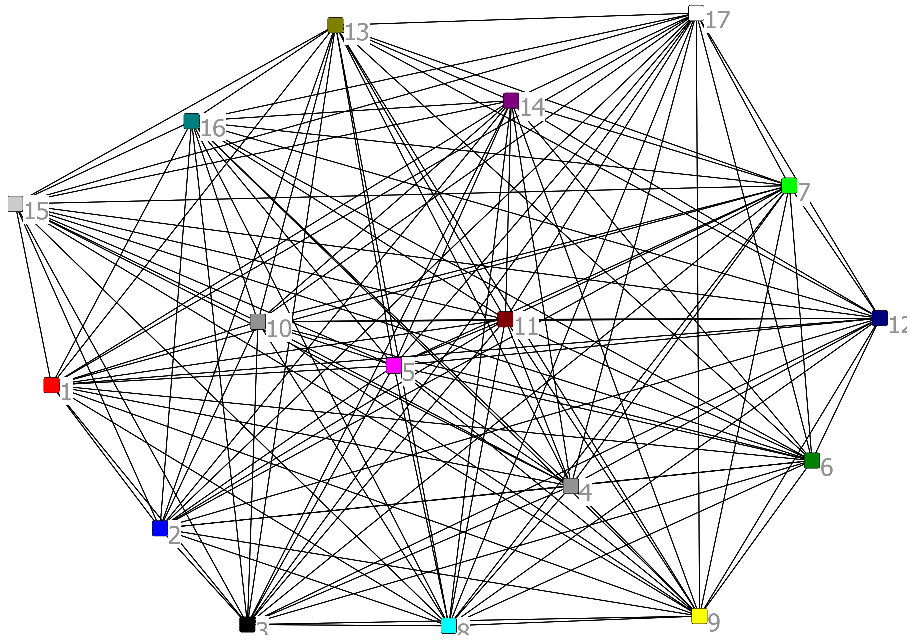

Figure 4.3: The network structure of intersectoral linkages.

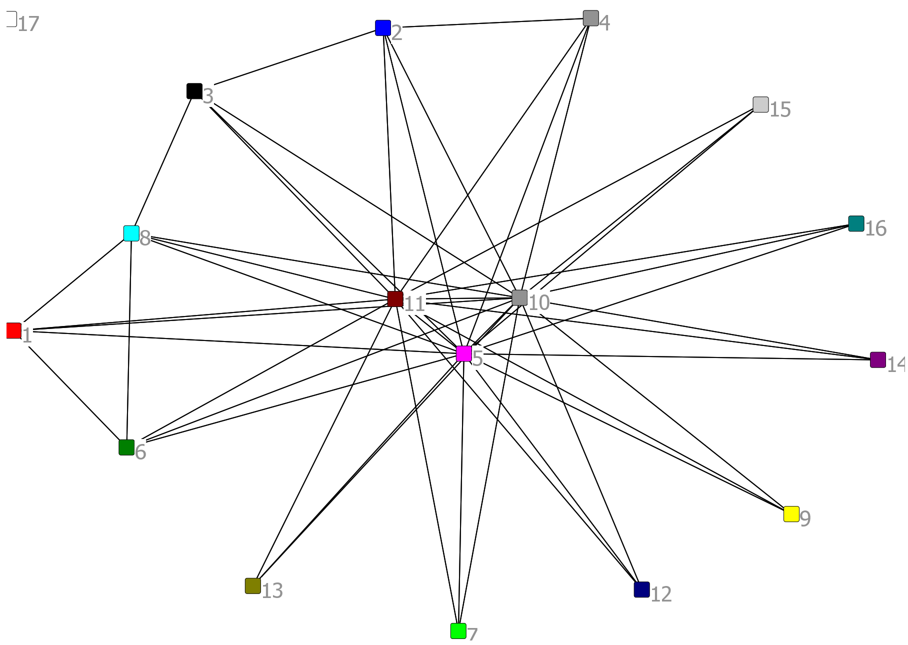

Figure 4.4: The dichotomized network structure of intersectoral linkages.

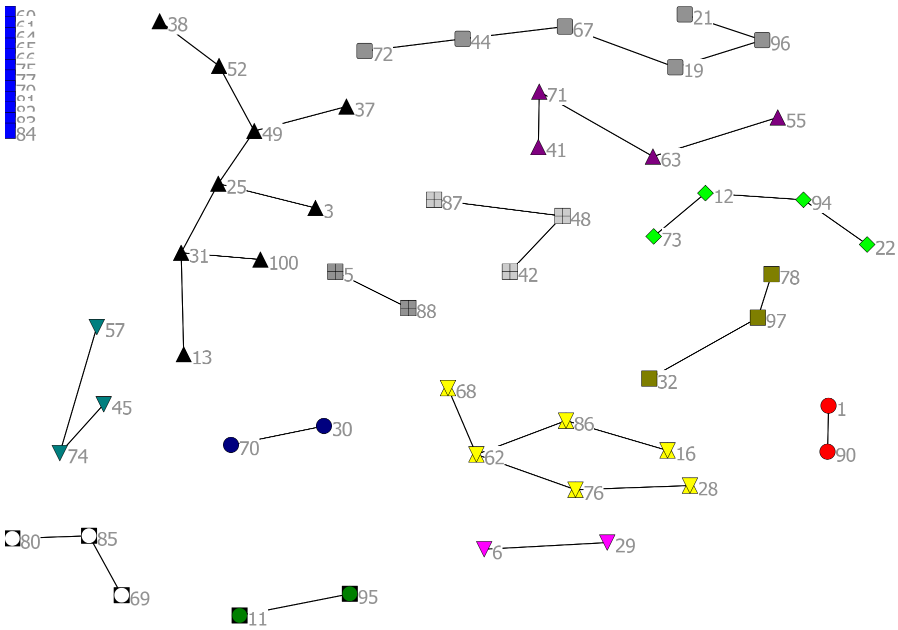

Figure 4.5: An example of the network structure of conglomerations.

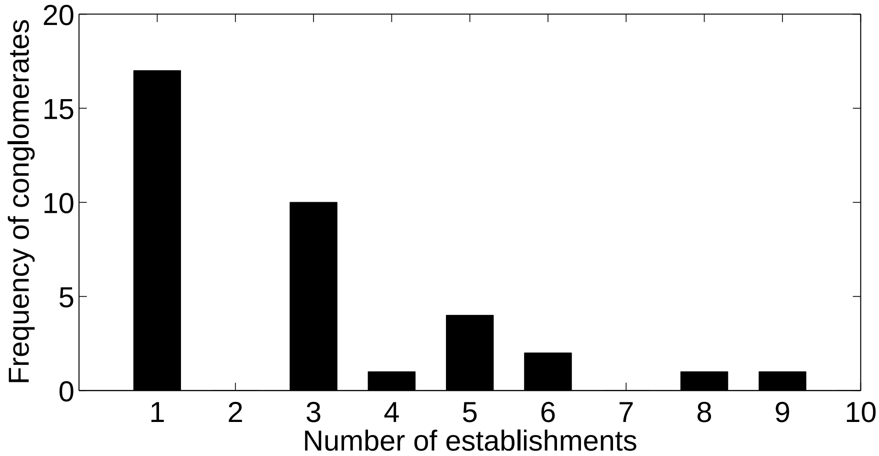

Figure 4.6: An example of diversification distribution of conglomerates generated according to Definition 17.

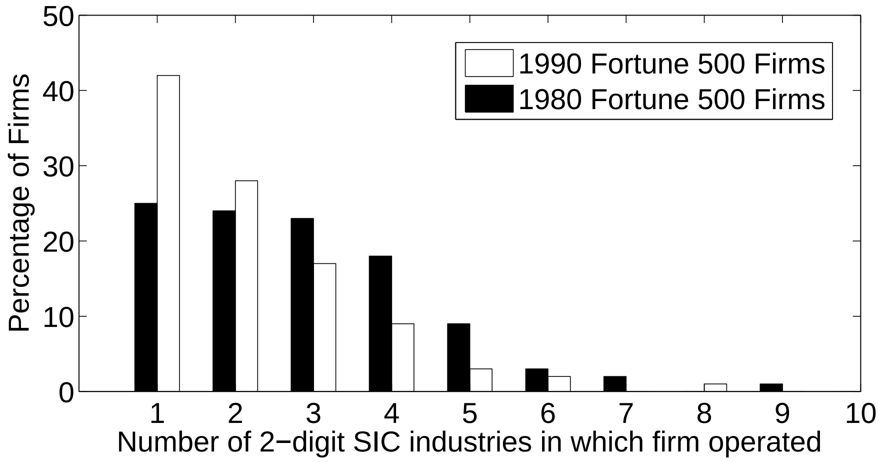

Figure 4.7: Frequency distribution of diversification: *Fortune* 500. Source: Davis, Diekman, and Tinsley [11, Figure 2].

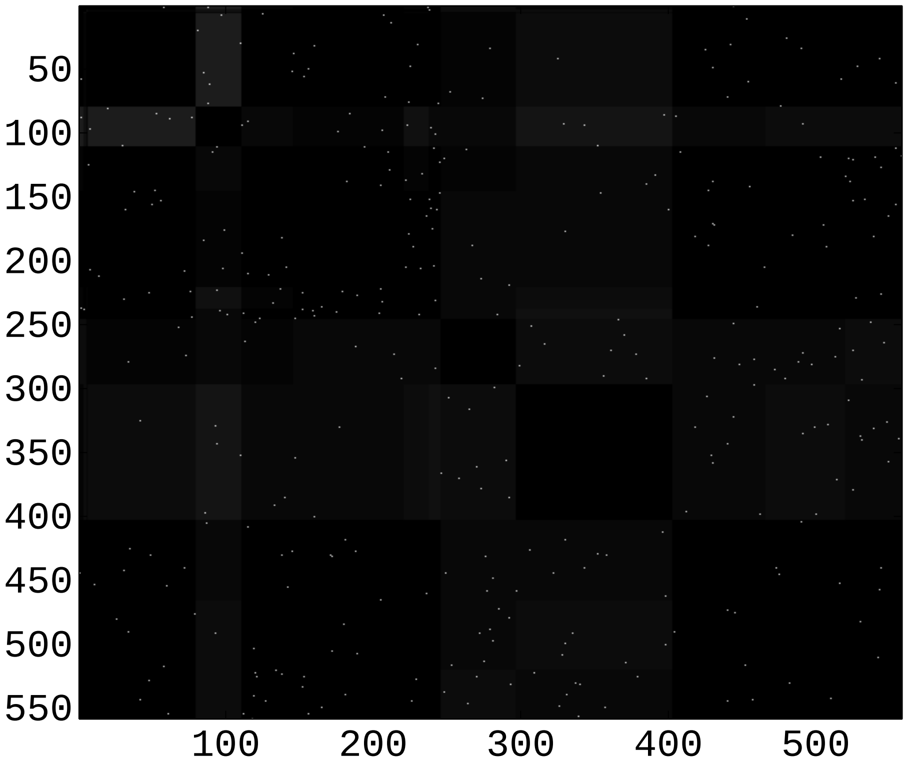

Figure 4.8: A representation of the network matrix **G** of the economy. Data sources: BEA's direct requirements tables for intersectoral linkages, number of establishments per sector from the US Census Bureau's County Business Patterns. Year: 2002. On the horizontal and vertical axis there are 558 firms ordered by sector of activity. These represent the 5524784 establishments distributed across the 14 sectors of the US economy, expressed in tens of thousands and rounded up within each sector. Elements in the matrix represent whether there is a connection or not. The different shading represent the different degree of complementarity, from low complementarity (darker) to high complementarity (lighter). The blocks represent the 14 different sectors. A dark block means that between block s's sector and block s''s sector there is low complementarity, a light block that there is high complementarity. The white dots correspond to the existence of a conglomerate relation between column i's firm and row j's firm. These dots are almost white because the intensity of the link between two firms within the same conglomerate is much higher with respect to any other pair of firms that do not share a link. The most important feature of this representation is that it highlights the sparseness of the matrix of conglomerations Γ with respect to the matrix of intersectoral linkages $\bar{\mathbf{B}}$.

IO code	**Sector**	**Gross Output**	**Employer establishments**	**Companies**
1	Agriculture, forestry, fishing, and hunting	240.8	29250	249290
2	Mining	188.7	19324	102029
3	Utilities	320.4	6223	18896
4	Construction	970.6	729842	2780323
5	Manufacturing	3848.3	310821	601181
6	Wholesale trade	894	347319	711083
7	Retail trade	1030.9	745872	2584689
8	Transportation and warehousing	579.2	167865	976826
9	Information	959.6	76443	309117
10	Finance, insurance, real estate, rental, and leasing	3438.4	507281	3047522
11	Professional and business services	1780.6	1061706	4877023
12	Educational services, health care, and social assistance	1295.7	629550	2430839
13	Arts, entertainment, recreation, accommodation, and food services	704.9	538265	1645857
14	Other services, except government	464	392656	2677613
	Total	16716.1	5524784	22974655

Table 4.2: Gross output (in billions of dollars), number of employer establishments, and number of nonemployer companies by industry. Year: 2002. Sources: BEA (accounts), US Census Bureau (County Business Patterns and Survey of Business Owners). We report the number of nonemployer companies only for illustrative purposes. In fact, these companies constitute three quarters of all establishments in the economy but account for only around 3% of total sales and receipts data. Hence, we consider only employer establishments in the analysis. We do not consider the residual category "Industries not classified" in the list of industries because it is not present in the list of industries used by the BEA.

IO code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	560	722	567	6250	1156	137	1008	336	3279	1716	223	444	360	219	612	096
2	560	0	3032	1033	3891	519	180	759	379	2953	3018	166	358	302	263	546	147
3	722	3032	0	706	2427	430	317	1399	454	1486	2114	328	721	402	375	403	116
4	567	1033	706	0	6187	867	840	788	518	1734	2760	143	313	476	396	276	082
5	6250	3891	2427	6187	0	2529	1243	4127	2447	2230	4787	2492	3481	3239	2832	5245	173
6	1156	519	430	867	2529	0	309	1084	575	1403	2557	409	685	617	559	779	170
7	137	180	317	840	1243	309	0	662	352	2179	2078	081	228	317	163	059	060
8	1008	759	1399	788	4127	1084	662	0	790	2297	2995	335	658	725	850	642	485
9	336	379	454	518	2447	575	352	790	0	1843	3658	451	851	706	736	552	248
10	3279	2953	1486	1734	2230	1403	2179	2297	1843	0	3257	2288	2122	3076	1170	1378	126
11	1716	3018	2114	2760	4787	2557	2078	2995	3658	3257	0	2413	3118	2530	2804	3028	116
12	223	166	328	143	2492	409	081	335	451	2288	2413	0	276	274	237	077	071
13	444	358	721	313	3481	685	228	658	851	2122	3118	276	0	536	449	281	074
14	360	302	402	476	3239	617	317	725	706	3076	2530	274	536	0	374	305	070
15	219	263	375	396	2832	559	163	850	736	1170	2804	237	449	374	0	094	173
16	612	546	403	276	5245	779	059	642	552	1378	3028	077	281	305	094	0	166
17	096	147	116	082	173	170	060	485	248	126	116	071	074	070	173	166	0
Mean	1040	1065	908	1040	3152	862	542	1153	876	1931	2526	604	859	842	688	850	140
Std	1557	1273	878	1486	1815	724	677	1046	931	961	1155	866	1033	1032	854	1341	106

Table 4.3: Complementarity matrix. All the entries are in 10^{-5}. Year: 2002. Source: BEA commodity-by-commodity direct requirements tables. We do not consider in the simulation exercises the following categories: "Government" (15), "Scrap, used and secondhand goods" (16), and "Other inputs" (17). The reason is that they are not reported in the list of industries of US Census Bureau's County Business Patterns.

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