Universitat Autònoma de Barcelona

INCENTIVES FOR UNIVERSITY STUDENTS: A THEORETICAL MODEL

MARCEL BOIXADER TOMÁS

GRAU EN ECONOMIA

TUTOR: AMEDEO PIOLATTO

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Abstract

The goal of this project is to analyse if, using economic analysis tools, it's feasible to consider an incentive scheme for students and universities in which universities would discount an amount from the tuition in exchange for strong academic results, and the idea is to analyse if said universities could benefit from something like this. Even though it is set in a completely theoretical framework, it would be interesting to see further research done based on this idea after having collected some data regarding tuition programmes and building a regression model to try and estimate something similar to what in this model is referred to as α and β.

When looking into it in a real world, it would probably have more application in students who are postgrads, rather than graduate students, since that's the academic research that it is thought could benefit a university. Regardless, publications are just used as an idea since it's what public universities use nowadays to measure "output" when evaluating professors and candidates.

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0. PRELUDE

This project has been written whilst in the middle of a "what should I do next year" crisis. Having been a university student for quite a while, it was kind of a big issue I had to deal with: should I apply for a master's and keep living off of my parents as a student? Should I take a break and get a job? Should I travel the world? The biggest issue I had (and still have) with staying as a student is money. Obviously, earning higher education brings future wealth and more happiness than one could ever hope for (or so I've been told), but as of now what it means is asking for student loans and living precariously or having a part-time job (in my case, teaching English) and having some money but also living precariously because, in that case, you don't get to sleep.

So there I was, happily awake in the middle of the night, trying to decide the topic on my project and what to do next year… magically, it all came together. "Hey, what if I find a formal way to convince universities to pay an average student like myself to go and study there? I could promise them I'll try my best. But how will they verify it, you ask? Well, they can't, but I can tell them exactly what I should be offered to accept and work hard." And since information economics has always been one of my interests, it was kind of obvious what I needed to do.

Special thanks

I would like to thank my project supervisor, Amedeo Piolatto, for guiding me through the steps of writing this Bachelor Project, and for his quick response to all of my requests, and for attempting to teach me how to use LaTex: even though I did not succeed, I will for sure learn how to use it in the near future, when time constraint isn't an issue.

I would also like to thank my father, Dionís Boixader, for helping me plot the graphs that appear in this Bachelor Project.

Marcel

1. INTRODUCTION

1.1. Motivation and goals

The reason for which this topic has been chosen is a mix of genuine academic interest and the personal situation that aligned with the realization of this project.

In an increasingly competitive world, education plays a crucial part in human development and general welfare improvement. Given the increase in universities and the common assumption that young people should study as much as possible and for as long as they can afford, it is certainly an area that deserves analysis and dedication from the academic community.

Although traditionally university has been mostly available to those born in higher income households, this is now less true in most developed countries. However, studying does not usually pay for itself at the moment of doing so: one studies with the goal of earning future wealth, but it comes with the sacrifice of present income. That is the reason for which a study such as the one exposed in this bachelor thesis makes sense: Could subsidizing university studies be a way of benefitting both parties involved? Do asymmetries of information change the answer to that first question?

1.2. References and previous work

The first literature that was read were papers written by some of the pioneers in the field of economics of information, such as Stigler (1961), Akerlof (1970), Arnott and Stiglitz (1990), Holmstrom (1982), and Holmstrom and Milgrom (1991). Once the idea for the project was decided, some digging through jstor.org[1] and sciencedirect.com[2] showed that vast research in this field already exists. According to Greenaway and Haynes (2003) the number of universities in the UK tripled between 1960 and 2000, while the number of students went from around 400,000 to 2,000,000. Besides that, universities followed a privatization trend: do these factors affect universities' behaviour? Do they have access to better quality students?

In a similar line of empirical studies, but in the USA in this specific case, Dynarski (2002) analyses the effect price discrimination in university tuition has on welfare and university efficiency. It is also mentioned that universities assign scholarships, although those are

[1] http://jstor.org
[2] http://sciencedirect.com

based not only on academic merit but also on athletic ability (college sport is a big industry in the USA) and others. However, American universities are mostly private, so one could argue that they're run in a more "profit-oriented" way, which is the reason for which sports scholarships and others are such a big deal.

In a more theoretical framework, and more in the likes of this project, Rothschild and White (1995) set a basic problem with a model that describes the students' as the universities input and the human capital it generates as an output. In their paper, a case of symmetric and complete information is analysed, suggesting further research that takes informational asymmetries, which are a reality, into account. This was followed up by Gary-Bobo and Trannoy (2008), who wonder why would universities set admission requirements and tuition fees at the same time since that seems counterintuitive.

An ideal model, that was well out of the authors reach due to its complexity and the time constriction to which this project is submitted, would have been derived from the open suggestion made by Rothschild and White (1995), Gary-Bobo and Trannoy (2008) that would take into account family income and skill premium such as was done in De Fraja (2009). It would be of the author's interest, as well, to follow up with an empirical study to try and estimate university utility functions and what students actually consider when deciding to go to university, and how that affects their utility and expected welfare.

1.3. Methodology

The functions chosen to represent the specific cases reflect what I believe would define a real-world situation in general terms: a situation in which the student is willing to give up more in order to attend university than the university is willing to give up in order to have that student.

This can be interpreted as the student having more willingness to take risks than the university is. For that, the university's utility function is set as concave to define risk aversion and the student's utility function is set as a plane to show risk neutrality.

Even though the idea of this project is to define a general case, it's hard to give answers without setting specific utility functions once the optimal values for y and e have been decided in general terms. Therefore, throughout the project, all the cases are solved assuming that the university's preferences are defined by a sigmoid function[3], and a student's preferences are defined by a plane.

Sigmoid functions were chosen because of their properties[4]:

- Real-valued
- Monotonous
- Differentiable

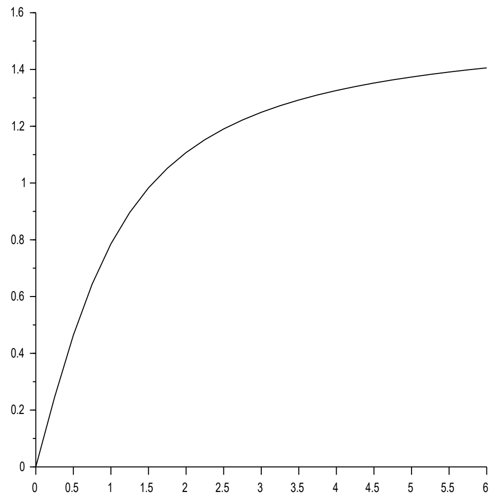

Figure 1. f(x)=arctan(x) when x>0

However, as it will be seen and commented upon in the conclusions, the preferred payments to students will depend largely on the university's preferences and what they give most importance to: either they give more importance to the publications from the students, yielding results that would favor hiring "good students" and induce them to produce high effort, or more importance to the monetary variables, in which case they will likely prefer bad students who exert low effort and, therefore, represent a higher monetary income.

1.4. Analysed cases

The problem can be analysed from two different perspectives:

- Complete information
- Incomplete information

The first one, complete information, analyses what happens when all the players are fully informed on the other players' preferences and utility.

The second one, incomplete information, analyses what happens when that is not the case. This has been looked into for two types of information incompletion: moral hazard and adverse selection. Detailed analysis and solutions have been given for both these cases,

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[3] As seen in Figure 1.
[4] See (Wikipedia, 2017).

and then there is the last section which tries to set the maximization problem a university would face when under both adverse selection and moral hazard.

2. SETTING THE BASE PROBLEM

2.1. Main problem and notation

The main problem that needs to be solved is the following: how will a university decide how much to offer a student on which they have no information? For that, a general utility function for the university and a general utility function for a student have been set. The university's goal is to maximize its utility, and the student's goal is to obtain a minimum amount of utility from accepting the contract. Since we're solving the general case first, the utility function for the university and the student don't really go into specifics about the quality of the student and its work (which will be evaluated as the number of publications). The notation used throughout the project is shown in Table 2-1. Bear in mind that h, l, g, and b are used as subindexes for U, p, x, w, y, e, and q. For example, e_h will refer to high effort.

Symbol	Meaning
B	University's expected utility
U	Student's expected utility
u^0	Student's reservation utility
α, β	Constants
p	Number of publications
x	Tuition
w	Amount university pays the student
y	w-x
e	Effort exerted by student
h	High
l	Low
g	Good
b	Bad
q	Probability

Table 2-1

The three variables on which the functions depend can be grouped into two independent variables: a monetary variable (*x* and w which will be substituted by *y*=*w*-*x*) which indicates the effect of a variation in monetary income on the utility function, and a non-monetary variable (e or p), which indicates the effect of a variation in effort on the utility function.

2.2. Functions to be maximized

The following step is to define the university's utility function B and the student's utility function U, which are set as:

$$B(p,x,w) = \alpha f_1(p) + \beta f_2(x-w) \tag{2.2.1}$$

$$U(x,e,w) = v_1(w-x) - v_2(e) \tag{2.2.2}$$

The functions f_1, f_2, v_1 and v_2 are assumed to be of a general character, and they are only required to be differentiable and monotonically increasing functions. The parameters α and β are positive numerical constants. The utility functions B and U will be determined as soon as f_1, f_2, v_1 and v_2 are chosen from a specific family of functions and the parameters take specific values.

Note that an optimal choice of the specific family of functions and the parameters are not within the goals of this work, and the assumption made is that they're given *a priori*. The parameters have an easy interpretation since they express the relative importance each university gives to the value of publications and to the value of money, meaning they define each university's preferences. Different universities have different α and β, but it is set as a condition that under all circumstances $\alpha + \beta = 1$.

Now, the university's utility function B and the student's utility function U can be written as functions of y, which is the difference between the payment received (w) and the tuition (x).

$$B(p, y) = \alpha f_1(p) - \beta f_2(y) \tag{2.2.3}$$

$$U(y, e) = v_1(y) - v_2(e) \tag{2.2.4}$$

Note that (2.2.3) is increasing with respect to p and decreasing with respect to y, while (2.2.4) is increasing with respect to y and decreasing with respect to e. This is due to the properties of the functions defined as f_1, f_2, v_1 and v_2.

The student will never accept a contract unless his expected utility is at least as high as his reservation utility u^0. Therefore, the university's maximization problem is restricted to an inequality in which the student's expected utility needs to be equal to (or higher than) the reservation utility.

$$\max_{p,y} \alpha f_1(p) - \beta f_2(y) \tag{2.2.5}$$

s.t.

$$v_1(y) - v_2(e) \geq u^0 \tag{2.2.6}$$

Although this may be easily solved, it lacks a practical interpretation, since the university may set only one of the two variables, y, but has no control of any kind over the remaining one, e. In order to address this issue, it will be assumed that the university has two possible outcomes, noted as p_h and p_l, depending on the number of publications, p_h being a high number of publications and p_l being a low number of publications. There is a probability distribution assigned to each of these outcomes depending on the effort exerted by the student, and by its type. These probabilities are shown in Table 2-2 and Table 2-3.

	p_h	p_l
e_h	q_h	$1-q_h$
e_l	$1-q_l$	q_l

Table 2-2. Probability of each outcome for a good student

	p_h	p_l
e_h	$1-q_h$	q_h
e_l	0	1

Table 2-3. Probability of each outcome for a bad student

The interpretation of these two tables could be read as a good student has a probability q_h of generating p_h when his effort is e_h .

It's worth noting that $e_h > e_l$, $p_h > p_l$ and $(q_h, q_l) > (0.5, 0.5)$.

These probabilities need to be considered to calculate the expected utility for both the university (which acts as the principal) and the student (who is the agent).

As mentioned in the introduction, a specific utility function has been set for the university and the same thing has been done for students. Since both functions are separable on y and p (for the university) and on y and e (for the student), the assigned values for each of the functions mentioned above are the following:

$$f_1(p) = \arctan(p) \qquad (2.2.7)$$

$$f_2(y) = \arctan(y) \qquad (2.2.8)$$

$$v_1(y) = y \qquad (2.2.9)$$

$$v_2(e) = \begin{cases} e & \text{if student is good} \\ ke & \text{if student is bad} \end{cases} \quad \text{where } k>1 \qquad (2.2.10)$$

Given these values, the university's utility function will now be represented by (2.2.11) and the student's utility will be represented by (2.2.12) when the student is good and (2.2.13) when the student is bad. See Figure 2 for a visual representation of the university's preferences given specific values for parameters.

$$B = \alpha \arctan(p) - \beta \arctan(y) \quad (2.2.11)$$

$$U = y - e \quad (2.2.12)$$

$$U = y - ke \quad (2.2.13)$$

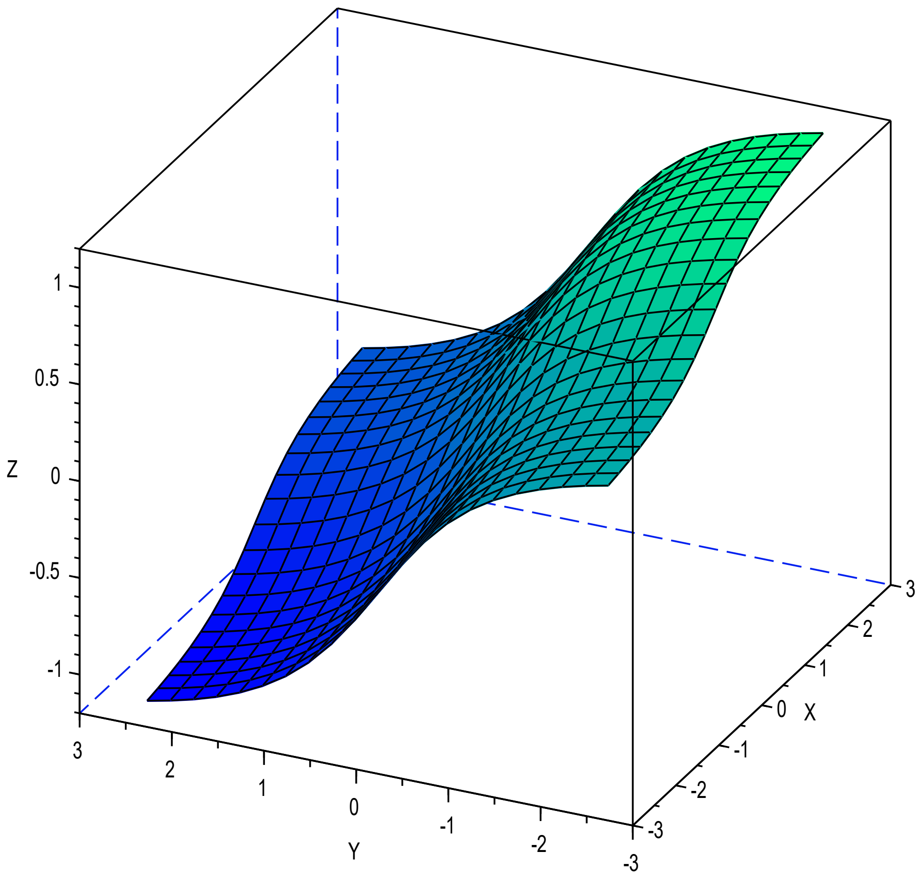

Figure 2. Utility function for the university given parameters alpha=0.33 and betha=0.67

Throughout the project, graphs have been plotted in order to set an example of how this could be solved graphically. To do this, values have been assigned to all the q, h, y and e, and these values are shown in Table 2-4.

(α, β)	(0.33, 0.67)
(p_h, p_l)	(1.5, 0.5)
(e_h, e_l)	(0.75, 0.25)
(q_h, q_l)	(0.75, 0.55)
u^0	-0.5

Table 2-4. Values assigned in order to draw functions and preference maps

3. SOLVING UNDER COMPLETE INFORMATION

3.1. What is complete information?

"In economics and game theory, complete information is a term used to describe an economic situation or game in which knowledge about other market participants or players is available to all participants. The utility functions, payoffs, strategies and "types" of players are thus common knowledge."

Source: Complete information - https://en.wikipedia.org

Figure 3. Complete Information definition

Complete information is described as "an economic situation in which knowledge about other market participants or players is available to all participants"[5] (Wikipedia, 2018).

In the problem stated above, complete information would imply that the university knows exactly which is the student's type (good or bad) and the effort exerted by the student is verifiable: that means that the university knows the likelihood of each outcome and it can verify if the student exerted high effort or low effort. Therefore, the restriction for the university is binding (university knows the minimum amount that should be offered and would still be accepted) and is subject to the student's utility being equal to its reservation utility.

3.2. Setting the maximization problem

In this case, the university will maximize its expected utility taking into account the probabilities assigned in Table 2-2 and in Table 2-3 depending on the type of student. That means that there are now 4 possibilities that the university needs to compare:

- Good student and high effort
- Good student and low effort
- Bad student and high effort
- Bad student and low effort

The university will offer a contract to whichever of these four maximizes the objective function B.

[5] See Figure 3.

3.2.1. Good student and high effort

$$\max_{y_h, y_l} \; q_h[\alpha f_1(p_h) - \beta f_2(y_h)] + (1-q_h)[\alpha f_1(p_l) - \beta f_2(y_l)] \quad (3.2.1)$$

s.t.

$$q_h[v_1(y_h) - v_2(e_h)] + (1-q_h)[v_1(y_l) - v_2(e_h)] \geq u^0 \quad (3.2.2)$$

3.2.2. Good student and low effort

$$\max_{y_h, y_l} \; q_l[\alpha f_1(p_l) - \beta f_2(y_l)] + (1-q_l)[\alpha f_1(p_h) - \beta f_2(y_h)] \quad (3.2.3)$$

s.t.

$$q_l[v_1(y_l) - v_2(e_l)] + (1-q_l)[v_1(y_h) - v_2(e_l)] \geq u^0 \quad (3.2.4)$$

3.2.3. Bad student and high effort

$$\max_{y_h, y_l} \; (1-q_h)[\alpha f_1(p_h) - \beta f_2(y_h)] + q_h[\alpha f_1(p_l) - \beta f_2(y_l)] \quad (3.2.5)$$

s.t.

$$(1-q_h)[v_1(y_h) - v_2(e_h)] + q_h[v_1(y_l) - v_2(e_h)] \geq u^0 \quad (3.2.6)$$

3.2.4. Bad student and low effort[6]

$$\max_{y_h, y_l} \; \alpha f_1(p_l) - \beta f_2(y_l) \quad (3.2.7)$$

s.t.

$$v_1(y_l) - v_2(e_l) \geq u^0 \quad (3.2.8)$$

3.3. Solving the problem for each case under complete information

To solve each of these cases we will be using the first order condition approach with a Lagrangian. However, since there is complete information and the restrictions are binding in all 4 cases, the optimal *v(y)* will be the same for y_h and y_l because of what is known as the efficiency condition. Nevertheless, we will prove that in all four cases by setting the Lagrangian and showing that $y_h=y_l=y$ when under complete information.

[6] Note that in Table 2-3 the probability of a high outcome with a bad student and low effort is set to 0.

3.3.1. Good student and high effort

The Lagrangian will be as shown in (3.3.1).

$$\begin{aligned} L = q_h[\alpha f_1(p_h) - \beta f_2(y_h)] + (1-q_h)[\alpha f_1(p_l) - \beta f_2(y_l)] + \\ +\lambda\left\{q_h[v_1(y_h) - v_2(e_h)] + (1-q_h)[v_1(y_l) - v_2(e_h)] - u^0\right\} \end{aligned} \tag{3.3.1}$$

The first order conditions (FOC) are:

$$\frac{\partial L}{\partial y_h} = \lambda q_h v'_1(y_h) - q_h \beta f'_2(y_h) \tag{3.3.2}$$

$$\frac{\partial L}{\partial y_l} = (1-q_h)\lambda v'_1(y_l) - (1-q_h)\beta f'_2(y_l) \tag{3.3.3}$$

$$\frac{\delta L}{\delta \lambda} = q_h[v_1(y_h) - v_2(e_h)] + (1-q_h)[v_1(y_l) - v_2(e_h)] - u^0 \tag{3.3.4}$$

When solving (3.3.2) and (3.3.3) we find (3.3.5) and (3.3.6) for λ:

$$\lambda = \frac{\beta f'_2(y_h)}{v'_1(y_h)} \tag{3.3.5}$$

$$\lambda = \frac{\beta f'_2(y_l)}{v'_1(y_l)} \tag{3.3.6}$$

Which means that $y_h = y_l = y$, provided that $\frac{f_2'(y)}{v_1'(y)}$ is an injective function. Substituting y_h and y_l for y in Eq. (3.3.4), which is the original restriction, the optimal function of y that the university offers a good student in exchange for high effort is:

$$v_1(y) = v_2(e_h) + u^0 \tag{3.3.7}$$

3.3.2. Good student and low effort

The rest of the cases under complete information will be resolved in the same way and can be referred to in the Annex, but only the Lagrangian and final solution will be shown here.

In this case:

$$\begin{aligned} L = {} & q_l[\alpha f_1(p_l) - \beta f_2(y_l)] + (1-q_l)[\alpha f_1(p_h) - \beta f_2(y_h)] + \\ & + \lambda\left\{q_l[v_1(y_l) - v_2(e_l)] + (1-q_l)[v_1(y_h) - v_2(e_l)] - u^0\right\} \end{aligned} \tag{3.3.8}$$

Which turns into:

$$v_1(y) = v_2(e_l) + u^0 \tag{3.3.9}$$

3.3.3. Bad student and high effort

$$\begin{aligned} L = {} & (1-q_h)[\alpha f_1(p_h) - \beta f_2(y_h)] + q_h[\alpha f_1(p_l) - \beta f_2(y_l)] + \\ & + \lambda\left\{(1-q_h)[v_1(y_h) - v_2(e_h)] + q_h[v_1(y_l) - v_2(e_h)] - u^0\right\} \end{aligned} \tag{3.3.10}$$

As expected, the general case gives the same solution as the one for a good student and high effort defined in (3.3.7). This will change when solutions for the specific functions are computed, but as of now:

$$v_1(y) = v_2(e_h) + u^0 \tag{3.3.11}$$

3.3.4. Bad student and low effort

Taking the probabilities defined in Table 2-3:

$$L = \alpha f_1(p_l) - \beta f_2(y_l) + \lambda[v_1(y_l) - v_2(e_l) - u^0] \tag{3.3.12}$$

The solution is the same as in (3.3.9):

$$v_1(y) = v_2(e_l) + u^0 \tag{3.3.13}$$

3.4. The university's choice

The university will choose which contract will offer and to whom it will do so. Since the solution has been found for a general case, it's hard to know what is the optimal setting for the university. For that reason, the set of functions shown in section 2.2. will be introduced and the results can be computed from there.

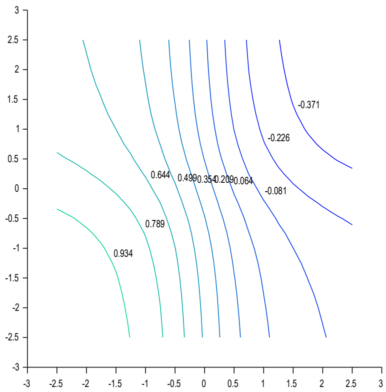

Figure 4 University preferences map

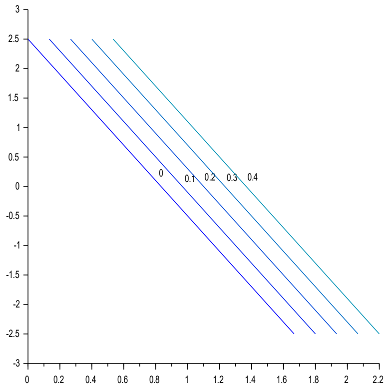

Figure 5 Student's utility function, which is the restriction

In the figures shown, one can see the university's preferences (Figure 4), the student's utility map (Figure 5) and the optimal solution, given the values in Table 2-4, is shown graphically in Figure 6. The solution will be where the university's indifference curves are tangent to the student's utility for the student's indifference curve in which U=u^0.

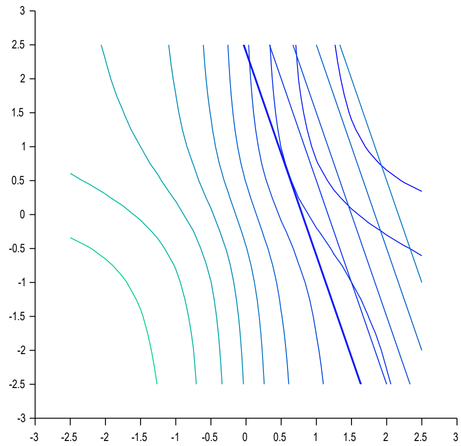

Figure 6

3.4.1. Good student and high effort

Eq. (3.3.7) now turns into:

$$y = e_h + u^0 \tag{3.4.1}$$

Generating the university and expected utility of:

$$B = q_h \alpha \arctan(p_h) + (1 - q_h)\alpha \arctan(p_l) - \beta \arctan(e_h + u^0) \tag{3.4.2}$$

3.4.2. Good student and low effort

Eq. (3.3.9) now turns into:

$$y = e_l + u^0 \tag{3.4.3}$$

Generating the university an expected utility of:

$$B = q_l \alpha \arctan(p_l) + (1 - q_l)\alpha \arctan(p_h) - \beta \arctan(e_l + u^0) \tag{3.4.4}$$

3.4.3. *Bad student and high effort*

Eq. (3.3.11) now turns into:

$$y = ke_h + u^0 \tag{3.4.5}$$

Generating the university an expected utility of:

$$B = q_h \alpha \arctan(p_l) + (1 - q_h)\alpha \arctan(p_h) - \beta \arctan(ke_h + u^0) \tag{3.4.6}$$

3.4.4. *Bad student and low effort*

Eq. (3.3.13) now turns into:

$$y = ke_l + u^0 \tag{3.4.7}$$

Generating the university an expected utility of:

$$B = \alpha \arctan(p_l) - \beta \arctan(ke_l + u^0) \tag{3.4.8}$$

Different universities will find different optimal contracts depending on the relative value assigned to p and y, that is, depending on the values of α and β which, in all cases, add up to 1. It can be seen, though, that the higher the value assigned to publications, the higher the utility derived from having a good student who exerts high effort, since in those cases the first part of the equation will be larger when the probability of a higher number of publications is larger than that of low publications, and a good student is more efficient than a bad student (meaning that a unit of effort from a good student has a lower impact on the student's expected utility than it would on a bad student).

In such a situation, the university only offers one type of contract depending on their expected utility derived from said contract, and it will only hire one type of student subject to a specific effort level.

3.5. Specific example and comparison

In order to compare results between different cases, a specific example has been calculated in each section. The values used for this example are always the same and are those given to draw the utility curves in Table 2-4, and *k=3.* Introducing said values into Eq. (3.4.1), Eq. (3.4.2), Eq. (3.4.3), Eq. (3.4.4), Eq. (3.4.5), Eq. (3.4.6), Eq. (3.4.7), and Eq. (3.4.8) the results are:

$$y_{gh} = 0.25 \tag{3.5.1}$$

$$B_{gh} = 6.72 \tag{3.5.2}$$

$$y_{gl} = -0.25 \quad (3.5.3)$$

$$B_{gl} = 22.58 \quad (3.5.4)$$

$$y_{bh} = 1.75 \quad (3.5.5)$$

$$B_{bh} = -29.15 \quad (3.5.6)$$

$$y_{bl} = 0.25 \quad (3.5.7)$$

$$B_{bl} = -0.63 \quad (3.5.8)$$

Where the first item in the subindices indicates the quality of the student and the second one indicates the effort level.

In both cases (good student and bad student), a higher level of effort proves more expensive to the university and therefore less desirable for this specific case. However, given y_{gl} is positive and the university's expected utility is still positive, one could argue that it could still be beneficial to pay a good student for high effort.

All in all, the university will offer a discount to the good student and ask for low effort: the student will still have to pay a certain amount of tuition (remember that $y=w-x$) but that's what yields the highest expected utility for the university.

4. SOLVING UNDER MORAL HAZARD

The rest of the cases that will be solved for this project are assuming that there is incomplete information and the goal is to analyse what happens to the players when asymmetries of information arise. This section focuses on Moral Hazard, a situation in which the university knows the type of student its dealing with but cannot verify the effort that said student will exert.

4.1. What is moral hazard?

Moral hazard arises when the principal cannot verify the agent's behaviour once the agent is covered by an insurance policy or, in our case, when the agent has signed a contract. In the base problem we set in the second part of the project[7] this would translate into the effort the student could exert, which can either be high or low, being non-verifiable by the university. Therefore, the university needs to find an optimal contract that incentivises the student to exert the desired level of effort and that maximizes the university's expected utility.

Bear in mind that, in this case, the university has information on the type of student but not on their effort. The probabilities of each outcome given each effort for each type of student stay the same as those shown in Table 2-2 and Table 2-3.

4.2. Setting the maximization problem for each case

As was done in the case with complete information[8], the university needs to find four optimal contracts. However, in the absence of full information, the salaries for each contract will depend on the final outcome, and the goal is to induce the student into exerting the desired effort by using these differences in y.

When working under the assumption of moral hazard, the objective function is subjected to two restrictions rather than just one. These restrictions are:

- Participation Constraint (PC): the expected utility from the desired effort level needs to be at least as high as the student's reservation utility.

[7] See 2. SETTING THE BASE PROBLEM.
[8] See 3.2. Setting the maximization problem.

- Incentive Compatibility Constraint (ICC): the expected utility from the desired level of effort needs to be at least as high as the expected utility from the alternative level of effort.

These restrictions tend to be binding: since the university knows the student's utility in each case, it will set y as low as the student will accept whilst making sure that the university still maximizes the objective function.

4.2.1. Good student when the university wants to induce high effort

The following procedure will be used to set all four cases that the university needs to maximize. The objective function and the participation constraint will be the same as those used in (3.2.1) and (3.2.2), which are indicated as (4.2.1) and (4.2.2) in this specific case. The incentive compatibility constraint needs to be taken into account and will be added (4.2.3).

$$\max_{y_h, y_l} \; q_h[\alpha f_1(p_h) - \beta f_2(y_h)] + (1-q_h)[\alpha f_1(p_l) - \beta f_2(y_l)] \tag{4.2.1}$$

s.t.

$$q_h v_1(y_h) + (1-q_h)v_1(y_l) \geq u^0 + v_2(e_h) \tag{4.2.2}$$

$$\begin{gathered} q_h v_1(y_h) + (1-q_h)v_1(y_l) - v_2(e_h) \\ \geq \\ q_h v_1(y_l) + (1-q_h)v_1(y_h) - v_2(e_l) \end{gathered} \tag{4.2.3}$$

4.2.2. Good student when the university wants to induce low effort

The objective function (4.2.4) and participation constraint (4.2.5) are once again taken from the complete information case, and the incentive compatibility constraint needs to be added.

$$\max_{y_h, y_l} \; q^l[\alpha f_1(p_l) - \beta f_2(y_l)] + (1-q^l)[\alpha f_1(p_h) - \beta f_2(y_h)] \tag{4.2.4}$$

s.t.

$$q^l v_1(y_l) + (1-q^l)v_1(y_h) \geq u^0 + v_2(e_l) \tag{4.2.5}$$

$$q_l v_1(y_l) + (1-q_l)v_1(y_h) - v_2(e_l) \\ \geq \\ q_h v_1(y_h) + (1-q_h)v_1(y_l) - v_2(e_h) \tag{4.2.6}$$

4.2.3. Bad student when university wants to induce high effort

$$\max_{y_h, y_l} \ (1-q^h)[\alpha f_1(p_h) - \beta f_2(y_h)] + q^h[\alpha f_1(p_l) - \beta f_2(y_l)] \tag{4.2.7}$$

s.t.

$$(1-q_h)v_1(y_h) + q_h v_1(y_l) \geq u^0 + v_e(e_h) \tag{4.2.8}$$

$$(1-q_h)v_1(y_h) + q_h v_1(y_l) - v_2(e_h) \geq v_1(y_l) - v_2(e_l) \tag{4.2.9}$$

4.2.4. Bad student when the university wants to induce low effort

$$\max_{y_h, y_l} \ \alpha f_1(p_l) - \beta f_2(y_l) \tag{4.2.10}$$

s.t.

$$v_1(y_l) \geq u^0 + v_2(e_l) \tag{4.2.11}$$

$$v_1(y_l) - v_2(e_l) \geq (1-q_h)v_1(y_h) + q_h v_1(y_l) - v_2(e_h) \tag{4.2.12}$$

4.3. Solving the problem for each case under moral hazard

At this point, we need to check if the restrictions are binding and show proof of it. If they are binding, there's no need to solve using Lagrange's method since there are two restrictions and two variables that need to be determined (y_h and y_l). Proof of binding restrictions will be provided for section 4.3.1 and for the other paragraphs in the annex.

In every low effort case, it is widely spread and assumed that the optimal payment scheme will have the same payoff scheme as the complete information version since it is assumed that when the agent accepts the contract his instinct is to exert low effort since that is what brings the least amount of disutility.

What that means is that if the principal tries to convince the agent to exert low effort, it'll offer only one possible salary regardless of the outcome, which is the minimum salary the agent will accept and that was computed in the complete information case.

In this case, the graphical representation of the problem has been done in 4 figures, which show the objective function, the two restrictions, and the graphical solution. These are Figure 7, Figure 8, Figure 9, and Figure 10.

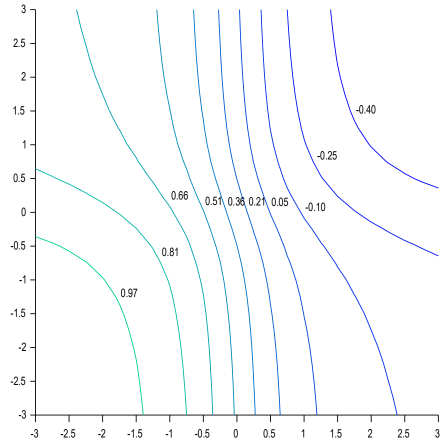

Figure 7 University's indifference curves

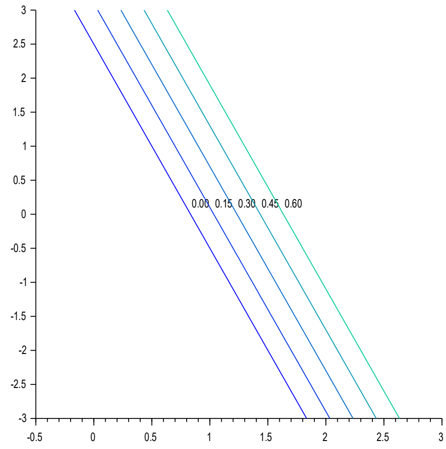

Figure 8 Curves representing the participation constraint

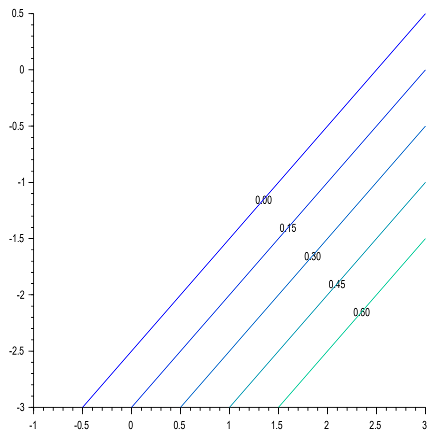

Figure 9 Curves representing the incentive compatibility constraint

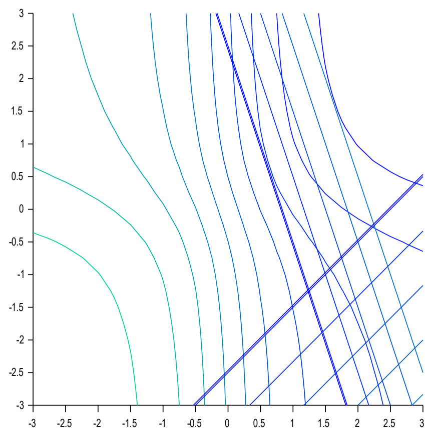

Figure 10 Solution

4.3.1. Good student when the university wants to induce high effort

Solve what was done in section 4.2.1. by using a Lagrangian. Since it was discussed that these restrictions need to be binding[9], given that the university knows the type of student and, therefore, the utility the student would receive from a given effort, the university can set a y that will bring the student just to the verge of accepting the contract without having to incur in any additional costs. Having two equations means that there's no need for the Lagrangian since it can be solved as a system.

With restrictions (4.2.2) and (4.2.3) both being binding, this can be solved as:

$$v_1(y_h) = \frac{q_l v_2(e_h) - (1-q_h)v_2(e_l)}{q_h + q_l - 1} + u^0 \quad (4.3.1)$$

$$v_1(y_l) = \frac{q_h v_2(e_l) - (1-q_l)v_2(e_h)}{q_h + q_l - 1} + u^0 \quad (4.3.2)$$

As mentioned previously, utility derived from effort in a good student will be defined by Eq. (2.2.10).

4.3.2. Good student when the university wants to induce low effort

Since the university observes the type of student, it can always ensure the lowest level of effort by offering each type of student a payment that is equal to the reservation utility. Because the student will minimize their cost, that will lead to the lowest level of effort. Therefore, the payment will be set by that defined in equation (3.3.9).

Even though one could make the effort to calculate the result of maximizing the problem shown in section 4.2.2, the complete information solution will guarantee that the student signs and have an even lower *y*.

$$v_1(y) = v_2(e_l) + u^0 \quad (4.3.3)$$

[9] Proof of binding restrictions can be found in the Annex.

4.3.3. Bad student when the university wants to induce high effort

The monotonicity of the functions involved, and by the same procedure as in section 4.3.1, allows to conclude that (4.2.8) and (4.2.9) are binding constraints. By the same procedure as in the good student, the results are:

$$v_1(y_h) = \frac{v_2(e_h) - q_h v_2(e_l)}{(1-q_h)} + u^0 \quad (4.3.4)$$

$$v_1(y_l) = v_2(e_l) + u^0 \quad (4.3.5)$$

4.3.4. Bad student when university wants to induce low effort

Following the same reasoning as in good student and low effort[10], the solution will be the same that was set when there was information completeness.

$$v_1(y) = v_2(e_l) + u^0 \quad (4.3.6)$$

4.4. The university's choice

The university will compute its expected utility from each of the previous cases and offer each type of student the contract that gives the highest expected utility. Just as what happened in section 3.4, the final decision by the university will depend on their utility function and the values given to α and β.

4.4.1. Good student when the university wants to induce high effort

Introducing (2.2.7), (2.2.8), (2.2.9), and (2.2.10) into (4.3.1) and (4.3.2) :

$$y_h = \frac{q_l e_h + (q_h - 1)e_l}{q_h + q_l - 1} + u^0 \quad (4.4.1)$$

$$y_l = \frac{q_h e_l + (q_l - 1)e_h}{q_h + q_l - 1} + u^0 \quad (4.4.2)$$

[10] See section 4.3.2.

Which, when introduced into the university's expected utility function (4.2.1) has the following output:

$$\begin{aligned} B = q_h \left[\alpha \arctan(p_h) - \beta \arctan\left(\frac{q_l e_h + (q_h - 1) e_l}{q_h + q_l - 1} + u^0 \right) \right] + \\ +(1 - q_h) \left[\alpha \arctan(p_l) - \beta \arctan\left(\frac{q_h e_l + (q_l - 1) e_h}{q_h + q_l - 1} + u^0 \right) \right] \end{aligned} \quad (4.4.3)$$

4.4.2. Good student when the university wants to induce low effort

Since the y is going to be the same as in section 3.3.2, the B will also be the same.

$$y = e_l + u^0 \quad (4.4.4)$$

$$B = q_l \alpha \arctan(p_l) + (1 - q_l) \alpha \arctan(p_h) - \beta \arctan(e_l + u^0) \quad (4.4.5)$$

4.4.3. Bad student when the university wants to induce high effort

Given equations (4.3.4) and (4.3.5), the solutions here would be:

$$y_h = \frac{k(e_h - q_h e_l)}{1 - q_h} + u^0 \quad (4.4.6)$$

$$y_l = k e_l + u^0 \quad (4.4.7)$$

And the university's expected utility from trying to induce high effort in a bad student:

$$\begin{aligned} B = (1 - q_h) \left[\alpha \arctan\left(p_h\right) - \beta \arctan\left(\frac{k(e_h - q_h e_l)}{1 - q_h} + u^0 \right) \right] + \\ + q_h \left[\alpha \arctan(p_l) - \beta \arctan(k e_l + u^0) \right] \end{aligned} \quad (4.4.8)$$

4.4.4. Bad student when the university wants to induce low effort

According to equation (4.3.6), the optimal y, in this case, would be:

$$y = k e_l + u^0 \quad (4.4.9)$$

And therefore, the university's expected utility is:

$$B = \alpha \arctan(p_l) - \beta \arctan(k e_l + u^0) \quad (4.4.10)$$

The university's preferred option will greatly vary depending on the value of parameters α and β since that's what defines their preferences. However, as discussed in the complete information case, it is easy to see that a university with a higher α will be more willing to hire good students and try and induce high effort from them since that would yield a higher expected utility.

4.5. Specific example and comparison

The same will be done in this section as was done in section 3.5, but using equations (4.4.1), (4.4.2), (4.4.4), (4.4.5), (4.4.6), (4.4.7), (4.4.8), (4.4.9), and (4.4.10).

4.5.1. Good student when the university wants to induce high effort

The contract will include two salaries, one for a high number of publications and one for a low number of publications, which can be observed after having exerted the effort.

$$\begin{aligned} y_h &= 0.67 \\ y_l &= -1 \end{aligned} \tag{4.5.1}$$

And the university's expected utility will be:

$$B = 0.39 \tag{4.5.2}$$

4.5.2. Good student when the university wants to induce low effort

As discussed, there will only be one salary when the desired effort is low. In this case, it will be:

$$y = -0.25 \tag{4.5.3}$$

And the university's expected utility will be:

$$B = 22.58 \tag{4.5.4}$$

4.5.3. Bad student when the university wants to induce high effort

In this case, the contract will also include two salaries, which are:

$$\begin{aligned} y_h &= 3.25 \\ y_l &= 0.25 \end{aligned} \tag{4.5.5}$$

And the expected utility:

$$B = -15.6 \tag{4.5.6}$$

4.5.4. Bad student and the university wants to induce low effort

In this case, there's only one salary, and it yields:

$$y = 0.25 \tag{4.5.7}$$

$$B = -0.63 \tag{4.5.8}$$

These results indicate that, in this specific case, the university prefers to accept a good student but induce low effort from them, since that yields the highest expected utility. Given that it happens to be an option with low effort, there is no distortion at the top. It would be expected, however, that in a case in which the university prefers a student who exerts high effort (regardless of if it refers to a good student or a bad student), said student would receive an additional amount of utility derived from the information asymmetry in comparison to the equilibrium case.

5. SOLVING UNDER ADVERSE SELECTION

In this section, the aforementioned problem will be set and attempted to be solved when the university does not know the type of student, that is when the university faces a situation of adverse selection.

5.1. What is adverse selection?

Adverse selection is a case of asymmetric information in which not all the parties involved in the contract have all of the information regarding the characteristics of the agent. In our specific case, the university knows that two types of students exist, and it also knows the probability distribution that describes their presence in nature, but it has no way of verifying the kind of student it's about to offer the contract. Since the university cannot verify the type of student, the possibility that a student might lie in order to increase his expected utility must be considered when computing the optimal contract.

For the sake of simplicity, we will assume that the effort is verifiable and will leave the more complex case (which is a situation with asymmetries arising from both adverse selection and moral hazard) for further on in the project[11].

5.2. Setting the problem

As in the previous cases, there will be a contract for good students and a contract for bad students. The student can choose either to reject both of them or to accept one of them. The tricky part here is to induce the student into signing the "correct" contract, meaning the university needs to offer such contracts that a good student will always sign a contract designed for good students and vice versa.

The university needs to maximize the objective function taking into account the salary that'll pay and the effort that it'll demand, and the two contracts offered will be those shown in Eq. (5.2.1) and Eq. (5.2.2):

$$(y_g, e_g) \tag{5.2.1}$$

$$(y_b, e_b) \tag{5.2.2}$$

[11] See 6.SOLVING UNDER MORAL HAZARD AND ADVERSE SELECTION in page 36.

Where Eq. (5.2.1) indicates the contract that, optimally, a good student would sign and Eq. (5.2.2) indicates the contract a bad student would sign.

It is important to analyse if the students have incentives to lie in order to gain utility (for example, good students might rather pretend to be bad students if the decrease in effort disutility is larger than the decrease in salary utility). Since the university now chooses the effort for each contract, and to simplify calculations, we will set $f_1(p_h) = f_1(e_g)$ and $f_1(p_l) = f_1(e_b)$ and introduce it to the objective function.

The university also knows that the probability of any given student to be of type good is q, and the probability of the student to be bad is *1-q*.

There are now four restrictions when maximizing the objective function, and the maximization problem is the following:

$$\max_{(y_g, e_g),(y_b, e_b)} q[\alpha f_1(e_g) - \beta f_2(y_g)] + (1-q)[\alpha f_1(e_b) - \beta f_2(y_b)] \tag{5.2.3}$$

s.t.

$$v_1(y_g) - v_2(e_g^g) \geq u^0 \tag{5.2.4}$$

$$v_1(y_b) - v_2(e_b^b) \geq u^0 \tag{5.2.5}$$

$$v_1(y_g) - v_2(e_g^g) \geq v_1(y_b) - v_2(e_b^g) \tag{5.2.6}$$

$$v_1(y_b) - v_2(e_b^b) \geq v_1(y_g) - v_2(e_g^b) \tag{5.2.7}$$

Where:

- Eq. (5.2.4) is the participation constraint for the good student
- Eq. (5.2.5) is the participation constraint for the bad student
- Eq. (5.2.6) is the incentive compatibility constraint for the good student
- Eq. (5.2.7) is the incentive compatibility constraint for the bad student

And where $v_2(e_g^b)$ is the disutility that a bad student receives from exerting the effort expected from a good student. That is, the super index indicates the type of utility function given Eq. (2.2.10), and the sub-index indicates the effort level that the student is exerting.

5.3. Solving the problem under adverse selection

To try and find the result, we'll use the specific functions $v_1(y)$ and $v_2(e)$, but not $f_1(e)$ or $f_2(y)$. This FOC approach looks simple enough to solve, but since the derivatives of arctan functions take the form $\frac{\delta \arctan(x)}{\delta x} = \frac{1}{1+x^2}$, to solve it you need to go through fourth-degree polynomial equations, which are difficult and painful to solve. The solution to this problem is found by using Lagrange multipliers on the binding restrictions in order to maximize the university's utility. Re-writing the maximization problem:

$$\max_{(y_g,e_g),(y_b,e_b)} q[\alpha f_1(e_g) - \beta f_2(y_g)] + \\ +(1-q)[\alpha f_1(e_b) - \beta f_2(y_b)] \tag{5.3.1}$$

s.t.

$$y_g - e_g \geq u^0 \tag{5.3.2}$$

$$y_b - ke_b \geq u^0 \tag{5.3.3}$$

$$y_g - e_g \geq y_b - e_b \tag{5.3.4}$$

$$y_b - ke_b \geq y_g - ke_g \tag{5.3.5}$$

It's easy to see that, in order for these restrictions to be all satisfied, not all of them can be binding. Given that $y_g > y_b$, $e_g > e_b$, and $k > 1$:

$$y_g - e_g \geq y_b - e_b > y_b - ke_b \geq u^0 \tag{5.3.6}$$

Which implies that the participation constraint of the good student Eq. (5.3.2) is not binding ($y_g - e_g > u^0$), but the participation constraint of the bad student Eq. (5.3.3) and the incentive compatibility constraint of the good student Eq. (5.3.4) can be. Regarding the ICC of the bad student Eq. (5.3.5), one needs to check if it's satisfied once the results have been computed.

The Lagrangian will look like this:

$$L = q\left[\alpha f_1(e_g) - \beta f_2(y_g)\right] + \\ +(1-q)\left[\alpha f_1(e_b) - \beta f_2(y_b)\right] + \\ +\lambda\left[y_b - ke_b - u^0\right] + \mu\left[y_g - e_g - y_b + e_b\right] \tag{5.3.7}$$

And it needs to be maximized with respect to e_g, e_b, y_g, and y_b. The FOC are the following:

$$\frac{\partial L}{\partial e_g} = q\alpha f_1^{'}(e_g) - \mu \tag{5.3.8}$$

$$\frac{\partial L}{\partial e_b} = (1-q)\alpha f_1^{'}(e_b) - k\lambda + \mu \tag{5.3.9}$$

$$\frac{\partial L}{\partial y_g} = -q\beta f_2^{'}(y_g) + \mu \tag{5.3.10}$$

$$\frac{\partial L}{\partial y_b} = -\beta(1-q) f_2^{'}(y_b) + \lambda - \mu \tag{5.3.11}$$

Unfortunately, solving this means going through fourth-degree polynomial equations and this is long and tedious to solve, added to the fact that I lack the tools to do so. However, relations between variables and functions were established and could be used in specific cases in which q, α, and β were known, and they could be solved if the effort were pre-set in order to answer the question "In an adverse selection situation, which would be the optimal y for each level of effort and type of student?". Doing this approximation is inefficient. So, after renaming the values $f_1^{'}(e_g)$, $f_1^{'}(e_b)$, $f_2^{'}(y_g)$, $f_2^{'}(y_b)$ as as E_g, E_b, Y_g, Y_b respectively (for visual purposes exclusively) the following were found:

$$\alpha E_g = \beta Y_g \tag{5.3.12}$$

$$\alpha(1-q)E_b + \alpha q E_g = k\beta(1-q)Y_b + k\beta q Y_g \tag{5.3.13}$$

$$y_b - ke_b - u^0 = 0 \tag{5.3.14}$$

$$y_g - e_g - y_b + e_b = 0 \tag{5.3.15}$$

5.4. Specific example and comparison

Given f_1 and f_2, deriving and solving has proven difficult. To solve the Lagrangian, what's been done is enter the 6 equations derived from the Lagrangian in a Wolfram-Alpha widget[12]. The values entered were those used for the other specific examples[13] but

[12] http://www.wolframalpha.com/widgets/view.jsp?id=a9536b3eddefb39312a81eeeb828835f

[13] See Table 2-4.

leaving the effort as a variable (because in this case it is) and assuming that *q=0.5*, which means that the likelihood of the student being good is the same as the student being bad. The output is shown in Table 5-1.

	Solution 1	Solution 2
e_g	-10.92	6.05
e_b	2.58	-1.06
y_g	-15.59	8.68
y_b	7.26	-3.69
λ	0.007	0.02
μ	0.0013	0.004

Table 5-1. Output when entered in wolfram alpha

As seen in the output, there are two solutions to the problem. These are likely two relative maximums of the function within the set constraints, without being able to tell for sure. Bear in mind that λ and μ are Lagrange multipliers and, therefore, they are not relevant to the solution that concerns us.

Since $v_2(e_b) > v_2(e_g)$ for any given value of *e*, it is reasonable to assume that the university will ask for higher effort to good students and lower effort to bad students: doing so is less costly to the university and has a higher output.

The expected utility from offering contracts is maximized when said contracts are $(y_g = 8.68,\ e_g = 6.05)$ and $(y_b = -3.69,\ e_b = -1.06)$. However, given that the value for effort is negative, we can consider that the contract offered for the bad student is such that only good students will decide to sign a contract and will choose their type. The expected utility for the university calculated from the objective function and with the obtained values is the following:

$$B = 2.72 \tag{5.4.1}$$

So it could be said that only the good student will accept the contract and the bad student will just have to pay tuition and exert whichever effort they prefer, but the tuition charged will have to compensate the discount (offered) to good students if the university wants positive expected utility.

Since it was mentioned that only the participation constraint of the bad student and the incentive compatibility constraint of the good student were binding, and there was formal proof that the participation constraint of the good student is satisfied, it needs to be checked if the incentive compatibility constraint of the bad student is satisfied. So, introducing the values in equation Eq. (5.3.5) one can find:

$$(-3.69)-3(1.06)\geq(8.68)-3(6.05)\rightarrow-0.51>-9.47 \qquad (5.4.2)$$

It is satisfied.

In this case, there is distortion at the top since the good student's expected utility is higher than it was in equilibrium, and there is what's known as informational rent, which is the difference between the university's expected utility under symmetrical information and in adverse selection: that is a loss of efficiency caused by uncertainty. The university, in this case, faces a corner solution in which the bad student doesn't want to engage in the proposed contract, or if paying tuition and exerting whichever effort it wants can be considered engaging, then doing that: bear in mind that the solution with negative effort satisfies the constraints, which means that if negative effort existed, it would be feasible. However, in a real setting negative effort is hard to imagine other than the bad student actively bothering the correct development of activity.

It could also be considered "how much does the bad student have to pay in exchange for $e_b=0$. In that case, the university would set the effort of the bad student to 0 and compute payments and effort for the good student.

6. SOLVING UNDER MORAL HAZARD AND ADVERSE SELECTION

After having found the maximization problem to which universities are submitted under complete information, moral hazard, and adverse selection, it seems only natural to try and set the problem when universities face both moral hazard and adverse selection. However, given the time constraint and the complexity of result interpretation, there has been no formal attempt to actually solve this case for the specific defined functions, leaving it to future research in case someone ever considered it worth their time.

In order to set this problem, there are different concepts that have appeared throughout the project that need to be taken into account. Until now, three different probabilities have appeared, those being q, q^h, and q^l. We face the situation in which the university cannot verify the effort exerted by the student (such as those cases studied in section 0) and also can't verify the type of student (see cases in section 5).

> q= Probability that the student of unknown type is good
>
> q^h= Probability of an outcome when the student exerts high effort
>
> q^l= Probability of an outcome when a student exerts low effort

Regarding the university's expected utility, the university has to choose which amount of effort it will attempt to induce and then would use q to assign the likelihood of a good student exerting high effort and a bad student exerting high effort. It would do the same to calculate the expected utility derived from inducing low effort on either type of student.

6.1. University prefers high effort

See Eq. (4.2.1) and Eq. (4.2.7) to find the expected utility when the university has no way of verifying the effort or the type of student but wants to induce high effort. Take those and the new maximization problem is:

$$\max_{y_h^g, y_l^g, y_h^b, y_l^b} q\left\{q_l[\alpha f_1(p_h) - \beta f_2(y_h^g)] + (1-q_l)[\alpha f_1(p_l) - \beta f_2(y_l^g)]\right\} + \\ +(1-q)\left\{(1-q_h)[\alpha f_1(p_h) - \beta f_2(y_h^b)] + q_h[\alpha f_1(p_l) - \beta f_2(y_l^b)]\right\} \tag{6.1.1}$$

And the restrictions will be:

$$q_h v_1(y_h^g) + (1-q_h)v_1(y_l^g) \geq u^0 + v_2(e_h^g) \tag{6.1.2}$$

$$(1-q_h)v_1(y_h^b) + q_h v_1(y_l^b) \geq u^0 + v_2(e_h^b) \tag{6.1.3}$$

$$q_h v_1(y_h^g) + (1-q_h) v_1(y_l^g) - v_2(e_h^g) \geq$$
$$\geq q_l v_1(y_l^g) + (1-q_l) v_1(y_h^g) - v_2(e_l^g) \qquad (6.1.4)$$

$$(1-q_h) v_1(y_h^b) + q_h v_1(y_l^b) - v_2(e_h^b) \geq v_1(y_l^b) - v_2(e_l^b) \qquad (6.1.5)$$

And to these four, there should be two additional restrictions that would prevent the student from lying about his type:

$$q_h \left[v_1(y_h^g) - v_2(e_h^g) \right] + (1-q_h) \left[v_1(y_l^g) - v_2(e_h^g) \right] \geq$$
$$\geq q_h \left[v_1(y_h^b) - v_2(e_{hb}^g) \right] + (1-q_h) \left[v_1(y_l^b) - v_2(e_{lb}^g) \right] \qquad (6.1.6)$$

$$(1-q_h) \left[v_1(y_h^b) - v_2(e_h^b) \right] + q_h \left[v_1(y_l^b) - v_2(e_h^b) \right] \geq$$
$$\geq (1-q_h) \left[v_1(y_h^g) - v_2(e_{hg}^b) \right] + q_h \left[v_1(y_l^g) - v_2(e_{\lg}^b) \right] \qquad (6.1.7)$$

6.2. University wants low effort

We would use the same procedure as we did for the case in which the university wants high effort but would change the utility functions and restrictions accordingly.

7. CONCLUSIONS

The project started with the idea of answering two sequential questions:

- Could an incentive program in which a university discounts tuition (and even pays) for students who excel academically?
- Would this still happen if the university lacks information regarding the student's ability and interest?

In a very simplified model, both questions have been answered in a theoretical framework in which it was assumed that it was both known what universities prefer and what students want. Even though it is a huge assumption, it was needed in order to set a reference.

Given the utility functions, both students and universities can benefit from such a program. It's proven beneficial for the university to have a certain type of student at a discounted tuition if that student produces certain results. Of course, the impact these results have on expected utility depend on the university's preferences, and the parameters used when estimating them have a big impact on the outcome.

What has been observed is that there is informational rent when situations of asymmetrical information are introduced into the analysis, since there is a difference between the amount that would be paid in symmetrical information (which is the benchmark situation, the equilibrium) and the optimal amounts found under moral hazard and adverse selection.

This seems to be the case in Moral Hazard when the optimal situation includes high effort on either type of student since the premium paid for good results needs to be larger than it would under complete information, while the payment for low results is the same as the unique payment when information completeness exists.

On the other hand, under adverse selection, where a general solution was not found (relations between variables were found, but not the solution), the specific calculated case needs to be taken into account and compared to the other cases with the same α and β. In this case, the contract designed to attract good students pays substantially more than the previous ones, although it also requires a larger amount of effort. In the case of the bad student, it yields results with negative effort and a negative payment, which basically means that the student will have to pay tuition but there won't be much of an effort requirement. It could also be interpreted as the bad student deciding not to engage in the contract and would result in the university having only good students accept the proposed contract.

Since in economics the idea is to interpret the theoretical results in a world that is real, one could interpret that in that case the bad student will have to decide upon going to university based on other elements which are likely to be in their utility function: bear in mind that the negative value of y means that the student pays tuition, and since that is within the participation constraint of the bad student it does not mean that said student doesn't want to go to university. However, the negative effort could be interpreted as a 0 and therefore the student's y would probably have to change a bit in order to still accommodate their participation constraint.

Since scholarships and subsidies to university are a reality, it would be of interest to follow up this project with a regression model that attempts an estimation of a university's utility function. It could also be of interest to compare the utility of a private university to that of a public university: public universities are thought of as public entities and have the goal of improving overall welfare while private universities are usually run with a financial goal in mind (although it's not usually the only goal these universities have).

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9. ANNEX

9.1. Proof that there's only one v(y) under complete information

For the high effort case when the student is bad:

$$\frac{\delta L}{\delta y_h} = -(1-q^h)\beta f_2^{'}(y_h) + \lambda(1-q^h)v_1^{'}(y_h) \tag{9.1.1}$$

$$\frac{\delta L}{\delta y_l} = -q^h \beta f_2^{'}(y_l) + \lambda q^h v_1^{'}(y_l) \tag{9.1.2}$$

Which means:

$$\lambda = \frac{\beta f_2^{'}(y_h)}{v_1^{'}(y_h)} \tag{9.1.3}$$

$$\lambda = \frac{\beta f_2^{'}(y_l)}{v_1^{'}(y_l)} \tag{9.1.4}$$

Provided that $\frac{f_2^{'}(y)}{v_1^{'}(y)}$ is injective, then $y_h = y_l$.

9.2. Proof of binding restrictions under Moral Hazard

For formal proof that restrictions are binding, Eq. (4.2.1) may be re-written as:

$$\max_{y_h, y_l} \quad -q^h \beta f_2(y_h) - (1-q^h)\beta f_2(y_l) + c \tag{9.2.1}$$

where k is a constant. Such function is:

- Strictly decreasing in both y_h and y_l (since f_2 is a strictly increasing function, $q^h \beta > 0$ and $(1-q^h)\beta > 0$).
- Continuous

On the other hand, the Eq. (4.2.2) may be re-written as

$$q^h v_1(y_h) + (1-q^h)v_1(y_l) + c' \geq 0 \tag{9.2.2}$$

Which is strictly increasing in both variables y_h, y_l .

Suppose that a maximum of Eq. (9.2.1) is reached on a point (y_h^0, y_l^0) such that Eq. (9.2.2) $(y_h^0, y_l^0) > 0$. In that case, the continuity of Eq. (9.2.2) guarantees that Eq. (9.2.2) $(y_h^1, y_l^1) > 0$ for any (y_h^1, y_l^1) which is close enough to (y_h^0, y_l^0) . By choosing (y_h^1, y_l^1) s.t. $y_h^1 < y_h^0$ and $y_l^1 < y_l^0$ we will have that $B(y_h^1, y_l^1) > B(y_h^0, y_l^0)$, showing that (y_h^0, y_l^0) cannot be a maximum. And by keeping y_l constant and using the same reasoning, we can prove that Eq. (4.2.3) is also binding.

9.3. Table of figures