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Population and Income Across Time and Space

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Chapter 1

The Globe as a Network

Over the past 1000 years, successive improvements in transportation technology have given people in every part of the world progressively easier access to goods, ideas, and people from every other part of the world. During this same period, the world has experienced gradually accelerating growth in population, and an abrupt increase in income per capita growth, first in Europe and then in other regions, after 1800 CE. This latest burst of growth is the proximate cause of the distribution of income across regions we see in the world today, with the great distance between rich and poor countries and all the challenges and opportunities this entails.

How big is the role of falling transport costs in these great shifts of population and income? Why did the growth rate of income per capita increase abruptly around 1800 CE, and why in some places and not in others? These are the questions I address in this paper. To that end, I build a quantitative dynamic spatial model, with an agricultural and a non-agricultural sector. In this model, I allow both population, through fertility and migration, and knowledge, through innovation and diffusion, to be fully endogenous. Bilateral transport costs between each pair of locations determine the cost of trade, the cost of migration, and the speed of the diffusion of ideas. These shape the networks of trade, migration and technology diffusion through which outcomes in distinct locations are linked. Productivity in the agricultural sector depends on exogenous factors such as climate and soil characteristics, while the productivity of the non-agricultural sector depends on access to stocks of ideas.

I find that this model implies the existence of a threshold for global transport costs, which can be characterized in terms of a simple network statistic. If transport costs are above this threshold, population growth drives down income per capita, and the world converges to a Malthusian steady state with no growth. If transport costs fall below this threshold, population growth leads to a structural transformation from the agricultural to the non-agricultural sector, and the world economy enters a process of sustained growth in population and income per capita. In general, a universal reduction in transport costs will impact some locations more than others, so the take-off into growth may occur in a subset of locations at first. Trade and technology diffusion imply that all locations will start to catch up eventually.

Taking this model to the data, I divide the world into 3° by 3° quadrangles. I exclude quadrangles that contain no land or that are in Antarctica, leaving 2,249 habitable locations. I assign each location an agricultural potential based on available evidence from ecological studies. I infer bilateral transport costs by calculating the cheapest route between each pair of locations, given the natural placement of rivers, oceans and mountains, and given the cost of traversing each of these topographical features.

I then conduct a quantitative exercise in two stages. First, I calibrate the handful of parameters that are not already taken from historical data or tied to specific targets so that model predictions for population density in all of the 2,249 locations match the data for 1000 CE as closely as possible, under the assumption that the world is in a Malthusian steady state. I then reduce the costs of water and land transport gradually, in a way that is consistent with historical evidence, and track the endogenous evolution of population and income in 50 year periods until 2000 CE.

Qualitatively, this exercise is able to match all of the salient features of the data. The model generates slow but accelerating growth for the first 800 years, an abrupt takeoff around 1800 CE with Europe in the lead, and a large increase in the dispersion of income per capita across regions after 1800 CE.

Quantitatively, the model is able to account for most of the variation in population density across 10 major regions in 1000 CE–55% in all. China, India and Europe were more densely populated than other regions because they had more land with better agricultural potential better-linked by water transport. Europe is particularly well-connected to water transport, and so it benefits from the water-biased transport cost reductions that occur before 1750 CE. This is why Europe starts growing first, and is what allows the model to account for nearly half (44%) of the variation in income per capita across regions in 1800 CE, the first year for which there exists meaningful data. The model tracks the sharp rise of dispersion in the distribution of income per capita during the 19th century almost perfectly, and ultimately generates 43% of the overall dispersion across regions in the 2000 CE.

There are also some patterns that the model is not able to match. In particular, the model does not predict enough growth in the United States, Canada, Australia and New Zealand after 1800 CE. Also, the model predicts too much convergence between Europe and the rest of the world during the 20th century. I believe that these observations indicate avenues for future research, and I discuss them in more detail in the conclusion of the paper.

This study breaks new ground in a number of areas. To the best of my knowledge, it is the first study to propose a theory of the take-off from stagnation to growth as a global phenomenon dependent on a reduction in transport costs. It is related to the theory of Desmet and Parente (2012), who examine the role of market size in the industrial revolution. I build upon this study by considering the role of transport costs, and expanding the analysis to a global scope. It is also related to Galor and Weil's (2000) unified growth theory. I build upon their study by considering the role of space, and by providing a particular rationale for the relationship between technological progress and population size that they propose. In my model, when transport costs are reduced, we might also say that the effective population size has increased, as people living in different locations have been brought effectively closer together. So when transport costs fall below the critical level, we could also say that a "critical mass" of connected people has been created, not unlike the threshold population size which emerges from Galor and Weil's model.[1]

[1]Galor and Mountford (2008) also analyze the effect of increased trade on the transition from stagnation to growth, and in particular on the divergence in income per capita between the richest and poorest

This is also the first study to leverage available data on topography and exogenous climate and soil characteristics in a quantitative model to assess their role in determining the distribution of population and income in the world today. Prominent among previous efforts to assess the impact of geographical features on the distribution of population and income are Henderson, Squires, Storeygard and Weil (2016) and Gallup, Sachs and Mellinger (1999). I confirm the main conclusions of these studies in finding an association between agricultural potential and high pre-modern population density, and between access to water transport and modern growth, and propose and test quantitatively specific mechanisms through which these features can have an impact. Also, these studies implicitly assume that the value of access to a river or to the coast is the same in every location in the world, regardless of how far away or how wealthy potential trading partners are. The method that I use, which, similar to Donaldson and Hornbeck (2016), calculates distances to trading partners and determines the value of the trading connection using a general equilibrium model, accounts better for this natural heterogeneity.

This study is also, to my knowledge, the first to allow for endogenous population growth in a spatial setting. A recent study which analyzes the global distribution of population and income using a spatial dynamic framework is Desmet, Nagy and Rossi-Hansberg's (2016). In contrast to my focus on understanding how we arrived at the distributions of 2000 CE, they take these distributions as a starting point, and run counterfactual scenarios for the future. Population growth plays no role in their model. Another related paper in this vein is that of Nagy (2017), which takes aggregate population and technology growth in the 19th century United States as given, and seeks to explain their distribution across space in the decades leading up to 1860.[2]

This study is also related to the literature which has looked at the relationship between between market access and the global distribution of income. Redding and Venables (2004) and Head and Mayer (2011) find important static effects, taking the current distribution of population and technology as given. The current study extends these efforts by investigating the role of market access in determining these distributions. There have also been a number of studies measuring the importance of market access within a single country, such as Donaldson and Hornbeck (2016).

My paper is also related to efforts such as Alcalá and Ciccone's (2004) and Pascali's (2016) to assess the impact of trade on growth. Pascali's study is particularly related, as he exploits heterogeneity in access to water transport in a similar fashion, although in his case he uses it to construct an instrumental variable. Whereas studies in this strain of literature have been primarily interested in establishing whether or not there is an effect of trade on growth, I build on their insights by proposing a particular model of this relationship and assessing its performance quantitatively.

Similarly to this paper, Buera and Oberfield (2015) propose diffusion as a dynamic gain from trade. I build upon their insights by modeling this mechanism in a spatial setting and assessing its impact on growth over the last 1000 years quantitatively. Comin, Dmitriev and Rossi-Hansberg (2013) propose a similar model of the diffusion of technology across space, and show that it is consistent with observed patterns of technology diffusion

countries. They argue that globalization accelerated the transition to sustained growth in more advanced countries, and delayed it in less-advanced countries.

[2] Similarly, the dynamic spatial framework of Caliendo, Dvorkin and Parro (2017) takes population growth as given.

over the past 150 years. My setup differs that of Nagy (2017) and Desmet, Nagy and Rossi-Hansberg (2016) in that I track to transmission of ideas to particular locations, which can then themselves transmit the idea, as if it were a virus.

Finally, my study builds on that of Acemoglu, Johnson and Robinson (2005), who document that western Europe's higher rate of growth between 1500 and 1800 is almost entirely due to the growth of a handful of countries on the Atlantic Ocean who were engaged in substantial overseas trade. While Acemoglu and coauthors emphasize the role of institutions in deciding which of the Atlantic traders were best able to take advantage of their ocean access, my paper confirms and deepens the significance of the first fact, by showing that falling water transport costs during this period benefited some locations more than others and can account quantitatively for a number of key patterns in population and income growth.

The remainder of the paper is organized as follows. Section 1.1 presents the model. Section 1.2 analyzes the long run outcomes of the model. Section 1.3 describes how I bring the model to the data in a quantitative exercise. Section 1.4 presents and discusses the results of the calibration of the initial 1000 CE steady state. Section 1.5 presents and discusses the results of the simulation of the evolution of global population and income per capita from 1000-2000 CE. Section 1.6 discusses possible extensions and concludes.

1.1 Theoretical framework

The basic building blocks are as follows. Time is discrete, and indexed by t. Each model period is intended to represent a span of about 50 years. There exist a finite number of discrete locations n, contained in the set $N \equiv \{1, 2, ..., n\}$. Each location, at each point in time, is distinguished by three permanent, exogenous characteristics, and two endogenous characteristics that evolve over time. The three exogenous, permanent characteristics are $\lambda_i > 0$ for $i \in N$, the quantity of available land, $\alpha_i \geq 0$ for $i \in N$, agricultural potential, and bilateral transport costs reflected in $\gamma_{ij} \in [0, 1]$ for $i, j \in N$. The two endogenous, time-varying characteristics for are $x_i(t) \geq 0$ for $i \in N$, the number of residents, and $m_i(t) \geq 0$ for $i \in N$, the stock of ideas.

Consumers are endowed with labor, from which they derive wage income, and value goods and housing. There are many types of goods, and firms produce each one using labor, land and other goods as inputs. There is one type of housing, and producing it requires land and goods. Housing production is more land-intensive than goods production, and the demand for it increases the negative welfare effects of having many consumers living in a single location.[3]

All the varieties of goods exist in a continuum, and are indexed between 0 and 1. Among these, there are two basic categories or sectors. All of the goods indexed between 0 and $A < 1$ (the span $[0, A]$) are agricultural goods. All of the goods indexed between A and 1 (the span $(A, 1]$) are non-agricultural goods.

All goods may be produced in all locations, but different locations are better at producing some goods than others. This means that consumers and firms in different locations can gain by trading with each other, each specializing in the type of production they excel at.

[3] Also known as congestion effects.

Locations may also differ in their average suitability for producing agricultural and non-agricultural goods. Average agricultural suitability is partly determined by the exogenous, time-invariant characteristics of each location which are summarized by agricultural potential α_i. This quantity is meant to represent all of the durable climatic and geological characteristics that make some places respond more fruitfully to the efforts of the farmer.

Average non-agricultural suitability does not directly depend on any fixed, exogenous feature of a location. Instead it depends on the endogenous, time-varying stock of ideas, $m_i(t)$. The way in which this stock evolves over time is as follows. Firms that produce goods employ labor and land in innovation, which gives them an immediate, private productivity boost. As an externality, this innovative effort also leads to the discovery of new ideas. These ideas are added to local stock of ideas, and may also diffuse between locations that are trading partners.

If the stock of ideas in a location is small, its overall productivity will be mostly determined by its agricultural potential. But if the stock of ideas grows, the importance of this exogenous characteristic will decline.

Trade is limited by the cost of transporting goods. Bilateral transport costs are embodied in the parameters $\gamma_{ij} \in [0, 1]$, which each represent the fraction of goods sent from i to j that arrive. It is assumed that transport within a location is costless ($\gamma_{ii} = 1$) and that the triangle inequality holds ($\gamma_{ij}\gamma_{jk} \leq \gamma_{ik}$ for $\forall i, j, k$). In the current section we postpone the analysis of time-varying transport costs, and assume that transport costs are constant over time.

Transport costs determine the trade opportunities available to consumers and firms in each location. They also determine, according to simple functions, the strength of two other types of bilateral links. These are the cost faced by consumers when migrating between locations, and the probability that an idea invented in one location will spread to another. Therefore, locations which have more trading opportunities will also learn about more new ideas, sooner, and be more easily reached by migrants.

Consumers are atomistic and live a single period. The number of consumers living in each location, $x_i(t)$, is determined according to two processes. The first process is fertility. For each consumer who lived in a location the previous period, a certain number will be born there the current period. Á la Hansen and Prescott (2002), the fertility rate is determined as a simple function of parents' real income, which is a measure of the abundance of goods and housing they enjoyed. If goods and housing are very scarce, net fertility will be negative and the local population will shrink. If they are abundant enough, it will be positive and the local population will grow.

The second process determining the distribution of consumers across locations is migration. Given their birthplace, each consumer chooses either that location or another in which to work and consume. They will tend to move towards locations where there is a greater abundance of goods and housing, but to do so they must pay a migration cost. They also have idiosyncratic preferences for specific locations, which may cause a minority of individuals to choose locations which are less desirable in terms of real income and migration cost.

In the following subsections, I will specify each component of the framework in greater detail. I will also derive the equilibrium conditions and laws of motion that jointly determine current real income in each location, population growth, and the invention and diffusion of technology over time.

Many of the choices and processes that will be described take place in the context of a single time period. Therefore, for simplicity, I will from here on omit t-indices except where doing so introduces ambiguity.

1.1.1 Consumers

Consumers are atomistic and live a single period. The number of consumers born in each location at time t is denoted by $x_{i,b}(t)$, hereafter referred to as $x_{i,b}$, except where ambiguous. The first decision they must make is where to live, work and consume. The number of consumers choosing each location at time t is denoted by $x_i(t)$, hereafter referred to as x_i, except where ambiguous.

A consumer born in $j \in N$ chooses a location $i \in N$ in which to live based on three factors. First, their beliefs about the real income they can enjoy in each location, u_i^*. Second, moving costs between their birthplace and each destination. It is assumed that in order to move, consumers must give up a certain fraction of the real income they will earn at their destination. The inverse moving cost, $\vartheta_{ji} \in [0,1]$, represents the fraction of real income that they get to keep. Third, consumers have random idiosyncratic preferences for each potential destination, represented by $\mu_i \sim F_i(\mu)$, drawn independently across individuals and locations from cumulative distribution function $F_i(.)$.

Formally, the location choice problem of a consumer born in $j \in N$ is given by

$$\max_{i \in N} \{\mu_i \vartheta_{ji} u_i^*\} \, | j \in N \tag{1.1}$$

In equilibrium, consumers' ex-ante beliefs must be true, and coincide with ex-post real income, $u_i^* = u_i$. Inverse moving costs are a simple function of transport costs,

$$\vartheta_{ij} = \zeta_{m,0} \gamma_{ij}^{\zeta_{m,1}},$$

for some $\zeta_{m,0} \in [0,1]$ and $\zeta_{m,1} \geq 0$. Idiosyncratic preference shocks are drawn from a Fréchet distribution, so that

$$F_i(\mu) = e^{-\mu^{-\varkappa}},$$

for $\varkappa > 1$.

Upon arriving in their destination $i \in N$, the choices consumers make of how much to consume of housing and of each good may be characterized in terms of a representative consumer. The representative consumer's real income is determined by their consumption of goods and housing as given by

$$u_i = \left(\int_0^1 c_{i,l} dl \right)^{\frac{\alpha}{\rho}} h_i^{1-\alpha}, \tag{1.2}$$

where $c_{i,l}$ represents the quantity consumed of good $l \in [0,1]$, and h_i represents the quantity of housing consumed. Parameter $\alpha \in [0,1]$ determines the importance of housing relative to goods consumption, and $\rho \in [0,1]$ determines the elasticity of substitution between different goods.

Each consumer is endowed with 1 unit of labor, which they provide inelastically to the local market in exchange for prevailing wage w_i. It assumed that the rights to land are distributed equally among all residents of location i, so that each owns a quantity $\frac{\lambda_i}{x_i}$. Given land rents $p_{i,\lambda}$, each consumer's income is equal to $w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}$. The representative consumer's budget constraint is then given by

$$\int_0^1 p_{i,l}c_{i,l}dl + p_{i,h}h_i = w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}, \tag{1.3}$$

where $p_{i,l}$ represents the local equilibrium price of good $l \in [0,1]$ and $p_{i,h}$ represents the equilibrium price of housing.

Given i, then, the problem of the consumer is maximize (1.2) subject to (1.3).

1.1.2 Goods Firms

Firms may enter freely into the production of any good $k \in [0,1]$ in any location $i \in N$ with zero fixed cost. Let us assume for the moment, as we will later confirm, that the problem of the producers of each good in each location may be characterized in terms of a representative producer.

The suitability of each location i, for producing each good $k \in [0,1]$, at each point in time t, is determined by productivity shock $s_{i,k}(t)$, hereafter referred to as $s_{i,k}$, which is drawn independently across locations, goods, and time periods. Each location has two distributions of productivity shocks, one for agricultural goods, and one for non-agricultural goods. For agricultural goods, $k \in [0,A]$, $s_{i,k} \sim G_{i,a}(s)$ with Fréchet cumulative distribution function $G_{i,a}(.)$ being given by

$$G_{i,a}(s) = e^{-\lambda_i \alpha_i^{\chi} s^{-\chi}}, \tag{1.4}$$

for $\chi > 1$. For non-agricultural goods, $k \in (A,1]$, $s_{i,k} \sim G_{i,n}(s,t)$, with

$$G_{i,n}(s,t) = e^{-\lambda_i m_i(t)^{\chi} s^{-\chi}}, \tag{1.5}$$

where $m_i(t)$ represents the time-t stock of ideas in location i and will hereafter be referred to as m_i. It is necessary to factor land area into the expectations of these distributions, because fundamentally they are meant to represent spatial variation in suitability for producing different goods, and locations in this model are allotted different amounts of space. The properties of the Fréchet distribution mean that $G_{i,a}(.)$ and $G_{i,n}(.)$ can be derived by assuming that a shock for each good is drawn for each tiny piece of land from $G^*_{i,a}(s) = e^{-\alpha_i^{\chi} s^{-\chi}}$ and $G^*_{i,n}(s) = e^{-m_i^{\chi} s^{-\chi}}$, and that the best draws are used. $G_{i,a}(.)$ and $G_{i,n}(.)$ then reflect the maximum of draws from $G^*_{i,a}(.)$ and $G^*_{i,n}(.)$ across the λ_i units of land available in i.

After observing $s_{i,k}$, the first choice made by the representative producer of k in i is how much to innovate. By employing labor $b_{i,k,I}$ and land $l_{i,k,I}$, the firm is able to improve its own efficiency in the current period. Final efficiency is given by

$$\hat{s}_{i,k} = s_{i,k}\left(b_{i,k,I}^{\eta} l_{i,k,I}^{1-\eta}\right)^{\kappa}, \tag{1.6}$$

where $\eta, \kappa \in [0,1]$.

Then, taking $\hat{s}_{i,k}$ as given, each firm chooses the quantity of labor, land and intermediate inputs to employ in production. The quantity produced $q_{i,k}$ is determined according to

$$q_{i,k} = \hat{s}_{i,k} \left(b_{i,k}^{\eta} l_{i,k}^{1-\eta}\right)^{1-\sigma-\kappa} \left(\int_0^1 z_{i,k,l}^{\rho} dl\right)^{\frac{\sigma}{\rho}}, \tag{1.7}$$

where $b_{i,k}$, $l_{i,k}$ and $z_{i,k,l}$ for $l \in [0,1]$ represent the quantities of labor, land and intermediate inputs employed, and $\sigma \in [0, 1-\kappa]$ is a parameter. Note that the production function, including the investment in innovation, exhibits constant returns to scale overall. This allows the representative firm characterization, and, together with the assumption of free entry and zero fixed cost, implies that firms must earn zero profits in equilibrium.

As firms must earn zero profits in the end, the firm's profit maximization problem can be fully represented as one of cost minimization, taking prices and the market-clearing quantity $q_{i,k}$ as given. Formally, the problem of the firm is

$$\min_{b_{i,k,I}, b_{i,k}, l_{i,k,I}, l_{i,k}, z_{i,k,l}} \left\{ w_i \left(b_{i,k,I} + b_{i,k}\right) + p_{i,\lambda} \left(, l_{i,k,I} + l_{i,k}\right) + \int_0^1 p_{i,l} z_{i,k,l} dl \right\}, \tag{1.8}$$

subject to (1.7) and (1.6).

The zero-profit condition implies that in equilibrium all firms must have a cost of production inversely related to their efficiency shock and equal to $\frac{P_i}{s_{i,k}}$, where P_i is defined as the efficiency price of a unit of output in location i. When selling its output to a buyer in some location $j \in N$, zero profits implies that the price charged will be $\frac{P_i}{s_{i,k}\gamma_{ij}}$, just covering the costs of production and transport.

1.1.3 Housing Firms

Firms may also enter freely into the production of housing in any location with zero fixed cost. The representative housing firm employs a quantity of land $l_{i,h}$ and quantities of intermediate inputs $z_{i,h,l}$ for $l \in [0,1]$ to produce a quantity of housing H_i according to

$$H_i = \left(\int_0^1 z_{i,h,l}^{\rho} dl\right)^{\frac{\varphi}{\rho}} l_{i,h}^{1-\varphi}. \tag{1.9}$$

In equilibrium, the profits earned by this housing producer must be zero. Taking the market-clearing quantity of housing H_i as given, the problem of the housing firm is

$$\min_{l_{i,h}, z_{i,h,l}} \left\{ \int_0^1 p_{i,l} z_{i,h,l} dl + p_{i,\lambda} l_{i,h} \right\} \tag{1.10}$$

subject to (1.9).

1.1.4 Market Equilibrium

When considering equilibrium outcomes in this economy, the first thing we need to know is the vector of real incomes, u_i for $i \in N$, which will result in a single period from any given allocation of population and idea stocks. To that end, let us define a market equilibrium as follows.

Given resident populations x_i and idea stocks m_i, a market equilibrium is defined as prices for goods, land, and labor, production decisions by goods firms and housing firms, and consumption decisions by consumers, such that markets for goods, land and labor clear, and all decisions are optimal.

As is shown in appendix A.3.3, these equilibrium conditions imply that real income in location i depends on two key quantities. The first is population density, $\frac{x_i}{\lambda_i}$. The second is a measure of location i's trade access to highly productive locations, which we will call market access. Market access is defined as $\mathbb{M}_i \equiv \left[\int_0^1 \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl\right]^{\chi\frac{1-\rho}{\rho}}$. In equilibrium it is equal to the following weighted sum:

$$\mathbb{M}_i = B_M \left[A \sum_{j\in N} \left(\frac{P_i}{P_j}\right)^{\chi} \gamma_{ji}{}^{\chi}\alpha_j + (1-A) \sum_{j\in N} \left(\frac{P_i}{P_j}\right)^{\chi} \gamma_{ji}{}^{\chi} m_j\right]. \tag{1.11}$$

In the above equation, B_M represents a constant equal to $\Gamma\left(1-\frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}}$, where $\Gamma(.)$ denotes the gamma function. What (1.11) means is that market access is improved by having low transport-cost access to locations that have high agricultural potential, large stocks of ideas, and low costs of production.

Equilibrium real income as a function of population density and market access is given by the following:

$$u_i = B_u \left(\frac{\lambda_i}{x_i}\right)^{\nu_2} \mathbb{M}_i^{\nu_1}, \tag{1.12}$$

where

$$\nu_1 \equiv \frac{\alpha + (1-\alpha)\varphi}{\chi(1-\sigma)},$$

$$\nu_2 \equiv 1 - \eta[\alpha + (1-\alpha)\varphi],$$

and

$$B_u \equiv \alpha^{\alpha} \left(\sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\right)^{\alpha+\varphi(1-\alpha)} \left(\frac{\varphi(1-\eta)}{1-\varphi}\right)^{\varphi(1-\alpha)} \frac{(1-B_{g,\lambda})^{1-\alpha}\left[\eta B_{g,\lambda} + 1 - \eta\right]^{\alpha}}{B_{g,\lambda}^{\eta[\alpha+\varphi(1-\alpha)]}}.$$

and $B_{g,\lambda} = \frac{(1-\eta)(\alpha+\varphi(1-\alpha))}{(1-\varphi)(1-\alpha)+(1-\eta)(\alpha+\varphi(1-\alpha))}$ represents the constant equilibrium fraction of land dedicated to goods production.

1.1.5 Evolution of population

Population evolves over time through two processes: fertility and migration. Following Hansen and Prescott (2002), net fertility is assumed to be a simple function $f(u)$ of parents' real income. It is assumed that $f(u)$ satisfies two properties: First, that if real income is low enough, population growth is negative. Second, that as real income increases without bound, that fertility approaches a finite positive limit. Formally, these two conditions can be represented as $\lim_{u \to 0} = 0$, and $\lim_{u \to \infty} = \bar{u}$, for some $\bar{u} \geq 1$.

The quantity of consumers born in a location, as a function of the number of consumers who lived there the previous period, is given by

$$x_{i,b}(t) = x_i(t-1) f\left(u_i(t)\right) \tag{1.13}$$

Migration occurs as the result of consumer choices of where to live, given their place of birth. The properties of the Fréchet distribution allow the following characterization of the fraction of consumers who will choose to move from i to j:

$$l_{ji} = \frac{\vartheta_{ji}^{\varkappa} x_{i,b} u_i^{\varkappa}}{\sum_{k \in N} \vartheta_{jk}^{\varkappa} x_{k,b} u_k^{\varkappa}} \tag{1.14}$$

The number of consumers living in i as a function of the numbers of consumers born in every location is then equal to

$$x_i = \sum_{j \in N} l_{ji} x_{j,b} \tag{1.15}$$

Combining (1.13) and (1.15) yields the following law of motion for $x_i(t)$:

$$x_i(t) = \sum_{j \in N} l_{ji}(t) x_j(t-1) f\left(u_j(t)\right) \tag{1.16}$$

1.1.6 Evolution of technology

Technological progress happens as a result of the resources that firms spend in innovation. Each firm benefits privately from its innovative effort by through an immediate increase in productivity. As an externality, the labor $b_{i,k,I}$ and land $l_{i,k,I}$ that each firm dedicates to innovation leads to the discovery of a number of ideas equal in measure to $b_{i,k,I}^{\eta\phi} l_{i,k,I}^{1-\eta\phi}$. The parameter $\phi > 0$ determines whether returns to density of innovative activity are increasing, decreasing, or constant. Aggregating across firms, the total number of ideas discovered in location i at time t is given by

$$\hat{m}_i(t) = B_m x_i(t)^{\eta\phi} \lambda_i^{1-\eta\phi}, \tag{1.17}$$

where $B_m \equiv B_{g,\lambda}^{1-\eta\phi} \frac{\kappa}{1-\sigma}$. These ideas are added to the location-i stock of ideas at time $t+1$.

Each period, any idea already in the stock of ideas at the start of period t in location i has a probability $\bar{\theta}_{ij}$ of diffusing to each other location j. If the idea diffuses and is not already known in location j, then it is added to location j's stock of ideas at time $t+1$. The diffusion probabilities are determined as a simple function of transport costs, according to

$$\bar{\theta}_{ij} = \gamma_{ij}^{\zeta_d} \tag{1.18}$$

for $\zeta_d > 0$.

A particular idea, discovered in a particular location i, may arrive in another location j after only one period, or after two, three, or more periods. It may be transmitted directly, or it may be transmitted through an intermediate chain of other locations that learn the idea first. To model this process, let $\theta_{ij,s}$ for $s \in \{0,1,2,...\}$ represent the probability that an idea invented in i is known in j after s periods. In the period of its discovery an idea is known in its home location and not in any other, so $\theta_{ii,0} = 1$ and $\theta_{ij,0} = 0$ for $i \neq j$. For $s \geq 1$, $\theta_{ij,s}$ is determined by the following recursive process:

$$\theta_{ij,s} = 1 - \underbrace{\underbrace{(1-\theta_{ij,s-1})}_{\text{Pr not known at } s-1} \underbrace{\prod_{k \in N} \left(1 - \theta_{ik,s-1}\bar{\theta}_{kj}\right)}_{\text{Pr no arrival in period } s}}_{\text{Pr not known at } s}. \tag{1.19}$$

In the above expression, $1-\theta_{ij,s-1}$ is the probability that the idea has not already reached j before s periods have passed; $\theta_{ik,s-1}\bar{\theta}_{kj}$ is the probability of transmission from k to j during the current period; and $\prod_{k \in N} \left(1 - \theta_{ik,s-1}\bar{\theta}_{kj}\right)$ is the probability that no location transmits the idea to j during the current period. Clearly, as long as there exists some sequence of $m+2$ locations, $\{i, l_1, l_2, \ldots, l_{m-1}, l_m, j\}$, such that the product $\bar{\theta}_{il_1}\bar{\theta}_{l_1 l_2} \ldots \bar{\theta}_{l_{m-1}l_m}\bar{\theta}_{l_m j} > 0$, then $\lim_{s\to\infty} \theta_{ij,s} = 1$. In other words, as long as there is a path of finite distance, however long and indirect, from i to j, then all ideas discovered in i will eventually arrive in j.

Each period after its discovery, each idea faces a probability $\delta \in [0,1]$ of becoming obsolete and no longer contributing to the level of technology in any of the locations in which it is known. Thus, the time-t level of technology in location i, $m_i(t)$, can be expressed as the following function of the ideas that have been discovered in each location in each previous period:

$$\begin{aligned} m_i(t) &= \hat{m}_i(t) + \sum_{s=1}^{\infty}(1-\delta)^s \hat{m}_i(t-s) + \sum_{s=1}^{\infty}(1-\delta)^s \sum_{j \neq i} \theta_{ji,s}\hat{m}_j(t-s) \\ &= \sum_{s=0}^{\infty}(1-\delta)^s \sum_{j \in N} \theta_{ji,s}\hat{m}_j(t-s) \end{aligned} \tag{1.20}$$

Rearranging (1.20), it is also possible to write the following law of motion:

$$m_i(t) = (1-\delta)m_i(t-1) + \hat{m}_i(t) + \sum_{s=1}^{\infty}(1-\delta)^s \sum_{j \neq i} \left[\theta_{ji,s} - \theta_{ji,s-1}\right]\hat{m}_j(t-s) \tag{1.21}$$

1.2 The Long Run

In the previous section, I constructed a model in which flows of goods, ideas and people between locations drives the evolution of population and productivity over time. Now it is natural to ask–what is the behavior of this system over the long run? Will population and technology continue to grow indefinitely, or will they stagnate? If the processes of population growth, migration, innovation and diffusion continue indefinitely in the absence of any changes to the transport network, where will people live, and how productive will they be?

As I will show in this section, each of these questions is liable to an analytical answer. In the long run, the economy must converge to a state in which the growth rates of population and technology are non-negative and the same in all locations, and in which all locations are populated. This state may be a Malthusian steady state in which growth rates are equal to zero, or an asymptotic balanced growth path with strictly positive growth. Which of these states comes to be depends on the overall level of transport costs, which are be summarized by a simple network statistic. In all cases, population and productivity agglomerate in central locations, according to a definition of centrality with clear roots in the network literature.

As a first step towards formalizing these statements, let us define the two possible types of long run states.

Definition 1 *A* ***Malthusian steady state*** *is a dynamic spatial equilibrium such that population $x_i(t) = x_i$ and idea stocks $m_i(t) = m_i$ in all locations $i \in N$ are both constant over time.*

Definition 2 *A* ***balanced growth path*** *is a dynamic spatial equilibrium such that population $x_i(t) = (1+g_x)^t x_i$ and manufacturing potential $m_i(t) = (1+g_m)^t m_i$ in all locations $i \in N$ grow at constant instantaneous rates $g_x > 0$ and $g_m > 0$, respectively.*

1.2.1 Real income under balanced growth

Labor is a key ingredient in innovation, and innovation drives non-agricultural productivity. Therefore it is no surprise that in the long run technology levels, as well as levels of real income, are a function of the distribution of population. If the growth rate of technology is constant, then the long run idea stock in location i is given by the following:

$$\begin{aligned} m_i(t)^{\frac{1}{\psi}} &= \sum_{j \in N} \sum_{s=0}^{\infty} (1-\delta)^s \theta_{ji,s} \hat{m}_j(t-s) \\ &= \sum_{j \in N} \hat{m}_j(t) \sum_{s=0}^{\infty} \left(\frac{1-\delta}{1+g_m} \right)^s \theta_{ji,s} \\ &= B_{m,2} \sum_{j \in N} \tilde{\theta}_{ji}^{\{g_m\}} x_j(t)^{\eta\phi} \lambda_j^{1-\eta\phi}, \end{aligned}$$

where $\tilde{\theta}_{ji}^{\{g_m\}} \equiv \frac{\delta+g_m}{1+g_m} \sum_{s=0}^{\infty} \left(\frac{1-\delta}{1+g_m} \right)^s \theta_{ji,s}$, and $B_{m,2} \equiv \frac{1+g_m}{\delta+g_m} B_m$. Note that the definition of $\tilde{\theta}_{ji}^{\{g_m\}}$ implies that it takes values only between zero and 1, as $\tilde{\theta}_{ii}^{\{g_m\}} = \frac{\delta+g_m}{1+g_m} \sum_{s=0}^{\infty} \left(\frac{1-\delta}{1+g_m} \right)^s = 1$ for $\forall i \in N$.

From this expression we can infer that if the amount of available labor (population) does not grow, technology will not grow, either. We can also infer that under balanced growth it must hold that $1 + g_m = (1 + g_x)^{\eta\phi\psi}$, and that

$$m_i^{\frac{1}{\psi}} = B_{m,2} \sum_{j \in N} \tilde{\theta}_{ji}^{\{g_m\}} x_j^{\eta\phi} \lambda_j^{1-\eta\phi} \tag{1.22}$$

Incorporating this information into a calculation of real income, we can apply equation (1.22) to (1.11) and (1.12) and write the following expression:

$$u_i \frac{(1+g_u)^t}{(1+g_x)^{\frac{\chi\phi\psi\nu_1}{\nu_2}t}} =$$
$$= B_{u,2}\left(\frac{\lambda_i}{x_i}\right)^{\nu_2}\left\{\sum_{j\in N}\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}^{\chi}\left[\frac{A\alpha_j^{\chi}}{(1+g_x)^{\eta\phi\psi t}}+(1-A)\left(B_{m,2}\sum_{k\in N}\tilde{\theta}_{kj}^{\{g_m\}}x_k^{\eta\phi}\lambda_k^{1-\eta\phi}\right)^{\chi\psi}\right]\right\}^{\nu_1},$$

where $B_{u,2} \equiv B_u\Gamma\left(1-\frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}}$ and g_u is defined as the constant growth rate of real income. From this expression we can infer that under balanced growth, $1+g_u = (1+g_m)^{\chi\psi\nu_1}$ must hold. We can also infer that that if growth rates are strictly positive, the contribution of agricultural potential α_i for $i \in N$ to real income approaches zero. Therefore, in a steady state, real income is given by the following expression:

$$u_i = B_{u,2}\left(\frac{\lambda_i}{x_i}\right)^{\nu_2}\left\{\sum_{j\in N}\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}^{\chi}\left[A\alpha_j^{\chi}+(1-A)\left(B_{m,2}\sum_{k\in N}\tilde{\theta}_{kj}^{\{g_m\}}x_k^{\eta\phi}\lambda_k^{1-\eta\phi}\right)^{\chi\psi}\right]\right\}^{\nu_1}. \tag{1.23}$$

In a balanced growth path with $g_m > 0$, real income is given by the following expression:

$$u_i = B_{u,3}\left(\frac{\lambda_i}{x_i}\right)^{\nu_2}\left\{\sum_{j\in N}\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}^{\chi}\left(\sum_{k\in N}\tilde{\theta}_{kj}^{\{g_m\}}x_k^{\eta\phi}\lambda_k^{1-\eta\phi}\right)^{\chi\psi}\right\}^{\nu_1}, \tag{1.24}$$

where $B_{u,3} \equiv B_{u,2}(1-A)^{\nu_2}B_{m,2}^{\chi\psi\nu_1}$.

The interpretation of these expressions is straightforward. In the long run, real income in each location depends first of all on the number of people living in that location (x_i), and the amount of land available to divide between them for use in housing and production (λ_i). Second of all, it depends on the number of people living in every other location (x_k), because a certain portion of those people are working every period to come up with new productivity-enhancing ideas. The ideas that are discovered in each location k accumulate not only in i, but also in each of location i's trading partners, at certain rates ($\tilde{\theta}_{kj}^{\{g_m\}}$). The resulting stocks of ideas in each of these trading partners j, along with the transport cost γ_{ji} and the equilibrium ratio of the costs of production $\frac{P_i}{P_j}$, determines the contribution of this trading partner to location i's real income.

If the world is in a Malthusian steady state, trade access to agriculturally fertile locations also contributes to real income. If the world is on a positive growth path, the contribution of agriculture in the long run is negligible relative to that of the non-agricultural sector.

1.2.2 The Network of Utility Spillovers

I will now show how the distributions under each of these long-run configurations, and the conditions for convergence to each of them, can be characterized in terms of a network of utility spillovers. For this end, it is convenient to state the system of equations represented by (1.23) and (1.24) using matrix notation. All of the analysis that follows will be conducted under the assumption that $\psi = \frac{1}{\chi}$.

Let $\mathbf{I}$ represent an n-dimensional identity matrix, and let us define $\boldsymbol{\alpha}$ as the $n \times 1$ vector such that the i^{th} element is equal to α_i; $\mathbf{x}^{\{k\}}$ as the $n \times 1$ vector such that the i^{th} element is equal to x_i^k; $\boldsymbol{\Lambda}$ as the $n \times n$ diagonal matrix such that the ii^{th} element is equal to λ_i; $\boldsymbol{\Xi}$ as the $n \times n$ diagonal matrix such that the ii^{th} element is equal to ξ_i; $\mathbf{G}$ as the $n \times n$ matrix whose ij^{th} element is equal to $\left(\frac{P_i}{P_j}\gamma_{ij}\right)^{\chi}$; $\boldsymbol{\Theta}^{\{\varsigma_m\}}$ as the $n \times n$ matrix such that the ij^{th} element is equal to $\tilde{\theta}_{ij}(\varsigma_m)$; and $\mathbf{U}$ as the $n \times n$ diagonal matrix such that the ii^{th} element is equal to u_i.

It is also convenient to define $\bar{u} \equiv \max\limits_{i \in N} u_i$, and $\tilde{u}_i \equiv \frac{u_i}{\bar{u}}$, and $\tilde{\mathbf{U}}$ as the $n \times n$ diagonal matrix such that the ii^{th} element equals $\tilde{u}_i$, so that $\mathbf{U} = \bar{u}\tilde{\mathbf{U}}$.

Now, (1.23) can be stated as

$$\bar{u}^{\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{\frac{1}{\nu_1}} \mathbf{x}^{\{\frac{\nu_2}{\nu_1}\}} = A\psi_{u,2}^{\frac{1}{\nu_1}} \boldsymbol{\Lambda}^{\frac{\nu_2}{\nu_1}} \mathbf{G}' \boldsymbol{\alpha} + (1-A)\psi_{u,2}^{\frac{1}{\nu_1}} \psi_{m,2} \boldsymbol{\Lambda}^{\frac{\nu_2}{\nu_1}} \mathbf{G}' \boldsymbol{\Theta}^{\{0\}\prime} \boldsymbol{\Xi} \boldsymbol{\Lambda}^{(1-\eta)\phi} \mathbf{x}^{\{\eta\phi\}}$$

and (1.24) can be stated as

$$\bar{u}^{\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{\frac{1}{\nu_1}} \mathbf{x}^{\{\frac{\nu_2}{\nu_1}\}} = \psi_{u,3} \boldsymbol{\Lambda}^{\frac{\nu_2}{\nu_1}} \mathbf{G}' \boldsymbol{\Theta}^{\{\varsigma_m\}\prime} \boldsymbol{\Xi} \boldsymbol{\Lambda}^{(1-\eta)\phi} \mathbf{x}^{\{\eta\phi\}}$$

There emerges from both of the above equations a key matrix, $\boldsymbol{\Omega}$, which may be thought of as the adjacency matrix of the network a utility spillovers:

$$\boldsymbol{\Omega} \equiv \boldsymbol{\Lambda}^{\frac{\nu_2}{\nu_1}} \mathbf{G}' \boldsymbol{\Theta}^{\{0\}\prime} \boldsymbol{\Xi} \boldsymbol{\Lambda}^{(1-\eta)\phi}.$$

The ij^{th} element of this matrix is equal to

$$\omega_{ij} = \lambda_i^{\frac{\nu_2}{\nu_1}} \left[\sum_{k \in N} g_{ki} \tilde{\theta}_{jk}(0)\right] \xi_j \lambda_j^{(1-\eta)\phi},$$

and represents the extent to which consumers in location j contribute to the utility of consumers in location i in the long run. This depends, naturally, on the amount of land available for productive use in location j, and on the product, for each location $k \in N$, of location j's contribution to that location's technology level through diffusion, and location i's trade connection to that location. In other words, the benefit that location i gets from population in location j depends on technology spillovers from j, not only to i directly, but also to each of i's trading partners.

Let the largest eigenvalue of this matrix be denoted π. π is a natural statistic to summarize the world's long-run *global potential*. Not surprisingly, this productive potential is strictly increasing in the land endowment of each location, λ_i, and strictly decreasing in the bilateral transport cost between each pair of locations, $\frac{1}{\gamma_{ij}}$. As we will see in the theorem that follows, the level of π is crucial to determining whether the world stagnates or achieves sustained growth.

1.2.3 Conditions leading to stagnation or sustained growth

Theorem 1 *In the environment that has been described, given a vector of starting conditions* $\mathbf{s} \in \mathbb{R}_{+}^{n^3(n-1)}$ *containing population* x_i *for* $i \in N$*, locally-invented ideas* $m_{i,I}$ *for* $i \in N$*, and diffused ideas* $m_{ij,D}$ *for* $i \in N$*,* $j \neq i$*, such that population* x_i *in at least one location is strictly positive:*

A. If $\frac{\nu_2}{\nu_1} > \eta\phi$*, the world will converge to a unique Malthusian steady state in which every location* $i \in N$ *has positive population.*

B. If $\frac{\nu_2}{\nu_1} = \eta\phi$*, then there exists a critical level of global productive potential* π^**, such that*

  i. if $\pi \leq \pi^*$*, the world will converge to a unique Malthusian steady state with positive population in each location* $i \in N$*, and*

  ii. if $\pi > \pi^*$*, the world will asymptotically approximate a unique balanced growth path with positive population in each location* $i \in N$*.*

C. If $\frac{\nu_2}{\nu_1} < \eta\phi$*, then there exists a critical level of global productive potential* π^* *and a critical frontier of starting conditions defined by a continuous, increasing function* $z(.)$ *mapping from* $\mathbb{R}_{+}^{n^3(n-1)}$ *to* $\mathbb{R}$*, such that*

  i. if $\pi \leq \pi^*$ *and* $z(\mathbf{s}) \leq 0$*, then the world will converge to a Malthusian steady state with positive population in each location* $i \in N$*, which may or may not be unique,*

  ii. if either $\pi > \pi^*$ *or* $z(\mathbf{s}) > 0$*, the world will asymptotically approximate a balanced growth path with positive population in each location* $i \in N$*, which may or may not be unique.*

Proof: See Appendix A.3.6.

What Theorem 1 states is that (a) if dispersion forces are stronger than agglomeration forces, sustained growth is not possible in the long run, (b) if the forces of agglomeration are equally balanced with the forces of dispersion, then sustained growth will occur as long as global productive potential is high enough, and (c) if agglomeration forces are stronger than dispersion forces, then sustained growth will occur if either global productive potential is high enough, or if starting levels of population and technology are high enough. This suggests at least three types of change, exogenous to the model developed here, that could push a system which is in a Malthusian steady state into a path towards sustained growth:

1. a major technological breakthrough that raises the level of technology in a discrete jump, leading to an increase in $z(\mathbf{s})$

2. the creation of additional land, leading to an increase in π

3. a reduction in transport costs, leading to an increase in π

So, if technological progress is essentially incremental, and if land-recovery efforts like those undertaken in the Netherlands are not a major force in economic growth globally, this leaves option number three. Indeed, a fall in transportation costs is one of the key economic facts of the past several centuries. Interpreted through the lens of the framework developed here, then, these reductions may have been a necessary condition for the take-off in global growth that has occurred.

1.3 Bringing the Model to the Data

To bring the model to the data, I divide the world into 3° × 3° quadrangles. I discard all quadrangles that do not contain land, and all of the quadrangles in Antarctica. This leaves 2,249 habitable locations. Figure 1.1 shows the 3 degree grid. It also shows the extents of 10 major regions, which play no role in the model or its computation, but are used to aggregate results up for comparison.

1.3.1 Agricultural potential from agricultural characteristics

I assign agricultural potential to each location based on the index of agricultural suitability provided by Ramankutty et al. (2002). To ensure that the index I use reflects only exogenous climate and soil characteristics which are stable over time, I regress the Ramankutty index on three variables which arguably do have these properties, and use the predicted values as my index of agricultural potential.

These three variables are the Normalized Difference Vegetation Index (NDVI), soil nutrient availability, and soil workability. NDVI is a measure of how "green" a location is when observed from a satellite.[4] This measure captures how favorable are basic climatic conditions, such as water availability and temperature, for the growth of vegetation.[5] I use indexes of soil nutrient availability and soil workability calculated by Fischer, et al (2008) for the United Nations Food and Agriculture Organization.

I specify a log-log relationship between these variables and the Ramankutty index, with a quartic polynomial in NDVI, quadratic terms for each of the soil quality measures, and a full set of interaction terms. Let the predicted values resulting from this projection be designated $\hat{a}_i$. Agricultural potential is then assigned according to

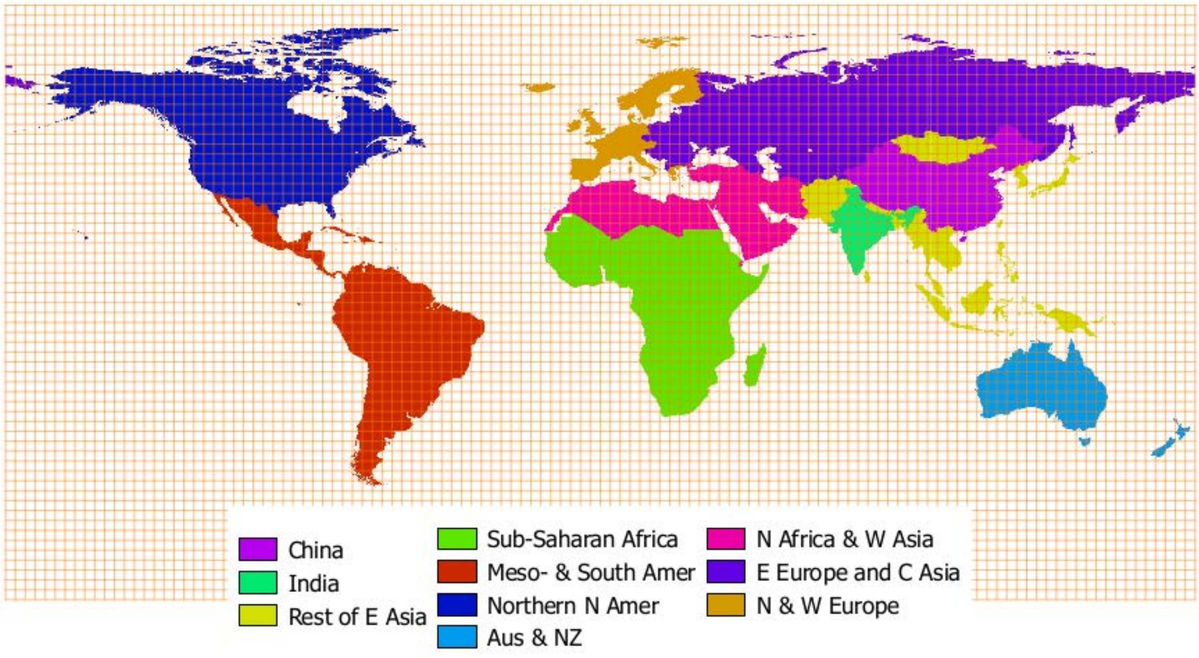
$$\alpha_i = \zeta_a \hat{a}_i,$$

where $\zeta_a > 0$ is a scale parameter which is calibrated to target the agricultural labor share in Europe in 1000 CE.

[4]Monthly observations for NDVI from February 2000 through January 2016 were taken from NASA LP DAAC (2016). The measure analyzed is the mean NDVI for each location over this entire time period.

[5]An alternative measure of water availability would be average rainfall. This measure has one key drawback, however: it cannot account for the lushness of certain river valleys, such as the Nile river delta, which in spite of having very little rainfall, are very "green," highly productive agriculturally, and very densely populated.

Figure 1.1: Major regions and 3° resolution grid



1.3.2 Transport costs from topography

I take information on the location of land, lakes, rivers and coastlines from the Natural Earth database. Navigable rivers are classified as those with a scalerank of 5 or lower in the Natural Earth data set, and this set is further pared by researching the navigability of the individual river systems that remain using a variety of sources, mimicking the methodology of Henderson et al (2016). I use Nunn and Puga's (2012) calculations of the Terrain Ruggedness Index proposed by Riley, DeGloria, and Elliot (1999). I use mean wave height calculations from Barstow, et al (2009).

Transportation costs between each pair of habitable locations may be carried out using land transport, river transport, sea transport, or a combination of all three. Transport is modeled as taking place on a network in which there are land, river and sea nodes. In each grid square, there exists one land node for each disjoint body of land which is at least partly inside the square, one river node for each navigable river system which is at least partly inside the square, and one sea node for each disjoint body of water that is at least partially inside the square. Each land node is directly connected to any land nodes in the eight adjacent grid squares which belong to the same body of land, and any river or sea nodes in the same grid square. Similarly, each river and sea node is directly connected to any river node, or sea node, respectively, in the eight adjacent grid squares, and any sea or land node, or river or land node, in the same grid square.

Land-land and sea-sea connections between two grid squares i and j, $i \neq j$, each face a mode-specific per-unit effective distance, $\tau_L(t)$ or $\tau_S(t)$ respectively, which is multiplied by the great circle distance d_{ij} between the centers (centroids) of the two grid squares (latitude-longitude quadrangles) to obtain the effective distance between the two nodes.[6]

[6] All distances are calculated taking the curvature of the Earth into account.

River-river connections face a per-unit effective distance of $\tau_S(1+\tau_V)$, where τ_V represents the increased cost which may be incurred due to the special difficulties of river navigation, relative to navigation on calm seas. Let the arc between the centers of squares i and j be divided into two segments, one, of length d_{ij}^i, running from the center of i to the border between the squares, and a second of length d_{ij}^j, running from the center of j to the border between the squares.[7] The effective distance of land-land connections is also multiplied by $1 + \tau_R \frac{d_{ij}^i \mathfrak{r}_i + d_{ij}^j \mathfrak{r}_j}{d_{ij}}$, where $\mathfrak{r}_i$ and and $\mathfrak{r}_j$ represent the average ruggedness of the terrain in grid squares i and j, respectively.[8] The effective distance of water-water connections $1 + \tau_W \frac{d_{ij}^i \mathfrak{r}_i^w + d_{ij}^j \mathfrak{r}_j^w}{d_{ij}}$, where $\mathfrak{r}_i^w$ and $\mathfrak{r}_j^w$ are indicator functions taking a value of 1 if the seas are "rough" in square i or j, respectively, and 0 otherwise. Seas are defined as being "rough" in a given square if mean significant wave heights in that square are greater than 1.5 meters.[9] The effective distance of land-river and land-sea connections, in either direction, is equal to the transshipment cost τ_T.

If a grid square has one land node, then the effective distances faced by that land node are those also faced by the habitable location in that grid square. If there is more than one land node in a grid square, the effective distances faced by the land node are equal to the arithmetic means of the effective distances faced by the various nodes.

The effective distance between each pair of habitable locations i and j, $\tau_{ij}(t)$, is then equal to the least-cost path between them through the network.[10] The inverse iceberg transport cost γ_{ij} is then given by $\gamma_{ij}(t) = e^{\tau_{ij}(t)}$, following Allen and Arkolakis (2014).

Given initial levels $\tau_L(0)$ and $\tau_S(0)$, let the basic cost of transport over land and water fall at constant rates ς_L and ς_S, such that

$$\tau_k(t) = (1 - \varsigma_k)^t \tau_k(0) \tag{1.25}$$

for $k \in \{L, S\}$ and $t \in \{0, 1, 2, ..., T\}$.

1.3.3 Net fertility

It is assumed that annual log net fertility is related to real GDP per capita, $\tilde{u}_i$, according to the following relation:

$$\begin{aligned} \log \tilde{f}_i(\tilde{u}_i) = \Big\{ & \left(1 + e^{\zeta_{f,0} + \zeta_{f,1}\tilde{u}_i}\right)^{-1} \zeta_{f,2} + \left[1 - \left(1 + e^{\zeta_{f,0} + \zeta_{f,1}\tilde{u}_i}\right)^{-1}\right] \zeta_{f,3} \\ & -2\left(1 + e^{\zeta_{f,4}\tilde{u}_i}\right)^{-1} (.5 - \zeta_{f,5}) - \zeta_{f,5} \Big\}. \end{aligned} \tag{1.26}$$

Real GDP per capita is assumed to correspond to utility according to $\tilde{u}_i = \tilde{\zeta}_f u_i$, where $\tilde{\zeta}_f \geq 0$ is a scalar multiplier. Parameters $\zeta_{f,k}$ for $k \in \{0, 1, ..., 5\}$ are estimated using data

[7] Due to the curvature of the globe, these two segments will never be exactly equal in length, as they would be if they were connecting centroids of true squares on a plane. Also note that it is a property of the longitude-latitude quadrangle grid that the arc between the centroids of two quadrangles that are adjacent diagonally will always pass through the point where the two corners of the quadrangles meet; so the arc is contained completely within the two quadrangles and does not pass through a third.

[8] I use Nunn and Puga's (2012) calculations of the Terrain Ruggedness Index proposed by Riley, DeGloria, and Elliot (1999).

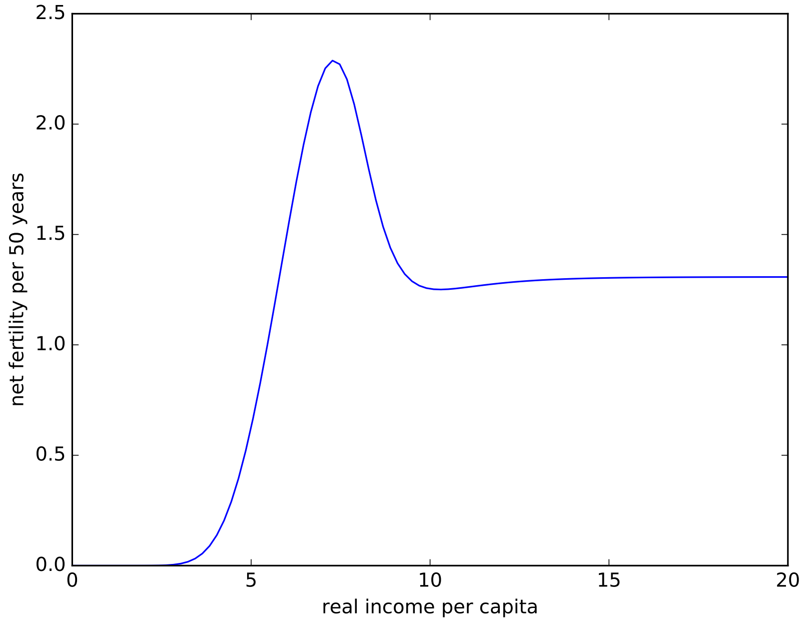
[9] I use mean wave height calculations from Barstow, et al (2009).

[10] I calculate least-cost paths using SciPy's highly-optimized implementation of Dijkstra's algorithm.

Table 1.1: Estimated Parameters for Fertility Process

$\zeta_{f,0}$	$\zeta_{f,1}$	$\zeta_{f,2}$	$\zeta_{f,3}$	$\zeta_{f,4}$	$\zeta_{f,5}$
-15.71	1.91	.03726	.01357	0.64	.00821

Figure 1.2: Net fertility as a function of real income



on rates of natural increase (birth rates minus death rates) and real GDP per capita borrowed from Delventhal, Fernández-Villaverde and Guner (2017). Table 1.1 shows the estimated parameters. Log net fertility per 50-year model period is then obtained by multiplying $\log \tilde{f}_i$ by 50. Figure 1.2 graphs the resulting function.

1.4 The world in 1000 CE

I conduct a quantitative exercise in two steps. First, I calibrate the model so that in the year 1000 CE the world is in a Malthusian steady state. Then, I reduce transport costs according to a pattern consistent with the existing historical evidence, and track the endogenous evolution of population and income per capita in 50-year periods until 2000 CE.

1.4.1 Calibration

Tables 1.2, 1.3 and 1.4 provide an overview of the values I choose for the parameters of the model and why. Some are set based on evidence provided by previous estimations or historical studies, and others are calibrated so that a moment of the model will exactly match a specific target which is independent of model outcomes. A small number of parameters are not tied down in either of these ways, and are set to achieve a better overall fit with the 2,249 population density moments of the initial steady state, or to achieve a better fit with qualitative features of the transition until 2000 CE.

All of the parameters that have the biggest impact on the fit of the model with the distribution of population density in 1000 CE are tied down by one of the first two methods. Those that remain to target this distribution explicitly are of secondary importance. I will now discuss each of these parameters in turn. The scale parameter on real income $\bar{\zeta}_f$, shown in Table 1.3, determines how real income in the model translates into fertility. In principle it would be possible to calibrate this parameter so that the model matched total world population in 1000 CE perfectly. The exact level of total world population in 1000 CE is, however, not known with great precision, so it makes more sense to allow this parameter to minimize the sum of squared errors between the model distribution of population density and the data.

The remaining parameters in this group are all initial transport cost parameters, shown in Table 1.4. The New World penalty, τ_{NW}, does not have a great impact on the Old World distribution of population, but it does improve model fit overall by reducing overall population density in the New World to close to the historical pre-Columbian levels. As discussed in the previous section, all indications are that New World regions lacked important transport technologies such as pack animals and sailing ships that were available throughout the Old World at this time. The penalty on rough and open seas seems to be rather well-identified, improving fit significantly when a high value is assigned to it. The penalties for traveling over rough terrain or over permafrost seem to be relatively weakly identified, though they do improve overall fit slightly when they take positive, but not very large, values.

I will now discuss each of the other parameters in the initial 1000 CE calibration. The first group of these parameters are shown in Table 1.2. I set $\alpha = 0.75$ according to evidence provided by Davis & Ortalo-Magné (2011), so that the share of income that consumers spend on housing is equal to 25%. By setting $\eta = 0.8$ and $\sigma = 0.2$, the land share in production is set to 16% and the intermediate input share is set to 20%, consistent with evidence provided by Desmet and Rappaport (2015) and Vandenbroucke (2008). Setting $\rho = 0.75$ implies an elasticity of substitution between goods of 4, consistent with the estimation of Bernard et al. (2003). Setting the elasticity of trade to distance $\chi = 6.5$ is consistent with evidence provided by Simonovska and Waugh (2014). Setting $\varphi = 0.5$ implies a land share in housing production consistent with Albouy and Ehrlich's (2017) study. My source for the value of κ, the elasticity of TFP to innovation effort, is Desmet and Rossi-Hansberg (2015). I set $\kappa = 0.5$.

The second group of this parameters is shown in Table 1.3. The elasticity of diffusion probability to distance, ζ_d, is calibrated so that the expected diffusion time from Baghdad to Pisa, Italy is 350 years. These two points and this length of time are chosen with reference to the diffusion of Indian numerals from the Middle East to Western Europe during the Middle Ages. In 825 CE Al-Khwarizmi, namesake of the word "algorithm", published a treatise on the use of Indian numerals. Knowledge of this method of numerical representation had recently spread to Al-Khwarizmi's city, Baghdad, from its place of origin in the Indian sub-continent.[11] In 1202 CE Fibonacci (of the "Fibonacci sequence") published

[11] Another mathematician, Al-Kindi, is known to have published a treatise on the same topic in either Baghdad or Basra in 830 CE.

Table 1.2: Calibration, Technology and preferences

(Parameters taken from the literature)

Par.	Par. Value	Target	Target value/source
α	0.75	housing expenditure share equal to 25%	*Davis & Ortalo-Magné (2011)*
η	0.8	land share in production equal to 16%	*Desmet & Rappaport (2015)*
σ	0.2	intermediate input share equal to 20%	20% *Vandenbroucke (2008)*
ρ	0.75	elast. of subst. btw. goods equal to 4	4 *Bernard et al. (2003)*
χ	6.5	trade elasticity to distance	*Simonovska & Waugh (2014)*
φ	0.5	land share, housing prod.	*Albouy & Ehrlich (2017)*
κ	0.5	elast. of TFP to innov.	*Desmet & Rossi-Hansberg (2015)*

his treatise *Liber Abaci*, the first known work by a Western mathematician comparing what Fibonacci now dubbed "Arabic numerals" to the Roman system of representation, and describing their use in performing calculations.[12]

The elasticity of migration to distance $\zeta_{m,1}$, is set so that in an idealized flat, homogeneous, endless plain in a steady state, the fraction of residents living at any given point who were born more than 50 kilometers away is equal to 15%. This is consistent with evidence on migration in rural 14th Century Nottinghamshire compiled by Whyte (2000). The scale parameter on agricultural potential ζ_a, is set so that the agriculture share of employment in Europe in 1000 CE is equal to 85%, consistent with evidence on Medieval European agriculture shares complied by Allen (2000). I normalize the elasticity of idea creation to the density of innovation effort ϕ, to equal 1, and set the elasticity of the effective technology level to the stock of ideas ψ equal to 2.63, which implies a balanced growth path income per capita will grow twice as fast as population. This is consistent with the data on population and income per capita growth in the United States from 1960 to 2010.

The last group of parameters is shown in Table 1.4. I set τ_L so that the price change per 111 kilometers (1° latitude) is 8%, which is near the middle of the range of price to distance elasticities that Masschaele (1993) finds for wheat being transported over land in

[12]The very first known reference to Indian numerals in Western Europe is contained in the *Codex Vigilanus* compiled by monks in Abelda de Iregua, Spain around 976 CE. I use the timing implied by the publication of *Liber Abaci* because then the event in Baghdad and the event in Pisa are like to like: both are treatises written by a well-known mathematician fully explaining the subject. Presumably knowledge of Indian numerals also existed in the Islamic world in a more obscure way for some decades or centuries before the publication of Al-Khwarizmi's treatise.

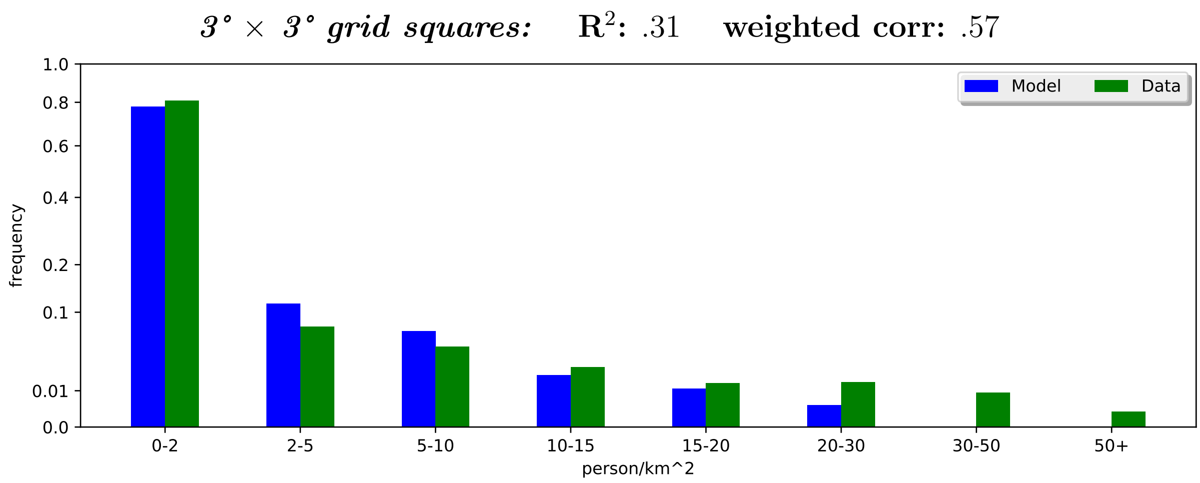
Table 1.3: Calibration: Diffusion & migration

Par.	Par. Value	Target	Source
ϕ	1	normalization	–
ζ_d	41.8	1000 CE expected diffusion time from Baghdad to Pisa equal to 350 years (diffusion time of Indian numerals)	*Devlin (2011) & Berggren (1986)*
$\zeta_{m,1}$	31.7	% of residents in idealized steady state from > 50km distant equal to 15%, as in migration in 14th C. Nottinghamshire	*Whyte (2000)*
ψ	2.63	BGP ratio of pop./income growth equal to 2, as in U.S. 1960-2010	*Maddison 2010 dataset*
ζ_a	8.87	1000 CE agriculture labor share in Europe equal to 85%	*Allen (2000)*
$\bar{\zeta}_f$	.1	1000 CE pop. densities	–
$\zeta_{m,0}$	0.1	evolution of population, 1000-2000 CE	–
ω	0.3	evolution of population, 1000-2000 CE	–

Table 1.4: Calibration, Initial Transport Costs

Par.	Par. Value	Target	Source
τ_L	.08	increase in wheat price of 8% per 111km in 14$^{\text{th}}$ C. Engl.	*Masschaele (1993)*
τ_S	$\frac{\tau_L}{8}$	ratio of coastal waters to land transport cost in 14$^{\text{th}}$ C. Engl.	*Masschaele (1993)*
τ_V	$2\tau_S$	ratio of river to coastal waters transport costs in 14$^{\text{th}}$ C. Engl.	*Masschaele (1993)*
τ_T	$1.47\tau_V$	ratio of transhipment cost per ton to river transport cost to move 1 ton 111 km in 19$^{\text{th}}$ U.S.	*Fogel (1962)*
τ_W	15	1000 CE pop. densities	–
τ_R	1	1000 CE pop. densities	–
τ_F	1	1000 CE pop. densities	–
τ_{NW}	24	1000 CE pop. densities	–

Figure 1.3: Distribution across population density levels in 1000 CE, model and data



14th Century England. Masschaele (1993) also finds a ratio of the average land transport cost to the average coastal waters transport cost of 8 to 1, and an average ratio of the river transport cost to coastal waters transport cost of 2 to 1; I use these numbers as is. Fogel (1962) estimates that the cost per ton of loading or unloading goods from a boat is 1.47 times the cost of transporting the same ton of goods on a river for 111 kilometers; I use this number as well.

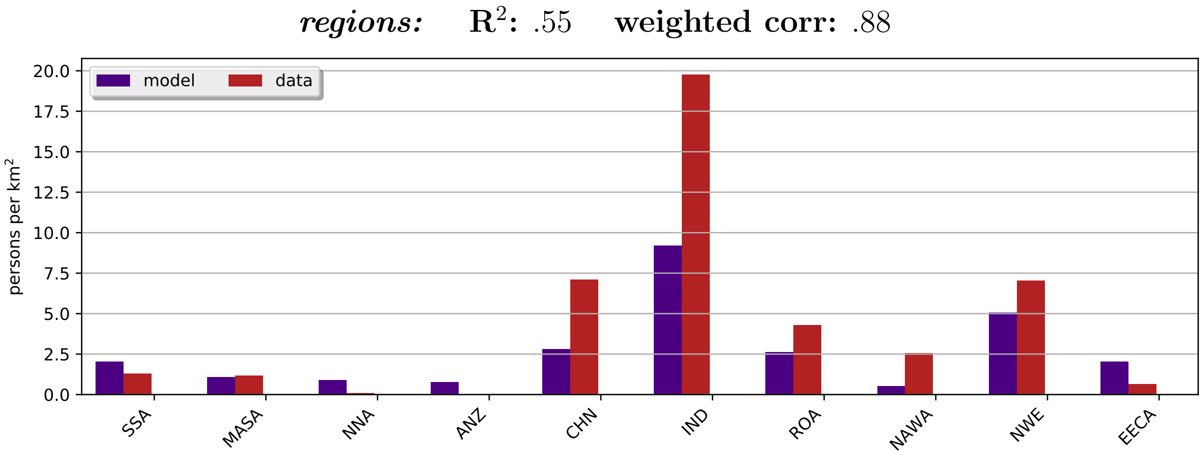
The parameters $\psi_{m,0}$ and ω, shown in Table 1.3, while they do have some effect on the distribution of population in 1000 CE, are set to match some qualitative features of the evolution of population between 1000 and 2000 CE and are discussed in Section 1.5.

1.4.2 Results, 1000 CE

The overall fit of the model with the data in 1000 CE is summarized in Figures 1.3 and 1.4. At the level of 3° by 3° quadrangles, the model is able to to account well for the distribution of locations across population density levels, though it is unable to generate the handful of locations with very high density which exist in the data. The model is able to do a good job accounting for which specific locations have low and high density as well, accounting for 31% of the overall variation.

As can be seen in Figure 1.4, the model also accounts well for which of the 10 major regions are densely and which are not as densely populated in 1000 CE. In the model, as in the data, India, China and Europe are the three most densely populated places in the world. The model is not able to quite match the same level of density as existed in India and China, in part because of its inability to generate very high density locations. Overall, it is able to account for most of the variation between these major regions–55%. The interpretation of this result is that agricultural potential and access to water transport, taken together, are able to account well for which regions were more and which were less developed in 1000 CE.

Figure 1.4: Mean population density of 10 major regions in 1000 CE, model and data



1.5 Falling Transport Costs

The second step of the quantitative exercise is to reduce transport costs according to a specified pattern and simulate the model until 2000 CE. This is done in two phases, as shown in Figure 1.5. First, between 1000 CE and 1500 CE, reduce water and land transport costs at constant rates, imposing a large reduction in water transport costs, and a much smaller reduction in land transport costs. Also over this period, the large penalty on traveling far from the coast or over rough seas is gradually removed. This is consistent with the broad pattern which has been found by Masschaele (1993) and others: that prior to the development of railroads, improvements in water transport were much more significant than any improvements in land transport. It is also consistent with the well-known developments in navigation technology over this period which culminated in the first cross-Atlantic voyages and the first circumnavigation of the globe.

From 1500 CE until 1750 CE there is a pause in the reduction of transport costs. Then from 1750 to 2000 CE transport costs are again reduced at a steady rate. This second

Figure 1.5: Falling transport costs

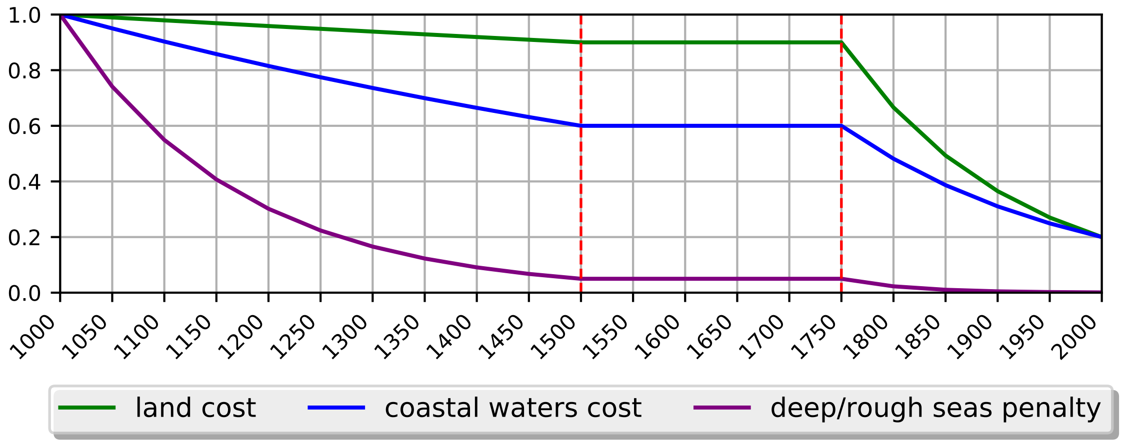


Table 1.5: Transport Cost Reductions

Par.	% in 1500 CE	% in 1750 CE	% in 2000 CE
τ_L	90%	90%	20%
τ_S	60%	60%	20%
τ_W	5%	5%	0.1%

phase of reductions is more land-biased than the first, to reflect the importance of land transport developments such as railroads and the automobile. The exact magnitudes of all of these transport cost reductions are chosen to approximate the qualitative features of the evolution of population and income between 1000 and 2000 CE. These values are shown in Table 1.5.

In addition to the aforementioned transport cost reductions, the penalty on transport in the New World, τ_{NW}, is removed linearly between 1450 and 1600 CE, reflecting the discovery of the Americas and Australia by Old World explorers and the spread of Old World transport technologies across the New World.

Two parameters from Table 1.3, $\psi_{m,0}$ and ω, are calibrated to improve the model fit with qualitative features of the evolution of population between 1000 CE and 2000 CE. $\psi_{m,0}$, which represents the inverse of the home bias exhibited by consumers in choosing migration, is chosen to ensure that a plurality of consumers stay in the locations they were born in, even in 2000 CE. ω, the elasticity of congestion to population density, is chosen to reduce the concentration of population growth in regions that take off early versus those that take off late.

1.5.1 Results, 1000-2000 CE

Figure 1.6 shows the evolution of total world population in the model and in the data. The model replicates well the overall pattern of accelerating growth in world population, with a sharp increase in growth rates after 1700 CE. The model starts with a total world population of 260 million people, which is inside the range of plausible historical estimates, and ends in 2000 CE with 6 billion, just as in the data.

As can be seen in Figure 1.7, the correlation between the model and the data distributions of population density, both across regions and across individual 3° by 3° locations, remains high for most of the simulation. Both of these correlations decline sharply as population growth accelerates after 1700 CE, ending in 2000 CE at lower but still positive levels.

Figure 1.8 compares the evolution of world mean real income per capita in the model and in the data, where the mean is taken of the natural log of real income and weighted by population. The discrepancy early in the simulation, when mean real income in the model is somewhat less than that in the data, is not particularly meaningful, as the numbers for the data during this period are themselves somewhat speculative. It is clear, however, that there is much more growth in income per capita after 1800 CE in the model than in the data. Aside from this, both the model and the data display the same basic pattern of accelerating growth, which is almost flat prior to 1800 CE, and increases sharply after 1800 CE.

Figure 1.6: Simulation results: world population

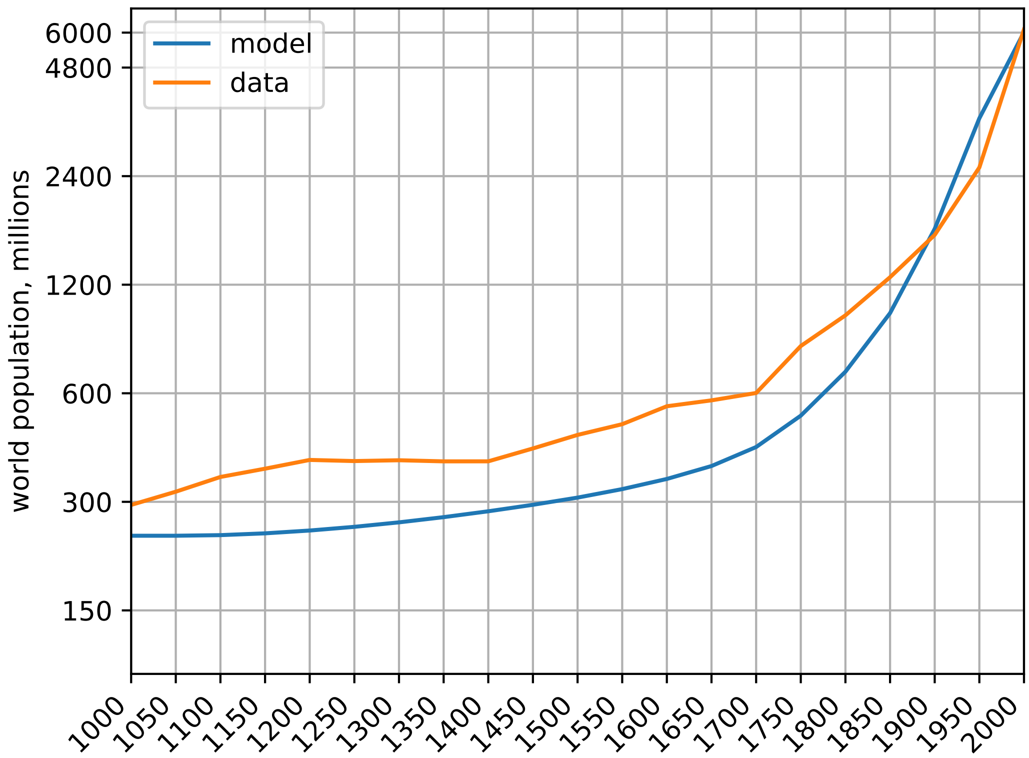


Figure 1.7: Simulation results: population density

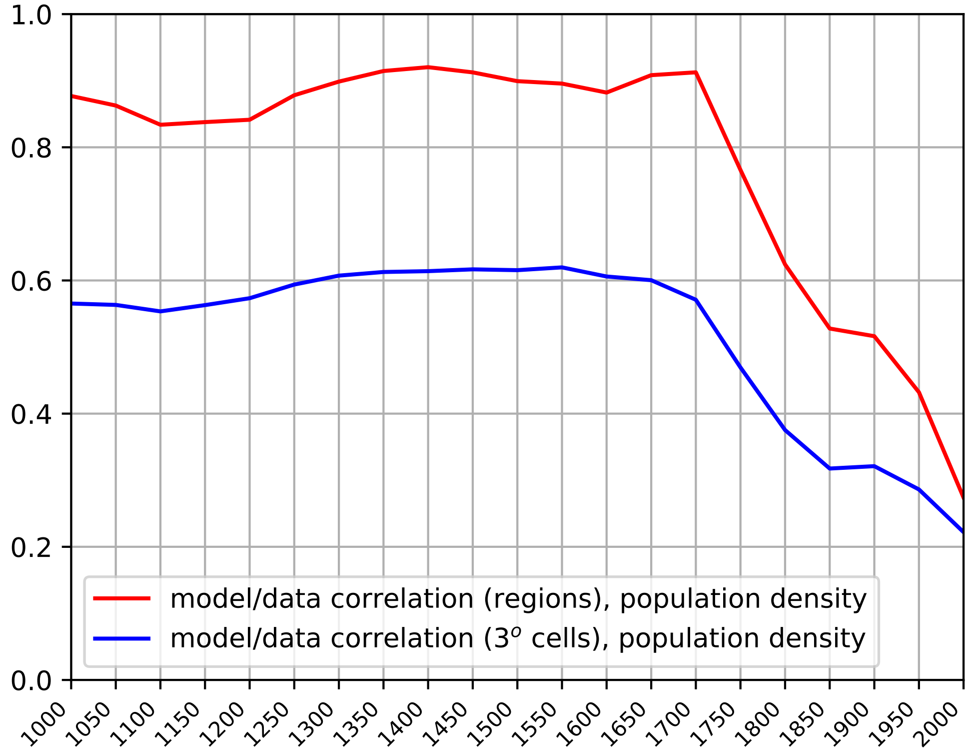
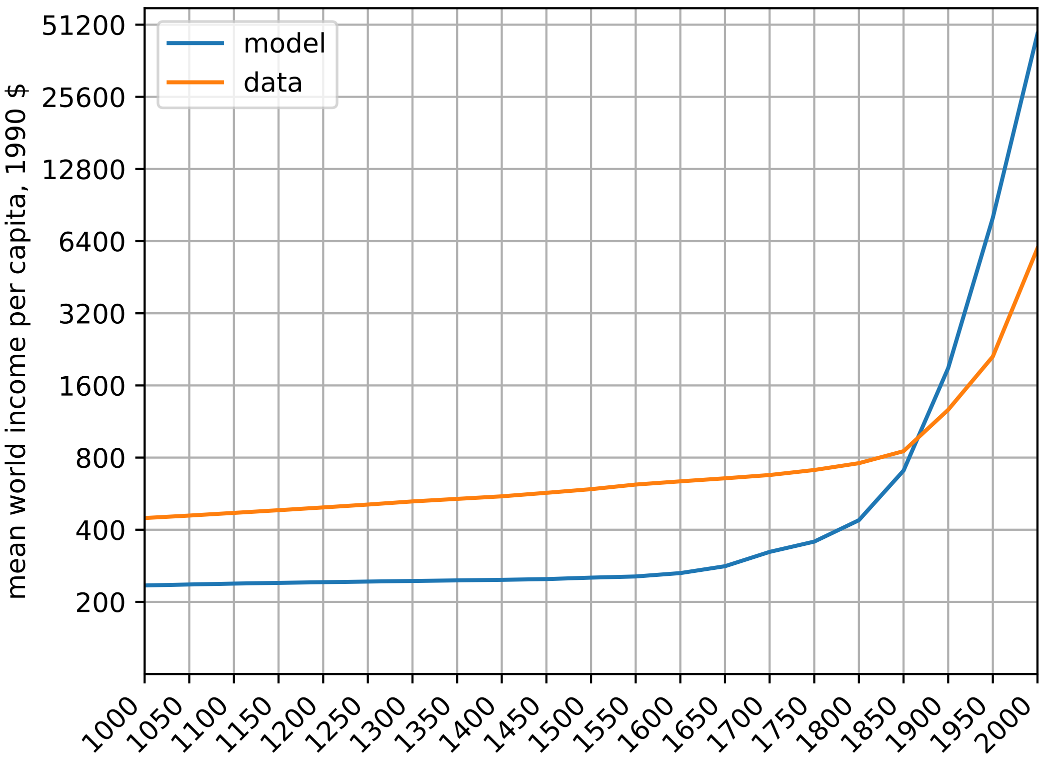


Figure 1.8: Simulation results: world income



The model matches very well the evolution of income dispersion across regions until 1900 CE, as can be seen in Figure 1.9. Income dispersion is measured as the variance in log real income per capita across the 10 major regions, weighted by population. They are at the same level in 1800 CE, and move together tightly for the next 100 years. From 1900 to 1950, the increase in dispersion in the model slows down slightly, while the increase in dispersion in the model accelerates. From 1950 to 2000, dispersion declines in both the model and the data, though this decline is considerably larger in the model than in the data. In the end, the variance across regions of log income per capita in the model is 43% of what is observed in the data in 2000 CE.

The simulation also matches well the evolution of Europe's lead in income per capita over the rest of the world. Figure 1.10, shows the evolution of the ratio between the population-weighted mean income per capita in Europe to the population-weighted mean income per capita for the entire world. This ratio in both the simulation and the data increase steadily, though the increase is not as big in the simulation as it is in the data. In Figure 1.11 we can see that the movements of this ratio in the model and in the data are highly correlated. This figure displays the evolution of the one-period growth rate of this ratio in the model and the simulation, which have a correlation of 0.51 for the entire simulation period. If we take into account the fact that the first meaningful observations for income per capita are really in 1800 CE, and so consider only the one period growth rates from 1850 onwards, this correlation is higher, at 0.66.

The first reliable data observations for income per capita begin in 1800 CE. As can be seen in Figure 1.12, the model at this point matches the distribution of income per

Figure 1.9: Evolution of income dispersion

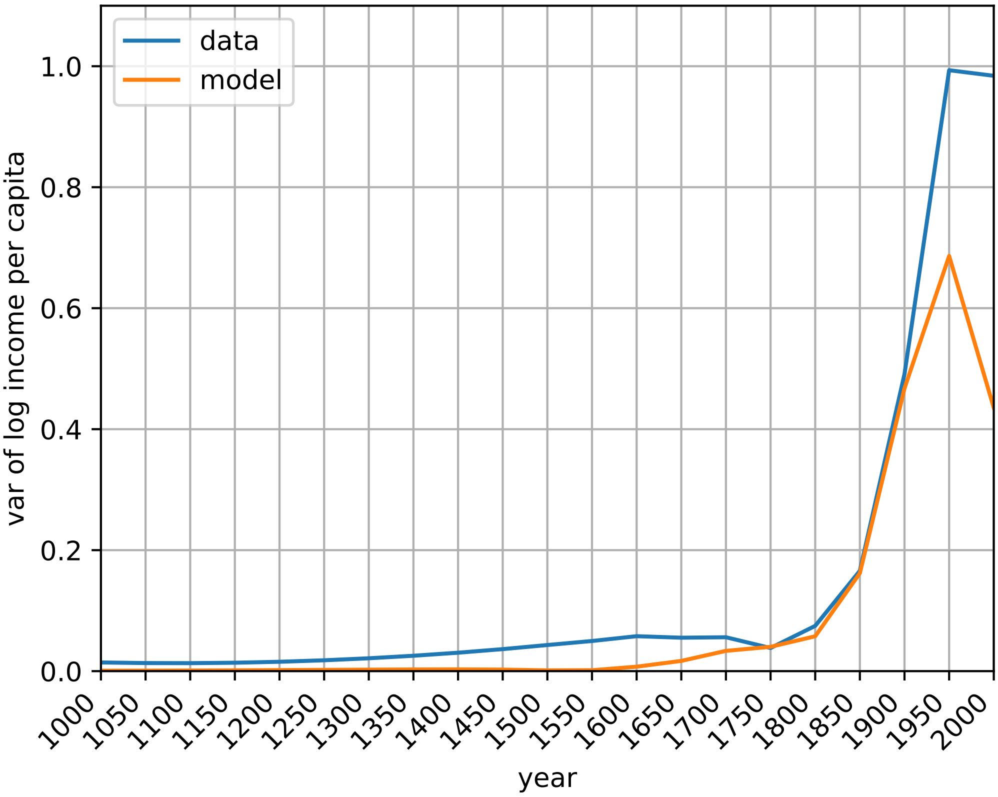


Figure 1.10: Europe/World Income Ratio

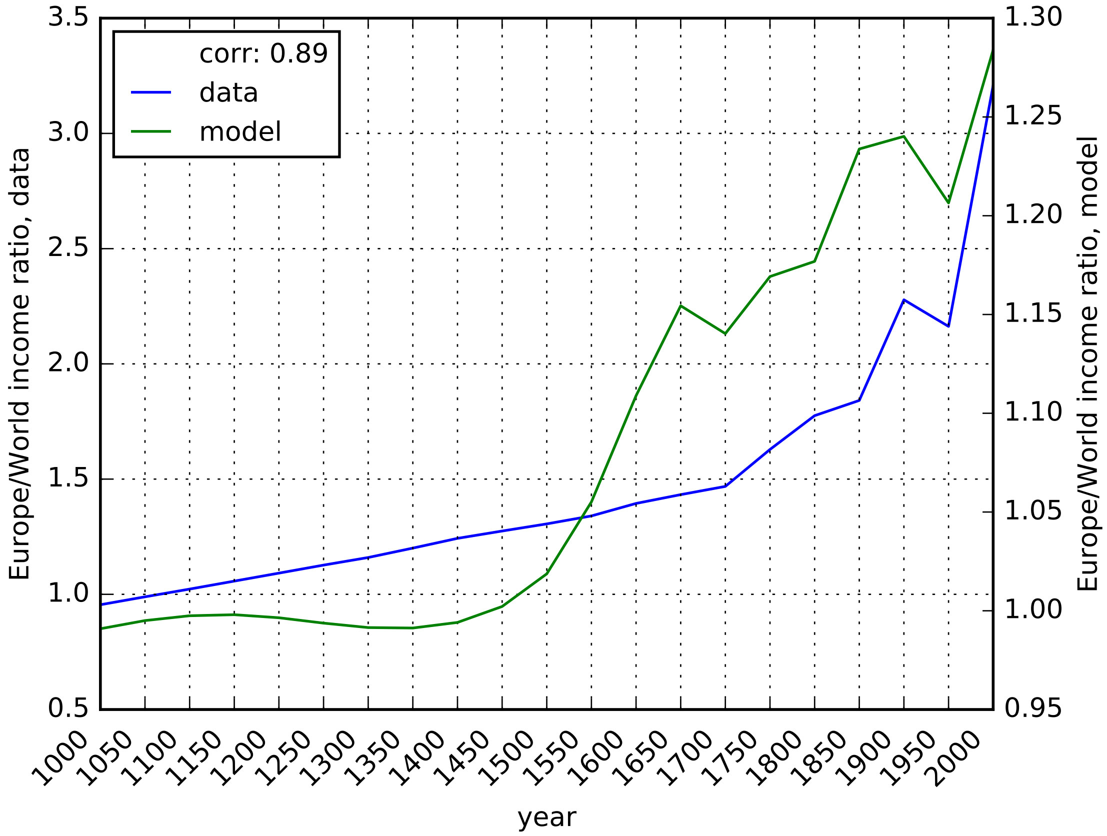


Figure 1.11: Growth Rate of Europe/World Income Ratio

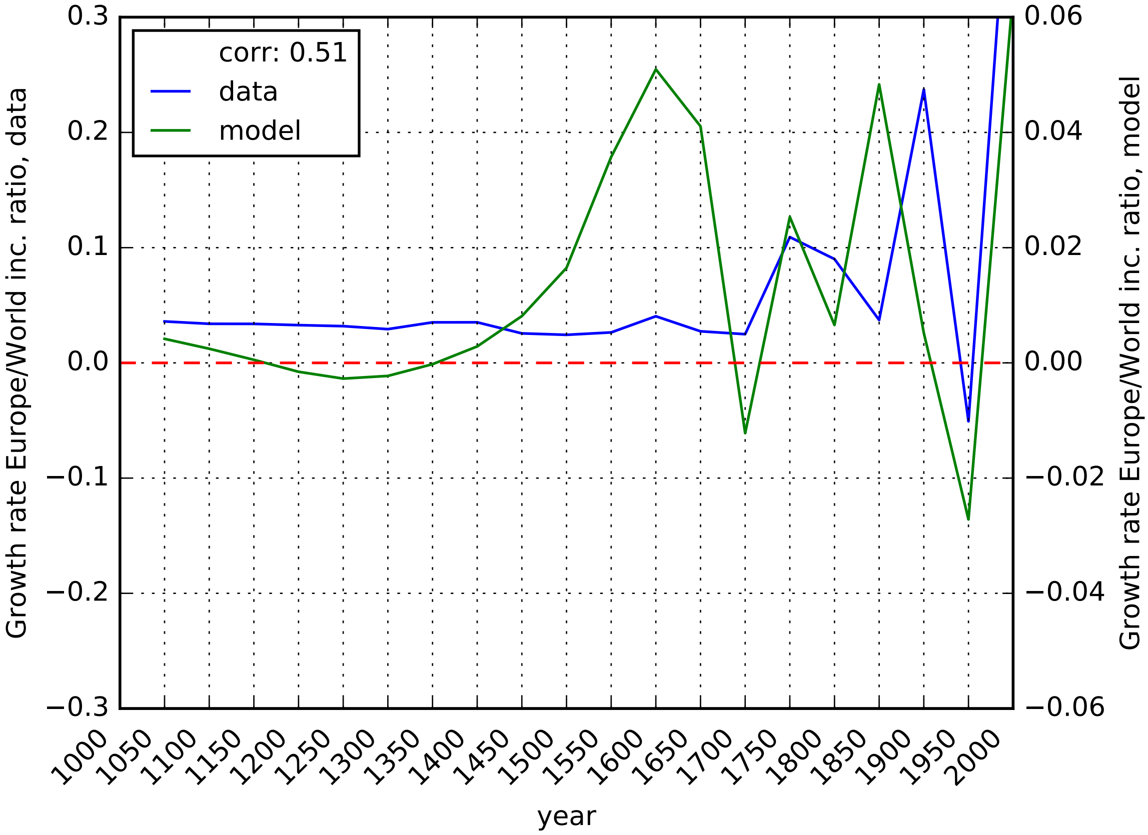
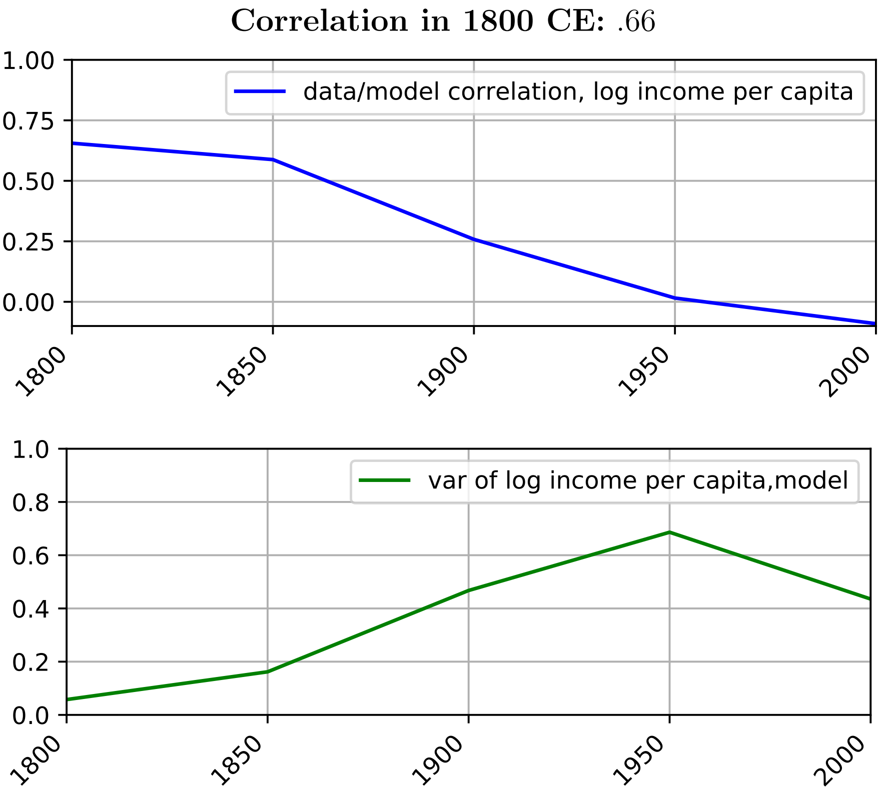


Figure 1.12: Simulation results: income per capita



capita across regions quite well. The correlation across regions between log income per capita in the model and in the data at this point is 0.66. As mean world income and the dispersion in income both increase after 1800 CE, this correlation declines.

What drives this decline in correlation? There are two main reasons: Northern North America and Australia and New Zealand do not grow enough between 1800 CE and 2000 CE, and Northern African and West Asia, Eastern Europe and Central Asia, and Sub-Saharan Africa grow too much. Figure 1.13 shows the correspondence between real income per capita relative to Northern and Western Europe in the data and in the model. The size of the marker for each region represents its total population. As can be seen in the figure, the distribution of income per capita across regions in the model lines up well with that in the data in 1800 CE, and the best linear fit line has a slope close to 1.

Looking next at Figure 1.14, we can see that over the intervening 200 years, we can see that the ratio between income per capita in Northern and Western Europe and in China the "Rest of Asia," Meso- and South America, and India have evolved in a manner more or less consistent with the data. Northern North America, comprising the modern countries of the United States and Canada, as well as Australia and New Zealand, however, have not grown nearly enough. Northern Africa and West Asia, and Eastern Europe and Central Asia have grown too much. And Sub-Saharan Africa has also grown too much, converging towards Europe more strongly than it does in the data.

Figure 1.15 compares the evolution of the correlation of log income per capita across regions between the model and the data, if the United States, Canada, Australia and New Zealand are included or excluded from the sample. We can see that excluding these four

Figure 1.13: Income relative to Europe - 1800 CE

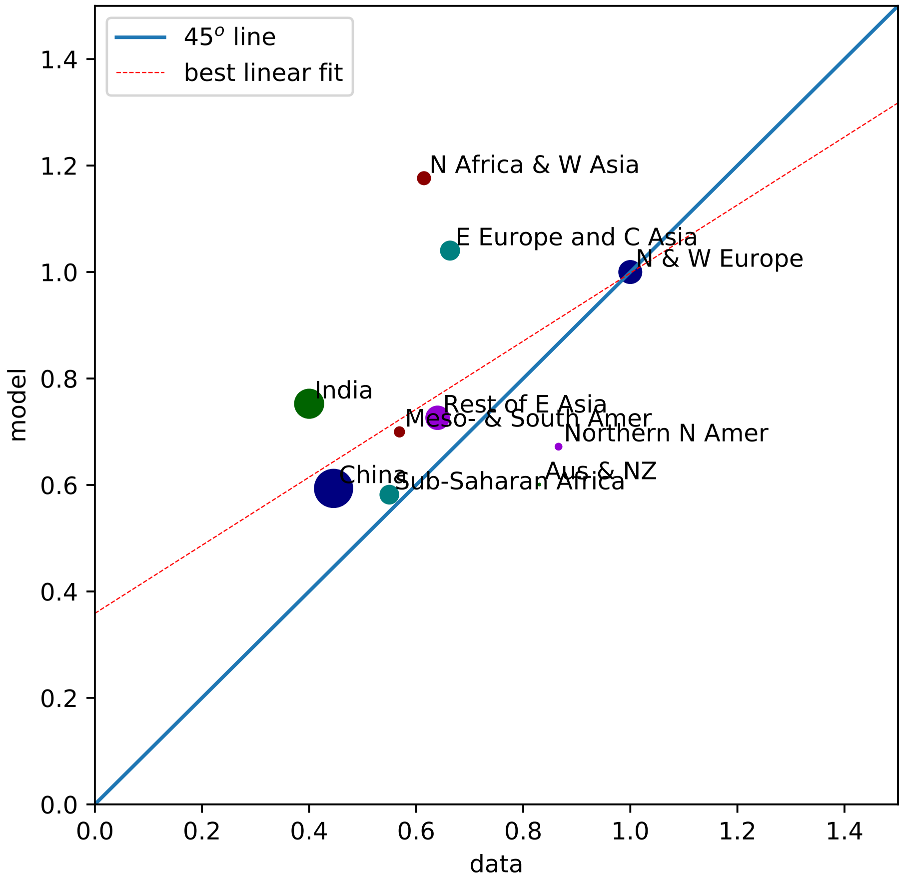


Figure 1.14: Income relative to Europe - 2000 CE

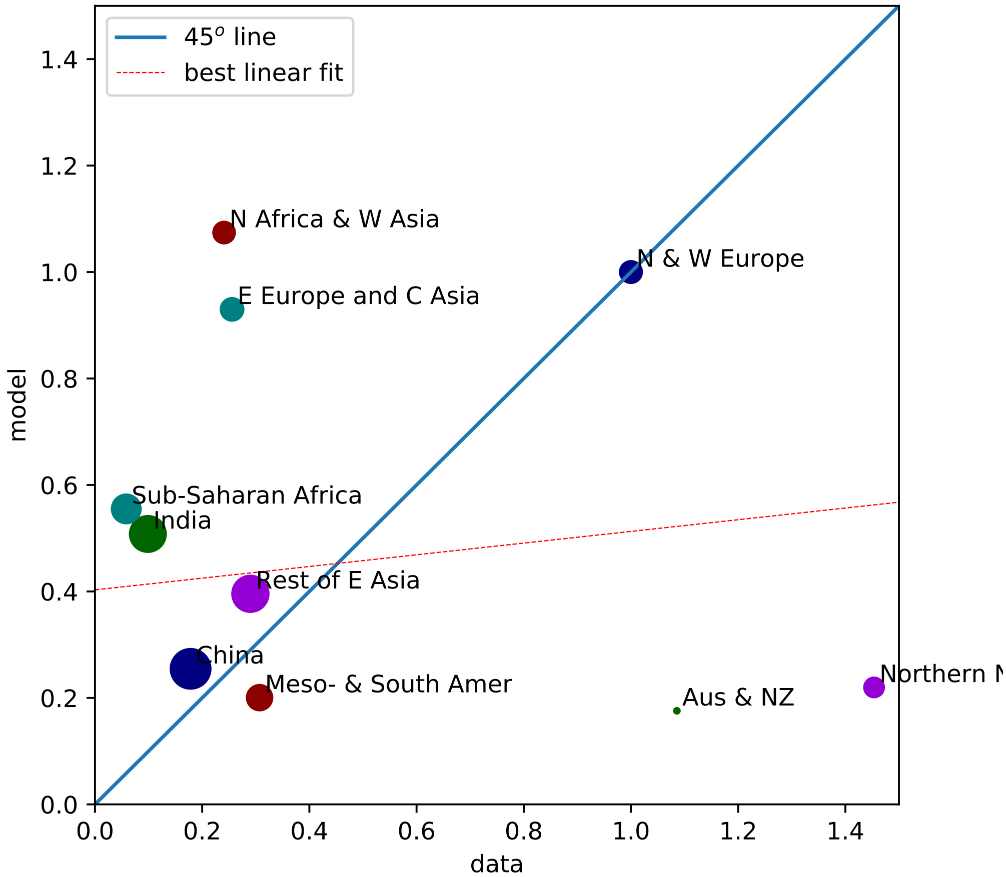
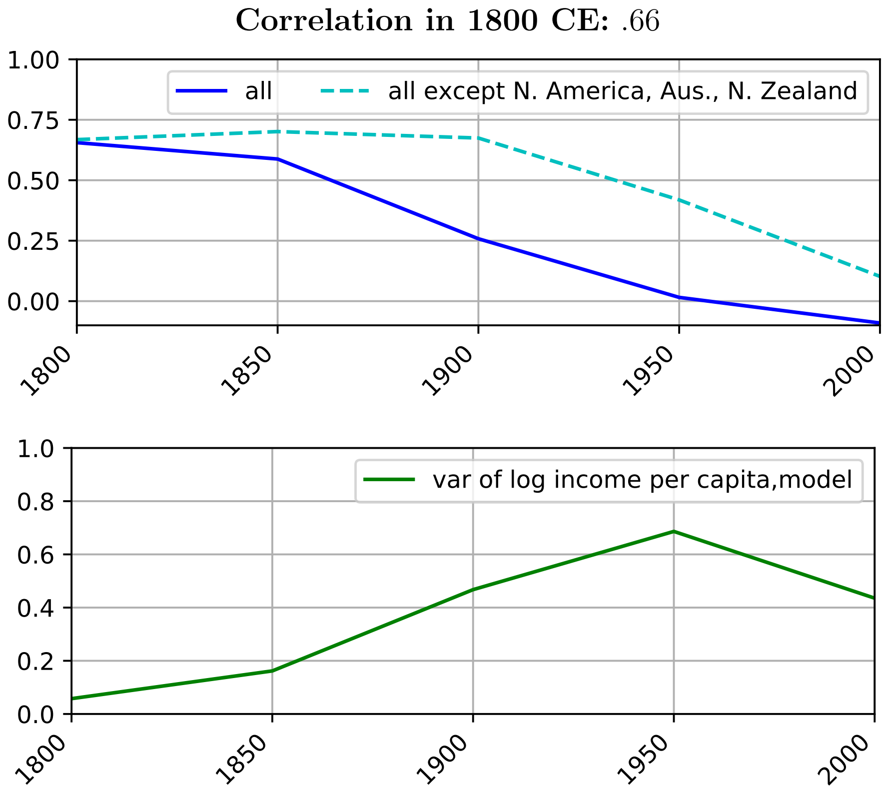


Figure 1.15: Simulation results: income per capita, U.S., Canada, Australia, N. Zealand excluded



countries improves the correlation with the data considerably, especially during the 19th century. During the 20th century, however, the correlation for the reduced sample still declines steadily. One reason for this is that there is too much convergence in general in the 20th century, as we saw when analyzing the evolution of income per capita dispersion.

1.5.2 Discussion

What might explain the inability of the model to match the fast growth in the U.S., Canada, Australia and New Zealand after 1800 CE, and the slow convergence generally after 1900 CE? Two possible explanations in particular spring to mind. First, we know that in reality trade costs and the speed of technology diffusion depend on other factors in addition to mere transport costs. By ignoring these factors, the model effectively imposes an average trade cost level and diffusion speed for the whole world. In reality, however, it may be that the costs should be lower, and the speed higher, between Europe and the United States, Canada, Australia and New Zealand, than they are between Europe and the rest of the world. One well-known fact that might justify such a difference is that in the 19th century all these countries were populated by people speaking the same language as the leading European industrial power, England.

A second possible explanation is that there is significant variation across regions in objective institutional quality. It may be, for example, that the United States, Canada, Australia and New Zealand have better property protections or constraints than other regions, for some reason that is not directly related to access to trade or technology diffusion.[13]

With this in mind, a fruitful way to extend the current exercise would be to impose additional restrictions on the model and perform counterfactual exercises to test each of these possible explanations. In this way it may be possible to determine whether either explanation is capable of reconciling the baseline model with the data, and which explanation seems to fit best.

1.6 Conclusion

In this paper we have seen that a pattern of falling transport costs consistent with historical evidence, applied to a spatial dynamic model in which the strength of bilateral connections is determined by the natural topography of the globe, can account for many of the important features of the evolution of the distribution of population and income over the last 1000 years. This modeling approach is able to generate initially slow, accelerating growth, with a sharp increase in population growth, income growth, and the dispersion of income across locations after1800 CE. Quantitatively, it is able to account for 55% of the variation across major regions in population density in 1000 CE, 44% of the variation across regions in income per capita in 1800 CE, and can generate 43% of the variation in income per capita across regions in 2000 CE.

This approach is also not able to match a number of facts, such as the rapid growth in income per capita in the United States, Canada, Australia and New Zealand after 1800 CE, and the slowness convergence of income per capita in the world in general during the 20th century. Future research could extend the framework presented here to test whether there are institutional or historical factors which can reconcile the model to the data on this and other points.

Another natural avenue for future research would be to try to explain the one key factor which this study has taken as exogenous–the evolution of transport costs. Why were key transport technologies developed at certain times and locations? What are the implications of allowing improvements in transport technology in some locations before others? The current framework, which is able to provide a quantitative approximation of the location-specific benefits and global aggregate consequences of transport technology changes, would be a natural starting point for such an investigation.

[13] This hypothesis would be consistent with the findings of an extensive literature in comparative economic development, of which Acemoglu, Johnson and Robinson (2001) is a prominent example.

Chapter 2

Demographic Transitions Across Time and Space

Demographic transition theory constitutes one of the most powerful ideas in economics and demography. The text book description of demographic transition is as follows:

> "The recent period of very rapid demographic change in most countries around the world is characteristic of the central phases of a secular process called the demographic transition. Over the course of this transition, declines in birth rates followed by declines in death rates bring about an era of rapid population growth. This transition usually accompanies the development process that transforms an agricultural society into an industrial one. Before the transition's onset, population growth (which equals the difference between the birth and death rate in the absence of migration) is near zero as high death rates more or less off set the high birth rates typical of agrarian societies before the industrial revolution. Population growth is again near zero after the completion of the transition as birth and death rates both reach low levels in the most developed societies." (Boongaarts 2009, page 2985).

In this chapter I put together and analyze data set on crude death rates (CDR) and crude birth rates (CBR) for 188 countries that spans more than 250 years. Following the text book description of the demographic transition, we then estimate for each country in our sample: i) initial (pre-transition) levels of the CDR and CBR, ii) the start dates of the mortality and fertility transitions, iii) the end dates of the mortality and fertility transitions, iv) final (post-transition) levels of the CDR and CBR. This procedure also allows us to estimate the length and the speed of each transition.

Looking at demographic transitions across time and space, I show that: 1) transitions are becoming faster, 2) the average level of GDP per capita at the start of a transition is more or less constant, 3) an important predictor of a country's transition is the prior transition of other countries which are "close" to it in a geographical and a linguistic sense, and which have similar legal systems.

Understanding the relationship between income and population is one of the oldest challenges in economics, going back to Malthus (1803) who developed a powerful model that links better technology with constant living standards. In a Malthusian world, technological change allows a higher income per capita which leads to higher population through higher fertility and lower mortality. In the presence of a fixed input such as land, this higher population translates into lower marginal productivities that decrease per capita income back to the stationary level previous to the technological advance. Malthus' model is quite successful at accounting for the main facts that prevailed until the nineteenth century, but it fails to explain the coexistence of growth in per capita income and low fertility. Becker (1960) and Becker and Lewis (1973) develop the idea of a trade-off between quantity and quality of children to show that higher per capita incomes and low fertility can go together. The interest in this mechanism was revived with the presentation of an operational dynastic model of fertility in Barro and Becker (1989) and Becker and Barro (1988).

Building on this initial work, Becker, Murphy and Tamura (1990), Lucas (1998), Jones (2001) and especially in an important contribution, Galor and Weil (1996, 1999, 2000) present models that try to capture the historical evolution of population and output. Several recent papers, e.g. Fernandez-Villaverde (2001), Kalemli-Ozcan (2003), Doepke (2004) and Bar and Oksana (2010), present quantitative versions of these models that can account for historical evidence on demographic transitions for specific countries. Jones, Schoonbroodt and Tertilt (2011) and Greenwood, Guner and Vandenbroucke (2017) provide recent reviews.

Few recent papers study the historical evolution of fertility. Spolaore and Wacziarg (2014) document that genetic and linguistic distance from France was associated with the onset of the fertility transition in Europe. De la Croix and Perrin (2017) focus on the fertility and education transition in France during the 19th century, and show that a simple quality-quantity model can do a decent job in explaining variations of fertility across time and counties in France. De Silva and Tenreyro (2017) focus on post-1960 transitions and emphasize the role of social norms and family planning programs in recent declines in fertility rates in developing countries. This study is also related to recent studies that provide an empirical analysis of demographic transitions across countries. Reher (2004) looks at a broad panel of countries and compares earlier with later demographic transitions, with a particular focus on the role of mortality in driving fertility changes. Murtin (2013) also constructs a panel and .finds evidence for a robust effect of early childhood education on fertility decline. Building on these earlier contributions, this study is the first to detect empirically a "demographic contagion" effect at a global scale, and to investigate it within a quantitative framework.

2.1 Demographic Transitions – Past and Present

In this section, we propose a methodology for analyzing country-level vital statistics and applies it to the historical data on the crude death rate (CDR) and the crude birth rate (CBR).

In the textbook case, a demographic transition has four stages:

- In Stage 1, both CBR and CDR are high and stable.
- In Stage 2, the CDR starts to decline while the CBR stays high.
- In Stage 3, the CBR also starts to decline.

- In Stage 4 both CDR and CBR are stable at a lower level.

We take these 4-stage benchmark seriously and try to fit it to available data for each country. In particular, for the CDR and the CBR, we estimate the following four variables that describe the data as good as possible: i) an initial (pre-transition) level, ii) the start date of the decline, iii) the end date of the decline, iv) a final (post-transition) level.

2.1.1 Econometric Model

Consider a dependent variable y_t observed for periods $t \in \{1, ..., T\}$ which can be represented as a linear function of a vector x_t of k regressors and a residual. Suppose that instead of being constant over time, the relationship between y_t and x_t evolves over time and can be broken into S distinct stages $s \in \{1, 2, ..., S\}$ connecting $S+1$ distinct endpoints represented by $\{\tau_1, \tau_2, ..., \tau_{S+1}\}$, such that $\tau_1 = 1$, $\tau_{S+1} = T$, $\tau_s \in \{2, ..., T-1\}$ for $s \in \{2, ..., S\}$ and $\tau_s < \tau_{s+1}$ for all $s \in \{1, ..., S\}$.

At each endpoint τ_s, $s \in \{1, ..., S+1\}$, the dependent variable is defined by an equation of the following form

$$y_{\tau_s} = x'_{\tau_s}\alpha_s + \nu_{s,\tau_s}\sigma_s, \tag{2.1}$$

where $\nu_{s,t} \sim N(0,1)$ for all s, α_s is a $k \times 1$ vector of regression coefficients, and σ_s is a scalar that determines the standard deviation at point τ_j.

Now suppose that in each stage s, i.e. when $\tau_s < t < \tau_{s+1}$, the dependent variable is defined by an equation of the following form:

$$y_t = x'_t f_s(\alpha_s, \alpha_{s+1}, t) + \varepsilon'_{s,t} g_s(\sigma_s, \sigma_{s+1}, t) \text{ for } \tau_s < t < \tau_{s+1},$$

where $\varepsilon_{s,t} \sim N(0,1)$ for all s, and f_s and g_s are continuous functions $f_s : R^k \times R^k \times R \to R^k$, $g_s : R^+ \times R^+ \times R \to R^+$ such that

$$f_s(\alpha_s, \alpha_{s+1}, \tau_s) = \alpha_s,$$

$$f_s(\alpha_s, \alpha_{s+1}, \tau_{s+1}) = \alpha_{s+1},$$

$$g_s(\sigma_s, \sigma_{s+1}, \tau_s) = \sigma_s,$$

and

$$g_s(\sigma_s, \sigma_{s+1}, \tau_{s+1}) = \sigma_{s+1}.$$

While it is possible to analyze the more general class of transition functions we have defined above, we will restrict our attention to the simplest case where f_s and g_s are linear transitions with respect to time between the parameters at τ_s and τ_{s+1} for all $s \in \{1, ..., S\}$, i.e.

$$f_s(\alpha_s, \alpha_{s+1}, t) = \frac{1}{\tau_{s+1} - \tau_s}\left[(\tau_{s+1} - t)\alpha_s + (t - \tau_s)\alpha_{s+1}\right], \tag{2.2}$$

and

$$g_s(\sigma_s, \sigma_{s+1}, t) = \frac{1}{\tau_{s+1} - \tau_s}\left[(\tau_{s+1} - t)\sigma_s + (t - \tau_s)\sigma_{s+1}\right]. \tag{2.3}$$

In order apply the theoretical framework described in the previous section to the specific context under study, suppose that the dependent variable y_t is either the CBR or the CDR for a particular country and that $S = 3$ (i.e. there is a stage where y_t is constant, another stage it is declining and a final stage it is constant again). Furthermore, we are interested in modeling a situation in which there is a transition between two stable regimes, so let us assume that $\alpha_s = \alpha_{s+1}$, $\sigma_s = \sigma_{s+1}$, and $\nu_{st} = \nu_{s+1,t} = \varepsilon_{st}$ for $s \in \{1, 3\}$.

Substituting in for f_1 and g_1 as given by equations (2.2) and (2.3), we can write the model for y_t as

$$\begin{aligned} y_t = {} & d_{1t}[x_t'\alpha_1 + \varepsilon_{1t}\sigma_1] \\ & + d_{2t}[x_t'\frac{1}{\tau_3 - \tau_2}\left[(\tau_3 - t)\alpha_1 + (t - \tau_2)\alpha_3\right] \\ & + d_{2t}[\frac{1}{\tau_3 - \tau_2}\left[(\tau_3 - t)\sigma_1 + (t - \tau_2)\sigma_3\right]\varepsilon_{2t} \\ & + d_{3t}[x_t'\alpha_3 + \varepsilon_{3t}\sigma_3], \end{aligned} \tag{2.4}$$

where $\{d_{st}\}_{s=1}^{3}$ are indicator functions given by

$$d_{1t} = 1\{t \leq \tau_2\}, \quad d_{2t} = 1\{\tau_2 < t < \tau_3\}, \text{ and } d_{3t} = 1\{t \geq \tau_3\}.$$

Equation (2.4) can then be rearranged in the following way

$$\begin{aligned} y_t = {} & \left[d_{1t} + d_{2t}\left(\frac{\tau_3 - t}{\tau_3 - \tau_2}\right)\right] x_t'\alpha_1 + \left[d_{3t} + d_{3t}\left(\frac{t - \tau_3}{\tau_3 - \tau_2}\right)\right] x_t'\alpha_3 \\ & + \left[d_{1t}\varepsilon_{1t} + d_{2t}\left(\frac{\tau_3 - t}{\tau_3 - \tau_2}\right)\varepsilon_{2t}\right]\sigma_1 + \left[d_{3t}\varepsilon_{3t} + d_{2t}\left(\frac{\tau_3 - t}{\tau_3 - \tau_2}\right)\varepsilon_{3t}\right]\sigma_3, \end{aligned} \tag{2.5}$$

where $\tau_2 \in \{1, ..., T-1\}$ and $\tau_3 \in \{\tau_2 + 1, ..., T\}$, with $\tau_2 \leq \tau_3$.

Estimation

The model, as we specified above, has $2k+2$ free parameters: the k parameters in α_1, the k parameters in α_3, plus τ_2 and τ_3. We choose these parameters according to the criterion of minimizing the unweighted sum of squared errors. This means that for a given (τ_2, τ_3) pair, estimation of (α_1, α_3) reduces to Ordinary Least Squares. The optimal (τ_2, τ_3) can then be located by a search algorithm across the possible values. To this end, let's define scalars

$$z_{1t} \equiv d_{1t} + d_{2t}\left(\frac{\tau_3 - t}{\tau_3 - \tau_2}\right)$$

and

$$z_{3t} \equiv d_{3t} + d_{2t}\left(\frac{t - \tau_2}{\tau_3 - \tau_2}\right).$$

Then given

$$\underset{1\times T}{y'} \equiv [y_1 \dots y_T],$$

and

$$\underset{2k\times T}{Z'} \equiv \left[\begin{bmatrix} z_{11}x_1 \\ z_{31}x_1 \end{bmatrix} \cdots \begin{bmatrix} z_{1T}x_T \\ z_{3T}x_T \end{bmatrix}\right],$$

the least-squares estimators of (α_1, α_3) given (τ_2, τ_3) have the following closed-form expression:

$$\begin{bmatrix} \widehat{\alpha}_1 \\ \widehat{\alpha}_2 \end{bmatrix} = [Z'Z]^{-1} Z'y.$$

Estimating σ_1 and σ_3 in this configuration is straightforward except for the fact that the contribution of each variance to the total variance differs across periods and so the errors must be weighted accordingly.

Define

$$e_t \equiv y_t - [z_{11}x_1 \ z_{31}x_1] \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_3 \end{bmatrix},$$

$$\underset{1\times T}{e_z^{1\prime}} \equiv [z_{11}e_1 \ldots z_{1T}e_T],$$

and

$$\underset{1\times T}{e_z^{3\prime}} \equiv [z_{31}e_1 \ldots z_{3T}e_T].$$

We calculate the following estimators for σ_1 and σ_3 given (τ_2, τ_3), which are asymptotically equivalent to the OLS estimators:

$$\widehat{\sigma}_1^2 = \left(\sum_{t=1}^{T} z_{1t}\right)^{-1} e_z^{1\prime} e_z^1$$

and

$$\widehat{\sigma}_2^2 = \left(\sum_{t=1}^{T} z_{3t}\right)^{-1} e_z^{3\prime} e_z^3.$$

Note that when $\sum_{t=1}^{T} d_{st} = 1$ and $\sum_{t=1}^{T} d_{2t} = 0$ for $s \in \{1, 3\}$, σ_s is not identified, but this is of little consequence as none of the estimators for the other parameters depend on the variance estimates.

While in general it may be interesting to include a larger number of regressors in x_t, the only specification of this model that we will use in the analysis that follows is the one where x_t contains only a constant term, $x_t' = 1$ for $\forall t$ and $k = 1$. Hence, before a transition start, i.e. while $t < \tau_2$, $y_t = \alpha_1$ (stage 1), between τ_2 and τ_3, y_t declines linearly (stage 2), and at τ_3, $y_t = \alpha_3$ (stage 3).

Restricted Cases

The econometric model that we have described so far addresses one question: assuming that a three-phase model of a linear transition between two constant means is a good description of the available data for a particular variable in a particular country, what should the parameters of this model be? It may be, however, that even if the three-phase model would be a useful description, one or more of the phases is not observed due to data limitations. To address these possibilities, in addition to estimating the parameters for the model described in the previous section, hereafter referred to as Case I, we also estimate the parameters of 4 other cases with additional restrictions, Cases II-V, as documented in Table 1.

Table 1: Different Cases of the General Model

	Parameter Restriction	Explanation	Number of Parameters
Case I	–	All 3 stages are observed	$2k + 2$
Case II	$\tau_2 = 1$	Only stages 2 and 3 are observed	$2k + 1$
Case III	$\tau_3 = T$	Only stages 1 and 2 are observed	$2k + 1$
Case IV	$\tau_2 = 1, \tau_3 = T$	Only stage 2 is observed	$2k$
Case V	$\tau_2 = 1, \tau_3 = T, \alpha_1 = \alpha_3$	Only stage 1 or 3 are observed	k

Case II will occur if we observe downward trends in the CBR and CDR starting from the first available observation and the transitions are completed. Case III will occur for transitions that are started but are not yet completed. Case IV are countries for which we only see downward trends in the CBR and CDR. Model V are anomalies.

The nesting structure of these models is as follows: Case I nests Cases II and III, Cases II and III both nest Case IV, and Case IV nests Case V. We select a model based on the following criteria: that a less-restricted case should be selected only if it does a significantly better job of fitting the data. We use a formal statistical test of model fit to inform this selection, but the final choice of a model incorporates additional information, such as whether the last level of the variable observed is more typical of a final steady state or of a continuing transition. The formal statistical test that is used to inform the process of case selection is an F-test at the 95% confidence level:

$$\frac{\frac{SSE^b - SSE^a}{m^a - m^b}}{\frac{SSE^a}{T - m^a}} \tag{2.6}$$

where a nests b, and, as mentioned in the previous section, $m^I = 2k + 2$. The case with the lowest sum of squared errors that is not rejected relative to a case that it nests is selected.

2.2 Data

2.2.1 Vital statistics and GDP per capita

In gathering data on crude birth rates and crude death rates, we have combined data from a number of different sources in order to obtain time series which are as long as possible for the greatest possible number of countries. From 1960 onwards we rely on the World Bank Development Indicators, and for many countries we fill in the decade of 1950-1960 with data from the UNData service of the United Nations Statistics Division. To fill in vital statistics prior to 1950, we begin with data from Chesnais' (1992) classic book on the demographic transition, and augment them with data from the Europe, Africa, Asia and Oceania, and The Americas volumes of B.R. Mitchell's (1998) International Historical Statistics. We also use aadditional sources for few countries.[1] The resulting data set on the CDR and the CBR covers 188 countries from 1735 to 2014.

[1]The State Statistical Institute of Turkey (1995) and Shorter and Macura (1982) for Turkey, the Swiss Federal Statistics Office (1998) for Switzerland, Haines and Steckel (2000) for the US, and Kingsley (1946)

We take data on purchasing power parity GDP per capita, given in constant 1990 Geary-Khamis dollars, from the 2013 version of Maddison's databases.[2] We extend the end of the panel from 2011 to 2015 using data from the World Bank Development Indicators. Table 2 shows the means and the standard deviations of the CBR, the CDR and the log GDP per capita in our sample. Table 3 shows the correlations across these three variables.

Table 2: Summary Statistics

Variable	sample mean	st. Dev.	N. Obs.
crude birth rate *(CBR)*, per 1000	30.4	11.9	15549
crude death rate *(CDR)*, per 1000	13.9	7.6	15496
ln GDP per capita *(lnGDPPC)*	7.75	1.02	15000

Table 3: Correlations among Key Variables

	CBR	CDR	lnGDPPC
crude birth rate *(CBR)*	1	0.51	-0.74
crude death rate *(CDR)*	0.51	1	-0.57
ln GDP per capita *(lnGDPPC)*	-0.74	-0.57	1

Projecting CDR Backwards

Vital statistics for only a few countries are available back into the 19th century and for a great many not until after 1950. As a result, there are numerous countries for which the start of either the CBR or CDR transition is not observed. Due to the fact that the CDR transition starts earlier than the CBR transition on average, there are many more "missing starts" for the CDR transitions than for CBR–in all there are 89 countries for which the start of the CBR transition is estimated to be observed but the start of the CDR transition is not.

One way to address this gap is to apply the 3-phase framework and project CDR backwards to an initial level which is predicted according to the country's initial level of CBR and perhaps other characteristics. To this end, we estimate the following equation:

$$\alpha_i^d = \beta_0 + \beta_1 \alpha_i^b + \beta_2 \left(\alpha_i^b\right)^2 + \beta_3 s_i^b + \beta_4 s_i^d + \varepsilon_i, \tag{2.7}$$

where $s^b \equiv \frac{\alpha_{3,i}^b - \alpha_{1,i}^b}{\tau_{3,i} - \tau_{2,i}}$ and $s_i^d \equiv \frac{\alpha_{3,i}^d - \alpha_{1,i}^d}{\tau_{3,i} - \tau_{2,i}}$ are the calculated slopes for CBR and CDR, respectively, during the transition, and ε_i is a mean-zero iid error term. The parameters of this equation are estimated using only the 24 countries for which it is estimated that both the CBR and CDR transitions are observed and that the CBR transition started prior to 1950. These earlier transitions are selected because the estimated gap of CBR over CDR is systematically higher for the later cohort, suggesting that these estimated initial levels may not reflect the true, long-run, "natural" pre-transition levels.[3] These

for India.

[2]This database can be accessed here: https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2013

[3]If they were representative of long-run pre-transition levels, the would imply a steady, high initial rate of population growth, which is counterfactual.

estimated parameters are then used to predict initial CDR levels for 89 countries for which the start of the CBR transition is observed, but the start of the CDR transition is not. After applying a set of criteria to remove outliers and unreasonable predictions, predictions for 77 countries remain.[4] This more than doubles the number of countries for which some estimate of the CDR transition start date is available, from 53 to 130 countries in total. Table 4 shows the results of the estimation of equation (2.7).

Table 4: CDR Projections

β_0	β_1	β_2	β_3	β_4
−9.2101	1.2796	−0.0078	−3.2453	3.0718
(17.1288)	(0.9473)	(0.0127)	(5.4868)	(1.7910)
$N = 24$	$R^2 = 0.773$			

Extension of GDP per capita data

The main source for GDP per capita data used in these estimations is the Maddison historical time series, 2013 version. While estimates are given for some countries going as far back as the year 1 CE, the time series for other countries does not start until the late 19th or early 20th centuries, long after many countries entered the CBR and CDR transitions. To facilitate analysis of the relationship between GDP per capita and demographic trends, we assign each country a value for GDP per capita in the year 1500, and fill in remaining gaps by interpolating linearly. Out of 158 countries that we assign a year 1500 GDP per capita value to, 17 have an observation in the Maddison 2013 database either in 1500 CE or in a preceding year, allowing interpolation. Values for the remaining 141 countries are imputed by projecting backwards from the earliest year for which data is available. If there exists a neighboring country for which data exists for the year 1500, we assign the first country a year 1500 value assuming that the ratio in per capita between the two countries has remained constant from 1500 until the first Maddison 2013 observation. If no such suitable comparison country exists, we assume that GDP per capita in the country in question remained constant until the first observation, and simply assign this value to the year 1500. Table A3 in the appendix provides details of how each year 1500 value of GDP per capita is assigned. Following this methodology we are able to assign conservative guesses, by which countries are assigned a GDP per capita that is similar to that of nearby countries for which data is available.[5]

2.3 Results

Figures 1 and 2 display time series of CBR and CDR, along with the fitted 3-phase transitions that we calculate, for few countries. Table A1 in the Appendix documents

[4]The criteria are: that the predicted initial level of CDR be higher than the estimated, truncated initial level; that the gap between the initial level of CBR and the initial level of CDR is not larger than the largest or smaller than the smallest gap observed in the sample of 24 countries used to estimate the parameters; and that the implied CDR transition length is not longer than the maximum obseved transition length in the 24 country estimation sample.

[5]It should also be noted that the year 1500 is prior to the advent of modern economic growth, and so income differences both across and within regions were small whem compared with the modern era, limiting the potential significance of any small errors introduced by this method of imputation.

the start and end dates, if they can be estimated, for each country in our sample.

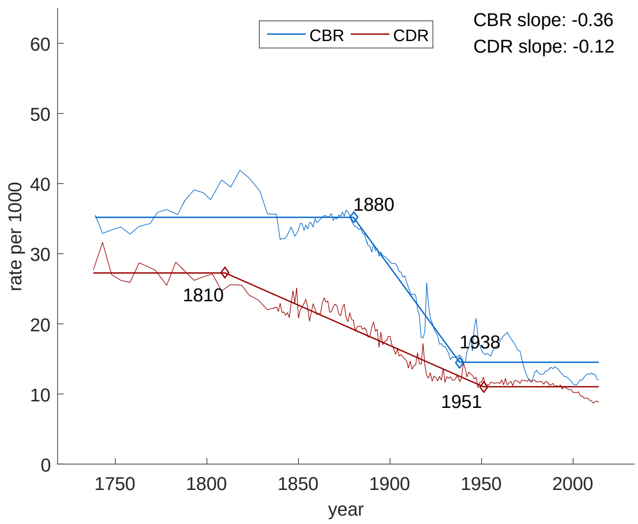


Figure 1-A: Britain

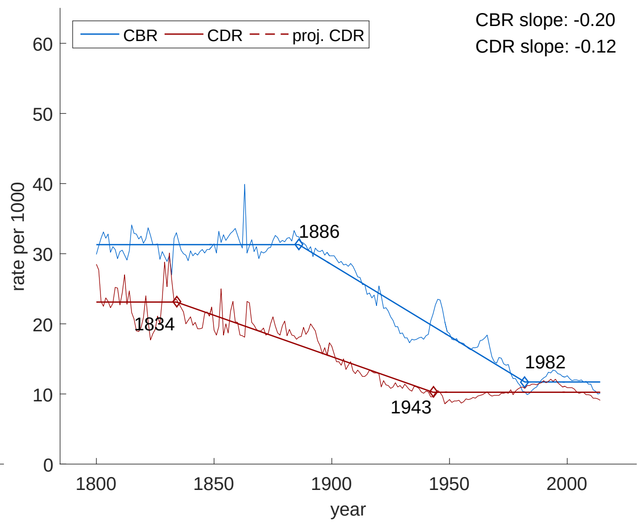


Figure 1-B: Denmark

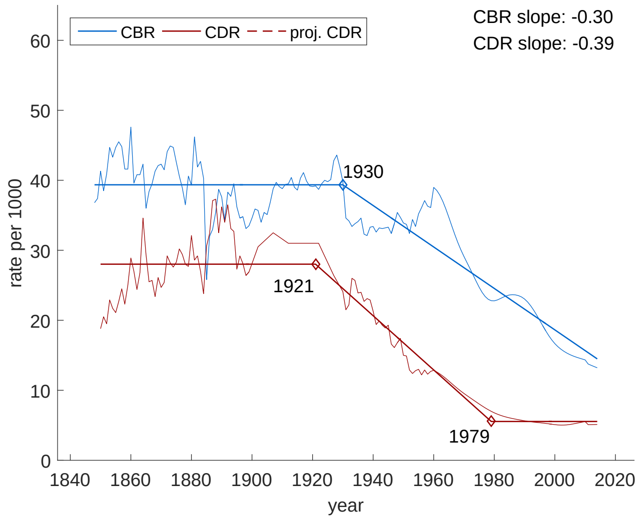


Figure 2-A: Chile

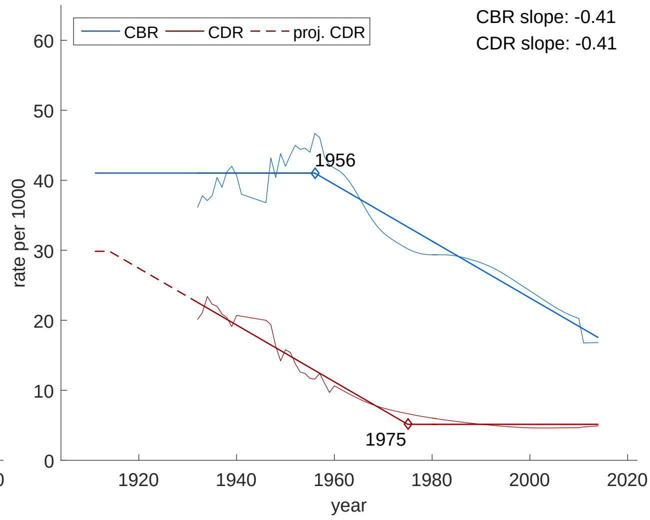


Figure 2-B: Malaysia

Figure 3 displays scatter plots of CDR and CBR, for every country in every year that they are observed, against log GDP per capita. Superimposed onto the plots is the best fit for a 3-phase transition as specified previously, with log GDP per capita taking the place of time.[6] According to this estimation, the "average" pre-transition CDR for the entire panel is 18.7 per year per 1000 people, and the pre-transition CBR for the entire sample

[6]As shown in Figure 3, this structure provides a reasonably good fit for the panel data with R^2 coefficients of .353 and .558, respectively.

is 43.9. The estimated post-transition CBR and CDR for the entire sample are 8.9 and 14.3, respectively. The crude death rate transition is estimated to start, on "average," when a country achieves a real GDP per capita of $1,224 constant 1990 international dollars. The "average" start of the CBR transition is estimated to be at $728. The end of the CDR and CDR transitions are placed at $4,964 and $14,472, respectively.

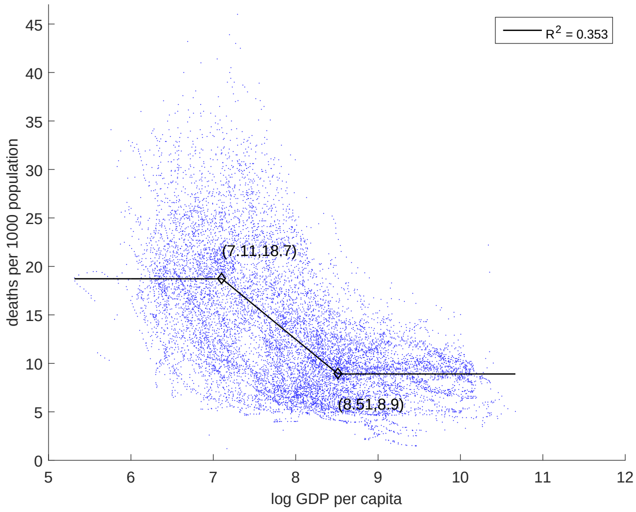


Figure 3-A: The CDR versus log GDP per capita

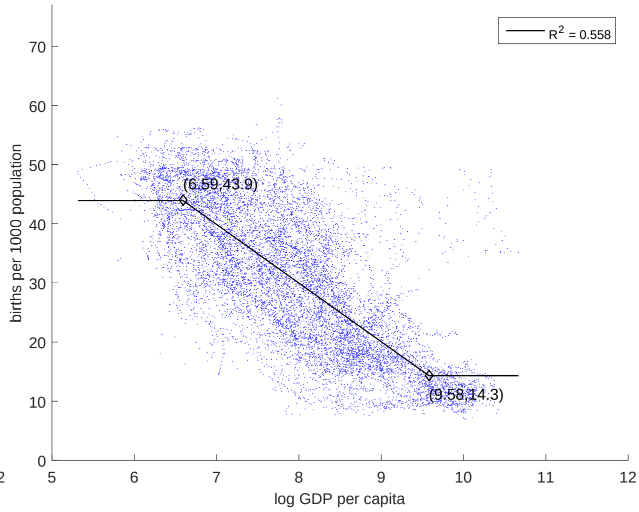


Figure 3-B: The CDR versus log GDP per capita

Table 5 documents the distribution of all countries in our sample according to different cases in Table 2. Overall, there are 28 countries for which we observe completed mortality and fertility transitions.

Table 5: Different Cases in the Data

CDR\CBR	Case 1	Case 2	Case 3	Case 4	Case 4	Total
Case 1	28	0	19	0	0	47
Case 2	15	11	60	13	3	102
Case 3	0	0	6	0	0	6
Case 4	1	0	13	1	2	17
Case 5	0	10	2	1	3	16
Total	42	21	100	15	8	188

The distribution of the log GDP per capita levels at the start of both the CBR and CDR transitions appear to have uni-modal distributions, which may be adequately approximated by a normal distribution. Figure 4 plots the empirical frequency of log GDP per capita at the start of each type of transition.

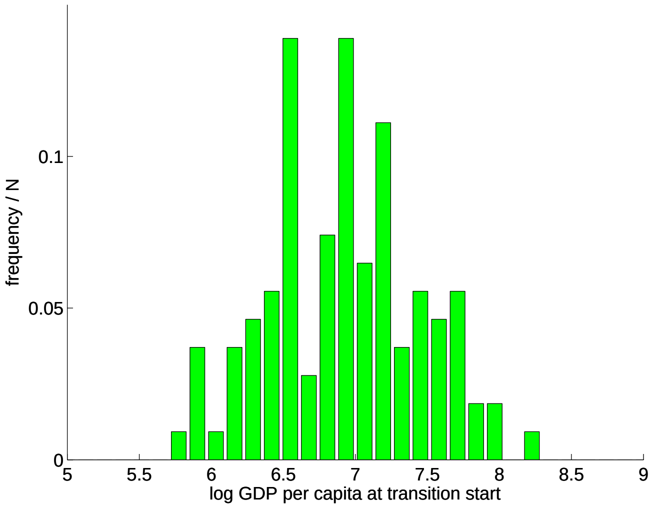


Figure 4-A: Log GDP per Capita at Transition Start (CDR)

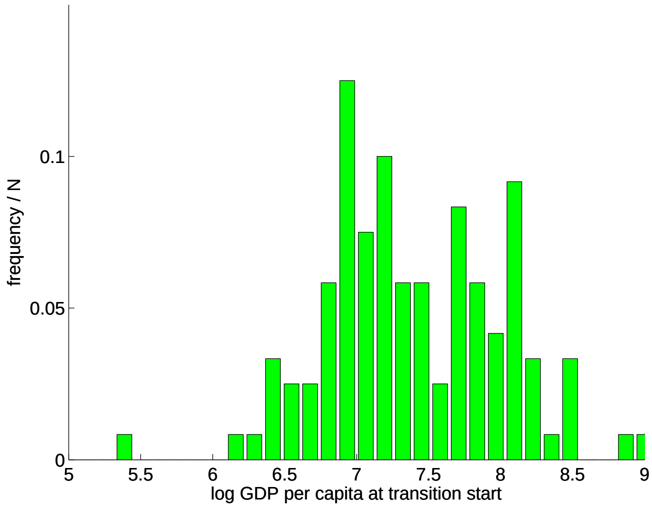


Figure 4-B: Log GDP per Capita at Transition Start (CBR)

2.3.1 Are Transitions Getting Faster?

Table 6 reports summary statistics for some features of countries as they enter the CDR and CBR transitions, broken into groups according to the time period in which their transition started. A number of patterns are evident in these numbers. First of all, the incidence of CDR transition starts is more spread out over time than the incidence of CBR transition starts, peaks sooner, and is almost completely over before 1960. The pattern of CBR transition starts, in contrast, peaks dramatically in the 30 years between 1960 and 1990, and there have also been several transition starts since 1990.

Table 6: Countries Entering Transitions

	<1870	1870-1900	1900-30	1930-60	1960-90	>1990	All
				Mortality Transition			
initial CDR	25.58	26.76	24.17	26.37	24.99	22.49	25.46
initial GDP pc ($)	4137	1503	1273	1006	854	940	1472
slope	-0.19	-0.26	-0.39	-0.50	-0.55	-0.80	-0.40
N	11	17	34	34	10	2	108
				Fertility Transition			
initial CBR	38.79	36.43	39.76	40.12	44.55	48.39	42.97
initial GDP pc ($)	1051	2419	1259	1987	2372	1080	2139
slope	–0.19	-0.31	-0.35	-0.47	-0.58	-0.55	-0.52
N	4	13	5	16	73	9	120

We can also detect in Table 4 a tendency for countries in later transition groups to transition faster, i.e. with a more negative slope, and also to transition at lower levels of

GDP per capita.[7] In the remainder of this section, we will explore in greater depth these two hypotheses–that countries that transition later do so more rapidly and from lower levels of income.

Are countries that started the demographic transition later completing it faster? We now address this question more thoroughly using six possible measures of transition speed: the slope of the reduction in crude death and crude birth rates during the transition, the total length of the crude death and crude birth rate transition measured from plateau to plateau, the total length of the crude birth rate transition measured from the the end of the initial crude birth rate plateau until the total fertility rate goes below 2.1, and the gap between the time elapsed from the start of the CDR transition to the start of the CBR transition. Figures 5 and 6 show scatter plots for each of these alternative measures against transition start date, and in each case a general downward trend is evident.

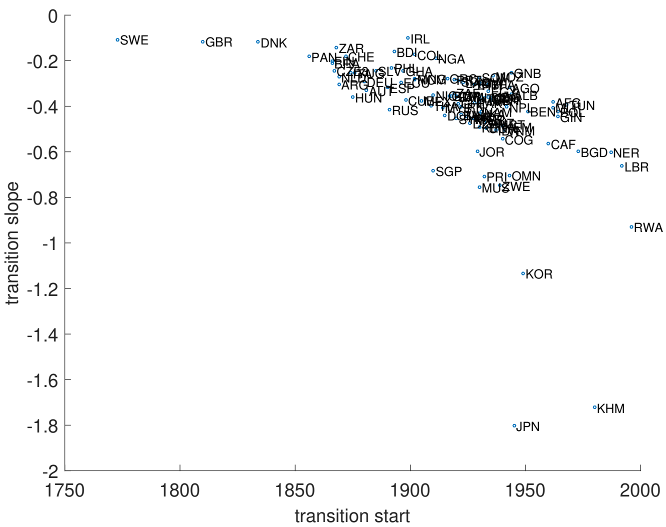


Figure 5-A: CDR Transition Slope

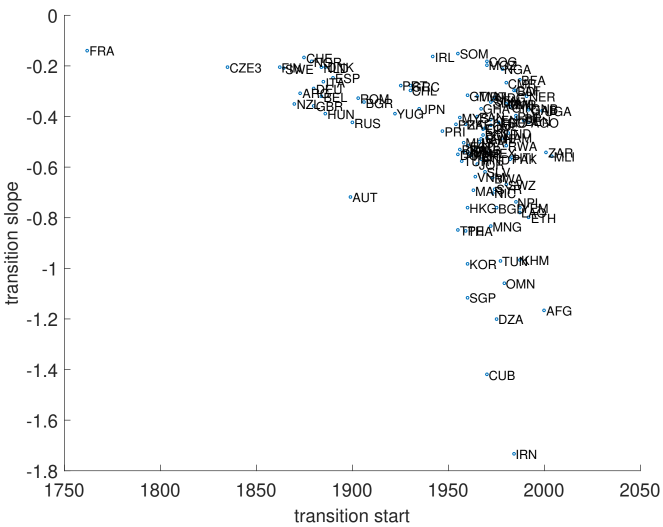


Figure 5-B: CBR Transition Slope

To measure the strength of this downward trend more precisely, we turn to a simple linear regression, which allows us to also control for some other factors in addition to timing, that may affect transition speed. We hypothesize that in addition to the timing of the transition start, the speed of the transition may also be affected by the level of GDP per capita at the transition start and by how high crude birth rates were initially.[8] Table 7 displays the results of simple linear regressions for each measure of transition speed including these three variables. In each case, transition start date is significantly related to transition speed.

[7]It also appears that countries entering the CBR transition later are doing so from higher initial birth rates. Interestingly, the corresponding trend is not clearly visible for CDR transitions.

[8]The initial level of crude birth rate is highly correlated with the inital level of crude death rate, and including the initial level of crude death rate in the regression does not significantly affect the results.

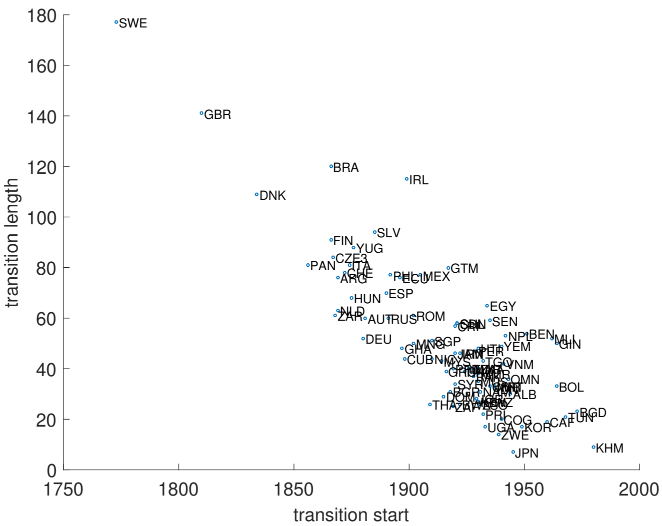


Figure 6-A: CDR Transition Lenght

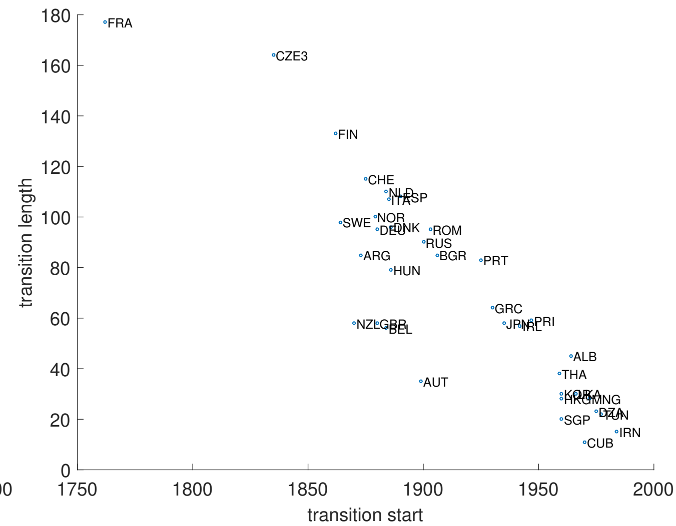


Figure 6-B: CBR Transition Lenght

Table 7: Transition Speed

	Dependent Variable			
	CDR Slope	CBR Slope	CDR Length	CBR Length
constant	0.45 (1.10)	0.77 (1.87)	160.05 (4.23)	353.27 (5.83)
ln GDPpc at the start	-0.06 (-1.39)	-0.99 (-2.18)	-0.95 (-0.23)	-20.63 (-2.97)***
starting CBR/10	0.06 (1.37)	-0.01 (-0.24)	-1.08 (-0.32)	-5.42 (-1.10)
start date/10	-0.05 (-6.95)***	-0.03 (-4.55)****	-6.90 (-12.40)***	-7.13 (-12.13)***
N	99	111	83	36
R^2	0.34	0.20	0.69	0.83

***indicates significance at 5% level.

Figure 7 shows scatter plots of log GDP per capita in each country at the start of its CDR and CBR transition, respectively.

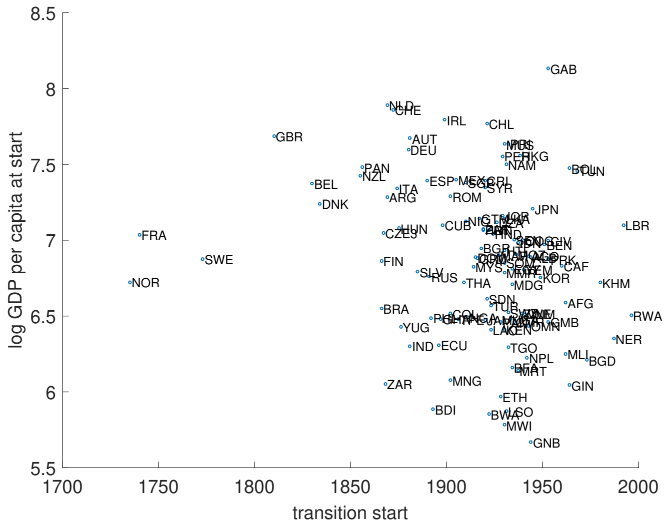


Figure 7-A: Log GDP per capita at the start of the CDR transition

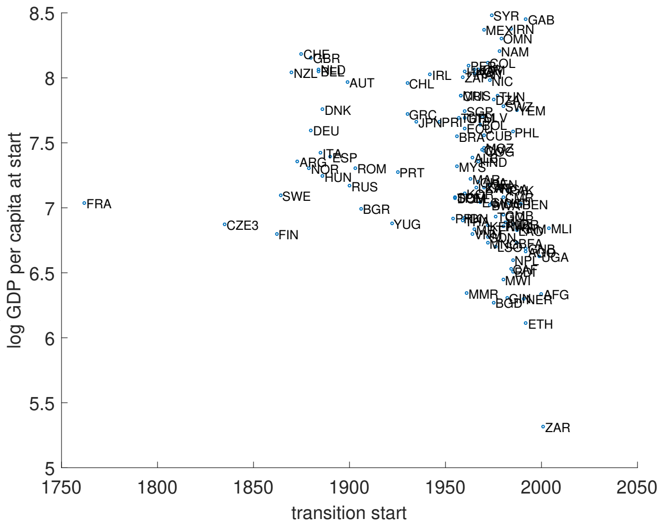


Figure 7-B: Log GDP per capita at the start of the CBR transition

2.4 An Empirical Analysis of Demographic Transitions

In the previous section we saw that the distributions of log GDP per capita levels at the start of transitions in crude birth rates or death rates are fairly stable over time and possess uni-modal distributions. This suggests that a modeling strategy which links the level of log GDP per capita to transition takeoffs may have some explanatory power. One possible approach is to model the start of each transition as a random event whose probability of occurring depends on log GDP per capita and possibly other variables. Let T represent the time at which a one-off event, such as a birth rate or death rate transition, occurs. Suppose that the probability of the event occurring at time t in country i, conditional on not having occurred previously, can be expressed in the following form

$$\Pr(T^i = t | T^i \geq t) = G\left(\sum_{l=0}^{k-1} x_{l,it}\beta_l\right), \tag{2.8}$$

where $G(.)$ is a function bounded between 0 and 1. In the exercise that follows, we will assume that $G(.)$ is the logistic cumulative distribution function in line with the well-known Logit model. In this specification $(x_{0,it}, x_{1,it}, ..., x_{k-1,it})$ are a set of k explanatory variables.

Consider a world populated with N different countries indexed by $i \in \{1, 2, ..., N\}$ for which a set of variables $x_{it} \in X$ is observed time $t = 1$ until T. Let T^i represent the time at which a given one-off event occurs in country i, and let d_{it} be an indicator function taking the value 1 if the event occurs in country i at time t and 0 otherwise. Let

$\Pr(T^i = t | T^i \geq t) = G\left(\sum_{l=0}^{k-1} x_{l,it}\beta_l\right)$ (according to 2.8). The parameters of this model can then be estimated by maximizing the log-likelihood, given below:

$$\log L_N = \sum_{i=1}^{N}\sum_{t=1}^{T_i} \log\left[d_{t,i} G\left(\sum_{l=0}^{k-1} x_{l,it}\beta_l\right) + (1-d_{t,i})(1-G\left(\sum_{l=0}^{k-1} x_{l,it}\beta_l\right))\right]. \quad (2.9)$$

Table 9 summarizes the results of the logit estimation where the only explanatory variable is log GDP per capita. This specification is able to replicate well the distribution of log GDP per capita at the start of the transition as can be seen in Figure 8-A. It does not perform well, however, in replicating the distribution of transition starts over time or in predicting transition start dates for individual countries, as can be seen in Figure 8-B and 8-C.

Table 9: The GDP per capita and Transitions

Variable	Estimates
cons	-23.40
	(1.04)
lnGDPPC	2.54
	(0.14)
LLn	-649.5
Psuedo-R^2	0.178
N. Obs.	50432

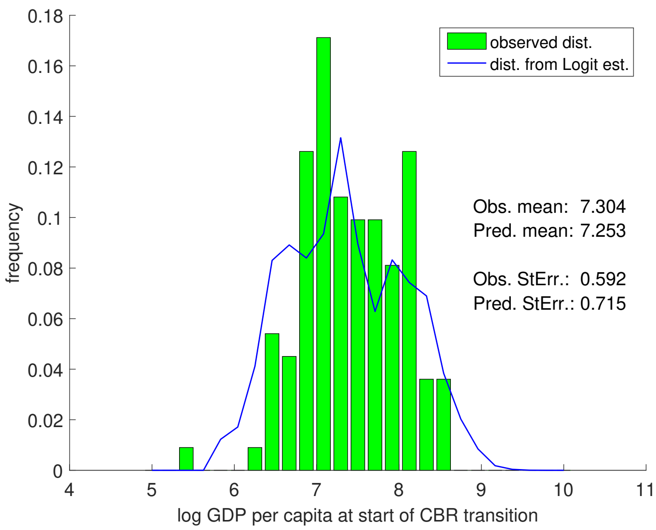


Figure 8-A: Distribution of Log GDP at the Start of the CBR Transitions

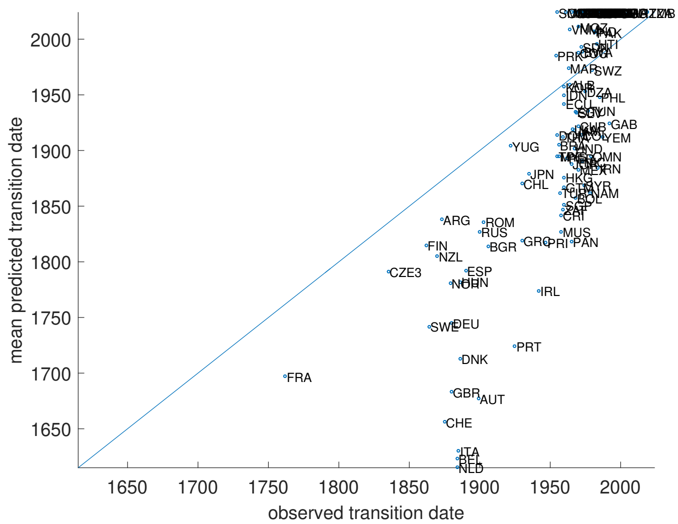


Figure 8-A: Within Sample Predictions

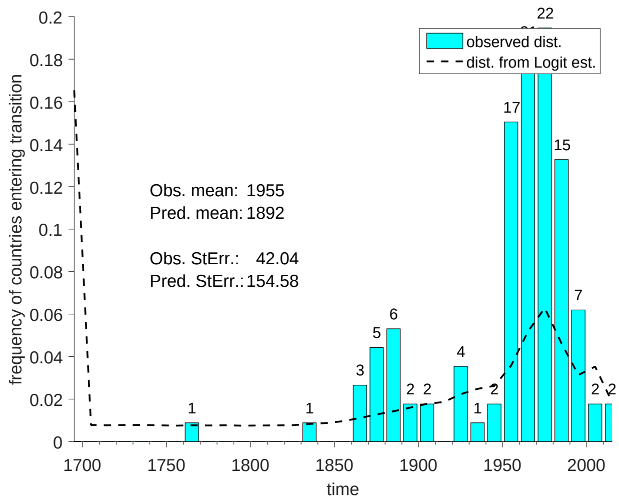


Figure 8-B: Distribution of Transtion Dates

Next we extend the Logit analysis by including network effect. In particular, we estimate the following expression,

$$\Pr(T^i = t | T^i \geq t) = G\left(\sum_{l=0}^{k-1} x_{l,it}\beta_l + \beta_k \left[\sum_{j=1}^{N} g_{ij} d_{t-1,j})\right]^{\psi}\right),$$

where $d_{t,j} = 1$ if the transition has already started in coutry j, and g_{ij} measures the inverse of *the distance* between country i and country j. Hence, if a country j is very far from country i, then g_{ij} is close to zero, and as a result, whether or not country j has already started its transition has no effect on the probablty that country i starts its transition. On the other hand, if country j is close to country i, then whether country $j's$ transition has a posityive efect on the probablity that country i also starts its transition.

Following Melitz and Toubal (2013), we assume that the distance between two countries is given by

$$g_{ij} = \exp\{\mathbf{z}'_{ij}\gamma\},$$

where $\mathbf{z}_{ij}$ is a column vector of bilateral distance measures and γ is a vector of coefficients. The parameter vectors β and γ and the parameter ψ can be estimated using log-likelihood.

2.4.1 Bilateral country distance measures

We borrow data on measures of bilateral geographic, linguistic, religious, and legal distance from Melitz and Toubal (2013), who investigate the effect of each of these distance measures on bilateral trade volumes in the second half of the 20th century.[9] The only

[9] Melitz and Toubal (2013) follow a long literature in international trade that estimates the effects of language and other factors on international trade volumes. Egger and Lassmann (2012) provide a useful overview.

measure of geographic distance we consider is great circle distance between capital cities. Melitz and Toubal (2013) construct and test several alternative measures of the degree of linguistic commonality between countries, ranging from the narrowest definition, simply recording whether the two countries share an official language or not, to more nuanced definitions based on the shares of population in each country that speak the same or similar languages. Here we use the measure based on the broadest definition of linguistic proximity which Melitz and Toubal call "LP2", and which they constructed using data on the distribution of spoken languages and Bakker's (2010) calculation of linguistic similarity. To reflect connections that may exist between countries for historical reasons independently of shared language, we also consider Melitz and Toubal's index of shared religion and a dummy variable for common legal origins. Table 9 displays summary statistics for these variables, and the correlation table is given in Table 10.

Table 9: Distance Measures

Variable	sample mean	st. Dev.	N. Obs.
ln Distance, km *(ldi)*	8.675	0.803	24668
Linguistic proximity *(lp2)*	0.635	0.722	24668
Common religion *(cmr)* $\in \{0,1\}$	0.160	0.218	24668
Common legal system *(cml)* $\in \{0,1\}$	0.211	0.408	24668

Table 10: Distance Measures, Correlations

	lp2	ldi	cmr	cml
Linguistic Proximity (lp2)	1	-0.36	0.25	0.40
ln Distance, km (ldi)	-0.36	1	-0.36	-0.28
Common religion (cmr)	0.25	-0.36	1	0.22
Common legal system (cml)	0.40	-0.28	0.22	1

The linguistic, religious, and legal *proximity* measures (*lp2*, *cmr*, and *cml*) are transformed into *distance* measures by calculating *distance* = 1 - *proximity*. Missing bilateral distances are imputed to take the maximum theoretical value for that distance–1 in the case of 1- *lp2*, 1- *cmr*, and 1- *cml*, and (the natural log of) 20,015 km in the case of great circle distance *(ldi)* between capital cities.[10] Finally, log geographical distance *ldi* is divided by ln 20,015 so that this distance measure, too, is normalized to fall between zero and 1.

[10] The circumference of the Earth is 40,030 kilometers, and so the maximum great circle distance between any two points on the globe is 20,015 kilometers.

2.4.2 Demographic Contagion

Table 11: Determinants of the Start of the CBR Transition

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
cons	-50.40	-89.92	-75.85	-73.18	-65.22	-71.95	-76.85	-73.74	-60.70	-61.81	-61.13
	(10.38)	(13.68)	(13.61)	(13.32)	(12.98)	(13.59)	(13.42)	(13.58)	(13.35)	(13.11)	(13.32
lnGDPPC	9.91	21.33	17.35	16.46	14.20	16.32	17.63	16.76	13.08	13.27	13.19
	(2.81)	(3.70)	(3.70)	(3.59)	(3.52)	(3.67)	(3.63)	(3.68)	(3.60)	(3.55)	(3.61
lnGDPPC ^{2}	-0.50	-1.35	-1.09	-1.04	-0.89	-1.03	-1.11	-1.05	-0.82	-0.83	-0.83
	(0.19)	(0.25)	(0.25)	(0.24)	(0.24)	(0.25)	(0.24)	(0.25)	(0.24)	(0.24)	(0.24
access		0.057	0.49	2.75	7.82	0.50	0.44	0.52	6.61	7.56	6.74
		(0.003)	(0.23)	(0.28)	(1.20)	(0.08)	(0.08)	(0.09)	(0.50)	(0.50)	(0.42
geo dist.				2.82							
				(0.05)							
$<$ 500km					2.34				2.29	2.14	2.25
					(0.32)				(0.27)	(0.33)	(0.30
500-1000km					1.72				1.46	1.64	1.46
					(0.31)				(0.28)	(0.31)	(0.29
1000-2000km					0.66				0.51	0.71	0.56
					(0.30)				(0.27)	(0.29)	(0.27
ling. dist						0.15			0.17		0.16
						(0.01)			(0.10)		(0.11
relig dist							-0.15				
							(0.02)				
legal dist								0.21		0.17	0.04
								(0.02)		(0.24)	(0.24
ψ, curv.			0.56	0.45	0.41	0.57	0.57	0.57	0.46	0.43	0.46
			(0.04)	(0.05)	(0.06)	(0.01)	(0.01)	(0.01)	(0.14)	(0.09)	(0.13
LLn	-645.5	-492.1	-487.1	-485.0	-470.0	-485.6	-487.0	-486.0	-467.8	-469.1	-467.7
Psuedo-R^2	0.183	0.377	0.384	0.386	0.405	0.385	0.384	0.385	0.408	0.406	0.408
N. Obs.	50432	50432	50432	50432	50432	50432	50432	50432	50432	50432	50432

Table 11 shows the results of the logit regression described in the previous section. Specification (1) shows the results of the regression without including any inter-country influence. Specification (2) adds a simple global count of the number of countries that have begun the transition, and specification (3) adds some curvature to that sum, which is still global. The estimated value of ψ, being less than 1, implies that there are diminishing returns–the more countries have already entered the transition, the smaller the effect of each additional country on other countries' odds of entering the transition. Specifications (4) through (11) weight the influence of one transitioned country on other countries according to the inverse distance between them, as determined by various measures of distance. When included by themselves, all 4 measures of distance–geographic, linguistic, religious and legal, have highly significant estimated coefficients, with geographic distance having somewhat more explanatory power (as reflected in the log likelihood sum) than the others. Religious distance has the wrong sign, which means that it is probably correlated with some excluded factor and thus the coefficient does not reflect the real effect of religious distance. Specifications (9), (10) and (11) include more than one measure of distance simultanously. Geographic distance retains a significant coefficient in all of these specifications, while linguistic and legal distance maintain positive, but not quite statistically significant point estimates.

In Figures 9 through 12, we take a closer look at the the Access to Transitions measure implied by specification 11. The distributions displayed in all of these figures are smoothed using a Gaussian kernel. Figure 9 shows the distribution of this measure at different points in time–not surprisingly, as more countries transition, this distribution moves steadily to the right. Figure 10 shows the transition probabilities implied if each country is assigned its actual Access to Transitions value and GDP per capita equal to $2000. Here we can see that in 1850, 1900 and 1950, Access to Transitions in the great majority of countries was such that their probability of transition at only $2000 GDP per capita would have been relatively small. In the year 2000 this situation changes dramatically, and the lowest yearly probability of transition for any country with $2000 GDP per capita would be 10%.

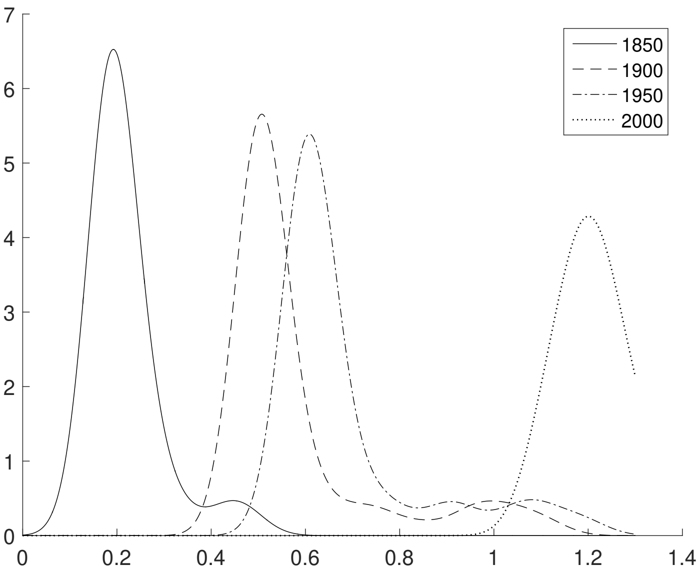


Figure 9: Distribution of "Access to Transitions" variable implied by Specification (11)

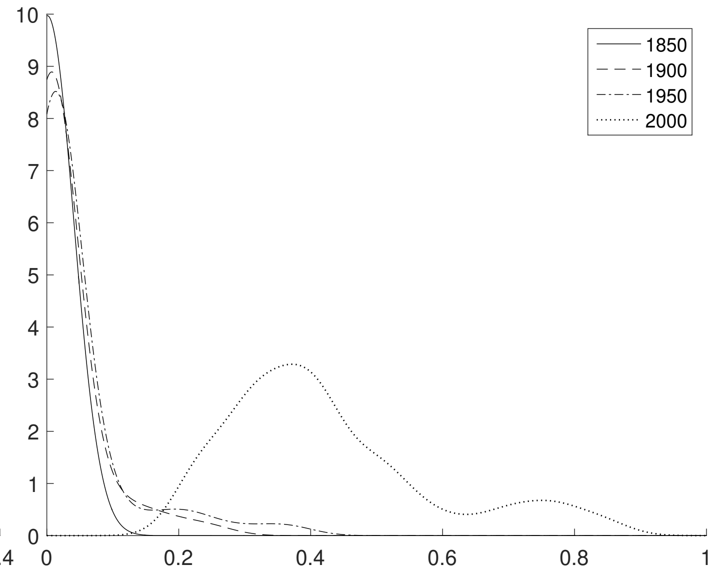


Figure 10: Transition Probability, given Access implied by Spec. (11) and GDP per capita = $2000

Figure 11 shows the evolution of the distribution of GDP per capita over time. Figure 12 shows the distribution of the probability of transition, given the observed GDP per capita for each country, assuming they have the mean level of Access to Transitions for the year 2000. Here we see that, not surprisingly, the distribution shifts right was time passes and more countries enjoy higher levels of GDP per capita. We also see the importance of the complementarity between a country's own level of development and the influence of its neighbors–even assigning all countries the mean level of access for the year 2000, even the richest country only in the year 2000 has a transition probability of 0.6. Compare this to Figure 10, where we see that even with a relatively low level of GPD per capita ($2000), in the year 2000 there are many countries whose probability of transition due to the influence of neighbors would be close to 1.

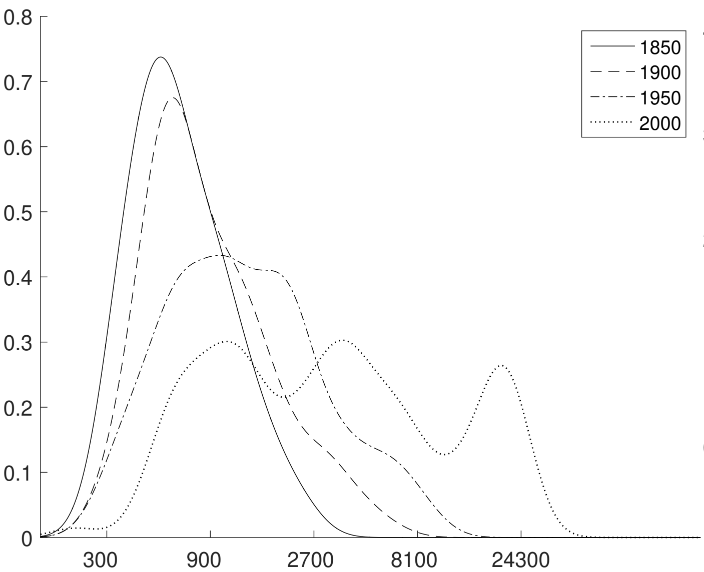


Figure 11: Distribution of log GDP per capita

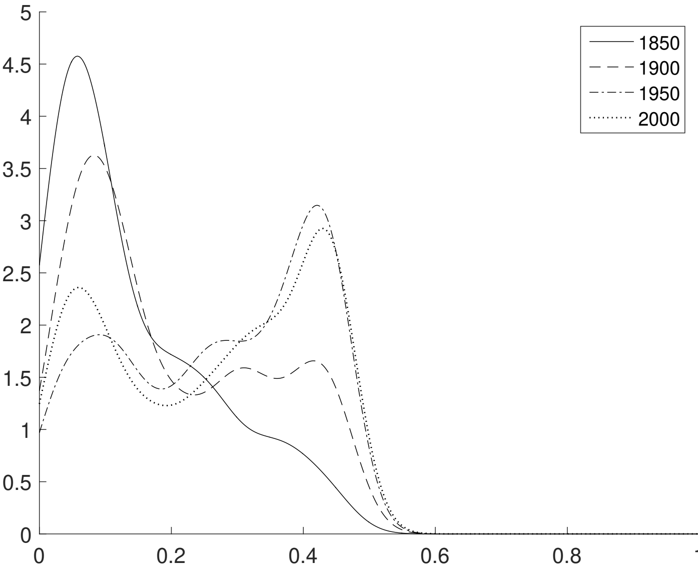


Figure 12: Transition Probability given log GDP per capita, with Access set to year 2000 mean

2.5 Conclusion

In summary, in this section we have documented three empirical findings. First, transitions in both fertility and mortality have been getting faster over time, according to all available measures. Second, in spite of this increase in the speed of the transitions, there is no clear trend in the level of GDP per capita at which countries enter the demographic transition. Finally, we have found suggestive evidence for a kind of "demographic contagion," whereby a transition in one country is statistically associated with following transitions in countries which are close to it geographically and linguistically, and have similar legal systems.

Chapter 3

The Diffusion of Demography: A Quantitative Exploration

In this chapter I build a model that can account for the facts about the global demographic transition that were documented in the previous chapter. I consider an economy with multiple locations. Each location is populated by a representative household that decides how many children to have and how much to invest in their education. Having and educating children is costly. A production technology combines unskilled and skilled labor. The economy is initially in a Malthusian steady state with high but constant levels of mortality and fertility. At a certain point in time, technological progress becomes skill biased. This occurs first in the frontier country, Britain in our analysis, and then diffuses slowly to other locations. Skill-biased technological progress makes investment in children more valuable and parents react by reducing the number of children but educating them better. We first calibrate the model economy to replicate the demographic transition in Britain. We then show that a simple mechanism of diffusion where skill-biased technological change travels from Britain to the rest of the world in a manner that only depends on geographic distance is able to generate sequences of demographic transitions, each happening faster than the previous one, exactly as we observe in the data.

Understanding the relationship between income and population is one of the oldest challenges in economics, going back to Malthus (1803) who developed a powerful model that links better technology with constant living standards. In a Malthusian world, technological change allows a higher income per capita which leads to higher population through higher fertility and lower mortality. In the presence of a fixed input such as land, this higher population translates into lower marginal productivities that decrease per capita income back to the stationary level previous to the technological advance. Malthus' model is quite successful at accounting for the main facts that prevailed until the nineteenth century, but it fails to explain the coexistence of growth in per capita income and low fertility. Becker (1960) and Becker and Lewis (1973) develop the idea of a trade-off between quantity and quality of children to show that higher per capita incomes and low fertility can go together. The interest in this mechanism was revived with the presentation of an operational dynastic model of fertility in Barro and Becker (1989) and Becker and Barro (1988).

Building on this initial work, Becker, Murphy and Tamura (1990), Lucas (1998), Jones

(2001) and especially in an important contribution, Galor and Weil (1996, 1999, 2000) present models that try to capture the historical evolution of population and output. Several recent papers, e.g. Fernandez-Villaverde (2001), Kalemli-Ozcan (2003), Doepke (2004) and Bar and Oksana (2010), present quantitative versions of these models that can account for historical evidence on demographic transitions for specific countries. Jones, Schoonbroodt and Tertilt (2011) and Greenwood, Guner and Vandenbroucke (2017) provide recent reviews.

Few recent papers study the historical evolution of fertility. Spolaore and Wacziarg (2014) document that genetic and linguistic distance from France was associated with the onset of the fertility transition in Europe. De la Croix and Perrin (2017) focus on the fertility and education transition in France during the 19th century, and show that a simple quality-quantity model can do a decent job in explaining variations of fertility across time and counties in France. De Silva and Tenreyro (2017) focus on post-1960 transitions and emphasize the role of social norms and family planning programs in recent declines in fertility rates in developing countries. This study is also related to recent studies that provide an empirical analysis of demographic transitions across countries. Reher (2004) looks at a broad panel of countries and compares earlier with later demographic transitions, with a particular focus on the role of mortality in driving fertility changes. Murtin (2013) also constructs a panel and .finds evidence for a robust effect of early childhood education on fertility decline. Building on these earlier contributions, this study is the first to detect empirically a "demographic contagion" effect at a global scale, and to investigate it within a quantitative framework.

Finally, by proposing technology diffusion as a mechanism linking the process of the demographic transition in different countries, this analysis also borrows from recent literature on technology diffusion, such as Lucas (2009) and Comin and Hobijn (2010).

3.1 Model

In this section we build a model of endogenous fertility, education, and technology diffusion with the goal of accounting for the trends we have documented. In this model there will be a quantity-quality trade-off between how many children to have and how much to educate them, following classic work by Barro and Becker (1989). We propose an economy with a skilled and an unskilled sector, as in Acemoglu (2002). An exogenous increase in the ratio of skilled to unskilled TFP raises the skill premium and induces parents invest in a smaller number of more educated children. In order to link fertility patterns across countries, we introduce technology diffusion in a manner similar to Lucas (2009), and allow the elasticity of catch-up growth to differ between the skilled and unskilled sectors. We show that if this elasticity is higher in the skilled sector, the skill premium will rise more sharply in countries that begin converging to the frontier later, leading to faster fertility transitions.

3.1.1 Consumer Preferences, Fertility, and Education Decisions

Consider a world that consists of different locations. Consumers in each location i live for two periods, one as children and one as adults. As children, consumers are under the care of their parents. As adults, they work, consume and choose how many children to have, n_{it}, and how much education, e_{it}, to provide for each of them. With an exogenous probability s_{it} a child survives to the adulthood.

Each unit of children requires a time commitment of τ_1, for a total time cost of $n_{it}\tau_1$. To achieve a level of education e_{it} for each child, parents must pay a total time cost of $n_{it}e_{it}\tau_2$. The level of education that children receive will determine their level of human capital when they are adults, given by

$$h_{i,t+1} = e_{it}.$$

Adults have a total time endowment of 1. They do not value leisure, and so supply $1 - \tau_1 n_{it} - \tau_2 n_{it} e_{it}$ units of time to the labor market. The income that parents receive per unit of labor depends on the equilibrium unskilled and skilled wages, w_{it}^U and w_{it}^S, and their level of human capital, h_{it}. In exchange for each unit of labor supplied, adults receive income

$$y_{it} \equiv w_{it}^U + h_{it} w_{it}^S.$$

Parents choose c_{it}, e_{it}, and n_{it} to maximize

$$\log(c_{it} - \bar{c}_i) + \gamma \log(s_{it} n_{it}) + \beta \log y_{i,t+1},$$

subject to

$$c_{it} = (1 - n_{it}(\tau_1 + \tau_2 e_{it}))y_{it},$$

and

$$y_{it+1} \equiv w_{it+1}^U + h_{it+1} w_{it+1}^S \text{ with } h_{it+1} = e_{it},$$

where $\bar{c}_i$ is a minimum consumption requirement..

Define the skill premium at time t as $\phi_{it} \equiv \frac{w_{it}^S}{w_{it}^U}$. Then the first order conditions of this problem are given by

$$\frac{[\tau_1 + \tau_2 e_{it}]}{1 - \frac{\bar{c}_i}{y_{it}} - [\tau_1 + \tau_2 e_{it}]\, n_{it}} = \gamma \frac{1}{n_{it}},$$

for n_{ii} and by

$$\frac{\tau_2 n_{it}}{1 - \frac{\bar{c}_i}{y_{it}} - [\tau_1 + \tau_2 e_{it}]\, n_{it}} = \beta \frac{1}{\frac{1}{\phi_{i,t+1}} + e_{it}},$$

for e_{it}. With simple algebra, the optimal decisions for e_{it} and n_{it} are given by

$$e_{it} = \frac{\frac{\beta}{\gamma}\frac{\tau_1}{\tau_2} - \frac{1}{\phi_{i,t+1}}}{1 - \frac{\beta}{\gamma}},$$

and

$$n_{it} = \frac{\gamma}{1+\gamma}\left(1 - \frac{\bar{c}_i}{y_{it}}\right)\frac{1}{\tau_1 + \tau_2 e_{it}}.$$

The human capital investment decision, e_{it}, is increasing in $\phi_{i,t+1}$ (the skill premium) and in $\tau_1$and is decreasing in τ_2. The number of children, n_{it} is decreasing in τ_1, τ_2 and e_{it}; and decreasing in $\bar{c}_i$

3.1.2 Production and Technology Diffusion

Time-t output for country i, Y_{it} is given by

$$Y_{it} = [(A_{it}S_{it})^{\rho} + (B_{it}[aL_{it}^{\omega} + (1-a)U_{it}^{\omega}]^{\frac{1}{\omega}})^{\rho}]^{\frac{1}{\rho}},$$

where where S_t represents the quantity of skilled labor employed and A_{it} represents the productivity of skilled labor, L_{it} represents the land endowment, U_{it} represents the quantity of unskilled labor employed, and B_{it} represents the productivity of the land and unskilled labor aggregate, and where $\frac{1}{1-\omega}$ represents the elasticity of substitution between land and unskilled labor and $\frac{1}{1-\rho}$ represents the elasticity of substitution between skilled labor and the land and unskilled labor aggregate.[1]

Factor shares for skilled labor and the land and unskilled labor aggregate are $\frac{A_{it}}{A_{it}+B_{it}}$ and $\frac{B_{it}}{A_{it}+B_{it}}$ respectively, and TFP $\tilde{A}_{it}$ can be defined as

$$\tilde{A}_{it} \equiv A_{it} + B_{it}.$$

Given this production technology, the skill premium is given by

$$\phi_{it} = \frac{w_{it}^S}{w_{it}^U} = \left(\frac{A_{it}}{B_{it}}\right)^{\rho} \frac{S_{it}^{\rho}}{[aL_{it}^{\omega} + (1-a)U_{it}^{\omega}]^{\frac{\rho}{\omega}-1}(1-a)U_{it}^{\omega-1}}.$$

The world is composed of 1 frontier country, indexed as country 0, and n following countries in the set $N \equiv \{1, 2, ..., n\}$. Time is discrete, indexed by $t \in \{0, 1, 2, ...\}$. The effective distance of each follower from the frontier country at each point in time, d_{it} is a function of a time-invariant geographic distance d_i^g, a time-invariant linguistic and/or cultural distance, d_i^c, and potentially time-varying idiosyncratic barriers to the diffusion of information represented by $\phi_{0i}(t)$:

$$d_{it} = \phi_{oi}(t) + \phi_1(t)d_i^g + \phi_2(t)d_i^c,$$

The parameters $\phi_l(t)$ for $l \in \{1, 2\}$ are shared across countries and may vary over time. In particular, it is assumed that these parameters decline at a constant rate from their initial values:

$$\phi_j(t+1) = \phi_j(t)(1 - g_{\phi_j}) \text{ for } j \in \{1, 2\}.$$

The idiosyncratic barriers term, $\phi_{0i}(t)$, can be thought of as reflecting how "open" or "closed" country i is in terms of its policies and other non-geographical, non-linguistic factors that might affect knowledge flows into country i.

There are frontier levels of skilled and unskilled productivity, denoted $\bar{A}_t$ and $\bar{B}_t$ respectively. These are assumed to have the constant values $\bar{A}_0$ and $\bar{B}_0$ for all periods $t \in \{..., -3, -2, -1, 0\}$. There is a frontier country, aka Great Britain, indexed as country 1, which has the lowest barriers to diffusion of the frontier levels of technology. It is assumed that they do coincide for all periods leading up to period 0, prior to the start of

[1] This production function follows the setup used in Fernandez-Villaverde (2001), with skilled and unskilled sectors as in Acemoglu (2002).

frontier technology growth, the technology levels in the frontier and the frontier country are the same: $A_{0t} = \bar{A}_0$ and $B_{0t} = \bar{B}_0$ for all $t \in \{..., -3, -2, -1, 0\}$.

At time 1, frontier skilled labor productivity makes an unanticipated discrete jump to $\bar{A}_1 > \bar{A}_0$, while frontier unskilled productivity retains its former value $\bar{B}_1 = \bar{B}_0$. Starting in period 2, the growth rates for both types of productivity experience an unanticipated, discrete jump from 0 to g_A, such that for periods $t \in \{2, 3, 4, ...\}$,

$$\bar{A}_t = (1 + g_A)\bar{A}_{t-1}$$

and

$$\bar{B}_t = (1 + g_B)\bar{B}_{t-1}.$$

For all time periods, productivities in each country grow at a rate that depends on their barriers to the frontier d_{it} and their distance from the frontier level of productivity, in accordance with the following laws of motion, inspired by Lucas (2009):

$$A_{i,t+1} = A_{it}\left(1 + g_B e^{-\zeta_B d_{it}} \frac{\overline{A}_t}{A_{it}}\right)^{\theta_1},$$

and

$$B_{i,t+1} = B_{it}\left(1 + g_B e^{-\zeta_B d_{it}} \frac{\overline{B}_t}{B_{it}}\right)^{\theta_2}$$

where $\zeta_A, \zeta_B > 0$ are parameters which determine the elasticity of the diffusion of each type of technology to distance. The elasticity of the growth rates to distance to the frontier is allowed to vary for the two types of producitivites–$\theta_1 \neq \theta_2$ in general.

3.1.3 Vital Statistics

Childhood survival rates are determined by the overall level of technology in a country, according to the following formula:

$$s_{it} = 1 - \frac{1 - s_i^0}{(A_{it} + B_{it})^{\zeta_s}}$$

where $\zeta_s > 0$. The CBR is given by

$$B_{it} = \frac{U_{it} n_{it}}{U_{it} + U_{it} s_{it} n_{it}} = \frac{n_{it}}{1 + s_{it} n_{it}}.$$

Similarly, the CDR is given by

$$D_{it} = \frac{U_{it} + U_{it} n_{it}(1 - s_{it})}{U_{it} + U_{it} s_{it} n_{it}} = \frac{1 + n_{it}(1 - s_{it})}{1 + s_{it} n_{it}}.$$

Finally, the population growth is given by

$$B_{it} - D_{it} = \frac{n_{it} s_{it} - 1}{1 + s_{it} n_{it}}.$$

3.2 Quantitative exercise

Now suppose we are in a world in which period 0 is 1775 and in which a model period lasts 25 years, and that there are 7 countries in the world: a frontier country (Great Britain), assumed to be on average effectively 50 kilometers from the notional "frontier" contained within its borders (in for example, London), a country that is 312.5 kilometers away (like Paris, France from London, England), a country that is 625 kilometers away (like Geneva, Switzerland), a country that is 1250 kilometers away (like Vienna, Austria), a country that is 2500 kilometers away (like Moscow, Russia), a country that is 5000 kilometers away (like Baghdad, Iraq), and a country that is 10000 kilometers away (like Manila, Philippines).

Distances d_{it} are a function of physical distance only:

$$d_{it} = \phi_t d_i^g,$$

where d_i^g represents the physical distance in kilometers between London, United Kingdom, and the capital city of country i.

Suppose that all of these countries are initially identical in all aspects other than their distance from the frontier, and that they are all initially in a population steady state in which total births equal total deaths. In period 0, frontier technology starts growing, and the importance of distance for diffusion starts falling, in the manner described in the previous section. Table 7 shows the parameter values. Figure 11 shows the model fit for Britain.

Table 7: Parameter Values

Parameter	Values
Preferences	$\beta = 0.8, \gamma = 1, \bar{c}_i = 2$
Cost of children	$\tau_1 = .12, \tau_2 = .02$
Technology	$\rho = .8, \rho = .8, \omega = .1$
Diffusion	$\theta_1 = .8, \theta_2 = .4, \zeta_A = 1, \zeta_B = .25, g_A = .5$
	$\phi_0 = \exp(2), g_{\phi_t} = .4895$
Mortality	$s_{i0} = .5$, $\xi_s = 2$

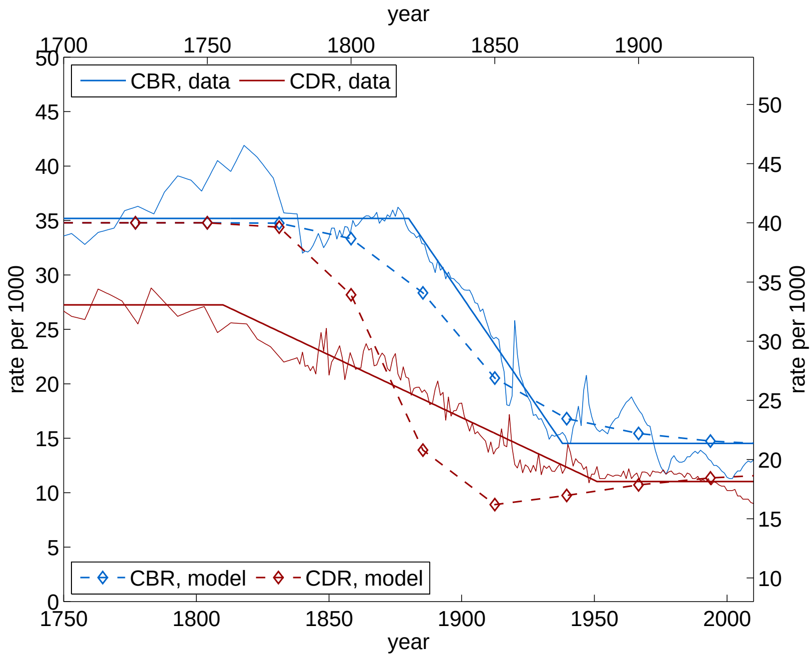


Figure 11: Great Britain, model vs. data

Figure 12-A plots the pattern of the evolution of technology in the frontier country described in Section 6.2, in which both types of TFP begin growing, but skilled-complementary TFP experiences an initial discrete jump. Figure 12-B shows how effective distance between the frontier country and the rest shrinks over time. As can be seen in the figure, the different countries become more and more similar in their levels of access to the frontier over time. Figure 13 shows the evolution of technology in two places, 625 km from London (Geneva) and 10,000 km from London (Manila). As can be seen figure, both countries initially experience no growth, even after growth has begun in the frontier. As the cost of distance falls, each country experiences a discrete growth takeoff, with the closer country taking off first. Catch-up growth induces a temporary oscillation of the ratio of skilled to unskilled TFP above its frontier, long-run level in each country. This is due to the assumption that $\zeta_A > \zeta_B$, so that the catch-up growth is more elastic in response to the gap to the frontier in skilled than unskilled technology. In Manila, which takes off later, catch-up growth is more rapid, and this oscillation is larger and of greater duration.

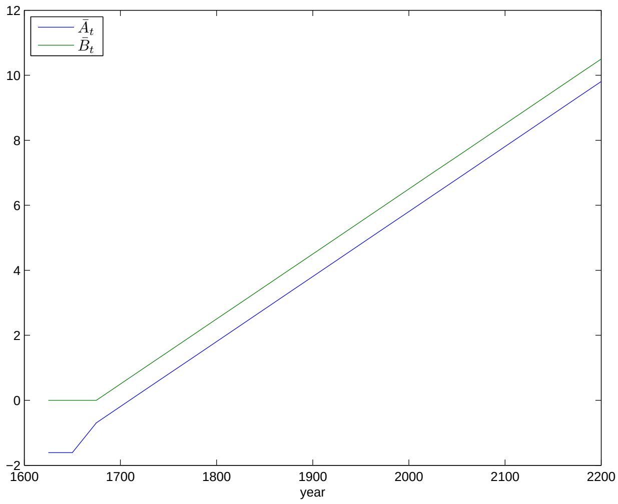


Figure 12-A: Technology Frontier

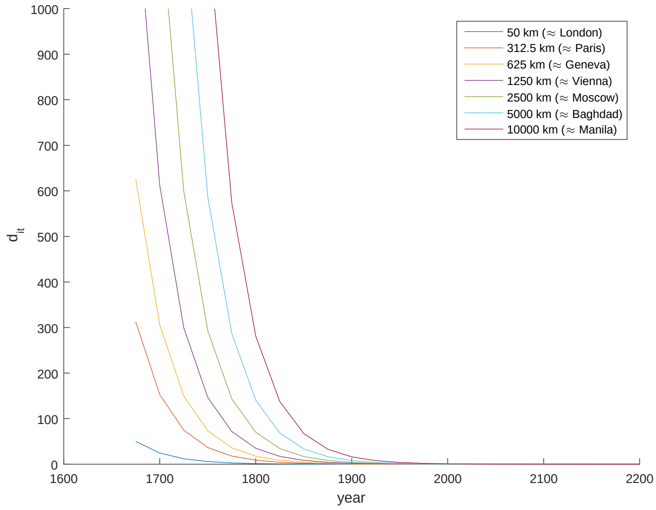


Figure 12-B: Distance from the Frontier

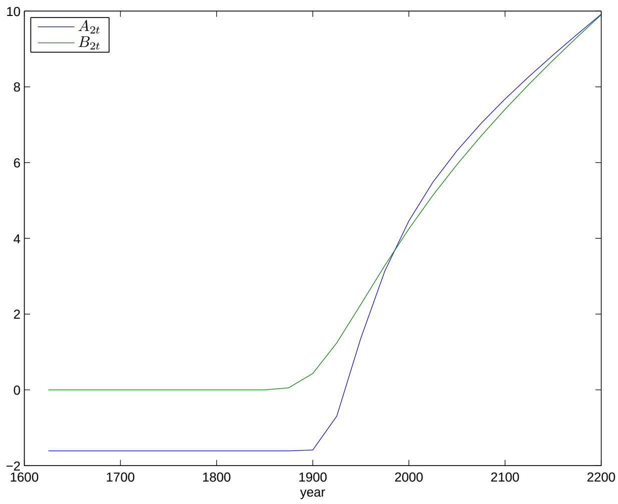


Figure 13-A: Technology in Geneva

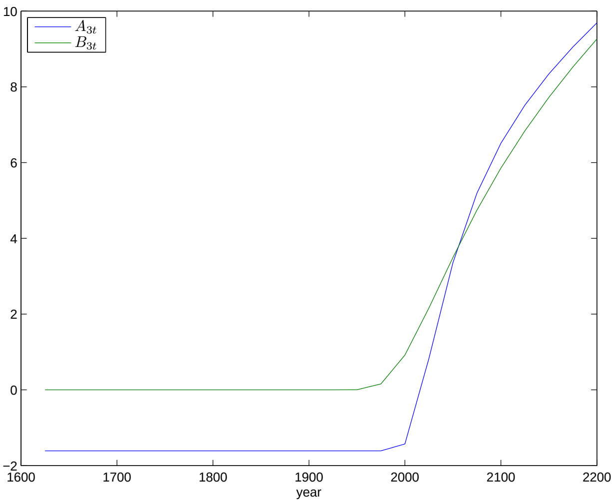


Figure 13-B: Technology in Manila

As technology improves and diffuses to other countries, skill premium start to rise in each of these locations. As a result, parents choose higher and higher levels education for their children. Figure 14-A plots the evolution of the skill premium in the various notional countries, and Figure 14-B plots the evolution of education levels. Because of higher elasticity of catch-up growth to technological gap in the skilled sector, the skill premium rises faster in later-transitioning countries, and so the increase in education levels is also more rapid.

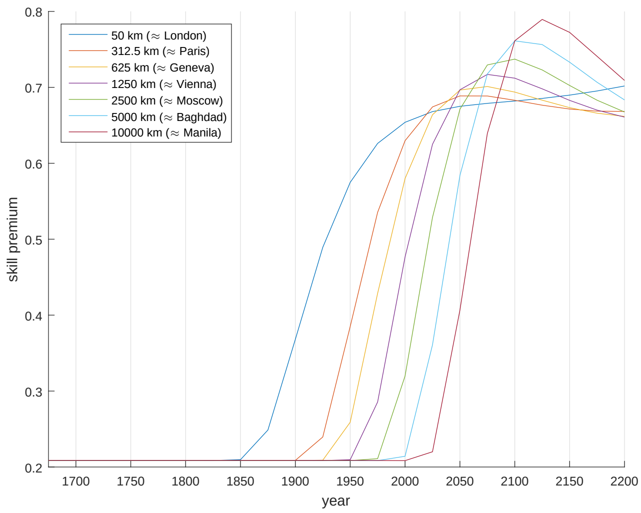


Figure 14-A: Skill Premium

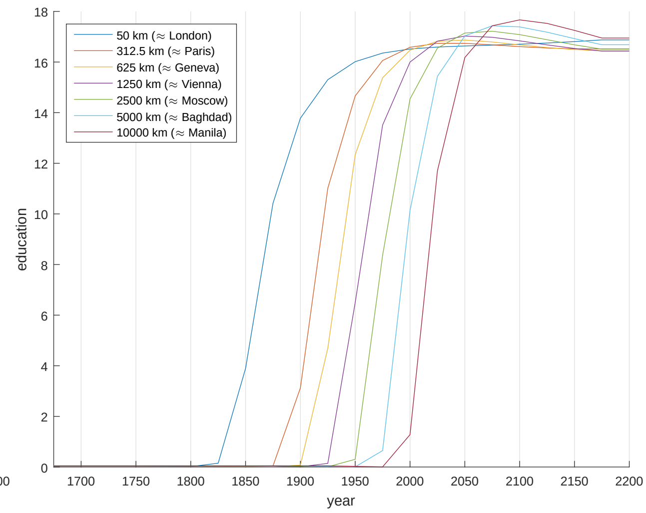


Figure 14-B: Education

As parents educate their children more, they also produce fewer children overall–the classic quantity-quality tradeoff. Figure 15-A shows the simulated path of the crude birth rate for the modeled countries. Because the rise in education levels is sharper in later-transitioning countries, so the fall in fertility is also more rapid, and the overall transition period shorter. Figure 15-B shows the length of each simulated transition. These vary in length from more than 120 years for the frontier country, to less than 80 years for the last model country to enter the transition.

Figure 16 compares the simulated transition lengths with transition lengths observed in the data. Here we see that this quantitative exercise is able to replicate the overall trend of accelerating transitions, and is able to account for roughly half of the overall decline in transition length over the observed period.

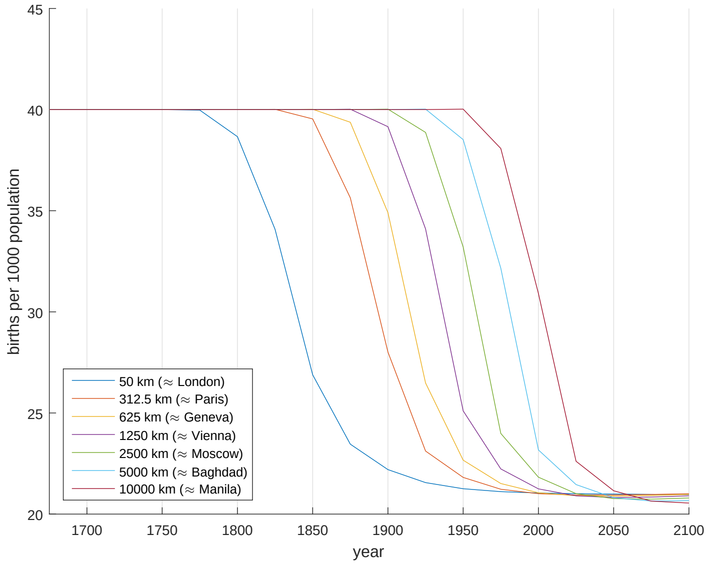


Figure 15-A: Fertility Transitions

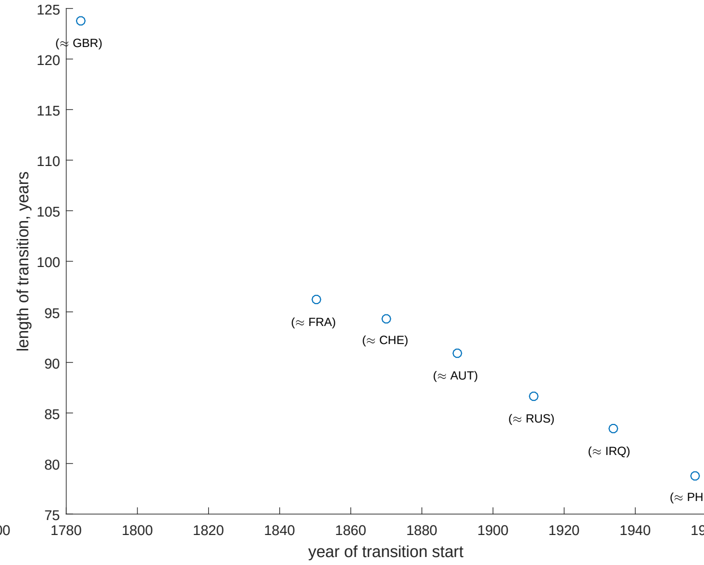


Figure 15-B: Transition Length

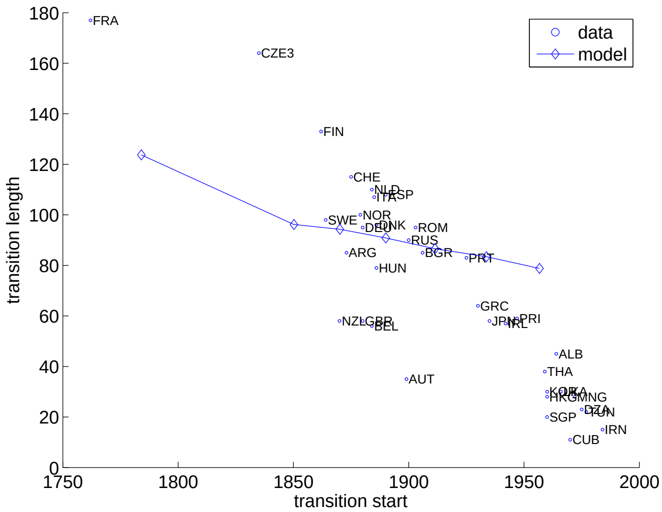


Figure 16: Transition Lenghts, Model vs. Data

3.3 Conclusion

In this paper we have constructed a dataset consisting of birth rates and death rates, and GDP per capita for a panel of 188 countries and spanning from 1735 until 2014. We have proposed a way of measuring demographic transitions which lets the data pick likely start and end dates for fertility and mortality transitions, and used our results to show that: 1) transitions are becoming faster, 2) the average level of GDP per capita at the start of a transition is more or less constant, 3) an important predictor of a country's transition is the prior transition of other countries which are "close" to it in a geographical and a linguistic sense, and which have similar legal systems.

We then build a model in the tradition of Barro, Becker and Lucas that can account for these facts. In addition to the standard quantity-quality trade-off between how many children to have and how much to educate them, there is also technological diffusion between countries. We conduct a quantitative exercise to show that a simple mechanism of diffusion where skill-biased technological change travels from Britain to the rest of the world in a manner that only depends on geographic distance is able to generate sequences of demographic transitions, each happening faster than the previous one, as we observe in the data, and the account for roughly half of the observed reduction in total transition time.

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Chapter A

Appendix

A.1 Supplementary Tables

⋆ indicates a CDR calculated by projecting backwards using the method described in section 2.

Table A1: Calculated Transition Start and End Dates

	CDR		CBR	
Country	Start	End	Start	End
Afghanistan	1962	n/a	2000	n/a
Angola	1943⋆	n/a	1992	n/a
Albania	1944	1974	1964	2009
United Arab Emirates	1938⋆	1977	1956	n/a
Argentina	1869	1945	1873	1958
Armenia	n/a	n/a	n/a	2001
Australia	n/a	1960	n/a	1987
Austria	1881	1941	1899	1934
Azerbaijan	n/a	1965	n/a	2000
Burundi	1893⋆	n/a	1985	n/a
Belgium	n/a	1955	1884	1940
Benin	1951⋆	2005	1989	n/a
Burkina Faso	1934⋆	n/a	1987	n/a
Bangladesh	1973	1996	1975	n/a
Bulgaria	1918	1949	1906	1991
Bahrain	1925⋆	1979	1959	n/a
Bahamas, The	n/a	1967	n/a	n/a
Bosnia and Herzegovina	n/a	1963	n/a	2000
Belarus	n/a	n/a	n/a	1999
Belize	1911⋆	1975	1982	n/a
Bolivia	1964	1997	1968	n/a
Brazil	1866⋆	1992	1956	n/a
Barbados	1923	1960	1952	1996
Brunei Darussalam	1912⋆	1976	1954	n/a
Bhutan	1942⋆	2006	1977	n/a

Calculated Transition Start and End Dates				
	CDR		CBR	
Country	Start	End	Start	End
Botswana	1922*	1977	1973	n/a
Central African Republic	1960	1979	1984	n/a
Canada	n/a	1955	n/a	2008
Switzerland	1872	1950	1875	1990
Channel Islands	1978	n/a	1962	n/a
Chile	1921	1979	1930	n/a
China	n/a	1972	n/a	2005
Cote dIvoire	n/a	1983	1969	n/a
Cameroon	1935*	1984	1980	n/a
Congo, Rep.	1940*	1973	1970	n/a
Colombia	1902	n/a	1972	n/a
Comoros	1921*	1997	1974	n/a
Cape Verde	1963	1999	1984	n/a
Costa Rica	1920	1977	1958	n/a
Cuba	1898*	1946	1970	1981
Cyprus	1922	1955	1945	2009
Czech Countries/Czechoslovakia	1867	1951	1835	1999
Germany	1880	1932	1880	1975
Djibouti	1927*	2000	1971	n/a
Dominica	1917*	1975	1961	2014
Denmark	1834	1943	1886	1982
Dominican Republic	1915*	1982	1955	n/a
Algeria	1926*	1993	1975	1998
Ecuador	1896*	1993	1960	n/a
Egypt, Arab Rep.	1934	1999	1968	n/a
Eritrea	1933*	n/a	1961	n/a
Spain	1890	1960	1890	1998
Estonia	n/a	n/a	n/a	n/a
Ethiopia	1928*	n/a	1992	n/a
Finland	1866	1957	1862	1995
Fiji	1867*	1977	1964	n/a
France	n/a	1990	1762	1939
Micronesia, Fed. Sts.	1845*	1987	1970	n/a
Gabon	n/a	1991	1992	n/a
United Kingdom	1810	1951	1880	1938
Georgia	n/a	n/a	n/a	2002
Ghana	1897*	2001	1967	n/a
Guinea	1964	2014	1982	n/a
Gambia, The	n/a	1993	1980	n/a
Guinea-Bissau	1944*	n/a	1992	n/a
Equatorial Guinea	n/a	n/a	n/a	n/a
Greece	1916	1955	1930	1994
Grenada	1914*	1966	1957	n/a

Calculated Transition Start and End Dates				
	CDR		CBR	
Country	Start	End	Start	End
Guatemala	1917	1997	1960	n/a
Guam	n/a	n/a	n/a	n/a
Guyana (British Guiana)	1919	1958	1969	n/a
Hong Kong SAR, China	n/a	1949	1960	1988
Honduras	1924*	1992	1967	n/a
Croatia	n/a	n/a	n/a	2002
Haiti	1930*	2001	1983	n/a
Hungary	1875	1943	1886	1965
Indonesia	1937*	1985	1960	n/a
India	n/a	2010	1981	n/a
Ireland	1899	2014	1942	1999
Iran, Islamic Rep.	1922*	1999	1984	1999
Iraq	n/a	1995	n/a	n/a
Iceland	n/a	n/a	1963	n/a
Israel	n/a	1945	n/a	n/a
Italy	1874	1955	1885	1992
Jamaica	1920	1966	1967	n/a
Jordan	1929*	1981	1965	n/a
Japan	1945	1952	1935	1993
Kazakhstan	n/a	1970	n/a	1996
Kenya	1930*	1980	1972	n/a
Kyrgyz Republic	n/a	1988	n/a	n/a
Cambodia	1980	1989	1987	n/a
Kiribati	n/a	1964	n/a	n/a
Korea, Rep.	1949*	1970	1960	1990
Kuwait	1848*	1982	1975	1992
Lao PDR	1923*	n/a	1987	n/a
Lebanon	n/a	1971	n/a	n/a
Liberia	1992	n/a	1985	n/a
Libya	1918*	1993	1980	1991
St. Lucia	1904*	1980	1969	2006
Sri Lanka	1930	1970	1966	1996
Lesotho	1931*	1979	1976	n/a
Lithuania	n/a	n/a	n/a	2003
Luxembourg	n/a	n/a	n/a	n/a
Latvia	n/a	n/a	n/a	n/a
Macao SAR, China	n/a	1970	n/a	1969
Morocco	1927*	1993	1963	n/a
Moldova	n/a	n/a	n/a	2008
Madagascar	1934*	n/a	1974	n/a
Maldives	1961	1998	1981	n/a
Mexico	1905	1982	1970	n/a
Macedonia, FYR	n/a	1966	n/a	2004

Calculated Transition Start and End Dates				
	CDR		CBR	
Country	Start	End	Start	End
Mali	1962	2014	2004	n/a
Malta	n/a	n/a	n/a	1983
Myanmar	1930*	1991	1961	n/a
Mongolia	1902*	2003	1972	2000
Mozambique	1936*	n/a	1970	n/a
Mauritania	1937*	1986	1965	n/a
Mauritius	1930	1965	1958	n/a
Malawi	1930*	n/a	1980	n/a
Malaysia	1914*	1975	1956	n/a
Namibia	1931*	1984	1978	n/a
New Caledonia	1843*	1995	1968	n/a
Niger	1987	n/a	1991	n/a
Nigeria	1911*	n/a	1978	n/a
Nicaragua	1910*	1997	1973	n/a
Netherlands	1869	1932	1884	1994
Norway	n/a	1954	1879	1979
Nepal	1942*	2006	1985	n/a
New Zealand	n/a	n/a	1870	1928
Oman	1943*	1989	1979	n/a
Pakistan	1928*	1990	1982	n/a
Panama	1856*	1983	1965	n/a
Peru	1929*	1989	1962	n/a
Philippines	1892*	1980	1985	n/a
Papua New Guinea	1948*	1986	1972	n/a
Poland	n/a	1957	n/a	2004
Puerto Rico	1932	1954	1947	2006
Korea, Dem. Rep.	n/a	1960	1954	n/a
Portugal	1919	1959	1925	2008
Paraguay	n/a	1995	n/a	n/a
French Polynesia	1870*	1988	1962	n/a
Qatar	n/a	1973	n/a	n/a
Romania	1902	1963	1903	1998
Russian Federation	1891	1951	1900	1990
Rwanda	1996	n/a	1980	n/a
Saudi Arabia	1938*	1988	1974	n/a
Sudan	1921*	2011	1972	n/a
Senegal	1935*	2012	1973	n/a
Singapore	1910	1961	1960	1980
Solomon Islands	1871*	n/a	1980	n/a
Sierra Leone	n/a	n/a	n/a	n/a
El Salvador	1885*	1996	1969	n/a
Somalia	1930*	n/a	1955	n/a
Suriname	n/a	n/a	1964	n/a

Calculated Transition Start and End Dates				
	CDR		CBR	
Country	Start	End	Start	End
Slovak Republic	n/a	n/a	n/a	2004
Slovenia	n/a	n/a	n/a	1998
Sweden	1773	1950	1864	1962
Swaziland	1932⋆	1980	1980	n/a
Seychelles	1872⋆	1980	1965	2001
Syrian Arab Republic	1920⋆	1987	1974	n/a
Chad	n/a	1985	n/a	n/a
Togo	1932⋆	1996	1976	n/a
Thailand	1909⋆	1979	1959	1997
Tajikistan	n/a	1984	n/a	n/a
Turkmenistan	n/a	1982	n/a	n/a
Tonga	1806⋆	1974	1963	n/a
Taiwan	n/a	1966	1955	n/a
Trinidad and Tobago	1897	1968	1962	2003
Tunisia	1968	1989	1977	1999
Turkey	n/a	1994	1957	n/a
Tanzania	n/a	n/a	n/a	n/a
Uganda	1933⋆	1970	1999	n/a
Ukraine	n/a	n/a	n/a	1999
Uruguay	n/a	1940	n/a	1942
United States	n/a	1955	n/a	1979
Uzbekistan	n/a	1989	n/a	n/a
St. Vincent and the Grenadines	1954	1983	1960	n/a
Venezuela, RB	1915	1976	1975	n/a
Virgin Islands (U.S.)	n/a	n/a	1964	n/a
Virgin Islands (U.S.)	n/a	1974	n/a	n/a
Vietnam	1941⋆	1995	1964	n/a
Vanuatu	n/a	1999	n/a	n/a
Samoa	n/a	1980	n/a	n/a
Yemen, Rep.	1940⋆	2002	1987	n/a
Serbia (Yugoslavia from 1900)	1876	1964	1922	n/a
South Africa	1919⋆	1978	1959	n/a
Congo, Dem. Rep.	1868⋆	2014	2001	n/a
Zambia	n/a	1969	n/a	n/a
Zimbabwe	1939⋆	1967	1970	n/a

Table A2: Country assignment to minor regions

Country	Min. Region #	Min. Region Name
Afghanistan	11	south asia
Angola	22	middle africa
Albania	33	central and eastern europe

(continued on next page)

Country assignment to minor regions (continued)		
Country	Min. Region #	Min. Region Name
UAE	12	middle east-asia minor
Argentina	42	southern south america
Armenia	12	middle east-asia minor
Australia	52	australia and pacific
Austria	33	central and eastern europe
Azerbaijan	12	middle east-asia minor
Burundi	22	middle africa
Belgium	31	northwestern europe
Benin	24	west africa
Burkina Faso	24	west africa
Bangladesh	11	south asia
Bulgaria	33	central and eastern europe
Bahrain	12	middle east-asia minor
Bahamas, The	41	central america and carribean
Bosnia & Herz.	33	central and eastern europe
Belarus	33	central and eastern europe
Belize	41	central america and carribean
Bolivia	42	southern south america
Brazil	42	southern south america
Barbados	41	central america and carribean
Brunei Dar.	51	southeast asia
Bhutan	60	east asia
Botswana	21	southern africa
CAR	22	middle africa
Canada	70	north america
Switzerland	31	northwestern europe
Channel Islands	31	northwestern europe
Chile	42	southern south america
China	60	east asia
Cote d'Ivoire	24	west africa
Cameroon	24	west africa
Congo, Rep.	22	middle africa
Colombia	41	central america and carribean
Comoros	23	east africa
Cape Verde	24	west africa
Costa Rica	41	central america and carribean
Cuba	41	central america and carribean
Cyprus	33	central and eastern europe
Czech C./-slvk/Rep.	33	central and eastern europe
Germany	31	northwestern europe
Djibouti	23	east africa
Dominica	41	central america and carribean
Denmark	31	northwestern europe

(continued on next page)

Country assignment to minor regions (continued)		
Country	Min. Region #	Min. Region Name
Dominican Rep.	41	central america and carribean
Algeria	80	north africa
Ecuador	42	southern south america
Egypt	80	north africa
Eritrea	23	east africa
Spain	32	southwestern europe
Estonia	33	central and eastern europe
Ethiopia	23	east africa
Finland	33	central and eastern europe
Fiji	52	australia and pacific
France	32	southwestern europe
Micronesia	52	australia and pacific
Gabon	22	middle africa
UK	31	northwestern europe
Georgia	12	middle east-asia minor
Ghana	24	west africa
Guinea	24	west africa
Gambia, The	24	west africa
Guinea-Bissau	24	west africa
Eq. Guinea	22	middle africa
Greece	33	central and eastern europe
Grenada	41	central america and carribean
Guatemala	41	central america and carribean
Guam	52	australia and pacific
Guyana	41	central america and carribean
Hong Kong	60	east asia
Honduras	41	central america and carribean
Croatia	33	central and eastern europe
Haiti	41	central america and carribean
Hungary	33	central and eastern europe
Indonesia	51	southeast asia
India	11	south asia
Ireland	31	northwestern europe
Iran	11	south asia
Iraq	12	middle east-asia minor
Iceland	31	northwestern europe
Israel	12	middle east-asia minor
Italy	32	southwestern europe
Jamaica	41	central america and carribean
Jordan	12	middle east-asia minor
Japan	60	east asia
Kazakhstan	14	central asia
Kenya	23	east africa

(continued on next page)

Country assignment to minor regions (continued)		
Country	Min. Region #	Min. Region Name
Kyrgyz Rep.	14	central asia
Cambodia	51	southeast asia
Kiribati	52	australia and pacific
Korea, Rep.	60	east asia
Kuwait	12	middle east-asia minor
Lao PDR	51	southeast asia
Lebanon	12	middle east-asia minor
Liberia	24	west africa
Libya	80	north africa
St. Lucia	41	central america and carribean
Sri Lanka	11	south asia
Lesotho	21	southern africa
Lithuania	33	central and eastern europe
Luxembourg	31	northwestern europe
Latvia	33	central and eastern europe
Macao, China	60	east asia
Morocco	80	north africa
Moldova	33	central and eastern europe
Madagascar	23	east africa
Maldives	11	south asia
Mexico	70	north america
Macedonia	33	central and eastern europe
Mali	24	west africa
Malta	33	central and eastern europe
Myanmar	51	southeast asia
Mongolia	14	central asia
Mozambique	23	east africa
Mauritania	80	north africa
Mauritius	23	east africa
Malawi	23	east africa
Malaysia	51	southeast asia
Namibia	21	southern africa
New Cal.	52	australia and pacific
Niger	24	west africa
Nigeria	24	west africa
Nicaragua	41	central america and carribean
Netherlands	31	northwestern europe
Norway	31	northwestern europe
Nepal	11	south asia
New Zealand	52	australia and pacific
Oman	12	middle east-asia minor
Pakistan	11	south asia
Panama	41	central america and carribean

(continued on next page)

Country assignment to minor regions (continued)		
Country	Min. Region #	Min. Region Name
Peru	42	southern south america
Philippines	51	southeast asia
Pap. New Guin.	52	australia and pacific
Poland	33	central and eastern europe
Puerto Rico	41	central america and carribean
Korea, D. Rep.	60	east asia
Portugal	32	southwestern europe
Paraguay	42	southern south america
Fr. Polynesia	52	australia and pacific
Qatar	12	middle east-asia minor
Romania	33	central and eastern europe
Russia	33	central and eastern europe
Rwanda	22	middle africa
Saudi Arabia	12	middle east-asia minor
Sudan	23	east africa
Senegal	24	west africa
Singapore	60	east asia
Solomon Is.	52	australia and pacific
Sierra Leone	24	west africa
El Salvador	41	central america and carribean
Somalia	23	east africa
Suriname	41	central america and carribean
Slovak Rep.	33	central and eastern europe
Slovenia	33	central and eastern europe
Sweden	31	northwestern europe
Swaziland	21	southern africa
Seychelles	23	east africa
Syria	12	middle east-asia minor
Chad	23	east africa
Togo	24	west africa
Thailand	51	southeast asia
Tajikistan	14	central asia
Turkmenistan	14	central asia
Tonga	52	australia and pacific
Taiwan	60	east asia
Trin. & Tob.	41	central america and carribean
Tunisia	80	north africa
Turkey	12	middle east-asia minor
Tanzania	23	east africa
Uganda	23	east africa
Ukraine	33	central and eastern europe
Uruguay	42	southern south america
United States	70	north america
(continued on next page)		

Country assignment to minor regions (continued)		
Country	Min. Region #	Min. Region Name
Uzbekistan	14	central asia
St. Vin. & Gr.	41	central america and carribean
Venezuela	41	central america and carribean
Vir. Is. (U.S.)	52	australia and pacific
Vir. Is. (U.S.)	52	australia and pacific
Vietnam	51	southeast asia
Vanuatu	52	australia and pacific
Samoa	52	australia and pacific
Yemen, Rep.	12	middle east-asia minor
Serbia/Yugo.	33	central and eastern europe
South Africa	21	southern africa
Congo, Dem. Rep.	22	middle africa
Zambia	22	middle africa
Zimbabwe	22	middle africa

A.2 Imputation of GDP per capita in 1500 CE

Table A3: Year 1500 real GDP per capita

Country	Value assigned	Source or method of imputation
Afghanistan	645	Projected backwards from 1950, assuming no change
Angola	332	Projected backwards from 1950 assuming constant ratio with Algeria
Albania	446	Projected backwards from 1870, assuming no change
UAE	609	Assumed to be the same as Jordan
Argentina	415	Assumed to be the same as Venezuela
Armenia	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Australia	518	Projected backwards from 1820, assuming no change
Austria	1,236	Projected backwards from 1820 assuming constant ratio with Centre-North Italy
Azerbaijan	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Burundi	360	Projected backwards from 1950, assuming no change
Belgium	1,467	Maddison 2013
Benin	342	Projected backwards from 1950 assuming constant ratio with Algeria

(continued on next page)

Imputation of GDP per capita in 1500 CE (continued)		
Country	Value assigned	Source or method of imputation
Burkina Faso	474	Projected backwards from 1950, assuming no change
Bangladesh	533	Assumed to be the same as India
Bulgaria	840	Projected backwards from 1870, assuming no change
Bahrain	12	middle east-asia minor
Bahamas, The	41	central america and carribean
Belarus	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Bolivia	415	Assumed to be the same as Venezuela
Brazil	415	Assumed to be the same as Venezuela
Barbados	41	central america and carribean
Brunei Dar.	51	southeast asia
Bhutan	60	east asia
Botswana	349	Projected backwards from 1950, assuming no change
CAR	577	Projected backwards from 1950 assuming constant ratio with Egypt
Canada	587	Assumed to be the same as the United States
Switzerland	1,016	Projected backwards from 1851 assuming constant ratio with England
Chile	415	Assumed to be the same as Venezuela
China	600	Projected backwards from 1820, assuming no change
Cote d'Ivoire	328	Projected backwards from 1950 assuming constant ratio with Algergia
Cameroon	501	Projected backwards from 1950 assuming constant ratio with Egypt
Congo, Rep.	378	Projected backwards from 1950 assuming constant ratio with Algeria
Colombia	415	Assumed to be the same as Venezuela
Comoros	419	Projected backwards from 1950 assuming constant ratio with Egypt
Cape Verde	336	Projected backwards from 1950 assuming constant ratio with Egypt
Costa Rica	415	Assumed to be the same as Venezuela
Cuba	415	Assumed to be the same as Venezuela
Czech C./-slvk/Rep.	849	Maddison 2013
Germany	1,146	Maddison 2013
Djibouti	473	Projected backwards from 1950 assuming constant ratio with Algeria
Denmark	755	Projected backwards from 1820 assuming constant ratio with the Netherlands

(continued on next page)

Imputation of GDP per capita in 1500 CE (continued)		
Country	Value as- signed	Source or method of imputation
Dominican Rep.	415	Assumed to be the same as Venezuela
Algeria	430	Projected backwards from 1820, assuming no change
Ecuador	42	southern south america
Egypt	680	Maddison 2013
Eritrea	390	Projected backwards from 1950, assuming no change
Spain	846	Maddison 2013
Estonia	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Ethiopia	390	Projected backwards from 1950, assuming no change
Finland	781	Projected backwards from 1820, assuming no change
France	1,151	Projected backwards from 1820 assuming constant ratio with Centre-North Italy
Gabon	378	Assumed to be the same as Republic of the Congo
UK	1,086	Maddison 2013
Georgia	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Ghana	354	Projected backwards from 1950 assuming constant ratio with Algeria
Guinea	303	Projected backwards from 1950, assuming no change
Gambia, The	454	Projected backwards from 1950 assuming constant ratio with Egypt
Guinea-Bissau	289	Projected backwards from 1950, assuming no change
Eq. Guinea	403	Projected backwards from 1950 assuming constant ratio with Egypt
Greece	660	Assumed to be same as Ottoman Empire
Grenada	41	central america and carribean
Guatemala	415	Assumed to be the same as Venezuela
Guam	52	australia and pacific
Hong Kong	615	Projected backwards from 1820, assuming no change
Honduras	415	Assumed to be the same as Venezuela
Haiti	415	Assumed to be the same as Venezuela
Hungary	796	Projected backwards from 1870 assuming constant ratio with Czechoslokakia
Indonesia	507	Projected backwards from 1815, assuming no change
India	533	Projected backwards from 1820, assuming no change
Ireland	877	Projected backwards from 1820, assuming no change
(continued on next page)		

Imputation of GDP per capita in 1500 CE (continued)		
Country	Value assigned	Source or method of imputation
Iran	608	Linear interpolation between Maddison 2013 values for 1 and 1820
Iraq	632	Linear interpolation between Maddison 2013 values for 1220 and 1820
Israel	609	Assumed to be the same as Jordan
Italy	1,533	Maddison 2013
Jamaica	415	Assumed to be the same as Venezuela
Jordan	609	Linear interpolation between Maddison 2013 values for 1 and 1820
Japan	542	Linear interpolation between Maddison 2013 values for 1450 and 1600
Kazakhstan	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Kenya	486	Projected backwards from 1950 assuming constant ratio with Egypt
Kyrgyz Rep.	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Cambodia	527	Assumed to be the same as Vietnam
Korea, Rep.	335	Projected backwards from 1820, assuming no change
Kuwait	609	Assumed to be the same as Jordan
Lao PDR	527	Assumed to be the same as Vietnam
Lebanon	609	Assumed to be the same as Jordan
Liberia	333	Projected backwards from 1950, assuming no change
Libya	821	Linear interpolation between Maddison 2013 values for 1 and 1950
Sri Lanka	550	Maddison 2013
Lesotho	355	Projected backwards from 1950, assuming no change
Lithuania	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Latvia	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Morocco	430	Projected backwards from 1820, assuming no change
Madagascar	300	Projected backwards from 1950 assuming constant ratio with Algeria
Mexico	415	Assumed to be the same as Venezuela
Mali	457	Projected backwards from 1950, assuming no change
Myanmar	504	Projected backwards from 1820, assuming no change
Mongolia	435	Projected backwards from 1950, assuming no change
Mozambique	357	Projected backwards from 1950 assuming constant ratio with Algeria

(continued on next page)

Imputation of GDP per capita in 1500 CE (continued)		
Country	Value assigned	Source or method of imputation
Mauritania	464	Projected backwards from 1950, assuming no change
Mauritius	785	Projected backwards from 1950 assuming constant ratio with Algeria
Malawi	324	Projected backwards from 1950, assuming no change
Malaysia	603	Projected backwards from 1820, assuming no change
Namibia	681	Projected backwards from 1950 assuming constant ratio with Algeria
Niger	461	Projected backwards from 1950 assuming constant ratio with Egypt
Nigeria	563	Projected backwards from 1950 assuming constant ratio with Egypt
Nicaragua	415	Assumed to be the same as Venezuela
Netherlands	1,111	Maddison 2013
Norway	921	Projected backwards from 1820 assuming constant ratio with Sweden
Nepal	397	Projected backwards from 1820, assuming no change
New Zealand	518	Assumed to be the same as Australia
Oman	609	Assumed to be the same as Jordan
Pakistan	533	Assumed to be the same as India
Panama	415	Assumed to be the same as Venezuela
Peru	415	Assumed to be the same as Venezuela
Philippines	584	Projected backwards from 1820, assuming no change
Poland	690	Projected backwards from 1870 assuming constant ratio with Czechoslokakia
Puerto Rico	415	Assumed to be the same as Venezuela
Korea, D. Rep.	335	Projected backwards from 1820, assuming no change
Portugal	1,103	Projected backwards from 1600 assuming constant ratio with Spain
Paraguay	415	Assumed to be the same as Venezuela
Qatar	609	Assumed to be the same as Jordan
Romania	679	Projected backwards from 1870 assuming constant ratio with Czechoslokakia
Russia	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Rwanda	409	Projected backwards from 1950 assuming constant ratio with Egypt
Saudi Arabia	609	Assumed to be the same as Jordan
Sudan	613	Projected backwards from 1950 assuming constant ratio with Egypt
Senegal	397	Projected backwards from 1950 assuming constant ratio with Algeria
(continued on next page)		

Imputation of GDP per capita in 1500 CE (continued)		
Country	Value assigned	Source or method of imputation
Singapore	603	Assumed to be the same as Malaysia
Sierra Leone	490	Projected backwards from 1950 assuming constant ratio with Egypt
El Salvador	415	Assumed to be the same as Venezuela
Somalia	613	Projected backwards from 1950 assuming constant ratio with Egypt
Slovak Rep.	849	Maddison 2013
Sweden	1,021	Projected backwards from 1820 assuming constant ratio with England
Swaziland	539	Projected backwards from 1950 assuming constant ratio with Egypt
Seychelles	603	Projected backwards from 1950 assuming constant ratio with Algeria
Syria	609	Assumed to be the same as Jordan
Chad	355	Projected backwards from 1950 assuming constant ratio with Egypt
Togo	429	Projected backwards from 1950 assuming constant ratio with Egypt
Thailand	570	Projected backwards from 1820, assuming no change
Tajikistan	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Turkmenistan	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Taiwan	606	Projected backwards from 1820, assuming no change
Trin. & Tob.	415	Assumed to be the same as Venezuela
Tunisia	477	Linear interpolation between Maddison 2013 values for 1 and 1820
Turkey	660	Maddison 2013
Tanzania	424	Projected backwards from 1950, assuming no change
Uganda	513	Projected backwards from 1950 assuming constant ratio with Egypt
Ukraine	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Uruguay	415	Assumed to be the same as Venezuela
United States	587	Projected backwards from 1650, assuming no change
Uzbekistan	692	Assumbed to be the same as the Former USSR estimate, which is projected backwards from 1885 assuming constant ratio with the Ottoman Empire
Venezuela	415	Projected backwards from 1800, assuming no change
Vietnam	527	Projected backwards from 1820, assuming no change

(continued on next page)

Imputation of GDP per capita in 1500 CE (continued)		
Country	Value assigned	Source or method of imputation
Yemen, Rep.	609	Assumed to be the same as Jordan
Serbia/Yugo.	551	Projected backwards from 1870, assuming no change
South Africa	539	Assumed to be the same as Swaziland
Congo, Dem. Rep.	426	Projected backwards from 1950 assuming constant ratio with Egypt
Zambia	494	Projected backwards from 1950 assuming constant ratio with Egypt
Zimbabwe	524	Projected backwards from 1950 assuming constant ratio with Egypt

A.3 Proofs

A.3.1 Proof of optimal land allocation

To start with, let us state the consumer's problem:

$$\max_{\{c_{il}\}_{l\in[0,1]},h_i} \left\{ \left(\int_0^1 c_{i,l}^{\rho} dl \right)^{\frac{\alpha}{\rho}} h_i^{1-\alpha} \right\}$$

such that $w_i + p_{i,\lambda}\frac{\lambda_i}{x_i} \geq \int_0^1 p_{i,l}c_{i,l}dl + p_{i,h}h_i$.

First order conditions with respect to consumption and housing imply the following two conditions:

$$c_{il} = \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i^{-\frac{\rho}{1-\rho}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} p_{il}^{-\frac{1}{1-\rho}},$$

implying

$$C_i = \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i} \mathbb{M}_i^{\frac{1}{\chi}},$$

and

$$h_i = (1-\alpha)\frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{p_{i,h}^{-\frac{\rho}{1-\rho}}} p_{i,h}^{-\frac{1}{1-\rho}} = (1-\alpha)\frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{p_{i,h}}.$$

The production function for a goods producer with efficiency shock $s_{i,k}$:

$$q_k = s_{i,k}\left(b_{k,I}^{\eta} l_{k,I}^{1-\eta}\right)^{\kappa} \left(b_k^{\eta} l_k^{1-\eta}\right)^{1-\sigma-\kappa} \left[\int_0^1 z_{kl}^{\rho} dl\right]^{\frac{\sigma}{\rho}}$$

The cost-minimization problem of a location-i goods producer is given by

$$\min_{b_k,l_k,b_{k,I},l_{k,I},\{z_{kl}\}_{l\in[0,1]}} \left\{ w_k\left(b_k+b_{k,I}\right)+p_{i,\lambda}\left(l_k+l_{k,I}\right)+\int_0^1 p_{kl}z_{kl}dl \right\}$$

It is straightforward to solve for the optimal allocation of labor and land between improvement and production, and write the following simplified problem in terms of production land and production labor only:

$$\min_{b_k,l_k,\{z_{kl}\}_{l\in[0,1]}} \left\{ \frac{1-\sigma}{1-\sigma-\kappa}\left[w_k b_k+p_{i,\lambda}l_k\right]+\int_0^1 p_{kl}z_{kl}dl \right\}$$

such that

$$q_k = s_{i,k}\left(\frac{\kappa}{1-\sigma-\kappa}\right)^{\kappa}\left(b_k^{\eta}l_k^{1-\eta}\right)^{1-\sigma}\left[\int_0^1 z_{kl}^{\rho}dl\right]^{\frac{\sigma}{\rho}}$$

First order conditions with respect to each type of input, land, intermediate inputs and labor, imply the following two conditions relating land and intermediate good inputs to the quantity of labor input:

$$l_k = \frac{1-\eta}{\eta}\frac{w_i}{p_{i,\lambda}}b_k$$

$$z_{kl} = \frac{\sigma}{\eta(1-\sigma-\kappa)}\frac{w_i}{p_{il}^{\frac{1}{1-\rho}}P_i^{-\frac{\rho}{1-\rho}}\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}b_k$$

These then imply the following relationship between the quantity of labor input and the quantity produced:

$$q_k = s_{i,k}b_k(1-\eta)^{(1-\eta)(1-\sigma)}\left(\frac{1}{\eta}\right)^{(1-\eta)(1-\sigma)+\sigma}\kappa^{\kappa}\sigma^{\sigma}\left(\frac{1}{1-\sigma-\kappa}\right)^{\kappa+\sigma}w_i^{(1-\eta)(1-\sigma)+\sigma}\left(\frac{1}{p_{i,\lambda}}\right)^{(1-\eta)(1-\sigma)}P_i^{-\sigma}\mathbb{M}_i^{\frac{\sigma}{\chi}}$$

It also implies the following minimized cost of production in terms of quantity of production labor:

$$\begin{aligned}&\frac{1-\sigma}{1-\sigma-\kappa}\left[w_ib_k+b_k\frac{1-\eta}{\eta}w_i\right]+\frac{\sigma}{\eta(1-\sigma-\kappa)}w_ib_k\\ &=\frac{1}{\eta(1-\sigma-\kappa)}w_ib_k\end{aligned}$$

This then implies the following efficiency cost of producing a single unit of good in location i:

$$P_i \equiv \frac{s_{i,k}}{\eta(1-\sigma-\kappa)}w_ib_k(1) = \frac{w_i^{\eta}p_{i,\lambda}^{1-\eta}\mathbb{M}_i^{-\frac{1}{\chi}\frac{\sigma}{1-\sigma}}}{\eta^{\eta}(1-\eta)^{1-\eta}\sigma^{\frac{\sigma}{1-\sigma}}}\left(\frac{1}{(1-\sigma-\kappa)^{1-\sigma-\kappa}\kappa^{\kappa}}\right)^{\frac{1}{1-\sigma}}$$

Note that the actual cost faced by the producer is $\frac{P_i}{s_{i,k}}$.

Flipping this expression around, we find that

$$\frac{w_i^\eta p_{i,\lambda}^{1-\eta}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}} = P_i \eta^\eta (1-\eta)^{1-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \left(\kappa^\kappa (1-\sigma-\kappa)^{1-\sigma-\kappa}\right)^{\frac{1}{1-\sigma}}$$

Applying this last formula to the expression for quantity produced in terms of quantity of labor employed yields the following:

$$q_k = s_{i,k} \left(\frac{w_i^\eta p_{i,\lambda}^{1-\eta}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}}\right)^\sigma w_i^{1-\eta} p_{i,\lambda}^{\eta-1} P_i^{-\sigma} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} b_k (1-\eta)^{(1-\eta)(1-\sigma)} \eta^{\eta-\eta\sigma-1} \sigma^\sigma \kappa^\kappa \left(\frac{1}{1-\sigma-\kappa}\right)^{\kappa+\sigma}$$

$$q_k = s_{i,k} b_k \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} \left(\frac{w_i}{p_{i,\lambda}}\right)^{1-\eta} \left(\frac{1-\eta}{\eta}\right)^{1-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}-1},$$

and finally:

$$q_k = s_{i,k} \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}-1} b_k^\eta l_k^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}.$$

Cost-minimization implies that all firms i location i must use the same ratio of land and labor, and aggregation implies that this must be equal to the aggregate ratio of land and labor used in goods production, $\frac{1-\sigma-\kappa}{1-\sigma} l_i$ and $\frac{1-\sigma-\kappa}{1-\sigma} x_i$, respectively. Then wages are given by

$$w_i = \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \eta \left(\frac{l_i}{x_i}\right)^{1-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

and land rents are given by

$$p_{i,\lambda} = \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} (1-\eta) \left(\frac{l_i}{x_i}\right)^{-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

Then,

$$w_i + p_{i,\lambda}\frac{\lambda_i}{x_i} = \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} \left(\frac{\lambda_i}{x_i}\right) \psi_{g,\lambda}^{-\eta} \left[\eta \psi_{g,\lambda} + 1 - \eta\right]$$

The production function of a housing producers is given by

$$H_i = \left(\int_0^1 z_{il,h}^\rho dl\right)^{\frac{\varphi}{\rho}} l_{i,h}^{1-\varphi}$$

The cost minimization problem of a housing producer is given by

$$\min_{\{z_{il,h}\}_0^1, l_{i,h}} \left\{\int_0^1 p_{il} z_{il,h} dl + p_{i,\lambda} l_{i,h}\right\}$$

First order conditions imply the following relationship between quantity of intermediate input used and quantity of land used as inputs:

$$z_{il,h} = \frac{\varphi}{1-\varphi} \frac{p_{i,\lambda}}{p_{il}^{\frac{1}{1-\rho}} P_i^{-\frac{\rho}{1-\rho}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} l_{i,h}$$

Using this, housing as a function of land employed is given by

$$H_i = \left[\frac{\varphi}{1-\varphi} \frac{p_{i,\lambda}}{P_i} \mathbb{M}_i^{\frac{1}{\chi}} \right]^{\varphi} l_{i,h}.$$

This then implies the following unit cost of production for housing, which in equilibrium will also be the housing price faced by consumers:

$$p_{i,h} = \frac{1}{1-\varphi} p_{i,\lambda} l_{i,h}(1) = \frac{P_i^{\varphi} \mathbb{M}_i^{-\varphi \frac{1}{\chi}} p_{i,\lambda}^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi}}$$

Flipping this around and plugging back into the previous expression for housing in terms of land use implies

$$\frac{p_{i,\lambda}}{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}} = \varphi^{\frac{\varphi}{1-\varphi}}(1-\varphi) P_i^{-\frac{\varphi}{1-\varphi}} p_{i,h}^{\frac{1}{1-\varphi}},$$

$$H_i = \left(\frac{p_{i,\lambda}}{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}} \right)^{\varphi} P_i^{-\varphi} \mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}} \left(\frac{\varphi}{1-\varphi} \right)^{\varphi} l_{i,h},$$

and

$$H_i = \varphi^{\frac{\varphi}{1-\varphi}} \left(\frac{p_{i,h}}{P_i} \right)^{\frac{\varphi}{1-\varphi}} \mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}} l_{i,h}.$$

Then, the following relationship can be derived between price of goods and price of housing:

$$p_{i,h} = P_i \mathbb{M}_i^{\frac{\sigma-\varphi}{\chi(1-\sigma)}} \frac{\left[\sigma^{\frac{\sigma}{1-\sigma}} (1-\eta) \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \left(\frac{l_i}{x_i} \right)^{-\eta} \right]^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi}}$$

$$\left(\frac{P_i}{p_{i,h}} \right)^{\frac{1}{1-\varphi}} = \frac{\varphi^{\frac{\varphi}{1-\varphi}}(1-\varphi)}{\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}} \left(\frac{l_i}{x_i} \right)^{\eta} \frac{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}}$$

Then,

$$w_i + p_{i,\lambda} \frac{\lambda_i}{x_i} = \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} \left(\frac{\lambda_i}{x_i} \right)^{1-\eta} \psi_{g,\lambda}^{-\eta} \left[\eta \psi_{g,\lambda} + 1 - \eta \right]$$

and

$$x_i h_i = x_i (1-\alpha) \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \frac{P_i}{p_{i,h}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} \left(\frac{\lambda_i}{x_i} \right)^{1-\eta} \psi_{g,\lambda}^{-\eta} \left[\eta \psi_{g,\lambda} + 1 - \eta \right]$$

Then setting demand equal to supply, $H_i = x_i h_i$,

$$\varphi^{\frac{\varphi}{1-\varphi}} \left(\frac{p_{i,h}}{P_i} \right)^{\frac{\varphi}{1-\varphi}} \mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}} \lambda_i (1 - \psi_{g,\lambda}) = x_i (1-\alpha) \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}$$

$$\frac{\varphi^{\frac{\varphi}{1-\varphi}}}{\sigma^{\frac{\sigma}{1-\sigma}}} \frac{1}{(1-\alpha)\kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}} \psi_{g,\lambda}^{\eta} \left(\frac{\lambda_i}{x_i} \right)^{\eta} \frac{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}} (1 - \psi_{g,\lambda}) = \left(\frac{P_i}{p_{i,h}} \right)^{\frac{1}{1-\varphi}} [\eta \psi_{g,\lambda} + 1 - \eta]$$

$$\frac{(1-\eta)}{(1-\varphi)(1-\alpha)} (1 - \psi_{g,\lambda}) = \eta \psi_{g,\lambda} + 1 - \eta$$

$$\psi_{g,\lambda} \left[\eta + (1-\eta) \frac{1}{(1-\varphi)(1-\alpha)} \right] = (1-\eta) \left[\frac{1}{(1-\varphi)(1-\alpha)} - 1 \right]$$

and, finally:

$$\psi_{g,\lambda} = \frac{(1-\eta)(\alpha + \varphi(1-\alpha))}{\eta(1-\varphi)(1-\alpha) + (1-\eta)}$$

$$= \frac{(1-\eta)(\alpha + \varphi(1-\alpha))}{(1-\varphi)(1-\alpha) + (1-\eta)(\alpha + \varphi(1-\alpha))}$$

A.3.2 Proof of equilibrium total revenue and wage

Quantity produced for a particular good in location i:

$$q_{i,k} = s_{i,k} \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma} - 1} b_{i,k}^{\eta} l_{i,k}^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

Revenue per unit of output for good k is given by $\frac{P_i}{s_{i,k}}$, so total revenue from good k, $y_{i,k}$, is given by

$$y_{i,k} = P_i \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma} - 1} b_{i,k}^{\eta} l_{i,k}^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

In equilibrium, each unit of resource will earn the same revenue no matter which good it is dedicated to producing. So, total revenue for location i, Y_i is given by

$$Y_i = \psi_y P_i x_i^{\eta} \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

where

$$\psi_y \equiv \psi_{g,\lambda}^{1-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \frac{(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}}}{1-\sigma}$$

Adapting the expression from section A.3.1 to account for the equilibrium fraction of land devoted to goods production, equilibrium wages are given by

$$w_i = \psi_{g,\lambda}^{1-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \eta (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} \left(\frac{\lambda_i}{x_i} \right)^{1-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

$$p_{i,\lambda} = \psi_{g,\lambda}^{-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \eta (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} \left(\frac{\lambda_i}{x_i}\right)^{1-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

Note that $w_i x_i = \eta(1-\sigma)Y_i$ and $p_{i,\lambda}\psi_{g,\lambda}\lambda_i = (1-\eta)(1-\sigma)Y_i$–each factor is paid exactly its CES share of revenue, as expected.

A.3.3 Proof of housing consumption and equilibrium utility

From appendix A.3.1, the following relationship between the price of goods and the price of housing:

$$\left(\frac{p_{i,h}}{P_i}\right)^{\frac{\varphi}{1-\varphi}} = \left(\frac{\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}}{\varphi^{\frac{\varphi}{1-\varphi}}(1-\varphi)}\right)^{\varphi} \psi_{g,\lambda}^{-\eta\varphi} x_i^{\eta\varphi} \lambda_i^{-\eta\varphi} \mathbb{M}_i^{\frac{1}{\chi}\left[\frac{\sigma}{1-\sigma}-\frac{\varphi}{1-\varphi}\right]\varphi}$$

Total housing production is then

$$H_i = \varphi^{\frac{\varphi}{1-\varphi}} \left(\frac{p_{i,h}}{P_i}\right)^{\frac{\varphi}{1-\varphi}} (1-\psi_{g,\lambda})\lambda_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\varphi}{1-\varphi}}$$

$$= \left(\frac{\varphi\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}}{1-\varphi}\right)^{\varphi} (1-\psi_{g,\lambda})\psi_{g,\lambda}^{-\eta\varphi} x_i^{\eta\varphi} \lambda_i^{1-\eta\varphi} \mathbb{M}_i^{\frac{1}{\chi}\frac{\varphi}{1-\sigma}}$$

Per-capita housing consumption, $h_i = \frac{H_i}{x_i}$, is then

$$h_i = \left(\frac{\varphi\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}}{1-\varphi}\right)^{\varphi} (1-\psi_{g,\lambda})\psi_{g,\lambda}^{-\eta\varphi} \left(\frac{\lambda_i}{x_i}\right)^{1-\eta\varphi} \mathbb{M}_i^{\frac{1}{\chi}\frac{\varphi}{1-\sigma}}$$

Appendix A.3.1 also provides the following expression for C_i:

$$C_i = \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i} \mathbb{M}_i^{\frac{1}{\chi}}$$

$$= \alpha\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \psi_{g,\lambda}^{-\eta}\left[\eta\psi_{g,\lambda}+1-\eta\right]\left(\frac{\lambda_i}{x_i}\right)^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi(1-\sigma)}}$$

Combining the expressions for per-capita consumption of housing and goods directly yields the following expression for equilibrium utility:

$$u_i = \psi_u \left(\frac{\lambda_i}{x_i}\right)^{\alpha(1-\eta)+(1-\alpha)(1-\eta\varphi)} \mathbb{M}_i^{\frac{\alpha+(1-\alpha)\varphi}{\chi(1-\sigma)}}.$$

Simplifying, this can be stated as

$$u_i = \psi_u \left(\frac{\lambda_i}{x_i}\right)^{1-\eta[\alpha+(1-\alpha)\varphi]} \mathbb{M}_i^{\frac{\alpha+(1-\alpha)\varphi}{\chi(1-\sigma)}},$$

where

$$\psi_u \equiv \alpha^{\alpha}\left(\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\right)^{\alpha+\varphi(1-\alpha)}\left(\frac{\varphi(1-\eta)}{1-\varphi}\right)^{\varphi(1-\alpha)}(1-\psi_{g,\lambda})^{1-\alpha}\,\psi_{g,\lambda}^{-\eta[\alpha+\varphi(1-\alpha)]}\left[\eta\psi_{g,\lambda}+1-\right.$$

A.3.4 Market Access

"Market access":

$$\mathbb{M}_i \equiv \left[\int_0^1 \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl\right]^{\chi\frac{1-\rho}{\rho}} = \left[\int_0^A \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl + \int_A^1 \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl\right]^{\chi\frac{1-\rho}{\rho}}$$

By definition, cost of production for a location-i producer of good k is $\frac{P_i}{s_{i,k}}$. Perfect competition implies that if good l is bought from location-j good l sold in location i will have a price equal to $p_{ij,l} = \frac{P_j}{s_{j,l}\gamma_{ji}}$.

The probability that the $p_{ij,l}$ is less than p, for $l \in [0, A]$, is given by

$$\begin{aligned}
\Pr\left(p_{ij,l} < p \middle| \, l \in [0,A]\right) &= \Pr\left(\frac{P_j}{s_{j,l}\gamma_{ji}} < pl \in [0,A]\right) \\
&= \Pr\left(s_{j,l} > \frac{P_j}{p\gamma_{ji}} l \in [0,A]\right) = 1 - \Pr\left(s_{j,l} \le \frac{P_j}{p\gamma_{ji}} l \in [0,A]\right) \\
&= 1 - e^{-\alpha_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}
\end{aligned}$$

By the same reasoning,

$$\Pr\left(p_{ij,l} < p \middle| \, l \in (A,1]\right) = 1 - e^{-m_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}$$

Then, the probability that $p_{ij,l}$ is less than p, unconditional on whether l is an agricultural or non-agricultural good, can be calculated as

$$\begin{aligned}
\Pr\left(p_{ij,l} < p\right) &= 1 - \left[\prod_{l\in[0,A]} \left(1 - \Pr\left(p_{ij,l} < p | l \in [0,A]\right)\right)\right]\left[\prod_{l\in(A,1]} \left(1 - \Pr\left(p_{ij,l} < p | l \in (A,1]\right)\right)\right] \\
&= 1 - \left[\prod_{l\in[0,A]} e^{-\alpha_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right]\left[\prod_{l\in(A,1]} e^{-m_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right] \\
&= 1 - \left[e^{-\alpha_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right]^A \left[e^{-m_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right]^{1-A} \\
&= 1 - e^{-\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}[A\alpha_j + (1-A)m_j]}
\end{aligned}$$

Then, by the properties of the Fréchet distribution, the probability that $p_{i,l} = \min_{j\in N} p_{ij,l}$ is less than p is given by

$$\begin{aligned}
\hat{G}_i(p) \equiv \Pr(p_{i,l} < p) &= 1 - \prod_{j\in N} \left[1 - \Pr(p_{ij,l} < p)\right] \\
&= 1 - \prod_{j\in N} e^{-\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}[A\alpha_j + (1-A)m_j]} \\
&= 1 - e^{-\sum_{j\in N}\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}[A\alpha_j + (1-A)m_j]}
\end{aligned}$$

$$\frac{d\hat{G}_i(p)}{dp} = \chi p^{\chi-1}\mathbb{P}_i e^{-\mathbb{P}_i p^{\chi}}$$

with

$$\mathbb{P}_i \equiv \sum_{j\in N}\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi}[A\alpha_j + (1-A)m_j]$$

Market access is therefore given by

$$\begin{aligned}
\mathbb{M}_i &= \left[P_i^{\frac{\rho}{1-\rho}}\int_0^1\left(\frac{1}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}}dl\right]^{\chi\frac{1-\rho}{\rho}} \\
&= \left[P_i^{\frac{\rho}{1-\rho}}\int_0^\infty p^{-\frac{\rho}{1-\rho}}\frac{d\hat{G}_i(p)}{dp}dp\right]^{\chi\frac{1-\rho}{\rho}} \\
&= \left[P_i^{\frac{\rho}{1-\rho}}\int_0^\infty p^{-\frac{\rho}{1-\rho}}\chi p^{\chi-1}\mathbb{P}_i e^{-\mathbb{P}_i p^{\chi}}dp\right]^{\chi\frac{1-\rho}{\rho}}
\end{aligned}$$

Change of variable: $x \equiv \mathbb{P}_i p^{\chi}$:

$$\mathbb{M}_i = \left[P_i^{\frac{\rho}{1-\rho}}\mathbb{P}_{i,a}^{\frac{1}{\chi}\frac{\rho}{1-\rho}}\int_0^\infty x^{1-\frac{1}{\chi}\frac{\rho}{1-\rho}-1}e^{-x}dx\right]^{\chi\frac{1-\rho}{\rho}}$$

Applying the definition of the gamma function, $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$:

$$\begin{aligned}
\mathbb{M}_i &= \Gamma\left(1-\frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}} P_i^{\chi}\mathbb{P}_i \\
&= \Gamma\left(1-\frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}}\sum_{j\in N}\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}^{\chi}[A\alpha_j + (1-A)m_j]
\end{aligned}$$

Note that the previous steps require that the restriction $\chi > \frac{\rho}{1-\rho}$ holds.

A.3.5 Goods Market Clearing and Prices Derivation

The probability that location j exports a given good l to location i, r_{ji}, is the same as the probability that location j can provide good l at the lowest cost:

$$\begin{aligned}
r_{ji} &= \Pr\left(p_{i,j,l} < \min_{k\neq j}\{p_{i,k,l}\}\right) \\
&= \int_0^\infty \prod_{k\neq j}[1-\Pr(p_{i,k,l} < p)]\frac{d\Pr(p_{i,j,l} < p)}{dp}dp \\
&= \int_0^\infty e^{-\sum\limits_{k\in N}\left(\frac{P_k}{\gamma_{ki}}\right)^{-\chi}p^{\chi}[A\alpha_k + (1-A)m_k]}\chi p^{\chi-1}\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi}[A\alpha_j + (1-A)m_j]\,dp
\end{aligned}$$

Change of variable $x = \mathbb{P}_i p^{\chi}$:

$$r_{ji} = \frac{\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi}[A\alpha_j + (1-A)m_j]}{\mathbb{P}_i}\int_0^{\infty} e^{-x}dx$$
$$= \frac{\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi}[A\alpha_j + (1-A)m_j]}{\mathbb{P}_i}$$

In terms of market access:

$$r_{ji} = \Gamma\left(1 - \frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}} \frac{\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}^{\chi}[A\alpha_j + (1-A)m_j]}{\mathbb{M}_i}$$

Aggregate expenditure on good l in consumption:

$$x_i p_{i,l} c_{i,l} = x_i \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i^{-\frac{\rho}{1-\rho}}\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} p_{il}^{-\frac{\rho}{1-\rho}}$$
$$= \alpha\psi_{g,\lambda}^{-\eta}[\eta\psi_{g,\lambda} + 1 - \eta]\,\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} x_i^{\eta}\lambda_i^{1-\eta} P_i^{\frac{1}{1-\sigma}}\mathbb{M}_i^{\frac{(1-\rho)(\sigma-\rho)}{\chi(1-\sigma)}} p_{il}^{-\frac{\rho}{1-\rho}}$$
$$= \alpha(1-\sigma)\left[\eta + \frac{1-\eta}{\psi_{g,\lambda}}\right]\left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}$$
$$= (1-\sigma)\frac{\alpha}{\alpha\varphi(1-\alpha)}\left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}$$

Aggregate expenditure on intermediate input l in goods production:

$$p_{i,l} z_{i,l} = \frac{\sigma}{\eta(1-\sigma)} \frac{x_i w_i}{P_i^{-\frac{\rho}{1-\rho}}\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} p_{i,l}^{-\frac{1}{1-\rho}}$$
$$= \psi_{g,\lambda}^{1-\eta}\frac{\sigma}{1-\sigma}\sigma^{\frac{\sigma}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}} x_i^{\eta}\lambda_i^{1-\eta}P_i^{\frac{1}{1-\rho}}\mathbb{M}_i^{\frac{(1-\rho)(\sigma-\rho)}{\chi(1-\sigma)}} p_{i,l}^{-\frac{1}{1-\rho}}$$
$$= \sigma\left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}$$

Aggregate expenditure on intermediate input l in housing production:

$$p_{i,l} z_{il,h} = \frac{\varphi}{1-\varphi} \frac{p_{i,\lambda}}{p_{il}^{\frac{1}{1-\rho}} P_i^{-\frac{\rho}{1-\rho}}\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} l_{i,h}$$
$$= (1-\psi_{g,\lambda})\psi_{g,\lambda}^{-\eta}(1-\eta)\frac{\varphi}{1-\varphi}\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} x_i^{\eta}\lambda_i^{1-\eta}P_i^{\frac{1}{1-\sigma}}\mathbb{M}_i^{\frac{(1-\rho)(\sigma-\rho)}{\chi(1-\sigma)}} p_{i,l}^{-\frac{1}{1-\rho}}$$
$$= \frac{1-\psi_{g,\lambda}}{\psi_{g,\lambda}}(1-\eta)(1-\sigma)\frac{\varphi}{1-\varphi}\left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}$$
$$= (1-\sigma)\frac{\varphi(1-\alpha)}{\alpha+\varphi(1-\alpha)}\left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}$$

Aggregate expenditure on good l in location i for all purposes, as a function of its price:

$$\tilde{y}_{i,l} \equiv x_i p_{i,l} c_{i,l} + p_{i,l} z_{i,l} + p_{i,l} z_{il,h} = \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}$$

From this expression, it follows immediately that total aggregate expenditure on goods equals total aggregate revenue of goods-producing firms:

$$\tilde{Y}_i \equiv \int_0^1 \tilde{y}_{i,l} dl = Y_i$$

Now, using export probabilities r_{ji}, it is possible to calculate $\tilde{r}_{ji}$, the share of i's aggregate goods expenditure that is spent on goods from location j. As it turns out, $\tilde{r}_{ji} = r_{ji}$:

$$\tilde{r}_{ji} = \frac{\int_0^1 r_{ji} \tilde{y}_{i,l} dl}{\tilde{Y}_i} = r_{ji} \frac{Y_i}{\tilde{Y}_i} = r_{ji}$$

In equilibrium, aggregate revenue of goods producing firms in location $i \in N$ must equal total expenditure from all locations $j \in N$ on goods produced in i:

$$Y_i = \sum_{j \in N} r_{ij} Y_j$$

Now let us substitute in for r_{ij} and develop this expression a bit further:

$$P_i x_i^\eta \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} = \Gamma\left(1 - \frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}} \sum_{j \in N} \frac{\left(\frac{P_j}{P_i}\right)^\chi \gamma_{ij}^\chi \left[A\alpha_i + (1-A)m_i\right]}{\mathbb{M}_j} P_j x_j^\eta \lambda_j^{1-\eta} \mathbb{M}_j^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

$$\begin{aligned} P_i x_i^\eta \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1} &\sum_{j \in N} \left(\frac{P_i}{P_j}\right)^\chi \gamma_{ji}^\chi \left[A\alpha_j + (1-A)m_j\right] \\ &= \sum_{j \in N} \left(\frac{P_j}{P_i}\right)^\chi \gamma_{ij}^\chi \left[A\alpha_i + (1-A)m_i\right] P_j x_j^\eta \lambda_j^{1-\eta} \mathbb{M}_j^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1} \end{aligned}$$

With transitive asymmetry, i.e., if $\frac{\gamma_{ij}}{\gamma_{ji}}\frac{\gamma_{jk}}{\gamma_{kj}} = \frac{\gamma_{ik}}{\gamma_{ki}}$, and taking market access as given, the following is a solution to the system of equations implied by the preceding expression:

$$\frac{P_i x_i^\eta \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1}}{A\alpha_i + (1-A)m_i} \left(\frac{P_i}{P_j}\right)^\chi \gamma_{ji}^\chi = \left(\frac{P_j}{P_i}\right)^\chi \gamma_{ij}^\chi \frac{P_j x_j^\eta \lambda_j^{1-\eta} \mathbb{M}_j^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1}}{A\alpha_j + (1-A)m_j}$$

and, thus:

$$\left(\frac{P_i}{P_j}\right)^{1+2\chi} = \left(\frac{\gamma_{ij}}{\gamma_{ji}}\right)^\chi \frac{x_j^\eta \lambda_j^{1-\eta}}{x_i^\eta \lambda_j^{1-\eta}} \frac{A\alpha_i + (1-A)m_i}{A\alpha_j + (1-A)m_j} \left(\frac{\mathbb{M}_i}{\mathbb{M}_j}\right)^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1}$$

If we apply the restriction that $\sigma = \frac{\chi}{1+\chi}$, then the above expression is a closed-form solution for relative prices for all locations. Given the types of values that are typically given to these parameters, in the literature, however, this is unlikely to be a reasonable restriction: it would imply, simultaneously, a very high share of intermediate inputs in production, and a very high elasticity of trade to transport costs. A smaller but still significant concern with this restriction is that it would also, due to the necessity that $\chi > \frac{\rho}{1-\rho}$, require a relatively high complementarity between goods. Assuming that $\sigma < \frac{\chi}{1+\chi}$, as is more reasonable, the interpretation of this expression is as follows: revenue per unit of input will be higher in locations that have less land and labor available for production, that have higher agricultural potential and technology levels, that have greater market access, and that face lower barriers to exporting than they do to importing.

One Period Spatial Equilibrium

In order to explore the basic properties of the mobility regime we have just specified, let us now define a one-period spatial equilibrium. Suppose the world exists for only a single period. A **one-period spatial equilibrium** consists of a static equilibrium summarized by u_i for all $i \in N$ and location choices by all consumers such that, given their starting locations, bilateral mobility costs, draws for idiosyncratic location preferences, and the location choices of other consumers, each consumer's choice maximizes his utility.

Following Redding (2016), the distribution of idiosyncratic preferences given by $M(.)$ implies that l_{ij}, the probability that a consumer with a starting location of i will choose to reside in j, will be given by the following:

$$l_{ij} = \frac{\vartheta_{ij}\mu_j^0 x_{j,b} u_j^{\varkappa}}{\sum\limits_{k\in N} \vartheta_{ik}\mu_k^0 x_{k,b} u_k^{\varkappa}}$$

$$x_j = \sum_{i\in N} l_{ij} x_{i,b} = \mu_j^0 x_{j,b} u_j^{\varkappa} \sum_{i\in N} \frac{\vartheta_{ij} x_{i,b}}{\sum\limits_{k\in N} \vartheta_{ik}\mu_k^0 x_{k,b} u_k^{\varkappa}} \tag{A.1}$$

These choice probabilities, aggregated over the distribution of starting populations $x_{i,b}$ for $i \in N$, imply the following ratios of basic utility that must hold for all $j, m \in N$:

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{\frac{x_j}{x_{j,b}}}{\frac{x_m}{x_{m,b}}} \frac{\sum\limits_{i\in N} \vartheta_{im}\tilde{l}_i}{\sum\limits_{i\in N} \vartheta_{ij}\tilde{l}_i} \right)^{\frac{1}{\varkappa}}, \tag{A.2}$$

where

$$\tilde{l}_i \equiv \frac{x_{i,b}}{\sum\limits_{k\in N} \vartheta_{ik}\mu_k^0 x_{k,b} u_k^{\varkappa}}.$$

The interpretation of this expression is the following: locations will have relatively higher utility in equilibrium which

1. have relatively low utility multipliers (i.e., $\mu_j^0 < \mu_m^0$),

2. are relatively costly for consumers in other locations to move to ($\sum_{i \in N} \vartheta_{ij} \tilde{l}_i < \sum_{i \in N} \vartheta_{im} \tilde{l}_i$), and

3. experience relatively larger inflows of resident population relative to their starting population ($\frac{x_j}{x_{j,b}} > \frac{x_m}{x_{m,b}}$).

The parameter $\varkappa$, then, determines the sensitivity of relative utilities to differences between locations of these three types. In the limit as $\varkappa$ approaches 1, a change in the ratio of amenity multipliers $\frac{\mu_m^0}{\mu_j^0}$ would be matched 1:1 by a change in relative utility $\frac{u_j}{u_m}$. In the opposite limit, as $\varkappa$ increases without bound, utility u_i is always equalized across locations in equilibrium regardless of the fundamentals.

Another interpretation of the parameter $\varkappa$ becomes apparent if we think about a *series* of one-period spatial equilibria indexed by t, such that $x_{i,b}(t+1) = x_i(t)$ for $t \in \{0, 1, 2, ...\}$. It can be shown that for an arbitrary distribution of starting population $x_{i,b}(0)$ or $i \in N$, such a series of equilibria is guaranteed to converge to a *stable* equilibrium, one in which $x_i(t) = x_{i,b}(t) = x_i$ for $i \in N$, as $t \to \infty$. $\varkappa$ determines the speed of this convergence, with a higher value implying faster convergence. In the limit as $\varkappa \to \infty$, the location choices of the very first equilibrium always yield the stable population distribution, regardless of the starting point.

To see how this specification of location preferences with idiosyncratic shocks nests the standard case of free mobility with no idiosyncratic shocks, consider a stable one-period spatial equilibrium. Consider the case in which all bilateral moving costs are zero: $\vartheta_{ij} = 1$ for all $i, j \in N$.[1] In this case, (A.2) implies that

$$\left(\mu_j^0\right)^{\frac{1}{\varkappa}} u_j = \left(\mu_m^0\right)^{\frac{1}{\varkappa}} u_m.$$

In other words, utility, controlling for location-specific amenity multipliers, is equalized. A spatial equilibrium under free mobility with no preference shocks would require exactly the same condition.

Now, to see how mobility restrictions between countries may play a role, let us consider the case where moving costs within each country are equal to zero, but moving costs between countries are infinite, as in the baseline model of Desmet, Nagy and Rossi-Hansberg (2016).[2] In this case, (A.2) implies that amenity-multiplier-controlled utility must be equalized within countries, and also that preference shocks play no role in pinning down the inequalities in utility which may exist between countries.[3] If, alternatively, moving costs between countries are positive but finite, then preference shocks do play a role in determining relative utilities between countries, and $\varkappa$ again plays its role of deciding how large the equilibrium inequalities will be and how fast a series of equilibria will converge to the stable distribution.

[1]In the context of the paramaterization specified by (**??**), this requires that $\zeta_3 = 1$, $\zeta_4 = 0$, and $\bar{\vartheta}(l, m) = 1$ for all country pairs l, m.

[2]In the context of (**??**), this requires that $\zeta_3 = 1$, $\zeta_4 = 0$, and $\bar{\vartheta}(l, m)0$ for all country pairs l, m such that $l \neq m$.

[3]The second point can be seen by noting that in a stable equilibrium, if ϑ_{ij} equals 0 whenever i and j belong to separate countries and 1 whenever they belong to the same country, and if j and m in (A.2) belong to separate countries, then (A.2) reduces to $1 = 1$, a condition which always holds and so cannot play a role in determining relative utilities between countries.

A.3.6 Proof of parameter regions for forces of agglomeration and dispersion and long-run outcomes

Theorem 2 *Given the environment that has been described, if* $\frac{\nu_2}{\nu_1} = \eta$ *and the world enters a balanced growth path, utilities corresponding to* $\bar{u}$, $\tilde{\mathbf{U}}$ *and population levels corresponding to* $\mathbf{x}$ *must jointly satisfy the following three conditions:*

1. $\bar{u}^{\frac{1}{\nu_1}}$ *must be the largest eigenvalue and* $\mathbf{x}^{\{\eta\}}$ *must be the corresponding right eigenvector of the matrix* $\xi\psi_4^{\frac{1}{\nu_1}}\tilde{\mathbf{U}}^{-\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}^{\{\varsigma_m\}}\mathbf{\Lambda}^{1-\eta}$

2. *Given* $\mathbf{x}$ *and* $\bar{u}$, $\tilde{\mathbf{U}}$ *must satisfy the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0}\frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j) + \kappa}\frac{\sum\limits_{i\in N}\vartheta_{im}\tilde{l}_i}{\sum\limits_{i\in N}\vartheta_{ij}\tilde{l}_i}\right)^{\frac{1}{\varkappa}}$$

   for all $j, m \in N$.

3. *The growth rate of population is equal to*

$$\varsigma_x = \frac{\sum\limits_{i\in N} x_i\left[f_i^0 f(u_i) + \kappa\right]}{\sum\limits_{i\in N} x_i} - 1,$$

   and so the growth rate of manufacturing potential is equal to

$$\varsigma_m = (1 + \varsigma_x)^{\eta} - 1$$

.

Corollary 2.1 *If* $\frac{\nu_2}{\nu_1} = \eta$, *and* $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, *and* $\mu_i^0 = \mu_j^0$ *for all* $i, j \in N$, *then, if the world enters a balanced growth path,* $u_i = u_j = \bar{u}$ *for all* $i, j \in N$ *and* $\bar{u}$ *and population levels corresponding to* $\mathbf{x}$ *are pinned down by the two following conditions:*

1. $\bar{u}^{\frac{1}{\nu_1}}$ *must be the largest eigenvalue and* $\mathbf{x}^{\{\eta\}}$ *must be the corresponding right eigenvector of the matrix* $\xi\psi_4^{\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}^{\{\varsigma_m\}}\mathbf{\Lambda}^{1-\eta}$.

2. *the growth rate of population must be equal to*

$$\varsigma_x = f^0 f(\bar{u}) + \kappa - 1,$$

   and the growth rate of manufacturing potential equal to

$$\varsigma_m = (f^0 f(\bar{u}) + \kappa)^{\eta} - 1$$

.

The interpretation of this characterization is as follows: $\mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{\mathbf{1}-\eta}$ is a matrix such that each ij^{th} element represents the access that the land area in location i which is being used to produce housing has to the land area in location j which is being used to produce goods. The largest eigenvalue of this matrix is, simply, a measure of how much land there is in the world and how well it is connected to other land. The dependence of the maximum utility level on this measure can be interpreted in the following way: land is a productive resource which is distributed across space, and people are better off when the locations holding this resource are better-connected. Similarly, the growth rate of the economy depends on this same measure: the economy grows faster when the world is better-connected.

The right eigenvector corresponding to the largest eigenvalue has in other contexts been interpreted as an *eigenvector centrality*, and this interpretation is appropriate here as well. This means that population agglomerates in locations that are *central*, in the sense of being well-connected, relative to the distribution of land.

Pre-multiplying the matrix $\mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{\mathbf{1}-\eta}$ by $\tilde{\mathbf{U}}^{\frac{-1}{\nu_1}}$ applies weights according to locations' relative utilities, with higher weights being placed on locations with relatively low utility. This makes sense as, relative to the homogenous, free-mobility case in which utility u_i equalizes across locations, a location that has lower utility will have higher population and thus more productive capacity, again relative to the equalized-utility case.

In order to specify the condition which determines whether the world will achieve sustained growth in the long run or will instead converge to a steady, state, it is convenient to introduce the concept of a "hypothetical" population growth rate–the population growth rate which would obtain in a hypothetical balanced growth path with a specified growth rate of manufacturing potential.

Definition 3 *Let the* ***hypothetical balanced growth path population growth rate****, $\tilde{\varsigma}_x(k)$, be defined implicitly as a function of k by the following three conditions:*

1.

$$\tilde{\varsigma}_x(k) = \frac{\sum_{i\in N} x_i\left[f_i^0 f(u_i)+\kappa\right]}{\sum_{i\in N} x_i} - 1$$

2. Given $\mathbf{x}$ and $\bar{u}$, $\tilde{\mathbf{U}}$ satisfies the system of equations given by

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0}\frac{f_m^0 f(u_m)+\kappa}{f_j^0 f(u_j)+\kappa}\frac{\sum_{i\in N}\vartheta_{im}\tilde{l}_i}{\sum_{i\in N}\vartheta_{ij}\tilde{l}_i}\right)^{\frac{1}{\varkappa}}$$

for all $j,m\in N$.

3. $\bar{u}^{\frac{1}{\nu_1}}$ is the largest eigenvalue and $\mathbf{x}^{\{\eta\}}$ is the corresponding right eigenvector of the matrix $\xi\psi_4^{\frac{1}{\nu_1}}\tilde{\mathbf{U}}^{-\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}^{\{k\}}\mathbf{\Lambda}^{1-\eta}$

Mirroring corollary 7.1, in the case where $\frac{\nu_2}{\nu_1}=\eta$, and $\vartheta_{ij}=1$, $f_i^0=f_j^0=f^0$, and $\mu_i^0=\mu_j^0$ for all $i,j\in N$, the definition of $\tilde{\varsigma}_x(k)$ given by definition 3 simplifies considerably, and depends on only two distinct conditions:

1.
$$\tilde{\varsigma}_x(k) = f^0 f(\bar{u}) + \kappa - 1$$

2. $\bar{u}^{\frac{1}{\nu_1}}$ is the largest eigenvalue of the matrix $\xi\psi_4^{\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}^{\{k\}}\mathbf{\Lambda}^{1-\eta}$.

In any case, the condition for long-run sustained growth is given by the following theorem:

Theorem 3 *Given the environment that has been described, if $\frac{\nu_2}{\nu_1} = \eta$, the world will asymptotically approach a unique balanced growth path if and only if $\tilde{\varsigma}_x(0) > 0$.*

Proof: See Appendix **??**.

Theorem 3 makes clear the dependence of growth on the level of connectedness: if transport costs are high enough, and thus the largest eigenvalue of $\mathbf{\Lambda}^{\eta}\mathbf{\Theta}^{\{0\}}\mathbf{\Lambda}^{1-\eta}$ is small enough, sustained growth is not possible, and the economy stagnates instead. Low-enough transport costs are a necessary condition for sustained growth.

Now let us examine the allocations of this steady state economy.

Theorem 4 *In the environment that has been described, if $\frac{\nu_2}{\nu_1} = \eta$ and the world converges to a Malthusian steady state, utilities corresponding to $\bar{u}$, $\tilde{\mathbf{U}}$ and population levels corresponding to $\mathbf{x}$ must jointly satisfy the following three conditions:*

1. $\dfrac{\sum_{i\in N} x_i \left[f_i^0 f(\bar{u}\tilde{u}_i) + \kappa\right]}{\sum_{i\in N} x_i} = 1$

2. $\mathbf{x}^{\{\eta\}} = \psi_4^{\frac{1}{\nu_1}}\bar{u}^{-\frac{1}{\nu_1}}\left(\mathbf{I} - \xi\psi_4^{\frac{1}{\nu_1}}\bar{u}^{-\frac{1}{\nu_1}}\tilde{\mathbf{U}}^{-\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{1-\eta}\right)^{-1}\tilde{\mathbf{U}}^{-\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\boldsymbol{\alpha}$

3. *Given $\mathbf{x}$ and $\bar{u}$, $\tilde{\mathbf{U}}$ must satisfy the system of equations given by*
$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0}\frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j) + \kappa}\frac{\sum_{i\in N}\vartheta_{im}\tilde{l}_i}{\sum_{i\in N}\vartheta_{ij}\tilde{l}_i}\right)^{\frac{1}{\varkappa}}$$
for all $j, m \in N$.

Corollary 4.1 *If $\frac{\nu_2}{\nu_1} = \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, and the world converges to a Malthusian steady state, then*

- $u_i = u_j = \bar{u} = f^{-1}\left(\frac{1-\kappa}{f^0}\right)$
- $\mathbf{x}^{\{\eta\}} = \psi_4^{\frac{1}{\nu_1}}\bar{u}^{-\frac{1}{\nu_1}}\left(\mathbf{I} - \xi\psi_4^{\frac{1}{\nu_1}}\bar{u}^{-\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{1-\eta}\right)^{-1}\mathbf{\Lambda}^{\eta}\boldsymbol{\alpha}$

Theorem 8 shows that the centrality interpretation of the equilibrium population distribution can be maintained in the steady state as well as in the balanced growth path, except that in this case it is not eigenvector centrality but the closely-related *Katz-Bonacich centrality.*[4] In a steady state, the maximum utility $\bar{u}$ is exactly at the level that is necessary for there to be zero population growth–in the simpler case treated by corollary 8.1 where utility equalizes across locations, this may be thought of as the "subsistence" level of utility.[5]

The characterization of balanced growth path and steady state allocations, as well as the condition that determines which type of allocation is the long-run destination of the economy, are similar for the case where $\frac{\nu_2}{\nu_1} < \eta$ and the forces of agglomeration are stronger than those of dispersion. The most important difference is that unlike in the previous case, where long-run utility for any single location was strictly decreasing in its own population, now the relationship is non-monotonic, with a downward-sloping portion followed by a an upward-sloping, concave portion. This means that even if transport costs are very high, so that an economy starting from nothing would quickly stagnate, a sustained-growth outcome can always be achieved if only the starting levels of technology and population are above a certain threshold.

In order to express this condition succinctly, it is convenient to define the population growth rate along a hypothetical transition path as a function of the population level in every location. It is convenient to abstract from the gradual adjustment of population in this hypothetical transition path, and assume that the population distribution in each period corresponds to the *stable distribution* associated with that level of total world population, where the *stable distribution* is defined formally as follows:

Definition 4 *Let the* ***stable distribution*** *associated with a total population level* $\bar{x} = \sum_{i \in N} x_i$ *be defined as a distribution such that* $x_{i,b} = x_i$.

In this hypothetical transition path, it is also convenient to abstract from the gradual gradual accumulation of ideas, and assuming that levels of technology instantly jump to the long-run levels associated with the stable population distribution.

Definition 5 *Let the* ***hypothetical transition path population growth rate****,* $\hat{\varsigma}_x(\bar{x}, t)$*, be defined implicitly as a function satisfying the following conditions:*

1. $\sum_{i \in N} x_i = \bar{x}$ *and the population distribution is* ***stable****.*

[4] See, for example, Bonacich (1987).

[5] If we label the potential balanced growth path utility level as determined by the matrix $\mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{1-\eta}$ as $\bar{u}_b$ and the "subsistence" level of utility as $\bar{u}_s$, the relation of the steady state to the balanced growth path can be illustrated in the following way. If $\bar{u}_s > \bar{u}_b$, i.e., if potential balanced growth path utility is lower than the level necessary to sustain growth, then by corollary **??** the world converges to a steady state, and also the matrix $\mathbf{I} - \xi \psi_4^{\frac{1}{\nu_1}} \bar{u}_s^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{1-\eta}$ is guaranteed to be invertible. As $\bar{u}_b \to \bar{u}_s$ from below, i.e., as transport costs become lower, the matrix $\mathbf{I} - \xi \psi_4^{\frac{1}{\nu_1}} \bar{u}_s^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{1-\eta}$ will come closer and closer to being singular. If $\bar{u}_b \geq \bar{u}_s$, then the population distribution implied by corollary 8.1 either does not exist or has negative elements–meaning that the only stable long-run outcome is the balanced growth path, with $\bar{u} = \bar{u}_b$, and allocations as given by corollary 7.1.

2.

$$\tilde{\varsigma}_x(\bar{x}) = \frac{\sum\limits_{i\in N} x_i \left[f_i^0 f(u_i) + \kappa\right]}{\sum\limits_{i\in N} x_i} - 1$$

3.

$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}\right]$$

4. Given $\mathbf{x}$ *and* $\bar{u}$*,* $\tilde{\mathbf{U}}$ *must satisfy the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j(t)) + \kappa} \frac{\sum\limits_{i\in N} \vartheta_{im} \tilde{l}_i}{\sum\limits_{i\in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$

for all $j, m \in N$*.*

In the special case where $\frac{\nu_2}{\nu_1} = \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, the conditions given in definition 5 are reduced to three:

1. $\sum\limits_{i\in N} x_i = \bar{x}$ and the population distribution is *stable*

2. $$\tilde{\varsigma}_x(\bar{x}) = f^0 f(\bar{u}) + \kappa - 1$$

3. $$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}\right]$$

Now let us define a *critical population level* as the threshold such that if the world starts with a stable population distribution and long-run levels of technology, and the total population level is higher than this critical level, it will achieve sustained growth.

Definition 6 *Considering the set of long-run, stable population and technology distributions, let the* ***critical population level*** $\bar{x}^*$ *be defined as follows for the following two cases:*

Case 1: *If* $\min\limits_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} > 0$*, then* $\bar{x}^* = 0$

Case 2: *If* $\min\limits_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} \leq 0$*, then* $\bar{x}^*$ *is the point such that* $\hat{\varsigma}_x(\bar{x}) = 0$ *and* $\frac{\partial \hat{\varsigma}_x(\bar{x})}{\partial \bar{x}} > 0$*.*

Making use of definition 6, the following theorem provides sufficient conditions for the economy to stagnate into a steady state in the long run.

Theorem 5 *Given the environment that has been described, if* $\frac{\nu_2}{\nu_1} < \eta$*, the following two conditions are sufficient for the world to converge to a Malthusian steady state in the long run:*

1. *The initial distribution of population* $\mathbf{x}(0)$ *is stable, with total population given by* $\bar{x}(0)$*, and the initial levels of manufacturing potential in each location are the long-run levels associated with* $\mathbf{x}(0)$*.*

2. $\bar{x}(0) \leq \bar{x}^*$

The implications of theorem 5 have an intuitive interpretation: if transportation costs are low enough, the economy will achieve sustained growth in the long run regardless of its starting point. If, however, transportation costs are high enough such that $\min_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} \leq 0$, then unless initial levels of population and technology are above a certain threshold, which for the case of stable, long-run starting conditions is given by $\bar{x}^*$, the economy will stagnate.

A full characterization of the necessary conditions for stagnation requires a consideration of all possible starting points for the economy, including population distributions that are not stable and arbitrary distributions of manufacturing potential.

Theorem 6 *Define* $\mathbf{s}(0)$ *as an* $n^3(n-1)$*-dimensional vector in* $\mathbb{R}_+^{n^3(n-1)}$ *composed of* $x_i(0)$, $m_{i,I}(0)$, $m_{ij,D}(0)$ *for* $i \in N$ *and* $j \neq i$*. There exists a function* $z(\mathbf{s})$ *in* $n^3(n-1)$ *arguments and a* $n^3(n-1)$*-dimensional hypersurface defined by the condition* $z(\mathbf{s}) = 0$ *such that the economy will converge to a Malthusian steady state if and only if* $z(\mathbf{s}(0)) \geq 0$*.*

Proof: See Appendix **??**.

What theorem 6 says is that there exists a frontier of initial population and technology levels such that, if initial levels lay within the limits of that frontier, the economy stagnates in the long run. In the case where transport costs are low enough that $\min_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} > 0$, this frontier collapses to the origin: $z(\mathbf{s}) = 0$ for all $\mathbf{s} \in \mathbb{R}_+^{n^3(n-1)}$. This theorem is also valid when $\frac{\nu_2}{\nu_1} \geq \eta$, though it is obviously not as useful for analyzing these cases as the preceding theorems. In cases where $\frac{\nu_2}{\nu_1} > \eta$, for example, the frontier defined by $z(\mathbf{s})$ expands outward from the origin without bound such that $z(\mathbf{s}) \geq 0$ for all $\mathbf{s} \in \mathbb{R}_+^{n^3(n-1)}$.

Theorem 7 *Given the environment that has been described, if* $\frac{\nu_2}{\nu_1} < \eta$ *and the world enters a balanced growth path, utilities corresponding to* $\bar{u}$, $\tilde{\mathbf{U}}$ *and population levels corresponding to* $\mathbf{x}$ *must jointly satisfy the following three conditions:*

1.
$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \xi \mathbf{\Theta}^{\{\varsigma_m\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}$$

2. *Given* $\mathbf{x}$ *and* $\bar{u}$, $\tilde{\mathbf{U}}$ *must satisfy the system of equations given by*
$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 \bar{f} + \kappa}{f_j^0 \bar{f} + \kappa} \frac{\sum_{i \in N} \vartheta_{im} \tilde{l}_i}{\sum_{i \in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$
for all $j, m \in N$*.*

3. *The growth rate of population is equal to*
$$\varsigma_x = \frac{\sum_{i \in N} x_i \left[f_i^0 \bar{f} + \kappa \right]}{\sum_{i \in N} x_i} - 1,$$

and so the growth rate of manufacturing potential is equal to

$$\varsigma_m = (1 + \varsigma_x)^\eta - 1$$

.

Corollary 7.1 *If $\frac{\nu_2}{\nu_1} = \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, then, if the world enters a balanced growth path, $u_i = u_j = \bar{u}$ for all $i, j \in N$ and $\bar{u}$ and population levels corresponding to $\mathbf{x}$ are pinned down by the following single condition:*

1.
$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \xi \mathbf{\Theta}^{\{\varsigma_m\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^\eta$$

where the growth rate of population is equal to

$$\varsigma_x = f^0 \bar{f} + \kappa - 1,$$

and the growth rate of manufacturing potential equal to

$$\varsigma_m = (f^0 \bar{f} + \kappa)^\eta - 1$$

.

Theorem 8 *In the environment that has been described, if either $\frac{\nu_2}{\nu_1} > \eta$ or $\frac{\nu_2}{\nu_1} < \eta$ and the world converges to a Malthusian steady state, utilities corresponding to $\bar{u}$, $\tilde{\mathbf{U}}$ and population levels corresponding to $\mathbf{x}$ must jointly satisfy the following three conditions:*

1. $$\frac{\sum_{i \in N} x_i \left[f_i^0 f(\bar{u}\tilde{u}_i) + \kappa \right]}{\sum_{i \in N} x_i} = 1$$

2.
$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^\eta \right]$$

3. *Given $\mathbf{x}$ and $\bar{u}$, $\tilde{\mathbf{U}}$ must satisfy the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j) + \kappa} \frac{\sum_{i \in N} \vartheta_{im} \tilde{l}_i}{\sum_{i \in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$

for all $j, m \in N$.

Corollary 8.1 *If either $\frac{\nu_2}{\nu_1} > \eta$ or $\frac{\nu_2}{\nu_1} < \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, and the world converges to a Malthusian steady state, then*

- $u_i = u_j = \bar{u} = f^{-1}\left(\frac{1-\kappa}{f^0}\right)$
-
$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^\eta \right]$$