
AUTHOR: Ruben Alonso Rodriguez

DEGREE: Economics

MENTOR: Valeri Sorolla Amat

DATE: 08/06/2015

*Acknowledgement:
A special thank you to Valeri Sorolla for his helpful insights and full availability.
Abstract

In the last decade the unemployment skyrocketed defining a dramatic landscape for the Spanish economy. In order to understand the root causes, I have revisited two theories widely extended in labor economics: The Classical Theory of Unemployment and the Keynesian Theory of Unemployment. Despite both conceptions are well known and supported by academic literature, in the Spanish case as in many other countries is still unclear what theory better adjust to reality. To solve this lack of clearness, I approach to this dilemma by considering the knowledge on the exposed theories and the behavior of the variables in the Spanish labor market. By means of this previous research I could build an econometrical analysis that provides evidence in favor of the Classical view.

Resumen

En la última década el desempleo se ha disparado de forma severa definiendo un escenario dramático para el mercado laboral español. Por tal de entender las causas de ello, he revisado dos de las teorías más extendidas en la economía del trabajo: La teoría clásica del desempleo y la teoría Keynesiana del desempleo. A pesar de que ambas concepciones tienen un claro soporte en la literatura académica, en el caso español y en muchos otros países, aún resulta difícil identificar que teoría se ajusta más a la realidad. Para despejar estas dudas, y atajar el dilema que ello conlleva, he considerado el conocimiento expuesto de ambas teorías y modelos así como el comportamiento de las variables económicas en el mercado laboral español. Mediante esta investigación previa he podido construir un modelo econométrico que provee evidencias a favor de la visión clásica.

Keywords:

Aggregate demand, labor supply, labor demand, real wages, prices, wage equation, shocks, Keynesian theory of unemployment, Classical theory of unemployment.
Index

1 - Introduction ........................................................................................................................................... 4

2 - What are these theories about? ........................................................................................................... 4

3 - Literature review for new extensions. ................................................................................................. 15

4 - Spanish labor market at a glance ......................................................................................................... 20

5 - Adaptation of the theories to the Spanish case. .................................................................................. 25

6 - Analysis: What theory fits most to the Spanish case? ......................................................................... 28

7 - Conclusions ........................................................................................................................................... 32

8 - References ........................................................................................................................................... 34

9 - Annex .................................................................................................................................................... 37
1 - Introduction

Along this essay I review two models that may help us to understand the current situation in the Spanish labor market. Both options have their foundation in strong mathematical background and empirical justification and they represent different alternatives for fighting unemployment. My goal is to obtain evidences to identify the best choice for Spain. Only if we detect the root causes of the problem we will be ready to carry out the right economic policies.

In order to achieve that goal, I firstly introduced in section 2 the theories, the ingredients, the graphical representation and the main differences between them. I followed with an extension of advanced models to shed some light on the next steps. Section 4 is focused on a brief study of the Spanish labor market, because is necessary to have a basic knowledge about our framework. The next chapter compares both models and explains graphically the potential cures. At last, is in the final analysis of section 6 where the main results emerge after having tested all the different scenarios. Concretely I have built and estimated an econometric model that provides evidences in favor of the classical view. I finish this work with some concluding remarks. In the annex are compiled all the additional content that did not have space enough in the essay and complement the explanations made on it.

2 - What are these theories about?

Nowadays, the extended literature of labor economics is composed by many theories and models. Among the topic of unemployment we can basically distinguish two approaches: the Classical theory of unemployment and the Keynesian theory of unemployment. In the following section I will review both presenting a short introduction with special attention to the basic ingredients (labor supply, labor demand and wage equation) as well as the effect of unemployment in each case.
Classical theory of unemployment

The Classical Theory of Unemployment has nothing to do with the classical view of employment that turned up by the most relevant economists in the 18\textsuperscript{th} century like Adam Smith or David Ricardo. They advocated for a full-employment labor market. However in this essay we will see it from another perspective:

- Labor demand\textsuperscript{1}:

The first ingredient, as mentioned above, is the labor demand. Its schedule determines the amount of labor that firms employ at a given real wage. The way to get the labor demand is by means of the neoclassical function of production:

Economic theory says production of goods and services (Y) have basically two factors: labor demand (L) and capital stock (K):

\[ Y = F(L,K). \tag{2.1} \]

Under the neoclassical function of production a couple of assumptions need to be taken into consideration:

\text{a) Constant returns to scale for the production function:} \\
\[ XY = F(XK, XL). \]

\text{b) Diminishing marginal returns to either factors of production:} \\
\[ F_L > 0; F_{LL} < 0. \]

On the other hand, firms select the level of labor that maximize their profits by taking prices of labor and capital also as given:

\[ \max_L \pi = pY - wL + rK = pF(K, L) - wL. \tag{2.2} \]

\textsuperscript{1} Refer to the appendix, section 9.2-B.
Prices ($p$), wages ($w$) and capital rents ($r$) represent the cost of the output and each factor of production respectively. Capital is an exogenous variable determined by the given inversion in the previous period.

If we want to identify the sign of the labor demand with respect to real wages, we can use the theorem of the implicit function that provides us the following equation:

$$\frac{\partial L^d}{\partial \frac{w}{p}} = -\frac{-1}{F_{LL}(K,L)} < 0.$$  \hfill (2.3)

We can demonstrate, thus, a negative correlation between employment and real wages which denotes a negative-sloped curve for the labor demand.

Once we know the slope of our labor demand’s curve, we may be interested in knowing the elements that affect the labor demand shifts. Generally is used the Cobb-Douglas function of production ($Y$) composed by an extra component of productivity ($A$) and the factor’s elasticity $\alpha$:

$$Y = F(K, L) = AK^\alpha L^{1-\alpha}.$$ \hfill (2.4)

Moreover, we will consider the firm as a monopoly and therefore we have to include a variable representing the monopolistic power, $m$. This indicator equals $\frac{\varepsilon}{\varepsilon-1}$, where $\varepsilon$ is the elasticity of product demand respect to prices.

Consequently, solving (2.2) under a scenario of imperfect competition we obtain that marginal product of labor equals wage equation multiplied by $m$:

$$F_L(K, L^d) = m \frac{w}{p}.$$ \hfill (2.5)

After some algebra and using the Cobb-Douglas equation we encounter the following equation in terms of variation of the variables and using logarithms:

$$\frac{\Delta L}{L} = -\frac{1}{\alpha} \frac{\Delta w}{w} - \frac{1}{\alpha} \frac{\Delta m}{m} + \frac{1}{\alpha} \left( \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} \right).$$ \hfill (2.6)
Which means from the equation above is that productivity and stock of capital are key factors to rise up employment while an increase in real wage or the monopolistic power have a negative effect on it.

- Labor supply\(^2\)

The second ingredient is the labor supply curve. It basically determines the size of the labor force: total individuals willing to work at a particular real wage. Derived from this sentence, we can consider as part of this labor force all individuals whose opportunity costs in terms of consumption of goods are lower than the real wage.

To obtain the labor supply in our Classical Theory of Unemployment, we will start from a microeconomics perspective by using the theory of consumption.

In this case individuals chose a certain level of consumption, \(C\), and labor, \(L\), in order to maximize their utility function \(U(C, \bar{L} - L)\).

Additionally, the amount of available hours is \(\bar{L}\) while \(\bar{L} - L\), to simplify, will be considered leisure (time during the day that an individual does not work).

The maximization of profits is also subject to a budget constraint. Specifically, this constraint ensures the consumption must equal labor and capital or non-labor rents, \(R\).

\[
pC = R + WL. \quad (2.7)
\]

Note the reader that the slope of the labor supply, it is to say the relationship between real wages and employment, will depend on the utility function’s form.

In the appendix, I use a logarithmic function and solve according to the constraint. The result shows that real wages affect positively to employment:

\[
L\left(\frac{w}{p}\right) = \frac{L}{2} - \frac{m}{2} \frac{w}{p}. \quad (2.8)
\]

\(^2\) Refer to the appendix, section 9.3-B
To sum up, we cannot take equations like (2.8) as conclusive. At the end what makes the difference is the empiric evidence.

Apart from that, the slope of the labor supply can also be positive or negative depending on the so-called income effect and substitution effect compiled in the Slutsky equation:

\[
\frac{\Delta H}{\Delta w} = \text{net effect} = \text{substitution effect} (+) - \text{income effect} (-). \tag{2.9}
\]

When the net effect is positive, and the substitution effect is bigger than the rent effect, the net effect is positive and workers will work more hours when their wages increase. But when rent effect is bigger, they workers will prefer to work less.

Real wages and the labor force (measured by the amount of work hours) are the axis of the curve. The Slutsky equation will then define a positive-sloped curve among real wages and the amount of hours when the substitution dominates (b) whereas a negative net effect will be accompanied by a negative-sloped curve (a).

\[3\text{ Substitution effect: If wages go up, leisure is more expensive due to a higher opportunity cost (in this theory the consumer consumes labor or leisure only) and leisure finally decreases, it is an increase in labor substituting leisure by consume. This effect is called substitution effect.}

\[\text{Income effect: If wages increase, rent increases as well and individuals prefer to consume the extra rent in leisure instead of labor. And therefore labor decreases.}
\]

\[\text{Additional note: The theory assumes leisure is a normal good (the more rent you have, the more you consume) but rent is not constant.} \]
Mc Connell, et al. in his manual of labor economics affirms evidence vary significantly by gender. While women have a stronger substitution effect showing a positive curve, Borjas and Heckman (1978) point for men a quasi-vertical curve ensuring a big increase in wages will affect slightly the amount of hours. However, the literature did not use to make distinction by gender and agree on a labor supply with positive slope, so we will consider this from now on.

- Wage equation

The last ingredient of the Classical Theory of Unemployment is the wage schedule or wage equation. This equation explains how the salaries are set up by external agents (like labor unions) and employees through collective or individual bargaining over the competitive level. Consequently the slope of this curve depends merely on the situation of the labor market and the ability of these agents to influence in the level of real wages. Generally, wages are fixed according to a given level of unemployment but they are also subject to other measures of the labor market like labor taxes or unemployment insurance.

Specifically, Blanchard (1998) remarks several key factors in the process of configuring a wage equation: the wage itself, productivity, reservation wage (minimum wage a worker is willing to accept) and the labor market conditions.

Classical perspective use a positive wage equation curve assuming there are higher wages when the more employment and the labor market is performing well\(\left(\omega = \bar{\omega}(L) \text{ where } \frac{d\bar{\omega}}{dL} > 0\right)\).

Like in the labor supply case, the slope of this curve depends on the model we choose. In the appendix I develop the monopolist union model\(^4\) that provides an example of wage equation in function of unemployment, but we cannot take this as something definitive. Nevertheless, as the lector can see in figure 2, the slope of the curve does not affect to determine unemployment, so we do not have to worry much about it.

\(^4\) Refer to the appendix, section 9.3-C
If we draw together the three ingredients already seen in this chapter, we can notice the following: first of all, we obtain an equilibrium level of employment, $n$, and real wage, $\frac{w}{p}$. At this point both have the intersection between wage equation and labor demand. The unemployment is determined, thus, by the gap between this intersection and the labor supply at a given real wage.

Under this perspective, unemployment appears because the real wage is above the competitive level, where labor supply and labor demand cross out. To reduce unemployment the solution is very intuitive: reducing the wage equation till it reaches equilibrium level (chapter 4).

![Figure 2: Unemployment in the classical theory of employment. Source: Galí (2013).](image)

**Keynesian theory of unemployment**

This theory has its origins in the publication *The General Theory of Unemployment, Interest and Money* of John Maynard Keynes’ (Keynes, 1936). This paper is a milestone in modern economy and promoted a new school of thought: the Keynesianism.

---

5 For the calculus, I use different notation than Galí (2013). Real Wage $\frac{w}{p}$, Unemployment = $L$ and Wage schedule = Wage equation.
6 In the Keynesian theory of employment, the labor supply curve is the same so it does not deserve further explanations.
A few decades ago, this theory was divided in several branches. One of them, the New-Keynesianism, started developing complex models to explain new conceptions in unemployment till nowadays.

In what concerns us, original Keynesians and New-Keynesianism declare: “employment is what determines the real wage, not the other way around like classical model predicts”. Consequently, real wage cannot be considered as a mechanism to adjust employment anymore.

In the next pages, following the same structure of the previous section, I introduce present three key ingredient with the new features of the Keynesian theory of Unemployment followed by a figure that depicts the main difference between both theories.

- Labor demand

Employment depends on the quantity of output (total income or production) that firms produce under the assumption prices are completely fixed. Moreover, the production of firms is given by the respective demand. As a result, the aggregate demand for goods sets up the income at a certain price, what finally leads to a new employment level. It is so because firms will hire new workers according to their specific production needs. Real wage is only determined by the wage equation when firms have already employed all the workers.

This theory also implies the aggregate demand is the mechanism whereby employment can be changed. This new conception forces us to revisit some other points.

The new mindset urges to focus on the monetary and fiscal policies as vehicles for changing the aggregate demand, and in the second instance, employment. And for explain this part I refer to the IS-LM model. It is defined by two equations:

IS (Investment-Saving):

\[
Y = C(Y - T(↓)) + I(i(↓))(↑) + G(↑) \tag{2.10}
\]
LM (Liquidity Preferences-Money Supply):

\[ \frac{M(t)}{P} = L(i(t), Y) \]  \hspace{1cm} (2.11)

Expansionary policies are expressed on the equation with red arrows. From the fiscal side, governments use to increase public expenditure or reduce the level of taxes. Monetary policies typically act on the money supply by lowering the interest rate. These expansionary policies have an implication in the IS-LM model. They move IS and LM respectively to the right what provokes a shift in the same direction of the aggregate demand fixing a new and higher level of output at a given price.

---

**Table 1**: Impact of expansionary policies in the aggregate demand. Source: Compiled by author.

By and large, this is a very important remark because permits new macroeconomic indicators such as interest rate to find a place in the models of the theory of employment. In the annex is available a short numerical model in which I have used the theory of the IS-LM model again to encounter a labor equation in function of interest rate. More

---

\(^7\) Refer to the annex, section 9.3-A
precisely, the real interest rate has a negative relationship with production because the higher the interest rate is, the more expensive the loans are which implies less inversion engaged by firms and less consumption by individuals. It affects at the end the level of output.

This theory finally says “the interest rate affects the level of production and in the second instance labor demand”. In other words, the relevance of monetary policies in the fixation of the labor demand is now a fact.

The inefficacy of direct wage adjustments on employment becomes a fact. It is measured by the wage flexibility, which has become almost a dogma in policy thinking. But under this view there is no way to overdue economic downturns by letting real wage to better adapt (moderate) to particular levels. Additionally, measures like subsidies, payroll taxes or, as said, cuts in nominal wages are no longer valid (Galí, 2013).

After reviewing the basics of the Keynesian Theory of Unemployment, there is no direct impact of real wages on labor demand and employment, labor demand curve has undoubtedly to be drawn as a vertical schedule.

- Wage equation

The wage equation is the total amount of wages determined over the competitive level. It can be fixed by unions, government, employers’ association or individuals.

New-Keynesian theory is generally conceived as a negative-sloped curve. It has a downward slope assuming a decrease in employment (firms fire workers massively) increments the productivity because there are less workers leaving the output unchanged (short run). A higher level of productivity expressed in terms of the marginal product of labor $F_L$, would increase to finally push for higher wages in the short run. In short, the more unemployment we have, the higher the productivity is and normally it also means the higher the wages become.

The relationship between marginal product of labor and wages, however, can more deeply explained: In a non-competitive market, in which wages hover over the competitive level, marginal product of labor equals the real wage multiplied by the monopolistic indicator $m^P_T$ (see (2.5)).
In order to shed a light on this explanation, let’s transform (2.5) into logarithms:

\[(\omega_t - p_t)(\uparrow) = F_{L_t}(\uparrow) - m_t^p.\]  

(2.12)

Now this relationship is clear as stated by the logarithmic expression. Depending on how much the marginal cost oscillates subject to price and wage stickiness, the variation in the price mark-up will be bigger or smaller than the variation in the marginal product of labor. This difference will establish the sign of the real wages (going up or down).

Concluding, in comparison with the wage equation with the one typically used in the classical model, we have just the opposite relationship \( \omega = \tilde{\omega}(L) \) where \( \frac{d\tilde{\omega}}{dL} < 0 \).

As in the previous theory, unemployment is also represented through the gap between labor supply and the intersection between labor demand and wage equation providing equilibrium levels of real wages and employment. Beware the reader that in this case, despite having a different slope for the wage equation, it does not affect when identifying the unemployment graphically. The gap (u) has the same limits.

The main difference resides in the way to increase employment. In order to achieve that, we need necessarily to shift the labor demand curve rightwards. This shift, as mentioned before, can only be conceived if the economy experiences positive fluctuations in the aggregate demand.

![Figure 3: Unemployment in the Keynesian theory of unemployment. Source: Galí (2013).](image)
3 - Literature review for new extensions.

The content sorted out so far has more to do with the first conceptions and fundamentals of both theories which have been evolved through more sophisticated models. William Bragg once said “the important thing in science is not to obtain new facts but to discover new ways to analyze them”. The background of this chapter is, thus, to present new paths of research.

For each theory I have selected two specific models that explain the new directions for research. The first is made by Fabiani et al. (2000) for the Italian economy. Secondly, we will analyze a couple of Galí’s models from a Keynesian perspective that will include the presence of unemployment and firms with mark-up.

The classical model has introduced new extensions to the original vision. In this section we study the price equation. Some authors like Blanchard et al. (2012) prefer an even simpler, completely horizontal. Others like Fabiani et al. (2000) go for a negative-sloped curve. It is similar to a new labor demand curve of the previous model (figure 2).

Fabiani et al. presents each ingredient of the model with the following array of equations with special remark to the price equation, inexistent in previous models:
Production function:

\[ y_t = n_t + \theta_t \]  \hspace{1cm} (3.1)

Price equation:

\[ p_t = \mu_t(↑) + w_t - \theta_t + \beta u_t \]  \hspace{1cm} (3.2)

Labor supply:

\[ l_t = \alpha E_{t-1}(w_t - p_t - \theta_t) + T_t \]  \hspace{1cm} (3.3)

Wage equation:

\[ w_t = E_{t-1}(p_t + \theta_t) + k_t(↑) - \sigma E_{t-1}u_t \]  \hspace{1cm} (3.4)

Unemployment:

\[ u_t = l_t - n_t \]  \hspace{1cm} (3.5)

Where \( y_t \) = production (output), \( n_t \) = employment, \( \theta_t \) = productivity, \( p_t \) = price equation, \( \mu_t \) = average price mark-up, \( w_t \) = wage equation, \( T_t \) = demographic factor \( u_t \) = log unemployment, \( k_t \) = average wage mark-up, \( l_t \) = labor supply (= labor force).

In this set of equation there are some points to highlight. First of all we have a mark-up for each wage and price equation. The wage mark-up is already known because represents the extra wage set up over the competitive level. Price equation has to do with the margin of prices over labor unit cost characteristic in any labor market. Finally, the model introduces expectations of previous year to define model variables. In short, old variables also affect present results.

After solving the model we obtain a complex formula of unemployment\(^8\):

\[ u_t = \frac{1}{\alpha-\beta}k_t + \frac{1}{\alpha-\beta}I_{t-1} + \frac{1}{\alpha-\beta}[\epsilon_i - \Phi\epsilon_i(a + \Phi - 1) + \Phi\epsilon_i] \]  \hspace{1cm} (3.6)

The equation states in order to successfully contract unemployment, both mark-ups need to be reduced. In addition, due to the introduction of expectations, the unexpected shocks play an important role to influence unemployment. Concretely, we talk about shocks in the aggregate demand (\( \epsilon_i \)), aggregate supply (\( \epsilon_i \)), labor participation (\( \epsilon_i \)) and non-competitive labor demand (\( \epsilon_i \)).

---

\(^8\) Refer to the appendix, section 9.2-D or to ECB working paper no.29 –September 2000.
The effect of a rise in the mark-ups is graphically represented in figure 5. When it happens, price and wage equation decrease with a shift to the left. In consequence, employment falls but the effect on real wage is unclear.

Doménech & Andres (2012) review this model for the Spanish economy to justify the importance of enforcing wage moderation policies. Also regret about the potential benefits that a prior and more accurate wage policies would have had on the real economy of Spain.

Concretely, they oriented the study to find out how many jobs could have been preserved if wage-moderation process would start at the beginning of the crisis in 2008.

On the side of new-Keynesians, in the last decades, these authors have been enhancing their models making them more and more sophisticated by aggregating key elements. In this section I focus on the new-Keynesian model with unemployment of Galí, (2013)⁹:

Dynamic IS equation:

\[ y_t = E_t \{ y_{t+1} \} - (i_t - E_t \{ \pi_{t+1} \} - \rho) \]  (3.7)

Price expected inflation equation:

\[ \pi_t^p = \beta E_t \{ \pi_{t+1}^p \} + \lambda_p \frac{1}{1-\alpha} (\bar{y}_t + \bar{\omega}_t) \]  (3.8)

Wage inflation equation:

\[ \pi_t^w = \beta E_t \{ \pi_{t+1}^w \} + \lambda_w \frac{\phi}{1-\alpha} (\bar{y}_t - \bar{\omega}_t) \]  (3.9)

Unemployment:

\[ \hat{u}_t = \frac{\bar{\omega}_t - (1+\frac{\phi}{1-\alpha}) \bar{y}_t}{\phi} \]  (3.10)

Monetary equation:

\[ i_t = \rho + \phi \pi_t^p + \phi_y \bar{y}_t + v_t \]  (3.11)

⁹ Refer to the annex, section 9.3-A
Where $\rho$=time discount rate$^{10}$, $i_t$ = nominal interest rate at period $t$, $\tilde{\omega}_t$ = output gap between real wage and natural wage ($\tilde{\omega}_t \equiv \omega_t - \omega_t^n$) $\tilde{\gamma}_t$ = output gap between real output and natural output ($\tilde{\gamma}_t \equiv \gamma_t - \gamma_t^n$), $\hat{y}_t$ = deviation of output from steady state, $\nu_t$ = exogenous monetary component, $\varphi$ = curvature of labor disutility, $1 - \alpha$ = labor demand elasticity. Let me remind all of these variables are expressed in log.

Note the lector in this equations we introduce the component of future expectations to determine current variables, in contrast with the classical models where the expectations were present ($X_{t-1}$). Moreover, the model consider output and wage gaps as a consequence of differences respect to the natural levels. The most outstanding novelty, thought, is the fact that not all the firms fix the prices but only some of them with a probability of $1 - \theta$.

As the models are extremely complex, they requires the use of specific software (Dinare). I am sharing here the results from two scenarios calibrated for the United States economy but that can be useful to understand the Spanish case: A positive shock in technology because of a 1 percent increase in the technology parameter ($a$), and a negative monetary shock due to an increase of a 25 basis point in the exogenous monetary component ($\nu_t$).

Firstly, I would like to start with a quick guidance to check these graphics. We focus on period 0 to figure out the immediate reaction of the variables. Then the curves continue till period 16 to show the evolution in each case.

The response to the first shock shows some predictive reactions such as the increase in the level of output or the decrease in inflation but, surprisingly, and in contrast with the standard models, the employment falls. It happens because a positive shock in technology is accompanied by a labor substitution. The classical view would argue this effect boost employment because a gain in productivity. Real wages rise a little bit because the existence of wage rigidities and the decline in prices.

---

$^{10}$ Relative valuation placed on a good at an earlier period compared with its valuation at a later period.
If we focus on the second shock, a contractive monetary policy (higher interest rates and lower money supply) lead to a reduction in output, employment and of course, in prices. This is also obvious. Nonetheless, in this case is relevant the fact unemployment is being affected by the behavior of interest rates. It rises more than 1 point in responses to tightening monetary strategy while under the classical view, employment should not been affected by the interest rate deviations. Real wage in this case, as expected does not experience a significant change because theoretically real indicators remain isolated towards this shocks in contrast with what classical literature predicts.
4 - Spanish labor market at a glance

In this section I present a brief description of the Spanish labor market with the main features and historical background of this economy.

The Spanish labor market has a chronic disease with employment since few decades ago. In the figure below is depicted the recent data base of Spain in contrast with the EU and OECD members.

![Figure 6: Evolution of the Unemployment rate (%) in the OCDE Countries (1999-2014). Source: OCDE](image)

Since the beginning of the democratic era, unemployment has been quite instable and above the average rates of the European Union member states. In the last 50 years Spain has suffered 3 peak periods on which the unemployment rate surpassed 20%, comparable with the job destruction in United States during the Great Depression. These peaks have to do with particular chaotic moments in the Spanish economy: In the mid-eighties with the end of the industrial restructuring, the early nineties, with the big public expenditure and global macroeconomic shocks and from 2008 onwards with the current financial recession and real state bubble.
As mentioned before, the aim of this work is not to undercover an essay of the causes of unemployment in Spain but, still, I think it is important to dedicate some time to the other features replicated in the literature about the topic:

- **Duality:** Spain has a very strong dual component. There are two differentiated groups: the one employed with good labor conditions and the one generally composed by un-skilled workers, immigrants and youngsters who suffer very low wages and vulnerability by the government. This effects deteriorate the implications of labor policies in employment and erode the welfare state.

- **Productivity:** Considered as the amount of output produced by a single worker in a certain period of time (normally hourly), has been always a weak factor in Spanish economy. The strongest recommendation on this side is to link wages to productivity. In general Spain shows lower levels than the northern economies of the Eurozone.

- **Temporality:** In the last years, the vast majority of new workers are hired through temporary contracts that provide less stability especially for the young people. It is one of the factors that fosters duality. Around 9 over 10 of total contracts in Spain have a short-term duration.

- **Education:** Due to the Spanish property bubble, an important sectorial shift took place impacting in a big mass of workers that did not have other skills to get hired in new sectors. In these environments, education plays an essential role to recycle unemployed force.
✓ Bureaucracy and law: This factor still pending to be solved is a really impediment to the creation of new enterprises because of bureaucratic bindings that spin out the process. The Spanish law continues neither to protect nor incentivize enough SMEs companies where in Spain they conform more than 99% share.

Now that employment and its historical background in Spain have been briefly studied, now it is time to pass to the other key elements: prices, nominal wages and real wages. To do so, I present a couple of graphs synthetizing the trends in these variables.

![Figure 8: Compensation rate](image1)
![Figure 9: CPI & GDP Deflator](image2)

Both graphs depicted above have a common denominator: Spain has responded differently to the financial crisis in contrast with the OCDE and euro zone countries on average. Its curves differ after experiencing similar trends along the nineties and early twenties. But in the previous days of the crisis we can perceive a higher increase of the nominal labor costs in Spain than the average of the Eurozone or OCED empowered by financial bubble and the high rates of economic growth.

---

11 Compensation rate include all the costs applicable for the firms. Monetary and non-monetary pays: Wages, social security, bonuses, overtime pay, sales commission, paid cars, stock options etc...
In terms of prices, for the same reason, Spain faced prices over the average only before the crisis. After it, CPI increased but GDP deflator performed a flat curve while in the Euro area both tendencies were similar to the same period of time.

![Figure 10: Spanish Real wage before and during the crisis in 2008-2013. Source: Compiled by author. Data: BBVA Research & OCDE.](image)

With the information collected in figures 9 and 10, we have the entire ingredients to build an approximation to the Spanish real wage. As depicted in this graphic, the calculus of real wages is much different, when the crisis started, depending on what type of price indicator is used. This difference occurs mainly due to nature of each price indicator. GDP deflator\(^{12}\) determines prices of all kind of goods, including both industrial (PPI) and consumer prices whereas CPI measures the prices of typical goods that consumers include in the shopping basket. Therefore, the first reflects in a better way price´s dimension as include a wider range of prices. Being that CPI has increased more than GDP deflator did after the outburst of the crisis, real wage deflated by CPI (red line) will be irremediably smaller as seen in figure 12.

Just before concluding this chapter, it is convenient to share the evolution of monetary policy indicators to have a clearer picture of all the ingredients and how can they contribute to the final output in the Keynesian thesis.

---

\(^{12}\) From now onwards this will be the price used in the models indicated by \(p_t\)
Both macroeconomic indicators are in hands of ECB officials. The European Central Bank is the only bank with the authority to control the monetary supply. Since the beginning of 2008 responded with expansionary policies injecting more liquidity by means of a constant reduction of the interest rate and more recently this policy turned even more expansionary under the direction of Mario Draghi, when the bank started to buy state members´ bonds to foster inflation and depreciate the Euro in order to be more competitive in the world trade thanks to a boost in exports.

The final picture among real wages and employment after merging figures 8 and 12 is the following.

Figure 11: Correlation between Real Wages and employment in the Eurozone (2002-2014) Annual variation. Source: Compiled by Author. Data: OCDE.
From a European perspective, we perceive euro zone members have had a long variety of reactions in the last crisis. For instance, during the crisis some countries have experienced a fall in the levels of employment despite a reduction of the real wages too (Southern Countries like Portugal or Greece), other increased from both sides (Northern Countries like Belgium and Germany).

If we focus on Spain, the spotter somehow defines a negative trend between real wages and employment in the periods before and after 2008, a year that became a truly turning point for the Spanish economy. This trend is more emphatically reproduced in Doménech et al. (2013). Nevertheless, as we will see in the next chapter, it does not help to distinguish which theory fits most because the data can be perfectly explained by both options.

5 - Adaptation of the theories to the Spanish case.

- Spanish situation in the models

The aim of these section is to build a simulation of what happens in Spain, it is to say, big decrease in employment but low increase of real wages, by changing the curves on figure 2 and 3 for each theory.

Theoretical literature and mere intuition says that such a big destruction of jobs can be mainly provoked by a contraction in the labor demand. So we assume from now
on, that it occurred. If job market loses workers, consumption and investment are reduced and these reduction are translated into lower output. When an economy produces less output, it impacts firms who have to set up adjustments in terms of cost reduction, which implies more job destruction and that the way we initiate a perverse vicious cycle.

Now that we know that a labor demand shock is a necessary ingredient, I dedicate the following paragraphs to explain the effect of this shock in a graph and the cure for it.

**Classical theory** advocates for a shift of wage equation and labor demand: The negative shift of the wage equation would move the curve leftwards provoking an increase in the level of real wages and unemployment.

As discussed before, we also need to take into account the negative fluctuation in the labor demand whose curve would shift downwards getting a fall in real wages and a bigger decrease in employment. Otherwise it is impossible to conceive the enormous loss of employment in the last years in Spain\(^{13}\). The result is, at the end, in figure 17 similar to Spain’s reality: slightly increase in real wages and big fall in employment.

For this case, I have drawn the shifts in two separated depictions. The first, in blue, show the shift of labor demand. The second, in red, the change in the wage equation.

---

\(^{13}\) In the simplified model appearing in BBVA Research – Report 2nd quarter 2013, the fluctuations happen due to negative shocks in the labor demand and supply demand. The result is the following: both shocks have a negative impact to employment but supply shock causes an increase in the real wages whereas the demand shock makes real wages go down.
New Keynesians would claim that a shock in the labor demand caused by a change in the aggregate demand was the main cause. In this case we have a negative-sloped wage equation curve, so is this curve the one that moved to the left (red). Aggregate demand, this time vertical, decreased in Spain a lot and that is why I also shifted it to the left in the graph (blue). Now we have a very similar representation to figure 18.

In this environment, the only solution to reduce the employment is through an expansion in the aggregate demand to the right. Wage equation can be monitored to control the level of real wages.
In both cases, the cure for unemployment lies in reverting shifts (opposite direction of the arrows).

For the classical theory of unemployment, real wage reduction or an employment subsidy which lowers compensations to the workers by firms can be a good measure to recoup the labor demand. We cannot forget, though, that in a situation like the one is suffering Spain, structural policies should additionally be imposed to reverse labor demand (i.e. labor reforms). In contrast, in economies with low unemployment rates and sporadic downturns a wage-moderation policy may be enough to come back to equilibrium levels of unemployment (Doménech et al., 2013).

Under the Keynesian view, it always goes through an aggregate demand so there is nothing we can do by adjusting the real wages. An expansionary fiscal and monetary policy could foster the aggregate demand and, then, force labor demand curve to return to its initial level.

6 - Analysis: What theory fits most to the Spanish case?

In this section we will focus on identifying what model better explains the Spanish labor market. We will follow with the main pillars to define the best theory. I run an econometrical analysis that recognize what variables affect most to unemployment. Depending on the influence of each dependent variable in the variation on the unemployment (looking at the coefficients) we will be able to take the first conclusions. Later on, we will do some additional research in order to complement the previous findings with empirical evidence.

1) Econometrical analysis.

In figure 15 we have seen the simple static relation between real wages and employment. However, it was a purely empirical relation based on the data without analytical justification. If we are interested in getting conclusive results, it is important to define the key ingredients by collecting all relevant data available. For this purpose an econometric model has been run takins the one done by Raurich et al. (2009) as reference.
I have concretely built a Spanish labor demand and wage equation along the last years fifty-five years with the following variables:

\[ n_t \]  
\[ \omega_t \]  
\[ y_t \]  
\[ \theta_t \]  
\[ k_t \]  
\[ l_t \]  
\[ r_t^s \]  
\[ r_t^l \]  
\[ b_t \]

- Log of employment
- Log of average real wage
- Log of real GDP
- Log of average total factor productivity
- Log of real net capital stock
- Log of labor factor productivity
- Log of real short-term interest rate
- Log of real long-term interest rate
- Log of real social security benefits per person

These array of variables let us to configure both employment and wage equations:

\[ n_t = \alpha_0 + \alpha_1 n_{t-1} + \alpha_2 \Delta n_{t-1} + \alpha_3 k_t + \alpha_4 r_t^l + \alpha_5 \omega_t + \alpha_7 y_t + u_{1t} \]  

(6.1)

\[ \omega_t = \beta_0 + \beta_1 \omega_{t-1} + \beta_2 n + \beta_3 \Delta n + \beta_4 \theta_t + \beta_5 b_t + u_{2t} \]  

(6.2)

Table 2: Definition of variables for the Spanish employment equation. Source: AMECO database

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>2.95113</td>
<td>0.693066</td>
<td>4.2581</td>
<td>0.0004  ***</td>
</tr>
<tr>
<td>l_employment_t-1</td>
<td>0.65684</td>
<td>0.0816345</td>
<td>8.0461</td>
<td>&lt;0.0001 ***</td>
</tr>
<tr>
<td>l_var-employ._t-1</td>
<td>0.0555378</td>
<td>0.0159446</td>
<td>3.4832</td>
<td>0.0022  ***</td>
</tr>
<tr>
<td>l_capital</td>
<td>0.209797</td>
<td>0.0815455</td>
<td>2.5728</td>
<td>0.0177  **</td>
</tr>
<tr>
<td>l_l_t_interest</td>
<td>-0.012188</td>
<td>0.0075653</td>
<td>-1.6110</td>
<td>0.1221</td>
</tr>
<tr>
<td>l_s_t_interest</td>
<td>0.00549497</td>
<td>0.0055536</td>
<td>0.9894</td>
<td>0.3337</td>
</tr>
<tr>
<td>l_wage</td>
<td>-0.292041</td>
<td>0.15169</td>
<td>-1.9252</td>
<td>0.0679  *</td>
</tr>
<tr>
<td>l_GDP</td>
<td>0.0148547</td>
<td>0.0422533</td>
<td>0.3516</td>
<td>0.7287</td>
</tr>
</tbody>
</table>

Table 3: Results of the employment equation for Spain. Source: Compiled by author via Gretl.

14 There are missing observations due to short-term and long-term interest rate data is shorter. (Only available since 1977). Then, Gretl clips the contrast with the shortest series for all variables.
Wage Equation: OLS, using observations 1961-2016 (T = 56)  
Dependent variable: l_rage

<table>
<thead>
<tr>
<th>Concept</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>-0.616482</td>
<td>0.240836</td>
<td>-2.5598</td>
<td>0.0135  **</td>
</tr>
<tr>
<td>l_rage_t-1</td>
<td>0.702075</td>
<td>0.0594154</td>
<td>11.8164</td>
<td>&lt;0.0001 ***</td>
</tr>
<tr>
<td>l_Productivity</td>
<td>0.393897</td>
<td>0.0960625</td>
<td>4.1004</td>
<td>0.0002 ***</td>
</tr>
<tr>
<td>l_Subsidy</td>
<td>0.00336804</td>
<td>0.00360557</td>
<td>0.9341</td>
<td>0.3547</td>
</tr>
<tr>
<td>l_employment</td>
<td>0.0161303</td>
<td>0.0142889</td>
<td>1.1289</td>
<td>0.2643</td>
</tr>
<tr>
<td>l_var-employment</td>
<td>-0.0539438</td>
<td>0.00955587</td>
<td>-5.6451</td>
<td>&lt;0.0001 ***</td>
</tr>
</tbody>
</table>

Mean dependent var 4.301374  S.D. dependent var 0.344275
Sum squared resid 168.7621  S.E. of regression 0.012577
R-squared 0.998787  Adjusted R-squared 0.998665
F(5, 50) 8232.255  P-value(F) 1.27e-71
Log-likelihood -313.3720  Akaike criterion -325.5241
Schwarz criterion -320.8128  Hannan-Quinn -320.8128
rho 0.238713  Durbin-Watson 1.505783

Table 4: Results of the employment equation for Spain. Source: Compiled by author via Gretl.

The first model reveals the results of running the model of the equation (6.1). The first remarkable point is that, as expected, we obtain a big and negative coefficient of the real wages (in bold). Furthermore, the impact of the long-term and short-term interest rates is irrelevant. Also the capital variable is important in this equation as is one of our factors of production along with the labor force. In table 5, the key variable is the total productivity of factors, which has a positive correlation with real wages.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Employment equation</th>
<th>Wage equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-Squared</td>
<td>0.9948</td>
<td>0.9988</td>
</tr>
<tr>
<td>Homoscedasticity1</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Autocorrelation2</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Multicollinearity3</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 5: Compilation of econometric contrast results. Source: Compiled by author.
1 Contrast: White, 2 Contrast: Breusch-Pagan (Durbin-Watson not conclusive), 3 Contrast: Variance Inflator Factor (VIF).

If we focus on the contrast done to prove the results, I have summarized it in the above table. In both cases, R-squared is almost 100, which denotes the correctness of the model. In the appendix, the lector will also find further clarifications of those contrasts of econometric phenomena: analysis of linearity, autocorrelation and homoscedasticity. In short, we have homoscedasticity which is good because it means all the random variables
have the same variance and is one of the assumptions for the regression of multiple variables. Luckily we do not have autocorrelation, known as the correlation between values of the process at different times. All the same I have to admit the model shows a severe multi-collinearity due to the fact this model is done by temporal series where the likelihood to experience correlation among the independent variables is generally high.

The outcome of this section is clear: according to the econometric evidence, the classical theory fits most as the data supports the negative correlation between real wages and employment, and productivity becomes crucial.

2) Additional evidence

In this section I compile a few evidences to complement the econometric analysis:

- **Real wage - employment correlation:*** As seen in figure 15 and 16 there is a little evidence that both variables have a negative correlation for Spain. Notwithstanding this finding gives us no added value because as shown in the previous chapter, this result can be obtained by both theories through different mechanisms. It is not helpful at all.

- **Price stickiness:** Galí in many of his papers concludes there is a micro-evidence on price setting behavior causing price rigidity in the short run. In other words, there is a fixed price in nominal terms for a relevant period of time. Several working papers like Alvarez et al. (2005) or Angeloni et al. (2006) provide evidences for the Euro-Area. Thus, this point is in favor of the Keynesian view.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Euro area</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI(^1)</td>
<td>15.1</td>
<td>24.8</td>
</tr>
<tr>
<td>Average duration ((\text{months}))</td>
<td>13.0</td>
<td>6.7</td>
</tr>
<tr>
<td>Median duration ((\text{months}))</td>
<td>10.6</td>
<td>4.6</td>
</tr>
<tr>
<td>PPI(^2)</td>
<td>20.0</td>
<td>n.a</td>
</tr>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surveys(^3)</td>
<td>15.9</td>
<td>20.8</td>
</tr>
<tr>
<td>Average duration ((\text{months}))</td>
<td>10.8</td>
<td>8.3</td>
</tr>
<tr>
<td>NKPC(^4)</td>
<td>13.5-19.2</td>
<td>7.2-8.4</td>
</tr>
<tr>
<td>Average durations ((\text{months}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internet prices(^5)</td>
<td>95.5</td>
<td>94.7</td>
</tr>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Payroll and subsidies: Nickell (1991) found that a great rise in tax wedge (10%), so payroll taxes, income and consumption taxes reduce significantly (1-3%) the amount of workforce. The same author in 1997 undertakes a study to explain the influence of variations in the unemployment subsidy on employment. The outcome highlights positive correlation of high or long subsidies with the rate of unemployment in OECD countries within 1983-1994. The impact of these measures in employment suggest this time a point in favor of classical thesis.

7 - Conclusions

In this work I have tried to find the model that better represents the Spanish labor market. To do so, I have selected two candidate theories: The Classical Theory of Unemployment and the Keynesian Theory of Unemployment. For each theory I have introduced the main features and ingredients as well as graphic representations. By a comparison among them, I highlighted the key differences and also covered the extensions used in the current research. Once all this information was collected in addition to the specific details of our framework in Spain, I started the main analysis of the essay.

It consists in an econometric analysis made through temporal series which shows a negative correlation between real wages and employment with a consistent coefficient in the employment equation. At the same time, productivity has also a strong positive coefficient in the wage equation. These results evidence a clear support to the classical model.

Additionally I brought to pass some research to complement the previous findings. I found elements in favor and against each theory. For instance, New-Keynesians advocate for the stickiness of prices in the short run and effectively, there are many evidences supporting this fact. However, other authors point that pay-roll taxes and subsidies have implications on employment, which contradicts this time the Keynesian view in favor of the Classical Theory of Employment.

---

15 Taxes on employment paid by firms
Even if some findings opt for the Keynesian theory and my personal opinion is close to the idea that aggregate demand provoked the labor demand shock in Spain, the output of the econometric test removes all doubt and puts me in a position to affirm the best option to explain the behavior of the Spanish labor market is the Classical Theory of Unemployment.

Because economics is not an exact science, further research could find new scenarios and findings that reassemble this essay. For instance, a deep inquiry on real state bubble may help to uncover the causes of the shrink in labor demand and could tilt the balance in favor of the other theory or reaffirm my first results.

In conclusion, I advocate for the Classical Theory of Unemployment. Notwithstanding this paper opens up to new extensions and paths of research to continue the work done so far.
8 - References


Doménech, Andrés and Javier Andrés (2013). “¿Puede la moderación salarial reducir los desequilibrios económicos?”. BBVA Research.


Sorolla, V (2014). “Modelos con determinación de salarios reales y paro”. UAB Class material of Macroeconomics II.

9 - Annex
9.1 Table of figures.

<table>
<thead>
<tr>
<th>New-Keynesian model</th>
<th>Classical Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 8.1</td>
<td>Labor Supply</td>
</tr>
<tr>
<td>Figure 8.2</td>
<td>Labor Demand</td>
</tr>
<tr>
<td>Figure 8.3</td>
<td>Equilibrium</td>
</tr>
<tr>
<td>Figure 8.4</td>
<td>Unemployment</td>
</tr>
<tr>
<td>Figure 8.5</td>
<td></td>
</tr>
<tr>
<td>Figure 8.6</td>
<td></td>
</tr>
<tr>
<td>Figure 8.7</td>
<td></td>
</tr>
<tr>
<td>Figure 8.8</td>
<td></td>
</tr>
</tbody>
</table>
9.2 Classical Theory of Unemployment

A) Labor demand

From the general expression of production we can answer the main principles of the classical theory of employment mentioned in page 3.

\[ Y = F (K, L) = A K^{\alpha} L^{1-\alpha}. \] (9.1)

a) When the exponents, known as the elasticity of each factor of production equals 1 \((\alpha + (1 - \alpha) = 1)\) is said the return to scale is constant. It also happens when a change in the total production \((Y)\) has the same effect in the factors:

\[ 2Y = A (2K)^{\alpha}(2L)^{1-\alpha} = 2^\alpha 2^{1-\alpha} (A K^{\alpha} L^{1-\alpha}) 2F (K, L). \] (9.2)

b) Partial derivatives of the factors of productions show positive sign in the first derivative whereas the second derivative is negative:

\[ F_L = (1 - \alpha) A K^{\alpha} L^{-\alpha} > 0; \quad F_{LL} = (1 - \alpha)(-\alpha) A K^{\alpha} L^{-\alpha-1} < 0 \]

\[ F_K = (\alpha) A K^{\alpha-1} L^{1-\alpha} > 0; \quad F_{KK} = (\alpha - 1)(\alpha) A K^{\alpha-2} L^{1-\alpha} < 0 \]

Graphically, these equations have the following implication:

![Graph showing output per head (Y) vs Capital-labour ratio (k)]
As a result, we have the following neoclassical function of production with Production (Y) in the vertical axis and the Factors (K,L) in the horizontal one.

c) Firms maximize their benefits taking prices of labor and products as a given and selecting a most efficient level of labor (L).

\[ \max_L \pi = P F(K, L) - WL \tag{9.4} \]

First order condition or Kuhn-Tucker condition is the equation of the partial derivative of labor equal to zero:

\[ \frac{\partial \pi}{\partial L} = 0 \rightarrow P F_L(K, L^d) - W = 0 \tag{9.5} \]

The partial derivative also known as marginal productivity of labor equals the real wage. This assumption becomes a milestone in the classical theory of employment:

\[ F_L(K, L^d) = \frac{W}{P} \tag{9.6} \]

In case of monopoly, the mark-up will emerge in (1.6) like that:

\[ F_L(K, L^d) = m \frac{W}{P} = \left( \frac{\sigma}{\sigma - 1} \right) \frac{W}{P} \tag{9.7} \]

The mark-up depends on the level of elasticity being the maximum (infinity) when elasticity is 1 and in perfect competence when elasticity is \( \infty \) and \( m=1 \).

d) Continuing with (9.7) we have:

\[ \Gamma \left( L, \frac{W}{P} \right) = F_L(K, L^d) - \frac{W}{P} = 0. \tag{9.8} \]

Using the theorem of the implicit function

\[ \frac{dy}{dx} = -\frac{f_x}{f_y} \]
\[
\frac{\partial L^d}{\partial W} = \frac{\partial^2 W}{\partial L^d} = -\frac{1}{F_{LL}(K, L)} < 0 < 0 \quad (9.9)
\]

e)

We need to calculate first \( F_L(K, L^d) \):

\[
F_L(K, L^d) = (1 - \alpha)Ak^\alpha L^{-\alpha} \quad (9.10)
\]

Starting from equation (1.7) and including component \( m \) of monopoly:

\[
(1 - \alpha)Ak^\alpha L^{-\alpha} = m \frac{W}{p} \rightarrow (1 - \alpha)Ak^\alpha L^{-\alpha} = m \omega \quad (9.11)
\]

We obtain:

\[
\omega_t = \frac{1}{m_t}(1 - \alpha)A_tK^\alpha t L^{-\alpha} \quad (9.12)
\]

Isolating \( L \):

\[
\omega_t = \frac{1}{m_t}(1 - \alpha)A_tK^\alpha t \frac{L^\alpha}{t} \rightarrow L^\alpha = \frac{1}{m_t}(1 - \alpha)A_tK^\alpha t \frac{1}{\omega_t} \quad (9.13)
\]

\[
L_t = \left(\frac{1}{m_t \omega_t}(1 - \alpha)A_tK^\alpha t \right)^{\frac{1}{\alpha}} \rightarrow \left[\frac{1}{m_t \omega_t}(1 - \alpha)A_tK^\alpha t \right]^{\frac{1}{\alpha}} \quad (9.14)
\]

In order to trace the variation of the variables we will use logarithms:

\[
\ln(p \cdot q) = \ln p + \ln q; \ln(p/q) = \ln p - \ln q; \ln p^n = n \ln p
\]

We get:

\[
\ln L = -\frac{1}{\alpha} \ln \omega - \frac{1}{\alpha} \ln m + \frac{1}{\alpha}(1 - \alpha) \ln A + \frac{1}{\alpha} \ln K \quad (9.15)
\]

In terms of variation of each variable we meet the same equation of the section 1:

\[
\frac{\Delta L}{L} = -\frac{1}{\alpha} \frac{\Delta \omega}{\omega} - \frac{1}{\alpha} \frac{\Delta m}{m} + \frac{1}{\alpha} \frac{\Delta A}{A} + \frac{\Delta K}{K} \quad (9.16)
\]
B) Labor Supply

The budget constraint is:

\[ pQ = M + WL \quad (9.17) \]

Divided by the price:

\[ Q = m + \frac{W}{p}L \quad (9.18) \]

From equation (A.1.2) we include \( \frac{w}{p} \bar{L} \) at both sides and divide all by \( p \). Thanks to this trick, we can add leisure in the equation as depicted below:

\[ Q + \frac{w}{p} \bar{L} = \frac{w}{p} L + \frac{w}{p} \bar{L} \rightarrow Q + \frac{w}{p} (\bar{L} - L) = \frac{w}{p} \bar{L} \quad (9.19) \]

With the use of the Lagrangian \( \mathcal{L} \) it would look like this:

\[ \mathcal{L} = U(c, \bar{L} - L) - \lambda \left( Q + \frac{w}{p} (\bar{L} - L) - \left( \frac{w}{p} \bar{L} \right) \right) \quad (9.20) \]

Khun-Tacker conditions:

\[ \frac{\partial \mathcal{L}}{\partial Q} = 0 \rightarrow 0 = U'_c - \lambda \]

\[ \frac{U_{L-L}}{U_Q} = \frac{w}{p} \quad (9.21) \]

\[ \frac{\partial \mathcal{L}}{\partial L} = 0 \rightarrow -U_{L-L} + \lambda \frac{w}{p} = 0 \]

This result poses the intra-temporal condition among consumption and leisure indemnifying the optimum amount of working hour. The slope of this curve becomes to be the real wage, with a negative slope.

None of less, in order to find a real labor supply, not depending on consumption, we have to substitute the intra-temporal condition in our budget constraint (A.2.1) given a certain utility function.

Let’s consider a logarithmic utility function: \( U (Q, \bar{L} - L) = \ln Q + \ln (\bar{L} - L) \).
In this case: \( \frac{\partial u}{\partial q} = \frac{1}{q} \) and \( \frac{\partial u}{\partial L-L} = \frac{1}{L-L} \).

Consequently, the inter-temporal condition would be:

\[
\frac{1}{q} = \frac{w}{p} \rightarrow \frac{Q}{L-L} = \frac{W}{p}.
\] (9.22)

Finally, isolate Q from (1.16) and substitute it to (1.12):

\[
\left( \frac{w}{p} \right) (L - L) = \frac{w}{p} L + m \rightarrow \left( \frac{w}{p} \right) L = 2 \left( \frac{w}{p} \right) L + m
\] (9.23)

If divide all by real wages and isolating L we get:

\[
L = \frac{\bar{L}}{2} - \frac{m}{2 \frac{w}{p}}
\] (9.24)

C) Model of the monopolistic union

In this model trade unions know labor demand \( \bar{L}^d(\omega) \) and labor supply is fixed \( \bar{L}^s \). The union equation is:

\[
(\omega - r)L
\] (9.25)

Subject to:

\[
L = \min(\bar{L}^d(\omega), \bar{L}^s)
\] (9.26)

Where L is employment, \( \omega \) are the wages and \( r \) is the rent of working elsewhere out of the company. Therefore, the benefit from working for the company is \( \omega - r \).

Let’s suppose \( L = \bar{L}^d(\omega) \), and A.3.2 turns to \( (\omega - r)\bar{L}^d(\omega) \). If we do the first order condition with the partial derivative of \( \omega \) we obtain:

\[
x \ast y' + x' \ast y \rightarrow 1\bar{L}^d(\omega) + \frac{d\bar{L}^d(\omega)}{d\omega} (\omega - r) = 0.
\] (9.27)

On the other hand, our labor demand based on Cobb-Douglas equation:
\[ Y = AL^\alpha \]  

(9.28)

Now as we have done before, we equal

\[
\frac{W}{P} = \omega = F_L \\
\rightarrow \omega = \alpha A \tilde{L}d(\omega)^{\alpha-1} \\
\rightarrow \tilde{L}d(\omega) = \left(\frac{\omega}{\alpha A}\right)^{\frac{1}{\alpha-1}} \\
\tilde{L}d(\omega) = \left(\frac{\alpha A}{\omega}\right)^{\frac{1}{1-\alpha}}. \tag{9.29}
\]

We also know the elasticity of labor demand respect to wages becomes the exponent. Knowing the formula of elasticity is easy to deduce that:

\[
-\frac{d\tilde{L}d(\omega)}{d\omega} \frac{\omega}{\tilde{L}d(\omega)} = \frac{1}{1-\alpha} = \varepsilon\tilde{L}d \omega. \tag{9.30}
\]

If we adequate equation A.3.3 multiplying and dividing all by \(\frac{\omega}{\tilde{L}d(\omega)}\), and substituting in A.3.6 we get:

\[
\left(\frac{\omega}{\tilde{L}d(\omega)}\right)\tilde{L}d(\omega) - \frac{d\tilde{L}d(\omega)}{d\omega} \left(\frac{\omega}{\tilde{L}d(\omega)}\right)(\omega - r) = 0 \\
\rightarrow \omega - \frac{1}{1-\alpha} (\omega - r) = 0 \\
\rightarrow \omega - \omega\alpha = (\omega - r) \\
\rightarrow \omega = \frac{r}{\alpha}. \tag{9.31}
\]

Now it is time to introduce the formula of \(r\). As said, it represents all the rents a worker perceives thanks for not being part of a firm. This equation has the following structure:

\[
r = (1 - u)\omega^e + ub. \tag{9.32}
\]

Where \(u\) is the unemployment rate, \(b\) is the subsidy of unemployment and \(\omega^e\) is the expected wage of working outside the company (competence). This equation seems to be very intuitive. We count all the wages of workers that are not working in a specific firm plus subsidy costs of all unemployed people. All together determines total external rents.
If we substitute A.3.8 in A.3.7 we have:

$$\omega = \frac{(1-u)\omega^e + ub}{\alpha}$$  \hspace{1cm} (9.33)

Assuming all the firms have same wages ($\omega = \omega^e$) and isolating $\omega$ finally obtain the equation A.1.1 of page 11:

$$w = \frac{ub}{\alpha(1-u)}$$  \hspace{1cm} (9.34)

D) Fabiani Model

First of all we have to write all the equations again, included the equations for the exogenous variables.

Aggregate demand in function of economic policy:

$$y_t = \emptyset (d_t - p_t) + a\theta_t.$$  \hspace{1cm} (9.35)

Production function:

$$y_t = n_t + \theta_t.$$  \hspace{1cm} (9.36)

Price equation:

$$p_t = \mu_t + w_t - \theta_t + \beta u_t.$$  \hspace{1cm} (9.37)

Labor supply:

$$l_t = \alpha E_{t-1}(w_t - p_t - \theta_t) + t_t.$$  \hspace{1cm} (9.38)

Wage equation:

$$w_t = E_{t-1}(p_t + \theta_t) + k_t - \sigma E_{t-1} u_t.$$  \hspace{1cm} (9.39)

Unemployment:

$$u_t = l_t - n_t.$$  \hspace{1cm} (9.40)

Aggregate demand shock in productivity:

$$\theta_t = \theta_{t-1} + \varepsilon^\theta_t.$$  \hspace{1cm} (9.41)

Labor force shock in demography:

$$t_t = t_{t-1} + \varepsilon^l_t.$$  \hspace{1cm} (9.42)

Non-competitive labor demand shock in the price mark-up:

$$\mu_t = \gamma \mu_{t-1} + \varepsilon^\mu_t.$$  \hspace{1cm} (9.43)

Non-competitive labor supply shock in the wage mark-up:
\[ k_t = \rho k_{t-1} + \varepsilon^k_t. \] (9.44)

Changes in economic policies (political economics):

\[ d_t = d_{t-1} + \varepsilon^d_t. \] (9.45)

➢ Steps:

1- Isolate \( p_t + \theta_t \) from A.2

\[ p_t = \mu_t + w_t - \theta_t + \beta u_t \]

\[ \Rightarrow p_t + \theta_t = \mu_t + w_t + \beta u_t \] (9.46)

2- Substitute 2.A.10 in A.5

\[ w_t = E_{t-1} (\mu_t + w_t + \beta u_t) + k_t - \sigma E_{t-1} u_t \]

\[ \Rightarrow \sigma E_{t-1} u_t - E_{t-1} (\beta u_t) = \mu_t + k_t + E_{t-1} (w_t) - w_t. \]

\[ \Rightarrow (\sigma - \beta) E_{t-1} u_t = \mu_t + k_t \]

\[ \Rightarrow E_{t-1} u_t = \frac{\mu_t}{\sigma - \beta} k_t \] (9.47)

3- Substitute 2.A.1.2 in 2.A.3.

\[ l_t = \alpha E_{t-1} (w_t - (\mu_t + w_t - \theta_t + \beta u_t) - \theta_t) + t_t \]

\[ \Rightarrow l_t = \alpha E_{t-1} (-\mu_t + \theta_t - \beta u_t) + t_t \]

\[ \Rightarrow l_t = \alpha E_{t-1} (-\mu_t - \beta u_t) + t_t \]

\[ \Rightarrow l_t = -\mu_t \alpha E_{t-1} - \alpha \beta (E_{t-1} u_t) + t_t \]

\[ \Rightarrow l_t = -\mu_t \alpha E_{t-1} - \alpha \beta \left( \frac{\mu_t}{\sigma - \beta} k_t \right) + t_t. \] (9.48)

4- Isolate employment from 2.A.1 and substitute \( y_t \) according to 2.A.0:

\[ y_t = n_t + \theta_t \]

\[ \Rightarrow n_t = y_t - \theta_t \]

\[ \Rightarrow n_t = (\emptyset (d_t - p_t) + a \theta_t) - \theta_t \] (9.49)
5- Then we substitute $p_t$ according to 2.A.2 in 2.A.13 and iterate:

\[ n_t = (\emptyset (d_t - \mu_t + w_t - \theta_t + \beta u_t + a\theta_t) - \theta_t \]

\[ n_t = (\emptyset d_t - \emptyset \mu_t - \emptyset w_t - \emptyset \theta_t - \emptyset \beta u_t + a\theta_t) - \theta_t \]

\[ n_t = \emptyset d_t - \emptyset \mu_t - \emptyset w_t - \emptyset \beta u_t + (a + \emptyset - 1) \theta_t \quad (9.50) \]

6- Substitute 2.A.14 in 2.A.5:

\[ u_t = l_t - (\emptyset d_t - \emptyset \mu_t - \emptyset w_t - \emptyset \beta u_t + (a + \emptyset - 1) \theta_t) \]

\[ \rightarrow u_t = \left( t_t - \alpha E_{t-1} \mu_t - \alpha \beta \left( \frac{\mu_t}{\sigma - \beta} k_t \right) \right) - \emptyset d_t + \emptyset \mu_t + \emptyset w_t - (a + \emptyset - 1) \theta_t \]

\[ \rightarrow (1 - \emptyset \beta) u_t - \left( \alpha \mu_t + \alpha \beta \left( \frac{\mu_t}{\sigma - \beta} k_t \right) \right) = t_t - \emptyset d_t + \emptyset \mu_t + \emptyset w_t - (a + \emptyset - 1) \theta_t \]

\[ \rightarrow u_t = -\frac{1}{(1-\emptyset \beta)}(\alpha \mu_t (\sigma - \beta) + \alpha \beta \mu_t k_t) + \frac{1}{(1-\emptyset \beta)}(t_t - \emptyset d_t + \emptyset \mu_t + \emptyset w_t - (a + \emptyset - 1) \theta_t) \quad (9.51) \]

7- Include shocks transforming $t_t, \mu_t, \theta_t$ and $k_t$ into $t_{t-1}, \mu_{t-1}, \theta_{t-1}$ and $k_{t-1}$:

\[ \rightarrow u_t(\downarrow) = \frac{1}{(1-\emptyset \beta)} \left[ -\alpha \mu_t(1) \left( \sigma - \beta \right) + \left( \beta k_t(1) \right) \right] \]

\[ + \left[ (t_{t-1} + \epsilon_t^1) - \emptyset (d_{t-1} + \epsilon_t^d) + \emptyset ((\gamma \mu_{t-1} + \epsilon_t^{\mu}) + \emptyset w_t - (a + \emptyset - 1)(\theta_{t-1} + \epsilon_t^s) \right] \quad (9.52) \]

8- Eliminate variables at period $t - 1$:

\[ \rightarrow u_t = \frac{1}{(1-\emptyset \beta)} \left[ (-\alpha \mu_t(\sigma - \beta) + \alpha \beta \mu_t k_t) + (\epsilon_t^1 - \emptyset \epsilon_t^d + \emptyset \epsilon_t^{\mu} + \emptyset w_t - (a + \emptyset - 1) \epsilon_t^s) \right] \quad (9.53) \]

16 If the mark-up increase, unemployment also does.

9.3. The Keynesian Theory of Unemployment.

A) New-Keynesian Model with unemployment

- Part 1: Presentation
To get started, this author defines a simulation for the United States economy, first of all the real wage equation equaled to a new conception of marginal product of labor:

\[
\frac{w_t(i)}{p_t} = \chi_t C_t L_t(i)^\alpha
\]  

(9.154)

Transforming into logarithms:

\[
\omega_t - p_t = \delta_t + c_t + \alpha l_t
\]  

(9.55)

This equation says workers will work if and only if real wage is not lower than his disutility of labor. This disutility is expressed in terms of labor supply shocks, consumption, and multiplied by the labor force with a marginal supplier of type \( i \) labor. In the equation above, we have that \( \delta_t \equiv \log \chi_t, c_t \equiv C_t \) and \( \alpha l_t \equiv \log L_t(i)^\alpha \).

Taking (2.10) as point of reference, the average wage mark-up includes the new marginal product of labor in the classical equation getting:

\[
\mu_t^\omega \equiv (\omega_t - p_t) - (c_t + \varphi n_t + \delta_t)
\]  

(9.56)

Assuming \( u_t = l_t - n_t \) like in the classical case it automatically implies:

\[
u_t = \frac{\mu_t^\omega}{\varphi}
\]  

(9.57)

In the absence of nominal wage rigidities, we obtain the natural rate of unemployment.

\[
u^n = \frac{\mu^\omega}{\varphi}
\]  

(9.58)

If there is market power in the labor market, the mark-up is positive and so that exist unemployment. Whereas in before unemployment is caused by wage rigidities, now is due to the changes in the wage mark-up.

In terms of inflation we obtain the so-called “New Keynesian wage Philips curve:

\[
\pi_t^\omega = \beta E_t\{\pi_{t+1}^\omega\} - \lambda_\omega \alpha (u_t - u^n).
\]  

(9.59)
This equation indicates that wage inflation is subject to current and expected future unemployment rates apart from future inflation as seen in before. We face again the importance of expectations in these models. We call it also the wage inflation equation.

In terms of prices, the novelty lies in the behavior of firms. They set different prices of their goods in any period of time. To remark this circumstance, independent probabilities across firms are required. The new logarithmic price equation has this look:

\[ p_t = \theta_t p_{t-1} + (1 - \theta_t)p_t^* \]  \hspace{1cm} (9.60)

Where \( p_t^* \), is the new price set by firms after adjusting \( p_t \) and \( \theta_t \) is the probability that a firm goes for a reset in the prices.

This new price \( p_t^* \), evolves from the classical equation (\( p_t^* = \delta_t \mu^p \)) and includes the probabilities of (2.15) to finally acquire:

\[ p_t^* = \mu^p + (1 - \beta \theta_p) \sum_{k=0}^{\infty} (\beta \theta_p)^k E_t \{ \delta_{t+k} \} \]  \hspace{1cm} (9.61)

being \( \delta_{t+k} = \frac{W_t}{F_L} \) using a Cobb-Douglas production function with only labor.

In other words, it points that firms decide to set the prices in each period adding a mark-up over current and future average marginal costs. But we also assume that not all the firms set the same level of mark-up and there are some of them that could potentially not establish anyone. That´s one of the major remarks of Galí from now onwards.

Compared with the previous inflation, this time is driven by current and expected average price markups (\( \mu_t^p \)) and desired markups (\( \mu^p \)):

\[ \pi_t^p = \beta E_t \{ \pi_{t+1}^p \} - \lambda_p (\mu_t^p - \mu^p) \]  \hspace{1cm} (9.62)

- Part 2. Proof
It comes from the fisher equation that defines real interest rate, \( r_t = i_t - E_t\{\pi_{t+1}\} \). In short:

\[
y_t = E_t\{y_{t+1}\} + \rho - r_t.
\]

\[
\rightarrow y_t = E_t\{y_{t+1}\} - (i_t - E_t\{\pi_{t+1}\} - \rho)
\]

It is to say, the expected output for the next period plus the difference between the time discount rate and the real interest rate.

- Price inflation equation:

\[
\hat{\mu}_t^P \equiv p_t - \frac{W}{mpn} - \mu^p
\]

\[
\rightarrow \hat{\mu}_t^P \equiv p_t - \frac{\omega}{p} \frac{p}{(1 - \alpha) \frac{N}{n}} - \mu^p
\]

Log-linearizing:

\[
\rightarrow \hat{\mu}_t^P \equiv p_t - [(\omega - p) - (\log(1 - \alpha) + y - n)] - \mu^p
\]

\[
\rightarrow \mu_t^P \equiv \log(1 - \alpha) + y_t - n_t - \omega_t - \mu^p
\]

\[
\rightarrow \hat{\mu}_t^P \equiv -\left(\frac{\alpha}{1 - \alpha}\right)(\bar{y}_t - \bar{\omega}_t).
\]

If we sum the price mark-up to the inflation expectations we have:

\[
\pi_t^P = \beta E_t\{\pi_{t+1}^P\} + \lambda_p \frac{1}{1 - \alpha}(\bar{y}_t + \bar{\omega}_t).
\]

- Wage inflation equation:

We first have

\[
\hat{\mu}_t^\omega \equiv \mu_t^\omega - \mu^\omega.
\]

If we substitute \( \mu_t^\omega \) (8.3) into this equation and assume \( c_t = y_t \) we obtain (in logarithms):
\[
\mu_1^\omega \equiv (\omega_t - (y_t + \varphi n_t + \delta_t)) - \mu^\omega.
\]

\[
\mu_2^\omega \equiv \tilde{\omega}_t - \left(1 + \frac{\varphi}{1-\alpha}\right) \tilde{y}_t.
\]  (9.67)

Now we add again the wage markup in the wage inflation equation (8.6):

As we have that:

\[
\mu_1^\omega = \varphi u_t.
\]  (9.68)

The new wage inflation equation can be written as:

\[
\pi_t^\omega = \beta E_t\{\pi_{t+1}^\omega\} - \lambda_\omega \varphi (\mu_1^\omega - \mu^\omega).
\]  (9.69)

If we add (8.15) in (8.17) we finally obtain:

\[
\pi_t^\omega = \beta E_t\{\pi_{t+1}^\omega\} + \lambda_\omega \frac{\varphi}{1-\alpha} (\tilde{y}_t - \tilde{\omega}_t).
\]  (9.70)

-Unemployment:

In this case we just isolate \(u_t\) from (8.16) and substitute (8.15):

\[
\hat{u}_t = \frac{\tilde{\omega}_t - (1 + \frac{\varphi}{1-\alpha}) \frac{\varphi}{1-\alpha}}{\varphi}.
\]  (9.71)

B) Labor equation.

The new labor demand is rewritten in terms of employment (L) from their new equation of production or output \(Y_t\) omitting factor of production K (but practically the same as 1.1).

\[
Y_t = A_t \bar{K}_t L_t^{1-\alpha} 17
\]  (9.72)

\[17\] We keep using Cobb-Douglas since is the most extended function of production and the one used in the classical theory which let us to better compare among both theories.
If we isolate $L$ from (1.10) we obtain a similar expression to (1.4) and (A.1.19):

$$L_t = \left[ \frac{Y_A}{A} \right]^{\frac{1}{1-\alpha}}$$

(9.73)

From this point we could keep going iterating this equation and equaling it to the real and proceed like in the previous chapter. However, in this case I would like to develop this equation a little bit more focusing on parameter $Y$.

From the AS-AD model (aggregate supply – aggregate demand) we extract a preliminary demand equation expressed like this:

$$Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \varepsilon_t.$$  

(9.74)

Where $Y_t$, is the total production of goods and services, $\bar{Y}_t$, the natural level of production, $r_t$, the real interest rate, $\alpha (> 0)$, the grade of sensitivity of the demand toward variations of the interest rate, $\rho (> 0)$, is the natural interest rate where the demand of goods and services equals the level of natural production and finally $\varepsilon_t$, is a random variable that measures exogenous shocks in the demand.

The real interest rate has a negative relationship with production because the higher the interest is, the more expensive the loans are which implies less inversion engaged by firms and less consumption by individuals. It affects at the end the level of output.

Additionally we have the fisher equation that poses that the real interest rate equals the nominal interest rate minus inflation. Considering two time periods $(t, t + 1)$ and expectation $(E)$, very important in new-Keynesian model, it is expressed like:

$$r_t = i_t - E_t \pi_{t+1}.$$  

(9.75)

If we substitute (1.14) and (1.13) in (1.12) the new employment equation is redefined in a more sophisticated one:
\[ L_t = \left[ \frac{\bar{Y}_{t} - \alpha (i_t - \pi_{t+1}, t - \rho) + \varepsilon_t}{\bar{A}} \right]^{\frac{1}{1-\alpha}}. \quad (9.76) \]

It states the interest rate affects the level of production and in the second instance labor demand. In other words, is demonstrated the relevance of monetary policies in the fixation of the labor demand.

The most relevant point here is that classical dichotomy poses real interest rate and natural interest rate do not differ \((r_t = \rho)\) and tend to be the same in the long run. Under this assumption \(Y_t = \bar{Y}_t\) which refuses the efficacy of monetary policies on employment.

9.4 - Econometric contrasts

- Employment equation\(^{18}\)

-Homoscedasticity (White method):

\(H_0\): Homoscedasticity  
\(H_1\): No Homoscedasticity.

\[ \text{Estadístico de contraste: } TR^2 = 21.068295, \]
\[ \text{con valor } p = P(\text{Chi-cuadrado}(14) > 21.068295) = 0.099895 \]

\[ \Rightarrow \text{The likelihood to refuse the hypothesis } H_0 \text{ of Heteroscedasticity is too high (≈10%). Therefore, there is Homoscedasticity #.} \]

-Multicollinearity (VIF\(^{19}\) method):

\[ \text{Mínimo valor posible = 1.0} \]
\[ \text{Valores mayores que 10.0 pueden indicar un problema de colinealidad} \]

\[
\begin{align*}
L_{n2} & \quad 28.118 \\
L_{n2var} & \quad 2.892 \\
L_k & \quad 103.171 \\
L_L_{t,i} & \quad 2.836 \\
L_s_{t,i} & \quad 4.162 
\end{align*}
\]

\(^{18}\)The autocorrelation cannot be measured in this equation via Gretl because some observations are missing because the data relative to the interest rate is shorter.  
\(^{19}\)VIF: Variance inflation factor.
With the exception of the interest rate, the rest of the variable have collinearity (VIF>10). We face the problem of collinearity.

- Wage Employment

-Homoscedasticity (White method):

- Autocorrelation (Breush-Godfrey method)

*Durbin- Watson method is not conclusive.

-Multicollinearity (VIF method):

With the exception of the employment and its variation, the rest of the variable have collinearity (VIF>10). We face the problem of collinearity.