Universitat Autònoma de Barcelona

Doctoral Thesis

# On the ranking of social alternatives

*Author:*
Riste Gjorgjiev

*Supervisor:*
Dr. Salvador Barberà

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# Contents

*To Helena,*
*my eternal Muse...*

# Chapter 1

# General Introduction

Being better than others has been an aspiration of every competitive individual, whether the benefits of this comparison are economic or simply bring in prestige. Decisions on who is better than whom may be reached through several methods: they may be based on voting, or on the evaluation of appropriately chosen criteria, which can then be measured and properly combined to reach a ranking of concerned individuals. This dissertation is about these methods and these criteria. We study how different factors interact and can be combined within a particular index,. We examine what are the conditions on the set of voters that, following a supermajority decision rule, give rise to a transitive social ranking. And finally, we investigate how can a group of peers rank themselves impartially, in a way that no individual can influence his own rank.

Each of these topics is elaborated in a different chapter of this dissertation. We devote Chapter 2 to the Human Development Index and its components. We investigate the importance of each factor in the substitution rates between the others. Chapter 3 addresses conditions guaranteeing transitivity of the social relation derived from supermajority decision rules. Finally, in Chapter 4 we provide a characterization of impartial ranking rules, preceded by series of examples and properties on that involved subject.

## 1.1 Complementarity in the Marginal Rates of Substitution of the Human Development Index

The Human Development Index (HDI) is a measure of a country's development level that considers components that go beyond income. In this chapter we first revise all past and present HDI together with their corresponding marginal rates of substitution (MRS). We find that according to the existing HDI, the implementation of a certain health policy in a country is independent of its education level and vice versa. On the other hand, there is a literature indicating that education and health are relevant in the production of one another. Motivated by these finding we propose an alternative HDI that would reflect with these findings when used to compare different societies.

## 1.2 Transitive Supermajority Rule Relations

This chapter is coauthored with Dimitrios Xefteris and here we establish a necessary and sufficient single-profile condition for obtaining a transitive relation under a supermajority decision rule. We first assume that the individual preferences are linear orders. We start by reducing the population by disregarding the individuals who have inverse preferences over the alternative set, and then we consider an equivalent supermajority rule that depends on the agents in the reduced population only. Our $a-$balancedness condition applies to the reduced population and it is both necessary and sufficient to guarantee transitive social preferences.

Our profile condition is composed of two distinct properties. The first one states that whenever there is an alternative that is preferred to the other two by a sufficient number of individuals, then transitivity is guaranteed. This happens since the relation between those two alternatives is irrelevant in this case. The second one discards the possibility of a cycle in the supermajority social preference relation when no alternative is preferred by a qualified majority.

Furthermore, we extend our result to the case in which individual preferences are given by weak orders. First, we notice that a supermajority rule relation of a preference profile with indifferences is equivalent to another supermajority rule relation of a transformation of the original profile which is composed of linear orders only. Then we apply the appropriate balancedness condition to the latter and the generalization of the result trivially follows.

## 1.3 Impartial Social Rankings

We consider situations where a set of individuals must rank themselves, on the basis of their opinions on what position should al others have in the ranking. Each agent provides an ordered list of the others, representing his opinion on others. We call the rule that determines the final ranking a *social ranking function* and we say that it is impartial if no change in any single agent's opinion can ever influence his own rank.

We present several impartial ranking functions, none of which is Paretian. We then prove that in fact no impartial social ranking function can be fully Paretian. However, we demonstrate that there exist impartial ranking rules satisfying a partial version of unanimity and some attractive additional properties. Finally, we characterize impartial ranking functions in terms of three axioms, which we prove to be sufficient and necessary for impartiality.

# Chapter 2

# Completarities in the Marginal Rates of Substitution of the HDI

## 2.1 Introduction

Every year the organization of the United Nations issues a report dedicated to Human Development (UNDP)[34]. Each of these reports contains the Human Development Index (HDI)[1], an aggregate measure of country's development level. It is generally agreed upon that health, education and income are the basic components that should enter any measure of human development. But the choice of variables to measure these components and of the formula to aggregate them into a single human development index is the subject of a debate, because it has many implicit implications.

A basic concern when designing any method to measure human development is the following: How much a country gain in one of the dimensions in order to compensate for a loss in some other, so that its index remains unchanged? This natural question can be rephrased in terms of the marginal rates of substitution (MRS) between pairs of variables defining the index.

Suppose that an index is defined in such a way that a one year decrease in a country's level of schooling can be compensated by a $100 increase in the yearly per capita income, while another index requires $300 in order to compensate for the same decay in the education of citizens. Clearly, these differences in the MRS would reflect a very strong, implicit value judgment regarding the cost of education. It is not surprising

[1]The index was initiated by the work of Sen (1985)[39] and Anand & Sen[1], stating that *the human development should concern with advancing the richness of human life, rather than the richness of the economy in which human beings live, which is only a part of it*

then, that the analysis of the MRS among these three components acquires, indeed, deep normative importance.

To complicate matters, there is an extensive literature regarding the correlation among some of the variables that are aggregated by any index of human development. In particular, there is a series of papers that investigate the relationship between education and health. Many authors[2] find a positive correlation between education and health. In addition, what we find more supporting for our work are the papers that relate education with health knowledge, see for instance Cutler and Lleras-Muney (2006)[11]. Their results find strong gradients where the better educated have healthier behaviors. People with more years of schooling are less likely to smoke, to drink a lot, to be overweight or obese, or to use illegal drugs. The better educated are more likely to exercise and to obtain preventive care such as flu shots, vaccines, mammograms. They also report having tried illegal drugs more frequently, but they gave them up more readily.

These results suggest that the education level matters whenever an individual acquires health. Consider a situation where an individual has to reduce his spending on his health insurance, i.e. he faces a reduction to his well-being status. Because of individual loss aversion, the individual has to make some adjustments in his health behavior in order to compensate for the well-being loss. According to the results from Cutler and Lleras-Muney (2006)[11], the more educated individuals could accomplish these health behavior adjustments with a lower cost. In other words, the cost of implementation of a government health policy depends on the peoples' education level. The effect of health in school behavior is also recognized, (Case, Fertig and Paxson 2005[8], Miguel and Kremer (2004)[30]), which allows us to consider health as a complement in the production of education.

In spite of these results, the MRS between health (education) and income in the all existing HDI are independent of education (health) level. We will later provide an extensive analysis of the MRS in the past and existing HDI. Here however, in Table[3] 2.1

[2]See for instance Lleras-Muney[27],[28],[18], Catherine E. Ross, Chia-ling Wu[36] find that the education gradients in health are positive on different health levels. On the other hand there is also literature that explains the causal effect of education on health. Older children that are sick or malnourished during childhood are more likely to miss school, see Case, Fertig and Paxson (2005)[9]. Behrman and Rosenzweig (2004)[43], Black, Devereux and Salvanes (2005)[6] show that children that are born with low or very low birth weight obtain less schooling that those born with higher weights.

[3]The data in the Table corresponds to Life Expectancy at Birth, Expected Years of Education and GNI per capita in PPP terms (constant 2005 international $) representing Health, Education and Income levels in Herrero at al (2012)[23]. The data sources for the three variables are *UN DESA*-UN Department of Economic and Social Affairs, *UNESCO* Institute for Statistics and *The World Bank* respectively, and it corresponds to 2011.

we show the value of some particular MRS in the HDI of Herrero at al (2012)[23], showing pairs of countries with "same" MRS between health (education) and income (as a consequence of similar levels in those two components), but with significant difference in the third component level. Following the arguments presented above, we propose

| Country | Health | Education | Income | MRS Income-Health ($) |
|---|---|---|---|---|
| Sudan | 61.5 | 4.4 | 1894 | 30.79675 |
| Mauritania | 58.6 | 8.1 | 1859 | 31.7235 |
| Austria | 80.9 | 15.3 | 35719 | 441.5204 |
| Andorra | 80.9 | 11.5 | 36095 | 446.1681 |
| | Health | Education | Income | MRS Income-Education ($) |
| Panama | 76.1 | 13.2 | 12335 | 934.4697 |
| Gabon | 62.7 | 13.1 | 12249 | 935.0381 |
| Luxembourg | 80 | 13.3 | 50557 | 3801.278 |
| Kuwait | 74.6 | 12.3 | 47926 | 3896.423 |

Table 2.1: The MRS in the latest HDI

an alternative measure of the human development, in which education and health are complements in the Income-Health and Income-Education MRS respectively. In order to distinguish among the different HD indices we introduce a notation for the past and present HDI. By $HDI$ we name the index used in the UNHD reports up to 2009, included. We call $HDI2010$ the index that was integrated in the UNHD reports since 2010[4]. By $newHDI$ we denote the latest HD index proposed by Herrero at al (2012)[23].

Before presenting the content of this paper, we explain the nomenclature of the human development ingredients. Each of the before mentioned indices is composed by the same three *HD components*: Health, Education and Income. Depending on the index, each of these components is represented by a certain *HD variable*. For instance, the HD variable that represents the Income component in the $HDI2010$ is the GNI per capita. Once the HD variable is normalized[5], we call it a *HD indicator* (or just an indicator). The rest of the paper is organized in the following way. In the next section we summarize the existing HD variables and indicators used in the three mentioned indices ($HDI$, $HDI2010$ and $newHDI$). In section 3 we introduce the our HD index, which we call $ourHDI$, we present its values and corresponding MRS using data from 2011. Section 4 concludes the paper.

[4]The HDI2010 results that we will refer later correspond to the data of 2011

[5]Different HD indices use different normalization procedures

## 2.2 A Summary of HD Measurement

Health, Education and Income are the three components that are common characteristic of the Human Development Index from its very beginning till today. However the variables that represent these components varied through time. Here we review the variables that formed part of the HDI by focusing on each component separately. Figure A.1 contains the summary of all such variables.

Life Expectancy at birth has been the variable that represents the Health component since the very beginning of the HDI. The health indicator was the normalized value of Life Expectancy obtained by the formula (2.1)

$$x^* = \frac{x - x_{\min}}{x^{\max} - x_{\min}}. \tag{2.1}$$

The normalization bounds were 85 and 25 years[6]. Herrero at al (2012)[23] show that the HDI country ordering is sensitive to the value of the lower bound and therefore they set it to zero.

In the first HDI (UNDP (1990)[34]), the Education component was represented by the Adult Literacy Rate only. Starting the following year another variable was included to the representation of this component. This variable was the mean years of schooling, later substituted by the combined gross enrollment ratio index. Using these two variables[7] the education indicator was the weighted sum computed by 2/3 of the literacy rate and 1/3 of the enrollment ratio. In 2010 the UN changed the education component variables to Expected and Mean Years of Schooling, UNDP (2010)[34]. Each of these variables was normalized using the formula (2.1). The upper bounds for mean and expected years of schooling were 13.2 and 20.6 years respectively whereas the lower ones were zero for both. In the UNDP (2011)[34] the expected years of education upper bound is set to 18 years, due to a change of data estimation methodology. Finally the education indicator was the geometric mean of the two normalized values.

In the latest work of Herrero, Martínez and Villar (2012)[23] there is another modification regarding the calculation of the education indicator. The authors exclude the Mean Years of Schooling variable from the Education component. They justify this step by the complexity of the interpretation and the importance of the former variable in the overall index. Moreover, the education indicator composed by expected years of schooling only focuses on the possibilities that people have, rather than their past

[6] Modified to 83.2 in 2010

[7] Note that both adult literacy rate and the combined gross enrollment ratio are indices themselves

achievement. Therefore, the education indicator in the new HDI is the normalized value of the Expected Years of Schooling. The upper and lower bounds used for the normalization are the same as in the HDI 2010.

The variable that represented the Income component of the HDI was real GDP per capita, substituted by real GNI per capita in 2010. Before moving to the normalization step, the variable of the Income component went through a logarithmic transformation that provided a decreasing effect of an additional unit of income on the HDI. This transformation was discarded between 1991 and 1999, when Atkinson's formula was applied to the GDP. Nevertheless, the logarithmic transformation to the GDP was again applied starting 2000, see Anand & Sen[2]. Finally, in the $newHDI$ Herrero at al (2012)[23] take out the *log*'s and discount the GNI by the GINI coefficient.

The normalization bounds for the Income variable changed through time. Until 1993 the upper and lower bounds were the observed ones in the current year. The dependence of the HDI ordering to the choice of the bounds was the reason to fix them in 1994. In 2010 the normalization upper bound is set to the actual observed maximum value of the indicator from the countries in the time series 1980-2010, whereas the lower one is maintained fix. In Table 4.1 we present the values of these bounds starting 1999 till today We note that the latest lower bound is zero because of the same reasons as in the

| | Income | |
|---|---|---|
| Year | Upper Bound | Lower Bound |
| 1999 | \$40000 | \$100 |
| 2010 | \$108211 | \$163 |
| 2011 | \$107721 | \$100 |
| 2011 new HDI | \$60000 | \$0 |

Table 2.2: Income normalization bounds used

normalization of the Health variable.

Before we summarize the functional forms used for calculation of the HDI, we introduce some notation for the variables of the HD components and their corresponding indicators. Let by $LE$ and $I$ we denote Life Expectancy at birth and Income variable. Whenever referring to the corresponding normalized value we write $LE^*$ and $I^*$[8]. Similar, let $EYS$ and $MYS$ be the Expected and Mean Year of schooling, with $EYS^*$ and $MYS^*$ being their normalized values.

Let $X^{\max}$ and $X_{\min}$ be the upper and lower bound for the variable $X$. Finally we

[8]Whenever we write $X^*$ we refer to the normalized value of the variable $X$, using the normalization bounds of the corresponding year

present the notation we use for the indicators of the HD variables. Let $E^*$ stands for the Education indicator of the corresponding year. For instance, its expression in 2010 would be

$$E^* = \sqrt{\frac{MYS}{MYS^{\max}}}\sqrt{\frac{EYS}{EYS^{\max}}} = \sqrt{MYS^*}\sqrt{EYS^*}.$$

To denote the Health and Income indicator we use the same letters as the ones for their normalized value $LE^*$ and $I^*$, since both indicators are computed using one variable only.

We end this section by presenting the aggregation formulas that we use to compute the HDI value. The first HDI formula was the equally weighted average of the three indicators $LE^*$, $E^*$ and $I^*$,

$$HDI = \frac{LE^* + E^* + I^*}{3}. \tag{2.2}$$

There is a series of literature that analyzes two important features of this aggregation formula, the marginal rates of substitution between the components and the index robustness to the changes of weights, see for instance Desai (1991)[14], Palazzi and Lauri (1998)[32], Sagar & Najam (1998)[38], Anand & Sen (2000)[2], Nathan et al (2008)[31], Herrero at al (2010)[22]. The additive form of the HDI implied perfect substitution across the normalized values, with constant marginal returns to improvements. This was against the idea that the worse the deprivation in a particular dimension, the more urgent the efforts to improve achievements in that dimension[9]. The other weakness of the averaging formula is the robustness of the index ordering whenever the normalization bounds are modified. We saw above that the upper and lower bounds used for normalization went through some changes, especially the ones for the Income variable, see Table 4.1. The country ranking that resulted from the HDI ordering was not robust to these changes, which made the traditional HDI dependent on the choice of the variable bounds.

These weaknesses of the traditional HDI led to a modification of its aggregation formula. Herrero at al (2010)[22] proposed and axiomatically characterized the aggregation formula first used in 2010, where the index is the geometric mean of the suitably calculated human development indicators

$$HDI2010 = (LE^* E^* I^*)^{1/3}. \tag{2.3}$$

[9] Although constant on the normalized values, the MRS between the Income and one of the other two components depended only on income. This dependency came from the logarithmic transformation of the GDP

$$MRS_{LE,I} = \frac{I[\log(I^{\max}) - \log(I_{\min})]}{(LE^{\max} - LE_{\min})}.$$

According to this formula the marginal rates of substitution between two indicators are now equal to their ratio. For instance, the MRS between the Education and Health indicator is given by

$$MRS_{E,LE} = \frac{LE^*}{E^*}.$$

This equation tells us that whenever the education level of a country is low, the substitution rate of education units is high. Therefore, the new HDI directs the governments to a efficient human development progress by improving their weakest component. In addition, the country ordering that results according to the aggregation formula (2.3) is robust to the upper normalization bounds (Herrero at al (2010)[22]).

Ravallion (2010)[35] examines the MRS in the HDI2010 between income and the other two components, in terms of the variables that represent them. He found that the value of year of schooling and life expectancy is very high for the countries with low income. For instance, the MRS between Life Expectancy and Income is given by

$$MRS_{LE,I} = \frac{I(\log(I) - \log(I_{\min}))}{LE - LE_{\min}}.$$

Considering the case of Zimbabwe, the country with the lowest income, Ravallion finds that the value of longevity is very low 0.51 per year, representing less than 0.3% of that countrys (very low) mean income in 2008. Thus the 2010 HDI implies that if Zimbabwe takes a policy action that increases national income by a mere 0.52 or more per person per year at the cost of reducing average life expectancy by one year, then the country will have promoted human development. Similar arguments can be found about the value of year of schooling.

The $newHDI$[23] corrects these troubling trade-offs by setting the lower bound to zero and excluding the $log$'s from the normalization of the Income variable. In Figures A.2 and A.4 we show the values of extra year of life and education according to this index. In the next section we compare these results with the corresponding ones of the index that we propose in what follows. We will see that our MRS do not exhibit the problematic trade-offs as in $HDI2010$, but instead they are consistent with the rates from $newHDI$.

## 2.3 HDI with MRS Complementarity, (ourHDI)

We begin introducing our HDI by presenting the variables that correspond to the three HD components, Health, Education and Income. These components have been divided into two types, known as commodities and capabilities Sen (1999)[39], means and ends Streeten (1994)[44], functioning and capabilities, Zambrano (2011)[47]. The idea behind this categorization is to divide the components into ones that explain people's possibility and ability to acquire. The Health and Education components form the group that describes people's possibilities for long and cultured life and therefore, their related variables should be an expectations of what an individual could achieve, while the income variable corresponds to their capacity to do so.

Our choice for the Health and Education variables is the same as in Herrero at al (2012)[23] (see also Figure A.1), i.e. life expectancy at birth *(LE)* and expected years of schooling *(EYS)*. The third component of the Human Development measurement represents the individuals ability to achieve the desired levels of health and education. Following $HDI2010$ and $newHDI$, we keep GNI per capita as the income variable.[10]

After choosing the component variables we continue with the normalization procedure. Using the arguments about the lower normalization bound for Life Expectancy in Herrero at al (2012)[23], we fix it to 0 and use equation (2.1) to obtain the Health indicator value. We maintain the upper bound at 83.2 as used in all HD indices since 2010. Regarding the normalization of $EYS$, we use the upper bound of 18 years as given in the latest data set at UNESCO Institute for Statistics, and 0 for the lower one.

We normalize GNI following the formula (2.1), choosing \$107 721 and 0 to be the upper and lower bounds. Later in this section we show that the MRS between the variables are invariant to the GNI upper bound.
Comparing with HDI2010 and HDI, we make two substantial changes in the treatment of the Income component variable, also applied by Herrero at al (2012)[23]. Anand & Sen (2000)[2] set up the idea of decreasing effect of an additional unit of income by making the logarithmic transformation of the GDP per capita. At the moment of this suggestion, the HDI was the arithmetic mean of the HD indicators and therefore, the marginal effect of an additional income unit was constant. Nevertheless, if the desired decreasing effect is embed in the aggregation formula the log transformation is not necessary, and therefore we do not apply it to the income variable. The second modification is setting the lower normalization bound equal to zero, motivated by the same reason as in the case of the health component variable.

Let us use the indicator notation we introduced before. By $LE^*$, $E^*$ and $I^*$ we denote the normalized values of the Health, Education and Income variables. We also maintain the notation for the actual values of the components as before, $LE$, $EYS$ and $GNI$. We say that the overall Human Development level of a country, $ourHDI$ is given

[10] The data about Life Expectancy, Expected Years of Education and GNI is taken from *UN DESA*-UN Department of Economic and Social Affairs, UNESCO Institute for Statistics and The World Bank

by

$$ourHDI = \frac{\sqrt{I^*LE^*} + \sqrt{I^*E^*}}{2} \tag{2.4}$$

We note that $ourHDI$ is a continuous function of the Human Development indicators. Its minimal and maximal values are 0 and 1 respectively. We note that $ourHDI$ and $newHDI$ differ on the variable levels where the index value is zero. According to both measures, the index of a country with no income is zero. We support this feature with the fact that income determines the people's ability to attain the desired levels of heath and education. No income means no possibility to acquire health and education.

Independently of the $GNI$ value, $ourHDI = 0$ when both $LE$ and $EYS$ are zero. This is not the case with $newHDI$ or $HDI2010$, which are zero if at least one of the indicators is on its minimal value. We think that a country with $EYS^* = 0$ but positive $LE^*$ should have a positive HD index, since $EYS^*$ is an expected value and it does not imply that children will have no education at all. Similar arguments hold when $LE^* = 0$ but $EYS^* \neq 0$.

Unlike the two existing aggregation formulas $ourHDI$ is not a symmetric function of the three indicators. In the formula (2.4) we see that the $I^*$ has a different treatment than $LE^*$ and $E^*$. When aggregating the indicators we use one part of the income in order to acquire health and the other for education. In this way the overall country's human development is the sum of the health and education levels that its citizens could achieve separately.

Aiming to achieve a decreasing effect of an extra unit from the income variable, we saw that $HDI2010$ and $HDI$ applied a logarithmic transformation to the $GNI$ and $GDP$ respectively. Since $\frac{\partial^2 ourHDI}{\partial I^*} < 0$[11], formula (2.4) implies a decreasing effect of an extra unit of income, and therefore the log transformation is not necessary.

### 2.3.1 The MRS's

In this subsection we analyze the properties of the MRS of $ourHDI$. We show that schooling and life expectancy are complementary in the Income-Health and Income-Education MRS respectively. Moreover, we show that our results are coherent with the MRS in $newHDI$, concerning the disparity in the longevity and schooling values raised by Ravallion (2010)[35].

[11]
$$\frac{\partial^2 ourHDI}{\partial I^*} = -\frac{1}{2}(I^*)^{-3/2}\left(\frac{\sqrt{LE^*} + \sqrt{E^*}}{4}\right) = -\frac{ourHDI}{4(I^*)^2}$$

#### 2.3.1.1 The Valuation of Life

By Value of Life (Longevity) we call the monetary rate at which a government is ready to drop a health policy that will lower the country's $LE$ for one year, while maintain the same HD index level. In The MRS between the Health and Income variables, corresponding to $ourHDI$ is given by

$$\frac{I^*}{LE^* + \sqrt{LE^* E^*}} = \frac{GNI}{LE + 2.14994\sqrt{LE * EYS}}, \tag{2.5}$$

whereas the corresponding MRS respect to $newHDI$ are

$$\frac{I^*}{LE^*} = \frac{GNI}{LE}.$$

We notice immediately that the Health-Income MRS from $ourHDI$ depends on the education level. Higher education implies lower substitution rate. This is in correspondence to the arguments about the complementarity in the MRS of the human development given in the introduction. Before we show some results we note that the right hand side in both equations are the MRS expressed as a function of the variables, not the indicators. The coefficient 2.14994 that appears in the denominator of the MRS in $ourHDI$ comes from the upper bounds used in the normalization of $LE$ and $EYS$, but not $GNI^{\max}$. In the appendix we formally show that the MRS as a function of the HD variables is independent of the GNI upper bound.

Table 2.3 contains the longevity values for some countries presented in the introduction, extended for the MRS between health and income respect to $ourHDI$. We see that unlike $newHDI$, the value of life respect to $ourHDI$ decreases with education. We

| Country | Health | Education | Income | $newHDI$ Inc.-Health ($) | $ourHDI$ Inc.-Health ($) |
|---|---|---|---|---|---|
| Sudan | 61.5 | 4.4 | 1894 | 30.79675 | 19.55 |
| Mauritania | 58.6 | 8.1 | 1859 | 31.7235 | 17.6 |
| Austria | 80.9 | 15.3 | 35719 | 441.5204 | 228.2 |
| Andorra | 80.9 | 11.5 | 36095 | 446.1681 | 246.1 |

TABLE 2.3: Comparison of some Longevity Values

present the full list of longevity values in Table A.2, whereas Table 2.4 summarizes those results, together with the corresponding ones of $HDI2010$ and $newHDI$. As a fraction of GNI, the longevity respect to $ourHDI$ and $newHDI$ is quite flat. The average value respect to these two indices is 0.8% and 1.4%, see Figure A.3. In the case of $HDI2010$, these percentages vary from 2.6% till 16.2%.

| Index | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| HDI2010 | 187 | 1227.809 | 1683.324 | 7.017889 | 12878.8 |
| newHDI | 187 | 168.7366 | 193.4582 | 4.665493 | 1373.992 |
| ourHDI | 187 | 87.66129 | 101.2142 | 2.397324 | 746.28 |

Table 2.4: The value of Life according to the three indices, a summary

The average valuation of life respect to $ourHDI$ is 54% of the corresponding one arising from $newHDI$. This proportion depends on the education level and its lower values is around 50%, as seen in the case of Norway, Australia, Denmark. On the other hand the percentage values that are higher than the average are typical for the countries with low expectation for schooling years, which goes along with the evidence that comparing to $newHDI$ the value of life in $ourHDI$ declines with the education. Figure A.6 plots these proportions.

As we mentioned before, Ravallion (2010)[35] questioned the disparity in MRS between health and income in $HDI2010$ of UNDP (2010)[34]. In order to check this issue with the data of 2011, in figure A.2 we plot the longevity values according to the three indices. We immediately notice that $ourHDI$ and $newHDI$ significantly reduce the large difference of the longevity emerging in the $HDI2010$. The maximal value of life of \$12 878 that corresponds to Qatar, is reduced to \$1 373 and \$746 by $newHDI$ and $ourHDI$ respectively. Moreover, according to $ourHDI$ the life value deviation is decreased to 101, the former being 1 683 and 193 in $HDI2010$ and $newHDI$.

Another issue that Ravallion (2010)[35] observe is the least-squares elasticity (the ordinary regression coefficient of $\log MRS$ on $\log GNI$). The estimated value according to HDI2010 is 1.14, which is found significantly greater than unity. This implies that the HDIs valuation of longevity as a proportion of mean income tends to rise with mean income. On the other hand, Viscussi and Aldy (2003)[45] conclude that this elasticity is lower than 1, and it is in the range 0.5-0.6. The estimated value of this coefficient in our model is 0.89 (st. error 0.005) and it is significant at any level. We consider this result as an improvement.

#### 2.3.1.2 The Valuation of Schooling

Here we compare the MRS between education and income variables among the three HD indices. As before, we call value of schooling the monetary rate at which a country could substitute a year of $EYS$ and still remain at the same HD level. We calculate the

MRS between education and income respect to $ourHDI$

$$\frac{I^*}{E^* + \sqrt{E^* LE^*}} = \frac{GNI}{EYS + 0.46513\sqrt{LE * EYS}}, \tag{2.6}$$

and $newHDI$

$$\frac{I^*}{E^*} = \frac{GNI}{EYS}.$$

As in the case of longevity, similar arguments hold about the influence of health in Education-Income MRS and about the independence of the MRS from the GNI upper normalization bound.

Table 2.5 contains the value of schooling of the countries mentioned in the introduction, whereas Table A.2 contains their full list. In Figure A.4 we see that as in the case

| Country | Health | Education | Income | $newHDI$ Inc.-Educ. ($) | $ourHDI$ Inc.-Educ. ($) |
|---|---|---|---|---|---|
| Panama | 76.1 | 13.2 | 12335 | 934.4697 | 441.45 |
| Gabon | 62.7 | 13.1 | 12249 | 935.0381 | 463.44 |
| Luxembourg | 80 | 13.3 | 50557 | 3801.278 | 1775.67 |
| Kuwait | 74.6 | 12.3 | 47926 | 3896.423 | 1816.1 |

Table 2.5: Comparison of some Schooling Values

of longevity, $newHDI$ and $ourHDI$ reduce the big disparity of the value of schooling that arises in $HDI2010$. In addition, we compare the values of schooling as a percentage of the GNI. In Table 2.6 and in Figure A.5 we summarize and plot these results. We

| Index | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| HDI2010 | 187 | 0.168704 | 0.039074 | 0.044298 | 0.335775 |
| newHDI | 187 | 0.087259 | 0.0285 | 0.055556 | 0.227273 |
| ourHDI | 187 | 0.0406353 | 0.0104972 | 0.0278871 | 0.0829782 |

Table 2.6: Value of Schooling as a % of GNI, a summary

notice that the schooling as of GNI is much smoother in ourHDI then the equivalent values respect to the other two indices. The variation of schooling as of GNI is smallest respect to $ourHDI$. Its minimum value is 2.6% (Iceland, Australia) and it goes up to 7.9% (Sudan). On the other hand, these variations are much higher according to the two other indices. Starting at 5.5% and 4.4%, they rise up to 22% and 33% in accordance with $newHDI$ and $HDI2010$ respectively.

### 2.3.2 MRS Sensibility to the Normalization Bounds

The expressions of the MRS in $ourHDI$ given in (2.5) and (2.6) depend on the coefficients 2.14994 and 0.46513 respectively, determined by the ratio between the upper bounds used in the normalization of $LE$ and $EYS$[12]. These values make Longevity and Schooling in $ourHDI$ conditional on normalization bounds, which we consider as a disadvantage to $newHDI$. The MRS give the information to governments about how should a certain policy effect health or education and therefore, being reliant on $LE^{\max}$ and $EYS^{\max}$ is not a desired property.

In order to see the effect of the normalization bounds to the MRS in $ourHDI$, we change the $LE^{\max}$ and $EYS^{\max}$ by 10% one at a time and compare the new values of longevity and schooling with the current once. We found that an increase of 10% in the $LE$ normalization bound decreases longevity between 2 and 2.4%, whereas it increases the schooling value for 2.5% for the countries with high $EYS$ and 3% for the once with low $EYS$. We obtained similar results when we modify $EYS^{\max}$. In this case the longevity and schooling are decreasing and increasing respectively, by similar amounts as before.

We made the analysis of the MRS sensibility assuming a 10% increase in the normalization bounds. We chose this amount since the variations of $LE^{\max}$ and $EYS^{\max}$ from 1985 till today are not higher than 10%. This implies that the MRS in $ourHDI$ would not vary more than 3% if in the aggregation formula (2.4) we consider one of the maximal value of $LE$ or $EYS$ from 1985.

We consider these effects as minor but not negligible. Notice that we made the sensibility analysis when there is a change in only one of two upper bounds. On the other hand, it is confirmed that health and education are highly correlated variables (Lleras-Muney[27],[28],[18], Catherine E. Ross at al (1995)[36]). We also found that the correlation coefficient between $EYS$ and $LE$ used for the calculation of $ourHDI$ is close to 0.8. Along with these arguments we expect that the maximum values obtained in life expectancy and expected years of schooling should move in the same direction. In case they do so, we should test the sensitivity of the MRS in $ourHDI$ to the changes of $LE^{\max}$ and $EYS^{\max}$ at the same time.

We check the data about the maximum values of $LE$ and $EYS$[13] starting 1985, and we found that the coefficients that appear in (2.5) and (2.6) do not vary more than 0.8%. Considering these variations of $LE^{\max}$ and $EYS^{\max}$ in $ourHDI$ we found that the longevity and schooling values, on avergage change in 1%.

[12] $2.14994 = \sqrt{\frac{LE^{\max}}{EYS^{\max}}}$ and $0.46513 = \sqrt{\frac{EYS^{\max}}{LE^{\max}}}$

[13] $EYS$ from *UN DESA*-UN Department of Economic and $LE$ from Social Affairs and UNESCO Institute for Statistics

## 2.4 Conclusion

The purpose of establishing $ourHDI$ is presenting a HD index that will allow complementarity in the MRS. The longevity and schooling in $ourHDI$ depend on the upper normalization values of $LE$ and $EYS$. To be more specific, they are a decreasing functions of $LE_{\max}$ and $EYS_{\max}$ respectively. However, the values of the MRS in $ourHDI$ experience some losses regarding the corresponding ones by $newHDI$.

By introducing $newHDI$, Herrero at al (2012)[23] made an index in which, the MRS between components are invariant to the normalization bounds. On the other hand, we have that the MRS in $ourHDI$ are invariant to the normalization bound of GNI, but not to $LE_{\max}$ and $EYS_{\max}$.

Despite the fact that the propose a fixed upper bound for the normalization of $LE$ and $EYS$ in $ourHDI$, we performed an analysis about the sensibility of the MRS when $LE^{\max}$ and $EYS^{\max}$ change. We can conclude that a modification in the upper bounds of $LE$ and $EYS$ would have a minor effect on the MRS in $ourHDI$. We think that we recompense this not so desired effect by the negative monotonicity of $ourHDI$ MRS respect to the Health and Education level.

# Chapter 3

# Transitive Supermajority Rule Relations

This chapter is coauthored with Dimitrios Xefteris[1]

## 3.1 Introduction

In many instances of collective decision making, supermajority rules are applied so that a social ordering of the alternatives is formed. An amendment of a country's constitution is one of the most common examples of such instances. Moreover, one encounters the necessity of a specified majority of votes in certain types of issues which are handled by hiring committees, boards of shareholders in private corporations (equity shares issues, merger decisions and others) and in federal judge appointment by the US Senate.

Consistent decision making under supermajority rules has been investigated so far through the existence of an equilibrium (a the set of maximal elements according to the prior determined majority quote). Slutsky(1979)[42], by the means of a multi-dimensional spatial model, provides a supermajority lower bound which is necessary and sufficient for the existence of a *supermajority equilibrium* for every possible profile in a society in which the number of voters exceeds by one or more the number of dimensions of the policy space. Coughlin (1981)[10] extends this result by obtaining the supermajority lower bound which is necessary and sufficient for any given society. Later on, Greenberg and Weber (1985)[21] show that although they differ, the supermajority lower bounds

[1] Department of Economics, University of Cyprus. E-mail: xefteris.dimitrios@ucy.ac.cy, Web-page: https://sites.google.com/site/dxefteris/

derived in Greenberg (1979)[20] and Coughlin (1981)[10] provide the same set of equilibria. As far as domain conditions are concerned, Austen-Smith and Banks (1999)[3] showed that single-peakedness (Black, 1948) is an equilibrium sufficient condition under a wide class of rules that include the supermajority rules.[2], Rothstein (1990)[37] and when they satisfy the single-crossing property (Gans and Smart (1996)[17]). Barberà and Moreno (2011)[4] suggest the top-monotonicity domain restriction, which includes the previous ones, and extend the result of Austen-Smith and Banks (1999)[3] to these types of preferences.

Although the existence of the set of maximal elements is indeed an issue of predominant importance, the existence of a transitive social ordering is also crucial in many circumstances. There are cases in which hiring committees need to produce an ordering of the set of candidates for multiple (and heterogeneous)[3] posts while military committees need to provide an ordering of alternative plans to a unit about to engage in a military operation. Despite the obvious importance of this issue, results regarding transitivity of the supermajority rules relations are scarce. To be fair, we should acknowledge that so far all the effort was rightfully directed in the understanding of the most interesting extreme of the supermajority spectrum; simple majority rule.[4]

The conditions that guarantee transitivity of simple majority are mainly related to restrictions in the domain of individual preference orders. Black (1958)[5], Sen (1966)[41], Inada (1969)[25], Sen and Pattanaik (1969)[40], Fishburn (1973)[16] and Barberà and Moreno (2011)[4] provide the most popular examples of such conditions. Inada (1969)[25] first notices that, as far as the simple majority rule is concerned, Sen's (1966)[41] value-restrictedness condition generates the widest domain of individual preference orders such that, if a preference profile is formed by a fraction of them, then a transitive simple majority rule relation is always guaranteed. According to this interpretation value-restrictedness is both sufficient and necessary for consistency of simple majority decision; no wider domain condition can ever be obtained. On the other hand, we know that a preference profile may yield a transitive simple majority rule relation even if it is not generated by any restricted domain of individual preference orders. That is, domain restrictions are not able to dichotomize the universal domain of preference profiles in a set composed of profiles which generate transitive social orders and in a set which

[2]For the case of simple majority rule in particular, existence of a maximal element is guaranteed when preferences are intermediate (Grandmont (1978) [19]

[3]Consider for example a university department which has an open tenure-track position and an open post-doc position and three candidates for both these positions.

[4]Simple majority has many good properties (Dasgupta and Maskin, 2008[12]) compared to other rules (including supermajorities) and has been characterized by May (1952)[29] and Campbell and Kelly (2000)[7].

contains profiles which generate intransitive social orders. For this reason, a single-profile approach was proposed (see Parks, 1976[33], for a comprehensive discussion of the single-profile approach). Kaneko (1975)[26] was the first to provide single-profile conditions for transitivity of the simple majority rule relation and was followed by Feld and Grofman 1983[15] and Xefteris (2012)[46].

In this paper we study the whole range of supermajority Social Preferences Aggregators (SPAs), from simple majority rule to unanimity rule, and we derive a necessary and sufficient single-profile condition for transitivity of each supermajority rule relation. This condition will describe properties that $a$) a certain preference profile should posses in order to yield a transitive supermajority rule relation (sufficiency) and $b$) if they are violated by a certain preference profile then this profile yields an intransitive supermajority rule relation (necessity).

Hence, our condition completely characterizes the subset of the universal domain of preference profiles which is composed out of profiles which produce a transitive supermajority rule relation and completely characterizes its complement too. We need to stress at this point that, obviously, the dichotomy of the universal domain will vary along with the exact supermajority rule under study. A preference profile may yield a transitive social preference relation under some supermajority rule and may yield an intransitive social preference relation under another one.

## 3.2 Analysis

Let $I = \{1, 2, \ldots, n\}$ be a finite set of the first $n$ integers. We call the elements of $I$ individuals and let us assume that $n \geq 3$. Let by $X$ we denote the set of alternatives and for the moment suppose that $X$ consists of three elements only, i.e. $X = \{x, y, z\}$. Further on we show that our result applies to any finite set of alternatives.

Let $\mathcal{R}$ and $\mathcal{P}$ be the sets of all weak and strict linear orders on $X$ respectively. Note that the three alternatives assumption implies that $\mathcal{P}$ contains six elements only. Let us assume for now that agent $i$ has a strict preferences $P_i$ over the set of alternatives $X$, i.e. $P_i \in \mathcal{P}$. Later we will extend our result to the case $R_i \in \mathcal{R}$. A profile of all individuals' preferences $P^n$ is an element of the $n-$dim set $\mathcal{P}^n$. Furthermore, let $\mathcal{C}$ be the set of all complete binary relations on $X$ respectively[5].

[5] Notice that $\mathcal{P} \subset \mathcal{R} \subset \mathcal{C}$

We are interested in the transitivity of a binary relation that represents a social ordering based on a supermajority. We say that $x$ is socially preferred to $y$ according to a supermajority rule if the difference between the number of individuals that prefer $x$ to $y$ and the ones that prefer $y$ to $x$ is greater then $a^*$, where $a^*$ is a positive number[6] that is not a multiple of 1/2,

$$|\{i \in I|xP_iy\}| > |\{i \in I|yP_ix\}| + a^*.$$

Notice that whenever the preference profile contains only linear orders, the previous condition reduces to $|\{i \in I|xP_iy\}| > \frac{n+a^*}{2}$ or, equivalently, to

$$|\{i \in I|xP_iy\}| > \frac{n+1}{2} + a,$$

with $a^* = 2a + 1$. Formally we have the following definition

**Definition 3.1.** For the preference profile $P^n \in \mathcal{P}^n$ we define a social relation $R \in \mathcal{C}$ through its strict $(P)$ and indifference part $(I)$ on the following way:

$$\forall x, y \qquad xPy \quad \text{if} \quad \big|\{i \in N|xP_iy\}\big| > \frac{n+1}{2} + a,$$

$$and \qquad xIy \quad if \quad \neg(xPy \ or \ yPx).$$

In other words, we say that the society chooses $x$ over $y$ if the number of individuals that prefers $x$ over $y$ exceeds the majority in $a$ votes. On the other hand, we see that the indifference part completes the binary relation with the pairs that do not satisfy the strict domination. We call this social decision function an $a - majority$ rule.

Notice that in the simple-majority case $a^* \in (0, 1)$ we have that $a$ belongs to $(-1/2, 0)$. Then in both cases $n = 2k$ and $n = 2k + 1$, the social dominance condition becomes $\big|\{i \in N|xP_iy\}\big| > k$.

In what follows we introduce some properties concerning profiles and individual preferences.

**Definition 3.2.** We say that the individuals $i, j \in N$ are mutually exclusive in the profile $P^n$ if

$$xP_iy \Leftrightarrow yP_jx, \quad \forall\, x, y \in X.$$

**Definition 3.3.** Given the preferences profile $P^n$, let $N_0$ be a subset of $N$ that *(a)* does not include any mutually exclusive individuals in $P^n$ and *(b)* for which $N_0' = N \setminus N_0$

[6] $a^* > 0$ and $a^* \neq \frac{1}{2}k$, $\forall k \in \mathbb{N}$.

is either the union of $\frac{|N_0'|}{2}$ disjoint pairs of mutually exclusive individuals or empty. We call the set $N_0 = N \setminus N_0^{'}$ a set of reduced population with respect to the $P^n$.

Let by $n_0$ we denote the number of individuals in $N_0$. Notice that when $n$ is an even number there is a possibility that the reduced population set is empty ($N_0' = N$). In this case the supermajority rule gives a social ordering with indifferences only, since for every $x$ and $y$ exactly half of the population prefers $x$ over $y$. This social ordering is transitive and hence we discard this possibility.

**Example 3.1.** *Consider the profile* $P^3 = (P_1, P_2, P_3)$ *given as*

| $P_1$ | $P_2$ | $P_3$ |
|---|---|---|
| $x$ | $y$ | $z$ |
| $y$ | $z$ | $y$ |
| $z$ | $x$ | $x$ |

*Then the set of reduced population would be* $N_0 = \{2\}$ *and* $N_0^{'} = \{1, 3\}$, *since* 1 *and* 3 *are mutually exclusive.*

In the definition of the $a-$majority rule we see that the decision for the social dominance of $x$ over $y$ depends only on the number of individuals that rank $x$ higher than $y$. Using the reduced population set, we can rewrite this condition depending only on the number of individuals in the reduced population that prefer $x$ to $y$.

**Definition 3.4.** Let $N_0$ be a reduced population with respect to the profile $P^n \in \mathcal{P}^n$. Then we define a social relation $R' \in \mathcal{C}$ through its strict ($P'$) and indifference part ($I'$) on the following way:

$$\forall x, y \qquad xP'y \quad \text{if} \quad \big|\{i \in N | xP_i y\}\big| > \frac{n_0 + 1}{2} + a,$$

$$and \qquad xI'y \quad if \quad \neg(xPy \text{ } or \text{ } yPx).$$

Without providing a formal proof, here we state the following proposition:

**Proposition 3.5.** *Let* $P^n \in \mathcal{P}^n$ *and let* $R$ *and* $R'$ *be the social binary relations obtained by the* $a-$*majority social rule respect to definition 3.1 and 3.4 respectively. Then* $R = R'$.

As a consequence of Proposition 3.5 in the rest of the paper when it comes to the $a-$majority we focus on the reduced preference profiles instead on the profiles of all population. Before stating our main definition, we introduce a reduced preference profile given a reduced population.

**Definition 3.6.** We call $P^{n_0} \in \mathcal{P}^{n_0}$ a reduced preference profile if it is the product of the preferences provided by the individuals in the reduced population,

$$P^{n_0} = P_{i_1} \times \ldots \times P_{i_{n_0}},$$

where $i_j \in N_0$ for every $j = 1 \ldots, n_0$.

**Definition 3.7.** We say that a preference profile $P^n$ is $a-$*balanced* [7] if

**i)** each alternative is ranked at the top (bottom) for less than $\frac{n_0+1}{2} + a$ individuals in the reduced population $N_0$, and

**ii)** there exists an alternative that is ranked at the top (bottom) for less than $\frac{n_0-1}{2} - a$ individuals in $N_0$

The simple-majority version of the $a-$balanced profile condition introduced here is equivalent to the balancedness condition employed by Xefteris (2012)[46], in order to provide transitivity conditions solely for simple-majority rule. Therefore, we see $a-$balancedness as a natural generalization of the balancedness concept to any supermajority rule.

The assumption of three alternatives implies that $\mathcal{P}$ consists of six linear orders and thus, we can split it into three pairs of mutually exclusive individuals. Therefore, if a preference profile is composed of four or more different linear orders then it must contain a mutually exclusive pair. As a result, we conclude that for any $P^n$, a reduced preference profile $P^{n_0}$ is formed at most by three distinct orders $Q_1$, $Q_2$ and $Q_3$.

**Theorem 3.8.** *Let $P^n$ be a preference profile and let $R$ be the social relation induced by the $a-$majority rule. Then $R$ is transitive if and only if $P^n$ is not $a-$balanced.*

*Proof:* Suppose that the preference profile $P^n$ is $a-$balanced. First we claim that $P^{n_0}$ can not be constructed by a single linear order. Assume the contrary, i.e. $\exists i$ s.t. $P^{n_0} = Q_i^{n_0}$. Then because of $i)$ we have that $n_0 \leq \frac{n_0+1}{2} + a$. This implies that $\frac{n_0-1}{2} - a \leq 0$, which makes $ii)$ false.

Assume now that there are $i, j \in \{1, 2, 3\}$ such that $P^{n_0} \in \{Q_i, Q_j\}^{n_0}$. The previous arguments allow us to say that both $Q_i$ and $Q_j$ are present in the reduced profile. Without loss of generality, we can assume that $Q_i$ and $Q_j$ could take one of the following

[7] Here we should note that our $a-$balancedness condition is completely distinct from the balancedness notion employed in cooperative game theory (a condition which guarantees that a game has a non-empty core).

four forms:

| | $Q_1$ | $Q_2$ | | $Q_1$ | $Q_2$ | | $Q_1$ | $Q_2$ | | $Q_1$ | $Q_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $a)$ | $x$ | $y$ | $b)$ | $x$ | $z$ | $c)$ | $x$ | $x$ | $d)$ | $x$ | $y$ |
| | $y$ | $x$ | | $y$ | $x$ | | $y$ | $z$ | | $y$ | $z$ |
| | $z$ | $z$ | | $z$ | $y$ | | $z$ | $y$ | | $z$ | $x$ |

Notice that $xQ_1yQ_1z$ is present in every of the four given possibilities. Then there are only four (out of the six elements in $\mathcal{P}$) candidates for $Q_2$, since $Q_1 \neq Q_2$ and $Q_2$ can not be the inverse of $Q_1$. The other choices for $Q_1$ are equivalent cases to the ones proposed and they can be obtained by permuting the names of the alternatives.

Notice that in $a)$ and $c)$ the requirement $i)$ implies that $n_0 \leq \frac{n_0+1}{2} + a$ and equivalently $\frac{n_0-1}{2} - a < 0$. This violates $ii)$ since every alternative appears as the first (last) choice at least in one $Q_i$.

Consider now the case $b)$. The fact that each alternative is at the top (bottom) for less than $\frac{n_0+1}{2} + a$ individuals (requirement $i)$) implies that $xIz$ and $yIz$. The non-violation of $ii)$ implies[8] that $n_0 > \frac{n_0+1}{2} + a$ and since all the individuals in the reduced population prefer $x$ to $y$, we have that $xPy$. This shows that the aggregate social relation $R$ is not transitive.
Case $d)$ is equivalent to $b)$.

We are now left with the case when $P^{n_0} \in \{Q_1, Q_2, Q_3\}^{n_0}$ where $\forall i \in \{1, 2, 3\}$ there exists $k \in N_0$ s.t. $P_k = Q_i$. Again, without loss of generality, $Q_1$, $Q_2$ and $Q_3$ could take one of the following two forms[9]:

| | $Q_1$ | $Q_2$ | $Q_3$ | | $Q_1$ | $Q_2$ | $Q_3$ |
|---|---|---|---|---|---|---|---|
| $a)$ | $x$ | $x$ | $y$ | $b)$ | $x$ | $z$ | $y$ |
| | $y$ | $z$ | $x$ | | $y$ | $x$ | $z$ |
| | $z$ | $y$ | $z$ | | $z$ | $y$ | $x$ |

Let by $n(Q_i)$ we denote the number of individuals in the reduced population whose preferences are represented by $Q_i$, i.e. $n(Q_i) = |\{j \in N_0 | P_j = Q_i\}|$. Consider the case $a)$. Since $x$ is at the top at the linear orders $Q_1$ and $Q_2$, condition $i)$ implies that

[8] On contrary we would have $\frac{n_0-1}{2} - 1 < 0$. This inequality, together with the fact that every alternative is at least once at the top (bottom) of some $Q_i$, violates the requirement $ii)$.

[9] Notice that the second ranked alternative has to be different in each of the $Q_i$'s. Otherwise we would have a preference profile with a mutually exclusive pair. This implies that the top alternatives could be different in every linear order $Q_i$ *(case (b))* or that there is one alternative that is top (last) choice in two linear orders *(case (a))*.

$n(Q_1) + n(Q_2) < \frac{n_0+1}{2} + a$. The same condition implies that $n(Q_3) < \frac{n_0+1}{2} + a$. Using these two inequalities we see that $xIy$. Using similar arguments we conclude that $n(Q_1) + n(Q_3) < \frac{n_0+1}{2} + a$ and $n(Q_2) < \frac{n_0+1}{2} + a$, i.e. $yIz$.

On the other hand, $n(Q_1) + n(Q_2) < \frac{n_0+1}{2} + a$ and $n(Q_1) + n(Q_3) < \frac{n_0+1}{2} + a$ imply that $n(Q_3) > \frac{n_0-1}{2} - a$ and $n(Q_2) > \frac{n_0-1}{2} - a$. Then since *ii*) is fulfilled we have that $n(Q_1) < \frac{n_0-1}{2} - a$, which implies that $n(Q_2) + n(Q_3) > \frac{n_0+1}{2} + a$. This means that the number of individuals in the reduced population that prefer $x$ to $z$ is greater than $\frac{n_0+1}{2} + a$ or $xPz$; transitivity is violated.

Let us focus on situation *b*). Since the requirement *ii*) is satisfied, without loss of generality we assume that $n(Q_1) < \frac{n_0-1}{2} - a$. This implies that $n(Q_2) + n(Q_3) > \frac{n_0+1}{2} + a$ or $zPx$. Now we distinguish between two cases: $n(Q_2) < \frac{n_0-1}{2} - a$ and $n(Q_2) > \frac{n_0-1}{2} - a$.

*Case 1:* Suppose that $n(Q_2) < \frac{n_0-1}{2} - a$. Then we have that $n(Q_1) + n(Q_3) > \frac{n_0+1}{2} + a$ or $yPz$. Finally,

- if $n(Q_3) < \frac{n_0-1}{2} - a$ then $n(Q_1) + n(Q_2) > \frac{n_0+1}{2} + a$ or $xPy$, and
- if $n(Q_3) > \frac{n_0-1}{2} - a$ then $n(Q_1) + n(Q_2) < \frac{n_0+1}{2} + a$ or $xIy$.

We see that both, $xPy$ and $xIy$, violate the transitivity condition.

*Case 2:* The arguments for this case are symmetric to the first one.

We have shown one direction of our theorem, i.e. we have proven that if $R$ is transitive then the profile $P^n$ is not $a-$balanced. Let us now assume that $P^n$ is not $a-$balanced. This means that at least one of the conditions *i*) and *ii*) is not satisfied, but not necessarily both.

Let us suppose first that *i*) is not satisfied, i.e. there is an alternative $x$ that appears on the top (bottom) of some $Q_i$ for more than $\frac{n_0+1}{2} + a$ individuals. By the definition of the $a-$majority rule we have that $xPy$ and $xPz$ ($yPx$ and $zPx$). This implies that the social preference $R$ is transitive independently of the relationship between $y$ and $z$.

For the rest of the proof we suppose that $P^n$ is not $a-$balanced through the violation of *ii*) only. The assumption that $P^{n_0} = Q_i^{n_0}$ together with condition *i*) requires that $n_0 < \frac{n_0+1}{2} + a$. This implies that $xIyIz$, i.e. $R$ is transitive.

Suppose now that $i, j \in \{1,2,3\}$ such that $P^{n_0} \in \{Q_i, Q_j\}^{n_0}$, i.e. there are two different linear orders that generate the profile of preferences. Similar as in the first part of the proof, these two linear orders could be one of the four cases:

a)

| $Q_1$ | $Q_2$ |
|---|---|
| $x$ | $y$ |
| $y$ | $x$ |
| $z$ | $z$ |

b)

| $Q_1$ | $Q_2$ |
|---|---|
| $x$ | $z$ |
| $y$ | $x$ |
| $z$ | $y$ |

c)

| $Q_1$ | $Q_2$ |
|---|---|
| $x$ | $x$ |
| $y$ | $z$ |
| $z$ | $y$ |

d)

| $Q_1$ | $Q_2$ |
|---|---|
| $x$ | $y$ |
| $y$ | $z$ |
| $z$ | $x$ |

Notice that in cases $a)$ and $c)$ non-violation of $i)$ implies $n_0 < \frac{n_0+1}{2} + a$, since in $a)$ $z$ appears at the bottom at $Q_1$ and $Q_2$ whereas in $c)$, $x$ appears at the top at $Q_1$ and $Q_2$. As in the previous case, we conclude that $R$ is transitive as $xIyIz$.

We continue by investigating case $b)$. The facts that $xQ_1z$ and $zQ_2x$, $yQ_1z$ and $zQ_2y$ together with condition $i)$ allow us to argue that $xIz$ and $yIz$. Furthermore, if $n_0 < \frac{n_0+1}{2} + a$ then we have that $xIz$, i.e. $R$ is transitive. On the other hand, the case $n_0 > \frac{n_0+1}{2} + a$ ($\frac{n_0-1}{2} - a > 0$) is not possible. On contrary, the condition $ii)$ is satisfied because $x$ appears at the bottom of $Q_1$ and $Q_2$ zero times. Case $d)$ is equivalent to $b)$.

On the end we have to consider the possibility that a reduced preference profile is build by three different linear orders, $P^{n_0} \in \{Q_1, Q_2, Q_3\}^{n_0}$. As before, these orders could take one of the following two forms:

a)

| $Q_1$ | $Q_2$ | $Q_3$ |
|---|---|---|
| $x$ | $x$ | $y$ |
| $y$ | $z$ | $x$ |
| $z$ | $y$ | $z$ |

b)

| $Q_1$ | $Q_2$ | $Q_3$ |
|---|---|---|
| $x$ | $z$ | $y$ |
| $y$ | $x$ | $z$ |
| $z$ | $y$ | $x$ |

Consider first the case $a)$. The non-violation of $i)$ implies that $xIy$ and $yIz$. On contrary $x$ or $y$ would appear at the top for more than $\frac{n_0+1}{2} + a$ times. If $n_0 < \frac{n_0+1}{2} + a$ we have that $xIz$, whereas $n_0 > \frac{n_0+1}{2} + a$ leads to completion of $ii)$ and hence, not possible.

Concerning the case $b)$, the violation of $ii)$ means than $n(Q_i) > \frac{n_0-1}{2} - a$, $\forall i \in \{1,2,3\}$. Then we have that

$$n(Q_2) + n(Q_3) = n_0 - n(Q_1) < n_0 - \frac{n_0 - 1}{2} + a = \frac{n_0 + 1}{2} + a.$$

In the same way we have that

$$n(Q_2) + n(Q_3) < \frac{n_0 + 1}{2} + a \quad \text{and} \quad n(Q_2) + n(Q_3) < \frac{n_0 + 1}{2} + a.$$

These inequalities show that for any pair of alternatives there are not enough individuals that prefer one over the other such that the society would do so as well. Hence, we have $xIy$, $yIz$ and $xIz$.

■

Notice that the $a-$balancedness is a condition that involves triplets of alternatives. In case that $X$ has more than three elements, we can apply this condition on every triplet of alternatives and extend our result to cases when $X$ is any finite set.

It is essential to point out that when $|X| \geq 3$ the definition of $a-$balancedness for any triplet $\{x, y, z\}$ applies to the restriction of the profile to this set. In other words, $P_i$ and $P_j$ need not be inverses of each other. However, if the restriction of $P_i$ on the triplet of elements is the inverse of the corresponding one of $P_j$ then they must be eliminated from the reduced profile.

### 3.2.1 Extension to profiles with Indifferences

In the analysis made each agent has strict preferences over the alternative set $X$. Very often individuals are indecisive between two alternatives and therefore we would like to extended our result to the case of preference profiles with indifferences. The idea behind this extension is to transform the preference profile with indifferences to a profile from $\mathcal{P}^m$. Suppose that the preference profile $R^n \in \mathcal{R}^n$ is such that there exists $i \in I$ with $R_i \notin \mathcal{P}$, i.e. $\exists x, y \in X$ s.t $xI_iy$. Without loss of generality assume that $xI_iyP_iz$. Then we substitute $R_i$ from the preference profile $R^n$ with the following two linear orders:

| $P_i'$ | $P_i''$ |
|---|---|
| $x$ | $y$ |
| $y$ | $x$ |
| $z$ | $z$ |

Notice that by making this substitution we have extended the preference profile to another profile from $\mathcal{R}^{n+1}$. Using this procedure we replace every individual preferences with indifference by two linear orders from $\mathcal{P}$. Furthermore, in the new extended profile we double every linear order from $\mathcal{R}^n$. For instance, if

| $R_1$ | $R_2$ |
|---|---|
| $x\,I\,y$ | $x$ |
| | $z$ |
| $z$ | $y$ |

then the extended profile would be

| $P_1$ | $P_2$ | $P_4$ | $P_5$ |
|---|---|---|---|
| $x$ | $y$ | $x$ | $x$ |
| $y$ | $x$ | $z$ | $z$ |
| $z$ | $z$ | $y$ | $y$. |

Let by $P^m$ we denote the extended profile $R^n$. Here we present the fundamental property that explains the relationship between the number of individuals in $R^n$ and $P^m$ that have strict preferences on one alternative over another.

**Property 3.1.** *For any $x, y \in X$ we have that*

$$|\{R_i \in \mathcal{R}^n | xR_iy,\ \neg(yR_ix)\}| > |\{R_i \in \mathcal{R}^n | yR_ix,\ \neg(xR_iy)\}| + a^* \tag{3.1}$$

$\Leftrightarrow$

$$|\{P_i \in \mathcal{P}^m | xP_iy\}| > |\{P_i \in \mathcal{P}^m | yP_ix\}| + 2a^*. \tag{3.2}$$

This property is true since the existing strict individual preferences in $R^n$ double in $P^m$. At the same time, for every $j$ such that there are $x$ and $y$ with $xI_jy$ we add $xP_j'y$ and $yP_j'x$. In other words, we increase the number of elements by one in the sets from the both sides of the inequality (4.2).

Notice that inequality (4.3) is equivalent to

$$|\{P_i \in \mathcal{P}^m | xP_iy\}| > \frac{2n+1}{2} + 2a + \frac{1}{2}$$

with $a^* = 2a + 1$. This equivalence allows us to enlarge the preference profile domains where our theorem applies.

**Theorem 3.9.** *Let $R^n$ be a preference profile from $\mathcal{R}^n$ and let $R$ be the social relation induced by the $a-$majority rule. Then $R$ is transitive if and only if the extended profile $P^m$ is not $(2a + 1/2)-$balanced.*

## 3.3 Concluding Remarks

The present paper establishes a necessary and sufficient single-profile condition for obtaining a transitive supermajority rule relation. Initially we assume that the individual preferences that form the profile are linear orders. We start by reducing the population by disregarding the individuals whose preferences over the alternative set are inverse, and then we consider an equivalent supermajority rule that depends on the agents in the reduced population only. Our $a-$balancedness condition applies to the reduced population and it is both necessary and sufficient to guarantee transitive social preferences.

Our profile condition is composed out of two distinct properties. The first states that whenever there is an alternative that is preferred to the other two by a sufficient number of individuals then transitivity is guaranteed. This happens since the relation between those two alternative is irrelevant in this case. The second one discards the possibility of a cycle in the supermajority social preference relation when no alternative is preferred by a qualified majority.

In the rest of the paper we extend our result to the case in which individual preferences are given by weak orders. First, we notice that a supermajority rule relation of a preference profile with indifferences is equivalent to another supermajority rule relation of a transformation of the original profile which is composed of linear orders only. Then we apply the appropriate balancedness condition to the latter and the generalization of the result trivially follows.

# Chapter 4

# Impartial Social Rankings

In this chapter we study methods to rank a finite set of individuals on the basis of the opinions that they themselves hold regarding the merits of each candidate. Each agent reports an ordered list of all others, representing his opinion about how they should be ranked. The collection of all such lists is called a profile, and it is the input for a social ranking rule that determines the final ranking. We do not assume that being classified higher need always be better. We allow rankings that are based on failure rather than success[1].

We assume that every agent favors a final ranking that is as close as possible to his ordered list, while giving him a best possible rank[2].

We call a *social ranking function* impartial if the contribution of any single agent to the social outcome can have no consequence on his own position in the final ranking. This inability of agents to affect their own position guarantees that the social ranking function can aggregate information without being manipulated by any agent.

Indeed, individuals may be tempted to act strategically along many collective decision-making processes, in order to get the social result as close as possible to their interests. An extensive literature on the manipulability of social choice rules and on the possibility to avoid manipulations under certain domain restrictions has developed for over forty years. The idea of impartiality was introduced more recently and is also in a similar spirit, but not quite the same. On the one hand, it only applies to cases when voters are also candidates to be elected or to be ranked. That makes it specific for some types of problems. But, once it applies, it is a stronger requirement than strategy proofness,

[1]A committee made of a company partners could rank themselves on the basis of failed projects

[2]As we explained before, "better rank" could mean first or last position, but not a placement in between.

because it does not only demand that agents should find it adequate to vote truthfully: it actually demands that the vote of an agent has no consequence for that agents election, or for its position in the social ranking[3].

Notice that this requirement also demands implicitly that the objectives of voters are limited. In some works, it is assumed that they only care whether they are elected or not. And in our case, it is also assumed that they only care about their own ranking. Otherwise, impartial rules could still induce strategic agents, if agents who cannot influence their own fate could change that of others they cared for.

Consider, as an example, that a ranking of universities is arrived at by letting all university representatives to submit their own ranking, and then aggregating all of them somehow. As a first approximation, we may assume that representatives would prefer to have their university ranked as high as possible, but actually not care about what others are above or below. Then, using an impartial rule would be useful, because no one would have a reason to hide her true valuation of others. Of course, if representatives would also attach importance to their positions relative to some other specific universities, then impartiality would not be sufficient to rule our strategic behavior. This is a limitation. But given that it is always hard to find positive results regarding these incentive issues, we consider our results to be a step forward.

There is a limited literature discussing various forms of impartiality. Impartiality was first introduced in de Clippel at al (2008)[13] studying how could partners divide a cash surplus, where each agent cares about his share only. Impartiality requires that the share of any participant is determined by the reports of the others. The paper characterizes a family of rules in which an agent's part is derived from aggregate reports on the share that other agents propose for him.

Holzman and Moulin (2013)[24] describe a situation when a group of peers must choose one of them to receive a prize in a way such that one's own message does not influence whether or not he wins the prize. After introducing a set of desirable properties, the authors obtain an impossibility result showing that under impartiality, two desirable properties cannot be satisfied at the same time: that the winner always gets at least one nomination, and that an agent nominated by everyone else always wins.

Holzman and Moulin (2013)[24] then propose a weaker set of requirements and are

[3]For instance, imagine that we have to rank a set of agents where everyone votes. Suppose that the ranking is fixed and that the voting is important only for the position of $x$ and $y$, who are up for the first and the second rank. Using the majority rule to determine the first place is strategy proof, but not impartial, assuming that x and y vote.

able to construct an impartial rule satisfying all of them . The agents are first partitioned in districts where a local winner is elected by plurality vote, provided they get sufficient support. The rule awards the prize to one of the local winners, if there is some. If there is none, then an a priori determined candidate wins. They show that such a nomination rule is impartial, but marred by the fact that one could eventually win without obtaining any vote. Notice that our assumption about agent's preferences is different than the one of Holzman and Moulin (2013)[24], where agents are selfish and only care whether they get the prize or not.This difference in individual preferences is due to the different nature of the final outcomes, which is a single winner in their case and a full ranking of a population in ours.

We begin by discussing a rule that is a variation of the Borda count and that is not impartial. This gives us a first taste of the difficulties we shall find later. Moreover, this rule itself will be instrumental in our next construction, this time of a rule that is indeed impartial. Under that rule, successive groups of individuals, whose rank has been already established in previous steps, decide who will be placed right above them. The main drawback of that procedure, and of several variants that use the same basic idea, is that the rank of at least one agent will be fixed. We further prove that rigidities of a similar type cannot be escaped, by showing that independently of preference profile, and under every impartial ranking function, there is a pair of agents such that one is always higher ranked than the other. This restricts the space of feasible rankings, and thus makes it impossible to get an impartial rule that is fully Paretian.

In view of that, we propose a weaker version of the unanimity property[4], which is a natural adaptation of the positive unanimity[5] given in Holzman and Moulin (2013)[24]. We require the existence of a special agent that we do not consider in the conditional statement of the property. Except for comparisons with that fixed agent, all other unanimous comparisons must be respected by the rule. Under this weaker notion, we prove that the rank of this special agent under any impartial and unanimous social ranking function must either be first or last. We may interpret this special agent as being a population member that need not be ranked but still has the right to vote.

Following this result we introduce two additional properties[6] and we show that there are impartial social rankings satisfying them. Moreover, we construct several examples demonstrating the independence of these conditions. In the rest of the paper we provide

[4]If every agent ranks one individual over another so does the social ranking.

[5]The agents who gets the votes from everybody else wins.

[6]*Monotonicity* - The improvement of an agent's position in some individual preferences should not result in the decline of his social rank, and *No Dummy* - For every agent there exists a profile where his preferences have an impact on someone else's rank.

a characterization result, by using three axioms that we prove to be necessary and sufficient for a social ranking function to be impartial. These axioms require that *(1)* no agent could change the set of individuals whose preferences matter for his rank, *(2)* any pair of mutually pivotal agents form an ordered groups[7] and *(3)* there is no so-called unilateral agents, who could one-sidedly decide whether some other individual is ranked right above or below them.

## 4.1 Notation and Results

Let $X = \{1, 2, \ldots, n\}$ be a finite set of agents. Each individual $i$ has a strict preference over $X \setminus \{i\}$ embodied by the linear order $R_i$. Let $R^n = (R_1, R_2, \ldots, R_n)$ be a profile of individual preferences. By $\mathcal{R}_i$ we denote the set of all $R_i$ whereas $\mathcal{D}$ stands for the set of all preference profiles $R^n$, $\mathcal{D} = \mathcal{R}_1 \times \ldots, \times \mathcal{R}_n$.

Let $P$ be a linear order over the set of agents $X$ and let $\mathcal{P}$ be the space of all such orders. In what follows we define a *social ranking function*:

**Definition 4.1.** The function $f : \mathcal{D} \to \mathcal{P}$ is called a social ranking function.

For any preference profile $f$ gives a population ranking represented by the linear order $P$. Before we look into some properties of $f$, here we introduce some useful notation concerning the preference profiles. By $(R^n_{-i}, R'_i)$ we denote a profile in $\mathcal{D}$ obtained from $R^n$ when individual $i$'s preferences $R_i$ are substituted by $R'_i \in \mathcal{R}_i$. Similarly, given the subset of agents $C \subseteq X$ by $(R^n_{-C}, R'_C)$ we denote a preference profile obtained from $R^n$ where the individual preferences $R_j$ of the agents in $C$ are substituted by $R'_j$.

Given the population ranking $P$, by $U_P(i)$ we denote the set of agents who are higher classified than $i$:

$$U_P(i) = \{j \in X | j \, P \, i\}.$$

We will also refer to $U_P(i)$ as the upper contour set of $i$ respect to $P$. Similarly, the lower contour set of $i$ respect $P$ is $L_P(i) = \{j \in X | i \, P \, j\}$.

Among all social ranking functions $f$ we are interested in the ones where no individual is influential enough to shift agents between his contour sets, which could cause change of his initial rank. We call these social ranking functions impartial.

[7] The agents belonging to one of them are always higher ranked than the members of the other.

**Definition 4.2.** Let $R^n \in \mathcal{D}$ and $f(R^n) = P$. We say that $f$ is impartial if $U_P(i)$ is invariant to changes in $R_i$, i.e.

$$U_P(i) = U_{P'}(i),$$

where $P' = f(R^n_{-i}, R'_i)$ for any $R'_i \in \mathcal{R}_i$.

We now examine the impartiality of several examples of social ranking functions. We start by introducing a social ranking function that is an adaptation of Borda Count to our model. Borda count assigns to each agent a certain number of points that depend on the agent's position in the individual preferences from $R^n$. The final ordering is determined by the points obtained by the agents. Let $B_i(R^n)$ be the number of Borda points obtained by agent $i$ in the profile[8] $R^n$. The following example demonstrates that Borda is not an impartial ranking:

**Example 4.1** (Borda)**.** *Consider the following preference profile $R^4 = (R_1, R_2, R_3, R_4)$ determined by the following preferences:*

| $R_1$ | $R_2$ | $R_3$ | $R_4$ |
|---|---|---|---|
| 2 | 1 | 2 | 3 |
| 3 | 4 | 4 | 2 |
| 4 | 3 | 1 | 1 |

*Then we have that $B_1 = 5$, $B_2 = 8$, $B_3 = 6$ and $B_4 = 5$ and hence the final ranking[9] would be*

| $P$ |
|---|
| 2 |
| 3 |
| 1 |
| 4 |

*Let agent 3 now changes his profile to $R'_3$,*

| $R_1$ | $R_2$ | $R'_3$ | $R_4$ |
|---|---|---|---|
| 2 | 1 | 1 | 3 |
| 3 | 4 | 2 | 2 |
| 4 | 3 | 4 | 1 |

[8]Unless necessary, we write $B_i$ instead of $B_i(R^n)$

[9]Using some tie-breaking rule for the placement of agents 1 and 4

*This modification of agent 3's preferences results*[10] *with the social ranking*

| $P'$ |
|---|
| 1 |
| 2 |
| 3 |
| 4 |

*where agent* 3 *lost one position respect to* $P$*.*

In our following example we construct an impartial social ranking function using the Borda points that an agents obtains from a profile that consists of a subset of individuals. To that end let us denote by $B_i(R_C)$ the Borda points obtained by agent $i$ in the profile $R_C$, where $C \subseteq X$. For notational convenience we write $B_i(R_{-C})$ instead of $B_i(R_{X \setminus C})$.[11]

Starting from a fixed linear order $P$ we shall construct a $P'$ according to the following algorithm:

**Example 4.2** (Borda Adjustment (BA))**.** *Let* $R^n \in \mathcal{D}$ *and let* $P$ *be a fixed linear order from* $\mathcal{P}$*. Without loss of generality assume that* $U_P(i) = \{1, 2, \dots, i-1\}$ *and* $U_P(1) = \emptyset$*, i.e. the rank of agent* $i$ *is* $i$*.*

*We leave* $n$ *at the bottom and in the first step we decide who will be ranked as second worst. We say that the individual* $i$ *is ranked* $(n-1)^{th}$ *if* $B_i(R_n) \leq B_j(R_n)$*,* $\forall j \in X \setminus \{n\}$[12]*. Let us denote this agent by* $i_{n-1}$*.*

*In the second stage we choose the* $(n-2)^{th}$ *ranked agent. Similar as in the first step, we say that the individual* $i_{n-2}$ *is on the* $(n-2)^{th}$ *position if*

$$B_{i_{n-2}}(R_n, R_{i_{n-1}}) \leq B_j(R_n, R_{i_{n-1}}), \qquad \forall j \in X \setminus \{i_{n-1}, n\}.$$

*We continue the same procedure so that at the last step we have that the individual* $i_2$ *is on the second position if* $B_{i_2}(R_{-\{i_1,i_2\}}) \leq B_{i_1}(R_{-\{i_1,i_2\}})$*. Figure 4.1 illustrate the* ***BA*** *social ranking function. We complete the example by calculating the resulting social ranking by BA given a preference profile in* $R^5$ *and an arbitrary* $P$*:*

[10] Now we have that $B_1(R^n_{-\{3\}}, R'_3) = 7$, $B_2(R^n_{-\{3\}}, R'_3) = 7$, $B_3(R^n_{-\{3\}}, R'_3) = 6$ and $B_4(R^n_{-\{3\}}, R'_3) = 4$

[11] For instance, according to the profile introduced above we have that $B_1 = 3 + 1 + 1 = 5$, but $B_1(\{2, 3\}) = 3 + 1 = 4$ but $B_1(-\{2, 3\}) = 1$.

[12] In case there is more that one candidate, we use the preferences of $n$ to decide for one of them

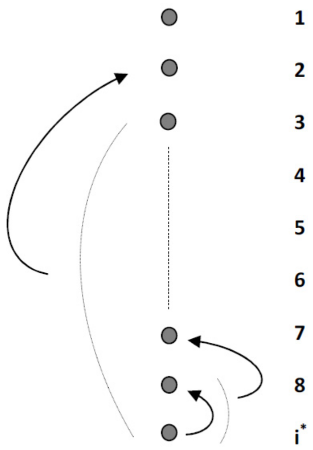



FIGURE 4.1: Diagram of BA

| $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|
| 2 | 3 | 4 | 3 | 1 |
| 3 | 1 | 1 | 2 | 2 |
| 4 | 4 | 2 | 5 | 3 |
| 5 | 5 | 5 | 1 | 4 |

*and*

| $P$ |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

*Along with the first step of BA we get that* $i_4 = 4$ *because* $B_4(R_5) < B_j(R_5)$ *for* $j = 1, 2, 3$. *Furthermore, since* $B_1(R_4, R_5) = 5$, $B_2(R_4, R_5) = 6$ *and* $B_3(R_4, R_5) = 6$ *we have that* $i_3 = 1$. *Finally along with the facts that* $B_3(R_1, R_4, R_5) = 9$ *and* $B_2(R_1, R_4, R_5) = 10$, *the social ranking provided by the BA is*

| $P'$ |
|---|
| 2 |
| 3 |
| 1 |
| 4 |
| 5 |

According to the definition of *BA*, a change of agents' individual preference could only influence the order of the agents from his upper contour set. Moreover, at the moment when agent's individual preferences become influential his rank has been established. This makes *Borda Adjustment* an impartial social ranking.

The existence of an impartial social ranking function leads to a necessity for additional investigation on some of its, one could nominate as, standard properties. The paper of

Holzman and Moulin (2013)[24] introduces notions such as monotonicity, unanimity[13] and no dummy, all of them regarding the impartial nominations for a prize. In the rest of this subsection we define these properties in the spirit of social rankings and provide several examples showing that there are impartial social ranking functions that satisfy them. Moreover, we will use the presented examples to show independence of the mentioned properties. We start by introducing the property of monotonicity requiring that an improvement of an agents position in some individual preference could not result with a decline of the same agent social rank. Here we state the formal definition:

**Property 4.1** (Monotonicity). *Let $R^n \in \mathcal{D}$ and $f(R^n) = P$. Let $R_i'$ be obtained from $R_i$ by improving $j$'s position while maintaining the other pairwise orderings the same. Then if $f(R_{-i}^n, R_i') = P'$ we have that $U_{P'}(j) \subseteq U_P(j)$.*

The next example of impartial social ranking shows the independence of monotonicity from impartiality. Given a preference profile, we choose an agent $n$ and consider the linear order from $\mathcal{P}$ where $n$ is at the bottom and the rest of the population is ordered in $P$ in the same way as it is in the individual preference of agent $n$. Then starting from the second-to-last position, the decision whether agent's position improves for one depends on the Borda points obtained by the agents ranked below him.

**Example 4.3** (Pairwise Borda (PB)). *Let $R^n$ be a preference profile and let $n$ be a fixed agent from $X$. Let $P \in \mathcal{P}$ be such that*

$$U_P(n) = X \setminus \{n\} \quad \text{and} \quad U_P(j) = U_{R_n}(j), \quad \forall j \neq n.$$

*Without loss of generality assume that $U_P(i) = \{1, 2, \ldots, i-1\}$ and $U_P(1) = \emptyset$, i.e. the rank of agent $i$ is $i$.*
*Then agent $n-1$ to take over the position of the agent $n-2$ if $B_{n-1}(R_n) > B_{n-2}(R_n)$. We denote the agent ranked at the $n-1$ position by $i_{n-1}$. Then the agent $i^*_{n-2}$ who is ranked right above $i_{n-1}$, which could be $n-2$ or $n-1$, improves his position for one if $B_{i^*_{n-2}}(R_n, R_{i_{n-1}}) > B_{n-3}(R_n, R_{i_{n-1}})$ etc.*

*Here we calculate the resulting social ranking given a preference profile in $\mathcal{R}^5$:*

| $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|
| 2 | 3 | 5 | 3 | 1 |
| 3 | 4 | 4 | 2 | 2 |
| 4 | 5 | 1 | 5 | 3 |
| 5 | 1 | 2 | 1 | 4 |

*and*

| $P$ |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

[13] In the mentioned paper the authors talk about positive and negative unanimity, which we integrate into a one single property.

*Then we have that* $B_4(R_5) < B_3(R_5)$ *and* $B_3(R_4, R_5) = 6 = B_2(R_4, R_5)$. *Finally since* $B_2(R_3, R_4, R_5) = 7 = B_1(R_3, R_4, R_5)$, $P$ *is the resulting population ranking respect to* ***PB***.

*Consider now a change in the preferences of agent* 4 *where he improves agent's* 1 *position respect to the others:*

| $R_1$ | $R_2$ | $R_3$ | $R'_4$ | $R_5$ |
|---|---|---|---|---|
| 2 | 3 | 5 | 3 | 1 |
| 3 | 4 | 4 | 1 | 2 |
| 4 | 5 | 1 | 2 | 3 |
| 5 | 1 | 2 | 5 | 4 |

*and*

| $P$ |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

*Then again* $B_4(R_5) < B_3(R_5)$ *but* $B_3(R'_4, R_5) = 6 > 5 = B_2(R'_4, R_5)$. *In accordance to* ***PB*** *agent* 2 *gets the third position. Furthermore, since* $B_3(R_2, R'_4, R_5) = 10 > 8 = B_1(R_2, R'_4, R_5)$, ***PB*** *provides the following population ranking:*

| $P$ |
|---|
| 3 |
| 1 |
| 2 |
| 4 |
| 5 |

Note that **PB** is an impartial social ranking function for similar reasons as **BA**. On the other hand, we see that the improvement of agent's 1 position in $R_4$ resulted with an enlargement of his upper contour set. This shows that **PB** violates monotonicity. On the other hand, in the following proposition we prove the monotonicity of **BA**:

**Proposition 4.3.** *Borda Adjustment is monotone.*

*Proof of Proposition 4.3:* Let $R^n \in \mathcal{D}$ and let $P'$ be the resulting ranking respect to **BA**. Assuming that agent $j$ increased the position of another agent $i$ in $R_j$ we could have two situations:

1. $i \in L_{P'}(j)$ that implies $U_{P'}(i) = U_{P''}(i)$, where $P'' = f(R^n_{-\{j\}}, R'_j)$.
2. If $i \in U_{P'}(j)$ then we have that $i$ obtains more Borda points in the new profile whereas a set of another agents in $U_{P'}(i)$ get less. This means that in the worst case $i$ could get his former position.

■

Notice that the rank of the last classified element in the initial $P$ can not be modified under any circumstances according to Borda Adjustment. This feature of **BA** suggests certain properties regarding the ranking of the agents obtained by some impartial social ordering. We state and prove the following proposition:

**Proposition 4.4.** *Let $f$ be an impartial social ranking. Let $R^n \in \mathcal{D}$ and $f(R^n) = P$. For any $i, j \in X$ let $S_{P_{i,j}}$ denotes the set of agents that according to $P$ are ranked between $i$ and $j$ i.e.*

$$S_{P_{i,j}} = L_P(i) \cap U_P(j).^{14}$$

*Then for every*

$$R'_{S_{P_{i,j}}} = (R'_{n_1}, \ldots, R'_{n_s}) \in \mathcal{R}_{n_1} \times \ldots \times \mathcal{R}_{n_s}.$$

*and $P' = f(R^n_{-S_{P_{i,j}}}, R'_{S_{P_{i,j}}})$ we have that*

$$i \in U_P(j) \Leftrightarrow i \in U_{P'}(j).$$

This proposition says that the pairwise ranking between any two agents according some impartial rule is invariant to changes in the individual preferences of the agents ranked between them.

*Proof of Proposition 4.4:* Suppose the opposite. Let $R'_{S_{P_{i,j}}} \in \mathcal{R}_{n_1} \times \ldots \times \mathcal{R}_{n_s}$ be such that $f(R^n_{-S}, R'_S) = P''$ and $i \notin U_{P''}(j)$.
Consider the following population ordering

$$P^{(1)} = f(R^n_{-\{n_s\}}, R'_{n_s}).$$

The impartiality of $f$ implies that the upper contour sets of $n_s$ respect to $P$ and $P^{(1)}$ coincide, implying that $i$ is higher ranked than $j$ in $P^{(1)}$. We get the same conclusion for the following sequence of population orderings

$$P^{(2)} = f(R^n_{-\{n_s,n_{s-1}\}}, R'_{n_s}, R'_{n_{s-1}}),$$

$$\vdots$$

$$P^{(n_s)} = f(R^n_{-\{n_s,\ldots,n_1\}}, R'_{n_s}, \ldots, R'_{n_1}).$$

On the other hand we notice that $P'' = P^{(n_s)}$ and hence we get a contradiction with our assumption.

[14] Notice that at least one of $S_{P_{i,j}}$ and $S_{P_{j,i}}$ is empty ($S_{P_{i,j}} = \emptyset$ or $S_{P_{j,i}} = \emptyset$).

■

Using Proposition 4.4 we obtain the following corollaries:

**Corollary 4.5.** *The pairwise ordering between $i$ and $j$ in $P$ obtained by some impartial ranking function $f$ is invariant to the changes of the individual preferences of the agents in $S_{P_{i,j}} \cup S_{P_{j,i}} \cup \{i, j\}$.*

This corollary says that $i$ will remain higher classified than $j$ not only if the agents between them change their preferences, but the agents $i$ and $j$ themselves.

**Corollary 4.6.** *Let $f$ be an impartial social ranking function and $P = f(R^n)$. Then there exists agents $i$ and $j$ such that $i$ is always better classified than $j$ in $P$ under any preference profile $R^n$.*

*Proof of Corollary 4.6:* Let $P = f(R^n)$ and let $U_P(1) = \emptyset$ and $U_P(n) = X \setminus \{n\}$. Then we apply Corollary 4.5 to agents 1 and $n$.

■

The last corollary implies that there is no impartial social ranking function that is surjective. In other words, no impartial social ranking function has $\mathcal{P}$ as its image. Moreover, in the examples of impartial social ranking functions given above we see that the image of $f$ consists of linear orders from $\mathcal{P}$ that contain an agent with a fixed position under any preference profile. Moreover, this agent in both examples is ranked at the bottom.[15] Considering the previous results, here introduce a variation of the standard unanimity property and than we will show that in any impartial ranking function that satisfies that additional property such an agent always exists.

**Property 4.2** (Unanimity). *Let $i^*$ be a fixed agent from $X$. If for any $i, j \neq i^*$ and any profile $R^n$ such that $iR_kj\ \forall k \in X \setminus \{i, j\}$ we have that $iPj$, where $P = f(R^n)$.*

The following proposition establishes the position of this special agent $i^*$ when unanimity is combined with impartiality:

**Proposition 4.7.** *Let $f$ be impartial and unanimous social ranking function. Then for every $R^n \in \mathcal{D}$ and $P = f(R^n)$ we have that either $U_P(i^*) = \emptyset$ or $U_P(i^*) = X \setminus \{i^*\}$.*

*Proof of Proposition 4.7:* Suppose the contrary. Let there be agents $i$ and $j$ such that $i \in U_{i^*}$ and $j \in L_{i^*}$. Then by the unanimity we have that if every agent places $j$ over $i$ in his individual preferences then $j$ must be socially better ranked than $i$. This is contradiction with proposition 4.4.

[15] We could easy modify **BA** and **PB** such that the individual with fix position is first ranked.

■

The interpretation or better said the benefit from the former results is that the existence of an agent whose rank is not of interest but his vote counts, allows us to construct an impartial ranking function that can generate any linear order over the new, restricted set of agents.

We conclude the set of properties of ranking function with so called No Dummy, which was also introduced by Holzman and Moulin (2013)[24], requiring that the vote of every agent must be important at some profile:

**Property 4.3** (No Dummy)**.** *For every agent $i \in X$ there exists a profile $R^n$ s.t*

$$f(R^n) \neq f(R^n_{-i}, R'_i)$$

*for some $R'_i \in \mathcal{R}_i$.*

In what follows we provide examples of simple impartial social ranking functions showing the independence of these properties. Table 4.1 summarizes these results.

**Example 4.4** (Partition Ordering PO)**.** *Let $X = X_0 \cup X_1$ be a fixed partition of the agent set. The partition is ordered, meaning that the elements of $X_0$ will always be higher classified than the ones in $X_1$. We write $X_0 \, P \, X_1$.*

*Given the profile $R^n$, for $i \in X_{1-j}$ by $B^j_i(R^n)$ we denote the Borda points of agent $i$ obtained by the preferences of the agents in $X_j$ in $R^n$, for $j = 0, 1$. In each of the partition sets we order the individuals using the points defined above.*

Clearly **PO** is an impartial ranking function since the change of an agents preference could only modify the ranking of the agents from the partition set where the agent himself does not belong. Moreover, notice that **PO** is also a monotonic social ordering given that improvement of agent $i$'s position in some individual preferences *(i)* would increase his overall points if the individual does not belong in the same partition set as $i$, or *(ii)* it would have no effect if both belong in the same partition set.

**Example 4.5** (Dictator)**.** *Let $i^* \in X$. Then for every preference profile $f(R^n) = P^*$ where $U_{P^*}(i^*) = \emptyset$ and $L_{R_{i^*}}(j) = L_{P^*}(j)$.*

The agent $i^*$ in this example is clearly a dictator. He is always best ranked and the other are ordered according to his individual preferences.

**Example 4.6** (Constant). *Let $P \in \mathcal{P}$ be a fixed linear order on $X$. Then for every profile $R^n$, $f(R^n) = P$.*

| | **Borda** | **BA** | **PB** | **PO** | **Dictator** | **Const** |
|---|---|---|---|---|---|---|
| Impartial | | ✓ | ✓ | ✓ | ✓ | ✓ |
| Monotone | ✓ | ✓ | | ✓ | ✓ | ✓ |
| No Dummy | ✓ | ✓ | ✓ | ✓ | | |
| Unanimity | ✓ | ✓ | ✓ | | ✓ | |

**Table 1:** Social Ranking Functions with the corresponding properties

### 4.1.1 Characterization

Notice that **BA** is an example of social ranking function that satisfies the four independent properties. The existence of such an example led us to the question if we could describe all the impartial social ranking functions. For that purpose we will first define a so-called set of pivotal agents that contains all individuals who could modify an agent's position by changing their individual preferences. Then we will use three axioms involving these sets in order to characterize the impartial social ranking functions satisfying them.

**Definition 4.8.** Let $f$ be a social ranking function and $f(R^n) = P$ and let $i, j$ and $h$ be three distinct agents from $X$. We say that agent $j$ is $i \leftrightarrow h$ pivotal at $R^n$ if there exists a $R'_j \in \mathcal{R}_j$ s.t. one of the following two cases occurs:

1. $i \in U_P(h)$ but $h \in U_{P'}(i)$
2. $h \in U_P(i)$ but $i \in U_{P'}(h)$

where $P' = f(R^n_{-j}, R'_j)$.

In other words, agent $j$ is pivotal for $i$ and $h$ at a given profile if a certain modification of his individual preferences can change the position of $i$ with respect to $h$. Furthermore, we say that the agent $j$ is influential for $i$ at $R^n$ if there exists $R'_j$ such that $U_P(i) \neq U_{P'}(i)$ where $P = f(R^n)$ and $P' = f(R^n_{-j}, R'_j)$. Whenever we need to specify the change of the individual preference of agent $j$ that causes the change of the rank of $i$ we will write $R'_j$-influential.

We start the characterization of the impartial social ranking functions by introducing axiom **A**, which requires that no individual could modify the set of agents that are pivotal for him:

**Axiom 4.9** (A). *If $j$ is $i \leftrightarrow h$ pivotal at $R^n$, it is also $i \leftrightarrow h$ pivotal at $(R^n_{-i}, R'_i)$ for every $R'_i \in \mathcal{R}_i$.*

The following proposition establishes the one way relationship between axiom **A** and an impartial social ranking function.

**Proposition 4.10.** *If $f$ is an impartial social ranking function then it satisfies **A**.*

*Proof:* Assume that there exists a $k \in X$ such that $j$ is $k \leftrightarrow i$ pivotal at $R^n$ but $j$ is not $k \leftrightarrow i$ pivotal at $(R^n_{-i}, R'_i)$.

The assumption that $j$ is pivotal for $i$ implies that

$$k \in U_P(i) \qquad \text{and} \qquad i \in U_{P'}(k), \tag{4.1}$$

where $P' = f(R^n_{-\{j\}}, R'_j)$ for some $R'_j$. On the other hand we have that

$$k \in U_{P''}(i), \quad \text{where} \quad P'' = f(R^n_{-\{i,j\}}, R'_i, R''_j)$$

for every $R''_j \in \mathcal{R}_j$, and in particular for $R''_j = R'_j$.

Then since $k \in U_{P''}(i)$, impartiality of $f$ implies that $k \in U_{P'}(i)$, which contradicts (4.1).

■

The following example shows that the invariance of the set of pivotal agents for $i$ to changes in his individual preferences is not sufficient for impartiality.

**Example 4.7.** *Consider the preference profile $R^3$ together with the corresponding social ranking according to Borda:*

| $R_1$ | $R_2$ | $R_3$ |
|---|---|---|
| 2 | 1 | 1 |
| 3 | 3 | 2 |

| $P$ |
|---|
| 1 |
| 2 |
| 3 |

*Notice that at $R^3$, 1 is $2 \leftrightarrow 3$ and 3 is $2 \leftrightarrow 1$ pivotal, since the position of 2 would descent or ascent following a change in $R_1$ or $R_3$ respectively*[16].

*Let agent 2 modifies his preferences to $R'_2$. Since the Borda points of every agent under the new profile $(R_1, R'_2, R_3)$ is the same*[17]*, the resulting social ranking is determined by a former established tie-breaking rules.*

| $R_1$ | $R'_2$ | $R_3$ |
|---|---|---|
| 2 | 3 | 1 |
| 3 | 1 | 2 |

| $P$ |
|---|
| 3 |
| 2 |
| 1 |

*However, we see that in the new profile 1 is $2 \leftrightarrow 3$ and 3 is $2 \leftrightarrow 1$ pivotal since a modification in one of $R_1$ or $R_3$ would alter the position of 2.*

The second axiom demands invariance of the relative position between $j$ and the agents he is influential for, to the changes of $j$'s individual preferences.

**Axiom 4.11** (B)**.** *Let $j$ is $i \leftrightarrow h$ pivotal at $R^n$. Then for every $R'_j \in \mathcal{R}_j$ and $P' = f(R^n_{-\{j\}}, R'_j)$ we have that*

$$hP'j \Leftrightarrow hPj.$$

**Proposition 4.12.** *If $f$ is an impartial social ranking function then it satisfies **B**.*

*Proof:* The result of this proposition is straightforward. Impartiality of $f$ excludes the possibility that the agent $k$ swaps from to $U_P(j)$ to $L_{P'}(j)$, where $P'$ is the social ranking as defined in axiom **B**.

■

Axiom **B** appears to be quite demanding and it seems sufficient for impartiality of a ranking function. However, the following example demonstrates the contrary:

**Example 4.8.** *Consider the social ranking function as described in figure 4.2, where the position of agent $i^*$ is fixed at the bottom and he determines who will be ranked right above him, which in this case it is agent 8. Furthermore, agent 8 determines the agents that will take $6^{th}$ and $7^{th}$ position, without specifying the pairwise ordering between them. To complete the ranking agents 6 and 7 decide who from the remaining agents will take the fifth position*[18]*, agent 5 determines the fourth etc. In the final step, agents 2, 3 and*

[16] In the case of three agents $|\mathcal{R}_i| = 2$ and therefore there is a unique possibility for a change in $R_i$
[17] $B_i(R_1, R'_2, R_3) = 3$ for every agent $i$.
[18] Except for $i^*$, we nominate the agents by the number that determines their rank.

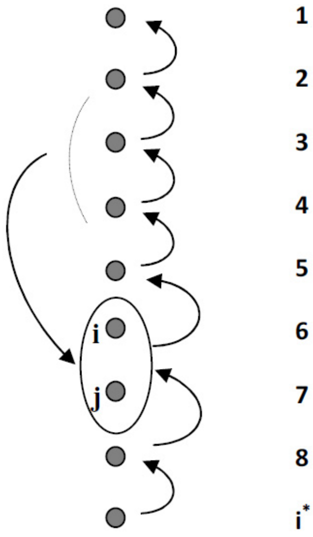



FIGURE 4.2: SRF satisfying Axiom B, but not impartial

4 *decide the pairwise ordering between* 6 *and* 7*.*

*Notice that for any preference profiles* $R^n$ *at which* 7 *is* $h \leftrightarrow k$ *pivotal, we have that* $h\,P'\,7$*, where* $P' = f(R^n_{-7}, R'_7)$ *for every* $R'_7$ *and* $h, k \in \{1, 2, 3, 4, 5\}$*. We obtain similar conclusion if we substitute agent* 7 *by* 6*. These arguments show that this example satisfies axiom* ***B*** *but it is not impartial since a change in the individual preference of agent* 6 *or* 7 *could change the set of his influential agents and hence, violate Axiom* ***A****.*

Finally using the third and the last axiom we discard some particular cases of impartial social ranking functions where an agent has the power to switch his own, with the position of another individual, without affecting though the individual's influential agents. We call the agent with such an authority unilateral. In the following definition we formalize this notion:

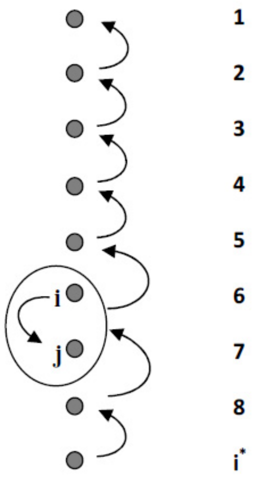
**Definition 4.13.** Let $j$ be $R'_j$-influential for $i$ at $R^n$. We say that $j$ is unilateral for $i$ at $R^n$ with $R'_j$ if

1. there is no agent $h$ s.t. $j$ is $i \leftrightarrow h$ pivotal with $R'_j$ and
2. $h$ is influential for $i$ at $R^n \Leftrightarrow h$ is influential for $i$ at $(R^n_{-\{j\}}, R'_j)$

We remark that according to the definition of unilateral agents, $j$ could be unilateral for $i$ at $R^n$ if at $P = f(R^n)$ they are ranked right next to each other. In any other case there would be an agent $h$ satisfying 1.. Notice that in Example 4.8 agent 7 could switch his own, with the rank of agent 6 in a such a way that there is no another agent $h$ s.t. 7 is $6 \leftrightarrow h$ pivotal. This however, is not a case of unilateral agent since this swap is due to the impact of 7 to the set of influential agents of 6.

In the following social ranking function we can see an example of unilateral agent:

**Example 4.9.** *Agent $i^*$, who is last ranked, determines agent 8 that will be placed right above him. Agent 8 decides which two individuals will be the ranked above him, without specifying the pairwise position.*




*As drawn in the diagram, we assume that these agent are $i$ and $j$. Furthermore, they both decide the fifth position etc. To complete the social ranking $i$ decides if $j$ will take the sixth or seventh position.*

*As we can see, $i$ is unilateral for $j$ since he could assign the fifth and sixth position to both of them, without affecting the set of influential agents for $j$, which in this case it consists of $i^*$ and 8.*

**Axiom 4.14** (C). *For any pair of individuals there is no unilateral agent at any preference profile.*

Before presenting our main result, with the help of the following table we remark that examples 4.7, 4.8 and 4.9 show the independence of the three introduced axioms:

| | Axiom **A** | Axiom **B** | Axiom **C** |
|---|---|---|---|
| Example 4.7 | ✓ | | ✓ |
| Example 4.8 | | ✓ | ✓ |
| Example 4.9 | ✓ | ✓ | |

**Theorem 4.15.** *The social ranking function $f$ is impartial at $R^n$ if and only if it satisfies* ***A***, ***B*** *and* ***C***.

*Proof of Theorem 4.15:* $\Rightarrow$ Let $f$ be an impartial social ranking function at $R^n$. Propositions 4.10 and 4.12 verify the validness of axioms **A** and **B**. Moreover, impartiality trivially fulfills axiom **C** as well.

$\Leftarrow$ Let $R^n \in \mathcal{D}$ and let $f$ be a social ranking function that satisfies **A**, **B** and **C** such that $P = f(R^n)$. Assume that $f$ is not impartial, i.e. there is an agent $j \in X$ who could change the set of individuals ranked above him at $R^n$, i.e.

$$U_P(j) \neq U_{P'}(j) \quad \text{where} \quad P' = f(R^n_{-\{j\}}, R'_j) \tag{4.2}$$

for some $R'_j \in \mathcal{R}_j$. This means that there exists another agent $i$ s.t.

$$i \in U_P(j) \quad \text{but} \quad i \in L_{P'}(j).$$

This implies that $j$ is $R'_j$-influential for $i$ at $R^n$. If there exists an agent $k \neq i, j$ s.t. $j$ is $i \leftrightarrow k$ pivotal with $R'_j$ then we have that

$$k \in U_P(i) \quad \text{and} \quad i \in U_{P'}(k), \tag{4.3}$$

which also implies that $j$ is $k \leftrightarrow i$ pivotal with $R'_j$ . This conclusion, together with (4.2) contradicts Axiom **B**. Therefore, it must be that such an agent $k$ does not exists.

In this case since $f$ is non-unilateral we have that there must be an agent $k \neq i, j$ s.t. $k$ is not influential for $i$ at $R^n$ but $k$ is influential for $i$ at $(R^n_{-\{j\}}, R'_j)$. Moreover, we have that $k$ is $i \leftrightarrow j$ influential, or equivalently $k$ is $j \leftrightarrow i$ influential. Then we have

$$k \text{ is } j \leftrightarrow i \text{ influential at } (R^n_{-\{j\}}, R'_j) \quad \overset{Axiom\,\mathbf{A}}{\Rightarrow} \; k \text{ is } j \leftrightarrow i \text{ influential at } R^n$$

$$\Leftrightarrow k \text{ is influential for } i \text{ at } R^n.$$

The last equivalence contradicts the assumption that $f$ is non-unilateral and hence we conclude the proof of the theorem.

■

Having characterized the impartiality of $f$, here we present a corollary that helps us to comprehend how actually an impartial rule operates:

**Corollary 4.16.** *Let $f$ be an impartial social ranking function. If for some $i, j \in X$ there exist $k_1, k_2$ such that $j$ is $i \leftrightarrow k_1$ pivotal and $i$ is $j \leftrightarrow k_2$ pivotal then for every*

*$R''_i \in \mathcal{R}_i$ and $R''_j \in \mathcal{R}_j$ we have that*

$$k_1 P k_2 \Leftrightarrow k_1 P' k_2, \quad \textit{where} \quad P' = f(R^n_{-\{i,j\}}, R''_i, R''_j).$$

Corollary 4.16 tells us that what could be the maximal position alteration the agent $i$ provoked by a modification of individual $j$ preferences. If $i$ not influential for $j$ then at most, he could be ranked right next to $j$. On the other hand, if $i$ is also influential for $j$ than the previous ranking is not possible. The minimal distance between the ranking of these two agents depends on the number of individuals that they are mutually pivotal between each other.

*Proof of Corollary 4.16* Let us assume the following positions of the concerned agents with respect to $P$:

| $P$ |
|---|
| . |
| $i$ |
| . |
| $k_1$ |
| $\vdots$ |
| $k_2$ |
| . |
| $j$ |
| . |

which for certain $R'_i$ and $R'_j$ could change to

| $P' = f(R^n_{-\{i\}}, R'_i)$ | | $P'' = f(R^n_{-\{j\}}, R'_j)$ |
|---|---|---|
| . | | . |
| $i$ | | $k_1$ |
| . | | . |
| $k_1$ | or | $i$ |
| $\vdots$ | | $\vdots$ |
| $j$ | | $k_2$ |
| . | | . |
| $k_2$ | | $j$ |
| . | | . |

We claim that at $(R^n_{-\{i,j\}}, R'_j, R'_j)$ we must have the following situation:

$$\begin{array}{c} f(R^n_{-\{i,j\}}, R'_j, R'_j) \\ \hline . \\ k_1 \\ . \\ i \\ \vdots \\ j \\ . \\ k_2 \\ . \end{array}$$

Otherwise we would have the following social ranking starting $P'$:

$$\begin{array}{c} P' = f(R^n_{-\{i\}}, R'_i) \\ \hline . \\ i \\ . \\ k_1 \\ \vdots \\ j \\ . \\ k_2 \\ . \end{array} \xrightarrow{\text{Assumption}} \begin{array}{c} f(R^n_{-\{i,j\}}, R'_i, R'_j) \\ \hline . \\ i \\ . \\ . \\ k_1 \\ \vdots \\ j \\ . \\ k_2 \\ . \end{array} \xrightarrow{\text{Impartiality}} \begin{array}{c} f(R^n_{-\{j\}}, R'_j) \\ \hline . \\ i \\ \vdots \\ k_1 \\ \vdots \\ j \\ . \\ k_2 \\ . \end{array}$$

which is contradictory with the result in $P''$.

However, according to Corollary 4.5 we have that pairwise ordering between $k_1$ and $k_2$ is invariant to changes of the individual preferences of the agent ranked between them and hence, to changes in $R_i$ and $R_j$.

In the cases when the initial social ranking $P$ is one of the followings:

$$\begin{array}{c} P \\ \hline . \\ k_1 \\ . \\ i \\ \vdots \\ k_2 \\ . \\ j \\ . \end{array} \quad \text{or} \quad \begin{array}{c} P \\ \hline . \\ i \\ . \\ k_1 \\ \vdots \\ j \\ . \\ k_2 \\ . \end{array}$$

we can get to a situation as in $(R^n_{-\{i,j\}}, R'_j, R'_j)$ by modifying $R_i$ and $R_j$ respectively.

■

## 4.2 Conclusion

In the present paper we introduce and investigate the consequences of a notion called impartiality of a social ranking function, which is a ranking rule having the property that by changing his initial preference an agent cannot influence his own rank. Our first result says that any impartial ranking rule is not fully Paretian. We also show that if we relax the Pareto condition to a weaker requisite of restricted unanimity, there must exists an agent who is either first or last ranked for any preference profile.

We provide some further examples showing that the introduction of additional properties like monotonicity and a No Dummy condition has no harmful effect on the existence of an impartial social ranking satisfying them all. Finally we introduce three axioms regarding the sets of pivotal and influential agents and show that these axioms are necessary and sufficient conditions for the impartiality of a social ranking function.

The main message from these axioms is that any impartial ranking function has one of the following two structures. One is such that there are consecutive groups of individuals with an already established rank, that decide who will be placed right above (below) them. The alternative structure is one where there is at least one pair of ordered subsets of agents such that the individuals from one of them are always higher ranked than those from the others, for any change in the preferences of any individual belonging in one of those two sets.

# Appendix A

# Tables and Figures

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<tr><td>1990</td><td rowspan="6">HDI traditional</td><td rowspan="9">Life Expectancy at Birth</td><td>Adult Literacy Rate</td><td>Log of Real GDP</td><td rowspan="3">Observed</td><td rowspan="6">Arithmetic Mean</td></tr>
<tr><td>1991</td><td rowspan="3">Adult Literacy Rate and Mean Year of Schooling (weighted average)</td><td rowspan="4">Real GDP with Atkinson formula</td></tr>
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<tr><td>2010</td><td>HDI2010</td><td>Expected and Mean Years of Schooling (geometric mean)</td><td>Log of Real GNI</td><td>Lower fixed, Upper observed</td><td rowspan="2">Geometric Mean</td></tr>
<tr><td>2012</td><td>newHDI</td><td>Expected Years of Schooling</td><td>Inequality Adjusted Real GNI</td><td>Lower=0, Upper Observed</td></tr>
<tr><td>2012</td><td>ourHDI</td><td>Expected Years of Schooling</td><td>Real GNI</td><td>Lower=0, Upper Fixed</td><td>Aggregation formula (4)</td></tr>
</table>

FIGURE A.1: HD variables through time

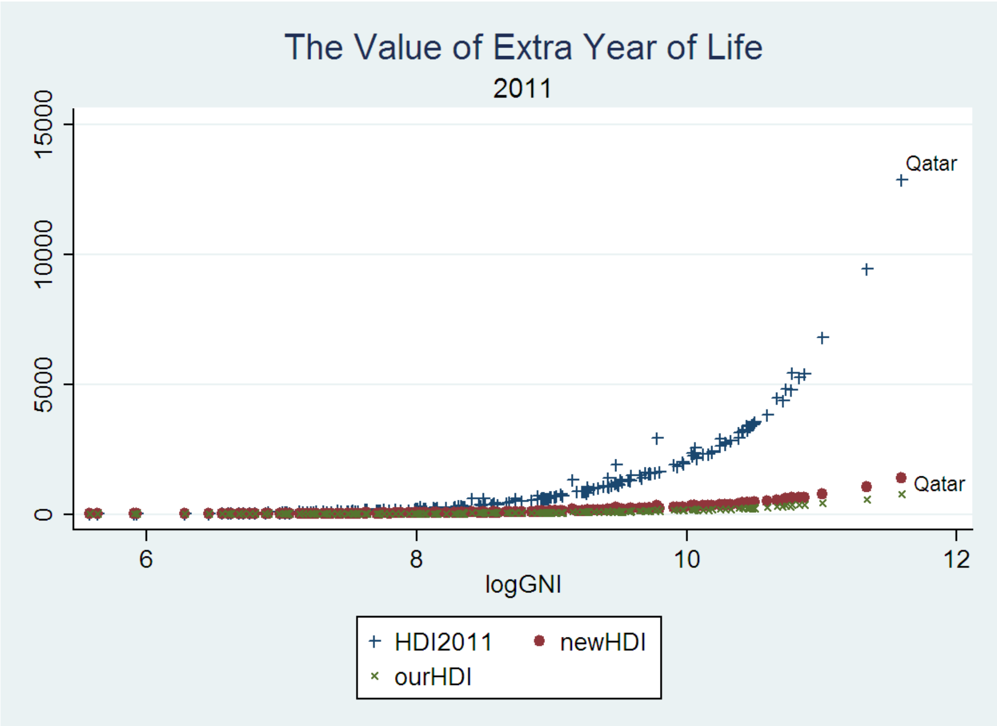


FIGURE A.2: Value of extra year of life, 2011

TABLE A.1: HDI Values and Rankings

| Country | HDI | | | Rank | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Norway | 0.942 | 0.787 | 0.654 | 1 | 2 | 5 |
| Australia | 0.928 | 0.700 | 0.563 | 2 | 5 | 13 |
| Netherlands | 0.909 | 0.692 | 0.567 | 3 | 7 | 12 |
| United States | 0.909 | 0.661 | 0.605 | 4 | 19 | 8 |
| New Zealand | 0.908 | 0.598 | 0.466 | 5 | 28 | 32 |
| Canada | 0.908 | 0.670 | 0.551 | 6 | 15 | 15 |
| Ireland | 0.908 | 0.664 | 0.518 | 7 | 18 | 24 |
| Liechtenstein | 0.905 | | 0.829 | 8 | | 2 |
| Germany | 0.905 | 0.681 | 0.547 | 9 | 10 | 18 |
| Sweden | 0.903 | 0.700 | 0.555 | 10 | 6 | 14 |
| Switzerland | 0.903 | 0.691 | 0.586 | 11 | 8 | 11 |
| Japan | 0.900 | 0.627 | 0.525 | 12 | 24 | 22 |
| Hong Kong | 0.897 | 0.642 | 0.623 | 14 | 21 | 7 |
| Iceland | 0.897 | 0.671 | 0.520 | 13 | 14 | 23 |
| Korea | 0.896 | 0.635 | 0.500 | 15 | 23 | 27 |
| Denmark | 0.895 | 0.681 | 0.548 | 16 | 11 | 17 |
| Israel | 0.887 | 0.578 | 0.470 | 17 | 31 | 30 |

Table A.1: HDI Values and Rankings

| Country | HDI | | | Rank | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Belgium | 0.886 | 0.670 | 0.536 | 18 | 16 | 19 |
| Austria | 0.885 | 0.683 | 0.549 | 19 | 9 | 16 |
| France | 0.884 | 0.640 | 0.515 | 20 | 22 | 26 |
| Slovenia | 0.883 | 0.615 | 0.468 | 21 | 27 | 31 |
| Finland | 0.882 | 0.677 | 0.534 | 22 | 13 | 21 |
| Spain | 0.878 | 0.619 | 0.484 | 23 | 25 | 28 |
| Italy | 0.874 | 0.616 | 0.482 | 24 | 26 | 29 |
| Luxembourg | 0.867 | 0.729 | 0.630 | 25 | 4 | 6 |
| Singapore | 0.865 | 0.678 | 0.657 | 26 | 12 | 4 |
| Czech Rep. | 0.864 | 0.572 | 0.423 | 27 | 32 | 36 |
| UK | 0.863 | 0.651 | 0.536 | 28 | 20 | 20 |
| Greece | 0.860 | 0.589 | 0.455 | 29 | 29 | 34 |
| UAE | 0.846 | 0.743 | 0.679 | 30 | 3 | 3 |
| Cyprus | 0.840 | 0.585 | 0.452 | 31 | 30 | 35 |
| Andorra | 0.839 | | 0.517 | 32 | | 25 |
| Brunei D. | 0.838 | | 0.604 | 33 | | 9 |
| Estonia | 0.834 | 0.509 | 0.372 | 35 | 38 | 46 |
| Slovakia | 0.834 | 0.545 | 0.401 | 34 | 35 | 43 |
| Malta | 0.831 | 0.561 | 0.418 | 36 | 33 | 39 |
| Qatar | 0.831 | 0.834 | 0.894 | 37 | 1 | 1 |
| Hungary | 0.817 | 0.517 | 0.366 | 38 | 37 | 47 |
| Poland | 0.814 | 0.507 | 0.378 | 39 | 40 | 44 |
| Lithuania | 0.809 | 0.486 | 0.364 | 40 | 42 | 48 |
| Portugal | 0.808 | 0.538 | 0.419 | 41 | 36 | 38 |
| Bahrain | 0.807 | 0.561 | 0.464 | 42 | 34 | 33 |
| Latvia | 0.805 | 0.461 | 0.337 | 43 | 44 | 53 |
| Chile | 0.804 | 0.416 | 0.330 | 44 | 54 | 55 |
| Argentina | 0.798 | 0.451 | 0.347 | 45 | 47 | 51 |
| Croatia | 0.795 | 0.476 | 0.351 | 46 | 43 | 50 |
| Barbados | 0.791 | | 0.372 | 47 | | 45 |
| Uruguay | 0.783 | 0.446 | 0.331 | 48 | 48 | 54 |
| Palau | 0.781 | | 0.276 | 49 | | 77 |
| Romania | 0.781 | 0.434 | 0.297 | 50 | 49 | 64 |
| Cuba | 0.776 | 0.371 | 0.220 | 51 | 65 | 99 |
| Seychelles | 0.773 | 0.379 | 0.355 | 52 | 64 | 49 |

Table A.1: HDI Values and Rankings

| **Country** | **HDI** | | | **Rank** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Bahamas | 0.772 | | 0.409 | 53 | | 40 |
| Montenegro | 0.772 | 0.416 | 0.282 | 54 | 55 | 73 |
| Bulgaria | 0.771 | 0.394 | 0.295 | 55 | 61 | 67 |
| Saudi Arabia | 0.770 | | 0.422 | 56 | | 37 |
| Mexico | 0.769 | 0.405 | 0.323 | 57 | 58 | 58 |
| Panama | 0.768 | 0.389 | 0.307 | 58 | 62 | 63 |
| Serbia | 0.765 | 0.418 | 0.280 | 59 | 53 | 74 |
| Ant.&Barbuda | 0.764 | | 0.345 | 60 | | 52 |
| Malaysia | 0.760 | 0.406 | 0.317 | 63 | 57 | 60 |
| Tr.&Tobago | 0.759 | 0.490 | 0.407 | 64 | 41 | 41 |
| Kuwait | 0.760 | 0.669 | 0.591 | 62 | 17 | 10 |
| Libya | 0.760 | 0.461 | 0.327 | 61 | 45 | 57 |
| Belarus | 0.756 | 0.460 | 0.321 | 65 | 46 | 59 |
| Russian F. | 0.755 | 0.430 | 0.330 | 66 | 50 | 56 |
| Grenada | 0.749 | | 0.242 | 67 | | 89 |
| Kazakhstan | 0.745 | 0.424 | 0.284 | 68 | 52 | 71 |
| Costa Rica | 0.742 | 0.361 | 0.278 | 69 | 70 | 76 |
| Albania | 0.739 | 0.351 | 0.236 | 70 | 76 | 94 |
| Lebanon | 0.738 | 0.412 | 0.315 | 71 | 56 | 61 |
| St.Kitts&Nevis | 0.735 | | 0.296 | 72 | | 65 |
| Venezuela | 0.735 | 0.401 | 0.288 | 73 | 59 | 70 |
| Bosnia&Herz | 0.734 | 0.366 | 0.243 | 74 | 67 | 88 |
| Georgia | 0.733 | 0.298 | 0.189 | 75 | 93 | 109 |
| Ukraine | 0.729 | 0.353 | 0.217 | 76 | 75 | 100 |
| Mauritius | 0.728 | 0.424 | 0.313 | 77 | 51 | 62 |
| Macedonia | 0.728 | 0.362 | 0.258 | 78 | 69 | 79 |
| Jamaica | 0.727 | 0.326 | 0.222 | 79 | 81 | 98 |
| Peru | 0.725 | 0.343 | 0.250 | 80 | 78 | 81 |
| Dominica | 0.725 | | 0.246 | 81 | | 83 |
| Saint Lucia | 0.723 | 0.355 | 0.249 | 82 | 74 | 82 |
| Ecuador | 0.720 | 0.346 | 0.244 | 83 | 77 | 87 |
| Brazil | 0.718 | 0.359 | 0.279 | 84 | 72 | 75 |
| St.Vincent&G. | 0.716 | | 0.244 | 85 | | 86 |
| Armenia | 0.716 | 0.314 | 0.193 | 86 | 85 | 106 |
| Colombia | 0.709 | 0.323 | 0.251 | 87 | 83 | 80 |

Table A.1: HDI Values and Rankings

| **Country** | **HDI** | | | **Rank** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Iran | 0.708 | 0.370 | 0.273 | 88 | 66 | 78 |
| Oman | 0.704 | 0.507 | 0.402 | 90 | 39 | 42 |
| Tonga | 0.704 | | 0.178 | 89 | | 112 |
| Azerbaijan | 0.700 | 0.360 | 0.246 | 91 | 71 | 84 |
| Turkey | 0.699 | 0.397 | 0.295 | 92 | 60 | 66 |
| Belize | 0.699 | 0.291 | 0.207 | 93 | 94 | 101 |
| Tunisia | 0.699 | 0.358 | 0.240 | 94 | 73 | 92 |
| Jordan | 0.697 | 0.310 | 0.199 | 95 | 89 | 104 |
| Algeria | 0.697 | 0.363 | 0.241 | 96 | 68 | 90 |
| Sri Lanka | 0.690 | 0.301 | 0.192 | 97 | 90 | 107 |
| Dominican Rep. | 0.689 | 0.328 | 0.240 | 98 | 80 | 91 |
| Samoa | 0.688 | | 0.168 | 99 | | 116 |
| Fiji | 0.688 | | 0.173 | 100 | | 114 |
| China | 0.687 | 0.331 | 0.230 | 101 | 79 | 96 |
| Turkmenistan | 0.686 | 0.325 | 0.224 | 102 | 82 | 97 |
| Thailand | 0.683 | 0.316 | 0.237 | 103 | 84 | 93 |
| Suriname | 0.679 | 0.313 | 0.233 | 104 | 87 | 95 |
| El Salvador | 0.673 | 0.299 | 0.205 | 106 | 92 | 102 |
| Gabon | 0.674 | 0.386 | 0.290 | 105 | 63 | 69 |
| Paraguay | 0.665 | 0.266 | 0.184 | 107 | 101 | 111 |
| Bolivia | 0.663 | 0.247 | 0.171 | 108 | 104 | 115 |
| Maldives | 0.660 | 0.313 | 0.198 | 109 | 86 | 105 |
| Mongolia | 0.653 | 0.272 | 0.159 | 110 | 99 | 121 |
| Moldova | 0.648 | 0.248 | 0.145 | 111 | 103 | 130 |
| Philippines | 0.644 | 0.247 | 0.155 | 112 | 105 | 126 |
| Egypt | 0.643 | 0.300 | 0.190 | 113 | 91 | 108 |
| Palestinian T | 0.640 | | 0.139 | 115 | | 134 |
| Uzbekistan | 0.641 | 0.242 | 0.141 | 114 | 109 | 133 |
| Micronesia | 0.636 | | 0.143 | 116 | | 131 |
| Guyana | 0.633 | 0.245 | 0.149 | 117 | 106 | 127 |
| Botswana | 0.633 | 0.312 | 0.282 | 118 | 88 | 72 |
| Syria | 0.632 | 0.283 | 0.173 | 119 | 96 | 113 |
| Namibia | 0.625 | 0.234 | 0.200 | 120 | 114 | 103 |
| Honduras | 0.624 | 0.228 | 0.155 | 121 | 115 | 125 |
| Kiribati | 0.623 | | 0.147 | 122 | | 129 |

Table A.1: HDI Values and Rankings

| **Country** | **HDI** | | | **Rank** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| South Africa | 0.619 | 0.281 | 0.245 | 123 | 97 | 85 |
| Indonesia | 0.617 | 0.276 | 0.164 | 125 | 98 | 119 |
| Vanuatu | 0.617 | | 0.161 | 124 | | 120 |
| Kyrgyzstan | 0.615 | 0.223 | 0.119 | 126 | 117 | 139 |
| Tajikistan | 0.607 | 0.214 | 0.114 | 127 | 119 | 142 |
| Viet Nam | 0.594 | 0.237 | 0.138 | 128 | 113 | 136 |
| Nicaragua | 0.590 | 0.221 | 0.129 | 129 | 118 | 138 |
| Morocco | 0.582 | 0.262 | 0.167 | 130 | 102 | 117 |
| Guatemala | 0.574 | 0.239 | 0.166 | 131 | 112 | 118 |
| Iraq | 0.574 | 0.243 | 0.142 | 132 | 107 | 132 |
| Cape Verde | 0.568 | 0.241 | 0.155 | 133 | 110 | 124 |
| India | 0.546 | 0.243 | 0.147 | 134 | 108 | 128 |
| Ghana | 0.541 | 0.185 | 0.100 | 135 | 125 | 151 |
| Equ. Guinea | 0.538 | 0.287 | 0.291 | 136 | 95 | 68 |
| Congo | 0.533 | 0.212 | 0.134 | 137 | 120 | 137 |
| Laos | 0.525 | 0.205 | 0.117 | 138 | 122 | 140 |
| Cambodia | 0.522 | 0.183 | 0.105 | 139 | 126 | 146 |
| Swaziland | 0.522 | 0.224 | 0.156 | 140 | 116 | 123 |
| Bhutan | 0.521 | 0.272 | 0.186 | 141 | 100 | 110 |
| Solomon Islands | 0.510 | | 0.104 | 142 | | 148 |
| Kenya | 0.510 | 0.174 | 0.095 | 143 | 131 | 154 |
| S.Tome&Principe | 0.507 | 0.181 | 0.107 | 144 | 127 | 144 |
| Pakistan | 0.505 | 0.198 | 0.116 | 145 | 123 | 141 |
| Bangladesh | 0.500 | 0.177 | 0.094 | 146 | 129 | 155 |
| Timor-Leste | 0.496 | 0.241 | 0.138 | 147 | 111 | 135 |
| Angola | 0.486 | 0.208 | 0.159 | 148 | 121 | 122 |
| Myanmar | 0.483 | 0.175 | 0.096 | 149 | 130 | 153 |
| Cameroon | 0.481 | 0.180 | 0.106 | 150 | 128 | 145 |
| Madagascar | 0.480 | 0.144 | 0.073 | 151 | 145 | 171 |
| Tanzania | 0.466 | 0.162 | 0.086 | 152 | 137 | 160 |
| P. N. Guinea | 0.465 | 0.158 | 0.104 | 153 | 139 | 147 |
| Yemen | 0.461 | 0.196 | 0.113 | 154 | 124 | 143 |
| Senegal | 0.460 | 0.163 | 0.094 | 155 | 135 | 156 |
| Nigeria | 0.459 | 0.174 | 0.103 | 156 | 133 | 149 |
| Nepal | 0.457 | 0.153 | 0.083 | 157 | 140 | 163 |

Table A.1: HDI Values and Rankings

| Country | HDI | | | Rank | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Haiti | 0.454 | 0.128 | 0.077 | 158 | 159 | 168 |
| Mauritania | 0.453 | 0.174 | 0.099 | 159 | 132 | 152 |
| Lesotho | 0.450 | 0.142 | 0.093 | 160 | 146 | 157 |
| Uganda | 0.446 | 0.153 | 0.081 | 161 | 142 | 166 |
| Togo | 0.436 | 0.141 | 0.067 | 162 | 147 | 177 |
| Comoros | 0.432 | 0.135 | 0.081 | 163 | 151 | 165 |
| Zambia | 0.429 | 0.132 | 0.077 | 165 | 155 | 169 |
| Djibouti | 0.429 | 0.159 | 0.101 | 164 | 138 | 150 |
| Rwanda | 0.428 | 0.153 | 0.082 | 166 | 141 | 164 |
| Benin | 0.428 | 0.163 | 0.086 | 167 | 134 | 159 |
| Gambia | 0.420 | 0.148 | 0.084 | 168 | 144 | 162 |
| Sudan | 0.408 | 0.135 | 0.090 | 169 | 152 | 158 |
| Cote d'Ivôire | 0.400 | 0.140 | 0.080 | 170 | 149 | 167 |
| Malawi | 0.399 | 0.129 | 0.063 | 171 | 158 | 179 |
| Afghanistan | 0.397 | 0.163 | 0.085 | 172 | 136 | 161 |
| Zimbabwe | 0.375 | 0.098 | 0.045 | 173 | 163 | 184 |
| Ethiopia | 0.363 | 0.149 | 0.073 | 174 | 143 | 172 |
| Mali | 0.360 | 0.141 | 0.075 | 175 | 148 | 170 |
| Guinea-Bissau | 0.354 | 0.139 | 0.071 | 176 | 150 | 175 |
| Eritrea | 0.349 | | 0.049 | 177 | | 183 |
| Guinea | 0.345 | 0.133 | 0.067 | 178 | 153 | 178 |
| CAR | 0.342 | 0.095 | 0.055 | 179 | 165 | 181 |
| Sierra Leone | 0.337 | 0.097 | 0.058 | 180 | 164 | 180 |
| Burkina Faso | 0.334 | 0.133 | 0.072 | 181 | 154 | 173 |
| Liberia | 0.328 | 0.100 | 0.040 | 182 | 162 | 186 |
| Chad | 0.328 | 0.132 | 0.071 | 183 | 156 | 174 |
| Mozambique | 0.321 | 0.130 | 0.068 | 184 | 157 | 176 |
| Burundi | 0.317 | 0.103 | 0.045 | 185 | 161 | 185 |
| Niger | 0.294 | 0.103 | 0.051 | 186 | 160 | 182 |
| Congo, Rep. of | 0.287 | 0.085 | 0.037 | 187 | 166 | 187 |

Table A.2: Value of Longevity and Schooling

| Country | Year of Longevity | | | Year of Schooling | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Norway | 4798.13 | 586.40 | 294.23 | 8473.00 | 2748.96 | 1369.64 |

Table A.2: Value of Longevity and Schooling

| **Country** | **Year of Longevity** | | | **Year of Schooling** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Australia | 3249.28 | 420.40 | 209.37 | 5586.95 | 1912.83 | 960.18 |
| Netherlands | 3536.58 | 451.08 | 227.71 | 6388.99 | 2166.79 | 1072.97 |
| United States | 4459.19 | 547.99 | 278.08 | 8151.96 | 2688.56 | 1324.24 |
| New Zealand | 2138.92 | 294.14 | 145.95 | 3606.46 | 1318.72 | 664.39 |
| Canada | 3379.78 | 434.15 | 222.01 | 6442.70 | 2197.88 | 1073.95 |
| Ireland | 2748.78 | 363.80 | 180.45 | 4627.11 | 1629.00 | 820.96 |
| Liechtenstein | 9453.32 | 1051.72 | 546.66 | 19163.87 | 5695.03 | 2734.89 |
| Germany | 3377.93 | 433.51 | 221.62 | 6415.94 | 2192.08 | 1071.43 |
| Sweden | 3432.86 | 440.26 | 226.45 | 6712.67 | 2282.61 | 1108.55 |
| Switzerland | 3838.32 | 485.10 | 250.57 | 7664.34 | 2559.23 | 1237.33 |
| Japan | 2942.97 | 387.23 | 202.23 | 6178.29 | 2138.74 | 1021.80 |
| Hong Kong | 4355.58 | 541.12 | 279.48 | 8711.16 | 2853.82 | 1379.88 |
| Iceland | 2698.87 | 358.85 | 178.66 | 4633.05 | 1630.78 | 818.85 |
| Korea | 2628.73 | 350.25 | 176.49 | 4713.05 | 1670.41 | 828.67 |
| Denmark | 3410.81 | 435.88 | 218.41 | 5933.60 | 2032.37 | 1013.97 |
| Israel | 2330.97 | 316.78 | 163.54 | 4631.86 | 1667.68 | 806.73 |
| Belgium | 3229.99 | 416.96 | 212.25 | 6018.61 | 2071.86 | 1017.20 |
| Austria | 3447.72 | 441.52 | 228.18 | 6861.63 | 2334.58 | 1128.06 |
| France | 2832.75 | 373.77 | 191.13 | 5410.38 | 1892.05 | 924.53 |
| Slovenia | 2318.31 | 314.17 | 157.68 | 4067.33 | 1474.20 | 734.33 |
| Finland | 3125.90 | 405.48 | 204.25 | 5581.96 | 1930.83 | 958.23 |
| Spain | 2409.05 | 325.65 | 165.23 | 4455.29 | 1596.87 | 786.64 |
| Italy | 2387.04 | 323.37 | 165.06 | 4532.44 | 1624.79 | 795.45 |
| Luxembourg | 5245.87 | 631.96 | 336.76 | 11832.78 | 3801.28 | 1775.67 |
| Singapore | 5390.01 | 648.20 | 340.10 | 11435.06 | 3650.63 | 1735.23 |
| Czech Rep. | 1990.71 | 275.48 | 140.31 | 3681.53 | 1372.12 | 673.25 |
| United Kingdom | 3212.36 | 415.16 | 211.46 | 6005.71 | 2068.07 | 1014.70 |
| Greece | 2168.57 | 297.21 | 150.33 | 3936.28 | 1439.21 | 711.23 |
| UAE | 6792.28 | 784.22 | 413.52 | 14427.22 | 4510.75 | 2132.21 |
| Cyprus | 2298.66 | 312.07 | 162.21 | 4659.87 | 1689.86 | 811.51 |
| Andorra | 3490.21 | 446.17 | 246.42 | 9241.48 | 3138.70 | 1405.17 |
| Brunei D. | 4832.34 | 586.58 | 306.45 | 9938.85 | 3244.89 | 1549.63 |
| Estonia | 1570.74 | 224.59 | 113.14 | 2741.29 | 1070.00 | 530.95 |
| Slovakia | 1912.52 | 265.23 | 135.61 | 3555.50 | 1342.15 | 655.88 |
| Malta | 1933.12 | 269.60 | 140.82 | 4000.48 | 1490.28 | 711.83 |

Table A.2: Value of Longevity and Schooling

| **Country** | **Year of Longevity** | | | **Year of Schooling** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Qatar | 12878.80 | 1373.99 | 746.28 | 31338.42 | 8976.75 | 4101.05 |
| Hungary | 1557.77 | 222.86 | 112.84 | 2769.38 | 1083.73 | 534.99 |
| Poland | 1605.74 | 229.32 | 116.76 | 2943.85 | 1140.59 | 559.84 |
| Lithuania | 1582.88 | 224.85 | 111.57 | 2566.03 | 1008.32 | 507.97 |
| Portugal | 1841.74 | 258.78 | 131.93 | 3446.02 | 1293.90 | 634.25 |
| Bahrain | 2883.77 | 375.09 | 196.57 | 5928.95 | 2102.16 | 1000.49 |
| Latvia | 1330.71 | 194.99 | 98.85 | 2364.23 | 952.87 | 469.81 |
| Chile | 1103.43 | 168.51 | 87.45 | 2218.11 | 906.73 | 436.15 |
| Argentina | 1293.81 | 191.40 | 96.62 | 2288.74 | 919.43 | 455.29 |
| Croatia | 1405.63 | 205.34 | 107.18 | 2861.82 | 1131.58 | 540.94 |
| Barbados | 1641.95 | 233.93 | 123.25 | 3479.95 | 1340.75 | 634.36 |
| Uruguay | 1135.09 | 171.97 | 87.54 | 2087.10 | 854.32 | 419.46 |
| Palau | 861.39 | 135.71 | 68.79 | 1517.69 | 662.86 | 326.86 |
| Romania | 962.36 | 149.27 | 75.98 | 1743.88 | 741.34 | 364.02 |
| Cuba | 365.83 | 68.47 | 34.04 | 617.72 | 309.49 | 155.61 |
| Seychelles | 1597.91 | 227.30 | 118.76 | 3219.85 | 1257.82 | 600.63 |
| Bahamas | 2252.92 | 304.62 | 164.08 | 5219.27 | 1919.08 | 885.40 |
| Montenegro | 880.62 | 138.89 | 72.29 | 1754.80 | 756.28 | 362.66 |
| Bulgaria | 1012.39 | 155.48 | 80.61 | 1973.05 | 832.99 | 401.13 |
| Saudi Arabia | 2353.27 | 314.94 | 163.55 | 4629.25 | 1698.83 | 816.64 |
| Mexico | 1135.40 | 172.01 | 89.90 | 2327.98 | 952.88 | 454.89 |
| Panama | 1058.71 | 162.09 | 85.52 | 2249.75 | 934.47 | 441.45 |
| Serbia | 869.31 | 137.40 | 71.49 | 1729.10 | 747.15 | 358.41 |
| Ant.&Barbuda | 1488.59 | 213.79 | 109.97 | 2796.43 | 1108.64 | 538.38 |
| Malaysia | 1241.97 | 184.43 | 97.79 | 2671.23 | 1086.11 | 510.21 |
| Tr.&Tobago | 2553.02 | 334.37 | 175.93 | 5199.44 | 1905.61 | 902.96 |
| Kuwait | 5417.78 | 642.44 | 343.00 | 12024.83 | 3896.42 | 1816.10 |
| Libya | 1115.93 | 168.94 | 83.93 | 1841.96 | 761.27 | 383.06 |
| Belarus | 1309.37 | 191.17 | 96.56 | 2255.52 | 920.48 | 455.54 |
| Russian F. | 1486.22 | 211.64 | 107.25 | 2571.89 | 1032.70 | 509.36 |
| Grenada | 529.38 | 91.87 | 46.25 | 926.41 | 436.38 | 216.70 |
| Kazakhstan | 1049.95 | 157.99 | 78.19 | 1634.02 | 700.99 | 354.08 |
| Costa Rica | 823.77 | 132.37 | 72.50 | 2087.59 | 897.18 | 405.79 |
| Albania | 597.51 | 101.47 | 55.63 | 1504.35 | 690.53 | 311.98 |
| Lebanon | 1211.49 | 180.11 | 92.97 | 2308.84 | 947.54 | 458.44 |

Table A.2: Value of Longevity and Schooling

| **Country** | **Year of Longevity** | | | **Year of Schooling** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| St.Kitts&Nevis | 1070.70 | 162.75 | 85.52 | 2203.65 | 922.25 | 437.66 |
| Venezuela | 914.52 | 143.23 | 73.86 | 1751.75 | 750.42 | 363.46 |
| Bosnia&Herz | 597.04 | 101.24 | 52.97 | 1222.61 | 563.53 | 268.68 |
| Georgia | 344.22 | 64.86 | 34.02 | 705.51 | 364.89 | 173.49 |
| Ukraine | 524.95 | 90.15 | 45.16 | 865.99 | 420.07 | 209.61 |
| Mauritius | 1175.98 | 175.99 | 91.40 | 2308.72 | 949.85 | 456.53 |
| Macedonia | 719.39 | 117.70 | 61.73 | 1482.05 | 661.95 | 314.76 |
| Jamaica | 509.72 | 88.74 | 45.88 | 980.66 | 470.07 | 227.03 |
| Peru | 688.13 | 113.36 | 59.74 | 1440.28 | 650.31 | 307.62 |
| Dominica | 599.30 | 101.79 | 53.94 | 1305.29 | 597.65 | 280.98 |
| Saint Lucia | 669.05 | 110.90 | 58.34 | 1394.28 | 631.53 | 299.31 |
| Ecuador | 590.92 | 100.38 | 52.14 | 1173.39 | 542.07 | 260.50 |
| Brazil | 877.78 | 138.26 | 71.58 | 1701.49 | 736.38 | 355.15 |
| St.Vincent&G. | 671.63 | 110.83 | 57.76 | 1330.54 | 607.05 | 290.65 |
| Armenia | 377.99 | 69.92 | 37.50 | 853.63 | 432.33 | 200.47 |
| Colombia | 684.50 | 112.82 | 58.65 | 1351.39 | 611.40 | 293.55 |
| Iran | 886.27 | 139.23 | 73.41 | 1849.30 | 800.32 | 378.37 |
| Oman | 2340.62 | 312.89 | 167.83 | 5256.47 | 1935.68 | 897.44 |
| Tonga | 298.89 | 57.90 | 29.91 | 570.51 | 305.55 | 147.71 |
| Azerbaijan | 762.68 | 122.57 | 65.26 | 1638.46 | 734.41 | 343.42 |
| Turkey | 1090.30 | 165.49 | 89.04 | 2494.75 | 1037.80 | 479.40 |
| Belize | 420.88 | 76.37 | 40.89 | 952.07 | 468.71 | 217.77 |
| Tunisia | 572.84 | 97.73 | 50.16 | 1076.55 | 502.14 | 244.43 |
| Jordan | 394.06 | 72.21 | 37.84 | 803.15 | 404.58 | 192.57 |
| Algeria | 625.67 | 104.76 | 54.36 | 1221.43 | 563.09 | 270.93 |
| Sri Lanka | 351.19 | 65.99 | 35.00 | 759.07 | 389.21 | 182.77 |
| Dominican Rep. | 665.26 | 110.18 | 59.06 | 1492.64 | 679.58 | 315.32 |
| Samoa | 275.43 | 54.30 | 28.79 | 586.69 | 319.59 | 150.15 |
| Fiji | 313.78 | 59.90 | 31.01 | 593.77 | 318.85 | 153.80 |
| China | 602.87 | 101.71 | 54.86 | 1390.24 | 644.48 | 296.88 |
| Turkmenistan | 696.71 | 112.40 | 57.85 | 1254.08 | 584.48 | 283.64 |
| Thailand | 617.66 | 103.83 | 55.35 | 1358.34 | 625.53 | 292.08 |
| Suriname | 643.94 | 106.77 | 55.95 | 1292.99 | 598.25 | 284.75 |
| El Salvador | 463.30 | 82.06 | 43.65 | 999.36 | 489.67 | 229.23 |
| Gabon | 1379.24 | 195.36 | 98.53 | 2247.85 | 935.04 | 463.44 |

Table A.2: Value of Longevity and Schooling

| **Country** | **Year of Longevity** | | | **Year of Schooling** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Paraguay | 347.18 | 65.20 | 34.71 | 753.17 | 390.66 | 182.68 |
| Bolivia | 322.08 | 60.87 | 30.82 | 547.78 | 295.91 | 146.09 |
| Maldives | 368.37 | 68.70 | 36.86 | 843.68 | 425.48 | 197.21 |
| Mongolia | 246.37 | 49.50 | 25.06 | 423.72 | 240.50 | 118.75 |
| Moldova | 212.16 | 44.13 | 23.34 | 439.47 | 256.97 | 121.07 |
| Philippines | 253.46 | 50.63 | 26.72 | 518.64 | 292.27 | 138.02 |
| Egypt | 392.64 | 71.98 | 39.26 | 949.48 | 479.00 | 217.74 |
| Palestinian T | 164.96 | 36.48 | 19.22 | 342.92 | 209.13 | 98.95 |
| Uzbekistan | 208.25 | 43.44 | 23.13 | 441.16 | 260.26 | 121.70 |
| Micronesia | 202.41 | 42.54 | 22.38 | 409.84 | 242.56 | 114.92 |
| Guyana | 221.54 | 45.67 | 24.20 | 464.48 | 268.24 | 126.09 |
| Botswana | 1914.63 | 245.28 | 120.85 | 2605.15 | 1069.59 | 542.58 |
| Syria | 284.48 | 55.90 | 30.56 | 703.64 | 375.49 | 170.25 |
| Namibia | 602.80 | 99.30 | 51.55 | 1104.27 | 535.00 | 257.25 |
| Honduras | 229.46 | 47.10 | 25.47 | 534.41 | 302.02 | 138.68 |
| Kiribati | 225.01 | 46.11 | 24.19 | 447.23 | 259.50 | 123.37 |
| South Africa | 1313.71 | 179.34 | 86.60 | 1644.65 | 722.82 | 373.78 |
| Indonesia | 271.95 | 53.54 | 27.63 | 508.87 | 281.52 | 136.23 |
| Vanuatu | 284.73 | 55.63 | 30.52 | 698.14 | 379.81 | 171.45 |
| Kyrgyzstan | 128.63 | 30.07 | 15.63 | 245.43 | 162.88 | 78.22 |
| Tajikistan | 120.86 | 28.70 | 15.24 | 251.79 | 169.91 | 79.70 |
| Viet Nam | 169.42 | 37.30 | 20.73 | 449.61 | 269.71 | 119.83 |
| Nicaragua | 143.57 | 32.84 | 18.03 | 358.93 | 225.00 | 101.46 |
| Morocco | 300.37 | 58.12 | 32.07 | 761.13 | 407.38 | 182.56 |
| Guatemala | 303.55 | 58.53 | 31.99 | 733.11 | 393.11 | 178.24 |
| Iraq | 224.24 | 46.04 | 25.43 | 560.60 | 324.18 | 145.10 |
| Cape Verde | 221.38 | 45.85 | 24.78 | 517.18 | 293.28 | 134.75 |
| India | 270.88 | 53.03 | 28.61 | 596.99 | 336.70 | 155.01 |
| Ghana | 99.00 | 24.67 | 13.20 | 208.37 | 150.86 | 70.16 |
| Equ. Guinea | 2927.65 | 344.58 | 187.83 | 5912.33 | 2286.75 | 1040.27 |
| Congo | 280.61 | 53.41 | 27.83 | 499.75 | 292.00 | 139.88 |
| Laos | 146.79 | 33.21 | 18.52 | 378.94 | 243.70 | 107.84 |
| Cambodia | 125.06 | 29.29 | 15.85 | 275.00 | 188.57 | 86.49 |
| Swaziland | 594.18 | 92.07 | 45.97 | 804.39 | 423.02 | 211.83 |
| Bhutan | 445.08 | 78.76 | 42.12 | 954.90 | 481.18 | 223.84 |

Table A.2: Value of Longevity and Schooling

| **Country** | **Year of Longevity** | | | **Year of Schooling** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Solomon Islands | 107.16 | 26.24 | 14.69 | 282.02 | 195.82 | 86.25 |
| Kenya | 108.69 | 26.13 | 13.44 | 183.29 | 135.64 | 65.85 |
| S.Tome&Principe | 115.70 | 27.70 | 14.75 | 239.42 | 165.93 | 77.59 |
| Pakistan | 181.91 | 38.99 | 22.96 | 598.45 | 369.57 | 151.96 |
| Bangladesh | 85.27 | 22.19 | 12.77 | 257.40 | 188.77 | 80.10 |
| Timor-Leste | 240.60 | 48.08 | 25.17 | 456.50 | 268.30 | 127.84 |
| Angola | 609.09 | 95.38 | 50.01 | 1040.81 | 535.60 | 254.78 |
| Myanmar | 92.75 | 23.54 | 13.02 | 227.84 | 166.85 | 74.54 |
| Cameroon | 193.53 | 39.36 | 20.08 | 296.87 | 197.18 | 96.61 |
| Madagascar | 37.21 | 12.35 | 6.64 | 81.21 | 77.01 | 35.63 |
| Tanzania | 89.91 | 22.82 | 12.33 | 188.71 | 145.93 | 67.06 |
| P. N. Guinea | 165.70 | 36.16 | 21.87 | 611.37 | 391.55 | 154.73 |
| Yemen | 150.63 | 33.79 | 18.99 | 398.46 | 257.33 | 112.68 |
| Senegal | 123.34 | 28.80 | 16.32 | 323.14 | 227.73 | 98.68 |
| Nigeria | 196.50 | 39.87 | 21.09 | 352.15 | 232.47 | 109.49 |
| Nepal | 58.26 | 16.86 | 9.53 | 161.54 | 131.82 | 57.30 |
| Haiti | 64.51 | 18.08 | 10.32 | 178.69 | 147.76 | 63.43 |
| Mauritania | 140.76 | 31.72 | 17.63 | 335.38 | 229.51 | 101.95 |
| Lesotho | 165.92 | 34.52 | 17.49 | 236.31 | 168.08 | 82.95 |
| Uganda | 79.75 | 20.78 | 10.60 | 125.90 | 104.07 | 50.99 |
| Togo | 44.67 | 13.98 | 7.43 | 86.32 | 83.13 | 38.95 |
| Comoros | 62.45 | 17.66 | 9.30 | 119.93 | 100.84 | 47.76 |
| Zambia | 109.35 | 25.59 | 13.73 | 200.71 | 158.73 | 73.54 |
| Djibouti | 194.11 | 40.33 | 24.62 | 721.24 | 457.84 | 178.34 |
| Rwanda | 77.69 | 20.45 | 10.42 | 123.89 | 102.07 | 50.06 |
| Benin | 98.73 | 24.31 | 13.00 | 193.70 | 148.26 | 69.00 |
| Gambia | 84.95 | 21.91 | 11.89 | 181.69 | 142.44 | 65.17 |
| Sudan | 134.24 | 30.80 | 19.55 | 633.04 | 430.45 | 157.16 |
| Cote d'Ivôire | 103.03 | 25.04 | 14.51 | 289.48 | 220.16 | 92.53 |
| Malawi | 44.45 | 13.89 | 7.42 | 85.41 | 84.61 | 39.39 |
| Afghanistan | 130.77 | 29.08 | 15.07 | 206.21 | 155.60 | 74.95 |
| Zimbabwe | 15.86 | 7.32 | 3.76 | 25.15 | 37.98 | 18.44 |
| Ethiopia | 56.16 | 16.37 | 9.03 | 129.84 | 114.24 | 51.26 |
| Mali | 86.50 | 21.85 | 11.72 | 163.62 | 135.30 | 62.71 |
| Guinea-Bissau | 81.24 | 20.67 | 10.68 | 125.43 | 109.23 | 52.78 |

Table A.2: Value of Longevity and Schooling

| **Country** | **Year of Longevity** | | | **Year of Schooling** | | |
|---|---|---|---|---|---|---|
| | UN | New | Our | UN | New | Our |
| Eritrea | 21.63 | 8.70 | 5.44 | 93.74 | 111.67 | 41.88 |
| Guinea | 54.54 | 15.95 | 8.59 | 108.14 | 100.35 | 46.32 |
| CAR | 48.69 | 14.61 | 8.14 | 104.76 | 107.12 | 47.41 |
| Sierra Leone | 52.95 | 15.42 | 8.41 | 102.23 | 102.36 | 46.56 |
| Burkina Faso | 78.47 | 20.60 | 11.94 | 220.46 | 181.11 | 76.12 |
| Liberia | 7.02 | 4.67 | 2.40 | 11.74 | 24.09 | 11.71 |
| Chad | 89.69 | 22.28 | 12.25 | 184.35 | 153.47 | 69.11 |
| Mozambique | 65.27 | 17.89 | 9.32 | 107.13 | 97.61 | 46.78 |
| Burundi | 15.77 | 7.30 | 3.69 | 22.83 | 35.05 | 17.36 |
| Niger | 34.32 | 11.72 | 7.13 | 121.52 | 130.82 | 51.22 |
| Congo, Rep. of | 10.15 | 5.79 | 3.07 | 17.58 | 34.15 | 16.03 |

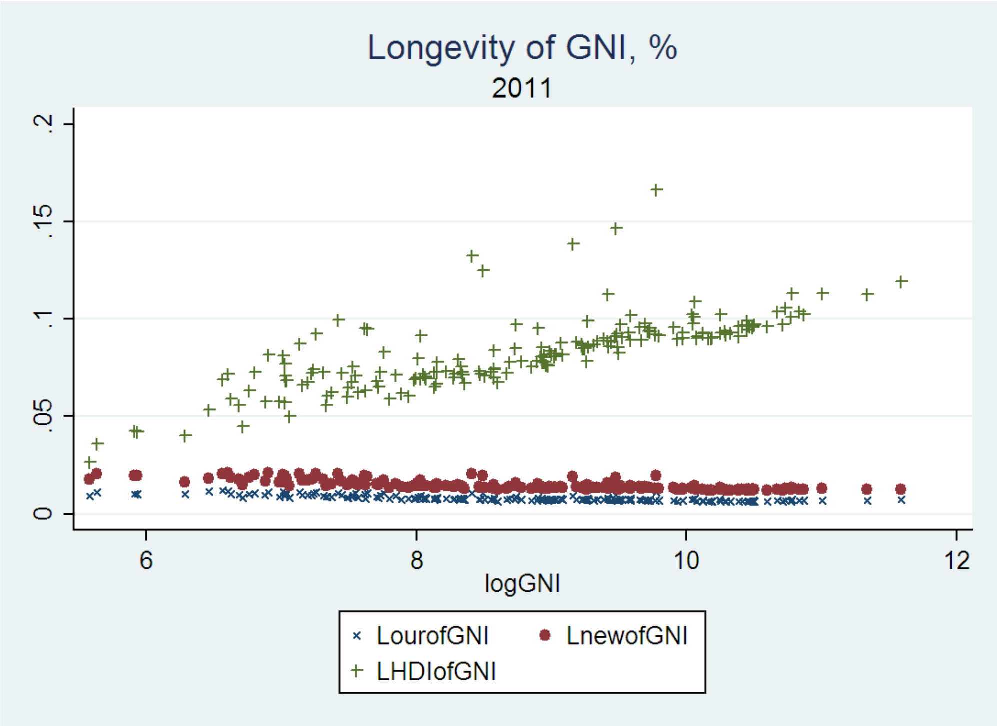


FIGURE A.3: Value of extra year of life, % of GNI

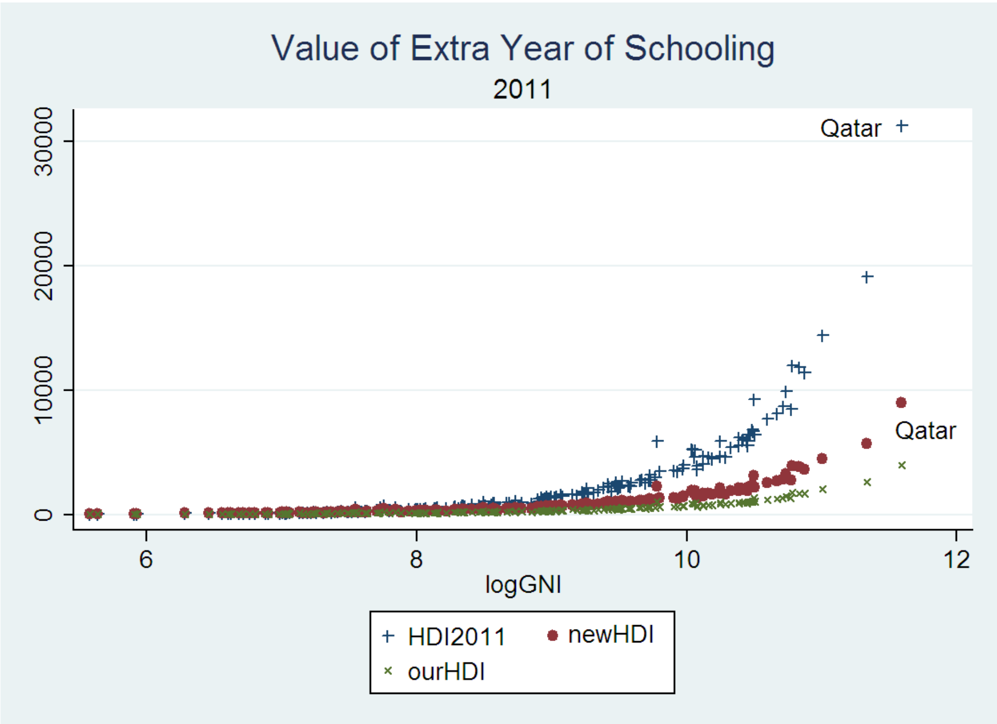


FIGURE A.4: Value of Schooling, 2011

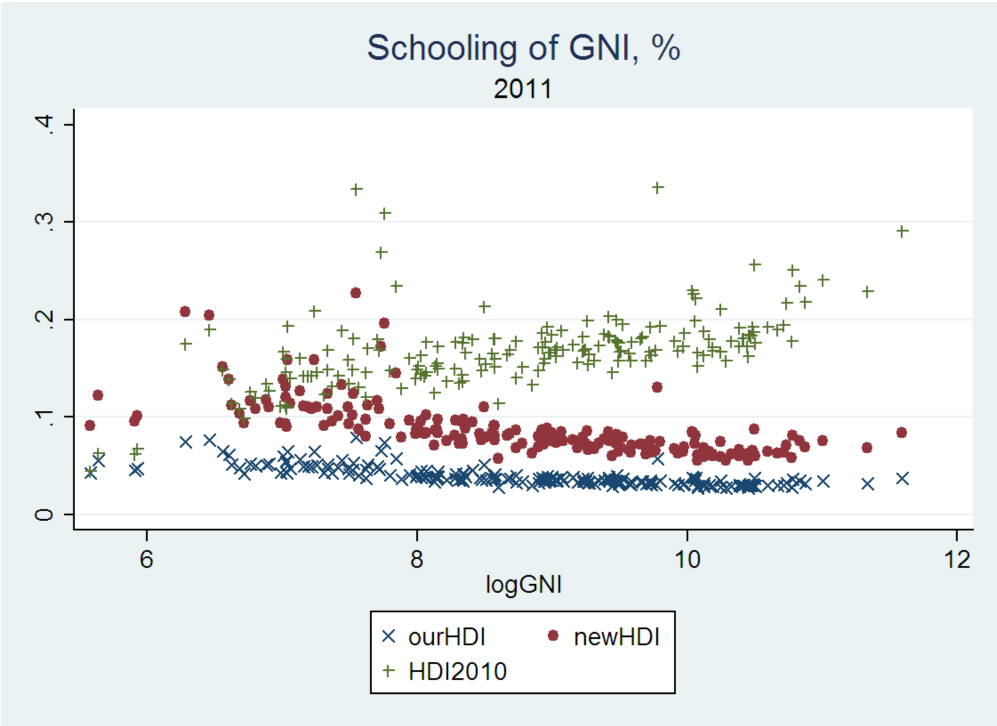


FIGURE A.5: Value of extra year of schooling, % of GNI

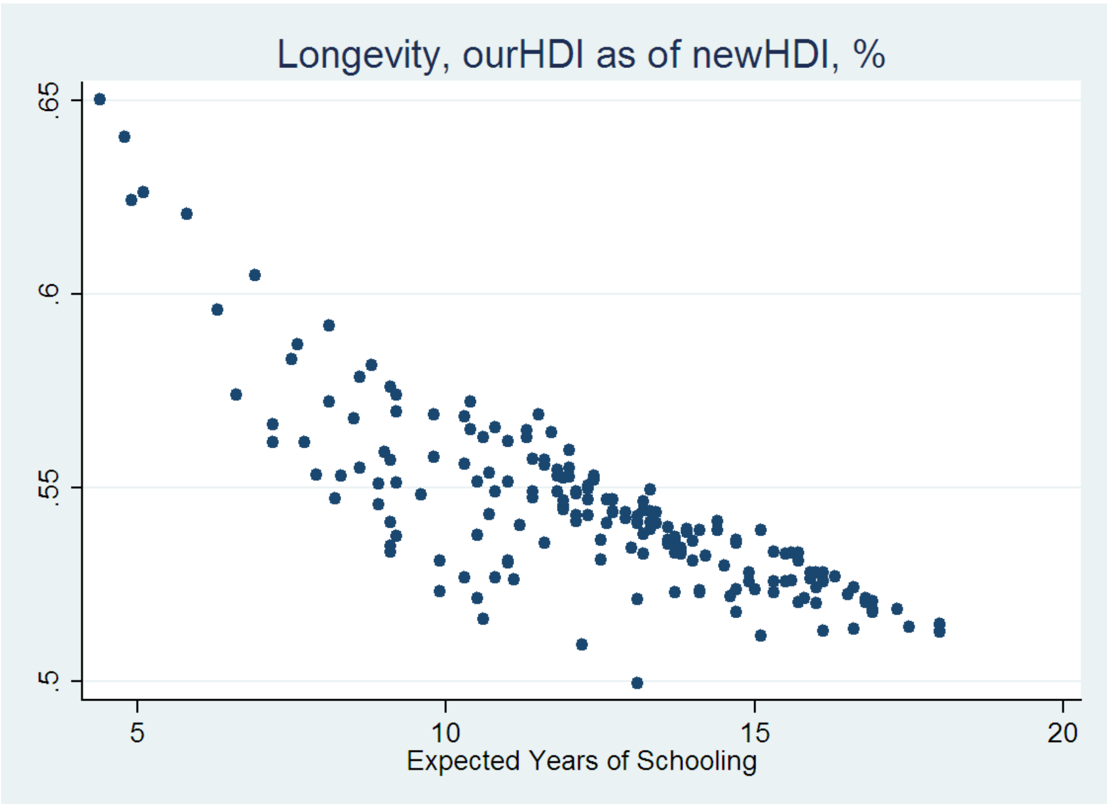


FIGURE A.6: The Value of Life, ourHDI as a proportion of newHDI

# Bibliography


[1] Anand, S., and Sen, A. Human development index: methodology and measurement. Tech. rep., Human Development Report Office (HDRO), United Nations Development Programme (UNDP), 1994.

[2] Anand, S., and Sen, A. The income component of the human development index. *Journal of Human Development 1*, 1 (2000), 83–106.

[3] Austen-Smith, D., and Banks, J. S. *Positive Political Theory: Collective Preference*, vol. 1. University of Michigan Press, 2000.

[4] Barberà, S., and Moreno, B. Top monotonicity: A common root for single peakedness, single crossing and the median voter result. *Games and Economic Behavior 73*, 2 (2011), 345–359.

[5] Black, D. *The theory of committees and elections*. Springer, 1986.

[6] Black, S. E., Devereux, P. J., and Salvanes, K. G. From the cradle to the labor market? the effect of birth weight on adult outcomes. *The Quarterly Journal of Economics 122*, 1 (2007), 409–439.

[7] Campbell, D. E., and Kelly, J. S. A simple characterization of majority rule. *Economic Theory 15*, 3 (2000), 689–700.

[8] Case, A., Fertig, A., and Paxson, C. The lasting impact of childhood health and circumstance. *Journal of health economics 24*, 2 (2005), 365–389.

[9] Case, A., Lubotsky, D., and Paxson, C. Economic status and health in childhood: The origins of the gradient. Tech. rep., National Bureau of Economic Research, 2001.

[10] Coughlin, P. J. Necessary and sufficient conditions for $\delta$-relative majority voting equilibria. *Econometrica 49*, 5 (1981), 1223–1224.

[11] Cutler, D. M., and Lleras-Muney, A. Education and health: evaluating theories and evidence. Tech. rep., National Bureau of Economic Research, 2006.

[12] Dasgupta, P., and Maskin, E. On the robustness of majority rule. *Journal of the European Economic Association 6*, 5 (2008), 949–973.

[13] de Clippel, G., Moulin, H., and Tideman, N. Impartial division of a dollar. *Journal of Economic Theory 139*, 1 (March 2008), 176–191.

[14] Desai, M. Human development: concepts and measurement. *European Economic Review 35*, 2 (1991), 350–357.

[15] Feld, S. L., and Grofman, B. Research note partial single-peakedness: An extension and clarification. *Public Choice 51*, 1 (1986), 71–80.

[16] Fishburn, P. C. *The theory of social choice*, vol. 264. Princeton University Press Princeton, 1973.

[17] Gans, J. S., and Smart, M. Majority voting with single-crossing preferences. *Journal of Public Economics 59*, 2 (1996), 219–237.

[18] Glied, S., and Lleras-Muney, A. Health inequality, education and medical innovation. Tech. rep., National Bureau of Economic Research, 2003.

[19] Grandmont, J.-M. Intermediate preferences and the majority rule. *Econometrica: Journal of the Econometric Society* (1978), 317–330.

[20] Greenberg, J. Consistent majority rules over compact sets of alternatives. *Econometrica: Journal of the Econometric Society* (1979), 627–636.

[21] Greenberg, J., and Weber, S. Consistent d-relative majority equilibria. *Econometrica 53*, 2 (1985), 463–64.

[22] Herrero, C., Martínez, R., and Villar, A. Multidimensional social evaluation: an application to the measurement of human development. *Review of Income and Wealth 56*, 3 (2010), 483–497.

[23] Herrero, C., Martínez, R., and Villar, A. A newer human development index. *Journal of Human Development and Capabilities 13*, 2 (2012), 247–268.

[24] Holzman, R., and Moulin, H. Impartial nominations for a prize. *Econometrica 81*, 1 (2013), 173–196.

[25] Inada, K.-i. A note on the simple majority decision rule. *Econometrica: Journal of the Econometric Society* (1964), 525–531.

[26] Kaneko, M. Necessary and sufficient conditions for transitivity in voting theory. *Journal of Economic Theory 11*, 3 (1975), 385–393.

[27] Lleras-Muney, A. The relationship between education and adult mortality in the united states. *The Review of Economic Studies 72*, 1 (2005), 189–221.

[28] Lleras-Muney, A., and Lichtenberg, F. R. The effect of education on medical technology adoption: are the more educated more likely to use new drugs. Tech. rep., National Bureau of Economic Research, 2002.

[29] May, K. O. A set of independent necessary and sufficient conditions for simple majority decision. *Econometrica: Journal of the Econometric Society* (1952), 680–684.

[30] Miguel, E., and Kremer, M. Worms: identifying impacts on education and health in the presence of treatment externalities. *Econometrica 72*, 1 (2004), 159–217.

[31] Nathan, H. S. K., Mishra, S., and Reddy, B. S. An alternative approach to measure hdi. *Indira Gandhi Institute of Development Research (IGIDR), Working Paper, WP-2008-001* (2008).

[32] Palazzi, P., and Lauri, A. The human development index: Suggested corrections. *QUARTERLY REVIEW-BANCA NAZIONALE DEL LAVORO* (1998), 193–221.

[33] Parks, R. P. An impossibility theorem for fixed preferences: a dictatorial bergson-samuelson welfare function. *The Review of Economic Studies 43*, 3 (1976), 447–450.

[34] Pro, U. N. *Human Development Report 2010, 2011*. Human Development Report, 2010, 2011.

[35] Ravallion, M. Troubling tradeoffs in the human development index. *Journal of Development Economics 99*, 2 (2012), 201–209.

[36] Ross, C. E., and Wu, C.-l. The links between education and health. *American sociological review* (1995), 719–745.

[37] Rothstein, P. Order restricted preferences and majority rule. *Social Choice and Welfare 7*, 4 (1990), 331–342.

[38] Sagar, A. D., and Najam, A. The human development index: a critical review. *Ecological economics 25*, 3 (1998), 249–264.

[39] Sen, A. Commodities and capabilities. *OUP Catalogue* (2011).

[40] Sen, A., and Pattanaik, P. K. Necessary and sufficient conditions for rational choice under majority decision. *Journal of Economic Theory 1*, 2 (2012), 178–202.

[41] Sen, A. K. A possibility theorem on majority decisions. *Econometrica: Journal of the Econometric Society* (1966), 491–499.

[42] Slutsky, S. Equilibrium under $\alpha$-majority voting. *Econometrica: Journal of the Econometric Society* (1979), 1113–1125.

[43] Strauss, J., and Thomas, D. Human resources: Empirical modeling of household and family decisions. *Handbook of development economics 3* (1995), 1883–2023.

[44] Streeten, P. Human development: means and ends. *The American Economic Review 84*, 2 (1994), 232–237.

[45] Viscusi, W. K., and Aldy, J. E. The value of a statistical life: a critical review of market estimates throughout the world. *Journal of risk and uncertainty 27*, 1 (2003), 5–76.

[46] Xefteris, D. A necessary and sufficient single-profile condition for transitivity of the majority rule relation. *Economics Letters* (2012).

[47] Zambrano, E. *Functionings, capabilities and the 2010 human development index.* United Nations Development Programme, 2011.