ESSAYS ON THE LABOR MARKET

by

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Introduction

Labor market is huge, important and interesting (at least for the labor economists). There are many aspects of the market, however, that we –despite our curiosity– do not know yet. In this thesis I aim to marginally contribute to our knowledge about the labor market. In particular, I aim at better understanding of the drivers of two main important variables of the labor market: unemployment and productivity. The former is a main concern of the workers, economists, journalists, politicians, labor force and their families, and therefore the whole society. The later is also widely used in the empirical and theoretical labor models. From the point of view of a macro-labor economist, productivity is particularly important because in the canonical workhorse model of study unemployment, Mortensen and Pissarides search and matching model, productivity is the driving force of the economy. Shortly, in this thesis I study three main topics: the sources of cross country differences in unemployment dynamics; the sources of rich set of employment flows at the firm level and its potential macroeconomic consequences, and the drivers of the dynamics of average labor productivity.

In the first chapter, Search, Rigidities and Unemployment Dynamics, I study the sources of cross-country differences in unemployment dynamics. Elsby, Hobijn and Sahin (2013) find that in Anglo-Saxon economies unemployment fluctuations are mainly driven by changes in the outflows out of unemployment, while in continental European and Nordic countries changes in inflows into unemployment are almost equally important. I provide evidence that a category of labor market regulations which I refer to as the restrictive regulations raise contributions of inflows into unemployment. On the contrary, higher firing costs reduce the importance of this driver of unemployment dynamics which is in line with literate. I introduce an aggregate regulatory rigidity into an otherwise standard search and matching framework. I calibrate the model to the US. Introducing rigidity and firing costs together explain about half of cross-country variations.

A growing body of search literature introduces large firms into Mortensen and Pissarides search models, however these models generally disregard the empirical evidence
that both growing and shrinking firms simultaneously hire and fire workers. In the second chapter, *Heterogeneous Workers, Firm Dynamics and the Countercyclicality of Productivity*, I develop a labor search model with both firm and worker heterogeneity. Random search and uncertainty about worker’s quality pre-hiring, create a rich set of employment flows, including simultaneous quits, hiring and firing at both growing and shrinking firms broadly consistent with thee mentioned empirical facts. Moreover, the model provides a mechanism to explain the substantial decline in the cyclicality of average labor productivity during the mid 80’s.

In the third chapter, *Was it the Fed or the heterogeneity that changed the cyclical pattern of productivity?*, I use the information in the factors of a large panel of macroeconomic variables to shed light on the main drivers of the structural change in cyclicality of productivity as well as the main drivers of unemployment. Using simple empirical techniques I show that financial institutions seems to be the best explanation for the decline in the cyclicality of productivity. I also find confirmative evidence in favor of the vanishing role of non-technology shocks in driving output, unemployment and productivity, which is in line with Barnichon (2010) argument. Meantime, I could not directly test the heterogeneity channels due to endogeneity, however, I find a structural change in the relation between separation and job finding probabilities: from negative correlation at pre-84 to positive correlation recently.
Chapter 1

Search, Rigidities and Unemployment Dynamics

1.1 Introduction

Why the dynamics of unemployment is so different among Anglo-Saxon and continental European countries? On average, inflows into unemployment in the latter countries contribute to the unemployment fluctuations almost three times more than in the former group (Elsby, Hobijn and Sahin, 2013). This pattern holds despite the fact that European countries typically are known to have relatively higher firing costs. Most of theoretical and empirical literature shows that higher firing costs dampen volatility of separation of workers at the marginal jobs, from which one may expect a lower contribution of inflows to the unemployment fluctuations in European countries.

The distinct behavior of those clubs of countries raises the question about possible sources and explanations of this pattern. Among different explanations, this paper focuses on a specific class of the labor market regulations. In particular, I show that a class of less studied regulations can be a potential explanation for the above mentioned cross country discrepancies. I construct an index of the restrictiveness of those regulations using the "Rigidity of Employment index" of World Bank’s Doing Business, which contains country specific information about various restrictions on the working arrangements. The constructed index exhibits a strong correlation with the dynamics of unemployment.

On the other hand, throughout the past decade the Diamond-Mortensen-Pissarides (DMP) search and matching model has been the dominant theoretical framework to study unemployment. Many recent dynamic stochastic models use this framework to study contributions of specific types of regulations such as unemployment benefits, hiring and firing
costs, employment protections, contractual environment, etc. to the labor market outcome. Nevertheless, the ability of the model to generate reasonable unemployment volatility has been questioned by the influential work of Shimer (2005a). Therefore the main question I try to address in this paper is “Whether the mentioned cross country differences can be explained by a DMP search and matching model?” To answer this question, I take a reduced form approach to introduce the described index of restrictive regulations into a DMP search model. Cross country variations in the index, in addition to the firing costs, explains about half of the cross country variations in the unemployment dynamics, as described earlier.

The remainder of the paper is organized as follows. I first provide an empirical motivation. In the third section I briefly address the related literature. The fourth section explains the model. The calibration methodology is described in section five. Section six presents the results. The last section concludes.

1.2 Empirical motivation

In a two-state worker model, where workers move between employment and unemployment, changes in either inflows into the employment pool (separations\textsuperscript{1}) or outflows from the unemployment (job findings) could generate fluctuations in the unemployment rate. In other words, unemployment goes up either because the probability that employed workers are losing their jobs is going up or because it is harder for the unemployed to find a job, or both. Despite quantitative disagreement among the researchers, the common message from decomposition of the U.S. unemployment is that in the U.S. the contribution of inflows to unemployment fluctuations is nontrivial, however outflows changes are the dominant driver of unemployment. Elsby, Hobijn and Sahin (2013), hereafter EHS, generalize Shimer’s (2007) measurement and decomposition methods and apply them to lower frequency data.

\textsuperscript{1}By the term “separation” I actually mean inflow to unemployment. These are not exact substitutes when either on the job search or the transmission from non-participation to unemployment is allowed. Since this paper abstracts from on the job search and non-participation, I use separation and inflow into unemployment interchangeably.
- quarterly and annually - of fourteen OECD countries, including the U.S., therefore provide a set of comparable cross country statistics. Table 1.1 reports their main findings. In the table $\beta_f$ and $\beta_s$ are contributions of job finding and separation rates to the unemployment volatilities, respectively. Their results show that variations in the job finding rates can explain around 85% of unemployment fluctuations in Anglo-Saxon economies, while in Continental Europe, Nordic countries and Japan generally separation rates are equally important. In the latter group, on average, job finding rates and separation rates contribute 55% and 45% to unemployment variations respectively. For brevity I refer to the latter set of countries as rigid economies.

There are couple of additional points, I would like to emphasis. First, I split the underlying time series of EHS for each country\footnote{I would like to thank Elsby, Hobijn and Sahin for making the detail data of their calculations available online.} and recalculate the decomposition for these subsamples. In almost all countries –except Japan-, the results do not substantially differ between the split subsamples. Second, the decomposition results show no clear relation neither to the unemployment levels nor to the flows rates. For example, with regard to the flows, as reported by EHS, the United States has by far the highest average outflows rate (56.5%) followed by Norway (38.5%) and Sweden (29.2%). On the other extreme, Italy with the lowest outflows rate in the sample (4.3%) has a $\beta_f$ very similar to Anglo-Saxon economies (0.85).

The focus of this paper is on the potential role of differences in labor market institutions. Several empirical papers study either steady state or business cycle effects of different regulations (e.g. Nickel and Layard (1999) and Gnocchi and Pappa (2011)). There are also numerous theoretical papers study contributions of specific regulations to the labor market. Throughout the past decade the DMP search and matching model has been the dominant theoretical framework to study unemployment. Many recent theoretical papers use this framework to study contributions of one particular regulation, such as unemployment benefits, hiring and firing costs, employment protections, contractual environment, etc. A
common feature of most of these policies, especially those used in general equilibrium models, is that they impose restrictions only on employment decision of the firm and not explicitly on the outcome of a match for the firm. I argue that, in general, regulations that restrict free working arrangement of a match may affect the dynamics of unemployment. Restrictive regulations refer to regulations that in one way or another prevent firms from freely choosing among different working arrangements. The idea is that restrictions on work arrangements have kind of asymmetric effects on a match during the business cycle. At good times the restrictions could reduce the profitability of a match; however the match still could be profitable enough to survive. At bad times, however, the small profits of less productive matches fade away by imposing restrictions. This triggers more job destruction than otherwise. Those restrictions, in general, include regulations that explicitly prohibit some working arrangements (e.g. explicit restrictions on weekly hours worked, forbidding temporary employment under certain conditions) or restrictions that makes some arrangements costlier for the firm (e.g. setting a premium for extra hours worker, or discriminating at firing cost among full time and temporary workers).

In this paper I calculate a proxy of these restrictions using "Rigidity of Employment index" of World Bank’s Doing Business. The World Bank’s index is a simple average of three subindices: hiring index, a rigidity of hours index and a difficulty of redundancy index, each takes a value between zero and one hundred, where a lower value indicates less restriction. These indices contain information on restrictive regulations such as working days, working hours and also restrictions on using different types of contracts. For example, a country receives a high index if it restricts weekend, night work and/or workday hours\(^3\). To construct the "Restrictive Regulation index" I calculate the first principal component of hiring and rigidity of hours indices. For comparability of the coefficients in coming regressions, through a linear transformation I scale the extracted components to the same range as the firing costs.

\(^3\)Difficulty of redundancy index has information on firing. Therefore is not appropriate for the purpose of restrictive regulation index.
Figure 1.1 compares indicators of the three categories of regulations (a measure of firing costs, an index of restrictiveness of regulations, and the level of unemployment insurance) for the fourteen OECD countries of table 1.1. The left, middle and right columns report the job security index of Heckman and Pages (2004), the constructed proxy for restrictive regulations (calculated using employment rigidity index of the World Bank Doing Business (2005)) and the unemployment replacement ratio of Nickel and Layard (1999). As one may expect, countries remarkably differ among all these dimensions, which extends to the subcategories of Angle-Saxons and the rigid economies, too. In the case of the job security index, among Anglo-Saxons the U.S., New Zealand and to some extend Canada have very low indices, while Australia can be considered having relatively a high index and the UK stands somewhere in the middle of the fourteen countries. The job security index of rigid economies varies from lower than Canada’s index for the case of Japan, to very high for Portugal, Spain and Italy. In the case of unemployment replacement ratio, the rigid economies generally show higher ratios than Anglo-Saxons, with the notable exception of Canada. In the case of the restrictive regulation index Anglo-Saxons have lowest indices. The lowest index among rigid economies (Norway) equates highest among Anglo-Saxons (the UK).

In general, Anglo-Saxon economies are usually known to be more flexible than Continental Europe and Nordic economies. In particular, they usually have relatively lower unemployment benefits, lower or no severance payment at the moment of employment reduction, more flexible regulations for signing and exerting different types of contracts and more flexible hours of working, etc. To investigate the potential relation among these variables and the illustrated fact by EHS, I regress $\beta_f$ of the countries in table 1.1 over the three variables reported in figure 1.1. I also regress over the possible combinations of these three variables. Table 1.2 provides the results. Since $\beta_s$ is almost equal to $1 - \beta_f$, regressions with $\beta_s$ instead of $\beta_f$ in the left hand side will provide very similar results. I emphasize that the aim here is not to provide a rigorous causality test, but to illustrate a potential relation between the right hand side variables and the $\beta_f$, especially considering the small
sample size. But still there are a couple of interesting results. First, despite small sample size, as shown in the first column (model I), in the regression with the three variables, all variables are significant at 5%. From the main regression (model I), one expects that countries with higher firing costs should have higher $\beta_s$ (lower $\beta_f$). This is in accordance with the literature, nonetheless cannot be found by regressing $\beta_f$ over firing costs only, such a regression (model V) gives a coefficient with the opposite sign and insignificant\(^4\) and generates an incredibly low R-squared of zero. Furthermore, regressing over any pair of the three variables leads to at least one insignificant\(^5\) coefficient. In particular, removing the rigidity index from the right hand side variables makes the coefficient of firing costs insignificant\(^6\). Moreover, model (I) produces by far the highest adjusted R-squared. Also the three models with highest adjusted R-squared share the rigidity index. Lastly, if instead of using the first principal component, one calculates an alternative index by averaging the two indexes of rigidity of hours and employment rigidity, the regressions give similar results but with marginally lower R-squared\(^7\). This goes in favor of using principal component analysis, and also in favor of the choice of these variables, because the principal component analysis reserves the information better than simple average.

All in all this primary study suggests that the restrictive regulations could play a significant role in driving the dynamics of unemployment. To my knowledge, however, no dynamic stochastic model of unemployment incorporates this family of variables as a whole. An important feature of this invented variable is that it represents a group of different regulations. Modelling each regulations separately makes the workhorse model of study unemployment, the DMP framework, very complex. That is why I choose a reduced form approach to introduce this class of regulations into the DMP search models.

It is worth noting that this paper aims to address the impacts of those restrictive regulations on unemployment dynamics only; measuring potential benefits or disadvantages

\(^4\)At 87% significance level compare to standard significance levels of 10%, 5% or 1%.
\(^5\)At 10% significance level.
\(^6\)At 39% significance level.
\(^7\)The results of regressions with the "simple averaged index" are not reported here, but are available upon request.
Figure 1.1: Regulations. Left: Job Security Index (Heckman and Pages, 2004). Middle: Restrictive Regulation Index (Own-calculation using World Bank Doing Business, 2005). Right: Unemployment Replacement Ratio (Nickell and Layard, 1999)
Table 1.1: Unemployment Fluctuations Decomposition

<table>
<thead>
<tr>
<th>Country</th>
<th>( \beta_f )</th>
<th>( \beta_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>93</td>
<td>10</td>
</tr>
<tr>
<td>Canada</td>
<td>79</td>
<td>23</td>
</tr>
<tr>
<td>New Zealand</td>
<td>88</td>
<td>13</td>
</tr>
<tr>
<td>U.K.</td>
<td>85</td>
<td>17</td>
</tr>
<tr>
<td>U.S.</td>
<td>85</td>
<td>16</td>
</tr>
<tr>
<td>France</td>
<td>54</td>
<td>45</td>
</tr>
<tr>
<td>Germany</td>
<td>56</td>
<td>47</td>
</tr>
<tr>
<td>Ireland</td>
<td>47</td>
<td>55</td>
</tr>
<tr>
<td>Italy</td>
<td>83</td>
<td>15</td>
</tr>
<tr>
<td>Japan</td>
<td>56</td>
<td>45</td>
</tr>
<tr>
<td>Norway</td>
<td>54</td>
<td>45</td>
</tr>
<tr>
<td>Portugal</td>
<td>68</td>
<td>32</td>
</tr>
<tr>
<td>Spain</td>
<td>57</td>
<td>43</td>
</tr>
<tr>
<td>Sweden</td>
<td>50</td>
<td>51</td>
</tr>
</tbody>
</table>

Note: \( \beta_f \) and \( \beta_s \) are contributions of job finding rates and separation to the unemployment volatilities, calculated using non-steady state decomposition method proposed by EHS. Source: Elsby, Hobijn and Sahin (2013).

associated with such policies is beyond the scope of this paper.

1.3 Related literature

A vast literature studies the gross flows of workers between unemployment and employment after the seminal work of Blanchard and Diamond (1990). However, more relevant to this study is the contribution of the flows into and out of unemployment in unemployment movements. Hall (2005a,b) and Shimer (2005b) measure unemployment flows and argue that separations are almost acyclical. The same is reported in early versions of Shimer (2007). A number of studies, e.g. Elsby, Michaels, and Solon (2009), Elsby, Hobijn, and Sahin (2011), Fujita and Ramey (2009), and Yashiv (2007), criticized the claim either by different measurement of flows or by different methods to measure the contribution of unemployment flows to the volatility of the U.S. unemployment rate. It is worth mentioning that different methodologies for assessing the relative contribution of flows in unemployment fluctuations gives different results even when applied to a similar time series of flows.
Table 1.2: OLS regression result

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
<th>(V)</th>
<th>(VI)</th>
<th>(VII)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firing costs</td>
<td>10.1**</td>
<td>8.2</td>
<td>3.6</td>
<td>-0.77</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Restr. Reg.</td>
<td>-10.3**</td>
<td>-12.6**</td>
<td>-5.3</td>
<td>-8.0**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.I.</td>
<td>-0.49**</td>
<td>-0.6**</td>
<td>-0.4*</td>
<td>-0.54**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Obs.</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>R²</td>
<td>0.71</td>
<td>0.46</td>
<td>0.42</td>
<td>0.49</td>
<td>0.00</td>
<td>0.31</td>
<td>0.38</td>
</tr>
<tr>
<td>R² adj</td>
<td>0.62</td>
<td>0.37</td>
<td>0.31</td>
<td>0.39</td>
<td>-0.08</td>
<td>0.25</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Notes: Firing costs: from Heckman and Pagès (2004) in percentage of monthly salary; Restrictive Regulation: own-calculation (see the text); U.I.: unemployment benefit replacement ratio from Nickell and Layard (1999). * and ** are significance levels at 10% and 5% respectively.

Shimer (2007) to decompose the contributions of flows to the unemployment movements applies a method originally proposed by Pissarides (1986) for the UK data. This decomposition method assumes unemployment at each period reaches its steady-state. The assumption mimics the data well when the flows are high and therefore after realization of new rates adjustment to the steady-state level of unemployment happens quickly. The assumption specially does a good job for the U.S. data, since the U.S. economy has an extraordinary high flows. While when it comes to the economies with low flows rates, the slow adjustment of unemployment is in contrast to the steady-state assumption of this decomposition method. EHS extend the model in this dimension and allow for sluggish unemployment adjustment. As a result, when applied to economies with low flow rates, EHS non-steady state decomposition method produces smaller residuals compare to the steady-state method.

Compare to relatively large literature on the U.S. data, cross country literature are few. I already talked about EHS. Petrongolo and Pissarides (2008) study the dynamics of unemployment in three European countries: the U.K., France and Spain. Since they use the steady-state decomposition, to deal with the problem of not fitting the data they drop
observations with large deviations from steady-state. Justiniano and Michelacci (2011) develop a real business cycle model augmented with search and matching frictions for six countries: the U.S. and the UK, France and Germany, and Norway and Sweden. They calibrate each country separately and allow for six different sources of shocks: neutral technology, Investment, job destruction, discount factor, matching function and aggregate demand. They introduce wage rigidity by allowing only a fraction of matches to negotiate the wage at each period. There are not other labor market rigidities, e.g. firing cost, in their model. They find that technology shocks are the main driver of labor market dynamics in the U.S. and some European economies like Sweden, however they affect comparatively weaker the labor market dynamics in France; they find mixed results for other countries in their study. Rogerson and Shimer (2011) survey the studies of three and four state model transitions.

My paper is also related to a branch of literature which incorporates labor market regulatory rigidities into the Mortensen and Pissarides search model. I briefly review some of the related studies. Garibaldi (1998) explores the equilibrium job destruction and job creation in a search and matching model incorporating firing costs and firing permissions. He shows that tighter firing restrictions make the job destruction less volatile. Mortensen and Pissarides (2003) study labor market policies effects to the steady state of search and matching models. Thomas (2006) finds that within the search and matching framework firing costs reduce the volatility of business cycle fluctuations. Veracierto (2008) reaches similar results with firing costs in a RBC model. Pries and Rogerson (2005) show that imperfect information about match quality in the presence of rigidities can generate observed lower worker turnover in Europe than in the U.S., despite similar job turnover. Guell (2010) theoretically discusses opposite effects of firing costs, depending on the modeling assumptions. Hobijn and Sahin (2013) study the effect of rigidities in terms of firing costs, entry costs, and a tax wedge between wages and labor costs on the firm-size distribution

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8 A growing literature, not mentioned in the main text, deals with wage rigidity. However this type of rigidity is not directly addressed in this paper. See Rogerson and Shimer (2011) for a recent survey.
and dynamics. Silva and Toledo (2011) show that introducing training costs besides separation costs improves the behavior of the Mortensen and Pissarides model in replicating the observed volatility of unemployment and vacancies. Sala, Silva and Toledo (2012) find that introducing temporary jobs with no firing cost, in presence of permanent jobs facing firing costs, increases the volatility of unemployment. Llosa et al. (2012) show that dismissal costs can explain the cross-country differences in intensive and extensive margin of labor supply over the business cycle. In a study parallel to this paper Murtin and Robin (2014) take a reduced form approach to introduce different labor market institutions simultaneously into a search model. They assume that changes in the policies affect the long run equilibrium through changing the structural parameters of the model, while the cyclical movements are driven by aggregate productivity shocks. Applying the model to a subsample of countries in the paper, they estimate the effect of different policies on the labor market outcomes.

1.4 The model

1.4.1 The economy

The economy consists of a unit mass of infinitively lived workers and an infinite mass of firms. Worker and firms discount future payoffs at a rate $\beta$. Workers are either employed or unemployed. An unemployed worker benefits from a constant payoff $b$ each period, which contains unemployment insurance, utility of leisure, and home production.

Each firm consists of an either filled or unfilled vacancy. A filled vacancy have access to potential production technology of $z_t x_t$, where $z_t$ is the aggregate productivity level, common to all matches, and $x_t$ is the match specific quality. The aggregate productivity follows an AR(1) process

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t$$

(1.1)

where $\varepsilon_t$ is i.i.d. normal disturbances with mean zero and standard deviation $\sigma_z$. 
Dynamics of $x_t$ is as following: all new matches start at highest possible match specific productivity level, say $x^N$. Each period with probability $\lambda$ the match specific productivity remains constant, and with the remained probability of $1 - \lambda$ the specific productivity switches to a new level; it is drawn randomly according to the c.d.f $G(x)$. A firm pays a cost of $c$ per period to post a vacancy. The number of matches is assumed to have a Cobb-Douglas form of

$$m(u_t, v_t) = A u_t^\alpha v_t^{1-\alpha}$$  \hspace{1cm} (1.2)$$

where $u_t$ is the number of unemployed and $v_t$ is the number of posted vacancies at period $t$. Thus, an unemployed worker finds a job each period with probability of

$$F_t = F(\theta_t) = m(u_t, v_t)/u_t = A\theta_t^{1-\alpha}$$  \hspace{1cm} (1.3)$$

where $\theta_t = v_t/u_t$ is the labor market tightness. On the other hand the probability that a firm finds a worker to fill its vacancy is

$$Q_t = Q(\theta_t) = m(u_t, v_t)/v_t = A\theta_t^{-\alpha} = F(\theta_t)/\theta_t.$$  \hspace{1cm} (1.4)$$

Thus when the labor market is tighter, it is easier for an unemployed worker to find a job, while it is harder for a firm to find a worker.

Figure 1.2 illustrates the timing of the events in the model. At the beginning of each period the aggregate productivity realizes. Then, non-productive firms may decide to post vacancies and the unemployed workers meet vacancies. Simultaneously, a fraction $s^x$ of matches from the previous period break exogenously and existing matches learn whether their match specific productivity has changed, and decide to produce or dissolve the match endogenously. Afterwards, production takes place.

*Modelling the rigidities.* There are two types of regulatory rigidities: firing costs and restrictive rigidities. The distinction stems from the way each of those rigidities affect the
decision of a firm. Firing costs realizes only at endogenous separations while restrictive rigidities could have direct effects during any period of production. In other words, as it is standard in the literature, I model the firing costs as firing taxes that firm has to pay; whenever a firm decides to dissolve the match endogenously it must pay the firing cost \( F \). Restrictive rigidities consist of those regulatory rigidities that prevent matches from responding flexibly to their economic state. More precisely, a worker-firm match in a fully flexible economy has the option to adjust features of the match, such as time and hours of work or type of contract, etc., to respond optimally to its state. However, regulations in a rigid environment may restrict an active firm to adjust those different features of the match. Adding all of these features together to a Mortensen-Pissarides search and matching model makes the model very complex, also one must take stand about the timing and the bargaining methods over each of this features. Since my goal here is not to study the allocative equilibrium of those features, I consider a reduced form approach to introduce an aggregate index of all those rigidity indices into the model. Another advantage of this approach is that it keeps the door open for using other indices fitting in the category of restrictive regulations.

I assume that in a flexible economy a match optimally arranges set up of production requirements, including the working hours, working days, type of contract, etc., corresponding to its state. The basic idea of modeling the restrictive rigidities is that the set of flexible economy arrangements nests those of a less flexible economy. Therefore, a firm in a rigid economy can do at most as good as its counterpart in the flexible economy. When the regulatory restrictions are binding the firm in the rigid economy might gain less profit relative to its counterpart firm in the flexible economy. I assume the regulations are not contradictory, in the sense that there is a non-empty set of production levels, \( p^{ref} \), from the point of view of the regulator such that the regulations are in favor of this reference point. In other words, if a firm produces at that reference level then it is not burdened by the regulations. One can think of it as a very regular match in a rigid economy with the typical work arrangements in terms of working hours, working days and the type of contract
which both parties would agree on the same arrangements if they were active in the flexible economy. However any deviation from this reference point is penalized by regulations in the rigid economy. In the following clarifying examples, for simplicity I assume that the rigid economy has no firing costs but it has other restrictive regulations. Suppose a match faces a low production state (a low combination of idiosyncratic and aggregate shock). If the firm operates in a flexible economy it may find it optimal to keep on producing, for example, with few hours of worked per period or under a temporary contract. However the optimal amount of work or temporary contracts in the rigid economy may be restricted by law. Therefore, if the firm is going to produce, it has to produce its state production by more paid hours than optimal, which translates to higher costs\(^9\). Since there is no firing cost, there would be levels of productivity that a firm in the flexible economy optimally decides to produce but counterpart firm in the rigid economy optimally decides to dissolve the match, because of excess cost implied by the law. Another example of binding regulations in bad times (in the real world, and not in the model) could be a situation where the production required a sunk cost, e.g. a stock of capital. Therefore, a dissolving match that generates slightly negative surplus may generate positive surplus by more days/hours of work, if hourly wages do not rise too much, however again this work arrangement could be prohibited by the law. On the other extend, in a very good productivity level, the firm in a flexible environment again can decide to produce through any optimal arrangements, but in the rigid economy the firm may not extract all potential profit by the restrictive regulations on night work, working days, working at holidays and temporary and fixed term contracts, and so on. I model this lost profit as a virtual cost deducted from the optimal profit in the flexible economy.

Denote by \(R(.)\) the (proportional) regressive effect of the restrictive regulations on the profit, therefore the revenue of a firm, with the potential output of \(z_t x_t\), net of the subtractive effects of the restrictive regulations is of the form

\(^9\)Wages generally could change by hours of work to a large extent, but the argument may hold if there is kind of wage stickiness. Particularly restrictive regulations may bind the negotiated wages also, which seems to be the case at least for many European countries.
\[ p(z_t x_t) = \begin{cases} 
  z_t x_t, & \text{if } z_t x_t \in p^{ref} \\
  (1 - R(\Delta(z_t x_t, p^{ref}))) z_t x_t, & \text{otherwise} 
\end{cases} \quad (1.5) \]

where

\[ \Delta(z_t x_t, p^{ref}) = \begin{cases} 
  z_t x_t - \max(p^{ref}), & \text{if } z_t x_t > \max(p^{ref}) \\
  z_t x_t - \min(p^{ref}), & \text{if } z_t x_t < \min(p^{ref}) 
\end{cases} \]

\( p(.) \) can be interpreted as the reduced or the virtual production function of the firm in the rigid economy. The implicit assumption is that \( R(.) \) is a function of the state of the firm, not the realized output, i.e. firm cannot reduce the implied restrictive cost by choosing a different production level. Therefore, if the match finds it optimal to produce, it also optimally decides to produce at \( z_t x_t \) and enjoys the net revenue of \( p(z_t x_t) \). Notice that if the \( p(.) \) is monotonically increasing, a firm in the rigid economy indeed decides to produce at the production frontier \( z_t x_t \). Also notice that equation 1.5 can be written as a single line equation with \( R(0) = 0 \), and \( \Delta = 0 \) if \( z_t x_t \in p^{ref} \).

I assume that \( R(\Delta) \) is second order differentiable at non-zero values of \( \Delta \), with the following characteristics

\[ 0 \leq R(\cdot) \leq 1 \]
\[ R' \geq 0, \ R'(0) = 0 \]
\[ R'' \geq 0 \]

The first condition insures that there would be a non negative left over after applying rigidities i.e. a positive production/profit is achievable after applying rigidities. The second condition tells that very close but outside of the \( p^{ref} \) the cost of applying rigidities is very
small, and the third condition means that the more the deviation of the firm potential production from the reference point, the larger the induced loss by rigidities is.

Figure 1.3 illustrates an arbitrary virtual production function in a rigid economy. The horizontal line is the state of productivity and the vertical line indicates the realized production according to the virtual production function. A firm operating in a flexible economy for any state of productivity fulfills all the production, hence its locus would be the 45 degree line. A firm in the rigid economy can extracts all the rent only if it is operating in the reference state, but for all other states the virtual production is less than the productivity state. The further the productivity deviates from the reference state, the more the deduction.

After explaining the environment, I write down the value functions. The Bellman value function of being an unemployed worker, $U_t$, and a new and old employed worker, $W_t^N$ and $W_t$ respectively, are as followings

\[ U_t = b + \beta E_t \{ F_t W_{t+1}^N + (1 - F_t) U_{t+1} \} \]  
\[ (1.6) \]

\[ W_t^N = w_t^N + \beta E_t \left\{ (1 - s) \left[ \lambda W_{t+1} (x^N) + (1 - \lambda) \int_{\xi_{t+1}}^{x^N} W_{t+1} (x) + (1 - \lambda) G(\xi_{t+1}) U_{t+1} \right] \right\} + s^x U_{t+1} \]  
\[ (1.7) \]

\[ W_t (x) = w_t (x) + \beta E_t \left\{ (1 - s) \left[ \lambda W_{t+1} (x) + (1 - \lambda) \int_{\xi_{t+1}}^{x^N} W_{t+1} (x) + (1 - \lambda) G(\xi_{t+1}) U_{t+1} \right] \right\} + s^x U_{t+1} \]  
\[ (1.8) \]

where $w_t^N$ and $w_t (x)$ are earnings of a new and old worker with specific productivities
Figure 1.2: Time line of events in the model

Figure 1.3: An arbitrary virtual production function in a rigid economy (green line) vs. production function in a flexible economy (blue line)
of $x^N$ and $x$ respectively. The match specific productivity threshold $x_{t+1}$ is such that a match with productivity below this threshold dissolves, since such a match produces negative surplus.

The value function of a new filled vacancy, $J_t^N$, an old match, $J_t$, and the vacancy value satisfy

$$V_t = -c + \beta E_t \{ Q_t J_{t+1}^N + (1 - Q_t) V_{t+1} \}$$

(1.9)

$$J_t^N = p_t (z_t x^N) - w_t^N$$

$$+ \beta E_t \left\{ (1 - s^x) \left[ \lambda J_{t+1} (x^N) + (1 - \lambda) E_{t+1} J_{t+1} (x) + (1 - \lambda) G(x_{t+1}) (V_{t+1} - F) \right] \right\}$$

(1.10)

$$J_t (x) = p_t (z_t x) - w_t$$

$$+ \beta E_t \left\{ (1 - s^x) \left[ \lambda J_{t+1} (x) + (1 - \lambda) E_{t+1} J_{t+1} (x) + (1 - \lambda) G(x_{t+1}) (V_{t+1} - F) \right] \right\}$$

(1.11)

I assume free entry condition for firms, therefore firms post vacancies while there is a positive rent. This gives

$$V_t = 0$$

(1.12)

**Wage setting.** Firms and workers each period negotiate over wages, and split the surplus of the match according to the Nash bargaining rule, where a worker enters into the bargaining with a bargaining power of $\pi$. Surplus of a match for the worker is $W_t^N(x) - U_t$,
if the match is new, and $W_t(x) - U_t$ otherwise. The surplus of a new match for a firm is $J_t^N$, while a continuing match provides a surplus of $J_t + F$, which indicates that in case of no agreement firm has to pay the firing cost $F$. The FOCs of maximization problems read

$$\pi J_t^N = (1 - \pi)(W_t^N(x) - U_t) \quad (1.13)$$

$$\pi (J_t + F) = (1 - \pi)(W_t(x) - U_t). \quad (1.14)$$

Substituting firm and worker’s value functions into the equations and solving for the wages one can derive

$$w_t^N = \pi ((1 - R) z_t x^N + c\theta_t) + (1 - \pi) b - \pi \beta (1 - s^r) F \quad (1.15)$$

$$w_t(x) = \pi ((1 - R) z_t x + c\theta_t) + (1 - \pi) b + \pi (1 - \beta (1 - s^r)) F \quad (1.16)$$

Remember that rigidities are non-linear functions of potential production, by deviating from the reference production the wages become smaller than the flexible economy. In the absence of the firing costs the two equations are similar. However if there are firing costs in the economy, as it is well-known in the literature, the firm and worker bargain over the firing costs as it is presented in the equations. At the first period, when no firing cost applies, workers agree on a lower wage, compensating future possible firing cost in case of endogenous separation. While later, any endogenous separation would cost the firm $F$, if no separation happens firm and worker share the amount, again considering the fact that a future break would cost $F$ for the firm. Therefore a worker in a continuing match enjoys the positive additional last term in his wage.

I close the model by unemployment dynamics

$$u_{t+1} = S_t (1 - u_t) - F_t u_t \quad (1.17)$$
where $S_t = s^x + (1 - s^x) G(x_t)$ is the total separation probability at time $t$.

1.4.2 Unemployment flows analysis

I apply EHS unemployment flows decomposition to the model. Here I summarize the basic assumptions and equations of the EHS model. The EHS decomposition links the discrete time observations to the unemployment rate which assumes to evolve in a continuous time frame work. Unemployment evolution in the EHS reads

$$\frac{du}{dt} = s_t (1 - u_t) - f_t u_t$$  \hspace{1cm} (1.18)

where $s_t$ and $f_t$ are the flows hazard rates corresponding the probabilities $S_t$ and $F_t$, respectively. Therefore the unemployment at the end of a period is

$$u_t = \lambda_t u_t^* + (1 - \lambda_t) u_{t-1}$$  \hspace{1cm} (1.19)

where

$$u_t^* = \frac{s_t}{s_t + f_t}$$  \hspace{1cm} (1.20)

denotes steady-state unemployment rate, and

$$\lambda_t = 1 - e^{-(s_t + f_t)}$$  \hspace{1cm} (1.21)

is the rate of convergence towards the steady-state. If the rates are high $\lambda_t$ is close to one, the unemployment adjusts quickly to its steady-state and $u_t^*$ is a good approximation of the unemployment rates at each period. This is the case of U.S. which motivates the use of steady-state decomposition as proposed by Fujita and Ramey (2009). However, for most of the countries the flows rates are not high enough to make $\lambda_t$ close to one, hence the $u_t$ for those countries depends on both $u_t^*$ and $u_{t-1}$ (eq. 1.19). This is the basic idea for non-steady state decomposition method of EHS. They decompose the unemployment fluctuations into three components, contribution of job finding rates, separation rates and
past unemployment, $\beta_f$, $\beta_s$ and $\beta_0$ respectively.

\[
\beta_f = \frac{\text{cov}(\Delta \ln u_t, C_{ft})}{\text{var}(\Delta \ln u_t)}, \quad \beta_s = \frac{\text{cov}(\Delta \ln u_t, C_{st})}{\text{var}(\Delta \ln u_t)}, \quad \beta_0 = \frac{\text{cov}(\Delta \ln u_t, C_{0t})}{\text{var}(\Delta \ln u_t)} \tag{1.22}
\]

where

\[
C_{ft} = \lambda_{t-1} \left[ - (1 - u_{t-1}^*) \Delta \ln f_t + \frac{1 - \lambda_{t-2}}{\lambda_{t-2}} C_{ft-1} \right]
\]

\[
C_{st} = \lambda_{t-1} \left[ (1 - u_{t-1}^*) \Delta \ln s_t + \frac{1 - \lambda_{t-2}}{\lambda_{t-2}} C_{st-1} \right]
\]

and

\[
C_{0t} = \lambda_{t-1} \frac{1 - \lambda_{t-2}}{\lambda_{t-2}} C_{0t-1}
\]

with $C_{f0} = 0$, $C_{s0} = 0$, and $C_{00} = \Delta \ln u_0$.

### 1.5 Calibration

#### 1.5.1 Flexible economy

At the first step, I target the U.S. as the benchmark of a flexible economy, since according to the data set the U.S. has no firing costs besides an employment rigidity index of zero. The frequency of the model is monthly, which coincides with the highest observed frequency of unemployment data (for the U.S.). This implies that the shortest period of unemployment is one month, later it is discussed how I treat this shortcoming of the model, in measuring the flows.

Table 1.3 summarizes choices of parameters for calibration of the flexible economy. The discount factor $\beta$ is chosen to be consistent with an annual interest rate of 4%. Hall and Milgrom (2008) estimate the unemployment benefit in the U.S. equal to 0.71. Following Fujita and Ramey (2012) $G(x)$ is lognormal with parameters $\mu_x$ and $\sigma_x$ when $x < x^N$,.
and \( G(x^N) = 1 \). \( \mu_x \) set to zero, representing that average of match specific productivities is approximately one. Standard deviation of match specific productivities, \( \sigma_x \) is taken from Sala et al. (2012). They set \( \sigma_x \) equal to 0.2 as an intermediate value within the range of 0.1 (den Haan et al., 2000) and 0.4 (Trigari, 2009) used in the literature. This is also within the range of 0.16 and 0.214 used by Fujita and Ramey (2012). The parameters of AR(1) process of aggregate productivity is chosen such that the quarterly average of monthly simulated data of model matches the variance and autocorrelation of the cyclical component of labor productivity data. Using the U.S. data I find a quarterly autocorrelation and variance of 0.745 and 0.0022 respectively. This requires \( \rho \) and \( \sigma_e \) equal to 0.995 and 0.001 respectively.

The matching function elasticity, \( \alpha \) is set to 0.7. This is close to Shimer’s (2005a) choice of 0.72. Some authors argue that this is too high (see for example Mortensen and Nagypal, 2007), however, recently Justiniano and Michelacci (2011) using a Bayesian approach estimate it 0.79 for the U.S. in a rich RBC model augmented by search. They also find that for their sample of six countries (France, Germany, Norway, Sweden, the U.K., and the U.S.) this parameter lies in the range of 0.69 (Germany) and 0.82 (France).

\( x^N \) in each case is set such that provides average productivity of 1. To calibrate the rest of parameters five statistics of the U.S. economy are targeted. First, I target average unemployment rate of 6%. Shimer (2005a) using CPS data calculates an average job finding probability of 0.45. I choose this as the second target. Third, Hagedorn and Manovskii (2008) calculate and target an average tightness of 0.6 for the U.S., I use this as the third target. And finally I target the contribution of job finding and separation rates in the unemployment volatilities, 85% and 16%, as reported by EHS. These provides the set of five targets. To match these facts I calibrate exogenous separation \( s^x \), the matching function multiplier \( A \), the cost of posting a vacancy \( c \), the bargaining power of workers \( \pi \), and the persistency of match quality \( \lambda \). I set the \( \lambda \) to 0.383, the monthly equivalent choice of Fujita and Ramey (2012) is about 0.7; for comparison their choice implies a mean waiting time of about three months between switches of match specific productivity while
this statistic turns out to be slightly less than two months in my calibration. By and large, the rest of four parameters are within the ranges used in the literature.

It is worth noting that data restrictions allow EHS to infer flows at annual frequency. To estimate the monthly rates $s_t$ and $f_t$ they assume that flow hazards are constant within years. Therefore eq. 1.19 and 1.21 change to

$$u_t = \lambda_t u_t^* + (1 - \lambda_t) u_{t-12}$$

$$\lambda_t = 1 - e^{-12(s_t + f_t)}$$

Substituting the later in the former gives

$$u_t = \left(1 - e^{-12(s_t + f_t)}\right) u_t^* + e^{-12(s_t + f_t)} u_{t-12}$$

(1.23)

EHS obtain the monthly job finding hazard rate from the monthly job finding probability, which in turn calculates from unemployment and short term unemployment data. Using the non-linear equation 1.23, they obtain the separation hazard rate as well. My model is monthly and allows to directly use eq. 1.19 and 1.21, however since I target EHS results I prefer to use eq. 1.23. Since the exact monthly flow rates from simulation of the discrete-time model are not compatible with this equation, I re-construct the corresponding monthly flow rates from simulation. For each year the monthly job finding rate is calculated by averaging rates of the first three simulated months$^{10}$, then using the unemployment data of start and end of that year, the separation rate calculated from eq. 1.23. Table 1.4 shows the performance of the calibrated model in matching the targets.

1.6 Results

$^{10}$The first three months averaging provides similar results of $\beta_f$ compare to case one applies the monthly data to equations 1.19 and 1.21.
Table 1.3: Calibration of the flexible economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.9967</td>
</tr>
<tr>
<td>Unemployment benefit</td>
<td>$b$</td>
<td>0.71</td>
</tr>
<tr>
<td>Persistence of the aggr. prod.</td>
<td>$\rho$</td>
<td>0.995</td>
</tr>
<tr>
<td>St. dev. of aggr. prod. shock</td>
<td>$\sigma_\varepsilon$</td>
<td>0.001</td>
</tr>
<tr>
<td>St. dev. of idiosyncratic shock</td>
<td>$\sigma_x$</td>
<td>0.2</td>
</tr>
<tr>
<td>Mean of the idiosyncratic prod.</td>
<td>$\mu_x$</td>
<td>0</td>
</tr>
<tr>
<td>Highest value of idiosyncratic prod.</td>
<td>$x^N$</td>
<td>1.22</td>
</tr>
<tr>
<td>Matching func. multiplier</td>
<td>$A$</td>
<td>0.525</td>
</tr>
<tr>
<td>Elasticity of matching func.</td>
<td>$\alpha$</td>
<td>0.7</td>
</tr>
<tr>
<td>Worker’s bargaining power</td>
<td>$\pi$</td>
<td>0.49</td>
</tr>
<tr>
<td>Exog. separation prob.</td>
<td>$s^x$</td>
<td>0.03</td>
</tr>
<tr>
<td>Vacancy posting cost</td>
<td>$c$</td>
<td>0.443</td>
</tr>
<tr>
<td>Persistence of idiosyncratic shock</td>
<td>$\lambda$</td>
<td>0.383</td>
</tr>
</tbody>
</table>

Frequency: Monthly.

Table 1.4: Flexible economy: Calibration targets and matches

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. Productivity</td>
<td>norm. to 1</td>
<td>0.994</td>
</tr>
<tr>
<td>Quarterly A.C. of Productivity</td>
<td>0.745</td>
<td>0.747</td>
</tr>
<tr>
<td>Quarterly St. dev. of Productivity</td>
<td>0.0022</td>
<td>0.0021</td>
</tr>
<tr>
<td>Ave. Unemployment</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Ave. J.F.P.</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Ave. Tightness</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>0.85</td>
<td>0.86</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Table 1.5: Flexible economy: Model performance

Panel A: Data (Source: Shimer, 2005a)

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
<th>v/u</th>
<th>f</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. dev.</td>
<td>0.190</td>
<td>0.202</td>
<td>0.382</td>
<td>0.118</td>
<td>0.075</td>
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<tr>
<td>Quart. A.C.</td>
<td>0.936</td>
<td>0.940</td>
<td>0.941</td>
<td>0.908</td>
<td>0.733</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>u 1</th>
<th>-0.894</th>
<th>-0.971</th>
<th>-0.949</th>
<th>0.709</th>
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<tbody>
<tr>
<td>v</td>
<td>1</td>
<td>0.975</td>
<td>0.897</td>
<td>-0.684</td>
<td></td>
</tr>
<tr>
<td>Corr.</td>
<td>v/u</td>
<td>1 0.948</td>
<td>-0.715</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>1</td>
<td>-0.574</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Model Performance

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
<th>v/u</th>
<th>f</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. dev.</td>
<td>0.068</td>
<td>0.012</td>
<td>0.012</td>
<td>0.004</td>
<td>0.008</td>
</tr>
<tr>
<td>Quart. A.C.</td>
<td>0.851</td>
<td>0.850</td>
<td>0.902</td>
<td>0.902</td>
<td>0.854</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>u 1</th>
<th>-0.345</th>
<th>-0.801</th>
<th>-0.801</th>
<th>0.314</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>1</td>
<td>0.674</td>
<td>0.674</td>
<td>0.285</td>
<td></td>
</tr>
<tr>
<td>Corr.</td>
<td>v/u</td>
<td>1 1.000</td>
<td>-0.155</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>1</td>
<td>-0.155</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.5 compares simulated flexible economy with data. Panel A reports the standard and deviations (auto)correlations in the data as reported by Shimer (2005a). Panel B shows the simulation results of my model. Quarterly variables are constructed by averaging monthly variables. Notice that the model, like other standard MP search and matching models, is not immune to Shimer (2005a) puzzle. As panel B of table shows the model generates too little volatility of unemployment, which is accompanied by little volatility in job finding and separation rates. However, the model is able to generate considerably higher volatilities than the basic model of Shimer (2005a), mainly because I chose a higher unemployment benefit and also because of idiosyncratic productivities.

Having the flexible economy model in hand, I run some experiments.
1.6.1 Adding firing costs to the flexible economy

In the first experiment I add different levels of firing costs to the benchmark flexible economy. Figure 1-4 depicts the evolution of $\beta_f$ for different levels of firing costs. Consistent with the statistical evidence at table 1.1, the model predicts that higher firing costs increase (decrease) the contribution of job finding (separation) rate to unemployment fluctuations. This is also consistent with empirical evidence of Messina and Vallanti (2007) that firing costs dampen the firm’s response of job destruction to the cycle. A decrease in volatility of job destruction and job creation after an increase in firing costs theoretically has already been shown by Garibaldi (1998) and Thomas (2006). However, I am not aware of any study that directly explores what happens to the relative contributions to unemployment fluctuations. According to figure 3, for the calibrated flexible economy, adding firing costs of only 0.04 percent of average wage is enough to make the job finding rates’ changes responsible for almost all fluctuations in unemployment.

Table 1.6 reports what happens within the simulated models. Consider a match with a productivity level slightly below endogenous separation threshold in an economy with no (trivial) firing costs. Everything equal, if the firing costs increases, the cost of dissolving the match increases more than the cost of production (wage). Hence, such a marginal match finds it optimal to produce if the firing costs increase. This dampens the volatility of separation rates, which in turn lowers the volatility of unemployment. On the other extreme, increasing the firing costs decreases the value of new matches as well, making firms less willing to post vacancies. Since in the model all matches start at highest idiosyncratic productivity level, given the parameters, whenever a worker meets a vacancy regardless of aggregate productivity shock they find it optimal to produce, however the volatility of job finding rates decreases.
Figure 1.4: Adding firing costs to the flexible economy

Table 1.6: Adding firing costs to the calibrated flexible economy

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{F}$ (% of average wage)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$u$ (%)</td>
<td>6.3</td>
</tr>
<tr>
<td>-ln $f$</td>
<td>0.524</td>
</tr>
<tr>
<td>-ln $s$</td>
<td>3.216</td>
</tr>
<tr>
<td>sd($u$)</td>
<td>0.16</td>
</tr>
<tr>
<td>sd(ln $f$)</td>
<td>107</td>
</tr>
<tr>
<td>sd(ln $s$)</td>
<td>206</td>
</tr>
</tbody>
</table>
1.6.2 Adding restrictive rigidities to the flexible economy

I assume $R(\Delta^-)$ of equation 1.5 takes the following functional form of

$$R(\Delta^-) = K(\Delta^-)^2 \quad (1.24)$$

For simplicity, I assume that the absolute value of loss is symmetric both sides of the $p_{\text{ref}}$, i.e. for any deviation $\Delta^+ = |\Delta^-|$ the proportional loss function is

$$R(\Delta^+) = \min\left(\frac{p_{\text{ref}} - \Delta^+}{\max(p_{\text{ref}}) + \Delta^+}R(\Delta^-)\right) \quad (1.25)$$

Absolute value symmetricity helps to have a monotonically increasing $p(.)$ for a wide range of $K$’s. $K$ is the parameter for entering different levels of rigidities. The case of $K = 0$ is no rigidity case. Assigning a positive number to $K$ generates a level of restrictive rigidities. For any positive $K$ the functional form implies that further deviations from $p_{\text{ref}}$ are restricted more by regulations, therefore accompany with higher lost profit and/or imposed operational cost. A larger $K$ represents more stringent laws.

Figure 1-5 shows the results of adding different levels of restrictive rigidities ($K$) into the flexible model. The larger the level of restrictive rigidities, the higher (lower) the $\beta_s$ ($\beta_f$). Adding a restrictive rigidity level of $K$ almost equal to 0.05~0.06 to the flexible economy gives relative contributions of an average European economy in the table 1.1. Figure 1-6 illustrates what a virtual production function with $K = 0.06$ stands for. It shows that for the calibrated flexible economy, relatively small rigidity is enough to generate the European case.

Table 1.7 provides a more detailed look into the simulations. Introduction/increase of restrictive rigidities pushes the marginal match, which before was indifferent between production and exit, to leave the market, because it works as a cost; However given the set up this situation prevails among more firms when the aggregate productivity shock is

---

11 The monotonicity is violated easier if I assume proportional symmetricity, i.e. $R^h(\Delta) = R^l(\Delta)$. 
Table 1.7: Adding restrictive rigidities to the calibrated flexible economy

<table>
<thead>
<tr>
<th>K</th>
<th>0</th>
<th>0.02</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>6.3</td>
<td>6.4</td>
<td>6.4</td>
<td>6.4</td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>(%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-lnf</td>
<td>0.524</td>
<td>0.524</td>
<td>0.525</td>
<td>0.527</td>
<td>0.527</td>
<td>0.528</td>
</tr>
<tr>
<td>sd(u)</td>
<td>0.16</td>
<td>0.18</td>
<td>0.20</td>
<td>0.26</td>
<td>0.28</td>
<td>0.30</td>
</tr>
<tr>
<td>sd(lnf)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\times e^{-4})</td>
<td>102</td>
<td>101</td>
<td>101</td>
<td>100</td>
<td>102</td>
<td>101</td>
</tr>
<tr>
<td>sd(lns)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\times e^{-4})</td>
<td>126</td>
<td>155</td>
<td>182</td>
<td>274</td>
<td>305</td>
<td>360</td>
</tr>
</tbody>
</table>

low, simply because in a bad aggregate state a larger mass of matches is located in the neighborhood of threshold idiosyncratic productivity level. Those discrepancies intensify the volatility of separations compare to benchmark set up, and magnifies the contribution of separations to the unemployment volatilities. It is worth noting that these restrictive rigidities increase the volatility of unemployment as well as its level.

1.6.3 Cross country performance

In this experiment I like to quantify the effects of restrictive rigidities on the unemployment dynamics. I proceed by introduce firing costs and rigidities simultaneously to the benchmark flexible economy and fitting the cross country data. I simply substitute \(\mathcal{F}\) in the model with the monthly amount of the firing costs. Entering the rigidity index is not a trivial job. To deal with introduction of indices of rigidity into the model, I consider the same functional form as equations 1.24 and 1.25. I assume that the multiplier, \(K\), is a linear transformation of the indexes, i.e. in equation 1.24, \(K = \zeta R\); hence, \(\mathcal{R} (\Delta^{-}) = \zeta R (\Delta^{-})^2\).

With regard to the reference production level, I assume that the restrictive regulations
Figure 1-5: Adding Rigidities to the flexible economy
Figure 1.6: Illustration of virtual production function.

(K = 0.06)
never bind an average worker with productivity one.

Figure 1.7 illustrates the results for $\zeta = 0.19$. Considering the simplicity of the model, the overall fit is reasonable; the least successful cases are Ireland and Norway, where the model overpredict the contribution of outflows rates, and the underprediction in the case of Italy. In particular, the model replicates the pattern observed in the data well; the correlation between the predicted contributions of job finding rates, $\hat{\beta}_f$, and the data is about 0.7. By introducing only two parameters, the model is able to explain about 48% of cross country variations. Recall that the firing costs are given, so the only calibrated parameter is the multiplier of the rigidity index, $\zeta$.

The role of unemployment benefit. It is worth noting the role of changing unemployment benefits in the model. Almost all countries in the sample have higher unemployment replacement ratio than the benchmark economy (the U.S.). Increasing the unemployment benefit in the model lowers $\beta_f$, since the surplus is relatively smaller. As the surplus is smaller, firms are more sensitive to changes in the surplus, and more low quality jobs dissolve. This all resembles Hagedorn and Manovskii (2008) remedy to the unemployment volatility puzzle of the Mortensen and Pissarides search model (Shimer, 2005a). Costain and Reiter (2008) investigate this behavior of MP search and conclude that the model exhibits the volatility puzzle at low values of unemployment benefit, while it is too sensitive to the policy for higher values. The model in this paper, which is basically a MP search model, has the same shortcoming. This is the main reason why I cannot provide a simple quantitative cross country analysis based on the differences in unemployment replacement ratio. As described in the calibration section, in the case of the flexible economy, I took Hall and Milgrom (2008) value of 0.71. With regard to using the same value for all the countries I should say that this value (0.71) lies well in the range of unemployment benefits for the six countries studied by Justiniano and Michelacci (2011)$^{12}$, as well as of the nine

$^{12}$From 0.69 (Germany) to 0.82 (France); with 0.79 for the US.
countries in Murtin and Robin (2014)\textsuperscript{13}. In earlier versions I used different values for the countries, for example using the value proposed by Hobijn and Sahin (2013) for France, but because I did not calibrate for each country separately, the model works well only around the targeted economy. This again resembles findings of Costain and Reiter (2008).

Finally, related to the last point, another important issue with this study is the interaction of unemployment benefit with the functional form of productivities at the firing threshold. In a sense, the results hinge on the functional form of productivities, especially the left tail of the productivity. If threshold productivities are closer to the center (i.e. higher average unemployment rate) then only because of using lognormal distributions model generates higher volatility of separations. As Thomas (2006) argues there is no reason to believe that all countries in this study have the same distribution of productivities. Estimations of worker’s heterogeneity in the calibration of Murtin and Robin (2014) seems to support cross country differences in workers distributions.

1.7 Conclusion

In this paper I studied how different categories of labor market regulations can affect the dynamics of unemployment. I find that restrictive regulations, a class of labor market regulations that disturb flexible adjustment of work arrangements, could increase the contributions of separations to unemployment fluctuations. I use cross-country data on restrictions on working time and hours, and on signing different types of contracts as a representative of this class of regulations. I use the first principal component of these variables as the restrictive regulative index. The more restrictions on the work arrangement, the higher of inflows into unemployment contributes to unemployment fluctuations. Higher unemployment benefits play the same role as restrictive regulations. On the contrary, higher firing costs tend to amplify the importance of inflow rates.

Simulations based on a standard Mortensen-Pissarides search and matching model sug-

\textsuperscript{13}From 0.683 (Germany) to 0.834 (Portugal); with 0.693 for the US.
gests that the main mechanism that firing costs affect the dynamics of unemployment is through termination decision. Firing costs dampen sensitivity of the response of a match to productivity shocks, since firms are less willing to layoff the redundant workers. This decreases the contribution of outflows to unemployment fluctuations. In contrast, a higher unemployment benefit provides a larger opportunity cost of employment. This makes a match more vulnerable to endogenous break, which in turn corresponds to a larger contribution of outflows to the fluctuations of unemployment. My proposed explanation for the observed effect of restrictive regulations –in the empirical part as well as in the simulations– has the same flavor. The restrictive regulations create more fluctuations in separations, since they impose additional operational costs to the threshold firms. In the bad times, a match may need to change some of its work arrangements to survive. However, restrictive regulations could make those arrangements too expensive -or even infeasible- for the match. Consequently, a firm bounded with those restrictive regulations, even in the presence of high firing costs, finds it optimal to terminate a match to not bear the excess costs implied by restrictive regulations.

Consistent with this explanation, I propose a reduced form framework to introduce the aggregate restrictive rigidities into Mortensen-Pissarides search and matching model. Introducing firing costs and the restrictions separately generates effects in the same direction as expected. Adding both firing costs and the restrictive rigidities simultaneously to model can explain about 48% of cross country variations in unemployment dynamics. As a result, despite of its shortcoming in generating reasonable unemployment volatility, the Mortensen-Pissarides search model is able to explain the described pattern of the sources of the cross country discrepancies in the unemployment dynamics.
Figure 1.7: Cross country performance of the model. Black: real data, Gray: Simulation
Chapter 2

Heterogeneous Workers, Firm Dynamics and the Countercyclicality of Productivity

2.1 Introduction

What explains the worker flows at the establishment-level? Do these micro-level factors have macroeconomic consequences? The non-linear relationship between the growth rate of establishment and the rates of hiring and separation are features of the data that many search models of Mortensen and Pissarides (1994) disregard in one way or another. To acknowledge the facts, I develop a random search model with heterogeneous agents. In addition to explaining the worker flows at the micro-level, at the aggregate level the model generates more realistic business cycle behavior of the average labor productivity (ALP).

Firms play an important role in the outcome of labor markets. Among other things, they have an impact on wage distribution, the employment level and job flows. Traditional search models of Mortensen and Pissarides (1994) and Pissarides (2000), however, by assuming either one-job-one-firm or firms with a constant marginal product of labor disregard the above mentioned facts. As Pissarides (2000) explains, with either of these assumptions the study of employment flows at the firm level is irrelevant. Therefore, there has recently been growing interest in introducing the notion of the firm into search models.\footnote{For examples of the interactions between firms and labor market outcomes, and for a non-exclusive list of recent literature incorporating the notion of firm, see section 2.} \footnote{An additional reason for introducing the notion of firm is due to the difficulty that traditional search models have in matching the aggregate behavior of unemployment (e.g. Shimer (2005), Costain and Reiter (2008)). For a detailed study of the effects of introducing large firms in search models see Hawkins (2011).}

Furthermore, Davis, Faberman, and Haltiwanger (2012) show that while the majority
of firms are inactive in terms of employment, other establishments exhibit such active employment policies that the three flows of hire, quit and layoff coexist at growing as well as contracting establishments. Figure 2-1 shows that shrinking establishments substantially hire even when they fire relatively many workers; analogously, growing firms shed workers even when their growth rate is high. These patterns are not consistent with what Davis, Faberman, and Haltiwanger refer to as the iron link assumption between job flows and worker flows in the canonical search and matching model of Mortensen and Pissarides (1994). Few models of search with large firms opt to explain the rich set of employment dynamics within the firm instead of using the iron link assumption.

On the other hand, and from a macroeconomic point of view, different studies point out that procyclicality of average labor productivity (ALP) vanished during mid-80’s. The ALP before mid-80’s shows significant positive correlation with the output while afterwards this correlation significantly declines such that the majority of measurements show a negative correlation, if any. At the same time, the correlation of ALP and unemployment substantially increases from significantly negative to significantly positive (Barnichon, 2010). Again canonical search models disregard these facts, by assuming that ALP is the driving force of the economy. In these models, when ALP is high, firms have more incentive to hire. A rise in the ALP boosts output and leads to a fall in the unemployment rate. As a result, the canonical models generate strongly procyclical ALP.

In this paper, I propose a labor search model which aims to reconcile the above mentioned facts; besides explaining the non-linear relationship between worker flows and the growth rate of the firm, the model substantially alleviates the strong procyclicality of ALP in the canonical models. I introduce firm and worker heterogeneity into a search model with large firms. Two-sided heterogeneity endogenously labels the employed as either matched

---

3The iron link assumption refers to the situation in which every separation (hire) reflects a destroyed (newly created) job.

4Using hp-filtered data, Gali and Gambetti (2009) find a cross correlation of 0.61 for pre-84 versus 0.03 for post-84; Gali and van Rens (2010) calculate correlations of 0.40~0.50 at pre-84 compare to -0.15~0.22 at post-84; Berger (2012) applies a rolling correlation and finds a regime change in mid-80’s from significant positive to insignificant (mainly negative); Hawkins finds a negative correlation of -0.13 for 2000-2010 period.
Figure 2-1: Worker flow rates as a function of establishment-level growth. (Source: Davis, Faberman, and Haltiwanger, 2012). Calculations using JOLTS establishment data pooled over 2001Q1–2010Q2. Estimates are employment-weighted averages of the establishment-level growth rates within intervals.
or mismatched. Mismatch is the driving force of quits. It also generates layoffs at growing, as well as hires at contracting firms. When there is a mismatch, either the worker or the firm is not qualified. Hence, the other party breaks the match. When the worker is weak point of the match, she is laid off. If the mismatch is on the side of the firm, the worker quits. More realistic cyclicality of ALP is a result of changes in the composition of employment. Mismatched workers exert low productivity. During expansions due to more hiring there are more mismatched, as a result, ALP does not increase one-to-one with output or employment.

2.2 Related Literature

Several studies underline the impact of firms on the labor market. Among others, Brown and Medoff (1989) and Oi and Idson (1999) show that larger firms pay higher wages (the well-known size-wage relationship). Moscarini and Postel-Vinay (2012) find that large firms have also a stronger negative correlation with unemployment than small firms at business cycle frequency. They show that large firms shed proportionally more jobs in the recessions and create more jobs later in expansions, both in gross and net terms. In a related paper, Kahn and McEntarfer (2012) find that net job creation in high quality firms is more responsive to the business cycle, while gross hires and separations are more responsive to business cycle at low quality firms. They use average pay as the main measure of quality, but also test for other definitions. Haltiwanger, Jarmin and Miranda (2013) document higher exit rates of young firms. Whereas, Brown and Medoff (2003) find that younger firms pay higher wages. Davis, Faberman, and Haltiwanger (2012) show that fast growing firms have higher vacancy filling rates. Faberman and Nagypal (2008) figure out that quit rate decline with establishment growth.

As I mentioned in the introduction, importance of firms motivated studies on different topics to introduce the notion of firms into search models. A non-exclusive list of papers that introduce firm into search models and their topics includes: unemployment and efficiency (Bertola and Caballero (1994), Smith (1999), Acemoglu and Hawkins (2006),

It is worth mentioning that the dominant framework of wage setting in the large firm literature is based on Stole and Zwiebel (1996) bargaining. However, a growing strand of multi-worker firm literature applies directed search in the manner developed by Menzio and Shi (2010, 2011) in order to exploit the convenient property of Block Recursivity. Kaas and Kircher (2011) develops a block recursive equilibria model to discuss efficiency of search models with large firms, they are able to replicate the fact that growing firms have higher job filling rates. To explain the jobless recovery Schaal (2012) develops a block recursive equilibria model with on-the-job search that exhibits a rich set of flows within firms. His model can account for layoffs in shrinking firms and quits in both expanding and shrinking firms and is consistent with a broad set of described facts about firms.

Faberman and Nagypal (2008) provide evidence that replacement of quitting workers could be an explanation of hires in shrinking firms. Their model also explains the dominance of quits over layoffs at small contractions.

There is a large literature of employment and job flows, however, abstracted from the notion of firm. Rogerson and Pries (2005) develop a learning model in which matches are both experience and inspection goods, to address higher worker turnovers in the US than in Europe, in spite of similar job turnovers. Their model abstracts from the notion of firms, however it has the ingredient of Jovanovic (1979) learning which is also the underlying mechanism for generating layoffs at growing firms in my paper, as pointed out by Davis, Faberman, and Haltiwanger (2012).

Galf and van Rens (2010), Barnichon (2010), Berger (2012) and Nucci and Riggi (2009) use different approaches, but with the common idea that cyclicality of productivity is a
result of effort variation, to explain cyclical behavior of ALP. Galí and van Rens (2010) show that variable effort, and wage rigidity can reduce Barnichon (2010) and Nucci and Riggi (2009) develop New-Keynesian models with variable effort. Regarding the business cycle implications, Berger (2012) is the closest paper to this paper. He uses ex-post heterogeneity in match quality to explain jobless recoveries and acyclicity of productivity. Combining preference shocks with the heterogeneity he is able to explain almost half of the decline in cyclicity of productivity. In comparison, my model generates a rich set of flows within firms, with distinguished quits and layoffs. My model also adopts wage bargaining, whereas he assumes a competitive wage to all workers regardless of their match quality. Hence, my model generates wage dispersion and size-wage effect.

2.3 Model

2.3.1 Environment

Time is discrete. The economy consists of a measure one mass of workers and a fixed mass of firms, $M_f$. Both firms and workers are risk-neutral and discount the future at rate of $\beta$. There are two types of workers with inherent high ability, $h$-workers, and low ability, $l$-workers, with respective mass of $M_h$ and $M_l = 1 - M_h$. I interchangeably refer to high and low ability workers as good and bad workers.

2.3.2 Production

Firms are multi-worker and own the technology to transform labor input into output. The labor input of a firm is the linear sum of efficiency units of its worker, hence the workers of different qualities are perfectly substitutable in production. There are two types of high quality (good) and low quality (bad) firms, $j \in \{h, l\}$, with masses $M_h^f$ and $M_l^f = M_f - M_h^f$ respectively. Denote by $x(i, j)$ the efficiency of an $i$-worker employed at a $j$-firm. Bad workers are equally efficient at good and bad firms, $x(l, h) = x(l, l)$. Good firms extract all ability of either type of workers hence efficiency of an $i$-worker hired by a $h$-firm, $x(i, h)$,
Table 2.1: Realized efficiency of workers at different firms

<table>
<thead>
<tr>
<th></th>
<th>h-firm</th>
<th>l-firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>h-worker</td>
<td>$x_h$</td>
<td>$x_l$</td>
</tr>
<tr>
<td>l-worker</td>
<td>$x_l$</td>
<td>$x_l$</td>
</tr>
</tbody>
</table>

cointides with her ability, whereas bad firms are unable to fully exploit ability of good workers. For simplicity, in line with Shi (2002), I assume that good workers in bad firms are as productive as bad workers, i.e. $x(h, l) = x(l, l)$. Therefore, $x(i, j)$ takes only two values of $x_h$ and $x_l$. Table 2.1 summarizes the realized efficiencies of workers in different firms.

The production technology of a type-$j$ firm which employs $n$ efficiency units of labor takes the form of

$$y = zF(n)$$

where $z$, productivity of the firm, is the composed of an aggregate productivity common to all firms, $z^t_a$, as well as an idiosyncratic productivity, $z^t_t$:

$$z = z^t_a z^t_t$$  \hspace{1cm} (2.1)

As in Hawkins (2011), I assume that all productivity components take their values from finite sets of productivities, i.e. $z^t_a \in (z_1 < z_2 < \ldots < z_{m_a})$ and $z^t_t \in (z_1 < z_2 < \ldots < z_{m_t})$. Aggregate productivity follows a first order Markov process, $G^a(z^t_{a+1}|z^t_a)$. The idiosyncratic productivity shock of a firm also follows a first order Markov process, $G(z^t_{t+1}|z^t_t)$; the shock is iid across firms and orthogonal to aggregate productivity.

$F(n)$ exhibits diminishing returns to labor, so that each firm decides to have a bounded size. Technically, I assume $F(n) = n^\gamma$ where $\gamma < 1$. As a result output of firm which has
units of $i$-worker takes the form

$$y(l_i, l_h, z; j) = zn^\gamma = z(x_i l_i + x(h, j) l_h)^\gamma$$  \hspace{1cm} (2.2)

### 2.3.3 Timing

The sequence of events is as follows. Firms start the period with the stock of employment inherited from past period, $(l_i, l_h)$. At the beginning of every period aggregate and idiosyncratic productivity shocks realize. Employed workers decide whether to quit their jobs. There is no on-the-job search in the model, hence to find a new job an employed worker must quit and join the job seekers, in which case she can look for a job within the same period. Then firms decide simultaneously on layoffs of each type of their workers and/or posting vacancies. Afterwards job seekers meet vacancies through a random search process. After hiring and at the onset of the production stage firms learn about the type of their new workers, wages are set accordingly and production happens.

### 2.3.4 Search and matching

Search is random and at the time of meeting neither the firms nor the worker can discover the type of the other party. Following den Haan et al. (2000) job seekers, $s_t$, meet vacancies, $v_t$, randomly through the matching function

$$m(s_t, v_t) = \frac{s_t v_t}{(s_t^\alpha + v_t^\alpha)^{1/\alpha}}.$$  \hspace{1cm} (2.3)

Given the matching function job finding and vacancy filling probabilities are

$$f_t = f(\theta_t) = \frac{m(s_t, v_t)}{s_t} = \theta_t (1 + \theta_t^\alpha)^{-1/\alpha}$$  \hspace{1cm} (2.4)

$$q_t = q(\theta_t) = \frac{m(s_t, v_t)}{v_t} = \frac{f(\theta_t)}{\theta_t} = (1 + \theta_t^\alpha)^{-1/\alpha}$$  \hspace{1cm} (2.5)

where $\theta = \frac{v}{s}$ is the labor market tightness.
One advantage of presumed matching function is that it always generates number of matches less than minimum of vacancies and job seekers; as a result this matching function always generates job finding and vacancy filling probabilities smaller than one. It is convenient to define the share of \( i \)-workers in the pool of job seekers, \( P_i \), and tightness associated with \( i \)-workers, \( \theta_i \), as

\[
P_i = \frac{s_i}{\sum_{k=\{h,l\}} s_k},
\]

\[
\theta_i = \frac{v}{s_i}.
\]

\( P_h \) represents the quality of pool of job seekers. The higher \( P_h \) is, the firm expects to eventually hire more of good workers. Finally, firms must pay a flow cost of \( c \) to post a vacancy.

### 2.3.5 Worker’s problem

All unemployed workers share the same unemployment benefit \( b \). In what follows, I skip the time index, instead superscript ‘ represents next period’s values. An unemployed worker of type-\( i \) enjoys the value of being unemployed as

\[
U_i = b + \beta \mathbb{E} \left\{ (1 - f(\theta')) U'_i + f(\theta') W'_i \right\} \quad i \in \{h,l\}.
\]

Value of having a job for a worker of type-\( i \) at a firm of type-\( j \) with \((l_l, l_h)\) workers (at the production stage) and productivity \( z \) is\(^5\)

\[
W_i \left(l_l, l_h, z; j\right) = w_i \left(l_l, l_h, z; j\right) + \beta \mathbb{E} \left\{ (1 - \delta_{ij}) W'_i \right\} + \delta_{ij} \left( (1 - f') U'_i + f' W'_i \right) \quad \right\}
\]

\(^5\)State of an employed worker contains more information but here I shortened for the sake of parsimony. The set of mentioned information here, besides \( \theta \) is enough to determine the wage.
where \( w_i(l_i, l'_i, z; j) \) and \( \delta_{ij} \) are wage and separation rate of an \( i \)-worker at the \( j \)-firm, respectively. Separations occur endogenously due to layoffs and/or quits. If the firm decides to fire workers of a given type, it randomly fires workers, i.e. the chance of being fired in the firm \( j \) is equal for all \( i \)-workers. Notice that separation rate is a function of state of the firm besides the type of the worker, \( \delta_{ij} = \delta_{ij}(l'_i, l'_h, z'; j) \), hence separation rates could be different for good and bad workers employed at the same firm.

The interpretations of the two value functions are straightforward, an unemployed worker enjoys its unemployment benefit, and next period she either remains unemployed or manages to find a job, in that case she enjoys the value of having a job. An employed worker of type-\( i \) receives her wage, and tomorrow she either remains employed or separates from the firm. In the latter case she looks for a new job and with probability \( f' \) she will find one in a firm \( k \).

**2.3.6 Wage setting**

I already mentioned that at hiring firm and worker cannot realize the type of the other party. In the search literature, Nash bargaining is a popular way of setting wages; particularly in multi-worker firm literature the dominant wage setting is Stole and Zwiebel (1996). Stole and Zwiebel’s method is an extension of single-worker firm Nash bargaining in which wage is outcome of bargaining over the marginal surplus; firm negotiates with its worker individually and simultaneously while treating each worker as the marginal worker. The marginal surplus of the match can be decomposed into a current and a continuation value of the marginal match. To have a closed from solution for the wages I depart from the literature of Nash bargaining and substitute the marginal surplus of a match.\(^6\) In particular I remain loyal to bargaining with the marginal worker over current value of the marginal

\(^6\)There is no closed form solution of wages of Stole and Zwiebel (1996) for the case of heterogeneous workers out of the steady state due to complexity of employment policies which in turn complicates the continuation value of the match. Imposing steady state conditions Cahuc, Marque and Wasmer (2008) are able to solve for wages. Among others, Elysh and Michaels (2012) and Hawkins (2011) find closed form solution for wages of multi-worker firms out of steady state in the case that workers are homogeneous.
match, however, I treat the continuation value of a match differently. Since workers do not search on the job every period firm saves the vacancy posting cost of its incumbent workers. I assume that every period workers ask for a share of what I refer to as full replacement cost, that resembles the unpaid hiring cost that firm gains due to its incumbent workers.\footnote{The results are qualitatively robust to reasonable alternative assumptions.} \footnote{The hiring cost that firm pays to hire new workers is sunk by the time of negotiation as it usually holds in canonical search models, therefore it does not appear in the bargaining.} \footnote{l-workers affect the value of the firm in opposite directions. On the one hand they are cheaper, on the other hand they are less efficient and also decrease marginal productivities of all other workers. All in all, the total benefits of hiring a l-worker is ambiguous.}

Full replacement reflects an ideal situation where a good firm posts vacancies to replace its own good worker with a newly hired one without adding any new l-worker. It is ideal in the sense that it is as if the firm is able to distinguish the type of the workers at meeting, hence to fully replace a h-worker, it does not have to hire newly met l-workers.\footnote{The hiring cost that firm pays to hire new workers is sunk by the time of negotiation as it usually holds in canonical search models, therefore it does not appear in the bargaining.} I calculate the full replacement cost as follows. First of all notice that by the time of negotiation the search season is over, therefore the marginal h-worker can only search for a job next period, in that case she will find a job with probability $f$, otherwise she will return to her firm of origin. On the other hand, if the firm is able to detect the type of new matches, it must post $1/(q P_h)$ vacancies at flow cost of $c$ per unit to replace the missed worker. Thus, today full replacement of that worker would cost firm $\beta f c/ (q P_h)$. It is worth mentioning that in the canonical one-job one-firm and multi-homogenous worker firm models, as in Elsby and Michaels (2013), a similar replacement term appears as a result of simplifications of future values of match.

The solution to Nash bargaining between a h-worker and a h-firm is
where \( \eta \in [0, 1] \) is the worker bargaining power. Left hand side bracket contains the marginal worker’s current surplus, i.e. wage net of unemployment benefit. Right hand side bracket includes two main components as discussed before. The first four terms are current surplus of a marginal \( h \)-worker to the firm: a marginal worker devotes marginally to output, receives her wage and also affects the wages of all other workers. The final term in the bracket is the full replacement cost. Hence \( w_h (., h) \) solves the following differential equation

\[
(1 - \eta) \left[ w_h (l_t, l_h, z; h) - b \right] = \eta \left[ \frac{\partial F (l_t, l_h, z; h)}{\partial l_h} - \frac{\partial w_h (l_t, l_h, z; h)}{\partial l_h} \right. - l_h \frac{\partial w_t (l_t, l_h, z; h)}{\partial l_t} - l_t \frac{\partial w_t (l_t, l_h, z; h)}{\partial l_t} \left. + \beta \theta_h \right]
\]

(2.10)

In general, differential equation (2.11) does not have a closed form solution. I assume \( l \)-workers have no bargaining power at any type of firm, hence they are paid \( b \) at any firm. Therefore eq. (2.11) simplifies according to proposition 1.

**Proposition 1**

The bargained wage of a \( h \)-worker at a \( h \)-firm solves the differential equation

\[
w_h (l_t, l_h, z; h) = \eta \left[ \frac{\partial F (l_t, l_h, z; h)}{\partial l_h} - l_h \frac{\partial w_h (l_t, l_h, z; h)}{\partial l_h} + \beta \theta_h \right] + (1 - \eta) b.
\]

(2.11)
Differential equation (2.12) can be solved using factor integral, thanks to Chebyshev theorem on the integration of binomial differentials.

**Proposition 2**

If \( 1/\eta - 1 \) is an integer, the solution for \( w_h(\cdot; h) \) in (2.12) is

\[
w_h = \sum_{k=0}^{1/\eta - 1} M(k) + \beta \eta c \theta_h + (1 - \eta) b
\]

(2.13)

where \( M(k) = z(-1)^k \left( \prod_{m=1}^{k} \frac{1/\eta - m}{\eta + m} \right) \frac{(x_h + x_h l_h)^{\gamma+k}}{l_h(x_h l_h)^k} \).

It is worth noting that in the special case of \( l_l = 0 \), the wage solution in (2.13) reduces to EM’s wage solution \([\text{equation (10)}]\)

\[
w_h(0, l_h, z; h) = \frac{\gamma}{1/\eta - 1 + \gamma} z x_h (x_h l_h)^{\gamma-1} + \beta \eta c \theta_h + (1 - \eta) b.
\]

\( h \)-workers are not distinguishable from \( l \)-workers in a \( l \)-firm, therefore \( l \)-firm pays all its worker the same. Table 2.2 summarizes the wage setting.

**Implications of Wages for Employment Flows**

Notice that incorporating the full replacement cost in the negotiations raises the wage of good workers. Considering bilateral blindness of agents at matching about the type of other party, mentioned wage rise helps to make the \( h \)-worker reluctant to search while employed at a \( h \)-firm, even at low idiosyncratic productivities; this may not hold true if one disregards the replacement cost in (2.10).

\( h \)-workers hired by \( l \)-firms quit after production because they are paid less than their reservation wage. Notice that those \( h \)-workers are not better of by quitting before production.
Table 2.2: Wage setting

<table>
<thead>
<tr>
<th>h-worker</th>
<th>h-firm</th>
<th>l-firm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_h(l_l, l_h, z; h)$</td>
<td>$b$</td>
</tr>
<tr>
<td>l-worker</td>
<td>$b$</td>
<td>$b$</td>
</tr>
</tbody>
</table>

2.3.7 Firm’s problem

Value of a $j$-firm with total and aggregate productivities $z$ and $z_a$ which inherits the stock of $(l_{-l}, l_{-h})$ workers from the past period is equal to expected present discounted value of the firm’s profits that is

$$
\Pi (l_l, l_h, z; j) = \max_{v, l'_l, l'_h} \left\{ zF(l'_l, l'_h; j) - \sum_{i \in \{l, h\}} l'_i w_i (l'_l, l'_h, z; j) - cv + \beta \mathbb{E} \left\{ \Pi (l'_l, l'_h, z'; j) \right\} \right\} \quad j \in \{h, l\}
$$

s.t. \quad l'_l = l_l - t'_l - l'_l + vq(1 - P_h) \\
      l'_h = l_h - t'_h - l'_h + vqP_h \\
      0 \leq t'_l \leq l_l - t'_l, \; \text{given} \; l'_l \; \text{and} \; l'_h

Every period the firm produces output combining labor with its technology. It pays the wages to each type of workers. It also must pay the cost of posted vacancies. Firms maximize their value by deciding on firing each type of their workers $(t'_l, t'_h)$ and also hiring through posting vacancies, $(v)$, given quits of either type of workers, $(l'_l, l'_h)$.

Knowing the wages one can use first order conditions of the firm’s problem (2.14) to achieve the employment policy of each firm.
Employment policy of a \( l \)-firm

By posting vacancies firms hire both type of workers. At \( l \)-firms, newly hired \( h \)-workers after revealing their productivities and negotiation participate in the production of that period only. As described before at the moment of negotiation the search season is over and \( h \)-worker is not better off if quits instead of participating in the production. Hence, the \( h \)-worker quits firm after the production, and \( l \)-firms are left with no \( h \)-worker at the moment of deciding about layoffs and posting vacancies, while \( l \)-workers have no incentive to quit. As a result, the problem of a \( l \)-firm which strats a period with inherited stock of \((l_t, l_h)\) workers is equivalent to the problem of its counterpart firm with \((l_t, 0)\) worker, because all its \( h \)-workers quit, \( l_h^0 = l_h \). With a slight abuse of notation the firm’s problem reduces from (2.14) to

\[
\Pi \left( l_t, l_h, z; j \right) = \Pi \left( l_t, 0, z; j \right) = \max_{v, l_t'} F( l_t', l_h'; j) - b \sum_{i \in \{l, h\}} l_i' - cv + \beta E \left\{ \Pi \left( l_t', l_h', z'; j \right) \right\} \quad (2.15)
\]

\[
\text{s.t.} \quad \begin{align*}
l_t' &= l_t - l_t^f - l_t^q + vq \left( 1 - P_h \right) \\
l_h' &= vq P_h \\
0 &\leq l_t^f \leq l_t - l_t^q,
\end{align*}
\]

given \( l_t^q = 0 \) & \( l_h^q = l_h \).

It is evident that firm does not simultaneously fire matched workers and post vacancies. One reason is that posting is costly. However, the heterogeneity of job seekers is also important; \( h \)-workers quit and firm must replace them. Conditional on hiring or firing, first order conditions are as follows

\[
\frac{\partial F(l_t', 0, z; l)}{\partial l_t} - b + \beta E \left\{ \frac{\partial}{\partial l_t} \Pi \left( l_t', 0, z'; l \right) \right\} = 0 \quad (2.16)
\]
\[
\frac{\partial F(l'_f, l'_h, z; l)}{\partial v} - bq - c + \beta E \left\{ \frac{\partial}{\partial v} \Pi (l'_f, 0, z'; l) \right\} = 0 \tag{2.17}
\]

Applying chain rule to (2.17) gives

\[
\frac{\partial F(l'_f, l'_h, z; l)}{\partial l_l} - b + P_l \beta E \left\{ \frac{\partial}{\partial l_l} \Pi (l'_f, 0, z'; l) \right\} = \frac{c}{q} \tag{2.18}
\]

**Proposition 3**

Employment policy of a \(l\)-firm, \((l'_f, l'_h) = (l'_f (l_t), l'_h (l_t, z; l))\), is of the form

\[
(l'_f, l'_h) = \begin{cases} (L_l (z), 0) & \text{if } z < Z^F_l (l_t) \\ (l_t, 0) & \text{if } Z^F_l (l_t) \leq z < Z^V_l (l_t) \\ (l_t, 0) & \text{if } Z^V_l (l_t) \leq z \\ + (P_l, P_h) q v^* 
\end{cases} \tag{2.19}
\]

where productivity thresholds of hiring and firing, \(Z^V_l (l_t)\) and \(Z^F_l (l_t)\) respectively, satisfy

\[
x_l Z^F_l \gamma (x_l l_t) \gamma^{-1} - b + \beta E (l_t, Z^F_l) \left\{ \frac{\partial}{\partial l_t} \Pi (l_t, 0, z'; l) \right\} = 0 \tag{2.20}
\]

\[
x_l Z^V_l \gamma (x_l l_t) \gamma^{-1} - b + P_l \beta E (l_t, Z^V_l) \left\{ \frac{\partial}{\partial l_t} \Pi (l_t, 0, z'; l) \right\} = \frac{c}{q} \tag{2.21}
\]

and the optimal responses of firing and hiring, \(L_l (z)\) and \(v^* (z)\) respectively, satisfy

\[
x_l z \gamma (x_l L_l) \gamma^{-1} - b + \beta E (L_l, z) \left\{ \frac{\partial}{\partial l_t} \Pi (L_l, 0, z'; l) \right\} = 0 \tag{2.22}
\]
\[ x_t z \gamma [x_t (l_t + q v^*)]^{\gamma - 1} - b + P_t \beta E \frac{\partial}{\partial l_t} \Pi (l_t + q v^* P_t, 0, z', l) = \frac{c}{q} \quad (2.23) \]

Figure 2.3 illustrates a schematic employment policy for a \( l \)-firm as a function of the realized productivity, \( z \). The intuition of employment policy is straightforward, if the productivity of the firm is lower than the firing threshold, the \( l \)-firm fires worker until the point that the value of the marginal \( l \)-worker equals zero to the firm. On the other hand, hiring threshold is the productivity level that the value of marginal \( l \)-worker coincides cost of hiring one unit of labor considering the fact that next period only a fraction \( P_l \) of new hires would stay at firm. For sufficiently high productivity levels firm posts vacancies such that (2.23) is satisfied. In line with other models of costly hiring there is an inaction productivity band; when productivity lies between the two thresholds neither firing nor hiring is optimal.

**Employment policy of a \( h \)-firm**

Heterogeneity of workers in both efficiency and wage makes the \( h \)-firms’ behavior dependant on parameters such as hiring cost, relative efficiencies and bargaining power of workers. As a result, \( h \)-firms’ policies in general are more complicated. Following lemmas help to discipline the policy of a \( h \)-firm.

**Lemma 1** Suppose the hiring is free, under presumed bargaining protocol good firms keep only good workers (i.e. fire all bad workers) iff \( \eta \) is sufficiently low.

Define supremum size of a \( h \)-firm, the employment level that maximizes the firm’s problem (2.14) at zero hiring cost. Denote the supremum size by asterisk, \((0, l^*_h(z))\).

**Lemma 2** Suppose there is no aggregate shock. There is a neighborhood of \( h \)-workers’ employment, \( \varepsilon > 0 \), such that for any \( l_h > l^*_h - \varepsilon \), \( \pi (0, l_h, z, \overline{w}_h; h, \overline{u}, P_h) \geq \pi (l_t, l_h, z, \overline{w}_h; h, \overline{u}, P_h) \) for \( \forall l_h > 0 \).

**Lemma 3** Suppose \( c \) is sufficiently low and \( P_h \) is correspondingly sufficiently high. If \( l_h \ll l^*_h \) firm fires all its \( l \)-workers.
Figure 2.2: Timing of events in the model

Figure 2.3: Employment policy of a l-firm
The first lemma states that assuming free employment adjustment, if good workers are cheap enough good firms always find it optimal to substitute their bad workers with good ones. Because of DRS production function adding a $l$ or $h$-worker to a firm decreases marginal productivities of all its workers; however a $h$-worker always exhibits higher marginal productivity than her $l$-type co-worker. The first consequence of DRS production function is that firms endogenously have bounded size. Moreover, if $h$-workers have relatively high bargaining power as opposed to lemma (1), loosely speaking, their high wages outweigh their superior marginal productivity and firm prefers to keep $l$-workers only; while for sufficiently low bargaining powers the firm is better off by substituting any marginal $l$-worker with an appropriate amount of $h$-workers.

Lemma (2) goes one step further and indicates that $h$-firms are not always better off filling their employment gap with $l$-workers. In another words, a $h$-firm may optimally decide to fire its $l$-workers, although it is below its supremum size. The underlying reason is that wage of $l$-workers is fixed at their reservation wage and therefore does not respond to their marginal productivity. Indeed, when $h$-workers are the desired workers, $l$-workers have such a low marginal productivity that they are too expensive for a firm at the neighborhood of the supremum size. It is as if $l$-workers exhibit spoiling effect in good firms: their small marginal contributions to close the output gap is outweighed by their deteriorating effects on profit through further rise of the wage bill.

The intuition for the last lemma is that when a good firm has relatively few $h$-workers, loosely speaking, it is optimum for firm to hire as much $h$-worker as possible to close the gap; however by not shedding $l$-workers firm deprives itself of higher profits of substituting –a fraction of- those $l$-workers with new $h$-workers. Mentioned spoiling effect prevails here, too.

Considering lemmas (1-3) in addition to the mentioned fact that $h$-workers quit from $l$-firms, one can conclude that a $l$-worker ($h$-worker) is mismatched if employed in a $h$-firm ($l$-firm). I use the notion $\sim j$ to indicate a mismatched worker in a $j$-firm.
In the following, I first write the $h$-firms’ problem in the interesting case that good workers have sufficiently low bargaining power. Later, I make a simplifying assumption that makes the $h$-firms’ problem akin to $l$-firms’ problem.

No worker quits a $h$-firm. However, if $l$-workers become mismatched, the problem of a $h$-firm reduces to choose among: "posting vacancies without any layoffs", "posting vacancies and fire mismatched workers (partly or totally)"; "inaction", “firing partly/totally the mismatched”, and “firing all mismatched plus part of the matched workers”.

\begin{equation}
\Pi (l_t, l_h, z; h) = \max_{v, l^l_t, l^l_h} \left\{ \begin{array}{l}
\left( l_t, l_h \right) \\
- \left( l^l_t, l^l_h \right) \end{array} , z - \mathbb{W} + \mathbb{E} \{ \Pi' \}, \\
F(l_t, l_h, z) - \mathbb{W} + \mathbb{C},
\right. \right. \right.
\end{equation}

\begin{equation}
\left. \begin{array}{l}
F \left( l_t, l_h, z \right) - \mathbb{W} + \mathbb{E} \{ \Pi' \}, \\
F \left( l_t, l_h, z \right) - \mathbb{W} + \mathbb{E} \{ \varphi' \},
\end{array} \right. \right. \right. \right.
\end{equation}

s.t. \quad 0 \leq l^l_i \leq l_i, \quad v \geq 0, \quad i \in \{ l, h \}

where $\mathbb{W} = \sum_{i \in \{ l, h \}} l_i w_i (l_t, l_h, z)$ is the wage bill.

The main problem to go further is that unlike the $l$-firm’s problem, here the orders of the productivity thresholds are not trivial. Indeed, it is possible that the ranking of thresholds changes at different employment levels. Therefore, I deal with (2.24) numerically.
As an attempt to proceed further and inspired by lemmas (1-3), I make the following assumption, to simplify the employment policy of a \( h \)-firm. Notice that I do not apply assumption 1 in the steady state exercise, and solve the global problem of the firm. I use this assumption only in the business cycle exercise that the problem is more complicated.

**Assumption 1**

\( \eta \) is sufficiently low, therefore there is a nonempty set of positive values of flow cost of posting vacancies such that good firms always fire the mismatched -i.e. only keep good workers-.

Notice that according to lemmas (2) and (3) statement of the assumption (1) is valid at employment levels close to and far from the supremum size. Assumption (1) generalizes the lemmas to all employment levels. Later I will check the strength of assumption in the case of no aggregate shock, and I show that the assumption holds true in the steady state of the economy.

Under the assumption (1), \( h \)-firms aggressively fire all mismatched as soon as they detect them. Using the assumption, one can make the same arguments as \( l \)-firm’s employment policy to conclude that only employed \( h \)-workers matter for firm’s future decisions, because firm fires all mismatched workers once they participated in production, \( l_i^f (; h) = l_i \). As a result \( h \)-firm knows that however none of its worker quits, \( l_i^q = 0 \), it will fire all the mismatched, \( l_i^f = l_i \). Therefore, the problem of a \( h \)-firm reduces to
\[ \Pi(l_t, l_h, z; h) = \Pi(0, l_h, z; h) = \max_{\nu l_h} F(l_t', l_h', z) - cv + \sum_i l_i w_i (l_t', l_h', z) + \beta \mathbb{E} \left\{ \Pi(0, l_h', z'; h) \right\} \]

s.t. \[ \begin{align*}
l_t' & = l_t - l_h' + v q P_h \\
l_h' & = v q P_l \\
0 & \leq l_h' \leq l_h - l_h'' \\
l_t'' & = l_t \\
given l_h'' & = l_h' = 0.
\end{align*} \]

Following the same steps described for a \( l \)-firm one can derive employment policy of a \( h \)-firm. The assumption is helpful to discipline the employment policy particularly because it rules out policies such as "keeping a fraction of the mismatched" and "posting vacancies without firing the mismatched". Hence, employment policy of a \( h \)-firm reduces to following proposition.

**Proposition 4**

Employment policy of a \( h \)-firm, \((l_t', l_h') = (l_t'(l_h, z; h), l_h'(l_h, z; h))\), is of the form

\[ (l_t', l_h') = \begin{cases} 
(0, L_h(z)) & \text{if } z < Z_h^f(l_h) \\
(0, l_h) & \text{if } Z_h^f(l_h) \leq z < Z_h^v(l_h) \\
(0, l_h) & \text{if } Z_h^v(l_h) \leq z \\
+ (P_t, P_h) q v^* & \text{if } Z_h^v(l_h) \leq z
\end{cases} \]

where productivity thresholds of hiring and firing, \( Z_h^v(l_h) \) and \( Z_h^f(l_h) \) respectively, satisfy
\[ x_h Z_h^\gamma (x_l h) \gamma^{-1} - w_h - l_h \frac{\partial}{\partial l_h} w_h + \beta E(l_h, z_h^l) \left\{ \frac{\partial}{\partial l_h} \Pi (0, l_h, z'; h) \right\} = 0 \]  \hspace{1cm} (2.27)

\[ c = q(x_l P_l + x_h P_h) Z_h^\gamma (x_h l_h) \gamma^{-1} - b q P_l - w_h q P_h \]

\[ -l_h \frac{\partial w_h}{\partial v} \Big|_{v=0} + \beta E(l_h, z_h) \left\{ \frac{\partial}{\partial v} \Pi (0, l_h, z'; h) \right\} \]

and the optimal responses of firing and hiring, \( L_h (z) \) and \( v^* (z) \) respectively, satisfy

\[ x_h z^\gamma (x_h L_h) \gamma^{-1} - w_h - L_h \frac{\partial}{\partial l_h} w_h + \beta E(L_h, z) \left\{ \frac{\partial}{\partial l_h} \Pi (0, L_h, z'; h) \right\} = 0 \]  \hspace{1cm} (2.29)

\[ c = q z^\gamma (x_l l_h + v^* q x)^\gamma^{-1} - b q P_l - w_h q P_h \]

\[ - (l_h + v^* q P_h) \frac{\partial}{\partial v} w_h + \beta E(l_h, z_h^v) \left\{ \frac{\partial}{\partial v} \Pi (0, l_h + v^* q P_h, z'; h) \right\} . \]

Notice that one can apply the chain rule to (2.28) and (2.30), although the subsequent equations are not simpler because workers are heterogeneous in efficiency and wage.

It is worth noting that inactivity region of a firm is determined by two factors. \( h \)-firms become unwilling to hire not only because hiring is costly, but also because workers are heterogeneous in efficiency and wage. \( l \)-workers have a spoiling effect on the value of \( h \)-firm as described earlier.

### 2.3.8 Market clearing

Finally, to close the model, markets must clear and firms and employment must evolve consistently. Denote by \( \Phi_j (l_j; z_i), j \in \{ h, l \} \) the mass of \( j \)-firms with idiosyncratic pro-
ductivity \( z_t \), that have less than or equal to \( l_j \) matched workers. \( \Phi_j (l_j; z_t) \) evolves as follows. Firms will remain in their former mass if hire sufficiently low or if shed matched workers. Some firms with the same idiosyncratic productivity join the mass because they fire workers such that they fit in the mass. Some other firms join the mass as they receive new idiosyncratic productivity of \( z_t \). Moreover, some firms leave the mass either because of hiring or because of changing idiosyncratic productivity.

\[
\Phi'_j (l_j; z_t) = \sum_{t'} G (z_t | z_{t'}) \int_{T_j (l_j; z_t)} d\Phi_j (m_j; z_{t'})
\]

\[
T_j (l_j, z_t) = \left\{ m_j | l_j \geq m_j + qP_jv (m_j, z_t; j) - l^f_j (m_j, z_t; j) \right\}
\]

\( j \in \{ h, l \} \)

The first term in the right hand side is the mass of all firms with any productivity level yesterday -including \( z_t \) itself- that today draw idiosyncratic productivity \( z_t \) and by their optimal employment policy are eligible to enter the mass. The second term is simply the former firms of the mass that today draw a different idiosyncratic productivity.

Total posted vacancies, \( V \), is the sum of vacancies posted by each type of firm, \( V_j \), and total unemployment, \( u^t \) is the sum of unemployment of each type

\[
V = \sum_{j \in \{ l, h \}} V_j = \sum_{j \in \{ l, h \}} \sum_{z} \int_{0}^{\infty} v_j d\Phi_j (l_j; z; j) .
\]

\[
u^t (z^t, u^{t-1}) = \sum_i u^t_i (z^t, u^{t-1}, P^t_{i-1})
\]

where the unemployed of each type are the job seekers who could not find a match,

\[
u^t_i (z^t, u^{t-1}, P^t_{i-1}) = s^t_i (u^{t-1}, P^t_{i-1}) (1 - f (\theta_t))
\]
The job seekers today are the unemployed past period, in addition to today’s separated workers, where the separated are either the mismatched or the fired due to low idiosyncratic shock.

\[
s_i^t (u_{i-1}^t, P_{i-1}^t) = u_{i-1}^t + V_{i-1}^t P_{i-1}^t q (\theta_{t-1}) + \sum_p \int_0^\infty G (\zeta_p | \zeta_p) (l_i - L_i (z_t)) d \Phi_i^{t-1} (l_i; z_i)
\]

where \( (\cdot) \) is the operator that returns the value of operand if positive and zero otherwise. Finally, the mass of employed of each type at equilibrium satisfies

\[
M_j - u_j \equiv L_j = \sum_z \int_0^\infty l_j d \Phi_j (l_j; z; j).
\]

### 2.4 Results

#### 2.4.1 Parameterization

I parameterize the model to reasonable values to see how the mechanisms in the model work. The time period I take to be one month. In Mortensen and Pissarides search literature usually it is assumed that separated workers have to wait one period, this assumption simplifies the surplus of having a job for the worker, which in turn simplifies Nash bargained wage. But also has a direct impact on the minimum unemployment duration, one way to deal with this problem is to consider weekly time periods (e.g. Hagedorn and Manovskii, 2008 and Elsby and Michaels, 2013). Because in my model workers start to search within the same period of separation, the model is able to generate zero unemployment duration between job transitions. However, the time period implies a minimum employment duration of one month.

I set the discount rate such that it matches a 4% annual interest rate, this requires \( \beta = 0.9967 \). In the benchmark calibration to pin down the unemployment benefit, I follow Hall and Milgrom (2008). They estimate unemployment benefit to be 0.71 of ALP in the
U.S. I normalize the efficiency of \( l \)-workers to one, and choose an efficiency of \( 1.25 \) for \( h \)-workers. The higher the relative efficiency of \( h \)-worker, the larger the relative size of a \( h \)-firm.

In the benchmark model I follow Schaal (2012) and set the elasticity of matching function, \( \alpha = 1.6 \). Following Hagedorn and Manovskii (2008) I target a steady state labor market tightness of 0.60, which implies a monthly job firing rate of 0.48. The implied job finding rate lies in the range of Shimer (2005) and Hall (2005).

The relative masses of workers, in addition to relative masses of firms determine the unemployment rate (together with the job finding rates) as well as the average micro-flows rate in the steady state. Notice that it is difficult for the model to quantitatively match all three flows of quits, layoffs and hiring, because in the model, there is interdependence between the flows, for example an attempt to decrease the rates of quits of \( h \)-workers from \( l \)-firms, through the variable \( P_h \) increases the number of mismatched at \( h \)-firms and therefore increases the firing rates at growing firms. Because the model abstracts from on-the-job search, and because I believe that on the job search plays an important role in explaining quits, and considering the fact that the main features of the workers flows that have been less explored are firing at the growing and hiring at the shrinking firms, I target these two rates, in addition to a steady state unemployment rate of 6%.

Following Silva and Toledo (2009), Elsby and Michaels (2013) and Hawkins (2011), I set the cost of vacancy such that hiring cost of a unit of labor to be about 4.3% of the quarterly compensation of a new hired worker. This implies a vacancy posting cost of about 10% of average monthly wages at steady state. In this exercise I do not distinguish between vacancy posting costs of bad and good firms, however the results are robust to use average wages specific to each type of firm.

As described in the \( h \)-firms’ problem, the interesting case of the economy is when the \( h \)-workers do not have too high bargaining power relative to \( l \)-workers. Because I assumed that \( l \)-workers have no bargaining power, therefore the bargaining power of \( h \)-workers must be relatively low, too. Moreover, according to proposition 2, \( 1/\eta \) must be an integer, hence
I choose $\eta = 1/7$. This value ensures that in the steady state with no aggregate shock $h$-firm prefers $h$-workers to $l$-workers. Recall that in the numerical exercises at the steady state, I do not impose assumption 1. The numerical solutions of $h$-firms without imposing assumption 1 reveal that with higher bargaining powers, even 0.5, still $h$-firms fire all their $l$-workers and keep only the $h$-workers. This is in line with the results of Cahuc, Marque and Wasmer (2008) that show that under Stole and Zwieble (1996) intra-firm wage bargaining firms at their steady state over-employ (under-employ) of workers with higher (lower) bargaining power.

Following Schaal (2012) I set the decreasing return to scale of production $\gamma = 0.85$. It is worth noting that as Schaal (2012) indicates targeting labor share induces a lower number to $\gamma$; moving around these values has little effect on the results, i.e. Assumption 1 holds.

I assume a two-state Markov process for the aggregate productivity with a persistency probability of $\lambda$, and deviation, $d$, from the normalized aggregate shock,

$$z_a^t \in \{1 - d, 1 + d\}$$

$$z_a^{t+1} = \begin{cases} 
  z_a^t, & \text{with probability } \lambda \\
  \text{switch state} & \text{with probability } 1 - \lambda 
\end{cases}$$

I choose $d$ and $\lambda$ such that the HP-filtered series of output matches the variance and autocorrelation of quarterly output of the U.S. These require $d = 0.005$ and $\lambda = 0.85$.

Elsby and Michaels (2013) and Hawkins (2011) assume that firms draw idiosyncratic shocks from a Pareto distribution, $z_t \sim pareto(k_t, m_t)$. These two variables affect the share of inactive firms, as well the distribution of the growth rates.

At calibration, Elsby and Michaels (2012) and Hawkins (2011) assume that idiosyncratic productivity has two components: a temporary shock and a fixed term. The fixed productivity in their model turns out to be irrelevant for the cross sectional log employment growth and for cyclical properties of aggregate variable; therefore they can use it to match the cross sectional distribution of firms. In the presented results I apply only one
fixed productivity to all firms to scale them up. I set it to 1.07, so that at steady state it generates average firm size 21.9.

Notice that to solve the model with aggregate shocks I utilize a method that resembles the Krusell and Smith (1998). Recently the trend in the large firm literature is to use the convenient assumption of free entry to pin down the tightness, as a result there is no need to limited-information techniques to approximate infinite dimension distributions. Menzio and Shi (2010, 2011) apply this concept to the directed search and develop block recursive equilibria models. Kaas and Kircher (2011) and Schaal (2012) use the block recursivity in their models. The trick may work in random search, too. But in my model, the entry is not enough to resolve the need for approximate distribution. I can allow for entry and it still pins down the tightness in the labor market given the $P_h$, however agents still need to forecast the evolution of $P_h$. Therefore I use the Krusell and Smith (1998) proposal. In the absence of entry, I assume agents forecast the evolutions of $\theta$ and $P_h$ using

$$u' = u^{ss} + c_1 (z_a' - z^{ss}) + c_2 (u - u^{ss})$$  \hspace{1cm} (2.35)$$

$$P'_h = P_h^{ss} + c_3 (z_a' - z^{ss}) + c_4 (P_h - P_h^{ss}) + c_5 (u' - u^{ss})$$  \hspace{1cm} (2.36)$$

$$\theta' = \theta^{ss} + c_6 (z_a' - z^{ss}) + c_7 (u' - u^{ss})$$  \hspace{1cm} (2.37)$$

I start with an initial guess of the coefficients and solve the equilibrium given the evolutions and then improve the coefficients in (2.35) to (2.37) by regressing over the simulated results, I iterate the process until convergence.

Tables (2.3) and (??) summarize the parameterization and performance of the model.

Following Davis and Haltiwanger (1992) I calculate the growth rate of a firm as the ratio of employment change over average employment at the start and at the end of the
measurement period,

\[ g = \frac{l' - l}{(l' + l)/2}. \]  

(2.38)

2.4.2 General model without aggregate shock

In this section, I assume there is no aggregate shock and I solve the general problem of a \( h \)-firm, i.e. eq (2.24). This also helps to have a sense of how strong Assumption 1 is. Without aggregate shock Assumption 1 holds with hiring costs up to three to four times higher than considered. To have comparable results with DFH, I convert monthly simulated data of the model into quarterly rates. As Figure 2-4 shows the model generates employment flows broadly consistent with the data. The three flows coexist at different growth levels. In particular, model generates layoffs in the growing and hires at shrinking firms. The main caveat with this preliminary exercise is the lower magnitude of quits rates. In the previous versions of the model that I do not target the flows rates model generates more reasonable
Table 2.4: Targets

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b/ALP$, (Hall-Milgrom, 2008)</td>
<td>0.71</td>
<td>0.70</td>
</tr>
<tr>
<td>Average market tightness, (Hagedorn-Manovskii, 2008)</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Average firing at growing firms (Davis et al., 2012)</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Average hiring at shrinking firms (Davis et al., 2012)</td>
<td>11%</td>
<td>5%</td>
</tr>
<tr>
<td>std. of growth rates of the firms (Davis et al., 2012)</td>
<td>0.22</td>
<td>0.09</td>
</tr>
<tr>
<td>Ratio of the firms with $</td>
<td>g</td>
<td>\leq 10%$ (Davis et al., 2012)</td>
</tr>
<tr>
<td>std. of growth rates of the firms (Davis et al., 2012)</td>
<td>0.22</td>
<td>0.09</td>
</tr>
<tr>
<td>Average firm size (BED, Elsby-Michaels, 2013)</td>
<td>17.3</td>
<td>21.9</td>
</tr>
<tr>
<td>Quarterly autocorrelation of output</td>
<td>0.842</td>
<td>0.738</td>
</tr>
<tr>
<td>Quarterly std of output</td>
<td>0.013</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Table 2.5: Business cycle performance

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model (with aggregate shock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarterly autocorrelation</td>
<td>$\gamma$</td>
<td>0.842</td>
</tr>
<tr>
<td></td>
<td>$ALP$</td>
<td>0.771</td>
</tr>
<tr>
<td></td>
<td>$u$</td>
<td>0.923</td>
</tr>
<tr>
<td>Quarterly std</td>
<td>$\gamma$</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>$ALP$</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>$u$</td>
<td>0.136</td>
</tr>
<tr>
<td>Average job finding rate</td>
<td>$u$</td>
<td>0.06</td>
</tr>
<tr>
<td>Correlation</td>
<td>$(\gamma, u)$</td>
<td>-0.911</td>
</tr>
<tr>
<td></td>
<td>$(ALP, u)$</td>
<td>0.486</td>
</tr>
<tr>
<td></td>
<td>$(ALP, \gamma)$</td>
<td>-0.357</td>
</tr>
<tr>
<td></td>
<td>$(ALP, \theta)$</td>
<td>-0.436</td>
</tr>
</tbody>
</table>

U.S. data for the period 2001Q1-2012Q4. GDP per capita, $\gamma$, is taken from National Income and Product Accounts (NIPA) Table 7.1; average labor productivity, $ALP$, is measured as seasonally adjusted output per hour in the non-farm business sector; unemployment rate, $u$, comes from BLS. All variables are reported as log deviations from an HP trend with smoothing parameters 1600.
quit rates but over-predicts hiring or firing at growing and shrinking firms. Introducing an inspection technology to the model improves performance of the model in that dimension. Figure 2.5 illustrates the distribution of establishment employment growth as simulated by the model. It broadly resembles the data, a mass point at around zero and relatively symmetric tails as reported by DFH.

Figure 2.6 shows the worker flows at each type of firms and helps to understand the sources of flows in the model. The left and right graphs demonstrate flows at good and bad firms respectively. First, all quits happens at bad firms; in the model no worker has incentive to quit a good firm. Second, the majority of layoffs at growing firms are the results of employment policy of good firms. There are small firings in growing bad firms which are due to aggregation over three periods. It could be case that a bad firm over the three periods of aggregation experiences a net growth, but in one or two of those periods faces a negative shock such that it optimally sheds some workers. And finally, both bad and good firms hire when contracting. They hire to replace part of the separated mismatched workers who either quit or being fired.

2.4.3 Business cycle

In the absence of aggregate shock, solving $h$-firm’s problem in the general form of (2.24) is very demanding due to dimensionality of the problem. Based on the results of the previous subsection, that validates in the steady state with no aggregate shock assumption 1 holds even at fairly higher hiring costs, I force $h$-firms to fire all mismatched workers. The simulation results are reported in table (1.4). Apart from the cyclical behavior of ALP that I discuss in more detail in the next subsection, perhaps the most interesting feature of the results is the relatively high volatility of unemployment. It seems that the model does not suffer the so-called Shimer puzzle in generating unemployment volatility. As Hawkins (2011) explains, the main reason could be the absence of extensive margin at the production side. When there is no entry, the wage bargaining assumption, increases motivation of firms to hire at booms. Because by hiring more, firm decreases wages of its
Figure 2.4: Employment flows rates as a function of firm growth

Figure 2.5: Distribution of employment growth: Data (blue) vs Model (green)
Table 2.6: Cyclicality of ALP in canonical MP search models

<table>
<thead>
<tr>
<th></th>
<th>Correlation with ALP</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data (2001Q1-2012Q4)</td>
<td>canonical M-P Search Models</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.486</td>
<td>-0.958</td>
<td>-0.892</td>
<td>-0.948</td>
</tr>
<tr>
<td>Output</td>
<td>-0.357</td>
<td>1.000</td>
<td>NA</td>
<td>1.000</td>
</tr>
<tr>
<td>Vacancy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>-0.436</td>
<td>0.999</td>
<td>0.967</td>
<td>1.000</td>
</tr>
</tbody>
</table>

other workers, and the rise in the wage due to the good shock is partially offset.

**Business cycle behavior of average labor productivity.** Canonical MP search models with homogenous workers imply a one-to-one mapping between ALP and output, and ALP and employment. This property can transmit to search models with heterogeneous large firms (e.g. a correlation of one between ALP and output in Hawkins(2011)). Here I present the cyclical behavior of ALP in three search models. Shimer (2005) influential paper that explores the unemployment volatility puzzle, Hagedorn and Manovskii (2008) that provides a solution, and Hawkins (2011) that develops a Mortensen and Pissarides model with large firms and matches many cross sectional features about the firms. As it is shown in the table the three models predict by far a different cyclical behavior for the productivity. Notice that in the M-P model productivity plays a crucial role by driving all economy. This prediction goes throw all canonical MP models.

Notice that strong procyclicality of ALP in the canonical models is somehow consistent with pre-84 data. Table 2.5 shows that the model breaks the one-to-one link between ALP and output to a correlation of around 0.36. Moreover, highly negative correlation (about -1) of ALP-unemployment in canonical models, enhanced to zero. In comparison to data, the 0.65 decline in ALP-output correlation (table 2.7) lies in the upper range of correlation changes (before and after mid-80’s) reported by Gali and Gambetti (2009), Gali and van Rens (2010) and Berger (2012). Gali and van Rens (2010) and Barnichon (2010) estimate
Figure 2.6: Employment flows rates separated by the type of firms: (left) h-firm, (right) l-firm.

Table 2.7: Model performance: Cyclicality of ALP

<table>
<thead>
<tr>
<th></th>
<th>corr(p, y)</th>
<th></th>
<th>corr(p, l)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Change</td>
<td>Level</td>
<td>Change</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pre-84</td>
<td>0.678</td>
<td>0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>post-84</td>
<td>0.087</td>
<td>-0.593</td>
<td>-0.40</td>
<td>-0.59</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no heterogeneity</td>
<td>1.000</td>
<td>0.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with heterogeneity</td>
<td>0.362</td>
<td>-0.638</td>
<td>0.00</td>
<td>-0.95</td>
</tr>
</tbody>
</table>
an increase in the correlation between ALP and employment from -0.59 to -0.91. Again the
difference between model results and the canonical model, lines up well with the change
in the data. The model presented in this paper with respect to canonical models induces
changes in business cycle properties of ALP that are comparable with data. In other words,
the underlying mechanism that drives discrepancies in the business cycle behavior of ALP
in my model and in the canonical models, could be a candidate for the source of regime
change of ALP during the mid-80’s. Here I briefly discuss what enables model to generate
more realistic correlations, with respect to high correlations predicted by canonical models.
I postpone exploring potential source of regime change of ALP to the end of this section.

A good aggregate productivity increases the productivity of the firm and its worker. As in the traditional MP models the rise of aggregate productivity motivates the firms to
post more vacancies and hire more. By hiring more, however, firms induce recruiting more
mismatched than before, in the \( h \)-firms introducing more \( l \)-workers with lower productivity
decreases average productivity, and attenuates the effect of the increase in aggregate pro-
ductivity on the output. In contrary, in the recessions firms decrease their hires, and more
firms remain inactive. Hiring less mismatched means that relatively more workers in the
recessions are matched to their firms, hence recessions are times of high productivity, and
very strong procyclicality of productivity in canonical models, declines to mild procycli-
cality. However the model, under presented parameterization, do not generate negative
correlation of ALP and output after the mid-80’s, but shows more consistent behavior of
ALP relative to canonical models.

Recall that assumption 1 required no firing costs and small hiring costs. Extending the
model in any manner that ultimately makes it more costly to replace a mismatched worker,
reduces the incentive of \( h \)-firms to fire the mismatched, and violates the assumption 1.
Introducing firing costs, training costs or making the learning time consuming are different
approaches to motivate the \( h \)-firm to keep \( l \)-workers longer. If \( l \)-workers stay longer in the
firm then in normal and good times it might be the case that firm fires no mismatched or
fires part of them. This makes the ALP less countercyclical than the former case, and it
also potentially can generate a jobless recovery. Notice that in this case, at recessions due to negative shock firms shed their workers, and start doing so by firing the mismatched. After the recession firm is left with only matched type, now with the recovery of the aggregate productivity, output rises, but firms are reluctant to hire because they know by hiring new mismatched join them, whom are costly to be laid off. In other words, costly firing makes the hiring less attractive to the firm especially after recessions. Hence, the model would generate a raise in ALP and productivity with little unemployment reduction.

**What explains the regime change of ALP?** So far I created a countercyclical ALP, but what can explain a drop in its procyclicality. In terms of model, it is straightforward to argue. Recall that traditional search models with homogeneous workers (even with the notion of firm) show very high correlation between ALP and output (e.g. a correlation one at Hawkins (2011)). In the model, heterogeneity matters to create mismatched and to provide incentives to firm to fire mismatched. Any scenario that either dampens the incentive of firm to respond to a mismatch or lowers the prevalence of mismatch creates high correlation of ALP and output, and resembles the pre-mid80’s era. At least there are two possibilities to generate the observed regime change of ALP. An interesting explanation proposed by Gali and van Rens (2010) and Berger (2012) is the decline of unionization which coincides the event well. In Gali and van Rens (2010) a decrease in unionization translates into more flexible labor markets, as a result less effort adjustment is required from the worker side, which in turn decreases the seemingly high correlation of ALP and output. In Berger (2012), however, the impact of decline in unionization is through the selective firing; lower union coverage allows more firms to fire workers based on the performance than the tenure. The latter explanation is consistent to my model, too. My model also provides a related but novel mechanism. Inability of firms in detecting the type of workers is a crucial ingredient; in a framework that the types are fully revealed pre-hiring, there is no mismatch to push down the ALP through the composition effect. The more the firm is able to detect workers’ type pre-hiring, the higher is the correlation of ALP and output.
Before 80’s when the unions had more power and coverage, firms had more incentive to discover the quality of workers pre-hiring, because after hiring it was difficult to shed the redundant workers. Higher effort of firms to realize workers’ type pre-hiring decreases the prevalence of mismatched, and leads to a procyclical ALP.

Another explanation is a change in the quality of jobs. If detecting the type of worker becomes harder just because of the nature of the jobs has changed -for example by spread of IT technologies in all industries or through faster technological progress-, then the mismatched prevails. Notice that it is not necessarily the case that the jobs become more complex per se; the argument holds true for the case that a new technology introduces to the firm. Perhaps, it is harder to assess qualifications of workers for the jobs that are new or unfamiliar to the firm. For example, a firm in pharmacy industry that for the first time decides to hire IT engineers for its network. It is relatively harder for the firm to assess the qualifications of the job applicants. Even it could be the case that the firm is not fully aware of its own needs at the first place. It is as if the firm learns about the quality of the new workers and the job requirements simultaneously. This channel is not appealing to explain the sudden drop in the cyclicality of ALP. Rather it lines up well with the recent data that productivity becomes more and more countercyclical.

Other features of the model. The model also predicts a positive size-wage relation of about 10% of the data, which is close to results of Elsby and Michaels (2013). At least two ingredients drive this result: intra-firm bargaining creates over-employment, which in turn substantially reduces the wages, and the hiring cost, that works in the opposite direction by making over-employment costlier to the firm.

In the former versions, that I allow firm entry (at the steady state), young firms pay higher wages mainly because young $h$-firms want to grow, and on average have more mismatched workers. The bargained wage of $h$-workers is an increasing function of $l$-workers, as a result $h$-workers at young firms are paid more.
2.5 Conclusion

In this chapter, I develop a labor search model that, without on-the-job search, is able to generate a rich set of flows broadly consistent with the data. Workers and firms are both heterogeneous, and there is partial assortative matching. However, random search and post-hiring learning about workers’ quality makes hiring of the mismatched workers inevitable. When hiring and firing costs are low, as soon as the firm detects a mismatched she will be laid off. Workers may also quit if they are not matched with their firm. Therefore in the model flows of hiring, quit and layoff prevails at wide ranges of growth rates.

The model alleviates the strong procyclicality implied by the standard search models, and provides an explanation for countercyclicality of average labor productivity observed from mid-80’s. Quantitatively, however, current calibration does not generate the countercyclicality of ALP. But with respect to the canonical models the magnitude of adjustments are comparable with observed changes in correlations of ALP and unemployment and output. The mechanism that drives these results is variations in the composition of employment. During expansions firms hire more, hence the share of mismatched in employment increases. Because the mismatched have lower productivity, the average labor productivity does not move one-to-one with employment or output. The opposite occurs in the recessions.
Chapter 3

Was it the Fed or the heterogeneity that changed the cyclical pattern of productivity?

3.1 Introduction

The cyclicality of average labor productivity substantially declined during the mid-80’s\(^1\). Before the mid 80’s the productivity was procyclical (positively correlated with output and negatively to unemployment), while afterwards the productivity seems to become acyclical, or even countercyclical (Hawkins, 2011; Zaveh, 2014). Productivity is the driving force of the workhorse model of study unemployment, Mortensen and Pissarides (1994) search model. In the canonical Mortensen and Pissarides models productivity is strongly procyclical. This pattern is somehow consistent with pre-84 data, while it is in sharp contrast to the recent behavior of productivity. In the search literature\(^2\) there are two approaches to explain the source of this behavior of productivity and also to resolve this inconsistency of MP models with the data. Barnichon (2010) develops a New-Keynesian model with nominal rigidities and variable effort. He shows that a sharp drop in the volatility of non-technology shocks in the mid-80’s, and a decline in the response of productivity to non-technology shocks can account for the observed decline in the cyclicality of productivity. On the other hand, Berger (2012) and Zaveh (2014) propose two other possible channels

\(^1\)See for example, Stiroh (2009), Barnichon (2010), Gali and Gambetti (2009), Gali and van Rens (2010)

\(^2\)In the business cycle literature, Gali and van Rens (2010) show that increasing flexibility in a model with endogenous variable effort and endogenous wage rigidity, declines the cyclicality of productivity. Nucci and Riggi (2009) develop a New-Keynesian model to study the cyclicality of productivity. In their model response of labor productivity to a demand shock switches sign from positive to negative when the sensitivity of effort compensation to individual work performance increases.
based on worker (and firm) heterogeneity: prevalence of selective firing and increase in the
difficulty of inspection of the quality of workers at hiring. Both of the above mentioned
heterogeneity channels reduce the cyclicality of productivity through a composition effect;
at good times there are more low-productive matches.

In this study I use the information in the latent factors of a large panel of macro-
variables to empirically investigate the sources of the structural change in the cyclical
behavior of productivity. The latent factors are good sources of information because they
represent (orthogonal) comovements in many variables. Another desirable property of the
latent factors is that they generally carry information about distinct categories of variables.
I use this characteristic of the factors to find the main drivers of the productivity as well
as output and unemployment pre- and post-84. The results shed some light on the sources
of the described change in the cyclicality of productivity. I also explore the sources of
dynamics of worker flows, as they directly determine the unemployment. I find (limited)
evidence in favor of the channels proposed by Barnichon (2010). The factors that are rep-
resentative of monetary and demand shocks drive the productivity and output pre-84, but
do not significantly contribute to these macro-variables post-84. This is consistent with
the mechanism proposed by Barnichon (2010) that documents a decline in the impact of
non-technology shocks on productivity, as a result technology shocks become the domi-
nant driver of the comovements between productivity and unemployment. In his model
technology shocks generate a positive correlation of unemployment and productivity.

It turns out that this methodology is not informative about the heterogeneity channels
due to endogeneity, however, the results suggests that (i) the relation between job finding
and separation structurally changed, and that (ii) the financial markets play a more role
in driving the unemployment, particularly through the job creation. The results also
open a third gate for the heterogeneity to affect the cyclicality of productivity: financial
innovations. Low cost financing could encourage firms to create low productive jobs or not
to shed the low productive workers.

It is worth noting that heterogeneity and structural break in response to non-technology
shocks are not necessarily mutually exclusive. Indeed, it is possible that decline in the
response of productivity to non-technology shocks, one of the building blocks of Barnichon’s
model, is (partly) driven by the rising importance of heterogeneity.

3.2 Econometric Framework

3.2.1 General specification

I focus on the linear specifications with the general form of

\[ y_t = c + \alpha y_{t-1} + \beta' Z_t + \varepsilon_t \]

where \( y_t \) is the variable of interest at time \( t \) and \( Z_t \) is a \( K \times 1 \) vector of predetermined
variables at time \( t \). \( Z_t \) generally contains a set of factors selected through a backward
selection in the presence of the AR component. Depending on the experiment, \( Z_t \) may
contain additional variables, such as job fining rates and separation rates. The process of
selecting factors/additional variables is described in more details in the next subsection.

3.2.2 Factor estimation and model selection

To estimate factors following Ludvingson and Ng (2009) I apply techniques of factor es-
timation for the case where both the number of economic time series used to construct
common factor, \( N \), and the number of time periods, \( T \), are large and converge to infinity.
The method is initially developed by Stock and Watson (2002a, 2002b) and Bai and Ng

Suppose one can observe a panel of variables \( X_{N \times T} \) with elements \( x_{it} \), where \( N \) the
number of variables is fairly large. The Factor model assumes all \( x_{it} \) are being driven by a
few numbers of “common” or “latent” factors, \( f_t \),

\[ x_{it} = \lambda'_i f_t + \epsilon_{it} \]
where again \( f_t \) is a vector of \( r \times 1 \) common factors with \( r \ll N \), \( \lambda_i \) is the vector of factor loadings, and \( \epsilon_{it} \) is the idiosyncratic errors. The methodology allows for a limited amount of cross-sectional correlation among \( \epsilon \)'s. Technically, \( f_t \)'s are estimated by principal component analysis (PCA).

A main advantage of applying factor model arises from the fact that –without having an a priori economic model in mind- since \( N \) is large, the dimensionality problem makes the selection of a subset of variables very difficult even impossible, given the limited number of observations and also because of the nature of problem which includes \( 2^N \) potential combination of variables.

To select the subset of factors that for each model, I apply a backward selection procedure. In the initial regression I put all factors at the right hand side. Then I remove the least significant factor from the regressors, and regress over the remained regressors. I repeat the process until all variables at the right hand side are significant at a predetermined level. Therefore in the final specification all factors are significant. In the above mentioned process I always force the model to keep an AR component of the left hand side variable with one lag.

Recall that the factors are orthogonal by construction. Given the orthogonality of the factors and interpreting the factors as the latent drivers of the large data set of observables, I interpret the selected model as a decomposition of the left hand side variable. i.e. I assume that the left hand side variable is mainly driven by the selected factors.

In the next experiments I introduce additional observables directly into the model. If necessary, I again apply the backward selection procedure –preserving the AR component–.

---

3I also do the robustness check with a forward selection algorithm. Forward selection does not affect the main results. In the text, however, when there are important discrepancies between the outcome of the two algorithms I discuss in detail.

4Usually it is not necessary to force the models to keep the AR component, because in almost all models the AR component is strongly significant.
3.3 Data

In this study I use three categories of data of the U.S. economy: the data of the main variable of interest in Mortensen and Pissarides search models, a large panel of macroeconomic variables to extract factors, and two flow rates of employment.

The large panel of macroeconomic variables. The large data set to extract the factors is the macroeconomic panel data of Ludvigson and Ng (2009b) which in turn their data set is based on Stock and Watson (2002b). The panel includes 130 monthly economic variables, each spanning the period 1959:1 to 2007:12. Following Stock and Watson (2002b, 2004, 2005), the series were selected to represent broad categories of macroeconomic time series: real output and income, employment and hours, real retail, manufacturing and sales data, international trade, housing starts, inventories and inventory sales ratios, orders and unfilled orders, compensation and labor costs, capacity utilization measures, price indexes, interest rates and interest rate spreads, stock market indicators and foreign exchange measures. After extracting monthly factors, I calculate the quarterly factors by averaging the monthly data. Before extracting factors, all variables were transformed so as to insure stationarity. In the appendix A I borrow and represent the detailed description of data and stationarizing transformations provided by Ludvigson and Ng (2009a,b). Finally I end up with 192 estimations for each factor from 1960:Q1 to 2007:Q4.

The information criteria introduced by Bai and Ng (2002), $IC_2$, indicates $r$ equal to 8, i.e. data is described well with 8 factors. Table 1 presents the summary statistics of a subset of factors that is relevant for this study, i.e. those are used in the final specifications.

To figure out what kind of information each of the estimated factors contain, following Stock and Watson (2002b) and Ludvigson and Ng (2009a), we calculate the marginal $R^2$s, obtained by regressing each of the 130 series on the eight factors, one at a time. Figures 3-1 to 3-8 show the marginal $R^2$ statistics from regressing the series number given on the x-axis onto the estimated factor named in the heading, since I use the same panel data as Ludvigson and Ng (2009a) -except the consumption series-, not surprisingly The factors
have very similar interpretation for the eight factors as well. $f_1$ is a real activity factor that loads heavily on employment and output data, and marginally on interest rates data. The second factor loads heavily on several interest rate spreads, especially the long term spreads. The third factor loads on prices. The fourth and fifth factors load mainly on level of interest rates. Factor 6 loads predominantly on the housing variables while factor 7 loads on measures of the money supply. Factor 8 loads mainly on variables relating to the stock market. In the discussion of results I provide more detailed information about the factors, if necessary.

Productivity, unemployment and output. For productivity I use output per hour in the Non-farm business sector, seasonally adjusted from Bureau of Labor Statistics (BLS). The unemployment data is available for all the period spanned by the factors, i.e. from 1960:Q1 to 2007:Q4. I also use the unemployment data of BLS calculated using Current Population Survey. This data also covers the 1960:Q1 to 2007:Q4 period. The output is GDP per capita from National Income and Product Accounts (NIPA) table 7.1. The data is available from 1969:Q1 to the last period covered by factors, 2007:Q4. The quarterly cyclical components of all these variables are calculated applying a HP filter with the smoothing parameter of 1600. To ease comparison of the regression coefficients I normalize all cyclical components by subtracting the mean and diving by its standard deviation. A summarization of the results is available at tables 3.1 and 3.2.

Employment flows rates. I use the quarterly job finding and separation rates of Shimer. This data covers 1951:Q1 to 2004:Q4. I calculate the cyclical components and normalize these variables as explained in the previous part. Tables 3.1 and 3.2 summarize the data.

3.4 Results and discussion

3.4.1 Benchmark models

Table 3.3 reports the pre- and post-84 specifications (with AR and latent factors) for the three variables of interest —productivity, output and unemployment— selected by
Table 3.1: Data (standard deviations)

<table>
<thead>
<tr>
<th>Variables</th>
<th>pre-84</th>
<th>post-84</th>
<th>Factors</th>
<th>pre-84</th>
<th>post-84</th>
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<td>0.78</td>
<td>$f_4$</td>
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<td>0.64</td>
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<td>$jfp$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$f_7$</td>
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<td></td>
<td>$f_8$</td>
<td>0.60</td>
<td>0.69</td>
</tr>
</tbody>
</table>

$p$: average labor productivity; $y$: output per capita; $u$: unemployment rate; $sp$: separation probability; $jfp$: job finding probability; $f_i$: $i$th factor of the large panel of macroeconomic data. All the variables at left are standardized (using the standard deviation of the whole sample) quarterly cyclical components of a HP-filter with smoothing parameter 1600.
Table 3.2: Data (correlation matrix)

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<td>u</td>
<td>sp</td>
</tr>
<tr>
<td>y</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>-0.44</td>
<td>-0.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sp</td>
<td>-0.63</td>
<td>-0.47</td>
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<tr>
<td>jfp</td>
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<td>0.70</td>
<td>-0.94</td>
<td>-0.40</td>
</tr>
</tbody>
</table>

For the abbreviations see notes on table 1.

For the abbreviations see notes on table 1.

Notes: The chart shows the R-square from regressing the series number given on the x-axis onto the estimated factor named in the heading. See appendix for a description of series. The factors are estimated using data from 1959:1-2007:12.

Figure 3.1: Marginal R-squares of factor 1
Notes: See figure 3.1.

**Figure 3.2:** Marginal R-squares of factor 2

Notes: See figure 3.1.

**Figure 3.3:** Marginal R-squares of factor 3
Figure 3.4: Marginal R-squares of factor 4

Notes: See figure 3.1.

Figure 3.5: Marginal R-squares of factor 5

Notes: See figure 3.1.
Notes: See figure 3.1.

**Figure 3.6:** Marginal R-squares of factor 6

Notes: See figure 3.1.

**Figure 3.7:** Marginal R-squares of factor 7
backward selection algorithm. $f_1$ which heavily loads on the real activities, employment and output data and interest rates, exhibits an strong and intuitive comovement with all the three variables at both pre- and post-84 subsamples. Interestingly the relation between $f_1$ and each of those variables shows very little change, since the estimated coefficients are surprisingly similar at both periods.

Let’s compare the two models of productivity pre- and post-84 in table 3.3. Apart from $f_1$ which is common between the two specifications, other determinants of productivity are different between pre- and post-84 specifications. To make a better understanding of the sources of changes in the behavior of productivity, I swap the specifications, i.e. I estimate pre-84 specification using post-84 data, and vice versa, and if I find insignificant variables I apply the backward selection to find a well-specified model. This procedure gives the model (I) of post-84 in table 3.4 that only contains the common factor of the two specifications, $f_1$. Model (II) represents an alternative specification. This is actually one of the intermediate specifications toward model (I). Model (II) is ruled out only because $f_6$ is significant at slightly higher than 0.10 (at 0.101), the advantage of this specification is that it contains $f_4$ and $f_6$ and reveals more information when compared with the benchmark specification. In the cross-fitted models (I) and (II) estimated coefficients of the common factor, $f_1$, show no sign switch with respect to the benchmark specifications at table 3.3. Considering the fact that this factor shows similar behavior at both periods with each of unemployment and output, $f_1$ is less likely to be the driver of the change in the cyclical behavior of productivity. In comparison, model (II) suggests that the sign of coefficients of $f_4$ and $f_6$ switched from pre-84 to post-84, making them better candidates than $f_1$ for the behavior change of the productivity. Hopes, however, fade away if one compares the results with the selected models of output. The estimated coefficients of $f_4$ switch sign in the selected models of output, too. $f_6$ also exhibits similar behavior as in the case of productivity; it negatively correlates to output pre-84 and does not significantly explain output afterwards. Another observation is that $f_7$ has no explanatory power post-84.

It is worth mentioning that forward selection algorithm (in which one starts with a null
Notes: See figure 3.1.

Figure 3.8: Marginal R-squares of factor 8

Table 3.3: Selected models with estimated factors at the right hand side

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<th>$f_3$</th>
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For the abbreviations see notes on table 1.
Table 3.4: Cross-fitting (regressing pre-84 sample on the selected post-84 model and vice versa)

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$^+$ $f_6$ in model post-84 (II) is significant at 0.101. For further details see notes on table 1.

Table 3.5: Selected models of flows probabilities

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<th>$jfp$</th>
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<td></td>
<td>(.06)</td>
<td></td>
<td>(.04)</td>
<td>(.06)</td>
<td>(.05)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>post-84</td>
<td>.62***</td>
<td></td>
<td>.43***</td>
<td>$-.45^{***}$</td>
<td>$-.61^{***}$</td>
<td>$-.12^*$</td>
<td></td>
<td></td>
<td>84</td>
<td>.79</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
<td></td>
<td>(.06)</td>
<td>(.09)</td>
<td>(.12)</td>
<td>(.07)</td>
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</tr>
</tbody>
</table>

$sp$: separation probability; $jfp$: job finding probability; $f_i$: $i$th factor of the large panel of macroeconomic data.
Table 3.6: Cyclicality of productivity (fitted vaues vs. data)

<table>
<thead>
<tr>
<th>Panel A.</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>$corr(p, y)$</td>
<td>$corr(p, u)$</td>
</tr>
<tr>
<td>pre-84</td>
<td>0.43</td>
<td>-0.44</td>
</tr>
<tr>
<td>post-84</td>
<td>-0.03</td>
<td>0.28</td>
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</table>

<table>
<thead>
<tr>
<th>Panel B.</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Model with factors only</td>
<td>$corr(\hat{p}, \hat{y})$</td>
<td>$corr(\hat{p}, \hat{u})$</td>
<td>$corr(\hat{p}, p)$</td>
</tr>
<tr>
<td>pre-84</td>
<td>0.45</td>
<td>-0.46</td>
<td>0.90</td>
</tr>
<tr>
<td>post-84</td>
<td>-0.16</td>
<td>0.35</td>
<td>0.80</td>
</tr>
</tbody>
</table>

For the abbreviations see notes on table 1. The variables with hats are the fitted values.

set of regressors and at each step adds the most significant variable to the model) provides similar results for all cases but for the output and productivity pre-84, with exactly no effect on the cross-fitting results. In the case of productivity $f_8$ (with the estimated coefficient of 0.21 and std.err. of 0.09) substitutes $f_6$ and $f_7$ in the selected model.

Finally $f_2$ shows no relation with productivity pre-84 (table 3.3 and 3.4); however, at the post-84 period $f_2$ is selected in the benchmark specification (table 3.3). As the table reveals $f_2$ is negatively correlated with unemployment and is positively correlated with output. Notice that during the post-84 $f_2$ is negatively correlated with productivity. In other words $f_2$, by itself, drives a countercyclical pattern of productivity. Put it differently, $f_2$ cancels out (partly) the cyclicality driven by $f_2$, and leads to a less cyclical productivity.

To summarize, so far I find that $f_2$ drives productivity and output in the opposite directions, meantime it generates a positive correlation of productivity and unemployment. Among other factors, $f_3$ and $f_5$ appeared in only one of the models, and $f_1$, $f_4$ and $f_6$ are less likely to drive a structural break in the cyclicality of productivity. Therefore, in addition to $f_2$ it remains to argue the possible contributions of $f_7$ and $f_8$.

---

5The selected model of output with forward selection selects all factors but $f_3$. Among the estimated coefficients all are positive except that of $f_6$, however $f_5$ and $f_8$ are not significant in the final model. Notice that by removing $f_5$ and $f_8$ I come back to the model selected by backward selection algorithm.
What are $f_2$, $f_7$ and $f_8$? Let’s have a closer look at the three candidates. $f_7$ represents money supply as well as inventory variables, and secondly PCE deflator. $f_8$ summarized information about the consumer credit, stock market and secondly the consumer expectations. It seems that these two orthogonal factors together represent demand and monetary shocks. In other words, they are representative of non-technology shocks among the factors. The fact that these two variables, especially $f_7$, are not selected in the post-84 period models is consistent with Gali et. al (2003). They argue the monetary policy of the Fed became more accommodating to technology shocks after 1982. Recall that Barnichon’s quantitative exercise relies on two observations: (i) a sharp drop in the volatility of non-technology shocks in the mid-80’s, and (ii) a decline in the response of productivity to non-technology shocks. The results reported here complement Barnichon’s VAR estimations about the second observation, as none of $f_7$ and $f_8$ is selected for the post-84 model of productivity. However, as table 3.1 shows the variance of $f_7$ in the post-84 is slightly below its pre-84 variance, meanwhile $f_8$ becomes more volatile post-84. This seems in contrast to the Barnichon (2010) and Gali et. al (2003). I list three reasons for the discrepancies. First, methodologically Barnichon (2010) identifies the non-technology shocks from the long run restrictions in a VAR model, while here I interpret the comovements in a set of reasonable candidates for demand and monetary shocks as non-technology shocks. These are not necessarily the same objects. Second, $f_7$ mixes monetary shocks with inventory shocks, and PCE deflator. Investing the source of this comovement requires further investigation, I postpone for future. Third, with regard to Gali et. al (2003) the time span is longer. Their sample ends at 1998:Q3, and particularly does not cover the Dot-com crash of 2000-2002. For comparison if I restrict my sample from 1984:Q1 to 1998:Q3, the standard deviation of the subsample reduces from 0.52 to 0.40 (compare to 0.54 pre-84). Notice that in Barnichon’s model the drop in the volatility is necessary to quantitatively explain the decline in the cyclicality of productivity. However, because his quantitative exercise abstracts from other potential causes, I do not see my results as contradicting but supplement to his.
As figure 3.2 shows, \( f_2 \) heavily loads on the interest rate spreads, especially the long run rates. A close economic counterpart is financial institutions, and (risky) investments. Dynan, Elmendorf, and Sichel (2006) provide evidence that financial innovations (such as lending practices and loan markets that have enhanced the ability of households and firms to borrow and changes in government policy such as elimination of Regulation Q, ceilings on interest rates on bank deposits) are a potential source of the great moderation. Assuming that \( f_2 \) represents the impacts of the financial institution, the results here suggest that they negatively affect productivity. One explanation could be that in the second period, firms have better access to the financial markets. They can raise funds easier, and therefore they afford keeping low productive workers as well as creating relatively lower quality jobs (consistent with the results in table 3.5). As a result, output goes up, unemployment decreases and the productivity goes down. The described mechanism introduces a third channel through which heterogeneity impacts productivity.

3.4.2 More on the sources of cyclicality

In the alternative explanation, heterogeneity can affect the cyclicality of productivity through two channels: selective firing (Berger, 2012 and Zaveh, 2014) and difficulty of inspection of the workers at hiring (Zaveh, 2014). It is difficult to link any of the eight estimated factors to the proposed two channels that heterogeneity can result in an acyclical or countercyclical productivity. One indirect way to test the two heterogeneity channels seems to use the information of the unemployment flows probabilities. If the selective firing become more pervasive in the economy, for example as a result of less union power/coverage, and if this is one of the channels that causes the observed pattern, then higher separation rates will increase productivity, because firms selectively fire the mismatched workers (i.e. the least productive workers) and productivity goes up as a result of the composition effect as described by Berger, (2012) and Zaveh (2014). On the other hand, if it has become more difficult to detect the mismatched workers at hiring, for example as Zaveh (2014) argues as a result of faster technological progress or spill over, a higher job finding rate results
in more mismatched employee, which in turn decreases the productivity, again due to the composition effect. The main obstacle to empirically test this channel is endogeneity, as productivity positively (negatively) affects the job finding (separation) rate. Indeed, one could expect the later dominates the former.

Instead I look at the factors that explain the two drivers of unemployment. As it is shown in table 3.2, the correlation between separation and job finding rates switched sign from negative at pre-84 to slightly positive post-84. The difference is strongly significant at 0.1% significance level. This sign change is consistent with the hypothesis that something has changed about the relation of the two variables. In particular, it is possible that a third factor impacts both variables post-84. To investigate more the phenomenon, I find the best specifications using backward selection. This time, for each variable the initial model includes the other flow rate in addition to the AR and factors. The results reported at 3.5 reveal that at the post-84 the rates contain important information about each other; where the two probabilities explain each other with a surprisingly positive load. Notice that the quality of the information is better than the information provided by the factors. Whereas, pre-84 these two rates do not exhibit such an interdependence. This is consistent with the hypothesis of a common driver of the variables at post-84.

The results also show that post-84, $f_4$ which is a second measure of (risk free) interest rates, positively affects job finding rates but not the separation. The estimated coefficients of $f_2$ are also consistent with my former discussions about the role of financial institutions on the unemployment.

As robustness check and to see whether this experiment tells us a consistent story about the sources of changes in the cyclicality of productivity, I calculate the correlation between the fitted values of the selected models. Table 3.6 compares the predicted correlation with the data. All in all, all models produce explain the correlations well, and the point estimates of the extended models are generally closer to data. I conclude that it is worth to look at factors and also the flows rates for the purpose of this study.
3.5 Conclusion

In this study I use the information in the factors of a large panel of macroeconomic variables to shed light on the main drivers of the structural change in cyclicality of productivity as well as the main drivers of unemployment. Using simple empirical techniques I show that real activities pre- and pot-84 have the impacts on unemployment, output and productivity. Financial institutions and risky investment seems to be the best explanation for the decline in the cyclicality of productivity.

I also find confirmative evidence in favor of the vanishing role of non-technology shocks in driving output, unemployment and productivity, which is in line with Barnichon (2010) argument. Meantime, I could not find direct evidence in favor of heterogeneity channels, however, I find a structural change in the relation between separation and job finding probabilities: from negative correlation at pre-84 to positive correlation recently. This is consistent with the explanation that a third factor drives both to the same direction.
Appendix A1.

Data sources, transformations, and definitions

Data from Global Insights Basic Economics Database, unless the source is listed as TCB (The Conference Boards Indicators Database) or AC (Ludvigson and Ng, 2000a,b) based on Global Insights or TCB data. ln denotes logarithm, Δln, Δ²ln, lv and Δlv denote the first and second difference of the logarithm, the level, and the first difference of the series. Row 3 is removed. Mnemonic (the series label used in the source database).
<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
<th>Base</th>
<th>Period</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>U 27+ wks</td>
<td>lhu27</td>
<td>Δln</td>
<td>Unemployed. By Duration: Persons Unempl. 27 Wks + (Thous. Sa)</td>
</tr>
<tr>
<td>32</td>
<td>UI claims</td>
<td>stm005</td>
<td>Δln</td>
<td>Average Weekly Initial Claims, Unemploy: Insurance (Thous.) (TCB)</td>
</tr>
<tr>
<td>33</td>
<td>Emp: total</td>
<td>ces002</td>
<td>Δln</td>
<td>Employees on Nonfarm Payrolls: Total Private</td>
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<tr>
<td>34</td>
<td>Emp: gds prod</td>
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<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Goods-Producing</td>
</tr>
<tr>
<td>35</td>
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<td>ces006</td>
<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Mining</td>
</tr>
<tr>
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<td>Emp: const</td>
<td>ces011</td>
<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Construction</td>
</tr>
<tr>
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<td>Emp: mfg</td>
<td>ces015</td>
<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Manufacturing</td>
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<tr>
<td>38</td>
<td>Emp: dble gds</td>
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<td>Δln</td>
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</tr>
<tr>
<td>39</td>
<td>Emp: nondble</td>
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<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Nondurable Goods</td>
</tr>
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<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Service-Providing</td>
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<tr>
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<td>Emp: TTU</td>
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<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Trade, Transportation, and Utilities</td>
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<td>Employees on Nonfarm Payrolls - Wholesale Trade</td>
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<td>Δln</td>
<td>Employees on Nonfarm Payrolls - Retail Trade</td>
</tr>
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<td>44</td>
<td>Emp: FIRE</td>
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<td>Employees on Nonfarm Payrolls - Financial Activities</td>
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<tr>
<td>45</td>
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<tr>
<td>46</td>
<td>Emp-hrs nonag</td>
<td>stm048</td>
<td>Δln</td>
<td>Employee Hours in Nonag. Establishments (AR, Bil. Hours) (TCB)</td>
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<td>47</td>
<td>Avg hrs</td>
<td>ces151</td>
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<td>Avg Weekly Hrs of Prod or Nonsup Workers on Private Nonfarm Payrolls - Goods-Producing</td>
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<td>Overtime: mfg</td>
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<td>Avg Weekly Hrs of Prod or Nonsup Workers on Private Nonfarm Payrolls - Mfg Overtime Hours</td>
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<td>Avg hrs: mfg</td>
<td>som001</td>
<td>Δv</td>
<td>Average Weekly Hours, Mfg. (Hours) (TCB)</td>
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<td>NAPM empl</td>
<td>pntemp</td>
<td>Δv</td>
<td>Napi Employment Index (Percent)</td>
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<td>51</td>
<td>Starts: nonfarm</td>
<td>hsftr</td>
<td>Δn</td>
<td>Housing Starts: Nonfarm(1947-58); Total Farm&amp;Nonfarm(1959-68) (Thous. Sa)</td>
</tr>
<tr>
<td>52</td>
<td>Starts: NE</td>
<td>hnr1e</td>
<td>Δn</td>
<td>Housing Starts: Northeast (Thous. U.S.A.)</td>
</tr>
<tr>
<td>53</td>
<td>Starts: MW</td>
<td>hsnwe</td>
<td>Δn</td>
<td>Housing Starts: Midwest (Thous. U.S.A.)</td>
</tr>
<tr>
<td>54</td>
<td>Starts: South</td>
<td>liwson</td>
<td>Δn</td>
<td>Housing Starts: South (Thous. U.S.A.)</td>
</tr>
<tr>
<td>55</td>
<td>Starts: West</td>
<td>hwsnt</td>
<td>Δn</td>
<td>Housing Starts: West (Thous. U.S.A.)</td>
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<tr>
<td>56</td>
<td>BP: total</td>
<td>hbr1t</td>
<td>Δn</td>
<td>Housing Authorized: Total New Priv Housing Units (Thous. Sa)</td>
</tr>
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<td>57</td>
<td>BP: NE</td>
<td>hsnbe*</td>
<td>Δn</td>
<td>Houses Authorized by Build. Permits: Northeast (Thous. U.S.A.)</td>
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<td>Houses Authorized by Build. Permits: South (Thous. U.S.A.)</td>
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<td>60</td>
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<td>Δn</td>
<td>Houses Authorized by Build. Permits: West (Thous. U.S.A.)</td>
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<td>PMI</td>
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<td>Δv</td>
<td>Purchasing Managers’ Index (Sa)</td>
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<td>NAPM new orders</td>
<td>pmnno</td>
<td>Δv</td>
<td>Napi New Orders Index (Percent)</td>
</tr>
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<td>NAPM vendor del</td>
<td>pndel</td>
<td>Δv</td>
<td>Napi Vendor Deliveries Index (Percent)</td>
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<tr>
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<td>NAPM Invent</td>
<td>pmi</td>
<td>Δv</td>
<td>Napi Inventories Index (Percent)</td>
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<td>67</td>
<td>Orders: cap gds</td>
<td>stm027</td>
<td>Δln</td>
<td>Mapi’ New Orders, Nondefense Capital Goods (Mil. Chain 1982 $) (TCB)</td>
</tr>
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<td>Description</td>
<td>Change</td>
<td>Notes</td>
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<td>M&amp;T invent</td>
<td>Manufacturing and Trade Inventories (Bdl. Chain 2000 $) (TCB)</td>
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<td>M&amp;T invent/sales</td>
<td>Ratio, Mfg. and Trade Inventories to Sales (Based on Chain 2000 $) (TCB)</td>
<td>Δlv</td>
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<tr>
<td>M1</td>
<td>M1</td>
<td>Δln</td>
<td></td>
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</tr>
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<td></td>
</tr>
<tr>
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<td>Money Supply - M2 in 1996 Dollars (Bci)</td>
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<tr>
<td>MB</td>
<td>MB</td>
<td>Δln</td>
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<tr>
<td>Reserves tot</td>
<td>Monetary Base, Adj. for Reserve Requirement Changes (Mill.$)</td>
<td>Δln</td>
<td></td>
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<td>Reserves nonbor</td>
<td>Depository Inst Reserves, Nonborrowed, Adj. Res Req Chgs (Mill.$)</td>
<td>Δln</td>
<td></td>
<td></td>
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<tr>
<td>C&amp;I loans</td>
<td>Commercial &amp; Industrial Loans Outstanding in 1996 Dollars (Bci)</td>
<td>Δln</td>
<td></td>
<td></td>
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<tr>
<td>C&amp;I loans</td>
<td>Weekly Rl Lg Coml Banks, Net Change Coml &amp; Indus Loans (Bdl. Sae)</td>
<td>Δlv</td>
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<td>Cons credit</td>
<td>Consumer Credit Outstanding – Nonrevolving (G19)</td>
<td>Δln</td>
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<tr>
<td>Inst cred/PI</td>
<td>Ratio, Consumer Installment Credit to Personal Income (Per.) (TCB)</td>
<td>Δlv</td>
<td></td>
<td></td>
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<tr>
<td>S&amp;P 500</td>
<td>S&amp;P’s Common Stock Price Index: Composite (1941-43=10)</td>
<td>Δln</td>
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<td>S&amp;P div yield</td>
<td>S&amp;P’s Composite Common Stock: Dividend Yield (% per Annum)</td>
<td>Δlv</td>
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<td>S&amp;P PE ratio</td>
<td>S&amp;P’s Composite Common Stock: Price-Earnings Ratio (% Nsa)</td>
<td>Δln</td>
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<td>Fed Funds ratio</td>
<td>Interest Rate: Federal Funds (Effective) (% per Annum, Nsa)</td>
<td>Δlv</td>
<td></td>
<td></td>
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<tr>
<td>Comm paper</td>
<td>Commercial Paper Rate (AC)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 mo T-bill</td>
<td>Interest Rate: U.S. Treasury Bills, Sec Mkt, 3-Mo. (% per Ann, Nsa)</td>
<td>Δlv</td>
<td></td>
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<td>6 mo T-bill</td>
<td>Interest Rate: U.S. Treasury Bills, Sec Mkt, 6-Mo. (% per Ann, Nsa)</td>
<td>Δlv</td>
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<tr>
<td>1 yr T-bond</td>
<td>Interest Rate: U.S. Treasury Const Maturities, 1-Yr. (% per Ann, Nsa)</td>
<td>Δlv</td>
<td></td>
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</tr>
<tr>
<td>5 yr T-bond</td>
<td>Interest Rate: U.S. Treasury Const Maturities, 5-Yr. (% per Ann, Nsa)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 yr T-bond</td>
<td>Interest Rate: U.S. Treasury Const Maturities, 10-Yr. (% per Ann, Nsa)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aaa bond</td>
<td>Bond Yield: Moody’s Aaa Corporate (% per Annun)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baa bond</td>
<td>Bond Yield: Moody’s Baa Corporate (% per Annun)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP-FF spread</td>
<td>Commercial Paper Spread (ACP)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 mo-FF spread</td>
<td>Interest Rate: Moody’s 3-Mo. FF Spread (ACP)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 mo-FF spread</td>
<td>Interest Rate: Moody’s 6-Mo. FF Spread (ACP)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 yr-FF spread</td>
<td>Interest Rate: Moody’s 1-Yr. FF Spread (ACP)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 yr-FF spread</td>
<td>Interest Rate: Moody’s 5-Yr. FF Spread (ACP)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 yr-FF spread</td>
<td>Interest Rate: Moody’s 10-Yr. FF Spread (ACP)</td>
<td>Δlv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aaa-FF spread</td>
<td>Bond Yield: Moody’s Aaa Corporate (% per Annun)</td>
<td>Δlv</td>
<td></td>
<td></td>
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<td>Baa-FF spread</td>
<td>Bond Yield: Moody’s Baa Corporate (% per Annun)</td>
<td>Δlv</td>
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<td>Ex rate avg</td>
<td>United States: Effective Exchange Rate (Merr) (Index No.)</td>
<td>Δln</td>
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<td>Ex rate: Switz</td>
<td>Foreign Exchange Rate: Switzerland (Swiss Franc per U.S. $)</td>
<td>Δln</td>
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<td>Code</td>
<td>Variable Description</td>
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<td>105</td>
<td>Ex rate: Japan exrjjan $\Delta ln$ Foreign Exchange Rate: Japan (Yen per U.S.$)</td>
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<td>106</td>
<td>Ex rate: UK exrjuk $\Delta ln$ Foreign Exchange Rate: United Kingdom (Cents per Pound)</td>
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<td>107</td>
<td>EX rate: Canada exrcan $\Delta ln$ Foreign Exchange Rate: Canada (Canadian $ per U.S.$)</td>
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<td>108</td>
<td>PPI: fin gds prfcsa $\Delta ln$ Producer Price Index: Finished Goods ($82=100, Sa$)</td>
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<td>109</td>
<td>PPI: cons gds prfcsa $\Delta ln$ Producer Price Index: Finished Consumer Goods ($82=100, Sa$)</td>
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<td>110</td>
<td>PPI: int mat ls pwmsa $\Delta ln$ Producer Price Index: Intermed Mat Supplies &amp; Components ($82=100, Sa$)</td>
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<td>111</td>
<td>PPI: crude mat ls pwcm sa $\Delta ln$ Producer Price Index: Crude Materials ($82=100, Sa$)</td>
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<td>112</td>
<td>Spot market price psccom $\Delta ln$ Spot market price index: bls &amp; crb: all commodities (1967=100)</td>
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<td>113</td>
<td>Sens mat ls price psni99q $\Delta ln$ Index Of Sensitive Materials Prices (1990=100) (Bci-99a)</td>
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<td>114</td>
<td>NAPM com price puncp lv NAPM Commodity Prices Index (Percent)</td>
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<td>115</td>
<td>CPI-U: all punew $\Delta ln$ Cpi-U: All Items ($82-84=100, Sa$)</td>
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<td>116</td>
<td>CPI-U: apparel ps83 $\Delta ln$ Cpi-U: Apparel &amp; Upkeep ($82-84=100, Sa$)</td>
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<td>CPI-U: transp ps84 $\Delta ln$ Cpi-U: Transportation ($82-84=100, Sa$)</td>
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<td>118</td>
<td>CPI-U: medical ps85 $\Delta ln$ Cpi-U: Medical Care ($82-84=100, Sa$)</td>
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<td>119</td>
<td>CPI-U: commn. puc $\Delta ln$ Cpi-U: Commodities ($82-84=100, Sa$)</td>
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<td>CPI-U: dbles pscd $\Delta ln$ Cpi-U: Durables ($82-84=100, Sa$)</td>
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<td>121</td>
<td>CPI-U: services pu $\Delta ln$ Cpi-U: Services ($82-84=100, Sa$)</td>
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<td>122</td>
<td>CPI-U: ex food puxf $\Delta ln$ Cpi-U: All Items Less Food ($82-84=100, Sa$)</td>
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<td>123</td>
<td>CPI-U: ex shelter puxas $\Delta ln$ Cpi-U: All Items Less Shelter ($82-84=100, Sa$)</td>
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<td>124</td>
<td>CPI-U: ex med puxmn $\Delta ln$ Cpi-U: All Items Less Medical Care ($82-84=100, Sa$)</td>
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<td>125</td>
<td>PCE defl gndc $\Delta ln$ Pce. Impl Pr Defl.Pce (1987=100)</td>
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<td>126</td>
<td>PCE defl: dbles gndcd $\Delta ln$ Pce. Impl Pr Defl.Pce: Durables (1987=100)</td>
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<td>127</td>
<td>PCE defl: nondble gndc $\Delta ln$ Pce. Impl Pr Defl.Pce: Nondurables (1996=100)</td>
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<td>128</td>
<td>PCE defl: service gndcs $\Delta ln$ Pce. Impl Pr Defl.Pce: Services (1987=100)</td>
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<td>129</td>
<td>AHE: goods ces275 $\Delta ln$ Avg Hourly Earnings of Prod or NonsupWorkers on Private Nonfarm Payrolls - Goods-Producing</td>
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<td>AHE: constr ces277 $\Delta ln$ Avg Hourly Earnings of Prod or NonsupWorkers on Private Nonfarm Payrolls - Construction</td>
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<td>AHE: mfg ces278 $\Delta ln$ Avg Hourly Earnings of Prod or NonsupWorkers on Private Nonfarm Payrolls - Manufacturing</td>
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<td>132</td>
<td>Consumer expect hhsnum $\Delta v$ U. of Mich. Index of Consumer Expectations (Bcd-83)</td>
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(Source: Ludvigson and Ng, 2009a,b)
Appendix A2.

Solution to Proposition 2. Using the factor integral method \( M = e^{\int \frac{1}{m_h} dl_h} = l_h^{\frac{1}{M}} \), wage equation reduces to

\[
wh = l_h^{-\frac{1}{\eta}} z x_h \int l_h^{\frac{\eta}{2} - 1} (x_l l + x_h l_h)^{\gamma - 1} dl_h + \eta \beta c \theta + (1 - \eta) b + l_h^{-\frac{1}{\eta}} C
\]

Keeping the wage bill constant requires that the constant (or actually the sum of constants) of integration, \( C \), must be equal to zero. Remainder integral is a binomial Chebyshev integral. According to Chebyshev theorem on the integration of binomial differentials: If \( \frac{1}{\eta} \) is integer, \( \int l_h^{\frac{\eta}{2} - 1} (x_l l + x_h l_h)^{\gamma - 1} dl_h \) is expressible in terms of elementary functions. Therefore to solve the integral I assume that \( \frac{1}{\eta} - 1 = K \) is an integer, i.e. \( \eta = \frac{1}{K + 1} \) therefore one can apply integration by part recursively, and derives the wage expression (2.13).
References


Guell, Maia (2010), "Firing Costs, Dismissal Conflicts and Labour Market Outcomes," Els Opuscles del CREI.


The World Bank, Doing Business, 2011,


