ABSTRACT OF THE THESIS: QUALITATIVE THEORY OF DIFFERENTIAL EQUATIONS IN THE PLANE AND IN THE SPACE, WITH EMPHASIS ON THE CENTER-FOCUS PROBLEM AND ON THE LOTKA-VOLTERRA SYSTEMS

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In the theory of ordinary differential equations we can find two fundamental problems. The *direct problem* which consists in a broad sense in to find the solutions of a given ordinary differential equation, and the *inverse problem* which consists in to find the more general differential system satisfying a set of given properties.

In this thesis we state and study the following inverse problem.

(I) *The inverse approach to the center-focus problem .*

We consider analytic (polynomial) vector fields $\mathcal{X} = (-y + X)\,\dfrac{\partial}{\partial x} + (x + Y)\,\dfrac{\partial}{\partial y}$, where $X = X(x, y))$ and $Y = Y(x, y))$ are analytic (polynomial) function without constant and linear terms defined in a neighborhood of the origin. It is well–known that $\mathcal{X}$ has a center at the origin if and only if $\mathcal{X}$ has in a neighborhood of the origin of a Liapunov-Poincaré analytic first integral $H = \dfrac{1}{2}(x^2 + y^2) + h.o.t.$. The center-focus problem consists in distinguishing when the origin of $\mathcal{X}$ is either a center or a focus. In this thesis we study the inverse problem: for a given Liapunov-Poincaré first integral H, we determine the analytic (polynomial) vector field $\mathcal{X}$ such that H is it a first integral. Moreover, given in a neighborhood of the origin an analytic function of the form $V = 1 + h.o.t.$, we determine the analytic (polynomial) vector field $\mathcal{X}$ such that V is it a Reeb integrating factor, and consequently the system has a center at the origin.

(II) *The inverse approach to the center-focus problem for differential systems with a Liapunov-Poincaré first integral of the form* $H = \dfrac{1}{2}(x^2 + y^2)\,(1 + h.o.t.)$. We find this new class of centers which we call weak centers, they contain the uniform and the holomorphic centers, but they do not coincide with the class of isochronous centers. We determine the conditions under which the vector field $\mathcal{X}$ has a weak center. By applying the obtained results we determine the necessary and sufficient conditions of the existence of a weak center for a wide class of polynomial differential systems.

(III) *Construction of the generalized* 3*-dimensional Lotka-Volterra systems having a Darboux invariant.*

We construct a generalization of 3-dimensional Lotka-Volterra model with the state space $\left\{(x, y, z) \in \mathbb{R}^3 : \quad x \geq 0, \quad y \geq 0 \quad z \in \mathbb{R}, \right\}$ and having the Darboux invariant $I = (x + y + z - 1)e^{-at}$, $a \in \mathbb{R}$. We characterize all the final evolutions of this model and we prove that this differential systems has 7920 different phase portraits.

The dynamics of the obtained differential equations on the invariant plane $x+y+z = 1$ is described by the two dimensional Lotka Volterra system and the dynamics at infinity produces a cubic planar Kolmogorov system.

The results of the thesis have been published in five papers.