

**Study of Monochrome and Color Polarimetric Cameras
to Complete Stokes Polarimetry**

Manal Ibrahim Ahmad Altaweel

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Study of Monochrome and Color Polarimetric Cameras to Complete Stokes Polarimetry

Manal Ibrahim Ahmad Altaweel

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the requirements for the degree of
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Thesis supervisors:

Dr. Àngel Lizana
Dra. Irene Estévez
Prof. Dr. Juan Campos

Departament de Física
Universitat Autònoma de Barcelona



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Conference Proceedings

3. **Complete Stokes Polarimetry Based on a Ferroelectric Liquid Crystal Panel and a Polarimetric Camera: Application to Biological Samples.** SPIE Proceedings, 2026. DOI: 10.1117/12.3099137.
4. **Liquid Crystal Technology for Polarimetric Imaging: Biomedical-Oriented Developments.** SPIE Proceedings, 2026. DOI: 10.1117/12.3099549.

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Abstract

Polarization is a key property of light, emphasizing its vectorial states. Animals such as bees, ants, and octopus utilize light polarization for navigation and communication, unlike humans. We sense the brightness and color of light but cannot tell if light is polarized or unpolarized, or how the polarization is oriented. Traditional color and intensity-based systems lack the capability of seeing polarization. To measure, visualize, and utilize this polarization information, polarimeters are needed.

The polarization state of the light is modeled by the Stokes vector with four components that represent the total incoming light intensity and the amount of light that is linearly and circularly polarized. All polarimeters incorporate one or more polarizers to analyze the polarization state of light. Most polarimeters also use retarders to provide diversity in the analyzed polarization states. These retarders may be waveplates that are rotated or retarders with adjustable retardance such as liquid crystal devices, electro-optical modulators, and other retardance modulators. However, ferroelectric liquid crystal (FLC) modulators are becoming more important due to their faster response in comparison to Twisted nematic liquid crystals. The ferroelectric liquid crystal introduces rapid modulation capabilities, surpassing the speed of conventional waveplates. Conventional imaging polarimeters often rely on sequential measurements, which can limit their temporal resolution and introduce artifacts in dynamic scenes. Furthermore, traditional waveplates used in polarimetric setups may lack the speed necessary for high-frequency applications, constraining their performance in real-time imaging scenarios. Two classes of polarimeters can perform simultaneous polarimetric measurement: division of aperture polarimeters and division-of-amplitude

polarimeters. Several polarimeters architectures have been analyzed in the literature, each one presenting its own drawbacks and strength. Recently, One can found on the market Monochrome polarization and RGB Polarization cameras that can capture both color and polarimetric state of the light in a single snapshot. This advanced new technology of polarized /RGB polarized sensors enables the simultaneous capture of linear polarization information without the need for sequential measurements, significantly enhancing temporal resolution. Despite these advancements, the design of the polarized camera introduces spatial gaps between pixels with identical polarizer angles, and color cross talk which can affect the reconstruction accuracy of polarization data.

In this thesis, we will study the polarimetric and spectral characteristics of micro-polarized and multispectral micro-polarized sensors. To overcome spatial gaps limitation on these sensors, we developed a Fourier interpolation method that reconstructs the polarization state with high precision, ensuring the reliability of these sensors to enable the simultaneous capture of linear polarization information without the need for sequential measurements, including experimental calibration, validation, measuring. Based on, we present a novel full Stokes imaging polarimeter that addresses the nowadays challenges through a unique combination of a polarized monochrome camera with a ferroelectric liquid crystal, and another polarimeter involves RGB polarized sensor with a ferroelectric liquid crystal.

This work demonstrates a significant step forward in the field of imaging polarimetry, offering faster and more reliable measurements while addressing practical limitations. The proposed system has the potential to unlock new possibilities for real-time polarimetric imaging in a wide range of applications.

Introduction

Different fundamental properties of electromagnetic waves, such as intensity, wavelength, phase and polarization, are commonly used to study light properties and light-matter interactions. Regarding polarization property, it has proved to be sensitive to samples characteristics, and the analysis of light polarization transformation after light-matter interactions leads to fundamental knowledge of samples. For instance, unpolarized sunlight transformation into polarized linear light occurs through Rayleigh scattering when interacting with the atmosphere gases, but polarization states are also modified by interaction with, for instance, anisotropic or dichroic materials.

Historically, scientists have described the phenomena of polarization through various theories and experiments. In 1669, Erasmus Bartholin observed the double refraction of light in Calcite crystals, and that Iceland spar crystals refract light and produce double images, which Huygens later explained through different light velocities inside the crystals as a function of different input light vibration directions. Thomas Young's wave-particle duality theory further demonstrated that split beams possess distinct polarizations. In 1808, Etienne Louis Malus observed that reflected light could become polarized, even from non-metallic surfaces, providing further evidence that polarization is a vectorial characteristic of light.

Currently, the technological development of polarimetric instrumentation, especially associated with polarimeters, basic instruments for polarimetric measurement, has led to numerous fields of research and industry using polarimetric techniques for their developments. One of these advanced technologies, of particular interest in this thesis, is the polarimetric camera. Ibn al-Haytham's work precedes these discoveries by about 600–800 years. The camera obscura of the Ibn Al-Haytham was the first controlled demonstration of image formation, laying the foundation for modern imaging systems. Then, the arrival of photographic plates in the 19th century led to the recording of the first photographs. The 20th century saw the development of more sophisticated optical systems, including spectroscopy and polarization optics, which extended imaging beyond intensity to include color, spectral content, and polarization states. Already in the 21st century, Willard S. Boyle and George E. Smith received the Nobel Prize in Physics in 2009 for the invention of the CCD camera, which had a significant social and technological impact due to its revolutionary role in digital imaging and various medical applications. The arrival of the CCD camera fostered the above-stated technologies to their full potential. The inclusion of microarrays of polarizers in CCD cameras, which led to the development of new polarimetric cameras in the 2000s, has taken polarimetric imaging technology to the next level.

Polarimetry serves as both a mathematical framework and an analytical tool to decode physical properties related to polarization and its transformation, linking physical principles with real-world phenomena. Imaging polarimetry reflects a shift from passive observation to active interpretation. Early optics was about reproducing the visible world, but modern imaging seeks to decode the unseen properties of materials, biological tissues, and even distant galaxies.

In this introductory chapter, we present an overview of the state of art of polarimetry imaging, especially in Stokes imaging polarimetry, in the polarized

monochrome and RGB polarized camera. In addition, we will describe some applications that motivated us to develop the research comprised in this thesis.

First, we describe in section 1.1 a collection of applications of polarimetry in main fields. Then, section 1.2 overview different imaging polarimetric methods described in the specialized literature, as well as positive results obtained when using different set-ups in other fields, such as remote sensing, astronomy, and material characterization. Afterward, an overview of the studies that built Stokes polarimetry and our proposed polarimeter.

1.1 Remote sensing applications

Polarimetric imaging has gained significant interest in remote sensing applications due to its capability to enhance the detection and characterization of objects and materials by providing additional information with respect to non-polarimetric images. This technology helps in studying atmospheric particles, reduces glare from water surfaces, enhances Earth surface imaging, and aids in monitoring vegetation health. By measuring polarization parameters of light emitted, transmitted, or scattered by various surfaces, physical and chemical surface properties may be retrieved through polarimetric signatures. Early uses of polarization in remote sensing were reported by Bernard Lyot when he published results of polarization measurements of the moon by his polarimeter in 1929. Replicating moon observations-measurements through the degree of polarization (DoP) channel by measuring a variety of substances make him conclude that the lunar surface was covered with small grains [1]. In much more recent studies into the security framework, the circular polarization component of light was found to provide a discriminating signature of landmines [2]. L.J. Denes demonstrates the increased polarization of a military vehicle compared to that of a vegetation background [3].

Another important area of application is environmental and atmospheric research. Polarimetry allows identification and quantification of different aerosol particle types (e.g. sulfate, nitrate, black carbon) which have distinct scattering and absorption properties. Since the measuring of the DoP is sensitive to particle size, shape, and refractive index, enhanced characterization compared to traditional radiometric sensors was provided [4]. For example, Rayleigh scattering dominates molecular gases which particles smaller than wavelength, while in sulfate aerosols Mie scattering occurs. In the former the scattered light is strongly polarized, and the DoP depends on the scattering angle, but the latter polarization patterns become more complex, with partial polarization. In addition, irregularly shaped or absorbing particles, like mineral dust, depolarize the light. In ref [5], the change in the polarization state reveals the particle's shape and composition.

Polarimetric measurements, particularly at multiple angles, reveal the degree to which light is absorbed versus scattered, thus providing direct insight into the absorption optical depth. In that sense, multi-angular polarimeters set-up is used to provide information enabling to estimate of the global aerosol absorption optical depth[6]. This knowledge is critical for understanding their impact on climate change, since Satellite-based instruments and lidar face challenges in directly measuring absorption, especially over complex surfaces like urban areas or oceans. Similarly, polarimetry has been contributing to cryosphere studies, current application includes ice sheet dynamics [7], where polarimetric techniques analyze scattering properties to monitor mass loss and predict seasonal freeze-thaw cycles, offering predictive insights into climate change impacts. In aquatic environments, polarimetric sensors provide high-resolution assessments of water quality by measuring the polarization characteristics and optical properties of suspended particles in seawater, chlorophyll, and pollutants, achieving over 90% accuracy in identifying different types of particles based on their optical properties [8], [9]. Furthermore, advancements in polarimetric radar technology, for instance, allows for

detailed land cover classification and mapping in urban environments changes with time [10], affected by disasters [11], enabling better urban planning and disaster resilience.

In object detection and classification, most of the research demonstrates the improved contrast which may be obtained with proper spectral and polarization selection. Figure 1-1 shows one example of the ability of polarization to show enhanced image contrast between different objects classes, as metallic cars or organic trees [12].

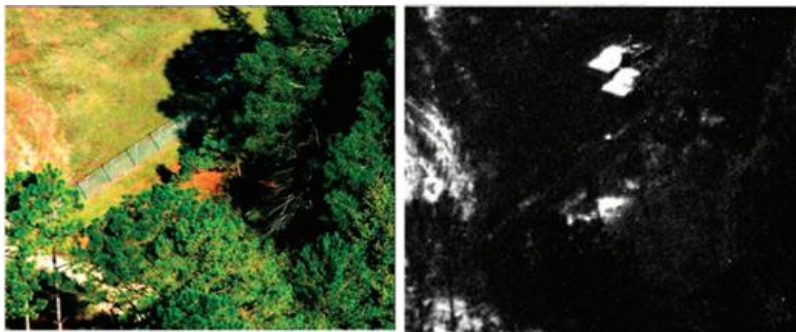


Figure 1-1. Visible picture of two pickup trucks in the shade (left), and polarization image (right). Enhanced contrast in the polarization image shows advantages for target detection using imaging polarimetry [12]. Note how trucks parked under the trees are revealed through polarimetric means.

1.2 Biomedicine applications

1.2.1 Polarimetric Imaging in Neurology and Brain Tissue Characterization

Biomedical polarimetry has attracted substantial attention for its ability to localize and visualize structural inhomogeneities in biological tissues. The polarization properties of living tissues complements other optical techniques, such as optical coherence tomography [13], hyperspectral imaging [14], and Raman spectroscopy

[15], enhancing the medical imaging and diagnostic noninvasively. It is offering innovative solutions to critical healthcare challenges.

Polarimetry is used to identify the orientation of tissue fibers [16], [17]. A recent advancement in neurology and brain imaging leverages the characteristic of polarimetric techniques in mapping the fiber tracts of white matter (WM), especially because the brain's WM consists of organized nerve fiber bundles that exhibit birefringence; a property where the refractive index varies with propagation direction. Polarimetry is suitable to detect birefringence characteristics of samples, providing rapid and detailed insights into brain tissue structure [18]. Moreover, polarimetry is helpful in detecting microstructural changes in brain tissue, because the diseased brain tissues, such as those affected by tumors or neurodegenerative conditions, often exhibit alterations in microstructure that affect their polarimetric properties. It is showing promise for minimally invasive mapping of microstructural features, which is crucial for understanding diseases like Alzheimer's or Parkinson's [19].

Even in the presence of complex surface topographies and surgical conditions, polarimetric techniques are aiding surgeons in making informed decisions during tumor resections [20], by differentiating healthy and diseased brain tissues. Recent research has focused on characterizing the polarimetric properties of various brain tumor types, particularly gliomas, using wide-field imaging Mueller polarimetry.

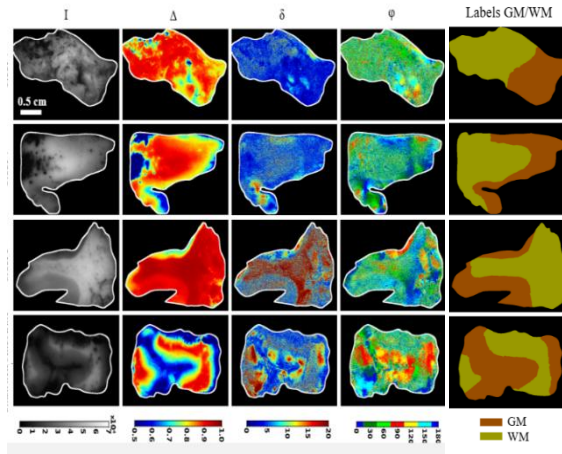


Figure 1-2. Polarimetric maps and neuropathological diagnosis samples [21]. While the first column refers to different tissues sections intensities images, columns two, three and four states for depolarization, linear retardance, and azimuth angle polarimetric channels, revealing different structures hidden in intensity channel which are of medical interest.

In Figure 1-2, by correlating polarimetric data with histological findings, significant differences in depolarization and linear retardance values (the columns 2 and 3) have been observed between healthy and tumor tissues. Notably, tumor regions exhibit pronounced randomization of the optical axis azimuth (column 4), highlighting the potential of polarimetry in enhancing intraoperative tumor margin delineation between grey matter (GM) and white matter (WM) as shown in Figure 1-2 in the last right column. These insights are pivotal in intraoperative settings, improving tumor margin delineation and preserving neurological function during surgery. This advancement demonstrates the growing interest in polarimetry for its non-invasive, sensitive, and detailed tissue characterization capabilities, essential in complex procedures like neurosurgery [21].

1.2.2 Expanding Applications in Biomedicine

Polarimetry applications in Biomedicine continues to expand due to the unique vectorial properties of polarization and the capability to address increasingly

complex media, from isotropic and homogeneous to anisotropic and inhomogeneous tissues. This progression is driven by the ability of polarimetric techniques, particularly Stokes-Mueller methods, to represent depolarization; a critical feature in biomedical studies involving multiple-scattering processes that alter the state and DoP [22]. The potential of using polarimetry for the study of organic tissues has been provided in literature [23], [24], [25] and provide a better performance of pathological diagnosis [26], [27]. For instance, a recent study focuses on creating a digital polarization tool to diagnose specimens remotely [28]. This innovation aims to enable pathologists to analyze polarized images of digitized slides, facilitating accurate diagnoses without the need for physical slide examination. In another hand, polarization is also used to determine the concentration of hemoglobin [29], oxygen [30], and glucose [31] in blood. Different concentrations of hemoglobin and oxygen vary the scattering properties of the sample thus changing light depolarization. In ophthalmology, polarimetric techniques are well established in monitoring retinal nerve fiber layers, facilitating in the early diagnosis of glaucoma [32]. A study on oral cancer revealed significant differences in birefringence between normal and cancerous tissues, indicating the potential for non-invasive diagnostics [33]. Reflection-mode polarimetry can guide laser mass spectrometry to target non-fat regions in breast cancer tissues, improving the identification of cancer markers [34].

Additionally, in burn assessment, polarimetry evaluates tissue scattering and collagen orientation, providing a precise measure of burn severity, aiding in treatment planning [35]. Polarization proved to be useful in dermatology for the detection of skin diseases, enhancing contrast between healthy and pathological tissues, especially in patients with darker skin types, where traditional examination methods often face challenges due to lower contrast [36], [37]. Imaging polarimeters described in [38] is used to aid in characterizing colon cancer stages and distinguishing between normal and tumor tissues, by analyzing depolarization and retardance measurements in tissue samples. Moreover, imaging polarimeters are

used as a tool for enhancing detection accuracy for pre-cancerous conditions by combined Mueller polarimetry with machine learning [39]. Thus, addressing challenges in visualizing early malignancies [40]. In dynamic tissue monitoring, rapid Stokes imaging allows continuous monitoring of tissue characteristics, enabling the study of dynamic physiological processes such as tissue clearing [41].

1.3 Astronomy

Polarimetry is a well-established technic in astronomic observation and analysis. Polarimetry helps in mapping the distribution and orientation of dust clouds in the galaxy, revealing multiple layers of dust along the line of sight [42]. Recent advancements in solar polarimetry have led to highly sensitive measurements of the Sun's polarization, particularly in quiet solar regions. This sensitivity is essential for probing the weak polarization signals in Fraunhofer spectral lines, which are often obscured by noise and stray light [43].

In Ref. [44], the design of a polarimeter for a stratospheric mission in Antarctica aims to analyze the polarization of light from solar spectral lines to study solar magnetic fields. In same vein, in Ref. [45] Stokes polarimetry was employed to extract information about magnetic fields, leveraging both the Zeeman and Hanle effects, which are essential for understanding the variability of solar magnetic fields.

Measuring the linear polarization of starlight is a crucial method for investigating the structure of the interstellar magnetic field [46]. Additionally, Polarimetry is increasingly recognized as a vital technique for detecting exoplanets, particularly due to its ability to discern polarized light from planetary atmospheres against the unpolarized background. This method leverages the unique polarimetric signatures produced by atmospheric molecules, which can indicate the presence of oceans or other features on exoplanets [47], [48]. Measuring the linear polarization of the interaction of supernovae with circumstellar material and the unique spectro

polarimetric signatures that supernovae exhibit may be correlated providing additional context for distance calculations to external galaxies [54], [55]

1.4 Material characterization

Polarimetric techniques are particularly valuable due to their ability to provide critical optical parameters such as film thickness and complex refractive indices of surfaces and interfaces. By measuring change in polarization upon reflection or transmission in terms of amplitude and phase differences, direct relation of the polarization states of light lead to different optical properties of the samples [49]. Many studies discussed ellipsometry as a technique for measuring complex refractive indices of organic solvents [50], emphasizing its effectiveness in characterizing materials by allowing for the determination of dielectric functions [51], and film thickness, all this information enhancing databases for various applications in materials science [52]. Furthermore, it is extensively applied in advanced plasmonic applications, including the analysis of nanoparticle arrays and metamaterials [50].

The recent development of Generalized Ellipsometry, the so-called Mueller polarimetry, uses Mueller matrix measurements of materials, representing the polarimetric transfer function of samples, for providing a powerful tool for materials characterization. For instance, it is used for probing the general anisotropic microstructure of complex media. As an example, Fig. (1.3) demonstrates how the proposed method in [53] successfully isolates the material's intrinsic properties from orientation-induced artifacts. This visual evidence underscores the potential of the technique to enhance the accuracy of polarization-based analyses in complex anisotropic media. In Fig. 1.3 (a), the individual elements of the Mueller matrix are visualized, revealing spatial variations in polarization response. The structured patterns indicate regions with different anisotropic properties, which are likely due to the sample's internal birefringence. In Fig. 1.3 (b), a pseudo-color representation

of the polarization properties is used, with different colors corresponding to distinct polarization parameters such as azimuthal orientation and degree of polarization. This approach enables precise characterization of birefringent materials, which is crucial in applications such as biological tissue analysis and stress-induced birefringence studies.

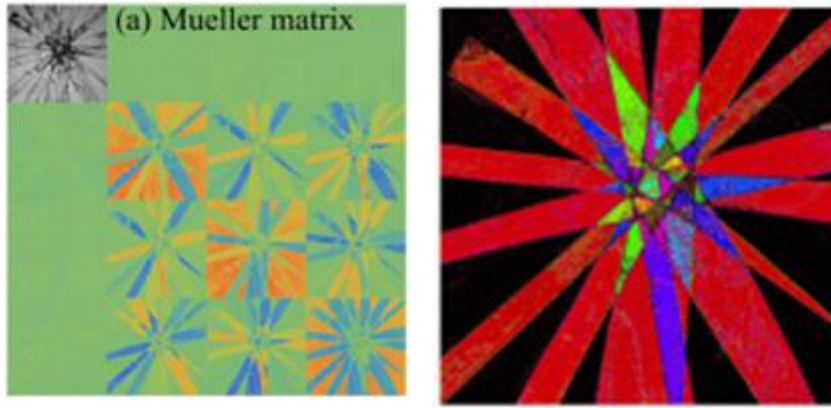


Figure 1-3. Mueller matrix representation and polarization-resolved visualization of an anisotropic sample. (a) Spatially resolved Mueller matrix elements, highlighting polarization-dependent transmission variations. (b) Pseudo-color image representing polarization properties such as retardance and orientation, emphasizing the anisotropic nature of the sample [53].

Another polarimetric technique is the so-called imaging Stokes polarimetry, which focusses on characterizing polarization spatial distribution existing from samples. It has been shown the ability of this technique to analyze real-time measurements as determination of surface anisotropies related to birefringence and dichroism [54], complementing traditional methods like ellipsometry. This technique enhances material science research by providing detailed insights into optical properties and internal structures of various specimens like chiral molecular ordering of liquid crystal droplets [55]. Recently, studies on materials like antiferromagnetic Weyl semimetals have demonstrated the utility of Stokes polarimetry in characterizing anisotropic optical and magneto-optical properties, which are essential for spintronic applications [56].

1.5 Methods for polarization measurements and polarimeter classes

Acquisition of polarimetric information is becoming increasingly important since there is an increasing number of applications where this information finds practical use, as has been highlighted in previous subsections 1.1-1.4. Our primary focus is on different classes analyzer's configurations that lead to passive Stokes-vector images. The complete polarization state of incoherent light can be expressed using the four Stokes parameters S_0 , S_1 , S_2 , and S_3 , indicating the overall light intensity component, the difference between horizontal and vertical linear polarization, the difference between 45° and 135° linear polarization, and the difference between right and left circular polarization, respectively.

When polarimetric images are necessary, image polarimeters are required. In imaging polarimeters we are interested in calculating the Stokes vector for each pixel of the image, so it can be categorized based on more complex architectures and data processing such as division of time polarimeters (DoTP), amplitude (DoAmP), aperture (DoAP), and focal plane (DoFP). In turn, in terms of time-dependence of the measurements, polarimeters have also been categorized into two primary classes [57]: division of time (or sequential) polarimeters, and simultaneous (snapshot) polarimeters. This classification is essential for understanding the evolution and application of polarimetric techniques across various fields.

1.5.1 Division of time

Division of time polarimeters utilize retarders whose effective retardance applied to the input state of polarization changes with time. As an example, this can be achieved by using liquid crystal panels (whose retardance can be controlled with the addressed voltage), by using an electro-optic retarder (a device that uses the electro-optic effect to change the polarization state of light passing through it) or by using rotating elements, such as rotated analyzers or rotated retarders. These time-dependent elements, followed by fixed analyzers, allow them to analyze the flux in

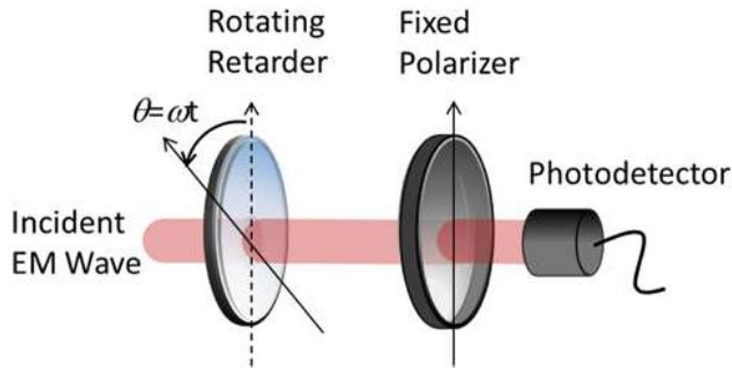


Figure 1-4. Scheme of a division of time polarimeter based on a rotating retarder and a fixed linear analyzer.

several states sequentially in time to measure polarization states. Three methods that make a series of polarimetric measurements over time to obtain the Stokes vector, are analytically outlined in Ref. [58] by Goldstein and in Ref. [59] by Tyo: rotating element polarimetry, oscillating element polarimetry, and phase modulation polarimetry. These systems were primarily point polarimeters, capturing polarimetric information at a single point of the light beam. Division of time polarimetric technique, while precise, had limitations in temporal resolution and was primarily suited for Mueller matrix polarimeters. These polarimeters are often chosen for Mueller matrix polarimeters [60] [61], which are opposed to the passive Stokes parameter system, and called active polarimeters that measure an object's polarizing properties. A classic example is the rotating retarder polarimeter, schematically illustrated in Fig. 1-4, where the temporal sequence of frames aids in Stokes parameters estimation [62] [63] [64]. However, this sequential nature can introduce artifacts in dynamic scenes with high polarization variations [65].

Numerous examples of polarimeters highlighted the role of liquid crystal retarders integrated into division of time polarimeters to enhance the time resolution.

They are also interesting devices because they may offer a more electronically controlled adjustments of the polarization state [65]. Variable-retardance (VR) polarimetry [66] for example, uses two mechanically fixed, variable retarders and a fixed linear polarization analyzer to make a set of measurements and modulate the phase of light from by which the Stokes vector can be reconstructed [66]. Moreover polarimeters based on photo elastic modulators [67], Faraday cells [68] are also well-known. The use of liquid crystal based and modulate phase based polarimeters is also interesting because such configurations reduce image wander, a common issue in traditional rotating systems. Advancements in this field have shifted focus from point detection to imaging polarimetry [69] [70], where spatially resolved polarization information is captured across an entire scene.

1.5.2 Simultaneous measurement polarimeters

There are different strategies that are used to analyze the flux in several states simultaneously, which is very important to approach the concept of real-time polarimetric measurements.

The first strategy is through beam amplitude division. It uses a single aperture but then splits the light using beam splitters to create multiple paths, and the different generated sub-beams are separately and instantaneously analyzed with different polarizing elements. An example of a division of amplitude polarimeter scheme is described in [71]. where a single system aperture is used, and four different polarization elements are arranged to create four displaced images on one focal plane.

Another approach to used to implement one-shot polarimeters is the division of wavefront approach. This technique involves splitting the light wavefront into multiple regions, each passing through different polarizing elements. The measurements from these regions are then combined to determine the overall

polarization state. An example of wavefront division polarimeter scheme can be consulted in Ref [72].

Thanks to recent advancements in polarimetric technology, micro-structured systems have significantly enhanced the potential of wavefront division polarimeters. Several integrated sensory devices that have been engineered for polarization imaging, for example a liquid crystal polarization camera. In Ref. [73] Oka et. al. presented a compact imaging polarimeter in which the azimuth and ellipticity angles spatial distribution can be determined from a single frame by using two pairs of birefringent prisms to spatially modulate the scene. Another type of sensor, the microgrid polarimeter, consists of an array of micro polarizers placed pixel by pixel on the focal plane array, this scheme leading to a revolution in the field of imaging polarimetry. A schematic of a microgrid is shown in Figure 1.5. This focal plane is then placed in a conventional imager. Despite the significant advantages and potential applications that this technology has introduced, its inherent structure and the need for macropixels (which involve analyzing multiple polarizations using individual pixels, each sensitive to a particular polarization) that are spatially repeated with certain period to measure the spatial polarimetric content, also bring certain drawbacks and measurement errors. For instance, since adjacent pixels are combined to estimate the Stokes parameters, estimation artifacts are introduced when there is spatial variation in the scene. In addition, typically microgrids are considered to have one quarter of the resolution of native detectors, and this considered to be a cost of the system. However, recall that other previously stated alternatives for real-time measurements, as the amplitude division polarimeters require four light paths, and therefore four times the number of detectors that the microgrid employs. Importantly, even knowing such resolution lost, the resolution obtained by microgrid polarimeters compared to other snapshot polarimeters is comparable [74].

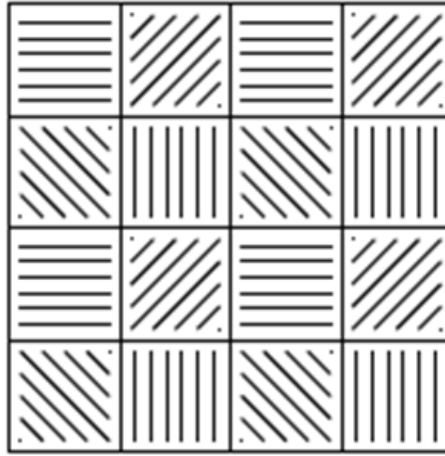


Figure 1-5. Scheme of the focal plane array for the microgrid polarimeter. Image taken from Ref. [75].

1.6 Imaging Polarimetry based on Polarimetric cameras

1.6.1 Monochrome and RGB Polarimetric Camera Architecture

Polarized sensors are used in polarimeter architecture, taking advantage of their ability to obtain polarimetric information in a scene with a single exposure. The main optical component depends on the idea of micro grid arrays, which is a pixelated wire grid polarizer [76], with each pixel being a wire grid polarizer oriented along a particular direction. Most often, the pixels are linear polarizers, being restricted to the linear content of polarization, but some other pixel designs have been proposed and fabricated for passing only the circular polarizations [77]. Few recent and residual proposals investigate the possibility to associate the polarizers-microarray with retarder-microarray for complete image Stokes metrology [78], [79], but the efficient the matching between the pixels of the retarder-based array, the linear polarizer-based array and the CCD sensor becomes an issue.

These new linear polarimetric cameras rely on microgrid polarizers matched on the sensor, effectively replacing rotating optical components in traditional polarimeters. In this way, architecture has no moving elements, so they avoid vibrations, beam wander and orientational alignment errors. In addition, this polarized sensor allows measuring the flux measurements simultaneously, leading to snapshot polarimeter.

Another very interesting feature for imaging applications is the ability to work with white light. To this end, the basic design of polarimetric cameras is combined with RGB filters to study the content of different frequencies separately. This results in macropixels composed of 16 pixels ($4 \times 4 = 16$ pixels, with one color filter, typically green, repeated twice) instead of the typical 4-pixel macropixels found in monochrome cameras. As an example, we review here the design of a commercial model: the Sony Polarsens sensors sketched in Fig. (1-6). The pixels are deposited with a micro polarizer array with four different directions (0° , 45° , 90° , 135°) and placed under the layer of micro lens array used to correctly focus the incoming light onto the pixel. The unit cell in this sensor is composed of 2×2 pixels. In the second sensor, Fig. (1-7), a Bayer color filter array is added, and the unit cell in this sensor is composed of 4×4 pixels. They can spatially resolve polarization information by taking flux measurements at each different polarized pixels through the scene. These types of polarimeters are suitable when dealing with samples and beams presenting significant polarization spatial distributions. The combination of a microgrid polarizer with a Bayer color filter array allows the simultaneous acquisition of color and polarization information.

The pixelated nature of the polarizer array results in spatial under sampling, leading to trade-offs between spatial resolution and polarization accuracy. Calibration is critical to correct polarization-dependent detector sensitivity and crosstalk between adjacent color filters in RGB polarized sensor.

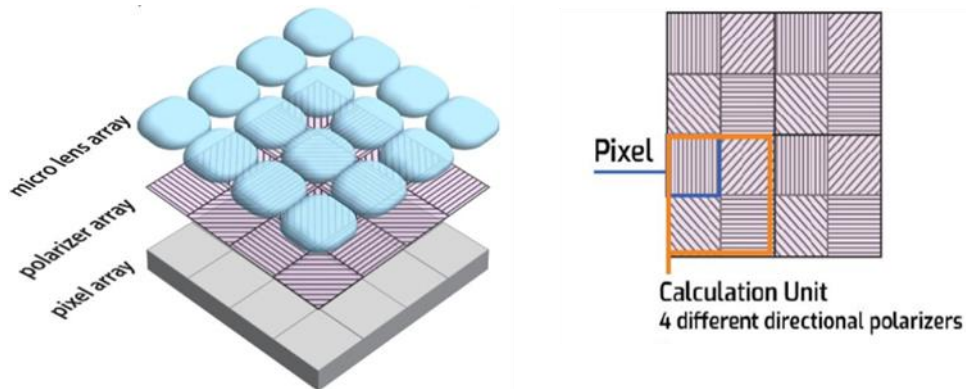


Figure 1-6. Monochrome polarized sensor [80]

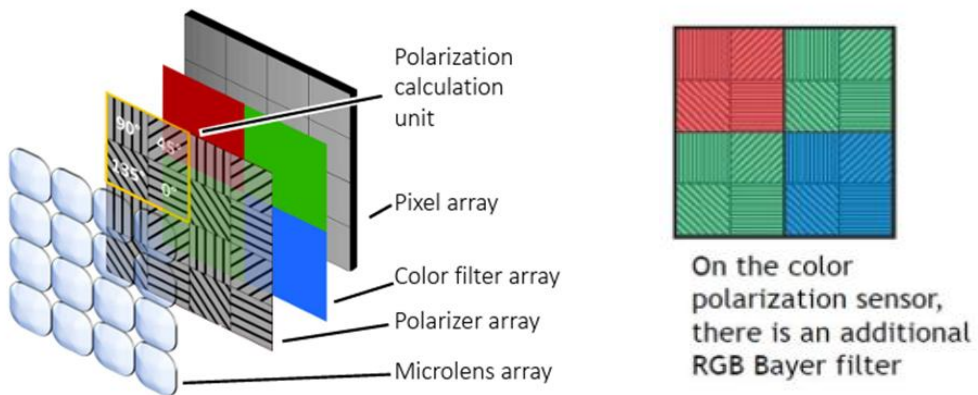


Figure 1-7. Color polarized sensor [80].

The need for an in-depth study on how to establish a calibration protocol for these devices, or to propose new prototypes capable of performing complete polarization measurements, are questions that motivate the realization of this thesis. The accuracy, precision and optical throughput of an imaging polarimetric system based on a micro polarizer array is ultimately limited by the performance of the two main

components in these compact devices which have an impactful effect on the working of this kind of imaging polarimeter. The first is the design of wire grid polarized sensors and the second is the color filters. Literature reviews about the problems in these types of polarimeters, as this technology suffers from spatial resolution limitation or color cross talk, and this thesis aims to offer solutions to these and other challenges. To further explore the motivation of this thesis, the following subsections review in more detail some limitations associated with both monochrome and color polarimetric cameras.

1.6.2 The Wire-Grid polarizer: Design and Limitations

A wire grid polarizer is composed of periodic thin, closely spaced, conducting wires. These polarizers are used to filter light waves. As shown in Fig. 1-8, when the incident unpolarized light impinges on the wire grid, the electric field can be decomposed into components parallel and perpendicular to the wires. Whereas the transmitted light is polarized perpendicularly to the wires, the parallel component to the wires is mainly absorbed. Of course, the performance of the polarizer (extinction ratio) is often highly influenced by the wavelength used in the illumination. Therefore, this parameter must be considered in system calibrations.

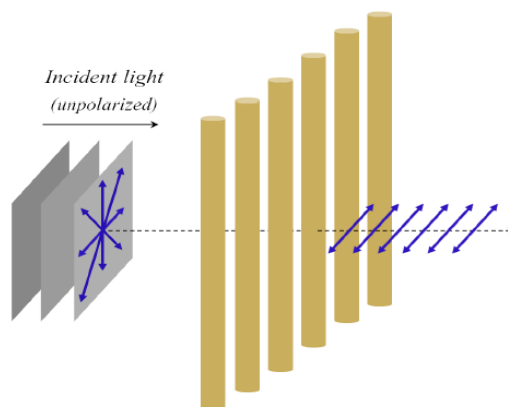


Figure 1-8. Wire grid polarizer is fabricated by aligning thin conducting wires, which preferentially absorb the electric field components parallel to the wire orientation [81].

A wire grid works as a polarizer, an anisotropic attenuator that attenuates the orthogonal components of an optical beam unequally. The two orthogonal transmission axes are designated by the amplitude attenuation coefficients along orthogonal transmission axes p_x and p_y ; where $0 < (p_x, p_y) < 1$. To characterize the wires grid polarizers and quantify the ability of polarizer to suppress one of the orthogonal electric field components while transmitting the other one, we use a key parameter called the extinction ratio. It is the ration between transmission intensities when the electric field is perpendicular to wires to intensities when the electric field parallel to wires. As said, this parameter depends on different factors such as the wire's fabrication quality, illumination wavelength and how the wire grids were manufactured and deposited to the sensor. In practice, there is not a perfect transmission along an orthogonal axis $p_x = 1$, or $p_y = 1$, or complete attenuation through the other axis, $p_x = 0$, or $p_y = 0$, but achieving the highest possible extinction ratio for a given wavelength range is the desired situation.

As an example, Fig. 1-9 show the transmission curve of aluminum (Al), gold (Au), and a perfect electrical conductor (PEC) substrate as an example of martials from which wire grids are fabricated. These polarizers are very efficient at transmitting the polarization states perpendicular to the wire direction (left) and blocking the state parallel to the wire direction (right). Note that the blocking intensities are different.

We see by these curves how some wire grid polarizers may be achromatic over large ranges within electromagnetic waves region. These curves show that the polarizer reaches optimum performance at wavelengths bigger than $0.5 \mu\text{m}$. The performance of wire grid polarizer is often characterized by their diattenuation performance or transmission curves as a function of wavelength, which is crucial for optimizing their efficiency across different spectral ranges.

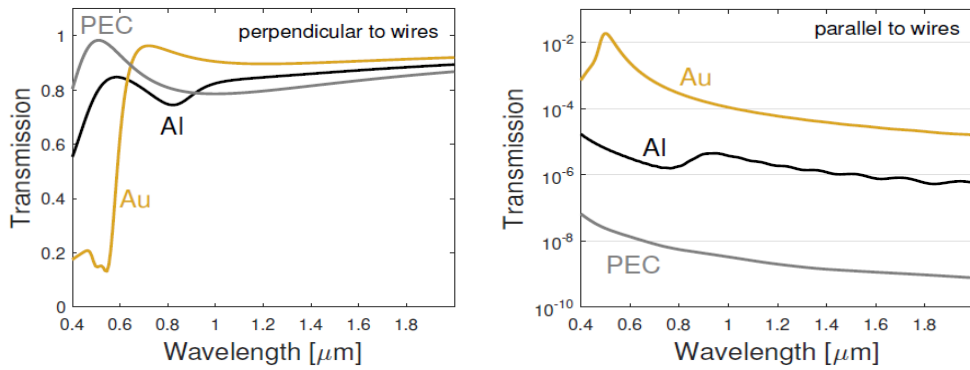


Figure 1-9. performance of a wire grid polarizers, made from aluminum (Al), gold (Au), and a perfect electrical conductor. The left curve is the transmission of the electric field perpendicular to wires, and the right curve is the transmission of the electric field parallel to wires [81].

Another factor that impacts on the performance and efficiency of the wire grid is the cross section. Figure 1-10 presents the scanning electron micrograph imaging the cross-section of a single pixel within a pixelated wire grid polarizer. One drawback of the wire grid polarizers is the cross talk due to diffraction at the edges of each micro polarizer, which decreases to certain extent the extinction ratio of micro polarizer wire grid. The specific materials and fabrication processes used can affect how the extinction ratio varies with wavelength.

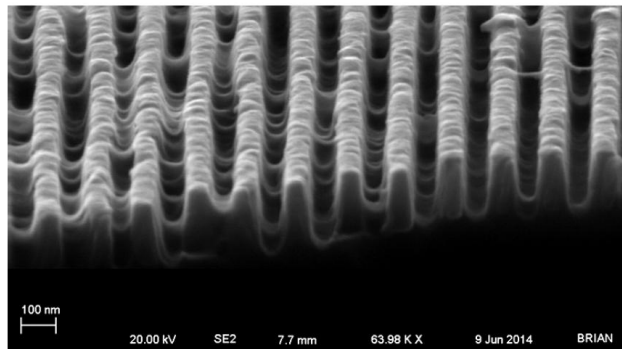


Figure 1-10. Scanning electron micrograph showing the cross- section of a single pixel within a pixelated wire grid polarizer [81].

The extinction ratio related to polarization image sensor can also be influenced by sensor's design. Following with the example of the Sony's polarimetric camera, we show the sensor's design in Fig. 1-11. It incorporates wire-grid polarizers formed directly on the chip, beneath the on-chip lens layer. This design minimizes the distance between the polarizer and the photodiode, improving the extinction ratio and reducing dependence on the incident angle.

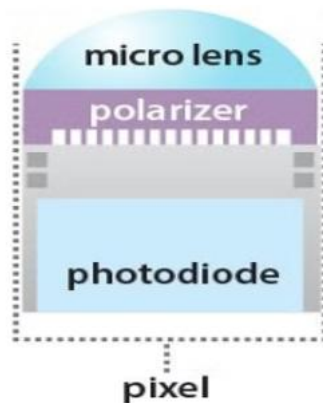


Figure 1-11. Polarized pixel structure in Sony sensor [80].

Complementary, Fig. 1-12 provides the extinction ratio of the polarization image sensor. It is noted an extinction ratio of 330 at a wavelength of 500 nm, which decreases to 130 at 650 nm. This indicates that while the extinction ratio decreases with increasing wavelength, the specific values and rate of decrease are determined by the sensor's design and materials. Therefore, the extinction ratio characteristics depicted in Sony's figure reflect the performance of their specific sensor design and materials across different wavelengths. While the general trend of decreasing extinction ratio with increasing wavelength holds, the exact performance is tailored by Sony's engineering choices to optimize the sensor's effectiveness across the visible spectrum. In this figure, the extinction ratio is actually higher in the blue (~400–450 nm) and decreases toward the red (~700 nm).

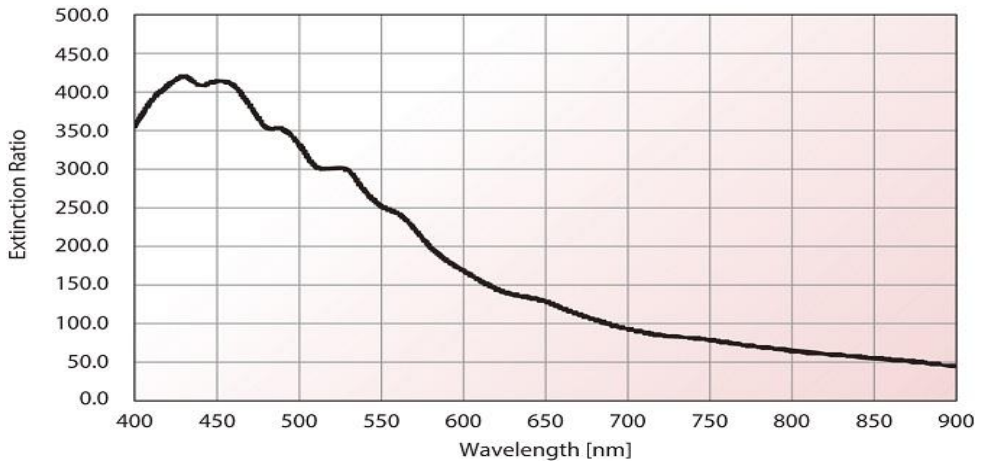


Figure 1-12. Extension Ratio of Sony sensor [80].

1.6.3 Color Filters: Design and Limitations

The spectral response of the color filters associated with RGB polarimetric cameras is another important parameter to be considered in calibrations for optimal device performance. The idea is that when measuring with a signal at a frequency suitable for a specific filter, due to the dispersion in the transmission of the filters, other filters optimized for different colors may also produce a non-zero response. This is also related to the nominal bandwidth of the color filters set in the camera pixels. As an example, the spectral response of the color filter of the polarimetric sensor studied in Ref. [82] is shown in Fig. 1-13. The filters have a non-zero response outside their peak optimized wavelength due to their bandwidth. Consequently, the tails of these non-zero transmissions can affect measurements in pixels with filters designed for other colors. This overlap leads to undesired color crosstalk effects, making it essential to implement calibration and removal methods to optimize the final measurements. A study provided in Ref. [83] proposed to use an external color filter in front of the beam splitter to narrow the response region of the camera color sensor. In our case, we will propose in this Thesis methods to correct the color cross talk in each channel. Our goal is to measure the polarization Stokes vector with several

discrete wavelengths at the same time with the highest resolution. For a proper measurement without the color cross talking, we will propose the application of demosaicking algorithms [84], [85] applied for the first time in framework of RGB polarimetric cameras. In addition, to obtain the maximum resolution some kind of interpolation will be necessary [86] .

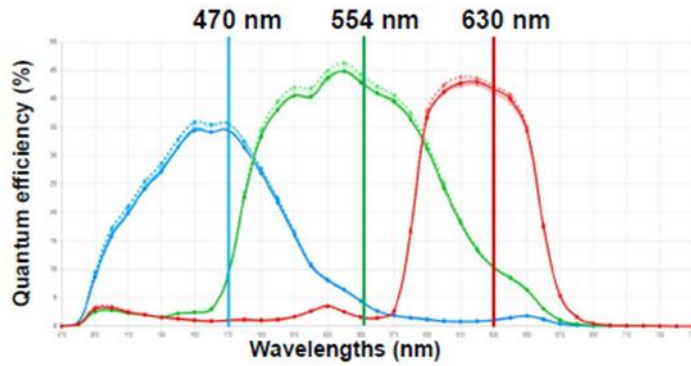


Figure 1-13. Spectral response of the color filters of RGB polarimetry sensor studied in Ref [80].

Chapter 2 Mathematical formalism

This chapter introduces the mathematical formalism used in this thesis to describe the polarization of the light, build complete Stokes polarimeter for measuring polarization state of light and to model the performance of polarimeter. First, the polarization of light is introduced, different mathematical models to describe the state of polarization of a light beam and its polarization characteristics. Next, it presents a general formulation of the measurement principle and the data reduction procedure of a polarimeter. In addition, the importance of optimizing the system in order to reduce the amplification of additive noise in the flux measurements during data reduction is addressed in this chapter. In particular, some figures of merit evaluating different aspects of the optimization of the system are reported. Moreover, some guidelines providing how to conduct a study of the system's robustness are also reviewed. Afterwards, it includes the methodology for determining the parameters tolerance in order to achieve a particular accuracy in the polarization measurements when considering experimental errors in the set-up. Furthermore, two methods for calibrating the polarimeter are explained.

2.1 Polarization of light

Light is an electromagnetic wave whose properties can be used to explain the behavior of the light. An electromagnetic wave is composed of two fields: an electric field, and a magnetic field. Both are described by vector fields with a magnitude and direction that are variables with the time and the position in space (x , y , z). Nonetheless, since in the case of visible light, the magnetic field amplitude is significantly lower than that of the electric field, we generally choose the vector of the electric field as reference for light propagation and features.

Christian Huygens was the first to suggest that light was not a particle but rather it behaves as a wave, based on his work on the propagation of light through crystals. In 1818 Fresnel and Arago carried out a series of fundamental investigations into Young's interference experiment, confirming the wave nature of light. The experiments of Fresnel and Arago led to the discovery that light consisted of two transverse components that are perpendicular to each other. In 1865, Maxwell first described light as an electromagnetic wave. According to electromagnetic theory, light propagates through free space as a transverse wave, where the electric field $\vec{E}$ and magnetic field $\vec{H}$ oscillate perpendicularly to each other and both are perpendicular to the propagation direction [82], [87]. which is given by the Poynting vector. Under this framework, polarization of light arises as a fundamental characteristic of transverse electromagnetic waves.

The polarization of light relates to the orientation of the electric field in the electromagnetic wave. Elliptical polarization is the most general form of the state of polarization for fully polarized light beams. In such a case, the electric field rotates uniformly in a helical pattern as the wave propagates.

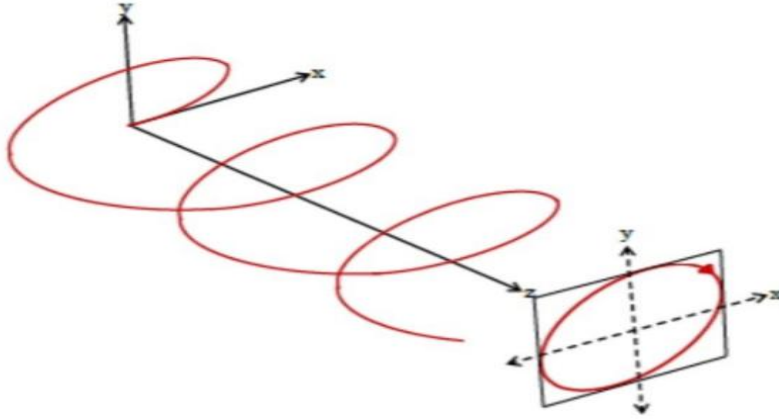


Figure 2-1. Electric field vector in space at any instant of time for an elliptically polarized light (rep) and the ellipse traversed by the vector in the xy-plane [88].

For the light propagating along the z-axis, it can be described as the superposition of two orthogonal linearly polarized components as shown in Figure 2-1.

$$\vec{E}(z, t) = E_x(z, t)\hat{x} + E_y(z, t)\hat{y} \quad 2.1$$

For light of angular frequency ω , wave number k and direction of propagation z , the electric field oscillates in the x-y plane such that it can be represented as a vector in Cartesian coordinates:

$$\vec{E}(z, t) = E_{0x} \cos(kz - \omega t + \varphi_x)\hat{x} + E_{0y} \cos(kz - \omega t + \varphi_y)\hat{y} \quad 2.2$$

where t represents time, $\hat{x}$ and $\hat{y}$ are the orthogonal unit vectors, E_{0x} and E_{0y} are the maximum electric field amplitudes in x and y directions, respectively, and φ_x and φ_y are the initial phases of orthogonal waves in x and y directions, respectively.

As the field propagates with time and through the space direction z of propagation, $E_x(z, t)$ and $E_y(z, t)$ give rise to a resultant vector at each time t and z position. This vector describes a locus of the tip of the electric field vector (at fixed position z) generally describes an ellipse with rotated main axis which is defined by [59], where

$$\frac{E_x^2(t)}{E_{0x}^2} + \frac{E_y^2(t)}{E_{0y}^2} - 2 \cos(\varphi) \frac{E_x(t)E_y(t)}{E_{0x}E_{0y}} = \sin^2(\varphi) \quad 2.3$$

($\varphi = \varphi_x - \varphi_y$) is the phase of the y component relative to the x component of the electric field.

Equation 2.3 describes the expression of a rotated ellipse described in the basis of the electric field orthogonal axis. (see Fig. 2-2(a)) with $\vec{E}$ as the radius vector. This description of light in terms of the polarization ellipse is very useful because it enables us to describe by means of a single equation any state of polarized light fully polarized. The time evolution of the equation is viewed along the direction of propagation of light, the tip of the electric field may rotate clockwise (or anticlockwise), and it is called right-handed (left-handed) elliptical polarization.

In addition, the reference frame can also be chosen to describe the polarization ellipse from its principal axes (see in Fig. 2-2(b) dashed rectangle where the ellipse is inscribed in its principal axes a and b , in the directions of the coordinate system x' and y' , which is rotated through an angle ψ with respect the coordinate system x - y).

Mathematically, we can apply a transformation (rotation) to the polarization ellipse form in Eq. 2.3 to write the ellipse in its principal axes, referred to the $\hat{x}$, $\hat{y}$ -coordinate system rotated by an angle Ψ with respect to xy -coordinate referring [2]). This is written as:

$$\frac{\hat{E}_x^2}{a^2} + \frac{\hat{E}_y^2}{b^2} = 1 \quad 2.4$$

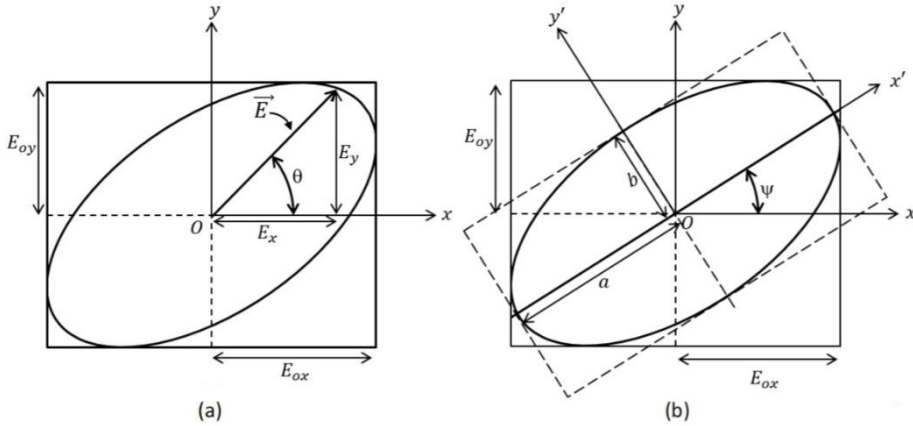


Figure 2-2. Representation of the Polarization ellipse (Eq. 2) : (a) in the x-y coordinate system at a given instant of time; (b) Described with respect the principal axes of the ellipse (Eq. 2.5) [88].

This description of light in terms of the polarization ellipse is very useful to study and describe fully polarized states of light, where the ellipse control parameters determine the particular state of polarization of the wave. In particular, Eq. 2.3 shows that only three parameters are needed to describe the polarization ellipse: the amplitudes for each component E_{0x} , E_{0y} , and the phase difference φ between them. The polarization ellipse degenerates to special forms of great interest for certain values of E_{0x} , E_{0y} and φ , as are the linear polarizations ($\varphi = 0$ or $\mp \pi$), or the circular polarizations, ($E_{0x} = E_{0y}$ and $\varphi = \mp \frac{\pi}{2}$). As an example, in the second and third columns in Table 2.1 there is a summary of the control parameters values associated with different cases of interest.

Polarization State	Amplitude	Phase	Equation	Stokes Parameter
Linear horizontal polarization	$E_{0y} = 0, E_0 = E_{0x}$	$\phi = \pm n\pi, n \geq 0$	$E = E_x + E_y = E_x, E_x(z, t) = iE_0 \cos(kz - \omega t)$	$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
Linear vertical polarization	$E_{0x} = 0, E_0 = E_{0y}$	$\phi = \pm n\pi, n \geq 0$	$E = E_x + E_y = E_y, E_y(z, t) = jE_0 \cos(kz - \omega t)$	$\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$
Linear polarization at $+45^\circ$	$E_0 = E_{0x} = E_{0y}$	$\phi = \pm n\pi, n \geq 0$	$E = E_x + E_y, E = (i + j)E_0 \cos(kz - \omega t)$	$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$
Linear polarization at -45°	$E_0 = E_{0x} = E_{0y}$	$\phi = \pm n\pi, n \geq 0$	$E = E_x + E_y, E = (i - j)E_0 \cos(kz - \omega t)$	$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$
Right circular polarization	$E_{0x} = E_{0y}$	$\phi = \pm n\pi/2, n \geq 0$	$E = E_0(i \cos(kz - \omega t) + j \sin(kz - \omega t))$	$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
Left circular polarization	$E_{0x} = E_{0y}$	$\phi = \pm n\pi/2, n \geq 0$	$E = E_0(i \cos(kz - \omega t) - j \sin(kz - \omega t))$	$\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Table 2-1. Different polarization states: second and third columns provide the values for the amplitudes and relative phases between light components related to polarizations noted in first column. The fourth column provides the associated electromagnetic equation. The last column provides corresponding Stokes vector forms.

Following the same example, some comments about information provided in Table 2.1 are given below:

- 1- Linearly horizontal polarized light and linearly vertical polarized light are set when one of the orthogonal components of the electric field, i.e. E_x or E_y are equal to zero (i.e., $E_{0x}=0$ or $E_{0y}=0$).
- 2- The polarization ellipse in the orthogonal electric field basis (i.e., in the x - y coordinate system) and in the principal ellipse axes (i.e., in the x' - y' coordinate system) coincides (i.e. the rotation angle ψ between reference systems is equal to 0, see Fig. 2.2) when $\varphi=\frac{\pi}{2}$ or $\varphi = \frac{3\pi}{2}$, and according to Eq. (2.4), it takes the form of:

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} = 1 \quad 2.5$$

The special case of left/right-handed circular polarization is obtained when maxim amplitudes of orthogonal components are equal ($E_{0x} = E_{0y} = E_0$):

$$\frac{E_x^2}{E_0^2} + \frac{E_y^2}{E_0^2} = 1 \quad 2.6$$

- 3- Arbitrary orientation for linearly polarized light is described with the equation of a straight line $E_y = \pm \left(\frac{E_{0y}}{E_{0x}}\right) E_x$, whose slope is given by the ratio of amplitudes, and which is obtained when $\varphi = 0$ or $\varphi = \pi$. The special case of linear +45 polarized light and linear -45 polarized light will generate if ($E_{0y}=E_{0x}$).

2.2 Polarization mathematical model

The polarization ellipse is an idealization of the true behavior of light; it is only a good approximation at any given instant of time. However, in numerous cases, as the beam of light propagates through space, the polarization ellipse parameters (relative amplitudes or relative phase between components) may vary with time, which give rise to changes on the ellipse ellipticity or/and orientation. This can be understood as changes of the light state of polarization, which in turn, give rise to partial or fully depolarized beams [59].

To consider depolarization content in light beams, a very proper mathematical formulation is the Stokes vector formalism. This account for temporal averages of the polarization ellipse parameters, which give rise to four intensity parameters which describe all polarization content of light beams, including depolarization. In addition, as these parameters are based on intensity magnitudes, they can be directly measured in laboratories. The Stokes vector are named after G.G. Stokes who first introduced them [89] in 1858.

The four Stokes parameters can be derived from proper combinations of polarized flux measurements. Let us assume monochromatic waves with constant amplitudes E_{0x} and E_{0y} and the phase difference φ . Since the electrical components E_x and E_y are implicitly time dependent as travelling waves, it is necessary to average over a single period of vibration $\langle \dots \rangle^T$. However, in view of the periodicity of $E_x(t)$ and $E_y(t)$, we need average the Eq. 2-3 only over a single period of oscillation. So, we rewrite Eq. 2-3 as:

$$\frac{\langle E_x^2(t) \rangle^T}{E_{0x}^2} + \frac{\langle E_y^2(t) \rangle^T}{E_{0y}^2} - 2 \cos(\varphi) \frac{\langle E_x(t) E_y(t) \rangle^T}{E_{0x} E_{0y}} = \sin^2(\varphi) \quad 2.7$$

After finding the average values and expressing the result in terms of intensity will lead to squares equation [87] :

$$\begin{aligned}
 (E_{0x}^2 + E_{0y}^2)^2 - (E_{0x}^2 - E_{0y}^2)^2 - (2E_{0x}E_{0y} \cos(\varphi))^2 \\
 = (2E_{0x}E_{0y} \sin(\varphi))^2
 \end{aligned} \tag{2.8}$$

The different terms in Eq. 2.9 can be labeled by the so-called Stokes parameters S_0 , S_1 , S_2 and S_3 , and subsequently, Eq. 2.9 is rewritten as:

$$S_1^2 + S_2^2 + S_3^2 = S_0^2 \tag{2.9}$$

These four magnitudes can be arranged in a column vector form, in the so-called Stokes vector [59]:

$$S = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y} \cos(\varphi) \\ 2E_{0x}E_{0y} \sin(\varphi) \end{pmatrix} \tag{2.10}$$

The condition in (2.9) and parameters in Eq. (2.10) reflects the condition for fully polarized monochromatic light, where S_0 states for the total intensity of the beam, and where the fully polarized content is completely described by the three independent polarized components S_1, S_2, S_3 . In particular, S_1 indicates the amount of horizontal or vertical linear polarization, S_2 quantifies the amount of diagonal or anti-diagonal linear polarization, and S_3 corresponds to the amount of right-handed or left-handed circular polarization.

In those cases where the beam also includes certain depolarization, the total intensity has certain non-polarized contribution and certain polarized contribution, and consequently, the equality in Eq. 2.9 becomes the following inequality:

$$S_1^2 + S_2^2 + S_3^2 \leq S_0^2 \tag{2.11}$$

On the other hand, whereas Stokes vectors describe the polarization characteristics of beams, within the Mueller-Stokes formalisms, Muller matrices describe the polarimetric properties of materials and are essential to modeling polarimetric light-matter interactions. Mueller matrices describe the linear

interaction of polarized light with an optically anisotropic (dichroic and/or birefringent) medium, as well as partially or fully depolarizing media. As they relate input and out polarizations, which are represented by 4x1 Stokes vectors, Mueller matrices are 4x4 matrices of real (intensity) elements. Under this approach, it is usually represented in the form:

$$M = \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix} \quad 2.12$$

where m_{ij} (with $j,k \in \{0,1,2,3\}$) are the Mueller matrix elements that encode polarimetric information of samples.

Under this scenario, the Mueller matrix of a system is defined as the matrix M which transforms an incident Stokes vector $\vec{S}_{in}$ into the exiting (reflected, transmitted or scattered) Stokes vector $\vec{S}_{out}$,

$$\vec{S}_{out} = M \cdot \vec{S}_{in} \quad 2.13$$

The state of polarization and its polarization ellipse can also be represented on the Poincare sphere, which is a very useful tool for visual descriptions (see Fig. 2.3). At the surface, the azimuth, ψ , and ellipticity, χ , angles, corresponding to the orientation and ellipticity (flattening) of the polarization ellipse, are associated to parallels and meridian lines, respectively. Importantly, the Stokes parameters can be thought of as coordinates on the surface of the Poincare sphere, so each axis is associated to S_1 , S_2 and S_3 parameters, respectively. It can be understood as a 3-dimensional plot of the Stokes vector, where the Cartesian coordinates correspond to the Stokes parameters, as depicted in Fig. 2.3.

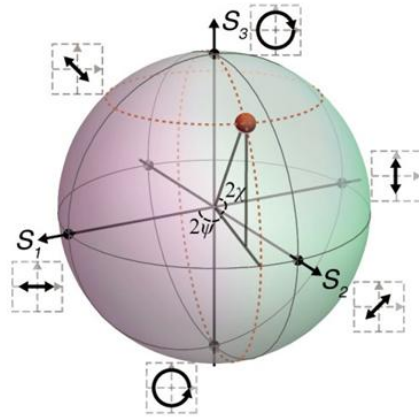


Figure 2-3. The Poincaré sphere. Each polarization state of light can be accurately characterized by a single point on the Poincaré sphere. The two parameters, main axis angle ψ and ellipticity angle χ , not only specify the polar coordinates on the sphere but also provide information about the shape of the polarization ellipse [90], [91].

Hence, any polarization state can be represented as a point on the unity-radius sphere. The points on the surface correspond to fully polarized states, while partially polarized states lie inside the Poincaré sphere at a distance $P = \sqrt{S_1^2 + S_2^2 + S_3^2}$ from the origin. Totally unpolarized light will be represented by a point at the origin of coordinates. The equator represents all linear polarization states since the S_3 parameter is null and no ellipticity is present. The "north" and "south" poles of the sphere represent purely right- and left-circular polarization, respectively. Any point lying between the equator and the poles corresponds to an elliptically polarized state. The angle of the ellipticity (χ) depends on the angle along the equator and the azimuth angle (ψ) accounts for the angle between S_1 and S_2 axes.

When the Stokes vector of a general polarization (partially or fully polarized) is normalized, by its first component S_0 , the resulting normalized Stokes vector, representing the fully polarized content of the beam, can be represented on the surface of a unitary Poincaré sphere. By considering the geometrical relations between a point on the Poincaré sphere described according cartesian coordinates

(S_1, S_2, S_3) or spherical coordinates (χ, ψ) , the Stokes vector in Eq. 2.10 can be written as a function of ellipticity and azimuth angles of the polarization ellipse:

$$S = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_0 \end{pmatrix} = \begin{pmatrix} P \cos(2\psi) \sin(2\chi) \\ P \sin(2\psi) \sin(2\chi) \\ P \sin(2\chi) \\ S_0 \end{pmatrix} \quad 2.14$$

Under this scenario, it is often useful to calculate polarization ellipse parameters as the angle of polarization ψ , and the ellipticity χ from Stokes parameters (that can be measured into laboratories by using Stokes polarimeters), as follows:

$$\text{Angel of Polarization} : \psi = \frac{1}{2} \arctan\left(\frac{S_2}{S_1}\right) \quad 2.14$$

$$\text{Ellipicity} : \chi = \frac{1}{2} \arcsin\left(\frac{S_3}{S_0}\right) \quad 2.15$$

Interestingly, the Stokes vector can be separated mathematically as the addition of into two vectors representing the fully polarized content (S_p) and the unpolarized content (S_{unpol}) of the light beam [75] .

$$S = S_p + S_{unpol} = \begin{pmatrix} \sqrt{S_1^2 + S_2^2 + S_3^2} \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} + \left(S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2} \right) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad 2.17$$

In addition, as light beams with certain content in depolarization accomplish the above-stated relation of $S_1^2 + S_2^2 + S_3^2 \leq S_0^2$, this allow us to stablish a criteria to quantify the fraction of depolarization light into a beam, by using the so-called Degree of Depolarization (DoP) [75], which is calculated as the ration between the

fully polarized contribution with the total intensity of the beam[1]: Mathematically, it is expressed in terms of the Stokes parameters in the following manner:

$$\text{Degree of Polarization: } DoP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad 2.18$$

A particular case of interest is to estimate the fraction of depolarized light with regards only linear polarization content in a beam, this being calculated with the so-called the Degree of Linear Polarization DoLP:

$$\text{Degree of Linear Polarization: } DoLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0} \quad 2.19$$

Analogously, if the fraction of circular polarization with respect depolarization is considered, the Degree of Circular Polarization (DoCP) can be used, which accounts for the S_3 parameter (circular polarization) versus total intensity given by S_0 :

$$\text{Degree of Circular Polarization: } DoCP = \frac{S_3}{S_0} \quad 2.20$$

2.3 Stokes Polarimeter: measuring principle and optimization

The main devices to characterize polarization features of light beams are Stokes polarimeters [75]. Stokes polarimeters can be divided into different categories as a function of some characteristics, such as their optical architecture, functionality, measuring capacity, etc. For instance, they may be divided into punctual or imaging polarimeters. Whereas punctual Stokes polarimeters measure polarization at a single point, imaging Stokes polarimeters captures polarization information across an entire image, this last case being the configuration of interest in this thesis.

Another category to divide Stokes polarimeters is on whether they provide complete polarimetric of light beams or focus on specific information: (1) Complete Stokes polarimeters if the four Stokes parameters can be determined from its

measurements, and (2) Incomplete Stokes Polarimeters which provide partial information (they measure a subset of the four Stokes elements).

In order for a Stokes polarimeter to conduct complete measurement of the Stokes vector corresponding to a light beam, it is necessary to implement a linearly independent system connecting different radiometric measurements obtained by projecting the analyzed Stokes vector over different measuring polarization configurations (polarization analyzers) [91], [92]. which are usually implemented by using combinations of well-known polarimetric elements, as retarders and polarizers. The methods to achieve this are referred to as the 'measuring principle'.

The Stokes vectors have four degrees of freedom (the four Stokes parameters), and thus, they can be obtained by taking at least four different independent radiometric measurements corresponding to four different polarization analyzers. In other cases, it is common to obtain the complete Stokes parameters from a larger number of polarization analyzers, which introduce data redundancy to the system and thus reduce statistical noise in the measurements. One very well-known measuring configuration, for its simplicity and well proved results in terms of noise amplification is based on the following six distinct intensity measurements [93]:

$$S = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} I_{0^\circ} + I_{90^\circ} \\ I_{0^\circ} - I_{90^\circ} \\ I_{45^\circ} - I_{135^\circ} \\ I_R - I_L \end{pmatrix} \quad 2.21$$

where the six intensities I_{0° , I_{45° , I_{90° , I_{135° , I_R , and I_L correspond to the intensities measurement when analyzing the horizontal, diagonal, vertical, and anti-diagonal linear polarization, and the right-handed and left-handed circular polarization, respectively.

As said, the Stokes polarimeters operate by acquiring intensity measurements with a set of polarization state analyzers (PSA). Given a number N of Polarization Analyzers that can be configured with a polarimeter, we can mathematically describe

the intensity measurement (I_q) corresponding to the projection of an arbitrary SOP over the q -th analyzer as:

$$I_q = A_q \cdot S \quad 2.22$$

The behavior of the whole N analyzers is characterized in the instrument matrix A , which is composed by staking vertically for each analyzer of the system (which is given by the first row of the Mueller matrix corresponding to the particular PSA configuration) [59], [94].

$$A = \begin{pmatrix} A_0^1 & A_1^1 & A_2^1 & A_3^1 \\ A_0^2 & A_1^2 & A_2^2 & A_3^2 \\ \vdots & \vdots & \vdots & \vdots \\ A_0^N & A_1^N & A_2^N & A_3^N \end{pmatrix} \quad 2.23$$

Hence, the intensity vector $\vec{I}$ detected in after the PSA is calculated by applying the instrument matrix to the incident Stokes vector:

$$\vec{I} = \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{pmatrix} = A \cdot \vec{S}_{in} = \begin{pmatrix} A_0^1 & A_1^1 & A_2^1 & A_3^1 \\ A_0^2 & A_1^2 & A_2^2 & A_3^2 \\ \vdots & \vdots & \vdots & \vdots \\ A_0^N & A_1^N & A_2^N & A_3^N \end{pmatrix} \cdot \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad 2.24$$

To calculate the unknown Stokes vector from a set of measured intensities consisting of the vector $\vec{I}$, the inverse of (A) is determined and applied to the measured data. The measured value for the incident Stokes parameters $\vec{S}_{in}$ is related to the data by the polarimetric data reduction matrix A^{-1} :

$$\vec{S}_{in} = A^{-1} \cdot \vec{I} \quad 2.25$$

In case of non-rectangular A matrices, the pseudo-inverse of (A) is calculated [94]. Note that under this scenario, for a proper performance of a Stokes polarimeter, a calibration procedure to determine the instrumental matrix (A) is mandatory.

2.3.1 Polarimetric calibration and data reduction matrix

In experimental implementations, polarization elements in the PSA are not ideal, and expected values of configuration parameters, as orientations, retardances or diattenuation values of polarization elements, deviate to a certain extent from the actual experimental values. This causes the experimental analyzers basis implemented in a polarimeter, always differ slightly from the theoretical estimated analyzers. If this fact is not considered, significant accuracy errors can propagate to the final measurements. To prevent this situation, it is crucial to calibrate the instrumental matrix (A) of the system. In this Thesis, the Data Reduction Matrix (DRM) [95] [96], [97], method is applied to calibrate matrix (A). This method consists in finding the (A) matrix directly solving Eq. 2.24 for a collection of know input states of polarization, arranged as different columns of the matrix $\vec{S}$, and by registering the corresponding exiting intensities, for a system of N polarization analyzers defining the unknown instrumental matrix (A). Under this scenario, if SOP are already known and the intensity vector $\vec{I}$ is measured, the only remaining parameter to be calculated is (A). This situation can be mathematically expressed as:

$$\vec{I} = \begin{pmatrix} I_1^1 & I_1^2 & \dots & I_1^M \\ I_2^1 & I_2^2 & \dots & I_2^M \\ \vdots & \vdots & \vdots & \vdots \\ I_N^1 & I_N^2 & \dots & I_N^M \end{pmatrix} = A_{calib} \cdot \vec{S} = \begin{pmatrix} A_0^1 & A_1^1 & A_2^1 & A_3^1 \\ A_0^2 & A_1^2 & A_2^2 & A_3^2 \\ \vdots & \vdots & \vdots & \vdots \\ A_0^N & A_1^N & A_2^N & A_3^N \end{pmatrix} \cdot \begin{pmatrix} S_0^1 & S_0^2 & \dots & S_0^M \\ S_1^1 & S_1^2 & \dots & S_1^M \\ S_2^1 & S_2^2 & \dots & S_2^M \\ S_3^1 & S_3^2 & \dots & S_3^M \end{pmatrix} \quad 2.26$$

where $\vec{S}$ is a $4 \times M$ matrix including the known input SOPs, A_{calib} is the instrumental matrix with dimensions of $N \times 4$ and accounting for all the polarimeter analyzers and $\vec{I}$ is an $N \times M$ matrix including the intensities corresponding to all possible combinations (projections) between input SOPs and analyzers.

The data reduction matrix A^{-1} is determined by performing the right pseudo-inverse of $\vec{S}$:

$$A_{calib} = \vec{I} \cdot \vec{S}^{-1} \quad 2.27$$

Note that if different wavelengths are used for illumination, or the beam is measured at different incident angles, each one of these configurations requires a particular calibration for the instrumental matrix, as the specific polarization analyzers depend on these parameters [98] .

2.3.2 The optimization in the Stokes polarimeter

Beyond calibration, a crucial issue for an optimal performance of polarimeters is the optimization procedure. In fact, it can be chosen an infinity number of combination of different polarization analyzers to construct the instrumental matrix (A) in Eq. 2.27, but their performance in terms of noise amplification to the final Stokes measurements may highly differ from one selection to another. This situation is closely related to the matrix inversions required to calculate the Stokes vectors, as described in the previous section. Therefore, there are multiple metrics in the literature that allow for an efficient selection to minimize the propagation of errors associated with these calculations.

One well-know approach for a Stokes polarimeter optimal performance is to study their associated measurement matrix (A) in terms of conditioning. The conditioning of (A) can be analyzed using singular value decomposition (SVD), which provides valuable insight into its numerical stability. This approach was introduced to polarimeter design by Tyo [99] and Sabatke et al. [100], offering guidance on constructing stable data reduction matrices.

Different indicators are obtained from the SVD of the matrix (A), which consists in a factorization of the form $A = UDV^T$, where U and V are two unitary matrices and D is a diagonal matrix. The diagonal elements of D are so called singular values (σ) of (A).

In the context of SVD, the transformation of (A) into diagonal matrix D using orthogonal basis U and V allows for easy identification of the intrinsic properties of A. By performing SVD, you can assess how (A) transforms different input states,

and the guiding the design of more robust and accurate polarimetric measurement systems. There are two categories of states: the first one as (A) a well-conditioned matrix, and this happen if all singular values are large and comparable, and in this case, this means that small measurement errors will not lead to disproportionately large variations in the Stokes parameter estimates during applying equation 2.25. On the other hand, the undesirable scenario is called ill-conditioned matrix, and when (A) has very small or zero singular values, it is ill- conditioned or nearly singular, indicating that it may not effectively differentiate between certain input states, leading to potential measurement inaccuracies. Understanding these concepts ensures that the measurement matrix (A) is designed to be as far from singular as possible, enhancing the reliability and precision of the polarimeter.

According to this, as a typical example of complete Stokes polarimeter setup, the measurement matrix (A) consists of four rows corresponding to four polarization analyzers, with a rank of 4. This rank indicator ensures that (A) is non-singular, meaning that the system can uniquely determine the Stokes vector. The rank of (A) is defined by the number of nonzero singular values, which is independent of the number of polarization analyzers used. The polarimetric measurement matrix (A) should be as far from singular as possible, ensuring it is well-conditioned.

When (A) has full rank (rank = 4), its inverse exists and is unique. Consequently, direct inversion can be used to compute the Stokes vector as in previous equation 2.26. However, in overdetermined systems where the number of polarization analyzers exceeds four the inverse of (A) is not unique. In such cases, the pseudoinverse is employed to determine the best least-squares solution for the Stokes vector, as described in the following equation:

$$\vec{S}_{in} = \tilde{A}^{-1} \cdot \vec{I} \quad 2.28$$

Where $\tilde{A}^{-1} = (A \cdot A^T)^{-1} \cdot A^T$.

In consequences, the rank of the polarimetric measurement matrix (A) identifies a polarimeter as complete or incomplete. The rank of (A) should be 4 for a complete Stokes polarimeter and smaller than 4 for incomplete polarimeter. After doing these steps, the Stokes polarimeters make number of measurements to reconstruct the Stokes parameters.

At this stage, the above-sated knowledge has been used to develop useful metrics to estimate the conditioning of instrumental matrices (A), and subsequently, the final performance of associated Stokes polarimeters in term of noise amplification.

Under this scenario, Tyo developed the Condition Number metric (CN), based on SVD of (A), to improve detection by means of choosing the optimal polarization analyzers. This figure of merit can be written in terms of the norms of the instrumental matrix, (A) [101]. It based on the norms of the measurement matrix, (A):

$$CN(A) = \|A\| \cdot \|A^{-1}\| \quad 2.29$$

And equivalently, it is commonly written in terms of the maximum and minimum singular values of (A):

$$CN(A) = \frac{\sigma_{max}}{\sigma_m} \quad 2.30$$

Although mathematically the minimum condition number of a matrix correspond to unitary matrices ($CN=1$), which do not amplify errors, in ref [102], it is demonstrated that the best condition number that can be achieved for an optimal polarimeter is $CN = \sqrt{3}$, due to constrictions associated to the form of Stokes vectors{Citation}. In other words, it is not possible to implement an instrumental matrix (A) equal to an unitary matrix, but CN metric allow to optimize (A) by

choosing instrumental matrices as away from singularity as possible, by selecting collection of analyzers going as close as possible to $CN = \sqrt{3}$. All this situation has a geometrical interpretation and it is proved that optimal polarimetric systems maximize the volume enclosed by analyzers when represented over the Poincare sphere [103].

While CN evaluates the conditioning of (A), there is another very well-establish metric in polarimetry field which focuses on equally distributing errors across all singular values, leading to the so-called Equally Weighted Variance (EWV) metric. In Sabatke [104], [105], [106] the EWV metric is derived based on SVD of the measurement matrix (A) and can be written as:

$$EWV = \sum_{j=1}^R \frac{1}{\sigma_j^2} \quad 2.31$$

where R is the rank of the matrix (A), and all its singular values σ_j are contributing to the summation with the same weight. Since singular values appear in the denominator, EWV diverges as (A) approach to a singular matrix and so, EWV should be minimized to optimize the polarimeter design.

Although the CN and EWV metrics tend to yield equivalent optimal solutions when analyzing the same instrumental matrix, (A), it is important to highlight that EWV, being based on the inverse sum of eigenvalues, as opposed to CN , which is calculated through the ratio of eigenvalues, offers a significant advantage. Specifically, EWV is sensitive to data redundancy improvements, whereas CN is not [103]. Optimization techniques such as minimizing CN and EWV guide the design of robust polarimeters and will be used in this Thesis to achieve such a goal.

2.4 Malus's law

In this section, we review a fundamental physical law, the well-known Malus's law, because it will be used as a tool for certain studies in this thesis. In the Mueller-Stokes formalism, it can be deduced in the following way. Let us suppose the Mueller matrix of a non-ideal linear polarizer is [59] :

$$M_{Pol}(P_x, P_y) = \frac{1}{2} \begin{pmatrix} p_x^2 + p_y^2 & p_x^2 - p_y^2 & 0 & 0 \\ p_x^2 - p_y^2 & p_x^2 + p_y^2 & 0 & 0 \\ 0 & 0 & 2p_x p_y & 0 \\ 0 & 0 & 0 & 2p_x p_y \end{pmatrix} \quad 2.32$$

where p_x and p_y are the transmissions along the x and y axis, respectively. Note that for the particular case of an ideal polarizer we have $p_x=1$ and $p_y=0$. Let us now review the normalized Stokes vector of a linearly polarized light oriented at an angle α , which is given by:

$$s_L(\alpha) = \begin{pmatrix} 1 \\ \cos(\alpha) \\ \sin(\alpha) \\ 0 \end{pmatrix} \quad 2.33$$

In this framework, the intensity after passing this linear polarization through the linear polarizer above stated, is given by the S_0 parameter of the exiting Stokes vector, this leading to:

$$I = \frac{1}{2} \{ (p_x^2 + p_y^2) + (p_x^2 - p_y^2) \cos(2\alpha) \} \quad 2.34$$

And considering that $\cos(2\alpha) = 2\cos^2(\alpha) - 1$, the intensity transmittance can be written as:

$$\begin{aligned} I &= \frac{1}{2} \{ (p_x^2 + p_y^2) + 2(p_x^2 - p_y^2) \cos^2(\alpha) - (p_x^2 - p_y^2) \} \\ &= p_y^2 + (p_x^2 - p_y^2) \cos^2(\alpha) \end{aligned} \quad 2.35$$

For an ideal polarizer, the transmitted intensity is given by the well-known expression of,

$$I = \cos^2(\alpha) \quad 2.36$$

Once the intensity expression for this system is derived, some metrics of interest in imaging applications, such as the image contrast, can be deduced.

The contrast is defined as:

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} = \frac{1 - C_{yx}^2}{1 + C_{yx}^2} \quad 2.37$$

Where we have used maximum and minimum values for Eq. 2.36, and where,

$$C_{yx}^2 = \frac{p_y^2}{p_x^2} \quad 2.38$$

By knowing Eq. 2.38, we can also express the ratio between the transmissions C_{yx}^2 in terms of the contrast C as:

$$C_{yx}^2 = \frac{p_y^2}{p_x^2} = \frac{1 - C}{1 + C} \quad 2.38$$

Chapter 3 Monochrome polarized camera, Calibration, Interpolation, Full Stokes expansion

In this chapter, we introduce a novel compact polarimetric imaging system that integrates a monochrome microgrid polarization camera with a Ferroelectric Liquid Crystal (FLC) modulator to achieve rapid, compact full-Stokes parameter measurements. Building upon the foundational concepts presented previously, this design allows quasi-real-time snapshot acquisition of spatially resolved polarization states, significantly reducing measurement time. Initially, we characterize and calibrate the intrinsic polarization response of the monochrome camera, whose sensor consists of microgrid polarizer arrays with four nominal polarization orientations (0° , 45° , 90° , and 135°). Experimental calibration methods are utilized to accurately determine critical polarization parameters, including Degree of Linear Polarization (DoLP), Angle of Linear Polarization (AoLP), and pixel contrast. Following this characterization, we extend the system capabilities through the integration of an FLC modulator, enabling dynamic modulation and retrieval of the full Stokes vector, including ellipticity and azimuth of polarization. The comprehensive optical configuration, mathematical formalism, and calibration procedures, a novel Fourier-based demosaicking method to overcome spatial resolution limitations, and to reconstruct high-fidelity polarization images from mosaiced sensor data are detailed, explores the advantages of using Fourier domain interpolation. By representing the demosaicking problem in the frequency domain, the proposed methodology offers a robust mechanism to mitigate aliasing and improve overall spatial resolution compared to traditional interpolation techniques. Along with rigorous experimental validation using controlled polarization states and practical/ simulated imaging scenarios. Our results underscore the accuracy, stability, and

robustness of this system, positioning it as a valuable tool for diverse real-time polarimetric imaging applications. This foundational work directly supports the subsequent extension to RGB polarimetric imaging discussed in the next chapter.

3.1 Introduction

Because polarimetric imaging can reveal polarization information that traditional intensity imaging cannot, it has become a powerful tool in many different fields. Imaging polarimetry improves contrast and offers sensitivity to subsurface organization, microstructural anisotropy, and surface texture by analyzing the incident state of polarized light. These capabilities have enabled advances in remote sensing [12], imaging through scattering environments such as fog and aerosols [107], non-invasive cancer diagnostics [39], tissue differentiation [108], and astrophysical observations [109]. As real-time and non-invasive diagnostic technologies continue to expand, the development of compact and fast Stokes imaging instruments has become increasingly important.

Polarimetric systems are typically based on either Mueller matrix imaging, which offers a complete description of depolarization and birefringence effects, or the more compact Stokes formulation, which captures the essential polarization parameters of the emergent field. While Mueller imaging provides comprehensive optical characterization, it requires complex and often bulky instrumentation. Stokes polarimetry, by contrast, offers a simpler and faster approach, making it particularly suitable for applications such as surgical guidance, endoscopic screening, and mobile or field-deployable diagnostics.

Despite these advantages, widespread adoption of Stokes polarimetric imaging has been limited by the performance constraints of existing technologies. Many conventional systems rely on mechanically rotating elements [110], [111] or sequential electro-optic retarders [112], which inherently restrict acquisition speed and introduce motion artifacts, preventing true real-time operation. Liquid crystal

variable retarders (LCVRs) alleviate mechanical motion but remain limited by thermal sensitivity and switching times on the order of tens of milliseconds [113]. Photo elastic modulators (PEMs) provide high-frequency modulation, yet require precise synchronization, short exposure times, and complex alignment, hindering their integration into imaging systems used in microscopy or clinical settings [114].

Division-of-focal-plane (DoFP) polarimetric cameras have recently enabled snapshot acquisition of the linear Stokes parameters (S_0 , S_1 , S_2) using pixel-level micro-polarizers [74] [115]. However, they are intrinsically unable to measure circular polarization (S_3). Prototype sensors with elliptical micro-polarizers [116] can measure S_3 but suffer from narrow bandwidth, wavelength-dependent calibration, and lack of commercial availability. Chip-scale meta surface polarimeters [117] offer compact full-Stokes operation but face challenges in fabrication uniformity, calibration stability, and limited spectral bandwidth.

Taken together, no existing approach offers a practical combination of speed, compactness, full-Stokes capability, wavelength flexibility, and calibration simplicity.

To address this gap, we present a compact and quasi-real-time full-Stokes imaging system that combines a division-of-focal-plane polarization camera with a fast-switching Ferroelectric Liquid Crystal (FLC) modulator.

The FLC acts as a fast binary modulator: it switches between two stable orientations of its fast axis at video rates, while keeping its retardance approximately constant ($\approx \lambda/4$). This enables the camera to record two independent polarization states within two frames, which is required to compute S_3 .

Combined with four micro-polarizer orientations at each superpixel, the system yields eight independent intensity measurements, forming an overdetermined

analyzer matrix that enables robust and noise-tolerant reconstruction of the full Stokes vector (S_0 – S_3) with minimal temporal modulation and no moving parts.

The proposed architecture offers a practical balance among compactness, speed, and full-Stokes capability. We perform wavelength-dependent calibration and evaluate system performance on both controlled polarization states and spatially varying samples, including radial polarizers and birefringent plastic films. The results demonstrate high reconstruction fidelity, stable condition numbers, and strong robustness across diverse polarization distributions, highlighting the system's potential for real-time biomedical imaging, remote sensing, and dynamic polarimetric inspection tasks.

3.2 Calibration of Monochrome polarimetric Camera

In Chapter 1, section 1.6, we detailed the inherent limitations of microgrid polarizer array sensors, emphasizing the necessity of accurate experimental calibration. The monochrome polarimetric sensor used in our study, Sony IMX250MYR, is provided with nominal pixel orientations at 0° , 45° , 90° , and 135° as have presented in chapter 1, figure (1-6). However, precise characterization of these micro-grids polarized pixels is essential, as even slight deviations can significantly degrade the accuracy of Stokes measurements.

In this section, we describe our camera's polarization performance experimentally at three different wavelengths. Malus' law calibration, which offers direct measurements of angular deviations and contrasts, and the Data Reduction Matrix (DRM) method, which produces an extensive instrumental calibration matrix, are two complementary calibration techniques used. When combined, these methods guarantee reliable and precise polarization state retrieval, which is necessary for superior polarimetric imaging

3.2.1 calibration by using Malus's Law

We used a collimated unpolarized light source and introduced a linear polarizer mounted on a motorized rotation stage as the input polarizer. The linear polarizer angle (α) was systematically rotated through 180° , taking measurements every 2° , producing 90 discrete angular positions. At each angle, intensity images were captured using the Sony IMX250MYR monochrome polarization camera. This measurement was repeated for three wavelengths: 470 nm, 550 nm, and 630 nm, enabling comprehensive spectral characterization. A schematic of the optical setup is provided in Figure 3.1.

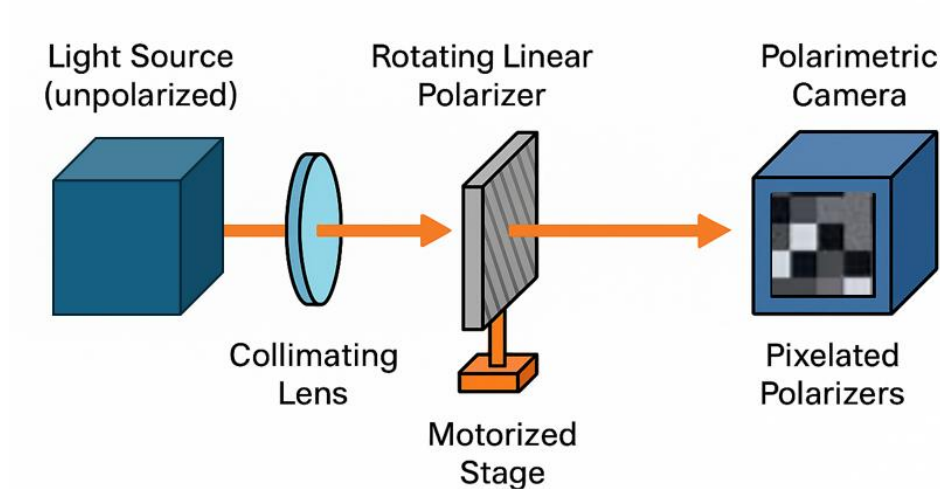


Figure 3-1. Schematic of the experimental setup used for polarimetric characterization based on Malus' law.

Each pixel within the camera sensor corresponds to one of four micro-grid polarizers (labeled P11, P12, P21, P22), arranged in a repeating pattern. Each of these micro-grid polarizers ideally has distinct polarization transmission axes (nominally at 0° , 45° , 90° , and 135°), although practical deviations exist.

Considering real polarization elements that are neither perfect nor ideal, each micro-grid pixel can be represented by a general polarizer Mueller matrix [59]:

$$M_{pol}(\theta) = \begin{pmatrix} A & B\cos(2\theta) & B\sin(2\theta) & 0 \\ B\cos(2\theta) & A\cos^2(2\theta) + C\sin^2(2\theta) & (A - c)\sin(2\theta)\cos(2\theta) & 0 \\ B\sin(2\theta) & (A - c)\sin(2\theta)\cos(2\theta) & A\sin^2(2\theta) + C\cos^2(2\theta) & 0 \\ 0 & 0 & 0 & C \end{pmatrix} \quad 3.1$$

with parameters defined by the transmission axes along the x and y directions (p_x, p_y). $A = \frac{p_x + p_y}{2}$, $B = \frac{p_x - p_y}{2}$ and $C = p_x p_y$. A, B and C in terms of transmission axes along x and y. For input linearly polarized light at an angle α , the detected intensity $I(\alpha)$ at a pixel with transmission axis θ is governed by Malus' law as:

$$I = \frac{1}{2} I_0 (A + 2B\cos^2(\theta - \alpha)) \quad 3.2$$

Figure 3.2 shows the intensity response curves, averaged across all pixels of each polarizer type (P11, P12, P21, and P22), for the green wavelength (550 nm).

Figure 3.2 (Left) illustrates the experimentally measured intensity response curves for each of the four-pixel types (P11, P12, P21, P22), averaged across all pixels of each polarizer type at 550 nm. To obtain smooth, continuous curves, intensity measurements at discrete polarizer rotation angles (every 2°) were directly fitted using Malus' law sinusoidal fitting. These resulting sinusoidal curves, approximately phase-shifted by 45° , validate the expected Malus' law behavior. The positions of maximum intensity correspond directly to the transmission axis angles (θ) of the micro-grid polarizers. This wavelength (550 nm) was explicitly selected as representative since similar responses were observed at 470 nm and 630 nm, indicating consistent polarization performance across the visible spectrum.

To increase the accuracy of angle determination, we applied a quadratic polynomial fit around each intensity peak, refining the angle estimate with sub-degree precision. Figure 3.2 (right) displays histograms of the experimentally measured transmission axis angles for each pixel type. The shift between histogram's angles is 45° , indicating equality with the nominal manufacturing angles. The offset is corresponding to the reference angle of our input polarizer. The width of each

histogram angle indicates deviations from the nominal angles (0° , 45° , 90° , and 135°). These results confirm the necessity of experimental calibration.

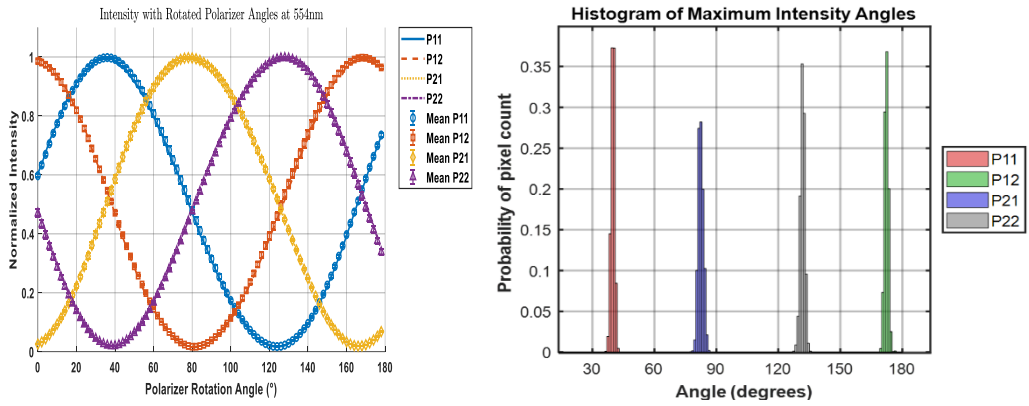


Figure 3-2. : (Left) Normalized intensity response curves for each type of micro-grid polarizer pixel (P11, P12, P21, P22) averaged across the sensor at 550 nm. Solid lines and symbols indicate experimental data fitted to Malus' law. (Right) Corresponding histogram showing the distribution of measured maximum intensity angles for each pixel type across the sensor.

Our measurements of transmission axis angles were constrained by a rotational increment of 2° , limiting angular resolution and preventing detection of subtle wavelength-dependent shifts. Attempts to refine these angles via quadratic polynomial fits around intensity peaks offered only minimal improvement, as the Malus-law intensity peaks are inherently smooth and broad, limiting the effectiveness of polynomial fits. Nevertheless, the angular deviations we were unable to resolve are typically negligible for practical polarimetric calibration purposes, given their small magnitude (expected $<1^\circ$). Future work could employ finer angular increments to enhance precision. Such small angular deviations ($<1-2^\circ$) from ideal values are generally within the expected manufacturing tolerances for micro-grid polarization sensors and do not substantially impact the overall polarization measurement accuracy, particularly after proper calibration.

After determining the effective transmission axis angle of each pixel type using Malus' law fitting, we next evaluate the polarization contrast of the micro-grid

polarizers. It quantifies the polarimetric discrimination ability, i.e., how effectively each pixel distinguishes between different polarization states. The contrast is essentially the polarization efficiency P_{pol} of each analyzer pixel, and it defined as the following equation [118]:

$$C \approx P_{pol} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad 3.3$$

Where I_{max} is the maximum intensity modulation, and I_{min} is the minimum intensity modulation, which corresponds to the maximum and minimum peaks in Figure 3-2 (left). In addition, the real wire-grid polarizers have finite leakage, and it usually characterized by extinction ratio (ER) [119]:

$$ER = \frac{I_{max}}{I_{min}} \quad 3.4$$

and relate it to polarization efficiency as [120]:

$$P_{pol} = \frac{ER - 1}{ER + 1} \quad 3.5$$

Figure (3-3) represents the average Contrast results vs wavelength, for each pixel type, the error bars are the standard deviation data. More importantly, the contrast is calculated at three visible wavelengths (470 nm, 550 nm, and 630 nm) for each pixel type across the sensor, so that it is good opportunity to characterize the spectral contrast of micro polarizer array experimentally.

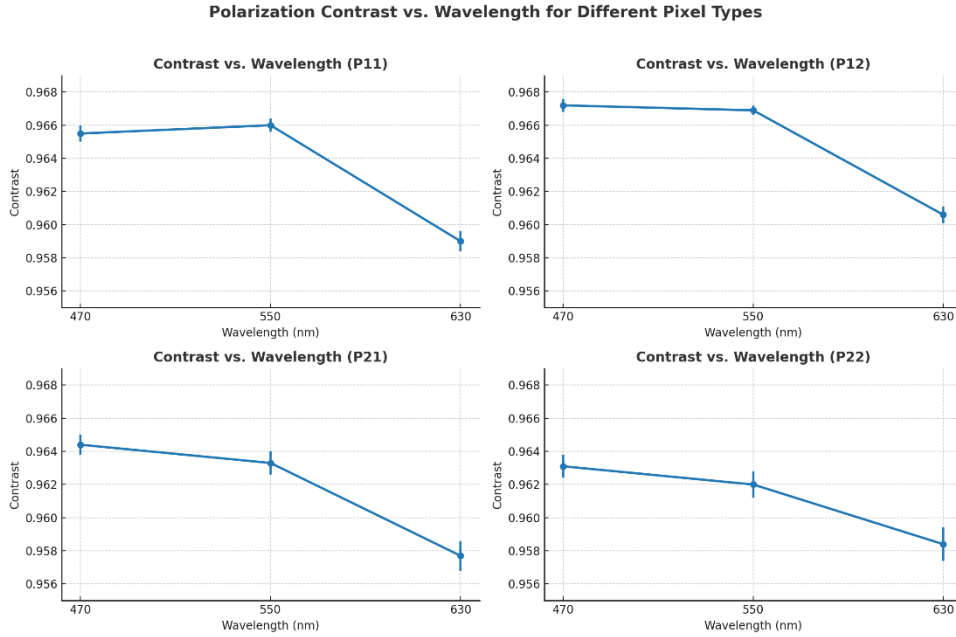


Figure 3-3. Polarization contrast vs. wavelength for the four-pixel analyzers (P11, P12, P21, P22). Note that the vertical axis range (0.956–0.968), to highlight sub-percent spectral variations.

Across all four-pixel orientations (P11–P22), the measured contrast is high, $C \approx 0.96$, at 470 nm and 550 nm and this indicate high quality micro polarizer. At 630nm a consistent small drop (0.005–0.008), and this indicate a nearly achromatic micro polarizer with a slight degradation at longer wavelength, but since, our values comparison occurs and lives in the third decimal place, we can conclude that the wavelength dependence is real, but exhibits only a weak spectral dependence between 470 nm and 630 nm. In our case, this systematic decrease which is of about 0.6-0.8% , and observed at 630nm, compared to 470nm, is in agreement with previously reported wavelength-dependent extinction ratios (ER) for Sony-type micro-polarizer arrays [121]. The mapping from extinction ratio to polarization efficiency (contrast), equation (3.5), strongly compresses changes when ER is large, which explains the slight variation seen in Fig. 3-3, when we comprised our sensor ER. According to the standard deviation shown by the error bars between types P11–P22 (<0.004), there is small pixel to pixel fabrication differences.

As a result, a mere $\approx 1\%$ absolute change in contrast results from even a factor-of-two reduction in ER. Other wire-grid micro-polarizer arrays in the visible range have reported similar high efficiencies with percent-level spectral variations [122]. Polarization efficiencies in the 0.9–0.99 range with only percent-level variation across the visible spectrum are generally reported by York et al. and subsequent DoFP calibration papers [123].

3.2.2 Calibration by using Data Reduction Matrix

We applied the Data Reduction Matric method (DRM) described in section 2.3.1 to calibrate our monochromatic polarimetric camera. To extract the calibrated instrument matrix A_{calib} , Eq. 2.27 must be applied. A commercial Thorlabs polarimeter was used to verify the values of six standard Stokes vectors (H, V, $+45^\circ$, -45° , right- and left-circular) that were generated using a monochromatic collimated beam, a rotating linear polarizer, and an achromatic quarter-wave plate. One image was recorded for each input state and digitally separated into the four micro-polarizer channels (I_{11} , I_{12} , I_{21} , and I_{22}), which correspond to the 2x2 super-pixel layout. The matrix I contained the intensities for every state, and the matrix S contained the known Stokes vectors.

The analyzer matrix of the polarized sensor for each wavelength (470, 554, and 630 nm) was obtained from the pseudo-inverse of S as $A_{calib} = I \cdot \tilde{S}^{-1}$. The first row of the Mueller matrix, A_{calib} , shows the effective analyzer for one each micro-polarizer type. From these calibrated analyzer vectors $A = (A_0 \ A_1 \ A_2 \ A_3)$ for each pixel group, the transmission-axis orientation, degree of linear polarization and ellipticity of each pixel group were computed using Eqs. (2.15), (2.16) and (2.18), enabling us to study wavelength-specific errors and deviations from the ideal design.

For instance, even though the microgrid array sensor is designed to detect only linear polarization components (S_1 , S_2), slight non-zero components in S_3 can

emerge. Wavelength-dependent birefringence or manufacturing tolerances are possibly the root cause of these irregularities.

3.2.3 Wavelength-Dependent DoP Behavior Across Micro-grid sensor

Figure 3.4 illustrates the observed DoP as a function of wavelength for the four-pixel orientations of the micro-polarizer array.

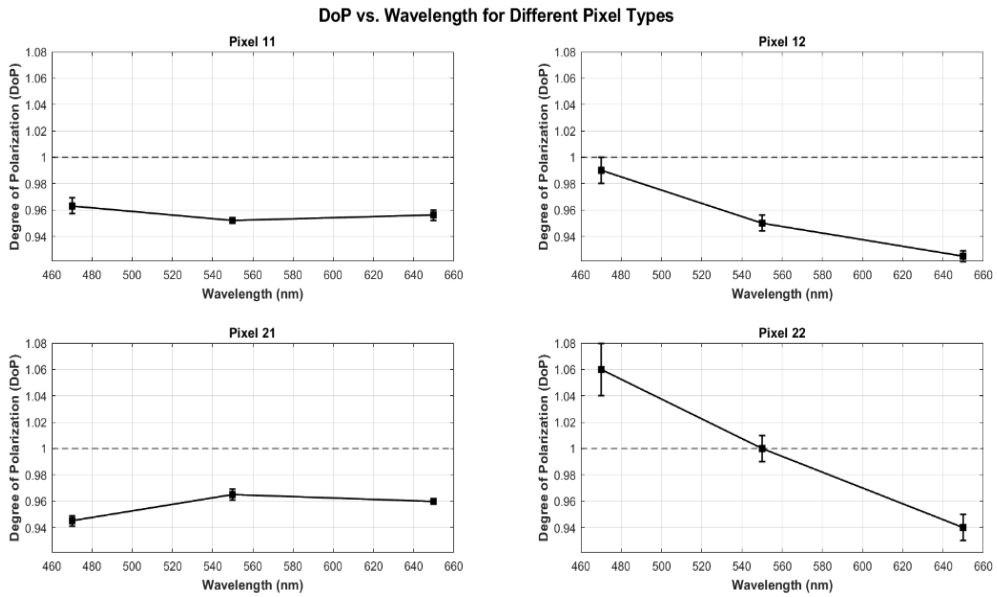


Figure 3-4. Measured Degree of Polarization (DOP) vs. Wavelength for each pixel group (P11–P22).

For P11 and P21 the DoP values remains relatively stable across three wavelengths, with a shallow maximum around 550 nm. In contrast, pixels P12 and P22 have an increased wavelength dependence, with a gradual decrease in DoP towards 630 nm, which suggests a higher sensitivity of these orientations to wavelength-dependent diattenuation and residual calibration errors, as reported for other wire-grid polarimeters [124].

At 470 nm the DoP measured for P22 is slightly above 1, which is not physically meaningful for a linearly polarized input. We attribute this small overshoot to numerical effects in the DRM inversion combined with detector noise and the limited

conditioning of the analyzer matrix, rather than to sensor physics. The horizontal line at DoP = 1 highlights this non-physical value. The error bars (the deviation across all pixels of the same orientation) are substantially lower than the change in DoP between wavelengths, so the observed wavelength trends are consistent, not random. Overall, the camera shows a weak but clear dependence of DoP on wavelength and pixel orientation, with 550 nm giving the most stable, nearly ideal response; therefore, we use 550 nm as the reference wavelength in the subsequent analysis.

To understand the physical limit of our sensor, we linked the measured DoP to the figure 1.12 in chapter 1, which is extinction ratio vs. wavelength. Table 3.1 compares the manufacturer-specified extinction ratio of the on-chip wire-grid polarizers with the measured DoP at the three calibration wavelengths. The extinction ratio of the IMX250MZR is highest in the blue (~400–430:1 at ~470 nm) and decreases towards the red (~100–150:1 at 630 nm), as reported in the Sony datasheet and independent characterizations [125]. For an ideal fully polarized input, the maximum achievable DoP is approximately $(ER - 1)/(ER + 1)$; this gives values close to 1 in the blue and slightly lower in the red.

Table 3-1. Comparison of Extinction Ratio and measured DoP for three Wavelengths.

<i>Wavelength</i>	<i>Expected Extinction Ratio [125]</i>	<i>Measured DoP Range</i>
470 nm	~400–430:1	0.94–0.99
554 nm	~230–250:1	0.95–0.97
630 nm	~100–150:1	0.92–0.96

Figure 3.5 summarizes the wavelength dependence of the sensor’s polarization performance by comparing the manufacturer-specified extinction ratio of the on-chip wire-grid polarizers with the experimentally measured DoP. The extinction ratio (left axis, red dashed line) decreases from the blue to the red, and the measured DoP (right axis, blue line with error bars) follows the same trend. Our measurements follow the same trend: DoP is highest at 470 nm, slightly reduced at 554 nm, and lowest at 630 nm.

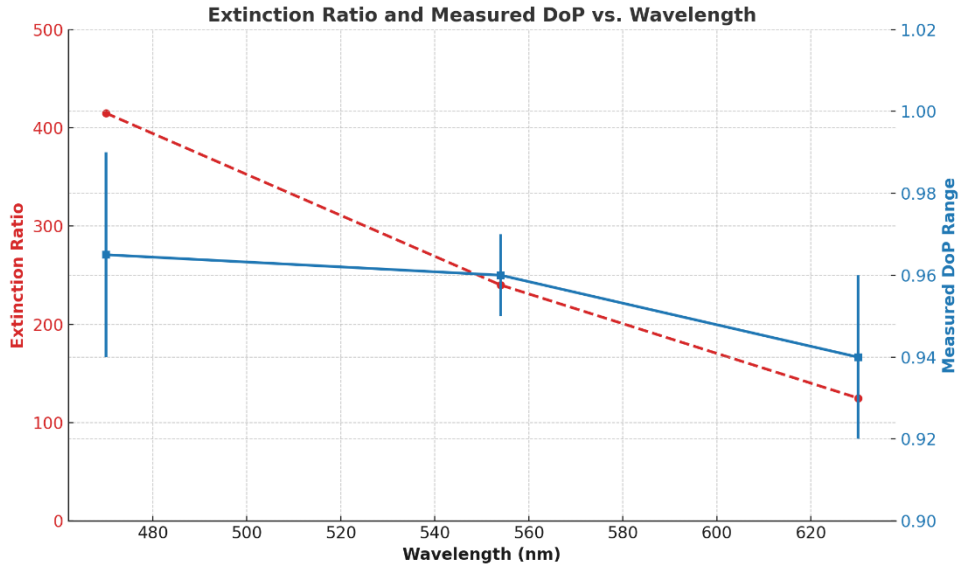


Figure 3-5. Comparison between expected extinction ratio (red dashed line) and experimentally measured Degree of Polarization (DoP, blue with error bars) across three wavelengths.

This agreement shows that the finite, wavelength-dependent extinction ratio of the micro-polarizers sets the effective ceiling for DoP, while the DRM calibration recovers the polarization response up to this physical limit.

3.2.4 Wavelength-Dependent Behavior of AoLP Deviations

The measured AoLP deviations from the ideal analyzer angles at the three calibration wavelengths are summarized in Table 3.2. Across all pixel types, the maximum AoLP error is approximately less than 1.5 degrees, with standard deviations ranging from 0.1 to 0.3 degrees. These values fall within the sensor's nominal specification of 1 degree and are comparable to those recorded for other wire-grid polarimeter sensors [126] [127].

Table 3-2. Comparison of the measured max AoLP and Measured Std deviation range for Three Wavelengths.

<i>Wavelength</i>	<i>Max AoLP Deviation</i>	<i>Std. Deviation Range</i>
470 nm	~0.5°–1.7°	0.21°–0.31°
554 nm	~0.4°–0.8°	0.12°–0.14°
630 nm	~0.8°–1.5°	0.13°–0.17°

Consistent with the DoP analysis, the AoLP is most accurate at 554 nm, where the polarizer extinction ratio and signal-to-noise ratio are close to their optimum values. At this wavelength, typical errors are less than 1 degree, and interpixel dispersion is minimal. At 470 nm, the lower quantum efficiency and higher relative noise lead to a slight increase in angular oscillation. At 630 nm, some trends (such as P22) exhibit a small systematic offset of order 1 degree, which is likely related to the weaker extinction ratio and the potential wavelength-dependent double refraction in the cap glass or micro polarizer array [107].

Finally, it should be emphasized that the accuracy of AoLP is only meaningful in regions where the DoP is sufficiently high; as mentioned in [126], the linear polarization angle becomes poorly defined when the degree of linear polarization is low. In our case, the polarization angle deviations listed in Table 3.3 are evaluated in regions with high polarization, so they constitute a reliable measure of the device's angle accuracy.

3.2.5 Wavelength-Dependent Ellipticity Behavior Across Micro-grid sensor

From Table 3.3, we observe that the analyzer's ellipticity remains very small at all wavelengths: the worst cases are: $|\chi| \leq 1.7^\circ$ at 470nm and $|\chi| \leq 1.4^\circ$ at 554nm and $|\chi| \leq 0.9^\circ$ at 630nm.

Table 3-3. Comparison of the measured max ellipticity and Measured Std deviation range for Three Wavelengths.

These values correspond to a circular component that does not exceed 6% in the

<i>Wavelength</i>	<i>Max Ellipticity (°)</i>	<i>Comment</i>
554 nm	$\sim -1.44^\circ$ (P11), mostly $< 0.2^\circ$	Almost purely linear
470 nm	$\sim -1.73^\circ$ (P11), some $\sim 0.6^\circ$	Small circular component
630 nm	$\sim +0.89^\circ$ (P22), others $\sim 0.4^\circ$	Slight residual ellipticity

case of analysis micro grid polarized sensor, so our analyzers are effectively linear. Ideally, this polarization parameter (ellipticity) should be zero. The small residual values are attributed to manufacturing variations in the wire grid polarizers, and to weak wavelength-dependent phase shifts in the glass cover and calibration optics. The smallest elliptic polarization values were obtained at 554 nm, while 470 nm and 630 nm showed slightly larger values for $|\chi|$, which corresponds to operation away from the optimal design wavelength. In general, the residual elliptic polarization is negligible for the purposes of this study.

3.3 Data interpolation

3.3.1 Introduction

The architecture of our polarimetric sensor consists of a 2x2 unit cell, in which each pixel detects intensity at one of the four different pixelated analyzers. Assume $I_{\theta k}(x, y)$ with $k \in \{0^\circ, 45^\circ, 90^\circ, \text{ and } 135^\circ\}$ is one of these four sub-images. The sensor measures a mosaic:

$$M(x, y) = \sum_{k=1}^4 \prod_k (x, y) I_{\theta k}(x, y) \quad 3.6$$

Where $\prod_k(x, y)$ is the binary mask for the k-th analyzer within each 2x2 unit cell. These intensities images $I_{\theta k}(x, y)$ are under sampled by a factor of 2. The design of

polarized sensor enables simultaneous snapshot acquisition of the four different analyzed intensities, but at the cost of spatial resolution.

Figure 3-6 visually illustrates this inherent limitation, (a) represents the full resolution of simulated cosine distribution image, (b) a sparsely sampled image after assigning intensity values (only) to the first pixel type within each repeating 2×2 -unit cell. The clearer depiction in (c) presents an enlarged view of a region, where each pixel is separated by four unfilled pixels (represented as zero values and visualized in gray in the color bar) and it is explicitly showing the checkerboard sampling pattern generated by the repeating 2×2 -unit cell structure in Figure 3.6 (d). This sparsity mirrors the actual output obtained when isolating polarization channels from the raw sensor data. The principal challenge here is the presence of these numerous empty values.

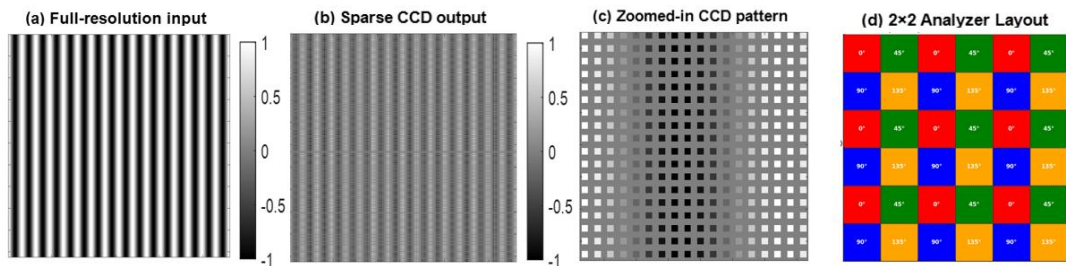


Figure 3-6. (a) Full-resolution synthetic intensity distribution (cosine pattern) as a test input. (b) sparse sampling due to microgrid polarizer layout. (c) Zoomed-in view of the sparse sampling pattern, revealing the checkerboard-like layout resulting from the 2×2 analyzer cell. (d) Schematic representation of the analyzer orientation pattern across the sensor, repeating every 2×2 pixels. Red in red, it is possible to read

In this section, we choose three classical, spatial-domain interpolation methods, bilinear, cubic spline, and Makima, which are commonly used as baselines in image reconstruction and demosaicking literature [128], [129] [130].

In the following sections, we will systematically analyze the nature and performance of commonly used classical interpolation techniques, subsequently motivating the development and evaluation of advanced methods tailored explicitly for high-fidelity polarization image reconstruction.

3.3.2 Bilinear interpolation

First-order (bilinear) interpolation is the simplest and most widely used demosaicking strategy for polarized sensors, and is usually used as a baseline when evaluating more advanced methods [130]. It estimates the intensity at a missing pixel by computing a weighted average of its two nearest known neighbors. In two dimensions, bilinear interpolation extends this concept by using the four nearest grid points in a 2×2 neighborhood. Bilinear interpolation assumes that the intensity varies approximately linearly between neighboring sample points in both x and y direction. As illustrated in Fig. 3.7, the intensity at an unknown position P (green point) is obtained as a weighted average of the four surrounding samples $Q_{11}, Q_{12}, Q_{21}, Q_{22}$ (red points), with weights proportional to the relative distances in x and y .

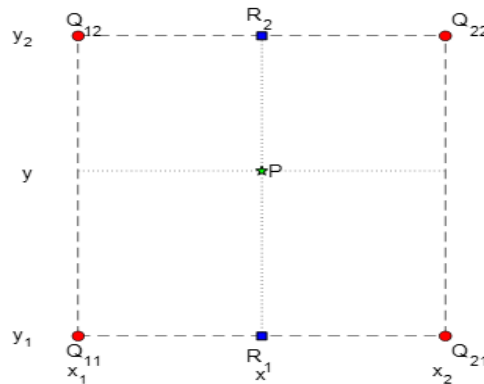


Figure 3-7. Schematic of Bilinear interpolation in 2D. The value at point P (green) is obtained as a weighted average of the four neighbouring samples $Q_{11}, Q_{12}, Q_{21}, Q_{22}$ (red), via intermediate linear interpolations along x to R_1 and R_2 (blue).

In our polarized sensor, the full resolution raw image is composed of a four different pixel type of intensity, which each corresponding to one analyzer orientation. Thus, a single exposure provides a 4 set of sub-sampled intensity images, each defined only on the pixel locations where that analyzer is present. To obtain full-resolution intensity images for each analyzer, we treat every sub-lattice as a coarse grid and bilinearly interpolate it onto the common detector grid. Formally, the interpolated intensity $\hat{Z}_k(x, y)$ for analyzer k inside a given cell is written as a convex combination of the four neighboring samples,

$$\hat{Z}_k(x, y) = \sum_{i=0}^1 \sum_{j=0}^1 w_{ij}(x, y) Z_k[m + i, n + j] \quad 3.7$$

Where $Z_k[m, n]$ represents the recorded signal (intensity) at the node of our lattice sensor (m, n) of the analyzer k , and the weights w_{ij} depend on the relative or fractional distances $\alpha, \beta \in [0, 1]$ from the upper left sample in both the horizontal and vertical directions. weights $w_{00} = (1 - \alpha)(1 - \beta)$, $w_{10} = \alpha(1 - \beta)$, $w_{01} = \beta(1 - \alpha)$, $w_{11} = \alpha\beta$ [131].

In practice, this is equivalent to applying standard two-dimensional linear interpolation (such as the `interp2` function in MATLAB with the "linear" option) to each image of the analyzer, so that we each sub-sampled image is first extracted on a sensor grid with spacing 2 pixels in the both directions, then we apply the bilinear interpolation resample onto the full resolution. In the next analysis of this thesis, we will show how the repeated process of all the four analyzers, then proceed to Stokes estimation. This could be valid if the spatial variation of the stokes vectors are slow, because it assumes the signal (or our intensity polarized analyzes images) varies smoothly and linearly across the local region.

3.3.3 Cubic-spline (bicubic) interpolation

On the other hand, the Cubic spline interpolation, which is consider as a second order approximation, fits a series of piecewise third-order polynomials to the known data points, ensuring continuity of the function and its first two derivatives, so that it produces smoother transitions than linear interpolation and is effective for capturing moderate spatial curvature in intensity variations.

In 2D on rectangular grid, the standard construction of this type of interpolation is separable, we apply a 1D cubic along rows, then along columns. To explain this method, assume we have data points along x-direction (x_i, f_i) . The idea of this type of interpolation located that instead of one huge polynomial that may cause wiggles, we fit one cubic per interval $[x_i, x_{i+1}]$ then stitch these cubics together so the curve is smooth:

- it goes through every data point (values match),
- the slope is continuous (first derivatives match at the joints),
- the curvature is continuous (second derivatives match at the joints).

To explain the method in 1D. on each interval $[x_i, x_{i+1}]$ we fit a cubic

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3 \quad 3.8$$

Four coefficients (a_i, b_i, c_i, d_i) per interval.

Such that it interpolates the samples $S_i(x_i) = f_i$, $S_i(x_{i+1}) = f_{i+1}$, and first and second derivatives are continuous at interior knots: $S'_i(x_{i+1}) = S'_{i+1}(x_{i+1})$ and $S''_i(x_{i+1}) = S''_{i+1}(x_{i+1})$.

The extension to 2D on the rectangular grid by MATLAB's `interp2(...,'spline')` implements this “bicubic (spline)” approach: first cubic spline across one dimension of the table, then spline across the other. MATLAB's 'spline' uses not-a-knot ends,

enforcing third-derivative continuity at the first and last internal knots, which avoids over-constraining edge slopes while keeping a smooth global curve.

3.3.4 Makima (Modified Akima) interpolation

Makima (Modified Akima Interpolation Method) is a compromise between cubic spline and piecewise linear interpolation. It avoids excessive oscillation by suppressing overshoots while maintaining reasonable smoothness. The method is particularly effective for preserving monotonicity and minimizing edge artifacts.

Now to explain the method, taking the 1D formulation in the signal level. Given samples $\{(x_i, f_i)\}_{i=0}^n$ with strictly increasing x_i , define the secant slopes:

$$S_i = \frac{f_{i+1} - f_i}{x_{i+1} - x_i}, \quad i=0, \dots, n-1 \quad 3.9$$

Makima constructs a piecewise cubic Hermite on each interval $[x_i, x_{i+1}]$:

$$S_i(x) = H_0(t)f_i + H_1(t)f_{i+1} + H_2(t)h_i m_i + H_3(t)h_i m_{i+1} \quad 3.10$$

Where:

$$t = \frac{x - x_i}{h_i}, \quad h_i = x_{i+1} - x_i \quad 3.11$$

With the standard Hermite basis

$$H_0(t) = 2t^3 - 3t^2 + 1 \quad 3.12$$

$$H_1(t) = -2t^3 + 3t^2 \quad 3.13$$

$$H_2(t) = t^3 - 2t^2 + t \quad 3.14$$

$$H_3(t) = t^3 - t^2 \quad 3.15$$

The key is how the tangent (derivative) estimates m_i are chosen.

Instead of global constraints, Makima uses local, difference-weighted blending of nearby secants to damp oscillations. A common formulation is:

$$d_i = |s_i - s_{i-1}|, \quad m_i = \begin{cases} d_{i-1}s_{i-1} + d_i s_i, & \text{if } d_{i-1} + d_i > 0 \\ \frac{s_{i-1} + s_i}{2}, & \text{otherwise} \end{cases} \quad 3.16$$

with appropriate one-sided variants near the ends. The weights d_i , grow when neighboring slopes change abruptly; that pulls m_i toward the more reliable side and suppresses overshoot across sharp changes. The result is a monotonicity-friendly cubic that stays close to the data without the ripples seen in natural splines. MATLAB's Makima implements this "modified Akima" family.

3.3.5 Fourier Domain interpolation

3.2.5.1 Basic Properties of the Fourier Transform

This section outlines key properties of the Fourier Transform (FT) [132], that form the theoretical foundation for the interpolation framework developed in the subsequent sections.

Let $G(\vartheta)$ be the Fourier Transform (FT) of $g(x)$:

$$g(x) \xrightarrow{TF} G(\vartheta) \quad 3.17$$

$$G(\vartheta) \xrightarrow{TF^{-1}} g(x) \quad 3.18$$

The FT of a comb of deltas is another comb of deltas with the inverse period (T) as shown in this equation:

$$\sum_k \delta(x - kT) \Leftrightarrow \frac{1}{|T|} \sum_m \delta\left(\vartheta - \frac{m}{T}\right) \quad 3.19$$

The FT of a shifted function is the same as the original function with an added linear phase proportional to the shift.

$$g(x - x_0) \Leftrightarrow G(\vartheta) e^{-i2\pi\vartheta x_0} \quad 3.20$$

Then, the FT of a shifted comb of deltas is given in eq. (3.21). The delta function is different from zero when the argument is equal to zero. So, we can introduce the phase factor inside the summatory.

$$\begin{aligned} \sum_k \delta(x - x_0 - kT) &\stackrel{FT}{\underset{FT^{-1}}{\rightleftharpoons}} \left\{ \frac{1}{|T|} \sum_m \delta\left(\vartheta - \frac{m}{T}\right) \right\} e^{-i2\pi\vartheta x_0} \\ &= \frac{1}{|T|} \sum_m \delta\left\{ \left(\vartheta - \frac{m}{T}\right) e^{-i2\pi\frac{m}{T}x_0} \right\} \end{aligned} \quad 3.21$$

The FT of the product of the two functions is the convolution between their Fourier transforms. So, the FT of the product of a function with a comb of deltas is given by Eq (3.22). We have the repetition of the FT function ($G(\vartheta)$) by a distance of $\frac{1}{T}$. We will assume in the next sections that the function $g(x)$ is band limited with a band-width smaller than the repetition rate. Under these assumptions the replicas will not overlap.

$$g(x) \cdot \sum_k \delta(x - kT) \stackrel{FT}{\underset{FT^{-1}}{\rightleftharpoons}} \frac{1}{|T|} G(\vartheta) \otimes \sum_m \delta\left(\vartheta - \frac{m}{T}\right) \quad 3.22$$

By combining the equations (3.21) and (3.22) one can obtain the FT of the product of a function with a shifted comb of deltas.

$$g(x) \cdot \sum_k \delta(x - x_0 - kT) \stackrel{FT}{\underset{FT^{-1}}{\rightleftharpoons}} \frac{1}{|T|} G(\vartheta) \otimes \left\{ \sum_m \delta\left\{ \vartheta - \frac{m}{T} \right\} e^{-i2\pi\frac{m}{T}x_0} \right\} \quad 3.23$$

If the shift is equal to the half of the comb period $x_0 = \frac{T}{2}$, we obtain:

$$\begin{aligned} g(x) \cdot \sum_k \delta\left(x - \frac{T}{2} - kT\right) &\stackrel{FT}{\underset{FT^{-1}}{\rightleftharpoons}} \frac{1}{|T|} G(\vartheta) \otimes \left\{ \sum_m \delta\left(\vartheta - \frac{m}{|T|}\right) e^{-i\pi m} \right\} \\ &= \frac{1}{|T|} G(\vartheta) \otimes \left\{ \sum_m \delta\left(\vartheta - \frac{m}{T}\right) (-1)^m \right\} \end{aligned} \quad 3.24$$

3.2.5.2 Digital Fourier Transform

Let us suppose that we have a sampled function in a matrix of (N×N) pixels with a sampling distance $(\Delta x, \Delta y)$. The minimum period that can be represented corresponds to the distance between two consecutive pixels is $2\Delta x$. This period will produce the maximum frequency. The sampling distance in the FT will be $(\Delta\mu, \Delta\vartheta)$, as shown in Figure 3-8, then the relation between the sampling distances in both domains will be:

$$\mu_{\max} = \frac{1}{2\Delta x} = \frac{N}{2} \Delta\mu \Rightarrow \Delta\mu \Delta x = \frac{1}{N} \quad 3.25$$

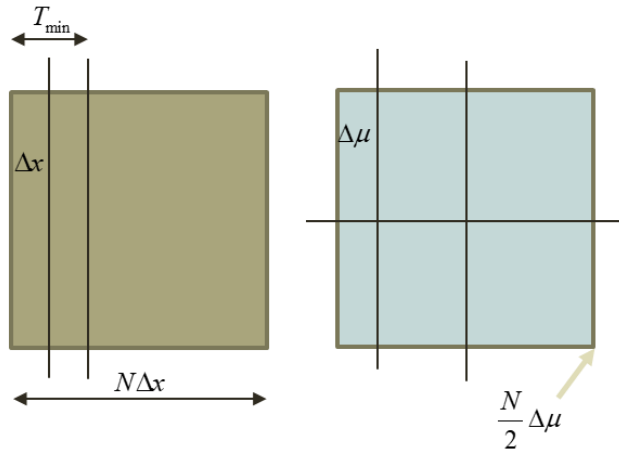


Figure 3-8. Relation between the sampling distances in spatial and Fourier domains

Let us assume that we sample a function $S(x, y)$ every two pixels:

$$S(2n\Delta x, 2m\Delta y) \equiv S(2n, 2m) = S(x, y) \cdot \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - 2m\Delta y) \quad 3.26$$

Its FT will consist in replicas of the original FT separated $\left(\frac{N}{2} \Delta\mu, \frac{N}{2} \Delta\vartheta\right)$ or, in pixels, $\left(\frac{N}{2}, \frac{N}{2}\right)$. If the samples are in the odd pixels in the x direction we obtain:

$$S(x, y) \cdot \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - 2m\Delta y) = S(2n+1, 2m) \quad 3.27$$

When the shift is in the y-direction we have:

$$S(x, y) \cdot \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - (2m+1)\Delta y) = S(2n, 2m+1) \quad 3.28$$

And if it is shifted in both directions:

$$S(x, y) \cdot \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - (2m+1)\Delta y) = S(2n+1, 2m+1) \quad 3.29$$

The period (sampling distance) is the same, but the delta functions are shifted to one pixel (half period), then, considering equation (3.24), each replica will be multiplied by:

$$\left\{ \begin{array}{ll} (1) & \text{whitout shift} \\ (-1)^p & \text{shift in the x direction} \\ (-1)^s & \text{shift in the y direction} \\ \left((-1)^p x (-1)^s \right) & \text{shift in both directions} \end{array} \right. \quad 3.30$$

where (p,s) are the order of the replica in the (x, y) directions.

3.2.5.3 Sampling Model of Polarization Cameras

The formalism developed in this section is inspired in the works by Tyo et al [12], [133].

The sampling with SONY sensor results in four different intensity distribution (I_k , $K=1\dots, 4$) with an interlaced sampling as shown in Eqs, (3.27-3.29). So, the intensity distribution captured by the sensor will be:

$$\begin{aligned}
I(m,n) = & (I_1) \sum_{n,m} \delta(x-2n\Delta x) \delta(y-2m\Delta y) \\
& + (I_2) \sum_{n,m} \delta(x-(2n+1)\Delta x) \delta(y-2m\Delta y) \\
& + (I_3) \sum_{n,m} \delta(x-2n\Delta x) \delta(y-(2m+1)\Delta y) \\
& + (I_4) \sum_{n,m} \delta(x-(2n+1)\Delta x) \delta(y-(2m+1)\Delta y)
\end{aligned} \tag{3.31}$$

When performing the FT of this distribution it will appear different replicas of the FT of the intensity distributions given by each analyzer ($\tilde{I}_k$), each one affected with the corresponding sign due to the comb shifting as given by Eq (3.30). As we have commented before, the replicas will be shifted ($p \frac{N}{2}, q \frac{N}{2}$) pixels. Figure 3-9 shows the location of the different replicas. The zero order (without shift) appears at the central part and is marked with 1. Note that the digital Fourier transformation is periodical, for this reason the first replica in the X direction appears in zones marked with 2. The first replica in the Y direction is marked with 3, and so on.

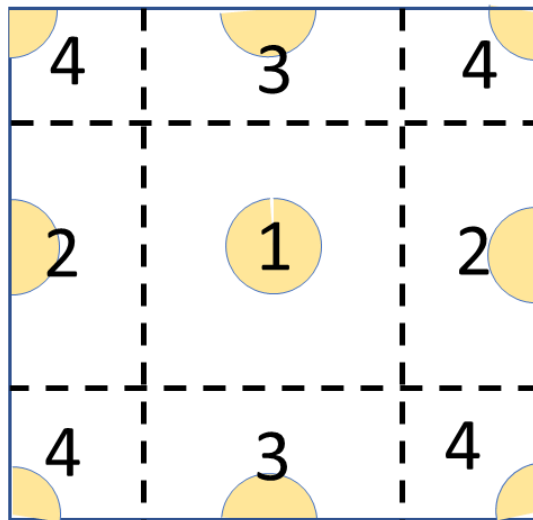


Figure 3-9. Location of each FT replica. The circles indicate the zones of low frequencies.

We consider that the first detector (A1) is centered, and the others are shifted. So, the Fourier transforms of the intensities corresponding to each detector will appear replicated with some sign changed. Figure 3-10 shows the sign of each replica for the four different analyzers. The FT of the intensity distribution will be the addition of the four FT shown in eq 3.24.

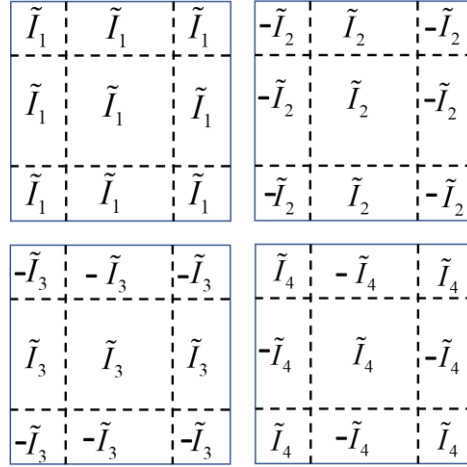


Figure 3-10. Signs of each replica correspond to the FT of the intensity distribution corresponding to the four detectors.

Numerically, the distributions of each zone of Figure 3-10 will be given by:

$$\left. \begin{aligned} \tilde{I}_{z1}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) + \tilde{I}_2(\mu, \nu) + \tilde{I}_3(\mu, \nu) + \tilde{I}_4(\mu, \nu) \\ \tilde{I}_{z2}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) - \tilde{I}_2(\mu, \nu) + \tilde{I}_3(\mu, \nu) - \tilde{I}_4(\mu, \nu) \\ \tilde{I}_{z3}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) + \tilde{I}_2(\mu, \nu) - \tilde{I}_3(\mu, \nu) - \tilde{I}_4(\mu, \nu) \\ \tilde{I}_{z4}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) - \tilde{I}_2(\mu, \nu) - \tilde{I}_3(\mu, \nu) + \tilde{I}_4(\mu, \nu) \end{aligned} \right\} \quad 3.32$$

Now it is necessary to extract these four FT distributions and center them appropriately as it is shown in 3-11.

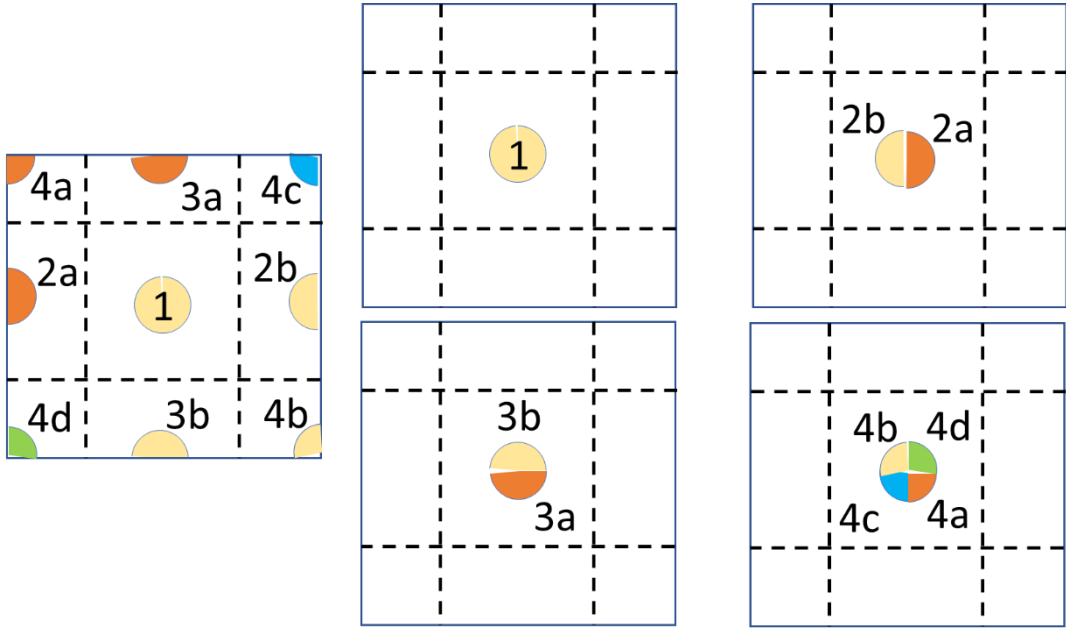


Figure 3-11. Extraction and centering of the four FT distributions

In four matrices filled with zeros, we insert the corresponding different zones. For the $\tilde{I}_{z1}(\mu, \vartheta)$, only it is necessary to move the central zone (1) to the central zone of the zero-filled matrix. For the $\tilde{I}_{z2}(\mu, \vartheta)$ zones (2a) and (2b) must be moved to the central part (1) of a zero-filled matrix. For the $\tilde{I}_{z3}(\mu, \vartheta)$ zones (3a) and (3b) must be moved to the central part (1) of a zero-filled matrix. Finally, For the $\tilde{I}_{z4}(\mu, \vartheta)$ zones (4a), (4b), (4c) and (4d) must be moved to the central part (1) of a zero filled matrix.

After this reorganization the inverse Fourier transform of each distribution is obtained.

$$\left. \begin{aligned} I_{z1}(x, y) &= I_1(x, y) + I_2(x, y) + I_3(x, y) + I_4(x, y) \\ I_{z2}(x, y) &= I_1(x, y) - I_2(x, y) + I_3(x, y) - I_4(x, y) \\ I_{z3}(x, y) &= I_1(x, y) + I_2(x, y) - I_3(x, y) - I_4(x, y) \\ I_{z4}(x, y) &= I_1(x, y) - I_2(x, y) - I_3(x, y) + I_4(x, y) \end{aligned} \right\} \quad 3.33$$

This can be written in matrix form as:

$$\begin{pmatrix} I_{z1} \\ I_{z2} \\ I_{z3} \\ I_{z4} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} \Rightarrow I_z = MI \quad 3.34$$

By inverting this equation, we obtain the interpolated intensity distributions due to the four detectors.

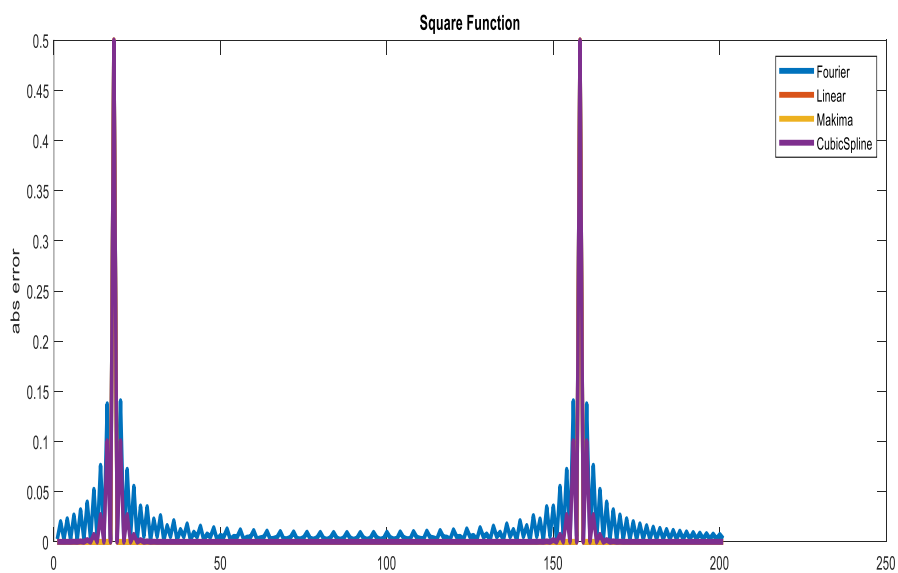
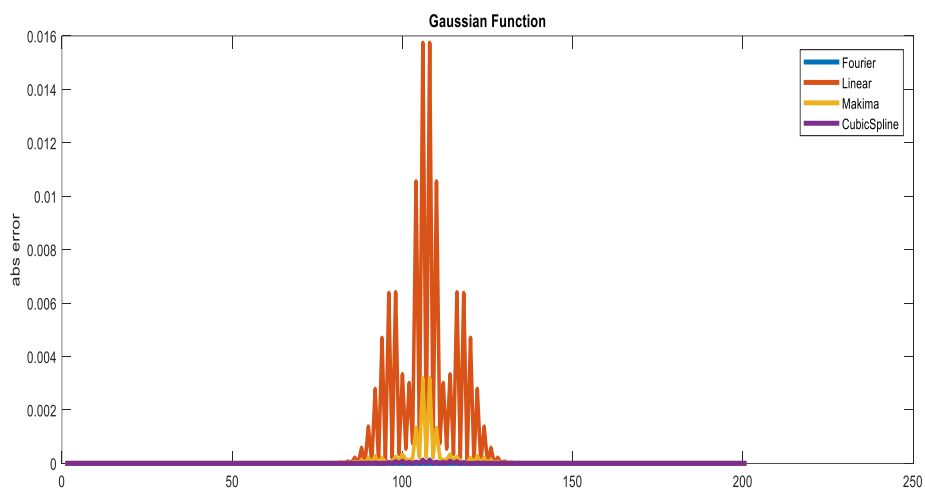
3.4 Interpolated Methods Performance

In this section, the performance of the four different interpolation methods (Linear, Makima, Cubic spline, and the Fourier domain method) will be assessed. We synthesis four test functions (F1-F4), each capturing a distinct spatial characteristic relevant to polarization imaging: a discontinuous rectangular function (F₁), Gaussian function (F₂), an-axis-aligned sinusoidal pattern (F₃), and a diagonally oriented sinusoidal pattern at 45° (F₄).

Functions F₂–F₄ were designed to be band-limited below the Nyquist frequency imposed by the 2×2 microgrid sampling, while F₁ intentionally violates this condition to assess interpolation behavior in the presence of sharp discontinuities.

Interpolation performance is assessed using one-dimensional absolute error profiles (center row), maximum absolute error, and computational time which measured for a 512×512 image using MATLAB. The 2D error maps are provided in Appendix A for completeness.

3.4.1 Core interpolation behavior: error profile analysis



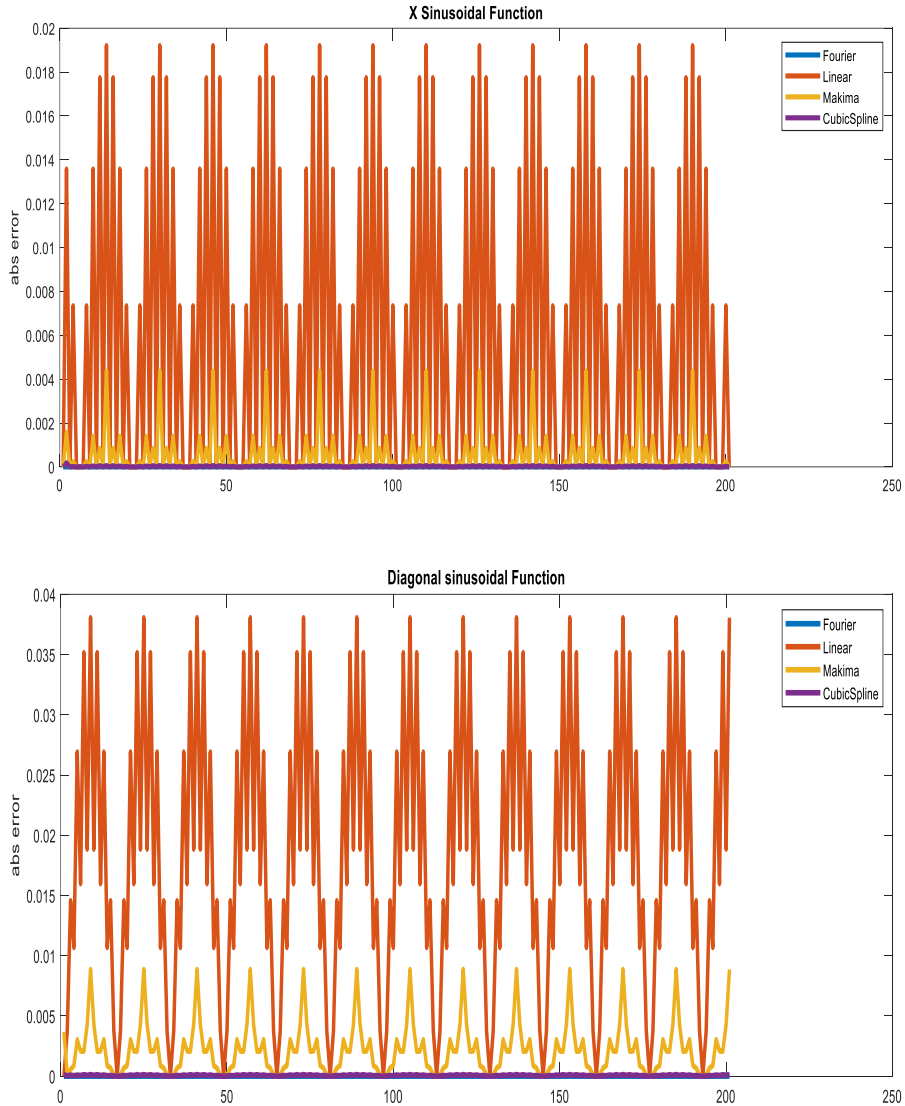


Figure 3-12. Absolute error profiles for interpolation methods. (first row) Gaussian, (second row) Rectangle, (third row) Sinusoidal (x-direction), (fourth row) Sinusoidal (diagonal).

The Gaussian test function's 1D absolute error profiles are shown in Figure 3.12, first row. Linear and Makima interpolation introduce oscillatory behavior, particularly in the region of presence a gradually varying intensity in the Gaussian function. This region has rich frequency. On the other hand, there are no oscillations

are showed in Fourier interpolation. This behavior reflects the ability of Fourier interpolation to maintain band-limited intensities, because Fourier is reflecting it is frequency-domain nature. Cubic spline interpolation still has residual oscillations and a larger peak error than Fourier, but somewhat reduced the oscillatory effect, this is because of it is higher order continuity.

The rectangular test function's 1D absolute error profiles are shown in Figure 3.12, second row. The maximum absolute error that all interpolation techniques achieve is (0.75). This behavior reflects the theoretical constraint imposed by the non-band-limited discontinuity. As smooth basis functions try to approximate a sharp transition, Fourier and cubic spline interpolations show oscillatory ringing near the edges, which is indicative of the Gibbs phenomenon. By preserving spatial positioning (locality), linear interpolation and Makima prevent overshoot, but at the cost of reduced structural accuracy and geometric distortion. This result shows that sampling theory, rather than algorithmic complexity, is the primary constraint on the interpolation performance of disconnected signals (un band limited).

The sinusoidal function (aligned with the axis) test function's 1D absolute error profiles are displayed in Figure 3.12, third row. Because Fourier interpolation is inherently compatible with periodic signals, it achieves near-perfect reconstruction while preserving both amplitude and phase. Curvature mismatch and phase distortion lead to systematic periodic residual errors in linear interpolation and Makima interpolation. Despite its relatively good performance, the interpolation of cubic functions still introduces minor phase errors. These results show that positional interpolation techniques, even when bandwidth is strictly restricted, are unable to reconstruct periodic signals accurately.

Figure 3.12, fourth row shows periodic curves of errors for the diagonal sine function. Again, Fourier interpolation produces the lowest error, demonstrating its resilience to spatially non-aligned frequencies. While linear and Makima methods

show marked periodic rises associated with direction mismatch, cubic spline interpolation maintains a relatively low error. Because intensity and polarization gradients rarely correspond to the pixel grid, this mode is of particular importance in polarization imaging. Fourier interpolation shows better performance in this case, highlighting its importance for accurately reconstructing spatially variable polarization information.

3.4.2 Comparison of maximum errors

Figure 3.13 summarizes the maximum absolute interpolation error (on a logarithmic scale) for each of the four test functions, while Table 3.4 provides the numerical values. For band limited functions, Fourier interpolation consistently produces the lowest reconstruction error, with values as low as 2.7×10^{-8} for sinusoidal and Gaussian functions.

All methods converge to the same maximum error of the rectangular function, showing that the nature of the function of non band limited, and not the interpolation technique which limits performance. Due to their limited ability to preserve phase and frequency information, linear interpolation and Makima interpolation show significantly larger errors for smooth and periodic functions.

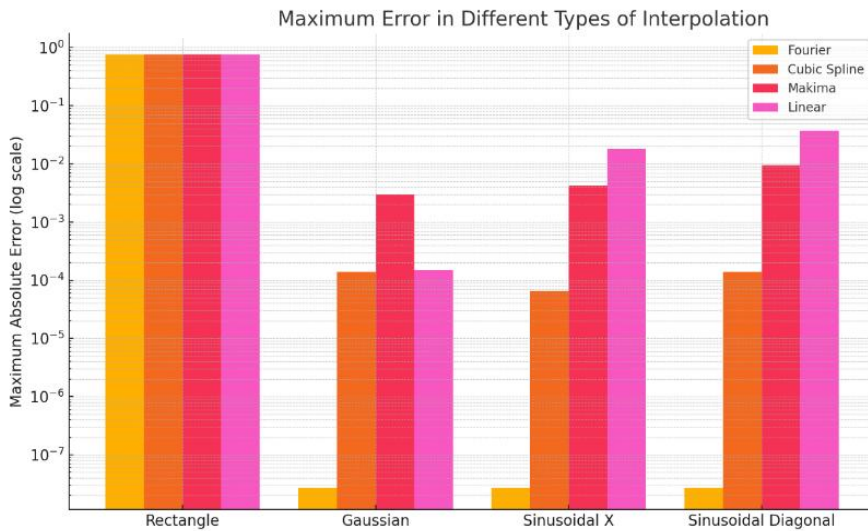


Figure 3-13. Maximum absolute interpolation error (log scale).

Table 3-4. Maximum absolute interpolation error for different simulated intensity.

Function Type	Fourier	Cubic Spline	Makima	Linear
Rectangle	0.75	0.75	0.75	0.75
Gaussian	2.7×10^{-8}	1.4×10^{-4}	3×10^{-3}	1.5×10^{-4}
Sinusoidal X	2.7×10^{-8}	6.5×10^{-5}	4.2×10^{-3}	1.8×10^{-2}
Sinusoidal Diagonal	2.7×10^{-8}	1.4×10^{-4}	9.5×10^{-3}	3.7×10^{-2}

3.4.3 Computational efficiency

Figure 3.14 shows the computational cost for each interpolation method, and Table 3.5 shows the corresponding values. Because linear interpolation is the fastest (0.018 seconds), it is suitable for real-time applications or applications with limited resources where accuracy is not critical.

Although Fourier interpolation requires forward and reverse fast Fourier transforms, it achieves an excellent balance between accuracy and efficiency, taking only 0.043 seconds. Cubic interpolation offers a moderate computational cost with

good accuracy, but Makima interpolation has the longest runtime (0.069 seconds), which limits its use in time-sensitive systems.

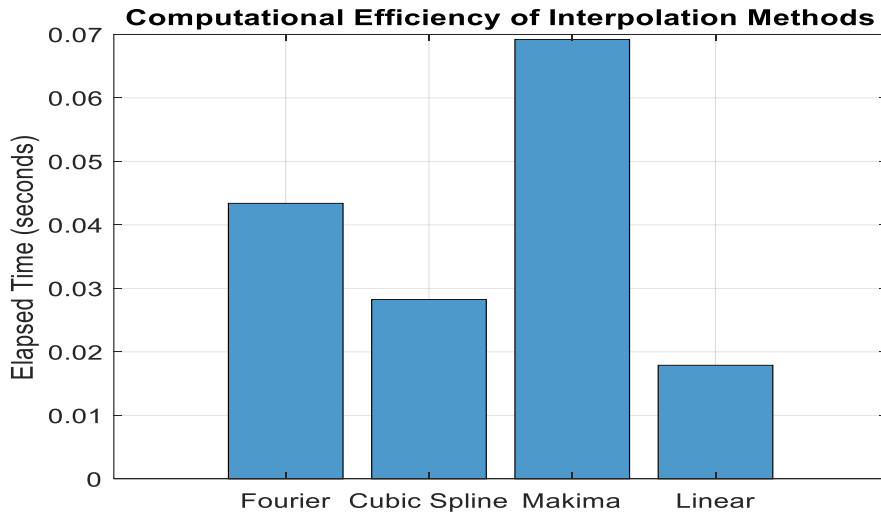


Figure 3-14. Computational Efficiency comparison for four interpolation methods.

Table 3-5. Elapsed Time of Interpolation Methods

Interpolation Method	Elapsed Time (seconds)
Fourier	0.043
Cubic Spline	0.028
Makima	0.069
Linear	0.018

3.5 Data reduction: from mosaic intensities to Stokes parameters

In this section, we will define the pipeline that maps raw mosaicked intensity measurements acquired by a microgrid-based polarimetric system into Stokes components and the derived polarization parameters.

The incident polarization field is described by a spatially varying Stokes vector as:

$$S(x, y) = [S_0(x, y), S_1(x, y), S_2(x, y), S_3(x, y)]^T \quad 3.35$$

Each pixel in the sensor acts as a polarization analyzer characterized by an analyzer vector a_k . The intensity measured at pixel (x, y) by analyzer k is given by:

$$I_k(x, y) = a_k^T S(x, y) \quad 3.36$$

Due to the spatial interleaving of analyzers, the recorded image is a mosaic intensity image in which each analyzer samples the scene on an offset sub-lattice. Each channel is therefore interpolated independently to obtain full-resolution intensities $I'_k(x, y)$.

The interpolated intensities are stacked into the measurement vector as:

$$I'(x, y) = [I'_1(x, y), I'_2(x, y), \dots, I'_N(x, y)]^T \quad 3.37$$

which is related to the Stokes vector through the analyzer matrix A :

$$I'(x, y) = A S(x, y), \quad S(x, y) = A^+ I'(x, y) \quad 3.38$$

where A is the analyzer matrix and A^+ its pseudo-inverse, where the dimension and rank of A depend on the available analyzer set.

$N=4$ for the monochrome microgrid sensor, enabling reconstruction of the linear Stokes vector $[S_0, S_1, S_2]^T$. For full-Stokes simulations, the analyzer set is extended by combining the sensor with a ferroelectric liquid crystal (FLC) modulator, yielding $N = 8$ independent intensity measurements and a full-rank analyzer matrix that enables reconstruction of all four Stokes components $[S_0, S_1, S_2, S_3]^T$.

Standard relationships are used to calculate derived polarization quantities, such as degree of linear polarization (DoLP) and angle of linear polarization (AoLP), from reconstructed Stokes components.

The output of the data processing operation consists of reconstructed Stokes coefficients and derived quantities. Because interpolation errors proliferate during the Stokes inversion process and subsequent polarization coefficient estimation, the interpolation step is essential to its accuracy.

3.6 Interpolation effects on Stokes and derived polarimetric parameters

In this section, our objective is comparing interpolation methods for polarimetric reconstruction, and structure around polarimetric quantities and physical sensitivity. We aim to determine which polarimetric quantities are affected by interpolation. Our Stokes images are scalar, AoLP is angular, and DoLP is magnitude.

3.6.1 Simulation-based evaluation of the data-reduction pipeline

To evaluate the impact of interpolation on polarimetric reconstruction, two analytically defined polarization scenes were simulated and used as ground truth: a linear polarization scene and an elliptical (full-Stokes) polarization scene.

The linear polarization scene is defined on a two-dimensional grid of size $N \times N$ with constant total intensity and spatially varying linear polarization:

$$S_0(x, y) = 1 \quad 3.39$$

$$S_1(x, y) = (d_M - d_m \cdot r) \cdot \cos(\pi \cdot x) \quad 3.40$$

$$S_2(x, y) = (d_M - d_m \cdot r) \cdot \sin(\pi \cdot y) \quad 3.41$$

Where r is the normalized radial distance from the image center, $d_M = 1$, and $d_m = 0.5$. This scene produces smooth, band-limited variations in polarization magnitude and orientation and is representative of linear polarization measurements achievable with the monochrome Sony microgrid sensor.

Figure 3.15 shows the corresponding ground-truth Stokes components S_0 , S_1 , and S_2 , together with the derived AoLP and DoLP maps. These ground-truth distributions are used to quantify reconstruction errors in linear Stokes parameters and derived polarization quantities.

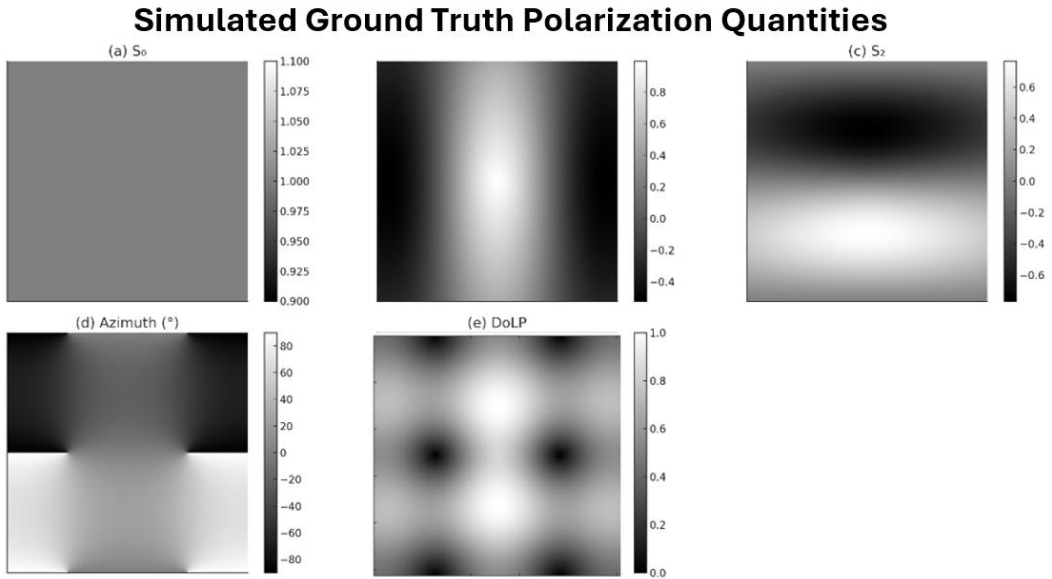


Figure 3-15. Simulated polarization quantities used as ground truth for interpolation evaluation. (a) S_0 : Total intensity (uniform). (b) S_1 : Horizontal variation in polarization. (c) S_2 : Vertical variation in polarization. (d) Azimuth map derived from simulated S_1 and S_2 . (e) DoLP: Degree of Linear Polarization.

Additionally, an elliptical polarization scene with full-Stokes ground truth was simulated in order to evaluate interpolation performance under more complicated circumstances:

$$S_0(x, y) = 1 \quad 3.42$$

$$S_1(x, y) = (d_m(\frac{X-Y}{2}) + d_M) \cdot \cos(2EL) \cdot \cos(90(x+y)) \quad 3.43$$

$$S_2(x, y) = (d_m(\frac{X-Y}{2}) + d_M) \cdot \cos(2EL) \cdot \sin(90(x+y)) \quad 3.44$$

$$S_3(x, y) = (d_m(\frac{X-Y}{2}) + d_M) \cdot \sin(2EL). \quad 3.45$$

with $d_M = 1$, $d_m = 0.5$, and spatially varying ellipticity El . This scene introduces oblique spatial frequencies and phase variations while remaining below the Nyquist limit.

Figure 3.16 presents the ground-truth full-Stokes distributions (S_0, S_1, S_2, S_3), which serve as reference for evaluating interpolation effects on full-Stokes reconstruction when the analyzer set is extended using an FLC-modulated configuration.

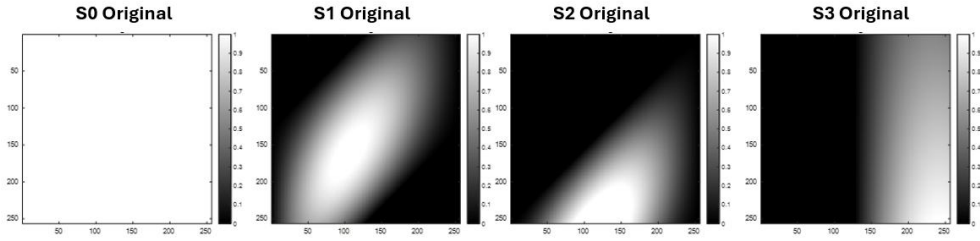
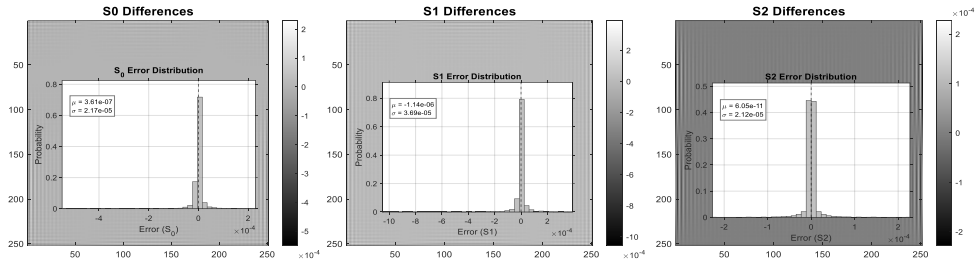


Figure 3-16. Simulated ground truth Stokes parameter images (S_0, S_1, S_2 , and S_3) used for evaluating interpolation accuracy.

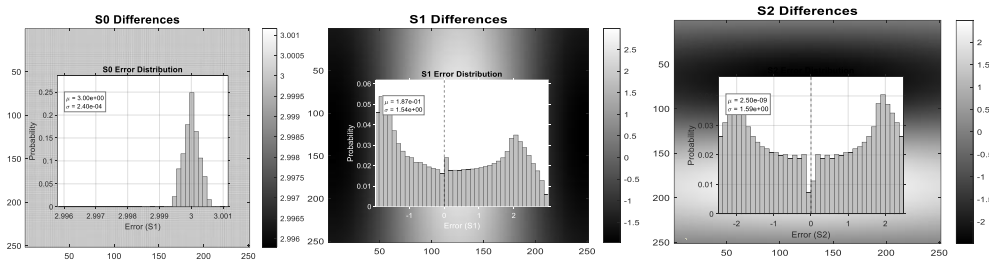
3.6.2 Evaluation of interpolation effects on Linear Stokes reconstruction

Figure 3.17 compares Stokes components reconstruction errors for four interpolation methods. Each panel shows the difference pixel wise images ($\Delta S_i = S_{i,\text{rec}} - S_{i,\text{GT}}$) with an embedded histogram reporting μ and σ . The Stokes errors are generally narrower for Fourier and cubic-spline, but Linear and Makima exhibit a visible S_0 offset and larger dispersion with bimodal error distributions in S_1 and S_2 , because they are a low-order demosaicking. These results are indicating that intensity reconstruction is not equally robust across methods in this simulated scene. Fourier-domain interpolation yields the most compact, near-zero-centered distributions, with weak spatial structure in the error maps, suggesting the best preservation of the band-limited polarization content in this simulated scene.

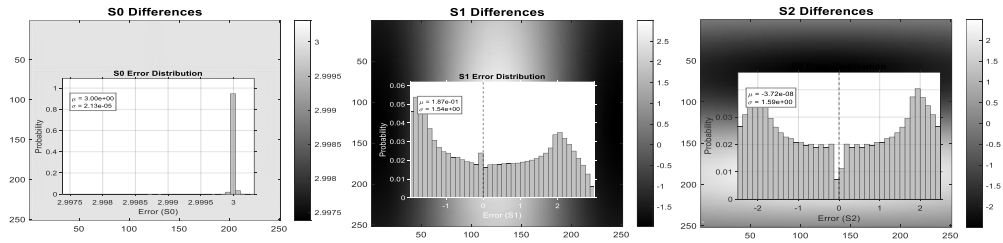
Difference Maps of Fourier Interpolation



Difference Maps of Linear Interpolation



Difference Maps of Makima Interpolation



Difference Maps of Cubic Spline Interpolation

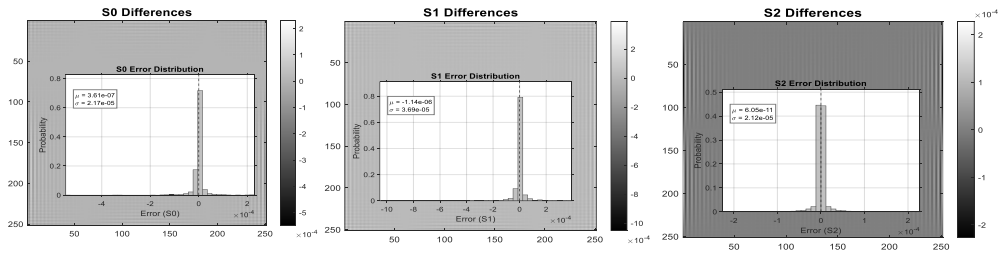


Figure 3-17. Stokes reconstruction errors for the simulated linear polarization scene using four interpolation methods.

3.6.3 Effect on Angle of Linear polarization (AoLP)

The AoLP error maps and embedded histograms summarizing the error distribution are displayed in Figure 3.18 for each of the four interpolation techniques. In general, all methods reconstruct AoLP with negligible residual errors centered on zero. The displayed error range is usually on the order of 10^{-3} degrees, and the lowest error levels are obtained with Fourier and cubic-spline interpolation.

Makima also provides very small AoLP errors, with a displayed range of about 10^{-4} degrees and a narrow histogram peak near zero. Conversely, linear interpolation shows the largest AoLP deviations, reaching approximately 2×10^{-2} degrees, and its histogram is noticeably broader, indicating greater error dispersion. These results confirm that all advanced interpolation methods considerably reduce AoLP reconstruction errors when compared to the linear baseline, and that the residuals that remain are insignificant over the whole field of view.

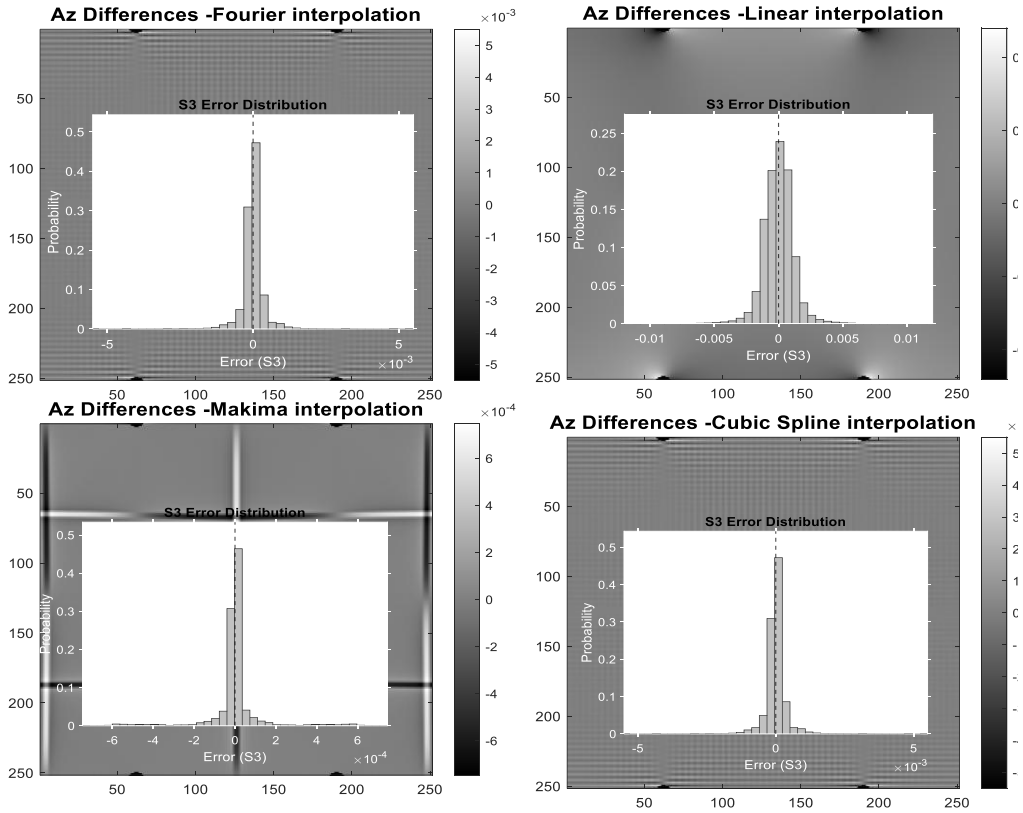


Figure 3-18. AoLP error maps for the four different interpolation methods, and embedded histograms.

3.6.4 Effect on DoLP

The DoLP error maps and associated embedded histograms for each of the four interpolation techniques are shown in Figure 3.19. The DoLP reconstruction errors are consistently very small, staying on the order of 10^{-4} and closely clustered around zero, indicating that DoLP is very resilient to the interpolation selection for this simulated scene. Even though the errors are still small in absolute value, the linear interpolation result exhibits the largest spread (broader histogram), indicating a slightly higher dispersion of DoLP errors.

Among all the interpolated methods, Makima yields the most concentrated histogram peak close to zero, indicating the most stable DoLP reconstruction. In this instance, the DoLP error distributions provided by cubic-spline and Fourier are extremely similar, indicating similar performance for this particular DoLP metric. This behavior is expected since the scalar DoLP map tends to reduce small interpolation deviations in S1 and S2 since DoLP is calculated from the magnitude of (S1, S2).

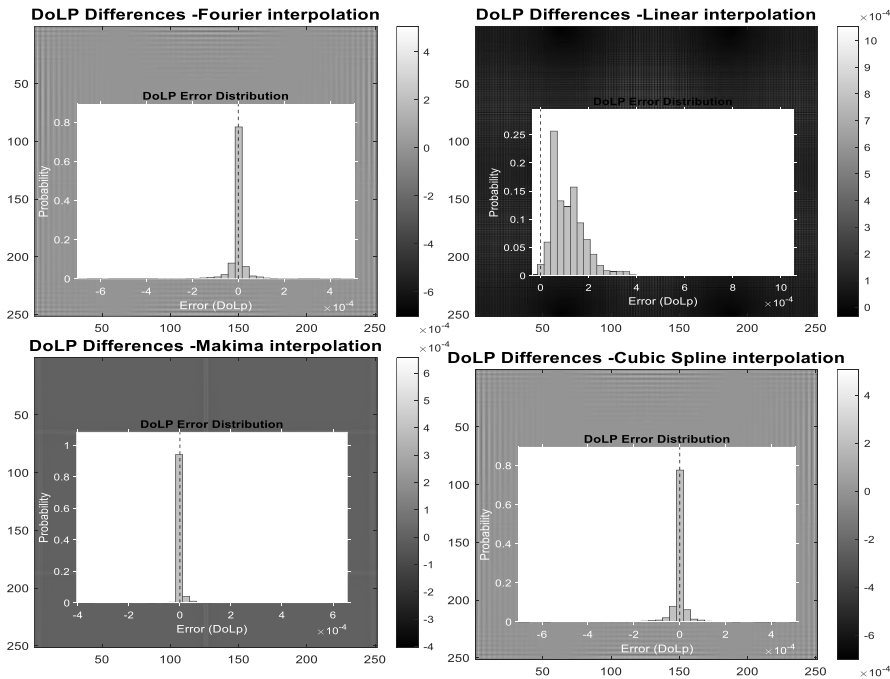


Figure 3-19. DoLP error maps for the four different interpolation methods, and embedded histograms.

3.6.5 Effect of interpolation on Full-Stokes elliptical scene

Figure 3.20 shows full-Stokes reconstruction error maps for the four interpolation methods together with their error histograms. For each Stokes component S_i , we display the difference $\Delta S_i = \hat{S}_i - S_i^{\text{GT}}$ using color limits adapted to the error range of that method. Because these limits are different across methods and components, the grayscale appearance of the maps is not intended for direct quantitative comparison. Instead, the embedded histograms summarize the full error distribution: the mean $\mu(\Delta S_i)$ indicates any systematic bias, and the standard deviation $\sigma(\Delta S_i)$ measures the spread of the errors, and these two metrics are used for the numerical comparison between methods.

Difference maps are visualized with each method of interpolation own limits which enhances low-level structure while preventing a few outliers from dominating the contrast. Under this scaling, Fourier interpolation reveals oscillatory residuals localized near the image boundaries. This behaviour is consistent with the implicit periodic boundary condition of FFT-based processing: when the scene is not periodic, the mismatch between opposite borders acts as a discontinuity, producing ringing patterns that concentrate at the edges. These boundary outliers broaden the histogram (increasing σ) even though the interior of the image remains almost unbiased, as reflected by the narrow central peak around $\mu \approx 0$.

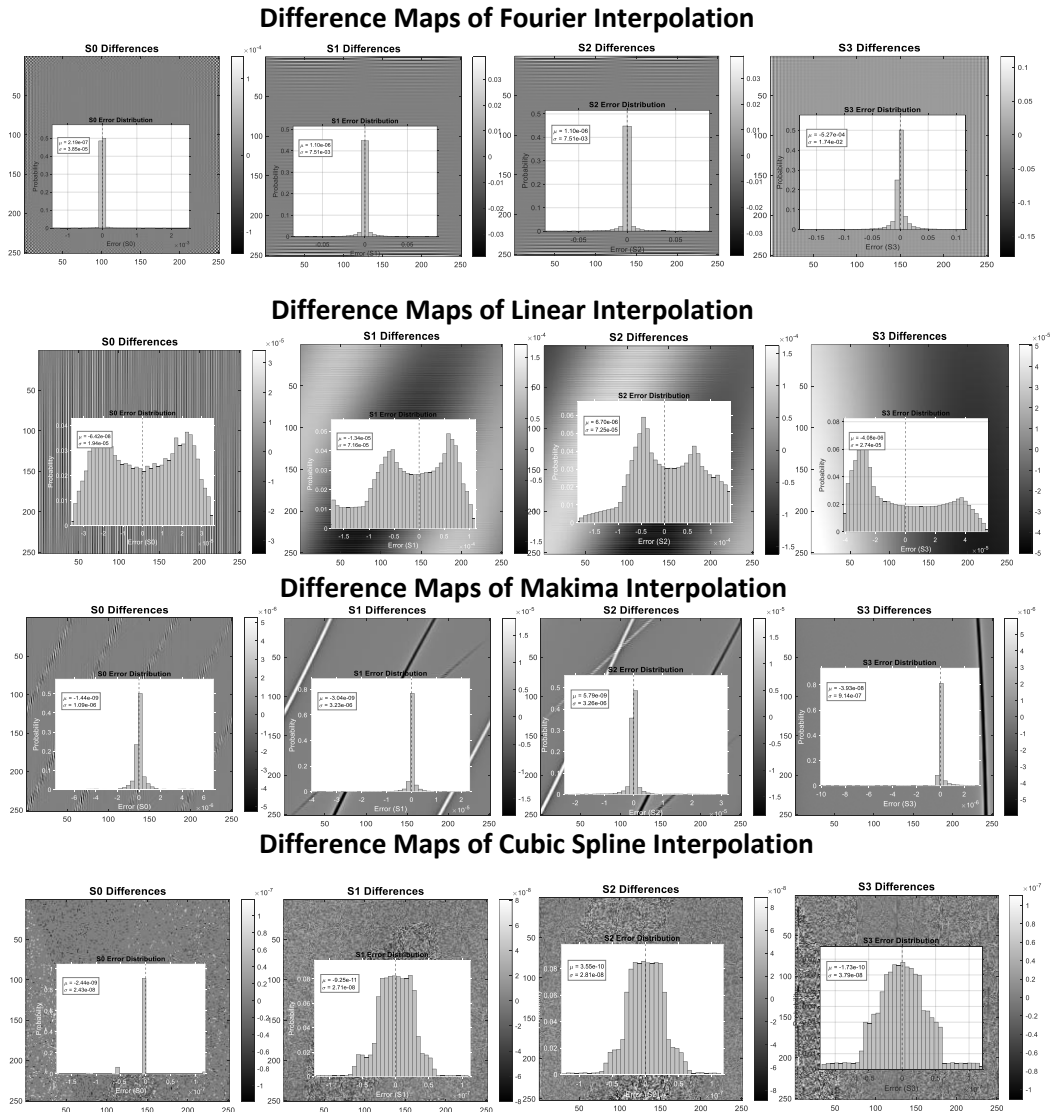


Figure 3-20. Difference maps and embedded error histograms for full-Stokes reconstruction using four interpolation methods: Fourier, Linear, Makima, and Cubic Spline.

Overall, the four methods exhibit clearly different error signatures. Linear interpolation produces the largest and most structured residuals, reflecting systematic demosaicking artifacts. Makima and cubic spline substantially reduce these structures and yield small, tightly centered histograms, with both μ and σ close to

zero. Fourier interpolation also achieves a near-zero mean error in the image interior, but its error maps show boundary-localized ringing due to the FFT's periodic extension, which inflates the global σ when statistics are computed over the full field of view. For this non-periodic elliptical scene, Makima and cubic spline provide the lowest overall variance, whereas Fourier is mainly limited by edge artifacts rather than by the accuracy of the reconstruction in the central region.

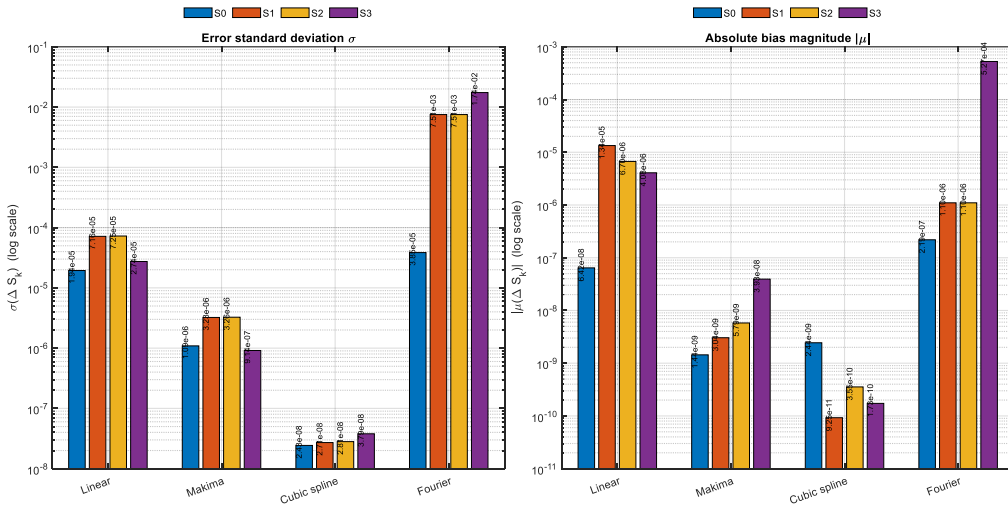


Figure 3-21. the mean error $\mu(\Delta S_i)$ and standard deviation $\sigma(\Delta S_i)$ of all Stokes components for the four interpolation strategies.

Using the same full-Stokes elliptical scene, Fig. 3-21 summarizes the mean error $\mu(\Delta S_i)$ and standard deviation $\sigma(\Delta S_i)$ of all Stokes components for the four interpolation strategies. Cubic spline delivers the best overall performance: it is essentially unbiased across $S_0 - S_3$ (with $|\mu| \leq 2.4 \times 10^{-9}$) and achieves the smallest dispersion ($\sigma \approx 2.4 - 3.8 \times 10^{-8}$). Makima is the second-best method, also nearly unbiased ($|\mu| \leq 3.9 \times 10^{-8}$) with standard deviations on the order of 10^{-6} . Linear interpolation shows substantially larger dispersion ($\sigma \approx 2.7 \times 10^{-5} - 7.3 \times 10^{-5}$) and non-negligible bias in S_1 and S_2 ($|\mu| \approx 10^{-5} - 10^{-6}$), consistent with the structured artifacts seen in the error maps.

Fourier interpolation, when evaluated over the full frame, exhibits markedly higher standard deviations for the polarization components ($\sigma(S_1) = \sigma(S_2) \approx 7.5 \times 10^{-3}$, $\sigma(S_3) \approx 1.7 \times 10^{-2}$) and a small bias in S_3 ($\mu(S_3) \approx -5.3 \times 10^{-4}$). As discussed above, this is mainly caused by boundary ringing introduced by the periodic extension in the FFT formulation. Because each difference map is displayed with its own 99th-percentile scaling, the grayscale contrast across methods is not directly comparable; the ranking is therefore based on the numerical values of μ and σ , which clearly favour cubic spline and Makima for this test scene.

3.7 Experimental validation on a radial polarizer.

In this section, we want to evaluate how four intensity demosaicking schemes (Bilinear, Bicubic Spline, Makima, and Fourier Transform) influence the polarimetric parameters AoLP, DoP, and El on real data. The scene is a Radial linear polarizer which was captured by our polarimeter (Monochrome polarimetric sensor and FLC modulator). We used the same two FLC recorded frames (images) and the same calibrated analyzer matrix to reconstruct Stokes S_0 , S_1 , S_2 , S_3 , for every kind of interpolation and the native super pixel (SP) reconstruction as a baseline to ensure a fair comparison. Results shown are for the red wavelength, because aside from any analyzer dispersion captured by calibration, green produced the same conclusions.

We quantify the impact on the calculated polarimetric parameters AoLP, DoP, and El using (i) circular profile on AoLP image, (ii) ring-mean versus radius for DoP image and El image, histogram distributions within a circular region of interest (ROI) of El. By these ways we can isolate the angular behavior and radial stability, and how much noise or aliasing differences across and between these chosen methods. SP, used as a low-resolution baseline.

Figure 3-22 show AoLP image of radial polarizer, with eight sections, with the principal values of AoLP (-90, +90), for Fourier transform demosaicking (Left), and

super-pixel resolution (SP) (right), and with the same white user-selected circular path. We used the same mapped circle drawn on the SP and all reconstruction AoLP methods with the same physical center and radius (R) in all reconstruction schemes. This figure only explains the geometry; comparisons are made in the next figures.

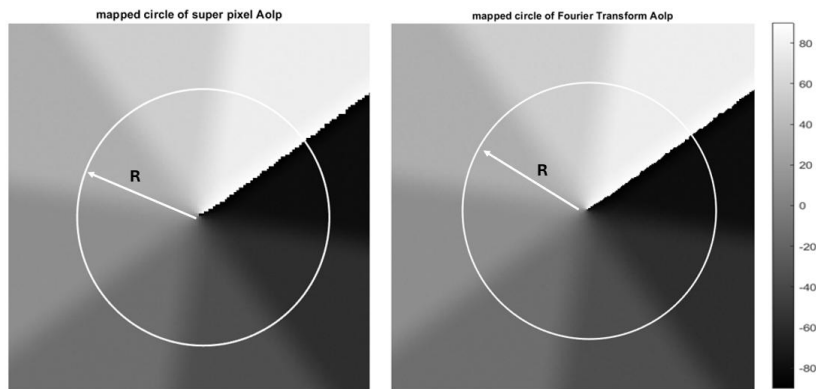


Figure 3-22. (Left): AoLP image of the radial polarizer with mapped circle for analysis. Left: the *same physical* circle mapped to the super-pixel (SP) grid. Circle of analysis chosen on the FT AoLP map (right).

Figure 3-23 (Top) represent the AoLP values versus angle along the analysis-selected circle for all methods. The staircase of $\sim 45^\circ$ steps correctly reflects the eight sections in the radial polarizer. (Bottom) show the AoLP differences between each method against super-pixel (SP), note that all the values are computed at the same angular samples, note how the AoLP differences remain close to 0° across the circle, and this at this range of differences is below than 1 will indicate that all demosaicking pipelines agree with SP to within a small fraction of a degree for this low-frequency target.

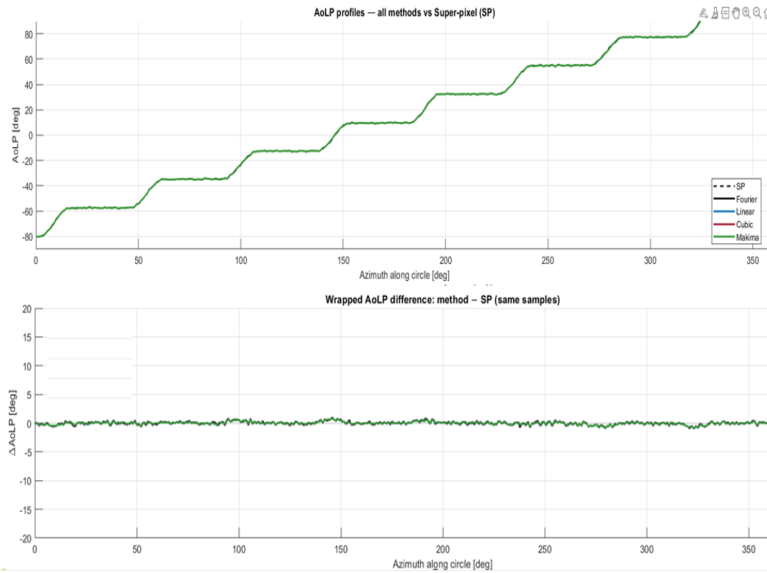


Figure 3-23. Circular AoLP profile for FT, bilinear, bicubic spline, and Makima. Super-pixel (SP) is plotted as discrete markers to emphasize it is a non-interpolated baseline. Differences of AoLP.

Figure 3.24 represent the DoP ring mean as a function of radius for all methods. For FT we also plotted a light-gray band showing the mean $\pm$ standard deviation. All methods track each other closely across radius. The DoP monotonically and rapidly rises from 0.25 near the center to then fixed level of 0.95 at large radii.

This behavior is expected for a radial polarizer: near the pole each ring integrates strongly varying analyzer orientations within a small footprint, which reduces the apparent polarization as the radius grows the local field varies more slowly across a super-pixel and the reconstructed DoP approaches the instrument's ceiling.

Because a radial polarizer's transmission angle varies as the azimuth, the polarization angle changes rapidly in the vicinity of the pole. Mean DoP as a function of radius. Light gray band shows FT mean $\pm 1\sigma$ across rings with sufficient pixel

support; all demosaicking methods track FT closely and converge toward the instrument ceiling away from the pole. Spatial averaging by the pixel/optics mixes many orientations within one footprint, reducing the polarization and therefore the measured DoP. The depolarization is strong near the center and vanishes with radius.

Inter-method differences are well within the FT $\pm\sigma$ band, and visually indistinguishable beyond $\sim 30\text{--}40$ px radius. This indicates that, for this scene and calibration, the choice of intensity interpolation has negligible impact on DoP, provided the Stokes inversion is properly normalized. The gray FT band in the figure is widest at small radii and tightens rapidly with radius. The larger inner variance is consistent with (i) higher angular gradient of the polarization field near the pole, (ii) finite pixel/aperture averaging, and (iii) demosaicking mixing across analyzer directions. After ~ 50 px the variance becomes small and flat, suggesting the reconstruction noise is dominated by sensor/shot noise rather than the interpolation scheme.

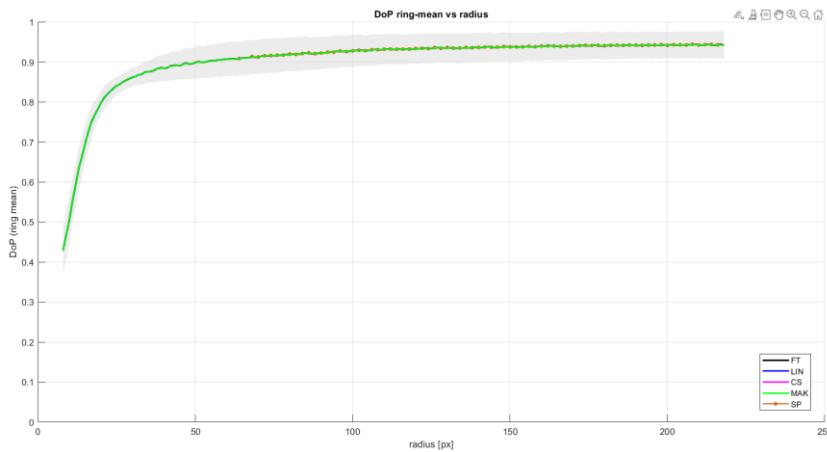


Figure 3-24. ring-mean Degree of polarization (DoP) versus radius for a radial polarizer.

light gray in the figure is the FT mean $\pm\sigma$ across each ring.

Another polarization polarimeter we evaluated is the ellipticity of a radial linear polarizer, for which the theoretical ellipticity equals zero. Figure 3-25 (Left) presents

the probability density of ellipticity inside the same mapped circular in figure 3-25. (Right) Ring-mean ellipticity versus radius. The mean remains within $\pm 0.2^\circ$ across most radii, and the ellipticity distribution is narrowly centered at zero for all methods.

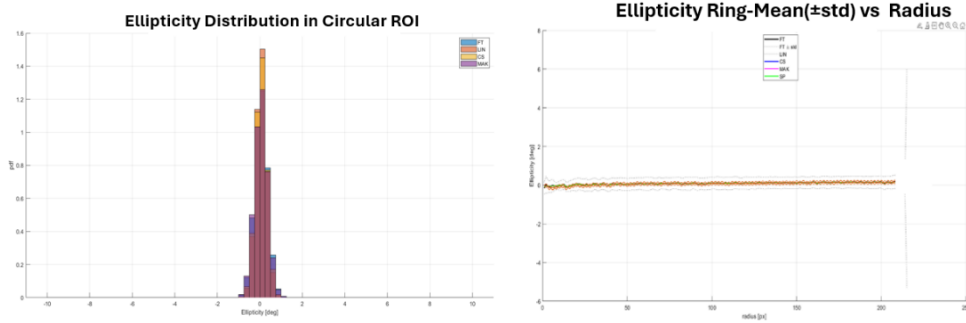


Figure 3-25. Ellipticity performance for a radial linear polarizer. (Left) Probability density of ellipticity inside a circular ROI centered on the polarizer. (Right) Ring-mean ellipticity versus radius. The mean remains within $\pm 0.2^\circ$ across most radii.

At the end of this section, the conclusion is that intensity demosaicking has little influence on the recovered polarization state for this class of radial polarizer target. These results indicate that, for this low-spatial-frequency target, the choice of interpolator mainly affects visual smoothness rather than the recovered AoLP, DoP and EI itself, consistent with prior DoFP demosaicking analyses [128], [130] [134].

3.8 Full Stokes Polarimeter

3.8.1 The Design of the Full-Stokes Imaging Polarimeter

In figure 3.26 we observe a photograph of the implemented Stokes imaging polarimeter. The setup integrates a Ferroelectric Liquid Crystal (FLC) modulator with a Sony IMX250MYR polarimetric camera.

The polarimetric camera (Sony IMX250-based sensor) incorporates four linear micro-polarizers (0° , 45° , 90° , 135°) arranged in a periodic super pixel.

To introduce the required retardance diversity, an FLC cell is placed before the camera.

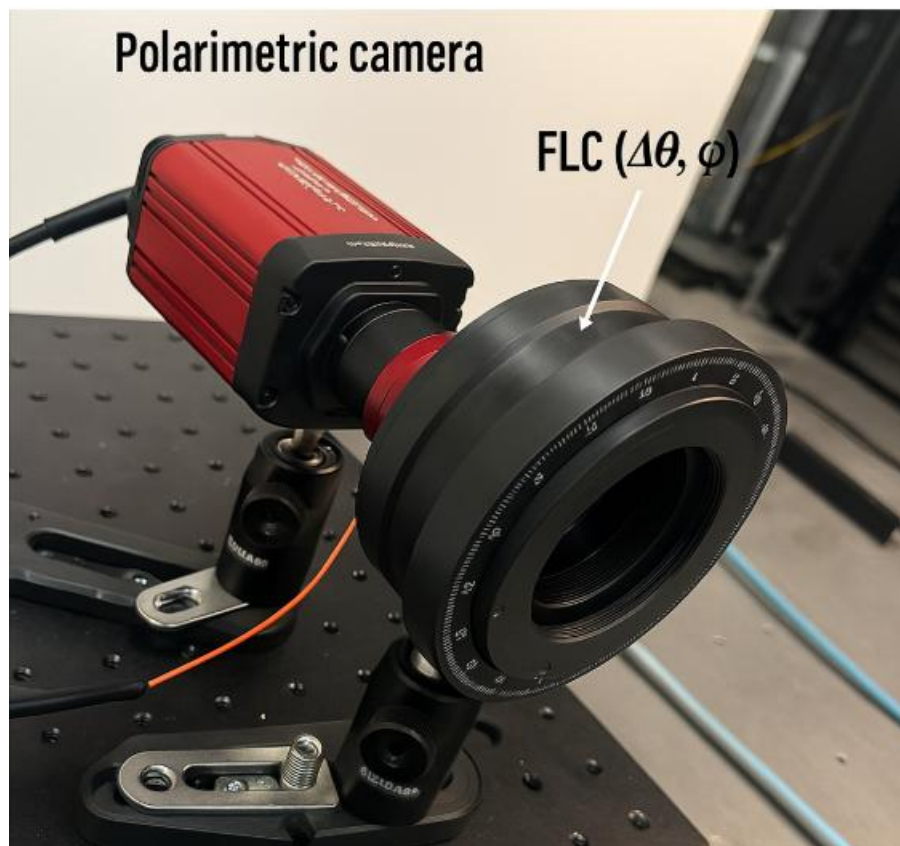


Figure 3-26. Photograph of the implemented Stokes imaging polarimeter. The setup integrates a Ferroelectric Liquid Crystal (FLC) modulator with a Sony IMX250MYR polarimetric camera.

3.8.2 Analyzer Matrix Formulation

The first element of the polarimetric setup is the polarized sensor. Each unit cell of this sensor contains four linear polarizers oriented at 0° , 45° , 90° and 135° . For each of these orientations, the corresponding Mueller matrix of a rotated linear polarizer $M_{pol}(\alpha)$ eq. (3.46) is considered, and the polarization state analyzer matrix A_{pol} is built from their first rows. In our case, the analyzer matrix is:

$$A_{pol} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad 3.46$$

FLC is the second element in our polarimetric setup. The Mueller matrix of the linear retarder with retardance φ and fast-axis orientation θ is given by:

$$M_{LR}(\theta, \varphi) = Rot(-2\theta)M_{WP0}(\varphi)Rot(2\theta) \quad 3.47$$

Where M_{WP0} represents a waveplate with its fast axis aligned along 0° , and $Rot(2\theta)$ is the rotation matrix, their matrices are:

$$M_{WP0}(\varphi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \quad \text{and} \quad 3.48$$

$$Rot(2\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) & 0 \\ 0 & -\sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In our setup, the FLC acts as a bistable quarter-wave retarder, switching between two optic axis orientations θ_1 and θ_2 , while maintaining constant retardance $\varphi = \frac{\pi}{4}$.

The temporal modulation is captured in the analyzer matrix:

$$A_{FLC} = \begin{bmatrix} M_{FLC}(\theta_1, \varphi) \\ M_{FLC}(\theta_2, \varphi) \end{bmatrix} \quad 3.49$$

For the whole system, for each FLC state, the effective analyzer of a given polarized pixel type is the first row of the product of Mueller matrix of the linear polarizer $M_{pol}(\alpha)$, and the rotated retarder Mueller matrix $M_{FLC}(\varphi, \theta)$. The total system analyzer, combining the polarimetric camera (modeled by A_{pol}) and the FLC modulation, is given by:

$$A_{Pol-FLC} = \begin{bmatrix} A_{pol}M_{FLC}(\theta_1, \varphi) \\ A_{pol}M_{FLC}(\theta_2, \varphi) \end{bmatrix} \quad 3.50$$

Now, for our compact system analyzer, we will call the $A_{Pol-FLC}$ as $A_{sys}(\phi)$, so that the resulting 8x4 system analyzer in term of ϕ is:

$$A_{sys}(\varphi) = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & \cos \varphi & \sin \varphi \\ 1 & 0 & -\cos \varphi & -\sin \varphi \\ 1 & \cos \varphi & 0 & -\sin \varphi \\ 1 & -\cos \varphi & 0 & \sin \varphi \\ 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix} \quad 3.51$$

By this, the incident Stokes vector with the measured intensities I are linking and can be written as:

$$I = A_{sys}(\phi) \cdot S \quad 3.52$$

According to $A_{sys}(\varphi)$ matrix, the first rows corresponding to the four directional polarized pixels for the FLC state of $\theta_1=0^\circ$, and the last four rows correspond to the same four directional polarized pixels for FLC state $\theta_2=45^\circ$.

This configuration yields 8 intensity measurements per super-pixel (2 FLC states $\times$ 4 micro-polarizer angles). These form a column vector:

$$I_n = [I(90^\circ, \theta_1), I(135^\circ, \theta_1), I(45^\circ, \theta_1), I(0^\circ, \theta_1), I(90^\circ, \theta_2), I(135^\circ, \theta_2), I(45^\circ, \theta_2), I(0^\circ, \theta_2)]^T \quad 3.7$$

In the absence of calibration errors, the Stokes vector can be retrieved from the intensity vector via the Moore–Penrose pseudoinverse, so that given the known system matrix A_{sys} , the measured Stokes vector S_m at each pixel is obtained by pseudo-inverse reconstruction:

$$S_m = \tilde{A}_{sys}^{-1} \cdot I_n \quad 3.54$$

The use of two fixed FLC orientations with four spatial analyzers has been previously shown to be sufficient for robust and well-conditioned Stokes reconstruction across varying polarization states [135], [136].

3.8.3 FLC Synchronization and Control of the Polarimetric Imaging System

Precise synchronization between the FLC and the polarimetric camera is achieved using a NI USB-6221 DAQ, controlled via LabVIEW. Two hardware-timed signals are generated: A ± 10 V analog square wave (20 ms period) to toggle the FLC between its two optic axis states. A TTL trigger pulse, delayed by ~ 5 ms, to initiate camera exposure only after the FLC has fully stabilized.

The camera's exposure time is tuned to avoid sensor saturation, and the trigger delay aligns with the FLC's sub-millisecond switching speed. This ensures that intensity measurements used for Stokes reconstruction are captured under stable modulation states. Figure 3.44 shows the measured and conceptual timing sequences between the modulation and acquisition signals.

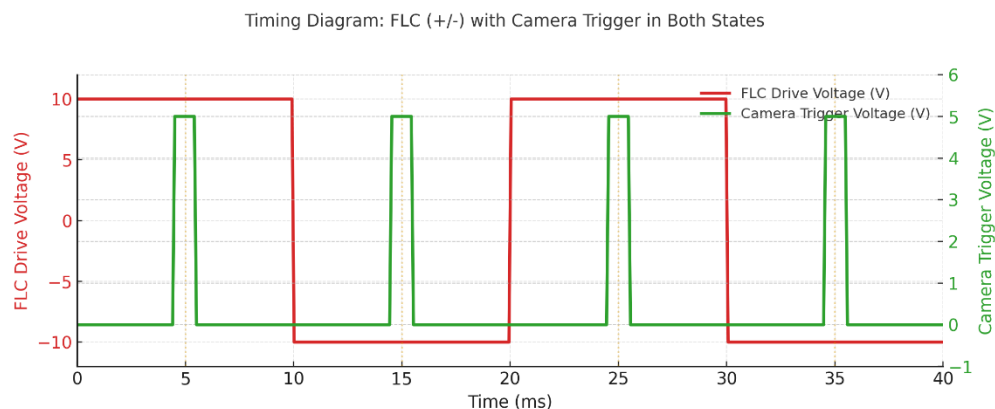


Figure 3-27. Timing diagram illustrates the synchronization between the FLC modulator and the snapshot polarimetric camera. The FLC switching signal (Analog Out from NI USB-6221) drives the FLC between two orthogonal fast-axis orientations (0° and 45°). A digital trigger pulse, delayed slightly with respect to the FLC transition, initiates the camera exposure to ensure the modulator has reached its stable state. This dual-channel synchronization ensures accurate and temporally consistent Stokes parameter acquisition with no moving parts.

The system is illuminated by a BlueBox (light source) emitting at 470 nm, 554 nm, and 630 nm, matching the spectral bands used in calibration section and validation Section. Figure 3-45 represents the optical configuration, side by side with the proposed polarimeter on the left, and the set-up to generate Poincare sphere wide range of state of polarization (SOP). This configuration is important in performing the calibration, validation, and measuring.

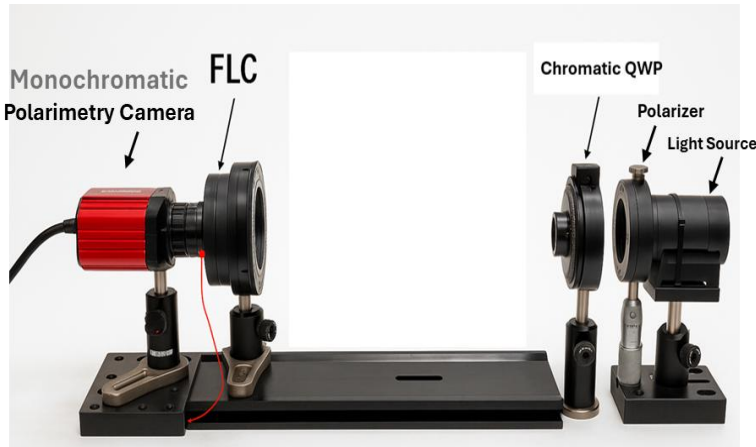


Figure 3-28. The optical configuration contains the light source, optical elements of polarizer, and achromatic quarter waveplate in the right, side by side with the compact full stokes polarimeter.

Having established the polarimeter's theoretical framework and practical configuration, we now turn our attention to a critical experimental step: calibration. Although the manufacturer provides nominal polarization orientations for the microgrid polarizer array (0° , 45° , 90° , 135°), practical measurements often reveal spectral-dependent slight deviations and non-ideal polarization performance. Such discrepancies, common in micro-polarizer array sensors, significantly impact the accuracy of polarization measurements, necessitating careful experimental spectral characterization. In the following section, we perform a detailed calibration to precisely determine the actual polarization characteristics, ensuring accurate and reliable Stokes vector reconstruction.

3.8.4 Calibration of the Full-Stokes Imaging Polarimeter

This step is standard practice in polarimetric measurements to determine the polarization state analyzer matrix for the internal optics of the polarimetric system. Polarization state analyzer in our setup case is $A_{sys}(\phi(\lambda))$, where Stokes

configurations involving FLC and polarized camera, and it is in term of ϕ , then it is wavelength dependent.

We calibrated the whole set-up using a well-defined set of reference SOPs, the configuration (right side) in fig (3-28) allows the generation of six standard SOPs (H, V, 45, 135, R, L), as we explained before that it is commonly used in Stokes polarimeter calibration.

During calibration, the FLC was driven between its two stable optic-axis orientations (0° and 45°), and for each FLC state the micro-polarizer array of the DoFP camera provided four intensity measurements. Each reference SOP therefore produced an-8-element intensity vector used to determine the system analyzer matrix.

Because both the FLC retardance and the micro-grid polarizers are wavelength-dependent, the calibration was performed independently at 470 nm, 554 nm, and 630 nm.

In determination of Data Reduction Matrix (DRM), for a given wavelength and the calibration SOPs, the measurement equation in () becomes:

$$I_{cal} = A_{sys} \cdot S_{cal} \quad 3.55$$

where A_{sys} is the 8×4 analyzer matrix that describes the combined action of the FLC (two orientations) and the four micro-polarizer directions of the DoFP sensor.

The DRM is obtained using the Moore–Penrose pseudoinverse of the reference matrix S_{cal} :

$$A_{sys} = I_{cal} S_{cal}^+ = I_{cal} \cdot S_{cal}^T (S_{cal} S_{cal}^T)^{-1} \quad 3.56$$

Example of the Calibrated Analyzer Matrix (554 nm)

Eq (3.36), shows the calibrated analyzer matrix at 554 nm. Each row corresponds to one combination of FLC state and micro-polarizer orientation.

The matrix clearly departs from the ideal theoretical model due to real-world effects such as FLC retardance errors, pixel-dependent extinction ratios, and spectral variations.

$$A_{green} = \begin{pmatrix} 1 & -0.174 & 0.636 & 0.572 \\ 1 & 0.771 & -0.388 & 0.416 \\ 1 & -0.894 & 0.257 & -0.365 \\ 1 & 0.024 & -0.734 & -0.511 \\ 1 & -0.082 & -0.723 & 0.553 \\ 1 & -0.833 & -0.473 & -0.354 \\ 1 & 0.735 & 0.386 & 0.420 \\ 1 & -0.078 & 0.623 & -0.537 \end{pmatrix} \quad 3.57$$

According to the condition number and wavelength dependence, the experimentally measured values in table below and were:

Table 3-6. Experimental Condition number vs Wavelength for the three calibrated wavelengths.

Wavelength (nm)	CN (A_{sys})
470 nm	11.90
554 nm	2.87
630 nm	2.71

The green and red channels exhibit excellent conditioning ($CN < 3$), consistent with their higher SNR and the near-ideal quarter-wave performance of the FLC around these wavelengths., while the blue channel shows a significantly larger condition number, consistent with reduced transmission and the well-known decrease in polarimetric performance at short wavelengths. This wavelength-dependent conditioning explains the larger errors observed in the validation experiments at 470 nm.

For Stokes Reconstruction, once A_{sys} calibrated, each Stokes vector is reconstructed as:

$$\hat{S} = A_{\text{sys}}^+ I \quad 3.58$$

where the pseudoinverse is computed as:

$$A_{\text{sys}}^+ = (A_{\text{sys}}^T A_{\text{sys}})^{-1} A_{\text{sys}}^T \quad 3.59$$

This operation is applied pixelwise to every super-pixel of the camera, enabling real-time full-Stokes imaging.

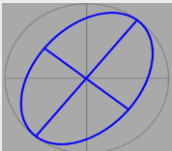
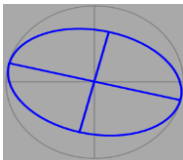
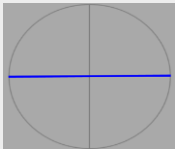
3.8.5 Validation of the Stokes Polarimetric Imaging System

To validate the accuracy of the calibrated polarimetric imaging system, a set of known polarization states was generated using a rotating linear polarizer and an achromatic quarter-wave plate. These inputs were sequentially projected onto the system at 470 nm, 554 nm, and 630 nm. The corresponding intensity frames were processed via the calibrated Data Reduction Matrix (A_{sys}) to reconstruct the full Stokes vectors,

To validate the polarimetric imaging system reconstruction at the three wavelengths, each Poincaré sphere for each wavelength in figures 3-29, 3-30, 3-31 show three reference polarization states measured with the Thorlabs polarimeter ($\times$) and the corresponding states retrieved by the proposed imaging polarimeter ($\bullet$). Solid lines connect each reference and measured pair. Insets illustrate the associated polarization ellipses in the laboratory frame for each state. The close agreement between reference and reconstructed SOPs confirms the accuracy of the calibrated analyzer matrix at this wavelength.

Results demonstrated high reconstruction fidelity in the green and red channels, However, the blue wavelength Blue (470 nm): Showed larger deviations errors.

Table 3-7. Validation of Stokes Parameters for Blue Wavelength (470 nm).

Numb. of Measurement		Thorlabs	Set-up Avg	Error	Set-up Std
Blue SOP 1 	Az	51,47	48,61	2.86	1,48
	Ellip	33,68	34,74	1.13	0,87
	DoP	97,19	95,60	1.59	0,06
Blue SOP 2 	Az	-14,09	-15,53	1.44	1,27
	Ellip	34,45	28,20	6,25	0,60
	DoP	97,99	94,57	3.42	0,05
Blue SOP 3 	Az	-0,28	-0,81	0.51	0,22
	Ellip	0,62	0,67	0.15	2.35
	DoP	98,27	93,47	4,8	0,02

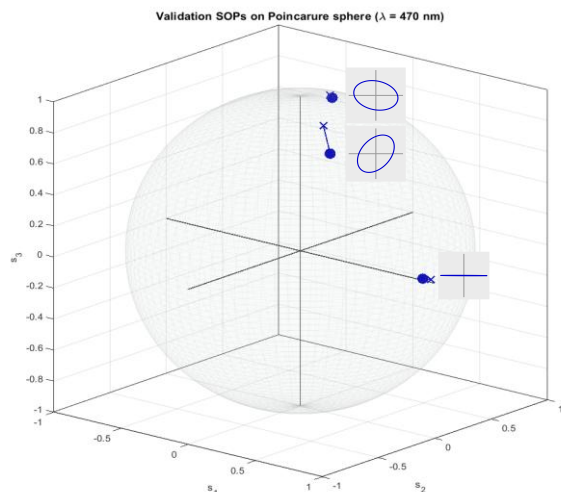
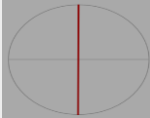
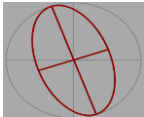


Figure 3-29. Validation SOPs on Poincaré Sphere ($\lambda=470\text{nm}$)

Table 3-8. Validation of Stokes Parameters for red wavelength (630 nm)

Numb. of Measurement		Thorlabs	Set-up Avg	Error	Set-up Std
Red SOP 1 	Az	89,77	89,40	0,37	0,27
	Ellip	-0,08	-0,09	0,01	0,30
	DoP	90,04	100,7	10,66	0,01
Red SOP 2 	Az	-70,58	-68,97	1,61	0,9
	Ellip	29,13	28,54	0,95	0,47
	DoP	93,94	93,37	0,57	0,01
Red SOP 3 	Az	51,60	53,87	2,27	0,84
	Ellip	-25,79	-25,91	0,12	0,30
	DoP	98,76	98,96	0,2	0,01

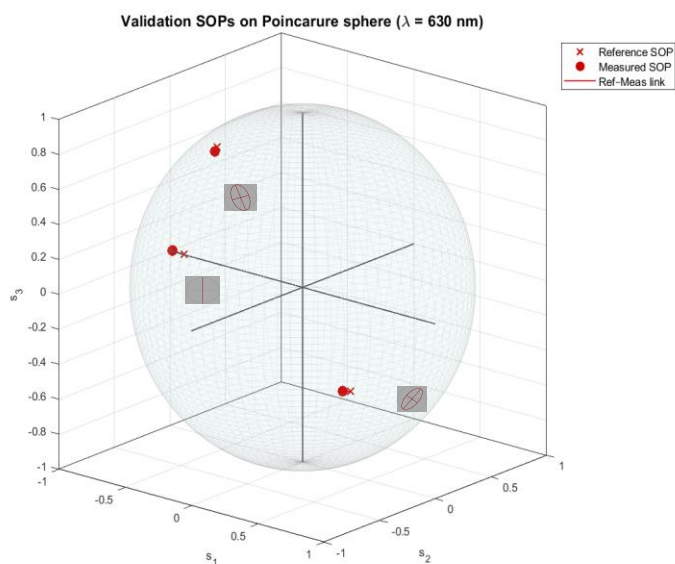


Figure 3-30. Validation SOPs on Poincaré Sphere ($\lambda=630\text{nm}$)

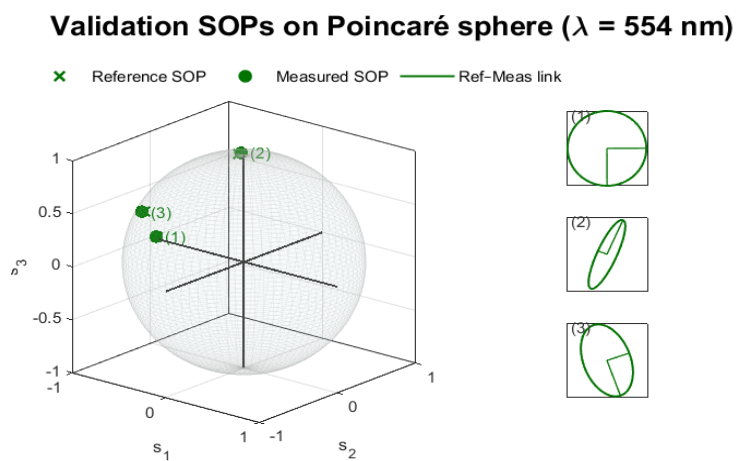
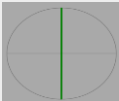
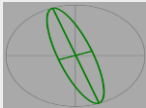


Figure 3-31. Validation SOPs on Poincaré Sphere ($\lambda=554\text{nm}$)

According to the tables, the green channel (554 nm) exhibited the highest measurement stability. Ellipticity standard deviation remained below 0.5° , and azimuth errors were consistently below 0.9° . DoP deviations remained under 0.3%.

Table 3-9. Validation of Stokes Parameters for Green Wavelength (554 nm)

Numb. of Measurement		Thorlabs	Set-up (Avg)	Absolute Error (%)	Set-up Std
Green SOP 1 	Az	-89,89	-89,8	0,09	0,2
	Ellip	0,77	0,69	0,08	0,5
	DoP	93,14	93,37	0.24%	0,01
Green SOP 2 	Az	67.86	66,60	1,26	0,98
	Ellip	31,56	31,43	0.13	0,5
	DoP	96,22	97,41	1.19	0,01
Green SOP 3 	Az	-70,42	-70,53	0,11	0,43
	Ellip	14,88	14.64	0,24	0,55
	DoP	94,21	95,98	1.88%	0,01

3.9 Polarization measuring

To assess system performance beyond calibration targets, we tested the polarimeter on a range of representative samples with known or measurable polarization features. These are chosen to demonstrate the versatility of the system under diverse polarization conditions and spatial structures.

Firstly, to evaluate our full-Stokes imaging polarimeter set-up to measure image with different polarization angles, with polarimetric accuracy, we measure the light that transmitted from radial test target consisting of eight sectors of linear polarizers, each oriented at discrete angles across 360° . The target was imaged under unpolarized illumination at three wavelengths: 470 nm (blue), 554 nm (green), and 630 nm (red).

Secondly, we will validate our full-Stokes imaging polarimeter set-up ability to resolve spatially varying retardance and orientation in a layered anisotropic sample, So that we measuring the polarization distribution in transmission mode of a stack of transparent polymer sheets with different thicknesses and orientations, examining our polarimeter ability to producing regions with distinct retardance and fast-axis angles, and separate different polarization information.

3.9.1 Polarimetric parameters Using Radial Polarizer Target

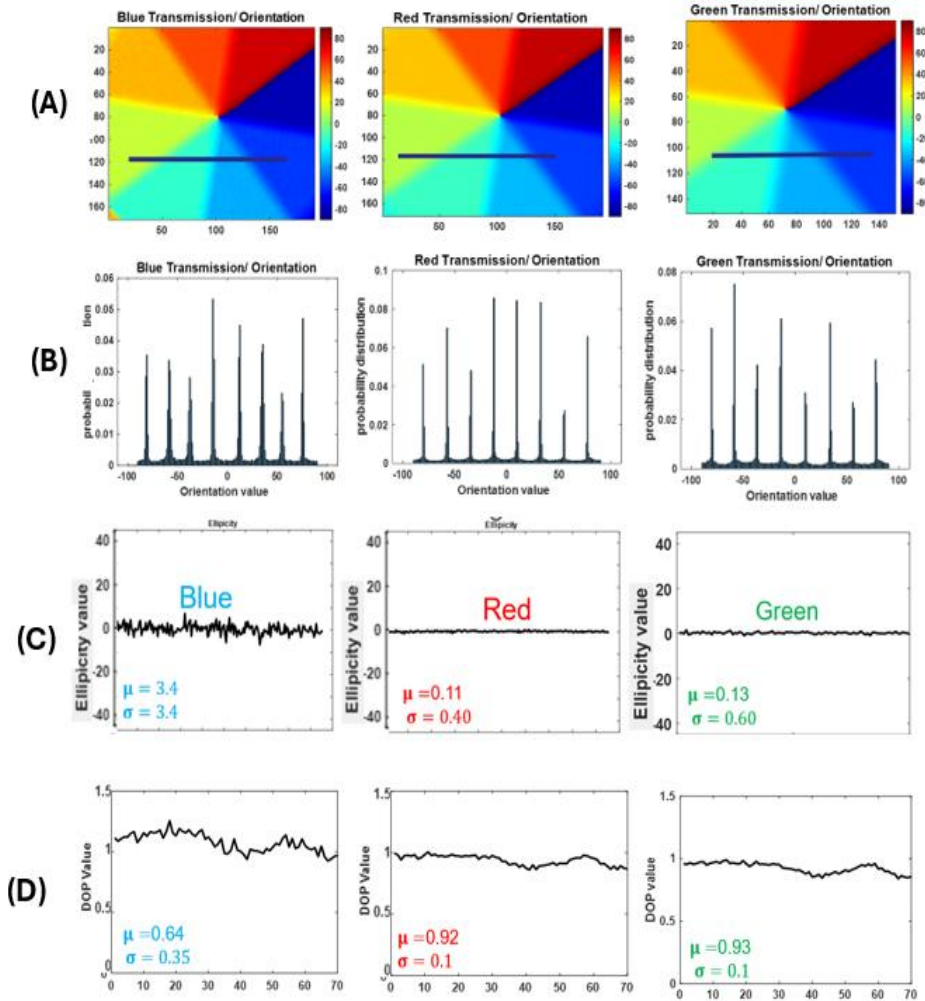


Figure 3-32. : (A) Orientation maps captured for blue (470 nm), red (630 nm), and green (554 nm) wavelengths using a radial polarizer composed of eight sectors at known angles. (B) Histograms of recovered orientation values across the radial pattern for each wavelength. (C) Ellipticity cross-sectional profiles for each wavelength. (D) Degree of Polarization (DoP) x-cross-sections.

Figure 3.32 (A) shows the reconstructed AoLP images of the radial polarizer at 470nm, 554nm and 630nm. These three images confirmed the correct recovery of

the AoLP parameter by their radial symmetry and 8 sector orientations for all wavelengths. The corresponding AoLP histogram across the images in Fig 3.32 (B) also quantify the AoLP accuracy, where they showed well separated peaks at the nominal sector angles. The blue channel shows slightly wider peaks, suggesting greater orientation standard deviation linked to the higher CN, whereas the red and green channels show narrow and well-defined distributions.

Figure 3.32 (C) showed the ellipticity values as a line profile across the ellipticity images of the radial polarizer. The value's range quantify the deviations of the measured ellipticity from the expected ideal ellipticity for linear polarization. The red and green channels show precise retrieval of linear polarization states by maintaining ellipticity values near zero ($\mu \approx 0.1^\circ$, $\sigma < 0.6^\circ$). The blue channel, on the other hand, exhibits higher ellipticity ($\mu = 3.4^\circ$, $\sigma = 3.4^\circ$), which is indicative of wavelength-dependent polarizer dichroism and decreased CN.

The third polarization parameter have been measured by our system is the DoP. Figure 3.32(D) showed the DoP values as a line profile across the DoP images of the radial polarizer. These results indicate high polarization fidelity and effective modulation contrast in these wavelengths. Strong polarimetric contrast and dependable modulation are indicated by the red ($\mu = 0.92$) and green ($\mu = 0.93$) channels, where high and stable DoP values with little standard deviations ($\sigma \approx 0.10$). The blue channel, on the other hand, exhibits a lower DoP ($\mu = 0.64$, $\sigma = 0.35$), which is in line with the combined effects of a lower detector sensitivity at shorter wavelengths and a lower extinction ratio. Overall, these findings highlight intrinsic wavelength-dependent limitations in the blue spectral band while confirming strong full-Stokes performance in the red and green spectral bands.

From a physical optics perspective, these results validate that the camera's analyzer micro-polarizer array, when combined with the FLC modulation states, can

From the standpoint of physical optics, these findings confirm that the camera's analyzer micro-polarizer array, when paired with the FLC modulation states, can precisely resolve the incident linear polarization content as long as the system's throughput and the extinction ratio of the test object (radial polarizer) stay constant. In red and green wavelengths, where the polarizer's dichroism and the detector's sensitivity are precisely matched to the modulation signal, the system exhibits outstanding DoP retrieval. In the meantime, the wavelength-dependent behavior of the optical materials used in the test scene and system directly affects the blue degradation, which is not only caused by camera noise.

3.9.2 Birefringent Layer Analysis with Plastic Films

A stack of birefringent plastic sheets was imaged by our set-up to study the system's sensitivity to stress-induced polarization and retardance effects. The observed ellipticity maps and DoP values exhibited consistent variations across the field, aligning with expected birefringent patterns. These measurements validate the system's ability to detect and quantify subtle polarization modulations from anisotropic materials.

The three ROIs correspond to the thick of birefringent plastic sheet in Figure 3-33 are represented. Mean polarization states of the three ROIs. Left: normalized Stokes vectors for ROIs R1–R3 plotted on the Poincaré sphere, showing three distinct linear–elliptical states. Right: corresponding mean $\pm$ standard deviation of DoP, AoLP and ellipticity for each region, illustrating high polarization purity in R1–R2 and increased depolarization in R3 together with clear differences in orientation and ellipticity across the stack.

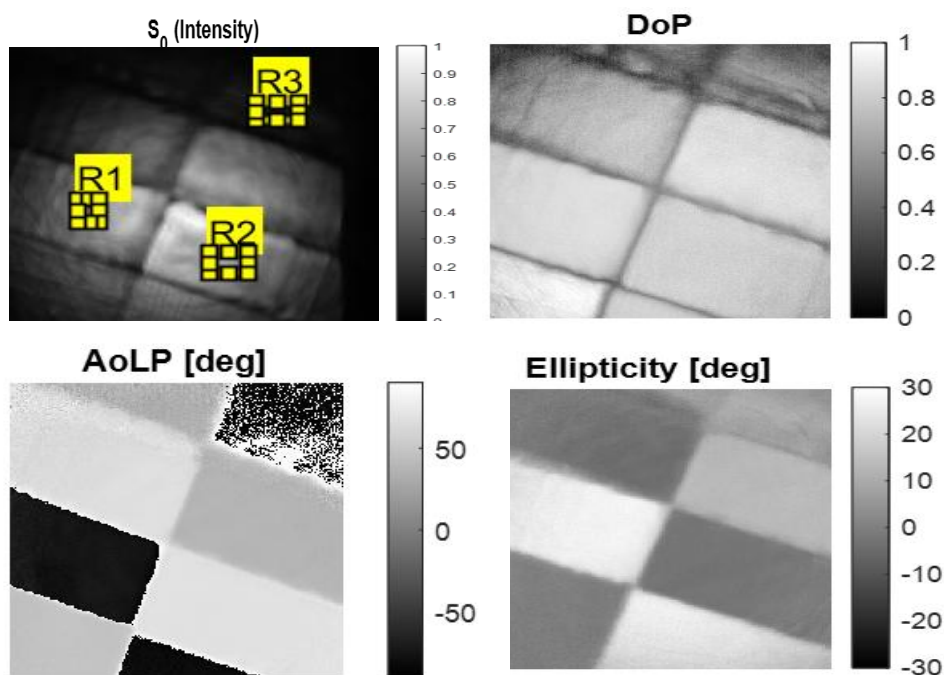


Figure 3-33. Normalized intensity S_0 image of the stacked polymer sample with three regions of interest (R1–R3) indicated, and corresponding maps of degree of polarization (DoP), angle of linear polarization (AoLP) and ellipticity.

They are with high Dop (≈ 0.9 –1) (Table 3-10) but clearly distinct AoLP and ellipticity values. On the Poincaré sphere Figure 2-34, the mean Stokes vectors of the ROIs form three compact clusters at different azimuths and latitudes, demonstrating that the instrument discriminates changes in both fast-axis orientation and retardance.

Table 3-10. The mean $\pm$ standard deviation of DoP, AoLP and ellipticity for each region R1–R3.

Region	DoP (mean $\pm$ std)	AoLP (mean $\pm$ std) [deg]	Ellipticity[deg] (mean $\pm$ std)
R1	0.862 ± 0.009	-74.7 ± 1.5	26.8 ± 0.5
R2	0.816 ± 0.017	68.5 ± 0.6	-2.6 ± 0.4
R3	0.547 ± 0.070	54.7 ± 6.8	9.4 ± 1.4

In this below figure, we can say that the plates exhibit high DoP inside each region, reduced DoP at the interfaces, distinct AoLP domains associated with different effective fast-axis orientations, and mainly small ellipticity values, indicating predominantly linear polarization with moderate variations in retardance across the stack.

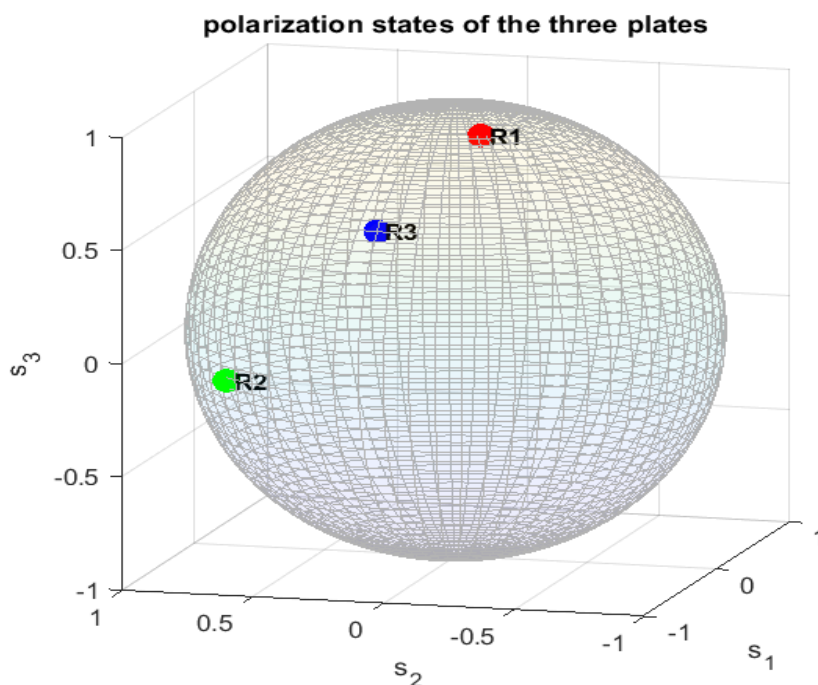


Figure 3-34. normalized Stokes vectors for ROIs R1–R3 plotted on the Poincaré sphere, showing three distinct linear–elliptical states with three regions of interest (R1–R3).

Chapter 4 RGB Polarimetry Camera: tri-wavelength polarization recovery, and Full-Stokes Expansion

In the previous chapter, the polarization behavior, calibration strategies, and spatial reconstruction of a monochrome microgrid polarization sensor were presented, both when used alone and when combined with a Ferroelectric Liquid Crystal (FLC) modulator. These methods are effective for monochrome sensors, where all pixels measure the same wavelength. However, they can not be directly applied to color polarimetric camera.

In color polarimetric sensors, polarization information and color information are measured simultaneously at the pixel level. Each pixel records only one wavelength-band and one polarization orientation, which means that the pixels are mosaiced. In addition, the color cross talk between RGB channels bias the reconstructed polarization information. The reconstruction methods designed for monochrome sensors cannot be directly applied to color-polarimetric systems without introducing systematic errors.

In this chapter, we address a fundamentally different and more challenging problem: the simultaneous reconstruction of polarization and spectral information at three wavelengths from a single mosaiced acquisition. To address the challenges introduced by this type of sensor, our reconstruction pipeline combines spatial Fourier-based demosaicking, color crosstalk unmixing technique, and polarimetric reconstruction to allow accurate polarization recovery at each wavelength from a single snapshot image.

While the RGB polarized sensor provides snapshot measurements of the linear Stokes parameters (S_0 , S_1 , and S_2) at each wavelength, it is not directly sensitive to circular polarization. To extend the system to full-Stokes imaging, a Ferroelectric Liquid Crystal modulator is placed in front of the RGB polarimetric camera. This modulator allows estimation of the circular Stokes parameter (S_3) in the RGB channels. The resulting system provides full-Stokes imaging polarimeter without the need for mechanical motion or sequential spectral and polarization acquisition.

4.1 Introduction

polarimetric color cameras are a forefront technology that simultaneously capture polarimetric and color information by analyzing polarization states across different color channels, commonly red, green, and blue.

In Figure (4-1), the sensor follows two periodic patterns:

1. Polarization pattern (DoFP):

A 2×2 super-pixel contains four linear analyzer orientations. The analyzer orientations as $\{0^\circ, 45^\circ, 90^\circ, 135^\circ\}$.

2. Color pattern:

RGGB Bayer type color filter array is used. Within a 2×2 color filter array unit there are two green filters, one red and one blue, reflecting the standard Bayer sampling strategy.

90°	45°	90°	45°
135°	0°	135°	0°
90°	45°	90°	45°
135°	0°	135°	0°

Figure 4-1. 16 physical pixels per superpixel; 12 channel images; green duplicated.

The polarization analysis and color filtering are performed directly at the pixel level. Each pixel is covered by (i) a linear micro-polarizer with a fixed orientation and (ii) a color filter (R, G, or B). Therefore, a single raw exposure records a mosaiced image in which different pixels correspond to different combinations of polarizer orientation and color filter. Consequently, the measured intensity in this type of

sensor is modeling as polarization analyzer and spectral response, that both are needing calibration.

Indeed, each of these color channels can carry different polarization information. Therefore, measuring the linear polarization at several discrete wavelengths simultaneously and with the highest possible resolution is of interest in multiple research areas[137]. Nonetheless, this situation has not yet been investigated in specialized literature, where it is still commonly assumed that all color channels transport the same polarization information.

In practice, polarimetric color cameras often come with the difficulty of color filter overlapping. As a special case, in our Sony RGB polarized sensor, Figure 4-2 illustrates the transmission responsivity of RGB color filters in this sensor. Because the broad range of wavelengths of the color filters are overlapping, each recorded color channel contains a non-zero contribution from wavelengths outside its nominal passband. For instance, the green filter partially transmits red and blue wavelengths, causing polarization-color crosstalk as shown in Figure 4-2.

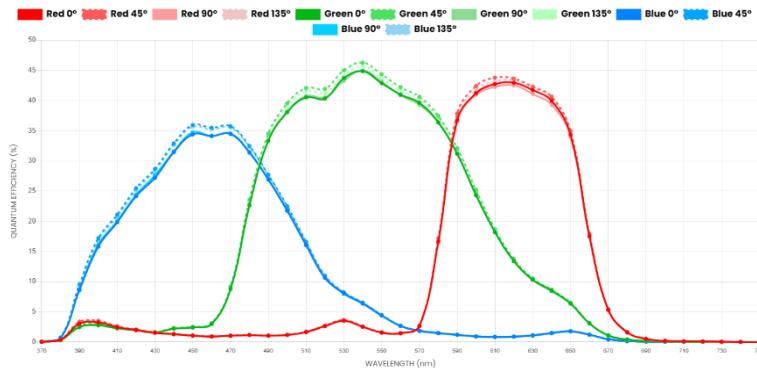


Figure 4-2. Relative spectral responses to different wavelengths of the SONY RGB sensors [80].

We will address the case of tri-wavelength polarization recovery using an RGB division-of-focal-plane (DoFP) color polarization sensor under narrowband

illumination at 470 nm, 554 nm, and 630 nm. Under this case of narrowband illumination and sensor architecture (sensor design), a recorded single mosaiced frame form a spatially multiplexed of different polarization and color information repeated by every 4×4 super pixel. As a result, each wavelength-dependent polarization image is strongly sub-sampled, and the measured RGB intensities are additionally affected by spectral overlap of the color filters, which produces wavelength mixing (color crosstalk).

The contribution of this chapter is an end-to-end, experimentally validated pipeline that combines:

(1) Fourier-domain demosaicking to recover 12 full-resolution channel images associated with the color–polarizer mosaic.

(2) An experimentally calibrated spectral unmixing model that corrects wavelength mixing caused by the RGB filter responses for the specific illumination wavelengths used in this thesis.

(3) Polarimetric reconstruction using wavelength-specific data reduction matrices (DRM) to obtain Stokes parameters and derived metrics (DoLP/AoLP) for each wavelength.

Fourier-domain demosaicking and frequency-domain formulations for mosaic sensors have been previously developed, including extensions to color–polarization cameras [138], [139]. The focus of this chapter is therefore not to claim a new demosaicking principle, but to demonstrate a practical and quantitative solution for accurate tri-wavelength polarization imaging on the specific RGB DoFP hardware sensor of our camera used in this thesis, where spectral crosstalk would otherwise bias the recovered Stokes parameters.

This ordering is essential: spatial demosaicking must precede spectral unmixing because the color filter array mixing occurs at the pixel level, while polarimetric

inversion must be applied only after wavelength separation to avoid systematic bias in polarization parameters under simultaneous multi-wavelength illumination.

In the validation step, we will demonstrate how the proposed method leads to a successful recovery of spectral and polarimetric information on both synthetic and real-world datasets. To test our approach, narrowband light beams at three wavelengths (470, 554, 630 nm), with different spatial polarization and degree of linear polarization distributions have been simulated and validated with experimental data. The results demonstrate the feasibility of the method for accurate three polarization channels measurements.

In addition, we will expand this sensor to Full-Stokes DoFP systems. It requires additional encoded retardance such as a micro retarder arrays or different type modulators [140] [137]. To this achieved, we built an RGB DoFP full-Stokes polarimeter using a FLC modulator atop of our RGB polarized sensor to reconstruct polarization across RGB spectral band in this sensor.

4.2 Theoretical formulation for Interpolation at the Fourier space (demosaicking)

As visualized in Figure 4.1, the RGB DoFP sensor multiplexes polarization and color at the focal plane, where the 4×4 super pixel contains the four micro-polarizer orientations combined with the RGGB color filter mosaic, resulting in 12 distinct analyzer color pixel classes (p, f), (four analyzer orientations $p \times$ three color channels f), distributed periodically across the array as represented in Figure 4-3.

To complete the camera sensor, this super pixel is replicated over the full sensor following a periodic pattern. Under this scheme, if an individual pixel has a size of Δx and Δy in the x and y directions respectively, when considering a particular pixel class, the closer pixel of the same class is found at distances $4 \cdot \Delta x$ and $4 \cdot \Delta y$ respectively (see Fig. 4-3). Consequently, for a camera sensor with $(N \times N)$ pixels

dimension, the 12 polarimetric images obtained by using the polarimetric color camera present a $N/4 \times N/4$ dimension, and thus, with reduced resolution with respect the camera sensor resolution. Under this scenario, to fill information in empty data spaces, we apply an interpolation process to obtain smother intensity images before being used for Stokes reconstruction step.

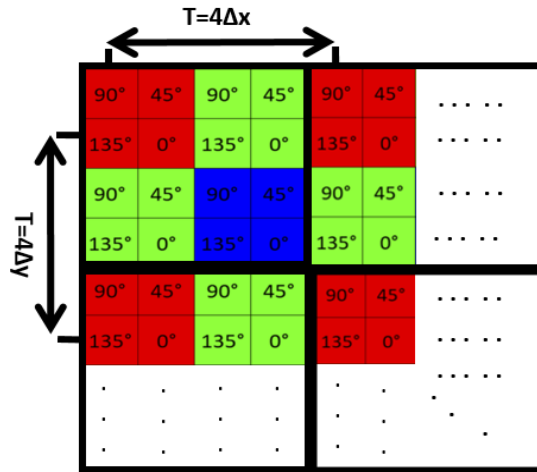


Figure 4-3. The full polarimetric color sensor consists of repeated 12 different analyzers (super pixel) following a periodic pattern. The closer pixel of the same class is found at distances $4 \cdot \Delta x$ and $4 \cdot \Delta y$.

This section is devoted to describing the interpolation method we applied to the 12 intensity images acquired by the polarimetric color camera, which is based on the Fourier Transform interpolation. We select this interpolation approach because it is the best interpolation according to the information theory when the sampled functions are bandlimited [57], [133]. Note that the proposed method must consider the different polarized pixels and color filter properties of analyzers (i.e., the 12 different analyzers).

We start by mathematically describing the intensity distribution $I_{pc}(4n, 4m)$ provided by the polarimetric camera for a particular pixel shift that will correspond to a specific pixel class (pc). Note that the subscripts (pc) denote for the specific

polarization analyzer type ($p=1, 2, 3, 4$; i.e., linear at $0^\circ, 45^\circ, 90^\circ$ and 135° respectively) and color filter ($c=1,2,3$ for R, G, B respectively). This intensity distribution can be written as the product of the complete intensity distribution arriving to the camera sensor, $I_{pc}(x, y)$, with a comb of deltas (in x and y directions) with the proper shift (k_x, k_y). Considering that each type of pixel is repeated itself with periods $T_x=T_y=T=4\Delta x$ (see Fig. 4-3) the sampled function and its Fourier transform can be written as:

$$I_{pc}(x, y) \cdot \sum_{r=0}^{\frac{N}{4}-1} \sum_{s=0}^{\frac{N}{4}-1} \delta\left(x - \frac{k_x T}{4} - rT\right) \delta\left(y - \frac{k_y T}{4} - sT\right) \xleftrightarrow[TF^{-1}]{TF} \frac{1}{|T^2|} \tilde{I}_{pc}(\mu, \nu) \otimes \left\{ \sum_{m=0}^3 \sum_{n=0}^3 \delta\left(\mu - \frac{Nm}{T}\right) \delta\left(\nu - \frac{Nn}{T}\right) e^{-i\frac{\pi}{2}(mk_x + nk_y)} \right\} \quad 4.1$$

where $k_x = \{0,1,2,3\}$ and $k_y = \{0,1,2,3\}$ account for the x and y shifts for each pixel class. Because the sampling mask is periodic with period $4\Delta x$, the Fourier transform of the sampled function contains $4 \times 4 = 16$ replicas (orders) arranged on a 4×4 grid in frequency. Each order is shifted to $(mN/4, nN/4)$ and is multiplied by a linear phase factor that depends on the spatial shift (k_x, k_y) :

$$e^{-i\frac{\pi}{2}(mk_x + nk_y)} \quad 4.2$$

This phase appears because spatial shifts in the sampling lattice correspond to linear phases in Fourier space. So, each replica will contain information of a combination of 12 images, each one multiplied by a different linear phase

The intensity distribution captured by the sensor will be the addition of the 16 images corresponding to the 16 possible shifts:

$$\begin{aligned}
I(m, n) = & \left(I_{1,1} \right) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - 2m\Delta y) \\
& + \left(I_{2,1} \right) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - 2m\Delta y) \\
& + \left(I_{3,1} \right) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - (2m+1)\Delta y) \\
& + \left(I_{4,1} \right) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - (2m+1)\Delta y) \\
& + \left(I_{1,2} \right) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - 2m\Delta y) \\
& + \left(I_{2,2} \right) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - 2m\Delta y) \\
& + \left(I_{3,2} \right) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - (2m+1)\Delta y) \\
& + \left(I_{4,2} \right) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - (2m+1)\Delta y) \\
& + \left(I_{1,2} \right) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - (2m+2)\Delta y) \\
& + \left(I_{2,2} \right) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - (2m+2)\Delta y) \\
& + \left(I_{3,2} \right) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - (2m+3)\Delta y) \\
& + \left(I_{4,2} \right) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - (2m+3)\Delta y) \\
& + \left(I_{1,3} \right) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - (2m+2)\Delta y) \\
& + \left(I_{2,3} \right) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - (2m+2)\Delta y) \\
& + \left(I_{3,3} \right) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - (2m+3)\Delta y) \\
& + \left(I_{4,3} \right) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - (2m+3)\Delta y)
\end{aligned} \tag{4.3}$$

Note that the intensity distributions $I_{p,2}$ $p=1, \dots, 4$ appear twice because the green sensor is repeated in two different locations. Therefore, for each possible pair of (k_x, k_y) values (12 possible combinations) and possible pair of replica order values (m, n) (16 possible combinations), a particular phase value will be obtained (i.e., $12 \times 16 = 192$). The two green lattices correspond to the same underlying green image per polarization, sampled at two offsets; therefore, the green sampling mask contains two impulses and produces the summed Fourier coefficients.

However, since these copies contain the original images information, but shifted to a unique frequency location corresponding to the periodic pattern, designing our polarimeter consists of obtaining non-overlapped images that can be separated and isolated. Note that if the frequency distribution corresponding to each replica is smaller than the distance between replicas (i.e., if replicas do not overlap), the intensity distribution of each one of the 16 replicas will consist of the addition of the 12 corresponding Fourier transformed images for each pixel class intensity.

Figure 4-4 shows the Fourier transform of the raw mosaic, where each diffraction order is a linear combination of the Fourier transforms of the 12 class images.

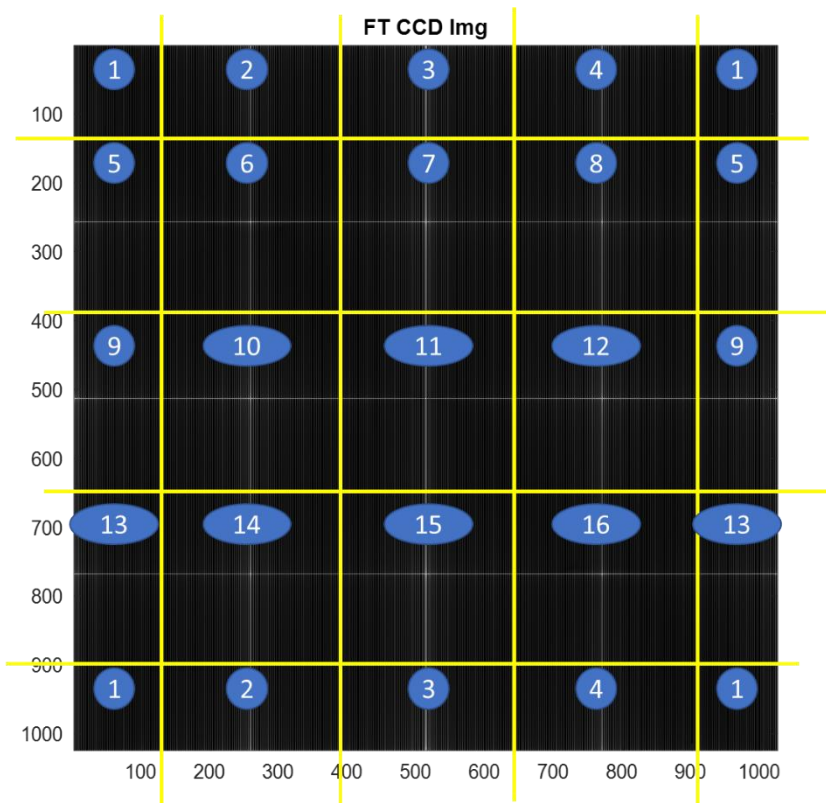


Figure 4-4. Fourier Transform of the CCD camera intensity distribution. The different diffraction orders appear numbered, being the marked with 11 the central one corresponding to $(m=0, n=0)$.

Operationally, we isolate each diffraction order and re-center it in a zero-padded frequency plane (Fig. 4-5), building a vector of measured order spectra.

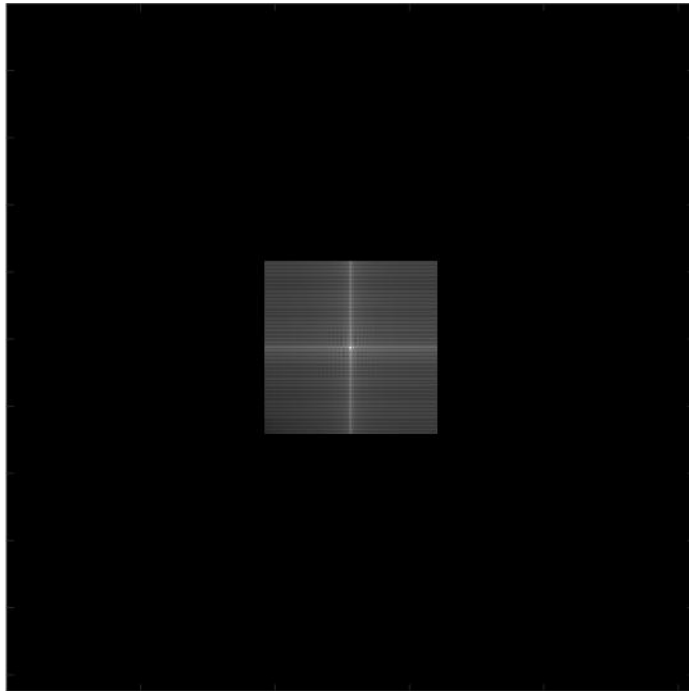


Figure 4-5. One of the diffraction orders isolated and centered in the full matrix

If the channel spectra are sufficiently band-limited, these orders can be isolated with a baseband mask without overlap. This non-overlap condition is what enables stable linear separation of the channels.

The resulting linear system can be written in matrix form with a mixing matrix M , that depends only on the known lattice shifts, as:

$$\tilde{I}_{\alpha} = M \tilde{I}_{pf}$$

4.4

Within each isolated order α , the spectrum is a linear combination of the 12 channel spectra. Stacking the 16 orders gives a linear system with a fixed mixing matrix M determined only by the macrocell geometry (phase terms from the shifts) as:

$$\begin{pmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \\ \tilde{I}_4 \\ \tilde{I}_5 \\ \tilde{I}_6 \\ \tilde{I}_7 \\ \tilde{I}_8 \\ \tilde{I}_9 \\ \tilde{I}_{10} \\ \tilde{I}_{11} \\ \tilde{I}_{12} \\ \tilde{I}_{13} \\ \tilde{I}_{14} \\ \tilde{I}_{15} \\ \tilde{I}_{16} \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & 1 & 2 & -2 & -2 & 2 & 1 & -1 & -1 & 1 \\ 1 & i & -1 & -i & 0 & 0 & 0 & 0 & -1 & -i & 1 & i \\ 1 & 1 & -1 & -1 & 2 & 2 & -2 & -2 & 1 & 1 & -1 & -1 \\ 1 & -i & -1 & i & 0 & 0 & 0 & 0 & -1 & i & 1 & -i \\ 1 & -1 & i & -i & 0 & 0 & 0 & 0 & -1 & 1 & -i & i \\ 1 & i & i & -1 & -2 & -2i & -2i & 2 & 1 & i & i & -1 \\ 1 & 1 & i & i & 0 & 0 & 0 & 0 & -1 & -1 & -i & -i \\ 1 & -i & i & 1 & -2 & 2i & -2i & -2 & 1 & -i & i & 1 \\ 1 & -1 & 1 & -1 & 2 & -2 & 2 & -2 & 1 & -1 & 1 & -1 \\ 1 & i & 1 & i & 0 & 0 & 0 & 0 & -1 & -i & -1 & -i \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & i & 1 & 1 \\ 1 & -i & 1 & 1i & 0 & 0 & 0 & 0 & -1 & i & -1 & i \\ 1 & -1 & -i & i & 0 & 0 & 0 & 0 & -1 & 1 & i & -i \\ 1 & i & -i & i & -2 & -2i & 2i & -2 & 1 & i & -i & 1 \\ 1 & 1 & -i & -i & 0 & 0 & 0 & 0 & -1 & -1 & i & i \\ 1 & -i & -i & -1 & -2 & 2i & 2i & 2 & 1 & -i & -i & -i \end{pmatrix} \begin{pmatrix} \tilde{I}_{1,1} \\ \tilde{I}_{2,1} \\ \tilde{I}_{3,1} \\ \tilde{I}_{4,1} \\ \tilde{I}_{1,2} \\ \tilde{I}_{2,2} \\ \tilde{I}_{3,2} \\ \tilde{I}_{4,2} \\ \tilde{I}_{1,3} \\ \tilde{I}_{2,3} \\ \tilde{I}_{3,3} \\ \tilde{I}_{4,3} \end{pmatrix} \quad 4.5$$

Coefficients with magnitude 2 occur because the green channel is sampled twice within the RRGB unit cell, producing two contributions with the same underlying (p, G) channel but different spatial shifts. We then invert this system to estimate the Fourier transforms of the 12 class images.

The estimation of the channels spectra is performed by solving the overdetermined linear system in the least-squares sense, using the Moore–Penrose pseudo-inverse, so the equation will be as:

$$\tilde{I}_{pc} = (M' M)^{-1} M' \tilde{I}_\alpha \quad 4.6$$

Finally, we recover the spatial-domain images by inverse Fourier transform as

$$I_{pc} = FT^{-1}(\tilde{I}_{pc}) \quad 4.7$$

At this stage, the reconstructed $I_{pc}(x, y)$ are full resolution but remain spectrally mixed due to RGB filter overlap; spectral unmixing step is applied next (Section 4.4).

Although the Fourier demosaicking strategy is consistent with the broader literature on microgrid polarimeters and focal-plane filter array[139], but the performance is strongest when the scene is sufficiently band-limited relative to the replica spacing [141].

4.3 Forward measurement model: polarization analysis and spectral mixing

This section defines a forward model for the raw measurements produced by an RGB DoFP color polarization sensor when the scene is illuminated by three narrowband sources, in our case at $\lambda_1 = 470nm$, $\lambda_2 = 554nm$, $\lambda_3 = 630nm$.

The measured intensity at each pixel is governed by two independent mechanisms. Firstly, the polarimetric analyzer response (micro-polarizer + optics), which maps the incident Stokes vector to an analyzer intensity (calibrated through the DRM for each wavelength). Secondly, the spectral mixing through the RGB color

filters, and this is because that each filter transmits a non-zero fraction of the other wavelengths (calibrated through an experimental transmission/mixing matrix).

4.3.1 Polarimetric analyzer response (per wavelength)

Let $S^{\lambda_i}(x, y) = [S_0, S_1, S_2, S_3]^T$ is the Stokes vector of the light field at wavelength λ_i incident in the polarized sensor, where each pixel contains a linear analyzer with one of the four orientations $p \in \{0^\circ, 45^\circ, 90^\circ, 135^\circ\}$. For an ideal linear polarizer at angle θ_p , the transmitted intensity is a linear function of the Stokes vector, and can be written as:

$$I_p^{\lambda_i}(x, y) = a_p^{\lambda_i} S^{\lambda_i}(x, y), \quad 4.8$$

where $a_p^{\lambda_i}$ is the analyzer row vector (1×4). In the ideal Mueller model, this row is proportional to $[1, \cos(2\theta_p), \sin(2\theta_p), 0]$, showing that a linear polarizer measures only the (S_0, S_1, S_2) components [59].

In practice, non-idealities (finite extinction ratio, wavelength dependence, optics, sensor effects) are captured by calibrating a wavelength-dependent data reduction matrix (DRM). We denote the vector of four analyzer intensities at wavelength λ_i as:

$$I^{\lambda_i}(x, y) = \begin{bmatrix} I_{0^\circ}^{\lambda_i}(x, y) \\ I_{45^\circ}^{\lambda_i}(x, y) \\ I_{90^\circ}^{\lambda_i}(x, y) \\ I_{135^\circ}^{\lambda_i}(x, y) \end{bmatrix} \quad 4.9$$

The Stokes estimate is obtained via a calibrated linear mapping:

$$\hat{S}^{\lambda_i}(x, y) = W^{\lambda_i} I^{\lambda_i}(x, y) \quad 4.10$$

where W^{λ_i} is the DRM calibrated separately for each wavelength.

4.3.2 Spectral mixing through RGB filters (simultaneous illumination)

Each pixel also contains a color filter $f \in \{R, G, B\}$. Let $t_f(\lambda_i)$ be the relative transmission (including sensor spectral sensitivity) of filter f at wavelength λ_i . Because real RGB filters are not ideal bandpass filters, $t_f(\lambda_i)$ is generally non-zero even when λ_i is not the nominal band of f . This produces spectral crosstalk between channels.

If the three wavelengths are turned on simultaneously, the sensor response is linear in optical power (within the non-saturated operating regime). Therefore, the measured intensity at a pixel with specific analyzer p and specific color filter f is the sum of the contributions from the three wavelengths:

$$I_{p,f}^{meas}(x, y) = \sum_{i=1}^3 t_f(\lambda_i) I_p^{\lambda_i}(x, y) \quad 4.11$$

The important point is that with three wavelengths on, each recorded RGB pixel is a linear mixture of the three wavelength-dependent analyzer intensities.

It is convenient to rewrite this mixing for each specific analyzer p in vector form, so that the RGB measurements (after reconstruction by interpolation to a common grid; see Section 4.3) will be defined as:

$$I_p^{ct}(x, y) = \begin{bmatrix} I_{p,R}^{ct}(x, y) \\ I_{p,G}^{ct}(x, y) \\ I_{p,B}^{ct}(x, y) \end{bmatrix}, \quad I_p^{\lambda}(x, y) = \begin{bmatrix} I_p^{\lambda_1}(x, y) \\ I_p^{\lambda_2}(x, y) \\ I_p^{\lambda_3}(x, y) \end{bmatrix} \quad 4.12$$

Then the spectral mixing model becomes:

$$I_p^{ct}(x, y) = C I_p^{\lambda}(x, y) \quad 4.13$$

With:

$$C = \begin{bmatrix} t_R(\lambda_1) & t_R(\lambda_2) & t_R(\lambda_3) \\ t_G(\lambda_1) & t_G(\lambda_2) & t_G(\lambda_3) \\ t_B(\lambda_1) & t_B(\lambda_2) & t_B(\lambda_3) \end{bmatrix} \quad 4.14$$

In our work of this thesis, the matrix C is measured experimentally using narrowband illumination at $\lambda_1, \lambda_2, \lambda_3$. Assuming narrowband illumination is very important because it allows each coefficient $t_f(\lambda_i)$ to be treated as a constant scalar for that wavelength.

4.3.3 Combined model and its role in the reconstruction pipeline

Combining Sections 4.3.1 and 4.3.2 gives a compact forward description of the measurement:

1. For each wavelength λ_i , the analyzer intensities are linear in the Stokes vector:

$$I_p^{\lambda_i}(x, y) = a_p^{\lambda_i} S^{\lambda_i}(x, y) \quad 4.15$$

2. The RGB filters mix the three wavelengths linearly:

$$I_{p,f}^{meas}(x, y) = \sum_{i=1}^3 t_f(\lambda_i) I_p^{\lambda_i}(x, y) \quad 4.16$$

Because the sensor and captured readout intensities are linear in intensity, in the non-saturated regime, simultaneous illumination produces additive contributions, so each RGB measurement is a linear mixture of the three wavelengths determined by the filter transmissions and sensor spectral sensitivity.

This next two sections explain the applied current measurement model, and how we experimentally estimate W^λ (polarimetric calibration) and C (spectral calibration).

4.4 Spectral calibration (RGB crosstalk / transmission matrix)

Under simultaneous tri-wavelength illumination (470, 554, and 630 nm), the measured RGB intensities therefore represent a linear spectral mixture of the three wavelength components. To illustrate this mechanism, Figure 4-6 represent the RGB spectra in our case of sensor: the selected narrowband wavelengths (vertical lines) intersect the three-color filter spectral response curves with different relative weights, which produces measurable leakage (color crosstalk) between channels. The partial overlap between the RGB channel responses indicates non-zero sensitivity outside each nominal band, which motivates the spectral mixing (color crosstalk) calibration and the subsequent spectral unmixing step prior to Stokes reconstruction.

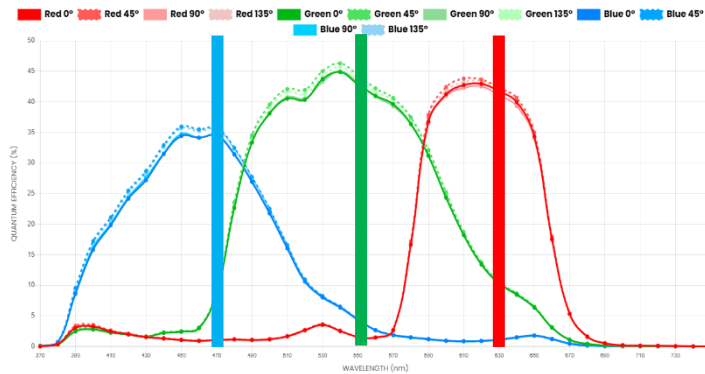


Figure 4-6. Spectral responsivity (quantum efficiency) of the RGB channels of the color-polarization sensor. The vertical lines mark the center wavelengths of the narrowband illumination used in this work (470 nm, 554 nm, and 630 nm).

To follow our pipeline to correct these color crosstalk intensities, we experimentally determined a 3×3 spectral mixing matrix C that maps the true per-wavelength intensities to the recorded color filter array channel intensities. This effect is described by the spectral mixing model introduced in Section 4.3. The objective of spectral calibration is to estimate the 3×3 mixing matrix C in eq.4.4 for

our camera and illumination wavelength and apply its inverse to recover per-wavelength analyzer intensities before Stokes reconstruction.

To measure C experimentally, the camera is illuminated with a polarization state for which the transmitted intensity through an ideal linear polarizer is independent of analyzer angle (i.e., a state with $S_1 = S_2 = 0$, such as circularly polarized light). This way the bias from the micro-polarizer orientation is removed. In that case, all four micro-polarizer orientations in a unit cell transmit the same fraction of the incident intensity, so the measured differences across color filter channels can be attributed to the color filter spectral response rather than to polarization angle effects. Mathematically, for an ideal linear polarizer, the transmitted intensity is:

$$I(\theta) = \frac{1}{2}(S_0 + S_1 \cos 2\theta + S_2 \sin 2\theta) \quad 4.17$$

So, if $S_1 = S_2 = 0$, the transmitted intensity is $I(\theta) = \frac{1}{2}S_0$, independent of analyzer angle. This follows directly from the Mueller-matrix expression for linear polarizers [59]. Circular polarization is more convenient in our setup in comparing to uncontrolled unpolarized light, where we can generate it repeatability.

For each illumination with circular polarization state at wavelength λ and each recorded color channel $f \in \{R, G, B\}$ by the sensor, the mean gray level GL_f^λ is recorded at several exposure times. Then, normalized by exposure time ET and by the incident optical intensity S_0^λ measured with an external polarimeter (Thorlab polarimeter) as in this equation:

$$Ft_f^\lambda = \frac{GL_f^\lambda}{ET \cdot S_0^\lambda} \quad 4.18$$

Then, the values Ft_f^λ averaged across the several exposure times (Appendix B). Finally, we normalized all coefficients by the reference response Ft_G^{550} because it corresponds to the matched filter–wavelength pair (green illumination measured

through the green filter) and, in our measurements, it produced the largest normalized camera response (highest signal level).

Using the maximum response as the reference improves numerical stability and provides a clear physical interpretation: all entries of C are reported as relative transmissions with respect to the green channel at 554 nm, so $C_{G,550} = 1$ by definition and the remaining coefficients quantify the fractional leakage into non-matched color filters. the full tables of ET , S_0^λ , gray levels, and intermediate values are listed in Appendix B.

Table 4.1 reports the resulting spectral mixing matrix C for our sensor at 470, 554, and 630 nm.

Table 4-1. Spectral Mixing matrix.

		Wavelength		
		R (630nm)	G (554nm)	B (470nm)
Transmission	R-filter	0.97	0,04	0,01
	G-filter	0.25	1	0,19
	B-filter	0,03	0.10	0,72

Figure 4-2 visually supports these coefficients: the vertical line at 470 nm intersects the blue response below its maximum, while the green response has non-zero value at 470 nm, consistent with the measured leakage terms in C . The spectral mixing is identical for each analyzer orientation p . Therefore, according to eq.4.13, to implement the measurements, we stuck the 12 analyzer images into a single vector, then, the 3×3 mixing matrix C extends to the full measurement model via a Kronecker product with the 4×4 identity:

$$I^{ct} = (T \otimes I_4) I^\lambda \quad 4.19$$

This notation ($\otimes$) denote the Kronecker product. The explicit expanded form is provided in Appendix C.

In the reconstruction pipeline following the measurement model (Section 4.3), the inverse matrix C^{-1} is applied to unmix the recorded RGB intensities into wavelength-resolved analyzer intensities, so that to recover per wavelength analyzer intensities I_p^λ , the spectral unmixing is performed by applying pseudo-inverse:

$$I_p^\lambda = C^{-1} I_p^{ct} \quad 4.20$$

Note that this step is performed in our pipeline prior to stokes reconstruction which based in DRM, which will be explained in the next section.

4.5 Polarimetric calibration (DRM) per wavelength

To follow our pipeline, the W^{λ_i} (polarimetric calibration) is needed to estimate the Stokes images (eq.4.8). In this section, a data reduction matrix W^λ is estimated at each wavelength $\lambda_i \in \{\lambda_R, \lambda_G, \lambda_B\}$.

Following polarimetric calibration principles in this thesis, which explained in chapter 2, this DRM calibration is performed independently at 470nm, 554nm, and 630 nm, because the response of sensor and the micro-grids are wavelengths dependent. To describe it briefly, the steps are:

1. Illuminating the sensor sequentially at $\lambda_R, \lambda_G, \lambda_B$.
2. A set of known input polarization states are generated and staked at a vector $\hat{S}^{\lambda_i}(x, y)$, for this input an intensities vector is staked in $I^{\lambda_i}(x, y)$.
3. Then, for each wavelength and matched color channel W^{λ_i} will be estimated by applying the pseudo inverse of eq. 4.10.
4. Store $W^{\lambda_R}, W^{\lambda_G}, W^{\lambda_B}$ for each color channel reconstruction.

This calibration captures wavelength-dependent analyzer behavior and any non-idealities of the optical path. This is standard in practical with DOFP polarimetry [140], [142].

Table 4.2 represent the averaged normalized analyzer components (S_1 , S_2 , S_3) for each pixel type (P11, P12, P21, P22) across red, green, and blue filters, calibrated at wavelengths (630 nm, 554 nm, and 470 nm). Each raw was normalized by its own S_0 value to allow direct comparison of their polarization sensitivity. The table reveals the spectral and spatial dependence of the analyzer orientations across the color-polarization sensor.

Table 4-2. Averaged normalized analyzer components (S_1 , S_2 , S_3) for each pixel type (P11, P12, P21, P22) across red, green, and blue filters.

Pixel Type	Filter	Dominant Component	S_1	S_2	S_3
P11	Red	S_1	-0.98	+0.05	-0.04
P11	Green	S_1	-0.94	+0.05	-0.02
P11	Blue	S_1	-1.00	+0.04	-0.03
P21	Red	S_2	-0.03	-0.96	+0.02
P21	Green	S_2	-0.04	-0.93	+0.03
P21	Blue	S_2	-0.03	-1.00	+0.02
P12	Red	S_2	+0.04	+0.95	-0.01
P12	Green	S_2	+0.04	+0.94	0
P12	Blue	S_2	+0.04	+1.00	-0.02
P22	Red	S_1	+0.97	-0.03	+0.03
P22	Green	S_1	+0.94	-0.04	+0.04
P22	Blue	S_1	+1.01	-0.04	+0.04

As expected from the micro-polarizer orientations, the dominant analyzer component remains in S_1 or S_2 , while small residual S_3 terms quantify departures from an ideal linear analyzer, as wavelength-dependent wire-grid behavior, as discussed in chapter 3. So that, a pure DoFP camera with linear micro-polarizers is fundamentally a linear-Stokes instrument (primarily S_0, S_1, S_2). The observed small S_3 component in our analyzer vectors, are non-ideal mixed (another types of

crosstalk) sensitivity such as manufacturing retardance, residual birefringence, or calibration noise. More importantly, they are not circular-polarization measurement [143].

In addition to the wavelength-matched calibration (e.g., 630 nm light incident on red-filtered pixels), we also evaluated the sensor response under wavelength–filter mismatch (e.g., 630 nm incident on green- and blue-filtered pixels). Figure 4.7 summarize the spectral evolution of the normalized analyzer components (S1–S3) for representative pixel types. It provides a direct experimental characterization of spectral leakage and proves the need for the spectral unmixing step in Section 4.3.

The dominant component magnitude is highest when the illumination wavelength falls within the nominal passband of the corresponding color filter array filter, while the green channel is comparatively less sensitive to wavelength mismatch due to its broader spectral response.

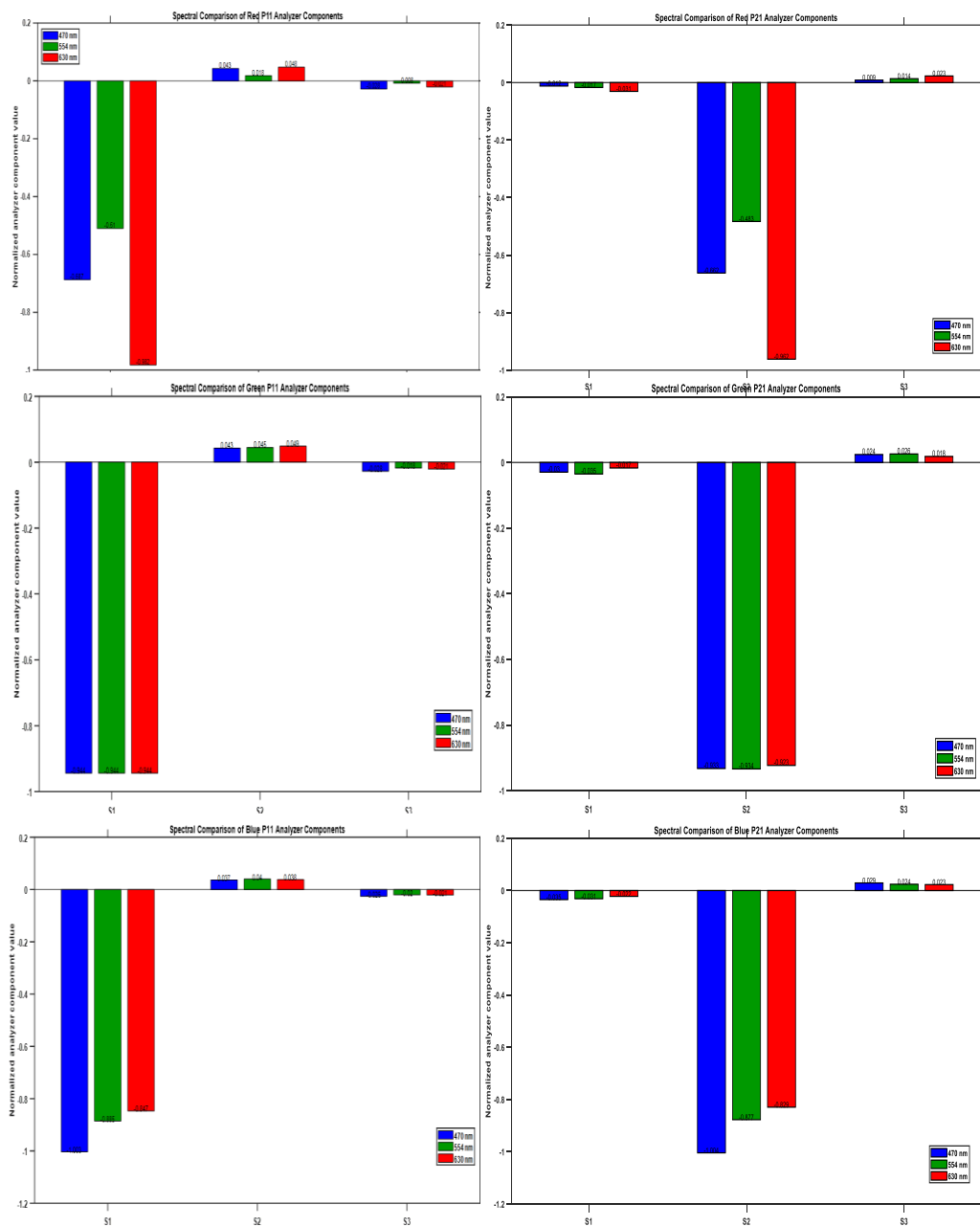


Figure 4-7. Spectral comparison of normalized average values of Stokes vector component (S1, S2, S3) for a given wavelength, for pixel P11 (Left column), and P21 (Right column) across the red filter (Top), green filter (Middle), and blue filters (Bottom).

To validate the polarimetric calibration, we evaluated pixel-wise distributions of key polarization observables derived from the calibrated analyzer vectors. Figure 4.8 shows AoLP histograms for each pixel type under matched calibration wavelength; the expected peaks near 0° , 45° , 90° , and 135° and their small standard deviations indicate angular consistency across the sensor.

Figure 4.9 shows ellipticity histograms, which cluster near zero as expected for predominantly linear analyzers, with small wavelength-dependent deviations. Figure 4.10 summarizes DoP (or DoLP) distributions, typically peaking near 0.95–0.98, consistent with high diattenuation of visible-band wire-grid micro-polarizers.

Occasional values slightly above unity can occur after Stokes reconstruction due to noise propagation and finite calibration errors (e.g., small biases in S_0 or amplification of noise in S_1 – S_3). Where required for visualization, DoP can be clipped to $[0, 1]$ without altering the underlying conclusions, as the relevant metric for sensor characterization is the distribution width and the separation between the four AoLP modes.

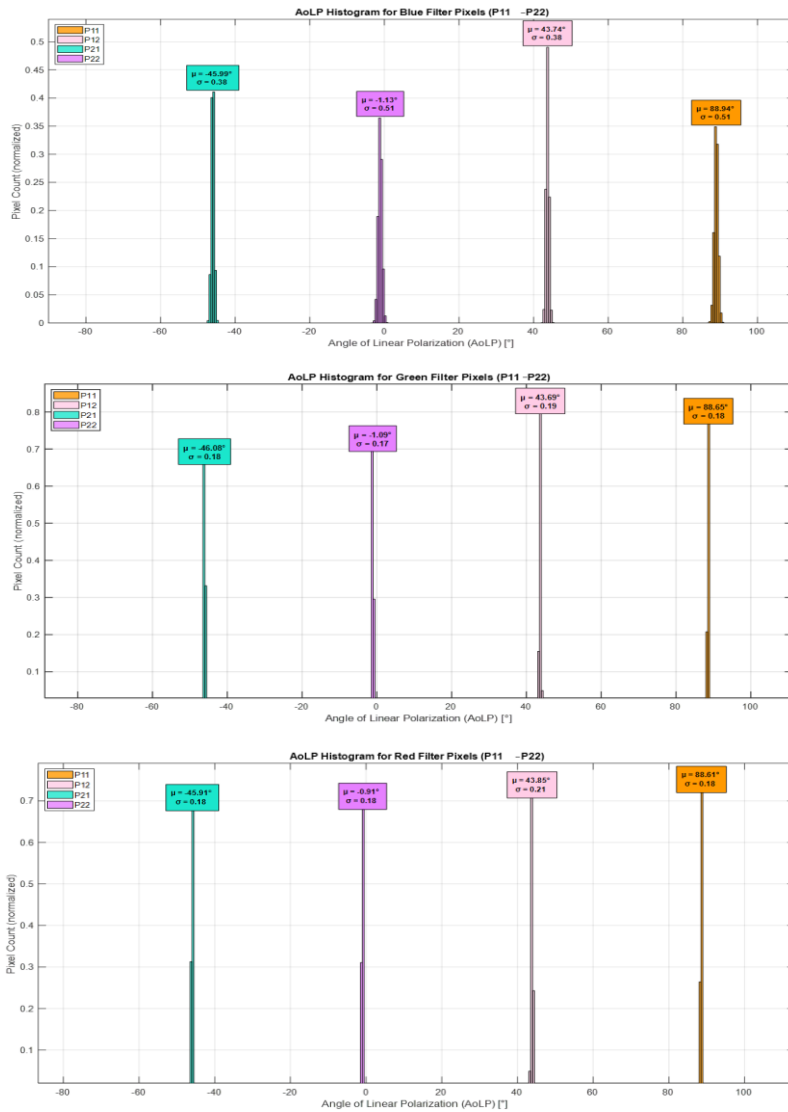


Figure 4-8. Normalized histogram of the Angle of Linear Polarization (AoLP) for the four micro-polarizer orientations under the green, red, and blue filter (calibrated with matched wavelength). Each histogram corresponds to a distinct pixel type (P11, P12, P21, P22). For direct quantitative insight, the plot incorporates the AoLP mean and standard deviation values.

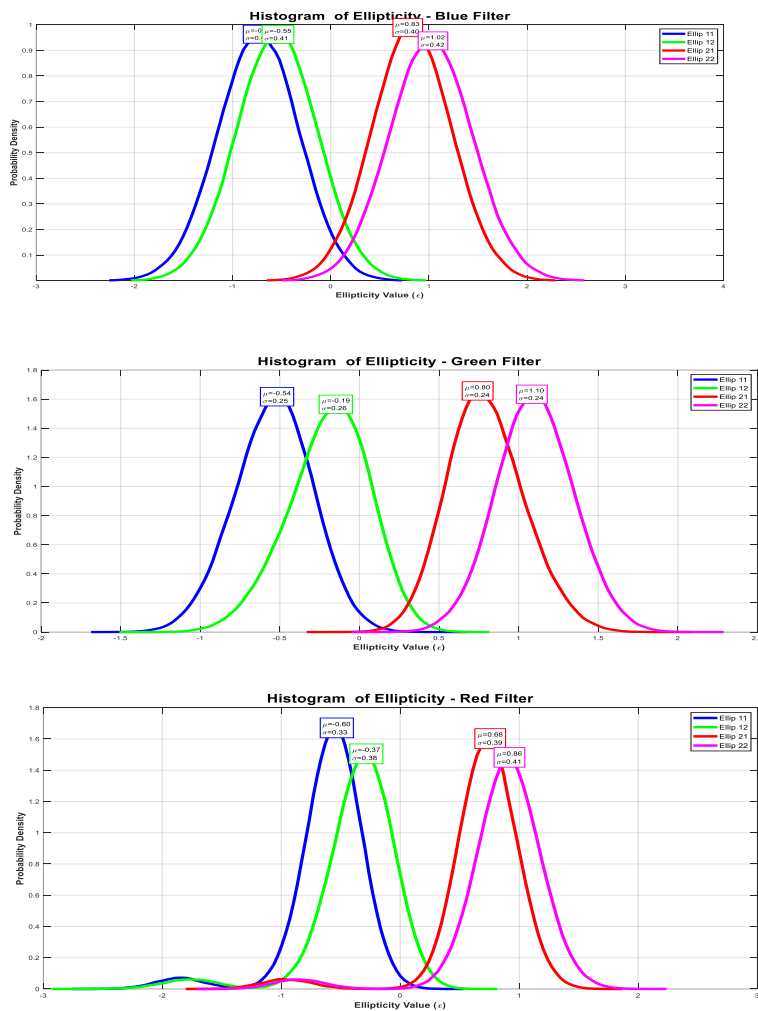


Figure 4-9. Pixel-level ellipticity histograms of four microgrid polarization directions under the green, red, and blue filters. Each distribution is annotated with the mean $\pm$ standard deviation to indicate polarization behavior and ellipticity uniformity per pixel class.

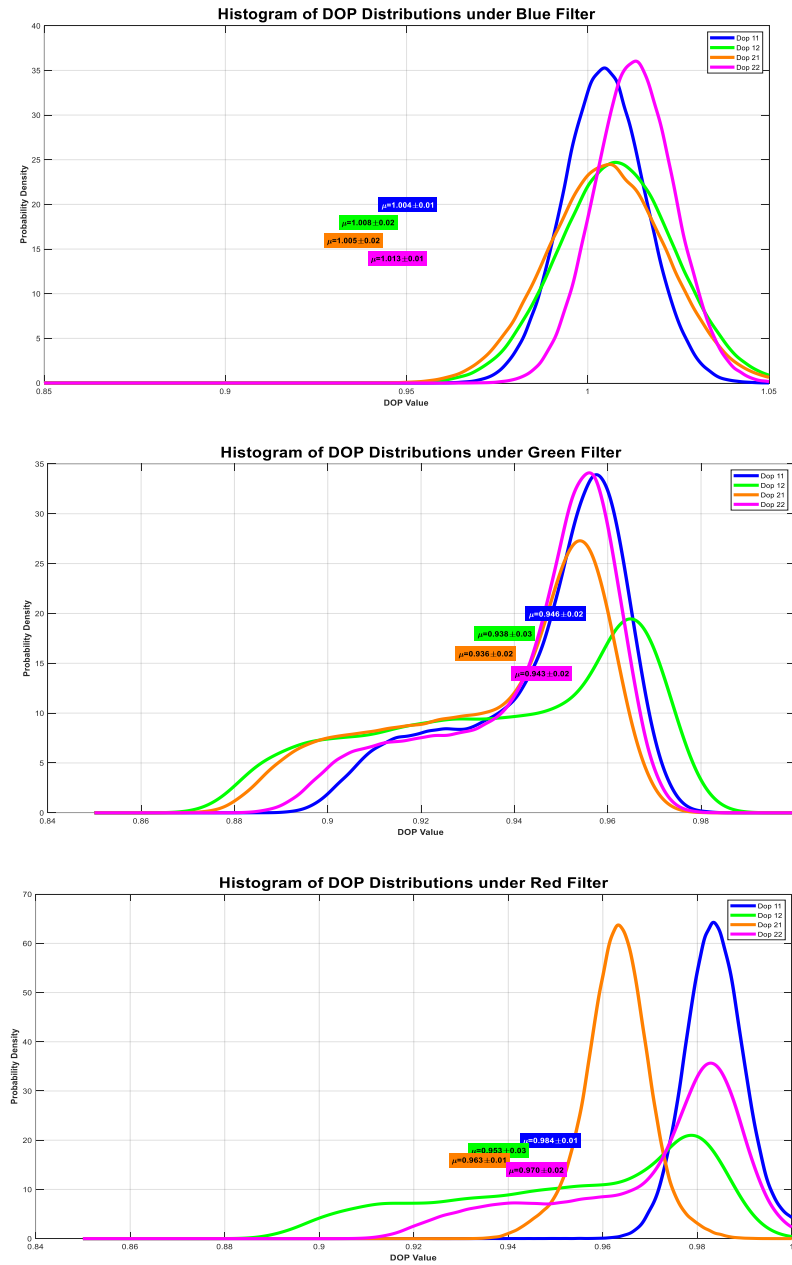


Figure 4-10. Pixel-level DoP histograms of four microgrid polarization directions under the green, red, and blue filters. Each distribution is annotated with the mean $\pm$ standard deviation to indicate polarization behavior and ellipticity uniformity per pixel class.

Finally, polarization is reconstructed independently for each wavelength λ_i using the wavelength-matched calibration W^{λ_i} . For each pixel (x, y) and each λ_i , the four analyzer intensities are stacked into a measurement vector $I^{\lambda_i}(x, y)$, and the Stokes vector estimate is obtained by applying the calibrated data-reduction matrix W^{λ_i} :

$$\hat{S}^{\lambda_i}(x, y) = W^{\lambda_i} I^{\lambda_i}(x, y) \quad 4.21$$

Here W_{λ_i} is obtained from the experimentally calibrated analyzer matrix for that wavelength, and it implicitly accounts for real analyzer imperfections and spectral dependence. Polarimetric reconstruction in this step contained obtain $\hat{S}^{\lambda_i}$ and derived metrics (DoLP/AoLP) for each wavelength.

If the FLC is used with the RGB DoFP camera, the same procedures are applied separately to each FLC state image, and the polarimetric reconstruction uses the corresponding full-Stokes system matrix (8 measurements per super-pixel).

4.6 Validation with simulation

4.6.1 Fourier interpolation analysis (function bandwidth study)

First, we will conduct an analysis of the Fourier Transform interpolation method applied to the signal captured by the color polarization camera. For that, we have generated 12 functions with different spatial frequency contents. In matrices with 1024x1024 pixels the following functions are generated: Five sinusoidal functions with different orientations. These are band limited functions. Two Gaussians with 50- and 100-pixels width that are almost band limited functions. A correlated random noise with a narrow spectral band, and four rectangular functions that are not band limited functions.

Each one of these functions are sampled following the scheme of polarization-color given in Figure 4-1 to simulate the image captured by the color polarization camera. A portion of this image is shown in Figure 4-11.

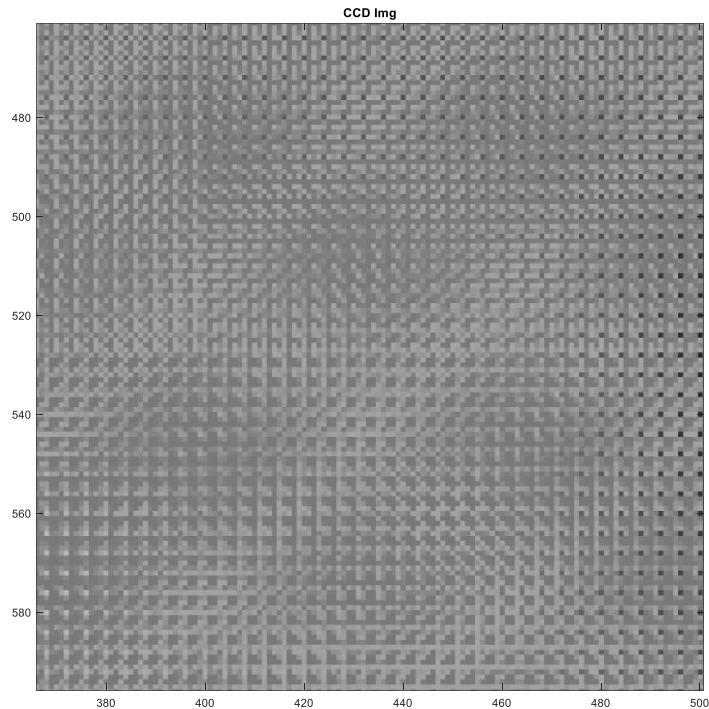


Figure 4-11. Enlarged portion of the simulated image captured by the polarization color camera containing 12 sampled functions.

In Figure 4-12 we show a summary of the Fourier transform interpolation method performance. Each row corresponds to a different function with different spatial frequency bandwidth. The left column shows the reconstructed function, the central column represents the error i.e. the difference between the original function and the reconstructed one, and the right column shows a histogram of the corresponding error.

In Figure 4-12(a) the results corresponding to one of the sinusoidal are shown. This function is band limited and with a spatial frequency that fall inside of the central “diffraction order”. In this case we perfectly recover the original function, the errors are of the order of $1e-7$, it means numerical error, as can be seen in the central and right column.

In Figure 4-12 (b) the results corresponding to a Gaussian are shown. As this is almost a band limited function with a bandwidth falling inside of one replica, the errors are also negligible.

Figure 4-12(c) corresponds to the case of a random correlated signal also with a bandwidth falling inside a FT replica. Here we can see that the error range is bigger. Here we must remark that the Fourier Transform algorithms assume that the functions are periodic (the matrix represented in the left column is repeated periodically). Then, when we look at the top/bottom or left/right borders we see that there is a discontinuity that create higher frequencies resulting in interpolation errors in these zones. This can be noticed if Fig 4-12 (c) central image were the value for a pixel not near the border, the error is very small. In this image the biggest errors are localized on the borders due to the commented discontinuity.

Finally, Figure 4-12(d) corresponds to a rectangle function. Because of the discontinuity on the borders of the rectangle, this function is not band limited, and here the errors are concentrated on the borders of the rectangle.

As a summary, we have shown that the Fourier transform based interpolation method gives very accurate results when the functions are band limited, and in general the errors will increase at the discontinuities or at the border of the image.

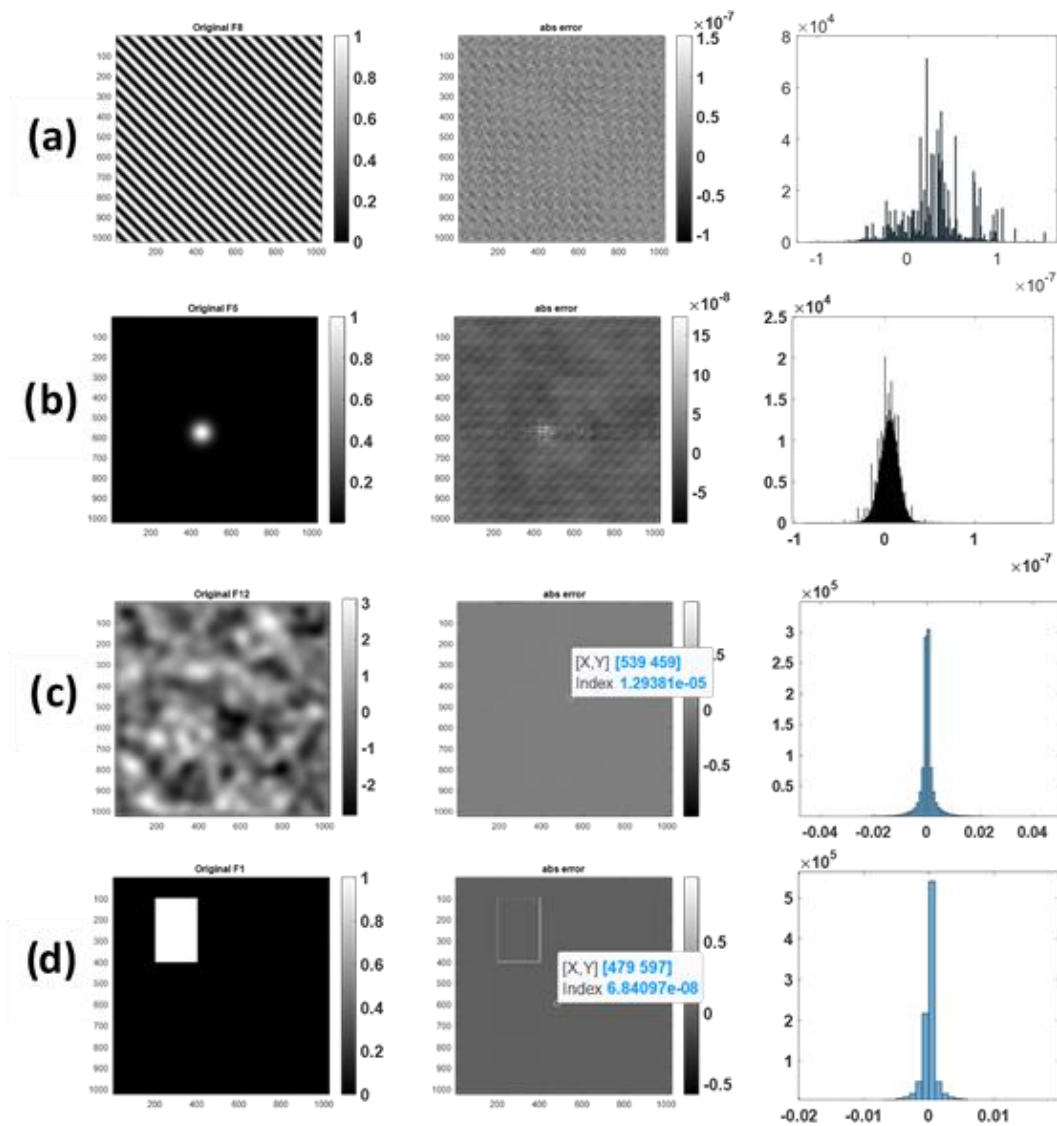


Figure 4-12. Rows correspond to some of the different functions: (a) sinusoidal, (b) Gaussian, (c) correlated random and (d) rectangular functions. The Left column corresponds to the reconstructed function after the interpolation. The central column is the error (difference between the original function and the interpolated one). The right column is a histogram of the corresponding error

4.6.2 Polarization and crosstalk analysis

In this subsection we will analyze the proposed method to measure the spatial polarization distribution in the three-color channels simultaneously. To this end we will assume that each wavelength transport different spatial polarization information. To describe the polarization state, we use three parameters (the intensity is assumed equal to one): Degree of polarization (DoP), Azimuth (Az) and Ellipticity (El). The corresponding components of the Stokes vector will be given by:

$$\begin{cases} S0 = 1 \\ S1 = DoP * \cos(2El) * \cos(2Az) \\ S2 = DoP * \cos(2El) * \sin(2Az) \\ S3 = DoP * \sin(2El) \end{cases} \quad 4.26$$

As we are measuring linearly polarized light the ellipticity (El) is equal to 0. The three linear partially polarized light spatial distributions are defined as:

RED Wavelength: The DoP varies with the radial coordinate between 0.5 and 1; the azimuth varies with the angular coordinate between -180 and 180 degrees.

GREEN Wavelength: The DoP varies with the radial coordinate between 0.5 and 1; the azimuth varies with the X coordinate.

BLUE Wavelength: The DoP is a correlated random distribution, and the azimuth varies with the Y coordinate.

In the first simulation we assume that there is no crosstalk, let us say, the elements outside the diagonal in the table 4.1 are equal to 0. Figures 4-13, 4-14, and 4-15 summarize the obtained results for the RED, GREEN and BLUE channels.

In the left column the Az (first row) and DoP (second row) parameters of the original functions are displayed. The central column shows the differences between the original and the reconstructed parameter after the intensity interpolation and

polarimetric data reduction. Finally, the right column shows the histogram of these differences.

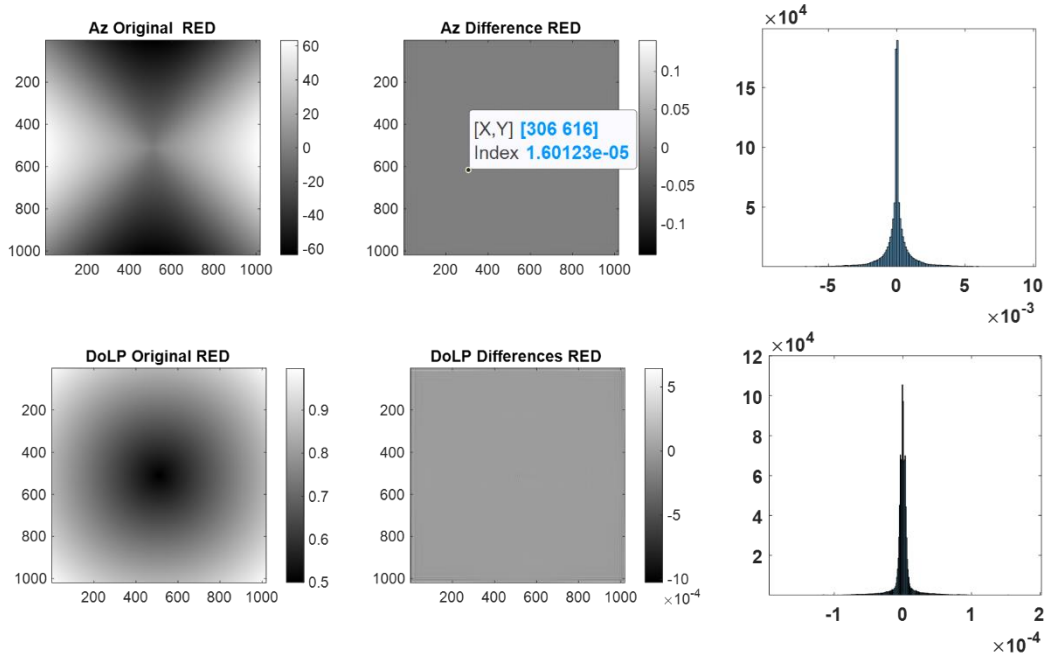


Figure 4-13. RED Channel, NO crosstalk. The first row corresponds to the Azimuth and the second to the DoP. The Left column corresponds to the original function. The central column is the error (difference between the original function and the interpolated and the reconstructed one). The right column is a histogram of the corresponding error

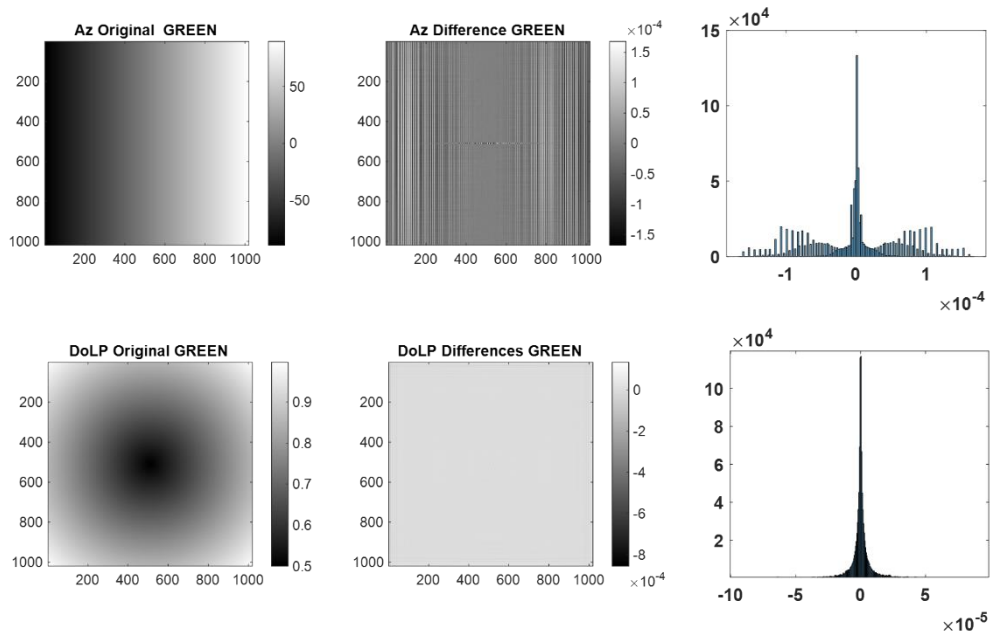


Figure 4-14. GREEN Channel, NO crosstalk. The first row corresponds to the Azimuth and the second to the DoP. The Left column corresponds to the original function. The central column is the error (difference between the original function and the interpolated and the reconstructed one). The right column is a histogram of the corresponding error

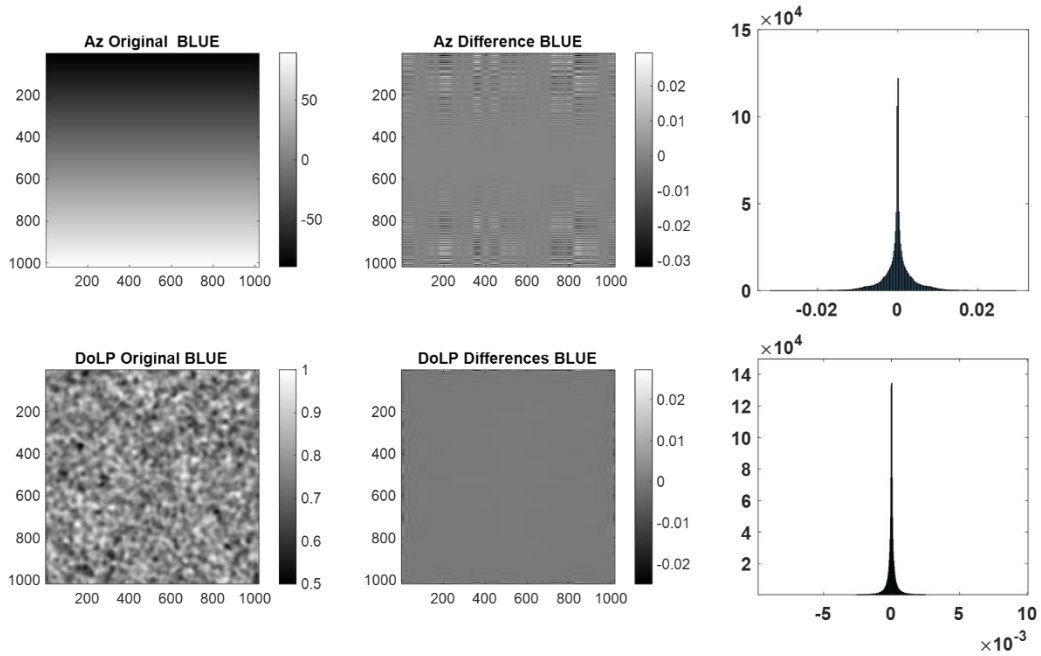


Figure 4-15. BLUE Channel, NO crosstalk. The first row corresponds to the Azimuth and the second to the DoP. The Left column corresponds to the original function. The central column is the error (difference between the original function and the interpolated and the reconstructed one). The right column is a histogram of the corresponding error.

As one can see in these figures the errors are almost negligible, being a little bigger in the case of discontinuities at the matrix borders as in the BLUE channel.

4.6.3 Impact of Color Crosstalk Correction on Polarimetric Interpolation

In the next simulation we will assume that there is cross talk between color channels, as described in table 4.1, but they are corrected in step 2 of the pipeline. The reconstruction results can be seen in Appendix D. As we have perfectly corrected the crosstalk the obtained results are the same as if there were no cross talk.

Finally, we have made the simulation with cross talk, but it is not corrected in the reconstruction. The results are shown in Figs 4-16, 4-17 and 4-18 for the RED, GREEN, and BLUE channels, respectively.

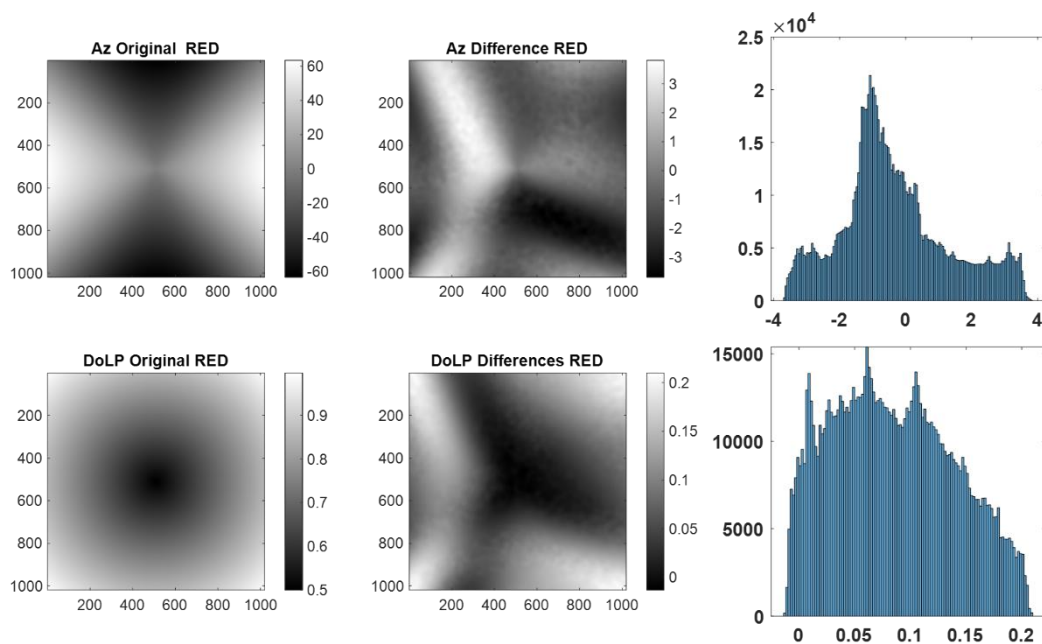


Figure 4-16. Red channel without correction of the color crosstalk.

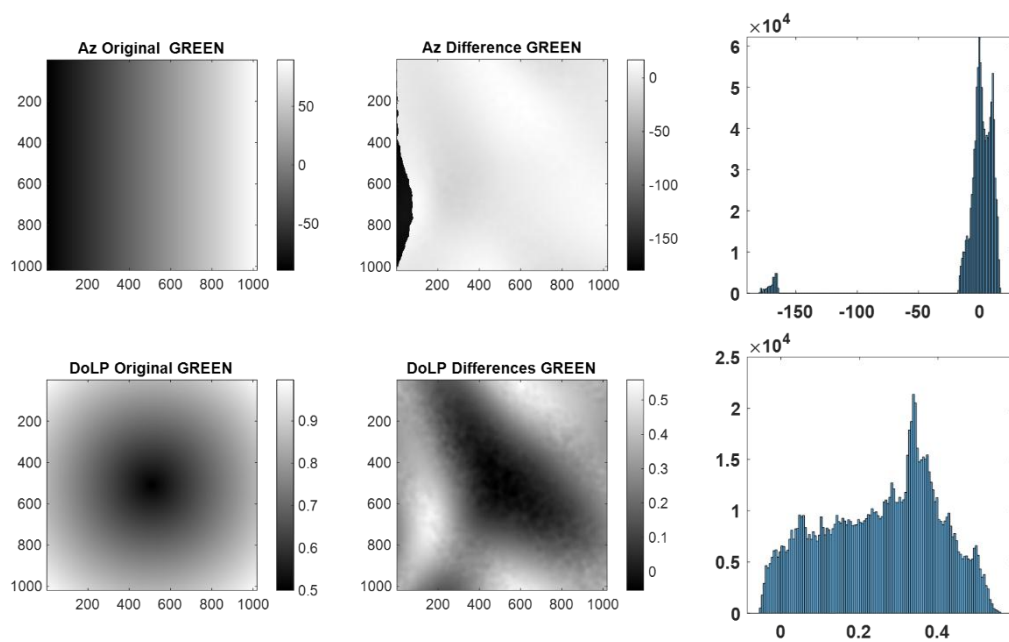


Figure 4-17. Green channel without correction of the color crosstalk.

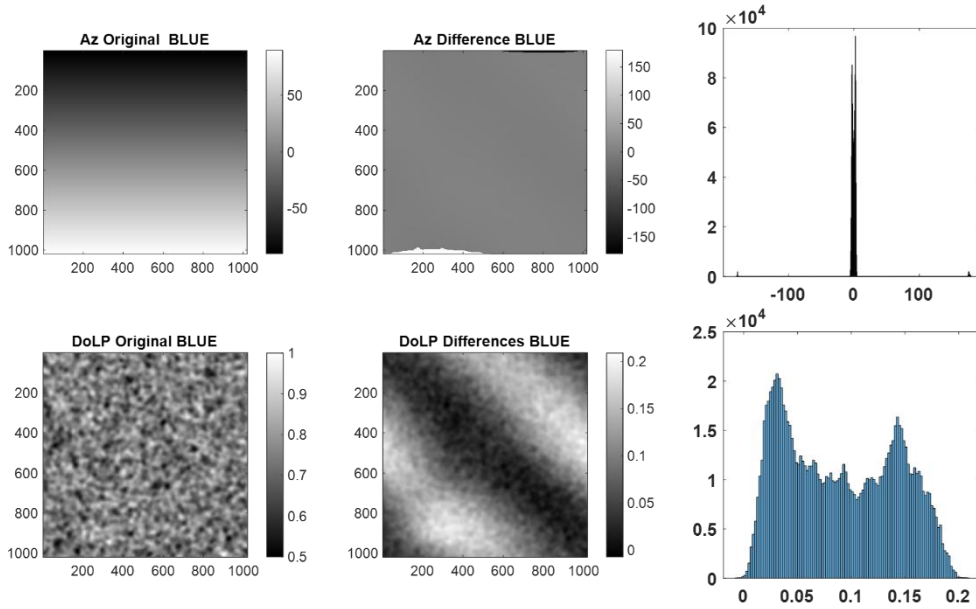


Figure 4-18. Blue channel without correction of the color crosstalk.

As we can see in these figures, the errors in the azimuth of the red channel lie in the range of $[-4, +4]$ degrees, being much larger in the green channel, and of course, much more significant in all cases than for the corrected case. The error in the DoLP could reach up to 20% on the red channel and 50% on the green channel. This shows the need to correct the crosstalk to accurately measure with three wavelengths simultaneously.

4.7 Experimental validation (radial polarizer + waveplate experiment)

To validate the method with experimental results we have performed the following experiment: a radial polarizer (from Codixx) with 8 sectors is followed by a non-achromatic wave plate, so, at the exit of the system, different elliptical polarizations are obtained for each sector and for each wavelength. The setup is illuminated sequentially with red, green, and blue wavelengths and measurements are taken. After that, we illuminated simultaneously with the three wavelengths. In

this case, we assume that the beams are fully polarized, so, by knowing (S_0 , S_1 , S_2) we can compute the S_3 component as:

$$S_3 = \sqrt{S_0^2 - S_1^2 - S_2^2} \quad 4.22$$

The combination of a linear polarizer with different orientations and the waveplate produces different elliptical polarizations (azimuth and ellipticity) for each sector. Furthermore, because the wave-plate is not achromatic, these parameters will be also different for each wavelength. To avoid prolonging the discussion, only the green channel results are displayed in Fig. 4-19 since it is the channel presenting the most significant error regarding the crosstalk effects.

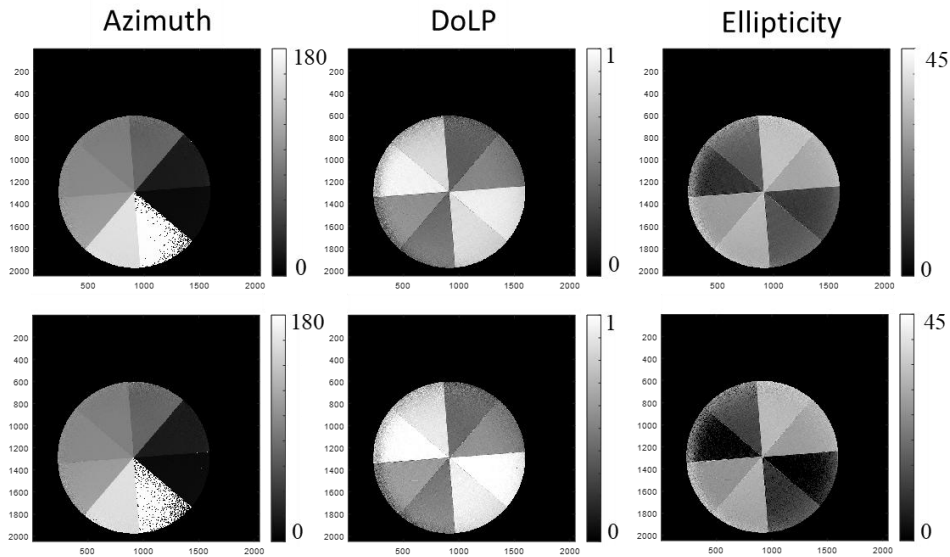


Figure 4-19. GREEN channel after interpolation and crosstalk correction. (Top row) system illuminated only with the green wavelength. (Bottom row) system illuminated with the tree wavelengths. (Left) Azimuth, (Center) DoLP, (Right) calculated Ellipticity

Top row of Fig. 4-19 shows the results obtained after interpolation and crosstalk correction for the green channel when the system is illuminated only with the green

wavelength. Bottom row of the figure shows the same results obtained when the system is illuminated with the three wavelengths at once.

The left column shows the azimuth of the obtained elliptical polarizations. Note that 0° and 180° correspond to the same azimuth, as can be seen on the bottom right sector (see white arrow). Each sector combined with the waveplate produces different ellipse orientations. The same results are obtained either with one or three wavelengths, showing that the proposed method is working well.

As we are measuring only with linear polarizers, only the components (S_0 , S_1 , S_2) can be measured, and then all the elliptical parts of the polarization produce linear depolarization. The Degree of Linear Polarization (DoLP) is shown in the central column of the figure to illustrate this effect. When the polarizer of a sector is parallel to the axis of the waveplate, the produced polarization will be linear, and the DoLP will be almost 1, while in the other cases the DoLP will decrease, with the limit case of ellipticities close to 45° , leading to the lowest DoLP values. Again, we can see that the top and bottom figures give the same results.

Finally, the column on the right shows the evaluated ellipticity assuming that light is fully polarized. We can see that when the DoLP is near to 1, the ellipticity is near to 0. These results demonstrate that with the proposed method, we can measure simultaneously with three different wavelengths that after interacting with the sample produce different polarization distributions.

4.8 FLC Integration for Full-Stokes Retrieval

In this section, we integrate FLC modulation into RGB polarized sensor. The system architecture and acquisition principle of the proposed full-Stokes polarimeter is shown in Figure 4-20.

In the top (Left), my photograph of the experimental setup combining an RGB division-of-focal-plane (DoFP) polarimetric camera with a ferroelectric liquid

crystal (FLC) modulator placed in front of the imaging lens. In the top (Right), polarization state generator, composed of a linear polarizer and a chromatic quarter-wave plate, is used for system calibration and validation under narrowband illumination. Building on the analyzer formulation introduced in chapter 3, the same calibration, measuring and validation's procedures and mathematical formalisms are followed. In the tables below, the validation tables are presented of our full Stokes outputs against a reference Thorlabs polarimeter.

In the bottom (Left), schematic of the RGB polarization mosaic of the DoFP sensor, which is also explained in section 4.1, enabling snapshot acquisition of wavelength resolved linear polarization information. In the bottom (Right) operating principle of the FLC modulation. As explained in chapter 3, the FLC switches between two discrete retardance states (a and b in the figure of FLC), introducing a controlled polarization rotation before the sensor.

In this section, the possibility of this combination of temporal modulation with the spatially multiplexed linear analyzers of the RGB DoFP camera is validated to enable a recovery of the complete Stokes vector, including the circular polarization component. The calibration outputs of analyzer set per wavelength for each matched color channel, and the validation tables based on external polarimeter as true ground truth state of polarization will be validated by Thorlabs polarimeter.

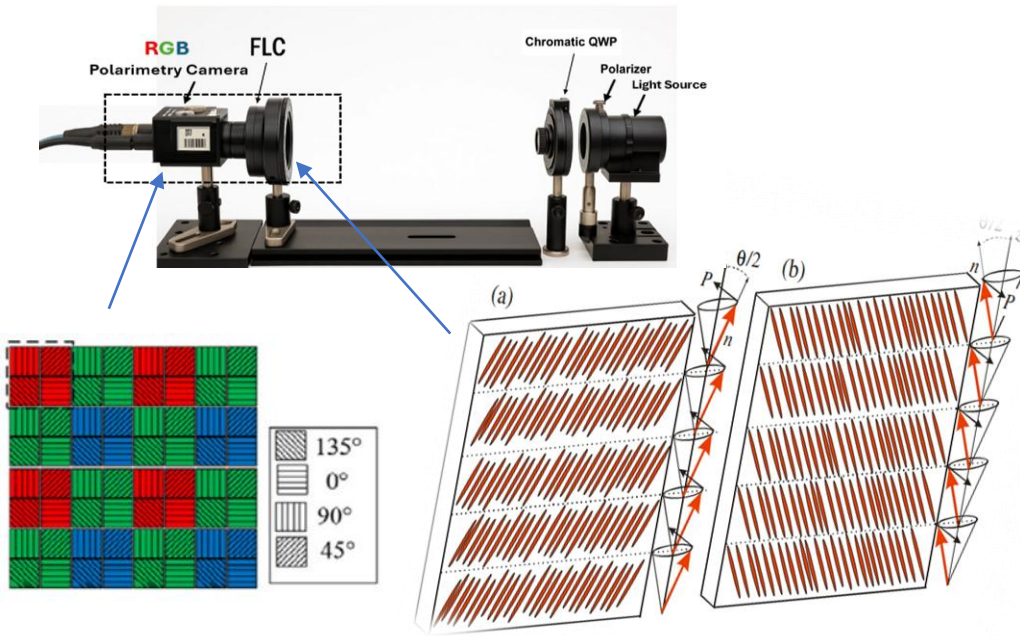


Figure 4-20. System architecture and acquisition principle of the proposed full-Stokes polarimeter

The same polarimetric principles applied in chapter 3 will be applied in this set-up.

The measured intensity associated with the analyzer state at wavelength λ_i is expressed in eq 4.8. The number analyzer rows vector $a_p^{\lambda_i}$ is upgraded to number $p=8$. Theoretically, this combination of two states FLC and the RGB polarized sensor results in analyzers with elliptical polarization sensitivity (rows with S_3 terms) as shown in eq 3.51.

I followed DRM calibration method, and the procedure presented in this thesis, to estimate the polarization analyzer matrix for each wavelength. To take into consideration wavelength-dependent effects resulting from the FLC retardance, sensor response, and optical components, the calibration is carried out separately for each wavelength for each color channel with maximize the transmission through the RGB filters to ensure accurate measurement and the retrieval of the instrumental matrix for this spectral band, and avoiding the saturation.

The experimental calibrated analyzer states are visualized using polarization ellipses representation in Figure 4.21, to validate the capability of the RGB FLC polarimetric system for full-Stokes imaging.

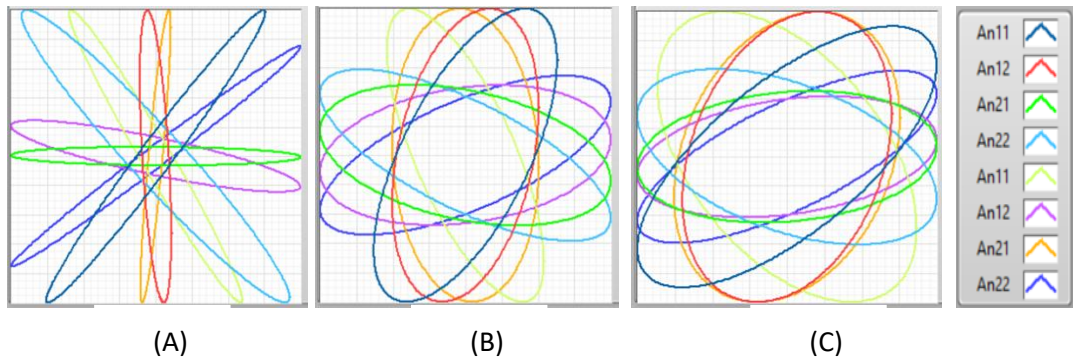


Figure 4-21. Polarization ellipse representation of the eight calibrated analyzer states for the blue channel (A), Red channel (B), and green channel (C).

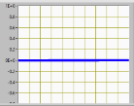
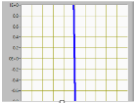
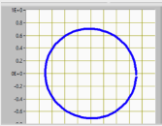
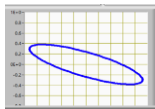
As can be seen in figure 4-21, the 8 analyzers corresponding to each wavelength are differently distributed away from the equatorial plane of the Poincare sphere. The calibrated analyzers for the blue channel with blue wavelength are almost confined to the equator of the Poincare sphere, indicating that they are near to linear rather than purely elliptical. As a result, the set-up is confirming higher sensitivity to circular polarization components for green and red wavelengths than to blue wavelength. This is expected when integrating a FLC with the microgrid sensor as

shown in chapter 3, where The FLC's phase at 470 nm deviates from nominal phase 90° , producing near to linear analyzer orientations.

Consequently, the different distributions between the three wavelengths confirm different ability of the set-up to sample both linear and circular polarization components at each color channel. In the next step, we will examine the possibility of the set-up to enable a robust reconstruction of all four Stokes parameters in a RGB channels.

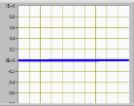
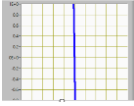
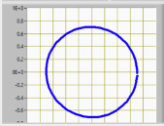
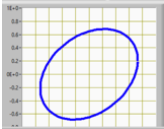
Following calibration, Stokes vector estimation is performed using a wavelength-dependent data reduction matrix (DRM) in eq. 4-10. The quantitative validation results for the blue, green, and red wavelength channels are summarized in Tables 4-3, 4-4, and 4-5 for blue, red, and green wavelengths, respectively.

Table 4-3. Validation of Stokes Parameters for Blue Wavelength (470 nm).

Numb. of Measurement		Thorlabs	Set-up (Avg)	Absolute Error (%)	Set-up Std
Blue Sop 1 	Az	0.898	0.496	0.402	0.596
	Ellip	-0.31	0.76	1.07	2.46
	DoP	99.58	97.8	0.02%	0.011
Blue SOP 2 	Az	-89.16	-83.23	5.93	31.12
	Ellip	0.01	-0.16	0.17	2.55
	DoP	98.19	100.1	0.02%	0.015
Blue SOP 3 	Az	-41.03	-35.82	5.21	13.97
	Ellip	-44.46	-43.22	1.24	0.92
	DoP	99.63	89.7	0.09%	0.103
Blue SOP 4 	Az	-18.87	-18.40	0.47	0.59
	Ellip	13.17	11.47	1.7	2.09
	DoP	99.58	89.7	0.09%	0.03

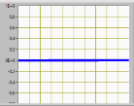
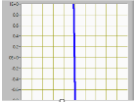
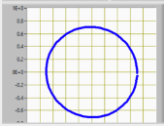
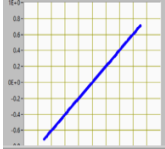
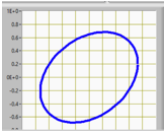
For blue channel (470 nm), most tested SOPs 's azimuth errors are usually less than 6° for the blue channel with smaller errors noted for states of linear polarization. For every tested state, ellipticity errors stay within a few degrees, demonstrating effective sensitivity to circular polarization. For elliptical and near circular polarization states, the degree of polarization is accurately estimated, but large relative deviations are seen indicating than noise sensitivity is amplified in these intensities' satates.

Table 4-4. Validation of Stokes Parameters for Red Wavelength (630 nm).

Numb. Measurement	of	Thorlabs	Set-up (Avg)	Absolute Error (%)	Set-up Std
Red SOP 1 	Az	-0.22	-0.72	0.5	0.58
	Ellip	-0.68	2.41	1.73	0.70
	DoP	99.72	85.46	14.26	0.009
Red SOP 2 	Az	89.81	89.28	0.53	0.48
	Ellip	-0.01	0.87	0.86	0.62
	DoP	96.80	79.51	17.29	0.008
Red SOP 3 	Az	-49.06	-45.68	3.38	2.10
	Ellip	42.36	41.72	0.64	0.33
	DoP	98.89	96.39	2.5	0.016
Red SOP 6 	Az	64.46	61.95	2.51	0.46
	Ellip	17.84	19.89	2.05	1.20
	DoP	98	93.47	4.53	0.018

In this table 4-4, the azimuth and ellipticity errors for the red channel (630 nm) stay within a few degrees in every tested state. Due to wavelength-dependent variations in sensor sensitivity, illumination power, and FLC retardance deviation from the nominal value, DoP errors are slightly higher than in the blue and green channels.

Table 4-5. Validation of Stokes Parameters for Green Wavelength (554 nm).

Numb. Measurement	of	Thorlabs	Set-up (Avg)	Absolute Error (%)	Set-up Std
Green SOP 1 	Az	0.85	0.65	0.2	0.41
	Ellip	0.015	1.15	1.14	0.48
	DoP	99.71	94.03	5.68	0.013
Green SOP 2 	Az	-89.19	-88.14	1.05	0.42
	Ellip	-0.30	1.09	0.8	0.51
	DOP	99.63	100.11	0.37	0.019
Green SOP 3 	Az	11.15	-24.80	13.65	6.46
	Ellip	-42.34	-40.80	1.54	0.75
	DoP	99.48	62.17	37.31	0.018
Green SOP 4 	Az	45	44.92	0.08	0.35
	Ellip	0.06	1.53	1.47	1.33
	DoP	100.23	100.007	0	0.018
Grren SOP 5 	Az	-44.86	-43.57	1.29	0.36
	Ellip	-0.16	0.81	0.65	1.26
	DoP	99.59	96.90	2.69	0.012
Green SOP 6 	Az	38.87	38.56	0.31	1.39
	Ellip	-35.66	-35.77	0.11	0.49
	DoP	99.77	99.90	0.13	0.028

Overall, the system shows the most stable performance for the green channel (554 nm). For most SOPs, azimuth errors are typically less than 2° and ellipticity errors stay near 1° . With a low standard deviation across repeated measurements, the DoP estimation exhibits high consistency. States with lower signal contrast are associated with larger DoP errors for specific SOPs, underscoring the inherent sensitivity of DoP estimation to noise and calibration flaws.

The polarimeter set-up shows precise recovery of the polarization azimuth and ellipticity for all three wavelengths for a broad range of input SOPs, demonstrating the capacity of the set-up to measure both linear and circular polarization components.

In general, although the non ideality of analyzer in blue wavelength, validation errors are small because we used a calibrated $A(\lambda)$ with 8 measurements per Stokes vector and high SNR exposures; that keeps even a non-optimal analyzer geometry reconstructs Stokes accurately.

In another hand, as an important characteristic of polarimeter, the reported standard deviations show good measurement repeatability across all wavelengths, indicating that the acquisition and reconstruction pipeline is stable. The limitation in this set-up is in the blue channel, because it often has lower transmission and counts, consequently a detected larger relative noise and bigger errors, especially for states near circular polarization.

Chapter 5 Conclusion

This thesis addresses the challenge of achieving accurate, compact, and quasi-real-time full-Stokes polarimetric imaging using division-of-focal-plane (DoFP) polarization sensors. While DoFP cameras enable snapshot measurement of the linear Stokes parameters (S_0 , S_1 , S_2), they are intrinsically unable to measure circular polarization (S_3) and suffer from spatial resolution loss due to pixel-level multiplexing. These limitations are further compounded by wavelength-dependent sensor behavior and analyzer non-idealities. In RGB DoFP polarimetric sensors, the problem is extended by spectral overlap of color filters, which introduces color crosstalk and biases the recovered polarization information. Taken together, these constraints prevent existing snapshot systems from simultaneously offering full-Stokes capability and have been the major motivation of this Thesis. Different results have been obtained to address these limitations.

5.1 Key contributions

In this thesis, we created a comprehensive calibration and processing pipeline for both monochrome and RGB micro-grid DOFP polarized sensor. To overcome the inherent drawbacks and limitations of snapshot polarimetry, this thesis combined different approaches as color crosstalk correction models, spatial reconstruction demosaicking techniques, and methods for the calibration of the wavelength effect in the measurements.

To extend snapshot polarimetry beyond linear polarization, a ferroelectric liquid crystal (FLC) modulator was integrated in front of both monochrome and RGB DoFP cameras. This configuration enabled full-Stokes imaging without mechanical motion and allowed estimation of the circular Stokes parameter (S_3) within a compact and fast-switching architecture.

For RGB DoFP polarimetric sensors, this thesis addressed the additional challenge of color crosstalk caused by spectral overlap of the RGB filters. A wavelength-specific calibration model was developed for each spectral channel side by side to the Fourier demosaicking strategy. This allowed implementing correction pipeline to mitigate spectral mixing before polarimetric reconstruction. This approach demonstrates the feasibility of snapshot RGB polarimetry polarization analysis for three wavelengths simultaneously.

Even though the suggested techniques greatly increase spectral accuracy and spatial fidelity, there are still issues with spectral mismatch and FLC wavelength-dependent behavior, especially when it comes to accurate S_3 retrieval. Future optimization of snapshot full-Stokes polarimetric systems can be clearly guided by these effects.

5.2 Scope and Methodological Limitations

This thesis demonstrates that it is feasible to build compact quasi real time full Stokes polarimeter by using the proposed scheme: combination of a polarized sensor with an FLC modulation. As polarimeters implementations always present experimental errors associated to the polarimetric base used and instrumental errors, an experimental calibration has been applied to the polarimeters, as well as an optimization, based on metrics as the condition number, to achieve an optimal operation of the system.

Second, full-Stokes recovery with a FLC is fundamentally sensitive to retardance accuracy and stability. The effective retardance depends on wavelength and can drift with temperature, drive conditions, and alignment. This indicates that S_3 is generally the component most prone to error: small model mismatches can map into systematic S_3 bias, particularly at wavelengths where the FLC deviates from quarter-wave behavior.

In addition, error sources as camera noise statistics, spatially varying signal levels, and sensor-dependent heteroscedastic noise are typically not incorporated into the Stokes estimation model. As a result, performance is frequently assessed using the condition number of the analyzer matrix alone, which characterizes sensitivity to model imperfections but does not fully capture noise propagation in imaging conditions.

This effect becomes especially relevant in RGB DoFP systems combined with FLC modulation, where different spectral channels exhibit markedly different intensity levels. Under photon-limited conditions, this leads to wavelength-dependent noise variances, so the assumption of uniform noise across channels could not be longer valid.

5.3 Scientific Impact

From a scientific perspective, one major scientific impact is the presentation of a new Stokes polarimeter prototype. This thesis contributes to the understanding of the capabilities and limitations of a snapshot full-Stokes image polarimeter scheme. The optimal operation of the polarimeter, based on a combination of a DoFP polarization sensors with FLC panel, is achieved by implementing wavelength-specific calibration together with methods of computational reconstruction of intensity measurements. The strength of this scheme is that it enables full spatial Stokes measure and is only limited by the switching speed of the ferroelectric element, typically in the kHz range, making it highly attractive for quasi-real-time applications.

The thesis highlights the practical consequences of spatial multiplexing, spectral mixing, and non-ideal modulation on Stokes reconstruction, and shows how these effects manifest in real imaging data. By experimentally analyzing monochrome and RGB polarimetric sensors across multiple wavelengths, the work provides a structured reference for evaluating snapshot polarimetric systems beyond idealized analyzer models. This contributes to more realistic interpretation and validation of polarimetric measurements in imaging conditions.

Taken together, the thesis provides computation, calibration and optimization methods for polarimetric systems, specifically designed for imaging architectures based on polarimetric sensors. These methods are suitable for implementation across multiple wavelengths simultaneously, making them particularly valuable for multichannel illumination setups.

5.4 Practical impact

From a practical standpoint, the proposed architecture demonstrates the feasibility of compact, quasi-real-time full-Stokes imaging in scenarios where

sequential polarization measurements are impractical or fast measurements are required. The absence of mechanical rotation and the snapshot nature of the DoFP sensors make the system suitable for dynamic scenes and time-varying illumination.

Under controlled conditions and wavelength-specific calibration, the system can support polarization-based analysis in applications such as laboratory biomedical imaging, remote sensing experiments, astronomy, advance materials characterization, botanical imaging, where relative polarization contrast and temporal consistency are more critical than absolute precision. Moreover, the spatial information that enables image enhancement in the previously discussed applications can be simultaneously obtained at different wavelengths. This is particularly valuable because the polarimetric response of materials varies significantly with wavelength and is closely linked to the internal structures of matter that interact selectively with specific spectral bands.

Experimentally, the system performs most stably under high-SNR conditions—high illumination without saturation—which aligns well with the fact that most practical imaging scenarios do not operate in extremely low-light regimes, making the propose polarimeter and methods broadly applicable.

5.5 Future Work

Future work can be organized around three natural extensions of this thesis.

First, in the direction to improve the stability of the optical calibration. The performance of the proposed system depends on the accuracy of the FLC retardance and its wavelength behavior. Future studies could introduce calibration methods accounting for temperature response dependence of the FLC panel, as well as simple verification routines that allow the system to be checked without repeating a full calibration each time.

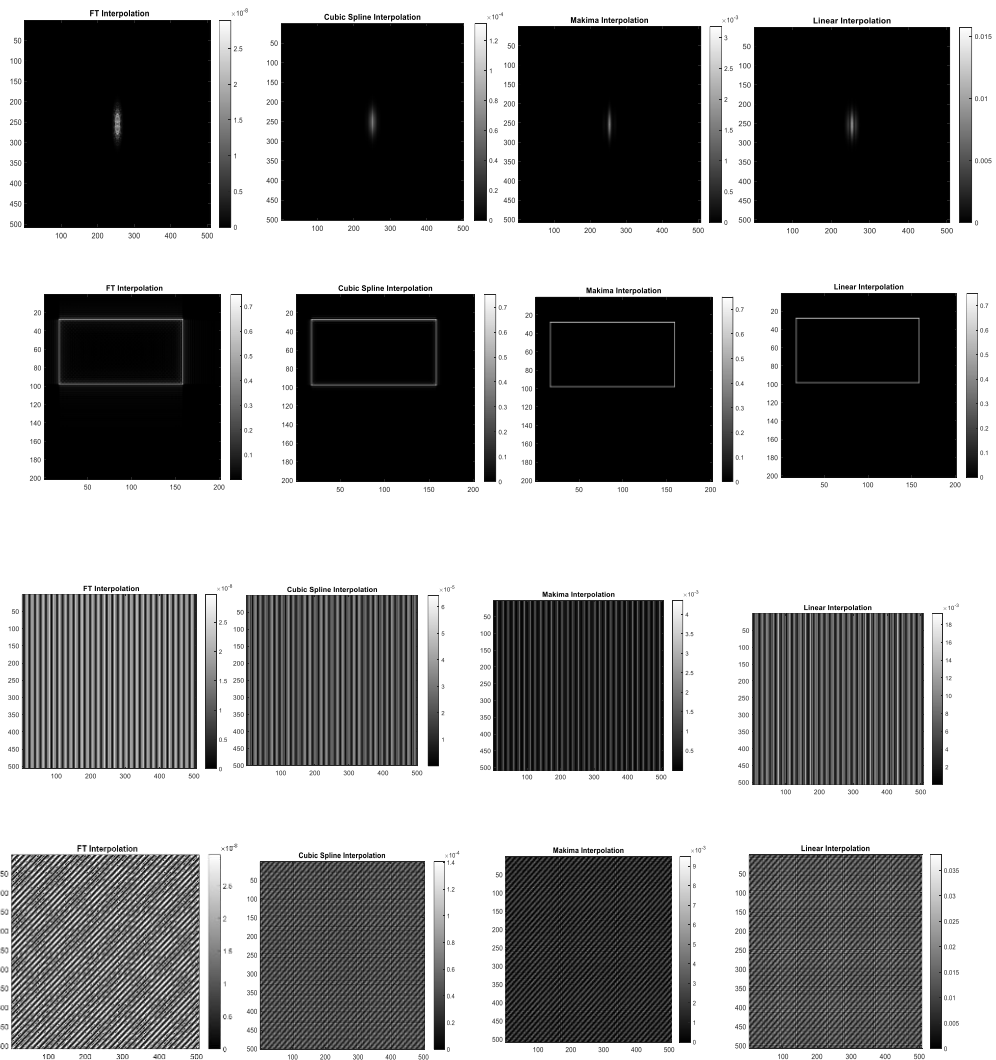
Second, improving the robustness of reconstruction under practical conditions. In RGB DoFP systems, spectral overlap between color channels limits polarization accuracy when the illumination or sensor response changes. Future work could focus on making the spectral correction more tolerant to small variations and on defining clear acquisition conditions, such as operating in a high-SNR but non-saturated regime, where the system performs most reliably.

Third, improving the interpretation and reliability of the measurements. Rather than assuming uniform accuracy across the image, future approaches could include simple indicators of measurement quality that reflect signal level and scene content. This would allow users to understand where polarization results are reliable and where they should be interpreted with caution.

Future studies will primarily examine the applicability of the proposed polarimetric measurement system. The scheme reduces the number of required instruments compared to a full Mueller polarimeter, enabling more compact and ergonomic configurations, real-time multichannel operation, and making it particularly appealing for biomedical imaging. The suitability of incorporating specific configurations of polarization gating will also be explored, as it can greatly enhance the visualization of certain pathologies—such as those in skin—through direct, non-invasive measurements. Altogether, these developments point toward a natural evolution of the system into a clinically oriented and transferable instrument.

Acronym	Meaning
CN	Condition Number
DoTP	Division of Time Polarimeters
DoAmP	Division of Amplitude
DoAP	Division of Aperture
DoLP	Degree of linear polarization
AoLP	Angle of linear polarization
Az	Azimuth
EI	Ellipticity
DoP	Degree of polarization
PSA	Polarization State Analyzer
PSG	Polarization State Generator
DOFP	Division of focal plan
FLC	Ferroelectric Liquid Crystal
SOP	State of Polarization
FT	Fourier Transform
DRM	Data Reduction Matrix
ROI	Region Of Interest
SNR	Signal to Noise Ratio

Appendix A



Columns from 1 to 4 are 2D absolute error maps for Fourier(FT), Cubic Spline, Makima and Linear interpolations for Gaussian function, rectangular function, Sinusoidal Function in the X-direction and Diagonal sinusoidal function.

Appendix B

Raw data used for RGB spectral crosstalk calibration

This appendix reports the raw gray-level measurements used to estimate the effective spectral mixing coefficients between the three narrowband illumination wavelengths (470, 554, 630 nm) and the camera's RGB channels. For each wavelength, three exposure settings were recorded. The quantity F_t is the exposure-normalized response used in Section 4.4.2 to compute the mixing matrix.

*Note: $ET \cdot S_0$ is reported as $\mu W \cdot s$ because ET is in seconds when computing this product; if ET is kept in ms, include the ms→s conversion explicitly.

Blue illumination (470 nm)

ET (ms)	S_0 (μW)	$ET \cdot S_0$ ($\mu W \cdot s$)*	GL_G	GL_B	GL_R	F_t _G	F_t _B	F_t _R
46882.5	1.4	65.64	580.8	2185.3	88.75	8.85	33.3	1.35
39998.5	1.78	71.2	508.9	1911.5	77.26	7.15	26.8	1.1
35007.5	2.24	78.42	541.72	2102.5	79.5	6.91	26.81	1.01
					Avg	7.6	29.0	1.2

Green illumination (554 nm)

ET (ms)	S_0 (μW)	$ET \cdot S_0$ ($\mu W \cdot s$)*	GL_G	GL_B	GL_R	F_t _G	F_t _B	F_t _R
46882.5	1.4	65.64	2930.6	282.42	124.3	44.65	4.3	1.9
39998.5	1.78	71.2	3227.7	309.4	131.2	45.33	4.3	1.8

35007.5	2.24	78.42	2798.6	267.22	113.55	35.69	3.41	1.4
					Avg	41.89	4.0	1.7

Red illumination (630 nm)

ET (ms)	S0 (μW)	ET·S0 ($\mu\text{W}\cdot\text{s}$)*	GL_G	GL_B	GL_R	Ft_G	Ft_B	Ft_R
46882.5	1.4	65.64	814.9	104.1	3235.8	12.4	1.59	49.3
39998.5	1.78	71.2	728.4	93.64	2912.2	10.23	1.31	40.9
35007.5	2.24	78.42	636.97	81.6	2557.8	8.12	1.04	32.62
					Avg	10.25	1.3	40.5

Derived effective spectral mixing matrix

Using the average Ft values above and normalizing by $F = \text{Avg}(\text{Ft_G at } 554 \text{ nm})$, the following effective mixing coefficients are obtained. Each column corresponds to one illumination wavelength; each row corresponds to the camera RGB channel. These coefficients are used as the 3×3 matrix in Section 4.4.2.

Effective mixing coefficients (un-normalized camera response)

	630 nm	554 nm	470 nm
R channel	0.967	0.041	0.029
G channel	0.245	1.000	0.181
B channel	0.031	0.095	0.692

Appendix C

$$\begin{pmatrix} I_{1,1}^{ct} \\ I_{2,1}^{ct} \\ I_{3,1}^{ct} \\ I_{4,1}^{ct} \\ I_{1,2}^{ct} \\ I_{2,2}^{ct} \\ I_{3,2}^{ct} \\ I_{4,2}^{ct} \\ I_{1,3}^{ct} \\ I_{2,3}^{ct} \\ I_{3,3}^{ct} \\ I_{4,3}^{ct} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0,04 & 0 & 0 & 0 & 0,01 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0,04 & 0 & 0 & 0 & 0,01 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0,04 & 0 & 0 & 0 & 0,01 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0,04 & 0 & 0 & 0 & 0,01 \\ 0,25 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0,19 & 0 & 0 & 0 \\ 0 & 0,25 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0,19 & 0 & 0 \\ 0 & 0 & 0,25 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0,19 & 0 \\ 0 & 0 & 0 & 0,25 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0,19 \\ 0,03 & 0 & 0 & 0 & 0,1 & 0 & 0 & 0 & 0,72 & 0 & 0 & 0 \\ 0 & 0,03 & 0 & 0 & 0 & 0,1 & 0 & 0 & 0 & 0,72 & 0 & 0 \\ 0 & 0 & 0,03 & 0 & 0 & 0 & 0,1 & 0 & 0 & 0 & 0,72 & 0 \\ 0 & 0 & 0 & 0,03 & 0 & 0 & 0 & 0,1 & 0 & 0 & 0 & 0,72 \end{pmatrix} \begin{pmatrix} I_{1,1} \\ I_{2,1} \\ I_{3,1} \\ I_{4,1} \\ I_{1,2} \\ I_{2,2} \\ I_{3,2} \\ I_{4,2} \\ I_{1,3} \\ I_{2,3} \\ I_{3,3} \\ I_{4,3} \end{pmatrix} \quad 0.1$$

Appendix D

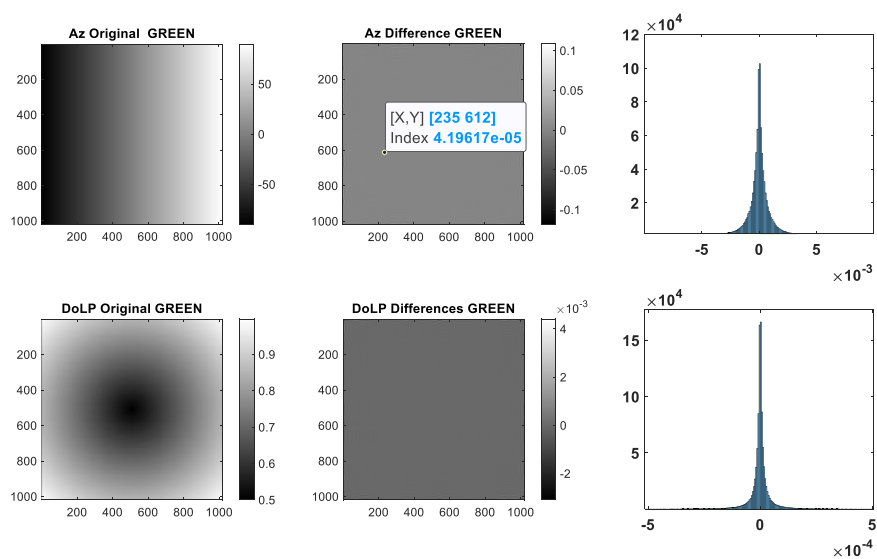
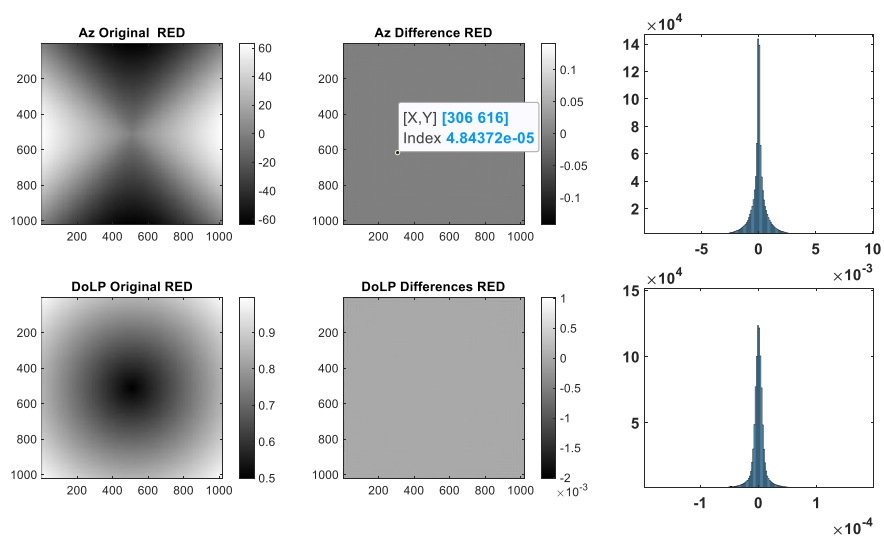


Figure 5-2. Crosstalk with corrections on Green channel.

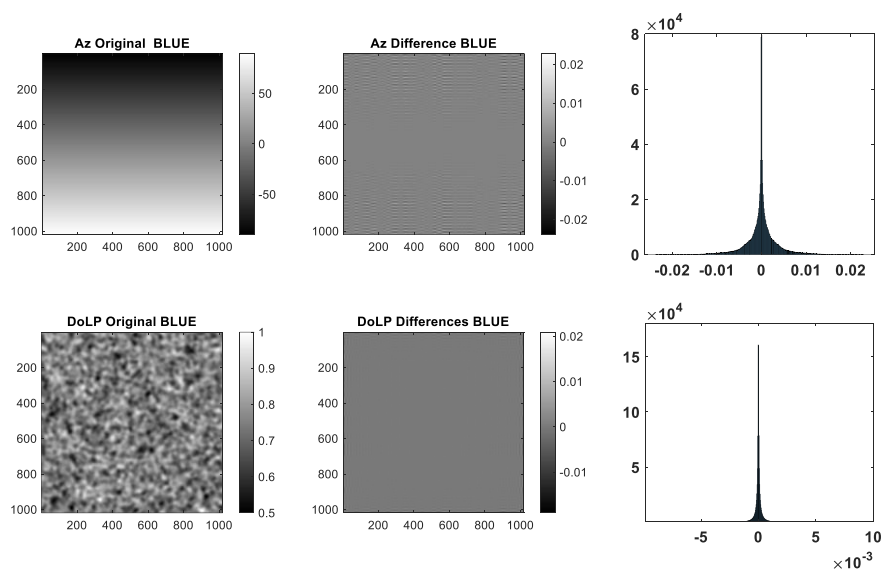


Figure 5-3. Crosstalk with correction on blue channel.

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