Universitat Autònoma de Barcelona

Essays in Contest Theory: Fostering competition in contests

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Contents

Abstract

My dissertation consists of three chapters which are devoted to Game Theory. Game theory is a discipline that studies mathematical models of conflict and cooperation between rational individuals. I mainly focus on situations of conflict, and my research consists of understanding the behavior and interactions of individuals who participate in a contest. Competition in contests might be mitigated for several reasons. For instance, individuals might exert less effort when competing with unskilled opponents. Also, in collective contests, individuals that belong to a group might reduce effort in the competition because they do not internalize the benefits that winning a public prize yields to all members in the group. For another example, consider the case of a contest organizer who fails to set the rules that maximize competition. My research focuses on finding solutions to the reduction of competition due to the previous reasons. Applications of my research include litigation, firms competing for market shares, job promotions or sport competitions among others.

The first chapter addresses conflicts between two groups when trying to win a group-specific public good prize. We analyze how these contests are affected when groups are led by an organizer with the capacity to impose transfers to share the costs of individual efforts within the group. Situations in which this problem is relevant abound. Consider the case of two law firms that compete to win a case. Winning a case is a public prize because lawyers increase their popularity when their firm wins cases. However, when individual effort to win a case is costly, lawyers may exert less effort than the one the boss of the law firm would like them to exert, causing underperformances. In order to implement a required level of effort, law firms may set a pay system that promotes effort. In particular, law firms may offer contracts in which lawyers that exert more effort receive a higher wage. Other examples are firms competing for market shares; they can design a pay system to implement optimal effort by its employees. In this chapter, I describe centralized levels of group effort as those that organizers in each group wish to attain, and I define underperformance (overperformance) as the situation in which effort exerted by individuals in a non-cooperative setting is smaller (larger) than the centralized levels of group effort. I show first that the larger group always underperforms, while the smaller one only does so if its size is sufficiently close to the larger group. Second, I show that organizers implement the centralized levels of group effort when sharing the effort costs in an egalitarian way. And third, I examine the game where organizers compete strategically in setting the cost-sharing scheme of their group. I show that the cost sharing rule set by the larger group is more egalitarian than the one of the smaller group.

In the second chapter I study the design of a contest between two possible budget constrained individuals when the organizer of the contest is not informed about the actual size of budgets and where the objective is to maximize competition. The problem of optimal contest design is relevant in areas such as R&D. R&D contracts that an investor sets to a researcher are based on absolute level of performance of the latter, which involves costly monitoring effort and could be complicated or even impossible to enforce. The use of contests solves the problem naturally, since a prize is going to be awarded to one of the participants by law enforcement. Contests create competition and eliminate costly monitoring effort. Examples of R&D contests in the area of artificial intelligence include the Netflix Prize, which is awarded to the algorithm that predicts better the rating for films or the Loebner prize, which is awarded to the most intelligent software. Researchers or firms that participate in the contest could be financially constrained. Spending resources is crucial to create a good product and win the contest. In this context, contest effort can also be understood as investment of resources (monetary effort) that the firm or the researcher invests. In these kinds of contests, organizers want to create a competitive environment to induce research in the particular area of knowledge the contest is focused on. This is the reason why I assume that the contest organizer wants to maximize competition. However, the existence of constraints is a problem for competition, since firms or researchers that are constrained cannot put competitive pressure on the rivals. Also, constraints of participants are normally unknown for the organizer. Thus, the problem of the organizer consists of designing the rules of the contests that increases competition taking into account this lack of information. I first define the optimal effort as the maximum levels of effort or competition achievable in a situation of complete information. When the organizer has complete information, competition is maximized when the contest is biased towards the constrained individual in case they have different budgets. Afterwards, I study the case where the organizer has incomplete information and asks individuals about their budgets. I show that the design of complete information fails to maximize competition, since unconstrained individuals lie about their budgets in order to ensure a favorable position in the contest. I show that there is a mechanism that implements the optimal effort. This mechanism consists of offering lower prizes to individuals that claim to be constrained to induce unconstrained players to report their true budget.

In the third chapter I study contests form a different point of view that usually the literature does. Most of the research on contests focuses on individuals competing for a desirable prize. However, there are situations in which individuals compete to avoid a bad, burden or punishment. To illustrate the problem suppose a government plans to develop a project that is useful and necessary for society. However residents oppose the project for being developed close to their homes because it causes a negative externality. Examples include airports, homeless shelters, prisons or toxic waste dumps among others. In this chapter I use the reverse lottery contest model to study such situations. I show first existence of equilibrium when individuals compete to avoid a bad using the reverse lottery contest model. Then, I study an application in which a government needs to allocate a dump in a region and wants to maximize lobbying. The government decides either to divide this dump in smaller pieces or not, considering that lobbies in each region influence the government to avoid such dumps. I show that lobbying is maximized when the government does not divide the dump. I also compare the properties of the reverse lottery contest with the conventional lottery contest. In a framework in which individuals compete to win many prizes and avoid many bads, I show that aggregate effort is higher using the conventional (reverse) lottery contest when there are more (less) prizes than bads.

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Chapter 1

Sharing the effort costs in group contests

1.1 Introduction

This paper addresses conflicts between two groups when trying to win a group-specific public good prize. We analyze how these contests are affected when groups are led by an organizer with the capacity to impose transfers to share the costs of individual efforts within the group. First we describe centralized levels of group effort - those that organizers wish to attain - and decentralized levels of group effort - those that individuals exert when they act non-cooperatively - and compare them. Throughout the paper we refer to them as simply centralized and decentralized (group) effort. Second, we characterize cost sharing schemes that induce centralized levels of group effort. And, third, we examine the game in which organizers compete strategically in setting the cost-sharing scheme of their group.

Economic situations such as sport competitions, R&D contests or litigation illustrate this problem. In all these situations, groups compete to win a prize that is public (or has a public component). All athletes that belong to a winning sport team, members of a successful research group in an R&D contest and all lawyers that belong to a winning law firm increase their popularity. However, in such situations group members might exert less effort than the group organizer (president of the sport team, head of the research group or boss of the law firm) would like them to exert because they expect their mates to compete more fiercely or because they do not internalize the benefits that winning a public prize yields to the rest of members of the group. For another example, think of cities competing to get the Olympic Games. Different levels of government (local council, region, State...) spend resources to get the games. All levels of government benefit from getting the games for tax collection or economic growth. Also, different levels of government might spend less resources to attract the games than what would be optimal for the country. Group organizers might not be able to force a level of effort. However, in many situations they have the authority to set cost redistributive policies to increase effort when it is observable. That is, sport team presidents, heads of research groups and bosses of law firms might pay higher wages to hardworking group members in deterrence to those that exert less effort. Effort can be observable and evaluated in terms of comparisons to previous individual performances. In the case of cities competing for the Olympic Games, transfers to the most investing level of government might take place. The Prime Minister observes investment of the different levels of government and might has the authority to set redistributive policies.

An important assumption of our model is that the prize is indivisible. We consider two types of contests: centralized and decentralized contests. In centralized contests there is an organizer in each group that chooses the effort of each individual. The goal of the organizer is to maximize the group payoffs. In decentralized contests there are no organizers and every individual chooses her effort non-cooperatively. If the effort exerted by a group in a centralized contest is larger than the effort exerted in a decentralized contest, the group underperforms with respect to the centralized contest (or simply, underperforms). Otherwise, the group outperforms (with respect to the centralized contest).

The objective of this paper is to determine when underperforming arises, and how we can implement the centralized levels of group effort through cost-sharing schemes. Also, we examine the game in which organizers compete strategically in setting the cost-sharing scheme of their group and compare these schemes with those that implement the centralized levels of group effort. The technology of conflict that we use is the *Tullock Contest Success Function* (*CSF*) because it reflects the applications that we study in this paper. Groups of athletes or lawyers that exert more effort do not necessarily win the match or the case. For tractability, we restrict our analysis to perfect substitute impact functions.

Our results are as follows.

First, we show that the larger group always underperforms while the smaller one only does so if its size is sufficiently close to the larger group. Otherwise, the smaller group outperforms. Underperforming arises because individuals in a group do not internalize the collective benefits that winning a public prize yields to the group. Outperforming arises because individuals of the smaller group take advantage of the underperformance of the larger group, thus exerting more effort than in the centralized setting.

Second, we examine situations where organizers, though observe effort, cannot force their individuals to exert a given effort but can design mechanisms to foster it. In particular, we assume that organizers can impose transfers among individuals within a group that depend on effort to promote it. We call this situation a decentralized contest with transfers. These transfers are not contingent on the outcome of the contest, might be positive or negative, and are balanced. We find that an equal sharing of the total cost in a group induces individual efforts that deliver the centralized level of group effort. Note that an egalitarian cost-sharing scheme is meritocratic, since individuals that exert low effort must contribute to paying the costs of those that exert high effort. We also show that other more complex transfer schemes implement the centralized effort.[1]

Third, we analyze a game in which organizers compete in the transfer they offer. Since in the previous set up organizers set transfers so that they commit to the centralized effort, in this game organizers choose transfers to maximize the group payoffs. This allows us to compare the effects of transfers that implement the centralized effort and transfers used to compete strategically in the contest. The game has two stages. In the first stage, organizers of both groups simultaneously set transfers strategically and in the second stage individuals exert effort. For analytical tractability, we focus on linear transfers. We show that the larger group sets a transfer that shares the effort costs in an egalitarian way, as it is the case in the implementation setup. The smaller group does not set an egalitarian cost-sharing scheme. It sets instead a less meritocratic transfer; that is, the transfer of the larger group promotes effort more than the transfer of the smaller group. With this scheme, members are induced to lower levels of effort, and the lower probability of winning is compensated by lower total costs. Thus, the smaller group set transfers differently. The reason is that when organizers behave strategically, the larger group has the dominant strategy of sharing the effort costs of the contest in an egalitarian way, while the smaller group best responds to this dominant strategy.

Though we find that transfers that implement the centralized effort are not optimal when organizers set transfers strategically in case groups differ in size, studying the implementation of the centralized effort still remains relevant for two reasons. First, aggregate effort in the contest

[1] In Appendix 1.C, we also provide a generalization to setups in which individual valuations of the public prize are heterogeneous.

is larger in the implementation case than in the strategic set up. Then, it is relevant from the point of view of a contest designer whose objective is to maximize aggregate effort and has the capacity to force group organizers to set transfers among group members. If transfers are the only tools that the contest designer has, the resulting transfers of the implementation case maximize competition. Situations where the Estate or organizations pin up wages that depend on effort or results to achieve a level of competition illustrate this problem. Second, it allows us to understand why the transfers that group organizers set in the strategic setting do not implement the resulting effort of a centralized contest.

Literature review

This paper is related to the literature in group contests. Group contests have been studied from two different perspectives depending on the *CSF* used in the study*:* the *Tullock CSF* and the all-pay auction. While the first one assigns a probability of winning to each group depending on exerted effort, the latter assigns the prize to the group that exerts more effort. The following research is focused on the *Tullock CSF* in group contests. Katz, Nitzan and Rosenberg (1990) study a contest between two groups competing for a public prize, with perfect substitute impact function, group-specific public good prize and linear costs. They find that the group size does not matter, since both groups exert the same effort. Baik (2008) analyzes the framework of Katz, Nitzan and Rosenberg (1990) when individuals value the prize differently, and shows that only those individuals that value the prize the most in each group exert effort in equilibrium. Lee (2012) analyzes the weakest link impact function and shows that all individuals in a group exert the same effort. Kolmar and Rommeswinkel (2013) generalize the previous research using a constant elasticity of substitution impact function. They find that the degree of complementarity does not affect the effort exerted by the group when individuals in each group are homogeneous. When individuals are heterogeneous, the higher the heterogeneity, the more similar the effort among individuals is, even though they are less likely to win the contest. Chowdhury, Lee, and Sheremeta (2013a) show that only one individual in each active group, that is not necessarily the individual in the group who values the prize the most, exerts effort using the best shot impact function. Other research adresses similar problems using the all-pay auction. The all-pay auction with an additive impact function is studied in Baik, Kim, and Na (2001) and Topolyan (2014). Chowdhury, Lee, and Topolyan (2013b) analyze a similar framework with the weakest link impact function. Barbieri, Malueg, and Topolyan (2014) study the best shot impact function. Finally, Chowdhury and Topolyan (2015) analyze the case in which one group uses a weakest link and the other group a best shot impact function. It is important to mention other research in group contests that focuses on the *Tullock CSF*. Esteban and Ray (2001) generalize the model of Katz, Nitzan and Rosenberg (1990) assuming groups compete for a prize with public and private characteristics and convex costs. They find that the larger group performs better than the smaller one the more convex costs are and the more public the prize is. See Cheikbossian (2008) and Nitzan and Ueda (2009,2013) for other papers in group contests for a public prize.

The major concern in the previous research is the performance of each group in the contest and the free riding problem within groups. Our contribution is to analyze group underperformances as the difference between the effort exerted by the group in a centralized and decentralized contest, to show how the centralized effort can be implemented using transfers among individuals and to analyze if these transfers are optimal for welfare maximizing organizers. We focus on the simplest framework based on the *Tullock CSF* with perfect substitute impact function and a group-specific public good prize. The analysis might become intractable with more complex impact functions.

Our research is also related to prize sharing in group contests. Nitzan (1991) assumes a prize sharing rule based on a convex combination of an egalitarian distribution and a distribution depending on the relative effort exerted by each individual with respect to the whole group. Lee (1995) generalizes Nitzan (1991) through the following two stage contest. In the first stage, an organizer in each group determines the sharing rule proposed by Nitzan (1991), while in the second stage the contest takes place, the rule being publicly known. Gürtler (2005) extends Lee (1995) analyzing the case in which groups determine their sharing rules simultaneously or sequentially. Nitzan and Ueda (2011) examine a general model in which the sharing rule is private information. Baik and Lee (2012) study whether a group reveals the sharing rule.

The papers mentioned in the previous paragraph study the effects of sharing a private and divisible prize in the contest. By definition, a public prize is non rival and non exclusive. Therefore, if a prize is public, it is non divisible. However, indivisible prizes may become somehow divisible by cost sharing. Some papers introduce the term of cooperation by splitting the effort costs of the contest in an egalitarian way in order to solve other problems. Ursprung (2012) compares the situations of incentives (as in prize sharing), cooperation and non incentives nor cooperation for a private prize. He finds that cooperation is preferred to incentives, though it is not maintainable in the long run. For a similar paper see Epstein and Mealem (2012). As far as we know, there is no previous research about deciding how to share the effort costs of the contest. Nitzan and Ueda (2014b), independently and simultaneously to our research, present a similar strategic set up but when sharing rules are private information, costs are convex and individual valuations of the prize are heterogeneous. In their strategic setting, convex costs allow every individual to be active in the contest and private information allows tractability, since the problem can be understood as a one stage game. Our paper not only differs to Nitzan and Ueda (2014b) with respect to private information in the strategic set up, but also in our aim to analyze the difference between a centralized and decentralized contest, and the implementation of the centralized effort.

An important difference between prize sharing and cost sharing is that while the former only allows for transfers among individuals within a group contingent on winning the contest, the latter is not contingent on the outcome.

This paper is also related to delegation in contests. These models study a two individual contest in which both of them decide either to delegate or not to a third party and the reward she should receive to overcome moral harzard. Warneryd (1998) shows that delegation is beneficial for individuals, at least if such delegation is compulsory. Baik (2007) focuses on the optimal contract and shows that the equilibrium contract is a no-win-no-pay contract. Baik (2008) studies fixed fees and contingent fees with caps for delegates, and shows that it is optimal to set a zero fixed fee and a contingent fee equal to the cap of the legal system. For other papers that study delegation see Schoonbeek (2002) and Brandauer and Englmaier (2006).

The main difference of this paper with the literature is that the organizer chooses a payment or transfer scheme to individuals to maximize the payoffs of the group, while in the delegation framework the objective of the principals is to maximize their own payoffs and not those of the delegates. Also, the main payment structure analyzed in these papers focuses on contingent fees, while our transfer is set independently on the outcome of the contest and depends in the effort of each individual relative the group effort.

The paper has the following structure. In the next section, the main framework is introduced. Section 1.3 analyzes centralized and decentralized contests and when underperformances and outperformances arise. Section 1.4 shows how organizers implement the centralized effort through transfers. Section 1.5 analyzes the case in which organizers set transfers strategically. Section 1.6 concludes. Proofs are relegated to the appendix.

1.2 The model

Consider a contest between two groups $l = 1, 2$ that compete in order to win a public prize. Each group is composed of n_l individuals and without loss of generality, $n_1 \geq n_2$. In particular, let $n_1 = an_2$, with $a \in [1, \infty)$. All individuals value the public prize v. Individuals of each group exert effort in order to win the prize. Effort exerted by individual i in group l is denoted by e_{li} and the total effort exerted for the group is $E_l = \sum_{i=1}^{n_l} e_{li}$. Let $E_{l-j} = \sum_{i \neq j}^{n_l} e_{li}$, $E_{l-jk} = \sum_{i \neq j,k}^{n_l} e_{li}$ and so on. The total effort exerted in the contest is $E = \sum_{l=1}^{2} E_l$.

The probability that group l wins the prize is:

$$p_l(E_l, E_m) = \frac{E_l}{E} \text{ for } l = 1, 2. \tag{1.1}$$

Note that $p_l(E_l, E_m)$, which is known as the *Tullock Contest Success Function*,[2] is twice continuously differentiable in $\mathbf{R}^2_{++}$, strictly increasing in E_l, strictly decreasing in E_m and satisfies homogeneity of degree 0.[3]

Effort is costly and the cost function is convex:

$$c(e_{li}) = \frac{e_{li}^{\phi+1}}{\phi + 1} \tag{1.2}$$

with $\phi \geq 0$. Thus, payoffs of individual i in group l are:

$$\pi_{li} = \frac{E_l}{E} v - \frac{e_{li}^{\phi+1}}{\phi + 1}, \; i = 1 ... n_l, \; l = 1, 2, \tag{1.3}$$

while group payoffs are:

$$\pi_l = \sum_{i=1}^{n_l} \pi_{li} = \frac{E_l}{E} v n_l - \sum_{i=1}^{n_l} \frac{e_{li}^{\phi+1}}{\phi + 1}, \; l = 1, 2. \tag{1.4}$$

Note that individuals in both groups value equally the public prize and have the same cost function. Groups only differ in size and its analysis is our main purpose. We distinguish between two types of contests: centralized and decentralized contests.

Definition 1.1 *A contest is centralized if there is an organizer in each group that chooses and forces the amount of effort that individuals in her group are going to exert to maximize group payoffs.*

[2] Skaperdas (1996) axiomatized the *Tullock CSF*. For the purposes of simplicity, we assume that efforts enter additively in the *CSF*. See Kohlmar and Rommeswinkel (2013) for a generalization with a CES as impact functions.

[3] This property implies that the probability of winning the prize does not depend on units of measurement.

We assume that organizers can force their individuals to exert such effort. The goal of each organizer is to maximize the payoffs of her group. Therefore, organizers internalize the benefits that winning a public prize yields to every individual in the group.

Definition 1.2 *A contest is decentralized if there are no organizers in groups and every individual in every group chooses the amount of effort that maximizes their own payoffs.*

In a decentralized contest individuals do not take into account the payoffs of the rest of the group. That is, individuals do not internalize the benefits that winning a public prize yields to every individual in the group when exerting effort.

Section 1.3 analyzes group underperformances and outperformances that arise in decentralized contests taking the centralized contest as a benchmark.

In Section 1.4, we analyze decentralized contests with transfers.

Definition 1.3 *A contest is decentralized with transfers if organizers, though can observe effort, cannot force their individuals to exert an amount of effort but can set transfers $t_l(e_{li}, E_l)$ among individuals within a group to provide incentives.*

We make the following assumption:

Assumption 1.1. *Transfers depend on effort, are not contingent on the outcome of the contest, might be positive or negative, are continuous and differentiable in both arguments and are balanced:*

$$\sum_{i=1}^{n_l} t_l(e_{li}, E_l) = 0 \text{ for } l = 1, 2. \tag{1.5}$$

The problem facing the organizers is to choose the transfers that implement the effort exerted in a centralized contest.

In Section 1.5, organizers set transfers strategically. We study the following two stage game. In the first stage, organizers choose non-cooperatively and simultaneously the transfer $t_l(e_{li}, E_l)$ to maximize the group payoffs. In the second stage, individuals exert effort. There is perfect information and the problem is solved by backward induction.

1.3 Underperforming and outperforming in decentralized contests

This section analyzes conditions for underperforming and outperforming in groups. To do so, we analyze two different types of contests: centralized and decentralized contest.

Let E_l^c and E_l^d be the effort exerted by group $l = 1, 2$ in a centralized and a decentralized contest respectively. Throughout the paper we refer to them as simply centralized and decentralized (group) effort.

Definition 1.4 *Group $l = 1, 2$ underperforms with respect to the centralized contest (or simply underperforms) if $E_l^c > E_l^d$. Group $l = 1, 2$ outperforms with respect to the centralized contest (or simply outperforms) if $E_l^c < E_l^d$.*

We will see next that underperforming in one group might induce outperforming in the other group.

We start by analyzing a centralized contest. In a centralized contest, payoffs of organizers are given by equation (1.4). The following first order conditions are necessary:

$$\frac{E_m}{E^2} v n_l = e_{li}^{\phi},\ i = 1...n_l,\ \ l = 1, 2. \tag{1.6}$$

Note that since the left hand side is equal for every individual in the same group and the right hand side is endogenous, every individual in each group exerts the same amount of effort in equilibrium when $\phi > 0$, so that $e_{li} = e_{lj} = e_l$ for every $i \neq j$.[4] Now, using the first order conditions of a representative individual for each group, it follows that $e_l = e_m = e$ for $m \neq l$. When introducing these conditions in equation (1.6), it follows that the effort exerted by any individual evaluated at the equilibrium is

$$e^c = [\frac{v n_l n_m}{(n_l + n_m)^2}]^{\frac{1}{\phi+1}}, \tag{1.7}$$

and the total effort of the group evaluated at the equilibrium is:

$$E_l^c = \sum_{i=1}^{n_l} e_{li}^c = n_l [\frac{v n_l n_m}{(n_l + n_m)^2}]^{\frac{1}{\phi+1}} \text{ for } l = 1, 2. \tag{1.8}$$

It is straightforward to see that since every individual in a centralized contest exerts the same amount of effort, the larger group exerts more effort and is therefore more likely to win the contest than the smaller group. Payoff function (1.4) exhibits a maximum since the second order condition is negative:

$$\frac{-2 E_m v n_l}{E^3} - \phi e_l^{\phi-1} < 0 \text{ for } l = 1, 2. \tag{1.9}$$

[4] In case $\phi = 0$, the marginal cost becomes unity. Then, there is a continuum of equilibria of effort exerted by individuals within the group. The sum of efforts of individuals in a given group are the same across all these equilibria, as the equilibrium winning probabilities are. For the remaining of the paper, in case $\phi = 0$, we will refer to the symmetric equilibrium within the group, in which every individual exerts the same amount of effort.

In a decentralized contest, there are no organizers and every individual chooses the amount of effort that maximizes her own payoffs, which are given by equation (1.3). The following first order conditions are necessary:[5]

$$\frac{E_m}{E^2}v = e_{li}^{\phi},\ i = 1...n_l,\ \ l = 1,2. \tag{1.10}$$

Again, when $\phi > 0$ individuals within the same group exert the same amount of effort so that $e_{li} = e_{lj} = e_l$ for every $i \neq j$. Using the first order conditions of a representative individual for each group, it follows that:

$$\frac{e_l}{e_m} = (\frac{n_m}{n_l})^{\frac{1}{\phi+1}}. \tag{1.11}$$

Then, the effort exerted by individuals depends on the size of both groups. Moreover, individuals in the larger group exert less effort than individuals in the smaller group. Multiplying both sides of equation (1.11) by n_l/n_m, we get:

$$\frac{E_l}{E_m} = (\frac{n_l}{n_m})^{\frac{\phi}{\phi+1}}. \tag{1.12}$$

It is easy to see that the larger group exerts more effort than the smaller group, being more likely to win the contest when $\phi > 0$. Otherwise, both groups exert the same amount of effort. Using equations (1.11) and (1.10) yields individual effort in each group:

$$e_l^d = [v\frac{n_m(\frac{n_l}{n_m})^{\frac{1}{\phi+1}})}{(n_l + n_m(\frac{n_l}{n_m})^{\frac{1}{\phi+1}})^2}]^{\frac{1}{\phi+1}}, \tag{1.13}$$

and group effort is:

$$E_l^d = \sum_{i=1}^{n_l} e_{li}^d = n_l[v\frac{n_m(\frac{n_l}{n_m})^{\frac{1}{\phi+1}})}{(n_l + n_m(\frac{n_l}{n_m})^{\frac{1}{\phi+1}})^2}]^{\frac{1}{\phi+1}}. \tag{1.14}$$

The following proposition states conditions for underperforming and outperforming.

Proposition 1.1 *The larger group always underperforms with respect to the centralized contest, while the smaller one does only if its size is sufficiently close to the larger group, and otherwise outperforms. That is, there exists an $\bar{a} \in [1, \infty)$ such that for $a < \bar{a}$, both groups underperform while for $a > \bar{a}$, Group 2 outperforms.*[6]

[5] Note that payoffs exhibit a maximum by condition (1.9).

[6] The results in Proposition 1.1, 1.2 and 1.3 can be extended to the general case in which the *CSF* has a general form $p_l(E_l, E_m)$ for $l = 1, 2$. This function must be twice continuously differentiable in $\mathbf{R}^2_{++}$, $p_l(0, E_m) = 0$ for $E_m > 0$ with $m \neq l$, strictly increasing in E_l, decreasing in E_m and must satisfy homogeneity of degree 0. The probability that group $m \neq l$ wins the prize is $p_m = 1 - p_l(E_l, E_m)$, and the function must be symmetric, i.e. $p_l(x, y) = p_m(y, x)$.

Underperforming and outperforming arise because in a decentralized contest individuals do not internalize the benefits that winning a public prize yields to the group, while organizers do in a centralized setting. In case $a < \bar{a}$, both organizers force their individuals to exert more effort than the one exerted in a decentralized contest since they take into account the benefits that every individual in the group obtains in case of winning the prize. Now analyze the case in which $a > \bar{a}$. In a centralized contest, the organizer in Group 2 knows that the effort that organizer in Group 1 chooses is high since her membership is larger. Then, the organizer in Group 2 chooses a small amount of effort, since lower costs compensate the smaller probability of winning of her group. However, in a decentralized contest, individuals in Group 2 take advantage of the lower amount of effort exerted in Group 1 due to the lack of internalization of the properties of the public prize and exert more effort than in the centralized contest, which yields outperforming. From a more technical point of view, the best replies of both groups shitf outwards in the centralized contest relative to the decentralized contest. The shift of the best reply of Group 1 is larger than the one of Group 2. It yields that for Group 1 (2), effort of Group 2 (1) is a strategic complement (substitute). In case the shitf of Group 1 is too large relative to the shift of Group 2, the best replies intersect in a point where effort of Group 2 is smaller in a centralized contest, which yields overperforming. Otherwise, underperforming arises.

1.4 Implementation of centralized effort

In the previous section, we assumed that individuals follow the instructions of their organizer obediently. In this section, we assume that organizers, though can observe effort, cannot force their individuals to exert an amount of effort, but can design mechanisms in order to implement the effort exerted in a centralized contest. In particular, we assume that organizers can set transfers among individuals within a group. We call this situation a decentralized contest with transfers. This case is specially relevant in situations that involve firms that cannot force their workers to exert a level of effort, but can design pay systems that implement it.

In a decentralized contest with transfers, individuals in each group might receive (pay) a transfer

We need to assume that $\partial p_l(x,x)/\partial E_l$ tends to 0 when x tends to infinity, and it tends to a number greater than $1/v$ when x tends to 0. Finally, it is also necessary to assume that $0 < y(\partial^2 p_l(x,y)/\partial E_l \partial E_m)/(\partial p_l(x,y)/\partial E_l) < 1$ for all $x > y > 0$. To see the analysis with the general form, see previous versions of this paper.

$t_l(e_{li}, E_l)$ from (to) other individuals in the same group. Payoffs of individual i in group l become:

$$\pi_{li} = \frac{E_l}{E}v - \frac{e_{li}^{\phi+1}}{\phi+1} + t_l(e_{li}, E_l),\ i = 1...n_l,\ \ l = 1, 2. \tag{1.15}$$

By condition (1.5), it follows that the group payoffs are the same as in equation (1.4).

Transfers redistribute the cost of the contest among individuals in a group. Organizers can compensate individuals that exert more effort in the contest and penalize those who exert less.

Throughout the paper, we will refer to meritocracy of transfers set by organizers, defined as follows:

Definition 1.5 *Transfer $t(e_{li}, E_l)$ is meritocratic if $\partial t(e_{li}, E_l)/\partial e_{li} > 0$. For any given vector of efforts $(\mathbf{e}_{l1}, ..., \mathbf{e}_{ln})$, transfer $t^1(\mathbf{e}_{li}, \mathbf{E}_{l-i})$ is more meritocratic than transfer $t^2(\mathbf{e}_{li}, \mathbf{E}_{l-i})$ at $(\mathbf{e}_{l1}, ..., \mathbf{e}_{ln})$ if $\partial t^1(\mathbf{e}_{li}, \mathbf{E}_{l-i})/\partial e_{li} > \partial t^2(\mathbf{e}_{li}, \mathbf{E}_{l-i})/\partial e_{li}$.*

The problem facing each organizer is to set a transfer $t_l(e_{li}, E_l)$ that implements the centralized effort under the assumption that the organizer in the other group does likewise.[7]

Proposition 1.2 *The following transfer implements the effort exerted in a centralized contest:*[8]

$$t_l^t(e_{li}, E_{l-i}) = c(e_{li}) - \sum_{i=1}^{n_l} \frac{c(e_{li})}{n_l} \text{ for } i = 1...n_l\ ,\ \ l = 1, 2. \tag{1.16}$$

This transfer is meritocratic. The organizer in Group 1 sets a more meritocratic transfer than the organizer in Group 2.

The upper-script t denotes a decentralized contest with transfers scenario. The proof consists of introducing (1.16) in (1.15) and checking that the transfer implements the effort of the centralized contest. The transfer that every individual in a group obtains is the difference between her cost of effort and the average of the cost of effort in the group.[9] This transfer is meritocratic and therefore

[7] In Appendix 1.B we show why this assumption is plausible. There, we study a game in which each organizer chooses either to monitor individuals or not.

[8] Transfers may not be the unique mechanism that implements the centralized effort. In particular, a severe punishment to individuals who deviate from the required effort would also implement the centralized effort. In equilibrium, the punishment would not be used since individuals would exert the required effort. However, such severe punishments would not be credible for individuals since the purpose of the organizer is to maximize group payoffs. Punishments, if they are not credible, might not achieve their main goal. We have only focused on continuous transfers for simplicity.

[9] If groups compete for a private prize, costs can also be shared. In case the prize were completely private and assuming that the prize is divided in per capita terms among the individuals of the group, the transfer that implements the effort in the centralized contest would be exactly the same. The intuition is the same as for the public prize case.

induces individuals to exert more effort than they would do without the transfers. Furthermore, the larger group sets a more meritocratic transfer than the smaller one, since the marginal cost of individuals decreases in the size of the group.

Although transfers are meritocratic and induce individuals in both groups to exert more effort, these transfers avoid the outperforming that might arise in the small group. The main reason is that by setting these transfers, the larger group is induced to exert a large amount of effort. In absence of transfers, the smaller group would exert less effort than in a centralized contest, since they are disincentivized by the agressivity of the larger group. Therefore, in order to implement the centralized effort the smaller group also needs meritocratic transfers.

Inserting transfer (1.16) in payoffs (1.15), it follows that:

$$\pi_{li} = \frac{E_l}{E}v - \sum_{i=1}^{n_l} \frac{c(e_{li})}{n_l},\ i = 1...n_l,\ \ l = 1, 2. \tag{1.17}$$

Therefore, the cost of any individual depends on the average of the cost of effort exerted by her group. By setting these transfers, organizers have introduced an egalitarian rule in the distribution of the costs of the contest. That is, every individual in the group pays exactly the same costs independently on the amount of effort made by the rest of the individuals in the group. The intuition is simple. In case one individual exerts more effort than another one in the group, the individual that exerts less effort obtains higher payoffs since every individual in the group benefits equally from a public prize. However, by setting this transfer, the individuals that exert less effort during the contest are forced to send a transfer to the hard-workers so that every individual obtains the same payoffs.

It is easy to see that with this transfer, the marginal cost of every individual is being substituted with the group marginal cost, implementing the centralized effort.

The transfer presented in Proposition 1.2 implements the centralized effort in case individuals within a group value the prize equally.[10] However, other transfers might implement the effort of the centralized contest. We show now the existence of other transfers that implement the centralized effort.

We first solve the problem of the decentralized contest with transfers. To do so, take the first order condition for every individual in each group of (1.15):

$$\frac{E_m}{E^2}v = e_{li}^{\phi} - \frac{\partial t_l(e_{li}, E_{l-i})}{\partial e_{li}} \text{ for } m \neq l,\ i = 1...n_l. \tag{1.18}$$

[10] Other transfers are also needed when individuals value the prize differently. This case is analyzed in Appendix 1.C for linear costs.

Denote $f(e_{li}, E_{l-i}) \equiv e_{li}^{\phi} - \partial t_l(e_{li}, E_{l-i})/\partial e_{li}$. Using (1.18) for any individual of both groups, we obtain the effort exerted in the group evaluated at the equilibrium in a decentralized contest with transfers:

$$E_l^t = v\frac{f(e_{mj}^t, E_{m-j}^t)}{(f(e_{li}^t, E_{l-i}^t) + f(e_{mj}^t, E_{m-j}^t))^2}, \; l = 1, 2, \; m \neq l. \tag{1.19}$$

The organizer of each group implements the centralized effort choosing the transfer $t_l(e_{li}, E_{l-i})$ such that $E_l^c = E_l^t$.

Proposition 1.3 *The following transfer implements the effort of a centralized contest:*

$$t_l^t(e_{li}, E_{l-i}) = c(e_{li}) - \sum_{k \neq i}^{n_l} \frac{c(e_{lk})}{(n_l - 1)} + A_l(\sum_{k \neq i}^{n_l} \frac{e_{lk}}{n_l - 1} - e_{li}), \text{ for } l = 1, 2, i = 1 \dots n_l \, . \tag{1.20}$$

$$\text{where } A_l = \frac{vn_m}{(n_l + n_m)^2}(\frac{vn_l n_m}{(n_l + n_m)^2})^{\frac{-1}{\phi+1}} \tag{1.21}$$

This transfer is meritocratic for $e_{li} > A_l^{\frac{1}{\phi}}$. *The organizer in Group 1 sets a more meritocratic transfer than the organizer in Group 2.*

The proof consists of introducing (1.20) in (1.15) and checking that the transfer implements the effort of the centralized contest. In the appendix, we show how to construct the transfer function. This complex functional form can be decomposed in two parts. First of all, the transfer that an individual obtains depends positively on the distance between her cost of effort and the average cost of the rest of individuals in the group. Additionally, it depends negatively on the distance between her effort and the average effort of the rest of individuals in the group, since A_l is positive. Therefore, A_l can be understood as a disincentivizing parameter. Parameter A_l is decreasing in n_l since organizers want their individuals to exert more effort the larger the group is. It is increasing in n_m, since the rival group becomes stronger and disincentivizes the organizer of the group.

Note also that the transfer is meritocratic for $e_{li} > A_l^{1/\phi}$. The reason is that the organizer of the smaller group sets a non meritocratic transfer to avoid outperforming.

Finally, note that contrarily to transfer (1.16), it is necessary to assume that the organizer of each group knows both the value of the prize and the size of the rival group when using (1.20). Thus, (1.16) is more useful in case that organizers are not aware of these parameters.

1.5 Strategic transfer setting by organizers

In this section, organizers choose transfers strategically in order to maximize the group payoffs. This analysis is interesting to determine under which circumstances the implementation of the effort of the centralized contest developed in the previous section is optimal for organizers. For analytical tractability, we assume organizers are constrained to use linear transfer functions as follows:

$$t_l(e_{li}, E_{l-i}) = a_l e_{li} + b_l E_l. \tag{1.22}$$

We also assume that $\phi = 0$. Linearity in costs allows the possibility of obtaining closed form solutions for efforts and the parameters of the transfer functions in this setting.

The game has two stages. In the first stage, each organizer chooses the parameters a_l and b_l simultaneously and non-cooperatively with the other group to maximize group payoffs. In the second stage, individuals in each group exert effort and the contest takes place.

Before solving the game, it is interesting to understand some properties of the linear transfer function. First of all, using equation (1.5) on the linear transfer function it follows that $b_l = -a_l/n_l$. Likewise, call $\alpha_l = (1 - a_l)$, and assume $\alpha_l \in [0, 1]$ for $l = 1, 2$.[11] Introducing these equalities in equation (1.22), and introducing the resulting transfer function in payoffs (1.15) yields:

$$\pi_{li} = \frac{E_l}{E} v - \alpha_l e_{li} - (1 - \alpha_l)\frac{E_l}{n_l} \text{ for } i = 1...n_l,\ l = 1, 2. \tag{1.23}$$

Therefore, payoffs of individuals in the contest are given by the expected value of winning the prize minus a convex combination between two rules of sharing the costs of the contest, where α_l is the parameter of the convex combination. In case $\alpha_l = 1$, each individual in a group faces her own cost, while in case $\alpha_l = 0$ each individual assumes the average effort cost of the group. Mixed rules are allowed, since $\alpha_l \in [0, 1]$ for $l = 1, 2$.

It is necessary to find the values of α_l that each organizer chooses in order to solve the game. We proceed using backward induction. Then, we start solving the second stage of the game. In the second stage, every individual in the contest chooses the effort that maximizes payoffs (1.23) taking parameters α_l for $l = 1, 2$ as given.

The following first order condition for any individual in group $l = 1, 2$ is necessary:

$$\frac{E_m}{E^2} v = \Gamma_l \text{ for } m \neq l, \tag{1.24}$$

where $\Gamma_l = \alpha_l(n_l - 1)/n_l + 1/n_l$. The marginal cost depends on the sharing rule chosen by the organizer of the group. The lower α_l is, the lower the marginal cost of each individual of group l for $l = 1, 2$ is (and viceversa). Note also that for a given $\alpha_l > 0$, the higher the size of group l,

[11] We assume $\alpha_l \in [0, 1]$ for $l = 1, 2$ to ensure an equilibrium in pure strategies. If we eliminate the lower bound of α_l, an equilibrium in pure strategies does not exist.

the lower the marginal cost of individuals is since part (or all) of the cost is shared among all the individuals in the group.

Using the first order conditions of any two individuals of different groups yields:

$$\frac{E_m}{E_l} = \Gamma \text{ where } \Gamma = \frac{\Gamma_l}{\Gamma_m}. \tag{1.25}$$

Equation (1.25) relates the effort exerted by group m relative to group l, which depends on parameter Γ. This parameter depends on α_l for $l = 1, 2$ and the size of both groups. Other things equal, the group whose α is lower is the group that exerts more effort; and again other things equal, the group whose size is larger exerts more effort than the other group for $\alpha_l > 0$.

Introducing equation (1.25) in (1.24) yields the effort exerted by each group as a function of Γ_l and Γ_m:

$$E_l = \frac{v\Gamma_m}{[\Gamma_l + \Gamma_m]^2},\ l = 1, 2,\ m \neq l. \tag{1.26}$$

We compute easily the probability of winning the contest of each group as a function of Γ_l and Γ_m:

$$p_l = \frac{\Gamma_m}{\Gamma_l + \Gamma_m},\ l = 1, 2,\ m \neq l. \tag{1.27}$$

Note that the equilibrium has not yet been characterized. The problem has been solved as a function of the decision rules that need to be set in the first stage of the game. Solving backwards, organizers set in a non-cooperative way the rules α_l such that they maximize their group payoffs. Introducing equations (1.27) and (1.26) in equation (1.23), and summing across individuals, we obtain the group payoffs as a function of α_l and α_m:

$$\pi_l(\alpha_l, \alpha_m) = p_l(\alpha_l, \alpha_m)n_l v - E_l(\alpha_l, \alpha_m). \tag{1.28}$$

Taking the first derivative of payoffs (1.28) with respect to α_l and making some algebraical arrangements:

$$\frac{\partial \pi_l}{\partial \alpha_l} = \frac{v(n_l - 1)\Gamma_m}{[\Gamma_l + \Gamma_m]^2}[-1 + \frac{2}{n_l[\Gamma_l + \Gamma_m]}] \text{ for } l = 1, 2,\ l \neq m. \tag{1.29}$$

Studying the properties of the group payoffs allows us to determine the decision rule set by each group and consequently characterize the equilibrium in the first and second stages. The following proposition shows the main results.

Proposition 1.4 *Suppose $\phi = 0$ and organizers set transfers strategically.*

a) If $n_1 = n_2$, then $\alpha_l^* = 0$ and

$$t_l^*(e_{li}, E_{l-i}) = e_{li} - \frac{E_l}{n_l} \text{ for } l = 1, 2. \tag{1.30}$$

Therefore, both groups in the contest exert the same amount of effort and are equally likely to win.

b) If $n_1 > n_2$, then $\alpha_1^* = 0$ and $\alpha_2^* = (n_1 - n_2)/((n_2 - 1)n_1)$. Therefore, organizer of Group 1 chooses the transfer (1.30), while organizer of Group 2 chooses:

$$t_2^*(e_{2i}, E_{2-i}) = \frac{n_1 n_2 - 2n_1 + n_2}{n_1(n_2 - 1)} e_{2i} - \frac{n_1 n_2 - 2n_1 + n_2}{n_1(n_2 - 1)} \frac{E_2}{n_2} \ . \tag{1.31}$$

The transfer set by Group 1 is more meritocratic than the transfer set by Group 2. Therefore, Group 1 exerts more effort than Group 2 and is more likely to win the contest. Also, the larger the difference of size between groups is, the less meritocratic the transfer set by Group 2 is.

The upper-script $*$ denotes the strategic transfer setting by organizers scenario. If groups have the same size, organizers split the effort costs of the group among individuals in an egalitarian way. This decision is a dominant strategy for both organizers. By setting this rule, individuals have a lower marginal cost, which induces them to exert more effort to be more likely to win the contest. Since groups are equal in size and have chosen the same rule, both groups exert the same effort in equilibrium and are equally likely to win. Furthermore, the transfers chosen by organizers when they behave strategically coincide with the implementation case when groups have the same size.

Effort is higher than in case the rule is $\alpha_l = 1$ (i.e. when there are no transfers) for both groups because the marginal cost of every individual in each group is lower due to the cost sharing rule. Since $\alpha_l = 0$ induces the same transfer as in Proposition 1.2, the best responses and the equilibrium when organizers set transfers strategically coincide with the case of decentralized contest with transfers.

In case groups differ in size, the organizer in the larger group splits the total costs among her individuals while the organizer in the smaller one sets a mixed rule. Furthermore, the higher the difference of size between groups, the less egalitarian the rule set by the smaller group is because the larger group has the dominant strategy of setting the egalitarian rule. The egalitarian rule for the larger group is more meritocratic than the egalitarian rule for the smaller one, since the marginal cost depends on the size of the group. Then, the smaller group sets a less meritocratic rule because the higher probability of winning that a more meritocratic rule yields does not compensate the cost of effort. Therefore, the transfer chosen by the organizer of the larger group coincides with the implementation case, while the transfer set by the organizer of the smaller group differs. It yields that the larger group exerts more effort than the smaller one and it is more likely to win the contest. In particular, for the larger (smaller) group, the effort of the smaller (larger) group is a strategic complement (substitute).[12]

While the transfers chosen by organizers when they behave strategically coincide with the implementation case when groups have the same size, they differ in case the size is different. The main reason is that when organizers set transfers strategically, the organizer in Group 1 has a dominant strategy of setting the transfer given in (1.30) while the organizer in Group 2 maximizes her payoffs subject to this dominant strategy. Then, the contest becomes sequential. However, in case organizers choose the transfer that implements the centralized effort, they are choosing a transfer restricting to a given effort which is the result of a simultaneous contest.[13]

A natural question is how setting transfers strategically affects group welfare comparing with transfers that implement the centralized effort. The answer to this question is that both groups would be better off in the case of organizers behaving strategically. The main reason is that the smaller group sets a less meritocratic transfer since the lower probability of winning the contest is compensated by a lower cost of effort. Furthermore, since the smaller group is weaker, the larger group exerts less effort in equilibrium. Therefore, both groups would obtain higher payoffs in case of setting transfers strategically. In a similar fashion, both groups would be better off in case both of them would agree not to set transfers. Transfers reduce the marginal cost of both groups and induce individuals to increase effort. This fact causes a reduction in individual and group payoffs.

All the results of this section are based on differences in group size. We might do a similar analysis assuming that all members in a group value the public prize equally but groups differ in how they value the prize. Let v_1 and v_2 be the valuation of each member of Group 1 and 2

[12] As in the previous sections, the problem has been solved for a public prize, but costs can also be shared with private prizes. Should the prize be private and divided in per capita terms among the individuals in the group in case of winning, the results would change. In particular, both groups would split the effort costs of the contest with the egalitarian rule. This is the case since, by dividing the private prize among individuals, the valuation of individuals of the prize falls. Then, the dominant strategy of the larger group is vanished, since the lower marginal cost by setting the egalitarian rule is compensated by a lower expected prize for each individual of the group. Then, both groups have the dominant strategy of splitting the effort costs in an egalitarian way. Should the prize have both public and private components, the larger group still has the dominant strategy, while the smaller one chooses a rule closer to the egalitarian rule the more private the prize is.

[13] Indeed, if the centralized contest solved in Section 1.3 were sequential, the resulting transfers that implement the centralized effort would coincide with the transfers of Proposition 1.4.

respectively, and assume $v_1 n_1 > v_2 n_2$. In this case, Group 1 splits the effort costs of the contest in an egalitarian way and exerts more effort than the other group, that sets a mixed rule of cost sharing. The main reason is that the group that sets an egalitarian rule is the one that has a larger valuation of the prize or smaller marginal costs (which are equivalent), while the other group sets a mixed rule. Thus, the combination of prize valuations and group sizes determines the cost sharing rules. [14]

1.6 Final remarks

This paper studies how to avoid over and underperforming in contests when the organizer has the capacity to impose transfers among individuals, and analyzes if these transfers are optimal when the objective of the organizer is to maximize group payoffs.

When organizers set transfers strategically, the model becomes intractable if costs are convex. Intuition suggests that cost convexity induces organizers to set meritocratic transfers to avoid the effects of higher marginal costs and induce individuals to exert more effort.

In Appendix 1.C, we study the implementation setup in which individual valuations of the public prize are heterogeneous. In the strategic setup with linear costs, only top individuals – those who value the public prize the most – exert effort. If costs are convex, the problem becomes intractable.

It is interesting to compare the effects of cost sharing presented in this paper with the literature of prize sharing. The problem of prize sharing consists of determining previously to the contest, how a divisible prize is shared among the individuals of the winning group according to a convex combination between two rules: divide the prize in per capita terms or depending on the effort of each individual relative the group effort. We compare it with Lee (1995) for being the original setting and for using similar assumptions to our model. He finds that when groups have the same size, organizers decide to share the prize depending exclusively on the relative effort. This result is in line with the results obtained in this paper, since groups set a rule that induces individuals to exert more effort in equilibrium.

However, in case groups differ in size, Lee (1995) finds that the organizer in the smaller group has the dominant strategy of choosing a rule based on relative effort, which induces to increase effort, while the organizer in the larger group chooses a mixed rule, such that both groups exert

[14] Previous versions of this paper develop the analysis.

the same effort in equilibrium. Thus, the rule set by the smaller group is more meritocratic than the rule of the larger group. This is because the organizer of the larger group, by setting a less meritocratic rule, reduces costs by inducing her individuals to reduce effort, and also induces their rivals to exert less effort, since efforts are strategic complements. These results constrast with our findings in cost sharing, since the smaller group is the one that sets a less meritocratic rule to induce the larger group to reduce effort by the strategic complementarity.

In general, the main difference between cost sharing and prize sharing in contests is the nature of transfers. In cost sharing, transfers are set independently on the result of the contest, while prize sharing could be considered as transfers that are contingent on winning the contest.[15] The interpretation is the following. When transfers are set independently on winning the contest or not, what is being set is a contract for every individual. However, when transfers are set contingent on the result of the contest, what is being decided is a premium for each individual.

In order to illustrate the problem recall again the case of sports. Sport teams pay the more hardworking athletes a higher wage, regardless of whether a match is won or not. In addition to this, in case of winning the match, athletes in the winning team obtain a premium which may likewise depend on the effort exerted by each athlete relative to the total effort exerted in the team, if this is the chosen rule.

Additionally, both problems differ since prize sharing can only be applied to the case in which the prize is divisible, while cost sharing applies both for divisible and indivisible prizes.

It is interesting to relate the results of Esteban and Ray (2001) to those found in this paper. Their purpose is to analyze the group size paradox in a decentralized contest framework, and they determine that the smaller group performs better in the contest the more private the prize is and the less convex costs are. When the prize is private and divisible, individuals in the larger group get a smaller part of the prize in case of winning, which induces them to decrease effort. Cost convexity reduces the incentives to exert large individual effort levels, which benefits the larger group. In this paper, we find that the larger group performs better in the contest both in the centralized setting and in the strategic scenario. In the first case, organizers internalize the benefits that winning a public prize yields to all individuals in the group so that the organizer in the large group induces her individuals to behave fiercely relative the small group. In the second case, the small group sets a less meritocratic transfer than the large group so that lowers costs of effort compensate the lower

[15] In previous versions of this paper we show how to derive the formulation of Lee (1995) from transfers contingent on winning the contest.

probability of winning the contest.

Finally, we relate the transfers of our framework with those of the Clark-Groves mechanism. These mechanisms are different because in the Clark-Groves case the planner does not know how individuals value the prize and in our set up organizers do. It would be interesting to study our framework in case individuals value the prize differently and organizers have incomplete information about it. We leave this for future research.

Chapter 2

Optimal contest design with budget constraints

2.1 Introduction

This paper studies the design of a contest between two possible budget constrained players when the organizer of the contest is not informed about the actual size of budgets and where the objective is to maximize aggregate effort. First we define the optimal effort as the maximum levels of effort achievable in a situation of complete information. We then propose a mechanism that implements the optimal effort when the organizer ignores the budget of players.

To illustrate the problem, suppose a research contest is held with the possibility of winning a prize. Some organizations hold contests in order to provide an incentive for research in certain specific fields of science. For instance, in the area of artificial intelligence, the Loebner prize is awarded to the most intelligent software, or the Netflix prize is awarded for the best algorithm for predicting user ratings for films. Competing firms may be financially constrained, and may not spend as much resources as desirable. Moreover, it may reduce the resources that unconstrained firms allocate to research.

For another application, suppose employees in a firm competing for a wage bonus. Employees may be constrained in the effort they can exert for having different abilities. The goal of the boss of the firm is to establish a contest in a way that encourages employees to exert as much effort as possible, since unconstrained employees may reduce effort in the firm when they compete with constrained employees.

While constrained players cannot exert more effort than their budget, unconstrained players reduce their effort when competing with a constrained opponent, since they find it easier to win the contest. To overcome this problem, we focus on the design of contests to maximize aggregate effort when the organizer ignores the budget of players.

In our model, two players compete by exerting costly effort in an imperfectly discriminating contest to win a prize. We focus on an imperfectly discriminating contest because in the proposed applications, the player that exerts more effort does not necessarily win the contest with probability 1.[1] We assume players have a budget constraint, which can be either high or low. If it is high, we assume the player does not face any constraint. Otherwise, she does. The contest is designed by an organizer whose objective is to maximize aggregate effort. To do so, the organizer can bias the contest in favor of any player, or set an unbiased contest. She also decides the prize each player competes for. We assume that the organizer is constrained in the prize she offers. In the application of employees competing for bonuses, the firm may be constrained in terms of the wages it offers. The organizer is not informed about the budget of players, but players know both their own budget and the budget of their opponent. In the application of employees competing for a wage bonus, bosses may not know the abilities of their employees, whereas employees normally know the abilities of their colleagues. In addition, in research contests, the organization may be unaware of the financial situation of the competing firms, although firms normally now the situation of their rivals in small markets. The game has three stages. In stage one, the organizer designs the contest by choosing and announcing a contest bias and a prize for each player for each possible vector of types of players. In stage two, each player sends a message to the organizer regarding her budget. In this stage, their strategy is either to report their true budget or to lie. In stage three players exert effort and the contest takes place.

We firstly analyze the benchmark case in which the organizer has complete information (i.e. when the organizer knows the type of players) and when the prize is not costly for the organizer. We first argue that the organizer offers to each player the highest possible prize. This is the case for two reasons. First, effort of players is increasing in the prize they compete for. Second, we assume the prize is not costly for the organizer and her objective is to maximize the sum of efforts. We show that the organizer maximizes aggregate effort by biasing the contest in favor of

[1] The firm that spends the greatest amount of resources on research does not necessarily achieve the best contribution to science. However, there is a positive relationship between expenditure and the probability of success. Something similar happens when employees compete in a firm for bonuses.

the constrained player if the players differ in type. This bias makes the effort of the constrained player more productive than the effort of the unconstrained player in the contest. This bias does not provide an incentive to the constrained player to increase the amount of effort (since she is constrained), but does encourage the unconstrained, which increases aggregate effort. When both players are either constrained or unconstrained, the organizer maximizes aggregate effort by setting an unbiased contest.

The previous set up fails to achieve optimal effort when the planner is incompletely informed. In particular, in stage two both players report they have a low budget in equilibrium regardless of their type. This is because reporting a low budget ensures that the contest is never biased against them. Thus, the organizer always sets an unbiased contest. This is not a problem if both players have the same budget, since effort maximization in these cases requires an unbiased contest. However, aggregate effort is not maximized if players have different budgets because the contest should be biased towards the player with a low budget.

We then propose a mechanism for implementing the optimal effort. We show that the organizer can implement the optimal effort by offering a lower prize to the players who report to be constrained and using the bias applied when there is complete information. The value of this prize and the bias depend on the type both players report to be in stage two. We find an interval of prizes that implement the optimal effort. When designing the contest, the organizer plays with two driving forces that affects the decision of players in stage two. The contest is potentially biased in favor of the player that reports to be constrained. However, she competes for a lower prize. The opposite happens for the player that reports to be unconstrained.

Using this mechanism, players report their true type in the case where both players have a high budget or in the case where they differ in type. In the event of both of them having a low budget, two equilibria arise depending on the prizes set by the organizer: both players report that they have a high budget or both report that they have a low budget. The first equilibrium always arises under this mechanism. The second one only does so if the prize the organizer sets is sufficiently large in the event of both of them reporting that they have a low budget. In both equilibria, the optimal effort is implemented because if both players spend their whole budget when they compete for a small prize, they do the same when competing for a higher prize. Though there is an equilibrium in which both players lie about their type, the organizer achieves her goal of maximizing aggregate effort.

We extend the problem when there is a continuum of possible budgets that players can have.

The problem becomes more complicated and additional assumptions must be made to implement the optimal effort using a similar mechanism. In particular, we assume that a player reports her true budget in case she is indifferent between reporting her true type and lying.

We then analyze the case in which the prize is costly for the organizer. In the application of employees competing for a bonus, the position that the winner gets may involve monetary rewards that are costly for the firm. The same may happen in research contests. Given complete information, the organizer maximizes effort subject to minimizing the costs of prizes offering the unconstrained players the highest possible prize, and a lower prize to the constrained players. Given incomplete information, we show that in order to implement the optimal effort, the organizer must offer a higher prize to players that claim to be constrained than when there is complete information. Thus, the first best, which corresponds to the optimal levels of effort and minimum prizes offered when there is complete information, cannot be achieved when there is incomplete information.

Other solutions exist to overcome the problem of constraints, for instance, giving transfers to constrained players. The use of transfers induces greater effort in the contest. This is because the constrained player is willing to use the resources of the transfer to exert effort, and it induces the unconstrained player to be more aggressive in the contest. However, transfers involve two problems. Firstly, unconstrained players may claim they are constrained in order to obtain a transfer, which they will not use in the contest, when the organizer ignores the budget of players. Secondly, transfers involves costs for the organizer and she may find that applying such a scheme is not cost-effective. For these reasons we do not focus on transfers, although we devote a section to discussing their implications.

Relationship to the literature

This paper is closely related to research that focuses on the optimal design of an imperfectly discriminating contest success function to maximize aggregate effort. Dasgupta and Nti (1998) study a contest in which the organizer designs the contest success function when she values the prize. They find that if the contest organizer is an expected utility maximizer, she uses a linear homogeneous contest success function. Nti (2004) designs the optimal contest success function when two players value the prize differently. He finds that it is optimal to give advantage to the player with a lower valuation in order to induce effort to both players. Franke (2012) obtains similar results for a contest among n-players, under an affirmative action perspective. He concludes that setting affirmative action to low-skilled players increases effort incentives of every player. His model is generalized by Franke et al. (2013), proving the existence of a contest success function that maximizes effort when players are heterogeneous, and also determining the set of active players.

To the best of our knowledge, this paper is the first one in analyzing the imperfectly discriminating contest success function that maximizes aggregate effort when players are constrained. We relate our results to this literature. The literature suggests that favoring the player with a lower valuation fosters effort of every player, which maximizes aggregate effort. When constraints are present, the optimal design also consists in favoring the low-skilled player (in this case, low-skilled is equivalent to be constrained). By doing so, the effort of the constrained player is more productive than the effort of the unconstrained. However, the constrained player cannot exert more effort than her budget. Then, this policy encourages only the unconstrained player to increase effort. Also, while previous research focuses on the case in which the contest organizer knows all the characteristics of players, this paper also differs in dealing with incomplete information. In particular, we assume that the contest organizer does not know the budget of players. Then, she designs a mechanism that implements the aggregate effort of complete information.

The literature of optimal contest design is vast and contains different areas. Some research has focused on the optimal prize structure in contests. This paper does not deal with more than one prize, but does with different prizes depending on the winner. In particular, we show that it is necessary to discriminate players depending on their budgets offering them different prizes to maximize aggregate effort under incomplete information. For a survey on optimal prize structure in contests, see Sisak (2009).

Some other research analyzes the role of information in contests and how affects aggregate effort. Wärneryd (2003, 2009) analyzes a contest for a prize of common but uncertain value, in an individual and group setting. He shows that asymmetric information reduces aggregate effort. Fey (2008) and Wasser (2011) analyze the role of private information regarding marginal costs. They focus on existence of closed form solutions, and the latter concludes that complete information maximizes aggregate effort. Serena (2014) studies a model in which a contest organizer decides either to reveal or not to contestants information regarding the marginal cost of their oponent. He finds that the organizer maximizes aggregate effort revealing private information when the distribution of types is skewed towards the low type.

While previous research focuses on how to use information to maximize aggregate effort, this paper deals with information as a problem the contest organizer faces. In particular, we show that when the contest organizer does not have information about the budget of players, she implements the effort of complete information by discriminating players using different prizes.[2]

This paper is also related to research on contests with budget constraints. Che and Gale (1997) show that when players are constrained, effort exerted in a Tullock contest is larger than in an all pay auction. Grossmann and Dietl (2012) show that in a two player contest, aggregate effort diminishes if at least one player is constrained.[3]

This paper contributes to the literature by designing a contest success function that overcomes the problem of budget constraints. In particular, by favoring the constrained player in the contest success function, the unconstrained player increases effort.

The problem of optimal design with constrained players is a concern in auctions since bidders with low budgets cannot put competitive pressure on bidders with high budgets. The optimal design of an auction involves setting an allocation rule and the payment each bidder faces to maximize revenue. Auctions are analyzed when there is incomplete information regarding the types of players; i.e. valuation and/or budget.

Laffont and Robert (1996) generalize the classical paper by Myerson (1981) by considering an auction in which all bidders face the same known constraint. They find that the probability of being allotted the good of each bidder must be increasing in the reported valuation of the good up to a point. From this point on, the probability of being allotted the good remains constant. This is called "pooling at the top".

Pai and Vohra (2014) study the case in which both valuation and constraints are private information. They define two thresholds v_H and v_L for the case in which there are two possible budgets b_H and b_L, where subscripts H and L denote high and low respectively.[4] They find that it is optimal to pool players and assign the same probability of being allotted the good to bidders with high budgets and a valuation higher than v_H and also to pool and assign the same probability of being allotted the good to bidders with low budgets and a valuation higher than v_L. Also, it is necessary to assign the same winning probability to bidders with a high budget whose valuations are slightly higher than v_L, which is strictly necessary to make low budget bidders to put competitive pressure on high budget bidders.

[2] It is worth comment some research that focuses on optimal timing structure of contests, as Gradstein (1999) and Fu and Lu (2009). Our set up is a one shot contest, but the organizers share the goal of maximizing aggregate effort. Also, for a survey of optimal sport contest design, see Szymanski (2003).

[3] Other research focuses on how players in a contest use a fix amount of resources. Matros (2006) studies an elimination tournament where players have fixed resources.

[4] Their model is general for k budgets, but for simplicity we refer here to the case of $k = 2$.

This paper transfers the problem analyzed in auctions to contests. However, the approach is quite different. We consider that the organizer does not know the types of players, but players know the type of their rival.

The paper is organized as follows. In Section 2.2 we present the model. In Section 2.3 we solve the problem under complete information. Section 2.4 proposes the mechanism to implement the optimal effort with incomplete information. In Section 2.5 we assume the prize is costly for the organizer. Section 2.6 discusses the use of transfers as an alternative mechanism. Section 2.7 concludes.

All the proofs are relegated to the Appendix.

2.2 Model

Consider an organizer that designs the rules of a contest between two players $i = 1, 2$ to maximize aggregate effort. The rules consist in biasing or not the contest in favor of any player and choosing the prize each player competes for.

Players exert effort e_i to win a prize $\alpha_i V$. Let p_i be the probability that player $i = 1, 2$ wins the contest. We assume that:

$$p_1 = \frac{\lambda e_1}{\lambda e_1 + e_2} \text{ and } p_2 = \frac{e_2}{\lambda e_1 + e_2}, \ \lambda \in (0, \infty). \tag{2.1}$$

Functions in (2.1), which are called *Contest Success Functions (CSF)*[5], are increasing in own effort, decreasing in the rival effort and homogeneous of degree 0. Parameter λ measures the bias of the *CSF* towards any player. In case $\lambda >(<)1$, the contest is biased favoring player 1 (2). In case $\lambda = 1$, the contest is unbiased.

The contest organizer chooses the value of λ and the prize each player competes for to maximize the sum of efforts. We assume that the organizer cannot offer a prize higher than V. Think that a higher prize may not exist or that the organizer is constrained in the prize she offers. However, we assume that the organizer could offer lower prizes. In particular, we allow the organizer to modify parameter α_i for $i = 1, 2$, where $\alpha_i \in [0, 1]$. We assume that the prize is not costly for the organizer, though we relax this assumption in Section 2.5.

Effort is costly, and the marginal cost of effort is 1. To afford the cost of effort each player has a budget $b_i \in \{b_L, b_H\}$, where $b_L < b_H$. This budget determines the type of each player $t_i = b_i$. We

[5] Skaperdas (1996) axiomatizes this function. In the Appendix we give microfoundations to this function.

assume that in case $t_i = b_H$, the budget of player i is so large that she is unconstrained (thus, the restriction can be omitted). Otherwise, she is constrained. Let $s \in S$ be a state of the world, and $s = (t_1, t_2)$. The set of states of the world is $S = \{(b_H, b_H), (b_H, b_L), (b_L, b_H), (b_L, b_L)\}$.[6]

The payoffs of each player are:

$$\pi_i = \alpha_i V p_i - e_i \text{ for } i = 1, 2. \tag{2.2}$$

We assume that the contest organizer does not know the type of players but is aware of the values of b_L and b_H.

While the organizer does not know the types of players, we assume that players know both their own type and their rival type.

The game has three stages.

In the first stage, the organizer announces for each posible vector of types a contest bias λ and a prize for each player $\alpha_i V$.

In the second stage, each player reports a message m_i to the organizer regarding her type, where $m_i \in \{b_L, b_H\}$, $m \in M$ is a message profile and $m = (m_1, m_2)$. The set of message profiles is $M = \{(b_H, b_H), (b_H, b_L), (b_L, b_H), (b_L, b_L)\}$. Thus, the strategy set of players in this stage is either reporting their true type or lying.

In the third stage, players exert effort and the contest takes place.

The equilibrium concept of the game is Nash equilibrium.

2.3 Equilibrium with complete information

In this section, we assume that the contest organizer knows the type of each player and therefore the state of the world. Thus, stage two of the game is omitted. We use this section as a benchmark. Denote as $x(t_1, t_2)$ variable x when the state of the world is $s = (t_1, t_2)$. W.l.o.g. assume that if players differ in type, $t_1 = b_H$ and $t_2 = b_L$. Before solving the problem, we simplify it arguing that the organizer setting $\alpha_i^*(s) = 1$ for all $s \in S$ and $i = 1, 2$ is an equilibrium. This is the case for two reasons. First, effort of players is increasing in the prize they compete for. Second, we assume the prize is not costly for the organizer and her objective is to maximize the sum of efforts. This is not the only equilibrium for $\alpha_i(s)$. We characterize all the equilibria in the next section, where the chosen prize takes a key role in the contest design. Studying the case where $\alpha_i^*(s) = 1$ for all $s \in S$

[6] In Section 2.4.1 we relax this assumption and assume a continuum of budgets (types).

and $i = 1, 2$ simplifies the following analysis, where the only remaining decision of the organizer is to set λ.

We solve the problem backwards. Let us begin by analyzing the third stage. Setting $\partial\pi_i/\partial e_i = 0$ for $i = 1, 2$, we get:

$$e_1 = \sqrt{\frac{Ve_2}{\lambda}} - \frac{e_2}{\lambda} \text{ and } e_2 = \sqrt{V\lambda e_1} - \lambda e_1. \tag{2.3}$$

When no player is constrained, the solution to (2.3) is:

$$e_i(b_H, b_H) = \frac{V\lambda}{(\lambda+1)^2} \text{ for } i = 1, 2. \tag{2.4}$$

Then, for a player to be constrained it is necessary that $b_L/V < \lambda/(\lambda+1)^2$. Let us analyze the first stage for each state of the world. When deciding λ, the organizer needs to set a bias such that players of type $t_i = b_L$ remain constrained. Otherwise, players of type $t_i = b_L$ exert less effort than their budget and induce players of type $t_i = b_H$ to reduce effort.[7]

2.3.1 State of the world $s = (b_H, b_H)$

In the first stage, the contest organizer chooses λ that maximizes the sum of efforts:

$$Max_\lambda \ \frac{2V\lambda}{(\lambda+1)^2}. \tag{2.5}$$

The first order condition is:

$$2V\frac{(\lambda+1)^2 - 2\lambda(\lambda+1)}{(\lambda+1)^4} = 0. \tag{2.6}$$

Solving for λ, we get that $\lambda^*(b_H, b_H) = 1$.[8] The contest organizer maximizes aggregate effort when the contest is unbiased. Finally, insert $\lambda^*(b_H, b_H) = 1$ in effort and payoffs to get $e_i^*(b_H, b_H) = \pi_i^*(b_H, b_H) = V/4$ for $i = 1, 2$.

2.3.2 State of the world $s = (b_H, b_L)$

Since player 2 is constrained, she exerts $e_2(b_H, b_L) = b_L$, while $e_1(b_H, b_L) = \sqrt{Vb_L/\lambda} - b_L/\lambda$. The contest organizer chooses λ that maximizes the sum of efforts:

$$Max_\lambda \sqrt{\frac{Vb_L}{\lambda}} - \frac{b_L}{\lambda} + b_L. \tag{2.7}$$

[7] This happens since effort of the constrained player is an strategic complement for effort of the unconstrained player.

[8] Note that the second derivative of (2.5) with respect to λ is negative for $\lambda < 2$, which ensures a maximum at $\lambda^*(b_H, b_H) = 1$. Note that the objective function is decreasing in λ for $\lambda > 1$ and there is a minimum at $\lambda^*(b_H, b_H) = \infty$.

The first order condition is:

$$-\frac{\lambda^{\frac{-3}{2}}\sqrt{Vb_L}}{2}+\frac{b_L}{\lambda^2}=0. \tag{2.8}$$

Solving for λ, we get that $\lambda^*(b_H,b_L)=4b_L/V$.[9] Using $\lambda^*(b_H,b_L)$, we get that $e_1^*(b_H,b_L)=V/4$, $e_2^*(b_H,b_L)=b_L$, $\pi_1^*(b_H,b_L)=V/4$ and $\pi_2^*(b_H,b_L)=V/2-b_L$. Finally, note that $b_L<\sqrt{V\lambda e_1}-V\lambda e_1$ whenever $V>(\lambda e_1+b_L)^2/\lambda e_1$. Introducing $\lambda^*(b_H,b_L)$ and $e_1^*(b_H,b_L)$ in such condition we get that player 2 is constrained whenever $b_L/V<1/4$. Then, the organizer biases the contest in favor of player 2.

2.3.3 State of the world $s=(b_L,b_L)$

The organizer chooses any value of λ such that $b_L/V<\lambda/(\lambda+1)^2$. Assume the organizer chooses $\lambda^*(b_L,b_L)=1$. By doing so, the contest remains unbiased and both players are constrained whenever $b_L/V<1/4$. It follows that $e_i^*(b_L,b_L)=b_L$ and $\pi_i^*(b_L,b_L)=V/2-b_L$ for $i=1,2$. Both players compete for prize V.[10]

All these results are gathered in the following proposition.

Proposition 2.1 *Suppose there is complete information. If $t_1=t_2$, the contest organizer sets an unbiased contest; i.e. $\lambda^*(t_1,t_2)=1$. Otherwise, the organizer biases the contest in favor of the constrained player; i.e. $\lambda^*(b_H,b_L)=4b_L/V<1$. Both players compete for prize V.*

When the organizer biases the contest in favor of one player, the effort of this player becomes more productive in the contest relative to the effort of her rival. We learn from Proposition 2.1 that under complete information, the organizer does not favor any player if both have the same type, while favors the constrained player in case of being different. In the first case, favoring one player would yield lower aggregate effort. In the second case, by favoring the constrained her effort becomes more productive in the contest relative to the effort of her rival. This bias does not incentivize her to increase effort, since she is constrained. However, it incentivizes the unconstrained to make more effort because she competes with a more productive player. Therefore, biasing the

[9] Note that the second derivative of (2.7) with respect to λ is negative for $\lambda<(64/9)b_L/V$, which ensures a maximum at $\lambda^*(b_H,b_L)=4b_L/V$. Note that $\lambda<(64/9)b_L/V$ is the important range of λ. While offering higher λ keeps player 2 constrained, it induces the player 1 to reduce effort.

[10] Any value of λ that satisfies $b_L/V<\lambda/(\lambda+1)^2$ yields the same aggregate effort. We assume $\lambda^*(b_L,b_L)=1$ since there is no reason to discriminate ex-ante between players.

contest increases aggregate effort relative to an unbiased contest when players differ in type.[11] In particular, players of type $t_i = b_L$ (b_H) exert effort $e_i^* = b_L$ ($V/4$) and get payoffs $\pi_i^* = V/2 - b_L$ ($V/4$). Then, players of type $t_i = b_L$ make less effort than players of type $t_i = b_H$ and get larger payoffs, since the contest is biased in their favor.

Before analyzing the incomplete information case, we define these levels of effort as a benchmark:

Definition 2.1 *For any state of the world $s \in S$, effort in state s is optimal if players exert effort $e_i^*(s)$.*

2.4 Incomplete information

In this section we assume that the contest organizer does not know the type of players. First, we explain that the optimal effort cannot be implemented with incomplete information under the contest design of Proposition 2.1. That is, if the organizer sets the contest bias according to the messages received in stage two, players may have incentives not to report their true type. In particular, player $i = 1, 2$ has incentives to report $m_i = b_L$ independently of her type.

This is a problem when players differ in type. To see this, suppose first that $s = (b_H, b_L)$. Both players reporting their true type is not an equilibrium. In this case, player 2 has the dominant strategy of reporting her true type, since in case of lying, the contest may be unbiased or biased against her. Given that, the best reply of player 1 is reporting $m_1 = b_L$ to play an unbiased contest.

[11] We have done the analysis for a very specific functional form of the *CSF*. In particular, this analysis can be generalized using the following *CSF*:

$$p_i = \frac{f_i(e_i)}{f_i(e_i) + f_j(e_j)} \text{ for } i = 1, 2,\ j \neq i. \tag{2.9}$$

Function $f_i(e_i)$ is called impact function, $f_i'(e_i) > 0$, $f_i^{''}(e_i) \leq 0$. Since the analysis becomes very complicated for a general impact function, the following linear form is normally used: $f_i(e_i) = \lambda_i e_i + \gamma_i$. For instance, see Nti (2004).

We encourage the reader to check that the results of Proposition 2.1 would not change using the linear form. In particular, in every state of the world, the organizer maximizes effort by setting $\gamma_i = 0$ for $i = 1, 2$ and the corresponding $\lambda = \lambda_1/\lambda_2$ set in Proposition 2.1. For the analysis when $s = (b_H, b_H)$, see Nti (2004). The more complex scenario is when players differ in type. Suppose $s = (b_H, b_L)$. Then, there is a continuum of equilibria regarding λ_1, λ_2 and γ_2. In particular, the organizer can implement the effort of Proposition 2.1 by combining λ_1, λ_2 and γ_2 such that $\lambda_1/(b_L\lambda_2 + \gamma_2) = 4/V$. Note that by setting $\gamma_2 = 0$, and isolating λ, we get the same *CSF* that in Proposition 2.1, which is a particular equilibrium of this general form. The intuition for the rest of states of the world is similar. The complete analysis is similar to the one done in Section 2.3, and thus omitted. We restrict to our simplest *CSF* to make clearer the analysis in the next sections.

By doing so, she reduces effort in equilibrium and her payoffs increase. Since by both players reporting the same type the contest is unbiased, aggregate effort is not maximized (maximization of aggregate effort requires that the contest is biased in favor of player 2).

However, both players lying when $s = (b_H, b_H)$ implements the optimal effort. Note first that both players reporting their true type is not an equilibrium. W.l.o.g. take player 2. Player 2 has incentives to deviate and report $m_2 = b_L$ since by lying, the contest organizer biases the contest in her favor choosing $\lambda < 1$. Since she is still unconstrained, she gets larger payoffs. Indeed, there is only one equilibrium in which players report $m_i = b_L$. If one player deviates, the contest organizer would bias the contest in favor of her rival. Note that both lying about their types is not a problem for the organizer in this state of the world, since an unbiased contest is set and the optimal effort is implemented.

Trivially, for $s = (b_L, b_L)$, both players report their true type and aggregate effort is maximized.

We propose another contest design such that the optimal effort is implemented. For the rest of this section, denote by $x(m)$ variable x when the message profile is $m = (m_1, m_2)$. Without loss of generality, assume that when $m_1 \neq m_2$, $(m_1, m_2) = (b_H, b_L)$.

In this design, the value of $\alpha_i(m)$ that the organizer chooses is important. It is straightforward to see that if a player $t_i = b_H$ competes for a prize $\alpha_i V$, with $\alpha_i < 1$, she always makes less effort than when competing for V. This is because the best reply of an unconstrained player is increasing in the prize for any effort of her rival. However, a player $t_i = b_L$ may not reduce her effort when competing for a prize lower than V, since she may be still constrained. Thus, the organizer must always offer a prize V to players that report $m_i = b_H$ and restrict to offer a prize $\alpha_i V$ to players that report $m_i = b_L$. Thus, from this point on and to ease notation, we remove the subscript of α_i. When the organizer sets different prizes she does not want to distort the decisions of effort.

Also, in equilibrium it must be the case that $\lambda(m_1, m_2)$ are those set in Proposition 2.1. That is, in case $m_1 = m_2$ the contest is unbiased and in case $m_1 \neq m_2$ the contest is biased in favor of the player that reports $m_i = b_L$. Otherwise, aggreagate effort is reduced.

By doing so, note that on the one hand, players of type $t_i = b_H$ may report $m_i = b_L$ to be favored in the *CSF* though competing for a lower prize if she gets larger expected payoffs. On the other hand, players of type $t_i = b_L$ may lie about their type to compete for a higher prize in case the organizer offers them a very small prize in case of winning though losing her favoritism in the *CSF*. The organizer has to design prizes to compensate these effects and consequently implement the optimal levels of effort.

To see the intuition of the effect of a prize reduction of player $t_i = b_L$, assume complete information. For a sufficiently small reduction in the prize the constrained player is competing for, she still spends her whole budget and the optimal levels of effort are achieved. This is due to two reasons. First, one player is constrained when she values the prize so much in relation to her budget. Then, a sufficiently small reduction of the prize does not deter her to spend her whole budget. Second, the best reply of the unconstrained player does not depend on the prize that the constrained player is competing for, but on her effort. Since a sufficiently small reduction in the prize of the constrained player does not induce her to reduce effort, the unconstrained player still exerts the same effort. The following Lemma shows under which conditions a reduction of the prizes for which constrained players compete does not affect the effort exerted in the contest.

Lemma 2.1 *Assume complete information. For any $\alpha \in [4b_L/V, 1]$, if players $t_i = b_H$ compete for prize V and players $t_i = b_L$ compete for prize αV, the contest organizer sets $\lambda^*(t_1, t_2)$ according to Proposition 2.1 and the resulting levels of effort are optimal.*

The following proposition explains the main result of this paper.

Proposition 2.2 *Suppose there is incomplete information. There exists $\alpha^*(b_H, b_L) < 1$ and $\alpha^*(b_L, b_L) < 1$ such that the following mechanism implements the optimal effort:*

a) If $(m_1, m_2) = (b_H, b_H)$, both players compete for prize V and $\lambda^*(b_H, b_H) = 1$.

b) If $(m_1, m_2) = (b_H, b_L)$, player 1 competes for prize V, player 2 competes for prize $\alpha^*(b_H, b_L)V$ and $\lambda^*(b_H, b_L) = 4b_L/V$.

c) If $(m_1, m_2) = (b_L, b_L)$, both players compete for prize $\alpha^*(b_L, b_L)V$ and $\lambda^*(b_H, b_H) = 1$.

Recall that the contest design depends on the messages of players regarding their type in stage two, and the design must be such that players exert the optimal levels of effort, independently if they report their true budget. Parameter $\alpha^*(b_H, b_L)$ must be sufficiently small to make players of type $t_i = b_H$ report their true type when $s = (b_H, b_H)$, and sufficiently large to make player of type $t_i = b_L$ report her true type when players differ in type. Also, $\alpha^*(b_L, b_L)$ must be sufficiently small to make $m = (b_H, b_H)$ be the unique Nash equilibrium when $s = (b_H, b_H)$ (otherwise, $m = (b_L, b_L)$ is another equilibrium) and to make player of type $t_i = b_H$ report her true type when players differ in type. Then, when $s = (b_H, b_H)$ or $s = (b_H, b_L)$, there is only an equilibrium under this mechanism which is truthtelling.

As we show in the Appendix, we prove that the mechanism of Proposition 2.2 implements the optimal effort for $\alpha^*(b_L, b_L) = 0$. When $s = (b_L, b_L)$, the only Nash equilibrium is $m = (b_H, b_H)$, excluding $m = (b_L, b_L)$. Obviously, $m = (b_L, b_L)$ cannot be an equilibrium. They get zero payoffs if both report their true budget, but positive payoffs if one of them deviates unilaterally since $\alpha^*(b_H, b_L) = \alpha^*(b_L, b_H) > 0$. The only Nash equilibrium is that both players report $m_i = b_H$. No player has incentives to deviate since a positive bias in the *CSF* does not compensate the lower prize. Thus, both players compete for V and the contest is unbiased. However, the optimal effort is implemented. Just note that if both players spend their whole budget when they compete for a small prize αV, the same happens when they compete for a higher prize V.

We choose value $\alpha^*(b_L, b_L) = 0$ for simplicity and to avoid a tedious analysis. However, we can extend the analysis to the case in which $\alpha^*(b_L, b_L) > 0$. Indeed, when $s = (b_L, b_L)$, the number of equilibria depend on the chosen value of $\alpha^*(b_L, b_L)$. In particular, if $\alpha^*(b_L, b_L)$ is sufficiently large, there are two Nash equilibria, $m = (b_H, b_H)$ and $m = (b_L, b_L)$. If $\alpha^*(b_L, b_L)$ is sufficiently small, the only Nash equilibrium is $m = (b_H, b_H)$. The complete analysis is provided by the author under request.

This mechanism implements the optimal effort for any distribution of types. Only in case both players are $t_i = b_L$ players may get information rents. This is because in equilibrium both players report $m_i = b_H$, they compete for a prize V and thus their payoffs are larger than in case both players report their true type.

2.4.1 Incomplete information with a continuum of types

We now generalize the model when there is a continuum of types. That is, players are endowed with a budget $b_i \in (0, \infty)$ and the type of each player is $t_i = b_i$.

In stage two, the set of strategies of players expands to $m_i \in (0, \infty)$.

Player i is constrained if $b_i/V < \lambda/(\lambda+1)^2 = \bar{b}$. We assume that if both players are constrained, then

$$\frac{b_1 + b_2}{\sqrt{V}} < \max\{\sqrt{b_1}, \sqrt{b_2}\}. \tag{2.10}$$

This assumption ensures that in case both players have a low budget, both players are constrained. That is, we assume that in case players have a low budget, their budgets are sufficiently similar. We require this assumption to ensure that our mechanism maximizes effort.

The optimal effort levels are the same than those of the two-type case. That is, optimal effort of players of type $t_i = b_i < \bar{b}$ are $e_i^* = b_i$ and optimal effort of player of type $t_i = b_i > \bar{b}$ are

$e_i^* = V/4$. Indeed, b_L and b_H are particular values below and above $\bar{b}$ respectively. Also, for the same reasons as in the two type case, the organizer offers prize V if both players are unconstrained. Then, $\alpha(m) = 1$ if $m = (\hat{b}_1, \hat{b}_2)$ where $\hat{b}_i > \bar{b}$.

Finally, we assume that a player reports her true budget in case she is indifferent between reporting her true type and lying.

The following proposition shows that there is an equilibrium in which effort is optimal when there is a continuum of types. Since the problem becomes very tedious, we show it for a reduced version of the mechanism in Proposition 2.2.

Proposition 2.3 *Suppose there is incomplete information, $b_i \in (0, \infty)$, and (2.10) holds. There exists $\alpha^*(m_1, m_2)$ such that the following mechanism implements the optimal effort:*

a) If $(m_1, m_2) = (\hat{b}_1, \hat{b}_2)$, where $\hat{b}_i > \bar{b}$ for $i = 1, 2$, both players compete for prize V and $\lambda^*(\hat{b}_1, \hat{b}_2) = 1$.

b) If $(m_1, m_2) = (\hat{b}_1, \hat{b}_2)$, where $\hat{b}_1 > \bar{b}$ and $\hat{b}_2 < \bar{b}$, player 1 competes for prize V, player 2 competes for prize $\alpha^*(\hat{b}_1, \hat{b}_2)V$ and $\lambda^*(\hat{b}_1, \hat{b}_2) = 4\hat{b}_2/V$.

c) If $(m_1, m_2) = (\hat{b}_1, \hat{b}_2)$, where $\hat{b}_i < \bar{b}$ for $i = 1, 2$, the contest is cancelled (the prize is 0).

As we show in the Appendix, the constrained player is indifferent between reporting her true type or any other type below her treshold $\bar{b}$ when competes with an unconstrained player. The organizer fails to set the required value of λ if this player reports a false budget. We assume that a player reports her true budget in case she is indifferent between reporting her true type and lying to overcome this problem. This assumption avoids the equilibria in which λ does not maximize aggregate effort.

Again, this mechanism does not induce a Nash equilibrium in which both players report their true type when both are constrained. However, the optimal effort is still implemented.

As in the previous section, players get information rents if both are constrained.

2.5 Costly prize

In this section, we assume that the prize the organizer offers in the contest is costly. In the application of employees competing for a wage bonus, the position that the winner gets involve monetary rewards that are costly for the firm.[12] The same may happen in research contests. For

[12] The assumption that the prize is not costly is still defensible. We may suppose that the organizer offers a prize which is not costly. For instance, in the application of employees competing for a bonus, we could consider they

the remaining of the paper, let q_s be the probability that the state of the world s arises. This will be useful to compare expected efforts and costs when the prize is costly with respect to the previous sections.

For simplicity, we assume that the payoffs of the organizer are

$$\pi_o = a(\sum_{i=1}^{2} e_i) - p_1\alpha_1 V - p_2\alpha_2 V, \tag{2.11}$$

where $a > 4$. This assumption ensures that the organizer values effort more than the prize.[13]

Since the organizer values more effort than the prize, she will offer prize V to players $t_i = b_H$ to maximize effort because effort of the unconstrained player is increasing in the prize. Thus, the objective of the organizer consists in implementing the optimal effort and minimizing the cost that supposes the prize of players $t_i = b_L$.

We start the analysis when there is complete information. We describe the design of the contest that implements the optimal effort and minimizes costs as follows.

Corollary 2.1 *Suppose there is complete information and the prize is costly. The organizer implements the optimal effort and minimizes costs when players $t_i = b_H$ compete for prize V, players $t_i = b_L$ compete for prize $\alpha V = 4b_L$ and sets $\lambda^*(t_1, t_2)$ according to Proposition 2.1.*

The proof follows from Lemma 2.1 and consists in choosing the minimum value of α that implements the optimal effort. The expected effort is $E(e_1 + e_2) = q_{HH}V/2 + q_{HL}(V/2 + b_L) + q_{LH}(V/2 + b_L) + q_{LL}2b_L$ and expected costs are $E(V + \alpha V) = q_{HH}V + q_{HL}(V + 4b_L)/2 + q_{LH}(V + 4b_L)/2 + q_{LL}4b_L$.

Now suppose there is incomplete information. We show that the organizer cannot implement the first best in Corollary 1. That is, using the mechanism of Proposition 2.2 and offering the lowest possible prizes that implement the optimal effort, she incurs in higher costs than in the complete information case.

Corollary 2.2 *Suppose there is incomplete information and the prize is costly. There exists values $\alpha^*(b_H, b_L)$ and $\alpha^*(b_L, b_L)$ such that the following mechanism implements the optimal effort and minimizes costs:*

a) If $(m_1, m_2) = (b_H, b_H)$, both players compete for prize V and $\lambda^*(b_H, b_H) = 1$.

compete for the opportunity of enjoying a facility which is property of the firm.

[13] Otherwise, the problem may involve not holding a contest when the organizer is risk neutral. This assumption ensures that the contest is always desirable for the organizer.

b) If $(m_1, m_2) = (b_H, b_L)$*, player* 1 *(*2*) competes for prize* V *(*$\alpha^*(b_H, b_L)V$*) and* $\lambda^*(b_H, b_L) = 4b_L/V$*.*

c) If $(m_1, m_2) = (b_L, b_L)$*, both players compete for prize* $\alpha^*(b_L, b_L)V$ *and* $\lambda^*(b_H, b_H) = 1$*.*

Expected costs will be higher than in the complete information case.

The mechanism consists in choosing the minimum values of $\alpha^*(b_H, b_L)$ and $\alpha^*(b_L, b_L)$ that implement the optimal effort. In the proof we choose value $\alpha^*(b_L, b_L) = 0$. This value induces the only equilibrium that both players report $m_i = b_H$ in case $s = (b_L, b_L)$. This equilibrium induces higher costs that in case we chose a value of $\alpha^*(b_L, b_L)$ that induces also the equilibrium in which both players report truthtelling when $s = (b_L, b_L)$. We choose $\alpha^*(b_L, b_L) = 0$ for two reasons. First, because when both players report $m_i = b_H$ they get higher payoffs, making the truthtelling equilibrium non-credible. Second, to avoid equilibria in mixed strategies.

Expected costs are always larger under incomplete information for several reasons. First, when $s = (b_H, b_L)$, the value of $\alpha^*(b_H, b_L)$ is larger in incomplete information, which is necessary to satisfy incentive compatibility and then implement the optimal effort, than in the complete information case. Also, when $s = (b_L, b_L)$, both players lie in equilibrium and compete for a higher prize, which increases the costs of the organizer.

Thus, a first best cannot be achieved under incomplete information when the prize is costly.

2.6 Transfers as a mechanism

Another tool to maximize aggregate effort when players are constrained are transfers. In a two player contest, when one of them is constrained, aggregate effort is reduced for two reasons. First, the constrained player cannot exert more effort than her budget. Second, the unconstrained player makes less effort when competes with a constrained player.

A possibility to overcome this problem is the use of transfers. If the contest organizer gives a transfer to the constrained player, she uses these resources to exert effort. This encourages the unconstrained player to increase effort, and then aggregate effort increases.

The use of transfers is costly, and the organizer may find no profitable to set transfers in case she does not value effort enough. For simplicity, we assume there is a one to one equivalence between a unit of effort and a unit of transfer. An example that illustrates the problem is rent seeking. Suppose two lobbies bribe the government to influence a policy. The government wants to maximize the money she receives from lobbies. It is profitable to give a transfer to the lobby that is constrained, since it induces the unconstrained lobby to spend more resources. It is reasonable to assume that a unit of money spent in lobby and transfer are equivalent.

We study now the use of transfers as an alternative method to the contest bias. Then, we study the model in Section 2.2 when the contest is unbiased and the organizer sets a transfer instead of a bias in stage one. We also assume that the prize V is exogenously given. First, we analyze the problem under complete information.

When $t_1 = t_2$, the organizer does not set any transfer. Note that when $s = (b_H, b_H)$, players are unconstrained and exert the maximum amount of possible effort $e_i = V/4$. Therefore, giving a transfer is costly and does not induce players to increase effort. When $s = (b_L, b_L)$, players exert their whole budget. Giving a transfer to one player induces her to make more effort, but does not fosters effort of the other player. For the assumption of the one to one equivalence, a transfer is not useful. The same happens when giving a transfer to both players.

Finally, we need to analyze the problem when only one player is constrained. Assume $s = (b_H, b_L)$. For player 2 to be constrained, it is necessary that $b_L < V/4$. In case transfers are not set, effort of player 1 and 2 are respectively $e_1 = \sqrt{Vb_L} - b_L$ and $e_2 = b_L$. We denote R_i transfer given by the organizer to player i and r_i transfer spent by player i. To implement the optimal effort, the organizer must give a transfer of resources $R_2 = V/4 - b_L$ to player 2. In such a case, effort is $e_1 = V/4$, $e_2 = b_L$, and player 2 spends additionally her whole transfer $r_2 = V/4 - b_L$.

To see that setting transfers is optimal to the organizer, define the following payoff function:

$$\pi_o = \sum_i (e_i + r_i - R_i). \tag{2.12}$$

In case no transfers are set, $\pi_o = \sqrt{Vb_L}$. Otherwise, $\pi_o = V/4 + b_L$. Giving a transfer is optimal since $V/4 + b_L > \sqrt{Vb_L}$ for $b_L < V/4$.

Now, assume incomplete information. Obviously, the optimal effort is implemented by setting transfers to the player that claims to be constrained. Since the organizer cannot distinguish between types, both constrained and unconstrained players report $m_i = b_L$ in stage two. In such a case, the constrained player always exerts effort $V/4$ and the unconstrained b_L, in addition to the transfer received. Then, the optimal effort is implemented. However, incentive compatibility is not satisfied and the organizer incurs in unproductive costs when gives a transfer to unconstrained players.

We analyze two different policies the organizer can set and compare them. In the first policy, the organizer does not set transfers. In the second policy, every player receives a transfer.

In case the organizer does not set transfers, her payoffs are:

$$\pi_o = q_{HH}\frac{V}{2} + (q_{HL} + q_{LH})\sqrt{Vb_L} + q_{LL}2b_L. \tag{2.13}$$

In case she gives a transfer to every player, independently of the state of the world, players of type $t_i = b_H$ exert effort $e_i = V/4$ and players of type $t_i = b_L$ exert effort $e_i = b_L$ and spend their transfer. Then, unconstrained players get a transfer that they do not use. In this case, the payoffs of the organizer are:

$$\pi_o = 2b_L. \tag{2.14}$$

Then, we compare the payoffs in both cases to determine either to set transfers or not. Transfers are set if $2b_L/V > q_{HH}\frac{1}{2} + (q_{HL} + q_{LH})\sqrt{b_L/V} + q_{LL}2b_L/V$. For any value of q_s or b_L/V, this inequality is not satisfied. Then, the organizer prefers not setting transfers.

This discussion illustrates the problem of using transfers. Designing a mechanism in which transfers implement the optimal level of effort is easy if we do not ask incentive compatibility. However, the problem becomes very complicated when designing a mechanism to reduce the cost of transfers. We cannot play with prizes or contest biases since affects the levels of effort.

Even in the case in which we derive a mechanism that implements the optimal effort through transfers, the organizer would be better off designing the contest through a contest bias since transfers are costly.

2.7 Conclusions

In this paper we studied the design of a contest between two possible budget constrained players when the organizer of the contest does not have information regarding their budgets and whose objective is maximizing aggregate effort. First we defined the optimal levels of effort as the maximum effort achievable under a complete information setting. We showed that aggregate effort is maximized when the contest is biased in favor of the constrained player in case they differ in type, and unbiased in case their types coincide. Second, we proposed a mechanism that implements the optimal effort when the organizer ignores the budget of players. This mechanism consists in setting lower prizes to players that report to be constrained.

The resulting equilibria of the mechanism implement the optimal effort, but not necessarily satisfy incentive compatibility. In particular, incentive compatibility is satisfied when both players have a high budget or when they differ in type. When both players have a low budget, both report a high budget in equilibrium. Then, when players have a low budget they can earn information rents since compete for larger prizes.

With this mechanism, when players differ in type, players with a low budget prefer to compete for a smaller prize and being positively discriminated in the contest, while players with a high budget prefer the opposite.

We also consider the case in which the prize is costly for the organizer. Suppose employees in a firm competing for a wage bonus. Offering a higher wage may be costly for the boss of the firm. We found that the prizes that implement the optimal levels of effort under incomplete information are higher than the prizes that minimizes costs under complete information. Thus, a first best cannot be achieved.

Finally, we discussed the use of transfers as a possible mechanism to maximize aggregate effort. We show that though the use of transfers implement the optimal levels of effort, the organizer deals with adverse selection problems and costs. Thus, we kept the design of the contest as our main objective.

Chapter 3

Contests for bads and applications

3.1 Introduction

Contests are situations in which individuals compete by spending resources or exerting effort to win a prize. Contests include several applications (litigation, political competition, wars, sports...) and for this reason they have been a main topic in economic research over the last two decades (see Konrad, 2009 for a survey). However, there are cases in which individuals compete to avoid a bad, burden or penalty. For instance, suppose a government plans to develop a project that is useful and necessary for society, but residents do not want the project to be developed close to their homes. Examples include airports, homeless shelters, prisons or toxic waste dumps among others. Negative reactions of residents are usually called "Not in my backyard". No research focuses on this view of contests. This paper studies situations in which individuals compete to avoid a bad, burden or penalty, using the reverse lottery contest model introduced by Fu et al. (2014).

Real life examples abound. For instance, a project in 2007 to build permanent terminal and passenger facilities in the Coventry airport, in the United Kingdom, was cancelled by the government because of public pressure. Also, in Hong Kong in 1998, the correctional school for drug addicts Christian Zheng Sheng College faced public opposition when was inaugurated for fear of an increase in delinquency around the area. These projects are good for society. An airport improves communications of the region and contributes to economic development. Also, a correctional school for drug addicts makes people aware of drug problems and contributes to reduce drug consumption in the area. Although these projects are good for society, they cause a negative externality to people whose homes are around the physical facilities of the project. Then, people tend to support these projects if they are not located around their homes but complain if they are developed in their neighborhood. Thus, competition to avoid the development of the project in their neighborhood naturally arises.

Most of the research in contests in which individuals compete to win a prize relies on the *Tullock Contest Success Function (CSF)*, which is the conventional lottery contest model. This *CSF* says that the probability of winning the contest of one individual depends on the effort this individual makes relative to the sum of effort exerted by all individuals in the contest. This *CSF* is based on a proportional sharing rule. Now suppose individuals exert effort to avoid a burden of negative value. It seems that the idea of a proportional sharing rule fails to give proper probabilities of getting the burden of each individual. To study these situations we use the reverse lottery contest model. Using this function, the probability of each individual of getting the burden is based on an inverse proportional sharing rule such that the probability of getting the burden for an individual is decreasing in her effort. Fu et al. (2014) present the model as a *Contest Elimination Function* (*CEF)*. They use the reverse nested lottery contest model, which is considered as the mirror image of the model of Clark and Riis (1996a), to study situations where individuals compete to win prizes of positive values. Using this model, they are able to determine the winner by selecting losers through a hypothetical sequence of lotteries based on effort that individuals make in a unique stage. Through the sequence of lotteries, each loser gets a prize and is excluded from competing in the next lottery, where the prize has more value. Our work differs since our focus is to study situations in which individuals compete to avoid a burden (prizes of negative value). To see the difference clearly, think that the reverse lottery contest fails to analyze situations where there is an only positive prize individuals compete for, while succeeds to study situations where there is only one burden to avoid.

We first present the game in which individuals compete to avoid a bad through the reverse lottery contest and show existence of equilibrium.

Afterwards, we analyze the effects on competition of dividing the bad of the contest in different pieces. To do so, we study an application where a central government decides either to set a big dump in one area or divide the dump in small pieces in different areas. We assume that there is an influence group in each area that lobbies the government to avoid the dump. The objective of the government depends on its nature. If it is corrupt, the objective is to maximize lobbying. Otherwise, the objective is to minimize lobbying and confrontation. We show that lobbying effort is maximized by setting a big dump. If the government divides the dump, lobbying effort diminishes for two reasons. First, the expected burden of getting one of the dumps is smaller. Second, if the number of dumps increases it is more likely that a region receives one dump, so the incentives of making lobbying effort decreases. This problem is related to multi-prize lottery contests, which have been studied in Clark and Riis (1996a,1998), Amegashie (2000), Yates and Heckelman (2001), Szymanski and Valletti (2005), Fu and Lu (2009,2012a,b), Fu et al. (2014) among others.[1] This work mainly differs from the previous literature since our purpose is to study multi-prize lottery contests with prizes of negative values, using the reverse nested lottery contest.

We also relate the reverse and the conventional lottery contest when competing to avoid bads. In a contest to avoid a bad only one individual gets the bad, which is considered as the loser, while the others are the winners. Indeed, a contest to avoid a bad among n individuals could be understood as a contest among n individuals that compete to win one of the $n-1$ prizes. The same happens with the conventional lottery contest when n individuals compete to win a prize. There is one winner of the prize and $n-1$ losers. We analyze which contest induces more aggregate effort under the same prize structure. That is, we study aggregate effort using the two models under two different scenarios: with two winning prizes and one losing penalty, and with two losing penalties and one winning prize. We show that in the first case aggregate effort is maximized under the conventional model while in the second case, with the reverse lottery contest. This is because, when there are two (one) winning prizes and one (two) losing penalty, the hypothetical sequence of lotteries is larger in the conventional (reverse) nested lottery contest model. The main difference between the formulations of the problem are based on where we focus the outcome of the contest. When we design the contest according to the conventional lottery contest, we focus on the effort that is required to win a prize, and indirectly, on avoiding a penalty. In other words, we focus on choosing the winner. The contrary happens under the reverse lottery contest. This study is relevant to determine the optimal contest design when competing both to win prizes and avoid penalties.

Finally, we propose a formulation that might be used as a tool to analyze tax competition when countries compete with taxes to attract firms. We show that using our formulation the race to the bottom does not happen.

[1] Sisak (2009) surveys the literature in multi-prize contests.

3.2 Reverse lottery contest and bads

We first study the usefulness of the reverse lottery contest to analyze situations where individuals compete to avoid a bad, burden or penalty.

Suppose a central government needs to allocate a dumping site for nuclear waste in one region. Let $N = \{1, ..., n\}$ be the set of lobbies, each of them allocated in one region, that compete to avoid the dump in their area. To do so, lobbies exert costly effort e_i, $i \in N$. Let $\mathbf{e} = (e_1, ..., e_n)$ be a vector of efforts. Effort can be understood as monetary resources that lobbies might use to influence the government.

Denote by $p_i(\cdot)$ the probability of each lobby $i \in N$ of getting the dump in its region. Function $p_i(\cdot)$ depends on $\mathbf{e}$. We use the reverse lottery contest introduced by Fu et al. (2014):

$$p_i = \frac{e_i^{-1}}{\sum_{j \in N} e_j^{-1}} \text{ if } e_j > 0 \text{ for all } j. \tag{3.1}$$

To avoid discontinuities, if $e_j = 0$ for all j, every lobby gets the dump with probability $1/n$. If only one lobby, w.l.o.g. lobby j, exerts $e_j = 0$, it gets the dump with probability 1. If k lobbies, $k \in N$ exert 0 effort, these lobbies get the dump with probability $1/k$.

Note that p_i is based on an inversely proportional sharing rule according to the effort exerted by each lobby. Then, the probability of getting the dump is decreasing (increasing) in own (rival) effort. It is also homogenous of degree 0 and it does not depend on the structure of the contest; i.e. the structure of the dump which lobbies try to avoid.

We study now the equilibrium effort when n lobbies compete to avoid a dump of value $-B < 0$. Cost of effort is linear. Payoffs of lobby i are:

$$\pi_i = -\frac{e_i^{-1}}{\sum_{j \in N} e_j^{-1}} B - e_i. \tag{3.2}$$

We show the following.

Proposition 3.1 *There exists a symmetric Nash equilibrium,* $e^* = B(n-1)/n^2$*. Individual payoffs are* $\pi_i^* = B(1-2n)/n^2$*.*

Proof. The first derivative of payoffs (3.2) with respect to e_i is:

$$\frac{\partial \pi_i}{\partial e_i} = B[\frac{e_i^{-2} \sum_{j \neq i} e_j^{-1}}{(\sum_{j \in N} e_j^{-1})^2}] - 1. \tag{3.3}$$

Equate (3.3) to 0. By symmetry, it follows that $e^* = B(n-1)/n^2$. We need to show now that payoffs of each lobby are concave in its own effort. The second derivative of (3.2) with respect to e_i is:

$$\frac{\partial^2 \pi_i}{\partial e_i^2} = \frac{2B}{e_i^3(\sum_{j\in N} e_j^{-1})}\left[\frac{-1}{e_i^2(\sum_{j\in N} e_j^{-1})^2} + \frac{2}{e_i(\sum_{j\in N} e_j^{-1})} - 1\right]. \tag{3.4}$$

This expression is negative if and only if $(1 - e_i \sum_{j\in N} e_j^{-1})^2 \geq 0$, which is always the case.

Insert e^* in (3.2) to get π_i^*. Finally, note that not participating in the contest yields payoffs $-B$, which are strictly smaller than π_i^*. ■

The equilibrium effort is the same that in the conventional lottery contest for a positive prize. Intuitively, equilibrium effort is increasing in the size of the dump and decreasing in the number of lobbies. Also, equilibrium payoffs are decreasing in the size of the dump and increasing in the number of lobbies. They differ from the conventional lottery contest due to the nature of the prize, which in this case takes a negative value.

It is important to remark that we have restricted our analysis to the symmetric case; i.e. lobbies value the dump equally and the effort of each lobby has the same impact in the reverse lottery contest function. Because of the functional form of the reverse lottery contest, there is not a Nash equilibrium in pure strategies when at least three lobbies differ to others in their characteristics.

3.3 Application: Division of a dump

In this section we analyze the effects on competition of dividing the bad of the contest in different pieces. We do so through the following application. Suppose a central government needs to allocate a dumping site for nuclear waste of value $-B$ in one region. The dump is divisible and the central government can divide it in $k \geq 1$ pieces of value $-b$, with $k \in \{1, n-1\}$, such that $B = kb$. The dump is necessary for society, but is a burden for the habitants of the region where is finally allocated. Suppose there is a lobby in each of the n regions that makes money contributions e_i to influence the central government not to allocate the dump in its place. The problem of the central government is to decide either to allocate just one dump in one region or to allocate several smaller dumps in different regions.

The objective of the central government depends on its nature. In case it is a corrupt government, its objective is to maximize revenue. Note that this application is the opposite of rent-seeking. Rent-seeking consists in a situation where entities want to get rents from the manipulation of a political environment. Political corruption is a phenomenon that arises naturally in this context, since lobbies might brive corrupt governments to get rents or influence a policy. Since we are dealing with bads, we might assume the same application with the difference that lobbies brive the corrupted government to influence a policy that does not hurt them. Contrarily, if the government is not corrupt, the objective is to reduce lobbying and confrontation. Clearly, if one policy is better to increase lobbying, the contrary is better to counteract it.

Assume that the dump is allocated according to the reverse lottery contest (3.1).

In case it allocates one dump of value $-B$ in one of the regions, using the general problem in equation (3.2), by Proposition 3.1, it follows that aggregate money contributions are $E = B(n-1)/n$.

Suppose it divides the dump in k pieces of value $-b$, with $k \in \{1, n-1\}$, such that $B = kb$. Lobbies send money contributions to avoid getting one of the k dumps. We use the formulation of Clark and Riis (1998) with the reverse lottery contest. That is, the probability of getting one of the dumps is the probability of getting one dump in the first place, plus the probability of not getting the dump in the first place times the probability of getting the second dump and so on. Cost of effort is linear. We focus on the symmetric Nash equilibrium. Assume that the payoffs of one lobby are:

$$\pi_i = -\frac{e_i^{-1}}{e_i^{-1} + (n-1)e^{-1}} b - \sum_{j=1}^{k-1} [\Pi_{s=1}^{j} (1 - \frac{e_i^{-1}}{e_i^{-1} + (n-s)e^{-1}})] \frac{e_i^{-1}}{e_i^{-1} + (n-j-1)e^{-1}} b - e_i. \quad (3.5)$$

Straightforward maximization yields the aggregate money contribution, which coincides with the one of Clark and Riis (1998)[2], and is the following:

$$E = \max\{0, \sum_{s=1}^{k} b(1 - \sum_{t=0}^{s-1} \frac{1}{n-t})\}. \quad (3.6)$$

Individual effort is $e = E/n$.

Proposition 3.2 *Aggregate contributions are larger when the central government allocates only one dump.*

It is straightforward to check Proposition 3.2. If the government divides the dump, aggregate contributions diminish for two reasons. First, the expected burden of getting one of the dump is smaller. Second, an increase in the number of dumps makes more likely getting one dump, so that decreases incentives of contributing.

[2] In Clark and Riis (1998), the contest consists of n individuals and k prizes of positive values using the conventional nested lottery contest.

A similar nested reverse lottery contest was used in Fu et al. (2014) and Lu et al. (2015a). The problem differs since they used the function to analyze the allocation of different positive prizes, and compare the results with the conventional nested lottery contest used in Clark and Riis (1998). In the Appendix we show the conditions for equilibrium effort (3.6) to be a maximum.

3.4 Choosing a technology: reverse vs conventional lottery contest

In this section we study which lottery contest (reverse or conventional) implies higher aggregate effort when the number of prizes and bads is the same. Choosing the conventional or reverse lottery contest when the prize and bad structure is the same are two different ways of promoting the same contest. In a contest that takes place according to the conventional lottery contest, we focus on the effort that is required to win a prize, and indirectly, on avoiding a penalty. In other words, we focus on choosing the winners. Contrarily, in a contest that takes place according to the reverse lottery contest, we are putting more emphasis on the effort that is required to avoid a bad, and indirectly, on winning a prize. In other words, we focus on choosing the losers. We study two scenarios. In the first scenario, there are two winning prizes of value $W > 0$ and one losing penalty of value $-L < 0$. In the second scenario, there is only one winning prize W and two losing penalties $-L$. In both scenarios, we assume there are only three individuals. This simple model allows us to study easily which contest structure implies higher aggregate effort. We focus on these simple cases because a generalization involves complicated algebra that does not add any new insight to the results.

Scenario 1. Two winning prizes and one losing penalty.

We study first the problem when the contest takes place according to a conventional nested lottery contest. We assume that, as in Clark and Riis (1998), the prize allocation is sequential given a vector of efforts exerted by individuals. We focus on the symmetric Nash equilibrium. Payoffs of one individual are:

$$\pi_i = W\frac{e_i}{e_i+2e} + W(1-\frac{e_i}{e_i+2e})\frac{e_i}{e_i+e} - L(1-\frac{e_i}{e_i+2e})(1-\frac{e_i}{e_i+e}) - e_i. \tag{3.7}$$

The first derivative with respect to e_i yields:

$$\frac{\partial \pi_i}{\partial e_i} = -\frac{e_i^4 + 6e_i^3 e + 13e_i^2 e^2 - 4e_i(L+W-3e)e^2 + 2e^3(-3L-3W+2e)}{(e_i+e)^2(e_i+2e)^2}. \tag{3.8}$$

Equate to 0 and by symmetry, $e_i = e$. Thus, isolating e we get that:

$$e = \frac{5(W+L)}{18}. \tag{3.9}$$

Note that effort is increasing in both the winning prize and losing penalty. Note also that this function exhibits a maximum since the second derivative of (3.7) is negative:

$$\frac{\partial^2 \pi_i}{\partial e_i^2} = -\frac{4(L+W)e^2(3e_i^2 + 9e_i e + 7e^2)}{(e_i + e)^3(e_i + 2e)^3}. \tag{3.10}$$

Individual payoffs are $\pi_i = (7/18)W - (11/18)L$. In case of making 0 effort, individual payoffs are $\pi_i = -L$. Thus, the participation constraint is satisfied.

We study now the problem when the contest takes place according to the reverse lottery contest. We focus on the symmetric Nash equilibrium. Assume that the payoffs of one individual are:

$$\pi_i = -L\frac{e_i^{-1}}{e_i^{-1} + 2e^{-1}} + W(1 - \frac{e_i^{-1}}{e_i^{-1} + 2e^{-1}}) - e_i. \tag{3.11}$$

The first derivative with respect to e_i yields:

$$\frac{\partial \pi_i}{\partial e_i} = -\frac{4e_i^2 + 4e_i e + (2L + 2W - e)e}{(2e_i + e)^2}. \tag{3.12}$$

Equate to 0 and by symmetry, $e_i = e$. Thus, isolating e we get that:

$$e = \frac{2(W+L)}{9}. \tag{3.13}$$

Note that again effort is increasing in both the winning prize and losing penalty. Note also that this function exhibits a maximum since the second derivative of (3.11) is negative:

$$\frac{\partial^2 \pi_i}{\partial e_i^2} = -\frac{8(L+W)e}{(2e_i + e)^3}. \tag{3.14}$$

Individual payoffs are $\pi_i = (4/9)W - (5/9)L$. In case of making 0 effort, individual payoffs are $\pi_i = -L$. Thus, the participation constraint is satisfied.

Then, we see that the aggregate effort in a conventional nested lottery contest is larger than when using the reverse lottery contest. This is because individuals have to put effort to compete in two allocations of the winning prize when competing under a conventional nested lottery contest, while only in one allocation of the penalty when competing under a reverse lottery contest. Because of this, individual payoffs are larger under the reverse lottery contest.

Scenario 2. Two losing penalties and one winning prize.

Again, we study first the problem when the contest takes place according to a conventional lottery contest. Assume that the payoffs of one individual are:

$$\pi_i = W\frac{e_i}{e_i + 2e} - L(1 - \frac{e_i}{e_i + 2e}) - e_i. \tag{3.15}$$

The first derivative with respect to e_i yields:

$$\frac{\partial \pi_i}{\partial e_i} = -\frac{e_i^2 + 4e_i e - 2(L + W - 2e)e}{(e_i + 2e)^2}. \tag{3.16}$$

Equate to 0 and by symmetry, $e_i = e$. Thus, isolating e we get that:

$$e = \frac{2(W + L)}{9}. \tag{3.17}$$

Note that effort is exactly the same that when the contest takes place according to a reverse lottery contest and there are two winning prizes and one penalty. Note also that this function exhibits a maximum since the second derivative of (3.15) is negative:

$$\frac{\partial^2 \pi_i}{\partial e_i^2} = -\frac{4(L + W)e}{(e_i + 2e)^3}. \tag{3.18}$$

Individual payoffs are $\pi_i = (1/9)W - (8/9)L$. In case of making 0 effort, individual payoffs are $\pi_i = -L$. Thus, the participation constraint is satisfied.

We study now the problem when the contest takes place according to a reverse nested lottery contest. Assume that the payoffs of one individual are:

$$\pi_i = -L\frac{e_i^{-1}}{e_i^{-1} + 2e^{-1}} - L(1 - \frac{e_i^{-1}}{e_i^{-1} + 2e^{-1}})\frac{e_i^{-1}}{e_i^{-1} + e^{-1}} + W(1 - \frac{e_i^{-1}}{e_i^{-1} + 2e^{-1}})(1 - \frac{e_i^{-1}}{e_i^{-1} + e^{-1}}) - e_i. \tag{3.19}$$

The first derivative with respect to e_i yields:

$$\frac{\partial \pi_i}{\partial e_i} = \frac{-4e_i^2 - 12e_i^3 e + e_i^2(6L + 6W - 13e)e + 2e_i(2L + 2W - 3e)e^2 - e^2}{(e_i + e)^2(2e_i + e)^2}. \tag{3.20}$$

Equate to 0 and by symmetry, $e_i = e$. Thus, isolating e we get that:

$$e = \frac{5(W + L)}{18}. \tag{3.21}$$

Note that effort is exactly the same that when the contest takes place according to a conventional nested lottery contest and there are two winning prizes and one penalty. Note also that the second derivative of (3.19) is negative if and only if:

$$\frac{\partial^2 \pi_i}{\partial e_i^2} = \frac{4(L + W)e(-6e_i^3 - 6e_i^2 e + e^3)}{(e_i + e)^3(2e_i + e)^2} < 0, \tag{3.22}$$

which is the case if and only if $e^3 < +6e_i^3 + 6e_i^2 e$. Note that it holds in equilibrium. Individual payoffs are $\pi_i = (1/18)W - (17/18)L$. In case of making 0 effort, individual payoffs are $\pi_i = -L$. Thus, the participation constraint is satisfied.

Then, we see that the aggregate effort in a conventional lottery contest is smaller than when using the reverse nested lottery contest. This is because individuals have to put effort to compete in two allocations of penalties when competing under the reverse nested lottery contest, while only in one allocation of the penalty when competing under a conventional lottery contest. Because of this, individual payoffs are larger under the conventional lottery contest.

We have proved the following.

Proposition 3.3 *When there are two winning prizes and one losing penalty, aggregate effort is maximized when the contest takes place according to a conventional nested lottery contest. When there are two losing penalties and one winning prize, aggregate effort is maximized when the contest takes place according to a reverse nested lottery contest.*

In this section we have shown that though we can express the same problem using both a conventional lottery contest and a reverse lottery contest, the effort outcomes are different. We remark that the main difference between the formulations of the problem are based on where we focus the outcome of the contest. When we design the contest according to a conventional lottery contest, we focus on the effort that is required to win a prize, and indirectly, on avoiding a penalty. Then, we focus on choosing the winners. The contrary happens under the reverse lottery contest. We also praise the utility and importance of the reverse lottery contest because of its tractability and its simplicity to solve problems that would be more tedious under a conventional lottery contest.

3.5 Related CSF and applications to tax competition

A general version of the reverse lottery contest used in the previous sections is:

$$p_i(\mathbf{e}) = \frac{f(e_i)}{(\sum_{j \in N} f(e_j))} \text{ for all } i \in N, \tag{3.23}$$

where $f(\cdot)$ is a decreasing function in its argument. Note that if $f(e_i) = \alpha e_i^r$, with $\alpha = 1$ and $r = -1$, the formulation used throughout the paper arises.

Function (3.23) can be easily axiomatized using four out of five axioms from Skaperdas (Probability, Anonimity, Consistency and Independency of Irrelevant Alternatives) and changing Monotonicity by Inverse Monotonicity. Inverse Monotonicity says that the probability of getting the bad is decreasing in own effort and increasing in the rival effort. Also, using Homogeneity of Degree Zero the reverse lottery contest used throughout the paper arises. The reader can find the details in previous versions of this paper and in Lu and Wang (2015b).[3][4]

We now introduce another form of CSF that is useful to analyze tax competition and follows from the previous general form in (3.23).

Application: Tax competition

In this application we explain that from the formulation in (3.23) we can get other functions that are useful to analyze problems outside the classical scope of contest theory, such as tax competition. Suppose a CEO plans to allocate her firm of value V in one of the $i = 1, ..., n$ countries. Countries compete among them to attract the firm by setting a tax rate $t_i \in (0, 1)$ that taxes V. Let $\mathbf{t} = (t_1, ..., t_n)$ be a vector of tax rates. Let $p_i(\mathbf{t})$ be the probability that county i attracts the firm, with $\sum_{i=1}^n p_j(t) = 1$ The contest takes place according to $p_i(\mathbf{t}) = f(t_i)/(\sum_{j=1}^n f(t_j))$ for all $i \in N$, where $f(t_i) = 1 - t_i$. The CEO allocates the firm according to the following *CSF:*

$$p_i(\mathbf{t}) = \frac{1 - t_i}{\sum_{j=1}^n (1 - t_j)}. \tag{3.24}$$

We assume that the decision of the CEO is noisy. That is, the CEO may take into account other factors than taxes when deciding where to allocate the firm, such as average wage or weather. We assume the expected tax income of one country is:

$$\pi_i = V \frac{1 - t_i}{\sum_{j=1}^n (1 - t_j)} t_i. \tag{3.25}$$

That is, expected tax income of one country is the probability of attracting the firm times the tax rate that taxes it, times the value of the firm. See that we are not dealing with a contest to avoid a bad. Indeed, we are dealing with a situation in which a reduction of the country tax rate, on the one hand, reduces the expected revenue from the tax base, but on the other hand, increases the probability of attracting the firm.

It is easy to show that this function has a maximum. Take the first derivative and equate to 0 to see that the symmetric equilibrium is $t^* = n/(2n - 1)$. Such a formulation could explain why taxes in equilibrium does not goes to 0 in absence of inmobile capital. Note that the payoff function has a maximum since the second derivative is negative:

$$\frac{\partial^2 \pi_i}{\partial t_i^2} = \frac{2V(\sum_{j \neq i}(1 - t_j))(1 + \sum_{j \neq i}(1 - t_j))}{(-\sum_{j=1}^n (1 - t_j))^3} < 0. \tag{3.26}$$

[3] This work has been developed simultaneously by Lu and Wang (2015b). They axiomatize the reverse lottery contest and the reverse nested lottery contest. For this reason, we have omitted the complete analysis of the axiomatization.

[4] We also remark that we can derive the reverse lottery contest through similar microfundations used in other research as in Corchón and Dahm (2010) for the conventional lottery contest.

3.6 Conclusions

This paper studies situations in which individuals compete to avoid a bad, burden or penalty, using the reverse lottery contest model. We show existence of equilibrium when individuals compete to avoid a bad. We also show that a contest organizer decreases competition when individuals compete to avoid several divisions of the original bad through a reverse nested lottery contest.

We argue that a contest among n individuals to avoid a bad can be understood as a contest among n individuals to win one of the $n-1$ prizes. This is because in a contest to avoid a bad only one individual gets the bad, which can be considered as the loser, while the others are the winners. Then, we analyze which contest structure induces more aggregate effort under the same prize and bad structure: the conventional or reverse lottery contest. That is, we study aggregate effort using the two contest models under two different scenarios: with two winning prizes and one losing penalty, and with two losing penalties and one winning prize. We show that in the first case aggregate effort is maximized under the conventional lottery contest while in the second case, with the reverse lottery contest.

Appendix A

Appendix of Chapter 1

A.1 Appendix 1.A

Proof of Proposition 1.1. Recall that $E_l^c = n_l[vn_l n_m/(n_l+n_m)^2]^{1/\phi+1}$, $E_l^d = n_l[vn_m(n_l/n_m)^{1/\phi+1})/(n_l+n_m(n_l/n_m)^{1/\phi+1})^2]^{1/\phi+1}$ and $n_1 = an_2$ with $a \in [1, \infty)$. First of all, we show that Group 1 always underperforms. Note that $E_1^c > E_1^d$ if and only if $1 > (1+a)^2/n_2 a^{\phi/\phi+1}(a^{1/\phi+1}+a)^2$. Let

$$h_1(a) = 1 - \frac{1}{n_2}\frac{(1+a)^2}{a^{\frac{\phi}{\phi+1}}(a^{\frac{1}{\phi+1}}+a)^2}. \tag{A.1}$$

Note that $h_1(a)$ is increasing for all $a \in [1, \infty)$ because:

$$\frac{\partial h_1(a)}{\partial a} = \frac{a^{-2+\frac{1}{1+\phi}}(1+a)[a^{\frac{1}{1+\phi}}(2+\phi(1-a)) + a(2+(3+a)\phi)]}{n_2(a+a^{\frac{1}{1+\phi}})^3(1+\phi)} > 0 \text{ for all } a \in [1, \infty). \tag{A.2}$$

See now that for all $a \in [1, \infty)$, $h_1(a)$ is positive, which implies that the larger group always underperforms. Take the limit of $h_1(a)$ when $a \to 1$. It follows that $h_1(a)$ tends to $1 - 1/n_2$, which is positive. Now, take the limit when $a \to \infty$. It follows that $h_1(a)$ tends to 1, which is positive. Since $h_1(a)$ is increasing for all $a \in [1, \infty)$, it follows that $h_1(a)$ is positive for all $a \in [1, \infty)$, which implies that the larger group always underperforms.

We now show that there exists an $\bar{a} \in [1, \infty)$ such that for $a < \bar{a}$, the smaller group underperforms, while for $a > \bar{a}$, the smaller group outperforms. To do so, note that $E_2^c > E_2^d$ if and only if $1 > (a+1)^2/n_1 a^{\frac{\phi}{\phi+1}}(a^{\frac{-\phi}{\phi+1}}+1)^2$. Let

$$h_2(a) = 1 - \frac{1}{n_1}\frac{(a+1)^2}{a^{\frac{\phi}{\phi+1}}(a^{\frac{-\phi}{\phi+1}}+1)^2}. \tag{A.3}$$

Note that $h_2(a)$ is decreasing for all $a \in [1,\infty)$ since:

$$\frac{\partial h_2(a)}{\partial a} = -\frac{1}{n_1}\frac{(a+1)\left(2a+\phi+3a\phi+a^{\frac{(2\phi+1)}{\phi+1}}\phi-a^{\frac{\phi}{\phi+1}}\phi+2a^{\frac{(2\phi+1)}{\phi+1}}\right)}{a^{\frac{(3\phi+1)}{\phi+1}}(\phi+1)\left(a^{\frac{-\phi}{\phi+1}}+1\right)^3} < 0 \text{ for all } a \in [1,\infty). \tag{A.4}$$

See now that for all $a \in [1,\infty)$, there exists an $\bar{a}$ such that $h_2(\bar{a}) = 0$. Take the limit of $h_2(a)$ when $a \to 1$. It follows that $h_2(a)$ tends to $1 - 1/n_1$, which is positive. Now, take the limit when $a \to \infty$. It follows that $h_2(a)$ tends to $-\infty$. Since $h_2(a)$ is decreasing for all $a \in [1,\infty)$, by the intermediate value theorem, there exists an $\bar{a}$ such that $h_2(\bar{a}) = 0$. Thus, for all $a < \bar{a}$, $h_2(a)$ is positive which implies that Group 2 underperforms, while for $a > \bar{a}$, $h_2(a)$ is negative which implies that Group 2 outperforms. ■

Proof of Proposition 1.2. For the first part, insert equation (1.16) in (1.15). The first order condition for every group coincides with (1.6).

For the second part, just note that $\partial t_l(e_{li}, E_{l-i})/\partial e_l = 1 - 1/n_l > 0$. Now suppose that $n_1 > n_2$. It follows that $1 - 1/n_1 > 1 - 1/n_2$, which implies that $\partial t_1(e_{1i}, E_{1-i})/\partial e_{1i} > \partial t_2(e_{2i}, E_{2-i})/\partial e_{2i}$. ■

Proof of Proposition 1.3. For the first part, equate (1.8) and (1.19) for both groups and solve the system of equations with unknowns $f(e^t_{li}, E^t_{l-i})$ for $l = 1, 2$. We obtain that

$$f(e_{li}, E_{l-i}) = e^{\phi}_{li} - \frac{\partial t_l(e_{li}, E_{l-i})}{\partial e_{li}} = A_l,\ l = 1, 2, \tag{A.5}$$

$$\text{where } A_l = \frac{vn_m}{(n_l+n_m)^2}\left(\frac{vn_l n_m}{(n_l+n_m)^2}\right)^{\frac{-1}{\phi+1}}. \tag{A.6}$$

Isolate and integrate $\partial t_l(z_{li}, E_{l-i})/\partial z_{li}$ with respect to z_{li} in the interval $[0, e_{li}]$. It follows that the transfer function of individual i in group l is given by:

$$t_l(e_{li}, E_{l-i}) = \frac{e^{1+\phi}_{li}}{1+\phi} - A_l e_{li} + Q(E_{l-i}). \tag{A.7}$$

Since $\sum_{i=1}^{n_l} t_l(e_{li}, E_{l-i}) = 0$, it follows that:

$$E_l A_l - \sum_{i=1}^{n_l}\frac{e^{1+\phi}_{li}}{1+\phi} = \sum_{i=1}^{n_l} Q(E_{l-i}). \tag{A.8}$$

In order to find the function $Q(E_{l-i})$, it is necessary to solve this functional equation. To do so, consider the vector of efforts in which every individual in the group exerts 0 effort. It follows that $Q(0) = 0$. Now, consider all the possible vectors of efforts in which all the components are 0 except for 1 individual, namely j. If follows that:

$$A_l E_{l-12\ldots(j-1)(j+1)\ldots n_l} - \frac{e^{1+\phi}_{lj}}{1+\phi} = (n_l - 1)Q(e_{lj}) + Q(0),\ j = 1 \ldots n_l. \tag{A.9}$$

which can be rewritten as:

$$Q(e_{lj}) = \frac{1}{n_l - 1}[e_{lj}A_l - \frac{e_{lj}^{1+\phi}}{(1+\phi)}],\ j = 1...n_l. \tag{A.10}$$

Now consider all the possible vectors of efforts in which every individual in the group exerts 0 effort except 2 individuals, namely j and k. It yields that:

$$A_l E_{l-12...(j-1)(j+1)...(k-1)(k+1)...n_l} - \frac{e_{lj}^{1+\phi}}{1+\phi} - \frac{e_{lk}^{1+\phi}}{1+\phi} = (n_l - 2)Q(e_{lj} + e_{lk}) + Q(e_{lj}) + Q(e_{lk}) + Q(0). \tag{A.11}$$

Introducing equation (A.10) for j and k and rearranging it is obtained that:

$$Q(e_{lj} + e_{lk}) = \frac{1}{n_l - 1}[(e_{lj} + e_{lk})A_l - \frac{e_{lj}^{1+\phi}}{(1+\phi)} - \frac{e_{lk}^{1+\phi}}{(1+\phi)}]. \tag{A.12}$$

Repeating this process, it is obtained that

$$Q(E_{l-i}) = \frac{1}{n_l - 1}[(\sum_{k\neq i}^{n_l} e_{lk})A_l - \sum_{k\neq i}^{n_l} \frac{e_{lk}^{1+\phi}}{(1+\phi)}]. \tag{A.13}$$

and introducing it in equation (A.7) the required transfer function arises.

The transfer is meritocratic if $\partial t_l(e_{li}, E_{l-i})/\partial e_{li} > 0$, which is the case if and only if $e_{li}^{\phi} - A_l > 0$. For the last part of the proposition, suppose that $n_1 > n_2$. It follows that $A_2 > A_1$, which implies that the transfer function of Group 2 penalizes effort more than the transfer function of Group 1. ■

Proof of Proposition 1.4. a) It is necessary to show that payoffs $\pi_l(\alpha_l, \alpha_m)$ are decreasing in α_l for all α_m, for $l = 1, 2$. This is the case if the derivative of $\pi_l(\alpha_l, \alpha_m)$ with respect to α_l is negative in the interval $\alpha_l \in [0, 1]$ for all $\alpha_m \in [0, 1]$. The derivative is negative if and only if $2 \leq n_l[\Gamma_l + \Gamma_m]$. Since the right hand side of the last expression is minimal if $\alpha_l = 0$, for $l = 1, 2$, it follows that $n_l[1/n_l + 1/n_m] \leq n_l[\Gamma_l + \Gamma_m]$. Making some computations, it follows that $2 = n_l[1/n_l + 1/n_m] \leq n_l[\Gamma_l + \Gamma_m]$. Therefore $\pi_l(\alpha_l, \alpha_m)$ is decreasing in the interval $\alpha_l \in [0, 1]$, with a maximum at $\alpha_l^* = 0$. Then, the dominant strategy of both groups is to set $\alpha_l^* = 0$. Insert the equilibrium rules α_l^* in a_l and b_l for $l = 1, 2$ in the linear transfer functions to obtain the transfers.

For the second part of the proof, since $\alpha_l^* = 0$, from equation (1.26) it can be deduced that $E_l^* = n_l v/4$ for $l = 1, 2$. It follows that $p_l^* = 1/2$ and $\pi_l^* = n_l v/4$.

b) First, it is necessary to show that $\pi_1(\alpha_1, \alpha_2)$ is decreasing in α_1 for all α_2. This is the case if the first derivative of $\pi_1(\alpha_1, \alpha_2)$ with respect to α_1 is negative for all α_2. The derivative is negative if and only if $1/n_1 \leq 1/n_2 + \alpha_2(n_2 - 1)/n_2 + \alpha_1(n_1 - 1)/n_1$. The right hand side is minimal in our domain of α_l if $\alpha_l = 0$, $l = 1, 2$. Then, under these rules, it follows that the derivative is negative if and only if $1/n_1 \leq 1/n_2 \leq 1/n_2 + \alpha_2(n_2-1)/n_2 + \alpha_1(n_1-1)/n_1$, which holds since $n_1 > n_2$. It follows that $\pi_1(\alpha_1, \alpha_2)$ is decreasing in α_1 for all α_2, with a maximum at $\alpha_1^* = 0$, which is the dominant strategy of Group 1. Now, it is necessary to show that $\pi_2(\alpha_1, \alpha_2)$ has a maximum at $\alpha_2^* = (n_1-n_2)/((n_2-1)n_1)$. Given that $\alpha_1^* = 0$, equating $\partial\pi_2/\partial\alpha_2$ to 0 and isolating α_2 yields that $\alpha_2^* = (n_1-n_2)/((n_2-1)n_1)$. In order to show that it is a maximum, it is sufficient to show that $\pi_2(\alpha_1, \alpha_2)$ is increasing (decreasing) in α_2 for all $\alpha_2 < (>)\alpha_2^*$. Payoffs of Group 2 are increasing (decreasing) in α_2 if and only if $\partial\pi_2/\partial\alpha_2$ is positive (negative). This is the case if and only if $(\alpha_2(n_2-1)/n_2 + /n_2 + \alpha_1(n_1-1)/n_1 + 1/n_1)^{-1} > (<) n_2/2$. Since we know from previous results that $\alpha_1^* = 0$, isolating α_2 we obtain from the previous inequality that this is the case if and only if $\alpha_2 < (>)\alpha_2^*$. Thus, $\alpha_2^* = (n_1-n_2)/((n_2-1)n_1)$ is the mixed rule chosen by the organizer of Group 2. Insert the equilibrium rules α_l^* in a_l and b_l for $l = 1, 2$ in the linear transfer functions to obtain the transfers.

For the second part, introducing $\alpha_1^* = 0$ and $\alpha_2^* = (n_1-n_2)/((n_2-1)n_1)$ in equation (1.25) and making some arrangements, it follows that $E_2^*/E_1^* = n_2/(2n_1-n_2)$, which is clearly lower than 1 since $n_1 > n_2$. Then, $E_1^* > E_2^*$ and as a result $p_1^* > p_2^*$. Finally, introducing the equilibrium values of α_1^* and α_2^* in equation (1.26), it follows that the effort exerted by Group 1 is $E_1^* = n_2 v(2n_1-n_2)/4n_1$, while the effort of Group 2 is $E_2^* = v(n_2)^2/4n_1$. It is straightforward to see that the transfer set by Group 1 is more meritocratic than the transfer set by Group 2, since Group 1 exerts more effort because its transfer yields a smaller marginal cost than Group 2. It is also straightforward to see that the larger the difference of size between groups is, the less meritocratic the transfer set by Group 2 is, because the resulting marginal cost is larger. ■

A.2 Appendix 1.B

In Section 1.4, we assume that each organizer sets a transfer that implements the centralized effort under the assumption that the organizer in the other group does likewise. In this appendix we show under which conditions this assumption is plausible. In particular, we assume that organizers decide either to monitor individuals or not.

For simplicity, assume linear costs. Suppose the following game. In the first stage each organizer decides either to monitor (M) or not (N) her individuals. Let $s = (s_1, s_2)$ be a vector of strategies of organizers, and $s_l = \{M, N\}$ for $l = 1, 2$. In the second stage, the contest takes place.

In case that both organizers monitor individuals, the centralized contest takes place. In case both organizers do not monitor individuals, the decentralized contest takes place. Otherwise, one group behaves as in a centralized contest and the other group as in a decentralized contest.

In Section 1.3 we analyze both centralized and decentralized contest. Recall that when both organizers monitor their individuals, both organizers maximize group payoffs:

$$\pi_l = \frac{E_l}{E} v n_l - E_l,\ l = 1, 2. \tag{A.14}$$

In equilibrium, $E_l^{MM} = vn_l^2 n_m/(n_l + n_m)^2$ and $\pi_l^{MM} = vn_l^3/(n_l + n_m)^2$. Superscript MM denotes that both organizers monitor their individuals.

Also recall that when both organizers do not monitor their individuals, they maximize their payoffs:

$$\pi_{li} = \frac{E_l}{E} v - e_{li},\ l = 1, 2. \tag{A.15}$$

In equilibrium, $E_l^{NN} = v/4$ and $\pi_l^{NN} = v(2n_l - 1)/4$. Superscript NN denotes that both organizers do not monitor their individuals.

Without loss of generality, suppose Organizer 1 monitors their individuals and Organizer 2 does not. Organizer in Group 1 maximizes (A.14) and individuals in Group 2 maximize (A.15). In equilibrium $E_1^{MN} = vn_1^2/(n_1 + 1)^2$, $E_2^{MN} = vn_1/(n_1 + 1)^2$, $\pi_1^{MN} = vn_1^3/(n_1 + 1)^2$ and $\pi_2^{MN} = v(n_1 n_2 + n_2 - n_1)/(n_1 + 1)^2$. The upper-script MN denotes the scenario in which Organizer 1 monitors her individuals and Organizer 2 does not.

We show now that there exists an $\mathring{a} \in [1, \infty)$ such that for $a < \mathring{a}$, the strategy profile $s = (M, M)$ is an equilibrium and for $a > \mathring{a}$, $s = (M, N)$ is an equilibrium.

To do so, we show first that Organizer 1 has the dominant strategy of monitoring her individuals. Her payoffs in case both organizers monitor their individuals can be rewritten as $\pi_1^{MM} = va^3 n_2/(1 + a)^2$. In case Organizer 1 deviates and sets $s_1 = N$, her payoffs are $\pi_1^{NM} = v(an_2^2 + n_2(a - 1))/(1 + n_2)^2$. Define

$$g_1(a) \equiv \frac{a^3}{(1 + a)^2 (a(1 + n_2) - 1)} - \frac{1}{(1 + n_2)^2}, \tag{A.16}$$

and note that $\pi_1^{MM} > \pi_1^{NM}$ whenever $g_1(a) > 0$. Note that $g_1(a)$ is increasing for all $a \in [1, \infty)$ since:

$$\frac{\partial g_1(a)}{\partial a} = \frac{a^2 (a + 2an_2 - 3)}{(a + 1)^3 (a + an_2 - 1)^2} > 0. \tag{A.17}$$

Take the limit of $g_1(a)$ when $a \to 1$. It follows that $g_1(a)$ tends to $1/4n_2 - 1/(1 + n_2)^2$, which is positive. Now, take the limit when $a \to \infty$. It follows that $g_1(a)$ tends to $1/(1 + n_2) - 1/(1 + n_2)^2$, which is positive. Since $g_1(a)$ is increasing for all $a \in [1, \infty)$, it follows that $g_1(a)$ is positive for all $a \in [1, \infty)$.

Now, see that payoffs of Organizer 1 in case both organizers do not monitor their individuals can be rewritten as $\pi_1^{NN} = v(2an_2 - 1)/4$. In case Organizer 1 deviates and sets $s_1 = M$, her payoffs are $\pi_1^{MN} = va^3n_2^3/(1 + an_2)^2$. Define

$$g_2(a) = \frac{a^3n_2^3}{(1 + an_2)^2(2an_2 - 1)} - \frac{1}{4}, \tag{A.18}$$

and note that $\pi_1^{MN} > \pi_1^{NN}$ if and only if $g_2(a) > 0$. Note that $g_2(a)$ is increasing for all $a \in [1, \infty)$

since:

$$\frac{\partial g_2(a)}{\partial a} = \frac{3a^2n_2^3(an_2 - 1)}{(an_2 + 1)^3\,(2an_2 - 1)^2} > 0. \tag{A.19}$$

Take the limit of $g_2(a)$ when $a \to 1$. It follows that $g_2(a)$ tends to $n_2^3/(1+n_2)^2(2n_2-1)-1/4$, which is positive. Now, take the limit when $a \to \infty$. It follows that $g_2(a)$ tends to $1/2$, which is positive. Since $g_2(a)$ is increasing for all $a \in [1, \infty)$, it follows that $g_2(a)$ is positive for all $a \in [1, \infty)$.

As a result, the organizer in the larger group has the dominant strategy of monitoring her individuals.

We show now that given the dominant strategy of Organizer 1, Organizer 2 monitors her individuals if and only if $a < \mathring{a}$. To see this, note that payoffs of Organizer 2 in case both organizers monitor their individuals can be rewritten as $\pi_2^{MM} = vn_2/(1 + a)^2$. In case Organizer 2 deviates, she get payoffs $\pi_2^{MN} = v(an_2^2 + n_2(1 - a))/(1 + an_2)^2$. Define

$$g_3(a) \equiv \frac{(1 + an_2)^2}{(1 + a)^2(a(n_2 - 1) + 1)} - 1, \tag{A.20}$$

and note that $\pi_2^{MM} > \pi_2^{MN}$ if and only if $g_3(a) > 0$. Note that $g_3(a)$ is decreasing for all $a \in [1, \infty)$

since:

$$\frac{\partial g_3(a)}{\partial a} = \frac{(n-1)\left(a^2n^2(1-a) + 2na(1-2a) - 3a + 1\right)}{(a+1)^3\,(a(n-1)+1)^2} < 0. \tag{A.21}$$

We show that there exists an $\mathring{a} \in [1, \infty)$ such that $g_3(\mathring{a}) = 0$. Take the limit of $g_3(a)$ when $a \to 1$. It follows that $g_3(a)$ tends to $(1+n_2)^2/4n_2-1$, which is positive. Now, take the limit when $a \to \infty$. It follows that $g_3(a)$ tends to -1. Since $g_3(a)$ is decreasing for all $a \in [1, \infty)$, by the intermediate value theorem, there exists an $\mathring{a}$ such that $g_3(\mathring{a}) = 0$. Thus, for all $a < \mathring{a}$, $g_3(a)$ is positive which implies that Organizer 2 sets $s_2 = M$, while for $a > \mathring{a}$, $g_3(a)$ is negative which implies $s_2 = N$.

This result explains that Organizer 1 always monitors her individuals while Organizer 2 does so only if the size of both groups is sufficiently similar. Otherwise, she does not monitor her individuals. Organizer 1 has the dominant strategy of monitoring her individuals because by doing so, her group becomes stronger. Also, since the size of her group is larger than the size of Group 2, Group 1 is always the strongest. Organizer 2 monitors her individuals only if the size of her group is sufficiently close to Group 1. In such a case, by monitoring her individuals, she encourages them to exert more effort and compete fiercely with Group 1. In case the size of both groups differ enough, Organizer 2 does not monitor her individuals and induces them to reduce effort. This is because the higher probability of winning that Group 2 obtains by exerting more effort does not compensate the cost.

A.3 Appendix 1.C

This appendix analyzes the implementation of the centralized effort setting when individuals have different valuations of the prize, and discusses the implications of this heterogeneity when organizers set transfers strategically at the end of the section. Assume that v_{li} is the valuation that individual i in group l has for the public prize. Assume without loss of generality that $v_{l1} > v_{l2} > ... > v_{ln}$. Assume also that $V_l = \sum_{i=1}^{n_l} v_{li}$. We assume linear costs for tractability.

We start the analysis by obtaining the effort that groups exert in a centralized contest when individuals in a group have different valuations of the prize. Payoffs of each organizer are:

$$\pi_l = \frac{E_l}{E} V_l - E_l \text{ for } l = 1, 2, \tag{A.22}$$

Each organizer chooses the amount of effort that maximizes the group payoffs given that she is competing with the other group. It follows that the centralized effort when individuals value the prize differently is:

$$E_l^{ch} = \frac{V_l^2 V_m}{(V_l + V_m)^2} \text{ for } l = 1, 2. \tag{A.23}$$

The upper-script ch denotes the centralized scenario where individuals value the prize differently. Therefore, the group whose sum of valuations is larger exerts more effort in equilibrium and is more likely to win the contest.

Now, we solve the decentralized contest with transfers with heterogeneous individuals. Payoffs of each individual are:

$$\pi_{li} = \frac{E_l}{E} v_{li} - e_{li} + t_{li}(e_{li}, E_{l-i}) \text{ for } i = 1...n_l, \ l = 1, 2, \tag{A.24}$$

Every individual chooses the effort that maximizes equation (A.24). The set of first order conditions are:

$$\frac{E_m}{E^2} = \frac{1 - \frac{\partial t_{li}(e_{li}, E_{l-i})}{\partial e_{li}}}{v_{li}} = w_{li}(e_{li}, E_{l-i}) \text{ for } m \neq l,\ i = 1...n_l. \tag{A.25}$$

Denote $w_{li}(e_{li}, E_{l-i}) \equiv (1 - \partial t_{li}(e_{li}, E_{l-i})/\partial e_{li})/v_{li}$. Note that in case that there are no transfers, only individuals whose valuation is larger in each group exert effort. In order to let the equilibrium be such that every individual in the contest exerts a positive amount of effort, let us use a transfer that allows that the first order condition of every individual is binding. Then, the right hand side of the equation for every individual is the same value. Using the first order condition of every individual in both groups, we obtain the group effort evaluated at the equilibrium:

$$E_l^{th} = \frac{w_{mj}(e_{mj}^{th}, E_{m-j}^{th})}{(w_{li}(e_{li}^{th}, E_{l-i}^{th}) + w_{mj}(e_{mj}^{th}, E_{m-j}^{th}))^2} \text{ for } l = 1, 2. \tag{A.26}$$

The upper-script th denotes the decentralized contest with transfers scenario where individuals value the prize differently. The organizer of each group implements the effort of the centralized contest when individuals value the prize differently choosing the transfer $t_l(e_{li}, E_{l-i})$ such that $E_l^{ch} = E_l^{th}$.

Proposition A.1 *When individuals have different valuations of the prize, the following transfer implements the effort of a centralized contest:*

$$t_{li}^{th}(e_{li}, E_{l-i}) = e_{li}[1 - \frac{v_{li}}{V_l}] + \sum_{j\neq i}^{n_l} \frac{e_{lj}}{n_l\ -1}[\frac{v_{lj}}{V_l} - 1],\ \textit{for}\ l = 1, 2, i = 1...n_l\ . \tag{A.27}$$

Proof. To check that this transfer implements the centralized effort, insert transfer (A.27) in (A.24) and see that the effort of the decentralized contest with transfers coincides with the centralized contest. To construct the transfer, equate (A.23) and (A.26) for both groups and solve the system of equations with unknowns $w_{li}(e_{li}^{th}, E_{l-i}^{th})$ for $l = 1, 2$. We obtain that

$$w_{li}(e_{li}, E_{l-i}) = \frac{1 - \frac{\partial t_{li}(e_{li}, E_{l-i})}{\partial e_{li}}}{v_{li}} = \frac{1}{V_l}, \text{ for } i = 1...n_l,\ l = 1, 2. \tag{A.28}$$

Isolate and integrate $\partial t_{li}(z_{li}, E_{l-i})/\partial z_{li}$ with respect to z_{li} in the interval $[0, e_{li}]$. It follows that the transfer of individual i in group l is given by:

$$t_{li}(e_{li}, E_{l-i}) = e_{li} - \frac{v_{li}}{V_l} e_{li} + Q(E_{l-i}). \tag{A.29}$$

Since $\sum_{i=1}^{n_l} t_{li}(e_{li}, E_{l-i}) = 0$, it follows that:

$$\sum_{i=1}^{n_l} \frac{e_{li} v_{li}}{V_l} - E_l = \sum_{i=1}^{n_l} Q(E_{l-i}). \tag{A.30}$$

In order to find the function $Q(E_{l-i})$, it is necessary to solve this functional equation. To do so, consider the vector of efforts in which every individual in the group exerts 0 effort. It follows that $Q(0) = 0$. Now, consider all the possible vectors of efforts in which all the components are 0 except for one individual, namely j. If follows that:

$$e_{lj}\frac{v_{lj}}{V_l} - e_{lj} = (n_l - 1)Q(e_{lj}) + Q(0),\ j = 1...n_l. \tag{A.31}$$

which can be rewritten as:

$$Q(e_{lj}) = \frac{e_{lj}}{n_l - 1}[\frac{v_{lj}}{V_l} - 1],\ j = 1...n_l. \tag{A.32}$$

Now, consider all the possible vectors of efforts in which every individual in the group exerts 0 effort except two individuals, namely j and k. It yields that:

$$e_{lj}[\frac{v_{lj}}{V_l} - 1] + e_{lk}[\frac{v_{lk}}{V_l} - 1] = (n_l - 2)Q(e_{lj} + e_{lk}) + Q(e_{lj}) + Q(e_{lk}) + Q(0). \tag{A.33}$$

Introducing equation (A.10) for j and k and rearranging we obtain that:

$$Q(e_{lj} + e_{lk}) = \frac{e_{lj}}{n_l - 1}[\frac{v_{lj}}{V_l} - 1] + \frac{e_{lk}}{n_l - 1}[\frac{v_{lk}}{V_l} - 1]. \tag{A.34}$$

Repeating this process, we obtain that

$$Q(E_{l-i}) = \sum_{j \neq i}^{n_l} \frac{e_{lj}}{n_l - 1}[\frac{v_{lj}}{V_l} - 1]. \tag{A.35}$$

And introducing it in equation (A.29) we get the transfer (A.27). ■

Individuals with lower valuations of the public prize have a more meritocratic transfer than individuals with higher valuations of the prize. By setting a more meritocratic transfer to individuals with lower valuations, these individuals are induced to exert the same amount of effort as the individuals with a higher valuation do. In particular, with these transfers every individual has the same marginal cost. It is worthy to note that although every individual in the group has the same marginal cost and since this problem is dealing with linear costs, the main importance falls in the aggregate of the effort exerted in the group. By setting these transfers, the marginal cost of every individual is being substituted with its valuation relative to the aggregate valuation of the group, implementing the centralized effort.

In the symmetric equilibrium in which every individual in the group exerts the same amount of effort, those individuals who have a higher valuation of the public prize face higher costs. Then, although these transfers induce all individuals to exert effort, they redistribute the cost in deterrence to those who value the public prize the most.

We finish by discussing the implications of heterogeneity when organizers set transfers strategically. Given the similarities with Section 1.5, we omit the analysis. First, the problem becomes excessively complicated if we allow a different transfer function for every individual. Then, if we focus on the same transfer function, only top individuals exert effort. The organizer of each group decides the transfer depending on the aggregate prize value of the group. In particular, an organizer has the dominant strategy of setting $\alpha_l = 0$ (1) if V_l is sufficiently large (small), independently on the rival group. This is because the organizer cares about the group payoffs and finds it convenient to set a meritocratic transfer function or not depending on this aggregate value. In case only one organizer has a dominant strategy, the rival group chooses a mixed rule $\alpha_l \in [0, 1]$. This is because the rival organizer maximizes her payoffs subject to this dominant strategy. Finally, if none of the organizers have a dominant strategy (their aggregate prize value is neither sufficiently large nor small), there is no equilibrium in pure strategies.

Appendix B

Appendix of Chapter 2

Proof of Lemma 2.1. In the state of the world $s = (b_H, b_L)$, player 1 is unconstrained and maximizes her payoff function (2.2). Her best reply is:

$$e_1 = \sqrt{\frac{Ve_2}{\lambda}} - \frac{e_2}{\lambda}. \tag{B.1}$$

Note that the best reply of player 1 does not depend on α. Player 2, who is constrained, maximizes her payoffs (2.2). The first derivative of (2.2) with respect to e_2 is:

$$\frac{\partial \pi_2(b_H, b_L)}{\partial e_2} = \frac{\alpha V \lambda e_1}{(\lambda e_1 + e_2)^2} - 1. \tag{B.2}$$

Note that the first derivative is positive if $\alpha V \geq (\lambda e_1 + b_L)^2/\lambda e_1$ and hence $e_2 = b_L$.

Inserting $e_2(b_H, b_L) = b_L$ in the best reply of player 1, $e_1(b_H, b_L) = \sqrt{Vb_L/\lambda} - b_L/\lambda$.

In the first stage, the contest organizer chooses λ that maximizes the sum of efforts. See that the aggregate effort does not depend on α. Then, choosing λ that maximizes the sum of efforts we get that $\lambda^*(b_H, b_L) = 4b_L/V$. Introducing $\lambda^*(b_H, b_L)$ in the condition for player 2 for being constrained, it follows that is necessary that $\alpha \geq 4b_L/V$. Insert $\lambda^*(b_H, b_L)$ in effort and payoffs to get that $e_1^*(b_H, b_L) = V/4$, $e_2^*(b_H, b_L) = b_L$, $\pi_1^*(b_H, b_L) = V/4$ and $\pi_2^*(b_H, b_L) = \alpha V/2 - b_L$.

Finally, suppose $s = (b_L, b_L)$. To ensure that players expend their whole budget, the organizer chooses a λ such that both of them are still constrained, which happens when $\alpha \geq b_L(\lambda + 1)^2/V\lambda$.

The organizer chooses $\lambda^*(b_L, b_L) = 1$. By doing so, the previous restriction is rewritten as $\alpha \geq 4b_L/V$. It follows that $e_i^*(b_L, b_L) = b_L$ and $\pi_i^*(b_L, b_L) = \alpha V/2 - b_L$ for $i = 1, 2$. ■

Proof of Proposition 2.2

The proof of Proposition 2.2 uses the following lemmas. To carry on this analysis, denote by $\pi(s; m)$ payoffs π when the state of the world is $s = (t_1, t_2)$ and the message profile is $m = (m_1, m_2)$.

Also, α depends on m (i.e. $\alpha(m)$) and $\alpha(b_H, b_L) = \alpha(b_L, b_H)$. The structure of the proof is as follows. First we derive conditions for $\alpha(b_H, b_L)$ and $\alpha(b_L, b_L)$ such that implement the optimal effort in each state of the world separately. Then, we show that there exists values for $\alpha(b_H, b_L)$ and $\alpha(b_L, b_L)$ that implement the optimal effort independently on the state of the world.

Lemma B.1 *Suppose $s = (b_H, b_H)$. There exists a value α^a such that for $\alpha(b_H, b_L) \in [0, \alpha^a)$ and $\alpha(b_L, b_L) = 0$ both players revealing their true type is the unique Nash equilibrium.*

Proof. We need a unique equilibrium in which both players report their true type. Since both players are unconstrained, different prizes affect the equilibrium efforts.

There exists a value α^a such that both players reporting their true type is a Nash equilibrium for $\alpha(b_H, b_L) \in [0, \alpha^a]$. To see this, recall that payoffs of players when both of them report their true type are $\pi_i(b_H, b_H; b_H, b_H) = V/4$ for $i = 1, 2$. Without loss of generality, suppose player 2 deviates from both players reporting their true type and reports $m_2 = b_L$. In such a case, player 2 competes for prize $\alpha(b_H, b_L)V$ and player 1 for prize V, $\lambda = 4b_L/V < 1$ and both of them are unconstrained. Solving the game, we get that payoffs of player 2 become $\pi_2(b_H, b_H; b_H, b_L) = (\alpha(b_H, b_L)V)^3/(4b_L + \alpha(b_H, b_L)V)^2$. Then, player 2 reporting her true type is part of the Nash equilibrium if and only if $\pi_2(b_H, b_H; b_H, b_H) \geq \pi_2(b_H, b_H; b_H, b_L)$, which holds if and only if $V/4 \geq (\alpha(b_H, b_L)V)^3/(4b_L + \alpha(b_H, b_L)V)^2$. Solving this inequality for $\alpha(b_H, b_L)$ we get that this is the case for $\alpha(b_H, b_L) \in [0, \alpha^a]$, where

$$\alpha^a \equiv A(\frac{b_L}{V}) + \frac{\frac{2b_L}{3V} + \frac{1}{144}}{A(\frac{b_L}{V})} + \frac{1}{12}, \text{ with } A(\frac{b_L}{V}) \equiv \sqrt[3]{\frac{b_L}{12V} + \sqrt{(\frac{b_L}{3V})^3 + 4(\frac{b_L}{V})^4 + 2(\frac{b_L}{V})^2 + \frac{1}{1728}}}\,. \quad \text{(B.3)}$$

It follows that equilibria in which one player reports her true type and the other player lies are impossible for $\alpha(b_H, b_L) \in [0, \alpha^a)$. Note that we do not include α^a in the interval to avoid multiplicity of equilibria.

If both players report $m_i = b_L$, they get $\pi_i(b_H, b_H; b_L, b_L) = 0$ because $\alpha(b_L, b_L) = 0$. This cannot be an equilibrium since they get positive payoffs by an unilateral deviation.

Thus, both players lying, or only one player lying and other reporting her true type, cannot be an equilibrium for $\alpha(b_H, b_L) \in [0, \alpha^a)$ and $\alpha(b_L, b_L) = 0$. The only Nash equilibrium consists in both players reporting $m_i = b_H$, and aggregate effort is maximized. ■

Lemma B.2 *Suppose $s = (b_H, b_L)$. There exists values α^b and α^c such that for $\alpha(b_H, b_L) \in (\alpha^b, \alpha^c)$ and $\alpha(b_L, b_L) = 0$ both players revealing their true type is the unique Nash equilibrium.*[1]

Proof. We first find the values of $\alpha(b_H, b_L)$ and $\alpha(b_L, b_L)$ that makes truthtelling an equilibrium and afterwards we restrict to those values where truthtelling is the unique equilibrium. There exists a value α^b such that both players reporting their true type is a Nash equilibrium for $\alpha(b_H, b_L) \in [\alpha^b, 1]$ and $\alpha(b_L, b_L) = 0$. To see this, recall from Lemma 2.1 that payoffs of players when both of them report their true type are $\pi_1(b_H, b_L; b_H, b_L) = V/4$ and $\pi_2(b_H, b_L; b_H, b_L) = V\alpha(b_H, b_L)/2 - b_L$. We first derive the payoffs in case one player deviates from truthtelling and then prove that truthtelling is an equilibrium.

Suppose player 1 deviates from both players reporting their true type and reports $m_1 = b_L$. Then, player 1 gets payoffs $\pi_1(b_H, b_L; b_L, b_L) = 0$. Thus, she does not have incentives to deviate.

Suppose player 2 deviates from both players reporting their true type and reports $m_2 = b_H$. Then, both players compete for prize V and $\lambda = 1$. Solving the game, it follows that $\pi_2(b_H, b_L; b_H, b_H) = \sqrt{Vb_L} - b_L$. Player 2 is constrained if $V > 4b_L$, which is always the case.

Then, both players reporting their true type is an equilibrium if and only if $\pi_1(b_H, b_L; b_H, b_L) \geq \pi_1(b_H, b_L; b_L, b_L)$ and $\pi_2(b_H, b_L; b_H, b_L) \geq \pi_2(b_H, b_L; b_H, b_H)$.

It is straightforward to see that $\pi_1(b_H, b_L; b_H, b_L) \geq \pi_1(b_H, b_L; b_L, b_L) = 0$.

Note that $\pi_2(b_H, b_L; b_H, b_L) \geq \pi_2(b_H, b_L; b_H, b_H)$ if and only if $V\alpha(b_H, b_L)/2 - b_L \geq \sqrt{Vb_L} - b_L$. Solving for $\alpha(b_H, b_L)$ we get that this is the case for $\alpha(b_H, b_L) \in [\alpha^b, 1]$, where

$$\alpha^b \equiv 2\sqrt{\frac{b_L}{V}}. \tag{B.4}$$

Note that $\alpha^b > 4b_L/V$ is always the case since $b_L < V/4$.[2]

Thus, both players reporting their true type is a Nash equilibrium for $\alpha(b_H, b_L) \in [\alpha^b, 1]$ and $\alpha(b_L, b_L) = 0$.

For all the analysis above, both players reporting $m_i = b_H$ or $m_i = b_L$ cannot be an equilibrium for $\alpha(b_H, b_L) \in (\alpha^b, 1]$ and $\alpha(b_L, b_L) = 0$. Note that we do not include α^b in the interval to avoid precisely these equilibria.

We need to define the conditions that ensure that both players reporting their true type is the unique Nash equilibrium. Then, we need to find additional conditions for $\alpha(b_H, b_L)$ that ensure that both players lying about their type is not a Nash equilibrium. When both players lie about their types, player 1 competes for prize $\alpha(b_H, b_L)V$, player 2 for V and $\lambda = V/4b > 1$. Solving the game, we get that $\pi_1(b_H, b_L; b_L, b_H) = \alpha(b_H, b_L)V - 4b_L\sqrt{\alpha(b_H, b_L)} + 4b_L^2/V$ and $\pi_2(b_H, b_L; b_L, b_H) =$

[1] The same applies for $s = (b_L, b_H)$.

[2] Recall that $b_L/V \in [0, 1/4]$ is the relevant range. For $b_L/V > 1/4$, player 2 becomes unconstrained. Thus, this is the range we study for the rest of the proof.

$2b_L/\sqrt{\alpha(b_H,b_L)}-b_L$. Player 2 is constrained if and only if $V > \alpha(b_H,b_L)V\lambda b_L/(\sqrt{\alpha(b_H,b_L)V\lambda b_L}-b_L)$, and solving for $\alpha(b_H,b_L)$ we get that this is the case if and only if $\alpha(b_H,b_L) > -4b_L/V - 2\sqrt{-4b_L/V+1}+2 \equiv \alpha^d$. Player 1 deviates from both players reporting a false type if and only if $\pi_1(b_H,b_L;b_H,b_H) > \pi_1(b_H,b_L;b_L,b_H)$, where $\pi_1(b_H,b_L;b_H,b_H) = V - 2\sqrt{Vb_L}+b_L$, which holds if and only if $V-2\sqrt{Vb_L}+b_L > \alpha(b_H,b_L)V - 4b_L\sqrt{\alpha(b_H,b_L)}+4b_L^2/V$. Solving for $\alpha(b_H,b_L)$ we get that this is the case if and only if $\alpha(b_H,b_L) < \alpha^c$, where

$$\alpha^c \equiv 5\frac{b_L}{V}+4(\frac{b_L}{V})^2 - 2\sqrt{\frac{b_L}{V}} - 4(\frac{b_L}{V})^{\frac{3}{2}}+1. \tag{B.5}$$

See that $\alpha(b_H,b_L) \geq \alpha^b$ implies $\alpha(b_H,b_L) \geq \alpha^d$. To see this, define $f_1(b_L/V) = 2\sqrt{b_L/V} - (-4b_L/V - 2\sqrt{-4b_L/V+1}+2)$ and see that this is the case whenever $f_1(b_L/V) \geq 0$. See first that $f_1(b_L/V) = 0$ only at $b_L/V = 0$ and $b_L/V = 1/4$ in the interval $b_L/V \in [0,1/4]$. Evaluate the first derivative of $f_1(b_L/V)$ at $b_L/V = \epsilon$ and $b_L/V = 1/4-\epsilon$, with $\epsilon \to 0^+$ to see that $\partial f_1(\epsilon)/\partial(b_L/V) > 0$ and $\partial f_1(1/4-\epsilon)/\partial(b_L/V) < 0$.[3] Since $f_1(b_L/V)$ is continuous at $b_L/V \in [0,1/4]$, it follows that $f_1(b_L/V) \geq 0$ in $b_L/V \in [0,1/4]$.

See also that $\alpha^c \geq \alpha^b$. To see this, define $f_2(b_L/V) = (5b_L/V + 4(b_L/V)^2 - 2\sqrt{b_L/V} - 4(b_L/V)^{\frac{3}{2}}+1) - 2\sqrt{b_L/V}$ and note that this is the case whenever $f_2(b_L/V) \geq 0$. See that $f_2(0) = 1$ and $f_2(b_L/V) = 0$ only at $b_L/V = 1/4$ in the interval $b_L/V \in [0,1/4]$. Since $f_2(b_L/V)$ is continuous in $b_L/V \in [0,1/4]$, $f_2(b_L/V) \geq 0$ for $b_L/V \in [0,1/4]$.

The analysis for $s = (b_L,b_H)$ is analogous. Then, for $\alpha(b_H,b_L) \in (\alpha^b,\alpha^c)$ and $\alpha(b_L,b_L) = 0$ both players reporting their true type is the unique Nash equilibrium when $s = (b_H,b_L)$. Note that we do not include α^b and α^c in the intervals to avoid multiplicity of equilibria. ■

Lemma B.3 *Suppose $s = (b_L,b_L)$. There exists values α^e and α^f such that for $\alpha(b_H,b_L) \in (\alpha^e,\alpha^f)$ and $\alpha(b_L,b_L) = 0$, both players reporting $m_i = b_H$ is the unique Nash equilibrium*

Proof. There exists values α^e and α^f such that both players reporting $m_i = b_H$ is a Nash equilibrium for $\alpha(b_H,b_L) \in [\alpha^e,\alpha^f]$. To see this, note first that when both players report $m_i = b_H$, both compete for prize V, $\lambda = 1$ and both are constrained. Payoffs of every player are $\pi_i(b_L,b_L;b_H,b_H) = V/2-b_L$. Without loss of generality, suppose player 1 deviates from both players reporting $m_i = b_H$ and reports $m_i = b_L$. Then, player 1 competes for prize $\alpha(b_H,b_L)V$, player 2 competes for prize V and $\lambda = V/4b_L$. Payoffs of player 1 are $\pi_1(b_L,b_L;b_L,b_H) = \alpha(b_H,b_L)V^2/(V+$

[3] Note that $f_1(b_L/V)$ is not well-defined for $b_L/V < 0$ and $b_L/V > 1/4$.

$4b_L) - b_L$. Player 1 remains constrained if $\alpha(b_H, b_L) \geq \alpha^e$, where

$$\alpha^e \equiv \frac{(1+4\frac{b_L}{V})^2}{4}, \tag{B.6}$$

and player 2 remains constrained if $4 \geq (1+b_L/V)^2$. This last inequality always holds since $b_L/V < 1/4$. Then, both players reporting $m_i = b_H$ is an equilibrium if and only if $\pi_1(b_L, b_L; b_H, b_H) \geq \pi_1(b_L, b_L; b_L, b_H)$ and $\pi_2(b_L, b_L; b_H, b_H) \geq \pi_2(b_L, b_L; b_H, b_L)$, and both inequalities hold if and only if $V/2 - b_L \geq \alpha(b_H, b_L)V^2/(V+4b_L) - b_L$. Solving the inequality for α we get that this is the case if and only if $\alpha(b_H, b_L) \leq \alpha^f$, where

$$\alpha^f \equiv \frac{1+4\frac{b_L}{V}}{2}. \tag{B.7}$$

It is necessary to show that $\alpha^e \leq \alpha^f$. Define $f_3(b_L/V) \equiv (1+4b_L/V)/2 - (1+4b_L/V)^2/4$. Note that $\alpha^e \leq \alpha^f$ whenever $f_3(b_L/V) \geq 0$. See that $f_3(0) = 0.25$ and $f_3(b_L/V) = 0$ only at $b_L/V = 1/4$ in the interval $b_L/V \in [0, 1/4]$. Since $f_3(b_L/V)$ is continuous in $b_L/V \in [0, 1/4]$, $f_3(b_L/V) \geq 0$ for $b_L/V \in [0, 1/4]$.

Only one player lying and other reporting her true type cannot be an equilibrium for $\alpha(b_H, b_L) \in (\alpha^e, \alpha^f)$. Note that the interval is open to avoid these equilibria.

If both players report $m_i = b_L$ they get $\pi_i(b_L, b_L; b_L, b_L) = 0$ because $\alpha(b_L, b_L) = 0$. This cannot be an equilibrium since they get positive payoffs by an unilateral deviation.

Thus, both players lying, or only one player lying and other reporting her true type, cannot be an equilibrium for $\alpha(b_H, b_L) \in (\alpha^e, \alpha^f)$ and $\alpha(b_L, b_L) = 0$. The only Nash equilibrium consists in both players reporting $m_i = b_H$, and aggregate effort is maximized. ■

Lemma B.4 *Condition $\alpha(b_H, b_L) < \alpha^a$ implies $\alpha(b_H, b_L) < \alpha^c$ and $\alpha(b_H, b_L) < \alpha^f$. Parameter $\alpha^a \geq \max\{\alpha^b, \alpha^e\}$ for $b_L/V \in [0, 0.25]$.*

Proof. We show first that $\alpha(b_H, b_L) < \alpha^a$ implies $\alpha(b_H, b_L) < \alpha^c$. To see this, define $f_4(b_L/V) = 5b_L/V + 4(b_L/V)^2 - 2\sqrt{b_L/V} - 4(b_L/V)^{\frac{3}{2}} + 1 - (A(b_L/V) + ((2/3)b_L/V + 1/144)/A(b_L/V) + 1/12)$. Note that $\alpha(b_H, b_L) < \alpha^a$ implies $\alpha(b_H, b_L) < \alpha^c$ whenever $f_4(b_L/V) > 0$. See that $f_4(0) = 0.75$, $f_6(0.25) = 2.5244 \times 10^{-29}$ and there does not exist $b_L/V \in [0, 1/4]$ such that $f_4(b_L/V) = 0$. Since $f_4(b_L/V)$ is continuous in $b_L/V \in [0, 1/4]$, $f_4(b_L/V) > 0$ for $b_L/V \in [0, 1/4]$.

We show now that $\alpha(b_H, b_L) < \alpha^a$ implies $\alpha(b_H, b_L) < \alpha^f$. To see this, define $f_5(b_L/V) = (4b_L/V + 1)/2 - (A(b_L/V) + ((2/3)b_L/V + 1/144)/A(b_L/V) + 1/12)$. Note that $\alpha(b_H, b_L) < \alpha^a$ implies $\alpha(b_H, b_L) < \alpha^f$ whenever $f_5(b_L/V) > 0$. See that $f_5(0) = 0.25$, $f_5(0.25) = 2.5244 \times 10^{-29}$

and there does not exist $b_L/V \in [0, 1/4]$ such that $f_5(b_L/V) = 0$. Since $f_5(b_L/V)$ is continuous in $b_L/V \in [0, 1/4]$, $f_5(b_L/V) > 0$ for $b_L/V \in [0, 1/4]$.

We show now that $\alpha(b_H, b_L) > \alpha^e$ implies $\alpha(b_H, b_L) > \alpha^b$ if and only if $b_L/V \in [0, 0.0218445)$ while $\alpha(b_H, b_L) > \alpha^b$ implies $\alpha(b_H, b_L) > \alpha^e$ if and only if $b_L/V \in (0.0218445, 0.25]$. To do so, we need to solve the following inequality, $\alpha^b > \alpha^e$, which can be rewritten as $2\sqrt{b_L/V} > (1+4b_L/V)^2/4$. Solving, we find that this is the case whenever $b_L/V \in (0.0218445, 0.25]$. Otherwise, $\alpha^e > \alpha^b$.

Finally, we need to show that $\alpha^a \geq \max\{\alpha^e, \alpha^b\}$ for $b_L/V \in [0, 0.25]$. To do so, define first $f_6(b_L/V) = (A(b_L/V)+((2/3)b_L/V+1/144)/A(b_L/V)+1/12)-(1+4b_L/V)^2/4$. Note that $\alpha^a \geq \alpha^e$ whenever $f_6(b_L/V) \geq 0$. See first that $f_6(b_L/V) = 0$ in $b_L/V \in [0, 0.0218445]$ only for $b_L/V = 0$. See also that $f_6(0.0218445) = 8.308\,4 \times 10^{-2}$. Since $f_6(b_L/V)$ is continuous in $b_L/V \in [0, 0.0218445]$, it follows that $f_6(b_L/V) \geq 0$ in this interval.

Define now $f_7(b_L/V) = (A(b_L/V)+((2/3)b_L/V+1/144)/A(b_L/V)+1/12)-2\sqrt{b_L/V}$. Note that $\alpha^a \geq \alpha^b$ whenever $f_7(b_L/V) \geq 0$. See first that $f_7(0.0218445) = 8.308\,4 \times 10^{-2}$ and $f_7(b_L/V) = 0$ in $b_L/V \in (0.0218445, 0.25]$ only for $b_L/V = 1/4$. Since $f_7(b_L/V)$ is continuous in $b_L/V \in (0.0218445, 0.25]$, it follows that $f_7(b_L/V) \geq 0$ in this interval. ■

With the previous Lemmas, we have already set a family of mechanisms that implement the optimal effort.

Proof of Proposition 2.2. By Lemmas B.1, B.2, B.3 and B.4, the organizer implements the optimal effort by setting $\alpha(b_H, b_L) \in (\max\{\alpha^b, \alpha^e\}, \alpha^a)$ and $\alpha(b_L, b_L) = 0$. When $s = (b_H, b_H)$ or $s = (b_H, b_L)$, players report their true type. When $s = (b_L, b_L)$, players report $m = (b_H, b_H)$. ■

We choose value $\alpha^*(b_L, b_L) = 0$ for simplicity and to avoid a tedious analysis. However, we can extend the analysis to the case in which $\alpha^*(b_L, b_L) > 0$. Indeed, when $s = (b_L, b_L)$, the number of equilibria depend on the chosen value of $\alpha^*(b_L, b_L)$. In particular, if $\alpha^*(b_L, b_L)$ is sufficiently large, there are two Nash equilibria, $m = (b_H, b_H)$ and $m = (b_L, b_L)$. If $\alpha^*(b_L, b_L)$ is sufficiently small, the only Nash equilibrium is $m = (b_H, b_H)$. The complete analysis is provided by the author under request.

Proof of Proposition 2.3

Throughout Lemmas B.2 to B.4 we set a family of mechanisms that implements the optimal effort. We restrict to a simpler mechanism to prove Proposition 2.3. We first extend Lemma B.4, again for the two type case, to show that the lowerbound α^e is not strictly necessary to implement the optimal effort. We set threshold α^e in Lemma B.4 to keep players constrained in equilibrium when $s = (b_L, b_L)$ in case they report different types not to confuse the reader; i.e. both players are constrained when they have low budgets. We show in the following Lemma that this threshold is not necessary to implement the optimal effort when $s = (b_L, b_L)$ for an specific value of $\alpha(b_H, b_L)$ and $\alpha(b_L, b_L)$.

Lemma B.5 *Suppose $s = (b_L, b_L)$ and let $\alpha(b_H, b_L) = \alpha^b + 0^+$ and $\alpha(b_L, b_L) = 0$. Both players reporting $m_i = b_H$ is the only Nash equilibrium. Also, the mechanism described in Proposition 2.2 works for these values.*

Proof. We prove first that when $s = (b_L, b_L)$, both players reporting $m_i = b_H$ is the only Nash equilibrium for $\alpha(b_H, b_L) = \alpha^b + 0^+$ and $\alpha(b_L, b_L) = 0$.

Recall from Lemma B.4 that $\alpha^b > \alpha^e$ for $b_L/V \in (0.0218445, 0.25]$. For this interval, from Lemma B.3 and Lemma B.4, the only Nash equilibrium is that both of them report $m_i = b_H$.

Suppose now $b_L/V \in [0, 0.0218445]$ so that $\alpha^e > \alpha^b$. Since $\alpha(b_L, b_L) = 0$, both players reporting $m_i = b_L$ is never an equilibrium. We show that the only equilibrium is both players reporting $m_i = b_H$. Recall from Lemma B.3 that when both players report $m_i = b_H$ they get payoffs $\pi_i(b_L, b_L; b_H, b_H) = V/2 - b_L$ for $i = 1, 2$. Without loss of generality, suppose player 1 reports $m_i = b_L$ and player 2 reports $m_i = b_H$. Then, player 1 competes for $\alpha(b_H, b_L)V$, player 2 competes for V and $\lambda = V/4b_L$. It is sufficient to show that $\pi_1(b_L, b_L; b_H, b_H)$ is strictly larger than $\alpha(b_H, b_L)V$ for $b_L/V \in [0, 0.0218445]$. Obviously, the payoffs of player 1 are strictly smaller than $\alpha(b_H, b_L)V$, since she does not win the prize with probability 1 in equilibrium and she exerts a positive amount of effort in equilibrium. Note payoffs of player 2 are strictly positive. Note that $\pi_1(b_L, b_L; b_H, b_H) \geq \alpha(b_H, b_L)V$ if $f_8(b_L/V) = 1/2 - b_L/V - 2\sqrt{b_L/V}$ is positive for this interval. To see that this is the case, note that $f_8(0) = 0.25$, $f_8(0.0218445) = 0.18256$ and $\partial f_8(b_L/V)/\partial(b_L/V) = -1 - 1/\sqrt{b_L/V}$ is decreasing. Thus, no player has incentives to deviate from both reporting $m_i = b_H$. ∎

The second part of the proof follows from Lemmas B.1, B.2, B.3 and B.4.

Proof of Proposition 2.3. Let $\alpha(b_1, b_2) = \alpha^b + \varepsilon$, with $\varepsilon \to 0^+$ when $b_1 > \bar{b}$ and $b_2 < \bar{b}$.

Suppose $s = (b_1, b_2)$ such that $b_i > \bar{b}$ for $i = 1, 2$. By Lemmas B.1 and B.4, in stage two, the only equilibrium consists of both players reporting messages $\{(m_1, m_2) : m_i = \hat{b}_i > \bar{b}\ for\ i = 1, 2\}$. Thus, the optimal effort is implemented in this case.

Suppose $s = (b_1, b_2)$ such that $b_1 > \bar{b}$ and $b_2 < \bar{b}$. By Lemma B.2, both players reporting $m_i = b_i < \bar{b}$ cannot be an equilibrium since they get zero payoffs. Also, by Lemma B.2, both players reporting $m_i = b_i > \bar{b}$, or player 1 reporting $m_1 = b_1 < \bar{b}$ and player 2 reporting $m_2 = b_2 > \bar{b}$

cannot be an equilibrium. We define and show now the set of possible equilibria, which consists of $\{(m_1, m_2) : m_1 = \hat{b}_1 > \bar{b} \; and \; m_2 \in (0, \bar{b})\}$. By Lemmas B.2 and B.4, player 1 in stage two reports $m_1 = \hat{b}_1 > \bar{b}$ and does not have incentives to deviate and report a lower budget. By Lemmas B.2 and B.4, player 2 does not report $m_2 = \hat{b}_2 > \bar{b}$. Suppose player 2 reports any $m_2 = \hat{b}_2 < \bar{b}$, then she gets payoffs $\pi_2(b_1, b_2; b_1, \hat{b}_2) = 2\sqrt{b_2/V} - b_2$ for all $\hat{b}_2 \in (0, \bar{b})$. There is only one equilibrium that maximizes effort. Since we focus on truthtelling equilibria, effort is maximized.

Suppose $s = (b_1, b_2)$ such that $b_i < \bar{b}$ for $i = 1, 2$. By Lemma B.3, B.4 and B.5, the only set of Nash equilibria in stage two is $\{(m_1, m_2) : m_i = \hat{b}_i > \bar{b} \; for \; i = 1, 2\}$. Thus, the optimal effort is implemented in this case. ■

Proof of Corollary 2.2. Let $\alpha^*(b_H, b_L) = 2\sqrt{b_L/V} + 0^+$ and $\alpha^*(b_L, b_L) = 0$. These values implement the optimal effort and minimizes costs.

It follows from Proposition 2.2 and consists in choosing the minimum value of α that implements the optimal effort. The expected effort coincides with the complete information case since the optimal effort is implemented. The expected costs are $E(V+\alpha V) = q_{HH}V + q_{HL}V(1+2\sqrt{b_L/V})/2 + q_{LH}V(1+2\sqrt{b_L/V})/2 + q_{LL}V$. From Lemmas B.2 and B.3, it is straightforward to see that expected costs are larger than in the complete information case. ■

Microfoundations of the Tullock CSF

In this appendix we give two microfoundations to the biased *Tullock CSF* we use throughout the paper. There is some critiscism towards the literature of the optimal design of the *CSF* arguing that it cannot be designed. In what follows, we describe two procedures that explain that giving advantage to a player with respect to another one is equivalent to design the value of λ of the *Tullock CSF*. We take the first microfoundation from Hillman and Riley (1987) and the second from Dahm and Porteiro (2005) and Corchón and Dahm (2010). The main difference between these microfoundations is that while in the former the organizer does not observe effort, in the latter she does. Each microfoundation consists of an additional stage (stage four) in the game described in Section 2.2 to motivate the use of our biased *Tullock CSF*. This additional stage consists of describing the procedure the organizer uses to allocate the prize.

Noise in the performance

We follow closely the microfoundation of Hillman and Riley (1987). Consider the model described in Section 2.2, with the additional assumption that there is an stage four in which the organizer awards the prize to the player whose performance is better. We assume that the organizer does not observe effort e_i but outcome z_i of each player. We assume that:

$$z_i = e_i \varepsilon_i. \tag{B.8}$$

That is, we assume that the outcome of each player depends on effort and a noise ε_i, with cumulative distribution function $H(\varepsilon_i)$ and density $h(\varepsilon_i)$. Funcion $H(\varepsilon_i)$ is strictly increasing and differentiable if and only if $\varepsilon_i \in (0, a)$. The noise has a lower support of 0 since only positive impact can produce a victory. The structure of the outcome is the product of effort and noise such that higher effort involves higher uncertainty.

The organizer can bias the contest giving different weights to the outcome of players. Let $\lambda_i \in (0, \infty)$ and $\lambda_j \in (0, \infty)$ be the weights the organizer gives to the outcome of player i and j respectively. Player i wins the contest if $z_i \lambda_i > z_j \lambda_j$. This can be rewritten as:

$$\varepsilon_j < \frac{\lambda_i e_i}{\lambda_j e_j} \varepsilon_i, \tag{B.9}$$

and the probability that it happens is $H(\lambda_i e_i \varepsilon_i / \lambda_j e_j)$. The probability of winning of player i is:

$$p_i = \int_0^a H(\frac{\lambda_i e_i}{\lambda_j e_j} \varepsilon) h(\varepsilon) d\varepsilon. \tag{B.10}$$

Suppose $h(\varepsilon_i) = \exp(-\varepsilon)$. Then, the previous expression can be rewritten as:

$$p_i = \int_0^a (1 - \exp(-\varepsilon \frac{\lambda_i e_i}{\lambda_j e_j}) \exp(-\varepsilon) d\varepsilon, \tag{B.11}$$

which can be rewritten as:

$$p_i = \frac{\lambda_i e_i}{\lambda_1 e_1 + \lambda_2 e_2}. \tag{B.12}$$

If $\lambda_1 / \lambda_2 = \lambda$, we get our formulation. Thus, the organizer biasing the outcome z_i of players is equivalent to the organizer biasing the *Tullock CSF.*

Noisy organizer

We follow closely the microfoundations of Dahm and Porteiro (2005) and Corchón and Dahm (2010).

Consider the model described in Section 2.2, with the additional assumption that there is an stage four in which the organizer decides the winning player. The organizer gets payoffs $U(w = i)$ if she awards the prize to player i, where w denotes the winning player. The organizer chooses the winning player that maximizes her payoffs.

Each player exerts costly effort e_i to influence the payoffs of the organizer, who observes these efforts. The impact of effort of each player is $\lambda_i e_i$, being λ_i a parameter that measures how the organizer values effort of each player. The organizer has preferences over players but players ignore them. We assume there is a parameter $k_i \in [0,1]$ that determines the preferences of the organizer for each player $i = 1,2$ from the point of view of contestants, and it is distributed according to some distribution function F. Define $k_1 = k$ and $k_2 = (1-k)$. The payoffs of the organizer if she awards the prize to player i are $U(w = i) = k_i \lambda_i e_i$. Player 1 wins the contest if:

$$U(w = 1) = k\lambda_1 e_1 \geq (1-k)\lambda_2 e_2 = U(w = 2) \quad \text{(B.13)}$$

$$\Leftrightarrow \quad k \geq \frac{\lambda_2 e_2}{\lambda_1 e_1 + \lambda_2 e_2} = \bar{k}. \quad \text{(B.14)}$$

Then, the winning probability of player 1 is

$$p_1(e_1, e_2) = 1 - F(\bar{k}). \quad \text{(B.15)}$$

Assuming $dF(\bar{k})$ is symmetric, we have that $1 - F(\bar{k}) = F(1 - \bar{k})$. This implies:

$$p_i(e_1, e_2) = F(\frac{\lambda_i e_i}{\lambda_1 e_1 + \lambda_2 e_2}). \quad \text{(B.16)}$$

If F is linear and $\lambda_1/\lambda_2 = \lambda$, we get (2.1).

Appendix C

Appendix of Chapter 3

We show now the conditions for function (3.5) to have a maximum. We follow a similar analysis used in Clark and Riis (1998) and Fu et al. (2014). Define $\lambda = e_i^{-1}/e^{-1}$. Then, equation (3.5) can be rewritten as:

$$\pi_i = -\frac{\lambda}{\lambda+(n-1)}b - \frac{(n-1)}{\lambda+(n-1)}\frac{\lambda}{\lambda+(n-2)}b \tag{C.1}$$

$$-\frac{(n-1)}{\lambda+(n-1)}\frac{(n-2)}{\lambda+(n-2)}\frac{\lambda}{\lambda+(n-3)}b - ... \tag{C.2}$$

$$-\frac{(n-1)(n-2)...(n-(k-1))\lambda}{(\lambda+(n-1))(\lambda+(n-2))...(\lambda+(n-k))}b \tag{C.3}$$

$$-\frac{e}{\lambda}. \tag{C.4}$$

Introducing $e = E/n$, where E is given in equation (3.6), the previous function can be rewritten as:

$$\pi_i = \frac{-b}{n}\sum_{s=1}^{k}[\lambda\prod_{t=0}^{s-1}\frac{n-t}{n-t+\lambda-1} + (1-\sum_{t=0}^{s-1}\frac{1}{n-t})\lambda^{-1}] \tag{C.5}$$

The first and second order condition for a local maximum are:

$$\frac{\partial\pi_i}{\partial\lambda} = \frac{-b}{n}\sum_{s=1}^{k}\{\prod_{t=0}^{s-1}\frac{n-t}{n-t+\lambda-1}(1-\sum_{t=0}^{s-1}\frac{\lambda}{n-t+\lambda-1}) - (1-\sum_{t=0}^{s-1}\frac{1}{n-t})\lambda^{-2}] \tag{C.6}$$

$$\frac{\partial^2\pi_i}{\partial\lambda^2} = \frac{-b}{n}\sum_{s=1}^{k}\{(\prod_{t=0}^{s-1}\frac{n-t}{n-t+\lambda-1})(\sum_{t=0}^{s-1}\frac{1}{n-t+\lambda-1})(-1+\sum_{t=0}^{s-1}\frac{\lambda}{n-t+\lambda-1}) \tag{C.7}$$

$$+(\prod_{t=0}^{s-1}\frac{n-t}{n-t+\lambda-1})[-(\sum_{t=0}^{s-1}\frac{1}{n-t+\lambda-1}) + \lambda\sum_{t=0}^{s-1}\frac{1}{(n-t+\lambda-1)^2}] \tag{C.8}$$

$$+2(1-\sum_{t=0}^{s-1}\frac{1}{n-t})\lambda^{-3})\}. \tag{C.9}$$

Note that $\lambda = 1$ satisfies the first order condition. Evaluate the second order condition at $\lambda = 1$ and see that there is a locat maximum whenever:

$$\frac{-b}{n}\sum_{s=1}^{k}\{(\sum_{t=0}^{s-1}\frac{1}{n-t})(-1+\sum_{t=0}^{s-1}\frac{1}{n-t})-(\sum_{t=0}^{s-1}\frac{1}{n-t})+\sum_{t=0}^{s-1}\frac{1}{(n-t)^2}+2(1-\sum_{t=0}^{s-1}\frac{1}{n-t})\}<0. \quad \text{(C.10)}$$

The participation constraint requires that:

$$\frac{-b}{n}\sum_{s=1}^{k}(1+(1-\sum_{t=0}^{s-1}\frac{1}{n-t}))>-b, \quad \text{(C.11)}$$

where the left hand side of the inequality are individual payoffs in the symmetric equilibrium when participating in the contest and the right hand side are the individual payoffs in case of making 0 effort. This expression can be rewritten as:

$$n>\sum_{s=1}^{k}(1+(1-\sum_{t=0}^{s-1}\frac{1}{n-t})). \quad \text{(C.12)}$$

Now, we show an example that satisfies these conditions.

Example C.1 *Let $n = 3$, $k = 2$ and $b = 1$. Then, individual symmetric effort is $e = 5/18$ and aggregate effort $E = 15/18$. The value of the second derivative evaluated at the symmetric equilibrium is: $-2.64 < 0$. Individual payoffs are $\pi_i = -17/18$, while in case they do not participate, individual payoffs are $\pi_i = -1$.*

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