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THE ROLE OF CROSS-SUBSIDIES IN THE INTERURBAN BUS SPANISH SYSTEM AND WELFARE EFFECTS OF LIBERALIZATION

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Abstract

The intercity bus market in Spain operates under a competition for the market system, granting a monopolistic concession to a company that functions without public assistance on various lines. Since some of them are profitable and other are loss-making, the operating firm runs assuming implicitly an internal cross-subsidization system, which ends up hurting lower income citizens. This study formalizes and solves the economic model of cross-subsidies, using bus demand data for the Madrid-Irun route to calculate the exact amount of profits used as subsidies for each line. Despite operating costs preventing profitability on some lines, the company's overall profits are positive and significant, indicating a failure of the current bidding system to promote fair competition. While liberalization could reduce prices, it does not ensure service continuity on unprofitable lines. An alternative approach would be the provision of direct subsidies to ensure connectivity. The study also estimates potential changes in demand and consumer surplus with liberalization. By highlighting the problems of the current cross-subsidization system and analyzing a proposal for liberalization with direct subsidies, this research addresses the reform needs of the Spanish intercity bus system.

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1. Introduction

Transportation is a fundamental aspect of modern society, allowing for the connection between different places and the mobility of people and goods, which is essential for the economy and social development: it improves people's accessibility to services and opportunities such as work, education, healthcare, etc. One of the most pressing issues in the field of transportation economics is the provision of public transportation as a necessity for individuals who do not have access to private vehicles, as well as a more environmentally sustainable travel alternative. With the increasing demand for transportation services, it is crucial to improve the efficiency and sustainability of the system. The aim of this TFG is to investigate the bus transportation system in interurban areas, currently characterized by cross-subsidization mechanisms, and explore potential strategies to enhance their performance.

The current bidding system that distributes the different routes in which interurban buses operate in Spain grants a monopolistic contract of the route to the winning company for a certain amount of time. As a result, the company establishes an internal system of cross-subsidies, which involve using profits from profitable lines (the ones with higher levels of demand/or lower operation costs) to cover losses from unprofitable lines (with low demand and/or higher costs). However, this approach is subject to criticism from both economic and social perspectives as it can lead to inefficient allocation of resources and also have a regressive impact, since most of the bus users belong to the lower parts of the income distribution.

In a European context of liberalization of the interurban transport market, and with a Spanish law expressing intentions to follow the example of our neighbors on the horizon, it is important to open the debate on the current impact of the cross-subsidy mechanism, which manages to sustain bus connection services despite the unsustainability of some of the lines. It is necessary for the economy to reconsider its current impact and evaluate the alternatives we have as a country, once the market is liberalized, in order to maintain the service on those unprofitable lines that predominantly connect rural low-density areas and ensure affordable mobility for the entire population.

In order to develop the work, the following research questions will guide the investigation:

- What is the context that explains the current intercity bus system in Spain?

- How does the mechanism of cross-subsidies work? How does it relate to transportation demand and various restrictions, whether from the government or the company, and cost?

- What is the empirical reality? What is the impact of costs on the profitability of transportation providers?

- What effect does this cross-subsidy system have on the population? What alternatives can be proposed and what impact would they have?

There is limited economic literature analyzing the system of cross-subsidies, and even less specialized in the transportation system, making this a novel project that aims to fill an existing knowledge gap. Research studies such as the one conducted by Asensio and Matas (2023), which analyze interurban road transport in Spain, or the study on cross-subsidies in the postal service by Cremer et al. (1997), will serve as guides and references for this work.

Regarding the methodology, a combination of theoretical and empirical methods will be used in the study to obtain and analyze the outcomes of the cross-subsidy system. Firstly, a simple partial equilibrium model will be developed to characterize the equilibrium under the assumption that each line belonging to the route represents a market. The objective is to understand the mathematical process that leads to the market allocation of the price of the ticket and number of bus users, as well as to try to understand how the subsidies are calculated. Once the economic model is solved, we will proceed with its empirical application. Based on the observed bus demand database for one of the routes (route 157, belonging to Madrid-Irún), which presents real data for some of the variables involved in the model, and using approximate calculations for the remaining ones, we will use R studio software to determine the values of all the unknowns and be able to solve the problem. We will also use this programming language to estimate the profitability of each line as well as to simulate the new demand, economic costs and impact on consumer's surplus of an alternative to the cross-subsidy system.

The remaining of the work is structured as follows. After this introduction, section 2 provides context of the interurban bus market situation through a literature review of the key concepts and previous background. Section 3 includes the formalization of the microeconomic model and its solving under different market situations. On section 4 the model is applied empirically, including explanations about the nature of the data used, our approach, and an analysis of the results. Section 5 explains the scenario of liberalization and its possible impacts on the structure and outcomes of the interurban bus market. Finally, in section 6 is proposed an alternative system and calculated its potential results in terms of demand increase, resources needed and consumer surplus variation. A final section of conclusions analyzes the main implications of the results obtained.

In conclusion, this study aims to shed light on unexplored aspects of the topic, providing valuable insights for future research and policy considerations.

2. Spain's intercity bus system

2.1. Importance of the bus in Spain

With a wide coverage of the national territory, intercity bus companies offer regular services connecting the main Spanish cities, as well as rural and remote areas that are often frequent in the country. This transportation market, which has a greater weight than in most European countries, was used by 206 million people in 2019, of which 30 belong to the state network of regular intercity buses (the subject of study of this work), according to Asensio and Matas (2023). Interurban bus transportation represents an economic and practical alternative to other means, such as airplanes or trains, which have a lesser weight in the Spanish transportation system. Among other things, this difference in weight is due to the fact that, contrary to what happened in other countries, interurban corridors were not reserved for trains (Van de Velde, 2014), and to a greater investment in road infrastructure until the expansion of high-speed rail. Therefore, the importance of buses in Spain is enormous, as it is the main mean of transportation for medium and long-distance passengers.

2.2. Regulation and outcomes

Currently, the intercity bus market in Spain is not liberalized, unlike the rest of Europe. While in other countries a "competition in the market" system has been advocated, in which companies compete with each other for their share of the market, trying to differentiate themselves from others either through price or service quality to gain a larger share of passengers, Spain has chosen a regulation in the form of "competition for the market". This system is characterized by companies competing for the exclusive operation of services for 20 years.

The Ley de Ordenación del Transporte Terrestre (1987) considers intercity bus service as a public service that will be offered under a competitive bidding system. Based on a regulatory framework, companies submit their proposals for operation to the competent authority, which in this case would be the Ministerio de Transporte (MITMA), with the hope of obtaining the concession or contract that allows them to operate the bus service in a monopolistic manner for a certain period of time. This proposal is conditioned by the so-called "pliegos", which are the variables specified by MITMA that are taken into account in each bidding process. Some of these evaluation criteria, which have varied over time, are price, frequency, safety, and vehicle quality and comfort. Each criterion is assigned a different weight, and the company that wants to win the bid must submit a proposal with a higher score than the others. However, in 2007 (when the contracts that had been extended to bus companies with the creation of the LOTT were about to expire), the pliegos were declared by the Comisión Nacional de la Competencia as anti-competitive. As a result, since then, elements that are contrary to competition have been eliminated from the bidding process, increasing the probability of a company participating in the bidding process and thus the number of bidders. Over the years, the importance of the two main competition variables, prices and frequency, has increased, but the comparison with intercity bus markets in other countries shows that liberalization (i.e., competition in the market) leads to lower prices and a better service adapted to demand. It is no coincidence that passengers on Spanish buses pay 88% more than French or Italian passengers, 41% more than Portuguese passengers, 36% more than German passengers, or 28% more than bus passengers in the UK (Asensio and Matas, 2023). Therefore, while the concession system could result in price and frequency combinations similar to those of a liberalized market, the defining structure of the Spanish market's bids has not yet achieved it.

2.3. Cross subsidies system

A crucial feature of concessions is that they are granted by routes, although a single route may (and often does) include more than one line.

We understand a route as a set of geographically nearby lines (e.g. Madrid-Irun). In contrast, a line refers to the sum of services operated by the company via bus, which follow a predefined and scheduled route to transport passengers from one origin to one destination (with internal stops), thus forming the entirety of the route. For example, the Madrid-Burgos, Burgos-Santander, or Bilbao-Madrid lines are part of the Madrid-Irun route. Services are equivalent to the frequencies at which the bus starts the route of the line, and they vary depending on how commercial the line is, as well as expeditions, which is the number of buses that leave on each service.

Once the key concepts relevant to our work have been defined, we can continue to explain the nature of concessions. The company that wins the tender operate the route without any direct economic support from the state, that is, without any type of direct subsidy. Moreover, the price per unit of distance (€/km) offered must be the same for all the lines. Within the same route, we can find commercial lines with high demand because they connect strategic cities, either due to their demographic or economic importance, as well as unprofitable routes, which generally offer services in rural territories with low population density and/or more mountainous or inaccessible areas. This polarity between the lines and the fact that the company operates at its own risk and fortune leads to the implicit implementation of an internal cross-subsidy system. Firm operate in profitable lines, which have higher profitability due to operating costs being lower than the price set during the competition for the route, but since the company is contractually obligated to operate all lines of the route regardless of their profitability, the profits of these lines are internally redistributed to cover the losses of the loss-making lines, where costs exceed the price. Cremer et al (1997) make a study of the cross subsidies in the postal sector, where they point out that if the operator had freedom to set its price to cover costs, service would be provided to all potential customers, but given the restriction of uniform prices (to make it "affordable"), they are likely to be below cost for some consumer types.

This system of cross-subsidies, like practically everything in economics, has its advantages and disadvantages.

On the one hand, since the government stays out of it, offering a service (on some lines) at a price below cost does not result in an increase in public spending, as direct subsidies from the state would. Additionally, the state ensures a service on lines that would not be possible with a total liberalization of the system, due to a lack of commercial appeal. Later on, we will evaluate this liberalization and its possible alternatives to ensure territorial connectivity.

However, the system of cross-subsidies results in significant welfare losses. Users of profitable lines face a bus ticket price higher than the cost, contrary to what would happen in a situation of perfect competition. This ends up causing the current system to have a significant regressive impact on the population, since regular bus users, who bear most of the costs, are mainly low-income citizens or students who do not have a private vehicle as an alternative. Additionally, as Asensio and Matas (2023) explain, buses face difficulties competing with other modes that do not have to bear the costs arising from the cross-subsidy system in profitable areas. It should also not be forgotten that since the characteristics of the itineraries are decided by the government, this is a rigid system in terms of adapting to changes in demand, creating new routes, or taking advantage of new technologies.

Overall, the impact of the cross-subsidy system on the public transport model and the population, and more specifically its lower strata, obliges us to study its economic functioning and impacts, as we will proceed to do next.

3. The microeconomic model

By formalizing the cross subsidies system into a mathematical model we intend to understand the intuition about the relation between variables such as price, quantity of consumers and costs. One may think on another crucial variable involved on the bus system, the frequencies of the buses, which are one of the points defined and inspected on the bus tenders. For the seek of simplicity, in this work we are going to keep the variable of frequencies out of the equations, and implicitly assume their no-variation. Next, you will find the solving of a simple partial equilibrium model that characterize the equilibrium of the market under different competing circumstances.

3.1. The model under perfect competition

The variables:

$\boldsymbol{c}$ is the marginal cost of the bus line, in km terms

$\boldsymbol{p}$ is the price/km paid by the consumer

$\boldsymbol{q}$ is the number of passangers-km to whom the service is provided

$\boldsymbol{p = \alpha - \beta q}$ is the consumers inverse demand function

$\boldsymbol{\alpha}$ is the maximum price the consumers are willing to pay

$\boldsymbol{\beta}$ is the slope of the consumers inverse demand function

We assume that:

- We are in perfect competition, such that p = mc
- Marginal cost is the only cost, and it is constant (mc = c)
- $\boldsymbol{\alpha, \beta}$ ***and*** $\boldsymbol{c}$ are exogenous variables

Then, the mathematical resolution of the problem in perfect competition is:

$$p = c$$

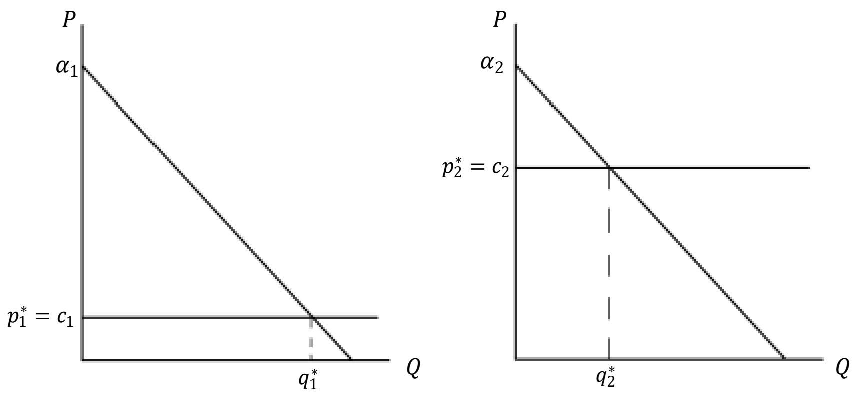
$$p = \alpha - \beta q$$

$$c = \alpha - \beta q$$

$$p* = c\,, \quad q* = \frac{\alpha - c}{\beta}$$

Where $\boldsymbol{p * and\ q *}$ are the equilibrium price and quantity.

The perfect competition graphical representation of situation in which there are 2 lines operated independently, idexed by $j = 1{,}2$, where 1 represents a line with low costs and 2 the one with high costs, would be as:



Low-cost line High-cost line

3.2. The cross subsidies model

Given a situation in which an intercity bus company wins a tender to operate a route in which there are profitable lines with costs being lower than competitive market price (in market equilibrium p=mc), and also unprofitable lines with costs being higher than market prices, a cross subsidies mechanism is implicitly applied. We describe it as follows:

3.2.1. Assuming linear demand functions

$\boldsymbol{p_1 = \alpha_1 - \beta_1 q_1}$ is the consumers inverse demand function for the line with prices higher than costs, we will call it the "Profitable line (1)"

$\boldsymbol{p_2 = \alpha_2 - \beta_2 q_2}$ is the consumers inverse demand function for the line with costs higher than prices, we will call it the "Loss-making line (2)"

$\boldsymbol{p_1 \text{ and } p_2}$ are the price/km paid by a consumer for the Profitable and Loss-making line, respectively

$\boldsymbol{q_1 \text{ and } q_2}$ are the passengers-km of the Profitable and Loss-making line, respectively

$\boldsymbol{\alpha_1 \text{ and } \alpha_2}$ are the maximum price the consumers are willing to pay on the profitable and loss-making line, respectively

$\boldsymbol{\beta_1 \text{ and } \beta_2}$ are the slope of the inverse demand functions of the Profitable line and Loss-making line

<u>Assumptions</u>

- Marginal cost is the only cost, and it is constant in both rutes (mc = c), and described as $\boldsymbol{c_1 \text{ and } c_2}$ (costs of vehicle/km from the Profitable and Loss-making line)
- $\boldsymbol{\alpha_{1,2}}$, $\boldsymbol{\beta_{1,2} \text{ and } c_{1,2}}$ are exogenous variables

<u>Restrictions</u>

The government imposes a restriction as to ensure that prices are the same in both lines and that there is no difference between consumers pays.

$$\boldsymbol{p_1 = p_2}$$

The intercity bus company self-imposes another restriction, a profitability restriction, by which the total revenue perceived by the two lines has to be at least equal to the total cost of providing services in both lines.

$$\boldsymbol{p_1 q_1 + p_2 q_2 \ \geq \ c_1 q_1 + c_2 q_2}$$

Revenue Costs

We will treat it as an equality for the seek of simplicity.

<u>Solving the model</u>

Our objective now is to find $\boldsymbol{p_1, p_2, q_1, q_2}$ in terms of the exogenous variables ($\boldsymbol{\alpha_1, \alpha_2, \beta_1, \beta_2, c_1, c_2}$).

We solve the problem for the following equations:

(1) $\boldsymbol{p_1 = \alpha_1 - \beta_1 q_1}$

(2) $\boldsymbol{p_2 = \alpha_2 - \beta_2 q_2}$

(3) $\boldsymbol{p_1 = p_2}$

(4) $\boldsymbol{p_1 q_1 + p_2 q_2 = c_1 q_1 + c_2 q_2}$

$$\boldsymbol{\beta_{1,2} > 0\ ,\ \ \alpha_{1,2} > 0\ ,\ \ p_{1,2} \geq 0\ ,\ \ q_{1,2} \geq 0}$$

We introduce (1) and (2) into (3) and isolate $\boldsymbol{q_2}$

(1) (2)

$$\boldsymbol{\alpha_1 - \beta_1 q_1 = \alpha_2 - \beta_2 q_2}$$

$$\boldsymbol{q_2 = \frac{\alpha_2 + \beta_1 q_1 - \alpha_1}{\beta_2}}$$

We introduce (1) and $\boldsymbol{q_2}$ from the previous equation into (4) and simplify:

(1) (1)

$$(\boldsymbol{\alpha_1} - \boldsymbol{\beta_1 q_1})\boldsymbol{q_1} + (\boldsymbol{\alpha_1} - \boldsymbol{\beta_1 q_1})\left(\frac{\boldsymbol{\alpha_2} + \boldsymbol{\beta_1 q_1} - \boldsymbol{\alpha_1}}{\boldsymbol{\beta_2}}\right) = \boldsymbol{c_1 q_1} + \boldsymbol{c_2}\left(\frac{\boldsymbol{\alpha_2} + \boldsymbol{\beta_1 q_1} - \boldsymbol{\alpha_1}}{\boldsymbol{\beta_2}}\right)$$

After a few simplification steps, the final (4) expression results in:

$$-\left(\boldsymbol{\beta_1}(\boldsymbol{\beta_1} + \boldsymbol{\beta_2})\right)\boldsymbol{q_1}^2 + \left(\boldsymbol{\beta_2}(\boldsymbol{\alpha_1} - \boldsymbol{c_1}) + \boldsymbol{\beta_1}(2\boldsymbol{\alpha_1} - \boldsymbol{c_2} - \boldsymbol{\alpha_2})\right)\boldsymbol{q_1} + (\boldsymbol{c_2\alpha_1} - \boldsymbol{c_2\alpha_2} + \boldsymbol{\alpha_1\alpha_2} - \boldsymbol{\alpha_1}^2) = \boldsymbol{0}$$

We can treat the expression as a second-degree function. Then, the solution for q1 is:

$$q_1^* = \frac{-\left(\beta_2(\alpha_1 - c_1) + \beta_1(2\alpha_1 - c_2 - \alpha_2)\right) \pm \sqrt{\left(\beta_2(\alpha_1 - c_1) + \beta_1(2\alpha_1 - c_2 - \alpha_2)\right)^2 + 4\left(\beta_1(\beta_1 + \beta_2)\right)(c_2\alpha_1 - c_2\alpha_2 + \alpha_1\alpha_2 - \alpha_1^2)}}{-2\left(\beta_1(\beta_1 + \beta_2)\right)}$$

By symmetry:

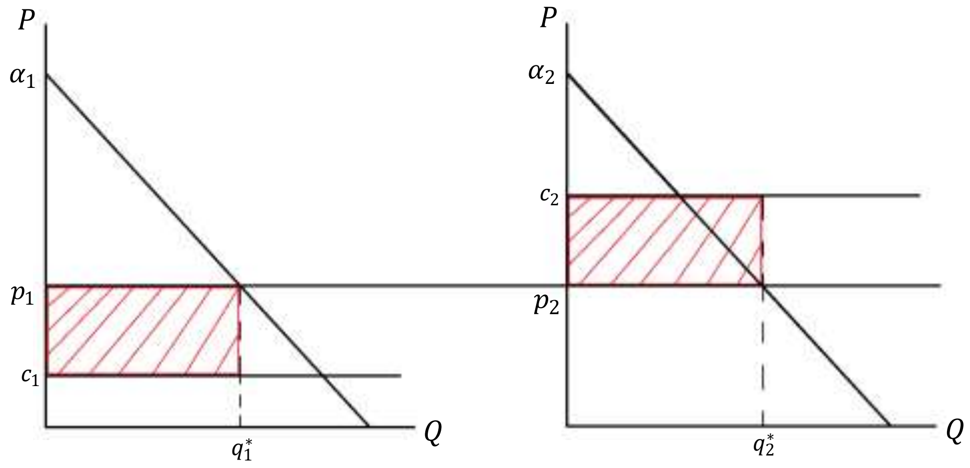
$$q_2^* = \frac{-\left(\beta_1(\alpha_2 - c_2) + \beta_2(2\alpha_2 - c_1 - \alpha_1)\right) \pm \sqrt{\left(\beta_1(\alpha_2 - c_2) + \beta_2(2\alpha_2 - c_1 - \alpha_1)\right)^2 + 4\left(\beta_2(\beta_2 + \beta_1)\right)(c_1\alpha_2 - c_1\alpha_1 + \alpha_1\alpha_2 - \alpha_2^2)}}{-2\left(\beta_2(\beta_2 + \beta_1)\right)}$$

Applying $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ into (1):

$$p_1^* = \alpha_1 - \beta_1 \frac{-\left(\beta_2(\alpha_1 - c_1) + \beta_1(2\alpha_1 - c_2 - \alpha_2)\right) \pm \sqrt{\left(\beta_2(\alpha_1 - c_1) + \beta_1(2\alpha_1 - c_2 - \alpha_2)\right)^2 + 4\left(\beta_1(\beta_1 + \beta_2)\right)(c_2\alpha_1 - c_2\alpha_2 + \alpha_1\alpha_2 - \alpha_1^2)}}{-2\left(\beta_1(\beta_1 + \beta_2)\right)}$$

$$p_2^* = \alpha_2 - \beta_2 \frac{-\left(\beta_2(\alpha_1 - c_1) + \beta_1(2\alpha_1 - c_2 - \alpha_2)\right) \pm \sqrt{\left(\beta_2(\alpha_1 - c_1) + \beta_1(2\alpha_1 - c_2 - \alpha_2)\right)^2 + 4\left(\beta_1(\beta_1 + \beta_2)\right)(c_2\alpha_1 - c_2\alpha_2 + \alpha_1\alpha_2 - \alpha_1^2)}}{-2\left(\beta_1(\beta_1 + \beta_2)\right)}$$

A graphical representation will help us to understand the outcome of the problem:



The red area represents the amount "subsidized" from one line to another. The area below prices and above costs from the profitable line graph is re-allocated by the firm to cover costs from the other. By doing so, the firm is securing its sustainability despite being obliged by contract of the route to provide services in unprofitable lines.

3.2.2. Assuming logarithmic demand functions

Linear inverse demand functions are useful for introducing and understanding the model, but the reality of the shape of the intercity bus demand function is very different. The price elasticity of the demand is constant for the intercity bus industry in Spain as indicate Asensio and Matas (2019); the percentage change in quantity demanded is constant for every percentage change in price, so realistically we will face logarithmic demand functions:

$\boldsymbol{Ln\, p_1 = \alpha_1 - \beta_1\, Ln\, q_1}$ is the consumers inverse demand function for the line with prices higher than costs, we will call it the "Profitable line (1)"

$\boldsymbol{Ln\, p_2 = \alpha_2 - \beta_2\, Ln\, q_2}$ is the consumers inverse demand function for the line with costs higher than prices, we will call it the "Loss-making line (1)"

Assumptions, restrictions and involving variables will be the same as on the linear demand function model, already explained.

Solving the model

Our objective is to find $\boldsymbol{p_2, q_1, q_2}$ in terms of the exogenous variables ($\boldsymbol{\alpha_1, \alpha_2, \beta_1, \beta_2, c_1, c_2}$).

We solve the problem for the following equations:

(1) $\boldsymbol{Ln\, p_1 = \alpha_1 - \beta_1\, Ln\, q_1}$

(2) $\boldsymbol{Ln\, p_2 = \alpha_2 - \beta_2\, Ln\, q_2}$

(3) $\boldsymbol{p_1 = p_2}$

(4) $\boldsymbol{p_1 q_1 + p_2 q_2 = c_1 q_1 + c_2 q_2}$

$$\boldsymbol{\beta_{1,2} > 0\,,\ \alpha_{1,2} > 0\,,\ p_{1,2} \geq 0\,,\ q_{1,2} \geq 0}$$

From (1) and (2) we get that

$\boldsymbol{p_1 = A\, q_1{}^{-\beta_1}}$, where $\boldsymbol{A = e^{\alpha_1}}$

$\boldsymbol{p_2 = B\, q_2{}^{-\beta_2},}$ where $\boldsymbol{B = e^{\alpha_2}}$

$\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ depends potentially on $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$, the determinant powers are $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$

The next step is to equate the previous expressions, given the $\boldsymbol{p_1 = p_2}$ condition. After that, we isolate $\boldsymbol{q_2}$ (or $\boldsymbol{q_1}$)

$$\boldsymbol{A\, q_1{}^{-\beta_1} = B\, q_2{}^{-\beta_2}}$$

$$\boldsymbol{q_2 = (\frac{B}{A})^{\frac{1}{\beta_2}} q_1{}^{\frac{\beta_1}{\beta_2}}}$$

Now that we have $\boldsymbol{q_2}$ in terms of the exogenous variables, we can substitute it (and $\boldsymbol{p_1}$) into (4):

$$\boldsymbol{A\, q_1{}^{-\beta_1}(q1 + (\frac{B}{A})^{\frac{1}{\beta_2}} q_1{}^{\frac{\beta_1}{\beta_2}}\ ) = c_1 q_1 + c_2 (\frac{B}{A})^{\frac{1}{\beta_2}} q_1{}^{\frac{\beta_1}{\beta_2}}}$$

Note that this is the expression that solves for $\boldsymbol{q_1}$. This is a polynomial equation of more than second degree, so we would need numerical values to find a solution. As it is not possible to find an analytical solution, this is the final expression that solves our problem.

By symmetry we solve for $\boldsymbol{q_2}$:

$$\boldsymbol{A\, q_2{}^{-\beta_2}(q2 + (\frac{B}{A})^{\frac{1}{\beta_1}} q_2{}^{\frac{\beta_2}{\beta_1}}) = c_2 q_2 + c_1 (\frac{B}{A})^{\frac{1}{\beta_1}} q_2{}^{\frac{\beta_2}{\beta_1}}}$$

Now we proceed to find $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$. From (1) and (2) we get that:

$\boldsymbol{q_1} = (\frac{\boldsymbol{p_1}}{\boldsymbol{A}})^{-\frac{1}{\beta_1}}$, where $\boldsymbol{A} = \boldsymbol{e^{\alpha_1}}$

$\boldsymbol{q_2} = (\frac{\boldsymbol{p_2}}{\boldsymbol{A}})^{-\frac{1}{\beta_2}}$, where $\boldsymbol{B} = \boldsymbol{e^{\alpha_2}}$

We can go directly to (4) and get the following expression, since $\boldsymbol{p_1 = p_2}$:

$$\boldsymbol{p_1((\frac{p_1}{A})^{-\frac{1}{\beta_1}} + (\frac{p_1}{A})^{-\frac{1}{\beta_2}}) = c_1(\frac{p_1}{A})^{-\frac{1}{\beta_1}} + c_2(\frac{p_1}{A})^{-\frac{1}{\beta_2}}}$$

By symmetry:

$$\boldsymbol{p_2((\frac{p_2}{A})^{-\frac{1}{\beta_1}} + (\frac{p_2}{A})^{-\frac{1}{\beta_2}}) = c_1(\frac{p_2}{A})^{-\frac{1}{\beta_1}} + c_2(\frac{p_2}{A})^{-\frac{1}{\beta_2}}}$$

As it happens for $\boldsymbol{q_1} and\ \boldsymbol{q_2}$, this is the final expression that solves for $\boldsymbol{p_1}\ and\ \boldsymbol{p_2}$.

4. Empirical application of the model

After formalizing and solving the theoretical model, we are now able to address empirically the cross subsidies question. Our goal will be, based on the existing literature and databases that collect some of the variables already mentioned that solve the model, to use computational methods to calculate the remaining elements that allow us to determine which lines are profitable and which ones are unprofitable. The purpose of this process is to understand the real market situation and conduct an analysis that explain the model and the situation of the companies operating in each (and all) of the lines on which they provide service. Moreover, we will be able to calculate whether the company is operating profitably and therefore assess the effectiveness of the current bidding system in its attempt to resemble the atmosphere of concessions to that of a liberalized market operating under perfect competition.

4.1. Data description and unknown variables computation

To carry out the empirical analysis of the intercity bus system, we will use the intermunicipal demand for the bus lines of route 157 in 2017, belonging to the "Madrid - Irún with hijuelas" connection. "Hijuelas" refers to all connections between nearby towns and cities that form a line of the route and to which the company is obliged to offer service as agreed upon in the conditions of the tender. The data for the VAC-157 (which is the technical name given to the Madrid - Irun route) is provided by the “Ministerio de Transporte, Mobilidad y Agenda urbana” (MITMA). One difficulty arises during data processing: the line to which each group of passengers belongs is not specified, only the location of the stop (it is common for two munpicipalies to be connected by more than one line under the same contract). In order to make an approximation, Asensio (2023) distributes the observed demand to each line proportionally based on the schedules of the VAC-157, where the precise itinerary and municipal code of the stops made by the buses are determined. The criteria to the allocation of passengers of each line is based on the quantity of services and their relative travel times, since each line may differ on the number of stops between the two municipalities. The data used is distributed in a matrix format, specifying the origin and destination of all the 49 lines that comprises the route, as well as the number of passengers, the number of passanger-kilometer, the total amount of money collected and the price paid by each consume.

Our data is both in annual format and broken down into 4 quarters; we will use the quarterly values, as the services offered by the company vary depending on the time of year and are commonly evaluated quarterly (the supply tends to increase during the summer months, for example). Let us remember that our objective is to determine the value of ($\boldsymbol{p}$, $\boldsymbol{q}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{c}$) for each of the lines.

4.1.1. Demand variables

After the previously explained data treatment we can find both $\boldsymbol{p}$ (collected under the name $\boldsymbol{pkm}$) and $\boldsymbol{q}$ ($\boldsymbol{nvikm}$), i.e., the price of the trip per kilometer and the number of passengers-kilometer.

As for $\boldsymbol{\beta}$, Asensio and Matas (2019) calculate, through an estimation of the aggregate demand function of the intercity bus market in 2016, that the price elasticity of demand is -0.69 with 99% confidence. Considering that we are operating with inverse demand functions, $-\frac{1}{0.69}$ will be the value of $\boldsymbol{\beta}$, with which we could calculate elasticity given a specific price and quantity in the market.

At this point we only remain to know the values of $\boldsymbol{\alpha}$. Once calculated from the previous variables (we will use R studio for this), we will have all the information on the demand function, which we remember was $\boldsymbol{Ln\,p = \alpha - \beta Ln\,q}$. Then, the formula for $\boldsymbol{\alpha}$ calculation is:

$$\boldsymbol{\alpha = Ln\,pkm + \beta\,Ln\,nvikm}$$

We will choose a line from VAC-157 and calculate its values in order to illustrate the meaning of each of them. For instance, the calculation for line 6 alpha (1st semester) is:

$$\alpha = Ln\ 0{,}07523 - \frac{1}{0{,}69} Ln\ 2818529{,}48 = 18{,}937$$

4.1.2. Costs

The accuracy in calculating the costs of each line is essential in the implementation of our model as the difference in values between the lines is where the sense of the system of cross-subsidies lies. However, it is impossible for us to know the exact costs of each line due to the lack of transparency from the companies, either for reasons of competition or negotiation with the administration, so we will have to make an estimation.

According to Alsa, one of the main operators of intercity routes in the country that offers service in more than 2.000 regular lines, including those comprised in the Madrid-Irún route (which is the subject of study in this work), most of their intercity buses have a capacity of 55 seats. The Observatorio de Costes del Transporte de Viajeros en Autocar (2017), that is the result of several studies carried out under the auspices of the Ministerio de Fomento, concludes that the total average cost/km of a bus with 39 to 55 seats is 1.363€/km (national average). This calculation considered both direct costs, whether for time (amortization and financing of the vehicle, driving personnel, diets, insurance, tax costs) or per kilometer (fuel, tires, repairs and maintenance), as well as indirect costs. However, in our empirical analysis we evaluate costs per passenger-kilometer, so we will have to adjust the cost per kilometer of the bus to the occupancy rate of the bus on each line and in a greater instance to the number of reinforcements that each line has, taking into account that it is common for more than one bus to leave simultaneously in the same service, especially on highly frequented lines, in order to cover all the demand. It is important to accurately determine these variables since a large part of the difference between loss-making lines, which usually connect villages where the population density is very low and therefore there is no demand for bus service to lower costs, and profitable lines, where buses are more crowded and therefore costs for the company are lower, depends on it.

There are also other sources of cost differentiation depending on the line, such as the geographical relief. Mountainous villages, which also usually have low population density, will cause higher operating costs and lower profitability due to an increase in fuel consumption caused by the elevation that the bus must overcome in order to guarantee connectivity to the municipality. However, the calculation of this additional cost is beyond the scope of this work.

The variables included on the cost computation are:

$\boldsymbol{avcost}$, represents the average cost/km of a 55-seat bus, and it is 1.363€/km as we already stated.

$\boldsymbol{serv}$ refers to the services, the number of times that the bus (or buses) operates for a specific line, computed from InfGuiaHorariosVAC-157, where the schedule and its variations depending on the moment of the year are explained. For instance, the line 6 has 212 services per trimester. An annual daily service departing from Madrid to San Sebastian for the outbound direction (90 per quarter), and another for the return direction also daily and annual (90 per quarter), as well as two additional services on Sundays and holidays (at 5pm and 8:30pm), representing 32 quarterly services.

$\boldsymbol{km}$ are the total kilometers of each route. The line 6 bus, for example, covers 455 km.

$\boldsymbol{npline}$ represents the number of passengers per complete line for each service. It is important as, if we have the passengers that completed the line (29 for the line 6), we can compute the total reinforces. $\boldsymbol{npline}$ comes from:

$$\boldsymbol{npline} = \frac{\boldsymbol{nvikm}}{\boldsymbol{km} * \boldsymbol{serv}}$$ [1]

$\boldsymbol{nbuses}$ is the number of buses that operate in each service, rounded up. It is common to more than one bus to leave simultaneously on the same service in order to cover all the demand, mostly on high frequented lines. Our computations show that the line 6 has 2 reinforcements (on average) for each service made, meaning that 2 buses depart at the same time. $\boldsymbol{nbuses}$ is calculated from:

$$\boldsymbol{nbuses} = \frac{\boldsymbol{npline}}{\boldsymbol{55} * \boldsymbol{ucap}}$$

55 is the total capacity of each bus, and $\boldsymbol{ucap}$ is the used capacity. According to the MITMA the average occupancy per vehicle is around 26

[1] Remember that $\boldsymbol{nvikm}$ represents the passenger-km, it is, the number of km travelled by each passenger multiplied by the total passengers. The denominator of the formula represents the total kilometers made by the bus. The numbers for the first trimester of the line 6, the example we are analyzing, are: $\frac{\mathbf{2818529}}{\mathbf{455*212}}$

passengers for a 55-seat bus. Then, the $\boldsymbol{ucap}$ will be 0.47 (a 47% of used capacity).

Finally, the formula that determines the cost per passenger-kilometer will be:

$$\boldsymbol{c = costs = \frac{avcost * serv * km * nbuses}{nvikm}}$$

Once the procedure to the cost computation has been explained, we are now able to compute the costs for all the 49 lines that comprise the VAC157. Line 6 in particular, which we are using as an example, bears the following cost per kilometer for the first semester:

$$c = \frac{1.363 * 212 * 455 * 2}{2818529} = 0{,}0932$$

4.2. Determining profits and subsidies

Right now, we have all the values of the variables belonging to the initial problem. From them it will be possible for us to identify which of the 49 lines operate at a loss and which are profitable. The objective is to calculate the total amount of profits that are redistributed by the company to cover the losses of the loss-making lines, or in other words, calculate the cross-subsidies. The mathematical formula is:

$$\boldsymbol{profits = pkm * nvikm - c * nvikm}$$

Regarding the case of the line 6, for the first semester:

$$profits = 0{,}0752 * 2818529 - 0{,}0932 * 2818529 = -50890€$$

Note that this line incurs losses, because the cost/km, calculated based on the level of occupancy, is higher than the price/km in this quarter. However, the situation varies in the following quarters since the demand increases. Nevertheless, the total amount of profits remains negative:

profits1(€)	profits2(€)	profits3(€)	profits4(€)	totalprofits(€)
-50890,61	2264,04	37484,27	9617,61	-1524,69

This example is repeated in many of the analyzed lines, as we will see below, due to seasonal variations in demand as well as in the services offered. It should be noted that, in order to consider a line profitable (or loss-making), we will take into account the total profits of the year, that is, the aggregate of the 4 quarters.

4.3. Results and analysis

Through the programming using R studio, we have been creating entries for each of the variables for each of the bus lines, which are added to the initial matrix of intermunicipal bus demand for the VAC-157. The results of the code are as follows:[2]

line	nvikm1	pkm1	alpha1	costs1 (€/vikn	profits1 (€)	profits2 (€)	profits3 (€)	profits4 (€)	totalprofits (€)
1	3115686	0,0732	19,06	0,0582	46708	50752	97805	44346	239611
2	3104304	0,0737	19,06	0,0569	52064	59991	97191	55479	264725
3	4403117	0,0736	19,56	0,0549	82134	49780	106310	45340	283564
4	1660038	0,0769	18,19	0,0672	16076	45850	-36686	-55523	-30282
5	481769	0,0728	16,34	0,0689	1899	10473	8015	10150	30537
6	2818529	0,0752	18,94	0,0933	-50891	2264	37484	9618	-1525
7	4044327	0,0732	19,43	0,0712	8400	85170	-20129	81478	154919
8	3969356	0,0729	19,40	0,0827	-38611	37797	50197	22670	72053
9	4225046	0,0693	19,44	0,0537	65750	-88280	-63936	-113628	-200094
10	995656	0,0694	17,35	0,0591	10251	-31109	-23339	-37209	-81406
11	852663	0,0695	17,12	0,0691	336	15377	-36476	10155	-10609
12	525823	0,0673	16,39	0,0982	-16249	-1166	-24077	-4241	-45733
13	926937	0,0697	17,25	0,1393	-64477	-39543	16842	-43239	-130417
14	2000571	0,0702	18,37	0,1033	-66189	-21873	36832	-37508	-88738
15	216385	0,0712	15,16	0,0802	-1949	4389	-36320	1066	-32814
16	273602	0,0652	15,41	0,1761	-30332	-26111	-20591	-27517	-104551
18	217775	0,0759	15,23	0,2024	-27551	-21182	-14335	-26472	-89540
19	147275	0,0752	14,66	0,2599	-27191	-21877	-17129	-25833	-92029
20	52555	0,0756	13,17	0,1502	-3921	-2360	2539	3305	-437
21	12872	0,0796	11,18	1,5968	-19529	-18361	-17958	-18869	-74717
22	37016923	0,0749	22,66	0,0780	-116130	177838	339826	285905	687440
23	1875239	0,0720	18,30	0,0539	33985	-29807	9456	-39260	-25625
24	530905	0,0697	16,44	0,1109	-21862	-9699	-4059	-12782	-48402
25	583616	0,0712	16,60	0,0617	5588	-20000	-14180	-22399	-50991
26	2572162	0,0731	18,78	0,0558	44456	-46481	14409	-67245	-54861
27	72075	0,0775	13,65	0,0964	-1363	318	-3368	-685	-5098
28	30069	0,0803	12,42	0,5467	-14023	-14161	-14114	-14141	-56438
29	80301	0,0788	13,83	0,3319	-20321	-18773	-20241	-17127	-76462
30	36989	0,0816	12,74	0,1403	-2173	-461	-1950	6	-4578
31	0	NA	NA	NA	NA	NA	-2213	NA	-2213
32	36970	0,0816	12,74	0,1217	-1482	242	-6450	684	-7007
33	162246	0,0785	14,84	0,1753	-15700	-9888	3305	-13490	-35773
34	29740	0,0763	12,35	0,1490	-2161	-2119	-1843	-2072	-8194
35	0	NA	NA	NA	NA	NA	NA	NA	0
36	1403112	0,0733	17,90	0,0648	11991	31143	12827	30519	86480
37	0	NA	NA	NA	NA	NA	-3204	NA	-3204
38	1	0,2179	-1,88	524,2308	-409	-409	-409	-409	-1635
39	13	0,2054	2,13	39,0028	-504	NA	NA	NA	-504
40	3027404	0,0738	19,02	0,0583	46728	52468	87245	48342	234783
42	1861448	0,0704	18,27	0,0555	27848	19193	27744	2612	77397
43	4100549	0,0735	19,46	0,0590	59605	83468	79211	75528	297812
44	764252	0,0699	16,97	0,0695	283	-32357	14415	17797	140
45	625015	0,0677	16,65	0,0826	-9305	8451	-9393	4545	-5702
46	615808	0,0678	16,63	0,0839	-9873	8632	-8737	4881	-5096
47	2396928	0,0703	18,63	0,0862	-38014	18272	-4548	-1435	-25725
48	858243	0,0700	17,14	0,0602	8434	-19221	33396	-22857	-248
49	5013667	0,0749	19,77	0,0587	81085	120901	44705	37402	284092
50	149652	0,0630	14,50	0,1752	-16799	-15276	NA	-15175	-47251
51	3574525	0,0765	19,30	0,0625	50093	-1871	55465	23217	126904

[3]

[2] The table only shows first quarter values for the variables nvikm (in passanger-km terms), pkm (price/km), alpha and cost, in order to make it legible. Profits are shown quarterly and with its total sum.
[3] The NA values in some lines for certain semesters originate from the bus not operating during that period.

At first glance, disparate values in the costs of the lines can be appreciated, some of which exceed the price, thereby causing negative values in the profit columns on the right. This is the case with Line 6, where profit values vary throughout the quarters, resulting in both negative and positive figures. Demand fluctuates depending on the time of year, such as during vacation periods, for example, while the static supply, determined by the administration, fails to accommodate these changes. Consequently, there is a variation in the occupancy rate of the buses, leading to fluctuations in the cost per kilometer over time. Therefore, it is necessary to look at the total profits column to know if a line is profitable or loss-making.

However, let us remember that the importance of the cross-subsidy system lies in its ability to internalize the losses of those unprofitable lines at the expense of the profits from the more commercial ones, while seeking a similar climate to that of liberalization (which commonly implies less benefits for the firm) through the "competition for the market" system. Therefore, to carry out a more thorough analysis of the market, it would be necessary to calculate the total profits of the company, by adding up the “totalprofits” column.

According to the findings, the company running route 157 has a favorable financial outcome, with profits totaling 1.392.557€. The income from the profitable lines and from the loss-making lines are 2.840.456€ and -1.447.900€, respectively. Under a situation that simulates perfect competition, the losses of the unprofitable lines should be equal to the profits of the remaining ones. In other words, the company would operate the route with a zero-profit margin, as stated by the restriction $\boldsymbol{p_1 q_1 + p_2 q_2 = c_1 q_1 + c_2 q_2}$ in our simple model with only two lines. However, empirical evidence contradicts this proposition and reveals a different reality in which the company enjoy a significant profit margin. From this, an important conclusion can be drawn, which is the lack of effectiveness of the current bidding system, using the “pliegos”, to simulate a liberalized market situation with perfect competition. A better adjustment of the “pliegos” should reduce the prices of the proposals from applicant companies in order to bring them closer to the marginal cost (overall). Currently, despite the existence of loss-making lines, the effect of a higher margin on the profitable lines prevails. Therefore, there is still room for improvement in the industry, either through a change in the “pliegos” or the bidding system, or through market liberalization, which we will discuss next.

5. Liberalization of the market

Users of liberalized interurban bus markets, at least in Europe, enjoy lower ticket prices than in Spain, where we have a bidding system that has already been demonstrated along this work to fail in their intentions to simulate perfect competition. Let us recall the social importance of a bus market with low mark-ups in prices, given the nature of its users. Regular interurban bus travelers tend to be low-income individuals or students who cannot afford a private vehicle, or elderly people who, due to their limitations, see the bus as a safer transportation option. Therefore, the bus plays a crucial role in ensuring sustainable mobility for the lower strata of society, a mobility that is undermined by the overpricing practices of companies. This situation has a regressive impact on the population, as it affects those population groups with fewer resources. For this reason and many others, it is necessary to explore a scenario of liberalization of the intercity bus market.

5.1. Expected impacts

A liberalization of the market would imply changes in the fundamentals of the bus market in Spain, potentially causing various effects that are discussed below.

Regarding the reorganization of services, the termination of concession contracts would mean that companies are no longer obligated to operate on unprofitable lines. The network would be rearranged to seek greater efficiency, reducing overall service costs and average travel times. Companies would compete on the more commercial lines, both existing ones and potential new ones, which would connect major provincial capitals and tourist destinations. Stops near large cities could be absorbed by the regional concession network. However, stops on loss-making lines, those connecting rural areas or areas with lower population density, would be in serious danger of disappearing. Without the support of cross-subsidization mechanisms, many connections between remote areas and towns in Spain would be lost, resulting in territorial disconnection, as companies would not find it profitable to operate on these lines. An alternative, which will be discussed later, would be the introduction of direct subsidies by the state, the consideration of those lines as a

Universal Service Obligation, which, as Cremer et al (1997) define, is the obligation to provide all users with a range of basic services at an affordable price.

As for the market structure, the study on interurban bus passenger transport conducted by the National Commission of Markets and Competition in 2022 predicts that market liberalization would increase concentration. However, despite the concentration, the power of companies could be reduced due to the presence of a large number of smaller operators with regional coverage and competition from the railway as an intermodal alternative.

The main effect of liberalization would be the reduction of fares on commercial services, particularly on what we have referred to as "profitable lines" throughout this study. Under suitable competitive conditions, prices (no longer influenced by cross-subsidization) would decrease. Consequently, vulnerable individuals using buses on these lines would no longer have to bear the additional cost associated with supporting loss-making lines (neither the higher price arising from economic ambitions of companies) and could travel at lower fares. Furthermore, according to the CNMC(2022), efficiency gains resulting from economies of scale and scope acquired by operators after liberalization would allow for further fare reductions.

The CNMC also anticipates improvements in frequencies, service quality, and variety, as well as an increase in demand. This potential demand increase, which we have also computed, is particularly relevant due to its environmental implications. According to the MITMA, CO_2 emissions per passenger from a bus are six times lower than those from a car. In fact, buses are three times more fuel-efficient than private vehicles in terms of liters of fuel per passenger/kilometer transported. It is estimated that a bus replaces between 14 and 30 private vehicles. In the context of the climate crisis, this change of transport demand makes liberalization an environmental necessity.

Overall, liberalization presents a plausible improvement for the market in many ways. However, it also brings forth a series of aspects that could be contentious and that the administration should address, such as the management and access regime to stations or the provision of service on unprofitable lines, which is relevant to our work and will be discussed further.

6. An alternative approach: estimating the liberalized system with direct subsidies

Liberalizing the intercity bus market, as we have mentioned before, would have an impact on the demand, which would increase on the liberalized lines. The new equilibrium point (for low-cost lines) will resemble to that described on section 3.1, where the model under perfect competition is solved. Additionally, under a market liberalization situation that replaces the current bidding system and thus eliminates the mechanism of cross-subsidies, companies will not be willing to provide service on unprofitable lines. Therefore, the government, if it wants to guarantee the mobility of current users of these lines (remember, people who cannot use private vehicles as an alternative) as well as territorial cohesion, must introduce a direct subsidy to these lines, it is, consider them as Universal Service Obligations.

6.1. Increase in demand approximation

We have calculated the potential increase in quantity demanded, in terms of passengers-km, the metric we have been using to compute the equilibrium until now (nvikmlib), but also in terms of new total passengers of each line (nviajlib), as it is easier to quantify the increase. To calculate the latter, we first needed to determine the average kilometers traveled by passengers on each line (meankm). This was done by dividing the total passenger-kilometers by the length of each line (we assume that there's no change on mean of km travelled by each line's consumer). It is necessary to mention that the following results are not taking into account the impact on demand that intermodal competition developed after liberalization could have, thus giving higher values than estimated now.

line	totalprofits (€)	nviaj	nvikmlib1	meankm1	nviajlib1	totalnviajlib
1	239611	55681	3649226	276	13212	65660
2	264725	50804	3709967	302	12298	60985
3	283564	71871	5387128	299	18007	82423
4	-30282	17547	1660038	473	3509	17547
5	30537	8394	500634	278	1803	9645
6	-1525	39666	2818529	354	7952	39666
7	154919	66607	4125427	295	14005	72768
8	72053	66063	3640883	282	12903	68888
9	-200094	81192	4225046	243	17403	81192
10	-81406	19521	995656	241	4138	19521
11	-10609	16799	852663	240	3554	16799
12	-45733	7182	525823	425	1238	7182
13	-130417	13065	926937	412	2248	13065
14	-88738	37122	2000571	270	7398	37122
15	-32814	5055	216385	259	835	5055
16	-104551	11304	273602	117	2332	11304
18	-89540	9890	217775	115	1900	9890
19	-92029	6496	147275	126	1167	6496
20	-437	4391	52555	159	330	4391
21	-74717	1857	12872	49	264	1857
22	687440	400089	35983512	401	89640	417020
23	-25625	40280	1875239	236	7953	40280
24	-48402	16114	530905	161	3306	16114
25	-50991	20225	583616	133	4375	20225
26	-54861	131092	2572162	96	26848	131092
27	-5098	5520	72075	65	1105	5520
28	-56438	3424	30069	32	954	3424
29	-76462	3730	80301	88	908	3730
30	-4578	2031	36989	80	465	2031
31	-2213	419	0	NA	NA	419
32	-7007	3755	36970	80	465	3755
33	-35773	12669	162246	77	2097	12669
34	-8194	1484	29740	85	350	1484
35	0	0	NA	NA	NA	0
36	86480	22677	1528338	315	4851	25782
37	-3204	56	0	NA	NA	56
38	-1635	0	1	6	0	0
39	-504	1	13	13	1	1
40	234783	49258	3559598	303	11759	58053
42	77397	35597	2194750	266	8255	38746
43	297812	69278	4773590	291	16401	80652
44	140	10792	767062	412	1863	11142
45	-5702	9273	625015	389	1607	9273
46	-5096	8666	615808	417	1477	8666
47	-25725	44265	2396928	276	8670	44265
48	-248	12336	858243	408	2106	12336
49	284092	54189	5930010	401	14772	62097
50	-47251	5060	149652	98	1527	5060
51	126904	38175	4110342	462	8894	40998 [4]

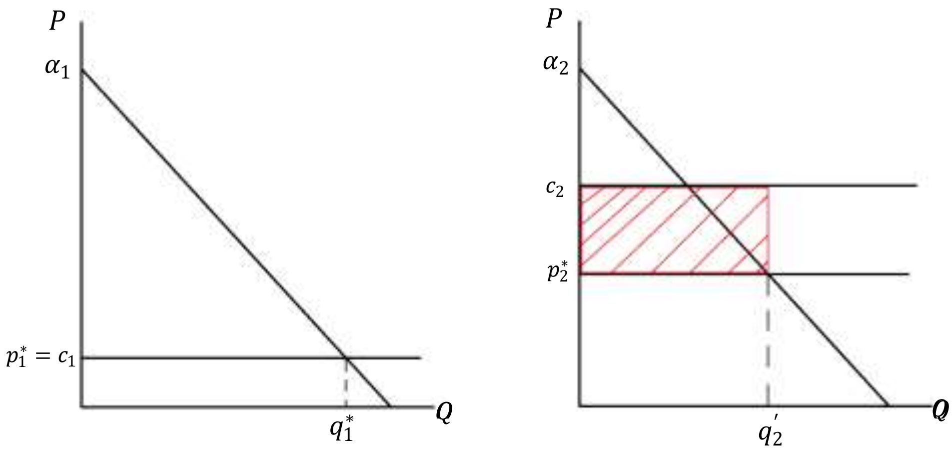
[4] The subscripts 1 in some of the variables indicate that the values shown are for the first quarter. For the rest of the variables in the table, the values are annual.

As we can see in the table, the liberalized lines experience an increase in the number of passengers: the route 157 is expected to accommodate 95.384 new travellers yearly (from 1.590.965 to 1.686.349), which represents a 6% increase on the total demand. Note that the calculation of the new demand has been made under the assumption that the new price/km of those lines liberalized would be equal to the cost/km of the line, something that it would be easy to differ in the real world, even if the competition is free to enter. However, even if such values were not reached we can safely assume that intercity buses would gain a significant number of consumers with liberalization. A prove of that is the reduction on prices and then demand increases that followed the liberalization on all other European countries.

Thanks to the table, it is also easier for us to see how the number of passengers does not vary in those lines where there are losses (the value in the totalprofits column is negative). The example of line 6 is illustrative, which has 39.666 passengers in both situations. This is due to our assumption that under a liberalized system with direct subsidies the administration would only cover the costs of the lines without allowing for a decrease in consumer price that would cause an increase in the equilibrium quantity.

6.2. Computation of the direct subsidies

Liberalization would imply the elimination of both the profitability restriction and the government restriction described throughout section 3.2. If the government wants to ensure the operation of high-cost lines, it will need to provide a subsidy equivalent to the difference between the cost and the price that the users of the line are expected to pay $(\boldsymbol{p_2^*})$. To better understand this, let's look at the following graphical representation:



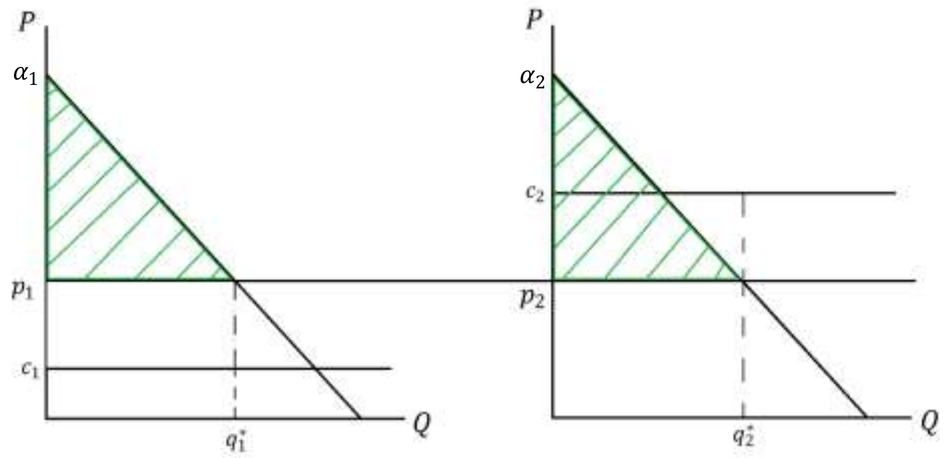
The red area represents the subsidy that the state must pay, and which is calculated as follows:

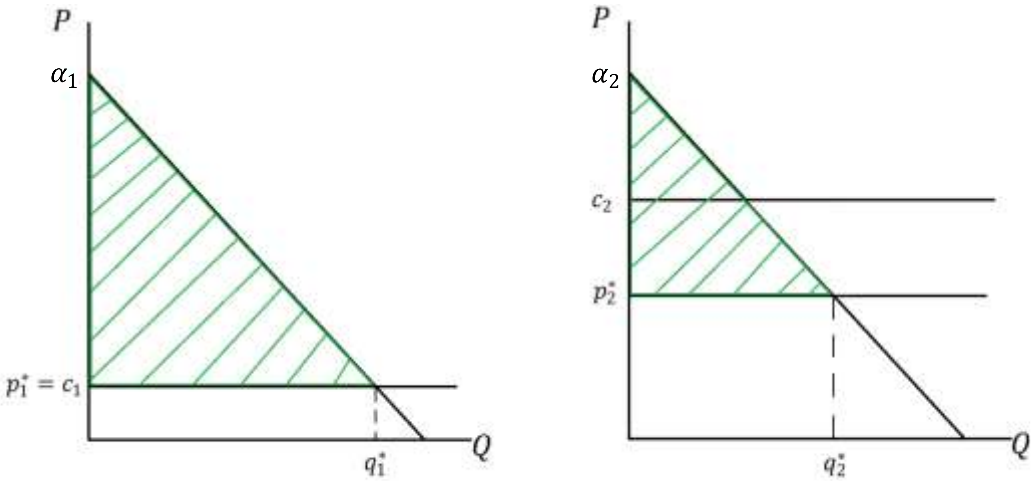
$$\boldsymbol{q_2(c_2 - p_2) = S}$$

If we assume a final consumer price equivalent to the price prior to liberalization, the total sum of direct subsidies would be equivalent to the losses from the deficit lines, which we have already calculated during the empirical application of the model, and that is 1.447.900€.

6.3. Consumer surplus variation

The reduction in prices on some lines, resulting from market opening, also translates into a variation of consumer surplus. This effect, since we are not assuming any price increase under the new system in this study and assuming a continuity of services in all lines with direct subsidies, will clearly be positive. Next, we can visually observe the increase in the consumer surplus area through a graphical representation:





In the first figure, the market is in the initial stage with an established system of cross subsidies. In contrast, the second figure represents the liberalized system with direct subsidies on lines with high costs. We can clearly observe an increase in the area, represented by the green color.

Furthermore, it would be of interest to calculate and quantify this increase. The ultimate objective is to compare the social benefits of introducing subsidies with the social costs incurred by taxpayers, in order to assess the social impact of the project. It is worth noting that transitioning from a system of cross subsidies (where the affected individuals belong to the lower part of the income distribution) to one funded by the entire population (or even higher-income earners) would eliminate its regressive nature. Although this comparative analysis goes beyond the scope of the study, we have utilized computational methods to calculate the variation in consumer surplus.

While the consumer surplus under cross subsidies is 4.850.486.208, the new approach (again, assuming price/km equal cost/km on liberalized lines) makes it increase until 5.167.036.081, which represents an increase of 6,52%. Next, there's included a table that resumes the total increase for each line:

line	totalCScross	totalCSlib
1	149582217	176510535
2	148635229	178582189
3	214686513	246272747
4	76103828	76103828
5	19342055	22256121
6	133013460	133013460
7	192726177	210791275
8	183725330	191763388
9	193685367	193685367
10	41175940	41175940
11	34871586	34871586
12	25749724	25749724
13	47767842	47767842
14	96194389	96194389
15	11805977	11805977
16	10570489	10570489
18	8805214	8805214
19	6013387	6013387
20	5342664	5342664
21	581071	581071
22	1822687985	1900367589
23	86465210	86465210
24	21621454	21621454
25	23110848	23110848
26	121778001	121778001
27	2692558	2692558
28	716319	716319
29	2434912	2434912
30	1193594	1193594
31	236294	236294
32	2457589	2457589
33	7735340	7735340
34	773019	773019
35	0	0
36	66618975	75530575
37	3872	3872
38	-3	-3
39	13	13
40	144335458	170242297
42	90209169	98177186
43	199759074	232739321
44	38773917	40051397
45	30969294	30969294
46	30965425	30965425
47	119428234	119428234
48	44355650	44355650
49	215393753	246985792
50	3650565	3650565
51	171741228	184496538

7. Conclusions

The current intercity bus market in Spain, the main medium and long-distance transportation, is characterized by following a competition for the market system in which a monopolistic concession for a route is granted to a company that operates without public help on different lines, whether profitable or unprofitable, creating a system of cross-subsidies among them. This system has a regressive impact on the population because there are consumers, the most, belonging to lower social strata, who pay a higher price than they should as to maintain it.

In this study, the economic model of cross-subsidies has been formalized and solved. Thanks to this, using bus demand data for the Madrid-Irun route and based on a cost approximation, we have been able to calculate the exact amount of profits that the company uses as subsidies for each of the lines. The results of the empirical application are clear, despite there being lines where operating costs do not allow for profitability, the total profits of the company are positive and significant. We can then conclude that the current bidding system fails in its intention to simulate an atmosphere of open competition through auctions, where companies see their profits diminished by having to present a competitive uniform price.

Liberalization is an economic option that could be considered for the intercity bus market in Spain. However, although it would reduce the prices of commercial lines, it does not ensure the continuation of service on loss-making lines. In fact, companies would stop operating on these routes due to the unprofitable nature of the line. An alternative would be the provision of Universal Service Obligation to those lines, and their financing through direct subsidies from the public budget to companies operating.

In addition to calculating the necessary amount of these direct state subsidies, this study also attempts to estimate the potential variation in demand and consumer surplus with liberalization, without considering the impact of intermodal competition. The findings clearly demonstrate an increase in passengers welfare.

Overall, we have highlighted the problems of the current system of cross-subsidies and analyzed a proposal for liberalization with direct subsidies that would eliminate the progressive nature and lack of competition in the Spanish bus industry.

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Annex:

```
rm(list=ls())

#Empirical application of the cross subsidies model
# Constant parameters
beta= 1/0.69
avcost = 1.363
buscap = 55
ucap = 0.47



# Upload modified database.
setwd("C:/Users/Tester/Desktop/TFG/Dades/Dades Definitives")
library("readxl")

## Warning: package 'readxl' was built under R version 4.2.3

bus157 <- read_excel("vac157.xlsx")
View(bus157)



# Convert it into a data frame.
bus157 = as.data.frame(bus157)
head(bus157)

##   line   nviaj1    nvikm1 collects1   nviaj2    nvikm2 collects2
nviaj3
## 1    1 11280.07 3115686.3 228136.63 14091.28 3968376.8 292657.40 16
584.69
## 2    2 10290.03 3104303.8 228709.02 13035.78 3981299.1 295517.67 14
697.80
## 3    3 14718.22 4403117.0 324039.23 18309.41 5572204.9 412637.50 20
821.61
## 4    4  3508.68 1660038.3 127706.07  4312.28 2036761.5 157479.68  5
098.11
## 5    5  1735.39  481769.3  35068.82  2123.36  596747.5  43642.74  2
442.78
## 6    6  7951.62 2818529.5 212059.35  9925.72 3524541.8 265214.00 11
658.37
##      nvikm3 collects3   nviaj4    nvikm4 collects4       pkm1
pkm2
## 1 4559777.8 339710.42 13724.89 3840781.2 286251.57 0.07322195 0.073
74738
## 2 4423902.5 332717.18 12780.14 3880158.8 291005.06 0.07367482 0.074
22644
## 3 6255258.8 469167.52 18022.01 5443237.0 408198.17 0.07359315 0.074
05282
## 4 2393810.7 186573.70  4627.86 2159532.3 167736.52 0.07692959 0.077
31867
## 5  669730.1  49477.07  2092.81  586696.6  43320.35 0.07279173 0.073
13434
```

```
## 6 3952917.7 300434.23 10129.92 3575054.9 272567.57 0.07523758 0.075
24779
##         pkm3       pkm4    nviaj    nvikm collects serv1 serv2 serv
3 serv4  km
## 1 0.07450153 0.07452952 55680.93 15484622  1576532    90    90    9
0    90 493
## 2 0.07520898 0.07499823 50803.75 15389664  1412978    90    90    9
0    90 480
## 3 0.07500369 0.07499181 71871.25 21673818  1524078   180   180   18
0   180 493
## 4 0.07794004 0.07767261 17546.93  8250143   471343   180   180   18
0   180 455
## 5 0.07387613 0.07383774  8394.34  2334944  1385967    13    13    1
3    13 468
## 6 0.07600316 0.07624151 39665.63 13871044   898612   212   212   21
2   212 455



# We create a loop to calculate alpha (first trimester).
bus157$alpha1 = NA
for(i in 1:dim(bus157)[1]){

  bus157$alpha1[i]= log(bus157$pkm1 [i]) + beta * log(bus157$nvikm1[i]
)

}# End of de loop.


# We create a loop to calculate alpha (second trimester).
bus157$alpha2 = NA
for(i in 1:dim(bus157)[1]){

  bus157$alpha2[i]= log(bus157$pkm2 [i]) + beta * log(bus157$nvikm2[i]
)

}# End of de loop.


# We create a loop to calculate alpha (third trimester).
bus157$alpha3 = NA
for(i in 1:dim(bus157)[1]){

  bus157$alpha3[i]= log(bus157$pkm3 [i]) + beta * log(bus157$nvikm3[i]
)

}# End of de loop.


# We create a loop to calculate alpha (fourth trimester).
bus157$alpha4 = NA
for(i in 1:dim(bus157)[1]){

  bus157$alpha4[i]= log(bus157$pkm4 [i]) + beta * log(bus157$nvikm4[i]
```

```r
)

}# End of de loop.




# We create a loop to create the variable npline (first trimester).
bus157$npline1 = NA
for(i in 1:dim(bus157)[1]){

  bus157$npline1[i]= bus157$nvikm1 [i] / (bus157$km [i] * bus157$serv1
[i])

}# End of de loop.


# We create a loop to create the variable npline (segon trimester).
bus157$npline2 = NA
for(i in 1:dim(bus157)[1]){

  bus157$npline2[i]= bus157$nvikm2 [i] / (bus157$km [i] * bus157$serv2
[i])

}# End of de loop.


# We create a loop to create the variable npline (third trimester).
bus157$npline3 = NA
for(i in 1:dim(bus157)[1]){

  bus157$npline3[i]= bus157$nvikm3 [i] / (bus157$km [i] * bus157$serv3
[i])

}# End of de loop.


# We create a loop to create the variable npline (fourth trimester).
bus157$npline4 = NA
for(i in 1:dim(bus157)[1]){

  bus157$npline4[i]= bus157$nvikm4 [i] / (bus157$km [i] * bus157$serv4
[i])

}# End of de loop.




# We create a loop to create the variable nbuses (first trimester).
bus157$nbuses1 = NA
for(i in 1:dim(bus157)[1]){
```

```
  bus157$nbuses1[i]= ceiling(bus157$npline1 [i] / (buscap * ucap))

}# End of de loop.


# We create a loop to create the variable nbuses (segon trimester).
bus157$nbuses2 = NA
for(i in 1:dim(bus157)[1]){

  bus157$nbuses2[i]= ceiling(bus157$npline2 [i] / (buscap * ucap))

}# End of de loop.


# We create a loop to create the variable nbuses (third trimester).
bus157$nbuses3 = NA
for(i in 1:dim(bus157)[1]){

  bus157$nbuses3[i]= ceiling(bus157$npline3 [i] / (buscap * ucap))

}# End of de loop.


# We create a loop to create the variable nbuses (fourth trimester).
bus157$nbuses4 = NA
for(i in 1:dim(bus157)[1]){

  bus157$nbuses4[i]= ceiling(bus157$npline4 [i] / (buscap * ucap))

}# End of de loop.




# We create a loop to calculate costs (first trimester).
bus157$costs1= NA
for(i in 1:dim(bus157)[1]){

  bus157$costs1[i]= ((avcost * bus157$serv1[i] * bus157$km[i] * bus157
$nbuses1[i]) / bus157$nvikm1[i])

}# End of the loop.


# We create a loop to calculate costs (second trimester).
bus157$costs2= NA
for(i in 1:dim(bus157)[1]){

  bus157$costs2[i]= ((avcost * bus157$serv2[i] * bus157$km[i] * bus157
$nbuses2[i]) / bus157$nvikm2[i])

}# End of the loop.
```

```
# We create a loop to calculate costs (third trimester).
bus157$costs3= NA
for(i in 1:dim(bus157)[1]){

  bus157$costs3[i]= ((avcost * bus157$serv3[i] * bus157$km[i] * bus157
$nbuses3[i]) / bus157$nvikm3[i])

}# End of the loop.


# We create a loop to calculate costs (fourth trimester).
bus157$costs4= NA
for(i in 1:dim(bus157)[1]){

  bus157$costs4[i]= ((avcost * bus157$serv4[i] * bus157$km[i] * bus157
$nbuses4[i]) / bus157$nvikm4[i])

}# End of the loop.




#We create a loop to calculate profits for each line (first trimester)
.
bus157$profits1= NA

for(i in 1:dim(bus157)[1]){

  bus157$profits1[i]= ((bus157$pkm1[i] * bus157$nvikm1[i])-(bus157$nvi
km1[i]*bus157$costs1[i]))

}
#End of the loop.


#We create a loop to calculate profits for each line (second trimester
).
bus157$profits2= NA

for(i in 1:dim(bus157)[1]){

  bus157$profits2[i]= ((bus157$pkm2[i] * bus157$nvikm2[i])-(bus157$nvi
km2[i]*bus157$costs2[i]))

}
#End of the loop.


#We create a loop to calculate profits for each line (third trimester)
.
bus157$profits3= NA

for(i in 1:dim(bus157)[1]){
```

```
  bus157$profits3[i]= ((bus157$pkm3[i] * bus157$nvikm3[i])-(bus157$nvi
km3[i]*bus157$costs3[i]))

}
#End of the loop.


#We create a loop to calculate profits for each line (fourth trimester
).
bus157$profits4= NA

for(i in 1:dim(bus157)[1]){

  bus157$profits4[i]= ((bus157$pkm4[i] * bus157$nvikm4[i])-(bus157$nvi
km4[i]*bus157$costs4[i]))

}
#End of the loop.




#We create a loop to calculate total profits for each line.
bus157$totalprofits = NA

for (i in 1:dim(bus157)[1]) {
  bus157$totalprofits[i] = ifelse(is.na(bus157$profits1[i]), 0, bus157
$profits1[i]) +
    ifelse(is.na(bus157$profits2[i]), 0, bus157$profits2[i]) +
    ifelse(is.na(bus157$profits3[i]), 0, bus157$profits3[i]) +
    ifelse(is.na(bus157$profits4[i]), 0, bus157$profits4[i])
}
#End of the loop.



#Total profits
sum(bus157$totalprofits, na.rm=TRUE)

## [1] 1392557



#Computation of profits from profitable lines
profitable_lines = sum(bus157$totalprofits[bus157$totalprofits>0], na.
rm=TRUE)
profitable_lines

## [1] 2840456



#Computation of losses from loss-making lines
loss_making_lines = sum(bus157$totalprofits[bus157$totalprofits<0], na
```

```
.rm=TRUE)
loss_making_lines

## [1] -1447900


#Quantity of loss making lines
num_loss_making <- sum(ifelse(bus157$totalprofits < 0, 1, 0),na.rm=TRU
E)
num_loss_making

## [1] 34




#Consumer surplus with cross subsidies

#CS on first trimester.
for(i in 1:dim(bus157)[1]){

    bus157$CScross1[i]= ((bus157$alpha1[i] - bus157$pkm1[i]) * bus157$
nvikm1[i]) / 2
  }#End CS calculation


#CS on second trimester.
bus157$CScross2= NA

for(i in 1:dim(bus157)[1]){

  bus157$CScross2[i]= ((bus157$alpha2[i] - bus157$pkm2[i]) * bus157$nv
ikm2[i]) / 2
}

#CS on third trimester.
bus157$CScross3= NA

for(i in 1:dim(bus157)[1]){

  bus157$CScross3[i]= ((bus157$alpha3[i] - bus157$pkm3[i]) * bus157$nv
ikm3[i]) / 2
}

#CS on fourth trimester.
bus157$CScross4= NA

for(i in 1:dim(bus157)[1]){

  bus157$CScross4[i]= ((bus157$alpha4[i] - bus157$pkm4[i]) * bus157$nv
ikm4[i]) / 2
}
```

```r
#Total CS with cross subsidies calculation
bus157$totalCScross = NA

for (i in 1:dim(bus157)[1]) {
  bus157$totalCScross[i] = ifelse(is.na(bus157$CScross1[i]), 0, bus157
$CScross1[i]) +
    ifelse(is.na(bus157$CScross2[i]), 0, bus157$CScross2[i]) +
    ifelse(is.na(bus157$CScross3[i]), 0, bus157$CScross3[i]) +
    ifelse(is.na(bus157$CScross4[i]), 0, bus157$CScross4[i])
}

sum(bus157$totalCScross)

## [1] 4850486208




#We compute the new equilibrium quantity under liberalization.
#1st trimester
bus157$nvikmlib1= NA

for(i in 1:dim(bus157)[1]){
  if(bus157$totalprofits[i] >= 0){
    bus157$nvikmlib1[i]= exp((bus157$alpha1[i] - log(bus157$costs1[i]))
)/ beta )

  } else {
    bus157$nvikmlib1[i] = bus157$nvikm1[i]
    }
} #End of the loop


#2nd trimester
bus157$nvikmlib2= NA

for(i in 1:dim(bus157)[1]){

  if(bus157$totalprofits[i] >= 0){
  bus157$nvikmlib2[i]= exp((bus157$alpha2[i] - log(bus157$costs2[i]))/
beta )

  } else {
    bus157$nvikmlib2[i] = bus157$nvikm2[i]
  }
} #End of the loop


#3rd trimester
bus157$nvikmlib3= NA
```

```
for(i in 1:dim(bus157)[1]){

  if(bus157$totalprofits[i] >= 0){
    bus157$nvikmlib3[i]= exp((bus157$alpha3[i] - log(bus157$costs3[i]))
)/ beta )

  } else {
    bus157$nvikmlib3[i] = bus157$nvikm3[i]
  }
  } #End of the Loop


#4th trimester
bus157$nvikmlib4= NA

for(i in 1:dim(bus157)[1]){

  if(bus157$totalprofits[i] >= 0){
    bus157$nvikmlib4[i]= exp((bus157$alpha4[i] - log(bus157$costs4[i]))
)/ beta )

  } else {
    bus157$nvikmlib4[i] = bus157$nvikm4[i]
  }
  } #End of the Loop




#Remind that the quantity of equilibrium (nvikm) represents the total
kilometers travelled by the total consumers. It would also be interest
ing to find out the new quantity in terms of passengers. To do that, f
irst we compute the km travelled by each consumers by mean.
#First trimester
bus157$meankm1= NA

for(i in 1:dim(bus157)[1]){

  bus157$meankm1[i]= bus157$nvikm1[i] / bus157$nviaj1[i]

  }


#Second trimester
bus157$meankm2= NA

for(i in 1:dim(bus157)[1]){

  bus157$meankm2[i]= bus157$nvikm2[i] / bus157$nviaj2[i]

}
```

```r
#Third trimester
bus157$meankm3= NA

for(i in 1:dim(bus157)[1]){

  bus157$meankm3[i]= bus157$nvikm3[i] / bus157$nviaj3[i]

}


#Fourth trimester
bus157$meankm4= NA

for(i in 1:dim(bus157)[1]){

  bus157$meankm4[i]= bus157$nvikm4[i] / bus157$nviaj4[i]

}


#Assuming that the average km travelled by each consumer does not chan
ge with liberalization (there's simply more passengers doing the same
path), we can compute the quantity of new passengers per trimester.
#First trimester
bus157$nviajlib1= NA

for(i in 1:dim(bus157)[1]){

  bus157$nviajlib1[i]= bus157$nvikmlib1[i] / bus157$meankm1[i]

}


#Second trimester
bus157$nviajlib2= NA

for(i in 1:dim(bus157)[1]){

  bus157$nviajlib2[i]= bus157$nvikmlib2[i] / bus157$meankm2[i]

}


#Third trimester
bus157$nviajlib3= NA

for(i in 1:dim(bus157)[1]){

  bus157$nviajlib3[i]= bus157$nvikmlib3[i] / bus157$meankm3[i]

}


#Fourth trimester
```

```
bus157$nviajlib4= NA

for(i in 1:dim(bus157)[1]){

  bus157$nviajlib4[i]= bus157$nvikmlib4[i] / bus157$meankm4[i]

}




#Total new passangers under liberalization
bus157$totalnviajlib = NA

for (i in 1:dim(bus157)[1]) {
  bus157$totalnviajlib[i] = ifelse(is.na(bus157$nviajlib1[i]), 0, bus1
57$nviajlib1[i]) +
    ifelse(is.na(bus157$nviajlib2[i]), 0, bus157$nviajlib2[i]) +
    ifelse(is.na(bus157$nviajlib3[i]), 0, bus157$nviajlib3[i]) +
    ifelse(is.na(bus157$nviajlib4[i]), 0, bus157$nviajlib4[i])
}

sum(bus157$totalnviajlib)

## [1] 1686349



#Variation of nviaj
var_nviaj= ((sum(bus157$totalnviajlib) - sum(bus157$nviaj, na.rm= TRUE
)) / sum(bus157$nviaj, na.rm= TRUE)) *100
var_nviaj

## [1] 5.995378




#We create a loop to compute CS under liberalization
#1st trimester
bus157$CSlib1= NA

for(i in 1:dim(bus157)[1]){

  if(bus157$totalprofits[i] >= 0){
    bus157$CSlib1[i] = ((bus157$alpha1[i] - bus157$costs1[i]) * bus157
$nvikmlib1[i]) /2

  } else {
    bus157$CSlib1[i] = bus157$CScross1[i]
  }
} #End of the loop


#2nd trimester
```

```
bus157$CSlib2= NA

for(i in 1:dim(bus157)[1]){

  if(bus157$totalprofits[i] >= 0){
    bus157$CSlib2[i] = ((bus157$alpha2[i] - bus157$costs2[i]) * bus157
$nvikmlib2[i]) /2

  } else {
    bus157$CSlib2[i] = bus157$CScross2[i]
  }
}
#End of the Loop


#3rd trimester
bus157$CSlib3= NA

for(i in 1:dim(bus157)[1]){

  if(bus157$totalprofits[i] >= 0){
    bus157$CSlib3[i] = ((bus157$alpha3[i] - bus157$costs3[i]) * bus157
$nvikmlib3[i]) /2

  } else {
    bus157$CSlib3[i] = bus157$CScross3[i]
  }
}
#End of the Loop


#4th trimester
bus157$CSlib4= NA

for(i in 1:dim(bus157)[1]){

  if(bus157$totalprofits[i] >= 0){
    bus157$CSlib4[i] = ((bus157$alpha4[i] - bus157$costs4[i]) * bus157
$nvikmlib4[i]) /2

  } else {
    bus157$CSlib4[i] = bus157$CScross4[i]
  }
}
#End of the Loop




#New CS with liberalization.
bus157$totalCSlib = NA

for (i in 1:dim(bus157)[1]) {
  bus157$totalCSlib[i] = ifelse(is.na(bus157$CSlib1[i]), 0, bus157$CSl
```

```
ib1[i]) +
    ifelse(is.na(bus157$CSlib2[i]), 0, bus157$CSlib2[i]) +
    ifelse(is.na(bus157$CSlib3[i]), 0, bus157$CSlib3[i]) +
    ifelse(is.na(bus157$CSlib4[i]), 0, bus157$CSlib4[i])
}

sum(bus157$totalCSlib)

## [1] 5167036081

#Variation of CS with liberalization
var_CS = sum(bus157$totalCSlib) - sum(bus157$totalCScross)
var_CS

## [1] 316549873



#Variation rate of CS
ratevar_CS = ((sum(bus157$totalCSlib) - sum(bus157$totalCScross))/sum(
bus157$totalCScross))*100
ratevar_CS

## [1] 6.526147



write.csv(bus157, file = "results157.csv")
```

line	nviaj1	nvikm1	collects1	nviaj2	nvikm2	collects2	nviaj3	nvikm3	collects3
1	11280	3115686	228137	14091	3968377	292657	16585	4559778	339710
2	10290	3104304	228709	13036	3981299	295518	14698	4423903	332717
3	14718	4403117	324039	18309	5572205	412638	20822	6255259	469168
4	3509	1660038	127706	4312	2036761	157480	5098	2393811	186574
5	1735	481769	35069	2123	596748	43643	2443	669730	49477
6	7952	2818529	212059	9926	3524542	265214	11658	3952918	300434
7	13729	4044327	296157	17097	5068414	372926	18988	5521834	411506
8	14068	3969356	289526	17499	5001047	365934	17748	5055885	378333
9	17403	4225046	292771	21284	5186628	365763	22275	5466498	390107
10	4138	995656	69133	5083	1226564	86654	5469	1322205	94424
11	3554	852663	59218	4367	1050742	74258	4727	1137899	81287
12	1238	525823	35395	1719	728701	50478	2626	1109415	79211
13	2248	926937	64633	3038	1253984	89567	4879	1997677	145952
14	7398	2000571	140387	9444	2577621	184704	11690	3337541	243408
15	835	216385	15403	1097	299304	21742	2150	699881	51040
16	2332	273602	17836	2745	333775	22057	3545	422770	27578
18	1900	217775	16529	2539	296417	22897	3317	382958	29744
19	1167	147275	11082	1727	213376	16397	2191	270835	21144
20	330	52555	3973	446	71799	5534	2027	324621	25236
21	264	12872	1025	529	28200	2193	639	33675	2596
22	92214	37016923	2772339	99832	40105026	3066308	104481	41825284	3228296
23	7953	1875239	135066	10153	2340303	172353	12517	2852227	211616
24	3306	530905	37013	4190	673533	49176	4589	737631	54816
25	4375	583616	41579	5150	706449	51983	5656	775014	57803
26	26848	2572162	188029	33003	3227038	240665	40949	3992372	301555
27	1105	72075	5583	1370	94091	7263	1842	136716	10524
28	954	30069	2415	821	28990	2277	777	29633	2324
29	908	80301	6328	1026	101254	7876	818	81749	6409
30	465	36989	3017	595	60025	4729	437	40753	3241
31	0	0	0	0	0	0	419	37361	2977
32	465	36970	3018	596	60198	4742	2160	201715	16053
33	2097	162246	12743	3047	239019	18555	5024	405529	31748
34	350	29740	2269	360	30412	2311	409	34023	2587
35	0	0	0	0	0	0	0	0	0
36	4453	1403112	102903	5265	1631251	122055	7760	2635897	195905
37	0	0	0	0	0	0	56	1005	111
38	0	1	0	0	0	0	0	1	0
39	1	13	3	0	0	0	0	0	0
40	10001	3027404	223372	12662	3878502	287994	14189	4287201	322772
42	7001	1861448	131136	9036	2420056	174125	11479	3203228	234320
43	14088	4100549	301510	17655	5216043	385849	20353	5900564	442069
44	1857	764252	53399	2505	1032391	73876	4035	1649130	120648
45	1607	625015	42339	2218	863556	60095	3387	1314138	93895
46	1477	615808	41771	2086	866420	60276	3172	1317834	94552
47	8670	2396928	168563	11217	3131992	224848	14269	4174603	305317
48	2106	858243	60078	2885	1175215	84067	4605	1866621	136685
49	12490	5013667	375493	13522	5431930	415309	14151	5664924	437249
50	1527	149652	9425	1759	173656	10948	0	0	0
51	7735	3574525	273352	9354	4327729	333018	10932	5039561	390354

line	nviaj4	nvikm4	collects4	pkm1	pkm2	pkm3	pkm4	nviaj	nvikm	collects
1	13725	3840781	286252	0,0732	0,0737	0,0745	0,0745	55681	15484622	1576532
2	12780	3880159	291005	0,0737	0,0742	0,0752	0,0750	50804	15389664	1412978
3	18022	5443237	408198	0,0736	0,0741	0,0750	0,0750	71871	21673818	1524078
4	4628	2159532	167737	0,0769	0,0773	0,0779	0,0777	17547	8250143	471343
5	2093	586697	43320	0,0728	0,0731	0,0739	0,0738	8394	2334944	1385967
6	10130	3575055	272568	0,0752	0,0752	0,0760	0,0762	39666	13871044	898612
7	16793	4974528	369235	0,0732	0,0736	0,0745	0,0742	66607	19609102	1260551
8	16749	4732289	350807	0,0729	0,0732	0,0748	0,0741	66063	18758578	725752
9	20230	4867309	340414	0,0693	0,0705	0,0714	0,0699	81192	19745480	314238
10	4830	1150943	80554	0,0694	0,0706	0,0714	0,0700	19521	4695368	329722
11	4151	986115	69036	0,0695	0,0707	0,0714	0,0700	16799	4027419	330511
12	1599	674530	47403	0,0673	0,0693	0,0714	0,0703	7182	3038468	NA
13	2901	1187204	85871	0,0697	0,0714	0,0731	0,0723	13065	5365801	362284
14	8591	2367100	169068	0,0702	0,0717	0,0729	0,0714	37122	10282833	753808
15	973	255228	18419	0,0712	0,0726	0,0729	0,0722	5055	1470797	471460
16	2682	318843	20652	0,0652	0,0661	0,0652	0,0648	11304	1348989	106867
18	2135	232860	17607	0,0759	0,0772	0,0777	0,0756	9890	1130010	96573
19	1411	166050	12440	0,0752	0,0768	0,0781	0,0749	6496	797536	79013
20	1588	254150	19094	0,0756	0,0771	0,0777	0,0751	4391	703125	77283
21	424	21529	1685	0,0796	0,0778	0,0771	0,0783	1857	96276	8119
22	103561	41467083	3174375	0,0749	0,0765	0,0772	0,0766	400089	160414316	3612684
23	9657	2241708	162901	0,0720	0,0736	0,0742	0,0727	40280	9309477	4296142
24	4029	647018	46093	0,0697	0,0730	0,0743	0,0712	16114	2589087	208167
25	5044	686614	49584	0,0712	0,0736	0,0746	0,0722	20225	2751693	234645
26	30292	2992683	219901	0,0731	0,0746	0,0755	0,0735	131092	12784256	531560
27	1202	80869	6261	0,0775	0,0772	0,0770	0,0774	5520	383750	149849
28	872	27860	2296	0,0803	0,0785	0,0784	0,0824	3424	116552	9038
29	978	85868	9523	0,0788	0,0778	0,0784	0,1109	3730	349172	56606
30	534	45159	5196	0,0816	0,0788	0,0795	0,1151	2031	182925	NA
31	0	0	0	NA	NA	0,0797	NA	419	37361	NA
32	534	45308	5184	0,0816	0,0788	0,0796	0,1144	3755	344191	48786
33	2501	192657	14953	0,0785	0,0776	0,0783	0,0776	12669	999451	139187
34	364	30994	2358	0,0763	0,0760	0,0760	0,0761	1484	125168	26567
35	0	0	0	NA	NA	NA	NA	0	0	338
36	5200	1623992	121431	0,0733	0,0748	0,0743	0,0748	22677	7294252	4224925
37	0	0	0	NA	NA	0,1104	NA	56	1005	3232
38	0	1	0	0,2179	0,2500	0,2037	0,2255	0	3	25
39	0	0	0	0,2054	NA	NA	NA	1	13	NA
40	12406	3781692	283869	0,0738	0,0743	0,0753	0,0751	49258	14974798	1412978
42	8081	2199655	157544	0,0704	0,0720	0,0732	0,0716	35597	9684387	769746
43	17181	5049553	377910	0,0735	0,0740	0,0749	0,0748	69278	20266710	1576532
44	2396	978742	70914	0,0699	0,0716	0,0732	0,0725	10792	4424514	362284
45	2061	798866	56189	0,0677	0,0696	0,0715	0,0703	9273	3601574	278895
46	1931	800252	56526	0,0678	0,0696	0,0717	0,0706	8666	3600314	285107
47	10109	2862785	205141	0,0703	0,0718	0,0731	0,0717	44265	12566308	775958
48	2741	1109578	80431	0,0700	0,0715	0,0732	0,0725	12336	5009656	384433
49	14027	5616408	429946	0,0749	0,0765	0,0772	0,0766	54189	21726930	3612684
50	1774	176777	11049	0,0630	0,0630	NA	0,0625	5060	500086	51106
51	10154	4637551	358106	0,0765	0,0769	0,0775	0,0772	38175	17579367	601194

line	serv1	serv2	serv3	serv4	km	alpha1	alpha2	alpha3	alpha4
1	90	90	90	90	493	19,06	19,41	19,62	19,38
2	90	90	90	90	480	19,06	19,42	19,59	19,40
3	180	180	180	180	493	19,56	19,91	20,09	19,89
4	180	180	180	180	455	18,19	18,49	18,74	18,58
5	13	13	13	13	468	16,34	16,66	16,84	16,64
6	212	212	212	212	455	18,94	19,26	19,44	19,29
7	232	232	232	232	455	19,43	19,77	19,90	19,75
8	341	341	341	341	353	19,40	19,74	19,78	19,67
9	694	694	694	694	240	19,44	19,76	19,84	19,66
10	180	180	180	180	240	17,35	17,67	17,79	17,57
11	180	180	180	180	240	17,12	17,44	17,57	17,34
12	90	90	90	90	421	16,39	16,89	17,53	16,80
13	225	225	225	225	421	17,25	17,71	18,41	17,64
14	180	180	180	180	421	18,37	18,76	19,15	18,63
15	29	29	146	29	439	15,16	15,65	16,89	15,41
16	155	155	155	155	228	15,41	15,72	16,04	15,63
18	196	196	196	196	165	15,23	15,70	16,08	15,33
19	180	180	180	180	156	14,66	15,22	15,58	14,83
20	32	32	92	32	181	13,17	13,64	15,84	15,45
21	130	130	130	130	116	11,18	12,30	12,55	11,91
22	1766	1766	1766	1766	400	22,66	22,80	22,87	22,85
23	180	180	180	180	412	18,30	18,65	18,94	18,57
24	265	265	265	265	163	16,44	16,83	16,98	16,75
25	162	162	162	162	163	16,60	16,91	17,06	16,85
26	616	616	616	616	171	18,78	19,12	19,45	19,00
27	52	52	52	52	98	13,65	14,04	14,57	13,82
28	180	180	180	180	67	12,42	12,35	12,38	12,34
29	52	52	52	52	376	13,83	14,15	13,85	14,27
30	16	16	16	16	238	12,74	13,40	12,85	13,37
31	0	0	16	0	238	NA	NA	12,73	NA
32	13	13	65	13	254	12,74	13,41	15,17	13,37
33	188	188	188	188	111	14,84	15,39	16,17	15,08
34	26	26	26	26	125	12,35	12,38	12,55	12,41
35	130	130	130	130	16	NA	NA	NA	NA
36	29	29	146	29	460	17,90	18,14	18,83	18,13
37	0	0	32	0	76	NA	NA	7,81	NA
38	12	12	12	12	25	-1,88	-3,45	-2,48	-1,46
39	12	12	12	12	31	2,13	NA	NA	NA
40	90	90	90	90	480	19,02	19,39	19,55	19,36
42	90	90	90	90	421	18,27	18,67	19,09	18,53
43	90	90	90	90	493	19,46	19,81	20,00	19,78
44	90	90	90	90	433	16,97	17,43	18,13	17,37
45	90	90	90	90	421	16,65	17,14	17,78	17,04
46	90	90	90	90	421	16,63	17,15	17,79	17,05
47	180	180	180	180	421	18,63	19,04	19,48	18,91
48	90	90	90	90	421	17,14	17,62	18,31	17,55
49	180	180	180	180	400	19,77	19,90	19,97	19,95
50	130	130	0	130	148	14,50	14,72	NA	14,74
51	180	180	180	180	455	19,30	19,58	19,81	19,68

line	npline1	npline2	npline3	npline4	nbuses1	nbuses2	nbuses3	nbuses4
1	70	89	103	87	3	4	4	4
2	72	92	102	90	3	4	4	4
3	50	63	70	61	2	3	3	3
4	20	25	29	26	1	1	2	2
5	79	98	110	96	4	4	5	4
6	29	37	41	37	2	2	2	2
7	38	48	52	47	2	2	3	2
8	33	42	42	39	2	2	2	2
9	25	31	33	29	1	2	2	2
10	23	28	31	27	1	2	2	2
11	20	24	26	23	1	1	2	1
12	14	19	29	18	1	1	2	1
13	10	13	21	13	1	1	1	1
14	26	34	44	31	2	2	2	2
15	17	24	11	20	1	1	1	1
16	8	9	12	9	1	1	1	1
18	7	9	12	7	1	1	1	1
19	5	8	10	6	1	1	1	1
20	9	12	19	44	1	1	1	2
21	1	2	2	1	1	1	1	1
22	52	57	59	59	3	3	3	3
23	25	32	38	30	1	2	2	2
24	12	16	17	15	1	1	1	1
25	22	27	29	26	1	2	2	2
26	24	31	38	28	1	2	2	2
27	14	18	27	16	1	1	2	1
28	2	2	2	2	1	1	1	1
29	4	5	4	4	1	1	1	1
30	10	16	11	12	1	1	1	1
31	NA	NA	10	NA	NA	NA	1	NA
32	11	18	12	14	1	1	1	1
33	8	11	19	9	1	1	1	1
34	9	9	10	10	1	1	1	1
35	0	0	0	0	0	0	0	0
36	105	122	39	122	5	5	2	5
37	NA	NA	0	NA	NA	NA	1	NA
38	0	0	0	0	1	1	1	1
39	0	0	0	0	1	0	0	0
40	70	90	99	88	3	4	4	4
42	49	64	85	58	2	3	4	3
43	92	118	133	114	4	5	6	5
44	20	26	42	25	1	2	2	1
45	16	23	35	21	1	1	2	1
46	16	23	35	21	1	1	2	1
47	32	41	55	38	2	2	3	2
48	23	31	49	29	1	2	2	2
49	70	75	79	78	3	3	4	4
50	8	9	NA	9	1	1	NA	1
51	44	53	62	57	2	3	3	3

line	costs1 (€/vikn	costs2 (€/vikn	costs3 (€/vikn	costs4 (€/vikn	profits1 (€)	profits2 (€)	profits3 (€)	profits4 (€)	totalprofits (€
1	0,0582	0,0610	0,0531	0,0630	46708	50752	97805	44346	239611
2	0,0569	0,0592	0,0532	0,0607	52064	59991	97191	55479	264725
3	0,0549	0,0651	0,0580	0,0667	82134	49780	106310	45340	283564
4	0,0672	0,0548	0,0933	0,1034	16076	45850	-36686	-55523	-30282
5	0,0689	0,0556	0,0619	0,0565	1899	10473	8015	10150	30537
6	0,0933	0,0746	0,0665	0,0736	-50891	2264	37484	9618	-1525
7	0,0712	0,0568	0,0782	0,0578	8400	85170	-20129	81478	154919
8	0,0827	0,0656	0,0649	0,0693	-38611	37797	50197	22670	72053
9	0,0537	0,0875	0,0831	0,0933	65750	-88280	-63936	-113628	-200094
10	0,0591	0,0960	0,0891	0,1023	10251	-31109	-23339	-37209	-81406
11	0,0691	0,0560	0,1035	0,0597	336	15377	-36476	10155	-10609
12	0,0982	0,0709	0,0931	0,0766	-16249	-1166	-24077	-4241	-45733
13	0,1393	0,1030	0,0646	0,1088	-64477	-39543	16842	-43239	-130417
14	0,1033	0,0801	0,0619	0,0873	-66189	-21873	36832	-37508	-88738
15	0,0802	0,0580	0,1248	0,0680	-1949	4389	-36320	1066	-32814
16	0,1761	0,1443	0,1139	0,1511	-30332	-26111	-20591	-27517	-104551
18	0,2024	0,1487	0,1151	0,1893	-27551	-21182	-14335	-26472	-89540
19	0,2599	0,1794	0,1413	0,2305	-27191	-21877	-17129	-25833	-92029
20	0,1502	0,1100	0,0699	0,0621	-3921	-2360	2539	3305	-437
21	1,5968	0,7289	0,6104	0,9547	-19529	-18361	-17958	-18869	-74717
22	0,0780	0,0720	0,0691	0,0697	-116130	177838	339826	285905	687440
23	0,0539	0,0864	0,0709	0,0902	33985	-29807	9456	-39260	-25625
24	0,1109	0,0874	0,0798	0,0910	-21862	-9699	-4059	-12782	-48402
25	0,0617	0,1019	0,0929	0,1048	5588	-20000	-14180	-22399	-50991
26	0,0558	0,0890	0,0719	0,0959	44456	-46481	14409	-67245	-54861
27	0,0964	0,0738	0,1016	0,0859	-1363	318	-3368	-685	-5098
28	0,5467	0,5670	0,5547	0,5900	-14023	-14161	-14114	-14141	-56438
29	0,3319	0,2632	0,3260	0,3104	-20321	-18773	-20241	-17127	-76462
30	0,1403	0,0865	0,1274	0,1149	-2173	-461	-1950	6	-4578
31	NA	NA	0,1389	NA	NA	NA	-2213	NA	-2213
32	0,1217	0,0748	0,1116	0,0993	-1482	242	-6450	684	-7007
33	0,1753	0,1190	0,0701	0,1476	-15700	-9888	3305	-13490	-35773
34	0,1490	0,1457	0,1302	0,1429	-2161	-2119	-1843	-2072	-8194
35	NA	NA	NA	NA	NA	NA	NA	NA	0
36	0,0648	0,0557	0,0695	0,0560	11991	31143	12827	30519	86480
37	NA	NA	3,2977	NA	NA	NA	-3204	NA	-3204
38	524,2308	1703,7500	757,2222	400,8824	-409	-409	-409	-409	-1635
39	39,0028	NA	NA	NA	-504	NA	NA	NA	-504
40	0,0583	0,0607	0,0549	0,0623	46728	52468	87245	48342	234783
42	0,0555	0,0640	0,0645	0,0704	27848	19193	27744	2612	77397
43	0,0590	0,0580	0,0615	0,0599	59605	83468	79211	75528	297812
44	0,0695	0,1029	0,0644	0,0543	283	-32357	14415	17797	140
45	0,0826	0,0598	0,0786	0,0646	-9305	8451	-9393	4545	-5702
46	0,0839	0,0596	0,0784	0,0645	-9873	8632	-8737	4881	-5096
47	0,0862	0,0660	0,0742	0,0722	-38014	18272	-4548	-1435	-25725
48	0,0602	0,0879	0,0553	0,0931	8434	-19221	33396	-22857	-248
49	0,0587	0,0542	0,0693	0,0699	81085	120901	44705	37402	284092
50	0,1752	0,1510	NA	0,1483	-16799	-15276	NA	-15175	-47251
51	0,0625	0,0774	0,0665	0,0722	50093	-1871	55465	23217	126904

line	CScross1	CScross2	CScross3	CScross4	totalCScross	nvikmlib1	nvikmlib2	nvikmlib3	nvikmlib4
1	29571019	38372697	44571805	37066697	149582217	3649226	4525700	5763596	4313804
2	29463620	38518966	43165986	37486657	148635229	3709967	4656076	5614746	4489868
3	42903888	55262346	62597600	53922680	214686513	5387128	6089074	7468671	5903915
4	15035993	18754803	22331592	19981439	76103828	1660038	2036761	2393811	2159532
5	3919469	4948733	5613093	4860762	19342055	500634	721134	756583	705370
6	26581408	33810909	38267124	34354019	133013460	2818529	3524542	3952918	3575055
7	39149419	49902874	54742644	48931239	192726177	4125427	6061282	5342844	5908311
8	38362637	49178251	49809934	46374507	183725330	3640883	5391766	5577670	4955533
9	40924298	51051290	54044359	47665419	193685367	4225046	5186628	5466498	4867309
10	8602152	10792381	11712483	10068925	41175940	995656	1226564	1322205	1150943
11	7271032	9127712	9956227	8516615	34871586	852663	1050742	1137899	986115
12	4292084	6130120	9686348	5641174	25749724	525823	728701	1109415	674530
13	7962220	11060115	18314535	10430972	47767842	926937	1253984	1997677	1187204
14	18305721	24084291	31836833	21967545	96194389	2000571	2577621	3337541	2367100
15	1632669	2331477	5883931	1957900	11805977	216385	299304	699881	255228
16	2099682	2611671	3377869	2481266	10570489	273602	333775	422770	318843
18	1650637	2315337	3063371	1775868	8805214	217775	296417	382958	232860
19	1073947	1615362	2099124	1224954	6013387	147275	213376	270835	166050
20	344107	486990	2557979	1953587	5342664	52555	71799	324621	254150
21	71467	172287	209930	127387	581071	12872	28200	33675	21529
22	418102044	455693294	476695689	472196957	1822687985	35983512	41792935	45161641	44257501
23	17093678	21732796	26905331	20733405	86465210	1875239	2340303	2852227	2241708
24	4345936	5644057	6235809	5395652	21621454	530905	673533	737631	647018
25	4823333	5946863	6580904	5759747	23110848	583616	706449	775014	686614
26	24053145	30737392	38666351	28321113	121778001	2572162	3227038	3992372	2992683
27	489222	656684	991013	555640	2692558	72075	94091	136716	80869
28	185551	177829	182226	170712	716319	30069	28990	29633	27860
29	551982	712418	562798	607714	2434912	80301	101254	81749	85868
30	234066	399941	260272	299314	1193594	36989	60025	40753	45159
31	NA	NA	236294	NA	236294	0	0	37361	0
32	233948	401222	1522121	300299	2457589	36970	60198	201715	45308
33	1197705	1830262	3262234	1445139	7735340	162246	239019	405529	192657
34	182580	187140	212137	191162	773019	29740	30412	34023	30994
35	NA	NA	NA	NA	0	NA	NA	NA	NA
36	12506880	14733644	24716127	14662324	66618975	1528338	1998922	2761984	1982995
37	NA	NA	3872	NA	3872	0	0	1005	0
38	-1	0	-1	-1	-3	1	0	1	1
39	13	NA	NA	NA	13	13	0	0	0
40	28680782	37451548	41736676	36466451	144335458	3559598	4455860	5328492	4301599
42	16938871	22506049	30464765	20299484	90209169	2194750	2623140	3494225	2225177
43	39742481	51477868	58795450	49743275	199759074	4773590	6171436	6761819	5889351
44	6458637	8961088	14890970	8463222	38773917	767062	803514	1800469	1194718
45	5181856	7372675	11635498	6779265	30969294	625015	863556	1314138	798866
46	5099279	7399084	11673453	6793609	30965425	615808	866420	1317834	800252
47	22248830	29708949	40504006	26966450	119428234	2396928	3131992	4174603	2862785
48	7325829	10310946	17023190	9695685	44355650	858243	1175215	1866621	1109578
49	49365509	53851059	56358112	55819073	215393753	5930010	6887395	6102586	5980411
50	1080619	1272753	NA	1297194	3650565	149652	173656	0	176777
51	34353535	42204406	49717647	45465640	171741228	4110342	4311034	5601672	4857081

line	meankm1	meankm2	meankm3	meankm4	nviajlib1	nviajlib2	nviajlib3	nviajlib4	totalnviajlib
1	276	282	275	280	13212	16070	20963	15415	65660
2	302	305	301	304	12298	15245	18654	14788	60985
3	299	304	300	302	18007	20008	24861	19547	82423
4	473	472	470	467	3509	4312	5098	4628	17547
5	278	281	274	280	1803	2566	2760	2516	9645
6	354	355	339	353	7952	9926	11658	10130	39666
7	295	296	291	296	14005	20446	18373	19945	72768
8	282	286	285	283	12903	18866	19580	17539	68888
9	243	244	245	241	17403	21284	22275	20230	81192
10	241	241	242	238	4138	5083	5469	4830	19521
11	240	241	241	238	3554	4367	4727	4151	16799
12	425	424	422	422	1238	1719	2626	1599	7182
13	412	413	409	409	2248	3038	4879	2901	13065
14	270	273	286	276	7398	9444	11690	8591	37122
15	259	273	326	262	835	1097	2150	973	5055
16	117	122	119	119	2332	2745	3545	2682	11304
18	115	117	115	109	1900	2539	3317	2135	9890
19	126	124	124	118	1167	1727	2191	1411	6496
20	159	161	160	160	330	446	2027	1588	4391
21	49	53	53	51	264	529	639	424	1857
22	401	402	400	400	89640	104034	112816	110530	417020
23	236	231	228	232	7953	10153	12517	9657	40280
24	161	161	161	161	3306	4190	4589	4029	16114
25	133	137	137	136	4375	5150	5656	5044	20225
26	96	98	97	99	26848	33003	40949	30292	131092
27	65	69	74	67	1105	1370	1842	1202	5520
28	32	35	38	32	954	821	777	872	3424
29	88	99	100	88	908	1026	818	978	3730
30	80	101	93	85	465	595	437	534	2031
31	NA	NA	89	NA	NA	NA	419	NA	419
32	80	101	93	85	465	596	2160	534	3755
33	77	78	81	77	2097	3047	5024	2501	12669
34	85	84	83	85	350	360	409	364	1484
35	NA	NA	NA	NA	NA	NA	NA	NA	0
36	315	310	340	312	4851	6451	8131	6349	25782
37	NA	NA	18	NA	NA	NA	56	NA	56
38	6	6	6	6	0	0	0	0	0
39	13	NA	NA	NA	1	NA	NA	NA	1
40	303	306	302	305	11759	14547	17635	14111	58053
42	266	268	279	272	8255	9794	12522	8175	38746
43	291	295	290	294	16401	20888	23324	20039	80652
44	412	412	409	409	1863	1950	4405	2924	11142
45	389	389	388	388	1607	2218	3387	2061	9273
46	417	415	415	414	1477	2086	3172	1931	8666
47	276	279	293	283	8670	11217	14269	10109	44265
48	408	407	405	405	2106	2885	4605	2741	12336
49	401	402	400	400	14772	17145	15245	14936	62097
50	98	99	NA	100	1527	1759	NA	1774	5060
51	462	463	461	457	8894	9318	12151	10635	40998

line	CSlib1	CSlib2	CSlib3	CSlib4	totalCSlib
1	34662205	43790737	56400938	41656655	176510535
2	35243216	45082495	54847255	43409223	178582189
3	52542320	60415596	74803923	58510907	246272747
4	15035993	18754803	22331592	19981439	76103828
5	4073933	5986576	6345544	5850068	22256121
6	26581408	33810909	38267124	34354019	133013460
7	39938758	59729435	52958427	58164656	210791275
8	35170333	53040797	54978180	48574078	191763388
9	40924298	51051290	54044359	47665419	193685367
10	8602152	10792381	11712483	10068925	41175940
11	7271032	9127712	9956227	8516615	34871586
12	4292084	6130120	9686348	5641174	25749724
13	7962220	11060115	18314535	10430972	47767842
14	18305721	24084291	31836833	21967545	96194389
15	1632669	2331477	5883931	1957900	11805977
16	2099682	2611671	3377869	2481266	10570489
18	1650637	2315337	3063371	1775868	8805214
19	1073947	1615362	2099124	1224954	6013387
20	344107	486990	2557979	1953587	5342664
21	71467	172287	209930	127387	581071
22	406373345	474964824	514904653	504124767	1900367589
23	17093678	21732796	26905331	20733405	86465210
24	4345936	5644057	6235809	5395652	21621454
25	4823333	5946863	6580904	5759747	23110848
26	24053145	30737392	38666351	28321113	121778001
27	489222	656684	991013	555640	2692558
28	185551	177829	182226	170712	716319
29	551982	712418	562798	607714	2434912
30	234066	399941	260272	299314	1193594
31	NA	NA	236294	NA	236294
32	233948	401222	1522121	300299	2457589
33	1197705	1830262	3262234	1445139	7735340
34	182580	187140	212137	191162	773019
35	NA	NA	NA	NA	0
36	13629634	18073574	25905124	17922243	75530575
37	NA	NA	3872	NA	3872
38	-1	0	-1	-1	-3
39	13	NA	NA	NA	13
40	33750119	43056772	51928054	41507351	170242297
42	19988287	24405097	33247463	20536339	98177186
43	46300276	60956154	67422701	58060191	232739321
44	6482528	6961861	16265372	10341636	40051397
45	5181856	7372675	11635498	6779265	30969294
46	5099279	7399084	11673453	6793609	30965425
47	22248830	29708949	40504006	26966450	119428234
48	7325829	10310946	17023190	9695685	44355650
49	58435945	68356883	60736314	59456651	246985792
50	1080619	1272753	NA	1297194	3650565
51	39531889	42040661	55293961	47630027	184496538