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Abstract

The Solvency II standard formula imposes a quantitative SCR – Solvency Capital Requirement – to insurance companies, depending on risk-based components. As we want to assess the potential impacts induced by the regulation on insurers' asset allocation, we study in a first chapter the differences between the efficient frontier of portfolios solving a return-standard deviation optimization problem, as designed by (Markowitz, 1952), and the efficient frontier of portfolios satisfying an optimization of Solvency II capital requirements. We find that in a low interest rates environment, the two optimization models lead to different optimal allocations, as the SCR model inhibits investments in corporate bonds to the detriment of government bonds but does not disincentivize risky asset classes such a real estate or equity as suggested in other studies. However, when interest rates are higher, the SCR optimization tends to over allocate corporate bonds and under allocate equity and real estate.

In the second part we focus on the fixed-income part of insurance companies' asset allocation that is subject to spread capital requirements set out by Solvency II standard formula, taking into account only the spread component of risk and return. We find that the SCR models may prevent insurers from allocating more assets to corporate and covered bonds, while they are incentivized to invest in the riskiest available government bonds. However, Solvency II may also encourage slightly more conservative asset allocations between ratings inside the corporate class. Finally, the preference between spread durations is not significantly affected by the choice of one optimization method over the other.

One of the key shortcuts of the Solvency II standard formula is that all parameters are calibrated at a one-year horizon, which is a common yardstick for Value-at-Risk calculations. However, this fails to address the fact that most insurance companies face liabilities on a longer term, and as such they should calibrate their risk appetite on a longer time frame. The third chapter is structured in two main parts: in the first one, we study the differences between the efficient frontier of portfolios solving a return-standard deviation optimization problem and the efficient frontier of portfolios satisfying an optimization of Solvency II capital requirements, considering liabilities with different durations. In the second part, we cover how the Solvency II Delegated Acts respond to these concerns by considering some specificities to account for assets that are supposed to be held for the long term. We finally present the characteristics of another risk measure that could complement the one-year Value-at-Risk concept under certain assumptions of length and illiquidity of liabilities, so that insurance undertakings would not be prevented to invest in the most suitable assets for their own time horizon.

Keywords: asset allocation, portfolio optimization, asset-liability management, Solvency II, insurance regulation, risk capital.

JEL codes: G11, G12, G15, G18, G22, G28.

Contents

Introduction to this doctoral thesis

The insurance industry plays a vital role in ensuring the stability and security of economies by providing risk mitigation and financial protection with the premise that insurance companies can honor the claims of their policyholders at any time. For this reason, the sector has been the object of regulations for many years, but the global financial crisis of 2008 marked by the collapse of major financial institutions triggered a profound reassessment of regulatory frameworks across the financial sector. In the insurance industry, Fortis NV, which had an insurance subsidiary – Fortis Insurance, had to be bailed out by the Belgian and Dutch governments and its operations nationalized or sold off to other financial institutions as a consequence of its exposure to toxic assets. Aegon, a major Dutch insurance company, suffered significant losses during the crisis related to mortgage-backed securities and other risky assets so that the Dutch government had to step in and provide support to the company, injecting capital to ensure its solvency and stability.

The financial crisis exposed critical weaknesses in the supervision and risk management practices of banks, insurers, and other financial entities, leading to a significant overhaul of regulatory regimes worldwide. In the banking sector, the Basel Committee on Banking Supervision introduced Basel II and subsequently Basel III, frameworks that aimed to strengthen the capital adequacy and risk management practices of banks. In the insurance industry, the crisis paved the way for the emergence of Solvency II, a comprehensive regulatory framework aimed at ensuring the stability and solvency of insurance companies operating within the European Economic Area, harmonizing and strengthening the pre-existing regulations of the member states. The regulation is established on a risk-based approach to capital requirements, recognized as the 99.5% Value-at-Risk (VaR) stress of an insurer's own funds over a one-year time horizon. It follows a principle of fair valuation of assets and liabilities, taking into account their market value and future cash flows rather than their book value. Some qualitative requirements are also implemented on risk management and governance as well as transparency and disclosure to regulators and policyholders. Regulations similar to Solvency II were released in other countries with the same principles. In Switzerland, for instance, the Swiss Financial Market Supervisory Authority (FINMA) implemented the Swiss Solvency Test (SST) in 2011 that shares similar objectives in promoting risk-based supervision and strengthening capital adequacy requirements. Similarly, in Bermuda, a renowned global hub for the insurance and reinsurance industry, the Bermuda Monetary Authority (BMA) introduced the Bermuda Solvency Capital Requirement (BSCR) regime in 2012. The United States of America, Canada and China also reformed their insurance regulations in the same period.

In addition to Solvency II and Basel III, several other regulations were introduced during this period to address specific aspects of insurance company operations. For instance, the International Financial Reporting Standards (IFRS) 17 seeks to enhance the transparency and comparability of insurance company financial statements, making the principles of accounting standards more compatible with the fair value approach of Solvency II. The International Association of Insurance Supervisors (IAIS) developed the Common Framework for the Supervision of Internationally Active Insurance Groups (ComFrame) to facilitate global cooperation and oversight of insurance groups.

For the investment managers of the insurance undertakings subject to Solvency II, the Solvency Capital Requirements (SCR) represent a new constraint that they have to consider when they build a Strategic Asset Allocation (SAA), along with their pre-existing considerations such as for instance maximizing returns in accordance with their self-assessed risk appetite, matching their liabilities via cash flow matching or other ALM techniques and ensuring a proper diversification between asset classes, sectors, and geographies.

The goal of this doctoral thesis is to understand if these new regulations may change the investment decisions of the managers of the insurance industry. To this end, we develop different comparisons between the asset allocation of an insurer that optimizes its portfolio according to the classical mean-variance framework designed by H. Markowitz (1952) to the result of an alternative optimization where risk is measured by the SCR.

In the first chapter, we present the general methodology and compare the asset allocations derived from these two optimization methods across four asset classes: sovereign bonds, corporate bonds, equity, and real estate. We repeat this exercise with data from two different periods to see how the results vary between a low-yield environment and the year 2022, where yields have risen significantly closer to the historical averages.

The second chapter focuses specifically on the fixed-income component of insurance companies' portfolios and examines the allocation of spread risk. Under SCR criteria, we expect a strong incentive to invest in government bonds, particularly riskier ones, as they incur no additional capital charge. We delve into the different categories of bonds to understand the treatment of the different rating categories and spread durations.

In the third chapter, we integrate the liability profile of insurance companies to gain a comprehensive understanding of the effects of Solvency II on their asset allocation. We explore the differences between optimizations with standard deviation constraints and SCR constraints by considering liabilities with varying durations. We study the impacts of the Long-Term Guarantees measures allowed by Solvency II to finally propose an alternative approach to calculating capital requirements that takes into account the duration of liabilities, particularly for long-term holdings.

I. Portfolio Optimization - Asset Classes

1. Introduction

The introduction of Solvency II regulation has changed deeply the insurance industry in the European Economic Area since January 2016 when it came into effect. The Directive 2009/138/ec of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II)[1], detailed in the Commission Delegated Acts[2], harmonizes the various rules in force under each national authority. The new regulation is organized around three pillars. The first one – that triggers the challenges treated in this paper – establishes quantitative constraints including Solvency Capital Requirements (SCR) that represent the minimum amount of own funds that an insurer must hold to operate its business. Pillar II sets out qualitative requirements for the risk management and governance of the undertakings and specifies details of the supervisory process while pillar III focus on transparency requirements to supervisory authorities and the public.

The quantitative requirements set out in the Delegated Acts introduce a conceptual change as compared to the Solvency I regime, which did not intend to capture the market, credit and operational risks of the company but was rather based on general rules leading to requirements proportional to the size of the undertaking. The SCR set out by Solvency II follow risk-based principles as they represent the 99.5% Value-at-Risk (VaR) stress of the undertaking's Own Funds (OF) at a one-year horizon. The companies may submit for approval their own internal model to the Regulator for calculating the SCR, but the Delegated Acts provide a general guidance for a Standard Model (SM) that will apply to the majority of undertakings who do not want to bear the extra-burden of setting and maintaining an internal model. The SM had been previously debated with all stakeholders during five quantitative impact studies (QIS), as the parameters of the calculation should fit uniformly for all situations across undertaking types, national specificities, market conditions, etc. For the market risk module in particular, the challenge was to calibrate adequate shocks and correlations for the risks involved in the calculation in order to set higher requirements for the holding of riskier assets, but at the same time not to penalize unduly these assets to avoid unnecessary portfolio de-risking.

However, these quantitative capital requirements are disrupting the way that insurance companies manage their investment portfolios. All investment allocations designed through a risk-return, mean-variance framework based on modern portfolio theory as formalized by H. Markowitz (1952) and long-term asset-liability considerations, must now incorporate risk capital as a new constraint in the equation. As the capital requirements for market risk in Solvency II regulation were calibrated to account for a 99,5% Value at Risk (VaR) at a one-year horizon, they allegedly reflect the risk variable as variance or

[1] European Union (2009).
[2] European Union (2015).

standard deviation do. But as internally assessed variables used by the management of insurance companies are substituted by another measure, enforced by an external entity – European Insurance and Occupational Pensions Authority (EIOPA) –, the target asset allocation of insurers is expected to be altered.

As we want to assess the potential distortions to the asset allocation of insurance companies, this paper compares the asset allocation of an insurer that optimizes its portfolio according to the classical mean-variance framework to the result of an optimization where risk is measured by the SCR. The next section explores the current research on the subject. The market module of Solvency II standard model SCR is briefly explained in the third section, so that we can explain the data and assumptions of our simulations in the fourth section. The two asset allocation optimizations are subsequently analysed, and their results are discussed before some concluding remarks.

2. State of the research

The potential effects of Solvency II on insurers' investments have been discussed since the early drafting of the Delegated Acts. Some studies focus on particular asset classes that could be overly penalized by the important capital requirements that they involve under Solvency II. Thus, the riskiest assets such as real estate (Rehkugler & Schindler, 2012), equity (Braun et al., 2014; Schwarzbach et al., 2014) or infrastructure ((Gatzert & Kosub, 2014) have been assigned higher capital requirements than what was needed according to the researchers, so that investments in those assets could be reduced by the regulation. One of the common objectives of those papers is to show that investments in risky or illiquid assets that serve long-term liabilities should not be penalized by a capital requirement calibrated at a one-year horizon. Indeed, too high capital requirements would hinder insurers' ability to invest in assets that provide an adequate return to fund long-term liabilities. Some newer studies are a bit more balanced, as (Wontke & Balleer, 2018) find that Solvency II could be less restrictive than the previous BaFin stress test as regards equity investments. The risk-free treatment of sovereign bonds in Solvency II standard model is the elephant in the room studied by (Duell et al., 2017): they find significant effects of the transmission of sovereign risk to insurance companies and stresses that those effects are completely ignored by the European regulation.

These preoccupations are present in other studies that try to assess the potential impacts of Solvency II on insurers' asset allocation by integrating all the asset classes in one general model. (van Bragt et al., 2010) tried to assess the future capital requirements of Solvency II of a fictitious life insurer based on the Dutch regulatory framework (FTK), considered a pioneer of the new European regulation. They stress the importance of asset-liability matching to reduce capital charges in the new framework. (Hoering, 2013) challenges the idea that Solvency II capital requirements could negatively impact the optimal portfolio allocation of insurance companies, as it appears that the Standard & Poor's rating model that most insurers try to observe requires more capital that Solvency II standard model for a representative life insurer. (Fischer & Schluetter, 2015) present a model that try to maximize shareholder value by maximizing the theoretical default put option of the company, concluding that with the parameters currently used in Solvency II standard model, insurers should not invest in stocks.

The idea of using some kind of portfolio optimization methodology derived from (Markowitz, 1952) to optimize the expected return of an insurer's investments with a given SCR and other parameters such as the capitalization level of the company has already been explored in the literature with different variants. We summarize the methodology, assumptions and results of the studies of (Braun et al., 2017, 2018; Escobar et al., 2019; Heinrich & Wurstbauer, 2018; Kouwenberg, 2018) in Figure 1 below.[3] Even if the approaches may differ among those studies, some conclusions are almost unanimously shared by all the researchers:

- Solvency II prevents insurers from investing as much as they would do otherwise in risky asset classes such as equity and real estate;
- Solvency II offers an incentive to invest in sovereign bonds, disregarding the potential spread or concentration risks;
- The impact of Solvency II on asset allocation depends on the level of capitalization of the company: well-capitalized companies will be less affected than their peers.

However, most of those papers do not try to adjust the backwards-looking parameters of their model for changes in expectations due to the current market environment. Expected annualized returns of 3% to 6% for european government bonds as proposed by (Braun et al., 2017, 2018; Heinrich & Wurstbauer, 2018) are just highly improbable given the sovereign yields of the last years. The expected returns for equity and real estate could be somehow adjusted as well to take into account the high valuations attained in the last years as compared to the fundamentals of the markets calculated from expected earnings.

The liability profile of the companies is not really considered in those works, as they treat the evolution of the liabilities quite independently from the evolution of the asset side, when the reality of the market value-based Solvency II world is quite different: as liabilities are discounted at the risk-free rate, they are highly correlated to the performance of the government bonds in the asset side. Thus, it should be possible to integrate them in the optimization problem without adding too much complexity.

[3] In this thesis, all tables, graphs, and diagrams are interchangeably referred to as figures to facilitate understanding.

Figure 1: Asset Allocation Optimization under Solvency II in the literature

Title	Authors	Year	Optimization type	Insurer type	Scenario	Expected returns: Treasury bills	Government bonds	Corporate bonds	Equity	Hedge funds	Property
Portfolio Optimization under Solvency II	Escobar, Kriebel, Wahl, Zagst	2019	Two-steps optimization: 1) Find the optimal investment strategy 2) Find a suitable relative asset liability cushion constraint	Life		1,00%	1,10%	1,80%	6,00%	N/A	N/A
The Impact of Risk-based Regulation on European Insurers' Investment Strategy	Heinrich, Wurstbauer	2018	Minimize SCR for a given target return.	Life		3,03%	5,43%	7,52%	10,17%	10,90%	7,82%
Strategic Asset Allocation for Insurers under Solvency II	Kouwenberg	2018	Maximize expected return on own funds subject to a given market SCR	Life		0,25%	1,50%	2,40%	4,86%	N/A	3,50%
Return on Risk-Adjusted Capital Under Solvency II: Implications for the Asset Management of Insurance Companies	Braun, Schmeiser, Schreiber	2018	Maximizing RORAC = E(R)/SCR	Life	B	0,33%	3,93%	4,77%	7,58%	N/A	2,85%
					A	N/A	4,79%	6,55%	1,89%	N/A	4,33%
Portfolio Optimization under Solvency II: Implicit Constraints Imposed by the Market Risk Standard Formula	Braun, Schmeiser, Schreiber	2017	1) Minimize volatility subject to a given expected return. 2) Discuss the admissibility (OF>SCR) of optimal portfolios.	Life		3,14%	5,96%	6,99%	9,21%	9,65%	4,81%

Annualized volatility (s)						Durations	Own Funds	Upper limits	Short positions	Main conclusions
Treasury bills	Government bonds	Corporate bonds	Equity	Hedge funds	Property					
0,00%	6,00%	9,53%	29,95%	N/A	N/A	Fixed	Variable	None	Prohibited	The optimal investment strategy is highly dependent on RALC (~OF), i.e. the risk aversion of the company.
1,24%	3,78%	6,34%	19,02%	10,64%	10,16%	Fixed	Variable	On corporates, stocks, RE and hedge funds (German Investment Regulation)	Prohibited	SCR prevent insurers from constructing optimal portfolios by assigning too much capital to equity and alternative asset classes. This impact is more important for undercapitalized insurers.
N/A	N/A	N/A	N/A	N/A	N/A	Fixed	Fixed	None	Allowed	Insurers are incited to hedge their interest rate risk on the liability side. SCR promotes concentrated investments in government bonds rather than in stocks or real estate.
0,14%	2,99%	4,68%	16,57%	N/A	0,76%	Fixed	Fixed	On corporates, stocks, RE and hedge funds (German Investment Regulation)	Prohibited	Concentration in treasuries and govies for better RORACs. SCR is more relevant than expected return in this optimization setting. Diversified portfolios need higher SCR.
N/A	3,16%	5,94%	18,85%	N/A	1,88%					
0,50%	3,34%	5,55%	19,26%	7,08%	1,76%	Fixed	Fixed	On corporates, stocks, RE and hedge funds (German Investment Regulation)	Prohibited	SCR entails higher concentration in govies. Asset stresses are more relevant than duration mismatch. Mean-variance efficiency of portfolios is unrelated to Solvency II admissibility.

3. Solvency II Standard Model: Market module

The Solvency Capital Requirements representing the VaR of the basic own funds of the undertaking is calculated as the sum of the Basic Solvency Capital Requirement (BSCR) and the Operational Risk, adjusted for the loss-absorbing effect of technical provisions and deferred taxes. The BSCR is the most important item, calculated from the diversified aggregation of different risk modules: the market risk module (SCR_{Mkt}), the counterparty risk module, the life underwriting risk module, the health underwriting risk module and the non-life underwriting risk module.

$$\mathrm{BSCR} = \sqrt{\sum_i \sum_j \rho_{i,j} * \mathrm{SCR}_i * \mathrm{SCR}_j} \tag{1.1}$$

where i,j represents all pairs of risk module, and $\rho_{i,j}$ is the correlation between those pairs set by the regulator in Solvency II Delegated Acts, so that the BSCR takes into account some diversification effect between risk modules and is inferior to their sum.

The same scheme applies for the calculation of the market risk module on which we will center our analysis: the SCR_{Mkt} aggregates the interest rate risk (SCR_{IR}), the equity risk (SCR_{EQ}), the property risk (SCR_{RE}), the spread risk (SCR_{CS} for credit spread), the concentration risk (SCR_{Conc}) and the currency risk (SCR_{Curr}).

$$\mathrm{SCR}_{\mathrm{Mkt}} = \sqrt{\sum_i \sum_j \rho_{i,j} * \mathrm{SCR}_i * \mathrm{SCR}_j} \tag{1.2}$$

where i,j represents all pairs of market risks, and $\rho_{i,j}$ is the correlation between those pairs as set out in the Delegated Acts and reproduced in Figure 2 below:

Figure 2

	Interest	Equity	Property	Spread	Concentration	Currency
Interest	1,00	A	A	A	0,00	0,25
Equity	A	1,00	0,75	0,75	0,00	0,25
Property	A	0,75	1,00	0,50	0,00	0,25
Spread	A	0,75	0,50	1,00	0,00	0,25
Concentration	0,00	0,00	0,00	0,00	1,00	0,00
Currency	0,25	0,25	0,25	0,25	0,00	1,00

where A = 0 in case of interest rate shock up and 0.5 in case of interest rate shock down.

The interest rate risk is calculated as the worst case between an interest rate up scenario and an interest rate down scenario. The interest rate up simulates a stress where the swap curve – the risk-free benchmark used by the regulator – increases by the minimum of a percentage depending on the maturities and +100 basis points. Then, given the market conditions of the last few years, it is equivalent to a +100bp interest rate shock. Similarly, the interest rate down scenario simulates a downwards stress of the swap curve calculated as a percentage of the yield at each maturity, but negative interest rates are not shocked and there is no minimum stress as in the interest rate up scenario, so that the total stress can be extremely low.

The equity risk is the aggregation of two types of equity risks:

$$\mathrm{SCR_{EQ}} = \sqrt{\mathrm{SCR}_{Type1}{}^2 + 2 * 0.75 * \mathrm{SCR}_{Type1} * \mathrm{SCR}_{Type2} + \mathrm{SCR}_{Type2}{}^2} \tag{1.3}$$

calculated as $\mathrm{SCR}_{Type1} = (39\% + \mathrm{SA}) * \mathrm{MV_{EQ1}}$ and $\mathrm{SCR}_{Type2} = (49\% + \mathrm{SA}) * \mathrm{MV_{EQ2}}$ where SA stands for Symmetric Adjustment – a countercyclical factor that we will set to 0 for simplicity in our analysis – and MV represents the market value of the corresponding type of equity. Type 1 represents equity securities listed in EEA or OECD countries and type 2 gathers all other kinds of equity, including equities listed in emerging markets, private equity, investment funds for which a look-through approach is not possible, hedge funds, commodities, etc. In our analysis, we will only consider type 1 equities so that $\mathrm{SCR_{EQ}} = \mathrm{SCR}_{Type1}$.

The property risk calculation is comparatively straightforward:

$$\mathrm{SCR_{RE}} = 25\% * \mathrm{MV_{RE}} \tag{1.4}$$

The spread risk also applies a shock to the market value of fixed-income assets, but this shock depends on the type of bond or loan, on the credit quality of the issuer, and on the modified duration of the asset. Public sector European issuers such as governments, the European Central Bank and other multinational organizations are assigned a shock of 0%, and thus considered risk-free.

We will not consider concentration risk as it is possible to eliminate it by diversifying the portfolio, nor currency risk as it is possible to invest only in domestic currency, or to mitigate fully any foreign currency exposure by using derivatives.

The three main risks of the market module of the Solvency II Standard Model corresponding to the three main asset classes of insurers' portfolios – bonds, equity, and real estate – are in their essence calibrated to be proportional to the market value of each asset, so that the volatility of the capital requirements depend directly on the volatility of the underlying. This means that any shock applied to the market value of the assets, and therefore to the own funds of the company, would be mitigated by a lower value of the corresponding capital requirements, making the resulting solvency ratio less volatile than the own funds of the insurer. This mitigation of the ratio between the excess of assets over liabilities and the risk measure would persist in other risk models based for instance in the standard deviation of the portfolio as the one that we will study in this thesis. However, we can think of two other factors mitigating respectively the spread and equity shocks of the Solvency II Standard Model. If the spread of the corporate bonds rises causing a decrease in the market value of the portfolio, the modified duration of the bonds decreases as well, so that the percentage of spread risk of the Standard Model applied to the market value of the same assets will marginally diminish. For the equity risk, the symmetric adjustment as set out by EIOPA in the article 172 of the Delegated Acts of Solvency II introduces a counter-cyclical factor, as the difference between the current level of the equity index defined by the regulation and its average value over the last 36 months is compared with the average long-term increase of 8% and will respectively increase or decrease the equity capital charge if the current level of the equity index is higher or lower than the expected figure. These two counter-cyclical factors are not explicitly considered in this thesis as we focus our analysis on a single period.

4. Data and portfolio assumptions

4.1. Choice of benchmarks

We gathered market data for the four main asset classes where insurers invest: government bonds, corporate bonds, equity, and real estate. The benchmarks of the fixed-income categories are the ICE BofAML Indices used in (CEIOPS, 2010): ICE BofAML AAA Euro Government Index for the allocation to sovereign bonds and ICE BofAML 7-10 Year A Euro Corporate Index for corporate bonds. Both indices are composed of bonds with a similar modified duration (8,66 years for the sovereign benchmark and 7,85 years for corporates). The use of the full ICE BofAML Year A Euro Corporate Index with all durations was also tested and lead to quite similar findings as the ones presented in section 5 for our benchmark, which is logical as the correlation of both benchmarks with the other asset classes vary only by a tiny amount – less than 0.02 in absolute terms. The difference would be that the asset allocations are less sensitive to changes in expected returns along the efficient frontier as the risk – measured by standard deviation or SCR – of the benchmark with all durations would be smaller as consequence of its shorter average duration. The corporate index was restricted to a single rating class representative of the median rating of the corporate bonds of European insurers to make the spread risk calculations possible. The consequences of choosing other ratings are studied in the second chapter of this thesis.

We choose to use the same equity benchmark as in the Solvency II calibration paper of the Committee of European Insurance and Occupational Pensions Supervisors[4]: the MSCI World Developed Price Index. We keep the world index as it is easily investable even for small undertakings through indices and does not involve higher capital requirements compared to domestic securities, to the contrary of non-EEA sovereign bonds which are not treated as risk-free investments as their European peers. In any case, a European index such as the MSCI Europe Price Index would probably not change much our

[4] CEIOPS (2010)

results as the volatility of the returns would be only marginally lower and the correlations with the other indices would vary by less than 0,1 in absolute terms.

As there is no straightforward property index available with daily data, we propose to use the SXI Real Estate Funds Total Return Index, a Swiss index composed of closed-end real estate funds. The underlying securities are less traded than usual equities so that they keep some of the main characteristics of direct real estate investments. The chosen benchmark shares similar characteristics to those of the appraisal-based indices studied in (CEIOPS, 2010) and specifically to the UK IPD index finally used by the Regulator in the calibration of Solvency II. The volatility of our benchmark is only slightly inferior to the UK IPD index, and it also shows a positive correlation with government debt and equity returns as shown in Figure 3.

4.2. Time interval used

The study of the risk characteristics of asset returns often comes with a conundrum when the total timespan of available data is limited by the number of observations. We need to focus on yearly returns to allow for a fair comparison between the results of an optimal portfolio allocation using standard deviation constraints with an optimization using SCR constraints, as Solvency II contemplates a one-year horizon and is calibrated accordingly. However, the yearly returns series are quite limited for the indices chosen as benchmarks – forty-nine years of data for the equity benchmark down to twenty-two years for the fixed-income one. The parameter estimates of returns are likely to be misestimated, and more importantly critical information about tail risk is missing as there is no way to directly know what would be the 1-in-200 year loss for any of the benchmarks. The calibration of Solvency II by the CEIOPS circumvents this shortage of information by using rolling-window annualization of daily data, i.e., considering the return of an asset class between any trading day and the same trading day in the following year. This method multiplies the data points available for the analysis by the number of trading days per year of each benchmark at the price of several significant shortcomings highlighted by (Mittnik, 2011). The main issue with the rolling-window annualization that every daily data point is only slightly different from the previous one as it contains mostly the same information – the return of the index for one year less one trading day. Mittnik demonstrates that such method produces strongly autocorrelated data that deteriorates not only the accuracy of Pearson correlation estimates between indices but also tail correlations and as such gives unreliable inputs for Solvency II shocks and probably overestimates the correlation between asset classes.

A possible alternative to the rolling-window annualization would be to infer the behavior of the yearly returns data from the daily data by a simple annualization to avoid the use of overlapping information. The yearly standard deviation would be inferred from the daily figure by using the square root of time rule. One of the possible issues of this method is that the distribution of daily data does not represent a fair image of the corresponding yearly distribution as daily data is subject to different skewness and kurtosis parameters (see for instance (Neuberger, 2012) and (Fama & French, 2018)) that are important for the estimation of tail risks. However, as our study focuses on a comparison of a rule-based risk metric (SCR) with a variance risk metric, only the first two moments of the distribution are actually relevant. The standard deviation derived from a short period of time will not differ substantially from the one derived from monthly or yearly data. Another small problem with using daily data is that the correlations between asset classes could be affected by the closing hours of the underlying assets of the indices. In order to avoid such discrepancy, the choice of a longer time interval would be preferable. Weekly returns have been chosen in this study to gather enough data points and avoid the specificities of dealing with daily data.

4.3. Statistics of asset classes

Twenty-two years of weekly data are available for the four asset classes, from 2000 through 2021. The past average returns are calculated on this period, along with the standard deviations and correlations between the benchmarks. The results are displayed in Figure 3 below.

Figure 3

	Government bonds	Corporate bonds	Equity	Real Estate
Average returns	4,19%	4,92%	5,64%	6,19%
Expected returns (2021)	-0,36%	0,63%	5,53%	3,84%
Standard deviation	3,95%	5,19%	18,13%	8,00%
Correlations:				
Government bonds	1,000	0,674	-0,284	0,102
Corporate bonds	0,674	1,000	0,061	0,188
Equity	-0,284	0,061	1,000	0,236
Real Estate	0,102	0,188	0,236	1,000

As always, past performance is not indicative of future results: the impressive returns of sovereign bonds and corporate bonds have been driven by a general fall in yields in the last two decades. The yields of their benchmarks stand respectively at -0,36% and 0.63% as of end 2021, indicating that it would be almost impossible to earn higher returns unless yields dive into much deeper levels in the short term. As regards standard deviations and correlations, they appear to be much less volatile than returns, so we will consider them as best estimates for the optimization problem.

In order to get a more reliable estimate of expected returns, we take the yields of the bonds benchmarks as estimates for future performance of the corresponding asset classes and calculate the values of equity and property expected returns based on the government bonds figure. The study of (Jorda et al., 2019) wherefrom we extract the data of Figure 4 gives a broad overview of worldwide historical risk premia for equity and real estate corresponding to the post-WW2 period (1963–2015).

Figure 4

	E(R-Rbonds)		
	Equities	Housing	Total
AUS	5,05	4,44	4,75
BEL	6,57	4,72	5,65
CHE	6,7	2,38	4,54
DEU	10,39	7,31	8,85
DNK	2,55	4,79	3,67
ESP	4,28	2,05	3,17
FIN	1,63	3,23	2,43
FRA	4,45	4,02	4,24
GBR	6,2	5,1	5,65
ITA	6,34	7,5	6,92
JPN	4,26	5,24	4,75
NLD	5,95	4,65	5,3
NOR	9,11	6,45	7,78
PRT	6,26	4,38	5,32
SWE	6,19	4,5	5,34
USA	4,81	3,28	4,05
World	5,66	4,63	5,14
Europe	5,89	4,70	5,30
Adjustment for transaction costs	0	-0,5	
Risk premia	5,89	4,20	

We consider past risk premia over such a long period as reasonably fairer indicators of expected returns than the actual returns of our shorter sample. We adjust further the real estate risk premium according to the conclusions on the impact of transaction costs of the same study (Table 10, p. 23). Those risk premia are then added to the expected returns of government bonds to give an estimate for expected equity and real estate returns.

4.4. Assumptions used for SCR calculations

We consider a representative insurer with an asset side of the balance sheet worth 110% the liability side, so that the own funds represent 10% of the liabilities or 9.09% of the assets. The assets consist exclusively in financial instruments subject to the market risk module, and the liabilities are composed exclusively of the Best Estimate of Liabilities (BEL) as defined in Solvency II Delegated Acts, without risk margin as we do not want to consider underwriting issues in this doctoral dissertation. The BEL has a duration of 8,66 years like the bond benchmarks. As the BEL is the result of a sum of cash flows discounted at the risk-free rate, it will behave similarly to government bonds. The insurer cannot act on the liability side for optimizing the portfolio but will be able to use it to get a short exposure to government bonds, as the model begins with a negative exposure to the risk-free rate worth ten times the own funds. Any other short exposure is prohibited.

In the first simulation, the insurer minimizes the risk of its portfolio as measured by its standard deviation for a given expected return. The parameters are described above. In the second simulation, the insurance undertaking minimizes its market SCR for a given expected return.

The standard model considers two distinct scenarios for interest rate risk: it is calculated as the worst case between an interest rate up scenario and an interest rate down scenario. The interest rate up simulates a stress where the swap curve – the risk-free benchmark used by the Regulator – increases by the minimum of a percentage depending on the maturities and +100 basis points. Then, given the market conditions of the last few years, it is equivalent to a +100bp interest rate shock for all maturities. Similarly, the interest rate down scenario simulates a downwards stress of the swap curve calculated as a percentage of the yield at each maturity, but negative interest rates are not shocked and there is no minimum stress as in the interest rate up scenario, so that the total stress can be extremely low as it was the case in the first years of the implementation of Solvency II through 2021. The shocks expressed as a percentage of market value of the financial instrument resulting from the swap curve at the end of the year 2019 for maturities dated from 1 to 20 years are presented in Figure 5.

Figure 5

Year	EIOPA SWAP curve			Shock (% market value)	
	Central	Shock up	Shock down	Shock up	Shock down
1	-0,59 %	0,42 %	-0,59 %	-0,99%	0,00%
2	-0,40 %	0,61 %	-0,40 %	-1,97%	0,00%
3	-0,25 %	0,75 %	-0,25 %	-2,94%	0,00%
4	-0,15 %	0,86 %	-0,15 %	-3,90%	0,00%
5	-0,08 %	0,92 %	-0,08 %	-4,85%	0,00%
6	-0,03 %	0,97 %	-0,03 %	-5,80%	0,00%
7	0,03 %	1,03 %	0,02 %	-6,73%	-0,08%
8	0,09 %	1,09 %	0,06 %	-7,65%	-0,26%
9	0,15 %	1,15 %	0,10 %	-8,57%	-0,44%
10	0,21 %	1,21 %	0,14 %	-9,47%	-0,64%
11	0,25 %	1,25 %	0,18 %	-10,37%	-0,83%
12	0,30 %	1,30 %	0,21 %	-11,26%	-1,04%
13	0,35 %	1,35 %	0,25 %	-12,13%	-1,25%
14	0,38 %	1,38 %	0,27 %	-13,00%	-1,47%
15	0,40 %	1,40 %	0,29 %	-13,87%	-1,61%
16	0,41 %	1,41 %	0,29 %	-14,72%	-1,79%
17	0,41 %	1,41 %	0,29 %	-15,56%	-1,92%
18	0,41 %	1,41 %	0,30 %	-16,40%	-2,07%
19	0,43 %	1,43 %	0,30 %	-17,23%	-2,33%
20	0,46 %	1,46 %	0,32 %	-18,05%	-2,60%

To calculate the capital charges of our assets and liabilities, we consider that our benchmarks are made of only one cash flow dated at their average modified duration. We use a linear interpolation between the two closer maturities available in Figure 5 to derive the exact shock for each asset. The resulting shocks are described in Figure 6 below. The IR down shocks are expressed as a negative value as they will be assigned to the liabilities which already carry a negative sign.

Figure 6

	Government bonds	Corporate bonds
Modified Duration	8,66	7,85
Shock IR up	8,26%	7,51%
Shock IR down	0,38%	0,23%

Our corporate benchmark is rated A, so that the spread shock applied by the standard model is equal to 9.00%. The equity and property shocks are respectively equal to 39% and 25% as announced earlier.

5. Optimizing the asset allocation of an insurance company

5.1. Standard deviation optimization

We first run the classical quadratic portfolio optimization model, with the expected returns, and empirical standard deviations and correlations found in section 4. A sample of portfolios situated on the efficient frontier are described in Figure 7 below, while the efficient frontier is drawn in Figure 8. The actual asset allocations corresponding to the portfolios on the efficient frontier are depicted in Figure 9. The SCR required by the efficient portfolios is also provided in the table and the graph. To get a visual comparison between the two risk measures, the standard deviation of the optimal portfolios is multiplied by 3 to consider that it does not represent a 99.5% Value-at-Risk stress of the own funds. The number can be thought of an approximation of the number of standard deviations necessary to reach said probability threshold, if we consider that the data follows a distribution with fatter tails than a normal distribution.

Figure 7

E(R)	0,5%	5,0%	10,0%	15,0%	16,7%
Risk measures (% of OF)					
SCR	8,5%	31,2%	60,7%	90,1%	100,0%
σ	3,4%	9,3%	18,6%	28,0%	31,3%
3σ	10,2%	28,0%	55,7%	84,1%	93,8%
Portfolio allocation (% of assets)					
Government bonds	98,4%	85,1%	69,7%	54,3%	49,1%
Corporate bonds	0,0%	5,0%	11,3%	17,6%	19,8%
Equity	0,7%	1,3%	1,9%	2,6%	2,8%
Real Estate	0,9%	8,6%	17,0%	25,5%	28,3%

The efficient portfolios are constituted mainly of government bonds, but there is a noteworthy allocation to corporate bonds and real estate as well that increases with the required return. The allocation to equity also increases but at a slower pace when moving from low to high expected returns. The significantly lower standard deviation of property compared to equity makes it the best of the riskier assets, so that equity has a relatively lower weight.

The SCR is slightly lower than three standard deviations for the portfolio with the minimum standard deviation (E(R) = 0,5%, first column in Figure 7), but then increases at a faster pace than 3σ, following the expected return. This happens because the most conservative portfolios are almost exclusively exposed to interest rate risk, which is possibly underestimated by Solvency II standard formula. On the other side of the spectrum, the portfolios requiring an expected return higher than 16.7% are not acceptable according to our simplified Solvency II model, as the market module alone requires more capital than the available own funds of our insurer.

Figure 8

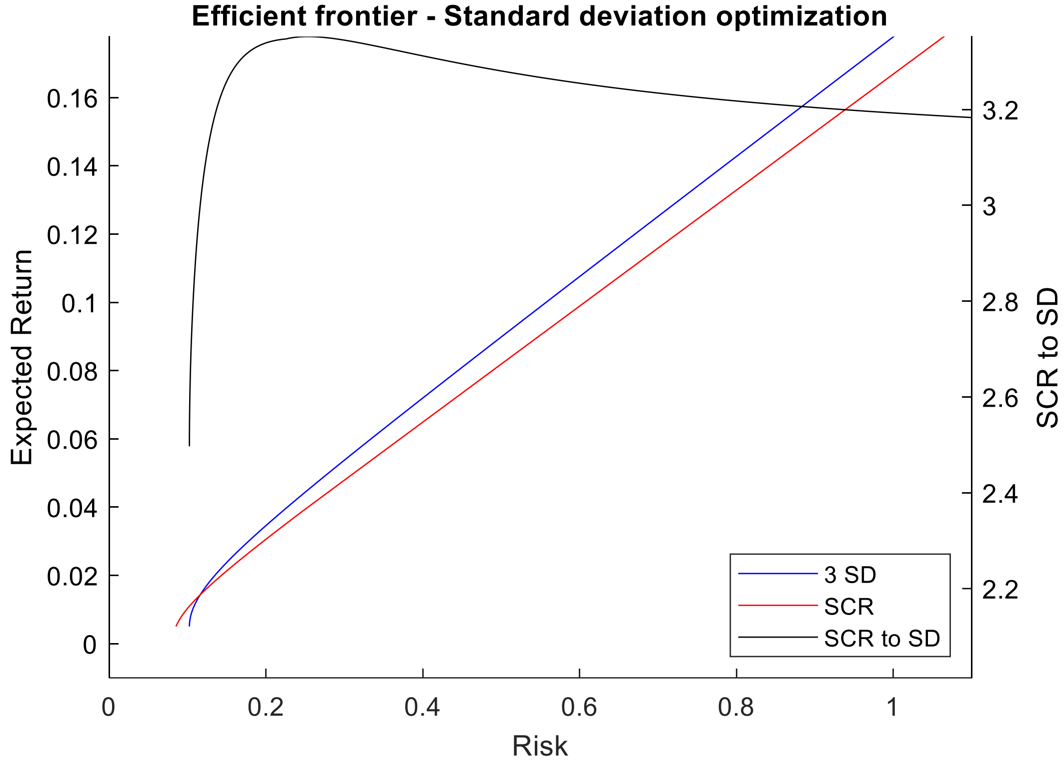
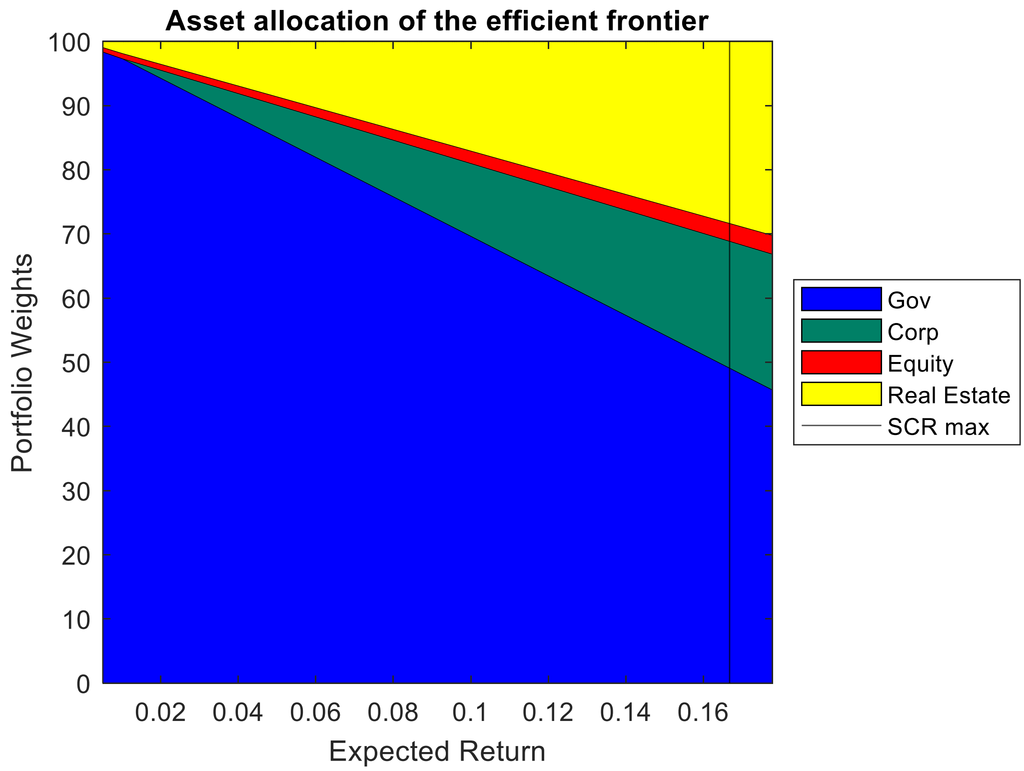


Figure 9



5.2. Solvency II Risk Capital Optimization

As a second step, we run the Solvency II optimization problem to find the efficient frontier of portfolios that minimize the SCR for a given level of expected returns. We share tables and graphs similar to the ones of the previous case.

In this setting, the allocation to government bonds dominates the picture even more than in the mean-variance simulation. Corporate bonds are completely inhibited as they are only allocated ca. ¼ of the weighting they had in the previous simulation for the corresponding expected returns across the efficient frontier. The small risk premium they provide does not compensate for the relatively high spread risk they entail. Real estate continues dominating the allocation to riskier assets as in the previous situation, but equities are assigned a material weight worth up to 6.9% for the riskiest portfolio. Interestingly, for each given expected return, the total weight allocated to riskier assets – equities and real estate – is bigger than in the classical optimization case. This fact may be partly explained by the low shock applied by the standard model in the case of negative interest rates, as it does not discourage asset-liabilities mismatches with negative duration gaps.

In most portfolios, the SCR represents just slightly more than 3σ of the own funds, so that both measures are not contradictory. The exception comes with the lowest values of expected returns and SCR, as the portfolio is overly concentrated in government bonds. As those portfolios have the highest duration gap of the simulation, they are not particularly incentivized by the 3σ measure, but as the interest rate shock of Solvency II standard model remains notably less significant than the property shock (8% vs. 25%), it is still worth broadening the interest rate gap in this case to decrease the overall market risk SCR.

In this simulation, a portfolio with a 16.8% expected return is still acceptable with the SCR constraint.

Figure 10

E(R)	0,1%	5,0%	10,0%	15,0%	16,8%
Risk measures (% of OF)					
SCR	7,8%	31,1%	60,4%	89,6%	100,0%
σ	3,7%	9,5%	19,2%	29,0%	32,6%
3σ	11,2%	28,5%	57,5%	87,1%	97,7%
Portfolio allocation (% of assets)					
Government bonds	99,1%	88,1%	77,0%	66,0%	62,1%
Corporate bonds	0,0%	1,6%	2,9%	4,2%	4,7%
Equity	0,0%	2,1%	4,2%	6,2%	6,9%
Property	0,9%	8,2%	15,9%	23,6%	26,3%

Figure 11

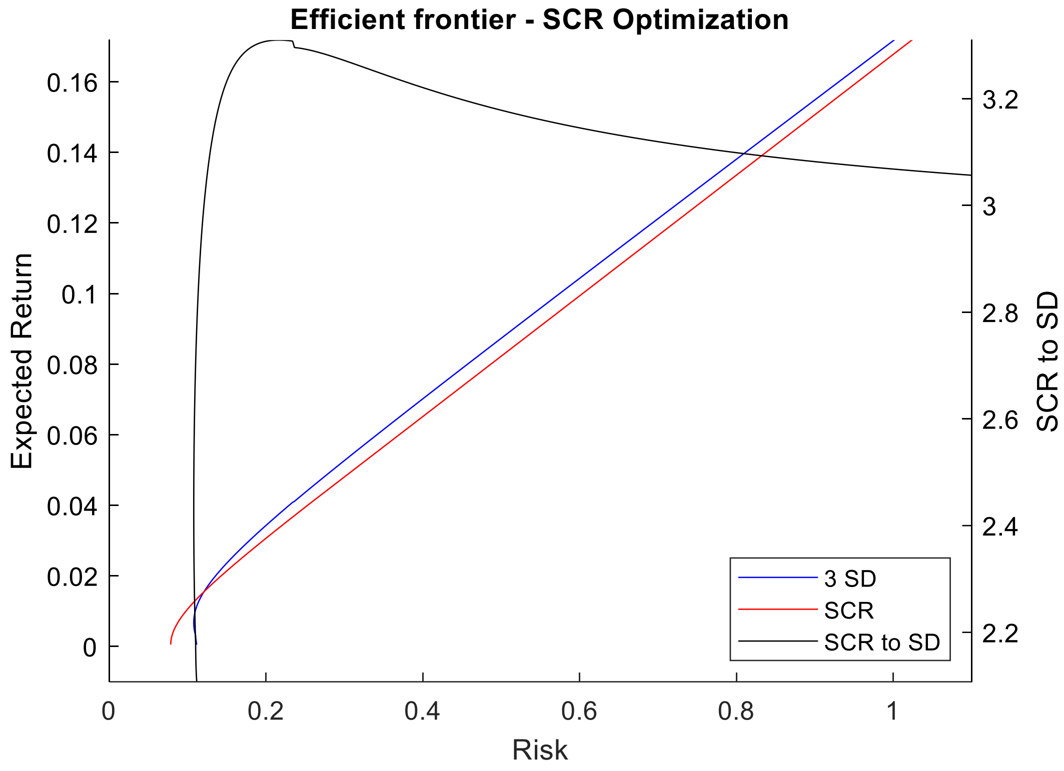
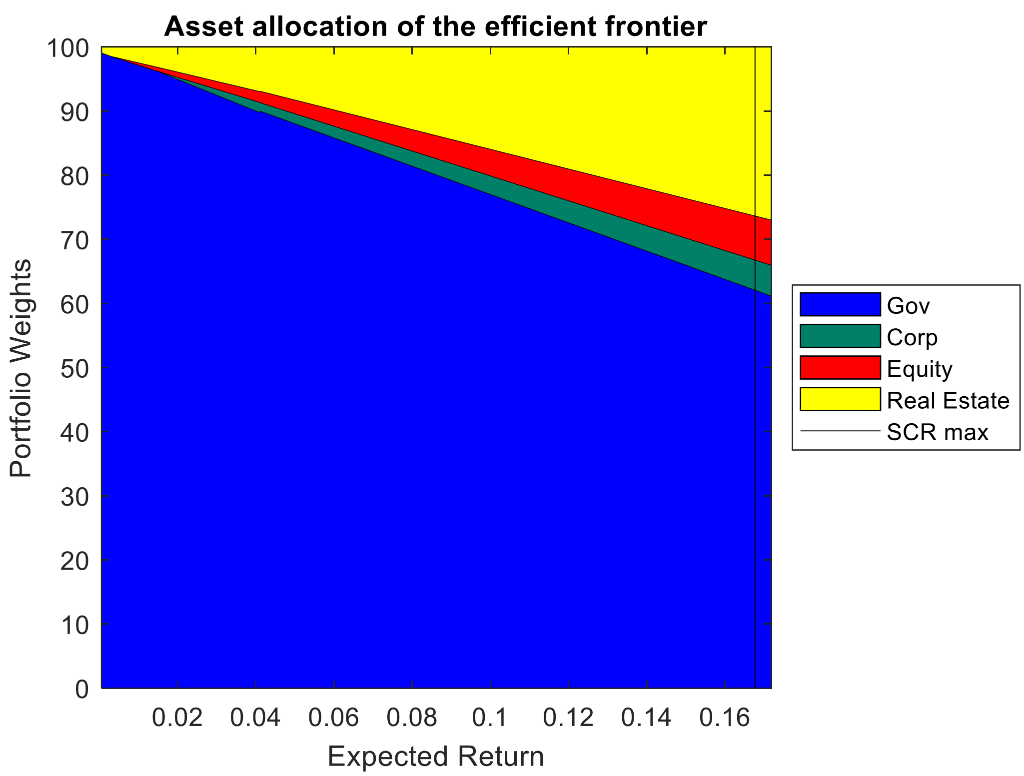


Figure 12



6. Asset allocation in a rising interest rates environment

The analysis presented in the previous section would give similar results with different dates around the second half of the 2010s until 2021 as the inputs did not change substantially. However, given the sharp rise in interest rates seen in the first half of 2022, the expected returns of the fixed-income asset classes increased dramatically as we infer them form the current yields, so that corporate bonds may be more appealing than they were previously. To a lesser extent, the capital charge of interest rate risk also changed as the Standard Model attributes a higher stress to the downward shock when rates are higher.

In this section, we reproduce the analysis of sections 4 and 5 including data of the first semester of 2022 to understand if the conclusions of our first analysis change when interest rates are higher.

6.1. Changes in parameters

The statistics of the four benchmarks calculated from the data series including the first half of 2022 are shown in Figure 13 and can be compared to those of Figure 3 to Figure 7 in the previous section. As expected, the main changes happen to be the expected returns of government bonds and corporate bonds, going from -0,36% and 0,63% to 1,34% and 3,30% respectively. More importantly, the additional return earned by corporate bonds over sovereign bonds – that we can interpret as the corporate spread that rewards an increased default risk for this class of fixed-income instruments, materialized by the spread risk in the Standard Model – almost doubles from 0,99% to 1,96%, whereas the risk premia earned by equity and real estate remain the same. Standard deviations and correlations do not vary materially from Figure 3.

Figure 13

	Government bonds	Corporate bonds	Equity	Real Estate
Average returns	3,45%	3,90%	4,60%	5,39%
Expected returns (June 2022)	1,34%	3,30%	7,23%	5,53%
Standard deviation	4,14%	5,42%	18,28%	8,38%
Correlations:				
Government bonds	1,000	0,695	-0,247	0,121
Corporate bonds	0,695	1,000	0,091	0,218
Equity	-0,247	0,091	1,000	0,250
Real Estate	0,121	0,218	0,250	1,000

The calculation of the capital charges remains the same, but one parameter changes quite considerably as interest rate rises. As the interest rate down shock is a fixed percentage of the absolute value of the risk-free interest rates, it becomes bigger when interest rates increase. The shocks expressed as percentage over market value applied to our fixed-income benchmarks increase from nearly 0% to more than 5%, so that any negative duration gap will have an actual impact in the total capital requirements of the portfolio, to the contrary of the previous simulations.

Figure 14

	Government bonds	Corporate bonds
Modified Duration	7,69	7,79
Shock IR up	7,37%	7,46%
Shock IR down	5,27%	5,32%

The spread shock barely changes to 8.96% as the duration of the corporate benchmark shortens slightly. The equity and property shocks remain equal to 39% and 25% as in the previous simulation.

6.2. Standard deviation optimization

The results of the portfolio optimization with standard deviation constraints including data until mid-2022 are depicted in Figure 15 to Figure 17 below.

As expected, the expected returns of all the portfolios of the efficient frontier are now higher than in the analysis done with 2021 data as expected returns have risen while expected risk has remained more or less constant. The portfolio of minimum risk is still heavily invested in government bonds, while the allocation to riskier asset classes increases with the standard deviation of the portfolio. The exception would be the allocation to equity, which becomes almost irrelevant with this simulation and decreases slightly when we go to riskier portfolios, most probably because the main advantage of this asset class is the diversification effect it provides and not the risk/return profile that is inferior to the one of the property asset class which offers an expected return of the same magnitude for less than half the risk. The most notable difference with the previous simulation lies in the allocation to corporate bonds that is now two to three times higher than in the previous example. The higher expected returns of the asset class make it more appealing while the risk involved remain similar to the one it had in the previous simulation. Government bonds cede roughly half of their allocation of the previous simulation to corporate bonds as they are proportionately less profitable.

When we compare the risk measured by the SCR and by three times the standard deviation of the portfolio, we observe that portfolios situated in the low-risk end of the efficient frontier and mostly invested in government bonds present a lower SCR just like in the previous exercise, but for riskier portfolios with a standard deviation higher than 6% the SCR becomes slightly higher than three standard deviations.

Figure 15

E(R)	2,2%	5,0%	10,0%	15,0%	22,4%
Risk measures (% of OF)					
SCR	7,9%	18,1%	40,2%	64,3%	100,0%
s	3,6%	6,0%	13,2%	21,0%	32,6%
3σ	10,9%	17,9%	39,6%	63,0%	97,9%
Portfolio allocation (% of assets)					
Government bonds	98,4%	88,9%	71,3%	53,7%	27,8%
Corporate bonds	0,0%	6,5%	19,1%	31,7%	50,2%
Equity	0,7%	0,7%	0,6%	0,5%	0,3%
Property	0,9%	3,9%	9,0%	14,1%	21,7%

Figure 16

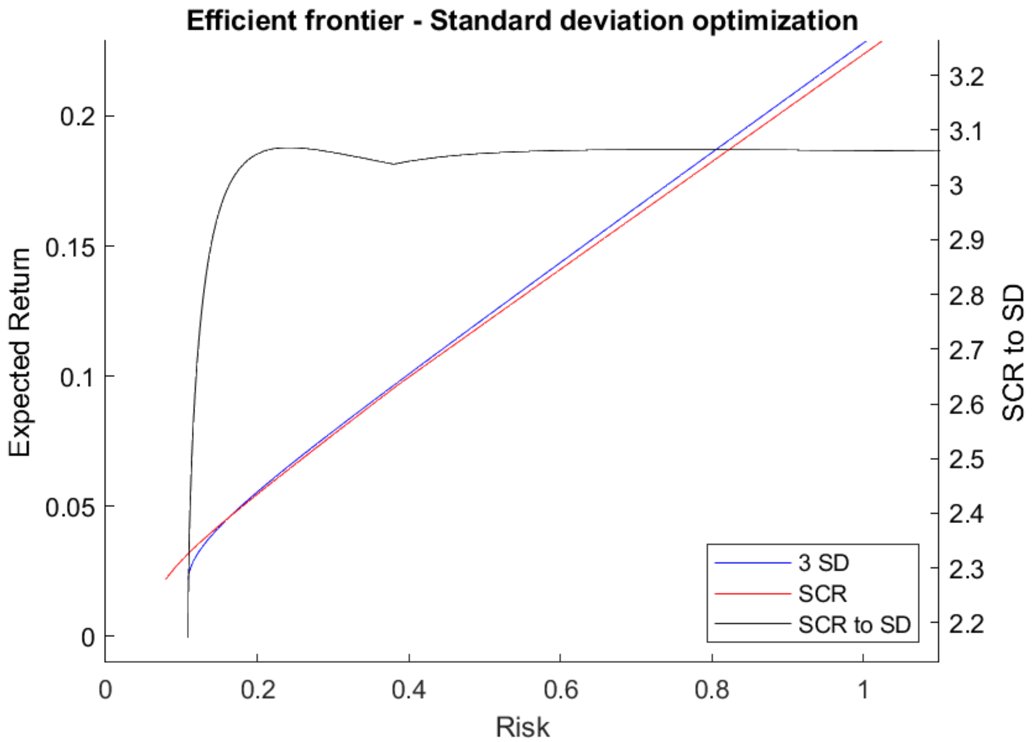
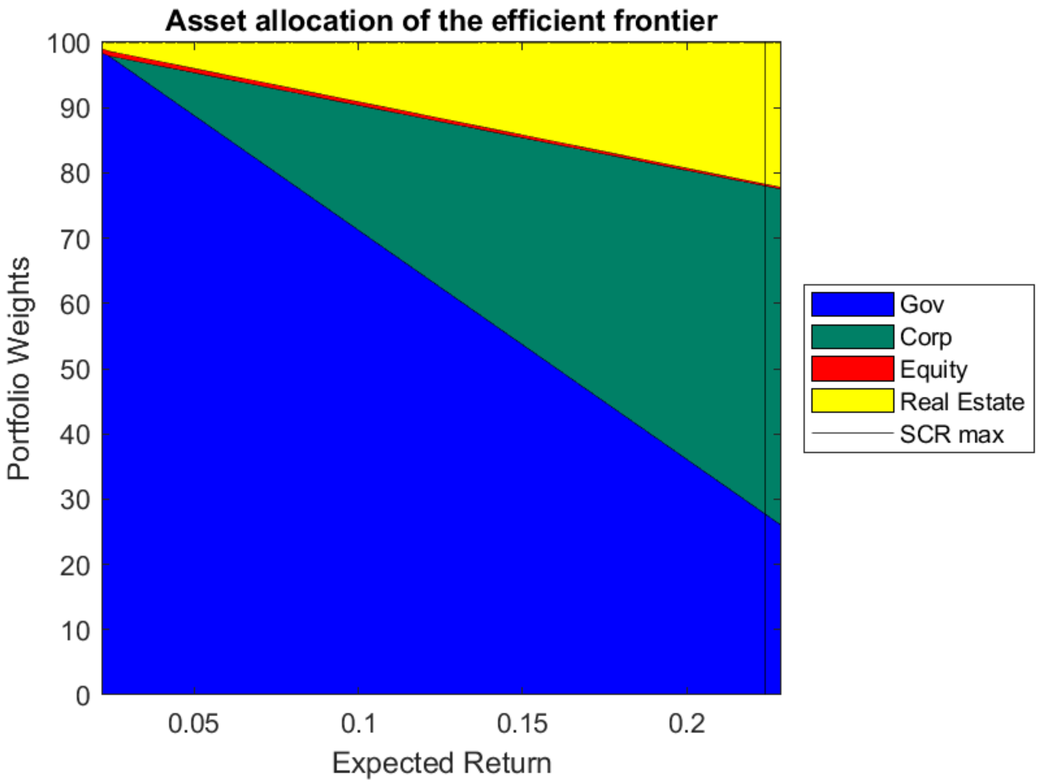


Figure 17



6.3. Solvency II Risk Capital Optimization

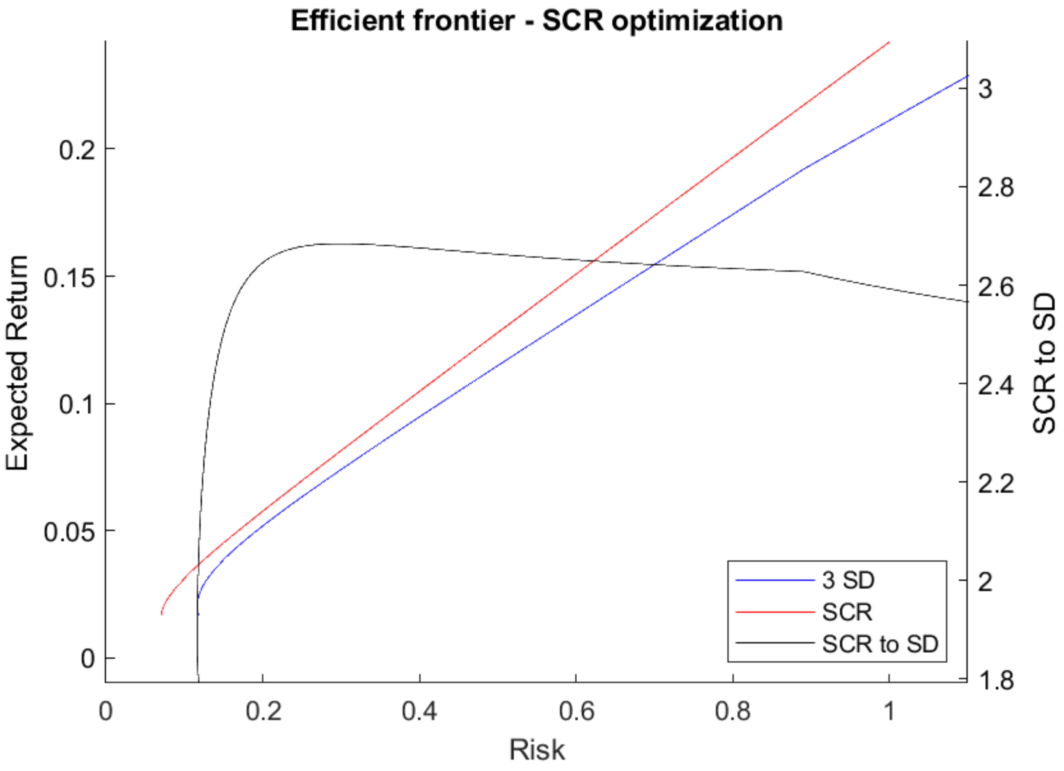
The efficient frontier of portfolios minimizing the SCR for a given expected return is described in Figure 18 to Figure 20. The main difference with the optimizations done with data until 2021 is that the allocation to corporate bonds increases with respect to the standard deviation optimization, whereas the SCR criterion inhibited investments in corporate debt in the optimizations of the 5th section. The respective shares of the three other asset classes decrease when compared to the ones of the standard deviation optimization, and equity is even fully removed from all the portfolios of the efficient frontier. Even government bonds are assigned a meager 5.2% for the portfolio of maximum risk allowed by the risk constraint (SCR = 100%) as corporate bonds represent a major part of the fixed-income allocation for riskier levels.

All the portfolios of the efficient frontier have a SCR that is less than three standard deviations, to the contrary of the optimization done with 2021 data. This is mainly due to the stand-alone risk characteristics of the two fixed-income categories that make most of the allocation of the optimal portfolios, as for instance the spread shock of the Standard Model for our A-rated corporate benchmark represents less than three times the standard deviation of said benchmark. To the contrary, real estate, which is allocated the biggest part of the portfolios of the optimization with 2021 data if we omit government bonds, has a stand-alone Solvency II shock roughly equal to three times the standard deviation of the corresponding benchmark.

Figure 18

E(R)	1,7%	5,0%	10,0%	15,0%	24,2%
Risk measures (% of OF)					
SCR	7,1%	16,9%	38,0%	59,6%	100,0%
σ	3,9%	6,4%	14,2%	22,6%	39,2%
3σ	11,8%	19,3%	42,7%	67,7%	117,7%
Portfolio allocation (% of assets)					
Government bonds	99,3%	86,1%	65,7%	45,4%	5,2%
Corporate bonds	0,0%	11,2%	29,1%	47,0%	84,9%
Equity	0,0%	0,0%	0,0%	0,0%	0,0%
Property	0,7%	2,7%	5,1%	7,6%	9,9%

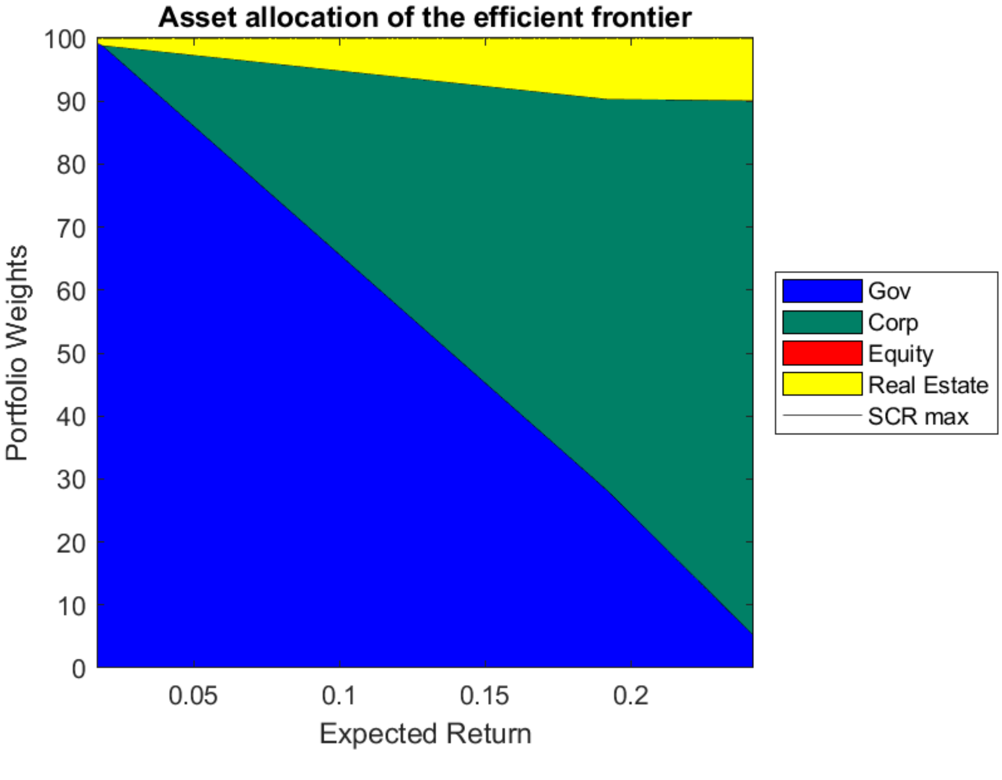
Figure 19



In order to understand why the optimization model with SCR constraints gives different outcomes depending on the timeframe used for the inputs, we could use one easy metric of capital efficiency to explain why an asset class is individually better in a certain environment than another: expected excess returns – understood as expected returns of the asset class minus expected returns of government bonds – over the SCR shock corresponding to the asset class. This would intentionally leave out a number of other less decisive factors including the correlations between asset classes or the fact that a single asset class such as corporate bonds may be subject to more than one shock, namely interest rate additionally to spread shock. In the optimizations of the fifth section, the corporate benchmark had a capital efficiency of 11.0%, lower than the capital efficiency of equity (15.1%) and real estate (16.8%), so that it had a relatively small allocation as compared to the other risky assets. However, when the expected returns are set higher according to the more recent data of the sixth section, corporate bonds become more capital efficient (21.9%), so that they dominate the asset allocation of risky benchmarks.

The same argument also applies to the optimal portfolios derived from the two optimizations with standard deviation constraints as we saw in the previous subsection, but the consequences of the expected risk-adjusted outperformance of an individual asset class are less disproportionate. This is mainly due to the treatment of the correlations between assets classes in the two risk calculations: the correlation matrix of the SCR market risk module makes the three risky asset classes – corporate debt, equity, and real estate – quite interdependent with correlations ranging from 0.50 to 0.75, while the standard deviation calculation takes into account lower figures ranging from 0.06 to 0.25[5], incentivizing the diversification between assets. Then, the asset allocation of the efficient portfolios derived by a minimization of the SCR amplifies the share of the most efficient risky asset class in each situation.

Figure 20



[5] Figures 3 and 13.

7. Conclusion

This paper compares two portfolio optimization methods – the classical expected return-variance setting and another one based on SCR – in order to understand if the solvency capital requirements set out in Solvency II Delegated Acts are likely to change the investment decisions of the managers of the insurance industry. Some previous studies use a similar methodology to show that investments in risky assets such as real estate and stocks may be hindered in favor of sovereign bonds. However, we find that the optimal asset allocation derived from an optimization of the Solvency II capital requirements is highly dependent on the values introduced as inputs, and especially the expected returns. If we take into account the low-yield environment of the first years following the implementation of Solvency II from 2016 to 2021 approximately, we have to adjust the high expected returns derived from average past returns used in previous research to more realistic values. In this case, we find that investments in sovereign bonds are effectively promoted by the new regulation, but not to the detriment of equities and real estate: corporate bonds are under allocated with these hypothesis and inputs. However, if we use data from 2022 where the yields begin to increase substantially, corporate bonds tend to have an outsized allocation when compared to the optimization with standard deviation constraints, whereas real estate have a smaller allocation and equity disappears. In both cases, the riskier portfolios of the efficient frontier derived from an optimization with SCR constraints tend to favor the most efficient asset of the three riskiest asset classes, when the optimization with standard deviation constraints promote a more diversified allocation. Another minor factor could be the treatment of negative rates in solvency II interest rate risk, as we see that there is no incentive to hold fixed-income securities in the balance sheet to balance the duration of liabilities when the company is chasing higher returns.

These optimization simulations remain at a very high level though, and more analysis are needed to understand more deeply the influence of durations in the investment process, the choice of a fixed-income security over another and the role of ratings. We should indeed bear in mind that insurance companies invest mostly in bonds for reasons that are not exclusively linked to Solvency II, and that the investment possibilities are various inside the asset class.

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II. Fixed-Income Asset Allocation Optimization

1. Introduction

We discussed in the previous chapter how the Solvency Capital Requirements (SCR) of the Solvency II standard formula laid out in the Directive 2009/138/ec of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance[6] may have introduced new incentives – or biases – for insurers to hold certain asset classes on their balance sheets, additionally to other existing criteria such as minimising the deviations of investment returns and abiding to internal capital efficiency targets or to the capital calculations of rating agencies.

We found that government bonds were favoured in the whole spectrum of risk appetite and that riskier assets like equity and property were relatively less negatively impacted than corporate bonds in a low-yield environment, as an insurance company that would seek to optimise its expected return in function of its capital requirement would invest in the most efficient of the three riskiest asset classes. However, fixed-income instruments including corporate bonds constitute the biggest part of the asset allocation of insurance undertakings as they must cover known or predictable liabilities that are sensitive to interest rate movements. As a consequence, it is paramount to understand the consequences of Solvency II on the asset class to see if the incentives depend only on the bond categories or if there are further distortions depending on country of investment, ratings, and durations.

This paper explores the asset allocation of an insurer that optimizes its fixed-income portfolio according to the classical return-standard deviation framework to the result of an optimization where risk is measured by the SCR. The next section summarises the current research on the subject. The spread risk component of the market risk module of Solvency II standard model is briefly explained in the third section, so that we can explain the data and assumptions of our simulations in the fourth section. The fixed-income asset allocation optimizations are subsequently analyzed and their results are discussed in the fifth section before some concluding remarks.

[6] European Union (2009).

2. State of the research

As we exposed in the previous chapter, the idea of using some kind of portfolio optimization methodology derived from Markowitz (1952) to optimize the expected return of an insurer's investments with a given SCR has already been explored in the literature with different variants for instance by Braun et al. (2017, 2018), Heinrich & Wurstbauer (2018), Kouwenberg (2018) and Escobar et al. (2019). They find that Solvency II offers an incentive to invest in sovereign bonds rather than in corporate bonds, disregarding the potential spread or concentration risks, but they do not investigate if those incentives and disincentives uniformly apply on all types of government and non-government debt or if they are more material on certain rating classes or durations.

Their models are perfectible if we have to study specifically fixed-income securities, as general models accounting for all asset classes make difficult to segregate the risks and returns that come from holding a risk-free security – interest rate risk and pure time value of money – and from holding a risky asset – additionally to the formers, spread risk and excess returns. They cannot take into account different ratings, and durations are mostly fixed, both on the asset side and the liability side, introducing some arbitrary results as they have to analyse portfolio allocation with artificial asset-liability mismatches. Moreover, they do not adjust the backwards-looking parameters of their model for changes in expectations due to the current market environment. For instance, expected annualized returns of 3% to 6% for european government bonds as proposed by Braun et al. (2017, 2018) and Heinrich & Wurstbauer (2018) are improbable given the sovereign yields of the period of study.

Outside the scope of portfolio optimization papers but specific to fixed-income securities, an interesting study of the significant effects of the transmission of sovereign risk to insurance companies comes from Duell et al. (2017): they conclude that the risk-free treatment of sovereign bonds in Solvency II standard model completely ignores the true risks of the asset class.

3. Solvency II standard model: spread risk

As we explained in our previous chapter, the market risk module (SCR_{Mkt}) of Solvency II aggregates the interest rate risk (SCR_{IR}), the equity risk (SCR_{EQ}), the property risk (SCR_{RE}), the spread risk (SCR_{CS} for credit spread), the concentration risk (SCR_{Conc}) and the currency risk (SCR_{Curr}).

$$\mathrm{SCR}_{\mathrm{Mkt}} = \sqrt{\sum_i \sum_j \rho_{i,j} * \mathrm{SCR}_i * \mathrm{SCR}_j} \qquad (2.1)$$

where i,j represents all pairs of market risks, and $\rho_{i,j}$ is the correlation between those pairs as set out in the Delegated Acts.

In this analysis, we want to focus on the returns of fixed-income securities that bear any kind of spread risk, additionally to interest rate risk. As the latter can be addressed by a separate exercise of asset-liability matching, we will be able to study the spread risk separately from all other market risks. We will not consider concentration risk as it is possible to eliminate it by diversifying the portfolio, nor currency risk as it is possible to invest only in domestic currency, or to mitigate fully any foreign currency exposure by using derivatives.

SCR_{CS} is calculated as the sum of three capital requirements:

$$\mathrm{SCR}_{\mathrm{CS}} = \mathrm{SCR}_{\mathrm{Bonds}} + \mathrm{SCR}_{\mathrm{Securitisations}} + \mathrm{SCR}_{\mathrm{CD}} \quad (2.2)$$

As securitizations and credit derivatives used for investment purposes rather than for hedging make up a small part of insurers' balance sheets and of the total investible universe as compared to bonds, we will focus our analysis on $\mathrm{SCR}_{\mathrm{Bonds}}$. In any case, the capital requirements set by Solvency II for spread risk on securitization positions as well as credit derivatives are significant and therefore unlikely to tilt insurers' asset allocation towards such instruments.

For all bonds that do not fall into any specific category, SCR_{Bonds} is calculated as a percentage ($stress_i$) of the market value of the instrument. $stress_i$ depends on the modified duration and the credit quality step which represents the corresponding better-known ratings of the major agencies (where 0 = AAA, 1 = AA, etc.).

Credit quality step		0		1		2		3		4		5 and 6	
Duration (dur_i)	$stress_i$	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
up to 5	$b_i \cdot dur_i$	—	0,9 %	—	1,1 %	—	1,4 %	—	2,5 %	—	4,5 %	—	7,5 %
More than 5 and up to 10	$a_i + b_i \cdot (dur_i - 5)$	4,5 %	0,5 %	5,5 %	0,6 %	7,0 %	0,7 %	12,5 %	1,5 %	22,5 %	2,5 %	37,5 %	4,2 %
More than 10 and up to 15	$a_i + b_i \cdot (dur_i - 10)$	7,0 %	0,5 %	8,4 %	0,5 %	10,5 %	0,5 %	20,0 %	1,0 %	35,0 %	1,8 %	58,5 %	0,5 %
More than 15 and up to 20	$a_i + b_i \cdot (dur_i - 15)$	9,5 %	0,5 %	10,9 %	0,5 %	13,0 %	0,5 %	25,0 %	1,0 %	44,0 %	0,5 %	61,0 %	0,5 %
More than 20	$\min[a_i + b_i \cdot (dur_i - 20);1]$	12,0 %	0,5 %	13,4 %	0,5 %	15,5 %	0,5 %	30,0 %	0,5 %	46,5 %	0,5 %	63,5 %	0,5 %

Some exposures receive a specific treatment, as established in the article 180 of the Delegated Acts. Of those, the two most relevant in the investment universe of insurance companies are covered bonds and European sovereign bonds – including supranational bonds and bonds issued by local authorities.

Covered bonds which have been assigned a credit rating of AAA or AA – that is, a large majority of those instruments – benefit from a more favorable $stress_i$ than the generic one:

Duration (dur_i) \ Credit quality step	0	1
up to 5	0,7 %. dur_i	0,9 %. dur_i
More than 5 years	$\min(3{,}5\% + 0{,}5\% \cdot (dur_i - 5); 1)$	$\min(4{,}5\% + 0{,}5\% \cdot (dur_i - 5); 1)$

European sovereign bonds are assigned a risk factor $stress_i$ of 0 %.

4. Data and portfolio assumptions

We chose to find the same series of market indices as those used by the CEIOPS[7] in the Solvency II calibration paper in the assessment of spread risk, so that the parameters of our models are fully comparable to those of Solvency II. Thus, we gathered the weekly excess return data of ICE BofAML Indices already calculated in the series – total returns are decomposed in risk-free returns, that is returns that would be earned from an index of German Bunds securities with the same duration characteristics as the index, and excess returns – to derive daily standard deviations and correlations between indices. Option-adjusted spreads over government bonds are used as an indicator of expected excess returns, rather than an average of past returns.

The analysis is built on twenty-two years of weekly data, from 2000 through June 2022. The standard deviations and correlations between the benchmarks are calculated on this period, while the expected excess returns are derived from the last data point of weekly option-adjusted spreads, as these represent the rewards that an investor can expect to get for bearing additional risk. However, as this spread risk is not only a risk of volatility of the market value of the asset but can also materialize in a default at some point, the expected excess returns should be adjusted for the expected credit loss of the instrument.

We choose to use some parameters from the calculation of the volatility adjustment laid out in Solvency II Delegated Acts to account for this expected loss. In Solvency II, the volatility adjustment is an additional discount to the liabilities of an undertaking used to mitigate the volatility of the own funds that comes from non-fundamental – i.e., non-credit related – spreads of the assets. To calculate this discount, EIOPA discloses quarterly a calculation of the non-fundamental spread of the portfolio of a representative insurer, according to an average asset allocation between sovereign and corporate bonds of different countries and ratings. We use the calculation of the fundamental spread made by EIOPA to assign to each corporate benchmark of this chapter a risk correction factor to their corresponding spread calculated as the sum of the probability of default – that can be considered an expected loss from default in this case – and of the cost of downgrade. The financial and non-financial values given by EIOPA are averaged into a single corporate matrix according to the weights of the representative European insurer.

7 (CEIOPS, 2010)

Thus, the risk correction applied to corporate bonds depends only on their maturity and credit quality step. Long-term averages are not considered in our calculations as they offer neither theoretical nor empirical basis to the calculation of the expected loss and are used by the Regulator as an extra step to make sure that the volatility adjustment is conservatively calculated. The country-specific parameter is also ignored.

The results of the general indices by bond category that we will use in the first analysis are displayed in Figure 21 below.

Figure 21

	A Euro Corporate	Euro Covered Bond	Euro Government
Option-Adjusted Spread	1,69%	0,77%	0,40%
Risk Correction	0,13%	0,04%	0,00%
Expected Excess Returns	1,56%	0,73%	0,40%
Standard deviation	2,35%	1,21%	3,81%
Correlations:			
A Euro Corporate Index	1,0000	0,5382	0,2836
Euro Covered Bond Index	0,5382	1,0000	0,4484
Euro Government Index	0,2836	0,4484	1,0000

We had to proxy the corporate benchmark with a rating-specific benchmark – ICE BofAML A Euro Corporate Index – to be able to calculate an exact SCR_{CS}, as the historical rating breakdown of the general Euro Corporate Index is unavailable and is likely to be quite volatile. On the contrary, the ratings of the Euro Covered Bond Index are deemed to be less volatile, so the corresponding SCR has been calculated based on the latest ratings available – 75% AAA, 21% AA, 4% A.

5. Optimizing the fixed-income allocation of an insurance company

5.1. Optimizing across bond categories

The first step of our analysis is an optimization of a portfolio made of three indices representing the main categories of fixed-income securities of an insurer's asset allocation.

5.1.1. Return-Variance Optimization

We first run the classical portfolio optimization model, with the expected excess returns, and their empirical standard deviations and correlations found in section 4: the portfolio standard deviation is minimized for a given expected excess return.

A sample of portfolios situated on the efficient frontier are described in Figure 22 below, while the efficient frontier is drawn in Figure 23. The actual asset allocations corresponding to the portfolios on the efficient frontier are depicted in Figure 24. The SCR required by the efficient portfolios is also provided in the table and the graph. To get a visual comparison between the two risk measures, the standard deviation of the optimal portfolios is multiplied by 3 to consider that it does not represent a 99.5% Value-at-Risk stress of the own funds. The number can be thought of an approximation of the number of standard deviations necessary to reach said probability threshold, if we consider that the data follows a distribution with fatter tails than a normal distribution.

Figure 22

E(R)	0,73%	1,00%	1,20%	1,40%	1,56%
Risk measures					
SCR	3,78%	4,79%	5,56%	6,32%	6,94%
σ	1,21%	1,38%	1,67%	2,03%	2,35%
3σ	3,63%	4,14%	5,01%	6,09%	7,05%
Portfolio allocation (% of assets)					
A Corporate Bonds	0,0%	32,0%	56,4%	80,5%	100,0%
Covered Bonds	100,0%	68,0%	43,6%	19,5%	0,0%
Government Bonds	0,0%	0,0%	0,0%	0,0%	0,0%

As explained above, government bonds are assigned the minimum weight of 0%, as they have the lowest expected excess returns – even when considering a 0% risk correction – but incur the highest risk as measured by the standard deviation. As expected, covered bonds dominate portfolio allocations in the lower range of expected excess returns while corporate bonds are more present in the higher range of expected excess returns.

The Solvency Capital Requirement is slightly higher than the 3σ comparable measure for most required expected returns and especially in the lower range where covered bonds dominate the asset allocation, reflecting a more conservative treatment in Solvency II for this bond category.

Figure 23

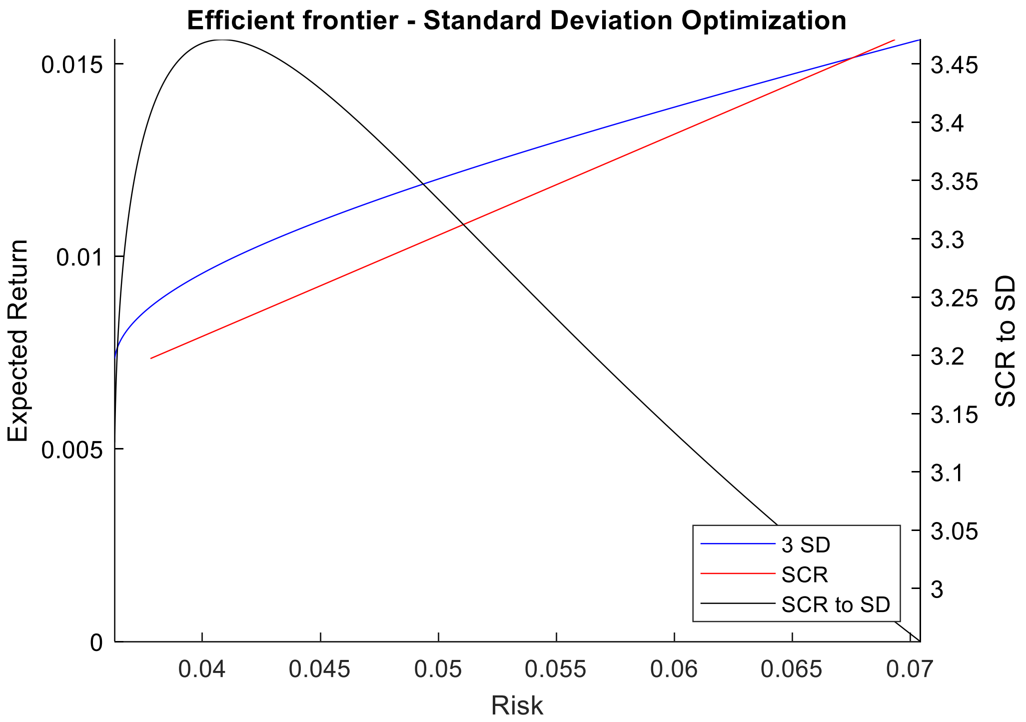
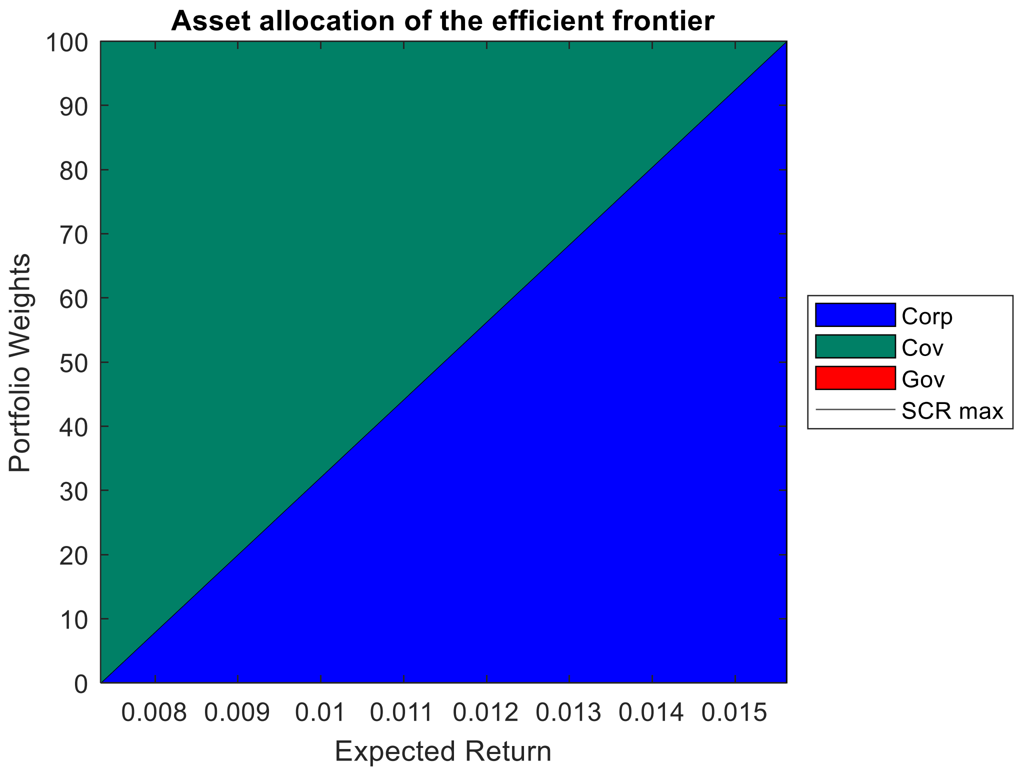


Figure 24



5.1.2. Solvency II Risk Capital Optimization

The second optimization consists in minimizing the portfolio SCR for a given expected excess return. As the standard model does not recognize that correlations between bond classes are less than 1, the optimal portfolios constitute a straight line between an all-government portfolio with no SCR and an all-corporate portfolio, as it is more capital-efficient than the covered bond index (Figure 26).

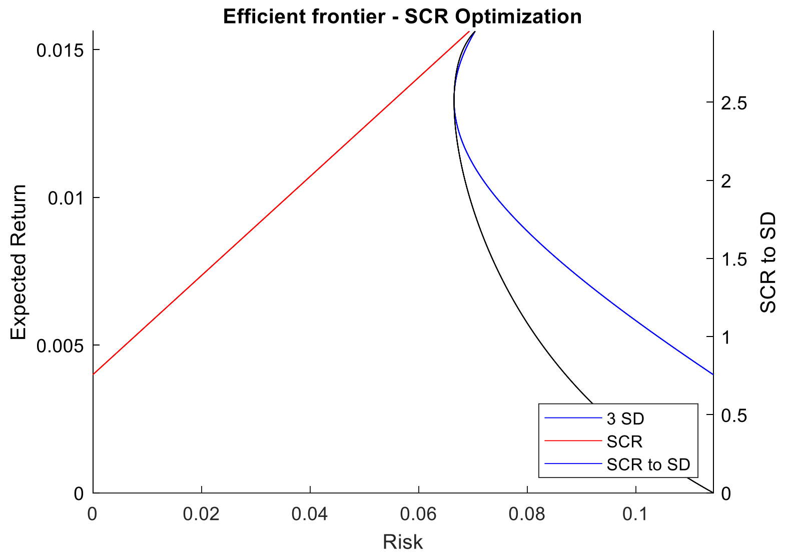
Figure 25

E(R)	0,40%	1,00%	1,20%	1,40%	1,56%
Risk measures					
SCR	0,00%	3,55%	4,75%	5,94%	6,89%
σ	3,81%	2,48%	2,26%	2,23%	2,34%
3σ	11,43%	7,44%	6,78%	6,69%	7,02%
Portfolio allocation (% of assets)					
A Corporate Bonds	0,0%	51,2%	68,4%	85,6%	100,0%
Covered Bonds	0,0%	0,0%	0,0%	0,0%	0,0%
Government Bonds	100,0%	48,8%	31,6%	14,4%	0,0%

Figure 26

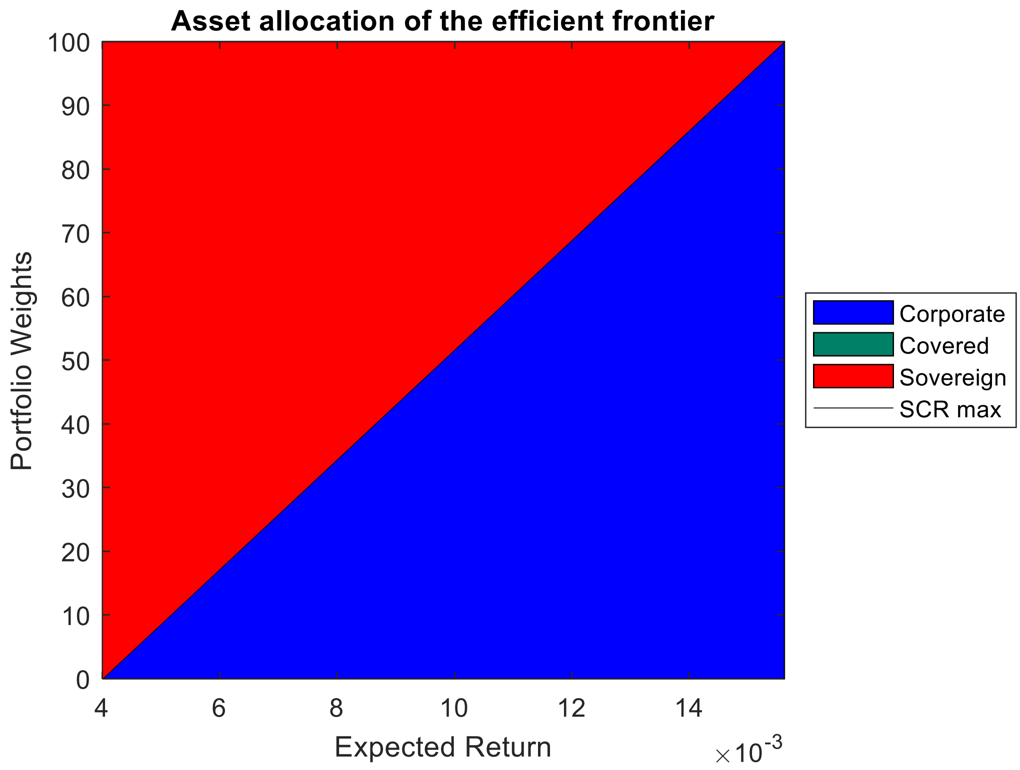
	A Euro Corporate	Euro Covered Bond
Expected Excess Returns	1,56%	0,73%
SCR	6,94%	3,78%
E(R)/SCR	22,5%	19,4%

Figure 27



The results of this return-SCR optimization are contradictory with the return-variance optimization. In this case, covered bonds are substituted by government bonds in the portfolios that are supposed to be less risky. However, when measured with the standard deviation of the expected excess returns, it appears that the portfolios with lower SCRs are riskier. More importantly, an insurance company with risk aversion that would follow our 3σ rule may avoid such portfolios as the SCR would be lower than three times the standard deviation of the expected excess returns of the portfolio.

Figure 28



5.2. Optimizing between ratings

To understand the effects of Solvency II at a more granular level, the comparison between the results of the optimizations with standard deviation criteria and with SCR criteria can be applied inside each category of bond, according to two dimensions: the rating profile and the duration profile. When we reach this level of granularity, the empirical correlations between the rating-specific benchmarks get increase significantly, and even more in the case of the correlations of the Standard Model that does not contemplate any diversification effect between ratings. Thus, the portfolio optimizations come down to a comparison of the capital efficiency – expressed as the ratio of the expected excess return over the relevant risk measure – of each of their constituents as correlations are no longer meaningful.

5.2.1. Corporate Bonds

The risk and return characteristics of the three main rating classes of corporate bonds calculated from the respective ICE BofAML indices are shown in Figure 29. Below-investment grade bonds are omitted as investment companies are not allowed to invest in this type of securities, and the AAA benchmark is excluded as it is less significant than the other three benchmarks and it does not offer a full data history because it lacks enough constituents in certain periods. As expected, the quality of the ratings is inversely related to the risk inherent to each benchmark measured by their respective standard deviation and SCR, and consequently to the expected excess returns. However, the relation between risk and return is not perfectly proportional, so that some ratings reward more efficiently the risk taken. When measured by the standard deviation, the most efficient rating is AA as it offers the most expected excess return per unit of risk taken, followed by A and BBB that have a similar ratio, while the most efficient rating with the SCR criterion is A that is clearly better than BBB. This is mainly due to the Solvency II spread shock applied to A-rated corporates that is approximately equal to three standard deviations, while the shocks corresponding to AA-rated and BBB-rated benchmarks represent more standard deviations.

Figure 29

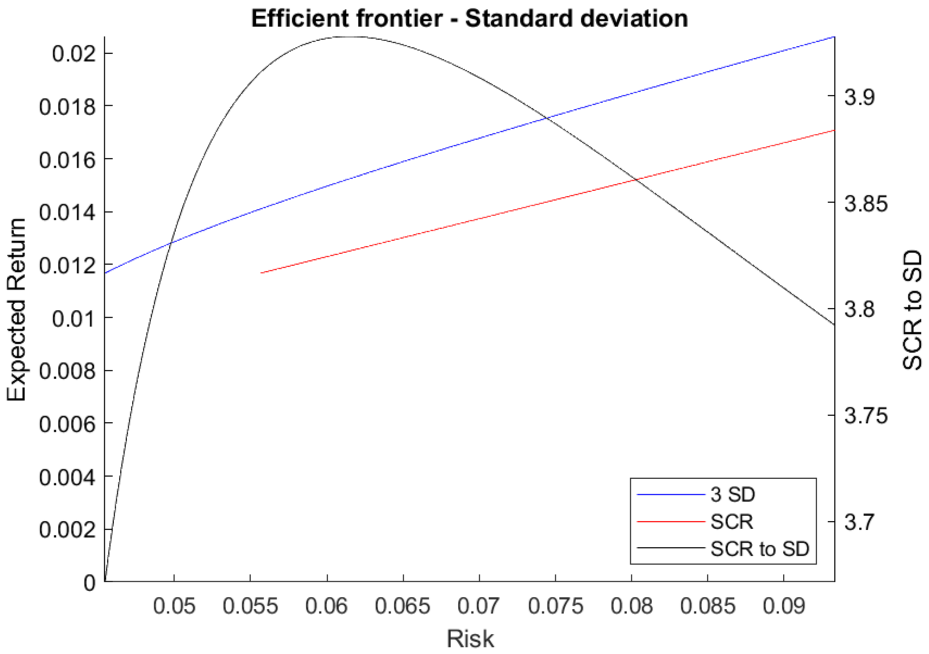
All maturities Corporates	AA-Rated	A-Rated	BBB-Rated
Option-Adjusted Spread	1,23%	1,69%	2,28%
Risk Correction	0,06%	0,13%	0,22%
Expected Excess Returns	1,17%	1,56%	2,06%
Standard deviation	1,52%	2,35%	3,11%
SCR	5,56%	6,94%	11,80%
E(R)/3σ	25,68%	22,18%	22,09%
E(R)/SCR	20,98%	22,52%	17,48%
SCR/3σ	122,38%	98,48%	126,40%
Correlations:			
AA-Corporate	1,0000	0,9250	0,8097
A-Corporate	0,9250	1,0000	0,8941
BBB-Corporate	0,8097	0,8941	1,0000

The efficient frontier of the portfolio optimization of these three benchmarks is described in Figure 30 and Figure 31 below. As the A-rated benchmark is less efficient than an equivalent mix of AA-rated and BBB-rated benchmark for the same level of risk and as it adds little diversification benefits considering its high correlations with the two other benchmarks, the efficient frontier is made of a linear combination of the AA and BBB benchmarks. The SCR derived from this type of asset allocation would be significantly higher than three times the standard deviation.

Figure 30

E(R)	1,17%	1,39%	1,62%	1,84%	2,06%
Risk measures					
SCR	5,56%	7,15%	8,68%	10,22%	11,80%
σ	1,52%	1,83%	2,22%	2,64%	3,11%
3σ	4,56%	5,49%	6,66%	7,92%	9,33%
Portfolio allocation (% of assets)					
AA-Corporate	100%	74%	50%	25%	0%
A-Corporate	0%	0%	0%	0%	0%
BBB-Corporate	0%	26%	50%	75%	100%

Figure 31



The portfolio optimization with SCR constraints gives a more balanced asset allocation as the A-rated benchmark is present in all portfolios between the two extreme points of the efficient frontier, either completed by the AA benchmark for the safer portfolios or by the BBB benchmark for the riskier ones. As a consequence, the relation between the SCR and the 3σ risk measure varies depending on the expected return of the asset allocation as the portfolios located near 1.56% of expected excess return are mainly composed of A-rated bonds and would then require a SCR lower than 3σ.

Figure 32

E(R)	1,17%	1,39%	1,62%	1,84%	2,06%
Risk measures					
SCR	5,56%	6,35%	7,45%	9,59%	11,80%
σ	1,52%	1,96%	2,40%	2,69%	3,11%
3σ	4,56%	5,88%	7,20%	8,07%	9,34%
Portfolio allocation (% of assets)					
AA-Corporate	100%	42%	0%	0%	0%
A-Corporate	0%	58%	89%	45%	0%
BBB-Corporate	0%	0%	11%	55%	100%

Figure 33

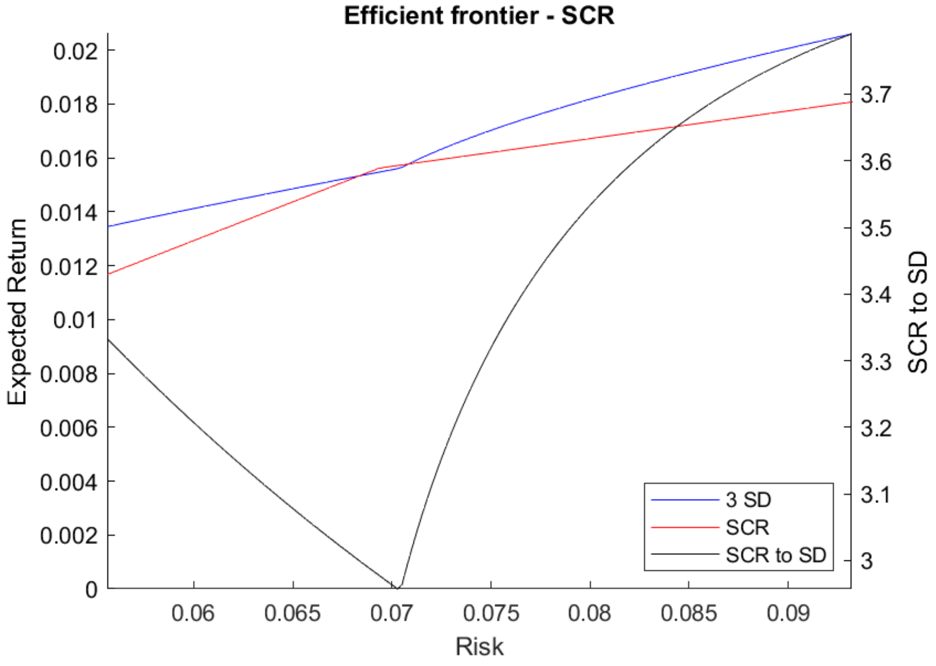
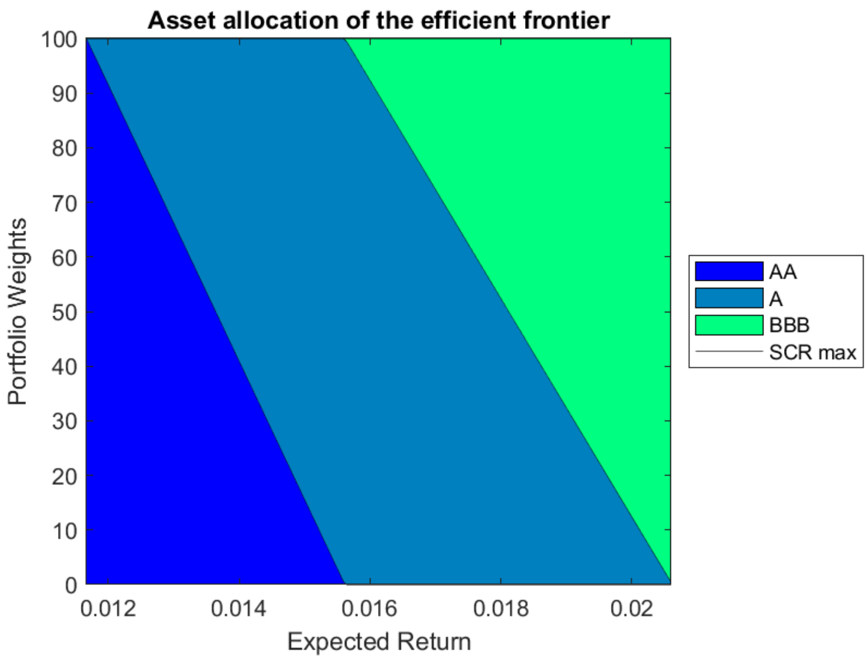


Figure 34



5.2.2. Government Bonds

The same exercise can be replicated for sovereign bonds with some limitations, as the number of different issuers inside each rating category is not sufficient to maintain an index in all periods. This problem can be circumvented by replacing the A-rated and BBB-rated benchmarks that lack data for the first decade of the studied period by two benchmarks specific to two countries – Spain and Italy – with a significant volume of debt outstanding in the financial markets and that represent two different degrees of more significant credit risk than the AAA and AA benchmarks. The risk and return characteristics of the four chosen benchmarks are described in Figure 35 below. We observe that the increased expected excess returns of the benchmarks with the lower ratings more than offset the increased risk measured by the standard deviation: the most efficient asset is by far the Italian benchmark. This tendency is exacerbated to an extreme when we look at the treatment of those bonds by Solvency II, as the spread risk faced by all sovereign benchmark is considered null. In this case, there is no real tradeoff between risk and reward but a simple ranking from the most rewarding asset to the least rewarding one irrespectively of the risk appetite of the insurer.

Figure 35

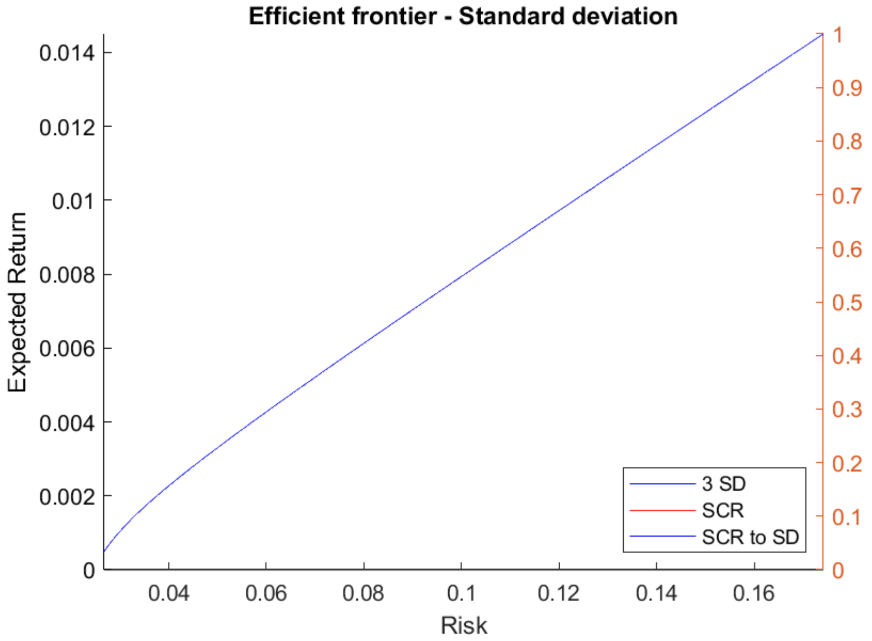
	AAA Sovereign	AA Sovereign	SP Sovereign	IT Sovereign
Expected Excess Returns	0,05%	0,40%	0,81%	1,45%
Standard deviation	0,89%	3,81%	5,61%	5,81%
SCR	0,00%	0,00%	0,00%	0,00%
E(R)/3σ	1,87%	3,50%	4,81%	8,33%
Correlations:				
AAA Sovereign	1,000	0,726	0,549	0,554
AA Sovereign	0,726	1,000	0,757	0,756
SP Sovereign	0,549	0,757	1,000	0,792
IT Sovereign	0,554	0,756	0,792	1,000

The efficient frontier of the portfolio optimization of the four benchmarks with a standard deviation criterion is described in Figure 36 and Figure 37. The efficient portfolios are exclusively composed of AAA-sovereigns and of Italian bonds as any combination of the two extreme assets is more efficient than the benchmarks with intermediary ratings, due to the outstanding risk efficiency of the Italian benchmark and its relatively low correlation with the AAA benchmark. In any case, the Solvency II framework undervalues completely the risk of this type of all-sovereign portfolios. The portfolio optimization would make no sense with the SCR constraint, as insurance companies are incentivized to invest in the issuers with the highest spread and consequently the highest risk. In our example, the insurer would invest 100% of the government bond portion of his fixed-income allocation in Italian bonds.

Figure 36

E(R)	0,05%	0,40%	0,75%	1,10%	1,45%
Risk measures					
SCR	0,00%	0,00%	0,00%	0,00%	0,00%
σ	0,89%	1,89%	3,15%	4,46%	5,81%
3σ	2,67%	5,67%	9,45%	13,38%	17,43%
Portfolio allocation (% of assets)					
AAA Sovereign	100%	75%	50%	25%	0%
AA Sovereign	0%	0%	0%	0%	0%
SP Sovereign	0%	0%	0%	0%	0%
IT Sovereign	0%	25%	50%	75%	100%

Figure 37



5.3. Spread duration

We can go a step deeper inside each corporate rating class to determine if certain duration buckets represented by the corresponding indices are more efficient than others to optimize the risk-return profile of the portfolio from a standard deviation and a SCR perspective. As the data from the year 2022 was at some points an exception to the long-term tendency and the general understanding by which longer maturities should command higher spreads for the same rating category, we chose to use the data as of the end of the year 2021.

5.3.1. AA-rated Corporates

The performance ratios of the two optimization models yield the same conclusion: it is more efficient – E(R)/ 3σ and E(R)/SCR are higher – to invest in the shorter-dated securities if one wants to invest in AA-Rated Corporate Bonds. The SCR measure emphasizes this especially in the first bucket as it is lower than three times the standard deviation, as opposed to most categories with longer durations – except the 7-10y one. The correlations between benchmarks are significantly lower than 1 for the duration buckets that are situated far from each other.

Figure 38

AA Corporates	1-3y	3-5y	5-7y	7-10y	10+y
Option-Adjusted Spread	0,45%	0,65%	0,74%	0,71%	1,04%
Risk Correction	0,04%	0,05%	0,07%	0,08%	0,15%
Expected Excess Returns	0,41%	0,60%	0,67%	0,63%	0,89%
Standard deviation	0,70%	1,25%	1,84%	2,53%	3,48%
SCR	2,00%	4,25%	5,89%	7,18%	10,61%
E(R)/3σ	19,8%	15,9%	12,2%	8,2%	8,5%
E(R)/SCR	20,7%	14,1%	11,4%	8,7%	8,4%
SCR/3σ	96,0%	113,0%	106,5%	94,5%	101,7%
Correlations:					
AA 1-3y	1,0000	0,8772	0,8182	0,7590	0,6102
AA 3-5y	0,8772	1,0000	0,9517	0,9209	0,7749
AA 5-7y	0,8182	0,9517	1,0000	0,9537	0,8385
AA 7-10y	0,7590	0,9209	0,9537	1,0000	0,8654
AA 10+y	0,6102	0,7749	0,8385	0,8654	1,0000

The portfolios of the efficient frontier are composed of a mix of all different duration buckets, minus the 7-10y one as it is less capital efficient than the previous and next duration benchmarks. In most cases, the optimal portfolios are composed of only two duration benchmarks but for those situated between 0.73% and 0,82% of expected excess returns there is a third benchmark even if the 5-7y benchmark is less efficient than the combination of the 3-5y and the 10+y ones on a standalone basis. In this case, the diversification effect induced by the correlation matrix allows for a more diverse exposure to the spread yield curve. Almost all portfolios have a SCR higher than three times the standard deviation, with the exception of the safest portfolios mainly composed of the 1-3y benchmark.

Figure 39

E(R)	0,41%	0,53%	0,65%	0,77%	0,89%
Risk measures					
SCR	2,00%	3,35%	5,25%	7,86%	10,61%
σ	0,70%	1,01%	1,52%	2,42%	3,48%
3σ	2,10%	3,03%	4,56%	7,26%	10,44%
Portfolio allocation (% of assets)					
AA 1-3y	100,0%	39,9%	0,0%	0,0%	0,0%
AA 3-5y	0,0%	60,1%	84,3%	33,7%	0,0%
AA 5-7y	0,0%	0,0%	0,0%	12,9%	0,0%
AA 7-10y	0,0%	0,0%	0,0%	0,0%	0,0%
AA 10+y	0,0%	0,0%	15,7%	53,4%	100,0%

Figure 40

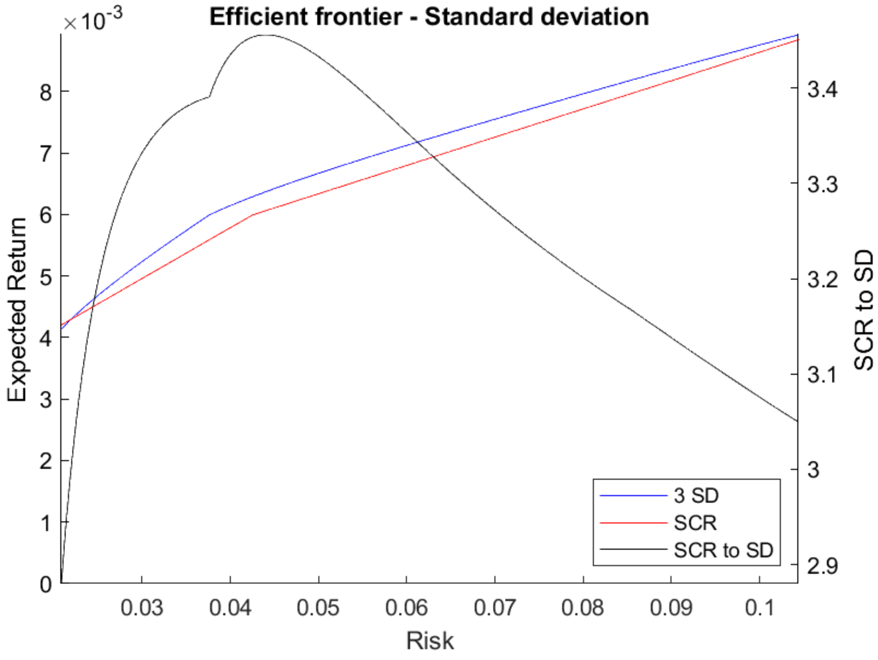
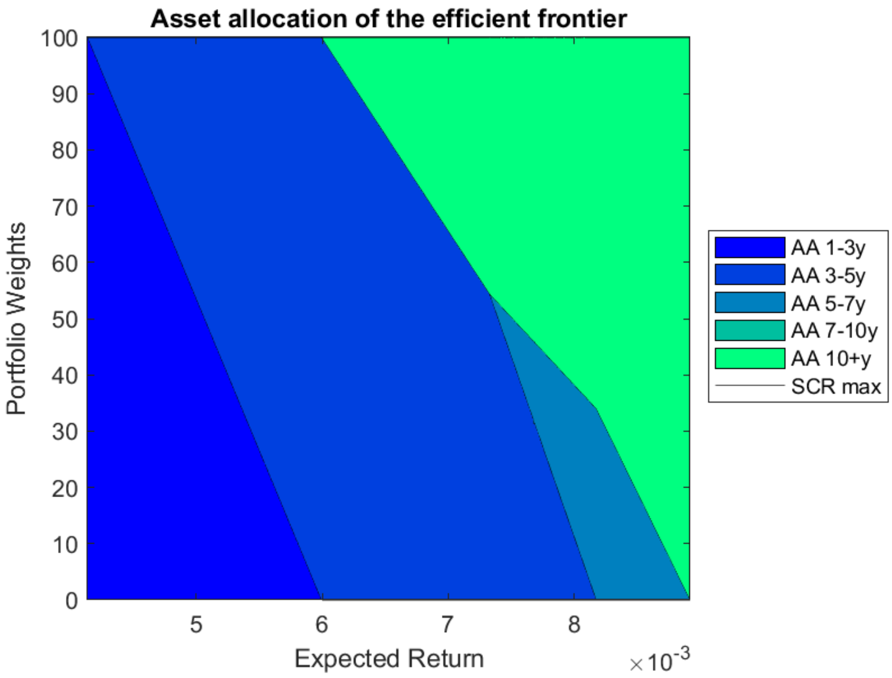


Figure 41



The portfolio optimization with Solvency II constraints gives quite similar results. The optimal portfolios are the same as with the standard deviation optimization, except for the portfolios situated between 0.73% and 0,82% of expected excess returns where the asset mix is exclusively composed of the 3-5y and 10+y benchmarks as they are collectively more efficient than the 5-7y one – there is no benefit from diversification. The SCR is higher than the comparable standard deviation measure in exactly the same instances as in the previous case.

Figure 42

E(R)	0,41%	0,53%	0,65%	0,77%	0,89%
Risk measures					
SCR	2,00%	3,35%	5,25%	7,85%	10,61%
σ	0,70%	1,01%	1,52%	2,41%	3,48%
3σ	2,10%	3,03%	4,56%	7,23%	10,44%
Portfolio allocation (% of assets)					
AA 1-3y	100,0%	39,9%	0,0%	0,0%	0,0%
AA 3-5y	0,0%	60,1%	84,3%	43,5%	0,0%
AA 5-7y	0,0%	0,0%	0,0%	0,0%	0,0%
AA 7-10y	0,0%	0,0%	0,0%	0,0%	0,0%
AA 10+y	0,0%	0,0%	15,7%	56,6%	100,0%

Figure 43

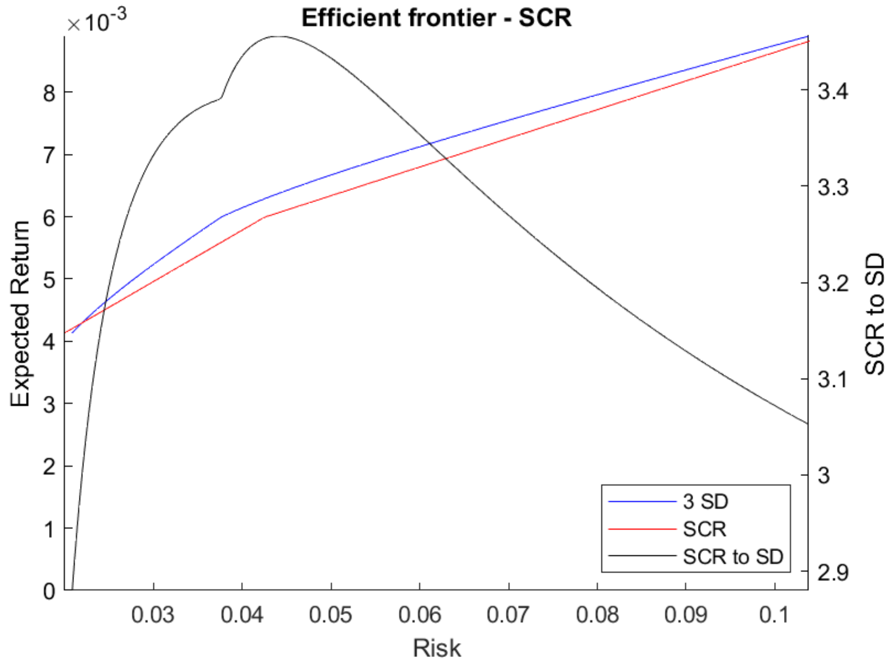
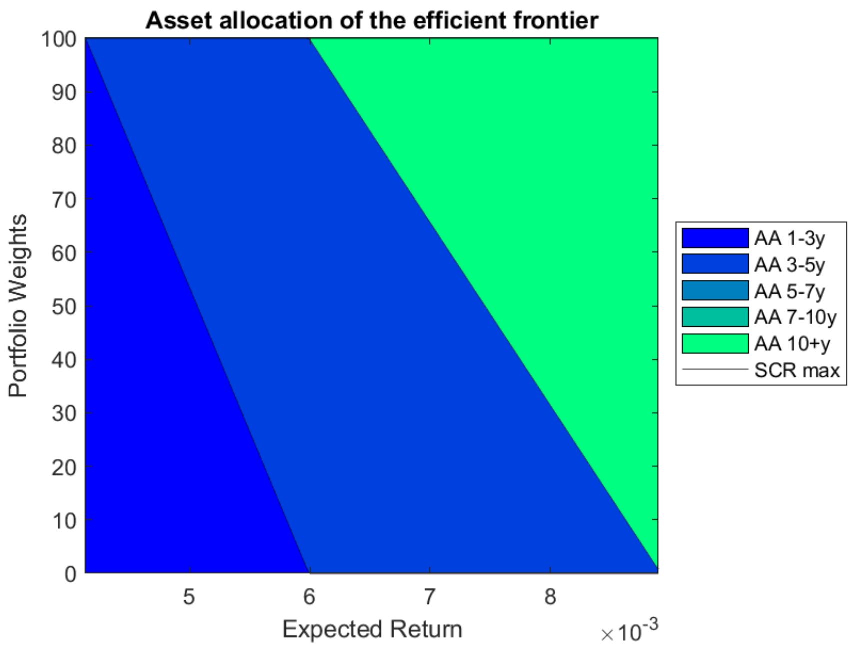


Figure 44



5.3.2. A-rated Corporates

The A-rated corporates share most risk and return characteristics of the previous benchmark. Longer durations entail higher expected excess returns and higher risks, measured by standard deviation and SCR. The performance ratios are more advantageous for shorter durations except for the difference between the E(R)/3σ ratios of the 7-10y and 10+y benchmarks, exactly as it was the case with AA-rated corporates. Once again, the SCR model treats more favorably the 1-3y and the 7-10y benchmarks than the model measured with standard deviation. The correlations between duration benchmarks are still lower than 1 but are comparatively a bit higher than the correlations between AA-rated duration benchmarks for most duration pairs.

Figure 45

A Corporates	1-3y	3-5y	5-7y	7-10y	10+y
Option-Adjusted Spread	0,60%	0,82%	0,90%	1,00%	1,16%
Risk Correction	0,07%	0,11%	0,14%	0,18%	0,25%
Expected Excess Returns	0,53%	0,71%	0,76%	0,82%	0,91%
Standard deviation	1,21%	1,94%	2,83%	3,89%	3,98%
SCR	2,70%	5,37%	7,44%	8,96%	11,51%
E(R)/3σ	14,7%	12,3%	9,0%	7,1%	7,6%
E(R)/SCR	19,8%	13,3%	10,2%	9,2%	7,9%
SCR/3σ	74,5%	92,4%	87,8%	76,8%	96,4%
Correlations:					
1-3y	1,0000	0,9104	0,8704	0,8368	0,6416
3-5y	0,9104	1,0000	0,9683	0,9449	0,7992
5-7y	0,8704	0,9683	1,0000	0,9595	0,8277
7-10y	0,8368	0,9449	0,9595	1,0000	0,8576
10+y	0,6416	0,7992	0,8277	0,8576	1,0000

The consequences of such characteristics can be appreciated in the composition of the portfolios of the efficient frontier in Figure 46 to Figure 48 below. The optimal portfolios are composed of the most efficient combination of two duration benchmarks situated on either sides of the duration axe. In this case, the 5-7y and 7-10y benchmarks are excluded from all portfolios as they are less efficient than the combination of the 3-5y and 10+y benchmarks, even after taking into account any diversification benefits. As we could expect from the relatively low capital applied by Solvency II as compared with the standard deviation measure on A-rated corporates (Figure 29 and Figure 45), the SCR of all efficient portfolios lies below 3σ, even after applying the correlations between benchmarks and even more so for the portfolios with lower expected excess returns.

Figure 46

E(R)	0,53%	0,63%	0,72%	0,82%	0,91%
Risk measures					
SCR	2,70%	4,06%	5,43%	8,59%	11,51%
σ	1,21%	1,55%	1,95%	2,88%	3,98%
3σ	3,63%	4,65%	5,85%	8,64%	11,94%
Portfolio allocation (% of assets)					
1-3y	100,0%	49,3%	0,0%	0,0%	0,0%
3-5y	0,0%	50,7%	99,1%	47,5%	0,0%
5-7y	0,0%	0,0%	0,0%	0,0%	0,0%
7-10y	0,0%	0,0%	0,0%	0,0%	0,0%
10+y	0,0%	0,0%	0,9%	52,5%	100,0%

Figure 47

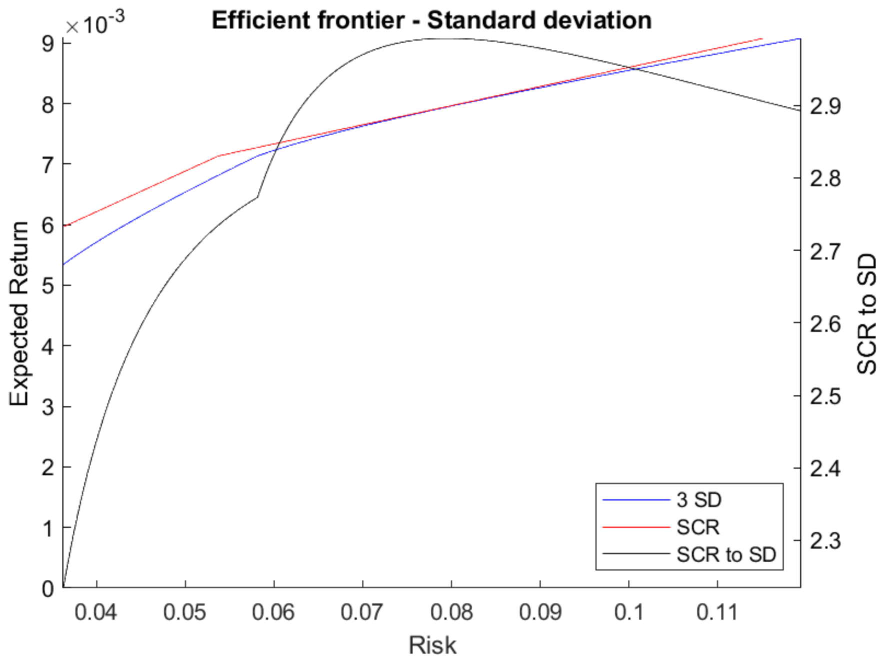
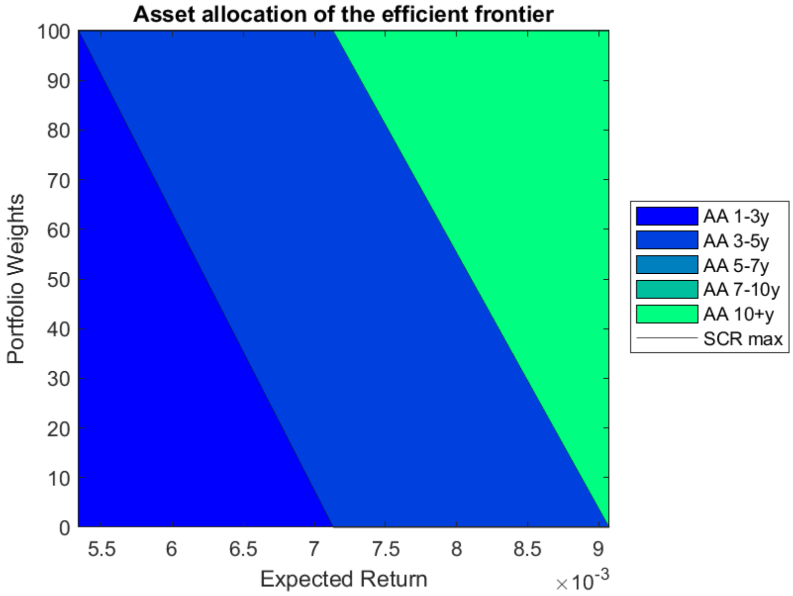


Figure 48



As a consequence of the quite similar characteristics shared by the standard deviation criterion and the SCR one, the portfolios of the efficient frontier are exactly the same when they are optimized following the SCR constraint. Neither the slight variations between the performance ratios with the SCR and the standard deviation denominator nor the diversification effect annihilated by the SCR method are sufficient to alter the choice of duration benchmarks.

Figure 49

E(R)	0,53%	0,63%	0,72%	0,82%	0,91%
Risk measures					
SCR	2,70%	4,06%	5,43%	8,59%	11,51%
σ	1,21%	1,55%	1,95%	2,88%	3,98%
3σ	3,63%	4,65%	5,85%	8,64%	11,94%
Portfolio allocation (% of assets)					
1-3y	100%	49,3%	0,0%	0,0%	0%
3-5y	0%	50,7%	99,1%	47,5%	0%
5-7y	0%	0,0%	0,0%	0,0%	0%
7-10y	0%	0,0%	0,0%	0,0%	0%
10+y	0,0%	0,0%	0,9%	52,5%	100%

Figure 50

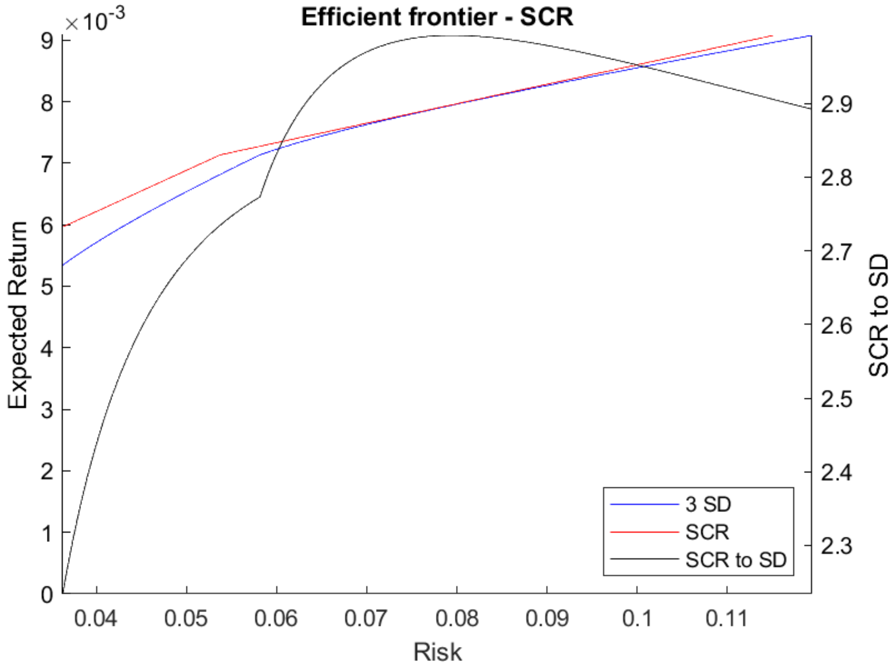
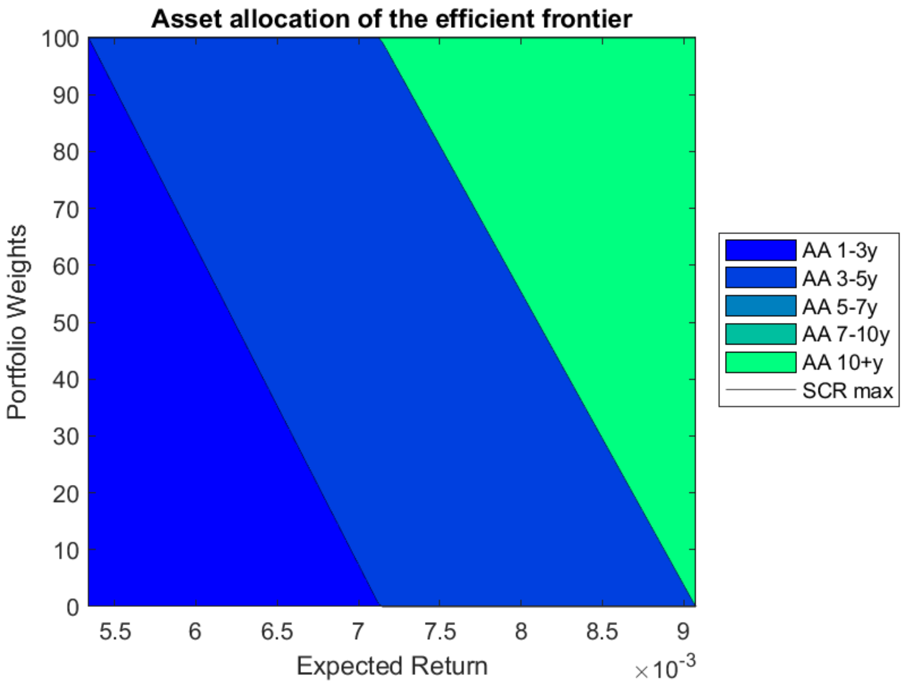


Figure 51



5.3.3. BBB-rated Corporates

The BBB-rated corporates have a smaller sample of duration benchmarks compared to their higer-rated counterparts, as the 10+y benchmark is not sufficiently populated to have a complete data history in our period of study. However, the same logics as with the higher-rated benchmarks seem to prevail. Higher durations entail more expected excess returns and more risk, and the performance ratios are better for the benchmarks of shorter duration. The SCR risk measure is relatively more favorable for the 1.3y benchmark, followed by the 7-10y one as in previous cases. An interesting fact is that the correlations between durations are actually lower than the correlations of the A-rated duration benchmarks, and also than the correlations of the AA-rated duration benchmarks for four out of six correlation pairs – especially the ones between durations situated close to each other.

Figure 52

BBB Corporates	1-3y	3-5y	5-7y	7-10y
Option-Adjusted Spread	0,78%	1,09%	1,22%	1,32%
Risk Correction	0,15%	0,19%	0,24%	0,29%
Expected Excess Returns	0,63%	0,90%	0,98%	1,03%
Standard deviation	1,73%	2,79%	3,69%	5,08%
SCR	4,83%	9,35%	13,17%	16,24%
E(R)/3σ	12,2%	10,7%	8,9%	6,7%
E(R)/SCR	13,1%	9,6%	7,5%	6,3%
SCR/3σ	93,2%	111,9%	118,8%	106,5%
Correlations:				
1-3y	1,0000	0,8417	0,8270	0,8090
3-5y	0,8417	1,0000	0,9111	0,9185
5-7y	0,8270	0,9111	1,0000	0,9363
7-10y	0,8090	0,9185	0,9363	1,0000

The portfolios of the efficient frontier of the standard deviation optimization are composed of combinations of two adjacent benchmarks, as in this case all duration benchmarks are used. There is no duration benchmark that would be improved by a combination of a riskier and a less risky duration benchmark. The diversification effect between benchmarks has no direct impact in this case as there is no portfolio composed of more than two benchmarks. As with the AA-rated benchmark, most portfolios have a SCR higher than three times the standard deviation, with the exception of the safest portfolios mainly composed of the 1-3y benchmark.

Figure 53

E(R)	0,63%	0,73%	0,83%	0,93%	1,03%
Risk measures					
SCR	4,83%	6,52%	8,20%	10,74%	16,24%
σ	1,73%	2,04%	2,46%	3,05%	5,08%
3σ	5,18%	6,11%	7,38%	9,14%	15,25%
Portfolio allocation (% of assets)					
1-3y	100,0%	62,5%	25,4%	0,0%	0,0%
3-5y	0,0%	37,5%	74,6%	63,6%	0,0%
5-7y	0,0%	0,0%	0,0%	36,4%	0,0%
7-10y	0,0%	0,0%	0,0%	0,0%	100,0%

Figure 54

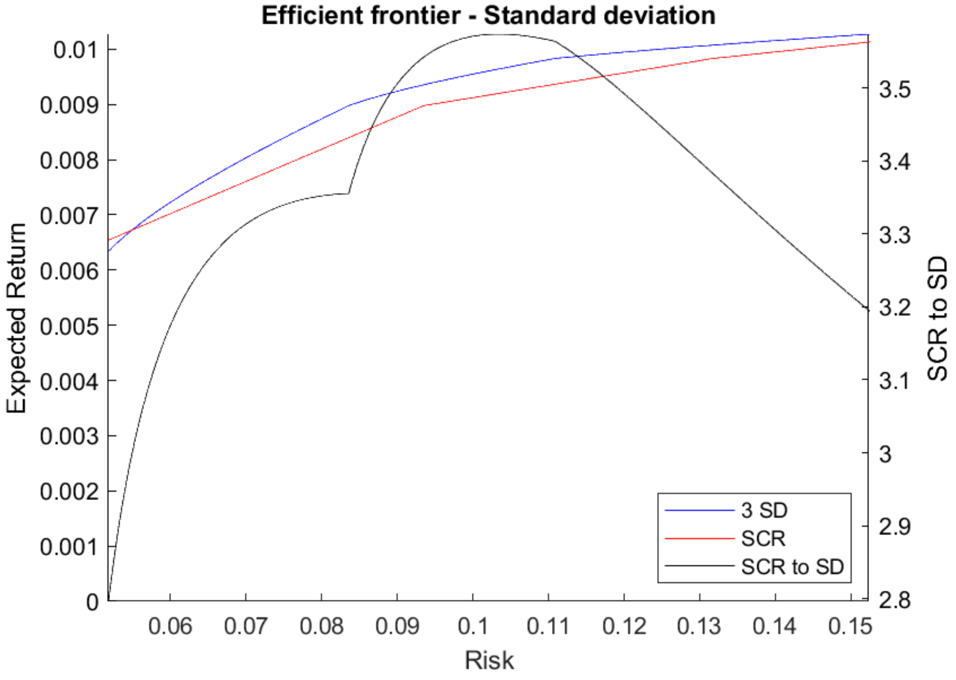
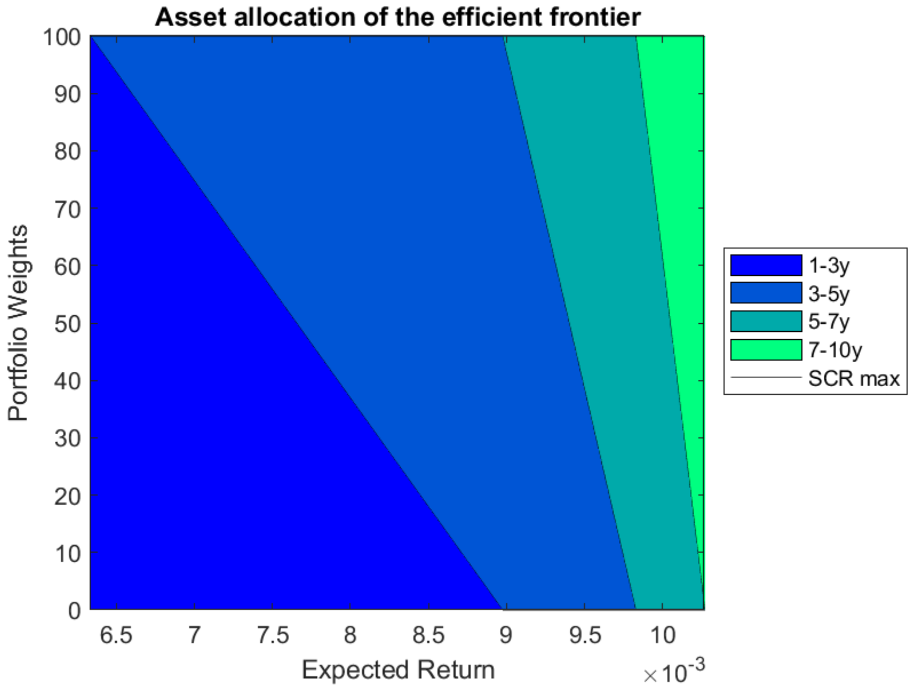


Figure 55



As the SCR risk measure does not alter fundamentally the order of preference as measured by the performance ratios between the different duration benchmarks, the efficient frontier of the SCR optimization remains the same as the standard deviation optimization.

Figure 56

E(R)	0,63%	0,73%	0,83%	0,93%	1,03%
Risk measures					
SCR	4,83%	6,52%	8,20%	10,74%	16,24%
σ	1,73%	2,04%	2,46%	3,05%	5,08%
3σ	5,18%	6,11%	7,38%	9,14%	15,25%
Portfolio allocation (% of assets)					
1-3y	100,0%	62,5%	25,4%	0,0%	0,0%
3-5y	0,0%	37,5%	74,6%	63,6%	0,0%
5-7y	0,0%	0,0%	0,0%	36,4%	0,0%
7-10y	0,0%	0,0%	0,0%	0,0%	100,0%

Figure 57

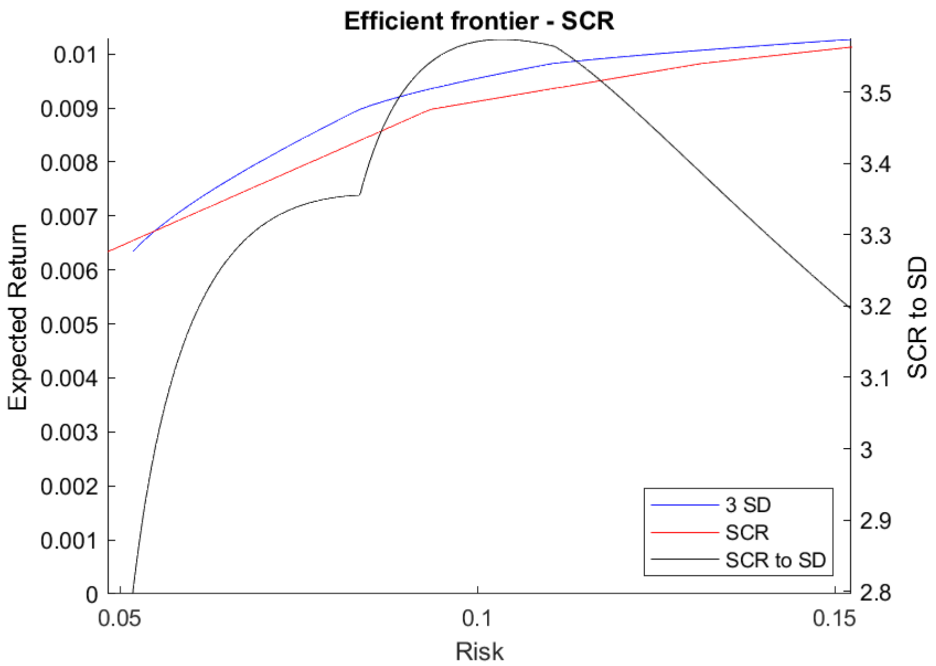
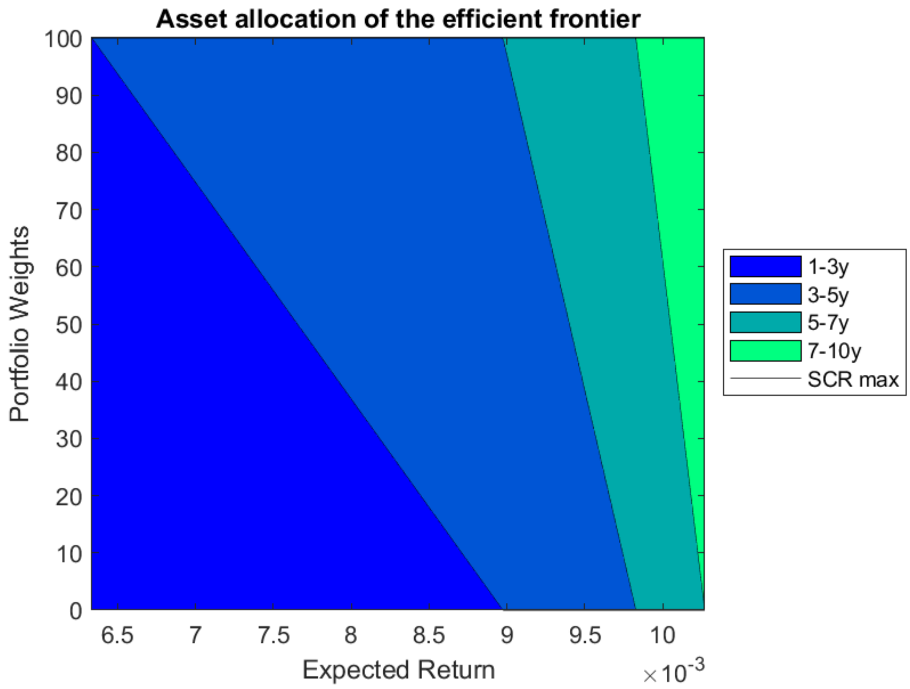


Figure 58


Asset allocation of the efficient frontier
Portfolio Weights
100
90
80
70
60
50
40
30
20
10
0
6.5
7
7.5
8
8.5
9
9.5
10
Expected Return
×10-3
1-3y
3-5y
5-7y
7-10y
SCR max

6. Conclusion

The optimal way to allocate spread risk in a fixed-income portfolio can be quite different when measured with the standard deviation of excess returns or with the SCR capital charge. When we look at the different bond categories, it would be natural for insurance companies to invest 100% of their portfolio in corporate and covered bonds to maximize returns. However, an optimization with SCR criteria gives a very strong incentive to invest in government bonds instead of covered bonds, and more specifically in the riskiest one as this would imply no additional capital charge.

When comparing the different rating-specific benchmarks of corporate bonds, the implementation of Solvency II is likely to encourage slightly more conservative asset allocations, as the A-rated benchmark is more efficient than the BBB-rated benchmark. But thus, it gives relevance to the A rating, which would be excluded from the portfolios of the efficient frontier of an optimization with standard deviation criteria.

If we deep dive into the different duration benchmarks inside each rating category, both models show that the capital efficiency of corporate bonds is the best in the lowest maturity buckets, irrespectively of the rating of the bonds. The optimal portfolios of the respective efficient frontiers of the two models are pretty similar, except in one instance where the diversification benefits of the standard deviation model introduce a third benchmark as compared to the SCR model with AA-rated duration benchmarks. Hence, even if in Solvency II the diversification between bond categories, credit ratings and durations is rendered useless as the total capital for spread risk is a mere sum of the capital charge of all the holdings and does not consider correlations, this fact is expected to be relatively insignificant in the building of the fixed-income part of the investment portfolio of an insurance company.

Our analysis has only taken into account the spread risk and return of fixed-income securities and not the pure interest rate risk. A combined analysis would require a discussion of the liability profile of insurance companies in order to get a more holistic picture of the effects of Solvency II on insurers' asset allocation.

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III. The Investment Horizon under Solvency II

1. Introduction

The general purpose of this thesis is to study how the Solvency Capital Requirements (SCR) of the Solvency II standard formula laid out in the Directive 2009/138/ec of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance introduces new constraints and incentives in the investment decisions of insurers.

In the first chapter, we simulated the portfolio of an insurance company seeking to minimise its capital requirements for a given level of expected returns. We found that government bonds were favoured in the whole spectrum of risk appetite no matter the economic conditions. For riskier assets, the portfolio optimization with Solvency II constraints accentuates the tendency given by the standard deviation optimization: in a low interest rates environment, equity and property are relatively more relevant whereas corporate bonds have almost less place in the optimal portfolios, but in an interest rate environment closer to the average the corporate bonds are given more allocation than real estate or equity.

In the second chapter, we focused on some of the aspects of fixed-income securities, which are the most important asset class in the balance sheet of an insurance undertaking. We segregated in our analysis the spread risk and return of fixed-income securities from the pure interest rate risk, and found that the optimal bond portfolios can be quite different when measured with the standard deviation of excess returns or with SCR. It would be better for insurance companies to invest 100% of their portfolio in corporate and covered bonds to minimize the standard deviation of excess returns, but the SCR optimization gives a very strong incentive to invest in government bonds. Within the corporate bonds category, Solvency II encourages a slightly more conservative asset allocation as compared with the standard deviation measure as A-rated bonds are more capital-efficient than BBB-rated bonds from the spread perspective. The Solvency II optimization supports low spread durations about as much as the standard deviation optimization does. However, Solvency II capital requirements do not incentivize diversification between bond categories, credit ratings or durations as the total capital for spread does not consider correlations different from 1.

Now that we know better the incentives and disincentives introduced by the spread module of Solvency II standard model, we would like to bring back the interest rate risk in our analysis to consider the total expected yield of the portfolio in order to understand how those conclusions hold with a realistic investment portfolio of an insurer. Indeed, one may think that the incentive to hold government bonds responds to a need to match the liabilities of the portfolio, that are discounted at a risk-free rate quite similar to the sovereign curves of investment grade European issuers. The investment universe of sovereign bonds offers securities with longer durations than the one of corporate bonds, so that the lack of spread capital charge for sovereign bonds could incentivize investments in longer-dated securities and therefore close the negative duration gap present in some undertakings with long-term liabilities.

We will study in the four following sections of this chapter if the SCR constraint restricts the optimal asset allocation of an insurance company looking to minimize the standard deviation of its returns for a given level of expected returns for different duration profiles. The sixth section goes beyond the discussions on the biases introduced by the capital charges of Solvency II standard model to debate the suitability of a one-size-fit-all Value-at-Risk model calibrated at a one-year horizon for all undertakings. Some other risk measures could be more helpful to grasp the specificities of the companies that assume long dated and illiquid liabilities, especially in life insurance.

2. State of the research

As explained in the previous chapters, some authors such by (Braun et al., 2017, 2018; Escobar et al., 2019; Heinrich & Wurstbauer, 2018; Kouwenberg, 2018) have already used a portfolio optimization model derived from (Markowitz, 1952a) to maximize the expected return of an insurer's investments for a given SCR. They find that Solvency II offers an incentive to invest in sovereign bonds rather than in corporate bonds or other riskier asset classes, disregarding the potential spread or concentration risks, but they do not investigate how much the conclusions of their analysis depend on their assumptions on the duration of the liabilities of the insurance company. In fact, they consider a fixed duration allegedly representative of an undertaking operating in the life segment, ranging from 6,7 years (Kouwenberg, 2018) to 12 years (Escobar et al., 2019) but then they pick benchmarks with lower durations for their assets so that their main concern as regards interest rate risk is the downwards shock – basically, they need to allocate as many assets as possible in the fixed-income category if they want to minimize this particular risk. One might think that this fact introduces a bias in their conclusions. Even if they say that the impact of interest rate risk is made quite irrelevant when compared to the other shocks, we would have to check how the optimal asset allocation changes if the duration of the liability changes, or if the benchmarks of the assets have different – longer – durations.

In the first chapter of this dissertation, we already adressed some discussable assumptions of their models, for instance by adjusting some backwards-looking measures like the mean returns of the asset benchmarks for some forward-looking estimates that come closer to the actual expectations of the market. We would like to go further in the analysis and study how dependent our results are on the duration of the liabilities of the insurer. We will use benchmarks of assets with a complete range of duration buckets so that the insurance company will be able to match the duration of the liabilities without committing its whole portfolio to the fixed-income asset class. This approach will also help us to understand how the conclusions we reached in the second chapter hold when we are studying together the spread risk and the interest rate risk.

3. Data and portfolio assumptions

This study reuses some of the data of our two previous chapters. The benchmarks for risky asset classes come from the first chapter: the MSCI World Developed Price Index is chosen as equity benchmark as it was used in the Solvency II calibration paper of the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS, 2010), and the SXI Real Estate Funds Total Return Index represents the Real Estate asset class. The benchmarks of the fixed-income categories are the ICE BofAML Indices already used in our second chapter because they were also the reference for (CEIOPS, 2010). We choose one set of five benchmarks for each maturity category of AA-rated government bonds (1-3y, 3-5y, 5-7y, 7-10y and 10+y) that represent the median rating of the sovereign investment universe of a European insurance company. As we found in our second chapter that the treatment of the spread risk of the SCR can vary according to the rating of the corporate benchmark, we build for this chapter custom composite benchmarks representative of a European insurer with the weights used by EIOPA for the calculation of the volatility adjustment as of June 2022, that we simplified by cutting the allocation to AAA-rated bonds – mostly covered bonds – and BB-rated bonds that can be held but cannot be purchased by insurers according to regulations. We rebalance weekly for each duration category the ICE BofAML indices of AA-rated, A-rated and BBB-rated bonds, except for the 10+y one that is exclusively composed of the AA-rated and A-rated categories due to the lack of available data for BBB-rated bonds.

Twenty two and a half years of weekly data are commonly available for those benchmarks from 2000 through June 2022. The standard deviations of the returns and correlations between the benchmarks are calculated over this period. We do not use a one-year rolling window of daily data to overcome the pitfalls denounced by (Mittnik, 2011) but we rather annualize the data with the square root of time rule. The expected returns are derived from the yield of the last data point for the fixed-income benchmarks to follow the approach of the first chapter and avoid assigning unrealistic future returns from the high mean of past data explained by the fall in interest rates of the last two decades. We adjust those expected returns by a risk correction factor corresponding to the sum of the probability of default – that can be considered an expected loss from default in this case – and of the cost of downgrade for each corporate benchmark, following the methodology of the second chapter inspired by the calculation of the volatility adjustment by EIOPA. The expected returns for equity and real estate follow the same approach as the first chapter, adding the risk premia identified by (Jorda et al., 2019) to the yield of the government bond category with the lowest maturity.

The data is summarized in Figure 59 below.

We consider a representative insurer with an asset side of the balance sheet worth 110% the liability side, so that the own funds represent 10% of the liabilities or 9.09% of the assets, like in our first chapter.

The assets consist exclusively in financial instruments subject to the market risk module, and the liabilities are composed exclusively of the Best Estimate of Liabilities (BEL) as defined in Solvency II Delegated Acts. As the BEL is the result of a sum of cash flows discounted at the risk-free rate, it will behave similarly to government bonds. Therefore, liabilities are modeled as a mandatory short exposure to one of the five categories of government bonds worth ten times the own funds.

Figure 59

		Government bonds					Corporate bonds					Equity	Real Estate
		1-3 y	3-5 y	5-7 y	7-10 y	10+ y	1-3 y	3-5 y	5-7 y	7-10 y	10+ y		
Modified Duration		2,02	3,84	5,74	8,23	16,10	1,90	3,81	5,57	7,69	12,86	-	-
Average return		2,15%	2,94%	3,59%	4,25%	5,21%	2,68%	3,30%	4,04%	4,28%	5,21%	4,60%	5,39%
Expected returns (2022)		0,92%	1,39%	1,64%	1,90%	2,37%	2,20%	2,95%	3,28%	3,41%	3,44%	6,82%	5,12%
Standard deviation		1,61%	3,12%	4,36%	5,76%	9,55%	1,46%	2,78%	4,06%	5,42%	6,99%	18,28%	8,38%
Correlations: Government bonds	1-3 y	1,000	0,954	0,898	0,833	0,663	0,503	0,510	0,469	0,418	0,399	-0,062	0,042
	3-5 y	0,954	1,000	0,981	0,931	0,783	0,529	0,577	0,562	0,530	0,525	-0,048	0,076
	5-7 y	0,898	0,981	1,000	0,975	0,857	0,519	0,592	0,600	0,589	0,603	-0,039	0,109
	7-10 y	0,833	0,931	0,975	1,000	0,922	0,461	0,549	0,578	0,594	0,631	-0,028	0,122
	10+ y	0,663	0,783	0,857	0,922	1,000	0,366	0,471	0,527	0,577	0,673	-0,046	0,130
Corporate bonds	1-3 y	0,503	0,529	0,519	0,461	0,366	1,000	0,933	0,891	0,827	0,687	0,114	0,151
	3-5 y	0,510	0,577	0,592	0,549	0,471	0,933	1,000	0,975	0,932	0,823	0,085	0,182
	5-7 y	0,469	0,562	0,600	0,578	0,527	0,891	0,975	1,000	0,973	0,886	0,094	0,204
	7-10 y	0,418	0,530	0,589	0,594	0,577	0,827	0,932	0,973	1,000	0,914	0,122	0,231
	10+ y	0,399	0,525	0,603	0,631	0,673	0,687	0,823	0,886	0,914	1,000	-0,022	0,218
Equity		-0,062	-0,048	-0,039	-0,028	-0,046	0,114	0,085	0,094	0,122	-0,022	1,000	0,250
Real Estate		0,042	0,076	0,109	0,122	0,130	0,151	0,182	0,204	0,231	0,218	0,250	1,000

4. Methodology

4.1. Solvency II standard model calculations

The analysis laid out in the next sections compares the standard deviation of a set of optimal portfolios to the corresponding Solvency Capital Requirement (SCR). As a reminder from the first chapter, we focus our thesis exclusively on the market module (SCR_{Mkt}) that aggregates the interest rate risk (SCR_{IR}), the equity risk (SCR_{EQ}), the property risk (SCR_{RE}), the spread risk (SCR_{CS}) and two other risks that are deemed to be totally diversifiable so that they will not be treated in our analysis (concentration risk and currency risk).

$$\mathrm{SCR}_{\mathrm{Mkt}} = \sqrt{\sum_i \sum_j \rho_{i,j} * \mathrm{SCR}_i * \mathrm{SCR}_j} \quad (3.1)$$

where i,j represents all pairs of market risks, and $\rho_{i,j}$ is the correlation between those pairs as set out in the Delegated Acts and reproduced in Figure 60.

The standard model considers two distinct scenarios for interest rate risk: it is calculated as the worst case between an interest rate up scenario and an interest rate down scenario. The interest rate up simulates a stress where the swap curve – the risk-free benchmark used by the regulator – increases by the minimum of a percentage depending on the maturities and +100 basis points. Then, given the market conditions of the last few years, it is equivalent to a +100bp interest rate shock for all maturities. Similarly, the interest rate down scenario simulates a downwards stress of the swap curve calculated as a percentage of the yield at each maturity, but negative interest rates are not shocked and there is no minimum stress as in the interest rate up scenario, so that the total stress can be extremely low as it was the case in the first years of the implementation of Solvency II through 2021. The shocks expressed as a percentage of market value of the financial instrument resulting from the swap curve at the end of the year 2019 for maturities dated from 1 to 20 years are presented in Figure 61.

Figure 60

	Interest	Equity	Property	Spread
Interest	1,00	A	A	A
Equity	A	1,00	0,75	0,75
Property	A	0,75	1,00	0,50
Spread	A	0,75	0,50	1,00

where A = 0 in case of interest rate shock up and 0.5 in case of interest rate shock down.

To calculate the capital charges of our assets and liabilities, we consider that our benchmarks are made of only one cash flow dated at their average modified duration. We use a linear interpolation between the two closer maturities available in Figure 61 to derive the exact shock for each asset. The resulting shocks are described in Figure 62 below. The IR down shocks are expressed as a negative value as they will be assigned to the liabilities which already carry a negative sign.

Figure 61

Year	EIOPA SWAP curve			Shock (% market value)	
	Central	Shock up	Shock down	Shock up	Shock down
1	0,74 %	1,74 %	0,18 %	-0,99%	-0,55%
2	1,27 %	2,27 %	0,44 %	-1,97%	-1,63%
3	1,45 %	2,45 %	0,64 %	-2,94%	-2,39%
4	1,61 %	2,61 %	0,81 %	-3,90%	-3,16%
5	1,69 %	2,69 %	0,91 %	-4,85%	-3,79%
6	1,78 %	2,78 %	1,03 %	-5,80%	-4,37%
7	1,86 %	2,86 %	1,14 %	-6,73%	-4,94%
8	1,94 %	2,94 %	1,24 %	-7,65%	-5,42%
9	2,02 %	3,02 %	1,35 %	-8,57%	-5,80%
10	2,09 %	3,09 %	1,44 %	-9,47%	-6,25%
11	2,18 %	3,18 %	1,53 %	-10,37%	-6,91%
12	2,21 %	3,21 %	1,57 %	-11,26%	-7,37%
13	2,24 %	3,24 %	1,61 %	-12,13%	-7,80%
14	2,27 %	3,27 %	1,63 %	-13,00%	-8,48%
15	2,29 %	3,29 %	1,67 %	-13,87%	-8,81%
16	2,28 %	3,28 %	1,64 %	-14,72%	-9,66%
17	2,25 %	3,25 %	1,62 %	-15,56%	-10,13%
18	2,22 %	3,22 %	1,60 %	-16,40%	-10,54%
19	2,19 %	3,19 %	1,56 %	-17,23%	-11,33%
20	2,17 %	3,17 %	1,54 %	-18,05%	-11,79%

Figure 62

	Government bonds					Corporate bonds				
	1-3 y	3-5 y	5-7 y	7-10 y	10+ y	1-3 y	3-5 y	5-7 y	7-10 y	10+ y
Modified Duration	2,02	3,84	5,74	8,23	16,10	1,90	3,81	5,57	7,69	12,86
Shock IR up	1,99%	3,74%	5,55%	7,86%	14,80%	1,88%	3,72%	5,39%	7,37%	12,01%
Shock IR down	-1,64%	-3,03%	-4,22%	-5,51%	-9,71%	-1,52%	-3,01%	-4,12%	-5,27%	-7,74%

4.2. Portfolio optimization

The first step of our analysis is to find the asset allocations that minimize the variance – hence the standard deviation – of the expected returns of the portfolio for each level or required return, as set out in (Markowitz, 1952b), which constitutes the ground of Modern Portfolio Theory.

Let us label the assets laid out in Figure 59 as the same number as their order of appearance in the figure and the liability proxy as 13. The total portfolio is composed of the assets and the liabilities, so that it is what is called in Solvency II Delegated Acts the OF (Own Funds) of the entity. The asset allocation of the portfolio depends on the weights of the assets represented by the scalar

$w = \begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_{13} \end{pmatrix}$ with the following constraints:

- $\sum_{i=1}^{13} w_i = 1$ (OF normalized to 1)
- $\forall$ i $\in [1,12],\ 0 \leq w_i \leq 11$ and $w_{13} = -10$
  (no short selling, the only short exposure is represented by the liabilities)

Let $\mathrm{R} = \begin{pmatrix} \mathrm{R}_1 \\ \mathrm{R}_2 \\ \dots \\ \mathrm{R}_{13} \end{pmatrix}$ be the expected returns of the assets and liabilities presented in Figure 59 and Σ the variance-covariance matrix of the returns of the series underlying the correlation matrix from Figure 59.

The expected returns of the portfolio are a linear product of the weights of the respective assets:

$$\mathrm{R_{OF}} = \mathrm{w}' * \mathrm{R} \qquad (3.2)$$

As the return-variance model assumes that the returns follow a normal distribution, we can write:

$$\sigma_{OF}^2 = \mathrm{Var}(\mathrm{R_{OF}}) = \mathrm{Var}(\mathrm{w}' * \mathrm{R}) = \sum_{\mathrm{i}=1}^{13} \sum_{\mathrm{j}=1}^{13} \mathrm{w_i w_j} \rho_{\mathrm{i,j}} \sigma_{\mathrm{i}} \sigma_{\mathrm{j}} = \mathrm{w}' \Sigma \mathrm{w} \qquad (3.3)$$

Then our optimization problem can be written as:

$$\min_w \mathrm{w}'\Sigma\mathrm{w}, s.t. \begin{cases} \mathrm{R_{OF}} = \mathrm{E(R)} \\ \sum_{i=1}^{13} \mathrm{w_i} = 1 \\ \forall \text{ i} \in [1,12], 0 \leq w_i \leq 11 \\ w_{13} = -10 \end{cases} \qquad (3.4)$$

This problem, that can theoretically be solved analytically, is at practical effects solved thanks to the Financial Toolbox of the software MATLAB. This allows us to reiterate the optimization for 1000 different required returns E(R), so that we can plot an efficient frontier of optimal portfolios as we will see in the next section.

We will then calculate the SCR of each of those optimal portfolios as explained in section 4.1, and plot the respective results in the same graph. In order to get a visual comparison between the two risk measures, the standard deviation of the optimal portfolios is multiplied by 3 to take into account that it does not represent a 99.5% Value-at-Risk stress of the own funds. The number can be thought of an approximation of the number of standard deviations necessary to reach said probability threshold, if we consider that the data follows a distribution with fatter tails than a normal distribution. This approach is convenient to compare both measures but remains a rule of thumb, so that we compute the ratio between the two to get further insights on the behavior of the SCR across the efficient frontier.

The weights of the optimal portfolios are represented in a second series of graphs in the next sections. The portfolios that cross the mark of SCR = 1 are marked with a dark line in the graphs to represent the subset of portfolios that do not comply with the solvency constraint.

The whole process is then repeated in a similar way for the SCR optimization, where the problem is to find the asset allocations that minimize the SCR of the portfolio for each level or required return. The asset allocation w is subject to the same constraints as set out for the return-variance model, and the expected returns of the assets and the liabilities remain the same. The only significant difference with the return-variance model where each asset is bound by only one risk measure is that in this case one asset can be affected by more than one shock – for instance, corporate bonds are impacted by the interest rate risk and by the spread risk – so that the formula involves one more step:

$$\mathrm{SCR_{Mkt}} = \sqrt{\textstyle\sum_i \sum_j \rho_{i,j} * \mathrm{SCR}_i * \mathrm{SCR}_j} \tag{3.1}$$

$$\text{Where } \mathrm{SCR}_i = \mathrm{w}' * \mathrm{CIR}_{i,k} \tag{3.5}$$

($\mathrm{CIR}_{i,k}$ is the shock applied to each asset k for each risk i)

So that we can write the optimization problem:

$$\min_w \textstyle\sum_i \sum_j \rho_{i,j} * (\mathrm{w}' * \mathrm{CIR}_{i,k}) * (\mathrm{w}' * \mathrm{CIR}_{j,k}) \,, s.t. \begin{cases} \mathrm{R_{OF}} = \mathrm{E(R)} \\ \sum_{i=1}^{13} \mathrm{w_i} = 1 \\ \forall \text{ i} \in [1,12], 0 \leq w_i \leq 11 \\ w_{13} = -10 \end{cases} \tag{3.6}$$

The optimization processes, figures and graphs are derived for each of the five maturity buckets of the government bond benchmark that depict the behavior of the liabilities.

5. Optimal asset allocation under different duration profiles

The efficient frontier of the portfolios minimizing the standard deviation for a given level of required return for the five different maturity buckets and the corresponding asset weights are reproduced in Figure 63 to Figure 72 below from a graphical and numerical perspective.

Figure 63

Standard Deviation Optimization

SCR Optimization

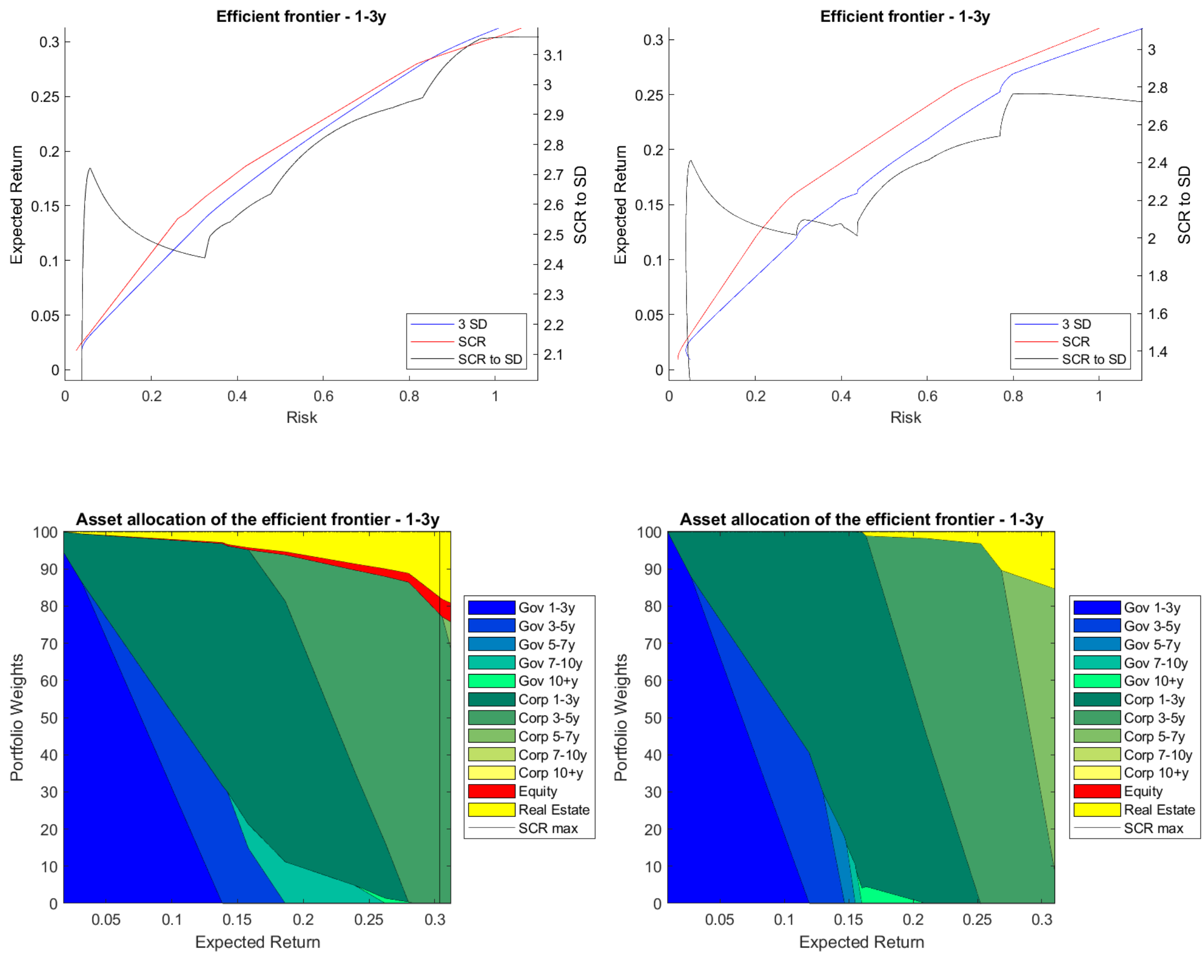


Figure 64

1-3y		Standard deviation optimization:					SCR Optimization:				
E(R)		5%	10%	15%	20%	30%	5%	10%	15%	20%	30%
Risk measures											
SCR		8,8%	18,8%	30,2%	48,0%	97,1%	7,9%	16,5%	26,4%	44,6%	93,4%
3σ		10,2%	22,9%	35,9%	52,7%	93,2%	10,8%	24,3%	38,3%	56,3%	102,3%
Duration gap		0,24	0,45	0,62	0,98	1,18	0,34	0,70	0,68	0,96	2,45
Portfolio allocation											
Government bonds	1-3 y	71,5%	30,7%	0,0%	0,0%	0,0%	66,3%	18,5%	0,0%	0,0%	0,0%
	3-5 y	5,4%	20,5%	22,3%	0,0%	0,0%	9,9%	31,9%	0,0%	0,0%	0,0%
	5-7 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	12,0%	0,0%	0,0%
	7-10 y	0,0%	0,0%	3,3%	9,5%	0,0%	0,0%	0,0%	2,4%	0,0%	0,0%
	10+ y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,0%	0,0%
Corporate bonds	1-3 y	22,0%	46,6%	70,1%	59,6%	0,0%	23,7%	49,5%	85,5%	56,3%	0,0%
	3-5 y	0,0%	0,0%	0,0%	23,6%	79,1%	0,0%	0,0%	0,0%	41,1%	28,5%
	5-7 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	57,4%
	7-10 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	10+ y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
Equity		0,1%	0,2%	0,5%	1,1%	4,2%	0,0%	0,0%	0,0%	0,0%	0,0%
Real Estate		0,9%	2,0%	3,8%	6,2%	16,6%	0,0%	0,0%	0,0%	1,6%	14,1%

Figure 65

Standard Deviation Optimization

SCR Optimization

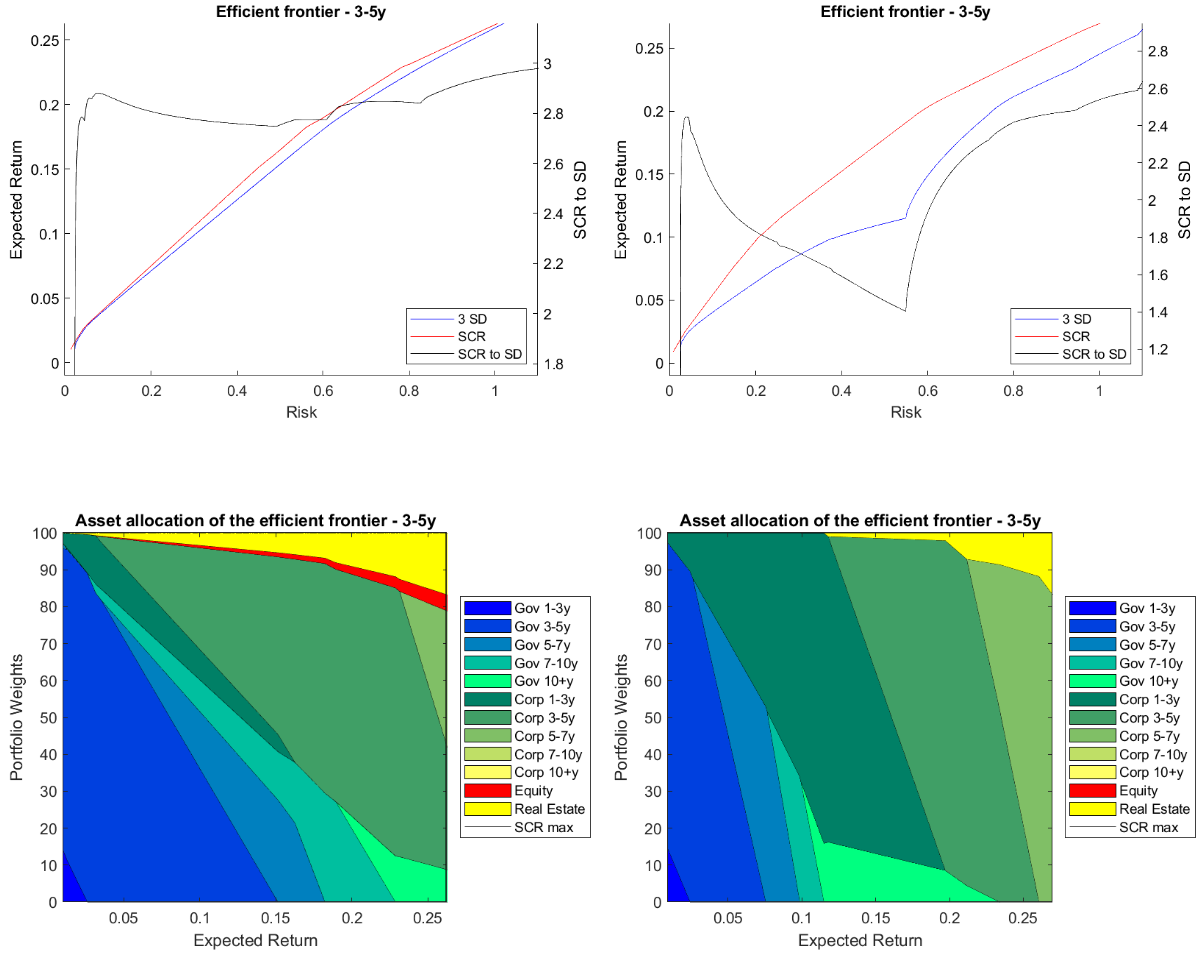
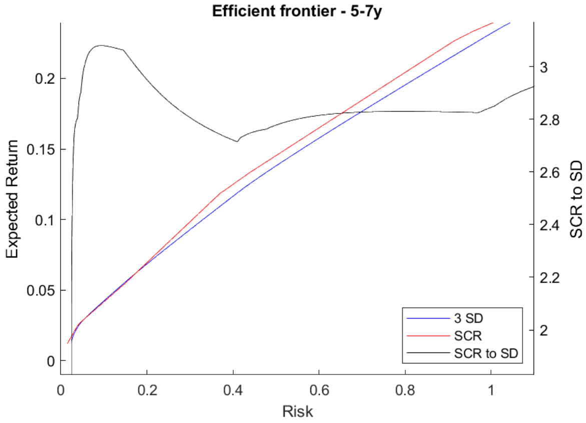


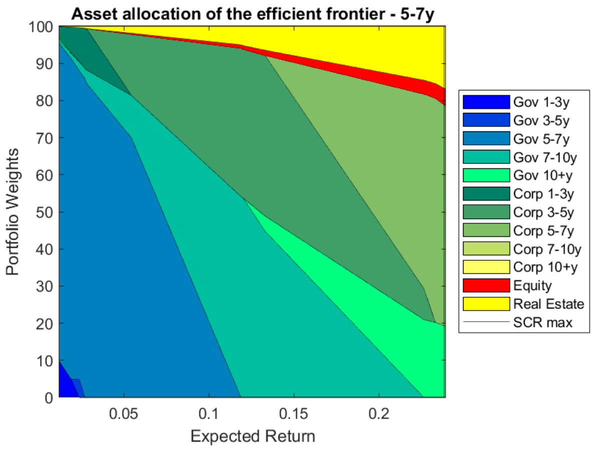
Figure 66

3-5y		Standard deviation optimization:					SCR Optimization:				
E(R)		5%	10%	15%	20%	30%	5%	10%	15%	20%	30%
Risk measures											
SCR		11,8%	28,2%	44,7%	65,0%	126,5%	9,1%	20,8%	39,2%	58,6%	124,7%
3σ		12,5%	30,5%	48,8%	68,6%	125,3%	14,0%	38,9%	60,3%	75,0%	126,9%
Duration gap		0,31	0,70	1,09	1,19	0,87	0,27	0,48	0,94	1,16	0,84
Portfolio allocation											
Government bonds	1-3 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	3-5 y	71,4%	36,3%	0,7%	0,0%	0,0%	45,4%	0,0%	0,0%	0,0%	0,0%
	5-7 y	3,5%	15,4%	27,4%	0,0%	0,0%	25,5%	0,0%	0,0%	0,0%	0,0%
	7-10 y	4,3%	8,6%	13,0%	19,7%	0,0%	0,0%	31,3%	0,0%	0,0%	0,0%
	10+ y	0,0%	0,0%	0,0%	3,5%	3,9%	0,0%	0,6%	13,1%	7,8%	0,0%
Corporate bonds	1-3 y	11,6%	8,2%	4,8%	0,0%	0,0%	29,1%	68,1%	49,2%	0,0%	0,0%
	3-5 y	7,5%	27,4%	47,7%	65,6%	0,0%	0,0%	0,0%	36,2%	89,2%	0,0%
	5-7 y	0,0%	0,0%	0,0%	0,0%	66,8%	0,0%	0,0%	0,0%	0,0%	64,2%
	7-10 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	9,7%
	10+ y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
Equity		0,3%	0,7%	1,2%	2,2%	6,3%	0,0%	0,0%	0,0%	0,0%	5,9%
Real Estate		1,4%	3,3%	5,3%	9,1%	23,0%	0,0%	0,0%	1,4%	3,1%	20,2%

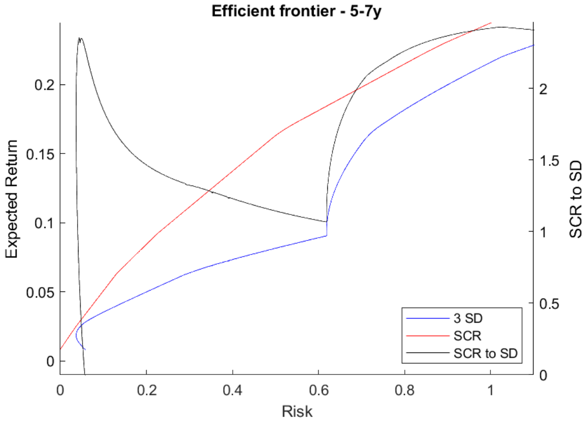
Figure 67

Standard Deviation Optimization





SCR Optimization



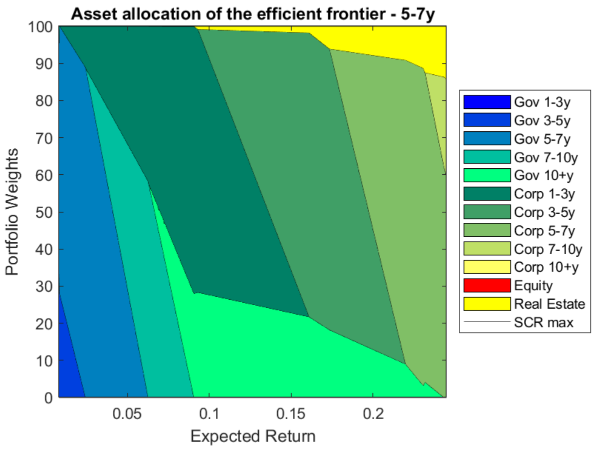
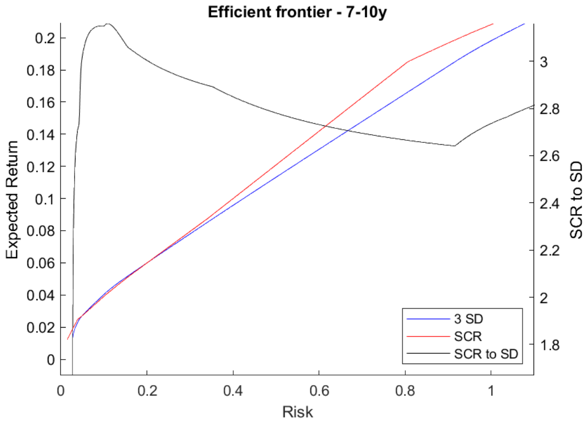


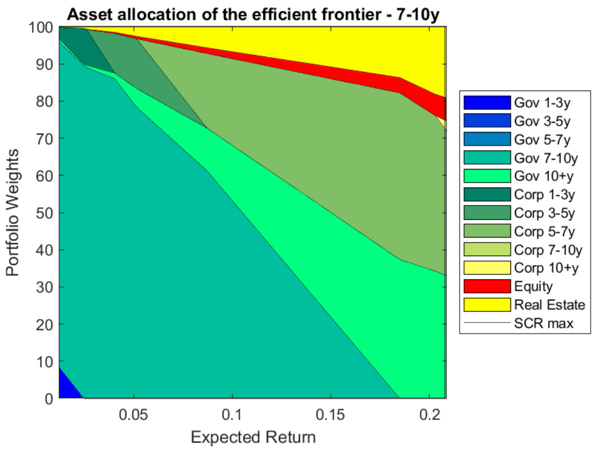
Figure 68

5-7y		Standard deviation optimization:					SCR Optimization:				
E(R)		5%	10%	15%	20%	24%	5%	10%	15%	20%	24%
Risk measures											
SCR		13,3%	30,7%	52,6%	78,1%	100,9%	9,8%	25,6%	44,9%	71,2%	96,6%
3σ		13,0%	33,2%	56,3%	82,8%	104,8%	20,1%	62,0%	68,4%	90,1%	119,4%
Duration gap		0,33	0,65	0,89	1,22	1,15	0,28	0,74	1,11	0,71	0,10
Portfolio allocation											
Government bonds	1-3 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	3-5 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	5-7 y	72,1%	20,0%	0,0%	0,0%	0,0%	29,3%	0,0%	0,0%	0,0%	0,0%
	7-10 y	10,3%	41,9%	36,5%	12,6%	0,0%	39,0%	0,0%	0,0%	0,0%	0,0%
	10+ y	0,0%	0,0%	7,2%	16,3%	19,1%	0,0%	27,6%	22,8%	12,9%	1,2%
Corporate bonds	1-3 y	1,7%	0,0%	0,0%	0,0%	0,0%	31,7%	63,6%	11,4%	0,0%	0,0%
	3-5 y	14,0%	33,2%	36,8%	18,1%	0,0%	0,0%	7,9%	64,2%	32,3%	0,0%
	5-7 y	0,0%	0,0%	9,7%	37,7%	59,2%	0,0%	0,0%	0,0%	47,0%	68,1%
	7-10 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	17,4%
	10+ y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
Equity		0,3%	0,8%	1,9%	3,1%	4,6%	0,0%	0,0%	0,0%	0,0%	0,0%
Real Estate		1,7%	4,1%	7,9%	12,2%	17,1%	0,0%	0,9%	1,6%	7,9%	13,4%

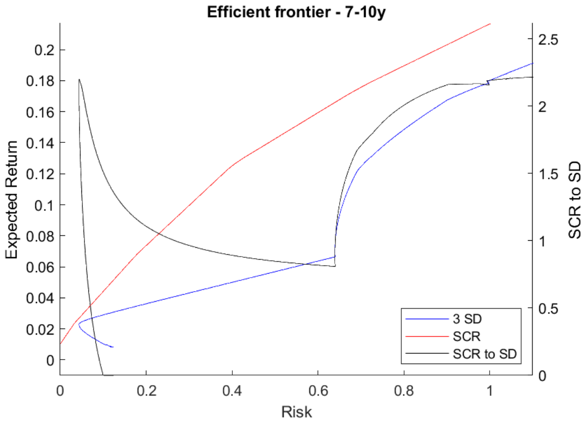
Figure 69

Standard Deviation Optimization





SCR Optimization



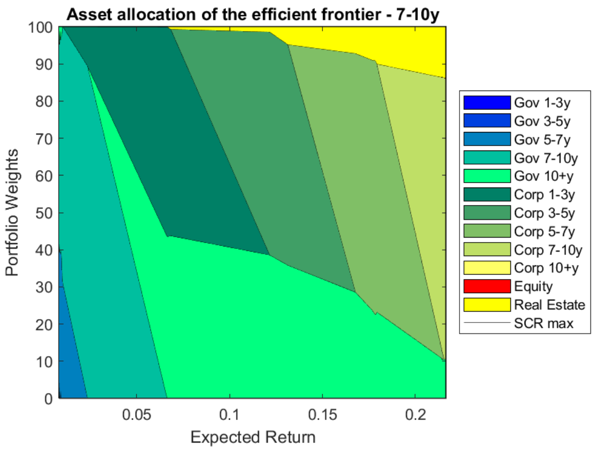
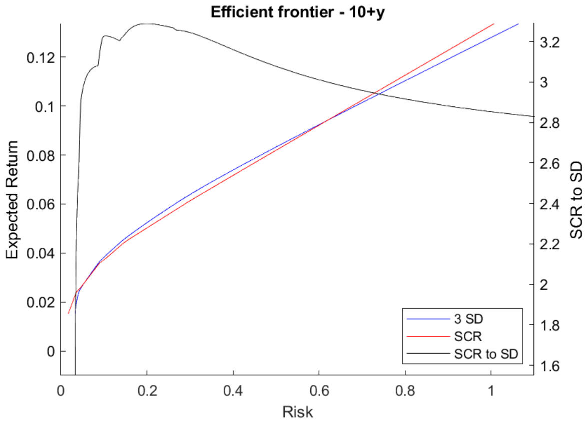


Figure 70

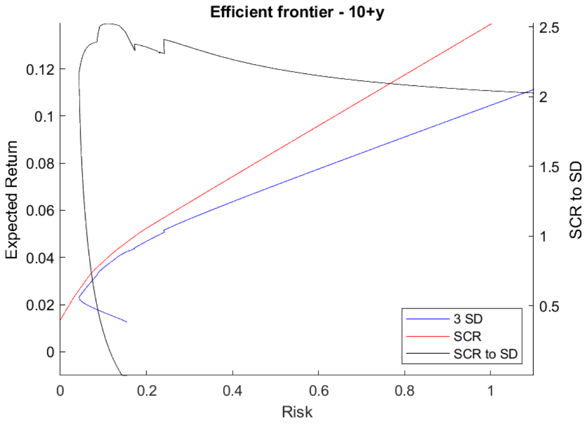
7-10y		Standard deviation optimization:					SCR Optimization:				
E(R)		5%	10%	15%	20%	25%	5%	10%	15%	20%	25%
Risk measures											
SCR		15,2%	40,1%	64,0%	93,3%	139,1%	11,9%	30,2%	54,5%	87,7%	130,1%
3σ		14,8%	42,6%	71,3%	101,7%	141,0%	40,0%	65,7%	80,8%	118,5%	161,8%
Duration gap		0,28	0,59	0,85	0,46	0,21	0,39	0,86	0,63	0,00	-1,29
Portfolio allocation											
Government bonds	1-3 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	3-5 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	5-7 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	7-10 y	79,3%	53,2%	21,9%	0,0%	0,0%	34,3%	0,0%	0,0%	0,0%	0,0%
	10+ y	4,6%	14,9%	28,1%	34,7%	23,6%	26,8%	40,7%	32,1%	15,9%	0,0%
Corporate bonds	1-3 y	0,0%	0,0%	0,0%	0,0%	0,0%	38,9%	22,6%	0,0%	0,0%	0,0%
	3-5 y	13,1%	0,0%	0,0%	0,0%	0,0%	0,0%	35,7%	29,0%	0,0%	0,0%
	5-7 y	0,0%	23,3%	36,0%	42,3%	13,4%	0,0%	0,0%	32,8%	28,7%	0,0%
	7-10 y	0,0%	0,0%	0,0%	0,0%	6,8%	0,0%	0,0%	0,0%	43,3%	80,5%
	10+ y	0,0%	0,0%	0,0%	0,0%	20,5%	0,0%	0,0%	0,0%	0,0%	0,0%
Equity		0,6%	1,9%	3,2%	5,5%	9,4%	0,0%	0,0%	0,0%	0,0%	5,8%
Real Estate		2,4%	6,7%	10,8%	17,5%	26,4%	0,0%	1,1%	6,0%	12,1%	13,7%

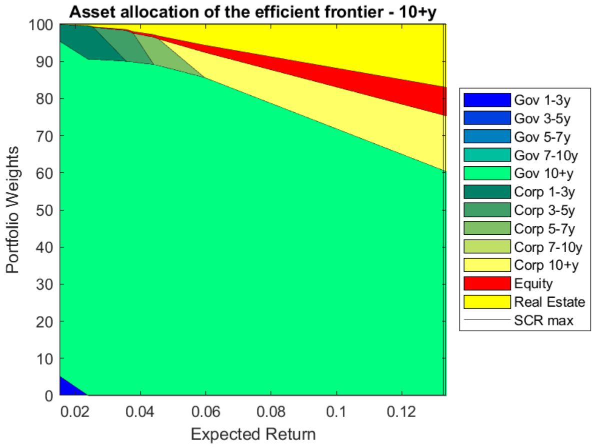
Figure 71

Standard Deviation Optimization



SCR Optimization





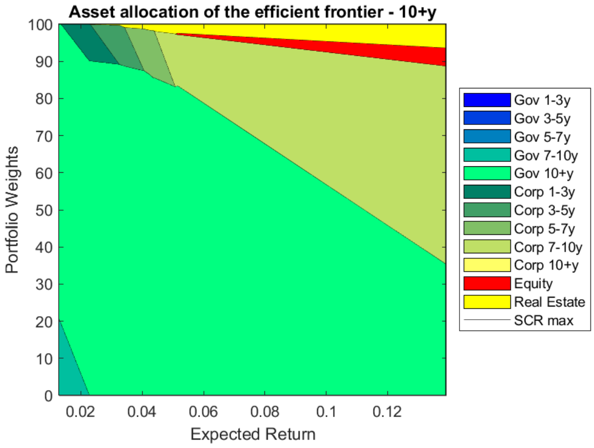


Figure 72

10+y		Standard deviation optimization:					SCR Optimization:				
E(R)		2,5%	5%	7,5%	10%	15%	2,5%	5%	7,5%	10%	15%
Risk measures											
SCR		4,2%	19,9%	43,7%	68,0%	116,8%	3,9%	18,0%	40,7%	63,8%	110,3%
3σ		4,5%	18,1%	41,7%	68,8%	125,2%	5,1%	23,4%	56,3%	93,2%	168,1%
Duration gap		0,11	0,06	-0,62	-1,63	-3,67	0,07	-0,15	-1,40	-2,74	-5,42
Portfolio allocation											
Government bonds	1-3 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	3-5 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	5-7 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	7-10 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
	10+ y	90,6%	87,9%	80,3%	71,7%	54,7%	89,9%	83,5%	70,6%	56,9%	29,4%
Corporate bonds	1-3 y	8,7%	0,0%	0,0%	0,0%	0,0%	7,9%	0,0%	0,0%	0,0%	0,0%
	3-5 y	0,0%	0,0%	0,0%	0,0%	0,0%	2,0%	0,0%	0,0%	0,0%	0,0%
	5-7 y	0,0%	4,9%	0,0%	0,0%	0,0%	0,0%	1,5%	0,0%	0,0%	0,0%
	7-10 y	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	12,6%	24,4%	35,7%	58,3%
	10+ y	0,0%	2,2%	8,5%	11,3%	16,8%	0,0%	0,0%	0,0%	0,0%	0,0%
Equity		0,1%	1,2%	3,2%	5,1%	9,0%	0,0%	0,0%	1,6%	2,9%	5,5%
Real Estate		0,6%	3,9%	8,0%	11,8%	19,5%	0,1%	2,5%	3,4%	4,6%	6,9%

At first glance, we observe the same general trends detected in the first two chapters between the optimal asset allocation of the portfolios satisfying the standard deviation constraint and the one satisfying the SCR constraint. The allocation to riskier asset categories such as equity and real estate is smaller in the SCR optimization than in the standard deviation optimization for the five categories of duration for liabilities as corporate bonds are a more risk-efficient way to get higher returns when yields are at a normal level. The equity allocation is even null for all categories of liabilities less the 10+y one as it is even less efficient than real estate.

In most cases the two risk measures are consistent with each other to the extent that both increase monotonously when the expected return of the optimal portfolio increases. There is however an exception found in the SCR optimization when the liabilities have a duration of 3-7y or higher: the safest portfolios according to the Solvency II requirements show a decreasing standard deviation when expected returns are increasing. For instance, in the 7-10y category, the optimal portfolio for a 1.5% expected return has a standard deviation of 2.37% and a SCR of 1.16% whereas for an expected return of 2%, the standard deviation goes down to 1.60% when the SCR increases up to 2.39%. This comes from the fact that some portfolios can have a very low SCR if they match the interest rate sensitivity of their liabilities with their fixed-income assets and do not invest much in assets subject to other risks such as spread, but they will have a material standard deviation as they will have to invest in assets with a lower average duration than the liabilities as the absolute value of the assets exceeds the value of the liabilities so that the risk measured by the standard deviation takes into account the imperfect correlation between assets and liabilities of different durations.

This exception explains the sharp increase of the SCR/σ ratio in the low end of expected returns of the SCR optimizations. The behavior of this ratio is a bit different between the standard deviation optimization and the SCR optimization: in the standard deviation optimization, the ratio remains relatively stable around 3 after the increase corresponding to the most conservative portfolios, but in the SCR optimization the ratio decreases again down to levels that are closer to 1 in the middle of most charts for all categories of liabilities except the 10+y one, and after a certain point they increase again sharply back to more normal levels between 2 and 3. Those points represent possible asset allocations where the Solvency II capital requirements are lowered proportionally to the standard deviation measure due to a simplification inherent to the calculation of the SCR standard model and related to the interest rate risk.

The two portfolio optimizations lead to similar results in terms of duration gap between the assets and the liabilities – expressed in years and scaled to the value of the assets in our figures. It remains positive but relatively small – lower than 1,5y – for the four shorter liabilities profiles, generally increasing with the expected returns to chase higher yields with longer durations in sovereign and corporate bonds, but for the portfolios with longer-dated liabilities the tendency then reverts, and the duration gap decreases when bonds are partially complemented by the two riskier assets. The duration gap is negative for most portfolios of the optimizations with the 10+y liabilities as the 10+y corporate benchmark has a lower duration than the corresponding sovereign benchmark used for the liabilities and as real estate and equity are allocated a bigger share of the portfolio.

If the duration gap is relatively good with the results of both optimization methods, there is an interesting difference when it comes to the concept of maturity or cash flow matching. Our benchmarks represent assets maturing and liabilities coming due in specific time ranges that would logically be slightly higher than the corresponding duration ranges (1-3y, 3-5y, 5-7y, 7-10y and 10+y). Insurance companies would be expected to match to some extent the maturity range of their liabilities with assets of similar maturity rather than with a mix of diversified maturities – even if the average duration of assets and liabilities matches. This would protect the insurance company against non-parallel shifts in the interest rates curve, convexity mismatch, reinvestment risk or liquidity issues. For instance, if a company invests in a combination of the 1-3y and 10+y benchmarks to finance their 5-7y liabilities matching the durations of the two sides of the balance sheet, it remains subject to a change of the shape of the interest rate curve by which the rates of the 1-3y and 10+y benchmarks rise while the 5-7y rate decrease. The portfolio optimization with standard deviation constraints takes indirectly into account some amount of maturity matching via the correlation matrix – the performance of the 5-7y benchmark is not perfectly correlated to those of the 1-3y and the 10y ones. However, the interest rate shock used by the Solvency II standard model is a parallel shift of +100 basis points on all points of the risk-free curve, and as such do not incentivize any kind of cash flow matching. This difference is clearly reflected in the portfolio optimizations: if we take for instance the portfolios with 10% of expected returns for liabilities of 3-5y, the portfolio satisfying the minimum variance allocates a total of 63.7% to sovereign and corporate bonds in the 3-5y range while the portfolio with a minimum SCR avoids completely this range and chooses to allocate more than 99% of the portfolio in a mix of 7-10y sovereign bonds and 1-3y corporate bonds. This example is representative of most portfolios of the efficient frontiers and applies to the other liabilities profiles.

The simplified view of the Solvency II standard model on interest rate movements, along with the higher capital efficiency of short-term spread durations that we commented in the previous chapter, opens the path to a possible strategy for the fixed-income part of the asset allocation of an insurance undertaking that seeks to minimize its SCR for a given expected return. Instead of using a strategic asset allocation with similar duration constraints for corporate and sovereign bonds to hedge the liabilities, the investment manager can split the exposure of the insurance company between spread risk and interest rate risk to maximize the returns of the portfolio: they will invest the corporate bond allocation in short-term instruments that are more capital-efficient, and the remaining part in long-term sovereign debt to close the duration gap with the liabilities.

The split between the allocation to sovereign and corporate bonds may seem surprising as the share of sovereign bonds is higher in the standard deviation optimization than in the SCR optimization for the same level of expected returns across all duration ranges of liabilities. The SCR would be expected to favor sovereign bonds as those assets are not subject to spread risk in Solvency II standard model. However, we must also consider another important assumption of Solvency II that is left unchanged in our optimization solving: the discount rate of the liabilities. The fact that liabilities are discounted at a risk-free curve – approximated by an AA-rated sovereign benchmark in our case – instead of at a curve that considers the actual yield of the portfolio makes it almost compulsory for the insurance undertaking to match it at least partially with an asset that will behave in the same way as the liabilities to lower the overall risk as measured by the standard deviation of the portfolio. The allocation of corporate bonds would be certainly higher if liabilities could be discounted at a rate incorporating a certain level of spread risk. Thus, the incentive to tilt asset allocations towards the sovereign sector does not only come from the risk-free treatment of such bonds in the SCR as already commented in the literature (Braun et al., 2017, 2018; Escobar et al., 2019; Heinrich & Wurstbauer, 2018; Kouwenberg, 2018) but also depends on the discount rate chosen for the liabilities.

6. The horizon of Solvency II standard model

In the previous section we have considered the investment horizon of the insurance company as an input variable of the optimization problem with standard deviation constraints and the Solvency II standard model. However, these two models share a common pitfall in that they are calibrated at a one-year horizon regardless of the investment horizon of the insurance undertaking. The Solvency II standard model represents the Value-at-Risk of the entity for a 99,5% occurrence probability at a one-year horizon, and the standard deviation of the returns of the portfolio also represents a yearly measure. (Estrada, 2014) explains that stocks, that are usually considered as a risky asset class based on the standard deviation of their annual returns, are actually less risky than bonds if we consider long-term periods of 10, 20 or 30 years. The study, based on a sample covering 19 countries and 110 years of data, shows that the terminal wealth of an equity portfolio will be higher than the one of a bond portfolio for all the percentiles and time periods of the terminal distribution that are analyzed in the work, so that the 99% VaR calibrated over a 10-year horizon is smaller for stocks than for bonds.

The concept of time diversification described for instance in (Kritzman, 2015) highlights the idea that over long time periods, above-average returns will compensate for below-average returns of stocks. The ratio of the annualized volatility of returns over the expected mean decreases continuously when the time horizon increases. Furthermore, the two yearly risk measures presented so far in our thesis do not consider a well-known phenomenon that is the mean-reverting behavior of most financial instruments. (Lu et al., 2018) verify the existence of mean reversion in major US equity indices. This can be easily extrapolated to assets bearing spread risk, as in the absence of default the non-fundamental spread of the bonds will converge towards zero at maturity. This is paramount for insurance companies with long-dated liabilities, as they might have the ability to weather a short-term stress in their own funds as long as they do not have to realize losses in that time frame and can focus on the date when their liabilities come due. The mean reversion behavior of the returns of the portfolio would make it so that for longer time frames, the probability of default derived by a naïve extension of the SCR would be higher than the actual probability of default, as noted by (Loois, 2015). Moreover, the existence of liabilities that require a minimum expected return on the assets side enhance the need of assuming long-term rewarded risks, as explained in (Hoevenaars et al., 2008).

6.1. Long-Term Guarantees measures

Solvency II Delegated Acts does consider under some circumstances certain adjustments in the standard model known as LTG – Long-Term Guarantee – measures to account for assets held for the long term.

Some examples are:

- Article 77b give the possibility to apply a matching adjustment in the calculation of the technical provisions, so that they are discounted at a yield that corresponds to the yield given by the assets adjusted for default risk. This immunizes the portfolio against spread shock, as any shock in the assets is absorbed in the liabilities as well. This rule comes with heavy requirements such as the necessity to treat each portfolio applying the matching adjustment as a separate entity when calculating own funds and SCR, so that the companies lose some diversification benefits in the calculation of the capital requirements.

- The volatility adjustment of the risk-free rate discount of Solvency II technical provisions implicitly acknowledges that liabilities are supported by risk-bearing assets. However, only a small part of the non-fundamental spread is applied in the discount of the liabilities, and the volatility adjustment is applicable by all companies regardless of the composition of their portfolio.

- Assets acknowledged as long-term equity holdings can benefit from a reduced capital charge of 22% in the equity risk submodule. The conditions to apply this rule are nonetheless very restrictive, as the average duration of the undertaking's liabilities should exceed 12 years and the decision to apply the reduced charge is subject to supervisory approval, so that only one undertaking was using this rule as of 2019.

Therefore, those adjustments do consider the time horizon of the undertakings but most of them come with impractical requirements. There might be some changes in the future to lessen these requirements following EIOPA's Opinion on the 2020 review of Solvency II, but the recommendations are still to be translated into law. In any case, the theoretical grounds of the one-year VaR will not change to allow variations in the time horizon considered for different undertakings under the current regulation.

6.2. The time horizon as an input variable

The answer to the problem could be to adapt the SCR framework to consider the time horizon of the undertaking. Following the ideas of (Loois, 2015) and (Devolder, 2018), we could propose an alternative measure of risk that distinguishes the concepts of probability of ruin, understood as the probability of the best estimate of the liabilities of the company becoming bigger than the market value of the assets as laid out in the current Solvency II framework, and probability of default, defined as the incapacity of the insurance company to pay a policy benefit to their policyholders in due time. The focus of the alternative risk measure on the probability of default should come with extra requirements regarding the illiquidity of the liabilities, to make sure that anticipated payments to policyholders do not occur before the intended maturity or are sufficiently penalized so that they do not exceed the market value of the corresponding assets. This measure exists for instance below the form of a long-term continuity test at a 15-year horizon for Pension Funds in the Netherlands.

(Campbell & Viceira, 2005) theorize a concept of term structure of the risk-return trade-off based on the annualized standard deviations of returns of different asset classes over different time horizons. The equity risk decreases monotonously when the horizon increases, but the risk of short-term bonds increases as they bear more reinvestment risk. Bonds with higher durations will incur less risk when their horizon matches their maturity. We could propose then an enhanced framework for Solvency II, in which the shocks applied to the different asset classes are not fixed but depend on the maturity of the liabilities, under certain conditions of eligibility. The calculated shock for each market risk i represented in Solvency II model would take the following form:

$$\mathrm{SCR_i} = \mathrm{f(D_L)} * \mathrm{Stress_i} * \mathrm{MV_i} \tag{3.7}$$

where $\mathrm{D_L}$ represent the duration of the liabilities of the portfolio and f is a function representing the term structure of the corresponding risk.

This would give the opportunity to insurance companies not only to lower their short-time measured risk but also to minimize their probability of default at maturity. We must keep in mind that financial risks come with an implicit term structure in which it can be riskier to hold short-dated assets for long-dated commitments, as there is more expected volatility in the consecution of the objective. An asset allocation tilted towards short-term instruments would be subject to reinvestment risk at maturity of the assets when the allocation in long-term instruments would not. The longer-dated instruments would probably give also higher expected returns to maximize the probability of covering the full amount of liabilities when they mature. This concern is even more relevant when risk-free interest rates are lower than the rate guaranteed to policyholders.

7. Conclusion

The comparison between the efficient frontier of portfolios solving a return-standard deviation optimization and the efficient frontier of portfolios satisfying an optimization of Solvency II capital requirements gives different results according to the duration of liabilities. The two risk measures are generally consistent with each other, and both optimizations lead to moderate duration gaps. Nonetheless, for mid- to long-term liabilities, the insurance company will choose to split its strategy between a short-term exposure to spread risk and a long-term exposure in sovereign bonds to manage interest rate risk.

However, both measures of risk should be adjusted to consider the time horizon of the company via parameters that depend on the term structure of risk. Indeed, Solvency II already contemplates some specific adjustments for long-term holdings but those are insufficient. We propose an alternative way of calculating the capital requirements expressed as a function of the duration of the liabilities of the undertaking, applicable under certain requirements of illiquidity.

The one-year calibration of Solvency II Value-at-Risk has more consequences than potential changes in the asset allocation of the insurers. As the companies cannot easily invest in riskier assets suitable for long-term commitments, they will have to change their product offering to unit-linked products where the policyholder retains all the investment risk. While this is a desirable outcome for the companies that can lower the level of market risk, this could also mark a turning point for the mission the industry and its service to clients. Not only will the policyholders depend increasingly on the returns of their own investments, but they will lack protection against a fundamental risk that is currently offered in products with long-term guarantees – such as savings products, annuities, etc. The insurance companies are in those cases supposed to assume a counter-cyclical role for the policyholders, are the former can cancel out periods of lower returns with period of higher returns thanks to their going concern and time diversification.

The short-term calibration of the Standard Model could also have unsought secondary effects on the capacity of insurers to invest in sustainable assets characterized by their long-term horizon. This would enter in contradiction with other objectives of EIOPA, as they have highlighted in several opinions and reports the importance of considering ESG factors in insurers' and pension funds' investment strategies and asset-liability management and try to promote green investments and green bonds. (EIOPA, 2019) aims to integrate ESG factors into the risk management frameworks of insurance companies and explores ways to consider the impact of sustainability risks into the Solvency II framework. However, the practical path chosen by the Regulator focuses on Pillars II and III of the Solvency II framework, enhancing primarily governance and reporting aspects of the insurance industry. The capital requirements of Pillar I would not be directly affected on the investment side, as EIOPA seems to study mainly the calibration of catastrophe risk of the underwriting modules. It will be interesting to see if the approach of the Regulator focused on Pillars II and III is sufficient to cover the deficiencies of the Solvency II Standard Model.

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General conclusions

Through the analysis conducted in this thesis, we have examined the impact of Solvency II on asset allocation decisions in the insurance industry. Our findings reveal that the optimization of portfolios based on SCR constraints results in asset allocations that are highly sensitive to input values, particularly expected returns. In a low-yield environment, investments in sovereign bonds are promoted, while corporate bonds are relatively underallocated. However, as yields increase, corporate bonds tend to receive a larger allocation, while real estate and equity allocations diminish. In both cases, the riskier portfolios of the efficient frontier derived from an optimization with SCR constraints tend to favour the most efficient asset of the three riskiest asset classes, while the optimization with standard deviation constraints promote a more diversified allocation.

Furthermore, our examination of fixed-income allocations and spread risk demonstrates some biases introduced by Solvency II. The regulation incentivizes investment in government bonds, particularly riskier ones, as they incur no additional capital charges. The treatment of different ratings under Solvency II encourages slightly more conservative asset allocations, favouring A-rated benchmarks over BBB-rated benchmarks. The analysis of duration benchmarks reveals that corporate bonds' capital efficiency is highest in the lower maturity ranges, regardless of their rating.

Integrating the liability profile of insurance companies provides a more comprehensive understanding of the effects of Solvency II on asset allocation. For insurers facing long-term liabilities, an optimal strategy could be to get a short-term exposure to spread risk and a separate, long-term exposure in sovereign bonds to manage interest rate risk, rather than trying to match the liabilities with assets of similar maturities. The limitations of the standard formula in addressing longer-term liabilities even with LTG measures highlight the importance of considering risk appetite on an extended time frame. Our proposed alternative approach to calculating capital requirements considers the duration of liabilities as input variable and could offer a valuable perspective for insurers managing long-term holdings.

Overall, the research conducted in this thesis underscores the relevance of Solvency II on investment decisions strategies within the insurance industry. The beneficial aspects of the regulation on the risk management of insurance companies are indisputable, as it has shaped risk-based constraints into a legal framework for all undertakings. Nonetheless, we detect in our analysis some biases that could negatively impact the investment decisions of insurance companies, if we consider the under- or over-estimation of some risks. For instance, the treatment of sovereign bonds as risk-free assets may not accurately reflect the underlying credit risk associated with certain government debt. This was particularly evident during the Eurozone debt crisis, where some EU countries faced significant financial difficulties and had their credit ratings downgraded. Treating all sovereign bonds as risk-free may give a false sense of security and fail to account for the potential default or downgrade risk, while strengthening the interdependence of the financial and sovereign sectors and, therefore, increasing systemic risks in an increasingly globalized world.

The internal models can correct these biases when they are used by insurance companies. Some general characteristics of these internal models are made public in the annual Solvency and Financial Condition Reports (SFCR) of the companies and studied at an aggregated level by (EIOPA, 2023). For instance, the internal models explained in the SFCR of 2022 of the three biggest insurance groups regulated by Solvency II – Allianz Group, Axa Group and Generali Group – consider that sovereign bonds are not exempted form spread risk capital charge to the difference of the Standard Model. Some insurance groups like Allianz and Axa mitigate the spread risk of their corporate bonds using a dynamic volatility adjustment that let them discount their liabilities according to the non-fundamental spread of their bond portfolio after applying the spread or credit shock, so that their liabilities behave more like an average of a corporate and sovereign benchmarks as we suggest in the section 5 of the third chapter of this doctoral thesis. However, these internal models remain bound by the general principles of Solvency II as they have to calibrate a 99.5% VaR calculated over a one-year time horizon, so that they cannot consider variations in the capital charges that depend on the duration of their liabilities as proposed in the section 6 of our third chapter. One must also consider that while they can be useful to get a better insight on the actual risk profile of an insurance undertaking, the calculations of internal models also imply a higher level of complexity and lessen the comparability of the SCR between different entities.

As for the issue of the time horizon, it is probably one of the most critical topics of the review of Solvency II as the insurance sector is probably the most suitable to invest in long-term assets, such as infrastructure projects or green investments that are necessary at a global scale to mitigate emerging risks derived from climate change, such as extreme weather events or natural disaster. Adapting the time horizon of the regulation could, then, let companies invest more in green projects, creating a positive feedback loop by reducing environmental risks.