Three Essays on Optimal Policy

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Three Essays on Optimal Policy

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To my parents

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Abstract

This thesis studies optimal monetary monetary and fiscal policy a government should implement under different scenarios.

In the first chapter, I study the optimal discretionary monetary policy under partial information (PI) where the central bank can only extract information from an endogenous signal, price inflation. The signal is determined in equilibrium by the policy rate and the unobserved supply and demand shocks. I solve for optimal policy in a non-linear model where the Phillips curve is bent by asymmetric wage adjustment costs and the “certainty equivalence" principal that prevails in linear models cannot be applied. Optimal policy prescribes that the central bank should raise the interest rate gradually when price inflation is low but respond strongly when it is high. This non-linearity arises because signal extraction interacts differently with optimal policy depending on the price inflation observed.

In the second chapter, I study the optimal fiscal policy in a model with two types of agents who are different in their access to the financial markets: Ricardian agents have full access to the financial markets while the hand-to-mouth agents are constrained and could only consume their labor income in each period. I find that the optimal labor-tax is more volatile compared with a representative-agent economy without physical capital and the volatility is captured by the equilibrium condition that these two types of agents are imposed with the same proportional labor tax. When capital is introduced to this economy, we find that in the long run capital tax should still be zero in the deterministic case. But the ex-ante capital tax in the stochastic economy is again disturbed by the same proportional labor tax condition, which makes it fluctuate around zero instead of staying there.

In the third chapter, I study the optimal debt issuance policy of a small open economy. To circumvent curse of dimensionality, long-term bonds are usually modeled as a perpetuity contract with coupon payments that decay geometrically in macroeconomic models. I argue that this simplification actually exacerbates the devaluation of the long bonds in the maturity analysis of sovereign debt. First, the front-loaded payment structure of the geometric bond is mismatched with the persistent income process when bad shocks hit; Second, the perpetual payment structure extends the scope for diluting the value of current issuance and this effect is priced in when the investors are rational. I find that zero coupon bond, which is a much closer instrument to the real world sovereign debt, could mitigate the problems and reduce the default probability.

Abstracte Esta tesis estudia la política monetaria monetaria y fiscal óptima que un gobierno debería implementar bajo diferentes escenarios. En el primer capítulo, estudio la política monetaria discrecional óptima bajo información parcial (PI) donde el banco central solo puede extraer información de una señal endógena, específicamente, la inflación de precios. La señal se determina en equilibrio por la tasa de interes y los choques de oferta y demanda no observados. Hago el calculo de la política óptima en un modelo no lineal donde la curva de Phillips obtiene su curvatura por costos de ajuste salarial asimétricos y no se puede aplicar el principio de "equivalencia de certeza" que prevalece en los modelos lineales. La política óptima prescribe que el banco central debe aumentar la tasa de interés gradualmente cuando la inflación de precios es baja, pero responde fuertemente cuando es alta. Esta no linealidad surge porque la extracción de señales interactúa de manera diferente con la política óptima dependiendo de la inflación observada.

En el segundo capítulo, estudio la política fiscal óptima en un modelo con dos tipos de agentes que son diferentes en su acceso a los mercados financieros: los agentes ricardianos tienen pleno acceso a los mercados financieros mientras que los agentes 'hand to mouth' están restringidos y sólo pueden consumir sus rentas laborales en cada período. Encuentro que el impuesto al trabajo óptimo es más volátil en comparación con una economía de agente representativo sin capital físico y que la volatilidad es capturada por la condicion de equilibrio que a estos dos tipos de agentes se les imponga la misma tasa proporcional del impuesto al trabajo. Cuando se introduce capital en esta economía, encontramos que, a largo plazo, el impuesto sobre el capital debería ser igual a cero en el caso deterministico. Pero el impuesto al capital ex-ante en una economía estocástica se ve nuevamente perturbado por la misma condición proporcional del impuesto al trabajo, que lo hace fluctuar alrededor de cero en lugar de permanecer allí.

En el tercer capítulo, estudio la política óptima de emisión de deuda de una pequeña economía abierta. Para eludir la maldición de dimensionalides, los bonos a largo plazo generalmente se modelan como un contrato a perpetuidad con pagos de cupones que decaen geométricamente en los modelos macroeconómicos. Argumento que esta simplificación en realidad exacerba la devaluación de los bonos largos en el análisis de vencimiento de la deuda soberana. En primer lugar, la estructura de pago

anticipada del bono geométrico no coincide con el proceso de ingreso persistente cuando se producen perturbaciones negativas; en segundo lugar, la estructura de pagos perpetuos amplía el margen para diluir el valor de la emisión actual y este efecto se valora cuando los inversores son racionales. Encuentro que el bono de cupón cero, que es un instrumento mucho más cercano a la deuda soberana del mundo real, podría mitigar los problemas y reducir la probabilidad de incumplimiento.

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Abbreviations

FI	**F**ull **I**nformation
MPC	**M**arginal **P**ropensity to **C**onsume
PI	**P**artial **I**nformation

Chapter 1

Introduction

The thesis focuses on how macro policy could maximize to households' welfare. To achieve this goal, I try to quantify the trade-offs faced by government in theoretical models and solve for the optimal monetary and fiscal policy under different scenarios. This thesis consists of 4 chapters, including the present chapter (Chapter 1) of introduction to the research topic.

Chapter 2 is motivated by a practical issue in policy circle: macro policies have to be made without knowing the state of economy in real time and policymakers have to infer the underlying states from some observable signals or indicators. The difficulties arise because the observed signals are, in general, endogenous to policy decisions and depend on multiple shocks hitting the economy. For example, the central bank may observe growing inflation but cannot be sure it is pushed by demand or supply shock; Meanwhile it needs to react to it by changing the nominal rate and the policy rate will in turn, affect the inflation observed. Therefore, the policy optimization and the signal extraction problem need to be solved simultaneously. This simultaneity problem is only addressed in linear model before and the method developed in the literature cannot be applied when the model features important nonlinear relationships, such as zero lower bound, financial frictions. I explore the optimal monetary policy under partial information when the Phillips curve is bent by asymmetric wage adjustment costs. I find that the central bank should raise interest rate gradually when price inflation is low but respond strongly when it is high, which is like Fed's reaction to inflation in post COVID times. I argue that the strength of monetary policy is determined by 2 factors: the uncertainty the central bank faces and the effectiveness of the monetary policy generated by nominal rigidity, both of which are changing along the Phillips curve.

Chapter 3 revisits a classical topic in fiscal policy: how should a government finance its expenditures, debt, or tax? Ramsey optimal tax theory prescribes that taxes on labor income should be smoothed and government should issue bonds to buffer the shocks while long- run capital tax should be set to zero. But this result assumes forward-looking representative agent, who could adjust their consumption and labor supply based on both tax and interest rates. However the strong response of aggregate consumption to interest rate changes is questionable in light of empirical evidence. The inclusion of hand-to-mouth agents, who have no access to the financial markets and cannot smooth their consumption, could explain the aggregate data better. In this chapter I explore the optimal fiscal policy when these two types of agents coexist in the economy. I find that the optimal labor-tax is more volatile compared with a representative agent economy without physical capital and the volatility is captured by the equilibrium condition that these two types of agents are faced with the same proportional labor tax. The long-run capital tax is again disturbed by the same proportional labor tax constraint, which makes it fluctuate around zero instead of staying there.

Chapter 4 focuses on the debt issuances policy in a small open economy where default could happen. I question the conventional way the literature models the long bond, as a perpetuity contract with coupon payments that decay geometrically at a constant rate. Although the curse of dimensionality could be circumvented by this simplification, it exacerbates the financial condition of the sovereign. First, the front-loaded payment structure of the geometric bond is mismatched with the back-loaded income process when bad shocks hit; Second, the perpetual payment structure makes the claims to the new debt overlap with the old debt such that new issuances “dilute” the old debt directly when the government cannot commit to its future policy. This lack of commitment is effectively penalized by rational investors, which makes the geometric bond price depreciate more than the short bond. I employ a dynamic model to show how geometric bond affects the issuance strategy and welfare of the sovereign compared with ordinary bonds in financial markets. The model indicates that modelling long bonds as geometric bond is not sufficient to analyze the maturity structure of sovereign debt.

Chapter 2

Optimal Monetary Policy with Signal Extraction

2.1 Introduction

In the recent studies of monetary policy, simple policy rules such as Taylor rules could achieve good results in simulated small macroeconomic models. At the same time, many empirical studies report that the policy specifications of this kind fit the actual behavior of the central banks in several countries.

But the central banks actually face much more complicated situation compared with the set-ups in the model. In particular, they do not know the underlying state of the economy in real time but only infer it from limited data set. This poses a challenge to the central bank, the information constraint. This challenge is twofold: how easy it is to get the relevant data and how accurate the indicators could reflect the fundamentals of the economy. For example, the central bank can only have very preliminary measure of current period GDP and update it at least one period later. Another key indicator for monetary policy, wage inflation, is difficult to observe because it is kept within employees and firms while the central bank can only rely on the survey data which is subject to large measurement error. Even though some indicators are much easier to get with accuracy, the central bank may still face great uncertainty when making monetary policy. A typical example is price inflation, which can be driven by a high demand shock or by a supply shortage or a combination of both. Different causes needs to be treated differently. The haziness would not disappear even though the central bank knows the price inflation perfectly. So the typical presumption in the macroeconomic models that policymakers know the state of the system at a point in time generally does not hold in policy field, policy decisions need to be made under partial information (PI) instead of full information (FI).

Apart from information barriers mentioned above, the central bank also needs to take into account that the signals it observes are endogenous to the policy rule. Indicators like inflation, output are determined not only by the fundamentals in the economy, but also by the monetary policy the central bank adopts. So the policy optimization and the signal extraction need to be solved simultaneously.

To our knowledge, all existing results in the literature on optimal monetary policy with PI circumvent this technical issue of simultaneity by introducing timing assumptions such that signal extraction and policy optimization can be solved separately. While this literature has led to many interesting applications, it can say nothing about how policy optimization and signal extraction interact with each other. What is emphasized in the literature is policy optimization under PI or uncertainty but what is missing is how policy choice affects the uncertainty the central bank faces. Svensson and Woodford (2003) and Svensson and Woodford (2004) address this simultaneity problem in linear model and develop a method called "certainty equivalence", which allow them to solve the signal extraction and optimal choice problems sequentially. But their method cannot be applied to many important non-linear scenarios in monetary policy analysis, such as zero lower bound, models with financial frictions etc.

In this paper, we focus on the non-linearity of new Keynessian Phillips curve, the substantial curvature in the relationship between money wage growth and unemployment. Phillips (1958) conjectures that this curvature owed to the fact that "... workers are reluctant to offer their services at less than prevailing rates when the demand for labour is low and unemployment is high so that wage rates fall only very slowly." As is supported empirically and modelled theoretically in the literature, We study the optimal discretionary monetary policy with signal extraction in such an economy where the Phillips curve is bent by the asymmetric wage adjustment cost. To the best of our knowledge, we are the first to study the optimal monetary policy with signal extraction in non-linear models.

To make the model more transparent and tractable, we assume the only nominal rigidity is asymmetric wage adjustment cost. When the central bank has full information on the economy, the optimal policy calls for strict wage inflation targeting and full stabilization of demand shocks. We introduce the information constraint through an identification problem: every period the economy is hit by two exogenous shocks, the supply and demand shocks, but the central bank can only infer the state of the economy from one single indicator, price inflation. We find that the responding rule of nominal rate to price inflation is quite non-linear: the central bank should raise the nominal rate gradually when price inflation is low but raise it sharply when price inflation is high. We argue that this non-linearity arises because the signal extraction problem interacts differently with optimal monetary policy depending the range of price inflation. In particular, the sluggish adjustment around the low realization of price inflation is justified by the strong real effects of monetary policy as small change of policy rate can make great influence on output. The inertial behaviour in the intermediate range of price inflation can be seen as policy cautiousness as the central bank faces more uncertainty in this regime. The strong response to high levels of price inflation is a choice under less uncertainty and the fading real effects of monetary policy.

Our model provides a plausible explanation of the Fed's monetary policy after COVID-19. When the inflation rate is below 3 per cent, the Fed was not sure about the cause of inflation, a supply shock or a demand shock. So it should move cautiously in such haziness. On top of that, the Fed also believes that the monetary policy is very powerful in inflation controlling as the Phillips curve is flat in this interval. But after January 2022, the inflation rate grew even higher, the Fed is more certain about the true cause behind it, the demand shock, so it raised the interest rates without hesitation, considering also the steep Phillips curve in this interval.

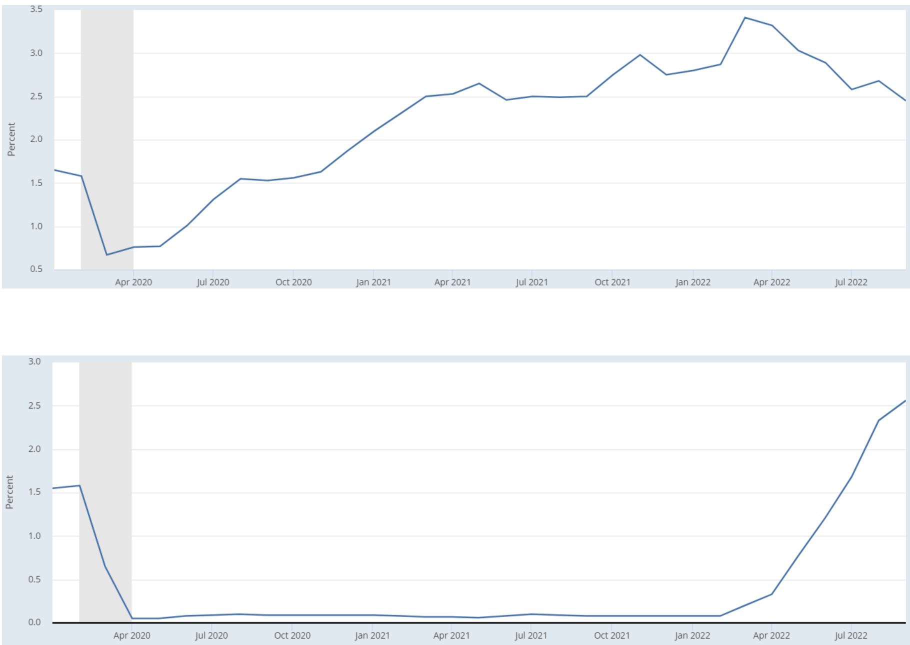


Figure 2.1: U.S. Inflation and Policy Rates

To highlight our contribution relative to the literature, we compare our results with some alternative policy choices and show that the endogeneity of the signal and the non-linearity of the wage Phillips curve bent by asymmetric wage adjustment cost are vital in monetary policy making and ignoring that could lead to great welfare loss.

The remainder of the paper is organized as follows. We discuss the related literature in Section 2.2. Section 2.3 introduces our main model and the solution under full information. In Section 2.4 we present the result and interpretation of the optimal monetary policy under partial information. Section 2.7 compares the optimal policy with some alternative policy rules and their welfare implications. For completeness, we present the solution for the case of serial correlated shocks in Section 2.8. Section 2.9 concludes.

2.2 Literature Review

Optimal monetary policy with signal extraction is often considered in linear models: Svensson and Woodford (2004) shows that in the case of a linear economic model with a quadratic welfare loss function, a principle of "certainty equivalence" applies: the government applies the policy under full information to its best estimate of the state of the economy. Aoki (2003) applies their results to optimal monetary policy with noisy indicators on output and inflation. Nimark (2008) applies them to a problem of monetary policy where the central bank uses data from the yield curve knowing that the chosen policy affects the very same data. Relatedly, Morris and Shin (2018) analyze the optimal weight on an endogenous signal in a linear policy rule.

But "certainty equivalence" cannot be applied to study the optimal monetary policy under partial information when the economy features some important non-linear relationships, a typical example of which is the substantial curvature in the relationship between money wage growth and unemployment. As documented by Phillips (1958), the curve is nearly vertical at high inflation and flattens out at low inflation, implying progressively larger output costs of reducing inflation. He conjectures that this curvature owed to the fact that "... workers are reluctant to offer their services at less than prevailing rates when the demand for Labour is low and unemployment is high so that wage rates fall only very slowly." The empirical evidence of downward nominal wage rigidity is provided in Akerlof et al. (1996) and Daly and Hobijn (2014). Kim and Ruge-Murcia (2009) and Benigno and Antonio Ricci (2011) study the optimal monetary policy in a dynamic stochastic general equilibrium model and find that the optimal inflation rate is positive. Fahr and Smets (2010) consider both nominal and real downward wage rigidity (DWR) in a monetary union and find that optimal grease inflation may be dampened by heterogeneity in the types of DWR in different regions.

We revisit Phillips' hypothesis that downward nominal wage rigidities bend the Phillips curve and consider how this non-linearity interacts with signal extraction under partial information. To the best of our knowledge, our paper is the first one to consider the optimal monetary policy with signal extraction in a non-linear model. The solution method to our model is based on the work of Hauk et al. (2021), which address the optimal fiscal policy with signal extraction problem from the first principal.

2.3 The structure of the economy

The model developed in this section is a small-scale, dynamic stochastic general equilibrium model with downward nominal wage rigidity.

2.3.1 Firms

Firms operate in a perfectly competitive goods market and produce output using the production function

$$Y_t = A_t N_t \tag{2.1}$$

Here A_t is average labor productivity, which evolves exogenously. The labor aggregate, N_t, is a Dixit and Stiglitz (1977) aggregate over a continuum of labor types $j \in [0, 1]$ and is of the form

$$N_t = \left(\int_0^1 N_t(j)^{1-\frac{1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2.2}$$

where $N_t(j)$ is the quantity of type-j labor employed by the firm in period t. The parameter ϵ represents the elasticity of substitution among labor varieties.

Let $W_t(j)$ denote the nominal wage for type-j labor prevailing in period t, for all $j \in [0, 1]$. As discussed below, nominal wages are set by workers of each type(or a union representing them) and taken as given by firms. Given the wages effective each period for different types of labor services, cost minimization by the firm yields the demand for each type of workers, given the firm's total employment N_t

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon} N_t \tag{2.3}$$

where

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \tag{2.4}$$

is an aggregate wage index. Because firms operate in a perfectly competitive goods market, they set the goods price P_t equal to the marginal production cost,

$$P_t = \frac{W_t}{A_t} \tag{2.5}$$

which yields the relationship between price inflation and wage inflation:

$$\Pi_t^p = \Pi_t^w \frac{A_{t-1}}{A_t} \tag{2.6}$$

where $\Pi_{t+1}^p = \frac{P_{t+1}}{P_t}$ and $\Pi_t^w = \frac{W_{t+1}}{W_t}$ denotes price inflation and wage inflation respectively.

2.3.2 Households

The economy is populated by a large number of identical households. Each household is made up of a continuum of infinitely-lived members specializing in a different labor service and indexed by $j \in [0, 1]$. Income is pooled within each household, which acts as risk sharing mechanism. The representative household chooses its path of consumption $\{C_t\}_{t=0}^{\infty}$ and wages and labor supply $\{W_t(j), N_t(j)\}_{t=0}^{\infty}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \tag{2.7}$$

and the utility function is defined by

$$U(C_t, N_t; Z_t) = Z_t \left(\ln C_t - \frac{\chi}{\eta + 1} \int_0^1 N_t(j)^{\eta+1} dj \right) \tag{2.8}$$

where $\chi \geq 0$ and $\eta \geq 0$ determine the dis-utility of labor supply. $\mathbb{E}_t$ is the expectation operator conditional on information at time t. $\beta \in (0, 1)$ is the discount factor. The preference shock Z_t shifts overall utility level and disturbs the household's intertemporal substitution of consumption. Households maximization problem is subject to a sequence of flow budget constraints, expressed in real terms as

$$C_t + \frac{B_t}{P_t} \leq \int_0^1 \frac{(1+\tau) W_t(j) N_t(j)}{P_t} dj - \int_0^1 \Phi(\frac{W_t(j)}{W_{t-1}(j)}) dj N_t + \frac{1 + i_{t-1}}{P_t} B_{t-1} + T_t \tag{2.9}$$

where B_t represents the quantity of one-period nominal riskless bonds purchased in period t and maturing in period $t+1$. The nominal interest paid during period t on the bonds held at the end of period $t-1$ is i_{t-1}. τ is an employment subsidy financed by means of lump-sum tax T_t that corrects the distortions caused by monopolistic competition in labor markets. τ is set to be equal to $\frac{1}{\epsilon-1}$ so that the marginal rate of substitution between leisure and consumption equals to the real wage under flexible wage setting.

As monopolistic competitors, households choose their wage and labor supply taking as given the firm's demand for their labor type. Labor market frictions induce a cost in the adjustment of nominal wages. We assume the wage adjustment cost takes the form of an altered linex cost function similar to Varian (1974):

$$\Phi(\frac{W_t(j)}{W_{t-1}(j)}) = \frac{\phi-1}{2}(\frac{W_t(j)}{W_{t-1}(j)} - 1)^2 + \frac{\exp(-\psi(\frac{W_t(j)}{W_{t-1}(j)} - 1)) + \psi(\frac{W_t(j)}{W_{t-1}(j)} - 1) - 1}{\psi^2} \tag{2.10}$$

The parameter ϕ determines the degree of convexity and ψ the degree of asymmetry in adjustment costs around zero wage inflation(W_t/W_{t-1}=1). When $\psi > 0$, adjustment costs for nominal wage increases are smaller than those for nominal wage cuts of the same size, capturing asymmetries in nominal wage adjustments. The specification nests a quadratic function, $\lim_{\psi\to 0}\Phi(\frac{W_t(j)}{W_{t-1}(j)}) = \frac{\phi}{2}(\frac{W_t(j)}{W_{t-1}(j)} - 1)^2$. Figure 2.2 gives a visual impression of a symmetric and asymmetric adjustment cost function. To simplify computations, we further assume the labor adjustment cost is proportional to the aggregate employment N_t, instead of $\int_0^1 \Phi(\frac{W_t(j)}{W_{t-1}(j)})N_t(j)dj$. The households utility maximization yields the following optimality conditions:

$$\frac{Z_t}{C_t} = \beta\mathbb{E}_t\frac{Z_{t+1}(1+i_t)}{\Pi^p_{t+1}C_{t+1}} \tag{2.11}$$

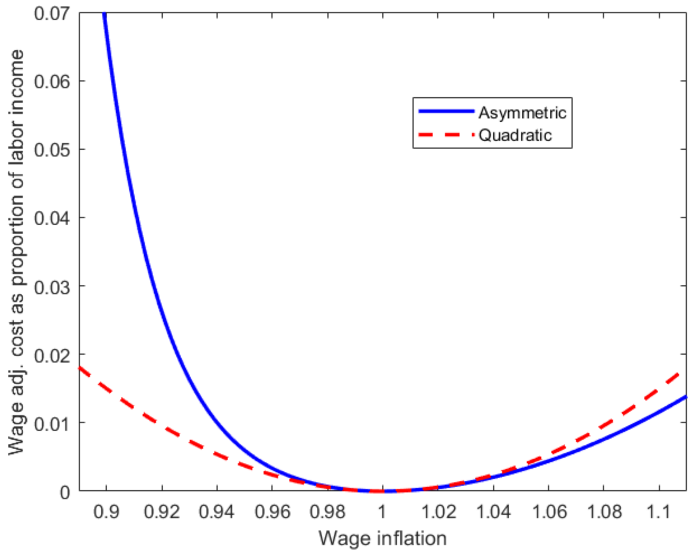


FIGURE 2.2: Adjustment cost functions

$$\frac{\epsilon\chi(\eta+1)Z_tN_t(j)}{W_t(j)}+\frac{(1+\tau)(1-\epsilon)Z_t}{C_t}-\frac{Z_t\Phi'(\frac{W_t(j)}{W_{t-1}(j)})N_t}{C_tW_{t-1}(j)}+\beta\mathbb{E}_t\frac{Z_{t+1}\Phi'(\frac{W_{t+1}(j)}{W_t(j)})N_{t+1}W_{t+1}(j)}{C_{t+1}W_t^2}=0 \tag{2.12}$$

2.3.3 Symmetric Equilibrium

The model incorporates multiplicity of equilibria and we pick up the symmetric case, where all households supply exactly the same amount of labor and demand the same level of nominal wage, i.e. $W_t(j) = W_t$ and $N_t(j) = N_t$. Dropping the index j, the households optimality condition yields

$$\epsilon\chi Z_tN_t^\eta+\frac{(1+\tau)(1-\epsilon)W_tZ_t}{P_tC_t}-\frac{Z_t\Phi'(\Pi_t^w)\Pi_t^w}{C_t}+\beta\mathbb{E}_t\frac{Z_{t+1}\Phi'(\Pi_{t+1}^w)\Pi_{t+1}^wN_{t+1}}{C_{t+1}N_t}=0 \tag{2.13}$$

The economy-wide resource constraint is

$$C_t=(A_t-\Phi(\Pi_t^w))N_t \tag{2.14}$$

Combining them with $P_t = \frac{W_t}{A_t}$, one can get the wage Phillips Curve of the economy:

$$\frac{\epsilon \chi Z_t C_t^2}{[A_t - \Phi(\Pi_t^w)]^2} + \frac{(1+\tau)(1-\epsilon)A_t Z_t}{A_t - \Phi(\Pi_t^w)} - \frac{Z_t \Phi'(\Pi_t^w)\Pi_t^w}{A_t - \Phi(\Pi_t^w)} + \beta \mathbb{E}_t \frac{Z_{t+1}\Phi'(\Pi_{t+1}^w)\Pi_{t+1}^w}{A_{t+1} - \Phi(\Pi_{t+1}^w)} = 0 \tag{2.15}$$

To illustrate how the downward nominal wage rigidity bends the Phillips curve, we plot the Phillips curve with different wage adjustment costs in the same figure. The point of zero inflation $\Pi^w = 1$ marked in Figure 2.3 represents the natural level of output and inflation. One can find that there is substantial curvature in the Phillips curve associated with asymmetric adjustment cost and it would be misleading to log linerize the model.

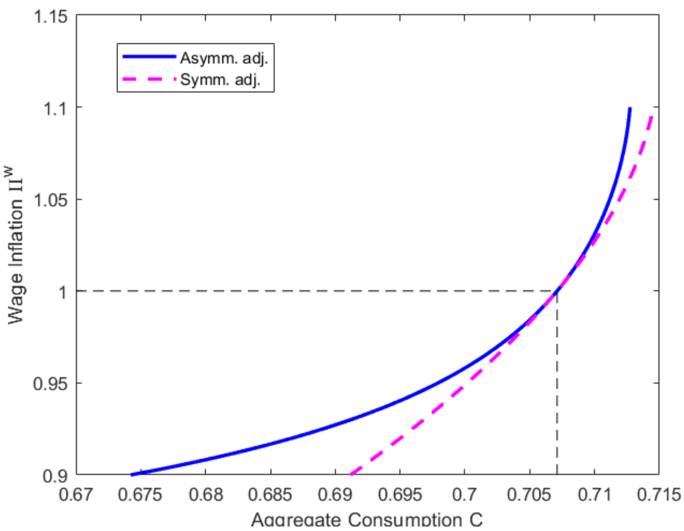


FIGURE 2.3: Wage Phillips Curve

2.3.4 Optimal discretionary monetary policy under full information

We consider the optimal monetary policy under discretion. In this case, the central bank cannot commit itself to any future action. The expectations in the Euler equation and the wage Phillips curve is taken as given by the monetary authority and will become a constant in equilibrium. We denote them as E and F respectively.

Without loss of generality, we set A_{t-1} and Z_{t-1} equal to their steady state value 1. One can write the Euler equation and the wage Phillips curve as:

$$\frac{A_t Z_t}{C_t} = \beta(1+i_t)E \tag{2.16}$$

$$\frac{\epsilon\chi Z_t C_t^2}{[A_t - \Phi(\Pi_t^w)]^2} + \frac{(1+\tau)(1-\epsilon)A_t Z_t}{A_t - \Phi(\Pi_t^w)} - \frac{Z_t \Phi'(\Pi_t^w)\Pi_t^w}{A_t - \Phi(\Pi_t^w)} + \beta F = 0 \tag{2.17}$$

Under this assumption the central bank's problem (2.18) becomes sequential optimization.

$$\max_{\{i_t, C_t, N_t, \Pi_t^w\}} U(C_t, N_t; Z_t) \tag{2.18}$$

$$\text{s.t. (2.14), (2.16), (2.17)}$$

choose the nominal interest rate i_t to maximize the household's utility, subject to the resources constraint, the dynamics IS curve and the wage Phillips curve.

The Lagrangian representation of the Ramsey problem is

$$\begin{aligned}\mathcal{L} =& U(C_t, N_t) + \lambda_t(C_t - (A_t - \Phi(\pi_t^w))N_t) \\ &+ \mu_t(\beta(1+i_t)EC_t - A_t Z_t) \\ &+ \nu_t(\epsilon\chi Z_t C_t^2 + (1+\tau)(1-\epsilon)A_t Z_t(A_t - \Phi(\Pi_t^w)) - Z_t\Phi'(\Pi_t^w)\Pi_t^w(A_t - \Phi(\Pi_t^w)) \\ &+ \beta F(A_t - \Phi(\Pi_t^w))^2)\end{aligned} \tag{2.19}$$

The first order necessary conditions associated with i_t is:

$$\mu_t \beta E C_t = 0 \tag{2.20}$$

and it is evident that $\mu_t = 0$. The intuition is that the central bank can always choose an interest rate level consistent with the Euler equation, in the absence of zero lower bound. The F.O.Cs associated with C_t, Π_t, N_t are:

$$[C_t] : U_C + \lambda_t + 2\nu_t \epsilon \chi Z_t C_t = 0 \tag{2.21}$$

$$[N_t] : U_N - \lambda_t \Phi(\Pi_t^w) = 0 \tag{2.22}$$

$$\begin{aligned}[\Pi_t^w] : &- \lambda_t N_t \Phi'(\Pi_t^w) - \nu_t[(1+\tau)(1-\epsilon)A_t Z_t \Phi'(\Pi_t^w) \\ &- Z_t \Phi'(\Pi_t^w)(1+\Pi_t^w)(A_t - \Phi(\Pi_t^w)) + Z_t \Pi_t^w \Phi'(\Pi_t^w)^2 - \beta F Z_t \Phi'(\Pi_t^w)(A_t - \Phi(\Pi_t^w))] = 0\end{aligned} \tag{2.23}$$

These three F.O.Cs together with the resources constraint and the Phillips curve characterize the solution $C_t, N_t, \Pi_t^w, \nu_t, \lambda_t$ and the nominal rate can be solved from the Euler Equation. When the distortions caused by monopolistic competition is corrected through an employment subsidy, one cannot do better than to set $\Pi_t^w = 1$ and $i_t = \frac{Z_t}{\beta} - 1$ so that there are no losses to wage adjustment and the economy achieves its first best.

2.4 Optimal discretionary policy with signal extraction

The implementation of the optimal monetary policy requires the central bank to have accurate measure of demand shock or the wage inflation. But the former one cannot be observed directly from the data and the latter one can only rely on survey data, which is subject to large measurement error. Prices are public data and the central bank have much easier access to it. So a natural problem is to explore the optimal monetary policy contingent on price inflation. Now we assume that the only signal the central bank could observe is the price inflation and the monetary policy is a feedback rule which vary interest rates responding to the observed signal, the inflation.

2.4.1 Information structure and the timing

We first consider the case where the supply and demand shocks are i.i.d and uniformly distributed on a support $[A_{\min}, A_{\max}]$ and $[Z_{\min}, Z_{\max}]$ respectively. Suppose the central bank observes at time t the price level P_t, simultaneously with the choice of nominal interest rate i_t. The output Y_t cannot be observed in the current period but will be revealed next period. This implies the information set I_t of the central bank at time t is given by

$$I_t = \{P_t, i_t, Y_{t-1}\} \cup I_{t-1} \tag{2.24}$$

where $I_{t-1} = \{P_{t-1}, i_{t-1}, Y_{t-2}, P_{t-2}, i_{t-2}, Y_{t-3} \cdots\}$ and $I_0 = \{P_0, i_0, Y_{-1}, P_{-1}, i_{-1}\}$.

Given this information set, the central bank could identify the supply and demand shocks A_{t-1}, Z_{t-1} of the previous period, In order to focus our analysis on the implications of policy making with partial information, we abstract from any information constraint faced by private agents. They are assumed to have complete knowledge about the states of the economy, including the realization of supply and demand shocks, consumption and wage inflation. The justifications for this assumption are twofold, as pointed out by Aoki (2003): on the one hand, consumption and wages are the choice variable of the private agents, which is based on the agents' own preference and the firm's production capacity. On the other hand, consumption and production decisions of private sectors are not as dependent on the availability of aggregate data as is the policy decision of the central bank.

To be precise, we assume the following time sequence. At the beginning of time t, the output of the previous periods Y_{t-1} is revealed. Combining this with the information in I_{t-1}, the central bank is able to identify the true value of past supply and demand shocks A_{t-1} and Z_{t-1}. Then the central bank announces its policy rule

$$i_t = \mathcal{R}(\Pi_t^p) \tag{2.25}$$

according to a policy function $\mathcal{R} : \mathcal{P} \to \mathcal{T}$. $\mathcal{P}$ and $\mathcal{T}$ denote the set of possible policy rate and the price inflation observed, i.e. $\Pi_t \in \mathcal{P}$ and $i_t \in \mathcal{T}$. Once the supply and demand shock A_t, Z_t are realized, the economy arrivesits equilibrium.

Now the central bank cannot observe the fundamental shocks, A_t and Z_t in real time, but only infer the underlying state of the economy from the endogenous signal Π_t^P, taking into account that the policy variable i_t maps into the endogenous signal Π_t^P through the following reaction function

$$\Pi_t^p = h(i_t, A_t, Z_t) \tag{2.26}$$

which is defined as an implicit function from Euler equation (2.16) and wage Phillips curve (2.17):

$$\frac{A_t Z_t}{\beta(1+i_t)E} = \sqrt{\frac{A_t(A_t - \Phi(A_t\Pi_t^p))(\Phi'(A_t\Pi_t^p)\Pi_t^p - (1+\tau)(1-\epsilon))}{\epsilon\chi} - \frac{\beta F(A_t - \Phi(A_t\Pi_t^p))^2}{\epsilon\chi Z_t}} \tag{2.27}$$

The utility function can be expressed in terms of policy variables, endogenous signals and fundamental shocks as:

$$\mathcal{U}(i_t, \Pi_t^p, A_t, Z_t) = U\left(\frac{A_t Z_t}{\beta(1+i_t)E}, \frac{A_t Z_t}{\beta(1+i_t)(A_t - \Phi(A_t\Pi_t^p))E}; Z_t\right) \tag{2.28}$$

The central bank's optimization problem can be stated formally as:

$$\max_{\mathcal{R}:\mathcal{P}\to\mathcal{T}} \mathbb{E}[\mathcal{U}(i_t, \Pi_t^p, A_t, Z_t)] \tag{2.29}$$

s.t. (2.25), (2.26)

Notice that, mathematically, the only difference with the FI-problem (2.18) is the presence of constraint (2.25), which requires monetary policy to be a feedback rules which vary interest rate responding to price inflation Π_t^p.

2.4.2 Solution of optimal monetary under discretion

We denote the solution to the central bank's optimization problem (2.41) as $\mathcal{R}^*$. According to Hauk et al. (2021), $\mathcal{R}^*$ satisfies the following necessary optimality condition:

$$\int_{\mathbf{A}(\Pi^p,\mathcal{R}^*(\Pi^p))} \frac{\mathcal{U}_i^* + \mathcal{U}_\Pi^* h_i^*}{|h_Z^*|} f_Z(\mathcal{Z}^*(\Pi^p,a)) f_A(a)\mathrm{d}a = 0 \tag{2.30}$$

for almost all $a \in [A_{\min}, A_{\max}]$. Here $\mathbf{A}(\Pi^p, i)$ denotes the support of $[A_{\min}, A_{\max}]$ conditional on observing Π^p and i, i.e. $\mathbf{A}(\Pi^p, i) = \{a \mid \Pi = h(i,a,z)$ for some $a \in [A_{\min}, A_{\max}]$ and $z \in [Z_{\min}, Z_{\max}]\}$. $\mathcal{U}_i^*$, $\mathcal{U}_\Pi^*$, h_i^* denote the functions evaluated at the optimal solution. $\mathcal{Z}^*(\Pi^p, a)$ is a function satisfying $\Pi^p = h(\mathcal{R}^*(\Pi^p), \mathcal{Z}^*(\Pi^p,a), a)$. Note that the value of the expectation E and F interacts with the optimal policy rule. To solve the model, we can apply the algorithm proposed in Hauk et al.(2021) with slight modification:Given the optimal policy under full information, $i^{FI}(A,Z) = \frac{Z}{\beta} - 1$, one can find the pairs (Π^p, i) for all possible realizations of (A, Z). Then follow the following steps:

Algorithm 1 (1) Discretize the set of possible values for Π^p, which is $[\frac{1}{A_{max}}, \frac{1}{A_{min}}]$ in our model;

(2) Guess an initial value for E and F, denoted as E^0 and F^0;

(3) Taking $E = E^0$ and $F = F^0$, for each value $\overline{\Pi^p}$ on the grid created in step 1, one can find the value i^* that solves the non-linear equation

$$\int_{\mathbf{A}(\overline{\Pi^p},i^*))} \frac{\mathcal{U}_i(i^*,\overline{\Pi^p},a,z^*) + \mathcal{U}_\Pi(i^*,\overline{\Pi^p},a,z^*) h_i(i^*,a,z^*)}{|h_Z(i^*,a,z^*)|} f_Z(z^*) f_A(a)\mathrm{d}a = 0 \tag{2.31}$$

where z^* denote the value of z solving the equation $\overline{\Pi^p} - h(i^*, a, z) = 0$ at a given a. $\mathbf{A}(\overline{\Pi^p}, i^*))$ is the set of realizations of A with positive density given a pair $(\overline{\Pi^p}, i^*)$; (4) Given the policy rule found in step 3, one can find the updated value for E and F, denoted as E' and F'; If $|E - E^0| < \varepsilon$ and $|F - F^0| < \varepsilon$, where ε is the convergence criterion, stop; If not, take $E^0 \leftarrow E'$, $F^0 \leftarrow F'$ and go back to step (3).

2.5 Calibration

In order to solve the model numerically, it needs to be calibrated and we summarize the parameter values in Table 2.1.

Preferences and Production The household's discount factor β is set to 0.99, reflecting a real interest rate of 3.3%. The elasticity of labor supply η takes value 1 and χ is chosen such that value of leisure in the non-stochastic steady state equals to 30% of time endowment. The demand shock Z_t ranges from 0.99 to 1.1, so that the lowest optimal nominal rate is 0 and the highest around 10%. The supply shock A_t is assumed to fluctuate between ±10% of the mean.

Labor Markets The elasticity of substitution among labor varieties ϵ is set to equal 4.5, to be consistent with an average unemployment rate of 5% when labor is indivisible, in line with Galí (2011). Wage rigidity is captured by the convexity parameter ϕ and the asymmetry parameter ψ in the adjustment cost function (2.10). ϕ is set to be 32, which can be translated in a Calvo probability of not changing wages of 0.76 per quarter. We set ψ equal to 1,077,970 (Kim and Ruge-Murcia (2009)).

Table 2.1: Parameters of New Keynessian Model

	value	Target
discount factor	$\beta = 0.992$	U.S. annual interest rate 3.3%
elasticity of labor supply	$\eta = 1$	
	$\chi = 2$	30% leisure time
supply shock	$A_t \in [0.9, 1.1]$	±10% from 1
demand shock	$Z_t \in [0.99, 1.1]$	$i^{FI} \in [0, 10\%]$
elasticity of substitution of labor	$\epsilon = 4.5$	natural unemployment rate 5%
Convexity in wage adj. cost function	$\phi = 32$	Calvo probability 0.76
Asymmetry in wage adj. cost function	$\psi = 1,077,970$	Kim and Ruge-Murcia (2009)

2.6 Results

We now show the computational solution of the optimal policy under partial information in the model of Section 2.4.

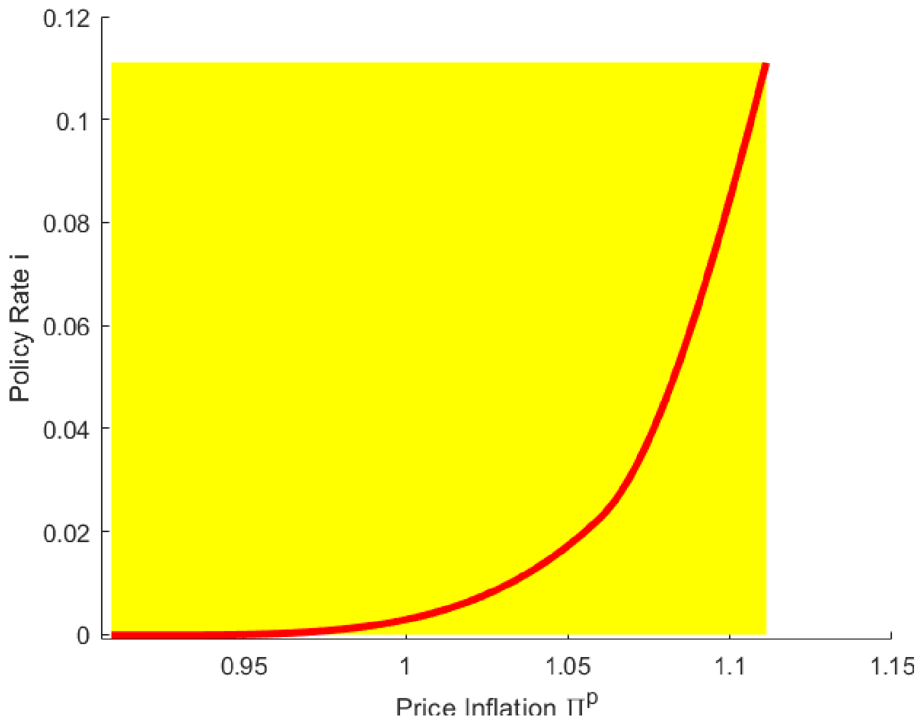


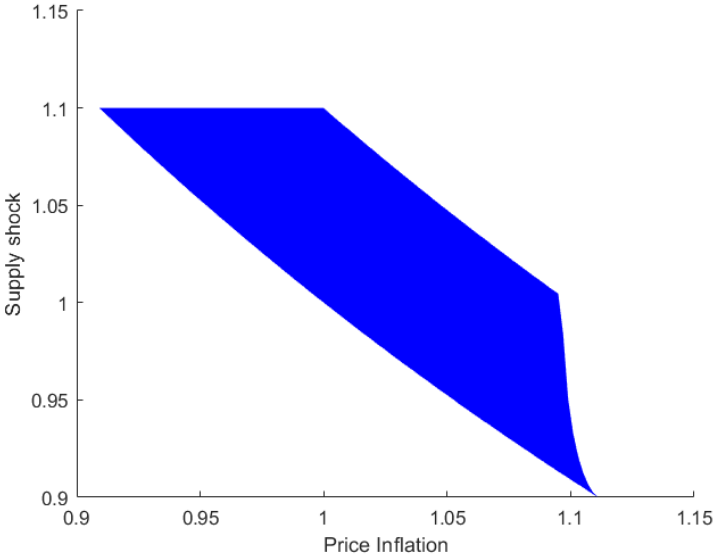
Figure 2.4: Optimal policy under FI and PI. Thick red line: $\mathcal{R}^*$; yellow region: set of FI pairs (Π_t^p, i_t) for all possible realizations of (A_t, Z_t); black dashed line: zero policy rate

We plot the optimal policy under full information and partial information in the same figure for comparison purpose. In Figure 2.4 the yellow region is the set of all equilibrium pairs (Π_t^p, i_t) that could have been realized under full information. The central bank adopt strict wage inflation target under full information, so $\Pi_t^w = 1$ always holds. Since the price setting is flexible, the firm always set $P_t = \frac{W_t}{A_t}$. As a result, Π_t^p ranges from $\frac{A_{t-1}}{A_{\max}}$ to $\frac{A_{t-1}}{A_{\min}}$, which means in equilibrium under full information, any price inflation(deflation) is purely caused by supply shock and demand shock plays no role in it. Then for any given level of price inflation, any level of demand shock could be realized, accompanied with the central bank's policy rate $i_t^{FI} = \frac{Z_t}{\beta} - 1$ to fully stabilize it. That is why the set of all equilibrium pairs (Π_t^p, i_t) consists of a rectangle.

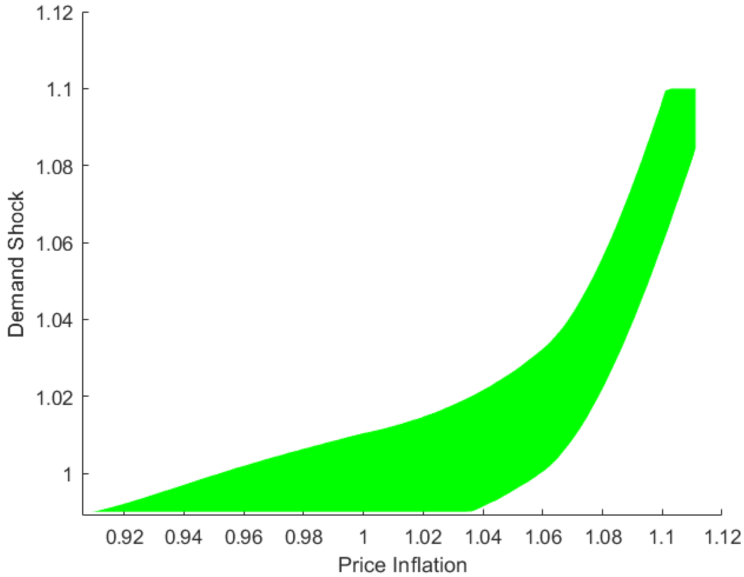
As for the case of partial information, the red line plots the policy rate i_t against Π_t^p according to $i_t = \mathcal{R}^*(\Pi_t^p)$, computed using Algorithm 1. The intuition for the results is as follows.

First, we can find that the policy rule is an increasing function of price inflation Π_t^p, i.e. $\mathcal{R}'(\Pi^p) > 0$. We know that the central bank should fully stabilize demand shock if it has full information about the economy. But under the scenario of partial information, the central bank can only infer it from the signal, Π_t^p. Therefore, the responding rule of nominal rate to price inflation is hinged on how the signal Pi_t^p reveals about the demand shock Z_t. From the reaction function defined from Equation (2.27), one can get $h_Z > 0$ by applying implicit function theorem. The mechanism behind this can be explained by the Euler equation (2.11) and the wage Phillips curve (2.15): higher demand shocks will boost aggregate consumption and move the wage inflation upwards along the wage Phillips curve hence increase price inflation. To stabilize the demand shock, $\mathcal{R}^*$ should also be an increasing function of Π_t^p.

Apart from being increasing, we see $\mathcal{R}^*$ is non-linear: the higher the price inflation is, the more forcefully the central bank will respond to it. We argue that how strongly the central bank should respond to the price inflation is decided by two



(a) Set of admissible supply shocks $\mathbf{A}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p)))$



(b) Set of admissible demand shocks $\mathbf{Z}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p)))$

FIGURE 2.5: Set of admissible fundamental shocks consistent with $\mathcal{R}^*$

factors: the informativeness of the signal and the effectiveness of the policy. The informativeness means how accurate the price inflation signals the demand shock. The more confidence the central bank has on the accuracy of the signal, the more determined it should respond to price inflation. The effectiveness means how easily the changes of policy rate can affect the economy. The more effective the policy rate is, the more cautious the central bank should move.

Figure (2.5) depicts the informativeness of the signal by showing the possible values of shocks that are compatible with each given level of price information Π_t^p and the policy rule $\mathcal{R}^*$. One can find that the set of the possible shocks that are compatible with a given signal, $\mathbf{A}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p))$ and $\mathbf{Z}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p))$ are narrowed down when Π_t^p moves towards its two extreme values. For the lowest and highest observation of price inflation, there is full revelation. The minimum value of Π_t^p is only consistent with the lowest possible Z_t and highest possible A_t. Given $\mathcal{R}^*(\Pi_t^p = \frac{1}{A_{\max}}) = \frac{Z_{\min}}{\beta} - 1$, any demand shock $Z_t > Z_{\min}$ will lead to $\Pi^w > 1$ hence $\Pi_t^p > \frac{1}{A_{\max}}$, which is inconsistent with the signal. But when $\Pi_t^p = \frac{\Pi_t^w}{A_t}$ increases, the central bank is uncertain about the true cause of the price inflation, an underact to the demand shock resulting higher Π_t^w or lower supply shock A_t. That is why we have more fundamental shocks that are compatible with the policy and the signal in the intermediate region. As Π_t^p gets high enough, the central bank becomes confident that wage inflation is happening and increase the policy rate sharply. When Π_t^p arrives its maximum $\frac{1}{A_{\min}}$, the central bank also raises the interest rate to the highest level $\frac{Z_{\max}}{\beta} - 1$. Conditional on observing $\Pi_{\max}^p$, $Z_{\max}$ is the only possible realization of demand shock. Any $Z_t < Z_{\max}$ will lead us to observe a lower price inflation Π_t^p.

Since the monetary policy aims to stabilize the demand shock, one can also interpret the monetary policy as a "mixed stategy" to the possible demand shocks. If we plot the optimal policy and the endogenous information set of demand shock, we can find that the nominal rate is more or less a weighted average of the possible demand shocks, as is shown in Figure 2.6.

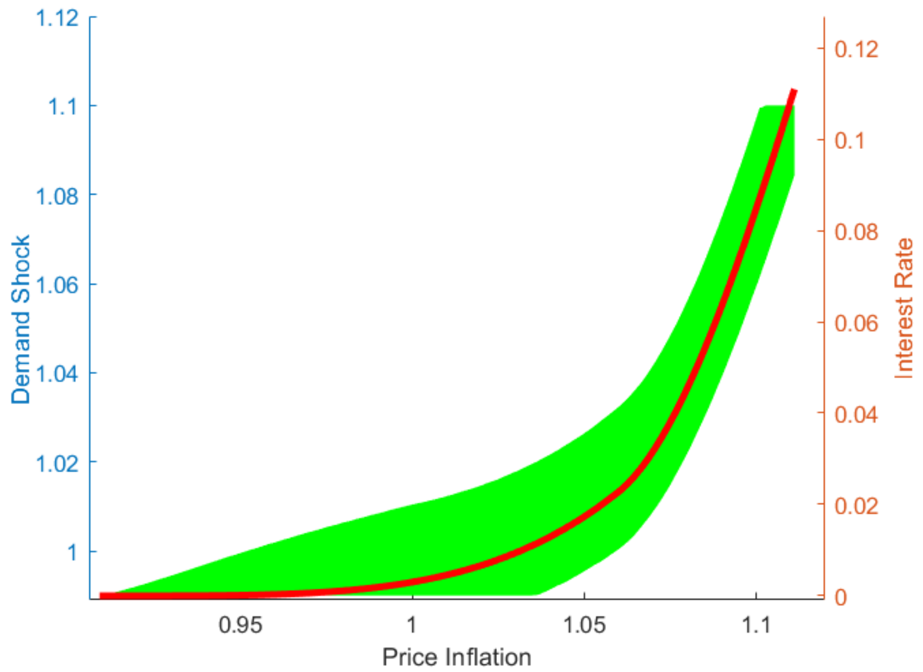


FIGURE 2.6: Optimal policy: a weighted average of possible demand shocks. The left y-axis (blue) is the scale for demand shocks and the right y-axis (red) is the scale for nominal rates

The other factor affecting the slope of the policy function is the effectiveness. The real effects of monetary policy is changing along the Phillips curve shown in Figure (2.3). When the wage inflation is low, the wage Phillips curve is flatten, the monetary policy is less transmitted as wage inflation and has a larger real effect on output. When the wage inflation is high, the wage Phillips curve is steep, the monetary policy is less effective and more transmitted as wage inflation. The real effect of monetary policy is determined by nominal wage rigidity. The higher the rigidity is, the more effective the monetary policy is. Downward nominal wage rigidity bends the Phillips curve and makes the monetary policy diminishing as wage inflation increases. So accordingly the central bank raises interest rate slowly when the monetary policy is very powerful but raises it more quickly when its real effect on aggregate demand is weak.

We can have a better understanding of the policy rule if we take both factors into account. When observing low level of price inflation, the central bank could extract a good signal about the demand shock Z_t, but still raise the policy rate slowly because the monetary policy has large real effects. In the intermediate region of Π_t^p, the monetary policy is less powerful but the central bank chooses to respond to price inflation mildly because it now faces much more uncertainty. When Π_t^p is high enough, the corresponding monetary policy rises sharply as there is less uncertainty now and the monetary policy is not so effective as it is in the low inflation regime.

2.7 Policy Comparison

In this section, we compare the optimal policy under partial information with some alternative policy rules, a simple Taylor rule, "certainty equivalence" and a "standard recipe" which can be seen as the expectation of the optimal policy rate under the probability measure of exogenous shocks. We plot these policies in Figure 2.7.

A simple Taylor rule $i_t = (\Pi_t^p)^\omega - 1$ where $\omega = 1.5$ captures how strongly the central bank responds to the inflation. Compared with optimal policy, the Taylor rule overreacts to price inflation and will lead to greater welfare loss as inflation gets higher.

The "standard" recipe,plotted in green line, can be seen as the expectation of the optimal policy rate under the measure of exogenous shocks

$$\int (\mathcal{U}_i^* + \mathcal{U}_\Pi^* h_i^*) f_Z(z) f_A(a) \mathrm{d}z \mathrm{d}a = 0 \tag{2.32}$$

One can find that its deviation from optimal policy becomes larger as inflation rate moves to the two extremes, which is not surprising as the "standard" recipe fails to take into account the endogeneity of the signal.

The certainty equivalence means that the central bank still adopts the function form the optimal policy under full information but replace the realizations of fundamental shocks with its best estimate, $i^{CE} = \frac{\mathbb{E}[Z_t]}{\beta} - 1$. One can find that the "certainty equivalence" prescribes an almost linear policy rule, which will cause great distortions in the middle region.

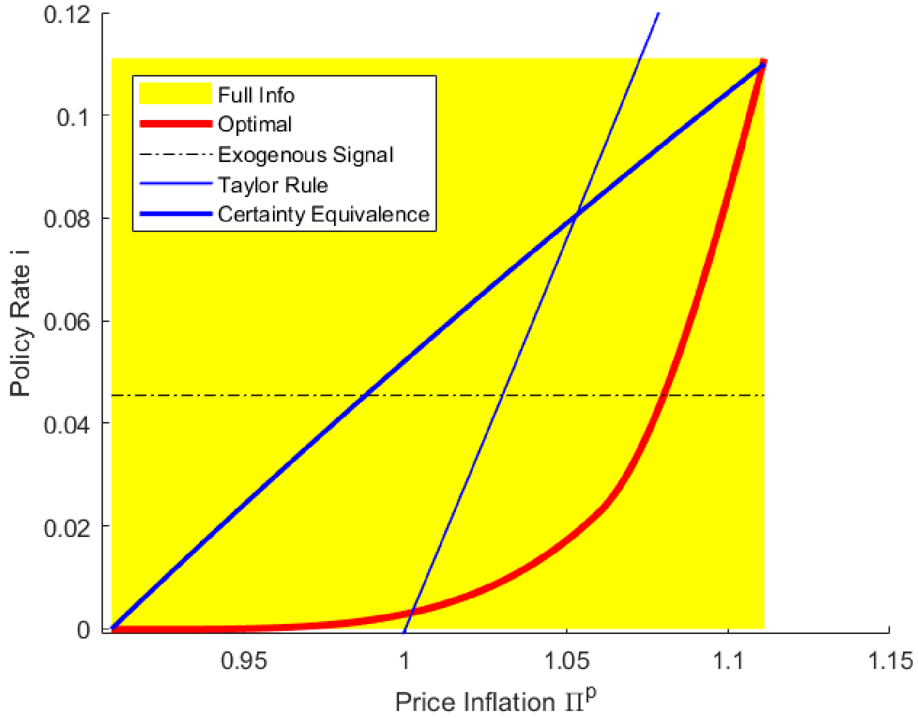


FIGURE 2.7: Policy comparison. Thick red line: $\mathcal{R}^*$; blue line: simple Taylor rule; dashed line: "Standard recipe", which treats signals to be exogenous; yellow region: set of FI pairs (Π_t^p, i_t) for all possible realizations of (A_t, Z_t); dotted line: zero policy rate

2.7.1 The endogeneity of the signal

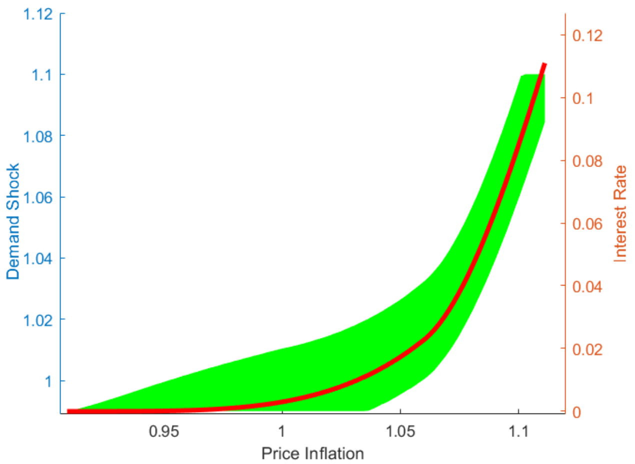
In this economy, the monetary policy aims for stabilizing demand shocks and can be seen as a weighted average of the latter. On the other hand, price inflation, the signal observed by the central bank is endogenous to the policy adopted, hence the set of possible of realizations of fundamental shocks is shaped by the monetary policy. In Figure 2.8, we plot the possible realizations of demand shocks that are consistent with the policy rules and the observations, represented by green color bars. At each given level of price inflation observed (x-axis), the height of the green bar represents the range of demand shocks could have been realized. The higher (wider) the bar is, the greater uncertainty the central bank faces. When the signal is taken as exogenous, policy choice has no effect on it and that is why we see a rectangle in the middle panel. As for the optimal policy and "certainty equivalence" principal, they have something in common: full revelation in extreme points, the signal of highest and lowest price inflation. But in the intermediate level of price inflation, the degree of uncertainty does not change much for the case of CE, in contrast to the case of optimal policy.

2.7.2 Welfare

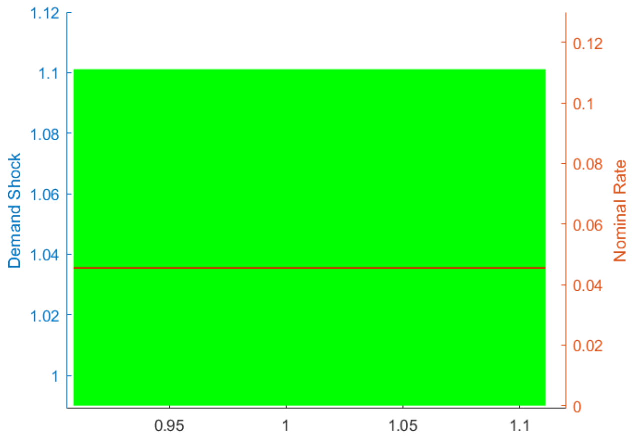
To further evaluate different policy rules, we now compute the unconditional mean of welfare for each policy rule (denoted as W) and their percentage differences with the welfare in an economy where the nominal rigidity is absent (denoted as W_{flex}), defined formerly as:

$$\Delta W = 100 \cdot [\exp(\mathbb{E}W_{flex} - \mathbb{E}W) - 1] \tag{2.33}$$

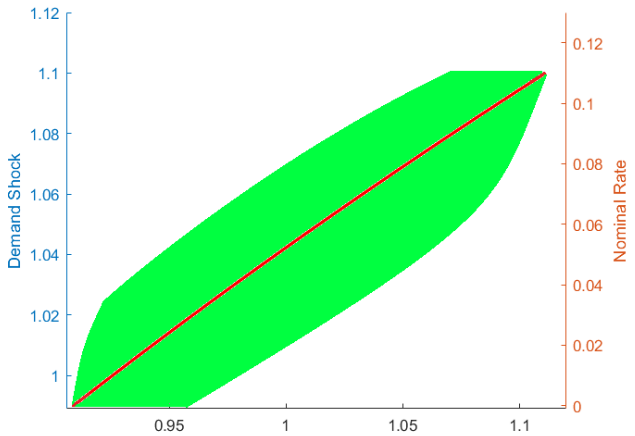
We can interpret this as a welfare loss from wage rigidity. The optimal policy under full information achieves its first best and therefore has 0 welfare loss. Table (2.2) presents the welfare loss for all the candidates policy rules under partial information. One can find that the optimal policy rule performs much better than the alternative choices.



(a) Set of admissible demand shocks $\mathbf{Z}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p))$



(b) Set of admissible demand shocks with "standard Recipe"



(c) Set of admissible demand shocks with "Certainty Equivalence"

Figure 2.8: Set of admissible demand shocks consistent with alternative policy rules

TABLE 2.2: Welfare loss

Full Information	Optimal	Certainty Equivalence	Taylor Rule	Standard Recipe
0	0.16	0.28	0.34	0.45

2.8 Serial correlated shocks

In our main example, the supply and demand shocks are independent identically distributed. The assumption of i.i.d. process for the exogenous variables is made for simplicity and illustrative purpose. To be consistent with empirical evidence, we solve the optimal monetary policy with serial correlated fundamental shocks.

2.8.1 Optimal policy under full information

To be more precise, we now assume the supply shock A_t and demand shock Z_t follow the AR(1) process with non-stochastic means normalized to unity:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t} \tag{2.34}$$

$$\ln Z_t = \rho_z \ln Z_{t-1} + \varepsilon_{z,t} \tag{2.35}$$

The autoregressive parameters, ρ_a and ρ_z, lie between zero and one. The innovations, $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ are drawn from normal distributions of mean 0 and standard deviations σ_a and σ_z.

Under rational expectations, one can find the dynamic IS curve and the new Keynessian Phillips curve:

$$\frac{A_t^{1-\rho_a} Z_t^{1-\rho_z}}{C_t} = \beta(1+i_t)\mathbb{E}_t \frac{\exp(\varepsilon_{z,t+1})\exp(\varepsilon_{a,t+1})}{C_{t+1}\Pi_{t+1}^w} \tag{2.36}$$

$$\frac{\epsilon\chi Z_t^{1-\rho_z} C_t^2}{[A_t - \Phi(\Pi_t^w)]^2} + \frac{(1+\tau)(1-\epsilon) A_t Z_t^{1-\rho_z}}{A_t - \Phi(\Pi_t^w)} - \frac{Z_t^{1-\rho_z}\Phi'(\Pi_t^w)\Pi_t^w}{A_t - \Phi(\Pi_t^w)} + \beta\mathbb{E}_t \frac{\exp(\varepsilon_{z,t+1})\Phi'(\Pi_{t+1}^w)\Pi_{t+1}^w}{A_t^{\rho_a}\exp(\varepsilon_{a,t+1}) - \Phi(\Pi_{t+1}^w)} = 0 \tag{2.37}$$

The optimal policy under full information can be stated formerly as:

$$\max_{\{i_t, C_t, N_t, \Pi_t^w\}} U(C_t, N_t; Z_t) \tag{2.38}$$

$$\text{s.t. } (2.14), (2.36), (2.37)$$

As shown in the appendix, the optimal policy sets $\Pi_t^w = 1$ and $i_t = \frac{Z_t^{1-\rho_z}}{\beta} - 1$.

2.8.2 Optimal policy under partial information

Note that the expectation part of the Phillips curve is not a constant any more but depends on the supply shock A_t. We denote this part as $F(A_t)$. The expectation part of the IS curve is still a constant in equilibrium, which is denoted as G. Now one can find the function mapping from the policy variable i_t to the signal Π_t^p:

$$\Pi_t^p = g(i_t, A_t, Z_t) \tag{2.39}$$

as an implicit function from:

$$\frac{Z_t^{1-\rho_z}}{\beta(1+i_t)G} = \sqrt{\frac{A_t(A_t - \Phi(A_t\Pi_t^p))(\Phi'(A_t\Pi_t^p)\Pi_t^p - (1+\tau)(1-\epsilon))}{\epsilon\chi} - \frac{\beta F(A_t)(A_t - \Phi(A_t\Pi_t^p))^2}{\epsilon\chi Z_t^{1-\rho_z}}} \tag{2.40}$$

and the optimal policy problem under partial information can be stated as

$$\max_{\mathcal{R}:\mathcal{P}\to\mathcal{T}} \mathbb{E}[\mathcal{U}(i_t, \Pi_t^p, A_t, Z_t)] \tag{2.41}$$

s.t. (2.25), (2.39)

The optimal policy rule $\mathcal{R}^*$ satisfies the first-order condition:

$$\int_{-\infty}^{+\infty} \frac{\mathcal{U}_i^* + \mathcal{U}_\Pi^* g_i^*}{|g_Z^*|} f_Z(\mathcal{Z}^*(\Pi^p, a)) f_A(a) \mathrm{d}a = 0 \tag{2.42}$$

To solve out the optimal policy, we use a linear function to approximate the expectation part in (2.40), i.e. $F(A_t) \approx F_1 \ln A_t + F_2$ and then apply the following algorithm:

Given the optimal policy under full information, $i^{FI}(A, Z) = \frac{Z^{1-\rho_z}}{\beta} - 1$, one can find the pairs (Π^p, i) for all possible realizations of (A, Z). Since now we have unbounded shocks A_t and Z_t, the pairs (Π^p, i) fill the whole $\mathbb{R}^2$ space. Then follow the following steps:

Algorithm 2 (1) Discretize the set of possible values for Π^p. Here we choose to discretize the interval $\mathbf{A} = [\exp((1-\rho_a)a_{t-1} - 3\sigma_a), \exp((1-\rho_a)a_{t-1} + 3\sigma_a)]$;
(2) Guess an initial value for G and F_1, F_2, denoted as G^0 and F_1^0, F_2^0;
(3) Taking $G = G^0$ and $F_1 = F_1^0, F_2 = F_2^0$, for each value $\overline{\Pi^p}$ on the grid created in step 1, one can find the value i^* that solves the non-linear equation

$$\int_{\mathbf{A}} \frac{\mathcal{U}_i(i^*, \overline{\Pi^p}, a, z^*) + \mathcal{U}_\Pi(i^*, \overline{\Pi^p}, a, z^*) g_i(i^*, a, z^*)}{|g_Z(i^*, a, z^*)|} f_Z(z^*) f_A(a) \mathrm{d}a = 0 \tag{2.43}$$

where z^* denote the value of z solving the equation $\overline{\Pi^p} - g(i^*, a, z) = 0$ at a given a;
(4) Given the policy rule found in step 3, one can find the updated value for G and F_1, F_2, denoted as G' and F_1', F_2'; If $|G - G^0| < \varepsilon$ and $|F_1 - F_1^0| < \varepsilon, |F_2 - F_2^0| < \varepsilon$, where ε is the convergence criterion, stop; If not, take $G^0 \leftarrow G'$, $F_1^0 \leftarrow F_1', F_2^0 \leftarrow F_2'$ and go back to step (3).

2.8.3 Results

Now we show the computational solution of optimal policy under partial information when the shocks are serial correlated in Figure 2.9. The yellow region

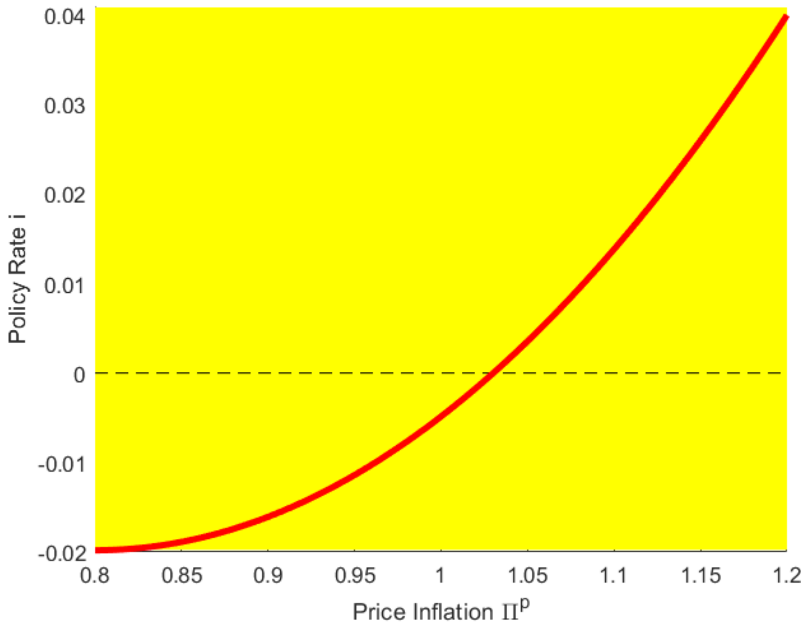


Figure 2.9: Optimal policy under FI and PI. Thick red line: $\mathcal{R}^*$; yellow region: set of FI pairs (Π_t^p, i_t) for all possible realizations of (A_t, Z_t); black dashed line: zero policy rate

represents the set of all equilibrium pairs (Π_t^p, i_t) that could have been realized under full information as before. Recalling that all equilibrium pairs (Π_t^p, i_t) consist of a rectangle when shocks are uniformly distributed. But this is not the case when we have serial correlated shocks. Now the equilibrium pairs (Π_t^p, i_t) fill the whole $\mathbb{R}^2$ space. For convenience, we plot the area when $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ falls within 3 standard deviations of their mean 0.

One can find that the optimal policy rate is still an increasing function of price inflation as before. This is because higher price inflation signals higher demand shock unambiguously as the i.i.d. case and the monetary policy aims for stabilizing the demand shock. The non-linearity of the policy rule can also be attributed to the changing nominal rigidity and uncertainty along the Phillips curve and lead inertial behavior responding to low inflation and strong reaction to high inflation.

2.9 Conclusion

This paper explicitly analyzes the optimal monetary policy with signal extraction in a non-linear model with asymmetric wage adjustment cost. We find that the asymmetric wage adjustment cost and the signal endogeneity are the two forces shaping the responsiveness of policy rate to price inflation. The asymmetric wage adjustment cost changes the effectiveness of the monetary policy along the Phillips curve via changing the nominal rigidity. The signal endogeneity changes the central banks certainty level about the economy as the price inflation varies. These two factors make the monetary policy exhibits non-linear behavior.

Chapter 3

Optimal Fiscal Policy with Ricardian and Hand-to-mouth Agents

3.1 Introduction

How should a government use the fiscal instruments when faced with shocks to the government expenditure? Ramsey optimal tax theory gives two important insights into this question: taxes on labor income should be smoothed and government should issue bonds to buffer the shocks (Barro (1979); Lucas Jr and Stokey (1983); Kingston (1991); Zhu (1992)), while long- run capital tax should be set to zero (Chamley (1986); Judd (1985)). These cornerstone results are all based on the assumption of a representative agent in the economy. Therefore they are all forward-looking and supposed to adjust their consumption and labor supply based on the tax and interest rates. However the strong response of aggregate consumption to interest rate changes that accounts for the large direct effects in representative agent models is questionable in light of empirical evidence. Macro-econometric analysis of aggregate time-series data finds a much smaller sensitivity of consumption to changes in the interest rate. The aggregate data should be viewed as generated by two types of agents: one forward-looking and consuming their permanent income; the other, behaving impatiently and spending its current income (Campbell and Mankiw (1989), Campbell and Mankiw (1991)).

If a significant fraction of agents are constrained in the financial markets, then they will only adjust their consumption to tax changes but not to the interest rate. Then what confidence can we have that tax recommendations obtained in a representative economy can minimize the total cost of distortion? Because equating taxes over time does not mean equating the marginal cost of distortions over time, which is not optimal any more. Then what is the optimal fiscal policy in an economy with forward-looking agents and hand-to- mouth agents? We try to answer this question in this chapter.

The model economy is inhabited by agents that differ in their access to the financial markets. Hand-to-mouth agents are constrained in the financial markets while Ricardian agents are not. Lump-sum tax is ruled out. In the first scenario, we study the optimal fiscal policy in an economy without capital described by Lucas Jr and Stokey (1983). Government uses flat-rate labor income tax and state-contingent bond to finance its expenditures. We find that, when government is not allowed to levy discriminatory labor tax, the optimal tax rate is not constant any more, even if we adopt the utility function that is homogeneous of consumption and labor supply and generates perfect constant tax rate in the representative economy. We also find that the more social planer cares about the hand-to-mouth agents, the more positively the optimal tax rate responds to the government expenditure. Government uses taxes to manipulate the prices of government bond and necessarily affects the inter temporal budget constraints of the Ricardian agents. If government is sided with the Ricardian agents, they will borrow at a low interest rate and lend at a high rate and vice versa.

In the second scenario, when capital is introduce to the model, we have indeterminacy of capital income tax and bond issuing. Follow Zhu (1992), we study the ex-ante capital tax rate in this economy and find that the fluctuations of capital tax is again captured by the equality condition of labor-income tax rates across agents.

My paper is related to two main strands of the literature. On the one hand, the paper builds on the earlier literature on the optimal policy, including Lucas Jr and Stokey (1983), Zhu (1992) ,Chari et al. (1994). The closest forebears to our framework is Bassetto (2014). He studies how the relative political power of "tax-payers" affect the fiscal policies of a country in peace time and war time. Werning (2007) focuses on the distributional effects of distortionary taxes. In his paper, the introduction of hand-to- mouth agents solves the indeterminacy problem arised by lump-sum tax. On the other hand, the literature that links high MPC with hand-to-mouth agents. Campbell and Mankiw (1989) provides empirical evidence of the hand-to-mouth agents. Kaplan and Violante (2014) show that uninsurable risk, combined with the co-existence of liquid and illiquid assets in financial portfolios leads to the presence of a sizable fraction of poor and wealthy hand-to-mouth households, as in the data. Cloyne et al. (2016) show that households with mortgage debt exhibit large and significant consumption responses to tax changes.Debortoli and Galí (2017) try to study the monetary transmission mechanism with a simple two-agent economy.

The rest of the chapter proceeds as follows. In Section 2, we study the optimal proportional labor tax in the complete market, which follows Lucas Jr and Stokey (1983) in an economy without capital; In Section 3, we solve the model numerically. In section 4, we study the optimal long run capital tax in a deterministic case (Chamley (1986)) and stochastic case (Zhu (1992)) respectively. Section 5 concludes.

3.2 The Model

We consider an economy with two types of households: The first type of households are hand-to-mouth. They have no access to the financial markets and consume their after-tax labor income every period, which are denoted by "K". The second type of agent have full access to the financial markets, which are denoted by "R". Both types of households have the same preferences, which are given by a utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{3.1}$$

where C_t is consumption and N_t is labor supply. We adopt the following utility function:

$$U(C, N) = u(C_t) + v(N_t) = \log(C_t) - D\frac{N_t^{\gamma+1}}{\gamma + 1} \tag{3.2}$$

so that the optimal labor tax rate is perfect constant in the representative agent economy described by Lucas and Stokey(1983). It would be convenient for us to compare the results.

The fraction of constrained households and unconstrained households are λ and $1-\lambda$ respectively.The technology follows the same spirit of Lucas Jr and Stokey (1983). Firms are operated in a perfect competitive market with linear production function of labor input $y_t = f(l_t) = l_t$. Let g_t denote government purchases at time t. Then the resources constraint is

$$g_t + \lambda C_t^k + (1 - \lambda)C_t^R = \lambda N_t^K + (1 - \lambda)N_t^R \tag{3.3}$$

The government could levy a proportional tax on the labor income τ_t^n and issue the government debt $b_t^g(g_{t+1})$ contingent on future spending. I also assume that the tax rate is constrained to be equal across both types of agents and the marginal tax rate is constant on all labor income. The government budget constraint is

$$g_t + b^g_{t-1}(g_t) = \lambda\tau^n_t N^K_t + (1-\lambda)\tau^n_t N^R_t + \sum_{g_{t+1}|g^t} p_t(g_{t+1})b_t(g_{t+1}) \tag{3.4}$$

The hand-to-mouth agents' budget constraint is:

$$C^K_t = (1-\tau^n_t)N^K_t \tag{3.5}$$

The Recardian agents could buy state-contingent government bonds, so their budget constraint is

$$C^R_t + p^g_t(g_{t+1})b^R_t(g_{t+1}) = b^R_{t-1}(g_t) + (1-\tau^N_t)N^K_t$$

Note that since the population of Ricardian agents is $1-\lambda$, the bonds held by them satisfies $(1-\lambda)b^R_t = b^g_t$.

3.2.1 Competitive Equilibrium and Ramsey outcome

The household first-order-condition require that the price of government bonds satisfies

$$p^g_t(g_{t+1}) = \beta\frac{u'(C^R_{t+1}(g^{t+1}))}{u'(C^R_t)}prob(g_{t+1}|g_t) \tag{3.6}$$

and that taxes satisfy

$$1-\tau^N_t = -\frac{v'(N^R_t)}{u'(C^R_t)} = -\frac{v'(N^K_t)}{u'(C^K_t)} \tag{3.7}$$

We use these expressions to eliminate the prices and taxes in the hand-to-mouth agents' budget constraints, i.e. $C^K_t = (1-\tau^N_t)N^K_t = -\frac{v'(N^K_t)}{u'(C^K_t)}N^K_t$

$$u'(C^K_t)C^K_t + v'(N^K_t)N^K_t = 0 \tag{3.8}$$

The special utility function (3.2) allows us to eliminate C_t^K and solve N_t^K explicitly from (3.8), which is invariant to tax rate change, $N_t^K = N^K = D^{-\frac{1}{\gamma+1}}$. They only adjust their consumption level to respond tax rate change. In other words, their marginal propensity to consume (MPC) equals 1. Since markets are complete, the Ricardian agents can choose their optimal contingent plans based on a single Arrow-Debreu budget constraint:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u'(C_t^R)C_t^R + v'(N_t^R)N_t^R] = b_{-1}^R(g_0)u'(C_0^R) \tag{3.9}$$

where $b_{-1}^R(g_0)$ is the amount of government bonds held by each Ricardian agent and the total quantity of government bond $b_{-1}^g(g_0) = (1-\lambda)b_{-1}^R(g_0)$.

DEFINITION 1 Given initial bond holdings $b_{-1}^R(g_0)$ by the Ricardian agents, a *competitive equilibrium* is a sequence of taxes τ_t^N, prices $\{p_t^g(g_{t+1}), w_t\}$, and non-negative quantities $\{c_t^K, N_t^K\}$, $\{c_t^R, N_t^R, b_t^R(g_{t+1})\}$ such that

(i) hand-to-mouth agents choose $\{c_t^K, N_t^K\}$ to maximize their expected utility (3.2) subject to the budget constraint (3.8), taking prices and taxes as given;

(ii)Ricardian agents choose $\{c_t^R, N_t^R, k_t\, b_t(g_{t+1})\}$ to maximize the same utility form (3.2), taking $\{p_t^g(g_{t+1}), w_t\}$ as given;

(iii) Firms maximize profits: the equilibrium wage $w_t = 1$;

(iv) the government budget constraint (3.4) holds;

(v) markets clear: the resource constraints (3.3) hold for all periods t and histories $\{g_t\}_{t=0}^{\infty}$.

The Lagrangian for the Ramsey problem can be represented as:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ & \alpha[u(C_t^K) + v(N_t^K)] + (1-\alpha)[u(C_t^R) + v(N_t^R)] \\ & + \nu_t[u'(C_t^K)C_t^K + v'(N_t^K)N_t^K] \\ & + \mu_t[u'(C_t^K)v'(N_t^R) - u'(C_t^R)v'(N_t^K)] \\ & + \theta_t[g_t + (1-\lambda)C_t^R + \lambda C_t^K - (1-\lambda)N_t^R - \lambda N_t^K] \\ & + \phi[u'(C_t^R)C_t^R + v(N_t^R)N_t^R - b_{-1}(g_0)u'(C_0^R)]\}\end{aligned}$$

The government budget constraint is not explicitly included because it is redundant when the agents' budget constraints are satisfied and the resources constraint holds. To avoid the time inconsistency problem and make model easier, I assume that the outstanding government debt in the initial period b_{-1} is 0. So the first order conditions for the Ramsey problem are:

$$[C_t^K] : \alpha u'(C_t^K) + \nu_t[u''(C_t^K)C_t^K + u'(C_t^K)] + \mu_t v'(N_t^R)u''(C_t^K) + \lambda\theta_t = 0 \quad (3.10)$$

$$[N_t^K] : \alpha v'(N_t^K) + \nu_t[v''(N_t^K)N_t^K + v'(N_t^K)] - \mu_t u'(C_t^R)v''(N_t^K) - \lambda\theta_t = 0 \quad (3.11)$$

$$[C_t^R] : (1-\alpha)u'(C_t^R) + \phi[u''(C_t^R)C_t^R + u'(C_t^R)] - \mu_t v'(N_t^K)u''(C_t^R) + (1-\lambda)\theta_t = 0 \quad (3.12)$$

$$[N_t^R] : (1-\alpha)v'(N_t^R) + \phi[v''(N_t^R)N_t^R + v'(N_t^R)] + \mu_t u'(C_t^K)v''(N_t^R) - (1-\lambda)\theta_t = 0 \quad (3.13)$$

We can solve the competitive allocation $C_t^i, N_t^i, i = K, R$ as a function of g_t and ϕ from these four first order conditions and equations (3.3), (3.8) and (3.9) . That means, if government purchases are equal after two histories g^t and $g^{\tilde{t}}$ for $t, \tilde{t} > 0$, i.e.,

$$g_{t+1} = g_{\tilde{t}+1}$$

then the Ramsey choices of consumption and leisure, $\{C^i_{t+1}, N^i_{t+1}\}$ and $\{C^i_{\tilde{t}+1}, N^i_{\tilde{t}+1}\}$, are identical, which asserts that the optimal allocation is a function of the currently realized government purchases g_t only and does not depend on the specific history preceding realizations of g^t. Combining Ricardian agents' F.O.C.s (3.12) and (3.13):

$$\begin{aligned}(1-\alpha)[u'(C^R_t)+v'(N^R_t)]+\phi[u''(C^R_t)C^R_t+u'(C^R_t)+v''(N^R_t)N^R_t+v'(N^R_t)]\\ +\mu_t[u'(C^K_t)v''(N^R_t)-v'(N^K_t)u''(C^R_t)]=0\end{aligned} \tag{3.14}$$

and the equilibrium conditions of labor market $1-\tau^N_t = -\frac{v'(N^i_t)}{u'(C^i_t)}$ for $i=\{N,K\}$. One can find a more intuitive expression for the optimal tax rate. Assume first $\mu_t = 0$, so that government can levy agent specific tax, then the optimal taxation is similar to the results in Lucas and Stokey economy:

$$\tau^K_t = \frac{\nu_t(1+\gamma)}{\alpha}$$

$$\tau^R_t = \frac{\phi(1+\gamma)}{1-\alpha}$$

i.e. the labor tax for Ricardian agents would still be a constant and the government use tax only to adjust the hand-to-mouth agents' consumption and labor supply. But equation (3.7) imposes equality of labor-income tax rates across agents, so the optimal tax rate is not constant any more and its volatility is captured by the second line of equation (3.14), where μ_t is the Lagrange multiplier associated with the equality constraint of labor income tax rate across 2 agents. Another way to analyze the problem is to find out the competitive allocation associated with a perfect constant tax rate. In this case, the hand-to-mouth agents achieves perfect consumption and leisure smoothing. Considering the resources constraint

(3.3) under this assumption:

$$g_t + \lambda C^K + (1-\lambda)C_t^R = \lambda N^K + (1-\lambda)N_t^R$$

All the shocks of government expenditures would be born by the Ricardian agents, which is not optimal from the perspective of a benevolent government.

3.3 Quantitative Analysis

3.3.1 Calibration

To provide a quantitative illustration of the role of heterogeneity, we consider a calibration of the model where the share of hand-to-mouth agents is set to $\lambda = 0.5$, following Campbell and Mankiw (1989). The parameter D is calibrated so that in the non-stochastic steady state with government debt and deficit equal to zero, the labor supply is 70 per cent of the time endowment.We assume the government spending follows an AR(1) process:

$$g_t = (1-\rho)\bar{g} + \rho g_{t-1} + \epsilon_t$$

The rest of the parameters are calibrated as following:

TABLE 3.1: Parameters of 2-agents Model

Parameters	Values
share of Keynessian agents λ	0.5
discount rate β	0.99
D	2
γ	1
time endowment	1
$\bar{g}$	0.175
ρ	0.95
variance of shock $\sigma^2(\epsilon)$	0.012^2

3.3.2 Fiscal policy of a benevolent government

First, we consider the fiscal policy of a benevolent government, which sets the Pareto weight of different agents equal to their population share. Figure 3.1 shows the simulated paths of government expenditures, competitive allocations and the tax rate, in contrast to two alternative "extreme" policy: one is to balance the budget period by period without issuing any bonds, the other is to impose a perfect constant tax rate, as the government does in a representative economy. When the

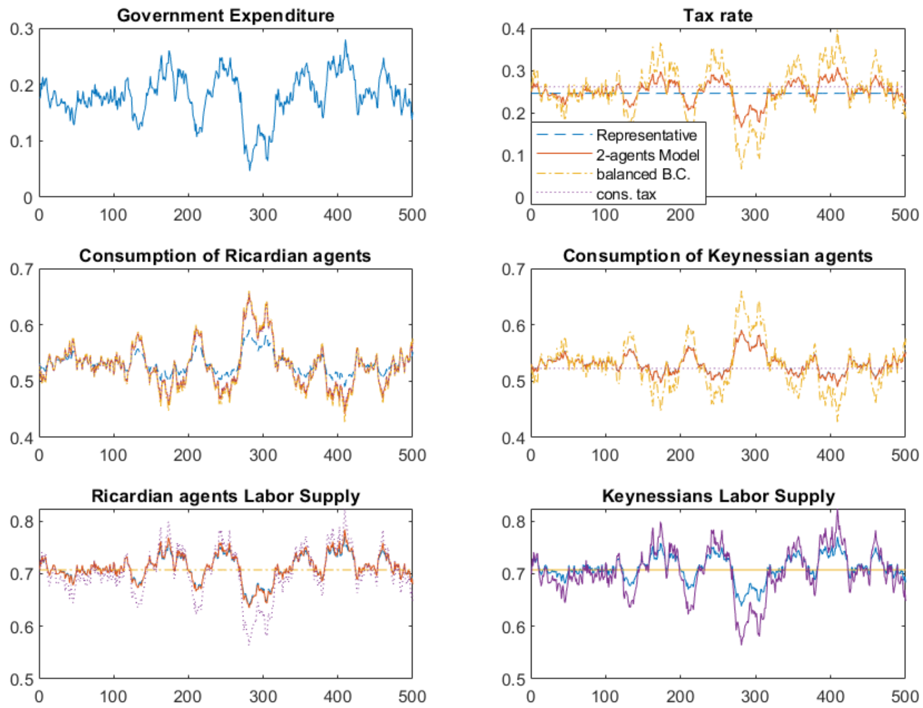


FIGURE 3.1: Competitive allocations under alternative policies

government balances its budget constraints period by period, it cannot issue public debt to buffer the expenditure shock. As a result, all the agents in the economy would be hand-to-mouth and they work for a fixed amount of time every period. In such an economy, the government expenditures perfectly correlate with tax rates and consumption, positive and negative respectively. We would observe the most volatile consumption in this no-bond world.

Now let's evaluate the constant tax rate policy. Since only a fraction of the population could hold the public debt which helps to buffer the expenditure shock, the government has to levy a slightly higher tax rate (0.2415-0.2398=0.0017) to achieve perfect insurance. Why is it not optimal? Because the government could use tax rate to change states prices and distribute the distortions more evenly across time. When the expenditure is high, government lowers labor tax rate and encourage the Ricardian agents to work more and lowers state price. That's why we could observe more volatile labor supply of the Ricardian agents when government adopt the optimal policy.

3.3.3 Debt or tax? a redistribution concern

Government will choose different strategies to buffer the expenditure shock when it favors different agents. Since only the Ricardian agents hold public debt, the government could affect their welfare by distorting the state prices when they save or disave. We can define the Ricardian agents' net savings as

$$S_t = (1 - \tau_t)N_t^R + b_{t-1}(g_t) - C_t^R$$

If the government sides with the Ricardian agents, it increases the state prices when Ricardian agents save ($S_t > 0$) and lowers state prices when $S_t < 0$, i.e. the Ricardian agents sell high and buy cheap. In Figure 3.2, I plot the reaction functions when the government put different weights on the agents. α is the Pareto weight on the hand-to-mouth agents. Lower α (yellow line) corresponds to the policy beneficial to the Ricardian agents, who save when the government expenditure g_t is low and vice versa. One can find that the consumption of Ricardian agents is relatively low when they save, which means higher state price determined by $u'(C_t^R)$ of their savings.

The implication of fiscal policy here is that the government employs taxes to distort inter-temporal prices to affect agents' wealth.

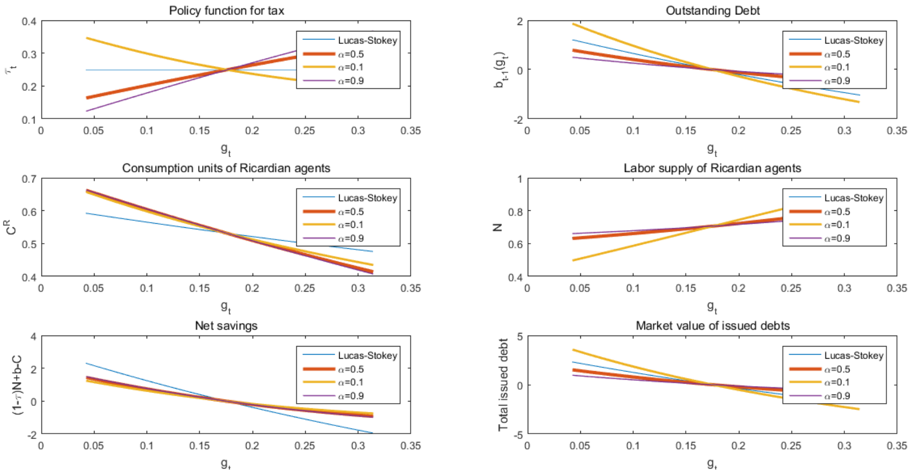


FIGURE 3.2: Reaction functions with different Pareto weights

3.4 Extensions to an economy with capital

This section extends the analyses of Ramsey taxation to an economy with capital accumulation. I use a stochastic version of a one-sector neoclassical growth model in discrete time and infinite horizon. The households' preferences are ordered by:

$$\sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi_t(g^t)(u(C_t) + v(N_t)) \tag{3.15}$$

We follow the same spirit before: the hand-to-mouth agents have no access to the financial markets and could only consume their after-tax labor income in each period. The Ricardian agents could either buy government bonds or invest in the capital market.

3.4.1 Endowment and Technology

The Ricardian agents bring the initial capital k_{-1} to this economy and they supply labor together with the hand-to-mouth agents to the production firm. There is only one final good which can be either consumed or invested. The production function is constant to scale:

$$y_t = F(K_{t-1}, N_t)$$

There is a government in this economy and the government expenditure in units of consumption good in period t is denoted by g_t, which is assumed to be an exogenous stochastic process and the only source of uncertainty. The technology constraint follows:

$$\lambda C_t^K + (1-\lambda)C_t^R + g_t + (1-\lambda)(k_t - (1-\delta)k_{t-1}) = F((1-\lambda)k_{t-1}, \lambda N_t^K + (1-\lambda)N_t^R) \tag{3.16}$$

where δ is the depreciation rate of capital.

There are three perfectly competitive markets in the economy: the labor markets, the capital market, and the government bond market. The firm rents capital from consumers and the government trades one-period state-contingent claims with consumers. Given the government expenditure $\{g_t\}_{t=0}^{\infty}$, the government finances its exogenous purchase and debt obligation by levying flat-rate taxes on earnings from capital labor, at rates τ_t^K and τ_t^N respectively, and by issuing state-contingent bonds. I also assume that the tax rate in labor income is constrained to be equal across agents. Then the government budget constraint follows:

$$g_t + b_{t-1}^g(g_t) = \tau_t^N w_t(\lambda N_t^K + (1-\lambda)N_t^R) + \tau_{t-1}^K r_t(1-\lambda)k_{t-1} + \sum_{g_{t+1}|g^t} p_t^g(g_{t+1})b_t^g(g_{t+1}) \tag{3.17}$$

The timing of trading is a crucial issue in this economy. In period $t = 0$, the supply of capital is inelastic and the tax on the capital income is therefore not distortionary. So the government wants to tax the capital income in the initial period as heavily as possible to minimize distortion caused by other distortionary taxes. If it happens that the revenue collected from this tax is big enough to finance all the current and future government expenditures, then there is no need to use distortionary taxes. To make the exercise interesting we impose an upper bound on the period 0 capital tax so that the government does need to tax labor and capital income in the future periods. To avoid policy indeterminacy, capital taxes are not state-contingent but decided one period in advance. Only Arrow securities are used to complete the markets.

The hand-to-mouth agents do not have access to capital markets and could only consume their labor income, so their budget constraints remain unchanged:

$$C_t^K = (1-\tau_t^N) w_t N_t^K \tag{3.18}$$

However the Ricardian agents' sequential budget constraints follows:

$$C_t^R + k_t + \sum_{g_{t+1}|g^t} p_t^g(g_{t+1}) b_t^R(g_{t+1}) = (1-\tau_{t-1}^K) r_t k_{t-1} + (1-\tau_t^N) w_t N_t^R + (1-\delta) k_{t-1} + b_{t-1}^R(g_t) \tag{3.19}$$

where $(1-\lambda) b_t^R(g_{t+1}) = b_t^g(g_{t+1})$ and $(1-\lambda) k_t = K_t$, i.e. they have equal share to the government bonds and the capital.

3.4.2 Competitive equilibrium

Firms Since the factors markets are perfectly competitive, the firm's F.O.C implies:

$$r_t = F_K((1-\lambda) k_{t-1}, \lambda N_t^K + (1-\lambda) N_t^R) \tag{3.20}$$

$$w_t = F_N((1-\lambda) k_{t-1}, \lambda N_t^K + (1-\lambda) N_t^R) \tag{3.21}$$

Households The households problem is to maximize their expected utility function under the budget constraints, the solutions are characterized by the following first order conditions:

$$(1-\tau_t^N)w_t = -\frac{v'(N_t^R)}{u'(C_t^R)} = -\frac{v'(N_t^K)}{u'(C_t^K)} \tag{3.22}$$

$$p_t^g(g_{t+1}) = \beta \frac{u'(C_{t+1}^R(g^{t+1}))}{u'(C_t^R)} prob(g_{t+1}|g^t) \tag{3.23}$$

$$u'(C_t^R) = \beta \mathbb{E}_t u'(C_{t+1}^R(g^{t+1}))[1+(1-\tau_t^K)r_{t+1}-\delta] \tag{3.24}$$

Under complete market condition, the Ricardian agents budget constraints could be summed into a single one:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u'(C_t^R)C_t^R + v'(N_t^R)N_t^R] = u'(C_0^R)[(1+r_0-\delta)k_{-1} + b_{-1}^R(g_0)] \tag{3.25}$$

where $\tilde{r}$ is the after-tax interest rate. The non-arbitrage condition implies

$$1 = \sum_{g_{t+1}|g^t} p_t^g(g_{t+1})[1+(1-\tau_t^K)r_{t+1}(g^{t+1})-\delta] \tag{3.26}$$

DEFINITION 3.3 Given initial capital and bond holdings $\{K_{-1}, b_{-1}(g_0)\}$, a *competitive equilibrium* is a sequence of taxes $\{\tau_t^K, \tau_t^N\}$, prices $\{p_t^g(g_{t+1}), r_t, w_t\}$, and non-negative quantities $\{c_t^K, N_t^K\}$, $\{c_t^R, N_t^R, k_t\}$ such that

(i) Hand-to-mouth agents choose $\{c_t^K, N_t^K\}$ to maximize their expected utility (15) subject to the budget constraint (18) taking prices and taxes that satisfy (21) as given;

(ii) Ricardian agents choose $\{c_t^R, N_t^R, k_t, b(g_{t+1})\}$ to maximize their utility, taking $\{p_t^g(g_{t+1}), r_t, w_t\}$ as given;

(iii) Firms maximize profits: the first-order conditions (20) and (21) hold;

(iv) Government budget constraint (17) holds;

(v) Markets clear: the resource constraints (16) hold for all periods t and histories

$\{g_t\}_{t=0}^{\infty}$

3.5 Analytical results

To simplify the problem, we further assume $b_{-1} = 0$ and $\tau_{-1}^K = 0$. The Lagrangian for the Ramsey problem is:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ & \alpha[u(C_t^K) + v(N_t^K)] + (1-\alpha)[u(C_t^R) + v(N_t^R)] \\ & + \nu_t[u'(C_t^K)C_t^K + v'(N_t^K)N_t^K] \\ & + \mu_t[u'(C_t^K)v'(N_t^R) - u'(C_t^R)v'(N_t^K)] \\ & + \theta_t[g_t + \lambda C_t^K + (1-\lambda)C_t^R + (1-\lambda)(k_t - (1-\delta)k_{t-1}) \\ & - F((1-\lambda)k_{t-1}, \lambda N_t^K + (1-\lambda)N_t^R)] \\ & + \phi[u'(C_t^R)C_t^R + v'(N_t^R)N_t^R]\} - \phi k_{-1}(F_{K,0} + 1 - \delta)u'(C_0^R)\end{aligned}$$

The first order conditions for $t > 0$:

$$[C_t^K] : \alpha u'(C_t^K) + \nu_t[u''(C_t^K)C_t^K + u'(C_t^K)] + \mu_t v'(N_t^R)u''(C_t^K) + \lambda\theta_t = 0 \quad (3.27)$$

$$[N_t^K] : \alpha v'(N_t^K) + \nu_t[v''(N_t^K)N_t^K + v'(N_t^K)] - \mu_t u'(C_t^R)v''(N_t^K) - \lambda\theta_t F_{N,t} = 0 \quad (3.28)$$

$$[C_t^R] : (1-\alpha)u'(C_t^R) + \phi[u''(C_t^R)C_t^R + u'(C_t^R)] - \mu_t v'(N_t^K)u''(C_t^R) + (1-\lambda)\theta_t = 0 \quad (3.29)$$

$$[N_t^R] : (1-\alpha)v'(N_t^R) + \phi[v''(N_t^R)N_t^R + v'(N_t^R)] + \mu_t u'(C_t^K)v''(N_t^R) - (1-\lambda)\theta_t F_{N,t} = 0 \quad (3.30)$$

$$[k_t] : \theta_t - \beta\mathbb{E}_t\theta_{t+1}(1 + F_{K,t} - \delta) = 0 \quad (3.31)$$

and $t = 0$:

$$[C_0^K] : \alpha u'(C_0^K) + \nu_0[u''(C_0^K)C_0^K + u'(C_0^K)] + \mu_0 v'(N_0^R)u''(C_0^K) + \lambda\theta_0 = 0 \quad (3.32)$$

$$[N_0^K] : \alpha v'(N_0^K) + \nu_0[v''(N_0^K)N_0^K + v'(N_0^K)] - \mu_0 u'(C_0^R)v''(N_0^K) - \lambda\theta_0 F_{N,0} = 0 \quad (3.33)$$

$$\begin{aligned}[C_0^R] :&(1-\alpha)u'(C_0^R) + \phi[u''(C_t^R)C_t^R + u'(C_t^R)] - \mu_0 v'(N_0^K)u''(C_0^R) + (1-\lambda)\theta_0 \\ &- \phi k_{-1}(F_{K,0} + 1 - \delta)u''(C_0^R) = 0\end{aligned} \quad (3.34)$$

$$\begin{aligned}[N_0^R] :&(1-\alpha)v'(N_0^R) + \phi[v''(N_0^R)N_0^R + v'(N_0^R)] + \mu_0 u'(C_0^K)v''(N_0^R) - (1-\lambda)\theta_0 F_{N,t} \\ &- \phi k_{-1}F_{KN,0}u'(C_0^R) = 0\end{aligned} \quad (3.35)$$

$$[k_0] : \theta_0 - \beta\mathbb{E}_0\theta_1(1 + F_{K,1} - \delta) = 0 \quad (3.36)$$

Steady state in the non-stochastic case Consider the special case in which there exists a $T \geqslant 0$ for which $g_t = g$ for all $t \geqslant T$, i.e. no more uncertainties after period T. Assume that there exists a solution to the Ramsey problem and that it converges to a time-invariant allocation, so that C, N, k are constant after some time. Then the steady state version of equation (3.31) implies:

$$1 = \beta(1 + F_K - \delta)$$

while the non-arbitrage condition for capital is $1 = \beta(1 + (1 - \tau^K)F_K - \delta)$, so the optimal capital tax in the long run is zero. Indeed it is not a surprising result if we look at Judd (1985), where the agents are divided into two class. Capitalists do not work and workers do not save. The result of this extreme case shows that the long-run capital tax should be zero even if the government only consider the workers.

Ex-ante capital tax in the stochastic case We consider the capital tax that is not contingent on the realization of current state but is already set in the previous period. We define the ex-ante capital tax:

$$\bar{\tau}^K_{t+1} = \frac{E_t p^g_t(g_{t+1})\tau^K_{t+1} r_{t+1}}{E_t p^g_t(g_{t+1}) r_{t+1}}$$

To study the ex-ante capital tax in a stationary equilibrium, we now assume that the process $\{g_t\}$ follows a Markov process with transition probabilities $\pi(g'|g) = Prob(g_{t+1} = g'|g_t = g)$. An economy converges to a stationary if the stochastic process $\{g_t, k_t\}$ is a stationary, ergodic Markov process on the compact set $\mathcal{G} \times \mathcal{K}$ and the allocations can be described by time-invariant rule $C(g,k), n(g,k), k'(g,k)$.
Propositon Let $P^\infty(\cdot)$ be the probability measure over the outcomes of the stationary equilibrium. If there exists a stationary Ramsey equilibrium allocation, the ex-ante capital tax rate satisfies $P^\infty(\tau^K > 0) > 0$ and $P^\infty(\tau^K < 0) > 0$

Proof By the definition of ex ante capital tax:

$$\begin{aligned}\bar{\tau}^K_{t+1} \geqslant (\leqslant) 0 \Longleftrightarrow \sum_{g_{t+1}} p^g_t(g_{t+1}|g_t)\tau^K_{t+1} r_{t+1} \geqslant (\leqslant) 0 \Longleftrightarrow \sum_{g_{t+1}} p^g_t(g_{t+1}|g_t)[r_{t+1} + 1 - \delta] \leqslant (\geqslant) 0 \\ \Longleftrightarrow u'(C^R_t) \leqslant (\geqslant) E_t \beta u'(C^R_{t+1})[1 + F_{K_{t+1}} - \delta]\end{aligned} \tag{3.37}$$

From the first order condition of equation (3.29) and (3.30), we can solve

$$-\theta_t = \frac{(1-\alpha)u'(C^R_t) + \phi[u''(C^R_t)C^R_t + u'(C^R_t)] - \mu_t v'(N^K_t)u''(C^R_t)}{1-\lambda}$$

Define

$$H_t \equiv \frac{-\theta_t}{u'(C_t^R)} = \frac{1-\alpha+\phi(1-\gamma_C)}{1-\lambda} - \frac{\mu_t v'(N_t^K)u''(C_t^R)}{(1-\lambda)u'(C_t^R)} \tag{3.38}$$

Then Equation (3.31) could be rewritten as:

$$u'(C_t^R)H_t = \beta E_t u'(C_{t+1}^R)H_{t+1}F_{K_{t+1}} \tag{3.39}$$

From the last equivalent condition of equation (3.37), we can get

$$H_t \geqslant (\leqslant) \frac{E_t \omega_{t+1} H_{t+1}}{E_t \omega_{t+1}} \tag{3.40}$$

where $\omega_{t+1} \equiv u'(C_{t+1}^R)(1+F_{K,t+1}-\delta)$

Since a stationary Ramsey equilibrium has time-invariant allocation rule for C, N, k, equation (3.37) could be rewritten as:

$$\begin{aligned}\bar{\tau}(g_t,k_t) \geqslant (\leqslant) 0 \Longleftrightarrow H(g_t,k_t) \geqslant (\leqslant) & \frac{\sum_{g_{t+1}} \pi(g_{t+1}|g_t)\omega_{t+1}(g_{t+1},k'(g_t,k_t))H_{t+1}(g_{t+1},k'(g_t,k_t))}{\sum_{g_{t+1}} \pi(g_{t+1}|g_t)\omega_{t+1}(g_{t+1},k'(g_t,k_t))} \\ & \equiv \Gamma H(g_t,k_t)\end{aligned} \tag{3.41}$$

Here the operator Γ is a weighted average of H with the property that $\Gamma H^* = H^*$ for any constant H^*. Under some regularity conditions, $H(g_t,k_t)$ attains its maximum H^+ and minimum H^- in the stationary equilibrium. Follow Zhu (1992) proof, there must exist a constant H^* such that $\Gamma H^* = H^*$

We can find that H consists of two part: the first part is a constant which is identical to that in the representative agent economy and implies zero long-run capital tax with probability 1; the second part comes from the equilibrium condition of same proportional labor tax for the agents again, which makes the ex-ante capital tax in the stationary economy fluctuate around 0. So if the planner are allowed to levy agent-specific proportional labor tax, the ex-ante long-run capital tax would be zero.

3.6 Conclusion

When two types of agents co-exist in the economy, homogeneous labor tax rate imposes an additional constraint to the government, apart from the implementability and resources constraints. Then the optimal tax prescription of constant labor tax and long-run 0 capital tax does not hold any more, which mirrors the classical result that incomplete tax system overturns the uniform commodity taxation.

Chapter 4

Misuse of Geometric-decaying Bonds in Sovereign Default Literature

4.1 Introduction

Short-term debt is often cast as the villain in sovereign debt crises, leaving the economy to sharp swings in interest rates and rollover crisis. However, as documented by Broner et al. (2013), emerging markets actively shift to shorter-maturity debt in a crisis and issue longer term debt in normal times. This favoritism towards short-term debt during periods of crisis appears puzzling to economists.

So why do emerging economies borrow short term in financial stress time? A spontaneous answer to this question is the relative cheaper borrowing cost associated with short-term debt. When a country is in bad economic condition, its sovereign debt depreciates, reflecting increasing default risk. Moreover the price of long-term debt falls more than the price of short-term debt, shifting the sovereign debt issuance to short term. There are two possible explanations for this relative price change in financial stress time. The first one, proposed by Broner et al. (2013), argues that shocks to lender's risk aversion raise the risk premium on long-term bonds more than on the short-term bonds. The second one, proposed by Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016), highlights the role of long-term debt "dilution", that is, new issuance reduces the value of outstanding stock of debt. Because of the pari passu clause of the sovereign debt, a bondholder's right to be repaid is not subordinated to the rights of others. When issuing long-term debt, the borrower cannot commit to a future sequence of debt issuances, and the probability of default increases with these future issuances. When the creditors anticipate this lack of commitment, the price of long-term debt effectively penalizes this "dilution" effect. What needs to be noted here is that the "dilution" of long bonds cannot alleviate the sovereign's debt burden but only increase the borrowing cost associated with long bonds. As long as the sovereign repays its debt, it needs to pay back the predetermined coupons or par value of the bonds. Depreciation of long bonds does not mean that it could repay less, on the contrary, is reflected in the issuance price, leads to higher borrowing cost.

But how the literature quantifies the "dilution" effect is questionable. Specifically, modelling long-term debts as a perpetual contract with geometric series of coupons changes the payment structure of long-term debt and exaggerates this effect. A typical long bond issued by emerging economies pays coupons (semi-)annually, and the principal payments arrive at the predetermined maturity dates. Because the coupon payments are much smaller than the principal, most long-term debt can be roughly seen as zero-coupon bonds. The geometrical bond, however, does not pay the principal but only a geometrical series of coupons. Such a payment schedule makes the claims of geometrical bonds issued in different periods overlap with each other, which means the claims of the new issuances crowd in to that of existing debts and increase the default probability directly. But the principal payments of long-term debts issued in different periods can be aligned to different periods. For example, a 10-year bond issued in 2022 matures in 2032 and this year's 10-year bond matures in 2033, which will not affect the default probability in 2022 directly as the overlapping case of geometrical bonds. So sovereign debts issued with ordinary instruments do not suffer so much as the geometric debt from the “dilution" effect.

The other problem brought by this special payment schedule is maturity mismatch between income process and liability process. The debt portfolio consists of 1-period short-term bond and geometric bond can only generate a descending debt profile, i.e. the outstanding debt is a decreasing function of time to maturity and it is front-loaded. But once the country is in recession, considering that GDP shocks are highly persistent, the income is expected to be low in the near future and revert in the far future, hence back-loaded. The mismatch between front-loaded liability and back-loaded income process makes it harder for the country to smooth consumption across states.

To illustrate how different debt instruments affect the emerging markets portfolio choice, I employ a dynamic model to study the optimal maturity of sovereign debt based on Arellano and Ramanarayanan (2012), which captures various essential elements of sovereign debt markets: an infinitely-lived sovereign with concave utility borrows by short- and long-term debts in global financial markets. Investors are risk neutral. The sovereign makes decisions sequentially, with no commitment to its future actions. Importantly, this includes both its decision to repay or default and its debt management decisions. The country can default on debt at any point in time but faces costs of doing so, in the form of lower income and exclusion from international financial markets. When the sovereign is highly indebted, a risk of default arises. In equilibrium, default tends to occur in low-income, high-debt times, when the cost of debt payments outweighs the costs of default. Bond prices are functions of the levels of each maturity of debt and income, which determine the borrower's probability of repaying in the future.

When the government only issues 1-period bond and the geometric bond. The price the latter is very sensitive and drops to zero quickly as the amount of issuance grows when the GDP is low. As a result, the total amount of funds raised by issuing long-term debt is very limited when the country is in recession.

But the pricing function of long-term debt looks totally different when the government issues zero-coupon bond. It is still a decreasing function of the issuance units, but less steep than before, which means the borrower can now raise more funds. With different debt instrument, the default probability is smaller and the sovereign's expected utility is higher under the same calibration. Therefore geometric bond actually makes the sovereign face more constraints in the financial market.

4.2 Model

The basic set-up for the model follows Arellano and Ramanarayanan (2012). They model the long-term debt as geometric bond and I will show the case of zero-coupon bond.

A small open economy receives a stochastic stream of income $\{y\}_{t=0}^{\infty}$ with compact support Y and follows a Markov process with transition function $f(y, y')$. The representative in this economy has preference:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

The borrower issues short- and long-term debt. Short-term debt is a one-period discount bond. If the country defaults, all outstanding debt is erased but there would be an output cost in the period of default:

$$y_t^{\text{def}} = \begin{cases} y_t & \text{if } y_t \leq (1-\lambda)\overline{y} \\ (1-\lambda)\overline{y} & \text{if } y_t > (1-\lambda)\overline{y} \end{cases}$$

where $\bar{y}$ is the mean level of income. In addition the economy would temporarily stay in financial autarky and θ is the probability that it could regain the access to international financial markets every period. The state of the economy at time t is given by $(\mathbf{B}, y)$.

4.2.1 Recursive Problem

we can define the economy's optimal problem recursively:

$$v(\mathbf{B}, y) = \max_{\{c,d\}}\{v^c(\mathbf{B}, y), v^d(y)\} \tag{4.1}$$

where v^c is the value associated with not default and stay in the contract:

$$v^c(\mathbf{B}, y) = \max_{\{c,\mathbf{B'}\}}\left[u(c) + \beta\int_{y'} v(\mathbf{B}', y')f(y, y')\,dy'\right] \tag{4.2}$$

where v^d is the default value and irrelevant of the current period outstanding debt:

$$v^d(y) = u\left(y^{\text{def}}\right) + \beta\int_{y'}\left[\theta v(\mathbf{0}, y') + (1-\theta)v^d(y')\right] f(y, y')\,dy' \tag{4.3}$$

Lenders are risk-neutral and the risk-less interest rate is r^*. So the short-term debt price is

$$q^1(\mathbf{B'}, y) = \frac{1}{1+r^*}\int_{R(\mathbf{B'})} f(y, y')dy'$$

where $R(\bullet)$ is the repayment set of y and defined as

$$R(\mathbf{B}) = \{y \in Y \mid v^c(\mathbf{B}, y) \geqslant v^d(y)\}$$

and the default set is the complement

$$D(\mathbf{B}) = \{y \in Y \mid v^c(\mathbf{B}, y) < v^d(y)\}$$

when $y \in R(\mathbf{B})$, optimal new issuance takes form $\mathbf{B'} = \tilde{B}(\mathbf{B}, y)$

Geometric Bond If the sovereign's debt instruments are 1-period bond and geometric bond, the outstanding debt portfolio should read $\mathbf{B} = (b^s_{t-1}, b^l_{t-1})$. Let the new issuance of geometric debt be l_t, which pays $\delta^{n-1} l_t$ units of consumption goods in every future period $t+n$, then the total stock of long-term debt, conditional on not defaulting, follows the law of motion:

$$b^l_t = \delta b^l_{t-1} + l_t$$

the sovereign's budget constraint is

$$c_t + b^s_{t-1} + b^l_{t-1} = y_t + q^s_t b^s_t + q^l_t l_t$$

the geometric bond price is defined

$$q^l_t(\mathbf{B'}, y) = \frac{1}{1+r^*} \int_{R(\mathbf{B'})} 1 + \delta q^l_{t+1} \left(\tilde{B}(\mathbf{B'}, y), y' \right) f(y, y') dy'$$

Zero-coupon Bond If both of the short- and long-term debts are zero-coupon bonds with maturity 1 and L, then the outstanding debt portfolio should read $\mathbf{B} = (b^1_{t-1} + b^L_{t-L}, b^L_{t-L+1}, b^L_{t-L+2}, \ldots, b^L_{t-1})$. The time-flow budget constraint conditional on not defaulting is :

$$c_t + b^1_{t-1} + b^L_{t-L} = y_t + q^1_t b^1_t + q^L_t b^L_t$$

the long-term debt price could be written in recursive form as:

$$q^2(\mathbf{B'}, y) = \frac{1}{1+r^*} \int_{R(\tilde{B}(\mathbf{B}', y))} q^1(\tilde{B}(\mathbf{B'}, y), y') f(y, y') dy'$$

$$q^3(\mathbf{B'}, y) = \frac{1}{1+r^*} \int_{R(\tilde{B}(\mathbf{B}',y))} q^2(\tilde{B}(\mathbf{B'}, y), y') f(y, y') dy'$$

$$\cdots$$

$$q^L(\mathbf{B'}, y) = \frac{1}{1+r^*} \int_{R(\tilde{B}(\mathbf{B}',y))} q^{L-1}(\tilde{B}(\mathbf{B'}, y), y') f(y, y') dy'$$

Government dynamically chooses its policies without commitment. They can default or issue new short and long bonds every period as a function of the pay-off relevant variables only, i.e. the outstanding debts $\mathbf{B}$ and income shock y. The government takes into account that its choices affect the future debt. Investors rationally anticipate future policies and their expectation are in turn reflected in current bond prices.

A **Markov Perfect competitive equilibrium (MPCE)** is a set of policy functions for consumption $c(\mathbf{B}, y)$, default decision $D(\mathbf{B}, y)$ and repayment and issuance decision $R(\mathbf{B}, y)$, $\mathbf{B'} = \tilde{B}(\mathbf{B}, y)$ and price functions $q(\mathbf{B'}, y)$ such that

1. Given the bond price functions $q_1(\mathbf{B'}, y)$, $q_l(\mathbf{B'}, y)$, the policy functions $D(\mathbf{B}, y)$, $R(\mathbf{B}, y)$, $b_1' = \tilde{b}_1(\mathbf{B}, y)$, $b_l' = \tilde{b}_l(\mathbf{B}, y)$ solve the borrower's optimization problem (1), (2) and (3);
2. The bond price functions satisfies the recursive form.

4.2.2 Equilibrium Condition

For simplicity, I assume the distribution function f is continuous and the bond price functions and the value of repay v^c are differentiable. The the first order conditions w.r.t. short and long issuances are:

Zero-coupon case:

$$u'(c_t)\left(q_t^1 + \frac{\partial q_t^1}{\partial b_t^1} b_t^1 + \frac{\partial q_t^L}{\partial b_t^1} b_t^L\right) = \beta \mathbb{E}_t\left[u'(c_{t+1}) \mathbb{I}_{\{y_{t+1} \in R(\mathbf{B'})\}}\right] \tag{4.4}$$

$$
\begin{aligned}
&u'(c_t)\left(q_t^L + \frac{\partial q_t^L}{\partial b_t^L} b_t^L + \frac{\partial q_t^1}{\partial b_t^L} b_t^1\right) \\
&\quad + \mathbb{E}_t \sum_{k=1}^{L-1} \left[\beta^k u'(c_{t+k}) \left(\frac{\partial q_{t+k}^1}{\partial b_t^L} b_{t+k}^1 + \frac{\partial q_{t+k}^L}{\partial b_t^L} b_{t+k}^L\right) \prod_{j=1}^{k} \mathbb{I}_{\{y_{t+j} \in R(\mathbf{B}_{t+j})\}}\right] \\
&\quad = \beta^l \mathbb{E}_t \left[u'(c_{t+L}) \prod_{j=1}^{L} \mathbb{I}_{\{y_{t+j} \in R(\mathbf{B}_{t+L})\}}\right]
\end{aligned}
\tag{4.5}
$$

Geometric case:

$$
u'(c_t)\left(q_t^s + \frac{\partial q_t^s}{\partial b_t^s} b_t^s + \frac{\partial q_t^l}{\partial b_t^s} l_t\right) = \beta \mathbb{E}_t \left[u'(c_{t+1}) \mathbb{I}_{\{y_{t+1} \in R(\mathbf{B'})\}}\right] \tag{4.6}
$$

$$
u'(c_t)\left(q_t^l + \frac{\partial q_t^l}{\partial b_t^l} l_t + \frac{\partial q_t^s}{\partial b_t^l} b_t^s\right) = \beta \mathbb{E}_t \left[u'(c_{t+1})(1 + \delta q_{t+1}^l) \mathbb{I}_{\{y_{t+1} \in R(\mathbf{B'})\}}\right] \tag{4.7}
$$

The optimal maturity structure equates the marginal gain in utility from issuing one more unit of debt at present to the marginal reduction in utility from repaying in the future. The marginal gain from issuance looks similar: q_t^m is the funds raised by issuing one unit of debt, $\frac{\partial q_t^m}{\partial b_t^n} < 0$ for $m, n \in \{1, L, s, l\}$ captures how issuance price changes with issuing quantity. The left hand side, as a whole, is the net effect of issuing one unit of debt, measured by the marginal utility of consumption at time t. One can find that the optimal conditions for short-term debt, Eq. (4.4) and (4.6), look similar in two cases and they quantify the trade-off between utility gain from issuance today t and disutility from repaying tomorrow $t+1$.

Eq. (4.7) gives us a hint on why geometric bonds are issued less in difficult times. Comparing the RHS of Eq. (4.6) and (4.7), one can find that issuing more debt not only shrinks the repayment set of $R(\mathbf{B'})$, but also lowers the future price of geometric debt q_{t+1}^l. Considering that the income process is persistent, lower income shock today implies lower income tomorrow, hence higher $u'(c_{t+1})$. The claims to geometric debt is fixed 1 unit of consumption, lower q_{t+1}^l only pushes the sovereign to borrow at higher costs in bad states.

Zero-coupon long bonds play a different role from others. On the one hand, issuing zero-coupon long bond not only affect the bonds' prices at current period t, but also affect the issuance prices afterward before its maturity, from $t+1$ to $t+L-1$, which is captured by the second line of Eq. (4.5). On the other hand, the disutility of repayment is measured by $u'(c_{t+L})$, which means the claims are aligned to the far future, L periods later, when recover is more likely to happen than the near future $t+1$.

The distinctive trade-offs of the geometric bond and zero-coupon bond shows that geometric bond is a bad approximation for the long bond in maturity analysis of sovereign debt, and I will show the numerical result in the following section.

4.3 Quantitative Analysis

Solving the long zero-coupon bond will encounter the dimensional curese, so here we only solve an infinite horizon model with 1 period and 2 periods bond.

4.3.1 Calibration

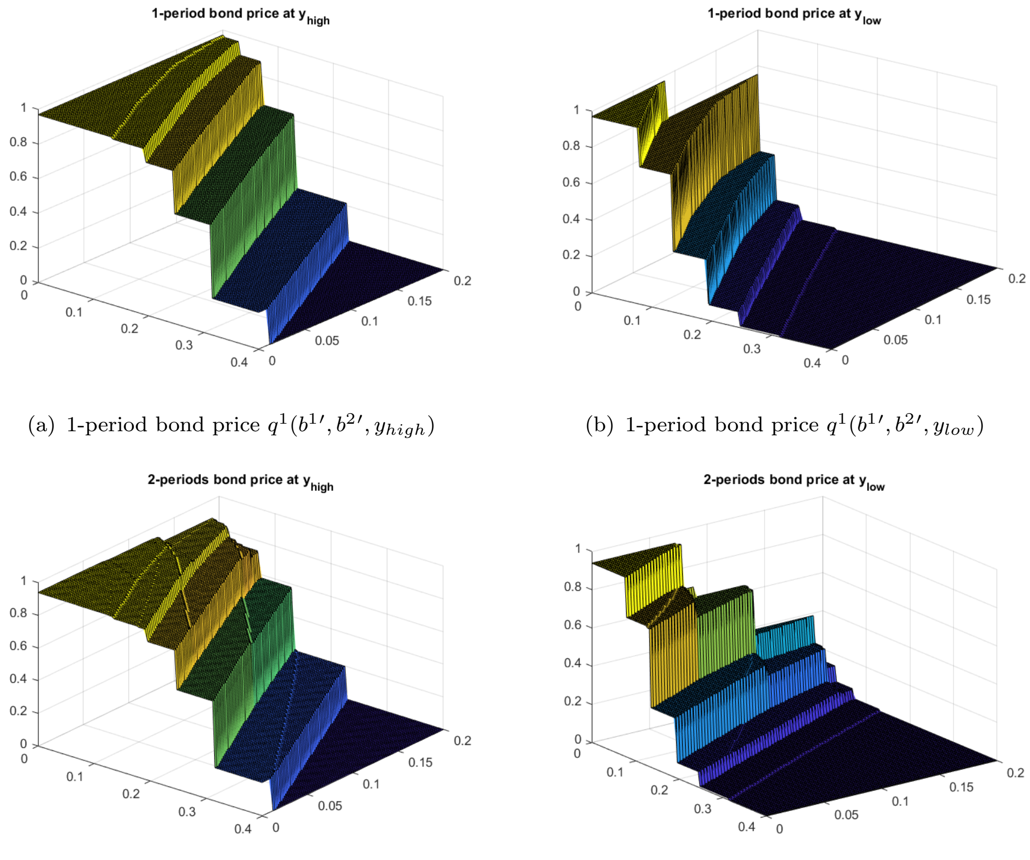
We follow Arellano and Ramanarayanan (2012) to calibrate the parameters.

TABLE 4.1: Parameters

	value	Target
Risk-free interest rate	$r^* = 3.2\%$	U.S. interest rate
Borrower's risk aversion	$\sigma = 2$	Standard value
Stochastic structure	$\rho = .9, \eta = .017$	Brazil GDP
Probability of Reentry	$\theta = .17$	Benjamin and Wright(2009)
Calibrated parameters:		
Output after default	$\lambda = .045$	
Borrower's discount factor	$\beta = .935$	spread and volatility of trade balance

4.3.2 Bond Prices and Policy Functions

First, I present the price functions of zero-coupon case in 3D model. Figure (4.1) shows the zero-coupon bond price as a function of 1-period and 2-periods issuances, controlling the current income shock y at high level and low level respectively. All the functions decrease with respect to the issuance, reflecting higher default probability when the debt burden is high.

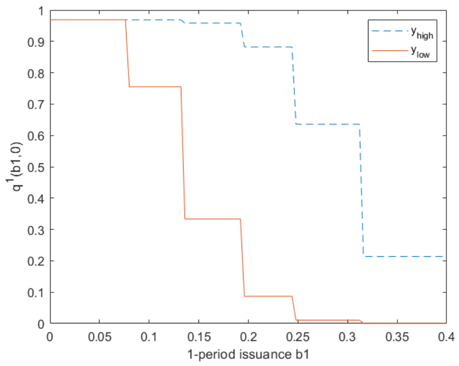


(a) 1-period bond price $q^1(b^{1\prime}, b^{2\prime}, y_{high})$ (b) 1-period bond price $q^1(b^{1\prime}, b^{2\prime}, y_{low})$

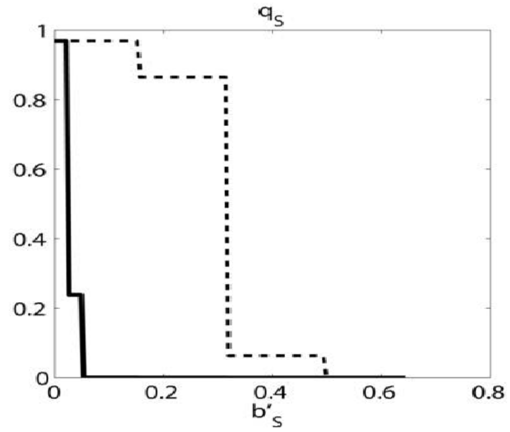
(c) 2-period bond price $q^2(b^{1\prime}, b^{2\prime}, y_{high})$ (d) 2-period bond price $q^2(b^{1\prime}, b^{2\prime}, y_{low})$

FIGURE 4.1: Zero-coupon bond price

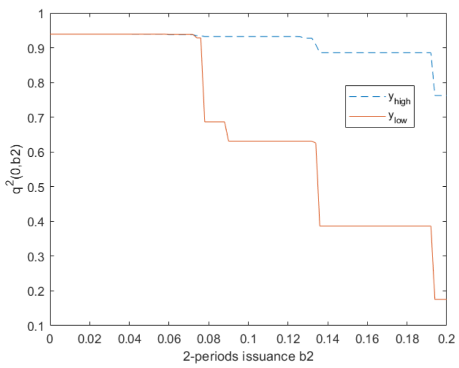
In Figure (4.2), I compare the price functions of zero-coupon bond with those in geometric bond model. What Arellano and Ramanarayanan (2012) do is to show a 2D graph, the short-term debt price as a function of short issuance, controlling long-term issuance at 0; the long-term debt price against geometric bond issuance, controlling short-term issuance at 0. So I show the graph in the same way for convenience. It is clear that the bond price depreciates more slowly as the issuance units grow if the government issues zero-coupon bonds, which means lower borrowing cost and default probability, 0.46% compared with 2.5% of geometric bond.



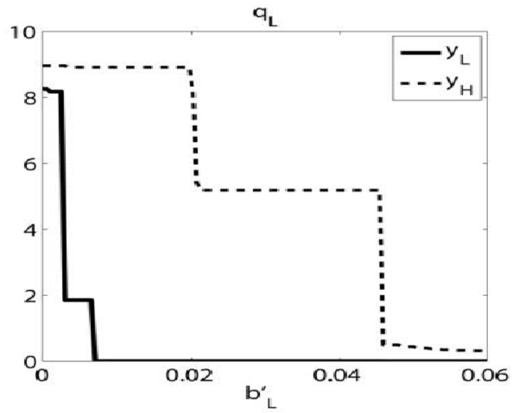
(a) 1-period bond price $q^1(b^{1\prime}, 0)$, 0-coupon bond



(b) 1-period bond price $q^1(b^{1\prime}, 0)$, geometric bond



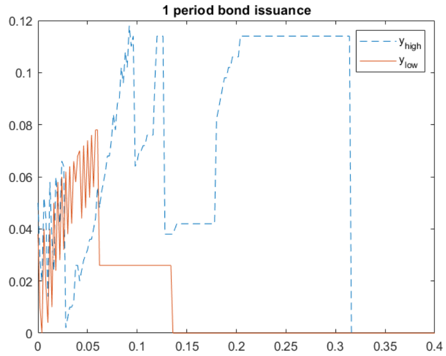
(c) 2-period bond price $q^2(b^{1\prime}, b^{2\prime})$



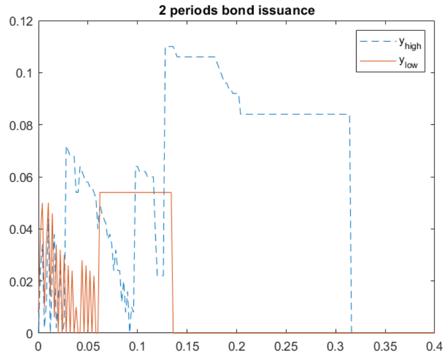
(d) Geometrical bond price $q^2(0, b^{2\prime})$

FIGURE 4.2: Price function comparison

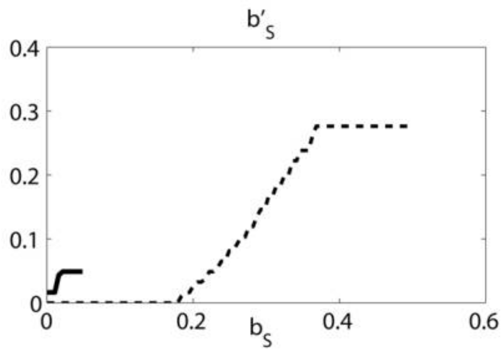
The price functions for long bonds are totally different for the two cases, so do the issuance strategies. I plot the policy function of short- and long-term issuances as a function of short-term outstanding debt in Figure 4.3, controlling long-term debt to be 0. One can find that geometric bond is hardly issued when the current income y_t is low. But when the debt instruments are zero-coupon bonds, both short- and long-term debts are traded more in the markets. The optimal policy even prescribes to issue more 2-periods bonds when relatively high level of short-term debt is matured.



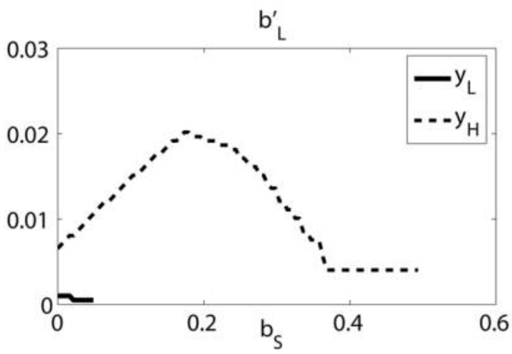
(a) $b^{1\prime}(b^1, 0)$, 0-coupon case



(b) $b^{2\prime}(b^1, 0)$, 0-coupon case



(c) $b^{1\prime}(b^1, 0)$, geometric bond case



(d) $b^{l\prime}(b^1, 0)$, geometric bond case

FIGURE 4.3: Issuance Policy Functions

4.3.3 Highlight the constraints brought by geometrical bond

The reason why geometrical bond increase the sovereign's borrowing cost and default probability is that it actually brings more constraints for the borrower. In real world, the fiscal authority can issue debts with different maturities, which can be seen as a portfolio of multiple zero-coupon bonds, say $b^1, b^2 ... b^n$ chosen freely by the government. Geometric bond actually imposes additional constraint on each maturity, which must satisfy $b_t^1 = \delta b_t^2 = \delta^2 b_t^3 ... = \delta^n b_t^n$. The issuance portfolio at time t has n degrees of freedom without this constraint but now has only one degree of freedom if issued in geometrical bond. When the geometric bond is applied to analyze the maturity structure of emerging market, it exacerbates two problems:

Maturity mismatch If a government is only allowed to issue 1-period short-term debt and geometrical bond, the outstanding debt portfolio can only be a decreasing function of time to maturity. $\mathbf{B}_t = (b_t^1 + b_t^L, \delta b_t^L, \delta^2 b_t^L, ...\delta^n b_t^L, ...)$, I plot this debt profile in Figure 4.4. One can see that the debt burden is always front-loaded, which means the sovereign can only borrow more from the near future than the far future. This is not the way those emerging economies want to align their payments when the economy is in recession. On the contrary, they want to borrow more from the far future than the near future when the current GDP level is low because income shocks are persistent and the recession is not likely to end in short time. But the geometrical bond actually imposes the government to borrow more from the near future than the near future, causing a mismatch between the maturity of debt and income, hence increases the default probability and borrowing cost.

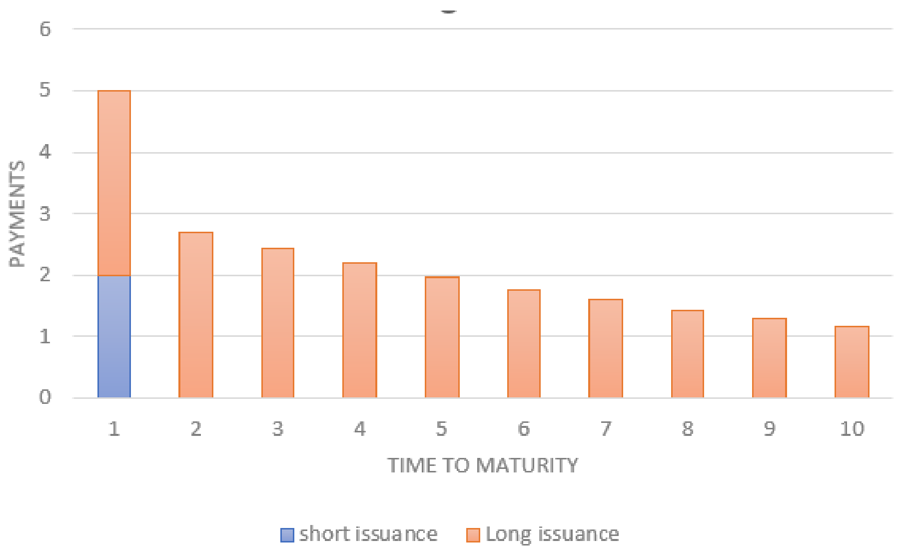


FIGURE 4.4: Outstanding debt as a function of time to maturity (2 units of 1-period short-term bonds and 3 units of geometrical bonds)

Overlapping of claims The second problem brought by the special payment schedule of geometric bond is the overlapping of claims of old and new debt, which makes new debt "dilutes" the existing debt directly. In table 4.2, I list the payment schedules of geometric debts issued in periods $t-1$ and t. One can find that the geometric bond issued in period t needs to be repaid with the same schedule as that issued in period $t-1$, which means issuances at period t increases the default probability of the bonds issued in $t-1$ directly. Actually all the geometric bond issuances overlaps, which means the long bonds issued in any period have to be diluted by the later issuance.

TABLE 4.2: Payment schedule of geometric bond

Periods	$t-1$	t	$t+1$	$t+2$	$t+3$	...
Old Debt	issue date	1	δ	δ^2	δ^3	...
New Debt		issue date	1	δ	δ^2	...

But the "dilution" effect of zero-coupon bonds is less severe. In Table 4.3, one can find the payment schedule for a 10-year zero-coupon bond. The long bond issued at period $t-1$ matures and needs to be repaid at $t+9$, the one issued 1 period later is supposed to be repaid 1 period later. Such debt instrument enables the sovereign to align the payments to different periods such that the claims of the later issuances will not crowd in to the maturity date of the old debt. But this does not mean the removal of "dilution" effect by issuing zero-coupon bonds. The effect still exists, since the accumulation of debt maturing at $t+1$ will lower the issuance price of short-term debt at t, increasing the default probability at t indirectly.

TABLE 4.3: Payment schedule of zero-coupon bond

Periods	$t-1$	t	...	$t+9$	$t+10$	...
Old Debt	issue date	0	0...	1	0	...
New Debt		issue date	0...	0	1	...

4.4 Conclusion

Geometric bond actually imposes additional constraints on government's debt issuance and will lead to greater default probability and welfare loss. The special payment schedule of it will exacerbate the debt "dilution" effect priced in long-term debt. Although modelling long-term debt as geometric debt could circumvent the curse of dimensionality in solving dynamic models, it is not sufficient to analyze the maturity structure of sovereign debt. More persuasive theorem is needed to explain why emerging economies borrow in short terms in financial stress time.

References

Akerlof, G.A., Dickens, W.T., Perry, G.L., Gordon, R.J., Mankiw, N.G., 1996. The macroeconomics of low inflation. Brookings papers on economic activity 1996, 1–76.

Aoki, K., 2003. On the optimal monetary policy response to noisy indicators. Journal of monetary economics 50, 501–523.

Arellano, C., Ramanarayanan, A., 2012. Default and the maturity structure in sovereign bonds. Journal of Political Economy 120, 187–232.

Barro, R.J., 1979. On the determination of the public debt. Journal of political Economy 87, 940–971.

Bassetto, M., 2014. Optimal fiscal policy with heterogeneous agents. Quantitative Economics 5, 675–704.

Benigno, P., Antonio Ricci, L., 2011. The inflation-output trade-off with downward wage rigidities. American Economic Review 101, 1436–1466.

Broner, F.A., Lorenzoni, G., Schmukler, S.L., 2013. Why do emerging economies borrow short term? Journal of the European Economic Association 11, 67–100.

Campbell, J.Y., Mankiw, N.G., 1989. Consumption, income, and interest rates: Reinterpreting the time series evidence. NBER macroeconomics annual 4, 185–216.

Campbell, J.Y., Mankiw, N.G., 1991. The response of consumption to income: a cross-country investigation. European economic review 35, 723–756.

Chamley, C., 1986. Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica: Journal of the Econometric Society , 607–622.

Chari, V.V., Christiano, L.J., Kehoe, P.J., 1994. Optimal fiscal policy in a business cycle model. Journal of polítical Economy 102, 617–652.

Cloyne, J., Ferreira, C., Surico, P., 2016. Staff working paper no. 589 monetary policy when households have debt: new evidence on the transmission mechanism .

Daly, M.C., Hobijn, B., 2014. Downward nominal wage rigidities bend the phillips curve. Journal of Money, Credit and Banking 46, 51–93.

Debortoli, D., Galí, J., 2017. Monetary policy with heterogeneous agents: Insights from tank models .

Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. The American economic review 67, 297–308.

Fahr, S., Smets, F., 2010. Downward wage rigidities and optimal monetary policy in a monetary union. Scandinavian Journal of Economics 112, 812–840.

Galí, J., 2011. The return of the wage phillips curve. Journal of the European Economic Association 9, 436–461.

Hatchondo, J.C., Martinez, L., Sosa-Padilla, C., 2016. Debt dilution and sovereign default risk. Journal of Political Economy 124, 1383–1422.

Hauk, E., Lanteri, A., Marcet, A., 2021. Optimal policy with general signal extraction. Journal of Monetary Economics 118, 54–86.

Judd, K.L., 1985. Redistributive taxation in a simple perfect foresight model. Journal of public Economics 28, 59–83.

Kaplan, G., Violante, G.L., 2014. A model of the consumption response to fiscal stimulus payments. Econometrica 82, 1199–1239.

Kim, J., Ruge-Murcia, F.J., 2009. How much inflation is necessary to grease the wheels? Journal of Monetary Economics 56, 365–377.

Kingston, G., 1991. Should marginal tax rates be equalized through time? The Quarterly Journal of Economics 106, 911–924.

Lucas Jr, R.E., Stokey, N.L., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of monetary Economics 12, 55–93.

Morris, S., Shin, H.S., 2018. Central bank forward guidance and the signal value of market prices, in: AEA Papers and Proceedings, pp. 572–77.

Nimark, K., 2008. Dynamic pricing and imperfect common knowledge. Journal of monetary Economics 55, 365–382.

Phillips, A.W., 1958. The relation between unemployment and the rate of change of money wage rates in the united kingdom, 1861-1957. economica 25, 283–299.

Svensson, L.E., Woodford, M., 2003. Indicator variables for optimal policy. Journal of monetary economics 50, 691–720.

Svensson, L.E., Woodford, M., 2004. Indicator variables for optimal policy under asymmetric information. Journal of Economic Dynamics and Control 28, 661–690.

Werning, I., 2007. Optimal fiscal policy with redistribution. The Quarterly Journal of Economics 122, 925–967.

Zhu, X., 1992. Optimal fiscal policy in a stochastic growth model. Journal of Economic Theory 58, 250–289.