Extra-mathematical connections: Connecting mathematics and real world

A practical application in a 7-8 year olds class

Núria Sabaté Quirós

nuria.sabateq@e-campus.uab.cat

Research mentor: Genaro de Gamboa

Universitat Autònoma de Barcelona

14 June 2016

Abstract: The learning of mathematics has been during many years isolated and separated from the real world. It has caused a lack of motivation and the recurring *"why am I learning this?"* question among many students. The approach based on extra-mathematical connections tries to reduce the separation between mathematics and reality in order to give learning meaning, as well as to make sense of the world through a mathematical perspective and foster the symbiotic relationship among these two worlds. This paper describes the process of design of a teaching unit based on extra-mathematical connections and analyses the learning opportunities that arise during its implementation in a second graders class in Primary Education.

Resum: *L'aprenentatge de les matemàtiques ha estat durant molts anys aïllat i desvinculat del món real, fet que ha provocat la desmotivació i la recurrent pregunta de "això per a què ho aprenc?" en molts estudiants. L'enfocament basat en les connexions extra-matemàtiques intenta reduir aquesta desvinculació de les matemàtiques i la realitat per tal de donar sentit tant a l'aprenentatge com al món que ens en volta a través d'una mirada matemàtica i afavorir el lligam simbiòtic entre tots dos mons. Aquesta recerca descriu el procés de disseny d'una seqüència didàctica basada en les connexions extra-matemàtiques i analitza les oportunitats d'aprenentatge que sorgeixen durant la seva implementació en una aula de segon de primària.*

Key words: mathematical connections, extra-mathematical, intra-mathematical, learning opportunities, real world.

Introduction

The extra-mathematical connections based approach tries to diminish the gap between mathematics and real world in classroom practises by searching for deep and necessary links between these two worlds. The National Council of Teachers of Mathematics, in the Common Core State Standards (2010), stresses the importance of establishing extra-mathematical connections in Primary Education settings to avoid the isolation of mathematics, especially when problem solving by stating that students should take into account the context from which the problem arose to find solutions and arguments, which illustrates the relationship mathematics-reality. The learning opportunities arisen from these connections enable the learning of both mathematics and world understanding.

Researchers have highlighted the importance of establishing extra-mathematical connections and suggested that usually, in mathematics classroom practices, the problem solving presented creates artificial contexts with only the necessary data to solve it regardless any other external factor (Gainsburg, 2008). Extra-mathematical connections are not made-up relations between mathematics and reality, but are deep, needed and real links that make the real world be understood through mathematics and at the same time, mathematics be better understood through reality. This pairing *mathematics-real world* is the basis for this research.

This paper analyses how extra-mathematical connections are established in a teaching unit implemented in a second grade class and what are the learning opportunities linked to them. The research seeks for deep extra-mathematical connections which arise learning opportunities for the students. It is important that the extra mathematical situation enables the understanding of the mathematical contents, which in this teaching unit is measurement and two-dimensional shapes' properties; and also that the mathematical learning helps students to better understand the external situation from which the problem arose. This situation is in this case the different starting positions in athletic races.

Results of this research led to understand which the important characteristics of classroom practises are to foster extra-mathematical connections and how they are created and used as learning opportunities.

The paper first presents the theoretical framework in which this research is based on, followed by the methodology used to implement and analyse the teaching unit and finally a deep analysis and discussion of the mathematical connections established during the development of the teaching practice.

Theoretical framework

Different contemporary researches stress the importance of keeping mathematics in contact with the real world by establishing extra-mathematical connections so that mathematics are not understood as an isolated area of knowledge but as part of everyday life and make actual sense to students (Cinzia, 2001; Gainsburg, 2008; Harvey & Averill, 2012; Verschaffel, 1997). Therefore, mathematical activities in classrooms should promote the process of mathematical modelling which, according to De Corte, Verschaffel and Greer (2000), builds “the bridge between mathematics as a set of tools for describing aspects of the real world, on the one hand, and mathematics as the analysis of abstract structures, on the other”. Mathematics should then be always related to the real world as well as the real world should be related to mathematics. Blum and Borromeo (2009) describe mathematical modeling as a bidirectional translation between the real world and mathematics.

Approaches based on extra-mathematical connections try to avoid the disconnection of both mathematic and real worlds and the artificial and separated from reality problems (Verschaffel, 1997) by working through contexts which satisfy the two following characteristics:

1. The context/situation cannot be understood without mathematics.
2. Realistic considerations need to be taken into account besides mathematics.

The first characteristic refers to the unconditional relationship between mathematics and the situation. Mathematical contents are essential for the understanding of the situation, and therefore, we ask questions about situations which will be answered through mathematics. They also need to be specific and unique for this situation so that the relation is close and rich. However, it does not mean that the learning outcomes cannot be extrapolated to other contexts.

With respect to the second feature, activities should foster the reflection on other aspects related to the context studied which could influence the processes or results. These influences could be related to both socio-cultural characteristics of the context and other areas of knowledge (Cinza, 2001). This is what Borromeo (2006) supports as the interpretation of the results by stating that mathematical results should be interpreted in relation to the context to determine if it can be considered as a real result.

Intra-mathematical connections are another type of mathematical connections that need to be considered. In this case, they do not relate mathematics to the real world but mathematics among themselves. Whereas the extra-mathematical connections are considered horizontal mathematising, intra-mathematical connections are considered vertical mathematisation (Kasier & Sriraman, 2006). In other words, these connections are a link from mathematical concepts to other mathematical concepts. Through intra-mathematical connections, mathematical knowledge is built since already known concepts are linked among them or to new concepts.

Both types of mathematical connections are interrelated. On the one hand, extra-mathematical connections foster the establishment of intra-mathematical connections to develop the mathematical analysis; on the other hand, intra-mathematical connections help to understand the extra-mathematical connections in a better way. Rossi and Rivera (2006) highlight the importance of both vertical and horizontal mathematization activities starting first from “real life situations or mathematical problems that are experientially real to students to engage them in activities that focus on vertical mathematization” to connect mathematical contents.

Methodology

This research has been carried out as part of a Final Degree Project on the Primary Education Teaching Degree for the Universitat Autònoma de Barcelona. The classroom experience was developed in *Els Arenys* primary school in Castellbisbal with one of the two 2nd year classes.

The target group was a 25-students class (15 girls and 10 boys) aged 7-8 years old, with a general average-high level in the mathematics area, according to the experienced classroom teacher. There were four students who have got an individualized plan regarding language, and three students who stand out in mathematical abilities. The individualized plans include methodological adaptations with regards to how information is given to the students, external support from specialists and content adaptation with regards to written language. It is important to highlight that despite the individualized plans are focused on the language area, they needed to be considered for the implementation of the lesson plan in terms of communication. Oral language was used over written language to ensure the understanding of these students. The group is used to work in cooperative groups since it has been the base for their learning throughout the initial cycle. This is a feature to consider which eased the development of the lesson plan for this research.

The lesson plan was implemented by the researcher during three 90-minutes sessions. The classroom teacher was in charge of recording the sessions as well as assisting the students when working in small groups. It was developed under the topic of athletics track races and aimed to work on three different blocks of the Mathematics Catalan Curriculum: numbering and calculus (numeració i càlcul), measure (mesura) and space and figure (espai i forma). It aimed to establish a relationship among these three contents so that students were able to create links between the mentioned blocks in regards to the situation presented. According to the contents, the objectives of the lesson plan were to identify the meaning of numbers in a real situation, to use different procedures to measure and compare distances, to analyze complex shapes and to describe the properties of shapes. All of these objectives laid under the main goal of connecting the situation presented to mathematic contents in order to understand the identified phenomena, which was the staggered starting position in some races.

The three sessions of the teaching unit followed a progression from identifying a problematic situation, through analysing and understanding it to finally applying the mathematical learning to a real situation. The first session started with presenting three videos of a 100m, 200m and 400m athletics races. From it, students could identify the staggered starting positions for the 200m and 400m races which were considered a problematic situation. After a discussion to make up hypothetical reasons for this phenomenon, in pairs they drew the starting position, the finish line and the trajectories of the different lanes of one of the three races. This activity was aimed as an introduction for the second session, which was more experimental.

During the second session, students were given a track model per group of four. They needed to measure with wool yarn the distances that each athlete would run if they all started on the same line and then compare them. They found out that the distances were not the same and looking at the shape of the track and at the different lengths of the yarns, they made sense of the staggered starting positions.

Finally, in the third session, they organized a race in the playground. To do so, each group was given 7 ropes which would guide the trajectory of their lane, so they all had the same distance to run. They started laying the ropes from the finish line and drawing a curve trajectory. Therefore, each group had a different staggered starting position, due to the shape of the track. This activity was the inverse procedure as the one with the model, so that students could create themselves the situation based on the mathematics they had learnt by experimenting with the model.

With regard to the data collection, two elements were used. Firstly, the sessions were fully video-recorded with one camera during the great group activities and an extra one was used when small group activities were carried out in order to collect more conversations among the students and among students and teacher. The sessions were not fully transcript but only those conversations in which there were significant and relevant mathematical conversations for the research. The criteria used for this first selection relates to the objectives of the research; only the conversations in which mathematical content or mathematical connections were identified were transcript as a first step for further analysis. Despite the research is carried out in English, the sessions were developed in Catalan to respect the school's procedures.

Secondly, students were asked to write a report at the end of each session in which they would take note of the key points of their activities and explain what they learnt during the session. They were free to use their own style, whether they wanted to write or draw.

Both transcriptions and students' reports were used as data to analyse the making of mathematical connections in the classroom and how they bring out learning opportunities during the practice.

The research has gone through three different stages: the first stage was the identification of the mathematical connections in the classroom activity; in second place it was the classification of the mathematical connection in extra-mathematical or intra-mathematical connections; and thirdly, the analysis of the learning opportunities related to the connections.

Analysis

According to the three research stages introduced above, the mathematical connections will be identified, classified and related to learning opportunities. In the following analysis, the relationships between an extra-mathematical situation and mathematical concepts which were established during the implementation of the lesson plan will be described and characterized explicitly. The different learning opportunities which were generated from each connection will also be described in relation to the initial objectives planned for the teaching unit lesson plan.

The teaching unit followed different extra mathematical connections, starting from a real situation considered a problem which led to mathematics and which went back to the problem itself after all.

Moreover, another type of mathematical connections was presented during the practise. This is the intra-mathematical connections, which will be also specified throughout the analysis and which relate mathematical contents to other mathematical contents. The intra-mathematical connections are closely related to the making of extra-mathematical connections since through them, different mathematical contents are related which is necessary to understand the relation between the real situation and mathematics.

The analysis is structured distinguishing the different connections arisen during the teaching unit and following a chronological order with which connections were established: connection of the problematic situation to mathematical contents, connection among already known mathematical contents, connection of already known mathematical contents to new ones and connection of the mathematical contents to the real world. Finally, other connections that were presented during the teaching unit but were not planned or deepen into are also analysed in the last part of the analysis.

Connection of the problematic situation to mathematical contents

The lesson plan started with a mathematical problem that the students came up with after they were shown three different videos of athletics races. The videos were specifically chosen so that the real situation was evident and so that they could identify it. In case they would have not done so, some questions about the race would have been asked to students to scaffold them on the identification of the problem. They were showed a 100m, 200m and 400m video. A girl spotted a problematic situation: "*they* *start differently. One is at the front and the other one is a little bit behind, and the other a little bit more, and the other more, and so on."* It was the starting point to all the following activities, which were aimed to understand this situation through mathematics.

The discussion about whether it was fair or not to start this way led to different interesting opinions on what lane was the best to start. Their perception was that the outer lane was the shortest because "*it is closer to the finish line because they start further in front*". Therefore, the first extra-mathematical connection was made through this observation. Distances and lengths of each lane were discussed regarding the athletics track. This related the mathematical contents of shapes and distances to the real world of athletics: the staggered starting positions on the different races are a consequence of the shape of the track; the outer the lane, the longer the perimeter, so starting from a further point equalises the distances to be covered.

A learning opportunity was arisen through this connection; students had the chance to identify the meaning of numbers in a real situation and discuss them in relation to what they meant on the distances of the races and the length of the lanes. The opportunity to identify shapes and distances on a real situation was enabled by the extra-mathematical situation of athletics races.

Connection among already known mathematical contents

This discussion connected the real situation about the starting positions in athletics to mathematical concepts already worked such as distances or shapes. The first step was to observe and try to describe the shape that an athletics track has. The decomposition of the figure into a rectangle and two semicircles was needed in order to understand the problem.

After that, they related the shape of the track to the different distances of the races. These connections were visible through some of the students' comments such as *"if the 100m is only the straight part, the 200m is one curve and a straight line and the 400m is one curve, one straight line, one curve and one straight line, then the 300m would be one curve, one straight line and one curve."* This student's comment is establishing a relationship between the complex shape of the track with the length of the races.

The second connection was then established, but in this case it is considered an intra-mathematical connection because, despite it is linked to the situation, what the student is relating is the content of shape and forms to the content of length and distances by linking each part of the track with a different shape to a certain length according to the races watched, and therefore, interpreting what the shape of an hypothetical 300m would be.

In the following conversation, a further relation is established between the starting positions and the trajectories of the different races, which is the first step to understanding the situation:

> *Student: They don't start staggered in the 100m.*
> *Teacher: In which ones do they start like this?*
> *S: In the 200 and 400m.*
> *T: And what is the difference between them?*
> *S: The 100m is only straight. The others have curves.*

At this point, students had had the learning opportunity to connect different already known mathematical contents among them and to the situation. They interpreted the shape of the track with regards to the simple figures and geometrical elements known (rectangle and circle, straight line and curves) and they also compared the different distances (shorter and longer). Connecting their knowledge about the complex shape of the track to the shapes of the races by decomposing the track into the simple figures of semi-circle and rectangle, allowed them to discriminate using mathematical knowledge which races started differently and what characteristics their trajectories had, despite not knowing yet the reasons why. Therefore, the learning opportunity of analyzing complex shapes was presented. Students had the opportunity to use and connect mathematical knowledge that they already had but putting it into the context of athletics to create a description of the races. They described the trajectories of the three different races using the terms *straight* and *curve*. This connection was essential to understand the first extra-mathematical connection already presented and it was also the starting point for the second intra-mathematical connection.

Connection of already known mathematical contents to new ones

During the experimentation with the model of an athletics track, several intra-mathematical connections were identified. It was aimed to enable the use of different procedures to measure and compare distances as a learning opportunity. The link was established between the different shapes and their exact length. Students were asked to measure the distance that three different athletes run with wool thread for the three different races, considering them all to start on the same line, as they had previously thought that it would be the "*fair"* position to start. Then they needed to compare the three threads of the same race in order to compare the distances run by each of them. They found out that only in the 100m the three threads were equally long whereas in the 200m and 400m they were different.

> *S1: We did the 100m and 200m.*
> *T: Let me see the threads...*
> *S1: The 100m's are okay, because they are all the same but... we did something wrong with the 200m...*
> *T: Why? What happened?*
> *S1: They are not the same.*
> *T: But you measured it well. Let's look at the 100m. If they are all equally long, it means that they all run... (waits for the students to answer)*
> *S2: The same!*
> *T: Exactly! Because the threads are the distance they run, and they are all equal.*
> *S2: But then they (pointing at the 200m) run different.*
> *T: Yes. Look at the threads, the yellow one (outer lane) runs more distance than the red (middle lane) and the red more than the purple (inner lane).(The teacher takes the ends of the threads to show the students the difference).*
> *S1: It's like the 400m, that they start like that (she does a staggered shape with her hands).*
> *S2: They run different.*
> *T: But if they all start on the same line and finish at the same line, how is it possible that they don't run the same? How do actual races start?*
> *S3: Staggered.*
> *T: And how did you start?*
> *S3: On the same line.*
> *T: And what happened?*
> *S2: That they finished staggered.*
> *T: And how is that possible? Is it fair? Would you agree on being the yellow one?*
> *S: No!*
> *T: Have a look at the track to see if you can find something…*
> *(The students think for a while and the teacher comes back)*
> *S1: In the 100m's the threads are the same because they don't have a curve, it's only a straight line. And then, if there is a curve, I think that they should be staggered because they are not all the same.*
> *T: Very good, but why are they not the same? What's the difference?*
> *S1: That this one is shorter (pointing at the inner lane).*

T: And then why do they start staggered?
S2: Because there are shorter and longer curves.

As this dialogue among a group of students and the teacher illustrates, by experimenting with the measures, they could actually visualize the difference of distances run by the athletes if they all start from the same line. After this identification and through the observation and manipulation of the materials, they made a connection between the different lengths of the threads with the starting positions in the 200 and 400m races. Therefore, they connected the concept of perimeter to the athletics track regarding the variable of the curves' sizes that influences the distance that each athlete needs to cover. The students concluded that the outer the lane is, the bigger the curve is, and the longer the distance to be covered.

According to Cinzia (2001), it would correspond to the third stage in which children put mathematical facts in relation, make conjectures about procedures and notice properties. From this, they understood the staggered starting positions of some specific races. In other words, they had the opportunity to explore learnt new mathematical concepts and skills aimed to understand the problem that was firstly identified through making connections with previous knowledge and building new ones.

Connection of the mathematical contents to the real world

Eventually, to apply the connections made, and as a mode of assessment, the students were asked to organize a race in the playground. Each group was given 7 equally long ropes and 2 cones. They named it the *"7-ropes race"* since they did not know how long it was. In fact, it was the inverse procedure to what they did on the track model. They had the distance but needed to establish the trajectory. Before doing it, a whole group discussion was held and they argued that if they put the ropes on a straight line, it would be *"more boring because they would all start at the same place"*, and they decided that they wanted to start staggered and that therefore, they would do a curved trajectory. This student's comment illustrates the learning opportunity that this activity enabled: to understand the properties of shapes in relation to their perimeter and its relationship with races' trajectories. They connected their knowledge about how curvilinear shapes affect the distance to cover by athletes to the real race that they were about to organize. They were given the opportunity to apply this mathematical connection between the trajectory and shape of their lanes to the organization of the real race. This is the second extra-mathematical connection, which is related to the first one but towards the other direction: from mathematical contents to a real situation. They had analyzed the situation mathematically and based on that, they discussed on how to start the race.

Once they finished setting up the ropes for the race, there were some children who were still not confident in understanding that they were all the same length as it is shown in the following transcript:

> *S: But this one is shorter and it will be easier to run.*
> *T: Let's see if it is actually shorter. Can you three please count each lane's number of ropes?*
> *S: They all have 7 ropes.*
> *S: They all run 7 ropes.*
> *T: Exactly. They will all run the same distance. They will all run the distance of 7 ropes.*
> *S: Yes, but because this lane (points at the outer lane) has a bigger curve, they ran out of ropes before. But they all run 7 ropes.*

With this activity, the connection was made the other way around, the students used the mathematics to recall and rebuild the situation that they were presented at the beginning. This dialogue tackles the mathematical concepts of perimeter, shapes and distances: all lanes are the same distance because they are all built with the same material but, because they follow a curvilinear trajectory, the outer lane's curve is longer and this is what causes the staggered positions. It was useful to consolidate the learning for those students who had already understood it with the previous activity and also to give another perspective of the problem to those who struggled to understand the issue on the model.

Therefore, besides the two extra-mathematical connections, two intra-mathematical ones were presented too. All four connections are interrelated as it will be presented in the results chapter.

Further connections presented during the teaching unit

In addition to the learning opportunities already described in relation to the connections and the initial objectives, other learning opportunities appeared regarding other mathematical contents. The concept of time and speed were arisen from the first connection during the first discussion. They were also linked to the length of the lanes. Students commented that the race was not fair because "*the ones on the shorter lanes will arrive first*". This connection that the students made between length and time (in relation to arrival positions) could have been a learning opportunity to introduce the concept of speed. Nevertheless, due to the fact that the research time was limited, no advantage was taken of these opportunities.

Moreover, during the first discussion, other learning opportunities related to other areas of knowledge appeared. The concept of fairness with regards to advantages and disadvantages and equality and equity, could have also been discussed, as well as all contents from the Physical Education area in relation to athletics.

Results

The analysis has highlighted the four connections which were established during the implementation of the lesson plan which belonged to two different groups: extra-mathematical and intra-mathematical connections. The chronological order in which they appeared has a close relation to the processes of mathematical modelling on which the lesson plan was based. This is, firstly, the extra-mathematical connection from reality to mathematics for the process of understanding the problematic situation of the staggered positions in athletics races; secondly, the two intra-mathematical connections for the processes of modelling, analysis and interpretation of the different elements that affect/cause such positions; and finally, extra-mathematical from mathematics to reality for the evaluation and communication of both the mathematical learning about shapes and distances and the learning about athletics races.

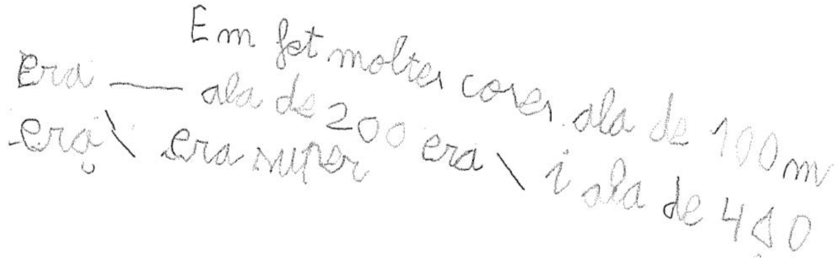
Both types of connections are as well linked among them. Extra-mathematical connections need from the intra-mathematical connections to be fully achieved. In other words, extra-mathematical connections require other mathematics to be developed and this requirement is what enables and makes richer the learning of mathematics through extra-mathematical connections. To illustrate, the first extra-mathematical connection regarded the distances and the length of the track and the first intra-mathematical connection was related to it since it used the ideas of lengths of the track and related them with the shapes of the tracks. Therefore, the second connection was linked to the first one since it took into account the observations from the first one to and related it to other contents.

Mathematical connections are not isolated phenomena, and constant referring to them will avoid the disconnection between mathematics and real world and so that students keep in mind which are the reasons for the mathematical learning and make sense of what they are doing. In the analysis, it can be seen that the establishment of connections allowed the students to produce explicit and significant relations between academic mathematics and an external situation, and therefore, the isolation of mathematics was avoided.

Besides the analysed connections, other connections were arisen by the students or by the activities. An intra-mathematical connection between the concepts of distance and time was brought up through, which could have derived to talking about the concept of speed. When students described an athlete as the "*best*", or as "*this one will go faster"* they were referring to the concept of speed. Therefore, despite not being the aim of the research, the real situation led to create other mathematical connections.

Finally, most of the connections established were the result of dialogues between both teacher-student and student-student. As seen in the transcripts, the use of mathematical questions from the teacher, encouraged students to reflect about the situation considering both mathematics and real world, and therefore, to establish connections. For instance, the question "*is it fair that they do not all have to run the same distance?*" made students think about distances taking into consideration the context of a race.

In relation to the communication, the students' communicative skills also affect the way in which they do the connections or in which they explain them. In this case, as it was a second grade class, students did not have all the specific vocabulary to explain the mathematical concepts they were relating. Alternative ways of explaining the connections they have made can be used to enable the process of reporting the connections. Therefore, to write the reports, students could use both writing and drawing. In the following example, the student chose to use the written language but realised that she did not have the words to explain it, so she included symbolic drawing to it.



Student's report: "in the 100m, it was ---, in the 200 it was / and in the 400 like /super"

As the report illustrates, the student's writing and communicative skills had an influence on how she explained the observations about the races' starting points. She may not had been able to explain it if only written language had been allowed to use, and the connection report would not have been completed.

Discussion

The aim of the research was to practically analyse how extra-mathematical connections can be used in the teaching and learning of mathematics and to analyse how learning opportunities arise from them. The lesson plan implemented in a 7-8 year olds class was analysed with regards to connections and two types were identified: extra and intra-mathematical connections.

Three main different specific characteristics from the implemented lesson plan can be used to conclude which features are important for enriching classroom activities to have and to promote the learning through mathematical connections. In first place, an activity based on extra-mathematical connections should have a real situation as a starting point. The real problem, which in this case was the starting position for real athletics races, made students understand why it was important to study and learn the concepts presented. It made sense for them as they could see a real application and did not have the feeling of learning something which is senseless. It is the starting point for connecting mathematics with real world. Therefore, the real situation is used as an artefact to introduce the mathematical problem and to arise the first extra-mathematical connection.

This aspect is much related to the second feature to be taken into account: the explicit objective or outcome. Students should be aware that what they are learning is worth it; in this case, the application in real life would be the reason why. The organization of the race was always in their minds and pushed them to work in order to achieve it. The presence of the real objective will indeed foster connections between the real world and mathematics and will allow students to create a sense of why mathematics are important to understand the world and become more competent citizens. It led to the second extra-mathematical connection. If there is not a real outcome, the extra-mathematical connection from mathematics to real world is more difficult to achieve, as there is no application of the mathematical learning.

The third main feature that will enrich mathematical classroom activities with regards to connections is the focus on competences. Being competent implies using knowledge and skills to solve real life situations, and that is what extra-mathematical connections are based on: combining what is usually understood as academic learning with reality to solve problems. With respect to this focus on competences rather than in academic content, the classroom teacher of the target group, after the lesson plan stated "*some students who usually do not stand out, did. In contrast to other students who are, academically speaking, very good who struggled to find reasons to explain the results*." This regards the objective of this activity, to understand the world through mathematics rather than just learning isolated mathematics or mathematics for an artificially created world which are not deepen related.

It is important to highlight that, despite the analysis has been focused on mathematical connections, I believe that they could also be considered and extrapolated to other school areas to enrich their activities. For instance, Akınoğlu and Özkardeş (2007) support in their article the positive effects of teaching the natural science concept of movement of matter through "real scenarios and problems chosen out of the real world in connection with daily life".

Reflecting on the implementation of the lesson plan, some aspects could have been improved in order to take better advantage of the learning opportunities. In first place, at the end of each lesson, the students were asked to do a report on what they had learnt during that lesson so that they could keep it and recall it at the next lesson since it is a part of the mathematical modelling process. However, I found out that some of the students did not have enough writing or drawing skills to communicate what they had learnt. Therefore, an improvement would be to provide them with scaffolding to do it or to find another way of communicating, such as recording a video in which they would explain their learning.

Furthermore, another element which could be improved is the description of the phenomena as a generalization. During the lesson plan only the phenomena in the athletics starting positions was analysed, but it was not applied or used in any other content. In order to make students aware that it was a mathematical phenomena and that it can also take place in other situations, a consolidation activity could have been done. The phenomena could have been analysed and described regarding the mathematical contents of perimeter and shape using other shapes and perimeters to see how it varies depending of the shape and discuss if it could also work in other contents.

The research has some limitations which need to be considered in terms of the methodology used. Firstly, the lesson plan was developed in just one target group and planned and adapted considering their specific needs. Therefore, despite the results could be extrapolated to other groups, considerations with regards to the group characteristics must be taken into account. Secondly, due to the fact that it was an isolated lesson plan implemented in the class for the research, it is unknown if the students consider extra-mathematical connections on their permanent and continuous learning. Another issue was the limited time and resources for the research, as well as the limited experience as a teacher of the researcher. This means that some learning opportunities could not be deepen into or expanded.

It is important to take into account that students or the activities themselves may foster the establishment of other connections at different points of the process and they should also be considered, as other learning opportunities may arise regarding other contents or other areas of knowledge. In a context not as limited as this research, these other mathematical connections could be taken more advantage and developed in depth.

Communication and interaction between the teacher and the students and also among students plays an important role with regards to the development of connections. Since most connections will be the fruit of conversations, teachers should be sharp when asking questions or making comments to foster these connections. Furthermore, teachers should also pay attention and listen actively to children’s interactions to spot and remark interesting comments or connections that the students may deal with during their conversations.

Linked to this research, further investigation can be done in the field of mathematical connections. The role that extra-mathematical connections play with regards to motivation and interests for mathematics learning in students could be a possible further research, comparing how interested students are in mathematics before and after a period of being taught on extra-mathematical connections. Moreover, how this connection-based approach in mathematics influences the students’ connections to real world when dealing with other areas of knowledge could also be a linked research.

Acknowledgements

Thank you to *Els* Arenys School for allowing me to carry on the research.

Thank you to all the students in the *Archeologists* class for taking part in the teaching unit.

Thank you to Maite, the class teacher, for making me become part of the class.

Thank you to Genaro, my mentor through this research, for guiding and giving me his best advice.

References

Akınoğlu, O & Özkardeş, R. (2007). The Effects of Problem-Based Active Learning in Science Education on Students' Academic Achievement, Attitude and Concept Learning. *Eurasia Journal of Mathematics, Science & Technology Education*, Vol 3(No 1), 71-81 .

Blum, W & Borromeo, R. (2009). Mathematical Modelling: Can It Be Taught And Learnt?. *Journal of Mathematical Modelling and Application* , Vol 1(No 1), 45-58.

Borromeo, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM*, Vol 38(No 2), 86-95.

Cinzia, P. (2001). How to connect school mathematics with students' out-of-school knowledge. *ZDM*, Vol 33(No 3), 75-84.

Council of Chief State School Officers [CCSSO] & National Governors Association [NGA]. (2010). Common Core State Standards for Mathematics. Retrieved from http://www.nctm.org/uploadedFiles/Standards_and_Positions/Common_Core_State_Standards/Math_Standards.pdf

Departament d'ensenyament de la Generalitat de Catalunya (2015). DECRET 119/2015, de 23 de juny, d'ordenació dels ensenyaments de l'educació primària. Retrieved from: http://portaldogc.gencat.cat/utilsEADOP/PDF/6900/1431926.pdf

De Corte E, Verschaffel L, Greer B (2000). Connecting mathematics problem solving to the real world. . International Conference on Mathematics for Living, Amman, Jordan, November 18-23.

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *J Math Teacher Educ*, Vol 11(No 1), 199–219.

Harvey, R & Averill, R. (2012). A lesson based on the use of contexts: an example of effective practice in secondary school mathematics. *Mathematics teacher education and development*, Vol 14(No 1), 41-59.

Kaiser, G & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM*, Vol 38(No 3), 302-310.

Rossi, J & Rivera, F. (2006). Sixth graders' figural and numerical strategies for generalizing patterns in algebra (1). *PME-NA Proceedings Algebraic Thinking*, Vol 2 (No 96), .

Verschaffel, L. (1997). Teaching realistic mathematical modeling in the elementary school A teaching experiment with fifth graders. *Journal for research in mathematics education*, Vol 28(No 5), 113-120.