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Networks in Finance:
Asset Pricing, Market Structure and Information

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UNIVERSITAT AUTÒNOMA DE BARCELONA

DOCTORAL THESIS

Networks in Finance: Asset Pricing, Market Structure and Information

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A meus pais, meu irmão, e meus avós.

Two roads diverged in a wood, and I

I took the one less traveled by,

And that has made all the difference."

(The Road not Taken - Robert Frost)

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I dedicate this doctoral thesis to the best part of me: my mom and dad, Andréa and Rômulo, and my brother, Rominho. Our unconditional love and faith are my anchor. They have taught me to be bold and humble, true to our values, and to "live life to its fullest".

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The completion of my PhD is much more than the culmination of my academic formation. It represents personal growth, a deeper knowledge of who I am and my aspirations. For me, being an academic is no different than being an athlete. Gradually, you learn to thrive and fail, to appreciate the sense of community, to try to take things seriously but lightly, and to know that there is always something more: to learn, to be curious about, to try, to enjoy. I look forward to my coming research projects and to running more marathons.

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Contents

Abstract

This doctoral thesis studies financial markets through the lens of networks theory. The three chapters are united by a theoretical approach, highlighting particular issues in finance: asset pricing, market structure, and information.

Chapter 1 is on market structure and financial networks. I study asset pricing when re-trade can take place in co-existing and interconnected markets. In my framework, there is a divisible asset and a finite set of traders. They are distributed over a trading network. Traders can acquire shares at a common price, and then they may trade with their connections at possibly different prices. I find that trading centrality, a novel network metric, is a sufficient statistic for the equilibrium. Trading centrality processes information about expected re-trade equilibria, and maps it to traders behavior before trade. A traders asset acquisition is proportional to his centrality, and the asset common price is defined by aggregating centrality globally. For the re-trades in the network, a trader demands the gap between his optimal level of asset and his centrality; while each price is defined by aggregating centrality locally in the sellers network. I investigate what market outcomes and welfare arise at different trading networks. Implications for asset issuance and interdealer markets are examined.

Chapter 2 is about the dynamics of stock prices and social influence. I develop a dynamic asset pricing model in which subject beliefs about stock price behavior are heterogeneous and susceptible to peer effects. Two types of traders optimally learn from past price realizations and share beliefs in a social network. I show that, at each period, the equilibrium price is a function of traders' beliefs and the network structure. As a result, booms and busts of the price-dividend ratio emerge. The most (least) speculative trader is the most influential during booms (busts). More connected networks exhibit less volatile price dividend ratio, booms and busts episodes last longer, and the average price realization is higher. Also, there is less disagreement in beliefs. However, if traders of the same type are highly interconnected stock market volatility is higher and booms and busts are shorter. The model captures relevant empirical features of stock prices and returns, and it is also consistent with the survey evidence on investor expectations.

Chapter 3, joint with Prof. Victoria Vanasco, investigates how social linkages interact with asymmetric information about asset fundamentals to shape market outcomes. We consider a market where traders are embedded in a social network and form beliefs about market size based on their connections. We distinguish two channels underpinning market outcomes: heterogeneous information about asset fundamentals and social linkages. In equilibrium, the network structure determines the information revealed and perceived by traders from price. What, in turn, pin down optimal demands. Asset price is determined by a single network coefficient capturing the direct effect of connections in traders beliefs. The model sheds new light on the implication of the increasing presence of retail investors in financial markets and the emergence of social platforms for engagement and investing.

Chapter 1

Asset Pricing and Re-Sale in Networks

1.1 Introduction

For a variety of financial assets, traders first acquire shares in the primary market of asset issuance and then go on to trade those shares in co-existing and interconnected secondary markets. This is the case, for example, for fixed-income securities such as corporate and government bonds. In the United States alone, issuance and trading values of fixed-income markets have surmounted $7,189.9 billion and $931.1 billion, respectively, in 2022 (SIFMA). Secondary markets typify the ubiquitous decentralized nature of modern financial markets in that assets move around by being re-traded among different counterparties at different prices. In contrast, primary markets are centralized in the sense that the traders bid for asset shares and pay a common price, the issuance price.

How asset price is affected by such dichotomy in trading configurations? This paper examines how the primary market is affected by the trading network of secondary markets. I study the decision of traders to acquire asset shares in anticipation of possibly being able to trade those shares later in the trading network. I reveal that traders act strategically in response to the interdependent terms of future re-trades. That's because asset shares are bought at issuance in anticipation of a variety of re-trade markets. But prices and demands of re-trades depend on how many shares traders already hold. As a consequence, I show that asset price in the (centralized) primary market is determined by the trading network structure of the (decentralized) secondary markets.

In the model, a finite set of traders must decide how much of an asset to purchase in two periods. The amount of asset shares available is exogenously-fixed. Traders have identical single-peaked preferences and are distributed over an exogenous *trading network*. At period one, everyone participates in the *primary market* (PM). Afterwards, trade happens in *local markets* (LM) described by the trading network.[1] With some positive probability, at most one trader is selected and forced to re-sell all his shares. This seller determines the *active local market* at period two where his connections are the buyers. All markets operate as a one-sided uniform-price auction.[2] Local markets can be thought of as meeting places where traders *can* trade, and the active local market as *when* exchanges are realized. The probability of being the seller is referred as the re-sell shock, and it captures a sudden need of liquidity.

My main contribution is to show that the equilibrium is described as a function of a unique simple measure, *Trading centrality*. Trading centrality is a novel network metric and a sufficient statistic for all market outcomes. Each trader's PM demand is given by his centrality and his LM demand is simply the gap between the his optimal holdings and his centrality. PM price is defined by aggregating trading centrality globally, and each LM price is defined by aggregating trading centrality locally in the network. Lastly, welfare is a weighted sum of traders' centrality with weights that are functions of the trading network structure.

At the core of the model are the conflicting incentives to acquire asset shares in anticipation of future re-trades. On the one hand, traders can secure a level of asset holdings in the PM. On the other hand, they face higher competition there. I show that this conflict is resolved by an endogenous substitutability of demands. A trader optimally decides to invest in the opposite way of his direct and indirect connections.

As a buyer in the PM,[3] a trader demand less when others demand more in expectation of a lower price in period two. That's because there will be i) more being sold - by his connections; and ii) less being demanded - by his competitors (i.e. they will be closer to their optimal holdings). Thus, traders defer asset consumption from the first to the second period, and the extent they do so depends on the local markets they can participate.

Traders use their network position to conjecture the set of local markets equilibria that could arise and, contingent on that, they strategically decide PM asset acquisition.

[1]The trading network describes secondary markets which are interliked through traders. It captures the fact that traders can participate in many different types of trading venues for possibly nondisjoint subsets of traders.

[2]As I explain later, traders are price-takers and thus truthful when submitting demand schedules in all markets they can participate. This means that traders ignore the *direct* impact of their bids on prices, as in Swinkels (2001) and Feldman et al. (2015). Price-taking assumption is the main departure from the well-known imperfect competition framework (as in Kyle (1989), Vives (2011) and Malamud and Rostek (2017)). However, it allows me to distill the equilibrium effects coming *only* from the structure of the trading network. And it is enough to guarantee the existence and uniqueness of equilibrium.

[3]Notice that, at period one, traders make decisions anticipating they can be both a buyer and a seller. However, as I show in the Section A.2, the selling motive is negligible in large enough networks - which I focus on.

Network position is the only dimension of ex-ante heterogeneity and it is the unique source delivering difference in demands. The trading network enables trade and, at the same time, constrains and correlates traders' behavior. The environment boils down to a one-shot, simultaneous-move network game of strategic substitutes played in the PM. Traders' best-respond to each others' demand schedule and the Nash equilibrium is given by the trading centrality.

With trading centrality in hand, I am able to investigate what market outcomes to expect across different and arbitrary trading networks. In term of prices, there are three main findings. First, symmetric networks exhibit higher PM price than core-periphery networks. In fact, PM price is bounded by its level on the extreme cases of such structures: the price is the highest on the complete network, and the lowest in the star. Second, changes in the degree distribution,[4] as stochastic dominant shifts, lead to monotonic changes in trading centrality and, in turn, in the PM price. Increasing connectivity unambiguously increases PM price, while increasing degree inequality unambiguously decreases PM price. Third, only the star trading network is stark enough to exhibit price increase over time. That's because its core trader is the unique one who obtains profits from re-selling. In all other network structures, PM price is higher than any LM price. And so re-selling is costly.[5]

Welfare, in terms of total expected utility, conversely echoes the findings of PM price. Thus, the network structure delivering lower (higher) PM price, the core-periphery (symmetric), has higher (lower) welfare.[6] All together, this reveals that reducing trading asymmetry, either by increasing network connectivity and/or reducing degree inequality, must not necessarily be welfare enhancing because of two opposing effects. On the one hand, more connected and equal trading networks are allocative efficient, as traders have the same (or complete) local market participation; on the other, prices are higher because competition is greater and uniform.

My paper speaks to the behavior of dealers, the financial intermediaries for assets traded off-exchange.[7] Dealers often absorb substantial inventory position at issuance or from their costumers, and then use the interdealer market to offload these positions. I attend to two unaddressed questions regarding how the interdealer network structure influences financial markets. I do so by leveraging the rationale that dealers' willingness to

[4]In a trading network, degree refers to how many connections (i.e. counterparties) a given trader (node) has. I define connectivity as the average degree, and degree inequality as degree variance.

[5]Duffie (2010) discusses the large body of research aiming to explain price reaction to supply (or demand) shocks observed in several financial markets. He argues that price concessions are given by those who have limited opportunities to trade with counterparties, in line with findings.

[6]Just as with PM price, I find that welfare is the highest (lowest) in the star (complete) network.

[7]Dealers are the backbone of trades for bonds (government, corporate and sovereign ones), derivatives, commodities, and currencies - to name a few. (See He et al. (2017) among others).

take on inventory is affected by their subsequent trading network; and prices and quantities in the interdealer network are influenced by their current inventory holdings.

First, my findings provides a justification for the often observed core-periphery trading network among dealers themselves.[8] Such structure coordinates dealers' inventory positions in a way that ensures the lowest average cost to take on asset shares, making them all better off. This suggests that the core-periphery interdealer network supports dealers' inventory management and guarantees market liquidity in secondary markets. Second, I reveal a novel effect of the interdealer network structure: it determines the issuance price of an asset, thus regulating credit provision for the issuers of securities (such as governments and firms) in the primary markets.

To gain further insights on how my framework maps to the data, I proceed in two distinct ways. I first layout the empirical implications to the interdealer market. Trading centrality helps rationalize the inconclusive evidence on whether central dealers[9] have better or worse terms of trade. I show that the trading centrality can induce both centrality premium and discount. But this ambiguity is resolved once we take into account the entire structure of the interdealer network. Trading centrality also provides a novel liquidity measure that only requires information about the trading network structure, and it is informative of prices and quantities.[10] Additionally, I illustrate the empirical application of the model with data on the US Corporate bonds secondary markets.[11] I document an interdealer network with a core-periphery structure, and then I analyze how it relates to the observed interdealer trades. As my theory predicts, I find central dealers sell more and at a higher price; and buy less and at a lower price.

Outline: The rest of the paper is organized as follows. section 1.1 concludes with the related literature. section 1.2 contextualizes my framework into the interdealer market for off-exchange securities. section 1.3 introduces the model and section 1.4 solves it. section 1.5 gives the main result. section 1.6 analyzes how the network structure affects equilibrium and section 1.7 studies welfare. section 1.9 discusses different extensions. section 1.10 concludes. Details and proofs are found in the Appendix.

[8] Recent empirical studies document that the interdealer market can be seen as a stable trading network with a core-periphery structure. Such interdealer network has high trading frequency and volume, with spillover effects on the overall market outcomes. See Section 1.2.

[9] In finance, essentially all studies so far define centrality by standard network metrics such as degree and eigenvector centrality.

[10] As I show in Section 1.5, this liquidity measure quantifies the re-sell cost: the difference between PM price and a LM price. It is similar to Kyle's lambda (Kyle (1985)) in that it measures the equilibrium price impact of a sell order flow.

[11] I use a restricted version of the TRACE data used by Friewald and Nagler (2019) which the authors made publicly available for replication purposes.

Related Literature: This paper is related to three areas of research: decentralized markets, network games, and over-the-counter (off-exchange) financial markets.

I share the perspective of Malamud and Rostek (2017) of modelling markets as incomplete and co-existing: traders cannot exchange with one another at all times and they can participate in more than one trading venue. The novelty of my paper is to combine both centralized and decentralized trades in a unified and dynamic framework.[12] Most importantly, decentralized markets are random realizations and only take place *after* the centralized one. Moreover, several other features set my paper apart from theirs. Here traders are price-takers, there is just one asset, and I restrict the pricing protocol to a one-sided uniform-price auction.

My paper allows for any market structure "between" centralized and bilateral trading.[13] A vast literature on decentralized markets makes the later extreme assumption. Here there are two modelling approaches. Search models, with the seminal contribution of Duffie et al. (2005) in finance; and network models, such as Kranton and Minehart (2001), Corominas-Bosch (2004), Manea (2016). While suitable for many markets such as the over-the-counter one between dealers and customers, in reality several assets are not traded as pairwise exchanges. Treasure securities for instance, are sold in auctions to a set of 20-30 dealers. Interdealer broker systems, electronic trading platforms for interdealer trades, also work as auctions.

The multi-lateral aspect of trades is also a natural implication of auction as a trading protocol. In my model, traders play a game by submitting demand schedules in each sub-graph of the network. Allowing (not assuming) strategic behavior is central in my framework. Here I build upon the game-theoretical view of decentralized trading with imperfect competition - as in Kyle (1989), Vives (2011) Rostek and Weretka (2012) and Rostek and Weretka (2015) to name a few. In this line of work, (finite) traders account for their impact on price and, because of that, they strategically "shade" their bids in the demand game.[14] My paper, in contrast, assumes that traders are price-takers and thus truthful.[15] Although a strong departure, price-taking renders great tractability of the model and ensures the equilibrium outcomes are driven solely by the trading network, without

[12] Somewhat related, Rostek and Yoon (2021) have unified imperfect competition and decentralized markets in a framework.

[13] In section A.8 I show that restricting the model to bilateral trades would miss all the interesting forces coming from the trading network that drives equilibrium outcomes.

[14] More specifically, imperfect competition means equilibrium is determined by a uniform-price (double) auction with traders submitting demand schedules taking into account their endogenously-determined price impact.

[15] My model accommodates imperfect competition, which I discuss in section 1.9. This is also work in progress and available upon request.

compromising the strategic aspect of trades.[16]

My paper is a novel application of games in networks. As I show later (section 1.4), the model is set of network games of global strategic substitutes and I rely on the findings of Bramoullé et al. (2014) and Bramoullé and Kranton (2016) to characterize equilibrium.

As aforementioned, a core motivation of this paper is the interdealer market and so my paper relates to the research on off-exchange markets. Given this particular interest, I devote section 1.2 to discuss recent empirical findings of the literature and my contributions

1.2 Interdealer Networks and Off-Exchange Securities

A substantial proportion of financial instruments - the so-called "off-exchange" assets such as Treasury and corporate bonds, debt securizations, currencies, etc. - are traded in primary and secondary markets. Primary markets are for asset issuance and serve to raise capital. They are centralized in the sense that dealers bid to acquire shares, usually in uniform-price auctions, and no trade happens among dealers themselves. Secondary markets is where trades actually happen in a decentralized way, and asset prices are dispersed. In the bonds market, for example, a firm or government (the issuer) creates a new bond and allocates it to dealers[17] at a common price; who then take the bond to secondary markets.

Primary and secondary markets are paramount for the well-function of financial markets and the economy. For instance, as of 2021, US fixed-income securities have issuance value of roughly $US\$13.5$ billion, outstanding value of $US\$52.9$ trillion and average daily traded value of $US\$969$ billion (SIFMA). The backbone of such trading activity are dealers,[18] the market-makers, who provide immediacy to other traders and ensure market liquidity. There are currently 3,394 dealers registered with Financial Industry Regulatory Authority (FINRA) in the US alone.

Dealers do not operate in a vacuum. Rather, when making markets, they rely on being able to trade with one another. Thus, the existence of a trading network among dealers

[16] Strategic behavior in my model arises purely from the interaction of re-sale risk and trading frictions imposed by the trading network. Not concerns about price impact as in the imperfect competition models.

[17] Usually investors do not participate in the primary market. The noteworthy example is the US Treasury securities market where a group of more than 20 primary dealers commit to buy large quantities of Treasuries every time the government issues debt, and stand ready to trade them in the otc market (FINRA). The use of primary dealers is common across many countries and it has been in force since 1960 (Arnone and Ugolini (2005)).

[18] A dealer, or broker-dealer, is a financial institution in the business of buying or selling securities on behalf of its customers or its own account or both. Dealers must be registered with a specific regulatory authority, such as Financial Industry Regulatory Authority (FINRA).

themselves.[19] Interdealer trades are useful because dealers often absorb substantial inventory position in primary markets or from their costumers, and then use the interdealer market to offload these position and rebalance their inventory.[20]

In reality, most interdealer networks have a stable core-periphery structure.[21] This is the case, for instance, for US corporate bonds (Dick-Nielsen et al. (2020), Goldstein and Hotchkiss (2020), Di Maggio et al. (2017b)); US foreign exchange (Hasbrouck and Levich (2020)); US debt securization (Hollifield et al. (2017)); US municipal bonds (Li and Schürhoff (2019)). This indicates that trading relationships exist and are persistent, and typically, a few large dealers (the core) are responsible for large share of the trading volume.

An extensive and growing empirical literature[22] relates the dealers' position in the network to their trading behavior. The recurring finding is that a dealer's centrality, his "importance" in the network, is a determinant for his: bid-ask spreads; trade volume and frequency; clientele characteristics; and trade execution speed. Moreover, there is heterogeneity at the linkage (relationship) level: a dealer's trading price, frequency and volume depends on the identity of his counterparty: if it is a customer or another dealer, and which customer/dealer is. Recent studies (Eisfeldt et al. (2018) and Di Maggio et al. (2017b)) also show that changes in the interdealer network, by the exit of a dealer, has significant impact in market outcomes.

However, evidence on whether centrality makes a dealer to have better or worse terms of trade, in terms of bid-ask spread, is still inconclusive. Li and Schürhoff (2019) and Di Maggio et al. (2017b) find a centrality premium: core (central) dealers charge a wider spread than peripheral dealers. Di Maggio et al. (2017b) also document that more central dealers pay lower spread. Meanwhile, Hollifield et al. (2017), Goldstein and Hotchkiss (2020) and Dick-Nielsen et al. (2020) find centrality discount: core dealers charge a narrower spread. Goldstein and Hotchkiss (2020) and Dick-Nielsen et al. (2020) are two exceptions who also look at interdealer trades, and they both find centrality premium.

[19] By forming trading relationships, dealers can reduce trading frictions, such as different forms of trading costs, funding constraints, search, and informational frictions. See Vayanos and Wang (2012).

[20] That's because inventory is risky and costly, and interdealer trades are the main tool to manage it.

[21] In fact, the observed majority of financial networks are core-periphery. See Bech and Atalay (2010) for evidence on US federal funds market;Boss et al. (2004), Craig and von Peter (2014), in 't Veld and van Lelyveld (2014) for interbank market in other countries. This is also true for the dealer-client networks. See Hasbrouck and Levich (2020), Hendershott et al. (2020), Hollifield et al. (2017), Li and Schürhoff (2019), Di Maggio et al. (2017b), Kondor and Pintér (2022) Czech et al. (2021) for otc markets; and Di Maggio et al. (2017a) for the stock market.

[22] This is possible due to the structure of financial data and the decentralized nature of secondary markets that allows us to identify different counterparties and construct trading networks.

The next figures show examples of real core-periphery interdealer networks. The more central dealers form the core, while the less central ones the periphery. Centrality is measured by the standard network metrics of degree and/or eigenvector centrality.[23] In the US municipal bonds markets, Li and Schürhoff (2019) infer the interdealer network with 2,238 dealers and a core size of 10 to 30 highly interconnected dealers (Figure 1.1). In the securization markets, Hollifield et al. (2017) document an interdealer network with 658 dealers (Figure 1.2). Both find that the most central dealers are more active and account for the vast majority of trades - what can be seen in the right-hand side network in those figures.

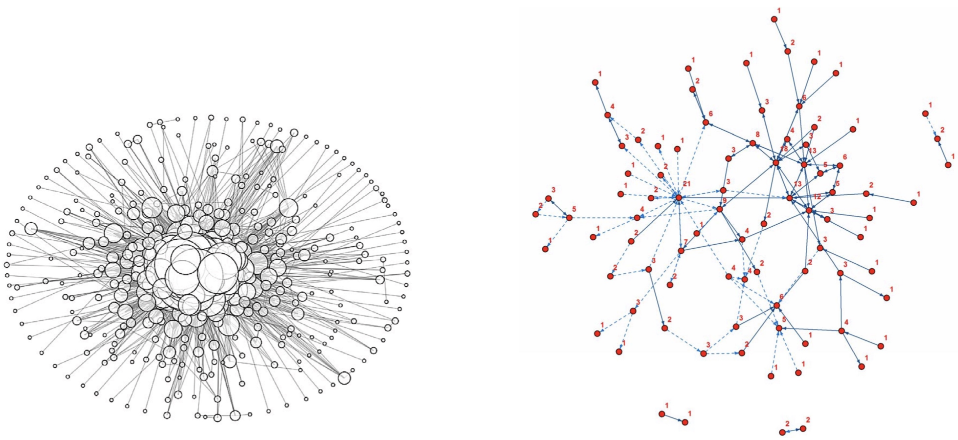


Figure 1.1: Hollifield et al. (2014)
On the left is the interdealer network: each node is a dealer and each arrow represents a directed order flow between a pair of dealers. On the right is the same network but only with the most active dealers. The active network only keep links with at least 50 trades and worth at least $\$10$ million. Dealers are labeled by their degree; links with trades worth more than $\$100$ million are shown in solid lines.

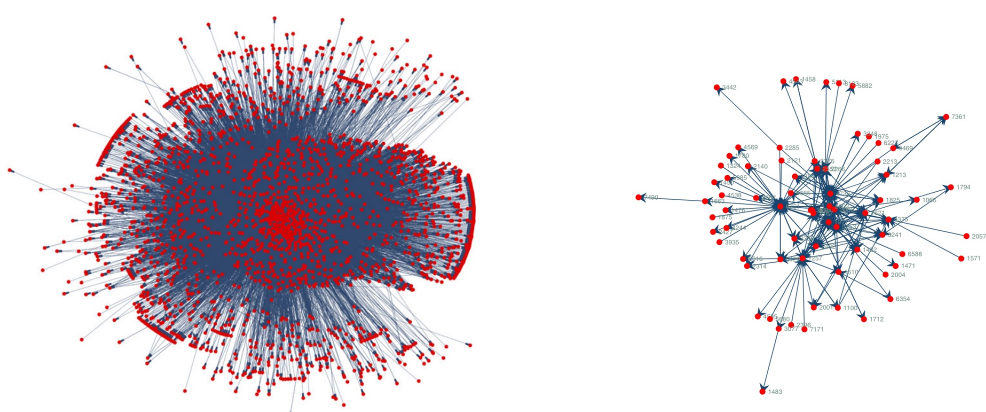


Figure 1.2: Li and Schürhoff (2019)
On the left is the interdealer network: each node is a dealer and each arrow represents a directed order flow between a pair of dealers. On the right is the same network but only with the most active dealers. The active network only keep links that exceeds 10,000 transactions over the sample period.

[23] Degree centrality means that a dealer is more central the more connection he has. Eigenvector centrality also accounts for indirect connections: a dealer is more central as he and his connections are more central.

Figure 1.3 and Figure 1.4 display different interdealer networks in the US Corporate bonds markets inferred by Dick-Nielsen et al. (2020) and Di Maggio et al. (2017b), respectively. Dick-Nielsen et al. (2020) investigate immediacy provision from 2002 to 2013 in a network with 3,499 dealers. In the right-hand side, the (inverse) distribution of eigenvector centrality clearly reveals that most dealers are less central (the periphery) and only a few (the core) are very central. The same is true for the interdealer network reported by Di Maggio et al. (2017b) who look at the entire universe of trades from 2005 to 2011. In the right-hand side, the cumulative distribution of trades as a function of a dealer's centrality shows that the top 50 dealers account for roughly 80% of all transactions.

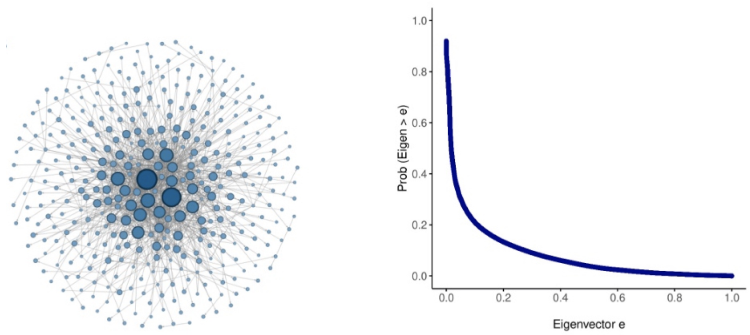


Figure 1.3: Dick-Nielsen et al. (2020)
Interdealer network where the size of a node (dealer) reflects his eigenvector centrality. The inverse distribution of eigenvector centrality is plotted on the right-hand side.

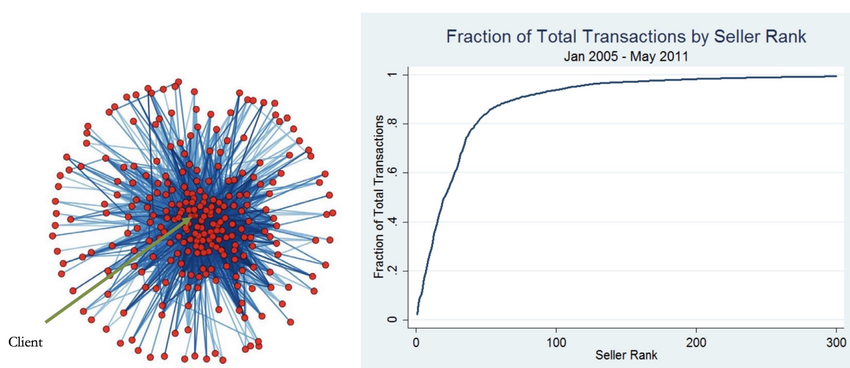


Figure 1.4: Di Maggio et al. (2017b)
On the left, the interdealer network; darker lines indicate higher number of transactions between a pair of dealers. On the right, the cumulative distribution of dealer's eigenvector centrality, measured by selling trades.

Even within the same market, the interdealer network and dealers' behavior vary greatly in the cross-section of assets. Hollifield et al. (2014) analyze various segments of the securitization market during eight months in 2011-2012. Figure 1.5 depicts the interdealer networks, dealer-customer trades, interdealer trades and prices for two securization instruments in their sample. The networks have notable different structures. The left one has roughly a monotonic evolution of prices, but the right one doesn't. Also, bid,ask and interdealer prices are similar in the left network; while in the right one interdealer prices can be above and below costumer-trade prices. It also seems that the volume in each dealer-customer and interdaler segments behave similarly: the right network with more customer trades also have more interdealer trades.

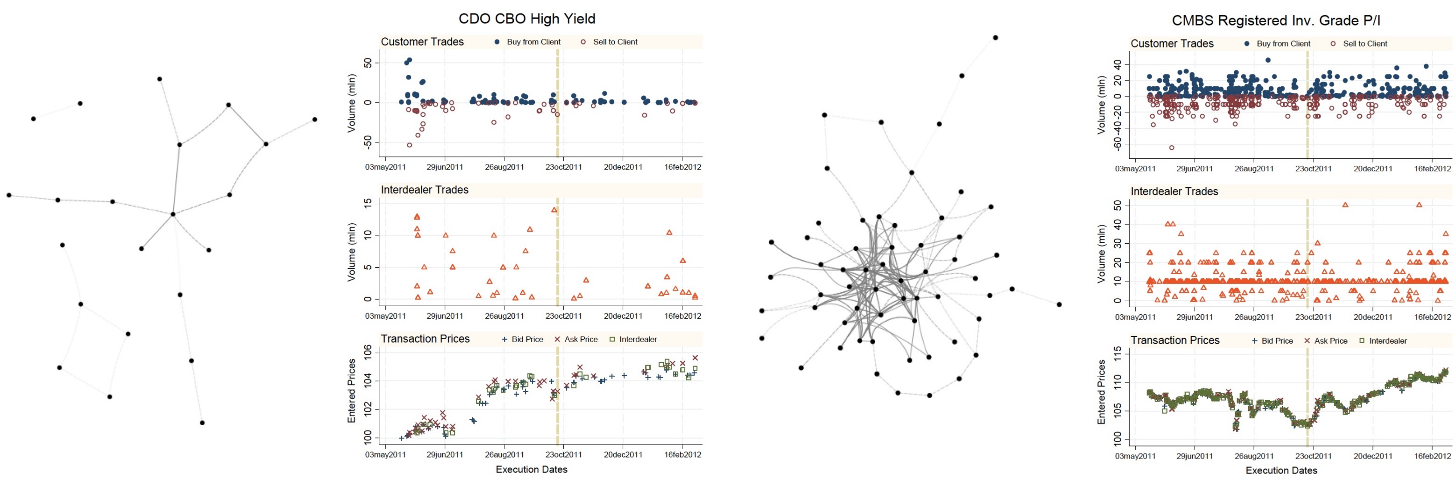


Figure 1.5: Hollifield et al. (2014)
At each panel, on the left is the interdealer networks: darker lines indicate higher number of transactions between a pair of dealers; on the right, the evolution of dealer-customer trade volume, interdealer trade volume and transaction prices.

The take-away so far is that the structure of the interdealer network shapes, to some extent, the market behavior of several financial assets. Perhaps surprisingly, the relationship between primary markets and interdealer networks has received scant attention.[24] After all, dealers are the link between primary and secondary markets. The extent that a dealer can successfully re-sell shares of the asset acquired at issuance relies on him being able to manage his inventory in the interdealer market. At the same time, how much inventory is accumulated at issuance influences liquidity need and provision in the interdealer trades. Hence, there is a two-way feedback effect between primary markets and the interdealer market.

Despite the vast theoretical and empirical literature,[25] two questions remain unaddressed regarding the interdealer network. The first is its relation with the issuance price of an asset. The second is how to explain the mixed evidence on dealers' centrality effects on market outcomes. My paper fills these gaps, and I show that both are related to how the structure of the interdealer network conveys information about *expected and correlated*

[24] I am not aware of any study, either theoretical or empirical, that explores it.

[25] Over the last two decades, the center of attention has been on the otc markets, with the seminal contribution of Duffie et al. (2005). Research has made great progress in advance our understanding of the bilateral trading behavior and its implication for prices and liquidity, both at the dealer-to-client and dealer-to-dealer segments.

terms of trades among dealers themselves and, thus, determines dealers' willingness to take on inventory and asset issuance price.

1.3 The Model

Markets and the Trading Network

There are two periods $t = \{1, 2\}$ and $N > 2$ traders. There exists a divisible asset in exogenous and fixed supply $Q > 0$[26] All traders can acquire asset shares at $t = 1$ in the *primary market* (PM) . Afterwards shares can be re-traded among traders themselves in *local markets*.

There exists a *trading network*[27] in which nodes are the traders and connections determine which traders have access to a particular local market jointly but not separately. A link between i and j means that trade between i and j is *possible*. Formally, the trading network is characterized by the adjacency matrix $\mathbf{G}$ such that $[G]_{ij} \equiv g_{ij} = 1$ if $i \in N$ and $j \in N$ are connected, and $g_{ij} = 0$ otherwise. By convention, $g_{ii} = 0$. The set of linkages of trader $i \in N$ is given by his neighborhood $N_i = \{j \in N : g_{ij} = 1\}$, and i's degree is the number of connections he has: $d_i = |N_i| = \sum_{j \in N} g_{ij} > 0$.[28]

Thus, the trading network summarizes the set of local markets at $t = 2$. There are N of them. Each is defined by a trader's neighborhood. I refer to the local market of trader $i \in N$ as when i sells his shares to his linked traders, his buyers, at an endogenously-determined uniform price (more details below).

Market participation at $t = 2$ is random. With a probability $\phi > 0$, which I refer as the *re-sell shock*, only trader $i \in N$ is selected and is forced to re-sell in his local market. This happens at the same probability ϕ for each trader. The seller establishes the *active local market* at $t = 2$. I make two assumptions:

Assumption 1. *At most one trader experiences the re-sell shock: $\phi < \frac{1}{N}$.*

Assumption 2. *Supply in any local market is inelastic: the seller does not choose his supply.*

[26] This specification may, for example, capture cases in which dealers face a common outside opportunity for customer sell order, or when they allocate a new security at issuance.

[27] The trading network is unweighted, undirected, fixed, exogenous and known.

[28] I focus my analysis on trading networks with a minimum degree of one, i.e. every trader has at least one connection. However, this is by no means a restrictive assumption, it is just the most interesting case. It also does not mean that the trading network must be connected. In the Appendix, I provide my main results for when there are "isolated" traders in the network. All the analysis and intuition presented in the main paper hold.

By Assumption 1, there is only *one* active local market or *none*.[29] Local markets can be thought of as meeting places where traders *can* trade, and the active local market as *when* exchanges are realized.[30]

Assumption 2 means that all markets are one-sided. The decision of a trader is how many shares to purchase in each market he has access to. The re-sell shock ϕ is interpreted as a sudden need to unload shares to exit the market. Re-trade in the local markets is for immediacy provision among traders themselves and, in turn, it enables the increase or decrease of asset holdings (i.e inventory management - more details below).

Traders

Traders have initial wealth $w > 0$, and no one is endowed with asset shares. Traders' goal is to build up asset inventory q_i.[31] Let $q_{i,1} \geq 0$ denote how many shares trader $i \in N$ acquires in the PM, and $q_{i,s} \geq 0$ the amount bought in the local market of seller $s \in N_i$. By assumption, if i and s are not connected they cannot trade and so $q_{i,s} = 0 \; \forall s \notin N_i$. Also, if i is the seller he must liquidate his position and so $q_{i,i} = -q_{i,1}$. At the end of period two, a trader's inventory q_i is the sum of the shares purchased at each period, $q_i = q_{i,1} + q_{i,s}$.

Each trader receives the net payoff of his trading activity. It is defined as the total utility derived from inventory q_i minus the total payment. Purchasing quantities $(q_{i,1}, q_{i,s})$, he enjoys a utility of

$$U(q_{i,1}, q_{i,s}) = (q_{i,1} + q_{i,s}) - \frac{1}{2}(q_{i,1} + q_{i,s})^2 \tag{1.1}$$

and pays $P_1 q_{i,1} - P_s q_{i,s}$ where P_1 is the PM price and P_s the price of seller s.

Preferences are single-peaked and traders have an optimal inventory of 1[32] Intuitively, a trader buys shares at each period to reduce the gap between the optimal inventory and his current one. But holding inventory entails a cost of $\frac{1}{2}q_i^2$. For dealers in financial markets, the costly inventory can be due to several reasons (e.g. regulatory capital or collateral requirements). Here it represents the expected cost of being forced to re-sell to raise

[29] In principle, there are 2^N possible states of the world: $\Omega = \varnothing \cup \{\omega_s : s \in \{1, 2..., 2^{N-1}\}\}$. I assign probability $\phi > 0$ to each state $s \in [1, N]$, which represents the identity of the single trader $i \in N$ who experiences the re-sell shock; probability $1 - N\phi$ to the empty state, in which no trader experiences the shock; and probability zero to the remaining $s \in [N+1, 2^{N-1}]$ states, which represents the possible combinations of more than one trader getting shocked.

[30] In other words, the re-sell shock "activates" a local network which is a subgraph of the entire network.

[31] For dealers in off-exchange markets, inventory is used to facilitate future trades with their customers in the otc market.

[32] This is a normalization. The more general setup with individual asset valuation α_i, possibly heterogeneous, is presented in section 1.9.

liquidity by quickly disposing inventory into a restricted, and possibly illiquid, local market (Duffie (2010),Duffie and Zhu (2017)).

The role of having two consecutive markets is best understood by analyzing the partial utility of the quantity traded in either period:

$$\frac{\partial U(q_{i,1}, q_{i,s})}{\partial q_{i,m}} = 1 - q_{i,1} - q_{i,s} \qquad m = \{1, s\} \tag{1.2}$$

This partial utility is the trader's marginal willingness to pay for the asset in market m given that he obtains $q_{i,-m}$ shares in the other market. It decreases in both quantities traded. Demands are then substitutes across markets. That's because more of the asset is preferred rather than less only up to the optimal inventory, as inventory is costly[33]

Pricing Mechanism

Traders are price-takers[34] and every market (local or otherwise) operates as a one-sided uniform-price auction. Each trader $i \in N$ submits a demand schedule $q_{i,m}(\cdot; P_m)$ in every market $m = \{1, \{s\}_{s \in N_i}\}$ he can participate. Equilibrium price in a market is determined by equating aggregate demand of the participant buyers with the asset inelastic supply.

The primary market features complete participation and a global market clearing condition holds. The PM price P_1, common to all traders, is given by

$$\sum_{i \in N} q_{i,1}(\cdot; P_1) = Q \tag{1.3}$$

The price of a local market is seller-specific. For a seller $s \in N$, his local market price P_s is given by the local market clearing condition

$$\sum_{i \in N_s} q_{i,s}(\cdot; P_s) = q_{s,1}(\cdot; P_1) \tag{1.4}$$

The pricing mechanism indicates that a feedback effect across traders' demands emerge. There are two reasons for that. First, even though the asset supply is exogenous in the PM, it is endogenous in every local market: the shocked trader re-sell his PM holdings. Second, the buyers in a local market may already have acquired shares in the PM what influences their willingness to pay for the seller's supply.

[33]In financial markets, a dealer's inventory is used in his intermediary activity with customers in the otc markets. Inventory allows dealers to provide immediacy and liquidity to costumers. I abstain from the discussion of dealer-customer trades as my focus is on inter-dealer trades.

[34]That is, traders are truthfull and ignore their direct price impact.

Figure 1.6 depicts the timeline of the model.

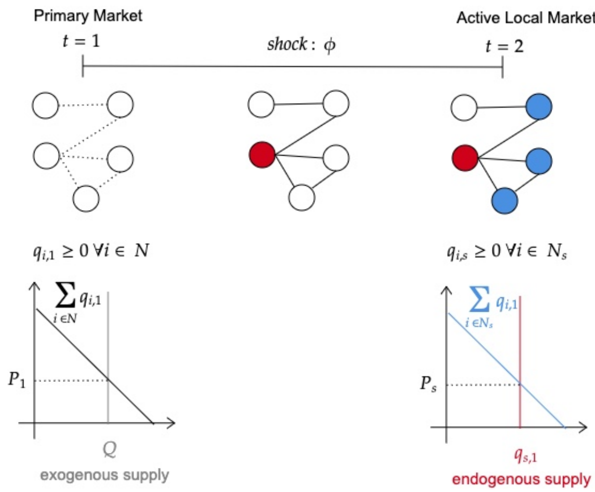


Figure 1.6: Timeline

One attractive feature of the baseline model is that it can be solved in closed form and is thus a parsimonious workhorse with which to develop intuition (section 1.4). The tractability relies on the core assumptions I make. Although not very general, they capture realistic features of the interdealer market, which I discuss in section A.3. In section 1.9 I present extensions of my framework that allows for: heterogeneity in asset valuation and risk preference; expected fundamental asset returns; and imperfect competition.

1.4 Equilibrium Analysis

Notation:

Bold lowercase letters refer to N-dimensional vectors and bold uppercase letter to N-square matrices. All Primary Market variables $(\{q_{i,1}(\cdot)\}_{i\in N}, P_1(\cdot))$ are conditioned on the primitives of the model: the trading network $\boldsymbol{G}$ and the shock parameter ϕ. I omit such notation for the sake of clarity

The model is solved backwards. I first characterize the local market equilibrium given a shock realization. Then, the equilibrium in the primary market is determined. In section A.4 I solve in detail traders' optimization problem.

Traders are rational and forward-looking. They decide their optimal demand schedules in anticipation of the re-sell shock and the different local markets that can take place at period two. Trader $i \in N$ chooses $(q_{i,1}(\cdot), q_{i,s}(\cdot))$ to maximize his expected net payoff,

$$\max_{q_{i,1}(\cdot),q_{i,s}(\cdot)} \quad E\left[(q_{i,1}+q_{i,s})-\frac{1}{2}(q_{i,1}+q_{i,s})^2-(P_1q_{i,1}+P_sq_{i,s})+w \;\middle|\; \boldsymbol{G},\phi\right] \tag{1.5}$$

Each trader faces a trade-off: how many shares to purchase at each period. On the one hand, he can acquire the asset with certainty in the PM but the price is likely to be higher - due to higher competition - and he faces the risk of re-selling. On the other hand, buying at period two is probably cheaper but he does not know if he will need to sell nor if he will participate in the active local market.

Jointly, ϕ and $\boldsymbol{G}$ imply that each trader has three levels of uncertainty: i) if he will participate in one or two markets; conditional on trading at $t=2$, ii) if he will buy or sell asset shares; and iii) if he is a buyer, with whom will he trade.

As it will be clear from the results (section 1.6), although buyers are not forced to provide liquidity to the seller they optimally decide to do so. For two reasons. First, no trader reaches his optimal inventory at $t=1$. Second, buyers guarantee a price concession to absorb seller's supply in any local market, in any trading network.[35] Thus, gains from trade arise because local markets provides more and cheaper shares to the participants buyers, and liquidity for the seller.

1.4.1 The Active Local Market

At $t=2$, both the PM and the re-sell shock have realized. The seller identity $s \in N$ is common knowledge and so it is the fixed asset supply $q_{s,1} \leq Q$ (recall Assumption 2). Notice that the PM at $t=1$ can be seen as endogenously determining individual asset endowment in the static market of a seller s.

Conditional on a level of current holdings $q_{i,1}$, each trader i as a buyer in period-two chooses his demand schedule $q_{i,s}(\cdot)$ for a local market price P_s. i's demand is given by[36]

$$q_{i,s}(P_s;q_{i,1}) = (1-q_{i,1}) - P_s \tag{1.6}$$

Since traders are price-takers, i's demand eq. (1.6) is the same in every local market he can participate. It is simply given by his willingness to pay more for the asset, i.e. his current marginal utility eq. (1.2).

Using eq. (1.6) and local market clearing conditions eq. (1.4), the set of equilibria at $t=2$ can be found, one for each possible seller $s \in N$.

[35]The unique exception is the star network, which is in itself an interesting finding that I discuss later on (section 1.6).
[36]This is simply the the first-order condition of (1.1) with respect to $q_{i,s}$ keeping $q_{i,1}$ and P_1 fixed.

Lemma 1. *Local Market Equilibrium*

Consider a Primary Market asset allocation $\{q_{i,1}\}_{i\in N}$*. The equilibrium in the local market of seller* $s \in N$ *with his network-induced set of buyers* $i \neq s : i \in N_s$ *is given by the selling price* P_s^**,*

$$P_s(\boldsymbol{q}_{N_s,1}) = 1 - \frac{\left(\sum_{i\in N_s} q_{i,1} + q_{s,1}\right)}{d_s} \tag{1.7}$$

and buyers' asset allocation,

$$q_{i,s}(\boldsymbol{q}_{N_s,1}) = \frac{1}{d_s}\left(q_{s,1} + \sum_{k\neq i, k\in N_s} q_{k,1}\right) - \frac{d_s - 1}{d_s} q_{i,1} \qquad \forall i \in N_s \tag{1.8}$$

where $\boldsymbol{q}_{N_s,1} \equiv (q_{s,1}, \{q_{i,1}\}_{i\in N_s})$*.*

Assumption 1 ensures there is a unique equilibrium at period two as there can be at most one shocked trader.

Corollary 0.1. *Active Local Market Equilibrium*

For a given shock realization, equilibrium at period two is unique. The interior equilibrium - that is, if trader $s \in N$ *is the one shocked - is given by Equation* (1.7) *and Equation* (1.8)*.*

If no trader is shocked, then no local market is active: $\boldsymbol{q}_{i,s} = 0 \forall i, s \in N$ *and* $\boldsymbol{q}_{i,1} > 0$*.*

The trading network directly affects the local market prices in two opposite ways. First, trough the positive "participation effect": the seller's degree d_s determines how large his market is. The higher d_s, the higher is $P_s(\boldsymbol{q}_{N_s,1})$. Second, through the negative "inventory effect": PM holdings in the seller's neighborhood, $q_{s,1}$ and $\{q_{i,1}\}_{i\in N_s}$, determine his supply and buyers' willingness to pay. The higher are PM holdings, the lower is P_s^*.

But that's not the whole story because these effects are interdependent what results in additional indirect effects. A larger pool of buyers (high d_s) may imply higher aggregate PM asset holdings just because there are more terms in the sum $(\sum_{i\in N_s} q_{i,1})$, what could drive $P_s(\boldsymbol{q}_{N_s,1})$ down. At the same time, a high d_s may imply lower aggregate PM asset holdings because the buyers anticipate the high competition and so ensured asset holdings in the PM, what could drive $P_s(\boldsymbol{q}_{N_s,1})$ up.

At the buyer level, equilibrium asset allocation $q_{i,s}(\boldsymbol{q}_{N_s,1})$ is decreasing in the seller's degree, as price increases in the later. And it is increasing in other buyers' (competitors) PM holdings $\left(\sum_{k\neq i,k\in N_s} q_{k,1}\right)$, since the higher these are the higher is i's residual supply and the lower is seller's price. Moreover, local markets allocate asset shares in accordance to who values it the the most at $t = 2$, to those who acquired the least shares in the PM. Thus, the buyers who provide more liquidity are those who would need less liquidity if they were to be hit by the re-sell shock.[37]

As I show next, traders understand that PM outcome determines liquidity supply and demand in local markets, what in turn influences traders' asset acquisition, and thus price, in the PM. And that's precisely why the trading network plays a crucial role in the PM. It incorporates the two-way feedback effects across markets and traders.

1.4.2 Primary Market as a Trading Game

Recall that the only source of ex-ante heterogeneity among traders is on network position. Different network positions mean that traders have different expected trades (i.e. local market participation) and, consequently, their asset acquisition decision will differ. More importantly, this decision depends on all other traders' decision as well since they determine the terms of trade in local markets.

In turn, the expected payoff (1.5) of each trader $i \in N$ in the PM, before any trade takes place, is only function of his and others PM holdings choice, $q_{i,1}$ and $q_{-i,1}$,[38] parameterized by the PM price P_1, the trading network $\boldsymbol{G}$, and the re-sell shock ϕ[39]:

$$\pi_i(q_{i,1}, \boldsymbol{q_{-i,1}}; P_1, \boldsymbol{G}, \phi) = w + (1-P_1)\cdot q_{i,1} - \frac{1}{2v_i(\boldsymbol{G},\phi)} q_{i,1}^2 - \phi \sum_j \tilde{g}_{ij}(\boldsymbol{G}) \cdot q_{j,1} q_{i,1} + \phi \sum_j \bar{g}_{ij}(\boldsymbol{G}) \cdot q_{j,1} \quad (1.9)$$

Perhaps surprisingly, equation (1.9) coincides with the payoff function of a network game of global strategic substitutes (notice that $\frac{\partial q_{i,1}}{\partial q_{j,1}} \leq 0 \quad \forall i \neq j$). The "network coefficients" $\{v_i(\boldsymbol{G},\phi), \tilde{g}_{ij}(\boldsymbol{G}), \bar{g}_{ij}(\boldsymbol{G})\}_{\forall i,j \in N}$ are endogenous, non-negative, and each is a function of the trading network structure. They encapsulate the local market effects on $q_{i,1}$ given i's network position (more details below).

Others' PM holdings $q_{-i,1}$ negatively impact trader i's optimal choice while also having positive externality. This is best understood if we put ourselves in the shoes of trader i conjecturing his local market trades. Consider first the PM demand of i's neighbors (i.e. his

[37]This apparent "coordination" between liquidity demand and supply of a trader reflects the strategic nature of the environment. This is going to be clear when in the upcoming analysis of traders' behavior in the PM (subsection 1.4.2)

[38]As usual, the notation $q_{-i,1}$ represents the demand of all traders but i, i.e. $\{q_{j,1}\}_{j \neq i, j \in N}$

[39]A quick way to see this result is just to replace local market variables in the expected payoff (1.5) with the equilibrium result of local markets (Lemma 1). With a little algebra, the "clean" representation in (1.9) is obtained. Full details are found in section A.4.

direct connections), $q_{N_i} \equiv \{q_{k,1}\}_{k \in N_i}$. As a seller, q_{N_i} pushes i's price down (the "inventory effect"). So i demands less if he expects his buyers to demand more in the PM. As a buyer, q_{N_i} pushes down the price i faces (also the "inventory effect"). So i also demand less (to afford more shares in local markets) if he expects his sellers and competitors to have high PM demand.

However these are just *first-order* effects. The connections of i's connections are also i's competitors and they offer alternative markets for i's neighbors, thus influencing i's PM decision. This is also true for the connections of the connections of i's neighbors, and so on. In all cases, the same reasoning holds: i takes into account the effect of every other trader when he acts a buyer and as a seller in local markets.

Bottom line is that the asset acquisition of each trader is negatively influenced by the same decision of each and every other trader, irrespectively if they are connected or not. This is the key insight of this paper because it reveals that the dynamic framework boils down to a one-shot, simultaneous-move network game played in the PM.

Since traders are price-takers, each PM price P_1 induces a game. In each game, the strategy for trader $i \in N$ is his asset acquisition decision in the PM. It is a mapping $q_{i,1} : q_{-i,1} \times P_1 \to \mathcal{R}$ where $q_{-i,1}$ is the strategy of all other traders different than i. Traders simultaneously choose their demand schedules by best-responding to the demand schedules of others. The equilibrium concept is pure-strategy Nash Equilibrium.[40]

Lemma 2. *Primary Market Trading Game*

For each PM game with price P_1, a trader i's asset demand schedule (best-response) is

$$q_{i,1}(\boldsymbol{q}_{-i,1}; P_1\phi, \boldsymbol{G}) = v_i(\boldsymbol{G}, \phi) \left[(1 - P_1) - \phi \sum_j \tilde{g}_{ij}(\boldsymbol{G}) \cdot q_{j,1} \right] \tag{1.10}$$

where

$$v_i(\boldsymbol{G}, \phi) \equiv \left[\frac{2\phi}{d_i} + 1 - \phi(d_i + 1) + \phi \sum_j g_{ij} \cdot \frac{(2d_j - 1)}{d_j^2} \right]^{-1} \tag{1.11}$$

and

[40] An important step in the paper is to formulate the model as a game. Network games of global strategic substitutes have been extensively studied - see Bramoullé et al. (2014) and Galeotti et al. (2010). I rely on the advances of this literature to characterize the equilibrium in every possible game and for *any* network graph $\boldsymbol{G}$.

$$\tilde{g}_{ij}(\boldsymbol{G}) = g_{ij} \cdot \left[\frac{1}{d_i} + \frac{(d_j - 1)}{d_j^2}\right] + \left[\sum_{\substack{z \neq j \\ z}} g_{iz} g_{jz} \frac{(d_z - 1)}{d_z^2}\right] \geq 0 \tag{1.12}$$

The trading network determines the influence among traders' strategies, while the re-sell shock ϕ regulates the global degree of substituability among PM demands: the higher ϕ, the greater is the chance of local market trading and thus the greater is the feedback effect between traders' demands.

The individual network effect, $v_i(\phi) > 0$, summarizes trader i's interactions in local markets, and it can be seen as the marginal benefit of acquiring shares in the PM.

Each global network coefficient $\bar{g}_{ij} \geq 0$ captures bilateral influences: it gives how influential is trader j on i's demand. Its value depends on how far apart i and j are[41] Notice that it is increasing in the number of overlapping connections i and j have. Implying that indirect connections can be more influential to i's decisions than i's neighbors. And that neighbors with the same degree can have different effects.

Lastly, the first term in the demand schedule, $(1 - P_1) \geq 0$, is common across all agents. It represents the optimal action absent network interactions: $(1 - P_1)$ is the individual demand in a Walrasian (competitive, static) market of size N.

In the section A.4 I derive the results above, and I discuss more deeply the functional forms of the payoff function and demand schedule. I also prove the existence of a unique interior equilibrium for each network game.[42]

It is useful to look at two traders' optimal PM demand schedules to understand how the Nash Equilibrium of each network game is found. The left-hand side of Figure 1.7 shows that, for a given P_1, the Nash Equilibrium is given by the intersection of traders' i and j best-responses. The right-hand side of Figure 1.7 depicts how the primary market equilibrium is one of the Nash equilibria such that equilibrium aggregate demand meets the exogenous asset supply $\bar{Q}$.

[41] That's because, as I discussed before,there are three channels through which $q_{j,1}$ impacts $q_{i,1}$: as a buyer from i or as a seller to i, if they are connected; and as a competing buyer to i if they share a common linkage to another agent z (i.e. if i, j have overlapping connections).

[42] The existence and uniqueness is guaranteed by Assumption 1, i.e. as long as the shock probability $\phi < 1/N$ (Proposition 21). Intuitively, $\phi > 1/N$ implies that traders expect more than one seller in the local market. The anticipation of "too much" local market trading may lead traders to either demand too much ($q_{i,1} \to 1$) or too little ($q_{i,1} \to 0$) in the PM. In the former case, local markets would collapse as buyers' willingness to trade would be virtually small. In the latter case, the PM would collapse as the market would not clear.

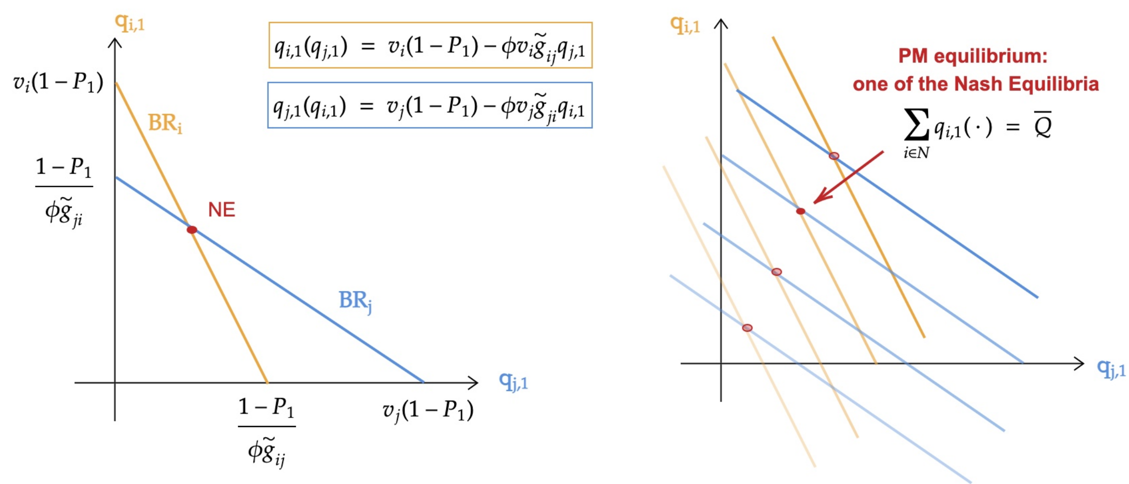


Figure 1.7: The demand schedules of two traders

Relative to the related literature of imperfectly competitive trading models (i.e. demand games, which Rostek and Yoon (2020a) provide an excellent review), the way I find the equilibrium in the model is different. The key feature of imperfect competition is that traders conjecture their endogenous (and unknown) price impact and have to do the same for others' price impact and demands, due to private information. As Rostek and Yoon (2020a) show, the equilibrium is characterized by two conditions: market clearing and correct price impacts. That is, each trader optimally chooses his demand schedule given his price impact such that his price impact equals the slope of his residual inverse supply function.[43] With price-taker traders (as in my model), finding equilibrium is simpler because traders only respond to each other demands and thus only one condition - market clearing - characterizes equilibrium. Although this is a strong assumption, I view it as plausible given the main goal of this paper: to distill the network effects on equilibrium. I obtain formal and closed formed solutions and I can study the implications of the structure of the trading network in isolation[44]

[43] More specifically, with imperfect competition, each trader chooses his demand as optimal pointwise for each price realization against a family of the residual supply (all other trader's demand schedule) with a deterministic slope (price impact) and random intercept (due to other traders private information).

[44] However, my main methodological contribution, which is to derive a sufficient network metric for the equilibrium (Theorem 1), is not limited by the price-taking assumption. In the Appendix, I derive and show the equilibrium in a setting where markets are imperfectly competitive (see section 1.9 for further discussion). As I will explain later on, if traders were strategic with respect to price, the endogenous price impact would depend on the network structure and would also influence the equilibrium. Then, the PM price would be determined by two related but distinct forces: price impact and network structure.

1.5 Trading Centrality, a sufficient statistic for Equilibrium

The analysis so far has two main conclusions. First, the presence of local markets leads to a substitution effect across PM demands $q_{i,1}(\cdot)$ (eq. (1.10)). Second, for each trader, how he reacts to others' behavior is determined by his network position in complicated ways.

The main contribution of this paper is that, by solving the model, I show that the unique equilibrium can be described as a function of a simple measure, *trading centrality*. Trading centrality, $\boldsymbol{c}(\boldsymbol{G}, \phi)_{N\times 1} : c_i(\boldsymbol{G}, \phi)\, \forall i \in N$, is a recursive network metric that produces a "score" for each trader. In Section A.6 I provide the formal definition of trading centrality and its analytical expression. However, the reason why trading centrality is a sufficient statistic is best understood by simply using economic intuition for what it does.

The trading centrality score measures the trader's endogenous valuation for the asset in the PM. The higher is c_i, the higher is i's marginal utility for asset holdings and thus the higher is he willingness to pay for shares in the PM. Moreover, it also implies that a trader is *more* central as his direct and indirect connections are *less* central.

That is, the PM demand schedule $q_{i,1}(\cdot)$ of each trader i (Equation 1.10) can be expressed in terms of his centrality.[45] c_i,

$$q_{i,1}(\boldsymbol{q}_{-i,1}; P_1, \boldsymbol{G}, \phi) = (1 - P_1)c_i(\boldsymbol{G}, \phi) \tag{1.13}$$

And c_i can be expressed in terms of other traders' centralities $\{c_j\}_{j\neq i}$,

$$c_i(\boldsymbol{G}, \phi) = v_i(\boldsymbol{G}, \phi)\left(1 - \phi \sum_j \tilde{g}_{ij} \cdot c_j(\boldsymbol{G}, \phi)\right) \tag{1.14}$$

The recursive form of trading centrality is illustrated in Figure 1.7 by noticing that the best-responses (i.e. demand schedules) depicted there are essentially Equation (1.14) replacing $c_i(\cdot), c_j(\cdot)$ with $q_{i,1}(\cdot), q_{j,1}(\cdot)$ and setting $P_1 = 0$.

The next figure depicts the demands of two traders with different centralities. The trader with higher centrality (in green) has a more elastic demand curve than the one with lower centrality (in orange). In turn, for any price P_1, the more central trader will acquire more

[45] I use "trading centrality" and "centrality" interchangeably.

shares than the less central one.

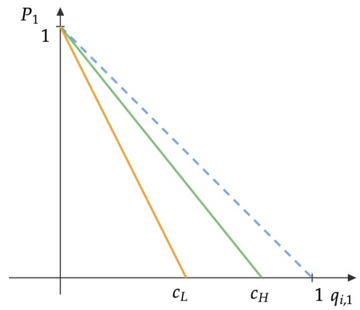


Figure 1.8: Demand schedules are determined by traders' centrality
The orange demand is of a trader with lower trading centrality; and the green demand if of a trader with higher trading centrality.
In blue is the demand schedule if the trading network did not exist.

Throughout the paper, with abuse of notation, I omit the functional arguments of trading centrality and write $\boldsymbol{c} = \boldsymbol{c}\left(\boldsymbol{G}, \phi, Q\right)$.

Since trading centrality determines traders' willingness to acquire the asset in the PM, it defines the equilibrium demands and, in turn, the equilibrium PM price. Thus, the unique equilibrium of the model is determined solely by the structure of the trading network and the shock parameter ϕ.

Theorem 1. *Primary Market Equilibrium and Trading Centrality*

Given an asset supply Q, the unique and interior equilibrium in the PM is determined by trading centrality $\mathbf{c}(\boldsymbol{G}, \phi)$.

Equilibrium PM price is given by

$$P_1^*(\boldsymbol{c}; Q) = 1 - \frac{Q}{c_A} \tag{1.15}$$

where $c_A = \sum_{i \in N} c_i$ is the aggregate trading centrality.

Each trader's equilibrium PM asset allocation is

$$q_{i,1}^*(\boldsymbol{c}; Q) = Q\frac{c_i}{c_A} \tag{1.16}$$

Or in matrix notation, $\boldsymbol{q}_1^* = \frac{Q}{c_A}\boldsymbol{c}$.

Trading centrality neatly characterizes equilibrium, and in an intuitive way. PM price is simply defined by aggregating trading centrality globally, as all traders can participate in that market. And each trader's asset acquisition is proportional to his centrality.

In the same spirit, for each local market, equilibrium price is defined by aggregate trading centrality locally in the seller's neighborhood. And each trader's demand schedule is simply determined by the gap between his optimal holdings and his centrality.

Corollary 1.1. *Local Market Equilibrium and Trading Centrality*

Given an asset supply Q, trading centrality $\boldsymbol{c}(\boldsymbol{G}, \phi)$ uniquely determines the equilibrium in a local market.

A trader's demand schedule is

$$q_{i,s}(\boldsymbol{c}; P_s) = \left(1 - \frac{Q}{c_A} c_i\right) - P_s \quad \forall i \in N \tag{1.17}$$

Each LM price is

$$P_s(\boldsymbol{c}) = 1 - \frac{Q}{c_A} \left(\frac{c_s + \sum_{i \in N_s} c_i}{d_s}\right) \quad \forall s \in N \tag{1.18}$$

The results above show how trading centrality is a sufficient statistic for market outcomes. Given an arbitrary trading network and a shock parameter ϕ, trading centrality can be computed. In turn, prices and demands in every possible market are found.

In equilibrium, traders *always* have a strictly positive demand in all markets they can participate at. That's due to three reasons. First, as the PM has more competing buyers, no trader is able to reach his optimal inventory at $t = 1$. Second, the anticipated yet uncertain re-sell shock also lowers PM demand to mitigate the risk of being a seller. Lastly, since the likelihood of being a buyer is weakly greater than of being a seller, traders command lower prices in local markets to compensate the shift from securing asset holdings in the PM at $t = 1$ to the risky asset allocation at $t = 2$.

All this leads to PM price being greater than any local market price in any network. The unique exception is for star network (Corollary 1.1).

Proposition 1. *Price Dynamics*

A trading network with a star structure is the unique one that can exhibit price increase over time. For any other network structure, asset price drops from $t = 1$ to $t = 2$.

Thus, liquidity (re-selling) is costly for the seller. Analysing price dynamics then means measuring traders' liquidity cost. Just as with equilibrium prices, liquidity cost is determined by the gap between the aggregate centrality in the seller's neighborhood and the price he would have in absence of PM holdings among his buyers (i.e. if his buyers had no asset endowment when trading with him):

$$P_i - P_1 = \frac{Q}{c_A} \left| 1 - \frac{c_i + \sum_{j \in N_i} c_j}{d_i} \right| \neq 0 \quad \forall i \in N \tag{1.19}$$

The next corollary of Proposition 1 reveals why price can increase in a star network. Its core trader is the only one who make profits from re-selling. That's because his price is higher than the price in any other market within the star network. In fact, the core's price is higher than the prices in any trading network structure of the same size.

Corollary 1.1. *Re-sell Cost*

For a trader $i \in N$, re-selling is profitable, i.e. $P_i - P_1 > 0$, if and only if i is the core of a star network.

Looking at the aggregate centrality in a trader's local market is advantageous because it has a straightforward and monotonic relation with liquidity cost. The next natural question is a trader's selling price P_i and liquidity cost $(P_1 - P_i)$ relate to his trading centrality c_i alone. It turns out that the answer is non-trivial because the relation between d_i with P_i and c_i ambiguous.[46] I find that in certain trading networks, c_i and d_i are positively correlated and, thus, so it is c_i and P_i. But other networks exhibit the opposite relation..[47]

The non-monotonic relation between trading centrality and degree has two crucial implications. First, in my model, central traders are not necessarily those highly connected. Second, central traders do not always have better terms of trades. Although these implications might be counter-intuitive, in reality they are observed in interdealer markets. The growing empirical literature on interdealer networks find mixed evidence on whether "central" dealers have higher or lower liquidity costs (section 1.2). My results suggest that a reason for this inconclusiveness could be that all these studies use standard network metrics such as degree and eigenvector centrality that are positively correlated to degree. Consequently,"centrality" only reflects the extensive margin of trades and imply that the centrality of a dealer is weakly increasing in the centrality of his connections (and connections' connections, etc.).[48] My novel trading centrality measure reveals that this might be misleading, particularly so when analyzing markets with endogenous and correlated terms of trade.

In my model, central traders emerge as market makers only for particular trading networks in which trading centrality and degree have a positive relationship. This is the case, for instance, for "nicely behaved" networks such as symmetric and core-periphery networks. In those cases, central dealers buy more in the PM to sell at a higher price in the local market, compared to others.

The fact that liquidity is costly is reminiscent of the long-standing theoretical literature

[46] In the appendix, Lemma 8 provides a formal statement of this result.

[47] I illustrate this on a simple example on section A.1. See also section A.6 for more details.

[48] This is how we usually think of centrality, as a proxy for "prestige" and "influential power".

of inventory behavior in the interdealer market, pioneered by Ho and Stoll (1981) and Ho and Stoll (1983). In these models, while interdealer trading enables inventory risk sharing, the initiating dealer must give up some portion of the spread to his counterparty.[49] It is also a common phenomena across a variety of securities and markets (Duffie (2010)).

The rest of the paper explores how the structure of the trading network itself affects PM price. I also study welfare. But in order to understand all my findings - including Theorem 1 - it is crucial to distill what information trading centrality encapsulates. Before proceeding, the next simple example distils what information trading centrality encapsulates. In sequence, I discuss the information content and mathematical form of trading centrality (subsection 1.5.2).

1.5.1 Example

The trading network structure is described by the $\boldsymbol{G}$ such that $g_{ij} = \{0,1\}, g_{ij} = g_{ji} \forall i,j \in N$. It is essential to keep in mind the substituability of demands that arises because of the feedback between the PM and local markets. This implies that the trading network induces endogenous ”trading costs" for holdings asset shares that reflect local market trades.

The *implied* trading network is given by the modified adjacency matrix $\tilde{\boldsymbol{G}}$ such that $\tilde{g}_{ij} \geq 0, \tilde{g}_{ij} \neq \tilde{g}_{ji} \forall i,j \in N$ (eq. (1.12)). $\tilde{\boldsymbol{G}}$ is a weighted and asymmetric matrix and a modified version of $\boldsymbol{G}$.[50] Take a trader $i \in N$. Each $\tilde{g}_{ij}$ is the marginal cost imposed by the holdings of a trade counterparty $j \neq i, j \in N$. It accounts for when i and j trade as a seller and a buyer to one another, and as competitors for a common linkage. On top of that, i incurs a marginal cost from his own holdings given his possible trade participations (i.e. every local market where he is a buyer, a seller or out of it), which is given by the individual coefficient $\frac{1}{v_i}$.

All these network-induced “trading costs” define i's marginal benefit of holding asset shares. Then, to optimally decide his PM demand, i equates it to the marginal cost of acquiring the asset, i.e. the PM price P_1. In matrix notation this gives,

[49] In these models, using interdealer trades to unwind inventory is a choice. However, as long as dealers are farther from their optimal inventory, the benefits of risk sharing via the interdealer market outweigh the trading costs. So dealers (almost) always choose to sell the inventory in the interdealer market if needed. This is the main conceptual difference with my paper since I assume that the seller (the initialing dealer) *must* sell. Interdealer trading in my model is a choice just for the buyers because they can demand zero.

[50] An interesting aspect of the forces in my model is that they turn the “plain” trading network, which is undirected and unweighted, into a weighted and directed network graph. Moreover, a graph that is weakly more connected than the trading network itself. See Figure 1.9

$$\underbrace{\mathbf{1}_N - \left(\boldsymbol{V} + \phi \tilde{\boldsymbol{G}}\right) \cdot \boldsymbol{q}_1}_{\text{asset holdings marginal benefit}} = \underbrace{\boldsymbol{P}_{\mathbf{1}N}}_{\text{asset holdings marginal cost}}$$

where $\boldsymbol{V}_{N \times N} \equiv diag(\frac{1}{v})$ is the diagonal matrix with entries $1/v_i$.

Notice that the above is simply the system of first-order conditions of traders' optimization problem in the PM (eq. (1.9)). Since all traders have the same optimal holdings of 1, the lower is the row-wise sum of $\left(\boldsymbol{V} + \phi \tilde{\boldsymbol{G}}\right)$ (i.e. trading costs), the higher is the trader's marginal utility and thus the higher is his willingness to pay. Then, in equilibrium, the trader's demand - and share allocation - is higher.

This operation is exactly what trading centrality does. But it gives the information in terms of marginal benefit instead of marginal costs:

$$\boldsymbol{c} \equiv \left(\boldsymbol{V} + \phi \tilde{\boldsymbol{G}}\right)^{-1} \mathbf{1}_N \tag{1.20}$$

Thus, the score of each trader is precisely his marginal utility of holdings asset shares. Or, in other words, the marginal benefit of acquiring shares in the PM.

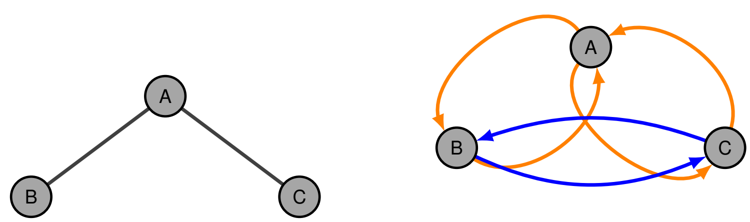


Figure 1.9: Orange links imply two trades (nodes) are buyers and sellers to one another (direct counterparties) in local markets. Blue links imply they are competitors (indirect counterparties).

Now let's look at a simple example. Consider $N = 3$ traders in the star trading network with structure described by the adjacency matrix $\boldsymbol{G}$ below (left graph of Figure 1.9). The implied star trading network is given by the modified adjacency matrix $\tilde{\boldsymbol{G}}$ (right graph of Figure 1.9):

$$\boldsymbol{G} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \tilde{\boldsymbol{G}} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1.25 & 0 & 0.25 \\ 1.25 & 0.25 & 0 \end{pmatrix}$$

The core trader A has the same interaction with both B and C, and that's why $\tilde{g}_{AB} = \tilde{g}_{AC} = 0.5$. However, B and C interact differently with one another and with A. Not only that, but they trade directly with A and indirectly with each other through A. That's why $\tilde{g}_{BA} = 1.25 > \tilde{g}_{BC} = 0.25$ and $\tilde{g}_{CA} = 1.25 > \tilde{g}_{CB} = 0.25$. The individual coefficients are

$\frac{\mathbf{1}}{\boldsymbol{v}} = (1, 1.1875, 1.1875)'$. A has a lower marginal cost of holding shares because he has greater local market participation.

Traders' marginal benefit of asset holdings is then[51]

$$\begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}1 & 0.125 & 0.125\\ 0.3125 & 1.1875 & 0.0625\\ 0.3125 & 0.0625 & 1.1875\end{pmatrix} \cdot \begin{pmatrix}q_{A,1}\\ q_{B,1}\\ q_{C,1}\end{pmatrix} \boldsymbol{q}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix} \cdot P_1$$

Thus, trading centrality is

$$\boldsymbol{c} = \begin{pmatrix}1.067 & -0.1067 & -0.1067\\ -0.267 & 0.871 & -0.0178\\ -0.267 & -0.0178 & 0.871\end{pmatrix} \cdot \mathbf{1}_N = \begin{pmatrix}0.853\\0.587\\0.587\end{pmatrix}$$

Letting the asset supply be one, the equilibrium asset allocation is $\boldsymbol{q}_1^* = (0.421, 0.289, 0.289)'$, which is proportional to trading centrality $\boldsymbol{c}$. The core trader is the most central and has the highest asset holdings. The equilibrium PM price is given by the aggregate trading centrality $c_A = 2.027$, such that $P_1^* = 1 - \frac{1}{c_A} = 0.5066$.

1.5.2 The Information Content of Trading Centrality

Now it is clear that trading centrality process information about each and every local market interaction, and maps it to the asset acquisition decision in the PM. Hence, trading centrality defines traders' behavior before trade and it implies that the centrality of a trader is decreasing in all other traders' centralities.[52]

The recursive representation in eq. (1.20) is reminiscent of the *negative* Bonacich centrality[53] with weights that are not a simple geometric series (as in the Bonacich measure), but instead are endogenous and capture trading incentives. They are the network-induced trading costs $\left(\{\frac{1}{v_i}\}_{\ \forall i \in N}, \{\tilde{g}_{ij}\}_{\ \forall i,j \in N}\right)$ that defines traders' payoff function eq. (1.9).

The strength of effect between two traders' centralities varies with how far apart they are. On one hand, a trader i respond *more* negatively to the demand of traders at most

[51]Notice that $\left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right) \cdot \mathbf{1}_N = (1.25, 1.5625, 1.5625)$. That is, the core has lower trading cost compared to the peripheries.

[52]This is makes sense as the framework is fundamentally a game of global strategic substitutes.

[53]The Bonacich centrality is a well-known network measure of node importance. Its common adoption in economics has been with a *positive* scalar as it means an agent is more powerful (central) the more powerful are his connections. This interpretation is meaningful in many economic scenarios that exhibits local complementarities and it was first invoke by Ballester et al. (2006). My paper contributes to the less explored network models in which Bonacich centrality with negative scalar is the appropriate measure of node influence.

two links apart (i.e direct connections or common connections). On the other, i respond *less* negatively to traders further away.

1.6 Trading Network Structure and Equilibrium

The overall takeaway so far is that the trading network induces a complex relationship between traders and markets what is encapsulated in the trading centrality, the sufficient statistic for equilibrium and my main contribution. Now I investigate how changes in the trading network structure affect equilibrium outcomes. In Section A.1 I illustrate the main findings of this section with a relatively simple example.

A first natural question is which network structure, if any, delivers a maximum or minimum value for PM price. I find that the complete trading network delivers the highest PM price while the star trading network the lowest possible PM price.

Proposition 2. *Bounds on Primary Market Price*

Consider an arbitrary trading network of size N. The equilibrium primary market price is bounded by its level on two specific networks of the same size: above by the complete network, and below by the star network.

There are no trading frictions in a complete network. All traders are connected with one another and everyone trades in both markets, either as a buyer or a seller. In equilibrium, demand schedules and asset allocation are homogeneous across traders in every market, and so are local market prices. From a buyer's perspective, trading in a local market is as competitive as in the primary market. For this same reason, the seller's price is likely to be higher. Both buyer and seller's effects combined induce traders to demand more in the primary market because i) as a buyer, higher PM demand lowers a seller's price; and ii) as a seller, a higher price will be obtained. Higher willingness to trade in the primary market pushes price up, even though the equilibrium allocation $q^*_{i,1} = \frac{\bar{Q}}{N} \forall i \in N$ is the same as in a frictionless market.

The bounds on PM price might indicate that price is monotonically affected by metrics about only the number of connections of each trader: connectivity, the average degree of the trading network; and degree inequality, the variance of traders' degree.[54] That's because the complete (star) network has the highest (lowest) connectivity and lowest (highest) degree inequality. However, this is not true.

Lemma 3. *PM price is non-monotonic in connectivity and degree inequality.*

[54]Notice that such effects are tightly interrelated. For instance, increasing the number of traders can affect how traders are connected. Or changing (re-arranging, deleting or adding) linkages can reduce or increase differences in traders' degree.

Put differently, much more information than just degree is needed to compare equilibrium outcomes in the cross-section of trading networks of the same size. Nonetheless, degree is the simplest and a straightforward network concept. And very often we just know the degree distribution of a network. In turn, I now show that the degree alone still brings novel insights. In particular, I investigate how PM equilibrium is affected by specific changes in the degree distribution of the trading network.

Consider, in particular, a change in the probability distribution over the degrees of traders that reflects an unambiguous increase in connectivity, as given by the criterion of First Order Stochastic Dominance (FOSD). Denote the degree distribution of two trading networks as P and P'. If P first-order stochastically dominates P', then the average degree under P is higher than under P', the reverse not true.

Proposition 3. *Changes in connectivity: FOSD*

Suppose that P' FOSD P. Then the PM price under P' is unambiguously higher than under P.

All traders' demand are higher after the change.

Now consider an unambiguous increase in degree inequality, while keeping average degree unchanged. This change is capture by a mean-preserving in the degree distribution. If P' is a mean-preserving spread of P, then the variance under P' is higher than in P, reverse not true.

Proposition 4. *Changes in degree inequality: Mean-preserving spread*

Now suppose P' is a mean-preserving spread of P. Than PM price is lower under P' than under P.

Traders' demand can increase or decrease depending on their network position. More (less) connected traders have an increase (decrease) in demand.

In other words, if we weakly increase all traders' degree - implying greater connectivity - PM price also increases (Proposition 3). And if degree inequality increases, keeping connectivity constant, PM price decreases (Proposition 4). It is worth pointing out that the reverse does not necessarily hold (i.e., the above propositions are not if and only if statements). As argued before, information on the number of traders' connections (i.e. degree distribution) is not enough to characterize equilibrium. Trading centrality - the key statistic for equilibrium - reveals that one must look at the pattern of connectivity.

Lastly, straightforward changes in the trading network are to add/remove a link or a trader. Both changes, when made alone and keeping everything else the same, lead to increase (decrease) in PM price. Moreover, changing the number of traders affects every local market price in the same direction as the PM one. Both facts are useful for the coming discussions.

Lemma 4. *Removal and addition of a link*

Everything else the same, adding (removing) a link from the trading network increases (decreases) PM price unambiguously.

Lemma 5. *Trading Network size*

Everything else the same, PM price and local market prices are increasing in N.

In the rest of this section, I study the equilibrium for particular trading network structures: symmetric and core-periphery. Doing so renders great tractability of the model because one just needs to keep track of one or two trading centrality scores respectively. In turn, I show that such networks contribute to understating the relationship between PM price and the trading network.

1.6.1 Symmetric Networks

In symmetric networks (or regular graphs), all traders have the same number of links and, thus, same network position. This means that a symmetric network is solely characterized by the number N of traders in it and their degree d. Examples are the complete network in which $d = N - 1$ and the ring network in which $d = 2$.

It immediate follows from Theorem 1 that prices and demands across traders and markets are homogeneous[55]

Proposition 5. *Equilibrium in Symmetric Networks*

Consider a symmetric network with N traders and degree d. Then, Primary Market equilibrium is

$$P_1^* = 1 - \frac{\bar{Q}}{N}\left(\frac{\phi + d(1+\phi)}{d}\right) \tag{1.21}$$

$$q_{i,1}^* \equiv q_1^* = \frac{Q}{N} \quad \forall i \in N \tag{1.22}$$

and the equilibrium in any Local Market is

[55]Notice that traders have the same demand and selling price if and only if they have the same network position. These two facts make it easier to study symmetric networks. There is just one unknown, a single trading centrality score; and four equilibrium values - the PM price, the single PM demand, the single LM price, and the single LM demand.

$$P_s^* = 1 - \frac{(d+1)}{d}\frac{Q}{N} \tag{1.23}$$

$$q_{i,s}^* \equiv q_2^* = \frac{Q}{Nd} \quad \forall i, s \in N \tag{1.24}$$

In any symmetric network, asset supply is divided equally among buyers and it is the as if no trading network exists, i.e in the perfectly (Walrasian) market. Although traders have the same willingness to pay, their demand schedule[56] $q_1 = (1 - P_1)\frac{d}{\phi + d(1+\phi)}$ is not the same as in perfect competition, $q_{ce} = (1 - P_1)$. Traders respond to the trading network and the re-sell shocked by submitting a more inelastic demand.

In contrast, PM price varies considerably across symmetric networks and it is never equal to the price of a perfectly competitive market. Thus, the distribution of asset shares alone is not informative about PM price nor about the network structure (since all symmetric networks have the same allocation). For financial market, this suggests that by looking only at dealers' inventories we could miss an important consideration for the cost of credit for the issuers.

Comparing price across periods, while PM price is increasing in degree the LM is not. In turn, more connected symmetric networks exhibits higher price drop.

Recall that the degree distribution of the trading network, although not fully informative about the equilibrium, is useful to compare market outcomes across network structures. It turns out that combining what the degree distribution tells us with the notion of symmetric brings further insights on PM price.

Fixing N, we already know that the complete network has the highest PM price (Proposition 2). Further fixing the number of linkages in the trading network, I find that the PM price in every symmetric network is higher than in any other structure.

Proposition 6. *Network Symmetry and Primary Market Price*

Suppose there are N traders and a fix number of connections among them. Then, the PM price in a symmetric network, if it exists, is higher than in any other network.

Benchmark cases:

The complete network is a special case of a regular network where no trading frictions exist since all traders are connected with one another. Equilibrium prices are $P_1^* = 1 - \frac{\bar{Q}}{N(N-1)}(N(1+\phi) - 1)$ and demand schedules are $q_1 = \frac{N-1}{N(1+\phi)-1}$.

[56] This follows because trading centrality is $c_i \equiv c = \frac{d}{\phi + d(1+\phi)} < 1$.

The other two extremes cases of symmetry are i) the empty network (i.e. without any trading relationships but with $\phi > 0$); and ii) the static, competitive market (i.e. without a trading network and/or $\phi = 0$).

The competitive market PM price equilibrium is $P_1^{ce} = 1 - \frac{Q}{N}$ and demand schedules are $q_1 = 1 - P_1$. In an empty network traders could still face the re-sell shock even though no local market actually exist. This risk makes demand schedules less elastic and drives PM price down. In equilibrium, PM price and demand schedules are, respectively, $P_1^* = (1-\phi)\left(1 - \frac{\bar{Q}}{N}\right)$ and $q_1(P_1) = 1 - \frac{1}{1-\phi}P_1$.

1.6.2 Core-Periphery Networks

Empirical evidence suggests that several financial markets exhibit a core-periphery structure (Section 1.2). We already know that the star network, a particular core-periphery structure, has the lowest PM price (Proposition 2). A core-periphery network consists of a well-connected set of nodes, the core, and the remainder nodes, the periphery, well connected to the core but sparsely connected internally (Figure 1.10). Motivated by the above, I now restrict my study of equilibrium in core-periphery trading networks. First, I look at the star network (left in Figure 1.10). Then, I analyze how price changes as the size of the core (right in Figure 1.10) and periphery grows.

I find that when the core consists of a single trader, he is the unique trader obtaining capital gains (Proposition 7). And that, even though PM price is increasing in the size of the core, the price drop monotonically decreases as the network grows (Proposition 8).

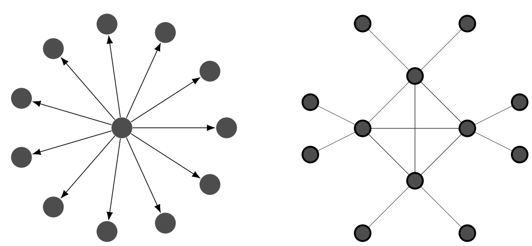


Figure 1.10: The figure depicts two core-periphery networks of the same size but with different cores

The star network is the simplest case of a core-periphery structure with a single core. An arbitrary core-periphery network can be understood as adding more traders to the core of the star network and keeping core's connectivity the same[57] The size of a core-periphery network is altered in two ways: by changing the number of core nodes and peripheral nodes.

[57]Core traders are fully connected among themselves, they have the same number of connections to peripheral nodes, and each periphery is linked to a single core node.

The core traders have incentives to increase their demand. They expect to sell at high price and not be able to buy a lot in any local market, even though their buying prices are likely to be low. The reverse logic holds for peripheral trader. The peripheries anticipate a very low selling price and low marginal utility for their supply since they would only be trading with a core trader. This drives their demand down. In equilibrium, the traders in the core (periphery) have the highest (lowest) asset holdings.

What drives apart equilibrium proprieties of the star and other core-periphery networks are two facts. First, deviating from a single core decreases core traders' trading centrality. Second, cores' centrality is more affected than peripheries' one. The reason being traders in the core also trade with each other, who themselves are relatively more central than the peripheries. Lower trading centrality drives core's demand down and, thus, their selling price decreases. At the same time, demand inequality decreases since the peripheries' demand is greater.

First, as the size of the core increases, core traders' trading centrality decreases. Second, cores' centrality is more affected than peripheries' one. The reason being they also trade with each other, who themselves are relatively more central than the peripheries. Lower trading centrality drives core's demand down and, thus, their selling price decreases.

The next proposition shows the asset pricing implication of such behavior.

Proposition 7. *Prices and the size of the Core*

Compared to any trading network with N traders, only the single core trader of the star network has a selling price higher than the PM price.

For any core-periphery network different than the star, selling prices are lower than the PM price for every trader. Although all prices are increasing in the size of the core.

The core trader of the star network, and only him, obtains capital gains from selling his shares. His price is the highest across all markets with N traders, including the PM. The deviation from a single core trader leads to higher PM price and lower local market prices, including for the core. Consequently, price drops over time - in stark difference with the star network.

Behind this result is the fact that cores' centrality is more affected than peripheries' once the network changes from a star to a core-periphery structure. Core traders' trading centrality decreases since they also trade with each other. Lower trading centrality drives their demand down and, thus, selling price decreases.

As the size of the core increases, competition among core traders intensify, reducing even further their centrality and demand. However, the market size effect dominates and all prices increases. Interestingly, the difference between core-periphery demands becomes smaller. The next proposition summarizes how PM equilibrium changes as the core-periphery network grows.

Proposition 8. *Equilibrium in Core-Periphery Networks*

As N and /or the number of core traders increase, the core-periphery network exhibits:

i) lower demand inequality $(q_{core,1} - q_{periphery,1})$, and

ii) smaller price change over periods $|P_1 - P_i|$ $\forall i \in N$.

If there is one core trader, there is lower price raise. For any other core size, there is lower price drop.

Summarizing, the star network has three interesting features: lowest PM price, highest demand inequality and possibility of capital gains. Such properties do not hold for any other core-periphery structure. Increasing the size of the core reduces demand inequality and no seller can obtain capital gain.

In A.10, I provide analytical solutions and further details on the equilibrium for core-periphery networks.

1.7 Welfare

I study welfare in terms of traders' expected indirect utility. Similarly to equilibrium prices (Theorem 1, Corollary 1.1), welfare is determined by aggregating trading centrality in a particular way (as I show below).

Even so, welfare comparison across different trading networks is challenging. I find that it is not necessarily true that welfare is enhanced by i) reducing the disparities between traders' number of connections and/or PM demand; or ii) increasing connectivity. That's because, since welfare depends on trading centrality, it inherits the non-trivial relation with network connectivity and degree inequality.

Formally, the expected indirect utility EU_i^* of a trader $i \in N$ can be written in terms of his and others' centrality,[58]

$$EU_i^*\left(\boldsymbol{c}; \boldsymbol{G}, \phi, \bar{Q}\right) = w + \left(\frac{Q}{Nc_A}\right)^2 \left[c_i - \frac{1}{2v_i}c_i^2 - \phi c_i \sum_j \tilde{g}_{ij} c_j\right] + \phi\left(\frac{Q}{Nc_A}\right)\left[\sum_j \bar{g}_{ij} c_j\right] \tag{1.25}$$

[58]Recall that i's expected indirect utility is given by $EU_i^* = w + (1 - P_1^*)q_{i,1}^* - \frac{1}{2v_i}(q_{i,1}^*)^2 - \phi \sum_j \tilde{g}_{ij} q_{j,1}^* q_{i,1}^* + \sum_j \bar{g}_{ij} q_{j,1}^*$.

and so EU_i^* is only a function of the trading network structure $\boldsymbol{G}$ and the parameters of the model (ϕ, Q).

Welfare of a trading network $\boldsymbol{G}$ is defined as the sum of traders' expected utility, $EU^* \equiv \sum_i EU_i^* \left(\boldsymbol{G}, \phi, \bar{Q}\right)$. Not surprisingly, it is also determined by traders' centrality and given by

$$EU^* \left(\boldsymbol{c}; \boldsymbol{G}, \phi, \bar{Q}\right) = Nw + \frac{Q^2}{2c_A} - \phi \frac{Q^2}{c_A^2} \sum_i \left[c_i \left(\frac{3\phi}{2} + \sum_{j \neq i} \bar{g}_{ji} \right) - \frac{3\phi}{2} \frac{c_i^2}{v_i} \right] \tag{1.26}$$

To gain some insights on welfare, I now focus on the extreme cases of symmetric and core-periphery networks[59] - i.e. the complete, the ring, and the star structure - plus the line network. All are depicted in fig. 1.11.

Welfare study in these network structures is facilitated because they all share the same property of trading centrality being monotonically increasing in degree.[60] I provide two welfare rankings: traders' expected utility within a trading network, and aggregate expected utility across trading networks of the same size. Thus, it holds that central dealers are those with i) higher PM demand (i.e. asset holdings); ii) lower liquidity costs (i.e. higher re-selling prices); and iii) who face lower local market prices as buyers.

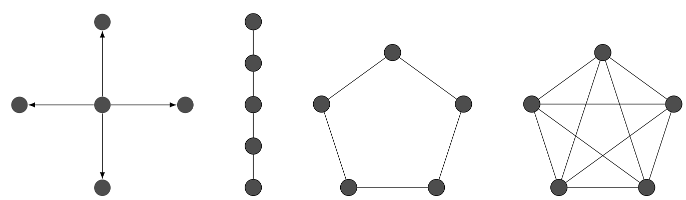


Figure 1.11: The star, the line, the ring and the complete trading network with five traders.

At the individual level, I find that a trader's expected utility EU_i^* is increasing in his centrality c_i.[61] Thus welfare ranking of traders is given by trading centrality: more central traders achieve a higher expected utility.

Proposition 9. *For $N > 3$ and if the trading network is either a complete, ring, line or star graph, then a trader's expected utility increases in his trading centrality.*

At the aggregate level, higher connectivity leads to higher aggregate trading centrality and, thus, higher PM price (eq. (1.15)). Not only that, but also higher local market prices

[59] In section A.12 I explicitly discuss the ring and the star networks. Their simplicity provides insights on how network size and degree distribution interact with one another, and how it affects the equilibrium.

[60] Recall Lemma 8 that this is not always the case.

[61] This results from the fact that trading centrality is increasing in degree See Section A.13 and Lemma 10.

since they are increasing in degree. As a consequence, I find that welfare decreases with aggregate trading centrality - and connectivity.[62]

Proposition 10. *For $N > 3$, welfare ranking across the following networks is:*

$$\textit{Star} > \textit{Line} > \textit{Ring} > \textit{Complete}.$$

Thus welfare ranking of trading networks is according to the aggregate trading centrality, and it is the reverse as the PM price rank. At a first glance this result seems odd. Welfare increases with degree (and asset allocation) inequality, and decreases with connectivity.

However, keep in mind that traders in the model are natural buyers of the asset by assumption. And each trader has a (weakly) greater probability of being a buyer of the asset in both periods. In turn, the opposite effect of the degree distribution on prices pushes them at such a greater level that is detrimental to traders' utility. On top of that, the likelihood of high local market price is higher in more connected and less unequal networks, what also drives utility down. Ultimately, greater welfare is determined by traders *expecting* lower prices in all markets - the PM and local markets.

It is worth emphasizing that asset allocation inequality is an equilibrium outcome. Traders, by taking into account their own and others' position in the trading network, optimally decide their holdings. That's why demand inequality is not necessarily detrimental for welfare, but a reflection of the strategic response between traders themselves.

My welfare analysis concludes that the trading network delivering the highest (lowest) PM price, the complete (star) structure, is the exact one delivering the lowest (highest) welfare. I stress that Proposition 10 only compares the four network structures illustrated in fig. 1.11. A more interesting exercise is to consider any arbitrary trading network and investigate which trader(s) or linkages should be removed or added to increase welfare. I leave this for future research.

In the literature of decentralized markets, a typical result argues that the absence of frictions would correspond to maximal welfare. This is not the case in my model and it's similar to the result obtained in Malamud and Rostek (2017), Wittwer (2021) andGlode and Opp (2020), who demonstrate that decentralized markets might be more efficient than centralized markets. In contrast to these previous work, my framework is the first so show that allocative efficiency in decentralized markets (i.e. local markets) does not lead to greater (or maximal) welfare. Moreover, intertemporally, welfare is maximized by having the two most extreme trading schemes: a centralized (PM) market followed by a market in which one trader intermediate all trading flows (star network).

[62]See section A.13. There I show that this result follows from the previous ones regarding PM price bounds (Proposition 2) and the price effect of the degree distribution (Proposition 3 and Proposition 4).

1.8 Empirical Exercise: Real-world Interdealer Network

An attractive feature of my model is that it has a straightforward empirical application to off-exchange assets, and it generates generates a rich set of empirical predictions. In this section, I give guidelines for future empirical work by an illustrative example on the US Corporate Bonds market. Namely, conditioning on the inferred interdealer network, I use trading centrality to compute dealers' inventory, and interdealer trade prices and quantities. Then I explore the sensitivity of various observable variables of interest with respect to trading centrality.

I use a sample of the TRACE data provided by Friewald and Nagler (2019)[63] with information on secondary markets of US Corporate bonds.[64] The TRACE database contains detailed transaction information on the prices and volumes both between dealers and customers (d2c trades)[65] and among dealers themselves (d2d trades). Crucially, the information is at the dealer-level. This allows me to recover one main ingredient needed for the empirical exercise: the interdealer network.

I document an interdealer network with 201 dealers and 452 links. The distribution of dealers' number of trading relationships and trading centrality are highly skewed. There is also great pair-wise heterogeneity of trading frequency and volume traded. I also document that dealers sell more often to the customer, instead of buying from him; and that the more central dealers have a greater volume of sell trades with the customer.

I find that, in the interdealer trades in the network, trading centrality has a positive effect in sell volume, and a negative effect in buy volume. Thus, as the model predicts, central dealers sell more and buy less in the interdealer market. For prices, trading centrality has a positive effect on sell price and a negative effect on buy price. That is, central dealers sell at a higher price and buy at a lower price, as implied by the model as well.

Section 1.8.1 depicts the inferred dealer network; and Section 1.8.2 discusses regressions outcomes. All details and regression outputs tables are found in the Section A.16.

[63]The authors' sample captures all trades executed by more than 2,600 dealers over the 2003 to 2013 period. In this empirical exercise, I use a restricted version of it which the authors made available for replication purposes.

[64]The ideal empirical exercise would combine data on primary markets, from the Mergent/FISD database in the case of US Corporate bonds, and the secondary markets, from the TRACE data. This is work in progress.

[65]Customers cannot be identified and d2c trades are all assigned to a representative "Customer".

1.8.1 The dealer network

There are 201 dealers in the sample, who trade 5 different bonds over 42 trading days (what corresponds to two months, May and June 2009).[66] I infer the interdealer network from the realized trades between any pair of dealers (d2d trades). I link two dealers if they trade with one another at least once during the sample period. I restrict the set of dealers to those who trade both at the d2d and d2c markets.

I document an interdealer network with 201 dealers and 452 links, depicted below (Figure 1.12).

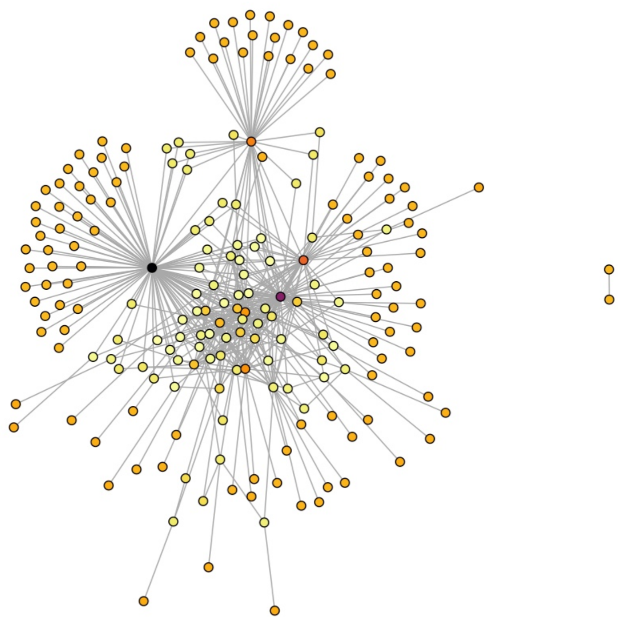


Figure 1.12: Dealer network: trading centrality increases from darker to lighter colored nodes

The table 1.1 shows some network characteristics. Both degree and trading centrality distribution skewed: the former is skewed to the right, and the latter to the left - as depicted in fig. 1.13. There is also high asymmetry in pair-wise trading frequency and volume traded. On average, two dealers trade less than six times. But there are few pairs that trade quite frequently. This relates to the core-periphery structure of the network, which suggests that just a few dealers intermediate most of the trades.

[66] My main analysis focuses on the trade data and model predictions using the full sample with all bonds.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
degree	201	4.498	9.473	1	1	4	93
trading cent.	201	0.873	0.097	0.000	0.836	0.944	1.000
#path-two connections	201	105.000	101.900	0	38	130	483
pair frequency	452	5.872	21.480	1	1	3	320

Table 1.1: Interdealer network statistics

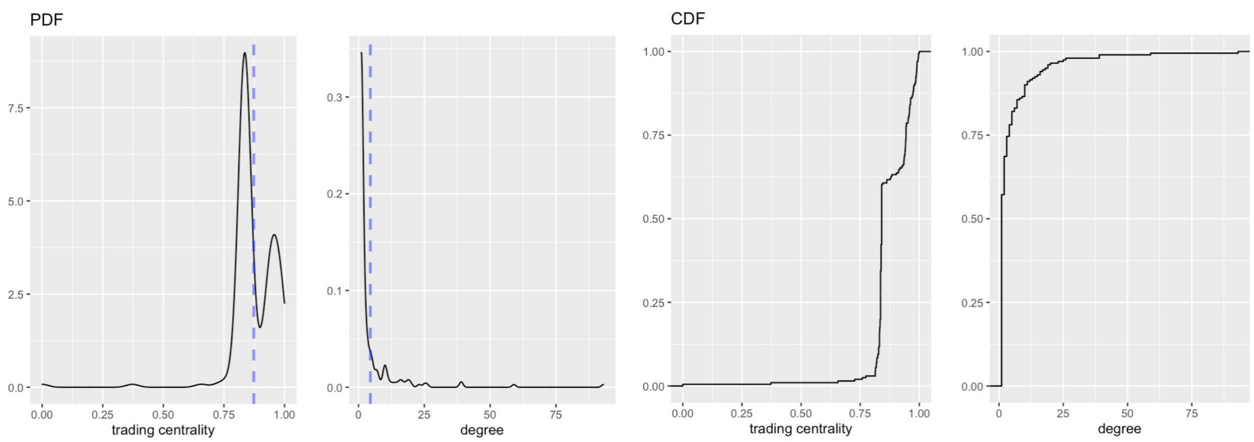


Figure 1.13: Trading centrality distribution, and degree distribution

Trading centrality and degree are negatively related. And so is my centrality with eigenvector centrality (fig. 1.14). This is not surprising as, for trading centrality, a dealer's centrality is deceasing in the centrality of his direct and indirect connections.

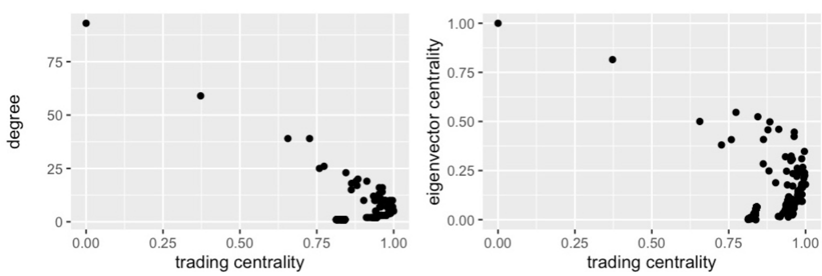


Figure 1.14: Trading centrality and degree relationship (left); and Trading centrality and eigenvector centrality relationship (right)

1.8.2 Trading Centrality and Interdealer Trades

Through the lens of my model, trading centrality determines and prices and quantities in the interdealer market, the empirical counterpart of period-two local markets. Is this the case in the data as well? I investigate the relationship between the trades in the interdealer network and trading centrality. According to the model, more central dealers sell more in the interdealer market. The relationship of trading centrality and selling price is ambiguous.

My analysis is at the daily level (i.e. a panel data). I estimate the following regressions for trade volume and price, respectively:

$$vol_{i,t} = \alpha_{i,t} + \beta_{ci} tc_i + \boldsymbol{\beta}_i' \boldsymbol{F}_i + \epsilon_{i,t} \tag{1.27}$$

$$pr_{i,t} = \alpha_{i,t} + \beta_{ci} tc_i + \boldsymbol{\beta}_i' \boldsymbol{F}_i + \epsilon_{i,t} \tag{1.28}$$

where $\boldsymbol{F}_i$ is the vector of dealer i's time-invariant observable characteristics such as degree deg, net total d2c qi (i.e. inventory), average price of inventory pi.[67] I also control for the transaction price pr in the volume regression.

Regressing trade volume and trade price on trading centrality delivers significant results. For trade volume, the sign of trading centrality coefficient varies depending on the set of controls. Trading centrality alone has a positive effect on volume, and so it does when controlling for inventory and trade price. However, adding degree and as control, turns the centrality coefficient negative (Section A.16.3). This implies that central dealers trade more volume in the network. Through the lens of my model, the change in sign when controlling for degree can be explained as follows. In the inferred interdealer network,degree and trading centrality are negatively related. Thus, since central dealers have less connections they unavoidably have less participation in the interdealer trade and thus less trade volume.

For trade price though trading centrality is only significant when controlling for degree. In this case, trading centrality has a positive effect on price: more central dealers face higher interdealer prices (Section A.16.3).

In my framework, an important distinction in local markets regards the side at which traders are, i.e., when traders are buyers and sellers. Because of that, I re-estimate eq. (1.27) controlling for the transaction side: if the dealer is buying or selling.

Trading centrality has a negative, significant coefficient in all buy volume regression specification. Thus, as the model predicts, central dealers buy less in the interdealer market. However, the sign of the centrality coefficient varies in the sell volume regressions. The coefficient is positive if degree is not added as a control. Thus, in general, central dealers sell more in the interdealer market. Again, as implied by the model. Recall that in the model, selling price decreases in centrality if one controls for the seller's degree. This

[67] In Section A.16 I discuss dealer-customer trades and how I calculate dealers' inventory.

suggests that a more central trader would want to sell less, just as the regression results with degree suggest (Section A.16.3).

Looking at prices, again centrality is only significant when degree is not added as control. In this case, trading centrality has a positive coefficient on sell price and a negative coefficient on buy price. Thus, central dealers sell at a higher price and buy at a lower price, as implied by the model as well.

Moreover, for both buy and sell trades, inventory price (from customer trades, i.e. primary market price) has a positive effect on prices. Through the lens of the model this makes sense. As a seller, higher inventory price means it is more costly to sell. And as a buyer, higher inventory price means it is relatively cheaper to buy in local markets, what increases the demand from the seller and so pushes his price up (Table A.5).

1.9 Discussion

With the intuition for my results in place, I now establish that the model accommodates pertinent extensions. In any of them, even though the model becomes less tractable, the main finding still prevails: primary market price is characterized by the trading network. The general formulation for trader's optimal demand schedule (1.10) is given by

$$q_{i,1}\left(q_{-i,1}; P_1, \phi, \mathbf{G}, \Psi\right) = \beta_i \left[a_i - bP_1 - \phi \sum_j \beta_{ij} q_{j,1}\right] \tag{1.29}$$

where Ψ is the set of model parameters apart from the shock ϕ. Coefficients $(\{\beta_i, a_i\}_{i\in N}, b, \{\beta_{ij}\}_{\forall i,j\in N})$ are endogenous and functions of one or all arguments $(\phi, \mathbf{G}, \Psi)$.

What differs across model specifications is, apart from *parameters*,[68] how the patterns of trading linkages map into trading centrality and, thus, prices and demands. I point out that the study of equilibrium outcomes and the trading network structure in section 1.6 and section 1.7 do not hold in general. I leave this exploration for a companion paper.

All details and full analytical solution for each extension are found in Section A.14.

Heterogeneity in Preferences:

Consider the more general set up in which traders have different individual valuation $\alpha_i > 0$ and risk-aversion γ_i in the quasilinear-quadratic utility,

$$U_i(Q_i) = \alpha_i q_i - \frac{\gamma_i}{2} q_i^2$$

[68]Namely, i) the different individual valuations and risk aversion, and ii) the different beliefs.

In financial markets, heterogeneity in α_i captures the different and persistent close relationships traders tend to form with their clients in OTC markets (Di Maggio et al. (2017b)). The different cost γ_i may be related to fund outside investments, regulatory capital or collateral requirements, which may vary across traders.

I find that the equilibrium is determined by a modified trading centrality that incorporates the different levels of risk-aversion and asset valuation. While risk-aversion affects the network effect across traders' PM demand, the individual valuations only affects the level of demand of each trader.

Expected Fundamental Returns

In reality, traders care about the fundamental return of an asset. They hold an asset not just for the sake of holding it (i.e. to enjoy utility flow) but because they expect that the asset itself is a good financial investment, with high intrinsic value. Suppose then the asset has uncertain return f which is normally distributed with mean μ and variance σ^2, and it is realized after all trading activities take place[69] traders have initial wealth w_0 and choose asset inventory q_i to maximize the expected CARA utility of final wealth $E\left[-\exp\left(\gamma W\right)\right]$ given by $W = f(q_{i,1} + q_{i,s}) - (P_1 q_{i,1} + P_s q_{i,s}) + w_0$.

I find that, in equilibrium, asset price reflects both the traders' beliefs on returns and the trading network. Importantly, the way the former is incorporated into price depends on the later. That's because, a trader i's PM demand depends on market price P_1, his information *and* the information and demand of *all other traders*, including those he is not directly connected to but who are connected with his connections. This is in stark difference with the canonical linear asset pricing model where individual demands depend on all agents' information set *but not directly* on other demands. That's because in such setting equilibrium price aggregate all useful information and so it is not necessary to know other demands. In my model, however, even an anticipated shock and the fact the it leads to different trading possibilities make agents to conditional on others demands, since this is informative about the market structure.

Price Impact

In my model, traders are price-takers and strategic in terms of demands in the sense that they understand that their PM demands directly affects prices in the trading network. In other words, they do take into account the effect of their first-period choices on the second period price, and vice-versa. In turn, traders' do have price impact in the PM which

[69]The normality assumption is standard in this literature. See, for instance, Kyle (1989), Vives (2011), Rostek and Weretka (2012), (Duffie and Zhu (2016)) and others.

arises endogenously precisely because of intertemporal demand dependence.

Notice that this is different than saying that traders try to manipulate prices by "shading their bids" in the PM, as in the models with imperfect competition such as Kyle (1989) and Rostek and Weretka (2015). Nonetheless, my framework can be used to study the economy with strategic traders that take into account their price impact, as Rostek and Yoon (2020b). I find that the equilibrium with imperfect competition is determined by a modified trading centrality that is only a function of the network structure - just as in my model.

1.10 Conclusion

This paper shows why and how future re-sale market structures affect asset pricing before trade, in what I call the primary market (PM). I develop a dynamic trading model where re-sale of a divisible asset takes place in local markets of limited and random participation, captured by a trading network. I show that to find the equilibrium is enough to look at the structure of the trading network.

Trading centrality, a novel network metric, is a sufficient statistic for equilibrium. Behind this result is the interdependency of demands across traders and markets due to the interaction of re-sell risk and the interconnected local markets. The key insight is that my network measure processes all the information driving traders' behavior.

My results are of interest to regulators, scholars and participants of financial markets alike. I argue that the interdealer network not just guarantee the well-functioning of over-the-counter markets but determines the cost of credit in the economy. Also, trading centrality offers a new measures of liquidity and "importance" in the interdealer market that only require information about the interdealer network structure. Both can be useful in empirical applications.

This paper allows for several and interesting extensions. Straightforward ones are having multiple sellers, and to allow for the choice of trading venues or contemporaneous access to all markets. One present limitation is that, due the two-period environment, I do not study intermediation chains which have been pointed outed as important in otc markets. However, having multiple periods natural extension of the model. Apart from financial market applications, the model I develop is suitable for any environment in which a good is initially priced by a large set of agents but subsequently it is only valued or can only be traded by a restricted subset of them.

Chapter 2

Social Beliefs and Stock Price Booms and Busts

2.1 Introduction

Motivated by recent booms and busts episodes in several asset markets, research has been devoted to better understand why and how such behavior emerges. At the same time, it is known that information and beliefs play a role in every model and every economic choice. The existing literature on asset pricing mainly focus on agents having subjective beliefs about market outcomes as an explanation for such phenomena of aggregate stock market prices. Yet, the social structures where these agents directly interact with one another have not been extensively explored. Social connections transmit information through communication, exchange of opinions, and observations of others' decisions. In this paper I argue that, along with heterogeneity in beliefs, information sharing dictated by investors' social ties are an important ingredient for understanding asset price dynamics.

In this paper I explore the effects of communication in an asset pricing model introduce by Adam et al. (2016) in which agents learn about stock-market price and are internally rational. I extend their work by allowing agents to hold heterogenous subjective expectations about asset price behavior and diffuse beliefs through connections captured by an exogenous social network of investors. At each period, agents hold private beliefs which are susceptible to social influece. After interacting with others in their neighborhood, they formulate public beliefs, which are a result of peer effects excerced by whom the agents are connected to. Hence, social interaction renders a conformism in public beliefs.

I focus on the role of social network as facilitating social influence, specifically through communication. People are constantly talking with each other, discussing opinions and seeking knwoledge. For instance, Calvó-Armengol et al. (2009) and Jackson (2010) argue that the behavior of individual agents is affected by that of their peers. This 'peer effect' can shape human behavior in the sense that a person holds certain public beliefs because his friends do so. Specifically in the financial markets, one can observe that investors tend to seek financial advice, search for stock-market news and discuss with their mates before making asset investment decisions. It is then natural to think that investors incorporate, at least partially, some of this social engagement into their conduct towards economic activity and tend to conform with those who they interact. Several studies in the network literature have explored these concepts under a wide range of research area (To name a few: Bala and Goyal (1998), DeMarzo et al. (2003), Banerjee et al. (2013), Banerjee and Fudenberg (2004), Golub and Jackson (2010)).

In the model, agents formulate beliefs about risk-adjusted stock price growth using optimal Bayesian filtering techniques and last-period observations of price and dividend. They adjust beliefs upward (downward) if they underpredict (overpredict) the realized price outcome. I characterize agents into two types, experienced (L) and inexperienced (H) investors. Both types have the same optimal belief-updating equation except that H-agents update their posteriors more heavily in response to forecast errors. Hence, inexperienced agents have more volatile expectations than experienced ones. Evidence from U.S. stock-market (Vissing-Jorgensen (2003), Adam et al. (2015)) shows that investors with less experience in the stock market (in number of years) updated expectations about future asset returns more strongly. This fact can be explained, for example, by investors having different information sources. In this case, experienced agents, who have been trading for longer, have access to certain pieces of valuable information before other investors; while the inexperienced type rely more on publicly available information such as past asset prices. Another plausible reason is that those with more experience have higher self-confidence to trade and so rely less on last period observations. Finally, investors can simply have different economic models in mind that lead them to interpret forecast errors diferently. Hong and Stein (2007) extensively discuss sources of disagreement in expectations in the stock market and corroborate their arguments with evidences from U.S. data.

After the individual learning scheme takes place, investors engage in direct, truthfull communication with others that are connected to them through an underlying social network. Each agent attaches a weight to each belief he receives, and these weights are type-specific. Hence, invidual weights are given by the proportion of H-type and L-type friends each investor has. This information sharing results in a new revision of subjective expectations. However, communication does not change agents' private belief about price growth, given by his type, and so agents agree to disagree forever.

Private and public beliefs interplay between each other. Each agent's private belief is given by his type and determines the way the agent filters out information from past observed market outcomes. This is an individual trait of the agent. In addition, each investor is influenced by his neighbors and he has incentives to conform to his friends' patterns of behavior. In the presence of such peer pressure, an agent formulates his public opinion by averaging his friends' beliefs. There is a large body of literature on the subject of peer effects in different areas, such as economics, sociology, education and crime. One challenge is how to capture such social influence. I choose a simple average of friends' beliefs for simplicity but the model accomodates other sorts of weighted averages. For instance, one alternative is De Groot Learning (DeGroot (1974)), in which each agent has a social influence weight.

The timing of the model is as follows. At the beggining of each period and using last-period observations, agents update private (initial) beliefs based on their forecast error. Then, agents share private beliefs with others who are part of their neighborhood and update their beliefs according to the weight their linkages imply. This results in investors' public beliefs. At the end of the period, equilibrium is determined by agents exchanging stock holdings and, since the stock market is perfectly competitive, the agent with the highest public belief holds the asset and determines it price.

I show that, because agents formulate and share beliefs about asset price, the equilibrium is then determined by investors' expectations. Differently from Adam et al. (2016), equilibrium price is a function of only the most optimistic agent's belief, defined as the one with the highest public belief.The only sources of heterogeneity among agents concern their perception about future stock-market behavior and who they are connected to. Since the stock market is perfectly competitive, in equilibrium the asset is held by the investor who is willing to pay the most for it which is precisely the agent with the highest expectation about asset price growth.

The model gives rise to a feed-back loop between beliefs and realized price, resulting in booms-and-busts phenomena of the price-dividend ratio. When asset price is increasing, both types expect it to keep increasing, what feeds back into a higher equilibrium price. This further increase in price makes agents revise beliefs upwards again, and realized price is again higher, and so on. The reverse holds during a bust. This self-referential aspect is similar to Adam et al. (2016). However, in my model the extent and frequency of booms and busts depend on the social network. I also obtain different behavior of agents' public beliefs which is a function of their network structure.

I show that booms episodes are determined by agents with more social connections to inexperienced investors. When price is increasing, and given that inexperienced agents are more reactive, H-type initial beliefs are higher than L-type beliefs. Thus, agents who interact more with inexperienced investors become more optimistic and so have higher public beliefs. The contrary holds during busts, marked by experienced agents being more optimistic. The periods inbetween boom and busts, referred as 'recovery periods', are characterized by the stock price staying around its mean value and experienced agents are the most influential, since H-type beliefs become too pessimistic after a bust.

The importance of modeling information sharing as weighting neighbors' beliefs is two fold. First, it captures the network structure. Second, equilibrium is a function of private beliefs. Even though only the most optimistic agent determines asset price, his public belief is a combination of others' beliefs who are socially linked to him. Hence, the fluctuations of the PD ratio hinge on the existence of correlated movements in agents beliefs and the social effects agents are exposed to.

I find that the structure of the social network matters for asset price fluctuations and price-dividend volatility. More connected networks exhibit less volatile price-dividend ratio and booms and busts episodes last for longer periods. At the same time, in such structures there is less dispersion in public beliefs and the average price realization is higher. Nonetheless, if agents of the same type are more connected to each other, stock market price fluctuates more and booms and busts have shorter duration.

I also evaluate the quantitative performance of the model by simulating the economy with different network structures. As Adam et al. (2016), I find that the model can replicate key empirical facts, specially the high volatility of the PD ratio and excessive return volatility. The equity premium puzzle, which they have difficulty in matching, I cannot also fully replicate. Overall, my model can account approximately half of the observed equity premium, what is slightly above Adam et al. (2016)' finding. Due to the lack of data, I am not able to confront simulation results with other countries' stock market, such as Japan and te Euro-Area. Nonetheless, Adam et al. (2010) report evidence that these countries, even though they all have experienced boom and busts, the frequency and timing of these cycles, as well as the variation of the PD ratio differ across them. I argue that one of the many reasons of why this is the case is their different social structures. I see this as a next research avenue.

As a final exploration, I relax the assumption of the social network being exogenously determined. I use a random graph approach, widely used in the network literature (Jackson and Wolinsky (1996), ERDdS and R&wi (1959)), to infer what network would emerge if the only knowledge one has is the probabilities of agents to connect with each other. I investigate the impact of homophily (Golub and Jackson (2012)) in stock market-equilibrium, that is, I assume agents of the same type tend to share more connections. I assume the network structure is captured by the homopholy index and that equilibrium is then a function of homophily and private beliefs. I find that asset price volatility and the frequency of booms and busts are positively related to homophily, whereas the duration of such episodes is negatively related.

I provide a simple way to introduce social interaction into a standard asset pricing model. The results show that stock market outcomes - namely, equibilibrium asset price, price-dividend behavior and agents' subjective beliefs - are all dependent on the underlying network topology. To my knowledge, this paper is the first to combine adaptive learning and social networks in the asset pricing literature. For instance, research on asset pricing and learning, in which Adam et al. (2016) fits in, has not yet incorporate any form of social dynamics or information sharing. Meanwhile, stock market models in information networks such as Ozsoylev and Walden (2011) and Xia (2007) all share the standard rational expectations assumption. Thus, this paper can serve as an initial step to better comprehend the interdependency of the stock-market behavior and the social structure of the economy.

Literature: My work relates to two fields of ongoing research about stock market behavior: adaptive learning and asset pricing in networks.

Under the adaptive learning literature, agents learn about stock prices. The underlying main hypothesis is that agents are not 'perfectly rational' in the sense of rational expectations (RE) but instead do not know exactly the pricing function that governs asset price and so they take rational decisions given the information they are assumed to possess. Adam and Marcet (2011) are the first to provide a microfoundation for these models and introduce the concept of 'internally rational' agents who maximize discounted expected utility under uncertainty given dynamically consistent subjective beliefs about the future. They study a simple asset pricing model with risk-neutral investors and show that learning about price behavior affects market outcomes, while learning about the discounted sum of dividends, the case under RE, is irrelevant for equilibrium prices.

In this same line, Adam et al. (2016) study a simple variation of the Lucas Jr (1978) asset pricing model in which agents hold subjective prior beliefs about risk-adjusted price growth. They show that internally rational agents update their beliefs about stock price behavior using observed stock price realizations. The setup gives rise to a model characterized by large asset price fluctuations and volatile price dividend ratio. They are able to replicate some stylized facts of U.S. stock price data that previous RE models have not been able to account for. Namely, the high persistence and volatility of the price-dividend (PD) ratio; high volatility of stock returns; and the predictability of long-horizon excess stock returns. As they explain, these results arise because there is a momentum of changes in beliefs around the RE value and a mean-reverting behavior of beliefs. Nonetheless, their model fail to reproduce the observed equity premium under reasonable degrees of risk aversion.

My model introduces an additional dimension to Adam et al. (2016). I let investors be heterogenous in their beliefs about asset price behavior and assume the existence of a social network connecting all agents. Given their linkages, agents are able to communicate and share individual beliefs about risk-adjsuted price growth. In this sense, the present framework captures how different stock price expectations and social interaction can affect equilibrium asset pricing outcomes and agents market behavior.

The asset pricing literature has documented the relevance of heterogeneity and social interaction among investors as influential to observed stock market outcomes. Vissing-Jorgensen (2003) surveys evidence on the stock market behavior and actions. She documents that an investor's belief about future stock-market returns depends on the investor's own experience measured by age, years of investment experience, and own past portfolio returns. She shows that expected returns are higher for those with low investment experience for a given age than for those with more years of experience. The evidence also suggests that investor beliefs do affect their stockholdings, and she concludes that that understanding beliefs is in fact useful for understanding prices.

Adam et al. (2010) report evidence that different stock-markets historically experienced substantial and sustained price increases that were followed by sustained and long lasting price reversals. They show time-series data for the U.S., Japan and the Euro-Area since 1970 and all markets present PD ratio booms and busts. However, the frequency and timing of these cycles, as well as the variation of the PD ratio differ across them. I argue that one of the many reasons of why such pattern of the PD ratio is different among those countries can be their social structure.

My work also fits in the strand of research about asset pricing in networks. Ozsoylev and Walden (2011) study the impact of the properties of information networks on asset pricing in a rational expectations equilibrium model. Agents communicate information to each other about asset payoffs according to an exogenous information networt and each agent has some information about her network neighbors payoff-related information. They provide closed form solutions and find that aggregate properties of the market - for example, price volatility, expected trading profits and agent welfare - are non-monotonic functions of network connectivity. Hong et al. (2005) use U.S. data on mutual fund holdings from the late 1990's and find that a mutual fund manager is more likely to buy (or sell) a particular stock in any quarter if other managers in the same city are buying (or selling) that same stock. They argue that this can be seen as the result of an epidemic model in which investors spread information about stocks to one another by word of mouth. However, their work is purely empirical and they do not estimate the underlying network per se. Xia (2007) develops an strategic rational expectations asset pricing model with one-way direct and truthful information transmission in circle and star networks. He finds that several aggregate market outcomes, including trading volume and price volatility, are all higher with communication than without.

The aforementioned studies share the same underlying assumption of rational expectations, which essentialy means that all agents have perfect knowledge about the pricing function that maps fundamentals to asset price. On the contrary, the agents in my model are 'internally rational' and hold a perceived law of motion for asset price while the realized price derives from an actual ('true') law of motion.

The role of communication, and more broadly social interactions, through social network in economic models has been recognized in other fields. Bala and Goyal (1998) show that communication in social networks play a major role in technology adoption. Banerjee et al. (2013) show, theoretically and empirically, that information diffusion influences people's adoption decision, in their case, in the participation of a microfinance program. Golub and Jackson (2012) explore how the speed of convergence of agents behaviors and beliefs depends on network structure. DeMarzo et al. (2003) consider the spread of information across a given network when individuals are subject to persuasion bias and show that agents with particular network positions can have disproportionate influence. Jackson (2010) presents a textbook on network economics.

I contribute to the stock-market behavior literature by aligning adaptive learning and social interaction. I introduce a simple model of communication among socially connected investors in an otherwise stadard asset pricing model where agents learn about prices. I derive the equilibrium outcomes of the model as a function of the communication strucutre. Also I quantitatively evaluate its perfomance to show that the model can replicate the observed asset market booms and busts and that the characteristics of such cycles vary considerably depending on social network.

The paper is organized as follows. Section 2.2, presents the model and Section 2.3 determines the equilibrium. Section 2.4 discusses agents' learning problem. Section

2.5 outlines the solution of the model and characterizes agents' behavior. Section 2.6 explores the implications for asset price dynamics. Section 2.7 discusses equilibrium as a function of network characteristics. Section 2.8 presents the results of simulating the model. Section 2.9 extends the model to the case with an unkown network structure. In Section 2.10 is a conclusion. The Appendix contains omitted proofs and discussions.

2.2 A Network Asset Pricing Model

I introduce an extended version of the model studied by Adam et al. (2016). It is a Lucas Jr (1978) asset princing model in which agents hold heterogenous prior beliefs about stock price behavior and are a part of an exogenously given social network. Those who are socially connect share beliefs. The information sharing is truthfull and credible (agents commit not to lie), and agents do not act stratigically towards price manipulation in ones' favor.[1] Agents are susceptible to peer effects from those thet are connected to.

2.2.1 Model Enviroment

Consider a discrete-time economy with a large number of N of infinitely-lived agents trading one unit of stock in a competitive stock market. At each period a stock yields a stochastic divend D_t and investors receive an exogenous endowment Y_t, both in the form of perishable consumption goods.

Given Y_t and D_t, aggregate consumption is such that economy's feasibility constraint holds: $C_t = Y_t + D_t$.

The exogenous processes for dividends and aggregate consumption are, respectively

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d, \quad \log \epsilon_t^d \sim_{ii} N(-\frac{s_d^2}{2}, s_d^2) \tag{2.1}$$

$$\frac{C_t}{C_{t-1}} = a\epsilon_t^c, \quad \log \epsilon_t^c \sim_{ii} N(-\frac{s_c^2}{2}, s_c^2) \tag{2.2}$$

where $a \geq 1$ and $\log \epsilon_t^d, \log \epsilon_t^c$ are jointly normal. Let $\rho = E\Big[(\epsilon_{t+1}^c)^{-\gamma}\epsilon_{t+1}^d\Big] = \exp\Big(\gamma(1+\gamma)\frac{s_c^2}{2}\Big)\exp\Big(-\gamma\rho_{c,d}s_c, s_d\Big)$.

There are two types of investors $K = \{h, l\}$: inexperienced (H) and experienced (L). Types differ in their private expectations about the behavior of future stock prices:

[1] That is, agents do not wish to formulate beliefs in order to influence price in such a way to make them hold asset in equilibrium.

$H-$agents have more volatile private beliefs than $L-$agents.[2] This characterization is motivated by Adam et al. (2015), who documented evidence that investors with less stock market experience are more heavily influenced by recent asset price realizations than those with more years as traders.

The investment problem

The risk-averse and internally rational agents have standard time-separable consumption preferences. Investor $i \in N$ of type $k \in \{h, l\}$ chooses consumption C_t^{ik}, bonds B_t^{ik} and stock holdings S_t^{ik} in order to maximize expected future utility[3]:

$$\max_{\{C_t^{ik}, S_t^{ik}, B_t^{ik}\}_{t=0}^{\infty}} E_0^{ik} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^{ik})^{1-\gamma}}{1-\gamma} \tag{2.3}$$

$$S_t^{ik} P_t + C_t^{ik} + B_t^{ik} \leq S_{t-1}^{ik}(P_t + D_t) + (1 + r_{t-1}) B_{t-1}^{ik} + Y_t \tag{2.4}$$

where $\gamma \in (0, \infty)$ is the coefficient of relative risk aversion; r_{t-1} is the real interest rate on riskless bonds issued in period $t-1$ and maturing in period t; and E_0^i denotes agent's subjective probability space that assigns probabilities to all external variables P_t, D_t, Y_t.

Initial endowments of individual stock holdings and bonds are $S_{-1}^{ik} = 1, B_{-1}^{ik} = 0$. To avoid Ponzi schemes and to guarantee a solution to the problem, assume the following bounds hold, $\forall i \in N, k \in K$:

$$\underline{\mathsf{S}} \leq S_t^{ik} \leq \bar{S} \tag{2.5}$$

$$\underline{\mathsf{B}} \leq B_t^{ik} \leq \bar{B} \tag{2.6}$$

with $\bar{S}, \bar{B} < \infty$, $\underline{\mathsf{S}} < 1 < \bar{S}$ and $\underline{\mathsf{B}} < 0 < \bar{B}$.

Below I will specify agents' subjective probabilities by a learining scheme describing their view about the evolution of (Y_t, D_t, P_t). Differently from what is implied by the RE assumption, these subjective probabilities may or may not coincide with the true probabilities governing the behavior of those variables.

Agent's optimal choices are given by the first order conditions of his utility maximization problem:

[2] I further disucss the characterization of types in Section 2.4

[3] For simplification, I assume agents have the same discount factor δ and coefficient of relative risk aversion γ.

$$(C_t^{ik})^{-\gamma}P_t = \delta E_t^{ik}\Big[(C_{t+1}^{ik})^{-\gamma}P_{t+1}\Big] + \delta E_t^{ik}\Big[(C_{t+1}^{ik})^{\gamma}D_{t+1}\Big] \tag{2.7}$$

$$(C_{t+1}^{ik})^{-\gamma} = \delta(1+r_t)E_t^{ik}\Big[(C_{t+1}^{ik})^{-\gamma}\Big] \tag{2.8}$$

As in Adam et al. (2016), I assume agent's individual income Y_t is high enough and that expected future PD ration is bounded.[4]

Assumption 3. *Given the individual maximization problem above, assume agents' private wealth Y_t is sufficiently large and that $E_t^{ik}\frac{P_{t+1}}{D_t} < \bar{M}$ for all i, k and for some $\bar{M} < \infty$, such that individual consumption choices can be approximated by the aggregate consumption: $\frac{C_{t+1}^{ik}}{C_t^{ik}} \approx \frac{C_{t+1}}{C_t}$*

Hence, the first-order conditions boil down to

$$(C_t)^{-\gamma}P_t = \delta E_t^{ik}\Big[(C_{t+1})^{-\gamma}P_{t+1}\Big] + \delta E_t^{i}\Big[(C_{t+1})^{\gamma}D_{t+1}\Big] \tag{2.9}$$

$$(C_{t+1})^{-\gamma} = \delta(1+r_t)E_t^{ik}\Big[(C_{t+1})^{-\gamma}\Big] \tag{2.10}$$

Define subjective expectations of risk-adjusted stock price growth and dividend growth to be, respectively

$$\beta_t^{ik} \equiv E_t^{ik}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_{t+1}}{P_t}\right] \tag{2.11}$$

$$\beta_t^{d,ik} \equiv E_t^{ik}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{D_{t+1}}{D_t}\right] \tag{2.12}$$

Stock holdings first-order condition (2.9) can then be written as

$$P_t = \delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma}E_t^{ik}\Big[P_{t+1} + D_{t+1}\Big] = \delta\beta_t^{i}P_t + \delta\beta_t^{d,ik}D_t \tag{2.13}$$

RE as a special case

When one assumes agents know the exact mapping from both exogenous processes (D_t, Y_t) into equilibrium asset pricing $P_t(Y^t, D^t)$, beliefs and equilibrium are given by

[4]This is the *Assumption 1* on Adam et al. (2016). Please refer to it for further details.

$$\beta_t^{ik} = \beta_t^{d,ik} = \beta^{RE} \equiv a^{1-\gamma}\rho \tag{2.14}$$

$$P_t = \frac{\delta\beta^{RE}}{1-\delta\beta^{RE}} D_t \tag{2.15}$$

Hence, under RE, beliefs about both risk-adjusted price growth and divdend growth are constant and so it is the price-dividend (PD) ratio. Asset price follows the same stochastic process as dividend and the PD ratio is constant.

2.2.2 Types' Private Beliefs

In order to focus on subjective expectations of stock market price, I endow agents with perfect knowledge of risk-adjusted dividend growth, that is, all agents hold rational expectations about these dividend processes:

$$\beta_t^{d,ik} = \beta^{RE} \quad \forall i \in N, k \in K, \forall t \tag{2.16}$$

Agents also know the consumption stochastic process. However, they do not know how the equilibrium asset pricing function and they must formulate beliefs about risk-adjusted price growth at every period. This implies that each agent has a subjective (and possibly different) view about the price dividend ratio at each period.

Heterogeneity is characterized by different types having different private beliefs. That is, $\beta_t^{ik} = \beta_t^{jk}$ for $i \neq j, \forall t$ and $\beta_t^{ik} \neq \beta_t^{jk'}$ for $i \neq j, k \neq k', \forall t$. Denote beliefs of inexperience and experience agents, respectively, as β_t^h and β_t^l.

2.2.3 The Social Network

Investors are located in an exogenously given network G of N nodes, each representing an agent. The relation $\mathcal{E} \subset N \times N$ describes which investors are connected in the network. $(i, j) \in \mathcal{E}$ means there is an edge (link) between agents i and j. I assume connections are bidirectional, $\mathcal{E}_{ij} = \mathcal{E}_{ji}$, so that $\mathcal{E}$ is symmetric.

The distance between two agents i and j is represented by a function $d^{\mathcal{E}}(i, j)$, which defines the number of edges in the shortest path between agents i and j. I use the convention that $d^{\mathcal{E}}(i, j) = \infty$ if there is no path between the two agents. An agent neighborhood N_{ik} is set of his closest nodes: $N_{ik} = \{j : d^{\mathcal{E}}(i, j) = 1\}$. I shall refer to a node in an agent's neighborhood as a friend. The degree of investor ik is defined as the investor's number of neighbors, including himself: $D_{ik} = |N_{ik}|$.

2.2.4 Social Dynamics

The main concern of this paper is to explore the effects of the network structure in stock market price behavior. Investors are susceptible to social influence. They exchange information about asset price through direct communication with those they are linked to. Agents conform with their peers and so revise private beliefs after social interaction.

At the beggining of each period, each agent ik updates his type-specific private belief β_t^k, $k = \{h, l\}$. Then, he interacts with others who belong to his neighborhood N_{ik}. Let the weight an investor ik assigns to type-H private beliefs β_t^h be λ^i and the one of type-L beliefs β_t^l be $(1-\lambda^i)$. Intuitivelly, λ^i captures the social influence of different types in investor ik's behavior. λ^i is a function of the investor's social network, not of his type k, and it is determined by the proportion of inexperienced friends he has:

$$\lambda^i = \sum_{j \in N_{ik}} \frac{1\{j = H\}}{D_i} = \frac{D_i^h}{D_i} \tag{2.17}$$

where $D_i^h = \{\#\text{H friends}\}$. Notice that λ^i accounts for investor's own type (self-loop).[5]

After communication, agent formulates a public belief by averaging their neighbors' expectations. Hence, at the end of each period agent's risk-adjusted price growth belief is given by

$$\tilde{\beta}_t^i = \lambda^i \beta_t^h + (1-\lambda^i)\beta_t^l \quad \forall i \in N, k \in K \tag{2.18}$$

$\tilde{\beta}_t^i$ is agent ik's public belief. Differently from his private belief, $\tilde{\beta}_t^i$ depends on the investor's set of friends, and not so much on his own-type. To make this point clearer, the superscript i from β_t^k , and the supersript k from $\tilde{\beta}_t^i$ are dropped.

Social dynamics is then incur peer effects and agents hold certain public beliefs because their friends do so. The challenge of the model is then how social interaction and influence affect stock price behavior.

2.3 Equilibrium

Since agents have heterogenous beliefs and do not hold rational price expectations, the stochastic process for equilibrium price is different from agents' perceived price process. The latter is given by individual stock-holdings first-order condition (2.13). Since the asset

[5]The λ^i measure includes agent's own type to capture the idea that individual beliefs matter. So that agents' subjective expectations are not solely determined by who they are connected to but also depends on they own type.

market is perfectly competitive, agents exchange stock holdings and the the asset is held by the agent who is willing to pay the most for it. Hence, equilbrium is determined by the marginal agent who holds the most optimistic public belief $\tilde{\beta}_t^i$..[6]

Denote the most optimitic's subjective belief about risk-adjusted stock price growth as

$$\tilde{\beta}_t^o = \max_{i \in N} \tilde{\beta}_t^i \tag{2.19}$$

The equilibrium pricing function is then[7]

$$P_t = \frac{\delta a^{1-\gamma}\rho}{1 - \delta \tilde{\beta}_t^o} D_t \tag{2.20}$$

where $a^{1-\gamma}\rho \equiv \beta^{RE} = \beta_t^d, \forall t$. This equation implies that agents' beliefs explicity determine equilibrium price. Asset price is increasing in both subjective expectations about risk-adjusted price growth and dividend growth.

For realized price to be well-defined at all periods, for any set of beliefs, I impose the existence of a maximum price-dividend value. This is justified by the fact that PD ratio will be bounded in equilibrium and it is consistent with the behavior of internally rational agents, and with asset pricing data..[8]

Assumption 4. *There exists a maximum equilibrium price-dividend ratio $\bar{PD} < \infty$.*

2.4 The Learning Problem

In order to formulate subjective beliefs about the stock price behavior, investors make use of two learning channels. Firstly, they use last-period price and dividends observations applied to Bayesian filtering techiniques to infer about realized stock market outcomes. Secondly, they share beliefs with their friends.

The former channel pins down private beliefs β_t^k, while the later determines public beliefs $\tilde{\beta}_t^i$. Social interaction has been introduced in the previous section, and to close up the model the individual learning scheme is specified below.

Recall that private beliefs are a function of agents' type and so there are as many

[6]Adam and Marcet (2011) extensively discuss this result under a similar framework. Please refer to them for more details.

[7]To see this, I can write

$$P_t = \max_{i \in N} \left[\delta E_t^i \big(P_{t+1} + D_{t+1} \big) \right]$$

and substitute out for $\tilde{\beta}_t^o$.

[8]See the discussion under Adam and Marcet (2011) and Adam et al. (2017).

β_t^k as existing types in the economy. Each type, and therefore agent, perceives that the process for risk-adjusted stock price growth is the sum of a persistent component b_t^k and a transitory component ε_t^k ,

$$\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}} = b_t^k + \varepsilon_t^k \tag{2.21}$$
$$b_t^k = b_{t-1}^k + \xi_t^k \tag{2.22}$$

for $k = h, l$ where $\varepsilon_t^k \sim_{ii} N(0, \sigma^2_{\varepsilon,k})$ and $\xi_t^k \sim_{ii} N(0, \sigma^2_{\xi,k})$, such that $\sigma^2_{\varepsilon,h} > \sigma^2_{\varepsilon,l}$ and $\sigma^2_{\xi,h} > \sigma^2_{\xi,l}$.

Let types' prior beliefs b_0^k be centered around the RE belief[9]

$$\begin{aligned} b_0^k &\sim_{ii} N(a^{1-\gamma}\rho, \sigma^2_{0,k}) \\ \beta_0^h &= \beta_0^l \\ \sigma^2_{0,k} &= \frac{-\sigma^2_{\xi,k} + \sqrt{(\sigma^2_{\xi,k})^2 + 4\sigma^2_{\xi,k}\sigma^2_{\varepsilon,k}}}{2} \end{aligned} \tag{2.23}$$

where $\sigma^2_{0,k}$ is the steady-state Kalman filter uncertainty about b_t^k for $k = h, l$. Type's posterior belief at any time period t is distributed as

$$b_t^k \sim N\Big(\beta_t^k, \sigma_{0,k}\Big) \quad \forall k = h, l \tag{2.24}$$

where $\sigma_{0,h} > \sigma_{0,l}$.

To learn about asset price growth, agents act as Bayesians and optimality filter out the persistent component b_t^k. Posterior beliefs are then given by

$$\beta_t^k = \beta_{t-1}^k + g^i\left[\left(\frac{C_{t-1}}{C_{t-2}}\right)^{-\gamma}\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}^k\right] \tag{2.25}$$
$$g^k = \frac{\sigma^2_{0,k} + \sigma^2_{\xi,k}}{\sigma^2_{0,k} + \sigma^2_{\xi,k} + \sigma^2_{\varepsilon,k}} \tag{2.26}$$

where g^k is the optimal Kalman gain and reflects how the type reacts to his forecast

[9]Following Adam et al. (2016), our framework constitute a small deviation from the RE case.

error $e_{t-1}^{k} \equiv \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-\gamma} \frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}^{k}$. Comparing types learning scheme, H-agents update more heavily their beliefs at each period and have more volatile expectations .

This setup means that both β_t^h, β_t^l constitute a small deviation from RE beliefs in the limiting case with vanishing uncertainty about the persistent component b_t $(\sigma_{0,k}^2 \to 0, \forall k)$. In such scenario types share the same belief which converges in distribution to β^{RE}. However notice that RE beliefs are never actually attainable and agents are consistently wrong on predicting price growth.

2.5 Solving the Model

It is important to keep in mind some important features of the model i) agents are internally rational and, at every period, make inferences about the stock market behavior by formulating subjective risk-adjusted stock price growth beliefs; ii) at each period, agents' learning problem consists of two stages: an individual Kalman filter problem that pins down private beliefs (type-specific) β_t^k; and communication, which pins down public specific (agent-specific) $\tilde{\beta}_t^i$; iii) equilibrium price is determined by the most optimistic agent, defined as the agent with the highest public belief $\tilde{\beta}_t^o$.

Henceforth, the model timeline is:

1. at t, last-period beliefs $\beta_{t-1}^i, \tilde{\beta}_{t-1}^k$ for $i \in N$, $k = h, l$ are determined and agents observe realized price P_{t-1} and dividends D_{t-1};
2. still at t and using past price observations, each type solves its learning problem by applying Kalman filtering techniques. This pins downs time t private beliefs β_t^h, β_t^l.
3. then and still at t, agents communicate and public beliefs are set: $\tilde{\beta}_t^i, \forall i \in N$;
4. at the end of period t, equilibrium price P_t is determined by the most optimistic agent: $\tilde{\beta}_t^o$;
5. at $t+1$, this process is repeated.

Notice that agents beliefs $\beta_t^k, \tilde{\beta}_t^i$ are predetermined at time t, so that the economy evolves according to a uniquely determined recursive process: the market-clearing price for period t is determined given time t beliefs; then in the next period $t+1$, beliefs are updated following this observation, communication takes place and $t+1$ PD ratio is realized.

Even though the most optimistic agent sets price, both types' private beliefs $\beta_t^k, k = h, l$ directly impact equilibrium PD ratio. This is a novelty introduced by this framework compared to the baseline model studied by Adam et al. (2016). Communication plays an important role since it determines the extent of each type's influence on realized risk-adjusted stock price growth: the type-belief with a higher weight (λ^i or $1-\lambda^i$) will have a higher impact on the price-setting equilibrium belief.

Solving the model consists on basically two stages, which are mutually influential: 1) the Kalman filter problems, which deliver β_t^h, β_t^l - this is essentially what Adam et al. (2016) do and I refer to them for the details of the results; 2) determine equilibrium weight λ_t^*, which delivers equilibrium beliefs and price $\tilde{\beta}_t^o(\lambda_t^*)$, $P_t(\lambda_t^*)$ - this is done by analyzing the network structure.

2.5.1 First Stage: Individual Learning Problem

The individual learning problem entails different evolution of belief depending on agent's type. In order to study the dynamics of the model it is then crucial to know how beliefs change through time and how distinct are these processes.

Bounded Beliefs

First-order condition (2.13) implies that subjective private beliefs must be bounded: to guarantee a solution for individual opimization problem it must hold that[10] $\beta_t^k < \delta^{-1} \quad \forall k, t$. Intuitively, private beliefs must not be too optimistic. Overly optimistic beliefs can give rise to a situation where subjective expected utility is infinite, so that problem agents' first order condition does not have a well defined solution. In turn, bounded beliefs guarantee the existence of a finite equilibrium PD ratio. Denote beliefs' upper bound by β_k^U.

For all $i \in N$, $k = \{h, l\}$ and t with $P_t < \infty, D_t < \infty, C_t < \infty$, subjective expected risk-adjused price growth is such that

$$\beta_t^{ik} < \beta^U \tag{2.27}$$

where $\beta_l^U = \beta_h^U \equiv \beta^U < \delta^{-1}$.

Comovement of Beliefs

Intial beliefs β_t^k are in a sense extrapolative expectations: if the stock market price has been rising, investors expect it to keep rising; and if it has been falling, they expect it to keep falling, that is,

[10] To see this notice that if $\beta_t^k = \delta^{-1}$, then $\delta\beta^{RE}D_t = 0$ which means there exists infinetely-many solutions to the first-order contion. On the other hand, if $\beta_t^k > \delta^{-1}$, then $\delta\beta^{RE}D_t < 0 \rightarrow D_t < 0$ which cannot hold.

$$\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}} > 1 \Rightarrow e_t^k > 0 \quad \forall k, t$$

where the same holds for reversed inequalities.

As a result, types' beliefs tend to comove and so either both experience and inexperienced agents believe stock price to increase or to decrease. Although this same expected qualitative change holds for most periods, the existence of an upper bound of beliefs implies that there could be times in which agents revise beliefs in opposite directions. That's because, depending on the difference between types' beliefs when PD ratio is approaching its upper limit, the price drop initiated by β_t^h is not enough to make realized price growth to be below β_t^l, resulting in $e_t^l > 0$ and L-type increasing its belief. In general, the comovement in beliefs holds as long as they are not sufficiently close to their upper bounds $\beta_t^k << \beta^U$ for $k = h, l$.

Regardless of its importance on shaping asset price behavior, the aforementioned case depends on a fine set of parameter values and beliefs range, and I abstein from it for now. For clarity, I impose the following assumption:

Assumption 5. *All agents, indepedently of their type, change their belief in the same direction. That is,* $e_{t-1}^h > 0$ *if anf only if* $e_{t-1}^l > 0\ \forall t$*. Hence, it must be that* $\Delta\beta_t^h > 0$ *if and only if* $\Delta\beta_t^l > 0\ \forall t$*. The same holds for reversed inequalities.*

Forecast Errors

Firstly, intial beliefs updating equations (2.25) imply that agents revise $\beta_t^k, \forall k, t$ in the same direction as the last forecast error: beliefs increase (decrease) if investors underpredict (overpredict) risk-adjusted asset price growth. To infer about the magnitude of changes in beliefs one needs to compare realized and subjective price growth[11]: $\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}} \times E_t^k\left\{\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}}\right\} \equiv \beta_t^k$.

Price equilibrium equation (2.20) shows that, for any equilibrium weight λ_t^*, a higher (lower) $\beta_t^k, k = h, l$ delivers a higher (lower) realized PD ratio. In abscence of further disturbances, this increases (decreases) risk-adjusted price growth. However, realized price growth will always strongly exceed the initial implied expected price growth. That's because actual risk-adjusted price growth is a steeper function of beliefs and its subjective expectations: when there is an increase in β_t^k, equilibrium price growth increase by more than 1 unit; the reverse holds for when subjective expectations decrease. This means that

[11] I will use the term price growth meaning risk-adjusted price growth for simplicity.

$|e_t^k| > 0$.[12]

To be precise, when both agents hold the RE belief it is true that their forecast errors will be zero. Since the focus of this paper is to consider deviations from rational expectations, I assume this scenario has probability zero of happening.

Forecast error across types differ. By the characterizion of experience and inexperienced agents (see equations (2.22)-(2.25)), the latter has consistently more extrapolative beliefs and thus exhibits greater errors. Formally.[13]

Proposition 11. *At any period, the forecast error of inexperienced investors (H) have greater magnitude when compared to experienced's (L) error:* $|e_t^h| > |e_t^l|$*. Since* $g^h > g^l$*, it holds then that*

$$|g^h e_{t-1}^h| > |g^l e_{t-1}^l| \quad \forall t \tag{2.28}$$

In other words, difference in type's reactions is suffciently high so that the absolute change in beliefs of inexperienced agents is greater than the absolute change of experienced agents at every period:

$$|\Delta\beta_t^h| > |\Delta\beta_t^l| \quad \forall t \tag{2.29}$$

Combining the previous Proposition 11 and Assumption 5, it is clear that agents agree in which direction they will revise their beliefs to but disagree in the magnitude of this change. The overall evolution of types' beliefs β_t^h, β_t^l is such that, at every period, change in beliefs are qualitatively the same but quantitatively different across types.

2.5.2 Second-Stage: Social Interaction

After intial beliefs β_t^h, β_t^l are set, agents communicate with their neighbors and formulate their end-of-period beliefs $\tilde{\beta}_t^i = \lambda^i \beta_t^h + (1-\lambda)^i \beta_t^l$. Equilibrium price is then determined by the agent(s) with the highest end-of-period belief $\tilde{\beta}_t^i$, also referred to as the most optmistic agent. Embodied in $\tilde{\beta}_t^o$ are equilibrium weights on type-specific beliefs β_t^h, β_t^l denoted as $\lambda_t^*, 1-\lambda_t^*$, respectively. Therefore, equilibrium belief is given by

[12] Proof in Appendix B.2.

[13] Proof in Appendix B.2

$$\lambda_t^* = \begin{cases} \max_i \lambda^i & \text{if} \quad \beta_t^h > \beta_t^l \\ \min_i \lambda^i & \text{if} \quad \beta_t^h < \beta_t^l \end{cases} \quad \text{equilibrium weight} \tag{2.30}$$

$$\tilde{\beta}_t^o = \lambda_t^* \beta_t^h + (1 - \lambda_t^*)\beta_t^l \quad \text{equilibrium belief} \tag{2.31}$$

Notice that even though λ_t^* changes over time, λ_i's are fixed. This is because the network G is assumed to be exogenous and fixed. To determine equilibrium one must know not only agents' type but their location and neighborhood in the social network. Besides agents' degree of connectivity, the identity of one's friends matter for pinning down realized PD ratio.

Combining equilibrium equations (2.20), (2.30), (2.31) and belief updating equations (2.25), realized risk-adjusted price growth is expressed as

$$\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{P_t}{P_{t-1}} = (a\epsilon_t^c)^{-\gamma}\left(a + \frac{a\delta\Delta\tilde{\beta}_t^o}{1-\delta\tilde{\beta}_t^o}\right)\epsilon_t^d \tag{2.32}$$

2.6 Price Dynamics

Together, the existence of a maximum for the PD ratio and beliefs are going to be key in giving rise to booms and busts in stock market asset price. Realized price and beliefs are consistently reinforcing each other and this constitutes a self-fulfilling mechanism that magnifies asset price changes: if stock market price goes up, this price increase feeds into investors' expectations about future price changes, which leads them to push the current price up even higher. This upward bid then makes realized asset price to increase further, resulting again in investors pushing the current price still higher, and so on.

Consider a situation in which agents become optimistic, meaning they increase their price growth expectations. This increase in expectations leads to an increase in the PD ratio, and consequently, realized price growth is greater and exceeds its initial expectations. The beliefs updating equations (2.25) then implies further upward revisions in price growth expectations and thus asset price increases further. In abscence of any fundamental shocks, this process leads to a sustained asset price boom in which the PD ratio and risk-adjusted price expectations jointly move upward. The price boom comes to an end when equilibrium PD function (2.20) gets closer to its maximum. At this point, realized price growth fails to fulfill agents' expectations which in turn leads to a reversal in the evolution of beliefs. Agents start to decrease their beliefs and the opposite dynamics above is set in motion.

Hence, an upper pound in the PD ratio implicates the asset price boom must come to an end. When this happen, price growth is compared to agents beliefs and such inconsistency will shoot $\beta_t^k, k = h, l$ a very low value. Booms and busts are detailed discussed in Section 2.62.6.2.

I examine the asset price behavior from three prespectives: PD ratio volatility, booms and busts dynamics and agents' disagreement. In order to focus in the extent of the causality between PD ratio fluctuations and agents' behavior, I abstein from major fluctations on the dividend process. That's to say that, even though it is true that a very negative/positive dividend realization could cause a significant decrease/increase in asset price, I rule out this scenario.

2.6.1 Price-Dividend Volatility

It is straightforward to see that changes in how agents perceive the stock market contributes to PD volatity. From equation (2.32), I have

$$Var\left(\ln \frac{P_t}{P_{t-1}}\right) \approx Var\left(\ln \frac{1-\delta\tilde{\beta}^o_{t-1}}{1-\delta\tilde{\beta}^o_t}\right) + \ln \varepsilon^d_t \tag{2.33}$$

Also, for a given λ_t^*, equilibrium belief volatility is given by

$$Var(\tilde{\beta}^o_t) = (\lambda^*_t)^2 Var(\beta^h_t) + (1-\lambda^*_t)^2 Var(\beta^l_t) \tag{2.34}$$

Thus, changes in type-beliefs β^h, β^l results in higher volatility of equilibium beliefs $\tilde{\beta}^o$, what contributes to greater price growth variation and thus, to higher PD ratio volatility.

2.6.2 Booms and Busts

I now show that a PD ratio boom and busts dynamics holds in the present framework emerging from agents optimally learning about equilibrium price process and social interaction. To begin with, suppose at time t investors expect a positive risk-adjusted price growth, that is $e^k_{t-1} > 0, k = h, l$. Then it holds that

$$\beta^k_t = \beta^k_{t-1} + g^i\left[\left(\frac{C_{t-1}}{C_{t-2}}\right)^{-\gamma}\frac{P_{t-1}}{P_{t-2}} - \beta^k_{t-1}\right] \Rightarrow \beta^k_t > \beta^k_{t-1} \quad \forall k = h, l$$

Both types' private belief increases and since experienced agents are more reactive to price growth observations $\beta^k_t > \beta^l_t$. Communication results in end-of-period beliefs $\tilde{\beta}^i_t$

$\forall i \in N$ and equilibrium is determined by $\lambda_t^* = \max_i \lambda^i$. Hence, realized PD ratio and price growth are, respectively

$$\frac{P_t}{D_t} = \frac{\delta\beta^{RE}}{1-\delta\tilde{\beta}_t^o} = \frac{\delta\beta^{RE}}{1-\delta[\lambda_t^*\beta_t^h + (1-\lambda_t^*)\beta_t^l]}$$
$$\frac{P_t}{P_{t-1}} = [a^{1-\gamma}(\epsilon_t^c)^{-\gamma}\epsilon_t^d]\left(\frac{1-\delta\tilde{\beta}_{t-1}^o}{1-\delta\tilde{\beta}_t^o}\right)$$

Since both β_t^h, β_t^l are greater than the respective last period beliefs, $P_t > P_{t-1}$ no matter the identity of the most optimistic agent in the the previous period $\tilde{\beta}_{t-1}^o$. At $t+1$, $e_t^k > 0$ $\forall k$, investors revise their beliefs upwards and asset price continues to increase. In the aftermath, $\beta_{t+s}^h > \beta_{t+s}^l$ for all $s \geq 1$, equilibrium is pinned down by $\lambda_{t+s}^* = \max_i \lambda^i$, and realized price growth is greater than one, increasing and exceeds subjective expectations $e_{t-s}^k > 0$. Hence, a boom dynamics is in place in which future expectations and asset price continues to increase, and the most optimistic agent is the one who has more inexperiend friends.

Due to its greater reactivenes, H-type will be the ones firstly disappointed by price realizations. As the PD ratio gets closer to its maximum value, equilibrium price growth increase will fail to fulfill β_t^h magnitude. At this point, H-expectation is bigger than realized price growth which means they start revising beliefs downwards. Consequently, price decreases and its drop is high enough to also make L-agents to revise their belief downwards.[14] A bust episode is in motion then.

The resulting price behavior is similar to what Adam et al. (2016) study but price realizations themselves differ since now communication is crucial for price setting and both types influence equilibrium. Nonetheless, the main mechanism behind the evolution of subjective risk-adjuted price growth beliefs is the same[15] As they show, the individual learning problem (2.21)-(2.26) results in beliefs β_t^h, β_t^l to stochastically oscilate the RE value and their upper bound β^U cannot be an absorbing point. This renders a momentum and mean-reversion behavior into the PD ratio. I refer to them for the details and analytical proofs of theses results.

The main difference between booms and busts is which type dictates equilibrium price. Booms are characterized by inexperienced agents being more influential whereas in busts experienced agents' beliefs are the main driving source. These episodes impart a high

[14]This follows from Assumption 5

[15]This follows because both model, ours and Adam et al. (2016)'s, share pratically the same individual learning scheme (2.21)-(2.26).

volatily of the PD ratio, in line with standard asset price evidences that motivate this paper.

2.6.3 Recovery Periods

In the periods between booms and busts events and in asbecence of further disturbances, stock price return to its mean deterministic value and oscilate much less around it. These recovery periods are marked by higher difference in private beliefs β_t^h, β_t^l such that $\beta_t^l > \beta_t^h$.

To see why that's the case, consider the bust following a boom. Stock price is continuosly decreasing and both types are revising beliefs downwards. H-type changes belief more aggresively and so β_t^h is smaller than L-type beliefs. Even when price start to increase again this continues to be true because of how further down inexperienced investors have pushed their expectations. It takes some periods of underpredictions - that is, $e_t^k > 0$ $k = h, l$ - for β_t^h to catch up with β_t^l.[16] When that happens, the optimistic behavior of investors has lead to such increase in asset price that a boom phase is in motion again.

The described stock price behavior implies that the speculative behavior of investors is an important mechanism behind it. The expectation of more speculative investors (here the H-type) influence others in the market and leads to a more rapidly increase/decrease in asset price, since beliefs are mutually reinforcing each other and realized price growth. Communication then results in they sharing similar expectations in booms/busts episodes. When asset price returns to its mean behavior, beliefs differ more and the experienced investors (L-type) are more influential in determining equilibrium price.

2.6.4 Duration of Booms and Busts

During a boom or a bust λ_t^* remains unchanged.[17] Let $\lambda_t^* = \lambda_{t-1}^* \equiv \lambda^*$ The effect of the social variable on risk-adjusted price growth is:

$$\frac{\partial\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}}}{\partial\lambda^*} = \delta a^{1-\gamma}\rho_t\left[\frac{(\beta_t^h - \beta_t^l)(1-\tilde{\beta}_{t-1}^o) - (\beta_{t-1}^h - \beta_{t-1}^l)(1-\tilde{\beta}_t^o)}{(1-\tilde{\beta}_t^o)^2}\right] \tag{2.35}$$

The above is positive whenever asset price and agents' expectations are increasing., and negative in the opposite case. During a boom phase, when price keeps getting bigger, a higher λ^* results in a higher realized price growth. Consequently, price grows faster the

[16] Proof in Appendix B.3

[17] See equation 27. Booms are characterize by $\beta^h > \beta_t^l$ and so $\lambda_t^* = \max_i \lambda^i$ for all in its duration. The reverse holds for a bust:$\beta^h < \beta_t^l$ and so $\lambda_t^* = \min_i \lambda^i$

higher is λ^* and the duration of the boom is smaller. In a bust episode the same result holds but a higher λ^* implies a lower realized price growth which implies price falls faster and the bust duration is also smaller.

Agents' social network is determinant of the duration of booms and busts. The greater the change in equilibrium belief between two periods the more intense (smaller duration) will be a boom episode since price growth realizations will be greater, and, consequently, the PD ratio function will get closer to its maximum quicker. Intuitively, when agents are heavily influenced by their more speculative friends - that is, when λ^i are relatively big - they incorporate more of the latter expectations into their end-of-periods beliefs $\tilde{\beta}^i$.

These results also imply that the magnitude of boom/busts, that is the volatility and maximum value of realized PD ratio, are also influenced by λ^*. A social network that results in a higher $\max_i \lambda^i$ when price is increasing will have shorter boom and bust durations, greater PD ratio volatility and higher PD ratio maximum.

Finally, the change in equilibrium beliefs $\Delta\tilde{\beta}^o_t$ is also a function of the communication varible. During a boom or bust episode,it holds that

$$\frac{\partial \Delta\tilde{\beta}^o_t}{\partial \lambda^*_t} = (\beta^h_t - \beta^h_{t-1}) - (\beta^h_t - \beta^l_{t-1}) \begin{cases} > 0 & \text{if} \quad e^k_t > 0 \\ < 0 & e^k_t < 0 \text{if} \end{cases} \tag{2.36}$$

Hence, during a boom (bust) equilibrium belief change is is greater (smaller) the bigger is λ^*_t, and consequently the higher is realized PD ratio.

Keep in mind that λ^i gives how agents are proportionally connected to inexperienced investors and λ^*_t is simple the maximum or minimum λ^i depending whether price is increasing or not. A higher λ^i means that an agent has relatively more H-friends. Since in booms episodes H-type private belief β^h is always higher than the L-type one, a higher λ^i (and consequently λ^*_t) results in a higher weight to those greater belief β^h_t and so equilibrium price will be greater. However, if λ^*_t is relatively small, a greater weight in equilibrium price is given to the smaller belief β^l_t and that's why realized PD ratio will be comparatively smaller. The same reverse reasoning holds for when price growth is below one.

2.7 Equilibrium and Network Topology

As discussed above, agents' degree and the network strucure are determinant of equilibrium prices and the characterization of booms and busts. To study price dynamics, I need to know for any agent at any point of time his type-belief $\beta^{i,k}_t$ and his social structure

λ^i. To know λ^i, all agents' type i_k $\forall i \in N$, their neighboors and these later identities $j_k \in N_i$ must be known. Therefore define the state of each agent as $x_i = (i_k, D_i^h, D_i^l)$ where i's degree is $D_i = D_{ih} + D_{il}$. Let the total degree of i be the actual number of friends of he has, $\bar{D}_i = D_i - 1$. Given individual states I can infer the stock market equilibrium outcome and price behavior. A natural outcome of our model is then an algorithm to compute agents' set of final beliefs and asset price outcomes (risk-adjusted price growth, price-dividend ratio, stock return) for a given parametrization.

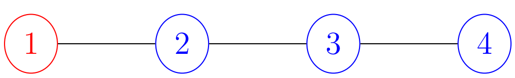
Next I present some simple examples of network graphs and how much information about asset price behavior I can get out of their structure.

2.7.1 Some Useful Examples

Consider four different networks structures: a tree, a complete network, a star and a circle. Apart from this, the economy is exactly the same in all of them, namely, equal shares of H and L agents.

Graphicaly, H nodes are colored in red and L nodes in blue. I report network mean degree, individual state, nodes' communication variable λ^i and equilibrium belief equation $\tilde{\beta}_t^o$.

Tree



$$E(d_i) = \frac{3}{2}$$

$$x_1 = (H, 1, 1), \quad x_2 = (L, 1, 3), \quad x_3 = (L, 0, 3), \quad x_4 = (L, 0, 3)$$

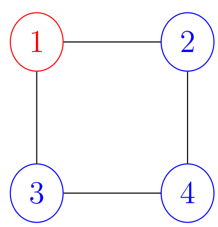
$$\lambda^1 = \frac{1}{2}, \quad \lambda^2 = \frac{1}{3}, \quad \lambda^3 = 0, \quad \lambda^4 = 0$$

$$e_{t-1} \begin{cases} > 0 & \lambda_t^* = \lambda^1 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^1 = \frac{\beta_t^h + \beta_t^l}{2} \\ < 0 & \lambda_t^* = \lambda^3 = \lambda^4 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^3 = \tilde{\beta}_t^4 = \beta_t^l \end{cases}$$

In a tree network, there is always two agents with just one link (those at the end poitns) and all the reamining nodes have total degree equal to 2. This is a network with a few connections. In general, for any network size N and any social structure (types and neighboords), there will be only three possible values for end-point nodes' weights $\lambda^i = \{0, 1/2, 1\}$, also three possible λ^i for nodes with degree 2: for H-type nodes $\lambda^i = \{1/3, 2/3, 1\}$ and for for L-type nodes $\lambda^i = \{0, 1/3, 2/3\}$. Hence end-of-period beliefs $\tilde{\beta}_t^i$ do not vary greatly across agents.

In the above example, equilibrium price is given either by agent 1 - when asset price is increasing - or by agent 4 - for decreasing price.

Circle

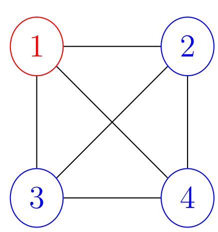


$$E(d_i) = 2$$
$$x_1 = (H,1,2), \quad x_2 = (L,1,2), \quad x_3 = (L,1,2), \quad x_4 = (L,0,3)$$
$$\lambda^1 = \frac{1}{3}, \quad \lambda^2 = \frac{1}{3}, \quad \lambda^3 = \frac{1}{3}, \quad \lambda^4 = 0$$
$$e_{t-1} \begin{cases} > 0 & \lambda_t^* = \lambda^1 = \lambda^2 = \lambda^3 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^1 = \tilde{\beta}_t^2 = \tilde{\beta}_t^3 = \frac{\beta_t^h + 2\beta_t^l}{3} \\ < 0 & \lambda_t^* = \lambda^4 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^4 = \beta_t^l \end{cases}$$

In a circle network, all agents have the same total degree of order 2 and this is also a network with a few connections. The set of values of λ^i is limited regarless the size of the network: for H-type nodes $\lambda^i = \{1/3, 2/3, 1\}$ and for for L-type nodes $\lambda^i = \{0, 1/3, 2/3\}$. Hence end-of-period beliefs $\tilde{\beta}_t^i$ not dot vary greatly across agents.

In the example, it is clear that most agents sharethe same communication structure λ^i, regardless of their types.

Complete

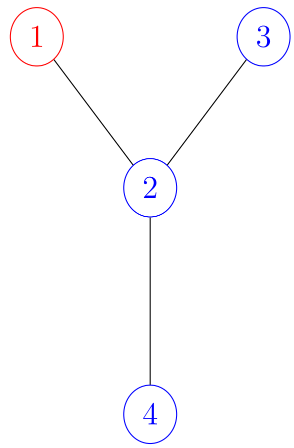


$$E(d_i) = 3$$
$$x_1 = (H,1,3), \quad x_2 = (L,1,3), \quad x_3 = (L,1,3), \quad x_4 = (L,1,3)$$
$$\lambda^1 = \frac{1}{4}, \quad \lambda^2 = \frac{1}{4}, \quad \lambda^3 = \frac{1}{4}, \quad \lambda^4 = \frac{1}{4}$$
$$\lambda = \lambda^1 = \lambda^2 = \lambda^3 = \lambda^4$$
$$\lambda_t^* = \lambda \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t = \frac{\beta_t^h + 3\beta_t^l}{4} \quad \forall e_{t-1}, t$$

The complete graph is characterized by all agents beign connected to one another. Because of this, they share the same degree of order $N-1$. In fact, λ^i is the same across agents and periods. Hence there is just one possible constant value for equilibrium weight regardless of price behavior. This implies that end-of-period beliefs $\tilde{\beta}_t^i$ are the same for all investors and I can think of this as a case of an representative agent with belief characterized by λ^*.

The graph above illustrates the aforementioned characteristics: the degree of all nodes is 3 and $\lambda^* = 1/4$.

Star



$$E(d_i) = \frac{3}{2}$$
$$x_1 = (H,1,1), \quad x_2 = (L,1,3), \quad x_3 = (L,0,2), \quad x_4 = (L,0,3)$$
$$\lambda^1 = \frac{1}{2}, \quad \lambda^2 = \frac{1}{4}, \quad \lambda^3 = 0, \quad \lambda^4 = 0$$
$$e_{t-1} \begin{cases} > 0 & \lambda^*_t = \lambda^1 \rightarrow \tilde{\beta}^o_t = \tilde{\beta}^1_t = \frac{\beta^h_t + \beta^l_t}{2} \\ < 0 & \lambda^*_t = \lambda^3 = \lambda^4 \rightarrow \tilde{\beta}^o_t = \tilde{\beta}^3_t = \tilde{\beta}^4_t = \beta^l_t \end{cases}$$

In a star network, all nodes have just one link except for the center which has a total degree of $(N-1)$. Thus, there are infinitely many possible λ^*_t realizations since the center's weight will depend on the size and structure of the network. For the end-nodes, there are just three possible values for $\lambda^i = \{0, 1/2, 1\}$ and depending on the center's type this set of values actually gets smaller: if the center is of H-type, $\lambda^i = \{1/2, 1\}$ and if it is of L-type $\lambda^i = \{0, 1/2\}$. Even though the center has the most connections, it will never be the unique optimistic agent: eitheir its λ^i is not a maximum/minimum or it is equal to at least another node's λ^i.

The example shows that since the center is an experienced agent, equilibrium is always given by the low-degree (equal to 1) nodes: when price is increasing, λ^* is from agent 1 , and when price is decreasing λ^* is given by agents 3 and 4.

2.7.2 On a Generic Network

Comparing events of a continuum increase and decrease of stock market price under the networks examplified above enable us to draw some conclusion of price dynamics as a function of the network structure.[18] For a given proportion of H and L agents in the economy, a more connected network results in lower disagreement - the dispersion of end-of-period beliefs $\tilde{\beta}^i_t$ across agents - and lower PD ratio volatility. In turn, in such social structure boom and bust episodes exhibits less strength - in terms of the magnitude of price increase - and last for longer periods.

That's because high connectivity increases the probability of different types sharing more linkages. Also, agents' types are less relevant the higher the mean degree. In the

[18] Refer to appendix B.4 for a detailed comparison of these examples.

limit of a fully connected network, equilibrium communication weight λ_t^* is constant and the same across agents. Equilibrium price is then determined solely by the proportion of H and L nodes in the economy.

The following proposition summarizes this discussion.

Proposition 12. *Consider a network (G, N) with N_h inexperienced investors and N_l experienced ones. Denote the individual degree of a type $k = \{h, l\}$ node as D_{ik}. The following are true:*

- *in the case G is fully connected, $\lambda^i = \lambda_t^* = \frac{N_h}{N}$ for all periods t, and all agents regardless of their type.*
- *If $E(D_{ih}) > N_h - 1$, then $Prob(\lambda_t^* = 1) = 0$. On the other hand, if $E(D_{ih}) \leq N_h - 1$, then $Prob(\lambda_t^* = 1) = \frac{1}{(N_h-1)N_l}$.*
- *if $E(D_{il}) > N_l - 1$, then $Prob(\lambda_t^* = 0) = 1$. On the other hand, if $E(D_{il}) \leq N_l - 1$, then $Prob(\lambda_t^* = 0) = \frac{1}{(N_l-1)N_h}$.*
- *as long as $D_{ik} > N_k - 1$, $\lambda^i \neq \{0, 1\} \; \forall i \in N$. Moreover, $\lambda^{ih} \in [\frac{1}{N_h}, 1)$ and $\lambda^{il} \in (0, \frac{1}{N_h}]$.*

As long as there are no agent only connect with others of its type, $\lambda^i \neq \{0, 1\} \; \forall i$ and so high network connectivity results in lower equilibrium weight λ_t^* at any period such that $\lambda_t^* \in \{0, 1\}$. In such case, investors of different types are more connected and thus share more beliefs. This is captured by weighting beliefs similarly, that is a low λ^i.

2.8 Simulation

To evaluate the quantitative potential of the model on replicating the relevant empirical asset price features, I simulate the model for different network structures.

I follow the calibration (see Table 2.1) and numerical algorithm of Adam et al. (2016).[19] I also take advantage of their data sources to evalute the match of empirical findings (Table 2.2) and the model outcomes.

parameter	δ	γ	a	s_d	s_c	$\rho_{c,d}$	β^{RE}	PD_{max}	g_h	g_l
value	1.0	5	1.001	0.0245	$\frac{s_d}{7}$	0.02	0.9961	500	0.0091	0.0061

Table 2.1: Simulation Calibration
For types' gain, I consider a 20% variation of the gain used in AMN.

[19]Details on the simulation algorithm can be found in Appendix B.5

Fact	Data
PD ratio Mean	123.91
PD ratio Variance	62.43
Stock return	2.25
Stock return Std. Dev	11.44
Bond return	0.15
Risk Premium	2.10
Dividend Growth Mean	0.41
Dividend Growth Std. Dev.	2.88

Table 2.2: U.S. Asset Pricing Facts, 1927:2 to 2012:2
Source: Adam et al. (2016)

I consider seven types of network structures by varying the social variable λ^i, given the assumption that all social connections are known by the econometritian. Since I know that only the maximum and minimum λ^i matter for pinning down equilibrium, it is enough to impose two values for λ^i to analyze stock market behavior. Table 2.3 reports the quantitative outcomes. The thinking exercise is to imagine the same economy - as described in the previous sections - with many individuals who can be rewired differently. The set of possible connections is captured by λ's values, so in a sense λ is a parameter and choosing its possible values means taking a stand on how the social connections look like. Throughout all the simulations, the economy is exactly the same - equal exogenous shocks, individual gains and aggregate proportion of types - the only parameter changing is λ.

Firstly, figures 2.1, 2.2 and 2.3 show realizations of time-series outcomes of our variables of interest generated from simulating the calibrated model specification of $\lambda_t^* = \{\frac{1}{2}, 1\}$. I display the evolution of the PD ratio, realized price growth, types' risk-adjusted price growth beliefs, equilbrium weight and belief. The simulated time series for the PD ratio reproduce booms and busts similar those I observe in the data, reported by Adam et al. (2016) and Adam et al. (2017) for example. Figure 2.1 display the fact that, even though dividend follow an stochastic process, its growth rate oscilates around its mean value of one which means a constant dividend growth process.[20] Comparing equilibrium belief and dividend fluctuations I see that the one responsible for the boom and busts are the former: time series of $\tilde{\beta}_t^o$ and PD_t move together.

Figure 2.2 displays some characteristics of the dynamics of our model: inexperienced agents are more reactive and have more volatile beliefs; and outside of booms and busts episodes the L-type agents is more optimistic. By looking at Figure 2.3 I see that

[20]This is clear by looking at the model dividend growth process speficiation (2.1)

experienced investors are the most influencial across the simulated periods. H-type beliefs matter the most for equilibrium under boom and busts. I do not report individual end-of-period beliefs $\tilde{\beta}_t^i$ but it is known they are bounded by β_t^h and β_t^l, and so they would be graphically located in between these time series in Figure 2.2.

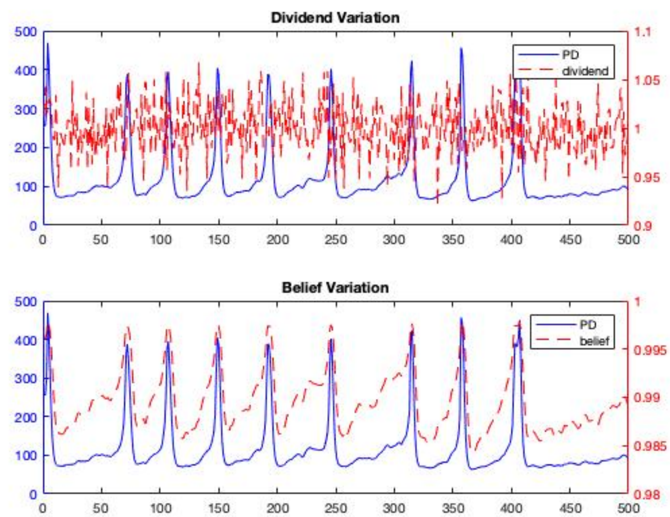


Figure 2.1: PD Ratio Volatility Sources
Figure compares model resulting processes for PD ratio, dividend growth and equilibrium belief under the specification of $\lambda = \{\frac{1}{2}, 1\}$. All x-axis are time periods in quarters.

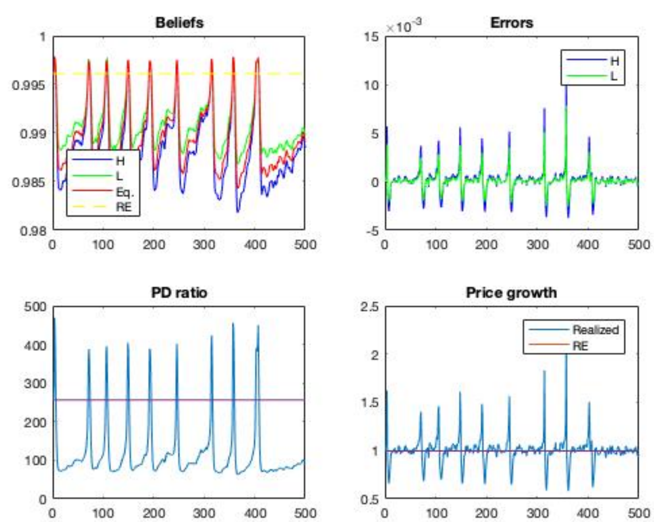


Figure 2.2: Simulation Outcome
Figure shows model resulting processes for types and equiblrium beliefs, types' forecasting errors, PD ratio and realized risk-adjusted price growth (in this order) under the specification of$\lambda = \{\frac{1}{2}, 1\}$. All x-axis are time periods in quarters.

The second exercise is to compare outcomes under different networks. The first four rows of Table 2.3 picture scenarios in which λ^* oscilates between two values and the

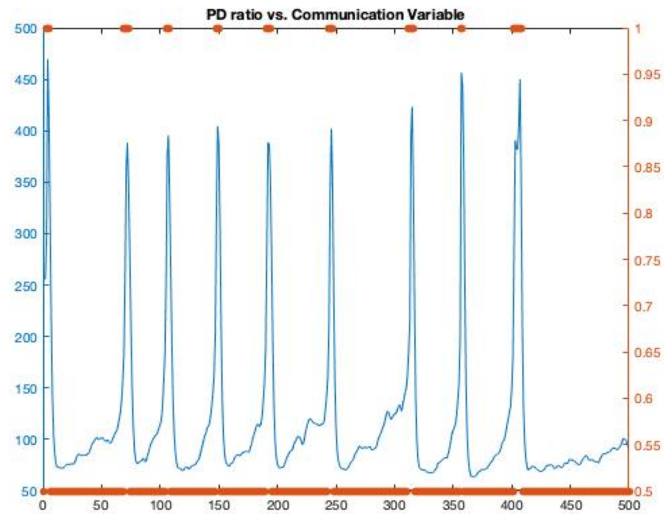


Figure 2.3: PD Ratio vs. Equilibrium Social Variable λ_t^*
Figure pictures the equilbrium λ_t^* at each PD ratio realization under the specification of $\lambda = \{\frac{1}{2}, 1\}$. The x-axis is time periods in quarters.

highest (lowest) one holds when price is continously increasing (decreasing). For example, $\lambda = \{\frac{1}{2}, 1\}$ implies that $\tilde{\beta}_t^o = \beta_t^h$ if $\beta_t^h > \beta_t^l$, that is during a boom, and $\tilde{\beta}_t^o = \frac{\beta_t^h + \beta_t^l}{2}$ if $\beta_t^h < \beta_t^l$, that is, during the bust. Consider first the cases where λ varies. Not surprisinly, the social structure exhibiting greater PD ratio volatility is the one assigning higher weight to β^h at any period (first and fourth rows). These same specifications delievers a lower PD ratio mean. Booms have shorter duration and recovery periods are greater for higher values of λ.

For the cases when λ is constant, PD ratio volatily increases significantly compared to the cases discussed above. Interestingly, the PD ratio is most volatile when both agent's types equally influece asset price and booms last the longest.

λ	PD_t	r_t^s	$r_t^s - r^b$	boom	recovery
$\{\frac{1}{2}, 1\}$	117.8340 (78.6245)	1.0161(0.1469)	1.0133 (0.1397)	3.7500	46.125
$\{0, 1\}$	132.9476 (70.4948)	1.0109 (0.1166)	1.0081 (0.1075)	4.7143	52.4286
$\{0, \frac{1}{2}\}$	140.0370 (73.9671)	1.0104 (0.1121)	1.0075 (0.1025)	5	44.8750
$\{\frac{1}{4}, \frac{2}{3}\}$	130.8282 (79.7531)	1.0126 (0.1236)	1.0098 (0.1150)	5	44.7500
0	139.0173 (73.5757)	1.0105 (0.1115)	1.0077 (0.1019)	5.1250	45.1250
$\frac{1}{2}$	125.1482 (85.4103)	1.0147 (0.1337)	1.0119 (0.1258)	5.5000	44.2500
1	106.4490 (82.6147)	1.0205 (0.1540)	1.0176 (0.1471)	4	45.7500

Table 2.3: Simulation Results
Risk-free interest rate is $r_t^b = 0.0048$ under all specifications. The implied dividend process have a mean dividend growth of 1.0012 and standard deviation of (0.0262). In brackets are the standard deviations of each varible. Boom and recovery variables are measured in quarters.

Comparing stock return across the different social structures I see that the major difference concerns its volatility. More volatile return coincides with more volatile PD ratio. There seems to be no evidence of neither higher PD ratio mean nor higher PD ratio volatility resulting in higher asset return. The resulting excess volatility of stock returns, defined as the much higher volatility of r_t^s compared to dividend growth volatility, matches the data. The equity premium outcome varies little across specifications and it is about half of its empirical mean value.

These results can be counterintuitive since one could expect that the more influential are inexperienced investors, what leads to a more volatile stock market, would require a higher equity premium to compensate for a higher risk. In fact, Barberis et al. (2015) find that, in a different framework where agents know the price process, the equity premium rises as the fraction of agents with more volatile beliefs about future asset price changes in the economy goes up. Under the baseline model of Adam et al. (2016), they are able to reproduce the empirical risk premium only at a sufficiently high level of risk aversion ($\gamma = 80$).[21] Thus, contrasting our result to the aforementioned ones suggests that a better understading of what are the roots of such observed high risk premium in the stock market is an important research avenue.

Notice that the dividend process in our framework has a lower mean value and variance, compared to the process implied by the data. Also, the simulated bond return is significantly smaller than its observed counterpart. The above suggests then the the main driving source of stock return outcome in the model is due to PD ratio behavior.[22]

Overall, the specifications replicate the empirical evidence of highly volatile PD ratio which can be found in Table 2.3. It is also in line with empirical PD ratio mean and excessive volatility of stock returns. Accounting for the existence of different types of investors seems to be an important feature of modeling stock market behavior.

2.9 Unkown Network Structure

The ongoing discussion has been focusing on what price dynamics would emerge for an exougenously given network. Some might argue that having the full knowledge of the underlying social connections of the stock market is overwhelming. Yet (partially) true, network theory comes in hand to overcome this obstacle and it enables the study of market outcomes when one does not know the full social structure. Instead of characterizing all existing edges among agents, I will assume that what is known are the probabilities

[21] Adam et al. (2016) quantitative results can be found in the Appendix B.5

[22] To see this notice that I can write $r_t^s = \frac{1+PD_t}{PD_{t-1}} \frac{D_t}{D_{t-1}}$.

of linkages between and across types. In this sense, I take a step back on the model framework and consider a more general setup.

The main idea behind random graph models is to suppose a random process is responsible for the formation of the links and then to randomly choose a network out of all the possible networks with such linkages, each network having an equal probability of being chosen. The properties of such random networks serve as a useful benchmark and provide some insights into the properties that some social and economic networks have.

I analyze a variation of the well-known Erdos-Reny Random Graph Model (ERDdS and R&wi (1959))[23] with heterogenous links. But instead of modeling the network formation as a purely random process, I assume it results from a strategic behavior of agents. In our context, agents tend to be connected with those who share the same type. Hence, this simplified version of so-called Islands-Connections Model of Jackson and Wolinsky (1996) and our goal is to study the impact of homophily on stock market equilibrium.

Homophilly is the tendency to disproportionally stabilish connections with those having similar traits. It has been the objective under analysis the network literature. For example, Golub and Jackson (2012) study a model of friendship formation and the pattern of linkages that emerges from homophily. Golub and Jackson (2012) study homophily under the context of social learning and investage it effects on the convergence of consensus.

An Equal-Sized Two-Island Model

Under a similar framework as above, let agents be divided in two groups $k = h, l$ each with mass μ_k, such that $\mu_h + \mu_l = 1$. For simplicity, assume groups are of the same size with $\mu_k = \mu = 0.5$ for all $k = h, l$. The network formation is such that links within a type are more probable than links across types, and the probability of those across types does not depend on the specifics of the types in question. An agent only distinguishes between agents of his own type and agents of a different type same-type; and they are all symmetric in how they do this. In a sense, other individual characteristics do not matter for a connection to exist.

The probability that an individual of type k has an (undirected) link to an individual of a different type k' is given by $p_{kk'} = p_{out}, \quad \forall k \neq k'$ and the probability that a link is formed with an individual of the same type is given by $p_{kk} = p_{in}, \quad \forall k$. All links are formed independently.

A measure of homophily is to compare the difference between same and different

[23] The Erdos-Reny model is the simplest random graph model in which edges are i.i.d. random varibales. All edges are independent and are formed with the same probability p. Hence, the distribtuion of all edges is a Binomial $Bin(n, p)$

linking probabilities to the average linking probability, with a normalization of dividing by the number of groups[24]:

$$p = \frac{p_{in} + p_{out}}{2} \tag{2.37}$$

$$\tilde{H} = \frac{p_{in} - p_{out}}{2p} \quad \in [0,1] \tag{2.38}$$

where $p_{in} \geq p_{out}$ If nodes only links to same-type nodes (so that p_{out} = 0), then $\tilde{H} = 1$ and if nodes do not pay attention to type when linking ($p_{in} = p_{out}$), then $\tilde{H} = 0.5$.

The proportion of agents' neighbors of a given type is then a function of network homophily. In turn, the social variables $\lambda_k, k = h, l$ are given by[25]

$$\lambda_h = \frac{p_{in}}{p_{in} + p_{out}} \quad \& \quad \lambda_l = \frac{p_{out}}{p_{in} + p_{out}} \tag{2.39}$$

The above can be written in terms of the homophily index $\tilde{H}$:

$$\lambda_h = \frac{1 + \tilde{H}}{2} \quad \rightarrow \frac{\partial \lambda_h}{\partial \tilde{H}} > 0 \tag{2.40}$$

$$\lambda_l = \frac{1 - \tilde{H}}{2} \quad \rightarrow \frac{\partial \lambda_l}{\partial \tilde{H}} < 0 \tag{2.41}$$

As homophily increases, communication variable λ_k increases for the H-type and decreases for the L-type.[26] Connections are mainly within groups, agents do not share their beliefs and the equilibrium is either determined by inexperienced investors in the case of a boom, or by experienced one, during a bust, that is, equilibrium weight tend to its extremum values: $\lambda^* \rightarrow 1$ if $e_t^k > 0$ and $\lambda^* \rightarrow 0$ if $e_t^k < 0$.

Proposition 13. *For any degree of homophily and any set of private beliefs β_t^h, β_t^l such that $\lambda_h \geq \lambda_l$, equilibrium belief $\tilde{\beta}_t^o$ is given by inexperienced investors network structure λ_h whenever asset price is increasing, and by λ_l when price is decreasing:*

$$\tilde{\beta}_t^o = \begin{cases} \lambda_h \beta_t^h + (1 - \lambda_h)\beta_t^l & \textit{if } P_t > P_{t-1} \\ \lambda_l \beta_t^h + (1 - \lambda_l)\beta_t^l & \textit{if } P_t < P_{t-1} \end{cases} \tag{2.42}$$

[24]This homophily measure is based on the work of
[25]Please refer to the Appendix for the derivation,
[26]This is a trivial result since $\lambda_h + \lambda_l$ must sum up to 1.

Homophily implication for stock market outcome easily follows then: PD ratio volatity and the frequency of boom/busts are positive related to $\tilde{H}$, whereas the duration of such episodes decreases as $\tilde{H}$ increases. This means that economies with segregated groups tend to exhibit a more espculative behavior in the stock market.

In fact, I can characterize asset price equilibrium behavior as a function of homophily.

Theorem 2. *Consider the learning asset pricing model characterized by equations* (2.1),(2.13), (2.25), (2.26) *with an unknown network structure. In the context of an equal-sized two-type random graph network formation model, define the homophily index $\tilde{H}$ as the normalized difference between same and different linking probabilities to the avarage linking probability and let $\lambda^h = \frac{1+\tilde{H}}{2}, \lambda^l = 1 - \lambda_h$. For any set of linkages probabilities and any degree of homophily, the price-dividend equilibrium equation is given by*

$$PD_t = \frac{\beta^{RE}}{\lambda_h^{\phi_t}\lambda_l^{1-\phi_t}\beta_t^h + \lambda_h^{1-\phi_t}\lambda_l^{\phi_t}\beta_t^l} \qquad \forall t, \beta_t^h, \beta_t^l \tag{2.43}$$

where ϕ_t is an indicator variable given by $\phi_t = 1_{\{\beta_t^h - \beta_t^l > 0\}}$ and zero otherwise, and $\beta^{RE} \equiv a^{1-\gamma}\rho$.

2.10 Concluding Remarks

This paper presents a simple asset pricing model with agents who have heterogenous subjective expectations about risk-adjusted price growth and socially interact, by sharing beliefs, through an exogenous given network. I show that the equilibrium asset price is a function of agents' beliefs and network structure. The model gives rises to boom and busts in the price-dividend ratio and such episodes characteristics, such as the duration and PD ratio volatility, depend on how communication among agents takes place. A key insight highlighted by the present framework is that booms (busts) are more influenced by agents who hold more (less) speculative beliefs.

One limitation of our study is our lack of asset pricing data apart from the U.S. stock-market. I also do not estimate the social network and take it as pre-determined. A next step of this research is then to estimate a network structure and apply its properties to the model, for different stock-markets.

An interesting avenue for future research could be to endogeneize the upper bound on agents' beliefs that it is needed in our model to guarantee a finite equilibrium price.

Chapter 3

Asset Pricing and Information on Social Networks

with Victoria Vanasco

3.1 Introduction

Recent years have seen a proliferation of social media platforms (e.g. Twitter and Reddit) and online trading platforms (eg. Robinhood and eToro) that allow investors to connect with each other, share information about financial markets, and even copy trading strategies. This has led to an increasing interest among academics and practitioners in understanding how social networks affect financial markets.[1] In this paper, we develop a model to investigate how social linkages among investors interact with asymmetric information about asset fundamentals to shape market outcomes. We propose that investors act upon the observation of who is trading in their social network: any given trader finds the market larger when more of his peers participate.

We build on the noisy rational expectation equilibrium (NREE) model but incorporate a social network connecting all agents. We introduce a new concept called "social market". A social market is the fraction of an investor's social network that trades the asset. Or, in other other words, an investor's belief about the market size. Each investor uses a private signal and his social market information to form his belief about the unknown value of the asset. The key feature of our framework is that the network pins down the importance

[1]For example, the FT recently wrote in Meme mania is reshaping US markets publish that "After all, that digital genie known as social media networks cannot be stuffed back into the bottle any time soon, in politics or finance. The real lesson for investors is that, in finance as in politics, they need to study attention patterns in cyber space as much as economic fundamentals to grasp where markets are moving, at least in the short term."

of each information source - private signals and social linkages - on the belief updating process and, ultimately, on asset demand decisions.

We find that investors' social markets determine the level and precision of information they extract from the public clearing price. The larger it is a trader's social market, the more informative (more precise) it is for him the public information. As a result, he places more weight on the clearing price because he believes it is a good reflection of the true value of the asset. Ultimately, individual demands are a deviation from the perfect information benchmark (when market size is known). Whenever the price is above (below) the prior expectation of the asset value, traders decrease (increase) their demand. Traders who deviate the most are those with smaller social markets.

In equilibrium, the asset price is determined by a unique coefficient that aggregates all traders' social markets. This coefficient is a measure of "collective precision" and it is given by the harmonic mean of traders' price signal precision.[2] This means that traders with higher information precision (i.e., larger social markets) are less influential in the harmonic mean compared to traders with lower information precision. Intuitively, this occurs because traders with lower social markets rely more on their private signals,[3] and therefore their beliefs are more informative and carry more weight in the equilibrium price.

A trader's "social market " is determined by two factors: his number of connections (degree) and how many of those are also traders and participate in the market.[4] That's because there exists "non-traders" in the social network who prevent traders to infer perfectly the (true) market size just by looking at their peers. Non-traders capture the idea that connections can still influence behavior even if they themselves do not actively trade.

We also explore how the equilibrium changes with the social network structure. We find that networks that lead to a higher level of collective precision are associated with lower price informativeness and higher price volatility. Intuitively, higher collective precision reflects that traders rely more on the public price information and less on their heterogeneous private signal about asset. This makes the equilibrium price itself less informative as it reveals to a lesser extent all the available information (i.e. private signals). This result suggests that a trade-off emerges when considering social networks: information diversity

[2]The harmonic mean can be thought of as a weighted average of individual price information precisions, where the weights are inversely proportional to the precisions.

[3]Traders' private signal on asset fundamental share the same exogenous precision; only the signal values differ.

[4]Specifically, in the model a finite number of agents are connected in a social network. And each agent can be classified as either a trader or a non-trader. Only traders are willing to trade the asset and the ones making demand decisions. Thus social connections do not necessarily reflect how the aggregate market looks like.

and precision. The former refers to the extent that social connections include both traders and non-traders. The latter refers to the size of social markets.

An interesting avenue for further exploration is to understand how changes in the degree distribution and the location of traders in the network influence market outcomes. This is not trivial as the collective precision is non-linear in network connectivity and it depends crucially on the location of non-traders in the network.

Literature: The traditional framework in asset pricing assumes decisions are made in a social vacuum in which agents only account for others information indirectly through observing equilibrium market prices or quantities. This view is being challenged by growing empirical literature that shows the significant impact of social interactions on the decisions of individual investors, professional investors, and firms. In his AFA Presidential Address, Hirshleifer (2020) calls attention to a new intellectual paradigm, social economics, and finance, which studies how social interactions can shape economic and financial outcomes. Indeed, the recent meme stock phenomenon showcases the power of social media platforms. These stocks have experienced rapid and dramatic price increases, driven in part by retail investors who are influenced by social media hype and coordination.

Our paper contributes to the literature by providing a novel and simple framework to study social interaction in the canonical noisy rational expectations (NREE) framework. The model yields closed-form beliefs, demands, and price. Closest to our paper, is Pedersen (2022). However, he studies a dynamic environment and focuses on how social networks affect the dissemination of information and how this, in turn, affects market outcomes. We instead model social network effects by simply arising from the observation of peers' market participation. Nonetheless, our model is flexible enough to accommodate other sources of network effects. We view this as a strength of our framework as social influence can take many forms. For example, information and belief sharing (communication), and conformism where investors change actions to behave as others do.

Several studies find evidence that social media can provide investment value (Chen et al. (2014),Jame et al. (2016),Farrell et al. (2022)), whereas other work suggests that social media may spread stale news or intensify behavioral biases (Heimer (2016), Cookson et al. (2022a), Bali et al. (2199)). Barber et al. (2022) and Eaton et al. (2022) document herding by Robinhood investors which are often inexperienced investors. Both studies find herding is associated with large price movements and reversals, and liquidity. Barber et al. (2022) also show that Robinhood investors are more likely than other retail investors to herd into certain stocks; while Eaton et al. (2022) that the market effect of Robinhood traders is different than traditional retail brokers Cookson et al. (2022b) measure retail investor disagreement using StockTwits, and they find that disagreement is associated with greater liquidity that facilitates trading by activist investors.

Our findings highlight the important role of social networks in shaping market outcomes and provide insights into the complex interactions between social and fundamental (economic) factors in these markets.

3.2 The Model

There is a short-lived, risky asset in the economy paying a random (unknown) return d at the future date. The asset is traded within a period and its supply is random and given by $n \sim N(0, \kappa_n^{-1})$.[5]

Investors and Preferences There are $i = 1, 2, ...N$ risk-averse agents, each with measure N^{-1}. Agents maximize expected utility over end-of-period wealth[6] $w_i = (p - d)x_i$ of buying x_i units of the risky asset at price p. They have CARA preferences with risk-aversion coefficient $\gamma > 0$,

$$u(w_i) = -e^{-\gamma w_i} \tag{3.1}$$

Only a subset of agents is "interested in the asset". We model this by assuming that agents have types. There are two types: a "trader" T and a "non-trader" NT. The type indicates an agent's interest in the risky asset: if $t_i = t_T$, i is a market participant and trades the asset, otherwise he is inactive. Individual types are exogenously determined.

Denote $\lambda_T \equiv \lambda$ such that $\lambda \equiv \frac{|i \in N : t_i = t_T|}{N}$ as the proportion of type-T in the economy, and so $\lambda_{NT} = 1 - \lambda$. For simplicity, normalize $t_T = 1$ and $t_{NT} = 0$ so that $t_i = \{0, 1\}$ and

$$\lambda = \frac{\sum_{i \in N} t_i}{N} \tag{3.2}$$

We are interested in the equilibrium market outcomes for the risky asset. Because of that, we focus on the behavior of type-T agents i.e. the market participants. We refer to them as investors and/or traders interchangeably.

Network There is a social network G that determines the connections among all agents

[5]For completeness, there is also a riskless asset with unitary return.

[6]The nonrandom initial wealth of traders is normalized to zero (this is without loss of generality with CARA preferences).

(regardless of their types). G is fixed, exogenous and not known. The variable $g_{ij} = \{0, 1\}$ describes network linkages between a pair i and j such that $g_{ij} = 1$ if i and j are connected. Let $N_i = \{j \neq i : g_{ij} = 0\}$ be i's neighborhood (the set of all the agents he's connected to), and $g_i = |N_i| = \sum_j g_{ij}$ be i's degree, i.e. the number of connections he has.

Traders do not know the market size λ. Instead, they form beliefs about the type distribution based on their local network. We assume that:

1. Traders believe they live in a full information world, and their inference about λ is given by a fraction of agents in their network that participate in market:

$$E[\lambda|N_i] = \lambda_i \tag{3.3}$$

   where

$$\lambda_i = \frac{\sum_{j \in N_i} t_i}{g_i} \tag{3.4}$$

2. The posterior about λ is perceived to be perfectly precise.

We refer to λ_i as trader i's "social market". The network conveys information only about *how many people* are trading. If $g_i = N \quad \forall i$ (or equivalently $N_i = G \quad \forall i$), then agents effectively know the market size. Since traders are "myopic", i.e. they do not know the entire network G, information about λ is "noisy" and heterogeneous.

Notice that the only relevant information from the other type NT-agents, who are not trading, is captured through $\lambda, \{\lambda_i\}_{t_i=1}$.

Information Let prior beliefs be given by $d \sim N(\mu, \kappa_d^{-1})$. Before markets open, traders (type-T agents) receive a noisy signal about asset fundamental returns (as they are interested in it). Mathematically, for all $i \in N$

$$\begin{aligned} t_i = 1 : \quad & s_i = d + \epsilon_i, \quad && \epsilon_i \sim N(0, \kappa_e^{-1}) \\ t_i = 0 : \quad & \text{no signal} \end{aligned} \tag{3.5}$$

There is no information sharing and each trader observes only his private signal.

Information sets are individual-specific. Each trader i's information set I_i consists of his social network, private information, and the price (public) information of the asset: $I_i = \{\lambda_i, s_i, p\}$.

3.3 Equilibrium with Socially Myopic Traders

Traders are rational and Bayesian. They optimally decide their asset demand conditional on all information they have. We define first the rational expectations equilibrium.

Definition A rational expectations equilibrium is a set of trades, contingent on the information that traders have, $\{x_i(s_i, \lambda_i, p)\}$, and a price functional $p(d, n, G)$ such that

(i) traders $i \in N : t_i = 1$ optimize given price p

$$x_i(s_i, \lambda_i, p) \in \arg\max E\left[-e^{-\gamma((p-d)\cdot x_i)}\middle| s_i, \lambda_i, p\right]$$

(ii) market clears:

$$X \equiv \sum_{i:t_i=1} x_i(s_i, \lambda_i, p) = n$$

Individual demand of i is given by

$$x_i = \frac{E[d|I_i] - p}{\gamma V[d|I_i]} \tag{3.6}$$

To formulate beliefs about fundamental return d, traders use their private signal s_i, social market size λ_i, and also public price p. The key characteristic of our environment is that each trader extracts different information from the price, which we denote by z_i. This is because they have different beliefs about market size λ depending on their social connections.

Specifically, we suppose that each trader i conjectures that the equilibrium price (perceived price) is as follows:

$$p_i = A_i + B_i \cdot \widehat{d}_i + C_i \cdot n \tag{3.7}$$

where

$$\widehat{d}_i \equiv \frac{1}{\lambda_i N} \sum_{j:t_j=1} s_j = d + \frac{1}{\lambda_i N} \sum_{j:t_j=1} \epsilon_j \tag{3.8}$$

is the private information aggregated by i. As a result, the price signal $z_i(p)$ extracted by i is

$$\begin{aligned} z_i(p) \equiv \frac{p - A_i}{B_i} &= \widehat{d}_i + \frac{C_i}{B_i} n \\ &= d + \frac{1}{\lambda_i N} \sum_{j:t_j=1} \epsilon_j + \frac{C_i}{B_i} n \end{aligned} \tag{3.9}$$

which is distributed normally distributed with mean d and variance $\kappa_{z_i}^{-1} \equiv \left(\frac{1}{\lambda_i \kappa_e \cdot N} + \left(\frac{C_i}{B_i} \right)^2 \cdot \frac{1}{\kappa_n} \right)^{-1}$.

Thus although p is public information, its interpretation is determined by traders' local networks. Traders understand the relationship between price and market size but they do not correctly conjecture the market size per se. In turn, the equilibrium typically will not be fully revealing.

Given the above, the return expectation and variance of trader i is

$$E[d|s_i, p] = \frac{\kappa_d \cdot \mu + \kappa_e \cdot s_i + \kappa_{z_i} \cdot z_i(p)}{\kappa_d + \kappa_e + \kappa_{z_i}} \tag{3.10}$$

$$V[d|s_i, p] = \frac{1}{\kappa_d + \kappa_e + \kappa_{z_i}} \tag{3.11}$$

Thus i's demand is then

$$x_i\left(s_i, \lambda_i, z_i(p)\right) = \frac{\dfrac{\kappa_d \cdot \mu + \kappa_e \cdot s_i + \kappa_{z_i} \cdot z_i(p)}{\kappa_d + \kappa_e + \kappa_{z_i}} - p}{\gamma(\kappa_d + \kappa_e + \kappa_{z_i})^{-1}} \tag{3.12}$$

3.3.1 Asset Price and Optimal Demands

See section C.1 for details on solving the Model.

To solve for the equilibrium, we follow the standard approach of the method of undetermined coefficients. First, given her expectation and market clearing condition, i finds coefficients $\{A_i, B_i, C_i\}$ by supposing that others extract the same information from price as she does (i.e., that others also conjecture price is given by (3.7)). This delivers i's demand.

Proposition 14. *Investors' demand Equilibrium demand is*

$$x_i(s_i, \lambda_i, p) = \kappa_e \cdot (s_i - p) - \frac{\kappa_e \cdot \kappa_d}{\kappa_e + \kappa_{z_i}} \cdot (p - \mu) \tag{3.13}$$

where individual price signal precision[7] *is*

$$\kappa_{z_i} = \lambda_i \kappa_e \left(\frac{1}{N} + \frac{\gamma^2}{(\lambda_i \kappa_e)\kappa_n} \right)^{-1} \tag{3.14}$$

Second, to find the equilibrium price, we conjecture that the price is actually $p = A + B\hat{d} + Cn$. Using market clearing condition and equilibrium demands Equation 3.13, true coefficients $\{A, B, C\}$ are determined.[8] This delivers the equilibrium asset price.

Proposition 15. *Asset Price*

Equilibrium price is

$$\begin{aligned} p &= \frac{1}{(\kappa_d + \kappa_h)} \left[\kappa_d \mu + \kappa_h \hat{d} - \kappa_h \frac{\gamma}{\lambda \kappa_e} n \right] \\ &= \frac{1}{(\kappa_d + \kappa_h)} \left[\kappa_d \mu + \kappa_h d + \kappa_h \left(\frac{1}{\lambda N} \sum_{t_i} \epsilon_i - \frac{\gamma}{\lambda \kappa_e} n \right) \right] \end{aligned} \tag{3.15}$$

where

$$\kappa_h \equiv \frac{1}{\lambda N} \left[\sum_{t_i} \frac{1}{\kappa_e + \kappa_{z_i}} \right]^{-1} \tag{3.16}$$

There are two channels through which the network influences asset price: i) market size λN; and ii) the endogenous coefficient κ_h. The latter is a measure of "collective

[7] An equivalent way to write is $\kappa_{z_i} = (\kappa_e \lambda_i)^2 \left[\frac{N \kappa_n}{(\kappa_e \lambda_i) \cdot \kappa_n + N\gamma^2} \right]$

[8] See the Appendix for the specific equations for coefficients $\{A_i, B_i, C_i\}$ and $\{A, B, C\}$.

precision", and it is the harmonic mean of the individual price information precision $\{\kappa_{z_i}\}_i$. We denote such harmonic mean[9] by[10]

$$h(G) = \left[\sum_{t_i} \frac{1}{\kappa_e + \kappa_{z_i}} \right]^{-1} \tag{3.17}$$

and so $\kappa_h = \frac{1}{\lambda N} h(G)$. Importantly, the collective precision κ_h (i.e. the harmonic mean) summarizes how the structure of the network affects aggregate outcomes.

The economic implication is that the traders that extract less precise information from price are more influential on price (higher impact). When calculating the harmonic mean of individual precisions, the reciprocal values are used as weights. Since traders with lower individual precision have higher reciprocal values, their weights are higher in the harmonic mean calculation. This implies that their beliefs contribute more significantly to the collective precision measure.

As it will become clear later on, the intuition behind this is that traders with lower individual precision rely more on their private signals when forming beliefs compared to traders with higher individual precision. As a result, their demands are relatively more informative and, consequently, exert a more substantial influence on the overall market.

In essence, traders with lower individual precision, despite having less precise price information individually, play a critical role in shaping market outcomes because their beliefs and demands have a larger impact on price discovery. Their influence in the collective precision of the market reflects the importance of incorporating heterogeneous information on fundamentals to capture the full range of information (private and public) in the marketplace.

Later we show that traders with less precise information from price are exactly those with smaller social markets (i.e. low λ_i).

[9]The harmonic mean appears from aggregation in market models and assigns higher weights to smaller elements.

[10]We could have alternatively defined the harmonic mean as $h(\kappa_{z_i}) = \left[\sum_{t_i} \frac{(\kappa_e + \kappa_{z_i})^{-1}}{\lambda N} \right]^{-1} = \lambda N \left[\sum_{t_i} \frac{1}{\kappa_e + \kappa_{z_i}} \right]^{-1}$ as $h(\kappa_{z_i})$. This definition does not alter our result as it only implies that κ_h would be scaled by $\frac{1}{\lambda N}$, i.e. $\kappa_h = \frac{1}{\lambda^2 N^2} h(G)$

3.3.2 Equilibrium Benchmark

Before proceeding, it is useful to look at the equilibrium when λ is known by all agents. That is when social networks do not matter. In this case, equilibrium price and demand are - respectively,

$$p^F = \hat{d} - \frac{\gamma}{\lambda \kappa_e} n \tag{3.18}$$

$$x_i^F = \frac{\kappa_e}{\gamma} (s_i - p) \tag{3.19}$$

With perfectly informed traders, price does depend on market size λ. But demands do not (only though prices).

Using (3.19), we can write equilibrium demand as a deviation of the perfect information case:

$$\left(x_i^F - \gamma x_i \right) = \frac{\kappa_d \kappa_e}{\kappa_e + \kappa_{z_i}} (p - \mu) \tag{3.20}$$

Proposition 16.

Social markets make traders deviate their behavior from the perfect information benchmark. Traders who deviate the most are those who myopically believe the asset market is small; those with low λ_i (smaller social markets).

Compared to the perfect information benchmark, traders decrease (increase) their demand whenever $p > \mu$ ($p < \mu$).

A low λ_i implies that the information extracted from price is less precise, leading to a greater change in demand compared to the case when market size is known. Traders decrease (increase) demand when the price is above (below) the prior expectation of the asset value.

Notice though that the result above does not depend on the true market size λ. That's because, at the individual level, a trader's behavior is only directly influenced by his social network - not the entire network structure.

3.4 Market Outcomes and the Network Structure

The network structure gives each investor "social market", λ_i. Two features of the network determine the social markets: the degree distribution $\{g_i\}_i$, and the type distribution

$\{t_i\}_i$. Market outcomes, in turn, are influenced by the social markets distribution $\{\lambda_i\}_i$.

3.4.1 Price Information

The price information extracted by each trader z_i is different from the (true) public information[11] z. The difference between the (true) public signal z and the perceived one z_i is given by,

$$\begin{aligned} z - z_i &= \kappa_d \left[\frac{1}{\kappa_h} - \frac{1}{(\kappa_e + \kappa_{z_i})} \right] (p - \mu) \\ &= \kappa_d \left[\lambda N \left(\sum_{j \neq i} \frac{1}{\kappa_e + \kappa_{z_j}} \right) + \frac{1}{(\kappa_e + \kappa_{z_i})} (\lambda N - 1) \right] (p - \mu) \end{aligned} \tag{3.21}$$

where $\lambda N - 1 \geq 0 \longleftrightarrow \lambda \geq \frac{1}{N}$.[12]

Although both signals are normally distributed around d, they have different - but similar - precisions. Individual price precision is $\kappa_{z_i} = \lambda_i \kappa_e \left(\frac{1}{N} + \frac{\gamma^2}{(\lambda_i \kappa_e)\kappa_n} \right)^{-1}$ (3.14). But true signal precision κ_z is

$$\kappa_z \equiv \lambda \kappa_e \left(\frac{1}{N} + \frac{\gamma^2}{(\lambda \kappa_e)\kappa_n} \right)^{-1} \tag{3.22}$$

Thus the difference relies on how the network is accounted for. That is, whether it is the true λ or individual λ_i that goes in the equation.

Inspection of the equilibrium reveals that the network plays a role only in the information revealed and perceived by traders from price, both the level $\{z_i\}_i$ and z and the precision κ_{z_i}, κ_z. Such influence comes about through the (true) market size λ and social markets $\{\lambda_i\}$. What, in turn, pin downs equilibrium demands and price.[13] Moreover, the way the network influences equilibrium is independent of signal realization.[14]

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[11] Similar to (3.9), public information z is defined as $z \equiv \frac{p-A}{B}$. More details on z_i, z_p in the Appendix.

[12] The only case when $\lambda N - 1 < 0$ is when none of the agents are traders. Obviously, this case is not interesting as it means no market exists.

[13] It is also straightforward to write equilibrium price (3.15) in terms of z,

$$p = \frac{1}{\kappa_d + \kappa_h} (\kappa_h z + \kappa_d \mu) \tag{3.23}$$

[14] This is because we assume private information has the same precision for all agents, $\kappa_{\epsilon_i} = \kappa_{\epsilon_j} = \kappa_\epsilon$.

3.4.2 The Role of Social Markets

Each trader is directly affected by his social market - and *not* by the entire network structure. A trader's social market λ_i is determined by two factors: how many friends i has, g_i, and how many of those are trading the asset, $t_{N_i} \equiv \sum_{j \in N_i} t_j$.

However, g_i and t_{N_i} affect λ_i in opposite ways. Keeping g_i fixed, the more friends i has that are trading the asset, the higher his λ_i. Conversely, keeping t_{N_i} fixed, the more friends i has, the lower is his λ_i. It turns out though that the former effect dominates: increasing both g_i and t_{N_i} leads to a higher λ_i. At this point, we focus on traders' social market λ_i. We discuss the role of g_i, t_{N_i} on market outcomes later on.

We find that the greater a trader's social market (higher λ_i), the more precise the information he extracts from price κ_{z_i}. This leads him to increase his demand x_i whenever realized price exceeds the expected return μ (i.e. $p - \mu > 0$). A higher λ_i also delivers lower variance in expected returns, $V[d|I_i]$.

Proposition 17.

- *Individual price signal precision κ_{z_i} is increasing in λ_i*
- *Individual price signal z_i is decreasing in λ_i*
- *If $p - \mu > 0$, individual demand is increasing in λ_i. If $p - \mu < 0$, the opposite holds.*

Traders trade for two reasons. First, they speculate on their private information, buying or selling depending on whether the price is larger or smaller than their private signal on return. The responsiveness to private information, given by the precision of private information κ_e, is independent of his social market. This is so since network connections are not for information sharing, learning or imitation. A trader simply observes the composition of his local network and is influenced by the extensive margin of trading, i.e. if his friends are traders or non-traders. The second component of trade is related to their market-making capacity with associated trading intensity given by a trader's social market. Such trading intensity depends negatively on the size of a trader's social market and, consequently, on the precision of his price signal. The larger is λ_i, the more precise information trader i extracts from price and thus with less intensity he trades for market-making purposes.

Thus, traders who myopically believe in a greater social market, demand more whenever price is greater than the expected fundamental return μ. These results may seem odd at first. However, this increase in demand is driven by relying more on the price signal (since the latter is more precise, i.e. κ_{z_i} is higher) and less on the private signal. In turn, this increase in demand pushes the price up - consistently with it being higher than μ.

Recall that social markets are encapsulated in the collective precision κ_h, which is the network-dependent coefficient of the pricing equation. Traders' social networks affect aggregate market outcomes through the true price signal precision κ_h. Since κ_h is the harmonic mean individual precisions $\{\kappa_{z_i}\}_{i:t_i=1}$, the following proposition holds.

Proposition 18.

The smaller a trader's social market λ_i, the more influential he is on the collective precision κ_h.

3.4.3 Price Volatility and Informativeness

In equilibrium, the way the network influences asset price and price information is captured by the market size λ and the harmonic mean $h(G)$. Both also affect the volatility of price $V(p)$:

$$V(p) = \left(\frac{\kappa_h}{\kappa_d + \kappa_h}\right)^2 \left[\frac{1}{\kappa_d} + \frac{1}{\lambda \kappa_e N} + \left(\frac{\gamma}{\lambda \kappa_e}\right)^2 \frac{1}{\kappa_n}\right] \tag{3.24}$$

Notice that $E(p) = \mu$ as expected since we assume on average a zero supply of shares, $E[n] = 0$.[15]

Proposition 19.

Asset price is increasing in market size λ and decreasing in the collective precision κ_h. For price volatility, the opposite holds. It is decreasing in λ and increasing in κ_h.

It also holds that the true price signal precision κ_z is increasing in λ.

The information content of the price can be defined as the difference between the equilibrium price and the information extracted from it (public signal), $p - z$,

$$p - z = \frac{\kappa_d}{\kappa_h}(\mu - p) \tag{3.25}$$

It follows then that price informativeness is positively affected by κ_h which in turn is increasing in both $\{\lambda_i\}_{i \in N}, \lambda$.

Proposition 20.

[15] See Vives (2008) pg 120

Price informativeness $|p - z|$ is increasing in market size λ and decreasing in the collective precision κ_h.

Thus networks that lead to a higher level of collective precision are associated with lower price informativeness and higher price volatility. Intuitively, higher collective precision reflects that traders rely more on the public price information and less on their heterogeneous private signal about asset. This makes the equilibrium price itself less informative as it reveals to a lesser extent all the available information (i.e. private signals). This result suggests that a trade-off emerges when considering social networks: information diversity and precision. The former refers to the extent that social connections include both traders and non-traders. The latter refers to the size of social markets.

3.5 Conclusion

This paper investigates the impact of social networks on financial markets by developing a model that incorporates heterogeneous information and social linkages among traders.

In our framework, a finite number of traders look at their social connections to infer the size of the market. We refer to this individual inference as a trader's social market. Traders form their beliefs about the true value of the asset based on a combination of a private signal and his social market.

The model yields closed-form solutions for beliefs, demands, and price. A trader's social market influences the level and precision of information they extract from the public clearing price, which in turn affects their demand decisions. Equilibrium asset price aggregates all traders' social markets - not only traders' private information.

We find that traders with larger social markets place more weight on the clearing price and have more accurate beliefs about the true state of the market. We also explore how the equilibrium changes with the social network structure, finding that networks with higher collective precision are associated with lower price informativeness and higher price volatility and that there is a trade-off between information diversity and precision in social networks.

Bibliography

Adam, K., Beutel, J., Marcet, A., Merkel, S. 2015. Can a financial transaction tax prevent stock price booms? Journal of Monetary Economics. 76, S90–S109.

Adam, K., Marcet, A. et al. 2010. Booms and busts in asset prices. Citeseer.

Adam, K., Marcet, A. 2011. Internal rationality, imperfect market knowledge and asset prices. Journal of Economic Theory. 146 (3), 1224–1252.

Adam, K., Marcet, A., Beutel, J. 2017. Stock price booms and expected capital gains. American Economic Review. 107 (8), 2352–2408.

Adam, K., Marcet, A., Nicolini, J. P. 2016. Stock market volatility and learning. The Journal of Finance. 71 (1), 33–82.

Arnone, M. M., Ugolini, M. P. 2005. Primary Dealers in Government Securities. International Monetary Fund.

Bala, V., Goyal, S. 1998. Learning from neighbours. The review of economic studies. 65 (3), 595–621.

Balasubramaniam, S., Gomes, A. R., Lee, S. 2020. Asset Reallocation in Markets with Intermediaries Under Selling Pressure.

Bali, T. G., Hirshleifer, D., Peng, L., Tang, Y. Attention, Social Interaction, and Investor Attraction to Lottery Stocks. 70.

Ballester, C., Calvó-Armengol, A., Zenou, Y. 2006. Who's Who in Networks. Wanted: The Key Player. Econometrica. 74 (5), 1403–1417.

Banerjee, A., Chandrasekhar, A. G., Duflo, E., Jackson, M. O. 2013. The diffusion of microfinance. Science. 341 (6144), 1236498.

Banerjee, A., Fudenberg, D. 2004. Word-of-mouth learning. Games and economic behavior. 46 (1), 1–22.

Barber, B. M., Huang, X., Odean, T., Schwarz, C. 2022. Attention-Induced Trading and Returns: Evidence from Robinhood Users. The Journal of Finance. 77 (6), 3141–3190.

Barberis, N., Greenwood, R., Jin, L., Shleifer, A. 2015. X-capm: An extrapolative capital asset pricing model. Journal of financial economics. 115 (1), 1–24.

Bech, M. L., Atalay, E. 2010. The topology of the federal funds market. Physica A: Statistical Mechanics and its Applications. 389 (22), 5223–5246.

Boss, M., Elsinger, H., Summer, M., Thurner 4, S. 2004. Network topology of the interbank market. Quantitative Finance. 4 (6), 677–684.

Bramoullé, Y., Kranton, R. 2016. Games Played on Networks. 82–112, Oxford University Press.

Bramoullé, Y., Kranton, R., D'Amours, M. 2014. Strategic Interaction and Networks. American Economic Review. 104 (3), 898–930.

Calvó-Armengol, A., Patacchini, E., Zenou, Y. 2009. Peer effects and social networks in education. The Review of Economic Studies. 76 (4), 1239–1267.

Chen, H., De, P., Hu, Y. J., Hwang, B.-H. 2014. Wisdom of Crowds: The Value of Stock Opinions Transmitted Through Social Media. The Review of Financial Studies. 27 (5), 1367–1403.

Condorelli, D., Galeotti, A., Renou, L. 2021. On the Efficiency of Large Resale Networks. SSRN Scholarly Paper 4026665, Social Science Research Network, Rochester, NY.

Cookson, J. A., Engelberg, J., Mullins, W. 2022a. Echo Chambers.

Cookson, J. A., Fos, V., Niessner, M. 2022b. Does Disagreement Facilitate Informed Trading?.

Corominas-Bosch, M. 2004. Bargaining in a network of buyers and sellers. Journal of Economic Theory. 115 (1), 35–77.

Craig, B., von Peter, G. 2014. Interbank tiering and money center banks. Journal of Financial Intermediation. 23 (3), 322–347.

Czech, R., Huang, S., Lou, D., Wang, T. 2021. Informed trading in government bond markets. Journal of Financial Economics. 142 (3), 1253–1274.

DeGroot, M. H. 1974. Reaching a consensus. Journal of the American Statistical Association. 69 (345), 118–121.

DeMarzo, P. M., Vayanos, D., Zwiebel, J. 2003. Persuasion bias, social influence, and unidimensional opinions. The Quarterly journal of economics. 118 (3), 909–968.

Di Maggio, M., Franzoni, F. A., Kermani, A., Sommavilla, C. 2017a. The Relevance of Broker Networks for Information Diffusion in the Stock Market. SSRN Scholarly Paper ID 2860118, Social Science Research Network, Rochester, NY.

Di Maggio, M., Kermani, A., Song, Z. 2017b. The value of trading relations in turbulent times. Journal of Financial Economics. 124 (2), 266–284.

Dick-Nielsen, J., Poulsen, T. K., Rehman, O. 2020. Dealer Networks and the Cost of Immediacy. SSRN Electronic Journal.

Duffie, D. 2010. Presidential Address: Asset Price Dynamics with Slow-Moving Capital. The Journal of Finance. 65 (4), 1237–1267.

Duffie, D., Gârleanu, N., Pedersen, L. H. 2005. Over-the-Counter Markets. Econometrica. 73 (6), 1815–1847.

Duffie, D., Zhu, H. 2017. Size Discovery. The Review of Financial Studies. 30 (4), 1095–1150.

Eaton, G. W., Green, T. C., Roseman, B., Wu, Y. 2022. Retail Trader Sophistication and Stock Market Quality: Evidence from Brokerage Outages.

Eisfeldt, A. L., Herskovic, B., Rajan, S., Siriwardane, E. 2018. OTC Intermediaries. SSRN Electronic Journal.

ERDdS, P., R&wi, A. 1959. On random graphs i. Publ. Math. Debrecen. 6, 290–297.

Farrell, M., Green, T. C., Jame, R., Markov, S. 2022. The democratization of investment research and the informativeness of retail investor trading. Journal of Financial Economics. 145 (2, Part B), 616–641.

Feldman, M., Immorlica, N., Lucier, B., Roughgarden, T., Syrgkanis, V. 2015. The Price of Anarchy in Large Games.

Friewald, N., Nagler, F. 2019. Over-the-Counter Market Frictions and Yield Spread Changes. The Journal of Finance. 74 (6), 3217–3257.

Gale, D. M., Kariv, S. 2007. Financial Networks. American Economic Review. 97 (2), 99–103.

Galeotti, A., Goyal, S., Jackson, M. O., Vega-Redondo, F., Yariv, L. 2010. Network games. The review of economic studies. 77 (1), 218–244.

Glode, V., Opp, C. C. 2020. Over-the-Counter versus Limit-Order Markets: The Role of Traders' Expertise. The Review of Financial Studies. 33 (2), 866–915.

Goldstein, M. A., Hotchkiss, E. S. 2020. Providing liquidity in an illiquid market: Dealer behavior in US corporate bonds. Journal of Financial Economics. 135 (1), 16–40.

Golub, B., Jackson, M. O. 2010. Naive learning in social networks and the wisdom of crowds. American Economic Journal: Microeconomics. 2 (1), 112–49.

Golub, B., Jackson, M. O. 2012. How homophily affects the speed of learning and best-response dynamics. The Quarterly Journal of Economics. 127 (3), 1287–1338.

Hasbrouck, J., Levich, R. M. 2020. Network Structure and Pricing in the FX Market. SSRN Electronic Journal.

He, Z., Kelly, B., Manela, A. 2017. Intermediary asset pricing: New evidence from many asset classes. Journal of Financial Economics. 126 (1), 1–35.

Heimer, R. Z. 2016. Peer Pressure: Social Interaction and the Disposition Effect. The Review of Financial Studies. 29 (11), 3177–3209.

Hendershott, T., Li, D., Livdan, D., Schürhoff, N. 2020. Relationship Trading in Over-the-Counter Markets. The Journal of Finance. 75 (2), 683–734.

Hirshleifer, D. 2020. Social Transmission Bias in Economics and Finance: Transcript of the AFA Presidential Address with Presentation Slides American Finance Association 2020 Annual Meetings. 39.

Ho, T. S. Y., Stoll, H. R. 1983. The Dynamics of Dealer Markets Under Competition. The Journal of Finance. 38 (4), 1053–1074.

Ho, T., Stoll, H. R. 1981. Optimal dealer pricing under transactions and return uncertainty. Journal of Financial Economics. 9 (1), 47–73.

Hollifield, B., Neklyudov, A., Spatt, C. 2017. Bid-Ask Spreads, Trading Networks, and the Pricing of Securitizations. The Review of Financial Studies. 30 (9), 3048–3085.

Hollifield, B., Neklyudov, A., Spatt, C. S. 2014. Bid-Ask Spreads, Trading Networks and the Pricing of Securitizations: 144a vs. Registered Securitizations. SSRN Electronic Journal.

Hong, H., Kubik, J. D., Stein, J. C. 2005. Thy neighbor's portfolio: Word-of-mouth effects in the holdings and trades of money managers. The Journal of Finance. 60 (6), 2801–2824.

Hong, H., Stein, J. C. 2007. Disagreement and the stock market. Journal of Economic perspectives. 21 (2), 109–128.

Hortacsu, A. Bidding Behavior in Divisible Good Auctions: Theory and Evidence from the Turkish Treasury Auction Market. 28.

in 't Veld, D., van Lelyveld, I. 2014. Finding the core: Network structure in interbank markets. Journal of Banking & Finance. 49, 27–40.

Jackson, M. O. 2010. Social and economic networks. Princeton university press.

Jackson, M. O., Wolinsky, A. 1996. A strategic model of social and economic networks. Journal of economic theory. 71 (1), 44–74.

Jame, R., Johnston, R., Markov, S., Wolfe, M. C. 2016. The Value of Crowdsourced Earnings Forecasts. Journal of Accounting Research. 54 (4), 1077–1110.

Kondor, P., Pintér, G. 2022. Clients' Connections: Measuring the Role of Private Information in Decentralized Markets. The Journal of Finance. 77 (1), 505–544.

Kranton, R. E., Minehart, D. F. 2001. A Theory of Buyer-Seller Networks. American Economic Review. 91 (3), 485–508.

Kyle, A. S. 1985. Continuous Auctions and Insider Trading. Econometrica. 53 (6), 1315–1335.

Kyle, A. S. 1989. Informed Speculation with Imperfect Competition. The Review of Economic Studies. 56 (3), 317–355.

Li, D., Schürhoff, N. 2019. Dealer Networks. The Journal of Finance. 74 (1), 91–144.

Lucas Jr, R. E. 1978. Asset prices in an exchange economy. Econometrica: Journal of the Econometric Society. 1429–1445.

Malamud, S., Rostek, M. 2017. Decentralized Exchange. American Economic Review. 107 (11), 3320–3362.

Manea, M. 2016. Models of Bilateral Trade in Networks. 697–732, Oxford University Press.

Ozsoylev, H. N., Walden, J. 2011. Asset pricing in large information networks. Journal of Economic Theory. 146 (6), 2252–2280.

Pedersen, L. H. 2022. Game on: Social networks and markets. Journal of Financial Economics. 146 (3), 1097–1119.

Rostek, M. J., Yoon, J. H. 2020a. Equilibrium theory of financial markets: Recent developments. Available at SSRN 3710206.

Rostek, M. J., Yoon, J. H. 2020b. Innovation in Decentralized Markets. SSRN Electronic Journal.

Rostek, M., Weretka, M. 2012. Price Inference in Small Markets. Econometrica. 80 (2), 687–711.

Rostek, M., Weretka, M. 2015. Dynamic Thin Markets. The Review of Financial Studies. 28 (10), 2946–2992.

Rostek, M., Yoon, J. H. 2021. Exchange Design and Efficiency. Econometrica. 89 (6), 2887–2928.

Swinkels, J. M. 2001. Efficiency of Large Private Value Auctions. Econometrica. 69 (1), 37–68.

Vayanos, D., Wang, J. 2012. Market Liquidity – Theory and Empirical Evidence.

Vayanos, D., Weill, P.-O. 2008. A Search-Based Theory of the On-the-Run Phenomenon. The Journal of Finance. 63 (3), 1361–1398.

Vissing-Jorgensen, A. 2003. Perspectives on behavioral finance: Does" irrationality" disappear with wealth? evidence from expectations and actions. NBER macroeconomics annual. 18, 139–194.

Vives, X. 2011. Strategic Supply Function Competition With Private Information. Econometrica. 79 (6), 1919–1966.

Wittwer, M. 2021. Connecting Disconnected Financial Markets? American Economic Journal: Microeconomics. 13 (1), 252–282.

Xia, C. 2007. Communication and confidence in financial networks.Technical report, Working Paper, University of Minnesota, Minneapolis, MN.

Appendices

Appendix A

Chapter 1

A.1 Illustrative Example of the Main Results

Consider five traders A, B, C, D and E, and six different trading networks depicted in Figure A.1. The networks are arranged in descending order of connectivity from left to right, as the first row of Table A.1 shows.

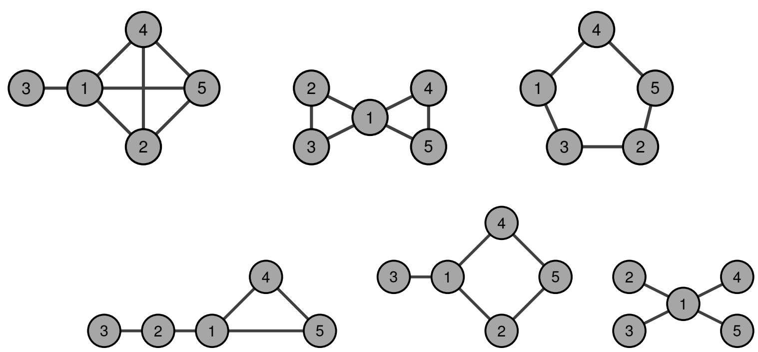


Figure A.1: Trading networks 1, 2 3, 4, 5 and 6 - respectively

	Net 1	Net 2	Net 3	Net 4	Net 5	Net 6
connectivity	2.8	2.4	2	2	2	1.6
inequality	1.2	0.8	0	0.5	0.5	1.8
PM price (P)	$P_2 > P_1 > P_3 > P_4 > P_5 > P_6$					
liquidity cost (PD)	$PD_6 > PD_5 > PD_4 > PD_3 > PD_1 > PD_2$					
welfare (EU)	$EU_6 > EU_3 > EU_5 > EU_4 > EU_2 > EU_1$					

Table A.1: Characteristics and equilibrium outcomes of Figure A.1. Connectivity is the average degree and inequality degree variance. Liquidity cost is the average difference between PM price and local market prices. Welfare is the aggregate expected utility.

We can first compare the structural proprieties of the trading networks - i.e. connectivity and degree inequality - and PM price. Network 2, the second most connected one, has the highest PM price. This reveals the non-monotonicity of PM price with respect to connectivity (Lemma 3). PM price is the lowest on network 6, the star network, what turns out to be the lower bound of PM price across all networks of the same size $N = 5$ (Proposition 2). Not depicted in fig. A.1 is the complete network, which has the highest PM price. Indeed, as Proposition 2 shows, the complete network imposes the upper bound on PM price level.

Comparison of networks 2 and 3 depicts that (weakly) increasing all traders' degree results in a higher price: every trader in network 2 is at least as connected as in network 3 (despite the former having higher degree inequality) and, consequently, $P_2 > P_3$ (Proposition 3).[1] Comparison of networks 3 and 4 (and/or 5) shows that, keeping connectivity fixed, the higher is degree inequality (network 4), the lower is PM price (Proposition 4).[2] Finally, notice that networks 4 and 5 have the same connectivity and degree inequality (degree distribution) but different PM prices.

We've also seen that asset price almost always drop over time (Proposition 1). Just as with PM price, such price dynamics is non-monotonic in connectivity and degree inequality. Interestingly, the rank of the trading networks with respect to PM price is the reverse to their rank with respect to price drop ($3rd$ and $4th$ rows of table A.1). This means that a higher price for acquiring asset shares does not lead to higher liquidity cost on average.

Welfare comparison offers additional insights (Section 1.7). Network 6, the star network, delivers the highest welfare. That's because its lowest PM price compensates its highest liquidity cost. Network 1, the most connected one, exhibits the lowest welfare. Actually, the complete network delivers the lowest welfare. Moreover, just as with trading centrality and PM price, connectivity and inequality are not sufficient information to analyze welfare - for example, look at networks 3, 4 and 5.

Lastly, we can compare traders within a network based on trading centrality and degree (Theorem 1, Lemma 8).[3] Network 2 is an example of when centrality and degree are negatively related: $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 < c_2 = c_4 = c_5 = c_3$. In turn, central dealers in network 2 have higher asset holdings and higher liquidity cost (lower re-selling

[1] Formally, the degree distribution of 2 FOSD the distribution of 3 and thus price in the former is greater.

[2] Formally, the degree distribution of network 4 is a mean-preserving spread of the degree distribution of network 3 and thus price in the former is lower.

[3] Recall that (as discussed when solving for the equilibrium) the relationship between trading centrality and degree plays an important role in shaping individual outcomes (i.e. demands and re-selling prices), although it is non-trivial. That's because certain network structures deliver a positive relationship between trading centrality and degree, while other structures deliver the opposite.

price). In the other networks of fig. A.1, central dealers are market makers - they acquire more shares in the PM and have lower liquidity cost - because centrality and degree are positively related. Even so, this relationship is not the same across networks. For instance, networks 1 and 5 are the only cases when centrality is monotonically increasing in degree..[4]

In sum, this example illustrates the non-trivial relationship between welfare, connectivity and degree inequality.

A.2 Buyer and Seller Effects

At period one, traders make decisions anticipating they can be both a buyer and a seller. However, I focus in the environment in which the seller effect is minor. This holds for all networks of at least size four. As the network grows, the probability of being a seller vanishes - and it does so at a higher rate than the probability of being a buyer (Figure A.2). Because of that, traders expect to be buyers, and this is what drives their behavior in the model - that is, why asset acquisition is postponed. The seller exists simply to create supply at period two. Indeed, the results showcase perfectly why the model is all about buying incentives. The highest welfare is on the network structures that exhibit the lowest prices in expectation. And, in equilibrium, re-selling is costly but for the core trader of a star network.

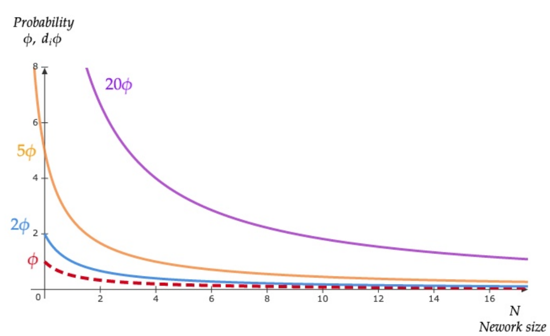


Figure A.2: Probability of being a seller ϕ versus being a buyer $d_i\phi$ as the network size N grows. I plot $\phi = \frac{1}{N+1}$.

In small networks (with two or three traders) the seller effect is not dominated by the buyer effect, an equilibrium might not exists. That's because the probability of being a seller is just as large as being a buyer for all traders. For example, in networks depicted in

[4]That is, in network 1 we have $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 > c_2 = c_4 = c_5 = c_3$; and in network 5 $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 > c_2 = c_4 = c_5 = c_3$. However, in network 3, centrality is positively related to degree, but non-monotonically: $d_1 > d_2 = d_4 = d_5 > d_3 \leftrightarrow c_2 > c_1 > c_4 = c_5 > c_3$.

fig. A.3. With two traders these probabilities are literally the same. With three traders, they can either be equal for all traders or be equal for most traders.

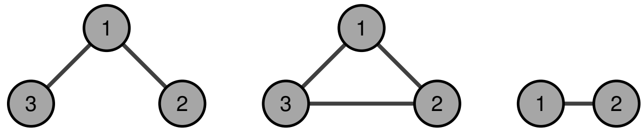


Figure A.3: Networks A, B and C (respectively) where the probability of being a seller ϕ and a buyer $d\phi$ are similar.

$$
\begin{aligned}
A &: \phi = \frac{1}{4}, \quad d\phi = \{\frac{1}{4}, \frac{1}{2}\} \\
B &: \phi = \frac{1}{4}, \quad d\phi = \frac{1}{2} \\
A &: \phi = d\phi = \frac{1}{3}
\end{aligned}
$$

A.3 Modelling Assumption and the Real-World Interdealer Market

In the context of off-exchange markets, the timing of the model captures the idea that dealers often absorb substantial inventory position in primary markets of asset issuance or from their costumers, and then use the interdealer market to offload these positions. Interdealer trades is how dealers provide liquidity to one another.

The re-sell shock[5] is interpreted as the risk of selling under pressure or the risk of having inventory imbalances resulting from unexpected and large customer orders. In either scenario, a dealer is forced to raise liquidity by quickly disposing his inventory in a possibly illiquid market (Duffie and Zhu (2017)), the interdealer market. This rationalizes why the shocked trader *must sell all* his holdings. And why I do not allow for the choice of being a seller, nor that two traders can be shocked at the same time. Such views of the re-sell shock have been empirically documented, and it turns out that they are not uncommon events.[6]

The quadratic utility function in the quantity traded[7]is microfounded in the mean-variance trade-off of a trader (CAPM theory), and it is equivalent for trading behavior to

[5]The use of a random shock to generate trade is common both in the finance (Duffie et al. (2005), Vayanos and Weill (2008)) and network literature (Gale and Kariv (2007), Condorelli et al. (2021)).

[6]See Di Maggio et al. (2017b), Balasubramaniam et al. (2020) among others.

[7]See, for instance, Kyle (1989) Vives (2011), Rostek and Weretka (2012), Rostek and Yoon (2020a) among others.

the the classic CARA-Normal setting. Apart from that, it is the standard in the literature and it contributes to the tractability of the model. Specifically, it leads to linear equilibria that have proved to be useful as a basis for empirical analysis and are supported in the empirical literature on single-sided multi-unit auction - which is the main mechanism of asset issuance in primary markets and electronic interdealer trades.[8] Moreover, this utility representations reflects the fact that dealers are risk-averse with respect to inventory (Ho and Stoll (1983)). Because of that, they often have a desired but costly inventory position, and they trade such to avoid large deviations from this target.

The trading network captures trading frictions that have been extensively explored in the OTC markets literature, both theoretical and empirically. The premise is that different traders must be sufficiently close on some dimension to be able to trade. This could be justified by having lower trading costs, or similar clientele so that both value the asset being issued. One possible interpretation is that linkages are due to previous investments in relationships: traders may invest time and resources to contact other traders and to know them better. An alternative view is that trading is costly, and linkages capture parties with an easiness to trade. In any case, I'm agnostic on how the network linkages have aroused and I assume it boils down to a pre-determined and fixed set of trading relationships, the trading network.

A.4 Solving the Model

A.4.1 Pricing Mechanism

It is convenient to visualize trades, i.e. the local market stage, as involving three steps. First, the shocked trader $s \in N$ - the seller - hands over his entire holding of the asset, $q_{s,1}$, to an auctioneer. Then, the auctioneer solicits bids from all *available* traders[9] - the buyers - in the form of demand schedules: combinations of price and quantity. A typical buyer i's trading strategy, as a function of the equilibrium seller's price and and his own pre-trading position, is the quantity that is awarded to him by the auctioneer. The equilibrium single price of the seller, P_s, is determined by equating demand and supply. After the auctioneer collects payment from all buyers, the total proceeds are returned to the seller and are his to keep. Note that, when the active local market closes, the seller's holding is zero while a buyer's total position is $q_{i,1} + q_{i,s}$. In other words, the seller exits the market while buyers

[8]For example, Hortacsu (2199), using data from Turkish Treasury auctions, shows that linear demands fit actual bidding behavior quite closely

[9]This means that the seller is inactive at the local market stage. Later in the Appendix I consider allowing the seller to choose his supply as well.

increase their asset holdings.

The assumption of a unique active local market ensures there is a single equilibrium price at period two, even though multiple prices are ex-ante possible. This is a distinct feature of the dealership market nowadays which are conducted through an inter-trader broker. The brokerage system guarantees anonymity and that a transaction takes place at a single price among those participating in it.

A.4.2 Optimal Demand Decisions

At $t = 1$, trader i submits demand schedule $q_{i,1}(\cdot)$ considering the probability of future local markets he could participate. His optimization problem is

$$\begin{aligned}\max_{q_{i,1}(\cdot):\mathcal{R}^N\to\mathcal{R}} \quad & \phi\left\{w + q_{i,1}\left(P_i - P_1\right)\right\} + \phi\sum_{s\in N_i}\left\{\left(q_{i,1}+q_{i,s}\right) - \frac{1}{2}\left(q_{i,1}+q_{i,s}\right)^2 + w - \left(P_1 q_{i,1} + P_s q_{i,s}\right)\right\} \\ & + \left(1-\phi(d_i+1)\right)\cdot\left\{q_{i,1} - \frac{1}{2}q_{i,1}^2 + w - P_1 q_{i,1}\right\}\end{aligned} \tag{A.1}$$

The first component of (A.1) is the what i gets when she is the seller in the local market. The second term accounts for every possible payoff i gets from trading as a buyer with each of her network-implied sellers. The last term is the payoff of just trading in the primary market, when i is not shocked nor connected to the shocked agent.

All traders face the same idiosyncratic shock ϕ and so they all have the same probability of being a seller. However, the likelihood of being a buyer is determined by how many connections i has. The network also dictates trader i's willingness to trade at $t = 2$, since buying in the PM brings i's closer to his target portifolio. Because of that, the choice of $q_{i,1}$ affects and is affected by local market demands $\{q_{i,s}\}_{i\cup s\in N_i}$ and prices $\{P_s\}_{i\cup s\in N_i}$.

The first order condition of (A.1) is

$$\begin{aligned}& \phi\cdot\left\{P_i - P_1 + q_{i,1}\frac{\partial P_i}{\partial q_{i,1}}\right\} \\ & + \phi\cdot\sum_{s\in N_i}\left\{1 + 1\frac{\partial q_{i,s}}{\partial q_{i,1}} - \left(q_{i,1}+q_{i,s}\right)\left(1 - \frac{\partial q_{i,s}}{\partial q_{i,1}}\right) - P_1 - P_s\frac{\partial q_{i,s}}{\partial q_{i,1}} - q_{i,s}\frac{\partial P_s}{\partial q_{i,1}}\right\} \\ & + \left(1-\phi(d_i+1)\right)\cdot\left\{1 - q_{i,1} - P_1\right\} = 0\end{aligned} \tag{A.2}$$

At $t = 2$, both the PM and the re-sell shock have realized. The seller identity $s \in N$

is common knowledge and, by assumption, he does not make any decision in the local market: he supplies all his PM shares, $q_{s,1} \leq Q$. Each buyer $i \in N_s$ chooses how many shares to buy from the seller s, $q_{i,s}$, taking into account his PM holdings:

$$\max_{q_{i,s}(\cdot):\mathcal{R}\to\mathcal{R}} \quad (q_{i,1} + q_{i,s}) - \frac{1}{2}(q_{i,1} + q_{i,s})^2 + w - (P_1 q_{i,1} + P_s q_{i,s}) \tag{A.3}$$

Notice that $(w - P_1 q_{i,1})$ is the capital available to invest after trading at $t = 1$. First-order condition delivers buyer i's demand schedule for seller $s \in N_i$:

$$q_{i,s}(P_s; \boldsymbol{q}) = (1 - q_{i,1}) - P_s \tag{1.6}$$

Buyer i's downward-slopping demand $q_{i,s}$ does not directly depend on the network, and it is negatively related to his PM holdings. That's because a buyer's willingness pay for q_s (i.e. his marginal utility from trading with the seller) is given only by how far he is from the target inventory, $(1 - q_{i,1})$ (See section 1.3). The higher is $q_{i,1}$, the more satisfied i is with his current amount of asset shares and so he will be less willing to trade with the seller for any possible price.

Demand (1.6) confirms that the local market is *perfectly competitive with heterogeneous valuations*: buyers' demands differ only by their asset holdings coming into $t = 2$.[10] As so, the local market can be interpreted as a static Walrasian market with heterogeneous asset endowment.

A.4.3 Active Local Market

The immediate corollary of Proposition regards positive price.

Corollary 20.1. *Seller's Price and Excess Demand*

Any seller $s \in N$ has positive equilibrium price if and only if there exists an excessive average demand in his neighborhood:

$$P_s > 0 \leftrightarrow \sum_{i \in N_s} \frac{(1 - q_{i,1})}{d_s} > \frac{q_{s,1}}{d_s}$$

In principle, P_s could be negative. To ensure weakly positive prices $P_s \geq 0$, there must be an *excessive average demand* in the seller's neighborhood, i.e the neighborhood's valuation of the asset is greater than the supply : $\sum_{i \in N_s} \frac{(1-q_{i,1})}{d_s} \geq \frac{q_{s,1}}{d_s}$. This constraint ensures that buyers are willing to trade with the seller since their asset holdings at $t = 2$,

[10]Two different buyers demand the same amount *if and only if* their first-period demand are them same.

$\{q_{i,1}\}_{i\in N_s}$, are low enough. To secure their optimal inventory - that is, to reduce $1-(q_{i,1}+q_{i,s})$ - a buyer's demand schedule is positive at the seller's equilibrium price.

From pricing equation (1.7) we can draw three conclusions. First, P_s^* is determined by the total average PM asset holdings, including the selling quantity. Equivalently, seller's price is given by the difference between the average of target inventories of his buyers and his per-link supply: $P_s = \frac{1}{d_s}\sum_{i\in N_s}(1-q_{i,1}) - \frac{1}{d_s}q_{s,1}$. Second, seller's price is constrained by the buyers' total amount of PM shares: $P_s^* \in [(1-q_{s,1}) - q_{N_s,1}^{\max}, (1-q_{s,1}) - q_{N_s,1}^{\min}]$, where $q_{N_s,1}^{\max}$ is the highest first-period consumption of a buyer and $q_{N_s,1}^{\min}$ is the lowest PM consumption of a buyer. Thus, the more heterogeneous buyers' asset holdings, the wider the range of possible prices for a given seller s. Finally, two cases can make P_s to be zero (negative): if buyers have high too much bond holdings (i.e., too high PM demand); and if the supply is much greater than seller's aggregate demand (i.e. excessive supply is large enough). In both cases, P_s has to be low enough to induce buyers to demand from the seller.

A.5 The Trading Network Game

It turns out that the model is in essence a network game of strategic substitute. This is the key insight of this paper as it allows the equilibrium characterization in the Primary and all Local markets. Moreover, the game belongs to a particular class of games: those with quadratic payoff function and linear best replies. The networks literature has extensively studied this type of games.[11] In this Appendix section, I derive in details the characterization of the game as a result of the model and prove the equilibrium outcomes. I start by describing the model as a network game, comprised of the degree distribution and each agent's action and payoffs. I then discuss the equilibrium concept, Nash Equilibrium, and provide equilibrium results for the game.

A.5.1 Actions, Links and Payoffs

Traders $i = \{1, 2, ..., N\}$ simultaneously choose actions: each trader i chooses his PM demand $q_{i,1} \geq 0$. Traders are embedded in the fixed Trading Network represented by the matrix $\boldsymbol{G} \in \{0,1\}^{N\times N}$ with $g_{ij} = 1$ implying a link between agents i, j.

Each trader's payoff is a function of own action, $q_{i,1}$, others' actions, $\boldsymbol{q_{-i,1}}$, the network $\boldsymbol{G}$, and the shock parameter $\phi \in (0, \frac{1}{N})$:

$$\pi_i\Big(q_{i,1}, \boldsymbol{q_{-i,1}}; \boldsymbol{G}, \phi\Big) = w + (1-P_1)\cdot q_{i,1} - \frac{1}{2v_i(\phi)}q_{i,1}^2 - \phi\sum_j \tilde{g}_{ij}q_{j,1}q_{i,1} + \phi\sum_j \bar{g}_{ij}q_{j,1} \qquad (1.9)$$

[11] See Bramoullé et al. (2014) and Galeotti et al. (2010).

where

$$v_i(\phi) \equiv \left[\frac{2\phi}{d_i} + 1 - \phi(d_i + 1) + \phi \sum_j g_{ij} \cdot \frac{(2d_j - 1)}{d_j^2}\right]^{-1} \tag{1.11}$$

$$\tilde{g}_{ij} = g_{ij} \cdot \left[\frac{1}{d_i} + \frac{(d_j - 1)}{d_j^2}\right] + \left[\sum_{\substack{z \neq j \\ z}} g_{iz} g_{jz} \frac{(d_z - 1)}{d_z^2}\right] \tag{1.12}$$

$$\bar{g}_{ij} = g_{ij} \cdot \frac{1}{2d_j^2} + \sum_{z \neq \{i,j\}} g_{iz} g_{jz} \cdot \frac{1}{d_z^2} \tag{A.4}$$

The endogenous coefficients $\{v_i(\phi), \tilde{g}_{ij}, \bar{g}_{ij}\}_{\forall i,j \in N}$, all non-negative, are determined by network graph $\boldsymbol{G}$ such that

$$\begin{aligned}
&\frac{\partial \tilde{g}_{ij}}{\partial d_i} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_i} = 0 \\
&\frac{\partial \tilde{g}_{ij}}{\partial d_j} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_j} \leq 0 \quad for \quad d_j \geq 2 \\
&\frac{\partial \tilde{g}_{ij}}{\partial d_z} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_z} \leq 0 \quad for \quad d_z \geq 2, \quad z \neq \{i,j\} : g_{iz} = g_{jz} = 1
\end{aligned}$$

Individual payoff function (1.9) is strictly concave in own-action $\frac{\partial^2 \pi}{\partial^2 q_{i,1}^2} = (2v_i(\phi))^{-1} > 0$ for all $i \in N$. The re-sell shock probability $\phi > 0$ regulates the global interaction effect among agents. As ϕ increases, the payoff externalities of agents' action become globally stronger. That's because, the higher ϕ, the greater is the chance of being both a buyer and a seller in the local market. And so, the greater is the feedback effect between markets.

The term multiplying individual action $q_{i,1}$, $(1 - P_1) \geq 0$ is common across all agents and represents an agent's optimal action absent network interactions. That is, $(1 - P_1)$ corresponds to the individual demand in a competitive market of size N, when future trading does not opportunity exists.

The individual network effect $v_i(\phi)$ is increasing in both i and his connections' degree, $d_i, \{d_j\}_{j \in N_i}$. The higher d_i, the higher is i's selling price and thus the more he consumes in the PM. The higher d_j, the higher the prices i will face in a local market, thus also inducing more demand in the PM.

The global network coefficients $\tilde{g}_{ij} \geq 0$ captures bilateral influences. Traders' actions are strategic substitutes since $\frac{\partial^2 \pi_i}{\partial q_{i,1} \partial q_{j,1}} = -\tilde{g}_{ij} \leq 0$.

The individual payoff equation (1.9) is exactly the individual expected utility (1.5) in the baseline model. Hence, payoff maximization is equivalent to trader's expected utility maximization problem.

A.5.2 Best Replies and Nash Equilibrium

The solution concept considered is pure-strategy Nash Equilibrium. Bramoullé et al. (2014) and Bramoullé and Kranton (2016) give the conditions that guarantee a unique interior equilibrium of a quadratic-linear network game of strategic substitutes. As I show, such conditions are met in the model and thus market outcomes are unique and interior.

Given the quadratic linear payoff function (1.9), an interior Nash equilibrium in pure-strategies $q^*_{i,1} > 0$ is such that $\partial \pi_i / \partial a_i(\boldsymbol{a}^*) = 0$ and $q^*_{i,1} > 0$ for all $i \in N$.

Lemma 6. *For a given price level P_1, each trader i's best-response to others' demands is given by the first order condition of* (1.9)*:*

$$q_{i,1}(q_{-i,1}; P_1, \phi, \boldsymbol{G}) = \max\left(0, v_i(\phi)\left[(1 - P_1) - \phi \sum_j \tilde{g}_{ij} q_{j,1}\right]\right) \tag{1.10}$$

Optimal demand $q_{i,1}(q_{-i,1}; P_1, \phi, \boldsymbol{G})$ is linear in others' demands and $q_{i,1}(\boldsymbol{q_1}) \in [0, 1 - P_1]$ Define the following matrices: $\boldsymbol{V}$ is the N-diagonal matrix with entries $\{v_i\}_i$; $\tilde{\boldsymbol{G}}$ is te N-square, not symmetric matrix with entries $\{\tilde{g}_{ij}\}_{i,j}$; $\mathbf{1}_N$ is the N-vector of ones.

The existence of a unique interior equilibrium for each game is guaranteed as long as the shock probability ϕ is no greater than $1/N$.

Proposition 21. *Primary Market Equilibrium Demand*

Denote $\boldsymbol{V}\tilde{\boldsymbol{G}}_S$ the symmetric part of the N-square matrix $\boldsymbol{V}\tilde{\boldsymbol{G}}$. For each price P_1, a unique and interior Nash equilibrium exists if and only if $\phi < -1/\lambda_{min}\left(\boldsymbol{V}\tilde{\boldsymbol{G}}_S\right)$. Then, the vector of optimal PM demands $\boldsymbol{q}_1$ is

$$\boldsymbol{q}_1 = (1 - P_1)\left(\boldsymbol{I} + \phi \boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1} \boldsymbol{V}\mathbf{1} \tag{A.5}$$

Traders' demands $\boldsymbol{q}_1$ is a linear function of the weighted adjacency matrix of global network effects $\tilde{\boldsymbol{G}}$ and the vector of individual effects $\boldsymbol{v}$.

The equilibrium PM price is determined by the market clearing condition (1.3). It is a particular Nash Equilibrium of the set of equilibria characterized by (1.10) such that aggregate demand meets exogenous asset supply $\bar{Q}$. Thus, equilibrium PM price follows directly from Proposition 21.

Theorem 3. *Assume conditions of Proposition 21 hold. Then, given asset supply $\bar{Q} > 0$, the unique and positive equilibrium primary market price is*

$$P_1^* = 1 - \bar{Q}\left(\boldsymbol{v}'\left(\left(\boldsymbol{I} + \phi \boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\right)'\mathbf{1}_{\boldsymbol{N}}\right)^{-1} \tag{A.6}$$

where $\boldsymbol{v}'\left(\left(\boldsymbol{I} + \phi \boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\right)'\mathbf{1}_{\boldsymbol{N}}$ is a strictly positive scalar.

As argued, the crucial demand decision is the primary market one and it allows us to determine i) traders' inventory at the end of period 2; ii) the PM (issuance) price of the bond, and so the cost of credit for the issuer; iii) all possible market outcomes in the local market.This last point is a corollary of Theorem 3.

Corollary 21.1. *Secondary Market Outcomes*

If conditions of Theorem 3 hold, equilibrium outcomes in the secondary market are determined by the network graph $\boldsymbol{G}$, the shock ϕ and issuer's asset supply $\bar{Q}$. They are characterized by the price vector $(\boldsymbol{p_d})_{N\times 1}$ and square matrix of demands $\boldsymbol{Q}_{N\times N}$:

$$\boldsymbol{p_d} = \mathbf{1}_N - \boldsymbol{D}^{-1}\Big(\boldsymbol{G} + I\Big)\boldsymbol{q_1^*} \tag{A.7}$$

$$\boldsymbol{Q}_d = diag\Big(\mathbf{1}_N - \boldsymbol{q}_1^*\Big)\boldsymbol{G} - \boldsymbol{G}diag\Big(\boldsymbol{p_d}\Big) \tag{A.8}$$

where $\boldsymbol{D} = diag\Big(\boldsymbol{d}\Big)$ is the diagonal matrix of individual degrees

The demand matrix $\boldsymbol{Q}_d$ gives the amount of asset shares traded between two agents at each possible market in period 2: rows indicate the buyer and columns the seller. For example, $[\boldsymbol{Q}_d]_{ij} = q_{i,j}$ is agent i's demand when j is the seller. Clearly, if i and j don't share a connection then $q_{i,j} = 0$. $\boldsymbol{Q}_d$ has a zero-diagonal as it does not include asset supply.

A.6 Trading Centrality

Two crucial features sets my trading centrality measure apart from the typical graph-theoretic measures of network centrality that are not suitable for the model. First, it captures not only first and higher-order inter-connectivity (friends, friends of friends, etc..) but also it encodes how such connections interact in local markets (as buyers, sellers, and competitors). Second, the relationship between centrality and individual degree depends on the structure of the trading network. Some networks - like symmetric and core-periphery graphs, and lines - have trading centrality increasing in individual degree. For arbitrary networks the reverse can hold. For example, if the network exhibits high-degree nodes connected to one another, than low-degree traders are the central ones.

Definition 4. *Trading centrality is N-dimensional vector $\boldsymbol{c}$ defined as*[12]

$$\boldsymbol{c}\left(\boldsymbol{G},\phi\right)=\left(\boldsymbol{V}(\boldsymbol{G},\phi)+\phi\tilde{\boldsymbol{G}}(\boldsymbol{G})\right)^{-1}\boldsymbol{1}_N \tag{1.20}$$

where $\boldsymbol{V}(\boldsymbol{G},\phi)$ is the N-diagonal matrix with entries $\{1/v_i\}_i$; $\tilde{\boldsymbol{G}}(\boldsymbol{G})$ is the N-square, not symmetric matrix with entries $\{\tilde{g}_{ij}\}_{i,j}$; and 1_N is the N-vector of ones.

Equivalently, trader i's trading centrality c_i is

$$c_i(\boldsymbol{G},\phi)=v_i\left((1-\phi\sum_j\tilde{g}_{ij}\cdot c_j(\boldsymbol{G},\phi)\right) \tag{1.14}$$

Trading centrality fixed point representation (1.20) and recursive representation (1.14) are related as follows:

$$\begin{aligned}
\boldsymbol{c}=\left(\boldsymbol{V}+\phi\tilde{\boldsymbol{G}}\right)^{-1}\boldsymbol{1}_N=\left(I+\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\boldsymbol{V}\boldsymbol{1}_N \\
or \\
\boldsymbol{c}&=\boldsymbol{V}\boldsymbol{1}_N-\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{c} \\
&=\boldsymbol{V}\boldsymbol{1}_N-\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\left(\boldsymbol{V}\boldsymbol{1}_N-\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{c}\right) \\
&=\boldsymbol{V}\boldsymbol{1}_N-\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{V}\boldsymbol{1}_N+\phi^2\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^2\boldsymbol{c} \\
&=\boldsymbol{V}\boldsymbol{1}_N-\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{V}\boldsymbol{1}_N+\phi^2\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^2\left(\boldsymbol{V}\boldsymbol{1}_N-\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{c}\right) \\
&=\boldsymbol{V}\boldsymbol{1}_N-\phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{V}\boldsymbol{1}_N+\phi^2\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^2\boldsymbol{V}\boldsymbol{1}_N-\phi^3\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^3\boldsymbol{c}... \\
&... \\
&=\left[\sum_{t=0}^{\infty}(-\phi)^t\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^t\right]\boldsymbol{V}\boldsymbol{1}_N
\end{aligned}$$

That is

[12] More precisely, trading centrality is a measure $\boldsymbol{c}:\boldsymbol{G}\rightarrow\mathcal{R}^N$, where $c_i(\boldsymbol{G},\phi)$ is the trading centrality of trader (node) i in the trading network $\boldsymbol{G}$.

$$c_i = v_i \left[1 + \sum_{t=0}^{\infty} (-\phi)^t \left(v_i \sum_j \tilde{g}_{ij} \right)^{t+1} \right]$$

The trading network adjacency matrix $\boldsymbol{G}$ describes all linkages among traders: it is symmetric and unweighted. But the crucial matrix for the model is the global network effect matrix $\tilde{\boldsymbol{G}}$ such that $\tilde{\boldsymbol{g}}_{ij} \geq 0$: it is asymmetric and weighted and it describes traders' interaction in any the Secondary Market. $\tilde{\boldsymbol{G}}$ itself induces a graph. Denote the trading network graph by $\mathcal{G}$ and the global network graph by $\tilde{\mathcal{G}}$, both undirected. It holds that $\mathcal{G} \subseteq \tilde{\mathcal{G}}$.

Matrix $\tilde{\boldsymbol{G}}$ has entries in the $[0, 1]$ interval except when $d_i = 1$ and $d_j > 1$. The centrality matrix $(I + \phi \boldsymbol{V} \tilde{\boldsymbol{G}})^{-1}$ has positive diagonal and negative off-diagonal. The diagonal is increasing in one's connectivity and neighbor's connectivity.

Lemma 7. *Centrality Matrix*

The centrality matrix $(I + \boldsymbol{V} \tilde{\boldsymbol{G}})$ *has diagonal equals to* 1 *and off diagonal elements in* $[0, 1)$*. Off-diagonal entry is zero iff* (i, j) *are not connected nor share a path-two link.*

The inverse centrality matrix $(I + \boldsymbol{V} \tilde{\boldsymbol{G}})^{-1}$ *is a* N*-square matrix with positive diagonal. Off-diagonal elements can be a) negative, if* (i, j) *are connected or share a path-two link; b) positive, if* (i, j) *are not connected nor share a path-two link*

To grasp the intuition behind Lemma 7, take trader $i \in N$. Traders j, k at most two link apart from i directly influence i's local market trading. If i is a buyer, he demands less in the PM when j, k demand more. In this way, i can buy more shares at a lower price in local markets. The same logic holds when i responds to j (or k) as a competitor for k (or j). As a seller, i also lowers his PM demand in response to higher PM demand from j, k. In this way, i sells less at a lower price. For a trader z further apart, i still responds negatively to z's demand. But this response is not as strong since z does not influence directly i's local market trading. Trader z only affects i because z's PM demand determines the terms of trade in other local markets (i has no access to) and, consequently, the PM demand of other traders. What ultimately determines the equilibrium PM price and asset allocation.

Trading centrality translates the above discussion into the matrix $\left(\boldsymbol{V} + \phi \tilde{\boldsymbol{G}}\right)^{-1}$. The *sign* of $\left(\boldsymbol{V} + \phi \tilde{\boldsymbol{G}}\right)^{-1}$ varies with how far apart traders are. If traders i, j are directly connected or have one common connection, then the (i, j) entry is *negative*. However, if i, j are more than two links apart, then (i, j) entry is *positive*.

A.6.1 Trading Centrality and Degree

As discussed in section 1.5, the relationship between trading centrality and individual degrees is non-trivial. The centrality measure encapsulates information above and beyond connectivity. And connectivity by itself is not enough to understand the feedback effect between the primary market and subsequent local markets. Meanwhile, as the network structure defines trading centrality, it also defines te relationship between centrality and degree. In some networks, being more central means being more connected. In others, the opposite holds.

Lemma 8. *Trading Centrality and Degree*

The relationship between trading centrality $\boldsymbol{c}$ and individual degree $\boldsymbol{d}$ depends on the structure of the trading network.

Consider an arbitrary trading network. If $corr(\boldsymbol{c}, \boldsymbol{d}) > 0$, then local markets (re-selling) prices $\{P_i\}_{\forall i \in N}$ are increasing in both $(\boldsymbol{c}, \boldsymbol{d})$.

If $(\boldsymbol{c}, \boldsymbol{d}) < 0$, the opposite holds.

A.6.2 Centrality and the Adjacency Matrix

Trading centrality is a function of the adjacency matrix $\boldsymbol{G}$ defining the trading network. To express trading centrality in terms of $\boldsymbol{G}$ alone, it is useful to introduce some notation. The vector of degrees is $\boldsymbol{d} = \boldsymbol{G}\mathbf{1}$. Define the following Nvectors which are functions of individual degrees:

$$\boldsymbol{d}^{\mathbf{1}} = (\boldsymbol{G}\mathbf{1})^{-1}$$
$$\boldsymbol{d}^{\mathbf{2}} = (\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}$$
$$\boldsymbol{d}^{\mathbf{3}} = 2(\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}$$

so that the entries of each vector are $d_i^1 \equiv \frac{1}{d_i}$, $d_i^2 \equiv \frac{d_i - 1}{d_i^2}$ and $d_i^3 \equiv \frac{2d_i - 1}{d_i^2} = \frac{d_i}{d_i^2} + \frac{(d_i - 1)}{d_i^2} = d_i^1 + d_i^2$.

Let $\boldsymbol{D}^{\mathbf{1}}, \boldsymbol{D}^{\mathbf{2}}, \boldsymbol{D}^{\mathbf{3}}$ denote the diagonal matrix with entries $\boldsymbol{d}^{\mathbf{1}}, \boldsymbol{d}^{\mathbf{2}}, \boldsymbol{d}^{\mathbf{3}}$, respectively:

$$
\begin{aligned}
\boldsymbol{D^1} &= diag(\boldsymbol{d^1}) = diag\left((\boldsymbol{G1})^{-1}\right) = \left((\boldsymbol{G1})^{-1}\right)\boldsymbol{1}^T\boldsymbol{I} \\
\boldsymbol{D^2} &= diag(\boldsymbol{d^2}) = diag\left(\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right) = \left((\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{I} \\
\boldsymbol{D^3} &= diag(\boldsymbol{d^3}) = diag\left(2(\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right) = \left(2(\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{I}
\end{aligned}
$$

The global network matrix $\tilde{\boldsymbol{G}}$ has zero-diagonal and non-negative entries everywhere else. It is useful to decompose it into direct and indirect effects:

$$
\tilde{\boldsymbol{G}} = \tilde{\boldsymbol{G}}_1 + \tilde{\boldsymbol{G}}_2
$$

where

$$
\begin{aligned}
\tilde{\boldsymbol{G}}_1 &\equiv \boldsymbol{D^1G} + \boldsymbol{GD^2} \\
\tilde{\boldsymbol{G}}_2 &\equiv \boldsymbol{GD^2G} - diag\left(\boldsymbol{GD^2G}\right)
\end{aligned}
$$

Then

$$
\begin{aligned}
\tilde{\boldsymbol{G}} &= \boldsymbol{D^1G} + \boldsymbol{GD^2}\left(I + \boldsymbol{G}\right) - diag\left(\boldsymbol{GD^2G}\right) \\
&= (\boldsymbol{G1})^{-1}\boldsymbol{1}^T\boldsymbol{IG} + \boldsymbol{G}\left((\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{I}(\boldsymbol{I} + \boldsymbol{G}) \\
&- \left(\boldsymbol{G}\left((\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{IG}\right)\boldsymbol{1}^T\boldsymbol{I}
\end{aligned}
$$

The matrix of indirect effects $\tilde{\boldsymbol{G}}_2$[13] is a weighted count of the paths of length two in the network. This path plays an important role in determining a trader trading centrality because it captures competition in the local market.

Now I turn into the individual effect vector $\boldsymbol{v}(\phi)^{-1}$,

$$
\boldsymbol{v}(\phi)^{-1} = (1-\phi)\boldsymbol{1}_N + 2\phi\boldsymbol{d^1} - \phi\boldsymbol{d} + \phi\boldsymbol{Gd^3}
$$

The diagonal matrix $\boldsymbol{V}$ has entries $\boldsymbol{v}(\phi)^{-1}$, and so in terms of $\boldsymbol{G}$ and ϕ,

[13]The matrix of path-two is given by $\boldsymbol{P}_2 = \boldsymbol{G^2} - diag(\boldsymbol{G}^2)$.

$$\begin{aligned}\boldsymbol{V} &= diag\left(\boldsymbol{v}(\phi)^{-1}\right) \\ &= \left((1-\phi)\mathbf{1} + 2\phi(\boldsymbol{G}\mathbf{1})^{-1} - \phi(\boldsymbol{G}\mathbf{1}) + \phi\boldsymbol{G}\left(2(\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right)\right)\mathbf{1}^T\boldsymbol{I}\end{aligned}$$

Hence, trading centrality $\boldsymbol{c}(\boldsymbol{G}, \phi)$ (eq. (1.20)) is given by

$$\begin{aligned}\boldsymbol{c} = \Bigg(&\left((1-\phi)\mathbf{1} + 2\phi(\boldsymbol{G}\mathbf{1})^{-1} - \phi(\boldsymbol{G}\mathbf{1}) + \phi\boldsymbol{G}\left(2(\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right)\right)\mathbf{1}^T\boldsymbol{I} \\ &+ \phi\Big((\boldsymbol{G}\mathbf{1})^{-1}\mathbf{1}^T\boldsymbol{I}\boldsymbol{G} + \boldsymbol{G}\left((\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right)\mathbf{1}^T\boldsymbol{I}(\boldsymbol{I}+\boldsymbol{G}) \\ &- \left(\boldsymbol{G}\left((\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right)\mathbf{1}^T\boldsymbol{I}\boldsymbol{G}\right)\mathbf{1}^T\boldsymbol{I}\Big)\Bigg)^{-1}\mathbf{1}\end{aligned} \tag{A.9}$$

A.6.3 Local Trading Centrality

From Lemma 1 and Theorem 1, the selling price of trade i is given by his and his buyers' centralities, $c_i, \{c_j\}_{j\in N_i}$ respectively:

$$P_i^* = 1 - \frac{\bar{Q}}{Nc_A}\frac{\left(c_i + \sum_{j\in N_i} c_j\right)}{d_i} \tag{A.10}$$

Then, i's trading cost is

$$\begin{aligned}P_1 - P_i &= \left(1 - \frac{\bar{Q}}{Nc_A}\right) - \left(1 - \frac{\bar{Q}}{Nc_A}\frac{\left(c_i + \sum_{j\in N_i} c_j\right)}{d_i}\right) \\ &= \frac{\bar{Q}}{Nc_A}\left(\frac{c_i + \sum_{j\in N_i} c_j}{d_i} - 1\right)\end{aligned} \tag{A.11}$$

It is useful to denote $\tilde{c}_i \equiv \frac{c_i + \sum_{j\in N_i} c_j}{d_i}$ as the local centrality of trader i, that is, the sum of his local market participants' centrality, including himself, controlled for his degree. With that, there is a straightforward relation between trading cost and local centrality.

Proposition 22. *Trading Cost and Centrality*

The trading cost of trader $i \in N$ is increasing in his local centrality $\tilde{c}_i$:

$$P_1 - P_i = \frac{Q}{Nc_A}\left(\tilde{c}_i - 1\right) > 0 \tag{A.12}$$

Moreover, $P_1 - P_i < 0$ *if and only if* i *is the core of a star network of size* $N \geq 3$*.*

Local centrality captures both participation and inventory effects. And that's the reason it's a sufficient statistic for liquidity cost.

A.7 Comparative Statics

Best-replies:

Trader i's response to changes in behavior of others is independent of PM price. In the best-replies space, varying price P_1 corresponds to parallel shifts of the demand schedules. As a trader $j \neq i$ changes his PM demand $q_{j,1}(\cdot)$, i reacts by changing his demand $q_{i,1}$ according to

$$\frac{\partial q_{i,1}(\cdot)}{\partial q_{j,1}(\cdot)} = -\phi v_i(\phi)\tilde{g}_{ij} \tag{A.13}$$

As expected, the partial derivative (A.13) is negative as the game is one of strategic substitutes. It is also asymmetric: the way j affects i is not the same as i affects j. That's because i and j can have different positions in the network and so they will face different sets of potential sellers, buyers and competitors. This is precisely the reason that just looking at how many connections (degree) of a trader is not enough to understand his behavior. Lastly, it only depends on the trading network and shock parameter since, as shown in Theorem 3, these are necessary and sufficient information to find PM equilibrium (along with the exogenous supply level $\bar{Q}$). Mathematically, the matrix $\tilde{\boldsymbol{G}}$ is asymmetric and individual coefficients $\{v_i(\phi)\}_i$ are heterogeneous.

The shock effect:

P_1 only affects how demands change in response to the shock. To see this, first notice that ϕ has a direct and indirect effect on $q_{i,1}$:

$$\frac{\partial q_{i,1}(\cdot)}{\partial \phi} = \frac{\partial v_i}{\partial \phi}\left[(1 - P_1) - \phi \sum_j \tilde{g}_{ij} q_{j,1}\right] - v_i \sum_j \tilde{g}_{ij} q_{j,1}$$

The indirect effect comes trough $v_i(\phi)$:

$$\frac{\partial v_i}{\partial \phi} = -v_i^2 \cdot \psi_i > 0 \qquad if \;\; \psi_i < 0 \Longleftrightarrow \underbrace{\frac{(d_i^2 + d_i - 2)}{d_i}}_{>0} > \sum g_{ij} \cdot \underbrace{\left(\frac{2d_j - 1}{d_j^2}\right)}_{>0}$$
$$< 0 \qquad if \;\; \psi_i > 0 \Longleftrightarrow \underbrace{\frac{(d_i^2 + d_i - 2)}{d_i}}_{>0} < \sum g_{ij} \cdot \underbrace{\left(\frac{2d_j - 1}{d_j^2}\right)}_{>0} \tag{A.14}$$

where, to simplify notation, I let

$$\psi_i \equiv \left[\frac{2}{d_i} - (d_i + 1) + \sum_j g_{ij} \cdot \left(\frac{2d_j - 1}{d_j^2}\right)\right]$$

ψ_i is a measure of relative connectivity between i and his neighbors. When i is, loosely, more (less) well-connected than his friends then $\psi_i < 0$ ($\psi_i > 0$). In turn, the total effect of ϕ on $q_{i,1}$ is positive (negative). Thus an increase in the shock probability makes a trader i to increase his demand only if he is more connected than his neighbors:

$$\frac{\partial q_{i,1}(\cdot)}{\partial \phi} = -v_i^2 \cdot \psi_i > 0 \qquad if \;\; \psi_i < 0$$

As the secondary market becomes more likely, i expects to sell relatively more asset shares at a high price if he is shocked, and to buy relatively few shares at low price is he is connected to the seller.

The individual degree effect:

Individual degrees $\{d_i\}_{i \in N}$ appear in all PM demand's network components $(v_i(\phi), \{\tilde{g}_{ij}\}_{j \neq i}$. Their effect is can be broken down by a trader's own degree, his neighbors' degrees, and his neighbors' connections degrees.

First, individual degree d_i has a positive effect on v_i and a negative effect on $\{\tilde{g}_{ij}\}_{j \neq i}$

$$\frac{\partial v_i}{\partial d_i} = -\left(-\frac{2\phi}{d_i^2} - \phi\right) \cdot v_i^2 = \left(\frac{2\phi}{d_i^2} + \phi\right) \cdot v_i^2 > 0$$
$$\frac{\partial \tilde{g}_{ij}}{\partial d_i} = -\frac{1}{d_i^2} < 0 \qquad \forall j \;\; s.t. \;\; g_{ij} \geq 1 \tag{A.15}$$

Both effects combined imply that $q_{i,1}$ is increasing in individual degree d_i.

Second, each direct connection's degree d_j has a direct and indirect effects, the latter coming from common friends:

$$\frac{\partial v_i}{\partial d_j} = -\phi\left(-\frac{2}{d_j^2}+\frac{2}{d_j^3}\right)\cdot vi^2 = \phi\left(\frac{2(d_j-1)}{d_j^3}\right)\cdot v_i^2 \geq 0$$
$$\frac{\partial \tilde{g}_{ij}}{\partial d_j} = -\frac{(d_j-2)}{d_j^3}\left(1+\sum_{z\neq\{i,j\}} g_{ij}g_{jz}\right)\begin{cases}\leq 0 & d_j \geq 2\\ > 0 & d_j = 1\end{cases} \tag{A.16}$$

Lastly, purely indirect connections' degrees, i.e. those who are connected to i only trough a common friend, have an effect on i's demand that depends only on he common friend's degree. That is, the effect of trader k's degree d_k such that $g_{ij}=1, g_{jk}=1, g_{ik}=0$ for all $j,k\in N$ is

$$\frac{\partial \tilde{g}_{ik}}{\partial d_k} = -\sum_{j\neq\{k,i\}} g_{jk}g_{ij}\frac{(d_j-2)}{d_j^3} \leq 0 \qquad since \quad d_k \geq 2 \tag{A.17}$$

The price effect:

The elasticity of demand of each trader is given by his individual network effect $v_i(\phi)$:

$$\frac{\partial q_{i,1}}{\partial P_1} = -v_i(\phi) \tag{A.18}$$

This has two implications. First, the higher $v_i(\phi)$, the more elastic is a trader's PM demand. Traders respond negatively but in different magnitude to changes in the PM price. Second, the slope of a trader's demand schedule changes as P_1 varies. This is precisely because each P_1 induces a different network game and traders' best-replies are game-specific.

Figure A.4 depicts the price effect. In the left-hand graph, each colored line is a trader's demand curve for a given price P_1. For example, the orange line is the schedule when $P_1 = 0.9$. Clearly, as P_1 increases the demand curve becomes steeper. The right-hand panel compares demand curves to two traders i (orange) and j (blue) such that $v_i(\phi) > v_j(\phi)$.

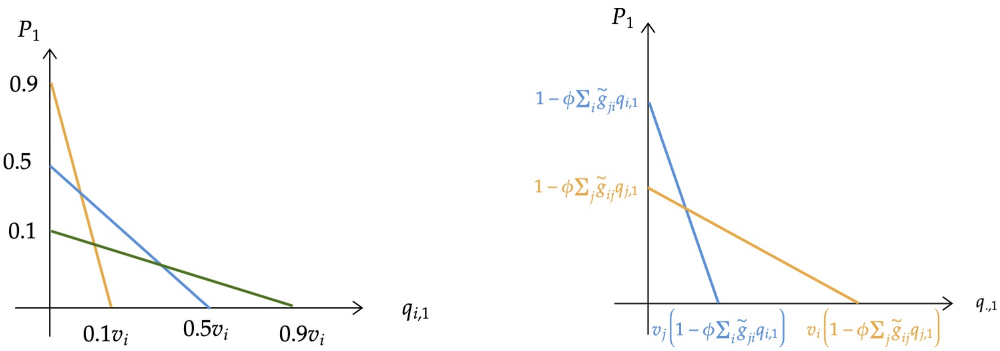


Figure A.4: Best-replies and Primary market price

A.8 Bilateral Trading Comparison

In this section, I discuss the equilibrium outcomes if only bilateral trade was allowed. Maintaining the assumptions of the model, this would be the case of a regular network with degree two, that is, a network with regular components of size two. Using the results for regular networks, PM equilibrium price and asset consumption are $P_1^* = 1 - \frac{\bar{Q}}{N}(1 + 2\phi)$ and $q_1^* = \frac{\bar{Q}}{N} \ \ \forall i \in N$, respectively. Any local market equilibrium is given by price and asset consumption, $P_2^* = 1 - 2\frac{\bar{Q}}{N}$ and $q_2^* = q_1^* \ \ \forall i \in N$.

Bilateral trade delivers the *lowest* possible prices in the primary market and all local markets. In equilibrium, asset consumption is the same in both periods even though their demands are different. A trader's demand schedule in the PM is $q_1 = \frac{1-P_1}{1+2\phi}$ and in the local market is $q_2 = \frac{P_1}{1+2\phi} - P_s$.

Traders' expected total asset holdings is simply the average supply $\frac{\bar{Q}}{N}$. Also, traders' expected utility is given by

$$
\begin{aligned}
EU &= w + (1 - P_1^*)q_1^* - \frac{1+2\phi}{2}(q_1^*)^2 \\
&= w + \frac{(1+2\phi)}{2}\left(\frac{\bar{Q}}{N}\right)^2
\end{aligned}
$$

With bilateral trade, if the restriction in the shock $\phi < \frac{1}{N}$ is discarded, local market price can be greater than the primary price.

Proposition 23. *Bilateral Trade and Difference in Prices*

If the trading network only allows for bilateral trades, equilibrium prices in the PM and *any local market are $P_1^* = 1 - \frac{\bar{Q}}{N}(1+2\phi)$ and $P_2^* = 1 - 2\frac{\bar{Q}}{N}$, respectively. Traders consume the same amount of asset shares in both periods.*

The negative demand shock ϕ controls the capital gain/loss of the seller. Selling bilaterally delivers capital loss (gain) if ϕ is less (greater) than $\frac{1}{2}$. If $\phi = \frac{1}{2}$, prices are equal.

It is useful to see the equilibrium in matrix notation for a direct comparison with my main result. First, demand schedule coefficients are $v_i(\phi) = (1+\phi)^{-1} \ \forall i$ and $\tilde{g}_{ij} = 1 \ \forall i, j : g_{ij} = 1$ and zero otherwise. This means that the global network matrix $\tilde{\boldsymbol{G}}$ equals the adjacency matrix $\boldsymbol{G}$ and that $\boldsymbol{V}\tilde{\boldsymbol{G}}$ is simply $(1+\phi)^{-1}\boldsymbol{G}$. Trading centrality simplifies to $\boldsymbol{C} = \frac{1}{1+\phi}\left(\mathbf{1} + \frac{\phi}{1+\phi}\boldsymbol{G}\right)^{-1}\mathbf{1}_N$.

A.9 Results

Degree Distribution

I study changes in the trading network structure by changing its degree distribution: that is, the number of connections of each trader. I do two exercises based on stochastic dominance: varying the average degree and the degree variance. Specifically, I compare two degree distributions such that i) one is a first-order stochastic shift of the other; and ii) one is a mean-preserving spread of the other.

Recall the stochastic dominance definition: a cumulative distribution F first-order stochastically dominates (FOSD) another distribution G iff $F(x) \leq G()$ for all x. Note that if F FOSD G, then F necessarily has a strictly larger expected value than G, $EF(x) > EG(x)$ (the reverse not true). Recall also the mean-preserving spread definition: F is a mean-preserving spread of G if and only if $EF(x) = EG(x)$ and $\int F(x)dx \leq \int_{\infty}^{c} G(x)dx$ for all c. Note that if F is a mean-preserving spread of G than F necessarily has a larger variance than G.

Denote the degree distribution of the trading network as P. Consider a change in the probability distribution over the degrees to P' that reflects an unambiguous increase in connectivity. In particular, suppose that P' FOSD P. Then, the average degree under P' is higher than under P. Moreover, each trader's degree under P' is at least as large as his degree under P'. I am interested in how traders' demands, and hence price, changes as the trading network shifts from P to P'.

Looking at network coefficients in (1.11) and (1.12), we know that $v_i(\phi)$ weakly increases while $\tilde{g}_{ij}$ weakly decreases for all $i, j \in N$ (see section A.7). Then, from the demand function (1.10), each trader demands more at each possible price level, i.e. his demand schedule becomes flatter. Since PM price is increasing in traders' demands, PM price increases as well.

Network Symmetry

A symmetric network, or regular graph, is such that degree of each node is equal. A graph is called k-regular if degree of each node is k. Regular graphs have useful properties. First, the necessary and sufficient conditions for a k-regular graph with N nodes to exist are that $N \geq k+1$ and that Nk is even. Second, the number of links E is given by $\frac{Nk}{2}$. It is also known that for any undirected graph E also relates the the sum of individual degrees such that $\sum_i d_i = 2E$. These four facts, combined to the findings regarding the degree distribution, lead to Proposition 6.

Take a k-regular network with N traders and, thus, $\frac{Nk}{2}$ links. Now consider any other network structure with the same number of traders and links. It must hold that the average degree in this network is such that $\frac{1}{N}\sum_i d_i = 2\frac{k}{2} = k$, i.e. the same as the in k-regular graph. And that its degree variance is positive, otherwise it would be a regular network (i.e. with zero variance). Thus, from Proposition 4, it holds the PM price in this network is lower than the one in the k-regular network.

A.10 Core-Periphery Networks

A core-periphery network structure typically consists of a well-connected set of nodes, the core, and the remainder nodes, the periphery, well connected to the core but sparsely connected internally. The most common example is the star network in which one node is fully connected to all other nodes, who themselves are only connected to the core.

Proposition 7 shows that the star network is the unique structure delivering capital gains for a seller while exhibiting the lowest primary market price. At the same time, empirical evidence has documented a core-periphery structure for different inter-trader markets. Motivated by these two facts, in this section of the Appendix I provide detailed results and proof for the class of core-periphery networks. In particular, I focus on the star graph, regular core-periphery networks, and the most extreme cases of a fully connected core (the complete case) and the sparsely connected core (the ring case).

A.10.1 Star Network

In this part of the appendix, I prove the following lemmas.

Lemma 9. *Across all markets and across networks of the same size N, the core's price of the star network is the highest.*

Another interesting feature of the star network is that it is the most unequal: it delivers the highest dispersion in asset allocation. Traders located in the periphery shift their asset consumption relatively more to the local market even though the seller's price is high. That's because, if a periphery is shocked, his selling price is so low that his capital loss would be larger than the difference in prices between the two markets he can act as a buyer. The next proposition state this result.

Proposition 24. *Inequality in a Star Trading Network*

Across trading networks of the same size N, the star structure delivers the highest dispersion in asset allocation. The core (periphery) has the highest (lowest) possible primary market demand.

Proposition 25. *Demand inequality is decreasing in the size of the star network N.*

Core's network coefficients are

$$v_c^{-1} = \frac{2\phi}{N-1} + N(1-\phi) = \frac{2\phi + (N^2 - N)()1 - \phi}{N-1}$$
$$\tilde{g}_{cp} = 1$$

and his demand function is then

$$q_c = v_c \left(1 - P_1 - \phi(N-1)q_p\right) \tag{A.19}$$

Peripheries' network coefficients are

$$v_p^{-1} = 1 + \phi\frac{(2N-3)}{(N-1)^2}$$
$$\tilde{g}_{pc} = 1 + \frac{(N-2)}{(N-1)^2} = \frac{N^2 - N - 1}{(N-1)^2}$$
$$\tilde{g}_{pc} = \frac{N-2}{(N-1)^2}$$

and their demand function is then

$$q_p = v_p \left(1 - P_1 - \phi\tilde{g}_{pc}q_p - \phi(N-2)\tilde{g}_{pp}q_p\right) \tag{A.20}$$

Now I can write the system of demands in 2-by-2 matrix format in which the first row/column refers to the core and the second row/column to a periphery. Matrices $\mathbf{V}, \tilde{\boldsymbol{G}}$ are given by

$$\left(\mathbf{V}^{-1} + \phi\tilde{\boldsymbol{G}}\right) = \begin{pmatrix} \frac{1}{v_c} & \phi(N-1) \\ \phi\tilde{g}_{pc} & \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} \end{pmatrix} \quad \text{(A.21)}$$

and trading centrality is then

$$\boldsymbol{TC} \equiv \begin{pmatrix} C_c \\ C_p \end{pmatrix} = \left(\mathbf{V}^{-1} + \phi\tilde{\boldsymbol{G}}\right)^{-1}\mathbf{1} = \frac{1}{\Delta}\begin{pmatrix} \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) \\ \frac{1}{v_c} - \phi\tilde{g}_{pc} \end{pmatrix} \quad \text{(A.22)}$$

where $\Delta \equiv det\left(\left(\mathbf{V}^{-1} + \phi\tilde{\boldsymbol{G}}\right)^{-1}\right)$ and it holds that $\frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) > \frac{1}{v} - \phi\tilde{g}_{pc}$ Equilibrium is determined by the weighted sum of centralities,

$$C_T \equiv \frac{1}{\Delta}\begin{pmatrix} \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) & \frac{1}{v} - \phi\tilde{g}_{pc} \end{pmatrix}\begin{pmatrix} 1 \\ N-1 \end{pmatrix} \quad \text{(A.23)}$$

Thus primary market price and asset allocation are, respectively

$$P_1^* = 1\frac{\bar{Q}}{C_T}$$
$$q_i^* = \frac{\bar{Q}}{C_T}C_i \quad i = \{c, p\}$$

A.10.2 Core-Periphery Networks

Growing the size of the core:

The next figures depicts Proposition 8. Figure A.5 shows the equilibrium of the model for growing core-periphery networks by increasing the number of N traders. The number of core traders, with the same connectivity to 2 peripheries, increases.

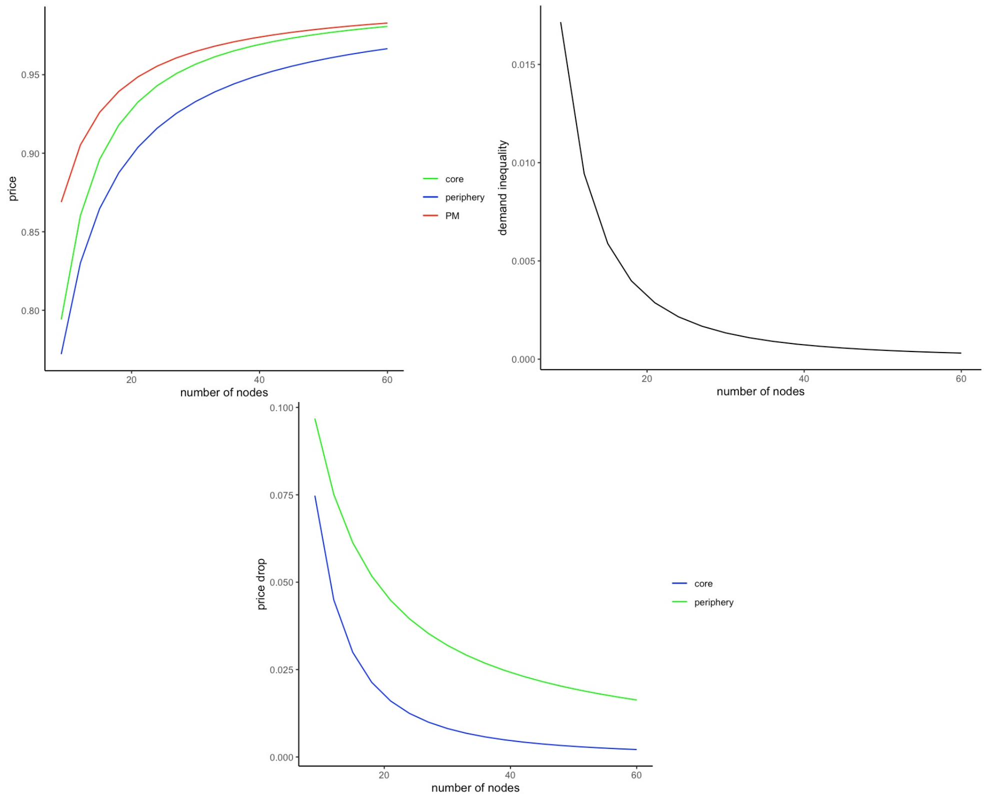


Figure A.5: Growing a Core-Periphery network: increasing N by adding more core traders with the same number of peripheral connections.

Similarly, Figure A.6 shows the equilibrium for different core-periphery network with the same number of $N = 12$ traders and different core sizes. The dashed lines are equilibrium outcomes for the star network of the same size $N = 12$.

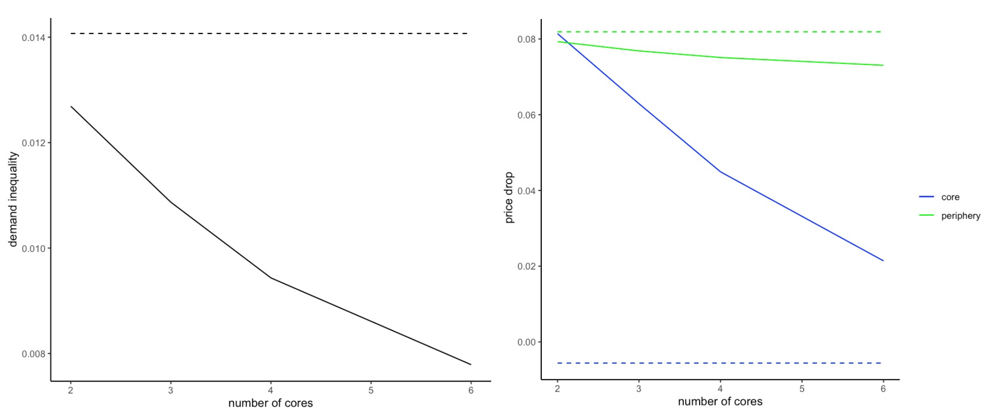


Figure A.6: Growing a Core-Periphery network: for a fixed $N = 12$ traders, the core size increases by moving a peripheral node to the core while keeping cores' connectivity to the periphery homogeneous

To make it clear the difference in the two exercises above, the first is moving from network B to C below. The second is moving from network A to B.

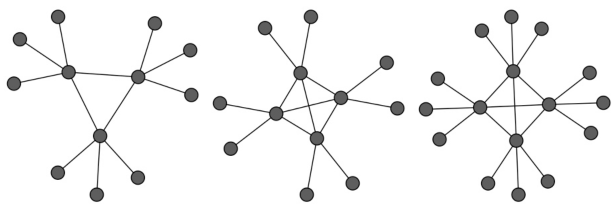


Figure A.7: Growing a Core-Periphery network: from left to right - network A, B and C

A.10.3 Regular Core-Periphery Network

A regular core-periphery network structure is quite tractable since cores' share the same demand, and so do peripheries. Thus I just need to keep track of two variables, a core's demand $q_{,1c}$ and a periphery's demand $q_{,1p}$.

The primitives for an arbitrary core-periphery structure are i) the number of cores n_c; ii) the number of peripheries *per core* n_p. So there are $(n_c n_p)$ peripheries and $N = n_c(1+n_p)$ nodes. Notice that core's degree is then $d_c = n_p + (n_c - 1) = N - (n_c-1)n_p$.

Suppose $n_c \geq 2$. Let $\tilde{g} \equiv \frac{(d_c-1)}{d_c^2}, \quad \tilde{g}_c = \tilde{g}d_{cc} + \frac{1}{d_c}$. Then, cores and peripheries' demand schedules (1.10) are, respectively

$$\begin{aligned} q_c &= v_c \left[1 - P_1 - \phi\tilde{g}_c\left(d_{cc}q_c + n_p q_p\right)\right] \\ q_p &= v_p \left[1 - P_1 - \phi\tilde{g}_p\left((d_{cc}+1)q_c + (n_p-1)q_p\right) - \phi q_c\right] \end{aligned} \quad \text{(A.24)}$$

The network-induced coefficients in (1.10), $\{(v_i, \tilde{g}_{ij})\}_{i,j\in N}$, become for a core

$$\begin{aligned} v_c &= \left[\frac{2\phi}{d_c} + 1 - \phi(d_c+1) + \phi d_{cc}\left(\tilde{g} + \frac{1}{d_c}\right) + \phi n_p\right]^{-1} \\ \tilde{g}_{cc} &= \tilde{g}_c d_{cc} \\ \tilde{g}_{cp} &= \tilde{g} n_p \end{aligned}$$

and for a periphery

$$\begin{aligned} v_p &= \left[1 + \phi\left(\tilde{g} + \frac{1}{d_c}\right)\right]^{-1} \\ \tilde{g}_{pc} &= \tilde{g}(d_{cc}+1) + 1 \\ \tilde{g}_{pp} &= \tilde{g}(n_p - 1) \end{aligned}$$

That is, demands are $q_c = v_c\left[1 - P_1 - \phi\tilde{g}_{cc}q_c - \tilde{g}_{cp}q_p\right] \quad \forall c \in N$ and and $q_p = v_p\left[1 - P_1 - \phi\tilde{g}_{pc}q_c - \tilde{g}_{pp}q_p\right] \quad \forall p \in N$.

Now I can write the system of demands in matrix format. Define 2-by-2 matrices Ψ_1, Ψ_2 in which the first row/column refers to a core and the second row/column to a periphery,

$$\Psi_1 = \begin{pmatrix} \psi_{1c} & 0 \\ 0 & \psi_{1p} \end{pmatrix} \quad \Psi_2 = \begin{pmatrix} 0 & \psi_{2c} \\ \psi_{2p} & 0 \end{pmatrix} \tag{A.25}$$

such that $\psi_{1c} \equiv \left[\frac{1}{v_c} + \phi\tilde{g}_c d_{cc}\right], \quad \psi_{1p} \equiv \left[\frac{1}{v_p} + \phi\tilde{g}(n_p - 1)\right]$, and $\psi_{2c} \equiv \tilde{g}_c n_p, \quad \psi_{2p} \equiv \tilde{g}(d_{cc} + 1) + 1$.

Then the system of demands is

$$\begin{aligned} \Psi_1\boldsymbol{q} &= (1 - P_1)\mathbf{1}_2 - \phi\Psi_2\boldsymbol{q} \\ (\Psi_1 + \phi\Psi_2)\,\boldsymbol{q} &= (1 - P_1)\mathbf{1}_2 \\ \boldsymbol{q} &= (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2(1 - P_1) \end{aligned} \tag{A.26}$$

(A.26) is the counterpart of (A.5). Since Ψ_1, Ψ_2 are 2-by-b2 taking the inverse is easy:

$$(\Psi_1 + \phi\Psi_2)^{-1} = \frac{1}{\psi_{1c}\psi_{1p} - \psi_{2c}\psi_{2p}} \begin{pmatrix} \psi_{1p} & -\phi\psi_{2c} \\ -\phi\psi_{2p} & \psi_{1c} \end{pmatrix}$$

Trading centrality (1.20) is now given by

$$\begin{aligned} \boldsymbol{c} &= (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2 \\ &= \frac{1}{\psi_{1c}\psi_{1p} - \psi_{2c}\psi_{2p}} \begin{pmatrix} \psi_{1p} - \phi\psi_{2c} \\ \psi_{1c} - \phi\psi_{2p} \end{pmatrix} \end{aligned} \tag{A.27}$$

where it holds that core's centrality is higher than periphery' centrality:

$$\psi_{1p} - \phi\psi_{2c} > \psi_{1c} - \phi\psi_{2p}$$

The next proposition gives the equilibrium for a regular core-periphery network with n_c

cores, $d_c n_p$ peripheries and connectivity among cores of d_{cc}.

Proposition 26. *Equilibrium in a Regular Core-Periphery Network*

Consider regular core-periphery network with n_c cores, $d_c n_p$ peripheries and connectivity among cores of d_{cc}. Suppose $n_c \geq 3$ and $d_{cc} \geq 2$. Then, trading centrality is given by

$$\boldsymbol{C} = (\Psi_1 + \phi \Psi_2)^{-1} \mathbf{1}_2 \tag{A.28}$$

Primary Market equilibrium price and demands are, respectively

$$P_1^* = 1 - \bar{Q} \frac{1}{C_c n_c + C_p n_p} \tag{A.29}$$

$$q_{i,1} = \bar{Q} \frac{C_i}{C_T} \quad i = \{c, p\} \tag{A.30}$$

where $C_T = C_c n_c + C_p n_p$.

To understand how equilibrium change as we change the structure of the core-periphery graph one must look at the trading centrality. One can show that: i) cores' (peripheries') centrality is decreasing (increasing) in cores' connectivity; ii) both centralities are increasing in the number of cores and/or peripheries.

Focusing on the extremes structures - complete and ring cores, cores' (peripheries') centrality is decreasing (increasing) in in the number of the cores and/or peripheries.

Complete and Ring Cases

The two most extreme cases of regular core-periphery networks are i) when the core is fully connected (complete); and ii) when each core is connected to other two (ring). I now compare primary market equilibrium in these two core-peripheries structure. In particular, I show that:

- Price is higher in the complete core than in the ring core, for any number of cores and peripheries (from the main result)
- Cores' (peripheries') centrality is lower (higher) in the complete structure
- Cores' (peripheries') demand is lower (higher) in the complete structure
- Demand dispersion (inequality) is lower (higher) in the complete (ring) structure

In addition, I study how the comparison changes as the number of peripheries changes. Increasing the number of peripheries results in

- Lower price difference between the complete and ring structures - even though price increases in both
- Lower difference in demand dispersion - inequality decreases in both

I obtain these results using Proposition 26 that shows that characterizing the equilibrium in any core-periphery structure is quite straightforward. One only needs to determine two variables: the demand schedules for a core and a periphery. For the complete case, notice that $d_{cc} = n_c - 1$ and $\tilde{g}_c = (n_c - 1)\tilde{g} + \frac{1}{d_c}$. Then, cores and peripheries' demands are, respectively

$$\begin{aligned} q_c &= v_c \left[1 - P_1 - \phi \tilde{g}_c \left((n_c - 1) q_c + n_p q_p\right)\right] \\ q_p &= v_p \left[1 - P_1 - \phi \left(\tilde{g} n_c + 1\right) q_c - \phi \tilde{g} (n_p - 1) q_p\right] \end{aligned} \tag{A.31}$$

For the ring case, $d_{cc} = 2$ $\tilde{g}_c = 2\tilde{g} + \frac{1}{d_c}$. Then, cores and peripheries' demand schedules are, respectively

$$\begin{aligned} q_c &= v_c \left[1 - P_1 - \phi \tilde{g}_c \left(2 q_c + n_p q_p\right)\right] \\ q_p &= v_p \left[1 - P_1 - \phi \tilde{g}_p \left(3 q_c + (n_p - 1) q_p\right) - \phi q_c\right] \end{aligned} \tag{A.32}$$

The next proposition shows that as the number of cores become too large, price in these structure converge to the same level.

Proposition 27. *As $n_c \to \infty$, then $P_1^{complete} - P_1^{ring} \to 0$ irrespective of the number of peripheries.*

A.11 Symmetric Networks

In regular networks, all nodes have the same number of links and position in the network. It immediate follows from Theorem 1 that traders have the same demand and selling price if and only if they have the same network position. These two facts make it easier to study regular networks. Equilibrium asset allocation is independent of the network structure, and it is the same as in the Walrasian market. PM and local market demands are, respectively, $q^*_{i,1} \equiv q^*_1 = \frac{Q}{N}, q^*_{i,s} \equiv q^*_2 = \frac{\bar{Q}}{Nd}$. The local market price is exclusively determined by traders' degree d, $P^*_s = 1 - \frac{(d+1)}{d}\frac{\bar{Q}}{N}$.

Proposition 28. *Primary Market in Regular Networks*

In regular networks, PM price is increasing in the size and degree of the network.

Differently, primary market equilibrium allocation is independent of the network. It is the same as in the Walrasian market: all traders demand $\frac{\bar{Q}}{N}$ in the primary market, and $\frac{\bar{Q}}{Nd}$ in any local market.

Proposition 28 shows that, for a fixed number of N traders, the higher is traders' degree the higher is the PM price. Similarly, for a fixed degree d, increasing the size of the network increases PM price.

The demand schedule in every market is homogeneous across traders: any asset supply is divided equally among buyers, as they have the same willingness to pay. In other words, asset allocation is the same as in a perfectly competitive market. As I show next, PM price varies considerably across regular networks and it is never equal to the price of a perfectly competitive market. The results in this section highlights that if we ignore asset issuance price and only look at traders' inventories in OTC we are missing important considerations of funding costs.

Using market clearing conditions, equilibrium asset allocation in the PM and local market are, respectively: $q^*_{i,1} \equiv q^*_1 = \frac{\bar{Q}}{N}$ and $q^*_{i,s} \equiv q^*_2 = \frac{\bar{Q}}{Nd}$. The local market price is exclusively determined by traders' degree as d: $P^*_s = 1 - \frac{\bar{Q}}{N}\left(\frac{d+1}{d}\right)$

Turning to primary market equilibrium, first notice that individual network coefficient v_i becomes $v_i^{-1} = v^{-1} = \frac{\phi + d(1+\phi-\phi d)}{d}$ and, in turn, trading centrality for every trader is $c_i = \frac{d}{\phi + d(1+\phi)} \forall i \in N$. Then, the traders' demand schedule is $q_1 = v(1 - P_1 - \phi d q_1) = \frac{d}{\phi + d(1+\phi)}$. And equilibrium PM is thus $P^*_1 = 1 - \frac{\bar{Q}}{N}\left(\frac{\phi + d(1+\phi)}{d}\right)$. As P^*_1 is increasing degree d, it is easy to compare different regular networks.

A.11.1 Complete Graph

The complete network is a special case of a regular network with degree $d = N - 1$. No trading frictions exist since all traders are connected with one another. Everyone trade in both markets, either as a buyer or a seller. Using the result from Proposition 28, primary market price is $P^*_1 = 1 - \frac{\bar{Q}}{N(N-1)}(N(1+\phi) - 1)$, and any local market equilibrium price is $P^*_s = 1 - \frac{\bar{Q}}{N-1}$. Demands in each market are, respectively,$q_1 = \frac{\bar{Q}}{N}$ and $q_2 = \frac{\bar{Q}}{N(N-1)}$.

It is easier to see that the complete network delivers the highest primary market price.

Proposition 29. *Complete Network*

For a fixed network size N and in comparison with other regular networks, the Complete Network exhibits the highest primary market price.

It is worth pointing out that the static, competitive market is different than an economy with an empty network, i.e. without any trading relationships Even though there is no local market in both scenarios, in an empty network agents still face the negative demands shock. This risk makes demand schedules less elastic and drives PM price down. In equilibrium, PM demand and price are, respectively, $q_{i,1}(P_1) = q_1(P_1) = 1 - \frac{1}{1-\phi}P_1 \;\; \forall i \in N$ and $P_1^* = (1-\phi)\left(1 - \frac{\bar{Q}}{N}\right)$.

A.12 Ring versus Star Trading Networks

The conclusion from the study of core-periphery networks (Section 1.6.2) is that prices and demands are tightly related to the number of traders in the core of core-periphery networks. Meanwhile, PM price is increasing the number of traders and it is determined by how they are connected among themselves. This suggest that the effect of growing the trading network on equilibrium outcomes depends on the way new traders and their linkages are added in the network. That is, there exists a size effect - increasing N, and a network effect - changing the degree distribution.

One way to differentiate between the size effect and network effect on PM price is to look on how price changes as we grow a ring network and a star network. The former is a regular network.[14] It exhibits no inequality in terms of degree and asset allocation: all traders have the same demands and selling price as they have the same degree and network position. The star network is the most unequal one with respect to both degree and demand.

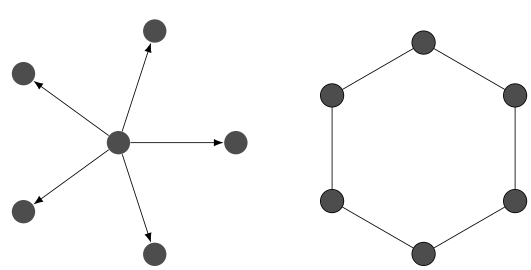


Figure A.8: A ring and a star network of the same size $N = 6$ traders.

The unique effect of growing the ring network is about market size. The growth of the star network, apart from capturing the size effect, carries network effect because the degree distribution changes. Adding one trader in either case means adding just one more

[14]section A.11 provides equilibrium outcomes for the general class of regular networks.

link. However, degree inequality and connectivity increase in the star network, while the degree distribution remains unchanged in the ring.[15]

It is useful to first compare market outcomes as the networks grow. By Lemma 5, we know that PM and local market prices in both networks increase. The difference is that in the ring price drops while in the star price can either increase or decrease: price increase if the core is selling and drops if one of the peripheries is the seller.

Even though PM price increases as the network becomes larger, it does so as diminishing rates. More importantly, it grows faster in the star network.

The results above are depicted in the next figure. It shows PM price drop and PM price growth rate as each structure grows.

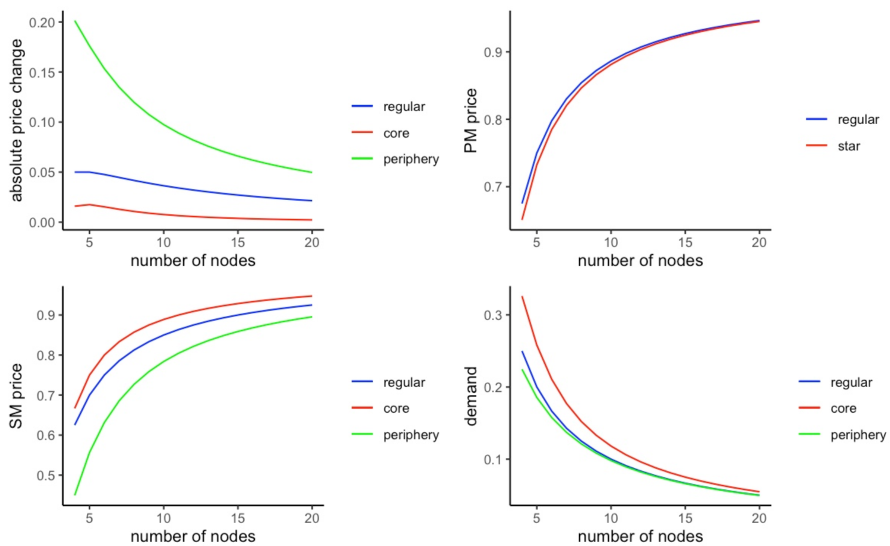


Figure A.9: Growing Networks: Ring versus Star structures

If we divide the star growth rate by the ring growth rate we isolate the network effect. I find that the network effect is positive any finite N, and that it is greater the smaller the size of the trading network.

[15]Notice that the number of linkages in the ring network is N and in the star network $N-1$, and connectivity is higher in the ring network.

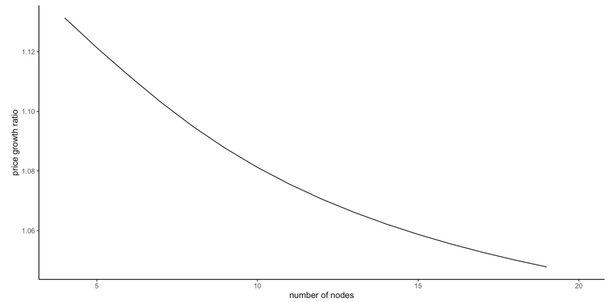


Figure A.10: PM price growth ratio: Star/Ring

This finding is important. The network effect in the star network makes its price diverge from the one in the ring. The importance of this effect though diminishes as the network grows.

More details

Even though PM price increases as both network structures becomes larger, its growth rate differs. To see this, look at how price grows as each structure grows:

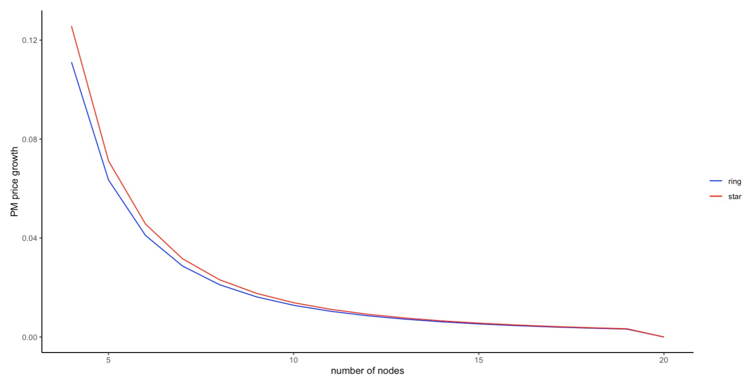


Figure A.11: PM Price growth as the network becomes larger: Ring versus Star structures

Another way to see this is to look at the price ratio of the PM price in star growth by the one in the ring:

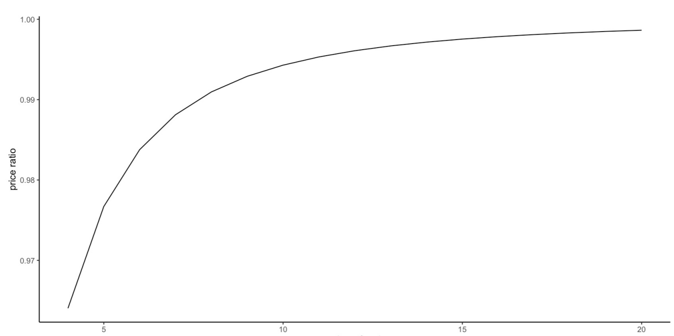


Figure A.12: PM price ratio: Star/Ring

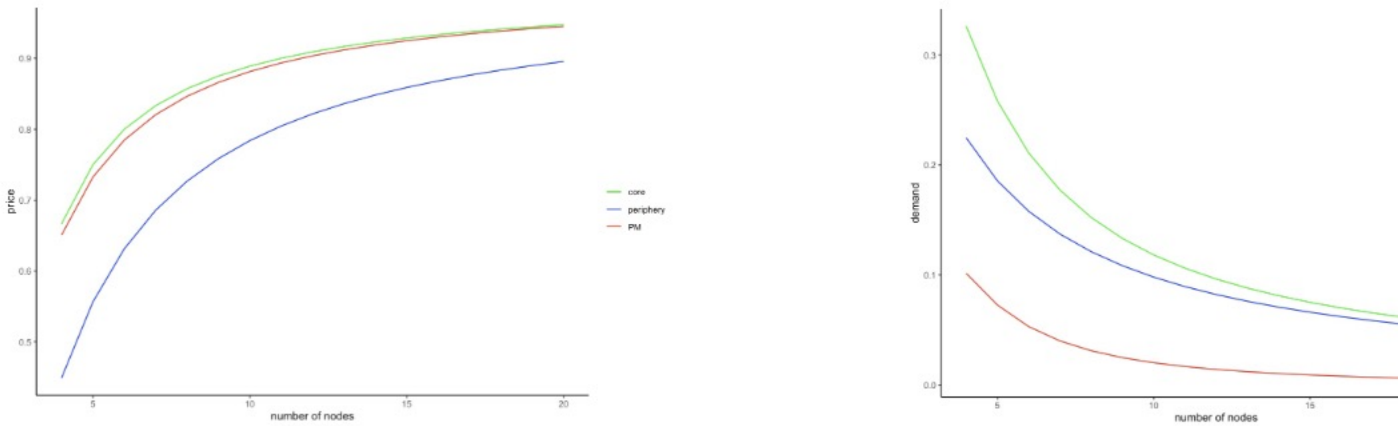


Figure A.13: Growing a Star Network

Mathematically, price growth ratio is

$$\left(\frac{P^{star}_{1,N+1}-P^{star}_{1,N}}{P^{star}_{1,N}}\right)\Big/\left(\frac{P^{ring}_{1,N+1}-P^{ring}_{1,N}}{P^{ring}_{1,N}}\right)=\frac{P^{star}_{1,N+1}-P^{star}_{1,N}}{P^{ring}_{1,N+1}-P^{ring}_{1,N}}\cdot\underbrace{\frac{P^{ring}_{1,N}}{P^{star}_{1,N}}}_{>1}$$

We already know that $P^{ring}_{1,N} > P^{star}_{1,N}$. We now find that, due to the change in the network degree distribution, price increases more in the star network than in the ring network as the structure grows. Notice that, in both structures, the price growth is positive but decreasing: price grows with the network at a diminishing rate.

A.12.1 Growing the Star Network

Even though growing the star leads to a more unequal network in terms of degree, the opposite occurs for demand inequality. The peripheral traders buy relatively more, i.e. become relatively more important in the PM, when this group is large. Difference in demands become smaller. Thus, core's capital gain decreases.

Proposition 30. *Growing a Star Trading Network*

For $N > 3$, core's capital gain and demand inequality decreases as the star network grows.

A.12.2 Comparing with the Complete network

I find that:

- Complete local market price and the core's price are the same. That's because,

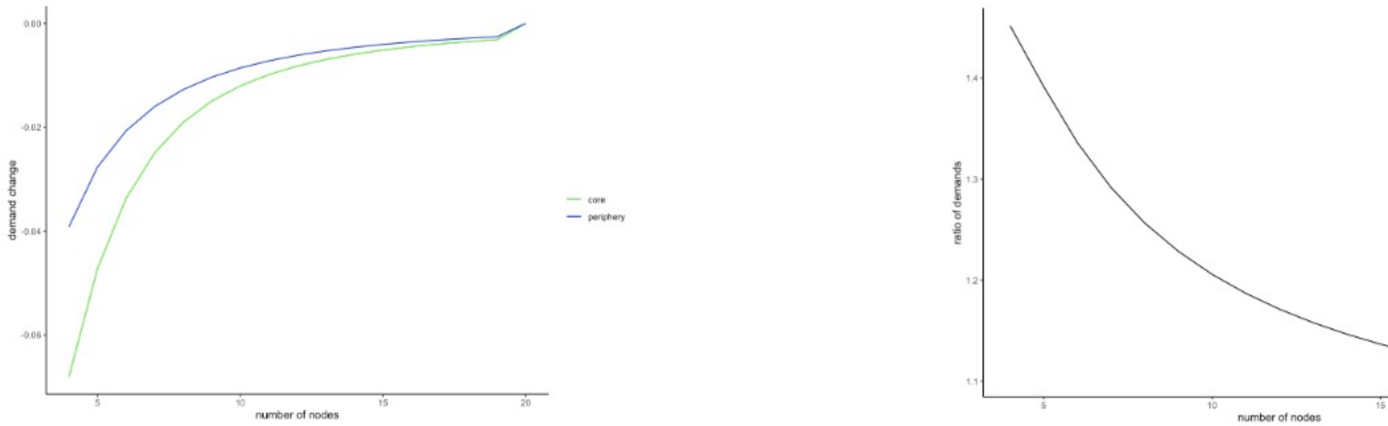


Figure A.14: Demand change and the size of the star network

the increase in core's demand and the reduction in peripheries' demand exactly compensate each other.

That is, for a network with N traders,

$$|q_{1,regular} - q_{1,core}| = (N-1) \cdot |q_{1,regular} - q_{1,periphery}|$$

- Complete and ring's demands are the same. And both less than the core's and higher than the peripheries'.

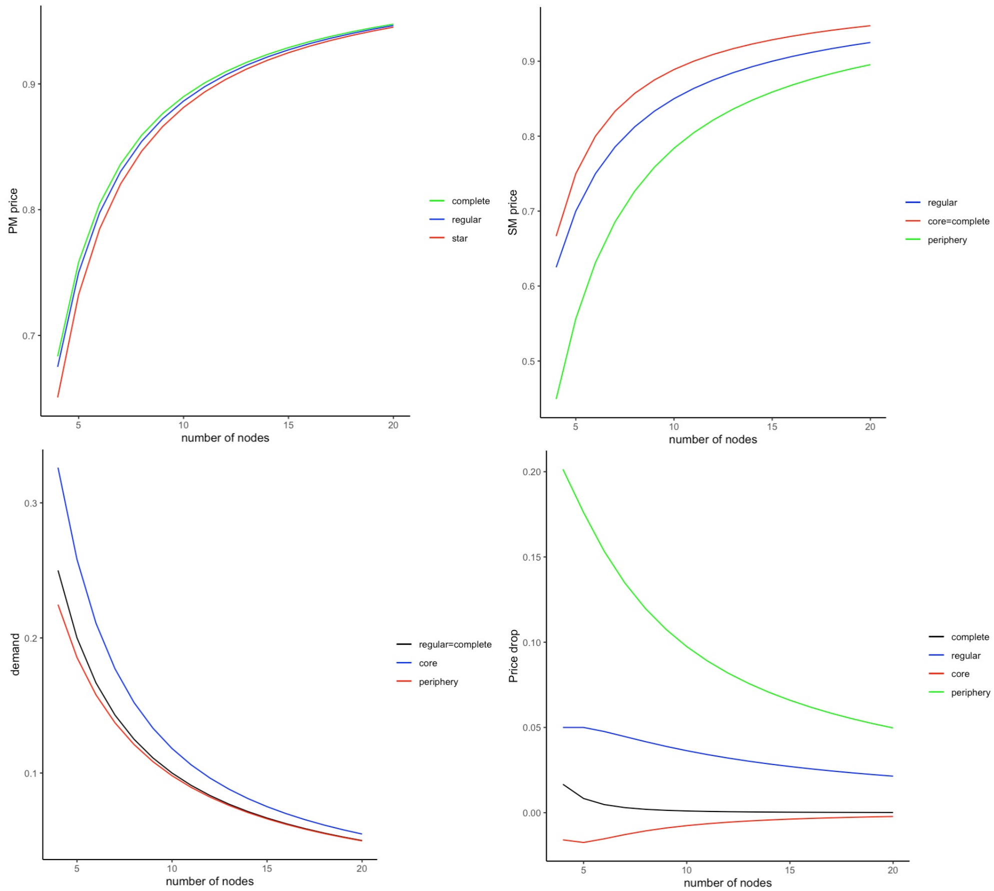


Figure A.15: Growing Networks: Ring versus Star structures

A.13 Welfare

The expected indirect utility EU_i^* of each trader $i \in N$ is given by eq. (1.25)

$$EU_i^*\left(\boldsymbol{c};\boldsymbol{G},\phi,\bar{Q}\right) = w + \left(\frac{Q}{c_A}\right)^2\left[c_i - \frac{1}{2v_i}c_i^2 - \phi c_i \sum_j \tilde{g}_{ij}c_j\right] + \phi\left(\frac{Q}{c_A}\right)\left[\sum_j \bar{g}_{ij}c_j\right] \qquad (1.25)$$

I analyze welfare as the sum of traders' indirect expected utility, $EU^* \equiv \sum_i EU_i^*\left(\boldsymbol{c};\boldsymbol{G},\phi,\bar{Q}\right)$:

$$EU^* = Nw + \left(\frac{Q}{c_A}\right)^2\sum_i c_i - \frac{1}{2}\left(\frac{Q}{c_A}\right)^2\sum_i \frac{c_i^2}{v_i} - \phi\left(\frac{Q}{c_A}\right)^2\sum_i\left(c_i\sum_j \tilde{g}_{ij}c_j\right) + \phi\left(\frac{Q}{c_A}\right)\sum_i\left[\sum_j \bar{g}_{ij}c_j\right]$$

The above can be simplified to

$$
\begin{aligned}
EU^* &= Nw + \frac{Q^2}{2c_A} - \frac{Q^2}{c_A^2}\sum_i \sum_{j\neq i} \bar{g}_{ij} c_j - \frac{3}{2}\frac{Q^2}{c_A^2}\phi \sum_i \left(c_i - \frac{c_i^2}{v_i}\right) \\
&= Nw + \frac{Q^2}{2c_A} - \frac{Q^2}{c_A^2}\sum_i c_i \sum_{j\neq i} \bar{g}_{ji} - \frac{3\phi}{2}\frac{Q^2}{c_A^2}\phi \sum_i c_i + \left(\frac{3\phi}{2}\frac{Q^2}{c_A^2}\right)\sum_i \frac{c_i^2}{v_i} \\
&= Nw + \frac{Q^2}{2c_A} - \frac{Q^2}{c_A^2}\sum_i c_i \left(\frac{3\phi}{2} + \sum_{j\neq i} \bar{g}_{ji}\right) + \left(\frac{3\phi}{2}\frac{Q^2}{c_A^2}\right)\sum_i \frac{c_i^2}{v_i} \\
&= Nw + \frac{Q^2}{2c_A} - \phi\frac{Q^2}{c_A^2}\sum_i \left[c_i \left(\frac{3\phi}{2} + \sum_{j\neq i} \bar{g}_{ji}\right) - \frac{3\phi}{2}\frac{c_i^2}{v_i}\right]
\end{aligned}
\tag{A.33}
$$

Hence, welfare $EU\left(\boldsymbol{c};\boldsymbol{G},\phi,\bar{Q}\right)$ in a trading network $\boldsymbol{G}$, given a shock and supply parameters $\phi, \bar{Q}$, is given by

$$
EU\left(\boldsymbol{c};\boldsymbol{G},\phi,\bar{Q}\right) = Nw + \frac{Q^2}{2c_A} - \phi\frac{Q^2}{c_A^2}\sum_i \left[c_i \left(\frac{3\phi}{2} + \sum_{j\neq i} \bar{g}_{ji}\right) - \frac{3\phi}{2}\frac{c_i^2}{v_i}\right] \tag{1.25}
$$

The welfare analysis is focused on four network structures depicted in fig. 1.11: the complete graph, the line, the star and ring. The following results are used to proof the main results in section 1.7.

Welfare comparison across these network structures is easier because we can invoke Proposition 3 and Proposition 4. Notice that the degree distribution of the complete network FOSD all the other ones; the degree distribution of the ring network FOSD the line and the star ones; and the degree distribution of the line FOSD the star one. Moreover, degree distribution of the star is mean-preserving spread of the line one. It is straightforward then that PM price rank is: $complete > ring > line > star$.

When trading centrality is increasing in degree, a trader's expected utility is increasing in his centrality. In turn, we can rank traders' welfare within a network: more central traders achieve higher expected utility.

Lemma 10. *Individual Welfare and Degree*

If trading centrality c_i of trader $i \in N$ is monotonically increasing in his degree d_i, then i's indirect expected utility EU_i^* is monotonically increasing in his centrality c_i and, thus, his degree d_i.

We can compare the complete network and the ring (or any other regular structure). The complete network has higher (average) trading centrality and thus prices. In turn, PM price in the complete network is higher than in the ring. And so is welfare.

For the structural effect, take the complete and the star. The trader in core of the star is fully connected in both networks. But in the star his connections are poorly connected, which pushes his centrality up and others' centrality down. Consequently, a trader as the core has higher expected utility in the complete network.

Lemma 11. *Welfare and Trading centrality*

Aggregate expected utility is decreasing in the aggregate trading centrality c_A*.*

High aggregate trading centrality implies high PM price. This is the main reason behind Proposition 10.

Another interesting question is to find the welfare maximizing trading network $\boldsymbol{G}^*$ delivering the highest aggregate expected utility. $\boldsymbol{G}^*$ is the solution to the following maximization problem,

$$\boldsymbol{G}^* = \arg\max_{\boldsymbol{G}} \quad EU^*\left(\boldsymbol{G}, \phi, Q\right) \tag{A.34}$$

Solving (A.34) is hard. Due to its dependence on trading centrality, welfare shares the property of non-monotonicity with respect to connectivity and degree inequality. Moreover, since trading centrality is a recursive measure (eq. (1.14)), it is affected by a trader's own degree and also the centrality and degree of other traders. It holds that, for all $i, j \in N$, $\frac{\partial c_i}{\partial c_j} < 0$ but $\frac{\partial c_i}{\partial d_j}$ can be positive or negative depending on all traders' degree, since trading centrality is determined by the distribution of individual degrees $\{d_i\}_{\forall i \in N}$. I leave this inspection for future research.

A.14 Extensions

A.14.1 Heterogeneous Preferences

The baseline model is the homogeneous version of the general setup of each trader $i \in N$ having quasilinear-quadratic utility over inventory with parameters of individual valuation $\alpha_i > 0$ and risk-aversion γ_i

$$U_i(Q_i) = \alpha_i q_i - \frac{\gamma_i}{2} q_i^2 \tag{A.35}$$

so that traders are ex-ante heterogeneous. From building up inventory q_i a trader obtains a marginal value α_i and has a marginal cost of $\frac{\gamma_i}{2} q_i$. Heterogeneity in α_i captures the different and persistent close relationships traders tend to form with their clients in OTC markets (Di Maggio et al. (2017b)). The different cost γ_i may be related to fund outside investments, regulatory capital or collateral requirements, which may vary across traders.

The unique and interior Nash Equilibrium is characterized by the demand schedule

$$q_{i,1} = \beta_i(\boldsymbol{\gamma};\phi,\mathbf{G}) \times \left[\underbrace{\left(m_i(\boldsymbol{\gamma};\phi,\mathbf{G})\,\alpha_i + \phi\sum_j m_{ij}(\boldsymbol{\gamma};\mathbf{G})\,\alpha_j \right)}_{a_i} - P_1 - \phi\sum_j \beta_{ij}(\boldsymbol{\gamma};\mathbf{G})\cdot q_{j,1} \right]$$

where now the endogenous network-induced coefficients also depend on the risk aversion of traders, and $b = 1$. In subsection A.14.1 give the full specification of coefficients above. An important observation is that individual valuations α_i only affects the level of demand.

The key feature of this extension is that it permits heterogeneous interdependencies among values $\{\alpha_i\}_{i\in N}$ that arises endogenously, as described next.[16]

Lemma 12. *local market Equilibrium*

Equilibrium price of seller s is

$$P_s^* = \left(\sum_{i\in N_s}\frac{1}{\gamma_i}\right)^{-1}\cdot\left[\left(\sum_{i\in N_s}\frac{\alpha_i}{\gamma_i}\right) - \left(\sum_{i\in N_s} q_{i,1} + q_{s,1}\right)\right] \tag{A.36}$$

and equilibrium allocation of each buyer $i\in N_s$ is

$$q_{i,s}^* = \left(\frac{\alpha_i}{\gamma_i} - q_{i,1}\right) - \frac{1}{\gamma_i}\cdot\left\{\left(\sum_{i\in N_s}\frac{1}{\gamma_i}\right)^{-1}\cdot\left[\left(\sum_{i\in N_s}\frac{\alpha_i}{\gamma_i}\right) - \left(\sum_{i\in N_s} q_{i,1} + q_{s,1}\right)\right]\right\} \tag{A.37}$$

Denote $\Gamma_s \equiv \sum_{i\in N_s}\frac{1}{\gamma_i}$. Then network coefficients become

[16] This setup follows Rostek and Weretka (2012). The difference is that in that paper the valuations are unknown and traders have private signals on their own and others valuations. Here, however, agents can infer others valuation in the PM through the network

$$v_i(\phi) \equiv \left\{\gamma_i(1-\phi-\phi d_i)+\phi\frac{2}{\Gamma_i}+\phi\sum_j g_{ij}\cdot\frac{1}{\gamma_i\Gamma_j^2}(2\gamma_i\Gamma_j-1)\right\}^{-1}$$

$$m_i(\phi) \equiv \left((1-\phi-\phi d_i)+\frac{1}{\gamma_i}\cdot\phi\sum_j g_{ij}\cdot\left[\frac{1}{\Gamma_j^2}+\frac{2(\gamma_i\Gamma_j-1)}{\gamma_i\Gamma_j^2}\right]\right)$$

$$\tilde{g}_{ij}^1 \equiv \left[g_{ij}\cdot\left(\frac{1}{\Gamma_i}+\frac{1}{\Gamma_j}-\frac{1}{\gamma_i\Gamma_j^2}\right)+\sum_{k\neq i,j} g_{ik}g_{jk}\cdot\left(\frac{1}{\Gamma_k}-\frac{1}{\gamma_i\Gamma_k^2}\right)\right]$$

$$\tilde{g}_{ij}^2 \equiv \left[g_{ij}\cdot\frac{1}{\gamma_j}\frac{1}{\Gamma_i}+\sum_{k\neq i,j} g_{ik}g_{jk}\cdot\frac{1}{\gamma_j}\left(\frac{1}{\Gamma_k}-\frac{1}{\gamma_i\Gamma_k^2}\right)\right]$$

The next lemma characterized PM equilibrium demands.

Lemma 13. *PM Equilibrium Demand*

$$q_{i,1}=v_i\left(\boldsymbol{\gamma};\phi,\boldsymbol{G}\right)\times\left[m_i\left(\boldsymbol{\gamma};\phi,\boldsymbol{G}\right)\alpha_i-P_1-\phi\sum_j\tilde{g}_{ij}^1\left(\boldsymbol{\gamma};\boldsymbol{G}\right)\cdot q_{j,1}+\phi\sum_j\tilde{g}_{ij}^2\left(\boldsymbol{\gamma};\boldsymbol{G}\right)\alpha_j\right]$$

Assuming further that traders have the same level of risk-aversion, $\gamma_i=\gamma\forall i\in N$, the equilibrium turns out to be quite similar to the baseline model. PM demand is given by

$$q_{i,1}=\frac{1}{\gamma}v_i(\phi)\times\left[m_i(\gamma,\phi)\alpha_i-P_1-\gamma\phi\sum_j\tilde{g}_{ij}^1\cdot q_{j,1}+\phi\sum_j\tilde{g}_{ij}^2\alpha_j\right] \qquad \text{(A.38)}$$

where $v_i(\phi),\tilde{g}_{ij}$ are the same as in the baseline model, and $m_i(\phi,\gamma=1-\phi-\phi d_i+\phi\sum g_{ij}\frac{1}{d_j^2}\left[\gamma+(2d_j-2)\right]$ and $\tilde{g}_{ij}^2=g_{ij}\cdot\frac{1}{d_i}+\sum_k g_{ik}g_{jk}\cdot\left(\frac{1}{d_k}-\frac{1}{d_k^2}\right)$.

A.14.2 Expected Fundamental Returns

In reality, traders care about the fundamental return of an asset. They hold an asset not just for the sake of holding it (i.e. to enjoy utility flow) but because they expect that the asset itself is a good financial investment, with high intrinsic value. My framework accommodates *asset-related information* and with that, as I show next, asset price reflects both the traders' beliefs on returns and the trading network. Importantly, the way the former is incorporated into price depends on the later.

To understand this results a brief description of this extension is enough (See appendix for all the details). Suppose the asset has uncertain return f which is normally distributed with mean μ and variance σ^2, and it is realized after all trading activities take place[17] traders have initial wealth w_0 and choose asset inventory q_i to maximize the expected CARA utility of final wealth $E\left[-\exp\left(\gamma W\right)\right]$ given by

$$W = f(q_{i,1} + q_{i,s}) - (P_1 q_{i,1} + P_s q_{i,s}) + w_0 \tag{A.39}$$

The counterpart Nash Equilibrium demand of Equation 1.10 is

$$q_{i,1} = \beta_i(\phi)\left[\underbrace{\mu \cdot \frac{(1+\phi m_i(\mathbf{G}))}{\gamma\sigma^2}}_{a_i} - \underbrace{\frac{1}{\gamma\sigma^2}}_{b} P_1 - \phi\sum_j \beta_{ij} q_{j,1}\right]$$

where the coefficients only depend on the trading network $\mathbf{G}$. As before, see subsection A.14.2 for a detail the full analytical solution.

The equilibrium implies that trader i's PM demand depends on market price P_1, his information *and* the information and demand of *all other traders*, including those he is not directly connected to but who are connected with his connections. This is in stark difference with the canonical linear asset pricing model where individual demands depend on all agents' information set *but not directly* on other demands. That's because in such setting equilibrium price aggregate all useful information and so it is not necessary to know other demands. In my model, however, even an anticipated shock and the fact the it leads to different trading possibilities make agents to conditional on others demands, since this is informative about the market structure.

Buyer i's demand from seller s is $q_{i,s} = \frac{\mu - P_s}{\gamma\sigma^2} - q_{i,1}$.

Lemma 14. *Local market Equilibrium*

[17]The normality assumption is standard in this literature. See, for instance, Kyle (1989), Vives (2011), Rostek and Weretka (2012), (Duffie and Zhu (2016)) and others.

$$P_s^* = \mu - \frac{\gamma\sigma^2}{d_s}\left(q_{s,1} + \sum_{i\in N_s} q_{i,1}\right)$$

$$q_{i,s}^* = \frac{1}{d_s}\left(q_{s,1} + \sum_{k\neq i, k\in N_s} q_{k,1}\right) - \frac{(d_s-1)}{d_s} q_{i,1}$$

Network coefficients become

$$v_i(\phi) \equiv \left[(1-\phi) - \phi\frac{(d_i-2)}{d_i} + 2\phi\cdot\sum_j g_{ij}\frac{1}{d_j}\right]^{-1}$$

$$\tilde{g}_{ij} \equiv g_{ij}\left(\frac{1}{d_i} + \frac{1}{d_j}\right) + \sum_{k\neq i,j} g_{ik}g_{jk}\frac{1}{d_k}$$

$$\tilde{g}_i \equiv -d_i + \sum_j g_{ij}\frac{1}{d_j} + \sum_j\sum_k g_{ik}g_{jk}\frac{1}{d_k}$$

Define vectors $\boldsymbol{v} = [v_i(\phi)]\tilde{\boldsymbol{g}}_{N\times 1} : [v_i(\phi)\cdot\tilde{g}_i]$ and matrices $\mathbf{V}_{N\times N} = diag(\boldsymbol{v}), \tilde{\boldsymbol{G}}_{N\times N} : [\tilde{g}_{ij}]$. Then the system of PM demands can be written in matrix form.

Lemma 15. *PM Equilibrium Demands*

$$\boldsymbol{q}_1^* = \left(\boldsymbol{I} + \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\cdot\left(\frac{\mu}{\gamma\sigma^2}\left(\boldsymbol{v} + \phi\tilde{\boldsymbol{g}}\right) - \frac{1}{\gamma\sigma^2}P_1\boldsymbol{v}\right)$$

A.14.3 Price Impact

My framework is essentially a static demand game because a trader's PM demand depends on expected local market trades, not on realized trade. This is the crucial feature of the model. And it is the reason local market trading induces a set of games in the PM. In equilibrium, prices and demands are not independent across markets. If they were, the markets would operate as independent venues and this is clearly not the case here.

Price impact in the PM arises endogenously in the model precisely because of intertemporal demand dependence. That is, it comes from *all the possible* local market exchanges in the trading network. Moreover, PM asset marginal utility is dictated by participation in the SM, and vice-versa.

Following the imperfect competition literature, the framework can be used to study the economy with strategic traders, as Rostek and Yoon (2020b).

Suppose $d_i \geq 3 \forall i \in N$. In every local market, a buyer $i \in N_s$ trades taking into account his price impact $\lambda_s \equiv \frac{\partial P_s}{\partial q_i}$. In equilibrium, I show that $\lambda_s = \frac{1}{d_s - 2}$ and so price impact is equal across buyers in a given local market. Buyer i's demand is $q_{i,s} = \frac{1}{\lambda_s + 1}(1 - q_{i,1} - P_s) = \frac{d_s - 2}{d_s - 1}(1 - q_{i,1} - P_s)$.

Lemma 16. *Local Market Equilibrium*

The local market or seller $s \in N$ has equilibrium price,

$$\begin{aligned} P_s^* &= 1 - \frac{1}{d_s}\sum_i a_i - \frac{1+\lambda_s}{d_s}a_s \\ &= 1 - \frac{1}{d_s}\sum_i a_i - \frac{d_s - 1}{d_s(d_s - 2)}a_s \end{aligned} \tag{A.40}$$

and asset allocation

$$q_{i,s}^* = \frac{1}{d_s(1+\lambda_s)}q_{N_s - i,1} + \frac{1}{d_s}q_{s,1} - \left(\frac{d_s - 1}{d_s(\lambda_s + 1)}\right)q_{i,1}$$

Lemma 17. *Primary Market Equilibrium*

$$q_{i,1} = \psi_i \left[(1 - P_1) - \phi \sum_j \psi_{ij} q_{j,1}\right] \tag{A.41}$$

where

$$\begin{aligned} \psi_i &= \left[1 + \phi(d_i + 1) + 2\phi \frac{(d_i - 1)}{d_i(d_i - 2)} + \phi \sum_j g_{ij}\frac{2}{d_j}\right]^{-1} \\ \psi_{ij} &= g_{ij} \cdot \left[\frac{1}{d_i} + \frac{1}{d_j}\right] + \sum_{z \neq \{i,j,k\}} g_{iz} g_{jz} \cdot \frac{(d_z - 2)}{d_z(d_z - 1)} \end{aligned}$$

A.15 Local Markets with Outside Traders

I assume that all traders participate in the primary market. However, this is not a restrictive assumption. The results are robust to incorporating outside traders who may only participate in a local market. In this section, I introduce a representative outside trader with demand q_t for each local market. This trader has the same quadratic-quasilinear preference but only one asset demand. The outside trader is interpreted as investors who only learn about the asset after the first trading round, or that due to financial constraints do not participate in the primary market.

The outsider demand schedule is $q_t = 1 - P_s$. By market clearing, seller s' price is $P_s^* = 1 - \frac{1}{d_s+1}\left(q_{N_s,1} + q_s\right)$.

From the local market equilibrium, one can already see that the only increasing price denominator by 1. In other words, the seller's effective degree is $d_s + 1$. Since this holds for every local market, all results remain unchanged.

The more realistic approach would be to include outside traders just in some local markets, or to allow for preference heterogeneity. The model accommodates all these extensions. Even though it remains tractable, it becomes harder to disentangle the network effects from preference and outside traders' effects. As I mentioned in the Introduction, the goal of this paper is to provide a benchmark framework in which the network effects are the *only driver* of equilibrium outcomes. The extensions presented in section 1.9 are pertinent are interesting. I leave them for future work.

A.16 Empirical Exercise

A.16.1 Bond-level Dealer Network

In Section 1.8 I explore the sample of trades of all bonds. But the same analysis can be done at the bond-level. As I show next, each bond in the sample has a different trading network. That's because the main take-away of my paper is that different assets have different prices because each has a different underlying trading network structure.

The next figure depicts the dealer network for each bond. Notice that, according to my definition, 2 bonds do not have a dealer network because only one dealer trade with the customer.

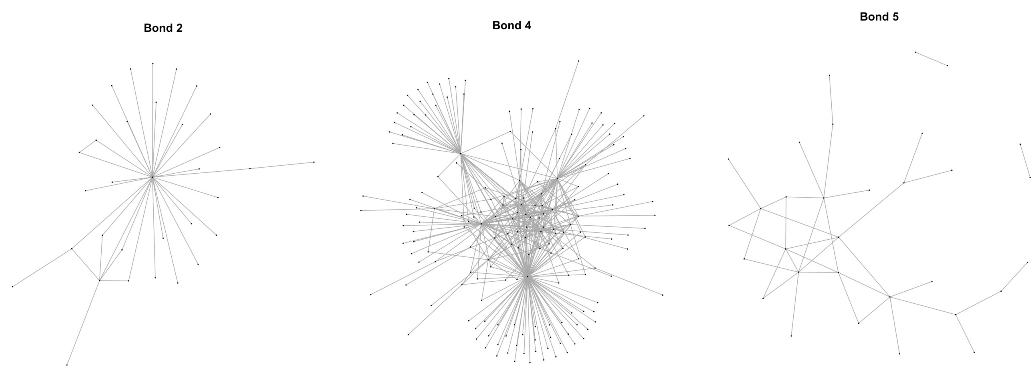


Figure A.16: Dealer networks for different bonds

Each bond in the data exhibits a different inventory price distribution, as the next figure shows.

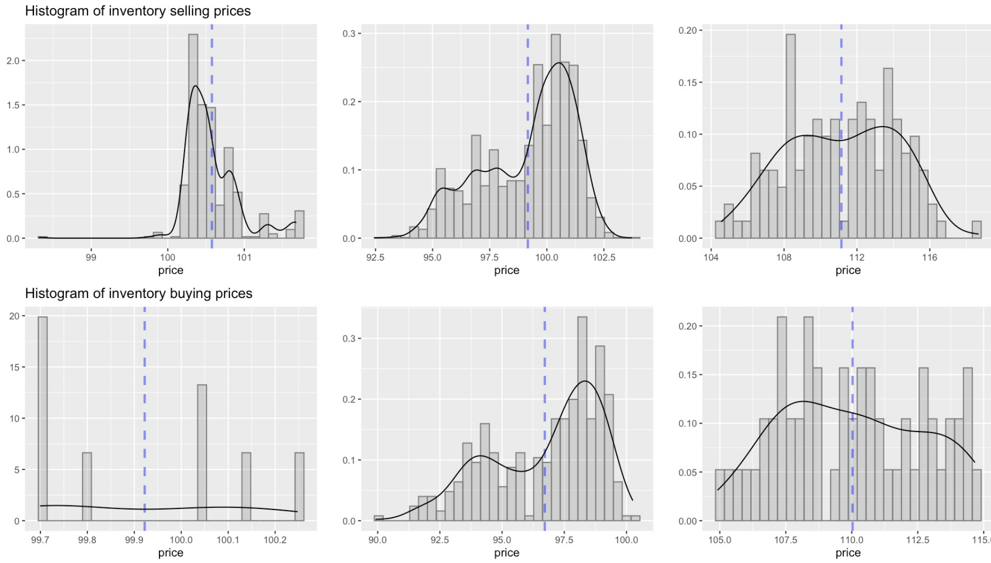


Figure A.17: Distribution of prices of dealer-customer trades

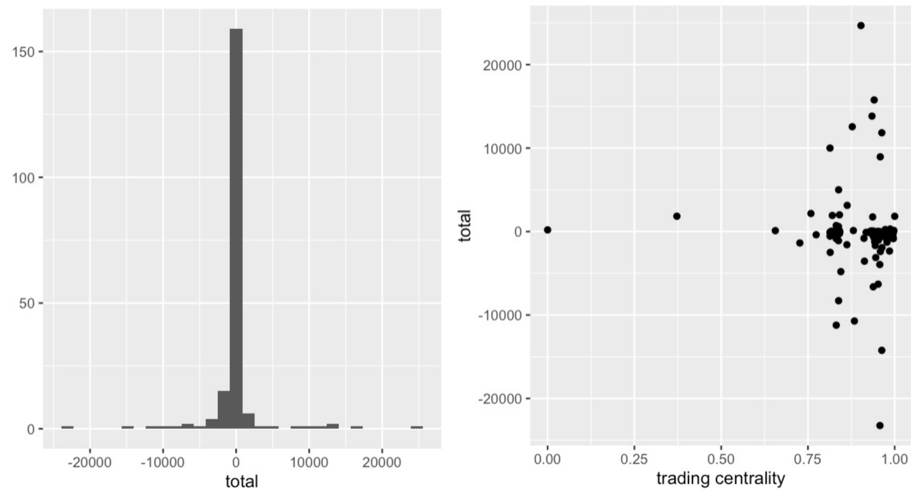
A.16.2 Dealers' Inventory

By trading with the customer, dealers accumulate inventory (bond holdings) over time. So, in my analysis, inventory is the empirical counterpart of PM demand. I compute inventory from net d2c trades. A positive (negative) inventory means that a dealer is a net buyer from (seller to) the customer. Through the lens of my model, a positive inventory would lead the dealer to trade in the interdealer network.

I calculate a dealer's (net) inventory at the end-of sample, referred as net total inventory (NTI), and its nominal and absolute values (qi, aqi).[18] NTI gives me a dataset with one observation per dealer with inventory as bond holdings at the end of the 42 trading days. As the figure below shows, inventory is concentrated around zero, indicative that most dealers trade in a way to off-set portfolio imbalances.

The relationship between dealers' inventory and trading centrality is depicted next. Since the attribution of trading centrality is highly skewed, at a first glance it does not have a clear relation with net bond holdings. Even so the graph reveals that the least central dealers have roughly zero net inventory - while the reverse is not true.

[18] I also calculate inventory at the daily level: net daily inventory (NDI) nominal and absolute values $(qid, aqid)$. This gives me an unbalanced panel data with multiple observations per dealer over 42 trading days. See appendix.



A.16.3 Regressions: Interdealer trades and Trading Centrality

Regressing trade volume or trade price on trading centrality delivers significant results. For trade volume, the sign of trading centrality coefficient varies depending on the set of controls. Trading centrality alone has a positive effect on volume, and so it does when controlling for nominal net total inventory (from customer trades) and trade price. However, adding degree and as control, turns the centrality coefficient negative.

Table A.2: D2D trade volume

	Dependent variable:					
	vol					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	38.6^{***}		-296.6^{***}	87.2^{***}	84.9^{***}	-150.2^{**}
	(11.7)		(59.6)	(12.4)	(12.4)	(61.0)
deg		-0.6^{***}	-3.9^{***}			-2.7^{***}
		(0.1)	(0.7)			(0.7)
qi				0.01^{***}	0.01^{***}	0.01^{***}
				(0.001)	(0.001)	(0.001)
pr					4.0^{**}	4.1^{**}
					(1.6)	(1.6)
Constant	38.3^{***}	84.1^{***}	395.1^{***}	23.7^{***}	-370.0^{**}	-133.2
	(8.7)	(6.4)	(62.8)	(8.7)	(162.6)	(173.2)
Observations	5,308	5,308	5,308	5,308	5,308	5,308
Adjusted R^2	0.002	0.003	0.01	0.02	0.02	0.03
Note:						*p<0.1; **p<0.05; ***p<0.01

For trade price though trading centrality is only significant when controlling for degree. In this case, trading centrality has a positive effect on price: more central dealers face higher interdealer prices.

Table A.3: D2D trade price

	Dependent variable:					
	pr					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	0.5^{***}		1.1^{**}	0.1	0.6^{***}	0.5
	(0.1)		(0.5)	(0.1)	(0.1)	(0.5)
deg		-0.01^{***}	0.01			0.01
		(0.001)	(0.01)			(0.01)
pi				0.2^{***}		0.2^{***}
				(0.02)		(0.02)
qi					0.000	
					(0.000)	
Constant	98.8^{***}	99.4^{***}	98.3^{***}	84.2^{***}	98.8^{***}	83.8^{***}
	(0.1)	(0.1)	(0.5)	(1.7)	(0.1)	(1.7)
Observations	5,308	5,308	5,308	5,308	5,308	5,308
Adjusted R^2	0.01	0.01	0.01	0.02	0.01	0.02
Note:						*p<0.1; **p<0.05; ***p<0.01

Conditioning on side

Table A.4: D2D trade volume conditional on side

	Dependent variable:					
	buy volume					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	-136.8^{***}		-194.2^{***}	-118.4^{***}	-118.9^{***}	-153.5^{**}
	(29.7)		(67.1)	(31.4)	(31.4)	(72.1)
deg		1.2^{***}	-0.7			-0.4
		(0.3)	(0.7)			(0.8)
qi				0.002^{*}	0.002^{*}	0.002^{*}
				(0.001)	(0.001)	(0.001)
pr					4.8^{**}	4.9^{**}
					(2.3)	(2.3)
Constant	182.2^{***}	43.9^{***}	243.5^{***}	176.2^{***}	-303.1	-268.4
	(26.5)	(8.0)	(69.5)	(26.7)	(232.8)	(241.8)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	0.01	0.005	0.01	0.01	0.01	0.01
	Dependent variable:					
	sell volume					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	120.5^{***}		$-1,939.0^{***}$	107.6^{***}	104.8^{***}	$-1,542.0^{***}$
	(16.5)		(154.8)	(16.0)	(16.2)	(157.4)
deg		-1.6^{***}	-24.0^{***}			-19.3^{***}
		(0.2)	(1.8)			(1.8)
qi				0.02^{***}	0.02^{***}	0.01^{***}
				(0.001)	(0.001)	(0.001)
pr					2.7	2.5
					(2.3)	(2.2)
Constant	11.0	154.2^{***}	$2,234.0^{***}$	1.2	-265.1	$1,537.0^{***}$
	(9.3)	(12.1)	(166.5)	(9.1)	(225.2)	(279.4)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	0.02	0.03	0.1	0.1	0.1	0.1
Note:						*p<0.1; **p<0.05; ***p<0.01

Table A.5: D2D trade prices conditional on side

	Dependent variable:					
	sell price					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	1.0***		0.9	0.7***	1.0***	2.1
	(0.1)		(1.3)	(0.1)	(0.1)	(1.3)
deg		−0.01***	−0.002			0.02
		(0.002)	(0.02)			(0.02)
pi				0.2***		0.2***
				(0.02)		(0.02)
qi					−0.000	
					(0.000)	
Constant	98.7***	99.9***	98.9***	84.1***	98.8***	82.1***
	(0.1)	(0.1)	(1.4)	(2.2)	(0.1)	(2.8)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	0.02	0.02	0.02	0.04	0.02	0.04
	Dependent variable:					
	buy price					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	0.2		0.7	−0.7**	0.1	−0.7
	(0.2)		(0.6)	(0.3)	(0.3)	(0.6)
deg		−0.001	0.01			−0.000
		(0.003)	(0.01)			(0.01)
pi				0.2***		0.2***
				(0.03)		(0.03)
qi					−0.000	
					(0.000)	
Constant	99.0***	99.2***	98.5***	80.1***	99.0***	80.1***
	(0.2)	(0.1)	(0.6)	(2.7)	(0.2)	(2.7)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	-0.000	-0.000	-0.000	0.02	0.001	0.02
Note:						*p<0.1; **p<0.05; ***p<0.01

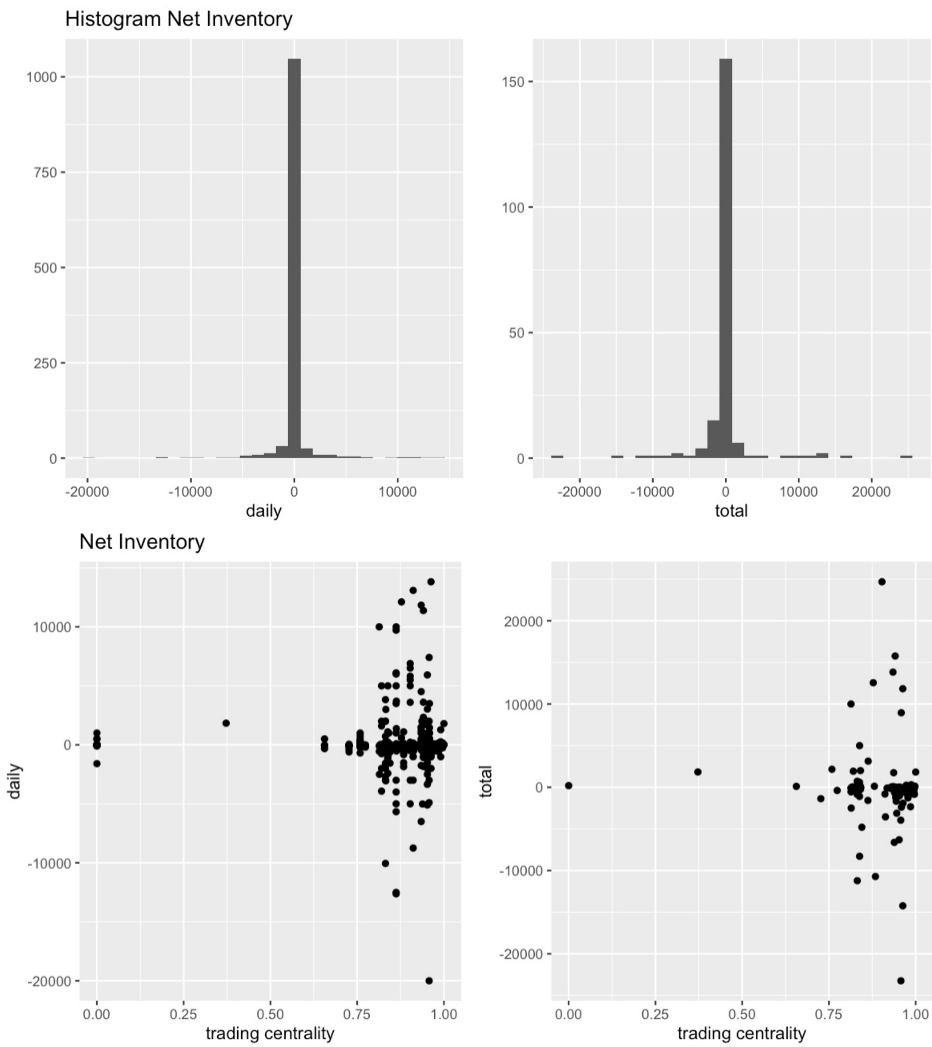
A.16.4 The Sample

Primary Market: D2C data

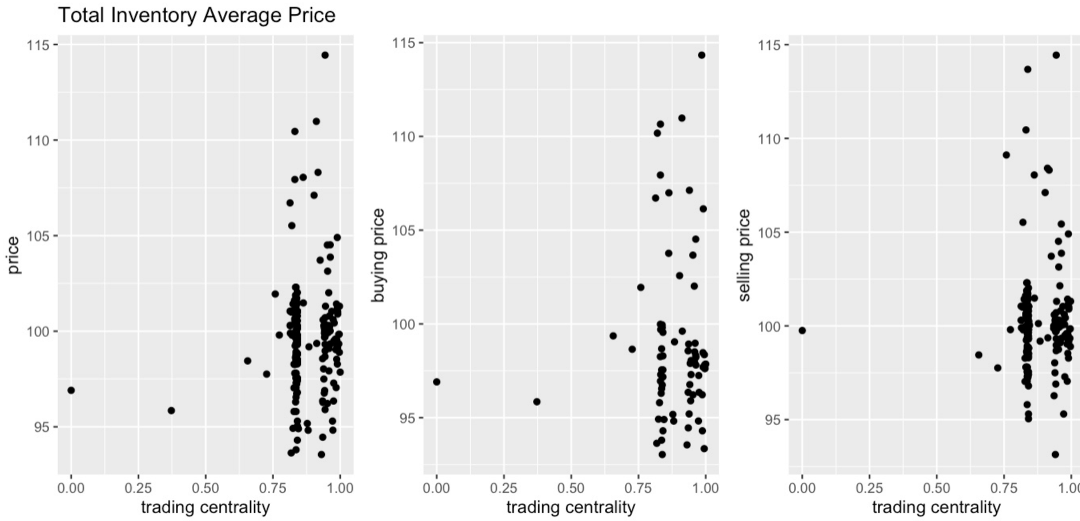
I calculate a dealer's (net) inventory at two different periods, daily and at the end-of sample, and their nominal and absolute values. That is, I compute:

- net daily inventory (NDI) nominal and absolute values ($qid, aqid$), which gives me an unbalanced panel data with multiple observations per dealer over 42 trading days;
- net total inventory (NTI) nominal and absolute values (qi, aqi), which gives me a dataset with one observation per dealer with inventory as bond holdings at the end of the 42 trading days.

As the figure below shows, both inventory measures are concentrated around zero, indicative that most dealers trade in a way to off-set portfolio imbalances.



The concentration of trading centrality values is also reflected in the graph relating it with the price of inventory. The lest central dealers have lower prices, although the reverse is not true.



Interdealer Market: D2D data

The following table summarizing trading frequency and market outcomes (prices and quantities) in the observed D2D trades. There are $2,654$ trades.

Table A.6: $N = 5,308$

Statistic	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
trd_dpair	5.619	6.180	1	1	8	30
trd_mpair	49.950	58.080	1	4	106	182
trd_pair	84.350	104.600	1	6	113	320
trd_ddealer	19.210	20.650	1	4	26	92
trd_mdealer	259.500	234.000	1	57	501	673
trd_dealer	466.400	387.600	1	135	849	1,006
vol	63.520	310.600	1	10	30	5,985
pr	99.190	2.571	91.230	98.300	100.300	116.600

Dealer-level information on transactions:

Table A.7: Buy and sell trades per dealer

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
qs	201	0.000	2,957.000	−24,843	10	151	14,884
qs.mean.all	201	188.300	611.400	3.000	16.250	75.000	5,000.000
ps.mean.all	201	98.990	2.232	93.550	98.020	100.000	112.400

Table A.8: Sell trades per dealer

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
qs.mean	85	−329.900	866.700	−5,000	−177.5	−22.5	−3
ps.mean	85	99.490	3.962	93.360	97.640	100.000	114.400
trd_side	85	31.220	122.800	1	1	12	870

Table A.9: Buy trades per dealer

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
qs.mean	176	114.300	435.000	4.000	15.000	63.540	5,000.000
ps.mean	176	98.930	1.875	94.300	98.020	100.100	107.000
trd_side	176	15.080	56.060	1	1	7	638

Appendix B

Chapter 2

B.1 RE as a special case

If the RE assumption holds, then all agents know exactly what price outcome will be for any given history of realizations of the exogenous processes (D^t, Y^t). In other words, $E_t^{ik} = E_t \; \forall t, i, k$ where $E_t(.)$ is the expectation of the 'true' stochastic price process. Agents then can interate forward on equation (2.9):

$$\begin{aligned} P_t &= \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} P_{t+1}\right] + \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} D_{t+1}\right] \\ &= \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left\{\delta E_{t+1}\left[\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\gamma} P_{t+2}\right] + \delta E_{t+1}\left[\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\gamma} D_{t+2}\right]\right\}\right] + \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} D_{t+1}\right] ... \\ &= E_t\left[\sum_{s=1}^{\infty} \delta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} D_{t+s}\right] + \lim_{s\to\infty} E_t\left[\left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} P_{t+s}\right] \end{aligned}$$

Assuming a no-rational-bubble requirement, it is common knowledge that

$$\lim_{s\to\infty} E_t\left[\left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} P_{t+s}\right] = 0$$

Hence, the equilibrium asset price is equal to the expected discounted sum of dividends

$$
\begin{aligned}
P_t &= E_t\Bigg[\sum_{s=1}^{\infty}\delta^s\Big(\frac{C_{t+s}}{C_t}\Big)^{-\gamma}D_{t+s}\Bigg]\\
&= E_t\Bigg[\sum_{s=1}^{\infty}\delta^s\Big(a^s\prod_{j=1}^{s}\varepsilon^c_{t+j}\Big)^{-\gamma}D_t(a^s\prod_{j=1}^{s}\varepsilon^d_{t+j})\Bigg]\\
&= D_t\Bigg[\sum_{s=1}^{\infty}\delta^s(a^{1-\gamma})^s\Bigg]\prod_{j=1}^{s}E_t\Big(\varepsilon^c_{t+j}\Big)^{-\gamma}E_t\Big(\varepsilon^d_{t+j}\Big)\\
&= D_t\Bigg[\sum_{s=1}^{\infty}\delta^s(a^{1-\gamma})^s\rho^s\Bigg]\\
&= \frac{\delta a^{1-\gamma}\rho}{1-\delta a^{1-\gamma}\rho}D_t
\end{aligned}
$$

where I use definition (2.12), $\rho = E[(\varepsilon^c_{t+1})^{-\gamma}\varepsilon^d_{t+1}]\ \forall t$, and exogenous processes (2.1) and (2.2).

Thus, defining $\beta^{RE} \equiv a^{1-\gamma}\rho$ I get the rational expectations equilibrium asset price in equation (2.15).

B.2 Proposition 11 - Inexperienced agents are more reactive

First, to see why forecast errors are different than zero at any time period, look at risk-adjusted price growth and its subjective expectations as a function of β^k_t:

$$
\frac{\partial\Big(\frac{C_t}{C_{t-1}}\Big)^{-\gamma}\frac{P_t}{P_{t-1}}}{\partial\beta^k_t} = \beta^{RE}\delta\lambda^{k,*}_t\frac{(1-\delta\tilde{\beta}^o_{t-1})}{(1-\delta\tilde{\beta}^o_t)^2} > 1 \tag{B.1}
$$

$$
\frac{\partial E^k_t\Big\{\Big(\frac{C_t}{C_{t-1}}\Big)^{-\gamma}\frac{P_t}{P_{t-1}}\Big\}}{\partial\beta^k_t} = 1 \tag{B.2}
$$

Clearly, a change β^k_t implies a one-to-one change in agents' beliefs (by definition) but results in a greater change for realized price growth. Hence, the later always exceeds its initial expectations.

Now, Proposition 11 easily follows from Assumption 5. To see this consider an arbitrary period t in which price is increasing. So it holds that $\Big(\frac{C_t}{C_{t-1}}\Big)^{-\gamma}\frac{P_t}{P_{t-1}} > 1$ and $e^k_{t-1} > 0$. By types' belief updating equation (2.25), it must be that agents revise beliefs updward at $t+1$: $\Delta\beta^k_{t+1} > 0 \; \forall k = h, l$.

Since $g^h e^h_t > g^l e^l_t$, it must be that $\Delta\beta^h_{t+1} > \Delta\beta^l_{t+1} > 0$. Consequently, $\beta^h_{t+1} - \beta^l_{t+1} > \beta^h_t - \beta^l_t > 0$ and thus it must be that $\beta^h_t > \beta^l_t$ and $\beta^h_{t+1} > \beta^l_{t+1}$.

On the other hand, consider a period when price is decreasing such that $\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{P_t}{P_{t-1}} < 1$ and $e^k_{t-1} < 0$. Again, by equation (2.25) it must be that agents decrease their beliefs next period: $\Delta\beta^k_{t+1} < 0 \; \forall k = h, l$.

Since $g^h e^h_t < g^l e^l_t < 0$, it must be that $\Delta\beta^h_{t+1} < \Delta\beta^l_{t+1} < 0$. That is, $\beta^h_{t+1} - \beta^l_{t+1} < \beta^h_t - \beta^l_t < 0$ and thus it must be that $\beta^h_t < \beta^l_t$ and $\beta^h_{t+1} < \beta^l_{t+1}$.

As pointed out, these results follow from the Kalman filter equation of each type. The gains are sufficiently different to guarantee that $|g^h e^h_t| > |g^l e^l_t| \; \forall t$. In fact, the difference in forecast errors $e^h_t - e^l_t \; \forall t$ is virtually small because, for both types, beliefs are just small deviations of the RE value (which is the same for both). Consequently, the range of difference in beliefs is also sufficiently small so that the comparison of each with realized price itself is virtually the same. The optimal Kalman gains then are what make H-type beliefs to be more reactive to the forecast error and so to guarantee $|\Delta\beta^h_t| > |\Delta\beta^l_t| \; \forall$ t.

This discussion is also clear by looking at simulation results in Section 2.8 (Figure 2.2). Importantly, despite the fact that beliefs oscilate around the RE value, these small deviations are sufficient to render the fluctuation in the PD ratio I obtain from the model. Adam et al. (2016) discuss this result in details and they argue that it is a strength of the model, since just slight departures from rational expectations are enough to reproduce stylized asset pricing facts.

B.3 Recovery Periods

Let t be the period after a sharp continuous decrease in asset price following the boom - that is, at the bottom of the bust - it holds that $e^h_{t-1} < e^l_{t-1} < 0$ and $\beta^h_t < \beta^l_t$. Price (and so the PD ratio) returns to its mean value. In absecence of further shocks, price growth is approximately constant which implies that agents find themselves too pessimistic and start revising beliefs upwards according to the updating equations (2.25):

$$\begin{aligned} \beta^h_{t+1} &= \beta^h_t + g^h e^h_t \\ \beta^l_{t+1} &= \beta^l_t + g^l e^l_t \end{aligned} \quad \text{such that} \quad e^k_t > 0 \quad k = h, l$$

Then $\Delta\beta^k_{t+1} > 0 \, \forall k = h, l$. By mean reversion of the PD ratio, $\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \approx 1$ during the recovery periods. Then, I can rewrite the above as

$$\beta^h_{t+1} \approx \beta^h_t(1 - g^h) + g^h$$
$$\beta^l_{t+1} \approx \beta^l_t(1 - g^l) + g^l$$

Since, $\beta^l_t > \beta^h_t$ and $g^h > g^l$ it holds that $(1 - g^l) > (1 - g^h)$. Because I consider gains to be small (see equation (2.26)), it must be that $\beta^l_{t+1} > \beta^h_{t+1}$. And so, for all period during recovery in which the PD ratio is approximately constant, $\beta^l > \beta^h$. After beliefs have consistently been increasing, price starts to increase sharply and so a boom episode is in motion.

B.4 Network Graphs examples

To the extent of comparison between the examples presented above, let private beliefs $\beta^h_0 = \beta^l_0 = \beta_0$ be the same across structures and assume dividend and consumption shocks are also equal across networks.

Consider first a price growth greater than one as initial condition so that $P_0 > P_{-1}$, $e_0 > 0$ and this constitues a boom episode. At $t = 1$, $\beta^k_1 \;\; k = l, h$ is the same for all networks and $\beta^h_1 > \beta^l_1$. Price is increasing and so equilibrium belief is pinned down by the highest λ^i. Realized PD ratio is $\frac{P_1}{D_1} = \frac{\delta\beta^D}{1-\delta\tilde{\beta}^o_1}$ where $\tilde{\beta}^o_1 = \lambda_{max}\beta^h_1 + (1 - \lambda_{max})\beta^l_1$. The maximum λ^i is greater under the tree/star structure, followed by the circle and the complete structures, respectively: $\lambda^*_{tree/star} > \lambda^*_{circle} > \lambda^*_{complete}$. Thus, equilibrium belief and realized price at $t = 1$ also follow this magnitude relation. That is

$$\tilde{\beta}^{o,tree/star}_1 > \tilde{\beta}^{o,circle}_1 > \tilde{\beta}^{o,complete}_1$$
$$\frac{P^{tree/star}_1}{D_1} > \frac{P^{circle}_1}{D_1} > \frac{P^{complete}_1}{D_1}$$

Since λ^*_t has a positive effect on price growth during a boom, the higher is the former the greater will be the forecast error $e^k_1 = \left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0} - \beta^k_1 > 0$ for $k = h, l$. Thus $e^k_{1,tree/star} > e^k_{1,circle} > e^k_{1,complete} > 0$. In the next period $t = 2$, it holds then that $\beta^{k,tree/star}_2 > \beta^{k,circle}_2 > \beta^{k,complete}_2 \;\; \forall k$ with $\beta^h_2 > \beta^l_2$. Equilibrium belief and price will follow the above relation across networks so that PD ratio is the highest in the tree/star graph and lowest in the complete structure.

These results holds for all periods whenever expected price growth is positive, and under this scenario I can conclude that:

- the lower the network mean degree (tree, star) the higher is realized price and equilibrium belief at any period;
- less connected networks (tree, star) have greater price volatility when compared to more connected ones (circle and complete graphs)
- higher network mean degree implies lower dispersion of agents' public beliefs $\tilde{\beta}_t^i$

The analysis is analogous for a bust episode but results are qualitatively different. Let initial conditions be of a negative realized and expected price growth $P_0 < P_{-1}$ and $e_0 < 0$. In the first period intital beliefs are the same across networks such that $\beta_1^h < \beta_1^l$. Price is determined by the lowest λ^i and so $\frac{P_1}{D_1} = \frac{\delta\beta^D}{1-\delta\tilde{\beta}_1^o}$ where $\tilde{\beta}_1^o = \lambda_{min}\beta_1^h + (1-\lambda_{min})\beta_1^l$. Equilibrium λ^* is lower in the tree/circle/star networks and higher in the complete graph and thus

$$\tilde{\beta}_1^{o,tree/star/circle} > \tilde{\beta}_1^{o,complete}$$
$$\frac{P_1^{tree/star/circle}}{D_1} > \frac{P_1^{complete}}{D_1}$$

Because λ_t^* has a negative effect on price growth when price is decreasing, the higher is λ_t^* the lower (or greater in absolute value) will be forescast errors $e_1^k = \left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0} - \beta_1^k < 0$ for $k = h, l$ since $\left[\left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0}\right]_{complete} < \left[\left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0}\right]_{tree/star/circle} < \beta_1^k < 0$. Thus $e_{1,complete}^k < e_{1,tree/star/circle}^k < 0$. In the next period $t = 2$ then, it holds that $\beta_2^{k,tree/star/circle} > \beta_2^{k,complete} \ \forall k$ with $\beta_2^h < \beta_2^l$. Equilibrium belief and realized price will follow the above relation across networks so that PD ratio decrease is greater under the complete network compared to the tree/star/circle graphs.

These results holds for all periods whenever price is expected to decrease and I can infer that:

- more connected networks have higher equilibrium social weight λ_t^* and thus exhibits lower realized price

- price decrease is greater the higher the mean degree and so it is PD volatility

Notice that even though the tree and star graphs have different structure, both share the same expected degree and consequently they face the same price dynamics. In this sense, individual nodes' degree is not as relevant as the overall degree of network connectivity.

B.5 Details on the Simulation

Under the calibrated parameters (Table 2.1), I simulated the model 1000 times throughout 500 quarters.

Our simulation method is close to Adam et al. (2016). The main difference is in our specification of agents' subjective beliefs. As explained by them, to guarantee beliefs' bound holds at all periods and equilibrium price always exist a differentiable projection facility in belief updating equation is introduced. Specifically, equation (2.25) becomes

$$\beta_t^k = \omega\left(\beta_{t-1}^k + g^k\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}} - \beta_{t-1}^k\right]\right) \quad \forall k = h,l \;\;\&\;\; t > 0 \tag{B.3}$$

$$\text{where} \quad \omega(x) = \begin{cases} x & x \leq \beta^L \\ \left(\frac{x-\beta^L}{x+\beta^U-2\beta^L}\right)\beta^U + \left[1 - \left(\frac{x-\beta^L}{x+\beta^U-2\beta^L}\right)\right]\beta^L & x > \beta^L \end{cases} \tag{B.4}$$

where $\beta^L = \frac{1}{\delta} - 2(\frac{1}{\delta} - \beta^U)$ and β^U is such that the PD ratio doesn't exceed 500. $\omega(x)$ is a dampening function that applies only to few observations - when beliefs are close tho their upper bounds. The upper bound on the PD ratio is chosen such that it is higher but close to its maximum value found in the U.S. data (which is approximately 375).

Booms are calculated as the number of time periods the PD ratio stays above its RE value at each cycle. Recovery periods are calculated as the number of time periods the PD ratio stays below its RE value at each cycle.

Since one of our study motivation is Adam et al. (2016), I specify their main set of results in Table B.1. I are able to repoduce similar empirical fact as them, but our model exhibit greater PD ratio volatility and mean depending on the social strcuture. Moreover, AMN do not discuss the lenght of booms and busts, but by looking at their results I can infer that the present model is able to account for the duration of these episodes. I consider this a relevant result since it implies that the presence of different investors in the market and how they behave are also determinant of the length and magnitude of booms and busts.

Fact	AMN	
	$\gamma = 5$	$\gamma = 80$
PD ratio Mean	122.50	115.75
PD ratio Std. Dev.	67.75	71.15
Stock return	1.27	2.11
Stock return Std. Dev.	10.85	16.31
Bond return	0.39	0.11
Risk Premium	0.88	2.0
Dividend Growth Mean	0.00	0.16
Dividend Growth Std. Dev.	2.37	4.41

Table B.1: Adam et al. (2016) Estimation Outcomes
Source: Adam et al. (2016)

B.6 Unkown Network Structure

Define the expected number of links between types k and k' as

$$Q_{kk'} = \mu_k \mu_{k'} p_{kk'} = \begin{cases} \mu^2 p_{out} & k \neq k' \\ \mu^2 p_{in} & k = k' \end{cases} \tag{B.5}$$

and the expected sum of degrees of nodes of type k as

$$d_k = \sum_{k'} Q_{kk'} = Q_{kk} + Q_{kk'} = \mu^2 (p_{in} + p_{out}) \tag{B.6}$$

By the definition then λ for each type is given by

$$\lambda_h = \frac{Q_{hh}}{d_h} = \frac{p_{in}}{p_{in} + p_{out}} \tag{B.7}$$

$$\lambda_l = \frac{Q_{lh}}{d_l} = \frac{p_{out}}{p_{in} + p_{out}} \tag{B.8}$$

Proposition

By definition $\lambda_h \geq \lambda_l$, due to homophily. Recall the equilibrium equations (2.30) and (2.31) for the model with a known network strucute. The propostion statement is simply the most-optimistic belief equation $\tilde{\beta}_t^o$ replacing the communication variable.

To make the point clear, notice that during an price increase $\beta_t^h \geq \beta_t^l$. Since $\lambda_h \geq \lambda_l$, it must be that the higher weight is given to the higher type-belief and so $\tilde{\beta}_t^o = \lambda_h \beta_t^h + (1 - \lambda_h) \beta_t^l$ during a boom. Whereas during a bust, $\beta_t^h \leq \beta_t^l$ and because $\lambda_l < (1 - \lambda_l)$, it must be that

$\tilde{\beta}_t^o = \lambda_l \beta_t^h + (1 - \lambda_l)\beta_t^l$.

Theorem

Look at equations (2.20), (2.30) and (2.31) along with the proposition above. Since $\lambda_h > \lambda_l$, it holds that

$$\phi_t = 1 \leftrightarrow \beta_t^h > \beta_t^l \Rightarrow \tilde{\beta}_t^o = \lambda_t^h \beta_t^h + (1 - \lambda_t^h)\beta_t^l$$
$$\phi_t = 0 \leftrightarrow \beta_t^h < \beta_t^l \Rightarrow \tilde{\beta}_t^o = \lambda_t^l \beta_t^h + (1 - \lambda_t^l)\beta_t^l$$

Writting price equilibrium equation (2.20) as a function of $\phi_t, \lambda_h, \lambda_l$ delivers the Theorem.

Appendix C

Chapter 3

C.1 Solving the Model

Individual Demands Trader i decides her demand using all information she has, i.e. s_i, λ_i, z_i. Given her beliefs on fundamental (3.10) and her perceived price (3.7), i's demand is determined by

$$\begin{aligned} x_i &= \frac{\dfrac{\kappa_d \cdot \mu + \kappa_e \cdot s_i + \kappa_{z_i} \cdot z_i}{\kappa_d + \kappa_e + \kappa_{z_i}} - p}{\gamma(\kappa_d + \kappa_e + \kappa_{z_i})^{-1}} \\ &= \frac{1}{\gamma}(\kappa_d + \kappa_e + \kappa_{z_i}) \cdot \left[\frac{\kappa_d \cdot \mu + \kappa_e \cdot s_i + \kappa_{z_i} \cdot \left(\frac{p - A_i}{B_i}\right)}{\kappa_d + \kappa_e + \kappa_{z_i}} - p \right] \\ &= \frac{1}{\gamma}(\kappa_d + \kappa_e + \kappa_{z_i}) \cdot \left[\frac{\kappa_d \cdot \mu + \kappa_e \cdot s_i - \kappa_{z_i} \cdot \frac{A_i}{B_i}}{\kappa_d + \kappa_e + \kappa_{z_i}} - \left(1 - \frac{\kappa_{z_i}/B_i}{\kappa_d + \kappa_e + \kappa_{z_i}}\right) p \right] \\ &= \frac{1}{\gamma} \cdot \left[\kappa_d \cdot \mu + \kappa_e \cdot s_i - \kappa_{z_i} \cdot \frac{A_i}{B_i} - \left(\kappa_d + \kappa_e + \kappa_{z_i} \cdot \frac{(B_i - 1)}{B_i} \right) p \right] \end{aligned} \tag{C.1}$$

Individual MUC Trader i finds coefficients $\{A_i, B_i, C_i\}$ using MUC by supposing that all traders extract the same information from price as she does. That is, i thinks others demand function $x_j(\cdot) \quad \forall j \neq i : t_j = 1$ depends on λ_i (not on λ_j, which we assume i does not know). Using her demand function (C.1) and market clearing condition (C.3), i computes price p by equating aggregate demand with the noisy supply n,

$$\frac{1}{N}\sum_{i:t_i=1}\frac{\frac{\kappa_d\cdot\mu+\kappa_e\cdot s_i+\kappa_{z_i}\cdot\left(\frac{p-A_i}{B_i}\right)}{\kappa_d+\kappa_e+\kappa_{z_i}}-p}{\gamma(\kappa_d+\kappa_e+\kappa_{z_i})^{-1}}=n$$

$$\frac{1}{\gamma N}\left\{(\lambda_i N)\kappa_d\mu+\kappa_e\sum_j s_j-(\lambda_i N)\kappa_{z_i}\frac{A_i}{B_i}-(\lambda_i N)\left[\kappa_d+\kappa_e+\kappa_{z_i}\frac{(B_i-1)}{B_i}\right]p\right\}=n$$

$$\left[\kappa_d+\kappa_e+\kappa_{z_i}\frac{(B_i-1)}{B_i}\right]p=\kappa_d\mu+\kappa_e\frac{1}{\lambda_i N}\sum_j s_j-\kappa_{z_i}\frac{A_i}{B_i}-\frac{\gamma}{\lambda_i}n$$

$$p=\left(\kappa_d+\kappa_e+\kappa_{z_i}\frac{(B_i-1)}{B_i}\right)^{-1}\left[\kappa_d\mu+\kappa_e\hat{d}_i-\kappa_{z_i}\frac{A_i}{B_i}-\frac{\gamma}{\lambda_i}\right]$$

The above must then equal her price guess (3.7), $p=A_i+B_i\cdot\widehat{d}_i+C_i\cdot n$:

$$\frac{\kappa_d\cdot\mu+\kappa_e\cdot\widehat{d}_i-\frac{A_i}{B_i}\cdot\kappa_{z_i}-\frac{\gamma}{\lambda_i}\cdot n}{\kappa_d+\kappa_e+\frac{B_i-1}{B_i}\cdot\kappa_{z_i}}=A_i+B_i\cdot\widehat{d}_i+C_i\cdot n \tag{C.2}$$

Matching coefficients

$$\frac{\kappa_d\cdot\mu-\frac{A_i}{B_i}\cdot\kappa_{z_i}}{\kappa_d+\kappa_e+\frac{B_i-1}{B_i}\cdot\kappa_{z_i}}=A_i\Rightarrow A_i=\frac{\kappa_d\cdot\mu}{\kappa_d+\kappa_e+\kappa_{z_i}}$$

$$\frac{\kappa_e}{\kappa_d+\kappa_e+\frac{B_i-1}{B_i}\cdot\kappa_{z_i}}=B_i\Rightarrow B_i=\frac{\kappa_e+\kappa_{z_i}}{\kappa_d+\kappa_e+\kappa_{z_i}}$$

$$\frac{-\frac{\gamma}{\lambda_i}}{\kappa_d+\kappa_e+\frac{B_i-1}{B_i}\cdot\kappa_{z_i}}=C_i\Rightarrow C_i=-\frac{\gamma}{\lambda_i\cdot\kappa_e}\cdot\frac{\kappa_e+\kappa_{z_i}}{\kappa_d+\kappa_e+\kappa_{z_i}}$$

With this, we can compute agent i's demand:

$$\begin{aligned}x_i\left(s_i,\lambda_i,p\right)&=\kappa_d\cdot\mu+\kappa_e\cdot s_i-\kappa_{z_i}\cdot\frac{A_i}{B_i}-\left(\kappa_d+\kappa_e+\kappa_{z_i}\cdot\frac{B_i-1}{B_i}\right)p\\&=\kappa_d\cdot\mu+\kappa_e\cdot s_i-\frac{\kappa_{z_i}\cdot\kappa_d}{\kappa_e+\kappa_{z_i}}\cdot\mu-\left(\kappa_d+\kappa_e-\frac{\kappa_{z_i}\cdot\kappa_d}{\kappa_e+\kappa_{z_i}}\right)\cdot p\\&=\kappa_e\cdot(s_i-p)-\frac{\kappa_e\cdot\kappa_d}{\kappa_e+\kappa_{z_i}}\cdot(p-\mu)\end{aligned} \tag{3.13}$$

Aggregate MUC

To find the equilibrium price, we conjecture that the price function is $p = A + B\hat{d} + Cn$. Given equilibrium demands (3.13), aggregate demand is

$$X(p) = \frac{1}{N}\sum_{i:t_i=1} \kappa_e \cdot (s_i - p) - \frac{\kappa_e \cdot \kappa_d}{\kappa_e + \kappa_{z_i}} \cdot (p - \mu) \tag{C.3}$$

Market clearing delivers

$$\begin{aligned} n =& \frac{1}{N}\sum_{i:t_i=t} \left(\frac{\kappa_e}{\gamma} \cdot (s_i - p) - \frac{\kappa_e \cdot \kappa_d}{\gamma\left(\kappa_e + \kappa_{z_i}\right)} \cdot (p - \mu) \right) \\ n =& \frac{\kappa_e}{\gamma} \cdot \left(\frac{1}{N} \sum i : t_i = t s_i - \lambda p \right) - \frac{\kappa_e \cdot \kappa_d}{\gamma} \left(\frac{1}{N} \sum i : t_i = t \frac{1}{\kappa_e + \kappa_{z_i}} \right) \cdot (p - \mu) \end{aligned} \tag{C.4}$$

Let[1]

$$\kappa_{\bar{z}} \equiv \frac{1}{\lambda N} \left[\sum_{t_i} \frac{1}{\kappa_e + \kappa_{z_i}} \right]^{-1} - \kappa_e \tag{C.5}$$

Then we can write

$$\begin{aligned} n =& \frac{\kappa_e}{\gamma} \cdot \left(\lambda \cdot \widehat{d} - \lambda p \right) - \frac{\lambda \cdot \kappa_e \cdot \kappa_d}{\gamma \cdot (\kappa_e + \kappa_{\bar{z}})} \cdot (p - \mu) \\ p =& \frac{\frac{\kappa_e}{\gamma} \cdot \lambda \cdot \widehat{d} + \frac{\lambda \cdot \kappa_e \cdot \kappa_d}{\gamma \cdot (\kappa_e + \kappa_{\bar{z}})} \cdot \mu - n}{\frac{\kappa_e}{\gamma} \cdot \lambda + \frac{\lambda \cdot \kappa_e \cdot \kappa_d}{\gamma \cdot (\kappa_e + \kappa_{\bar{z}})}} \\ p =& \frac{\lambda \cdot \widehat{d} + \frac{\lambda \cdot \kappa_d}{(\kappa_e + \kappa_{\bar{z}})} \cdot \mu - \frac{\gamma}{\kappa_e} \cdot n}{\lambda \left(\frac{\kappa_e + \kappa_{\bar{z}} + \kappa_d}{\kappa_e + \kappa_{\bar{z}}} \right)} \end{aligned} \tag{C.6}$$

and find the equilibrium price

$$p = \frac{\kappa_d}{\kappa_e + \kappa_d + \kappa_{\bar{z}}} \cdot \mu + \frac{\kappa_e + \kappa_{\bar{z}}}{\kappa_e + \kappa_d + \kappa_{\bar{z}}} \cdot \widehat{d} - \frac{\gamma}{\lambda \cdot \kappa_e} \cdot \frac{\kappa_e + \kappa_{\bar{z}}}{\kappa_e + \kappa_d + \kappa_{\bar{z}}} \cdot n \tag{3.15}$$

Matching the above with our conjectured price function, we can find the true price coefficients $\{A, B, C\}$:

[1] Another way to derive $\kappa_{\bar{z}}$ is to notice that $\frac{1}{\kappa_e + \kappa_{\bar{z}}} = \frac{1}{\lambda N} \sum i : t_i = t \frac{1}{\kappa_e + \kappa_{z_i}}$

$$
\begin{aligned}
A =& \frac{\kappa_d \cdot \mu}{\kappa_d + \kappa_e + \kappa_{\bar{z}}} \\
B =& \frac{\kappa_e + \kappa_{\bar{z}}}{\kappa_d + \kappa_e + \kappa_{\bar{z}}} \\
C =& -\frac{\gamma}{\lambda_t \cdot \kappa_e} \cdot \frac{\kappa_e + \kappa_{\bar{z}}}{\kappa_d + \kappa_e + \kappa_{\bar{z}}}
\end{aligned}
\tag{C.7}
$$

C.2 Network Effect

Connectivity $\&$ Social Market

To understand this result, we can first decompose how the network determines λ_i. That depends on how many friends i has, g_i and how many of those are trading the asset, $t_{N_i} \equiv \sum_{j \in N_i} t_j$. Moreover, g_i and t_{N_i} affect λ_i in opposite ways.

1. holding t_{N_i} fixed: $\uparrow g_i \Rightarrow \downarrow \lambda_i$

   "I have more friends"

2. holding g_i fixed: $\uparrow t_{N_i} \Rightarrow \uparrow \lambda_i$

   "More of my friends are trading"

3. $\Delta t_{N_i} \cdot \Delta g_i$?

   - $\uparrow t_{N_i} \downarrow g_i \Rightarrow \uparrow \lambda_i$

     "I have few but like-minded friends, all trading the stock" *(assumption: agent is trading)*

   - $\downarrow t_{N_i} \uparrow g_i \Rightarrow \downarrow \lambda_i$

     "I have lots but different-minded friends"

On (3) notice that the "type effect" dominates the "size effect":

$$
\left| \frac{\partial \lambda_i}{\partial g_i} \right| = \left| \frac{t_{N_i}}{g_i^2} \right| < \left| \frac{\partial \lambda_i}{\partial t_{N_i}} \right| = \left| \frac{1}{g_i} \right|
$$
$$
\text{since} \quad g_i > t_{N_i} \equiv \sum_{j \in N_i} t_j
$$

that is, if I have one more friend who is also trading then $\uparrow \lambda_i \longleftrightarrow$ "I myopically believe the stock market is greater (more participants)"

Demand $\&$ Social Market

1. holding t_{N_i} fixed: $\uparrow g_i \Rightarrow \downarrow \lambda_i \Rightarrow \downarrow x_i$

   "I have more friends thus I believe market is relatively smaller and thus I buy LESS" ?

2. holding g_i fixed: $\uparrow t_{N_i} \Rightarrow \uparrow \lambda_i \Rightarrow \uparrow x_i$

   "More of my friends are buying thus I believe market is larger and thus I buy MORE"?

3. $\Delta t_{N_i} \cdot \Delta g_i$

   - $\uparrow t_{N_i} \downarrow g_i \Rightarrow \uparrow \lambda_i \Rightarrow \uparrow x_i$

     "I have few but like-minded friends thus I buy MORE"

   - $\downarrow t_{N_i} \uparrow g_i \Rightarrow \downarrow \lambda_i \Rightarrow \downarrow x_i$

     "I have lots but different-minded friends thus I by LESS"

Harmonic Mean

Consider the individual coefficients $(\kappa_{z_i} + \kappa_e)$ that appear in each agent's equilibrium demand (3.13). These coefficients are positive real numbers. The harmonic mean of $\{(\kappa_{z_i} + \kappa_e)\}_i$ is defined as

$$H = \frac{\lambda N}{\sum_{t_i=1} \left(\frac{1}{\kappa_{z_i} + \kappa_e} \right)} \tag{C.8}$$

Looking at the definition of public information precision (3.16) it is clear that $\kappa_{\bar{z}}$ is a function of H:

$$\begin{aligned} \kappa_{\bar{z}} &:= \frac{1}{\lambda N} \left[\sum_{t_i=1} \left(\frac{1}{\kappa_{z_i} + \kappa_e} \right) \right]^{-1} - \kappa_e \\ &= \frac{1}{\lambda^2 N^2} H - \kappa_e \end{aligned} \tag{C.9}$$

For simplicity, we use the following equality to refer to the harmonic mean

$$h = \frac{1}{\sum_{t_i=1} \left(\frac{1}{\kappa_{z_i} + \kappa_e} \right)} \tag{3.17}$$

which is equivalent to $\frac{1}{\lambda N}H$. That is, we write

$$\kappa_{\bar{z}} = \frac{1}{\lambda N}h - \kappa_e \tag{C.10}$$

C.3 Comparative Statics

Network Effect

Individual level:

At the individual level, we find that λ_i positively affects price precision κ_{z_i}, $\frac{\partial \kappa_{z_i}}{\partial \lambda_i} > 0$.

$$\begin{aligned}
\frac{\partial \kappa_{z_i}}{\partial \lambda_i} &= -\left[-\frac{1}{\lambda_i^2}\frac{1}{N\kappa_e} - \frac{1}{\lambda_i^3}\frac{2\gamma^2}{\kappa_n \kappa_e^2}\right]\left[\frac{1}{\lambda_i N}\kappa_e^{-1} + \kappa_n^{-1}\gamma^2\left(\lambda_i \kappa_e\right)^{-2}\right]^{-2} \\
&= \left(\frac{1}{\lambda_i^2 \kappa_e}\right)^{-2}\left[\frac{1}{N} + \frac{1}{\lambda_i}\frac{2\gamma^2}{\kappa_n \kappa_e}\right]\kappa_{z_i}^2 \\
&= \lambda_i^4 \kappa_e^2 \left[\frac{1}{N} + \frac{1}{\lambda_i}\frac{2\gamma^2}{\kappa_n \kappa_e}\right]\kappa_{z_i}^2 \\
&> 0
\end{aligned}$$

Aggregate Level:

$$\begin{aligned}
\frac{\partial \kappa_{\bar{z}}}{\partial \lambda} &= -\frac{1}{\lambda^2 N}\left[\sum_{t_i}\frac{1}{\kappa_e + \kappa_{z_i}}\right]^{-1} \\
&= -\frac{1}{\lambda}\kappa_{\bar{z}} \\
&< 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \kappa_{\bar{z}}}{\partial \lambda_i} &= -\lambda N\left[\sum_{t_i}\frac{1}{\kappa_e + \kappa_{z_i}}\right]^{-2}\left(\frac{-1}{(\kappa_e + \kappa_{z_i})^2}\right) \\
&= \frac{1}{(\kappa_e + \kappa_{z_i})^2}\kappa_{\bar{z}} \\
&> 0
\end{aligned}$$

$$
\begin{aligned}
&\frac{\partial p}{\partial \kappa_h} = -\frac{1}{\kappa_d + \kappa_h}\left[p - \left(\hat{d} - \frac{\gamma}{\lambda \kappa_e} n\right)\right] < 0 \\
&if \quad p > \left(\hat{d} - \frac{\gamma}{\lambda \kappa_e} n\right)
\end{aligned}
$$

Individual Demand

A larger social market, and thus more precise price information, leads to higher demand when $p > \mu$.

$$
\begin{aligned}
&p > \mu : \\
&\quad \frac{\partial x_i}{\partial \kappa_{z_i}} = \frac{\kappa_d \kappa_e}{(\kappa_e + \kappa_{z_i})^2} (p - \mu) \\
&\quad > 0 \\
&\quad \updownarrow \\
&\quad \frac{\partial x_i}{\partial \lambda_i} = \frac{\partial x_i}{\partial \kappa_{z_i}} \frac{\partial \kappa_{z_i}}{\partial \lambda_i} \\
&\quad = (> 0) \cdot (> 0) \\
&\quad > 0
\end{aligned}
$$

C.4 Price Information

The price information extracted by each trader i, z_i (3.9), is different from the (true) public information z. On one hand, the individual price signal is

$$
\begin{aligned}
z_i &= d + \frac{1}{\lambda_i N} \sum_{t_i} \epsilon_i - \frac{\gamma}{\lambda_i \kappa_e} n \\
&= \frac{\kappa_d}{(\kappa_e + \kappa_{z_i})} (p - \mu) + p
\end{aligned} \tag{C.11}
$$

On the other hand, the "true" information content in price (i.e. the "true public signal") is given by[2]

[2]Recall (C.7) that gives coefficients (A, B, C): $z \equiv \frac{p - A}{B} = d + \frac{1}{\lambda N} \sum_{t_i} \epsilon_i + \frac{C}{B} n$

$$
\begin{aligned}
z &= d + \frac{1}{\lambda N}\sum_{t_i} \epsilon_i - \frac{\gamma}{\lambda \kappa_e} n \\
&= \frac{\kappa_d}{\kappa_h}\left(p - \mu\right) + p
\end{aligned} \tag{C.12}
$$

Individual Price Signal

When $p > \mu$, a larger social market, and thus more precise price information, leads to a lower level of inferred price (price signal). That is, z_i is decreasing in κ_{z_i} when $p > \mu$:

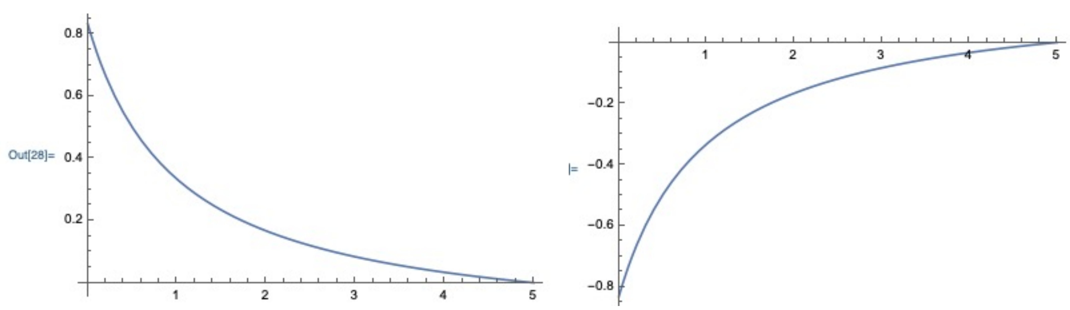
$$
\begin{aligned}
\frac{\partial z_i}{\partial \kappa_{z_i}} &= \frac{1}{\kappa_e + \kappa_{z_i}} \cdot p - \frac{1}{(\kappa_e + \kappa_{z_i})^2}\left[(\kappa_e + \kappa_d + \kappa_{z_i})p - \kappa_d \mu\right] \\
&= -\frac{\kappa_d}{(\kappa_e + \kappa_{z_i})^2}(p - \mu) \\
&= \frac{1}{\kappa_e + \kappa_{z_i}}\left(p - z_i\right)
\end{aligned} \tag{C.13}
$$

Alternatively, we can write z_i in terms of private signals $\{s_i\}$: $z_i = d + \frac{1}{\lambda_i N}\sum_{t_i} \epsilon_i - \frac{\gamma}{\lambda_i \kappa_e} n$ (C.11).Then

$$
\frac{\partial z_i}{\partial \lambda_i} = \frac{1}{\lambda_i^2}\left(\frac{\gamma}{\kappa_e} n - \frac{1}{N}\sum_{t_j} \epsilon_j\right) \tag{C.14}
$$

Public Information Distance

Now we look at the distance from the true price signal z.



x-axis: κ_{z_i}; L: $p - \mu < 0$, R: $p - \mu > 0$

Thus lower κ_{z_i} delivers a higher $|z - z_i|$.

$$
\begin{aligned}
\frac{\partial(z-z_i)}{\partial\kappa_{z_i}} &= \frac{\partial\kappa_{z_i}}{\partial\lambda_i} \times \left\{ \frac{1}{(\kappa_e+\kappa_{z_i})^2}\kappa_d \cdot \mu - \frac{(\kappa_{p_i}+\kappa_e+\kappa_{z_i})}{(\kappa_e+\kappa_{z_i})^2} \cdot p \right\} \\
&= \frac{\partial\kappa_{z_i}}{\partial\lambda_i} \times \underbrace{\frac{1}{(\kappa_e+\kappa_{z_i})^2}}_{\uparrow\kappa_{z_i}\rightarrow\downarrow(.)} \underbrace{[\kappa_d\mu - (\kappa_{p_i}+\kappa_e+\kappa_{z_i})p]}_{\uparrow\kappa_{z_i}\rightarrow\downarrow(.)} \\
&= p\left(\frac{\kappa_d+\kappa_e+\kappa_{z_i}}{(\kappa_e+\kappa_{z_i})^2} - \frac{1}{\kappa_e+\kappa_{z_i}}\right) - \frac{\kappa_d\mu}{(\kappa_e+\kappa_{z_i})^2} \\
&= \frac{\kappa_d(p-\mu)}{(\kappa_e+\kappa_{z_i})^2}
\end{aligned} \tag{C.15}
$$

From the previous results, we find that traders with **greater social market** extract **more precise** price information. When $p > \mu$ ($p < \mu$) this implies:

- higher (lower) demand
- lower perceived price
- price signal further to the truth

When $p > z_i$ ($p < z_i$)

- perceive higher (lower) price when $p > z_i$ ($p < z_i$)
- perceive a price signal closer to the truth

That is

$$
p > \mu : \uparrow \lambda_i \longleftarrow \uparrow \kappa_{z_i} \longleftarrow \begin{cases} \uparrow x_i \\ \downarrow z_i \\ \downarrow (z - z_i) \end{cases}
$$

At the same time,

$$
p > z_i : \uparrow \lambda_i \longleftarrow \uparrow \kappa_{z_i} \longleftarrow \uparrow z_i
$$