An Alternative Approach to Fundamental General Physics Concepts

MASTER IN
HIGH ENERGY PHYSICS, ASTROPHYSICS AND COSMOLOGY

BY

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DEDICATION

This work is dedicated to the following people:

Major Prosper K. Ahialey(rtd), Dr. Kwame Danso, Mr Philipson Adams, and Paster James Takyi ; for the happiness they brought into my life.

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First of all, I express my profound gratitude to the Almighty God(JEHOVAH) for His guidance, provision of good health, and His grace towards me. My Lord is good and His mercy endures forever, He saw me through my stay here.

Secondly, my gratitude goes to my supervisor Prof. Emilio Elizalde Ruis for his love, and all the assistance he gave me from the first day I met him till today. I have had fruitful discussions with him that helped to shape my ideas. I am also thankful to all professors and researchers of the Institute of Space Science . To all teachers who have made impact on me, I say thank you.

I could not have gone through this master degree without the financial, spiritual and moral support of my relatives and friends, especially my friend Oluwasesan Ademolu Lawal. May God bless you, I love you.

Finally, I wish to apologize to the following authors for copying parts of their books without any permission: Paul A. Tipler and Gene Mosca. The objective of this work is not to criticize their work nor that of any other existing physicist. My sole motive was to come out with a new way of solving the vacuum energy problem.

ABSTRACT

In this thesis, an alternative approach to the fundamental general physics concepts has been proposed. Starting by demonstrating that the electrostatic potential energy (energy stored) of a discrete system of charges would be stored by charges or would be the energy the charges possess by virtue of their positions relative to each other, we go on to demonstrate that the electrostatic potential energy (the energy stored) of a continuous system of charges should be stored by the charges and not the field. We also demonstrated that the electrostatic potential energy stored by the capacitor is equal to the work that the capacitor can deliver. This work is not in principle stored as energy in the capacitor. This work is equal to work that the electric field between the plates of capacitor would do if charges were to move from one plate to the other. It is then inferred that there is a possibility that the electric field between the plates of the capacitor or any electric field does not store energy or has no energy density, as well as magnetic field. It is found that there is no direct relation between the electric or magnetic energy and photons. We proceed to discuss the validity of equivalence between a “standing wave” derived from Maxwell's equations and the simple harmonic oscillator. An alternative derivation of the blackbody radiation formula is proposed. We apply the results obtained to the determination of the energy density of the vacuum (which is empty space and not the ground state of fields). It is also found that the solution of Maxwell's equations in vacuum do not describe a wave with amplitude. It is also found that the electromagnetic wave with two modes of polarization is not needed to explain the results the blackbody radiation and also that the oscillating modes in the blackbody cavity might not exist. Meaning the zero-point of energy of electromagnetic radiation may not exist. Finally, we suggested that a possible explanation of the energy of the vacuum (if it exits) causing the accelerating expansion of the universe might be the energy of virtual photons.

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CHAPTER ONE

1.0-Introduction

This chapter starts with an overview of the problem and ends with the statement of work to be carried out in the next chapters.

1-1 overview of the problem

More than a decade ago, astronomical observations have revealed that the expansion of the universe is in an accelerating phase (the scale factor obeys $\ddot{a}>0$). The observed accelerating expansion requires either a modified theory of gravity or if we want to remain in the context of the standard general relativity, the existence of a smooth component with positive energy density and negative pressure dubbed dark energy. As a simplest explanation of this phenomenon; the cosmological constant, a smooth component with positive energy density and negative pressure ie a component with an equation of state parameter ω equals to -1, has been suggested. This cosmological constant is to be regarded as the vacuum energy. However, the calculated value of the energy density of vacuum (empty space in classical physics) gave values that are from 40 to 120 orders of magnitude (depending on the cut off and on the approach) greater that the observed value of the vacuum energy density.

This wide range over which the values are scattered has led many people to question that if the standard model of particle physics is the towering edifice to modern physics why is it giving values that are widely scattered over a range of 60 orders of magnitude?

There are a lot people who believe that the energy of vacuum is to be taken into account in the Einstein theory of gravitation . Others even have doubts about the existence of this energy of the vacuum. While another class of people think that the vacuum really contains energy but it is not to be added to the Einstein equations. These doubts concerning the energy of the vacuum are due to the fact that the observed value of the component with the negative pressure does not agree with the value given by the physicists. The discrepancy in the value estimated through quantum field theory and that obtained through cosmological methods has led the Nobelist Frank Wilczek to note the fact that we get different values using subatomic and cosmological analysis means that there must be serious flaws in the reasoning involved in one or both of those analysis (Robert L. Olershaw). The Nobelist Frank Wilczek has characterized the situation as follows “We do not understand the disparity . In my opinion, it is the the biggest and the most profound gap in our current understanding of the physical world...[the solution to the problem] might require inventing entirely new ideas, and abandoning the old ones we thought to be well established … Since the vacuum energy density is central to both fundamental physics and cosmology, and yet poorly understood, experimental research into its nature must be regarded as top priority for physical science”.

Due to this unresolved problem concerning the energy of the vacuum, some theorists have postulated the existence of another fundamental force called quintessence with an equation of state parameter ω in the range $-1< \omega < -1/3$ and with variable energy density.

Quintessence is believed to be caused by a scalar field, and the energy density depends on the value of the kinetic energy and the potential energy of the universe. Other theorists even go further to postulate the existence of a phantom energy, a component with an equation of state parameter less that minus one ($\omega <-1$). A phantom type of dark energy often leads to unpleasant consequences such as a singularity in a finite future time called "Big smash"[1] when a first article appeared concerning the ultimate fate of a universe filled with a phantom energy. The name has been changed to Big Rip there after[2]. Big Rip is the fate of the universe in which all gravitationally bound structures from clusters of galaxies, galaxies, solar system to molecules, atoms and subatomic particles get destroyed in a finite future time. For more discussions on singularities see [3] and [4] .

In this thesis, we consider that the problem of the energy of the vacuum is a quantum problem that originated in classical physics. For that reason we proposed an alternative approach to the fundamental general physics concepts. Starting from different explanation of the nature of the electromagnetic field and of the electromagnetic waves, we go on to propose an alternative derivation of the blackbody radiation. And finally, we apply this results obtained to the determination of the energy density of the vacuum.

CHAPTER TWO

2.0-Waves.

This chapter starts with the definitions and descriptions of certain terms about the waves by Paul A. Tipler and Gene Mosca [5], and ends with my own definitions or descriptions.

2.1.Description and definitions of Paul A. Tipler and Gene Mosca

This section contains a part of the book written by the above mentioned authors with almost no modification.

2.1.1Periodic waves

If one end of a taut string is shaken back and forth in periodic motion, then a periodic wave is generated. If a periodic wave is traveling along a taut string or any medium, each point along the medium oscillates with the same period.

2.1.2.Harmonic waves.

Harmonic waves are the most basic type of periodic waves. All waves, whether they are periodic or not, can be modeled as superposition of harmonic waves. Consequently, an understanding of harmonic wave motion can be generalized to form an understanding of any type of wave motion. If a **harmonic wave** is traveling through a medium, each point of the medium oscillates in simple harmonic motion.

If one end of a string is attached to a vibrating tuning fork that is moving up and down with simple harmonic motion, a sinusoidal wave train propagates along the string. This wave train is a harmonic wave. As shown in Figure-2.1, the shape of the string is that of a sinusoidal function . The minimum distance after which the wave repeats (the distance between crests, for example) in Figure-2.1 is called **wavelength** λ.

As the wave propagates along the string, each point on the string moves up and down – perpendicular to the direction of propagation – in simple harmonic motion with the frequency of the tuning fork. During one period T of this motion the wave moves a distance of one wavelength so its speed is given by

$$v=\frac{\lambda}{T}=f\lambda \quad , \qquad (2.1)$$

where we have used the relation T=1/f.

Because the relation $v=f\lambda$ arises only from the definition of wavelength and frequency, it applies to all periodic waves.

The sine function that describes the displacements in Figure-2.1 is

$$y(x)=A\sin(2\pi\frac{x}{\lambda}+\delta) \quad , \qquad (2.2a)$$

where A is the amplitude , λ is the wavelength, and δ is a phase constant that depends on the choice of the origin (where x=0). This equation is expressed more simply as

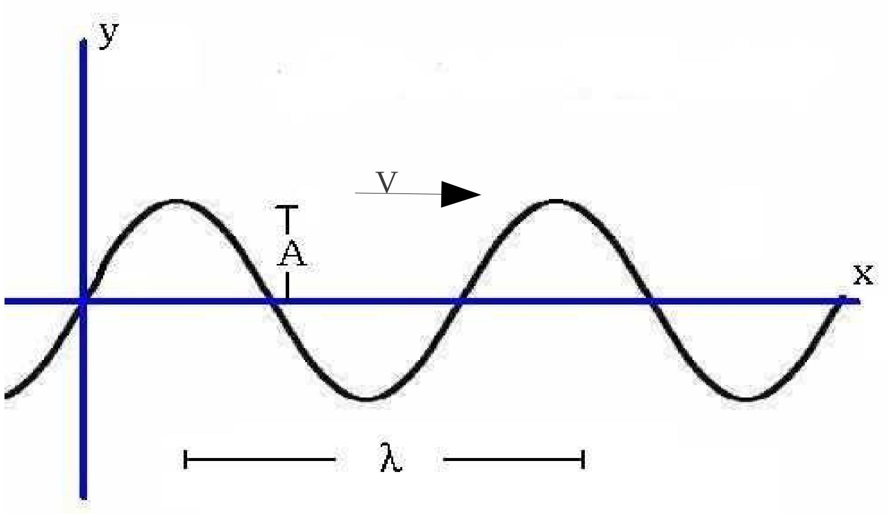


Figure 2.1 *Harmonic wave at some instant in time. A is the amplitude and λ is the wavelength.*

$$y(x) = A \sin(kx + \delta) \quad , \tag{2.2b}$$

where k, called the wave number is given by

$$k = \frac{2\pi}{\lambda} \quad .$$

Note that k has units of m^{-1}. (Because the angle must be in radians, we sometimes write the unit of k as rad/m.) When dealing with a single harmonic wave we usually choose the location of the origin so that δ=0.

For a wave traveling in the direction of increasing x with speed v, replace x in Equation(2.2b) with (x-vt). With δ equal to zero, this gives

$$y(x,t) = A \sin k(x - vt) = A \sin(kx - kvt) \quad ,$$

or

$$y(x,t) = A \sin(kx - \omega t) \quad , \tag{2.3}$$

called Harmonic Wave Function.

Where $\omega = kv$ is the angular frequency, and the argument of the sine function, (kx-ωt), is called the phase. The angular frequency is related to the frequency f and the period T by

$$\omega = 2\pi f = \frac{2\pi}{T} . \tag{2.4}$$

2.1.3.Harmonic Sound Waves

Harmonic sound waves can be generated by a tuning fork or a loudspeaker that is vibrating with simple harmonic motion. The vibrating source causes the air molecules next to oscillate with simple harmonic motion about their equilibrium positions. These molecules collide with the neighboring molecules, causing them to oscillate, which in turn collide with their neighboring molecules, causing them to oscillate and so forth, thereby propagating the sound wave. Equation(2.3) describes a harmonic sound wave if the wave function y(x,t) is replaced by s(x,t), which represents the displacements of the molecules from their equilibrium positions. Thus,

$$s(x,t)=s_0 \sin(kx-\omega t) \quad . \tag{2.5}$$

These displacements are along the direction of propagation of the wave, and lead to variations in density and pressure of the air.

2.2.My definitions and descriptions.

Though I am not in disagreement with the above definitions, I would like to redefine or reexplain certain terms in a different way for the purpose of my discussion later.

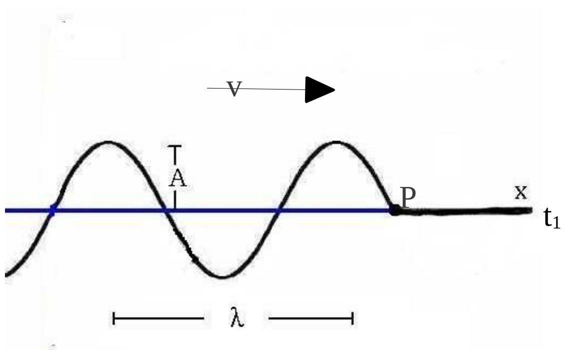
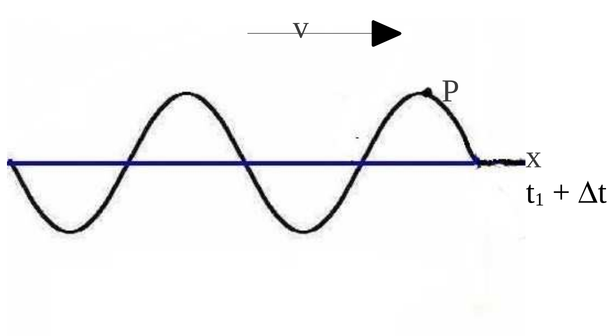


Figure 2.2 *The wave has reach point P at time t_1. During time Δt, the wave advanced past point P a distance $v\Delta t$.*

(a)



(b)

It is very important first of all to establish the fact that as the wave is propagating in a medium, the wave does not oscillate but the medium does oscillate. For this purpose let's consider a sinusoidal wave train as shown in Figure-2.2 propagating from left to right on a string. Let's focus our attention on the point P. we can remark that as the first single wave arrive at point P, the point will be rising up when the first half-wavelength of the wave is moving pass the point. When the first half-wavelength has fully passed the point, the point will be at the equilibrium position and will continue to move down because the next half-wavelength of the wave will be also passing. As soon as the trough has passed, the point will begin to rise up again till it reaches the equilibrium position immediately when the next half-wavelength has just passed. This therefore results in the up and down motion of very point on the string or the medium as already explained by Tipler.

In what follows when I mention a wave or the wave, I will be referring just a single wave as given in figure below. That is made up of one crest and one trough.

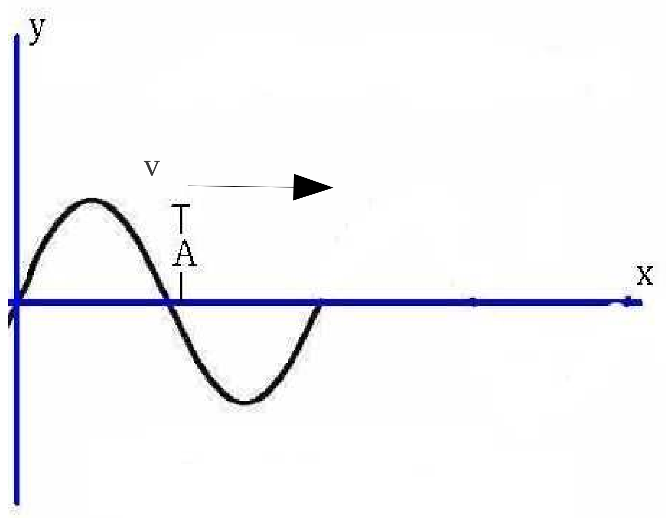


Figure 2.3 *Single wave traveling from left to right*.

I now define **the period of a wave as the time required for the wave to move a distance of one wavelength**. Of course for a transverse wave on a string for example, when the wave moves a distance of one wavelength, every point on the string where the wave passed will oscillate up and down once. But I just do not want to associate the oscillation of the medium or anything else to my definition.

When a harmonic wave train is moving from left to right for example at a speed let's say v, we can define the **frequency of the wave as the number of waves that would pass or pass a point, let's say P in our previous discussion , per unit time or second.** For example if 400 waves of the wave train would pass a given point per second, the frequency of the wave would be equal to 400 Hz . Again it is important to mention that if the 400 waves are moving, let's say, from left to right, each point of the medium will oscillate up and down 400 times. I do not want to associate the definition with the oscillation or the frequency of the source that produces the wave.

Or it could be defined simply as the inverse of the period (the time required for the wave to move a distance of one wavelength).

CHAPTER THREE

3.0.Electricity and Magnetism

This chapter starts with the answer to the question whether there is energy stored in the electric field, followed with the answer to another question about the validity of the differential form of Maxwell 's Equations, and ends with the question whether the solution of Maxwell's Equations in vacuum is really a wave function. The answers to each of the three questions contain the standard version, taken from a book written by Paul A. Tipler and Gene Mosca , and my opinion.

3.1. Is there energy stored in the electric field?

3.1.1 Standard approach or opinion. (from Tipler)

This section contains the standard or general opinion concerning the energy of the electric field. It is taken from a book written by the above mentioned authors[6], but it contains only some parts that would be important for my discussion later.

3.1.1.1.The storage of Electrical Energy.

When a capacitor is being charged, electrons are transferred from the positively charged conductor to the negatively charged conductor. This leaves the positively charged conductor with an electron deficit and the negatively charged conductor with an electron surplus. Alternatively, transferring positive charges from the negatively charged conductor to the positively charged conductor could also charge the capacitor. Either way, work must be done to charge the capacitor and some of this work is at least stored as electrostatic potential energy.

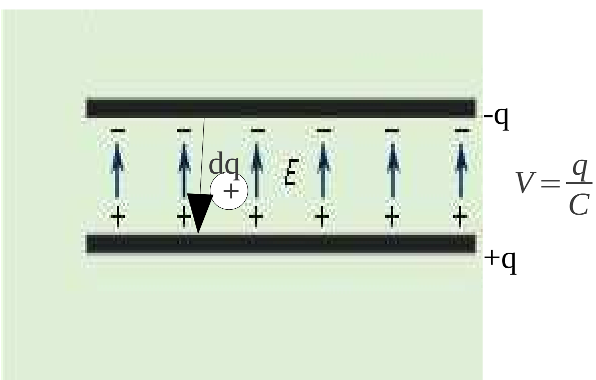


Figure 3.1 *When a small amount of positive charge is moved from the negative conductor to positive conductor, its potential energy is increased by dU=Vdq, where V is the potential difference between the conductors.*

We start with two uncharged conductors that do not touched each other. Let q be the positive charge that has been transferred during the initial stages of the charging process. The potential difference then is V=q/C. If a small amount of additional positive charge dq is now transferred from the negative conductor to the positive conductor through a potential increase V (Figure-3.1), the electrical potential energy of the charge, thus the capacitor, is increased by

$$dU = Vdq = \frac{q}{C} dq . \qquad (3.1)$$

The total increase in potential energy is the integral of dU as q increases from zero to its final value Q:

$$U = \int dU = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{1}{2} \frac{Q^2}{C} \quad . \qquad (3.2)$$

This potential energy is **the energy stored in the capacitor**. Using the definition of capacitance (C=Q/V),

we can express this energy in terms of either Q and V, C and V, or Q and C:

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2 \quad . \qquad (3.3a)$$

3.1.1.2.Electrostatic Field Energy

During the process of charging a capacitor, an electric field is produced between the plates. The work required to charge the capacitor can be taught as the work require to establish the electric field. That is, we can think of the energy stored in the capacitor as the the energy stored in the field, called **electrostatic field energy**.

Consider a parallel-plate capacitor. We can relate the energy sored in the capacitor to the electric field strength between the plates. The potential difference between the plates is related to the field strength by V=Ed, where d is the plate separation distance. The capacitance is given by $C=\varepsilon_0 A/d$. The energy stored is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \left(\frac{\epsilon_0 A}{d}\right) (Ed)^2 \quad . \qquad (3.4a)$$

The quantity Ad is the volume of the space between the plates of the capacitor. This volume is the volume of the region containing the electric filed. The energy per unit volume is called the energy density u_e . The energy density in an electric field of strength E is thus

$$u_e = \frac{energy}{volume} = \frac{1}{2} \epsilon_0 E^2 \quad . \qquad (3.5)$$

Thus, the energy per unit volume of the electrostatic field is proportional to the square of the field strength. Although we obtained Equation(3.5) by considering the electric field between the plates of a parallel-plate the capacitor, the result applies to any electric field. Whenever there is an electric field in space, the electrostatic energy per unit volume is given be Equation(3.5).

3.1.2 My Opinion.

In this section, we attempt to establish whether there is an energy stored in the electric field; alternatively, we investigate whether the electric field has an energy density.

To know whether there is energy stored in the electric field, let's consider the following cases:

3.1.2.1 Electrostatic Energy in a Discrete Charge distribution

Before considering the case of electrostatic potential, let's consider the gravitational potential. To lift a bowling ball from the ground and place it on a table, we need to do work against the gravitational force of attraction of the earth. The work needed to lift the ball is equal to the potential energy of the ball at that hight (in our case the top of a table). Allowing the ball to drop back to the floor, the potential energy of the ball is converted in its kinetic energy, if no other force is acting on the ball except the force of gravity. At the point before the ball impacts with the ground, the kinetic energy of the ball is equal to its initial potential energy at the top of the table and its potential energy is zero. **Some people even go on to say that the work done to lift the ball and put it on the table is stored in form potential energy by the ball. They also say that this potential energy is released when the ball falls to the ground.** But is it really the work done to lift the ball that is stored as potential energy , or the ball has a potential energy by virtue of its position relative to the earth? To try to give an answer to this question, let us consider an apple tree that has an apple hanging on one of its branches. The apple, though no one has done work to lift it to its actual position, has gravitational potential energy by virtue of its position in the earth gravitational field; that is by virtue of its position relative to the ground (earth). When the apple falls to ground, its potential energy is converted into its kinetic energy. At the point before the apple impacts with the earth, its kinetic energy is equal to its initial potential energy and its potential energy is zero. The potential energy that the apple possesses when hanging on the tree is not due to the energy of any work stored in it, but it is due to its position relative to the earth.

In the case of the ball, the work done to lift the ball from the ground to the top of the table is neither stored in the ball as potential energy nor lost. The essence of the work is to overcome the attractive force of gravity to lift the ball to the top of the table. The energy that is also released when the ball falls to the ground is not the potential energy, but its kinetic energy is released as heat and sound energies as the ball impacts the ground. Of course saying that the potential energy of the ball is released as the ball falls is not wrong, but when making such statement, we have to keep in mind that it is not the potential energy that is converted directly into thermal and sound energies as the ball falls to the ground but it is its kinetic energy.

Now let us consider the case of electrostatic field. The potential difference between two points a and b in an electrostatic field is the negative of the work per unit charge that the field would do on a **hypothetical** test charge as the charge moves from point a to point b. It is by definition given as

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}.\vec{dl} \quad , \tag{3.6}$$

where $\vec{E}$ is the electric field vector; that is the force per unit charge that acts on a test charge during every infinitesimal displacement $\vec{dl}$ from point a to point b. Alternatively, it is the force per unit charge at every point along the motion. It could be the same force per unit charge (uniform electric field) or it could vary from point to point.

So the electrostatic potential at a given point relative to infinity in an electric field is the negative of work per unit charge that the electric field would do on a hypothetical test charge as the test charge moves from infinity to that point. It is also equal to work required to bring the test charge from infinity to that point. Hence the potential energy of a charge Q_2 at point P_2 in an electrostatic field of charge Q_1 is equal to the work that the electric field due to charge Q_1 will do on charge Q_2 as it moves from infinity to that point. It is the work needed to bring charge Q_2 from infinity to that point .

Now let us consider the case of two charges Q_1 and Q_2 of the same sign.

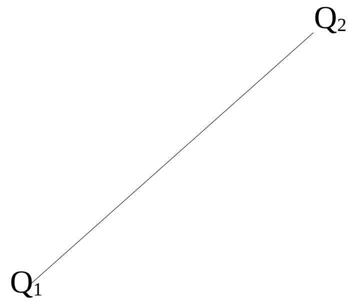


Figure3.2. *System of two point charges.*

To bring two charges together from an infinite separation against their electrostatic repulsion also requires work. Let's assume that the point charge Q_1 is located at point P_1, far from any other charge. Charge Q_1 has or stored no potential energy because it is not in the electric field of another charge. Let us consider the work done in bringing charge Q_2 from an infinite separation to point P_2 a distance R_{12} from charge Q_1 . The work required to overcome their repulsive force, that is the work required to bring Q_2 to point P_2 is

$$W_2 = Q_2 V_{12} \quad , \tag{3.7}$$

where V_{12} is the electrostatic potential due to charge Q_1 at the location of Q_2 relative to the potential at infinity(V=0)

The work required to bring Q_2 from infinity to its actual position is equal to the electrostatic potential energy of charge Q_2 at that point. This electrostatic potential energy is due to its position relative to charge Q_1. We generally say that the system of charges Q_1 and Q_2 has electrostatic potential energy equal to

$$U = Q_2 V_{12} \quad . \tag{3.8}$$

I**s it the work that is required to bring the two charges from infinite separation to their current separation that is stored as electrostatic potential of the system?**

The work that is done is not stored as their electrostatic potential energy. The work is not also lost. The work done (the work required) is to overcome the repulsive force between the two charges. When we are bringing charge Q_2 from infinity to point P_2, the electric field due to charge Q_1 is also doing work on charge Q_2, and this is equal but opposite in sign to work required to bring charge charge Q_2 from infinity to that point. The system of the two charges possesses electrostatic potential energy by virtue of their position relative to each other. Even if we consider that it is the work required to bring the charges together that has been stored as their potential energy, there is a very important question that we need to ask ourselves: **Is this energy stored in their electrostatic field?**

The answer to this question, I hope nobody will argue with me, is NO. The charge Q_2 has an electrostatic potential energy by virtue of its position in the electrostatic potential of charge Q_1. When charge Q_2 is allowed to recede away from Q_1, the force on charge Q_2 due to charge Q_1 will do work on charge Q_2 . When the electrostatic force is the only force acting on charge Q_2 , by the conservation of energy, when charge Q_2 is at an infinite separation from Q_1, its potential energy would be zero whereas its kinetic energy would be maximum (that is equal to its initial potential energy at point P_2). So if the electrostatic potential energy is stored somewhere, it is not in the electric field of the two charges but it is by one of charges or both charges.

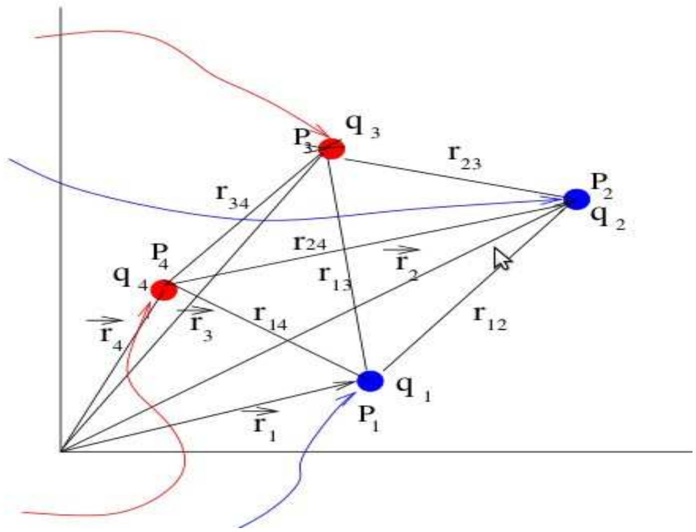


Figure3.3 *System of four point charges in space. The charges are brought into the system one by one.*

Let's now bring in from infinity another charge Q_3. The work required to bring Q_3 into the system is

$$W_3 = Q_3 (V_{13} + V_{23}) \quad , \tag{3.9}$$

where V_{13} is the electrostatic potential at point P_3 due to charge 1 and V_{23} is the electrostatic potential at point P_3 due to charge 2.

The total energy (work) required to assemble the system of three charges is

$$W_e = W_2 + W_3 \quad ,$$

$$W_e = Q_2 V_{12} + Q_3 (V_{13} + V_{23}) \quad . \tag{3.10}$$

This energy is also equal to the electrostatic potential energy of the system.

Let's bring again from infinity a fourth point charge Q_4. The work required to bring Q_4 into the system is

$$W_4 = Q_4 (V_{14} + V_{24} + V_{34}) \quad . \tag{3.11}$$

This also equal to the electrostatic potential energy of charge Q_4 at point P_4.

The total energy required to assemble the system of four charges is

$$W_e = W_2 + W_3 + W_4 \quad ,$$

$$W_e = Q_2 V_{12} + Q_3 (V_{13} + V_{23}) + Q_4 (V_{14} + V_{24} + V_{34}) \quad . \tag{3.12}$$

Let's ask ourselves an important question: Is it the work (energy) required in assembling the system of three or four charges that has been stored as the electrostatic potential energy of the system?

By analogy with the system of two charges, we can say that the work required to assemble the system is neither stored nor lost. The essence of the work is to overcome their mutual electrostatic repulsion to bring them closer. Their electrostatic potential energy is by virtue of their position relative to each other. When allowed to recede from each other one by one, their electrostatic potential energy will be converted into kinetic energy. For example, when the fourth charge is allowed to recede from the other three charges, their electrostatic field will do work on charge Q_4. By work- energy theorem, the work done will be equal to the change in kinetic energy of charge Q_4. And the change in kinetic energy is also equal to the change in potential energy. So when charge Q_4 recedes to infinity, where its potential energy is zero, its kinetic energy would be equal to its potential energy at its initial point (that is at point P_4)

To bring in from infinity an i^{th} point charge Q_i into the system of i-1 point charges, requires that we do work against the repulsive force of the i-1 charges. The essence of this work is to overcome the repulsive force of this system of i-1 charges. The work (energy) required to bring Q_i into the system is

$$W_i = Q_i \sum_{j=1}^{i-1} V_{ji} \quad . \tag{3.13}$$

The total energy required to assemble a system of N point charges is

$$W_e = \sum_{i=2}^{N} W_i = \sum_{i=2}^{N} Q_i \sum_{j=1}^{i-1} V_{ji} = \sum_{i=2}^{N} \sum_{j=1}^{i-1} Q_i V_{ji} \quad . \tag{3.14}$$

Note that

$$Q_i V_{ji} = Q_i \frac{Q_j}{4\pi\epsilon_0 R_{ji}} = Q_j \frac{Q_i}{4\pi\epsilon_0 R_{ij}} = Q_j V_{ij} \quad . \tag{3.15}$$

Physically, this means that the partial energy associated with two points charges is equal no matter in what order the charges are assembled. Hence

$$W_e = \sum_{i=2}^{N} \sum_{j=1}^{i-1} Q_i V_{ji} = \frac{1}{2} \sum_{i=2}^{N} \sum_{j=1}^{i-1} \left(Q_j V_{ij} + Q_i V_{ji} \right) \quad ,$$

$$W_e = \frac{1}{2} [Q_1 V_{21} + Q_2 V_{12} + Q_1 V_{31} + Q_2 V_{32} + Q_3 V_{13} + Q_3 V_{23} + Q_1 V_{41} + Q_2 V_{42} + Q_3 V_{43} + Q_4 V_{14} + Q_4 V_{34} ......]$$

$$W_e = \frac{1}{2} [Q_1 (V_{21} + V_{31} + V_{41} + ...) + Q_2 (V_{12} + V_{32} + V_{42} + ...) + Q_3 (V_{13} + V_{23} + V_{43} + ...) + ...] \quad ,$$

$$W_e = \frac{1}{2} [Q_1 V_1 + Q_2 V_2 + Q_3 V_3 + ...] = \frac{1}{2} \sum_{i=1}^{N} Q_i V_i \quad , \tag{3.16}$$

where $V_i = \sum_{j=1, j \neq i}^{N} V_{ji}$ is the potential at the location of the ith point charge due to other (N-1) charges.

Is the electrostatic potential energy of the discrete system of charges stored in their electric field?

To answer this question, we will go back to consider a system of two point charges. If the electrostatic potential energy of two point charges is not stored in their electric field as argued or explained above, then the electrostatic potential energy of a discrete system of N charges is not also stored in their electric field. This electrostatic potential energy as its name states is the energy the system possesses by virtue of their position relative to each other; whether N is equal to two, three, ten or 1000 point charges.

The electrostatic potential energy of a system of charges is the energy that will be released as the point charges are allowed to recede from each other one by one.

But how is this energy released? Is it released in space?

To answer this question, let us consider a system of two point charges separated by a distance R. The system has electrostatic potential energy by virtue of their position relative to each other. Let 's also assume that the charges are of the same sign as discussed above. When we keep one of the charges fixed and allow the other to move, its or their electrostatic potential is not released in space. It is converted into kinetic energy. And if no other force is acting on this charge except the electrostatic force of the other charge, the change in its kinetic energy would be equal to the chance in its potential energy. From this we can infer that when the charges in the system of N charges are allowed to recede from the system one by one, their electrostatic potential energy (that is the energy sored by the system) would be released as kinetic energy of the system.

3.1.2.2 Electrostatic Potential Energy of a Continuous System of charges.

On a microscopic scale, charge is quantized, there are often situations in which the charges are so close together that the charges can be thought of as continuously distributed. According to the standard approach, the electrostatic potential energy of a continuous charge distribution is given as follows:

If $\rho(\vec{r})$ is the density of charge distribution at $\vec{r}$, the above result can be generalized. Hence the work required to assemble a continuous charge distribution is

$$W_e=\frac{1}{8\pi\epsilon_0}\iint\frac{\rho(\vec{r})\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d\tau\, d\tau' \quad ,$$

$$W_e=\frac{1}{2}\int\rho(\vec{r})V(\vec{r})d\tau \quad . \tag{3.17}$$

(In case of a line charge or surface charge distribution, the integration is over the appropriate dimensions.)

Given that the integral is over the charge distribution, it may be extended over all space by defining the charge density to be zero outside the distribution, so that the contribution to integral come from the region of space where the density is non-zero. Hence Equation 3.17 can be written as

$$W_e=\frac{1}{2}\int_{all\ space}\rho(\vec{r})V(\vec{r})d\tau \quad . \tag{3.18}$$

Using the differential form of Gauss's law $\nabla\cdot\vec{E}=\frac{\rho}{\epsilon_0}$, we can write (3.18) as

$$W_e=\frac{\epsilon_0}{2}\int_{all\ space}(\nabla\cdot\vec{E})V(\vec{r})d\tau \quad . \tag{3.19}$$

On using the vector identity

$$\nabla\cdot(V\vec{E})=V\nabla\cdot\vec{E}+\vec{E}\cdot\nabla V \quad ,$$

we obtain

$$W_e = \frac{\epsilon_0}{2} \int_{allspace} \nabla \cdot (V \vec{E}) \, d\tau + \frac{\epsilon_0}{2} \int_{allspace} |E|^2 \, d\tau \quad , \tag{3.20}$$

where we have used $\vec{E} = -\nabla V$.

We can convert the first integral to a surface integral by using the divergence theorem. We get

$$W_e = \frac{\epsilon_0}{2} \int_S (V \vec{E}) \cdot d\vec{S} + \frac{\epsilon_0}{2} \int_{allspace} |E|^2 \, d\tau \quad .$$

But $V \propto R^{-1}$, $\vec{E} \propto R^{-2}$ and $dS \propto R^2$, we may therefore take the bounding surface to infinite distances, where the electric field is zero. The first integral vanishes and we have

$$W_e = \frac{\epsilon_0}{2} \int_{allspace} |E|^2 \, d\tau \quad . \tag{3.21}$$

This is the work (energy) required to put (assemble) the continuous distribution of charges together; hence the energy required to set up the electric field. **The question that we need to answer is: where is the electrostatic potential energy of this continuous system of charges stored? Is it stored in the Fields or by the charge density (that is the charges).**

The answer given by Prof Jim Stiles [19] of the University of Kansas is:

One equation for W_e suggest that the energy is stored by the fields, while other by the charges. It turn out that both interpretations are correct. The fields cannot exist without the charge density, and knowledge of field allows us to determine the charge density. In other words, charges and fields they create are "inseparable pairs", since both must be present, we can attribute the energy stored to either quantity.

This answer is a very interesting one. It is clear that charges and fields they create are inseparable pairs. If we can interpret one of the equations as if the energy is stored by the charges and the other equation as if the energy is stored in the fields, that means that there is a problem we need to solve. If the energy can be thought of as stored in either the field created by charges or by the charges, it is nevertheless clear that it must be stored in one of them. But the answer to this question can be given by considering the electrostatic potential energy of a discrete system of charges since the electrostatic potential energy of a continuous system of charges is the generalization of a discrete system.

Considering this option, it was explained that the electrostatic potential energy of a system of two charges is the energy the system of charges have by virtue of their positions relative to each other. We can say that this energy is stored by either of those charges (depending of the one we consider to be in the electrostatic potential of the other). The same argument was extended to a discrete system of charges, and it was inferred that the electrostatic potential energy of a discrete system of N charges is stored by the charges. It is the energy the charges possess by virtue of their position relative to each. If we all accept that the electrostatic potential energy of a continuous system of charges is a generalization of the discrete system, I do not think that some one will argue with me that the electrostatic potential energy of a continuous system of charges is stored by the charges even though Equation(3.21) can be interpreted as it was stored in the field. It is the energy the continuous system of charges possesses or stores by virtue of their position relative to each other. It is the energy required to construct the charge distribution. To elucidate this point, let us consider the electrostatic potential energy of a spherical conductor of radius R. When the sphere carries a charge q, its potential relative to infinity(V=0) is

$$V=\frac{kq}{R} \quad .$$

The work we must do to bring an additional amount of charge dq from infinity to the surface of conductor is Vdq. This work is equal to the increase in potential energy of the conductor:

$$dU=Vdq=k\frac{q}{R}dq \quad . \tag{3.22}$$

The total potential energy is the integral of dU as q increases from zero to its final value Q. Integrating, we obtain

$$U=\frac{k}{R}\int_0^Q qdq=\frac{kQ^2}{R}=\frac{1}{2}QV \quad , \tag{3.23}$$

where V=kQ/R is the potential on the surface of a fully charged sphere. Equation3.23 can be interpreted as $U=Q\times\frac{1}{2}V$ where $\frac{1}{2}V$ is the average potential of the spherical conductor during the charging process[8]. Since the electric field created by a conductor is perpendicular to the surface of the conductor, when charges are on the surface of the conductor, they must not experience the electrostatic repulsion of other charges. Now let's assume that the amount of charge dq in Equation3.22 is allowed to recede from the conductor. As soon as it leaves the surface, it will experience an electrostatic repulsion due to the field of charge q. The electric field due to charge q will do work on charge dq, thereby sending it to infinity again. If no other force acts on the charge element dq, this work done must be equal to the change in its kinetic energy. By the conservation of energy, we have

$$\frac{1}{2}mv_1^2+dqV_1=\frac{1}{2}mv_2^2+dqV_2 \quad , \tag{3.24}$$

where m equals the mass of the charge element dq. On the surface of the sphere the potential $V_1=V$ and the initial velocity at that point is zero. At infinity, the potential $V_2=0$ and the velocity is equal to v_2. Hence Equation3.24 becomes

$$dqV=\frac{1}{2}mv_2^2 \quad , \tag{3.25}$$

proving that the change in potential energy is equal to the change in kinetic energy. Here the kinetic energy of charge dq is equal to it initial potential energy on the surface of the sphere. We can say that the potential energy of the charge element dq has been converted into its kinetic energy. The purpose of this description is to confirm that the potential energy is stored by the charges and when it is released it is converted into kinetic energy.

We all agree that a point charge in space at an infinite distance from other charges has no potential energy and we can say that this point charge stores no potential energy. Since the electric field is inseparable from the charge, it is clear that this point charge has electric field around it. The electric field of this point charge will store no energy. This means that even admitting that the electric field of a continuous system of charges stores energy, then this cannot be generalized to the every electric field.

Also to arrive at Equation3.21, we started from Equation3.17, which is the potential energy of a continuous system of charges. We can therefore boldly say that Equation3.17 **is equal** to Equation3.21. Meaning that

$$\frac{1}{2}\int \rho(\vec{r})V(\vec{r})d\tau=\frac{\epsilon_0}{2}\int |E|^2 d\tau \quad . \tag{3.26}$$

Now let's evaluate the energy stored in a certain region R outside the charge distribution. Let's assume that the volume of that region is V_R . Using the right-hand side of Equation3.26, the electrostatic potential energy or the energy stored in this region is

$$E_R=\frac{1}{2}E^2 V_R \quad . \tag{3.27}$$

Using also the left-hand side of Equation3.26, the electrostatic potential energy of this region or the energy stored in this region is

$$E_R=\frac{1}{2}\int_{V_R} \rho(\vec{r})V(\vec{r})d\tau \quad . \tag{3.28}$$

It is clear that the electrostatic potential is non-zero in this region since there is electric field in this region due to the charge distribution outside the region, but there is no charge density in this region, which means that $\rho(\vec{r})=0$. Hence Equation 3.28 becomes

$$E_R=0 \quad . \tag{3.29}$$

This means that the electrostatic potential energy stored in this region is zero. How can we have at the same time and in the same region

$$E_R=0 \quad \text{and} \quad E_R=\frac{1}{2}E^2 V_R \quad ? \tag{3.30}$$

When moving from Equation3.18 to Equation 3.19, we made use of the differential form of Gauss's law, replacing the charge density at a point by the divergence of the electric field at that point. We then proceeded to Equation3.20 using a vector identity, and finally we arrived at Equation3.21. I find surprising that some people still will require that the charge density and the potential term appear before they can know that the electrostatic potential energy of continuous system of charges is stored by the charges or is the energy of the charges. What we forgot later is that in this process there is an implicit term of the electric field corresponding to charge density and another term corresponding to the potential difference. And a careful thinking with observation of the equations can reveal this to us.

From all these explanations, it is clear that the electrostatic potential energy of a discrete system of charges is not stored in their electric field but by the charges; as well as the electrostatic potential energy of a continuous system of charges even though the electrostatic potential energy of a continuous system can be evaluated using Equation3.21; giving the impression as if the energy was stored in the electric field. The electric field by itself should not have energy density, but in every electric field there is a potential field that can be defined. Meaning that at every point in an electric field there can be defined an electrostatic potential, and another charge found in that electric field should have (or if we want store) an electrostatic potential energy. The electric field, the magnetic field as well as the potential field, as everyone will agree with me, are not something visible or material that exist in space around charges or currents, whether they being static or time varying fields. We can nevertheless measure their strength in the region and know that they are present at any given point or in any given region. For example, we can determine the presence of the electric field at a point by the force it will exert on a charge placed at that point.

Let' s also consider the case of a parallel-plate capacitor given in the diagram below.

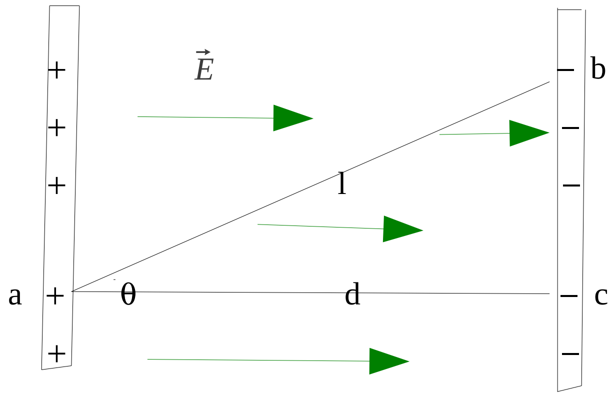


Figure-3.4 *A 2D diagram of a Parallel-plate capacitor. d=plates separation distance .θ= The angle the field makes with the finite displacement* $\vec{l}$ *as the hypothetical test charged particle moves from a to b. The electric field is parallel to the infinitesimal displacement at all points along the path from point a to point c. The green arrows denote the electric field lines between the plates.*

The potential difference of a fully charged parallel-plate capacitor is equal to the work per unit charge that the electric field between the plates of the capacitor will do on a hypothetical test charge as the test charge moves from point a to point c. It is given as

$$V=\int_a^c \vec{E}\cdot\vec{dl} \quad .$$

Given that $\vec{E}$ is uniform between the plates and is in the direction of the infinitesimal displacement vector $\vec{dl}$ at every point along the motion of the test charge, we obtain

$$V=\int_a^c E\,dl\cos\theta=E\int_a^c dl=E\,d \quad . \qquad (3.31)$$

If the test charge of charge q moves from point a to c, its potential energy will change by an amount equals to

$$\Delta u=q\,\Delta V=qV \quad . \qquad (3.32)$$

This change in potential energy of charge q is equal to the change in its kinetic energy, and it is also equal to the work done by the electric field on the charge q as it moves from a to c. When we write the electrostatic potential energy of the capacitor as

$$U=\frac{1}{2}QV \quad , \qquad (3.3b)$$

we can interpret $\frac{1}{2}V$ as the average potential during the charging process. The potential energy of the capacitor given by Equation3.3b is the energy stored by the capacitor. This energy stored is equal to work that the capacitor can deliver when it discharges. For example if the charges (electrons) where allowed to move from the negative plate to the positive plate, the electric field between the plates will deliver or do a work equals to the potential energy of the capacitor given by Equation3.3b. As the capacitor discharges (electrons moving from negative to positive plate) the potential difference between the plates decreases because the electric field between the plates also decreases. The potential difference between the plates decreases till it reaches zero when the capacitor is completely discharged. The electric field created between the plates is a force field, which does a work and this work is equal to the electrostatic potential energy of the charges and hence the capacitor. A force field should not store energy or have an energy density, but a force should be able to produce work. And that is what the electric field does. If we want to consider the work that the electric field between the plates of the capacitor can do as the energy of the field or the energy stored by the field, then we need to know that this energy is not in reality stored by the field but the field will produce this energy in form of work when charges moves from one plate to the other.

From Equation3.3b, when we use the definition of the capacitance (C=Q/V) to obtain

$$U=\frac{1}{2}CV^2 \quad , \tag{3.3c}$$

we should not forget that we have substituted Q by CV . And that the charge Q can also be written as

$$Q=CV=\epsilon_0\frac{A}{d}Ed=\epsilon_0 EA \quad , \tag{3.33}$$

where we have made the substitutions $C=\varepsilon_0 A/d$ and V=Ed.

Substituting the potential difference by V=Ed and the charge Q with its value given in equation3.33 into Equation3.3b gives us

$$U=\frac{1}{2}\epsilon_0 EA\times Ed \quad .$$

Rearranging we obtain the electrostatic potential energy of the capacitor is

$$U=\frac{1}{2}\epsilon_0 E^2 Ad \quad . \tag{3.4b}$$

Obtaining this equation should not lead us do deny that this energy is due to the charge Q whose value is given by Equation3.33 and to the potential difference V=Ed, and just make the conclusion that wherever there is electric field there is an energy density or there is an energy stored in the field there. Equations 3.34 and 3.21 are of course misleading; but as physicists we should be careful not to fall in a trap, because falling in a trap can sometime cost a lot to science.

To confirm that the energy stored in the capacitor is indeed stored by the charge, let 's consider a parallel-plate capacitor given in figure3.1 when the capacitor is fully charged, with a potential difference V between its positive and negative terminals. . Let 's connect its positive terminal to its negative terminal by a conducting-wire. The electrons that are the charge carriers will flow from the negative terminal (plate) to the the positive terminal. We shall assume that the resistance of the conducting wire is negligible and that the electrons move in the wire without collision with the lattice of the ions.

We also know that the potential energy of a positive charge +q increases if it is taken to region of higher potential and the potential energy of a negative charge -q decreases if it taken to region of higher potential.

Going back to case of a capacitor, we can boldly say that if the electric field is the only force acting on the electron, the kinetic energy of the electron will increase by the same amount as the electron moves from the negative plate to the positive plate; that is the electron moving from low potential to higher potential, hence a decrease in its potential energy. Let the charge (electron) has velocity v_1 at point

P_1 on the negative plate where the potential is V_1 . If it moves to point P_2 on the positive plate, where the potential is V_2 , then, its velocity v_2 at this point is given by the conservation of energy as

$$\frac{1}{2}mv_1^2+qV_1=\frac{1}{2}mv_2^2+qV_2 \quad . \tag{3.34}$$

If we consider that the velocity of the electron at point P_1 is zero, then the kinetic of the electron when it reaches the positive plate will be equal to the change in its potential energy. We obtain

$$v_2^2=\frac{2q(V_1-V_2)}{m} \quad , \tag{3.35}$$

where m is the mass of the electron.

The work done on a particle (electron) by the electric field in the conducting-wire in moving it from P_1 to P_2 is given by work-energy theorem

Work done = Change in K.E

$$Work\,done=\frac{1}{2}m(v_2^2-v_1^2)$$

$$= q(V_1-V_2) \tag{3.36}$$

We can therefore say that the energy stored by the capacitor is the electrostatic potential energy of its charges. This electrostatic potential energy is not dissipated in space as the capacitor discharges, but it is converted into the kinetic energy of the charges. As the electrons move from the negative plate to the positive plate (discharge of the capacitor), the potential energy of the electrons decreases. In this process, the potential difference of the capacitor also decreases till it reaches zero when all electrons have migrated to the positive plate. At this moment the capacitor is completely discharged. As the capacitor is discharging, its potential energy is of course decreasing. It is clear that the work required to charge the capacitor is the work required to establish the electric field. The work required to establish the electric field is also the work the electric field of the capacitor will produce when the capacitor is discharging. This work produced by the electric field is equal to the chance in potential energy of the charges (electrons) as they move from the negative plate to the positive plate. By Equation3.36, the change in potential energy of the charge is equal to the change in its kinetic energy. But it is clear that the change in potential energy of every charge will not be equal; given that the potential difference of the capacitor also decreases as it discharges. What I am trying to make clear is that the established electric field of the capacitor does work and this work is equal to the potential energy lost by the charges but the electric field in the plate of the capacitor does not store energy and therefore cannot have an energy density. The electric field is a invisible force field (per unit charge), as is every force or force field, that can do work as every force can. Every force field should be able to produce work but a force (or force field) by itself should not store energy or have an energy density.

In the real situation, when we connect the positive terminal to the negative terminal of a capacitor by a conducting-wire, thereby creating an electric field in a conducting-wire, the electrons that move from the negative plate to the positive plate gain kinetic energy due to work done by the field on free electrons or because of the potential difference. However, steady state is soon achieved as the kinetic energy gained by the electrons is continuously dissipated as thermal energy into the conductor, due to the interactions between free electrons and lattice of the ions of the conducting-wire. This mechanism for increasing the thermal energy of a conductor is the well known process called **Joule heating .** A process first studied by James Prescott joule in 1841. We generally say that the electrostatic potential energy of the charges are dissipated as thermal energy. But if we really want to be specific, the potential energy of the charges is first converted into their kinetic energy, and the latter dissipated as thermal energy. Hence we say energy is transferred from the electrical power supply to the conductor and any materials with which it is in thermal contact as thermal energy. Even though it is not the electrical energy of the power supply that is directly dissipated as thermal energy. Some of the applications of Joule heating are [18]:

1.An incandescent light bulb glows when the filament is heated by Joule heating, so hot that it glows white with thermal radiation (also called black body radiation).

2.Electric stoves and other electric heaters usually work by Joule heating.

3.Soldering irons and cartridge heaters are also very often heated by Joule heating.

In all these precesses we say that the devices dissipate electrical energy supplied by the power supply as thermal energy, even though it is not the electrical energy (electrostatic potential energy of the charge carriers) that is being dissipated directly. The electrostatic potential energy of the charges is first converted into kinetic energy and this kinetic energy is lost as thermal energy by collisions with the lattice of the ions.

For an AC current, the voltage keep changing polarity. The charge carriers that are often electrons are being accelerated and decelerated because the electric field in the conducting wires keep changing directions. This acceleration and deceleration causes the electrons to loose some of their kinetic energy as photons. These precesses tell us that it is not the electrostatic potential energy of the electrons that are directly converted into thermal energy or photons but it is their kinetic energy that is converted directly into thermal energy or photons. The electrostatic potential energy of the electrons is converted into kinetic energy and their kinetic energy is later converted into photons.

A charge particle found at a given in point in an electric field has potential energy , because at every point in an electric field we can define an electric potential or a potential field can be defined in an electric field. A charge particle at a given potential cannot loose its electrostatic potential energy if it does not move away from that point (assuming the potential at the point does not change). The energy radiated by charges is never directly their electric energy or their electrostatic potential energy. It is often their kinetic or vibrational (oscillatory) energy that is being converted to photons. They do also radiate their thermal energy. So saying that thermal energy (example blackbody radiation) is made up of electric energy is just inventing something that do not exist.

3.2.The differential forms of Maxwell's Equations

3.2.1. Standard opinion

This section contains a portion of the MIT , fall 2005, OpenCourseWare [7] . I have only copied the portion that is of interest for my discussion.

The differential form of Gauss's laws for electricity and magnetism are given as:

$$divr(\epsilon_o \vec{E}) = \rho \quad , \tag{3.37}$$

and

$$divr(\mu_o \vec{H}) = 0 \quad . \tag{3.38}$$

3.2.1.1 Stokes' Theorem

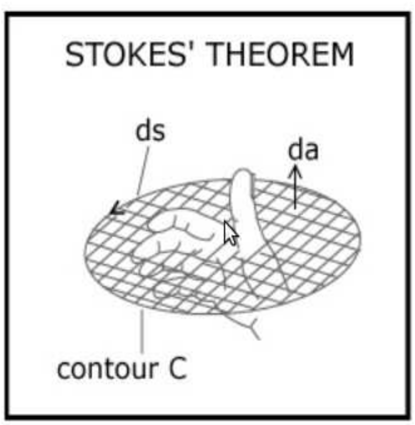
a-Curl Operation

Let's consider an arbitrary vector field $\vec{A}$, from Stokes' theorem we have

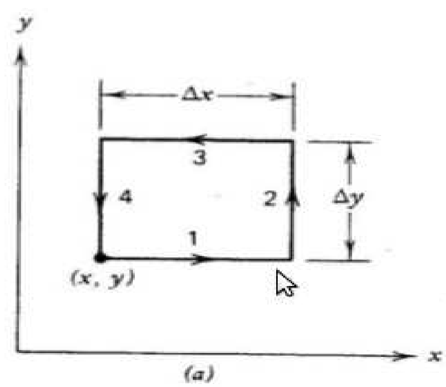
$$\oint \vec{A} \cdot \vec{ds} = \iint curl\, \vec{A} \cdot \vec{da} \quad , \tag{3.39}$$

where

$$curl(\vec{A})_n = \lim_{da_n \to 0} \frac{\oint \vec{A} \cdot \vec{ds}}{da_n} \quad . \tag{3.40}$$



$$\int_S \nabla \times \overline{A} \cdot \overline{da} = \oint_C \overline{A} \cdot \overline{ds}$$



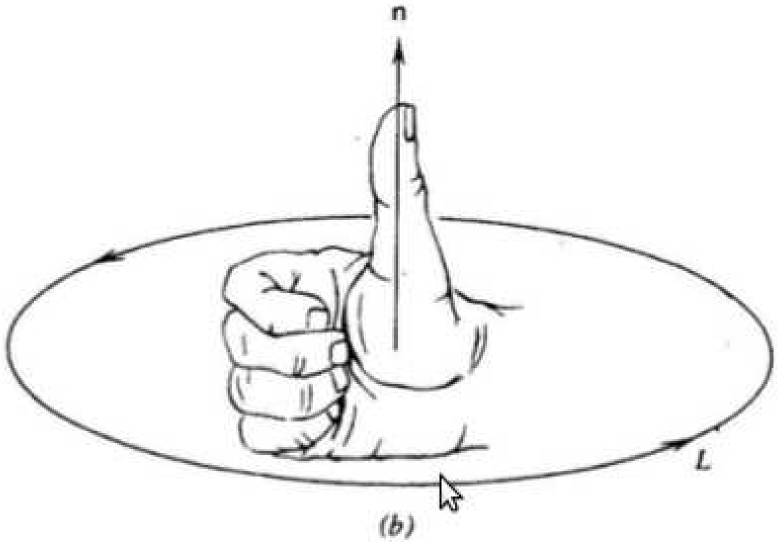


Figure 3.5 (a) Infinitesimal rectangular contour used to define the circulation. (b) The right-hand rule determines the positive direction perpendicular to a contour.

$$\oint \vec{\mathbf{A}}\cdot\vec{ds}=\int_{x}^{x+\Delta x} A_x(x,y)dx+\int_{y}^{y+\Delta y} A_y(x+\Delta x,y)dy+\int_{x+\Delta x}^{x} A_x(x,y+\Delta y)dx+\int_{y+\Delta y}^{y} A_y(x,y)dy \quad ,$$

$$\oint \vec{\mathbf{A}}\cdot\vec{ds}=\Delta x\Delta y[\frac{[A_x(x,y)-A_x(x,y+\Delta y)]}{\Delta y}+\frac{[A_y(x+\Delta x)-A_y(x,y)]}{\Delta x}] \quad ,$$

$$\oint \vec{\mathbf{A}}\cdot\vec{ds}=da_z[\frac{\partial A_x}{\partial y}-\frac{\partial A_y}{\partial x}] \quad . \tag{3.41}$$

Hence

$$curl(\vec{\mathbf{A}})_z=\frac{\oint \vec{\mathbf{A}}\cdot\vec{ds}}{\vec{da}_z}=\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y} \quad . \tag{3.42}$$

By symmetry,

$$curl(\vec{\mathbf{A}})_y=\frac{\oint \vec{\mathbf{A}}\cdot\vec{ds}}{\vec{da}_y}=\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x} \quad , \tag{3.43}$$

and

$$curl(\vec{\boldsymbol{A}})_x = \frac{\oint \vec{\boldsymbol{A}} \cdot \vec{ds}}{\vec{da}_x} = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \quad . \tag{3.44}$$

Which implies that

$$curl(\vec{\boldsymbol{A}}) = \nabla \times \vec{\boldsymbol{A}} \quad . \tag{3.45}$$

b-Stokes' integral theorem

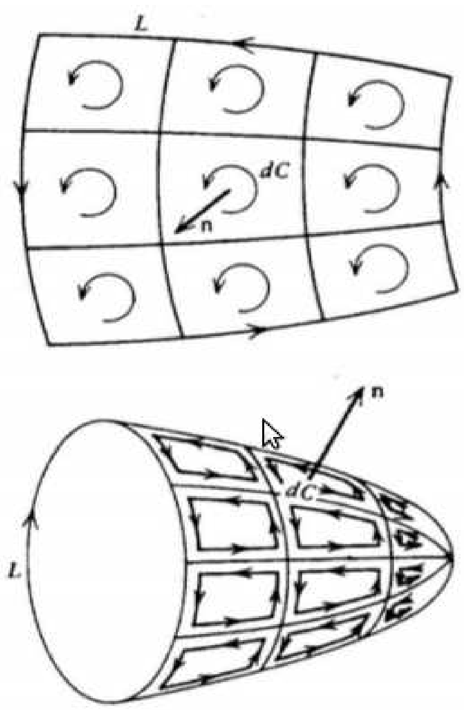


Figure 3.6 Many incremental line contours distributed over any surface, have nonzero contribution to the circulation only along those parts of the surface on the boundary contour L.

Courtesy of Krieger Publishing. Used with permission.

It is by definition given as

$$\lim_{N \to \infty} \sum_{i=1}^{N} \oint_{dc_i} \vec{\boldsymbol{A}} \cdot \vec{ds} = \oint_{C} \vec{\boldsymbol{A}} \cdot \vec{ds} \quad . \tag{3.46}$$

Hence,

$$\lim_{N\to\infty}\sum_{i=1}^{N}\oint_{dc_i}\vec{\boldsymbol{A}}\cdot\vec{ds}=\sum_{i=1}^{N\to\infty}(\nabla\times\vec{\boldsymbol{A}})\cdot\vec{da}_i=\int_S(\nabla\times\vec{\boldsymbol{A}})\cdot\vec{da} \quad . \tag{3.47}$$

c-Faraday's law in differential form

Combining Faraday's law and Stokes' theorem, we have

$$\oint_C\vec{\boldsymbol{E}}\cdot\vec{ds}=\int_S(\nabla\times\vec{\boldsymbol{E}})\cdot\vec{da}=\frac{-d}{dt}\int_S\mu_0\vec{\boldsymbol{H}}\cdot\vec{da} \quad . \tag{3.48}$$

Thus,

$$\nabla\times\vec{\boldsymbol{E}}=-\mu_0\frac{\partial\vec{\boldsymbol{H}}}{\partial t} \quad . \tag{3.49}$$

d-Ampere's law in differential form

Stating Ampere's law and using Stokes' theorem, we have

$$\oint_C\vec{\boldsymbol{H}}\cdot\vec{ds}=\int_S\nabla\times\vec{\boldsymbol{H}}\cdot\vec{da}=\int_S\vec{\boldsymbol{J}}\cdot\vec{da}+\frac{d}{dt}\int_S\epsilon_0\vec{\boldsymbol{E}}\cdot\vec{da} \quad , \tag{3.50}$$

hence,

$$\nabla\times\vec{\boldsymbol{H}}=\vec{\boldsymbol{J}}\cdot\vec{da}+\epsilon_0\frac{\partial\vec{\boldsymbol{E}}}{\partial t} \quad . \tag{3.51}$$

3.2.2 My Opinion

Maxwell's equations are the statements of the laws of electricity and magnetism. These laws include Gauss's laws of electricity and magnetism, Faraday's law of magnetic induction , and Ampere's law. These laws can be stated mathematically as

$$\oint_S \vec{E}\cdot\vec{da}=Q/\epsilon_0 \quad , \quad \text{Gauss's law for electricity} \tag{3.52}$$

$$\oint_S \vec{B}\cdot\vec{da}=0 \quad , \quad \text{Gauss's law for magnetism} \tag{3.53}$$

$$\oint_C \vec{E}\cdot\vec{ds}=\frac{-d}{dt}\int_s \vec{B}\cdot\vec{da} \quad , \text{ Faraday's law} \tag{3.54.a}$$

$$\oint_C \vec{B}\cdot\vec{ds}=\mu_0 I \quad . \text{ Ampere's law} \tag{3.55}$$

Ampere's law remains valid as long as currents are steady and continuous. Ampere's law is used simply by selecting any closed loop, traversing it with small element $\vec{ds}$, and solving the resulting equation. It is important to note that we are free to select any surface, a flat disc or perhaps a shape similar to a grocery bag as in Figure-3.7, and we should expect the same results.

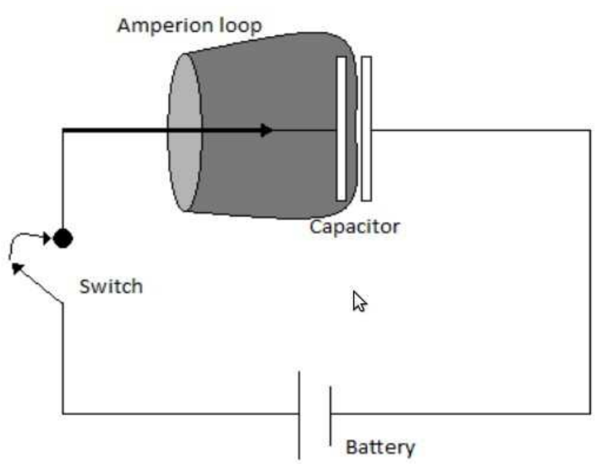


Figure 3.7 *Ampere's law for a disk like surface and another surface of the shape of a grocery bag.*

Ampere's law predicted the magnetic field very accurately but Maxwell noticed that there was something missing. He noted that a capacitor is made up of two conducting plates that are separated by a distance d, and that while that capacitor was charging, positive charges accumulate on one plate and negative charges accumulate on the other; but that no current passes between the plates. A capacitor is essentially a “gap” in the circuit, but due to its nature , the circuit remains complete. However if we use Ampere's law to find the magnetic field at a point in space, it is possible so select one closed loop with the surface of a bag shape passing through the capacitor, so that no current passes through the surface.

This would indicate that there is no magnetic field at the point. However, another surface defined by the same loop can be selected that passes through one of the wires connected to the capacitor and since current flows through the wire, the law would clearly indicate that there is magnetic field at the point (through the same loop). This could not be, something must be missing.

Maxwell called the missing term “displacement current” although it is not real current but rather a changing electric field within the capacitor. Since charges are accumulating on the plate of the capacitor, there is a changing electric field between the two plates. Maxwell completed the equation by adding the term $\mu_0 \epsilon_o \frac{d\phi_e}{dt}$, to Equation(3.55) to obtain

$$\oint_C \vec{\boldsymbol{B}} \cdot \vec{ds} = \mu_0 \epsilon_0 \frac{d\phi_e}{dt} + \mu_0 I \quad , \tag{3.56}$$

now called Maxwell-Ampere's law and which applies also to currents that are not steady.

When the first term on the right hand side of Equation(3.56) is not present, the equation reduces to Ampere's law.

The differential forms of Maxwell's equations are questionable for the following reasons:

a-First reason

Let's consider Equation(3.54) and let's derive the differential form of Maxwell's equation from Faraday's law.

Applying Stokes's theorem to the left-hand side of Equation(3.54), we have

$$\oint_C \vec{\boldsymbol{E}} \cdot \vec{ds} = \int_S curl\, \vec{\boldsymbol{E}} \cdot \vec{da} = \frac{-d}{dt} \int_s \vec{\boldsymbol{B}} \cdot \vec{da} \quad . \tag{3.57}$$

Which implies that

$$\int_S curl\, \vec{\boldsymbol{E}} \cdot \vec{n}\, da = \frac{-d}{dt} \int_s \vec{\boldsymbol{B}} \cdot \vec{n}\, da \quad , \tag{3.58}$$

where $\vec{n}$ is a unit vector normal to the surface element da, of course $\vec{n}$ varies from element to element but all $\vec{n}$'s must be on one side of the two-sided surface.

When we have the above Equation(3.58), there is no mathematical formula that permits us to conclude that

$$curl\, \vec{\boldsymbol{E}} = -\frac{\partial \vec{\boldsymbol{B}}}{\partial t} \quad . \tag{3.59}$$

The only conclusion that can be derived from it is that:

$$curl\, \vec{\boldsymbol{E}} \cdot \vec{n} = -\frac{\partial \vec{\boldsymbol{B}}}{\partial t} \cdot \vec{n} \quad . \tag{3.60}$$

Hence

$$\|curl\, \vec{\boldsymbol{E}}\| \cos\theta_1 = -\left\|\frac{\partial \vec{\boldsymbol{B}}}{\partial t}\right\| \cos\theta_2 \quad , \tag{3.61}$$

where θ_1 is the angle the vector $curl\,\vec{\boldsymbol{E}}$ makes with the unit vector normal to the surface element da, and θ_2 is also the angle the vector $\frac{\partial \vec{\boldsymbol{B}}}{\partial t}$ makes with the unit vector normal to surface element da.

To prove that when we have Equation(3.58) it does necessarily imply Equation(3.59), let us consider three arbitrary vector fields

$\vec{\boldsymbol{A}}(\vec{r},t)$, $\vec{\boldsymbol{B}}(\vec{r},t)$, and $\vec{\boldsymbol{C}}(\vec{r},t)$.

If $$\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{A}}=\vec{\boldsymbol{C}}\cdot\vec{\boldsymbol{A}} \quad , \tag{3.62}$$

this does not necessarily imply that

$$\vec{\boldsymbol{B}}=\vec{\boldsymbol{C}} \tag{3.63}$$

To prove this let

$$\vec{\boldsymbol{B}}=B_1(x,y.z,t)\vec{i}-B_2(x,y,z,t)\vec{j}+B_3(x,y,z,t)\vec{k} \quad , \tag{3.64}$$

$$\vec{\boldsymbol{C}}=C_1(x,y.z,t)\vec{i}+C_2(x,y,z,t)\vec{j}+C_3(x,y,z,t)\vec{k} \quad , \tag{3.65}$$

and

$$\vec{\boldsymbol{A}}=A_1(x,y.z,t)\vec{i}+A_2(x,y,z,t)\vec{j}+A_3(x,y,z,t)\vec{k} \quad , \tag{3.66}$$

where

$$C_1=-B_1 \quad , \quad C_2=B_2 \quad , \quad C_3=B_3 \quad , \quad B_1=B_2 \quad \text{and} \quad A_1=A_2 \quad .$$

We can rewrite $\vec{\boldsymbol{C}}$ as

$$\vec{\boldsymbol{C}}=-B_1\vec{i}+B_1\vec{j}+B_3\vec{k} \quad , \tag{3.67}$$

$$\vec{\boldsymbol{B}}=B_1\vec{i}-B_1\vec{j}+B_3\vec{k} \quad , \tag{3.68}$$

also

$$\vec{\boldsymbol{A}}=A_1\vec{i}+A_1\vec{j}+A_3\vec{k} \quad . \tag{3.69}$$

Multiplying (that is taking the dot product of the vectors) Equation(3.68) with Equation(3.69), we obtain

$$\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{A}}=B_1A_1-B_1A_1+B_3A_3=B_3A_3 \quad . \tag{3.70}$$

Also multiplying Equation(3.67) with Equation(3.69), we obtain

$$\vec{\boldsymbol{C}}\cdot\vec{\boldsymbol{A}}=B_1A_1-B_1A_1+B_3A_3=B_3A_3 \quad . \tag{3.71}$$

Hence

$$\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{A}}=\vec{\boldsymbol{C}}\cdot\vec{\boldsymbol{A}} \quad \text{but} \quad \vec{\boldsymbol{B}}\neq\vec{\boldsymbol{C}} \quad . \tag{3.72}$$

In particular if for example we have

$$\vec{\boldsymbol{B}}=2\vec{i}-2\vec{j}+3\vec{k} \quad , \tag{3.73}$$

$$\vec{\boldsymbol{C}}=-2\vec{i}+2\vec{j}+3\vec{k} \quad , \tag{3.74}$$

and

$$\vec{\boldsymbol{n}}=\vec{i}+\vec{j}+\vec{k} \quad . \tag{3.75}$$

Thus,

$$\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{n}}=3 \quad , \tag{3.76}$$

and

$$\vec{\boldsymbol{C}}\cdot\vec{\boldsymbol{n}}=3 \quad . \tag{3.77}$$

We therefore have

$$\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{n}}=\vec{\boldsymbol{C}}\cdot\vec{\boldsymbol{n}} \quad \text{, but} \quad \vec{\boldsymbol{B}}\neq\vec{\boldsymbol{C}} \tag{3.78}$$

What I want to establish is that Equation(3.60) tells us that the normal components on the surface elements da's of $curl\,\boldsymbol{E}$ and $-\{\frac{\partial\vec{\boldsymbol{B}}}{\partial t}\}$ are equal, and if their normal components are equal that does not necessarily imply that they are the two vectors are equal. They could be directed in different directions.

The vector would be equal if and only if Faraday's law was given by

$$\oint_C \vec{\boldsymbol{E}}\,ds=\int_S curl\vec{\boldsymbol{E}}\,da=\frac{-d}{dt}\int_s \vec{\boldsymbol{B}}\,da \quad , \tag{3.79}$$

that is the scalar da and not the vector $\vec{da}$ but that is not so.

Let's also consider the Maxwell-Ampere's law given by Equation(3.56), applying Stokes' theorem to this equation we have

$$\int_S curl\vec{\boldsymbol{B}}\cdot\vec{da}=\mu_0\epsilon_0\frac{d}{dt}\int_S \vec{\boldsymbol{E}}\cdot\vec{da}+\mu_0\int_S \vec{\boldsymbol{J}}\cdot\vec{da} \quad . \tag{3.80}$$

As already demonstrated above this equation does not necessarily imply that

$$curl\,\vec{\boldsymbol{B}}=\mu_0\epsilon_0\frac{\partial\vec{\boldsymbol{E}}}{\partial t}+\mu_0\vec{\boldsymbol{J}} \quad , \tag{3.81}$$

but it does imply that

$$curl\,\vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{n}}=\mu_0\epsilon_0\frac{\partial\vec{\boldsymbol{E}}}{\partial t}\cdot\vec{\boldsymbol{n}}+\mu_0\vec{\boldsymbol{J}}\cdot\vec{\boldsymbol{n}} \quad . \tag{3.82}$$

Which in turn implies that

$$\|curl\,\vec{\boldsymbol{B}}\|\cos\theta_1=\mu_0\epsilon_0\|\frac{\partial\vec{\boldsymbol{E}}}{\partial t}\|\cos\theta_2+\mu_0\|\vec{\boldsymbol{J}}\|\cos\theta_3 \quad , \tag{3.83}$$

where θ_1 is the angle the $curl\,\boldsymbol{B}$ makes with the unit vector normal the surface element da,

θ_2 , the angle the vector $\frac{\partial\vec{E}}{\partial t}$ makes with the unit vector normal to the surface element da

and θ_3 is the angle the vector $\vec{J}$ makes with the unit vector normal to the surface element da.

All the $\theta's$ of course vary from point to point on the surface S.

b-Second reason

Secondly, let's consider the "Amperian" loop given in Figure-3.8 where the attention is focused on the disk shape surface bounded by the loop. On that surface the current I passes through it. When we apply Stokes' theorem to this surface, we can partition the surface into infinitesimal surface elements given in Figure-3.9 below.

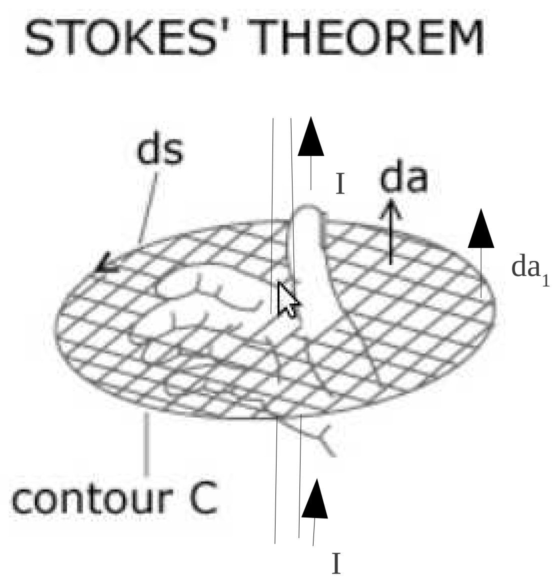


Figure-3.8 *Stokes' theorem applied to the circulation of the magnetic field around a current-carrying wire.*

Let us consider the surface element da_1 where no current passes through. In fact no current passes through the majority of the infinitesimal surface elements. According to Stokes' theorem when we take the circulations $\oint_{\delta S}\vec{B}\cdot\vec{ds}$ around the infinitesimal surface elements da, the infinitesimal circulations are not in general zero, but it is all the interior line integrals that cancel each other along a border between two da's, where the integrals are in opposite directions. If the infinitesimal circulations are not zero that means that $curl\,\vec{B}$ will be defined at every point on the surface S; including points where no current passes. The consequence is that $curl\,\vec{B}\neq 0$ on the element da_1 . But using the differential form of Maxwell-Ampere's law we have $curl\,\vec{B}=0$ at the same point because no current passes through that surface element. How can we have at the same time $curl\,\vec{B}\neq 0$ and $curl\,\vec{B}=0$ at the same point?

Also my interpretation of Faraday's law

$$\epsilon = \oint \vec{\boldsymbol{E}} \cdot \vec{ds} = -\frac{d\,\phi_m}{dt} \quad , \tag{3.54.b}$$

is not that there are always electric field lines that encircle a changing magnetic field, but it is that a changing flux through a surface bounded by a loop of conducting wire would induce a current and therefore a emf in the loop. And since the current is the rate of flow of charges and the flow of charges is the consequence of the electric field doing work on the charge carriers, the changing flux set up an electric field in the loop whether the loop is circular or square in shape it does not matter. What matters is the the loop should be closed. The current in the loop of wire is not of course due to the electric field outside the conducting wire but it is due to the electric field inside. So I would say that when the loop is not closed or when there in no wire bounding the surface, no electric field would be observed surrounding a changing magnetic flux. I therefore do not think that Stokes' theorem is even applicable to Faraday's law.

3.3. Are the solutions of Maxwell's equations in vacuum wave functions?

3.3.1 Standard opinion

This section contains a part a the book written by Paul Tipler and Mosca [8], but it contains only the summary of some parts that will be important for my discussion later.

Maxwell's equations in vacuum leads to wave equations for electric and magnetic fields. We consider only empty space (space in which no charges or currents exist) and assume that the electric and magnetic fields $\vec{E}$ and $\vec{B}$ are functions of time and one space coordinate only, which we take to be x coordinate. Such a wave is called **plane wave,** because $\vec{E}$ and $\vec{B}$ are uniform throughout any plane perpendicular to the x axis. For a plane electromagnetic wave traveling parallel to the x axis, the x components of the fields are zero, so the vectors $\vec{E}$ and $\vec{B}$ are perpendicular to the x axis and each obeys the wave equation:

$$\frac{\partial^2 \vec{\boldsymbol{E}}}{\partial x^2}=\epsilon_0 \mu_0 \frac{\partial^2 \vec{\boldsymbol{E}}}{\partial t^2} \quad , \tag{3.84}$$

this is the wave equation for $\vec{E}$.

$$\frac{\partial^2 \vec{\boldsymbol{B}}}{\partial x^2}=\epsilon_0 \mu_0 \frac{\partial^2 \vec{\boldsymbol{B}}}{\partial t^2} \quad , \tag{3.85}$$

which is the wave equation for $\vec{B}$.

Solving for the y component of Equation(3.84), we obtain

$$E_y=E_0 \sin(kx-\omega t) \quad . \tag{3.86}$$

If we substitute this solution into $\frac{\partial E_y}{\partial x}=\frac{-\partial B_z}{\partial t}$, we have

$$\frac{\partial B_z}{\partial t}=\frac{-\partial E_y}{\partial x}=-kE_0 \cos(kx-\omega t) \quad .$$

To solve for B_z , we take the integral of $\partial B_z/\partial t$ with respect to time. Doing so yields

$$B_z=\int \frac{\partial B_z}{\partial t}dt=\frac{k}{\omega}E_0 \sin(kx-\omega t)+f(x) \quad , \tag{3.87}$$

where f(x) is an arbitrary function of x.

We next substitute the solution (Equation3.86) into $\frac{\partial B_z}{\partial x}=-\mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$ and obtain

$$\frac{\partial B_z}{\partial x}=-\mu_0 \epsilon_0 \frac{\partial B_z}{\partial t}=\omega \mu_0 \epsilon_0 E_0 \cos(kx-\omega t) \quad .$$

Solving for B_z gives

$$B_z=\int \frac{\partial B_z}{\partial x}dx=\frac{\omega \mu_0 \epsilon_0 E_0}{k}\cos(kx-\omega t)+g(t) \quad , \tag{3.88}$$

where g(t) is an arbitrary function of time. Equating the right hand sides of Equations (3.87) and (3.88) gives

$$\frac{k}{\omega}E_0\sin(kx-\omega t)+f(x)=\frac{\omega\mu_0\epsilon_0 E_0}{k}\cos(kx-\omega t)+g(t) \quad . \tag{3.89}$$

Substituting c for ω/k and $1/c^2$ for $\mu_0\epsilon_0$ gives

$$\frac{1}{c}E_0\sin(kx-\omega t)+f(x)=\frac{1}{c}E_0\cos(kx-\omega t)+g(t) \quad , \tag{3.90}$$

which implies that f(x)=g(t) for all values of x and t. These remain equal if and only if f(x)=g(t)=constant (independent of both x and t). Thus, Equation(3.87) becomes

$$B_z t=\frac{k}{\omega}E_0\sin(kx-\omega t)+constant=B_0\sin(kx-\omega t) \quad , \tag{3.91}$$

where $B_0=(k/\omega)E_0=(1/c)E_0$. The integration constant was dropped because it plays no part in the wave. It merely allows for the presence of a static uniform magnetic field. Because the electric and magnetic fields oscillate in phase and have the same frequency, we have the general result that the magnitude of the electric field is c multiply by the magnitude of the magnetic field for an electromagnetic wave:

$$E=cB \quad . \tag{3.92}$$

The direction of propagation of an electromagnetic (em) wave is always the direction of the wave vector product $\vec{E}\times\vec{B}$. For the wave described in the preceding discussion, the electric and magnetic fields are given by

$$\vec{\boldsymbol{E}}(x,t)=E_0\sin(kx-\omega t)\hat{j} \quad , \tag{3.93}$$

and

$$\vec{\boldsymbol{B}}(x,t)=B_0\sin(kx-\omega t)\hat{k} \quad . \tag{3.94}$$

Thus,

$$\vec{E}\times\vec{B}=[E_0\sin(kx-\omega t)\hat{j}]\times[B_0\sin(kx-\omega t)\hat{k}]=E_0B_0\sin^2(kx-\omega t)\hat{i} \quad . \tag{3.95}$$

The term on the right is a vector in the +x direction, so we have verified that $\vec{E}\times\vec{B}$ is in the direction of the propagation for this electromagnetic wave.

We see that Maxwell's equations imply wave equations(3.84) and (3.85) for the electric and magnetic fields; and that if E_y varies harmonically as in Equation(3.86), the magnetic field B_z is in phase with E_y and has an amplitude related to the amplitude of E_y by $B_z = E_y/c$. The electric and magnetic fields are perpendicular to each other and to the direction of propagation of the wave.

30-4 ELECTROMAGNETIC RADIATION

Figure 30-6 shows the electric and magnetic field vectors of an electromagnetic wave. The electric and magnetic fields are perpendicular to each other and perpendicular to the direction of propagation of the wave. Electromagnetic waves are thus transverse waves. The electric and magnetic fields are in phase and, at each point in space and at each instant in time, their magnitudes are related by

$$E = cB \qquad 30\text{-}18$$

where $c = 1/\sqrt{\mu_0 \epsilon_0}$ is the speed of the wave. The direction of propagation of an electromagnetic wave is the direction of the cross product $\vec{E} \times \vec{B}$.

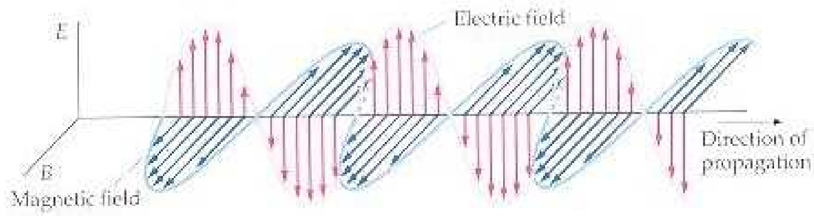


FIGURE 30-6 The electric and magnetic field vectors in an electromagnetic wave. The fields are in phase, perpendicular to each other, and perpendicular to the direction of propagation of the wave.

3.3.2.My opinion

3.3.2.1 Vector Fields

Vector fields arose originally in classical field theory in 19th century physics, specifically in magnetism. They were formalized by Michael Faraday, in his concept of lines of force, who emphasized that the field itself should be an object of study, which it has become throughout physics in the form of field theory. In addition to the magnetic field, other phenomena that were modeled as vector fields by Faraday include the electrical field and light field.

In vector calculus, a vector field is an assignment of a vector to each point in a subset of Euclidean [17] space. A vector field in the plane, for instance, can be visualized as a collection of arrows with a given magnitude and direction each attached to a point in the plane. Vector fields are often used to model, for example, the speed and direction of a moving fluid throughout space, or the strength and direction of some force, such as the magnetic or gravitational force, as it changes from point to point.

From the above description, it is clear that a vector field used to model the the speed and direction of a moving fluid throughout space by trying to represent a vector (an arrow) to each point is space is not something physical that actually exist in space. In space, when we have a moving fluid, what we actually have is the moving fluid (made of many particle that can be described as continuous), such that we can assign a speed and direction to every point in the fluid. In other terms, what we have in space are particles moving with given speeds in some particular directions. We then combine the speed and the direction of the moving fluid by representing the velocity vector field in space. Though in reality what exists in space is not the velocities, but points (particles) in the fluid moving in given directions. The reality is that at every point and at any given time, we can assign a speed and direction to each point in a moving fluid.

For the case of a force such magnetic, electric or gravitational force, we also try to model the strength and direction in which the force will act on particle if found at some points in the region by vectors that often represent the force per unit charge (electric field) or the force per unit mass (gravitational field). We combine our knowledge of the strength of the force at a given point and the direction in which the force will act to represent it by a vector. We then try to combine the two knowledge that we have to draw vectors in space pointing in different or the same directions. For example for the electric field, what exits actually in space at every point around any given charge is the strength of attraction or repulsion. In the same way that for a fluid it is not the velocities that exist in space but each point in fluid having a given speed and direction, we do not also have force per unit charge vectors (electric field vectors) at every point in space pointing either upwards or downwards or in any direction, but at every point in space around a charged particle, another charged particle will experience a given strength of the force and move in a given direction. We are using our knowledge of strength of the force that some particle will experience and the direction in which it will move to model the two by a vector in space.

3.3.2.2 Solutions of Maxwell's Equations in vacuum.

Though the solutions given by Equations(3.93) and (3.94) have the mathematical forms of wave functions, it is very important that we investigate the physics meaning that is hidden behind these two equations, because I do believe that even though we need Mathematics to express our ideas in Physics, Physics is not Mathematics. So just drawing conclusions based on a mathematical solution we obtain may sometime lead us astray.

Before establishing whether the solutions given by Equations(3.93) and(3.94) are wave functions or not, let us consider the following problem:

A positive point charge q_1 = +12 nC is on the x axis at x=-3 m, and a second positive point charge q_2 =+12 nC is also on the x axis at x= 3 m, as given in the diagram below. Find the resultant electric field due to the two charges at point A on the y axis where y= 4m.

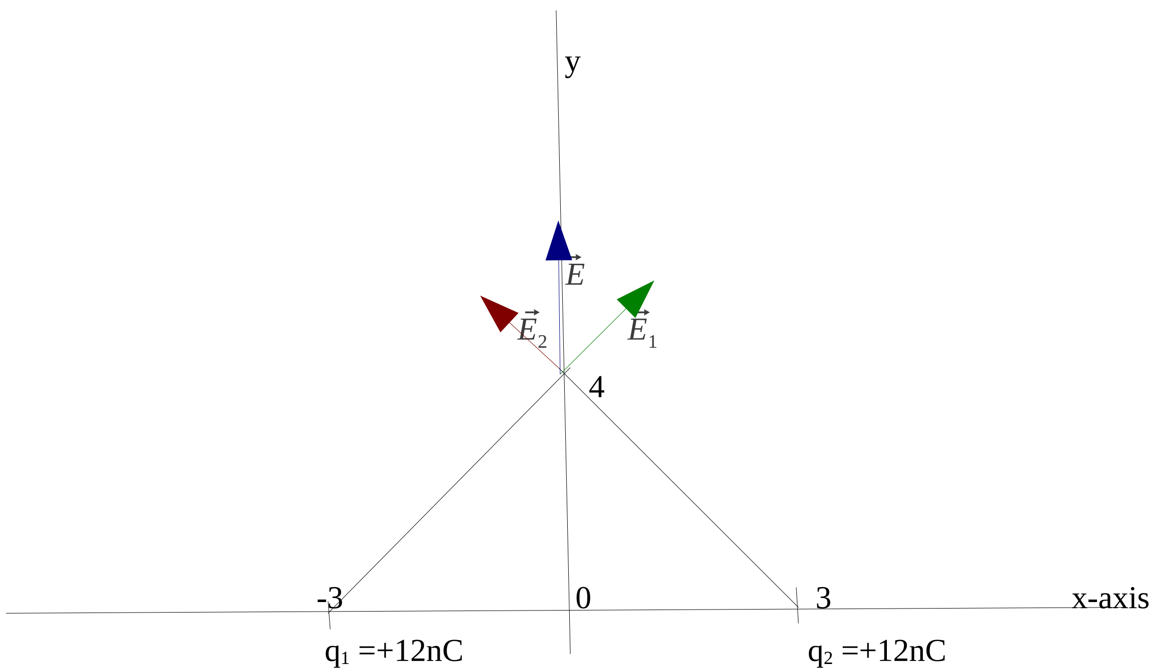


Figure 3.9 *Resultant electric field of two identical point charges.*

Let's find the resultant electrostatic field at point y= 4m on the y-axis, that is the point A(0,4,0). To achieve this, we need to resolve the electric fields at point A due to the two charges.

The electric field due to charge q_1 can be resolved into the vertical and horizontal components as

$$E_{1x}=E_1\cos\theta \quad , \qquad (3.96)$$

and

$$E_{1y}=E_1\sin\theta \quad . \qquad (3.97)$$

The field due to charge q_2 can also be resolved into the vertical and horizontal components as

$$E_{2x}=-E_2\cos\theta \quad , \tag{3.98}$$

and

$$E_{2y}=E_2\sin\theta \quad . \tag{3.99}$$

The magnitude of the electric fields at point A due the two charges are

$$E_1=k\frac{q_1}{r^2} \quad \text{and} \quad E_2=k\frac{q_2}{r^2} \quad , \tag{3.100}$$

since the two point charges are at the same distance to the field point A. Also $q_1=q_2=q$, hence

$$E_1=E_2=E'=k\frac{q}{r^2} \quad , \tag{3.101}$$

Their numerical value is given as

$$E_1=E_2=E'=\frac{8.99\times10^9\times12\times10^{-9}}{5^2}=4.32\,N/C \quad , \tag{3.102}$$

where we have used $k=8.99\times10^9\,N.m^2/C^2$.

The x-component of the resultant field due to the two point charges is therefore equal to

$$E_x=E'\cos\theta-E'\cos\theta=0 \quad , \tag{3.103}$$

and the y-component of the resultant field is

$$E_y=E'\sin\theta+E'\sin\theta=2E'\sin\theta \quad . \tag{3.104}$$

Hence the resultant electric at point A is equal to

$$\vec{E}=2E'\sin\theta\,\vec{j} \quad . \tag{3.105}$$

Its numerical value is also given as

$$\vec{E}(0,4,0)=2\times4.32\times\frac{4}{5}=6.91\,N/C\,\vec{j} \quad . \tag{3.106}$$

When representing the vector field $\vec{E}$ at point A(0,4,0), we draw an arrow starting from A and we extend it as long as we desire as in Figure-3.9 . By representing the vector $\vec{E}(0,4,0)$ with an arrow, this does not mean that the electric field at point A given by $\vec{E}=2E'\sin\theta\,\vec{j}$ which is numerically equal to $\vec{E}=6.91\,N/C\,\vec{j}$ extends from point A to another point in space. But it tells us that the field strength at point A(0,4,0) is equal to $E=|2E'\sin\theta|$, which also tells us about the magnitude of the force (per unit charge) that another charged particle would experience at that point, and that the charged particle would be accelerated along the y-axis ie in the direction of the unit vector $\vec{j}$. We cannot therefore say that because we have drawn the field vector $\vec{E}$ extending from point (0,4,0) to another point that the electric field at point (0,4,0) given by Equation(3.106) extends from that point to the point where the vector ended. Mathematically, the electric field at a point is viewed as a vector $\vec{E}$ that extends from one point to the other. But when we comeback to physics the length of electric field vector at that point gives us the information about the field strength at that point and the direction in which the vector points gives the information about the direction in which a test charge will be accelerated if found at that point. Physically, there is no electric vector in space pointing from one point to the other. Physically, an electric field is a vector function, and similarly to any function defined in space, it has a specified value at every point in space. Therefore its value at a given point in space cannot extend to another point, but it can have the same value everywhere (uniform electrostatic field).

Now let us consider the solution given by Equation(3.93) at for example at time t=0. It is given as

$$\vec{\boldsymbol{E}}(x,0)=E_0 \sin(kx)\hat{j} \quad , \tag{3.107}$$

or

$$\vec{\boldsymbol{E}}(x,0)=E_0 \sin(\frac{2\pi}{\lambda}x)\hat{j} \quad , \tag{3.108}$$

where $k=\frac{2\pi}{\lambda}$.

At point $x=\frac{\lambda}{4}$, Equation(3.108) gives as

$$\vec{\boldsymbol{E}}(\frac{\lambda}{4},0)=E_0 \sin(\frac{2\pi}{\lambda}.\frac{\lambda}{4})\vec{j}=E_0 \sin(\frac{\pi}{2})\vec{j}=E_0\vec{j} \quad , \tag{3.109}$$

hence at point $x=\frac{\lambda}{4}$,the field strength is equal to E_0 and the field vector points in the $\vec{\boldsymbol{j}}$ direction. This does not mean that the electric field extends from point $(\frac{\lambda}{4},0,0)$ to another point with coordinates $(\frac{\lambda}{4},E_0,0)$, but it tells us that the field strength at point $x=\frac{\lambda}{4}$ is E_0 and that a test charge found at that point at that instant would experience an acceleration upward ie in the $+\vec{\boldsymbol{j}}$ direction. Alternatively, it tells us the force per unit charge that a test charge would experience; that is its magnitude and the direction in which the test charge would be accelerated. It is therefore located at that point and does not extend to another point contrary to the general belief.

Let's consider another point $x=\frac{\lambda}{8}$, at that point we have

$$\vec{\boldsymbol{E}}(\frac{\lambda}{8},0)=E_0 \sin(\frac{2\pi}{\lambda}.\frac{\lambda}{8})\vec{j}=E_0 \sin(\frac{\pi}{4})\vec{j}=\frac{2}{\sqrt{2}}E_0\vec{j} \quad . \tag{3.110}$$

This also tells that at point $x=\frac{\lambda}{8}$ and time t=0 the electric field strength is equal $\frac{2}{\sqrt{2}}E_0$. Thus when drawing the field vector, we start from point $(\frac{\lambda}{8},0,0)$ and extend it upward in the $+\vec{\boldsymbol{j}}$ direction , but this should not be taken or assumed or inferred that the electric field extends from point $(\frac{\lambda}{8},0,0)$ to another point $(\frac{\lambda}{8},\frac{2}{\sqrt{2}}E_0,0)$. It just tells us that the field strength at point $(\frac{\lambda}{8},0,0)$ is $\frac{2}{\sqrt{2}}E_0$ and that a test charge if found at the point at that instant would be accelerated in $+\vec{\boldsymbol{j}}$ direction.

At point $x=\lambda$, Equation(3.108) gives

$$\vec{E}(\lambda,0)=E_0\sin(\frac{2\pi}{\lambda}.\lambda)\vec{j}=E_0\sin(2\pi)\vec{j}=0 \quad . \tag{3.111}$$

Equation(3.111) also tells us that the field strength at that point is zero. Meaning that a test charge if found at that point would experience no acceleration.

From point x= 0 to x=λ the electric field $\vec{E}(x,0)$ at every point is located at that point and does not extend to another point or upward. This means the electric field of our so called electromagnetic wave will just be points located on the x-axis that are traveling at the speed of light, and this implies a wave without amplitude, if the amplitude of the wave is define to have the usual meaning as: The amplitude of wave is the maximum displacement of the wave from equilibrium position . Secondly, when we viewed Equation3.93 mathematically, the electric field vectors are perpendicular to the direction of the wave. But physically there is no electric field vector in space pointing perpendicular to the direction of propagation of the wave. The electric field vector at a point gives just one information in physics: The field strength at that point and the direction in which a test charge will be accelerated if found at that point. When solving problems in physics, we always use the mathematical idea, but that should not be translated physically. As in the above example, we used the mathematical idea to find the resultant electric field due to the two charges. When I obtained the answer that is $\vec{E}=6.91N/C\,\vec{j}$, I should be able to translate it into physics to say that the field strength at that point is 6.91N/C, and a test charge if found at that point will be accelerated along the y-axis. **I should not say that there is vector $\vec{E}=6.91N/C\,\vec{j}$ at that point pointing along the y-axis**. **The physics is the reality.**

Thirdly, if this wave has no amplitude then is it not a transverse wave as people claim. **Physically there is no vector pointing somewhere in space and that can oscillate up and down.**

At point $x=\frac{3\lambda}{4}$, Equation(3.108) gives us

$$\vec{E}(\frac{3\lambda}{4},0)=E_0\sin(\frac{2\pi}{\lambda}.\frac{3\lambda}{4})\vec{j}=E_0\sin(\frac{3\pi}{2})\vec{j}=-E_0\vec{j} \quad . \tag{3.112}$$

When we draw the vector at point $(\frac{3\lambda}{4},0,0)$ the arrow starts at that point and has the same length as the vector at point $(\frac{\lambda}{4},0,0)$ but this time it is directed downward. This again tells us that the field strength at point $(\frac{3\lambda}{4},0,0)$ is E_0 and that a positive test charge if found at that point and at that instant would be accelerated downward ie in the $-\vec{j}$ direction.

So considering the solution given by Equation(3.93) at time t=0 and from point x=0 to point x=λ,

we obtain the figure below:

Figure 3.10 *Plot of the function* $\vec{E}(x,t)=E_0\sin(kx-\omega t)\hat{j}$ *at time t=0 for one period of the sine function , that is from x=0 to x=λ.*

This representation is the mathematical idea, but when we come back to physics the vector pointing from point to point do not exist. It will just be a straight line. When we describe the wave on string, we are using mathematics to describe something that is physical. The wave-function describes the displacement of the string from the equilibrium position as the wave propagates through it. But when we come back here, we are making for ourselves a physical idea from a mathematical idea that do not exist physically.

My argument that it is not wave function arises from two points:

Firstly, if the proof given in section3.1.2 that the electric field is a force field and does store energy or has an energy density holds, that will imply a wave that does not carry energy. It can only do work on a charged particle that is found on its path, and this work will be its energy. Without a charged particle found in it, the “wave” would have no energy.

Secondly, the field vectors cannot oscillate because they do not exist physically and therefore can not extend from one point to another point in space. We cannot therefore say that the wave is transverse wave meaning the field vectors oscillate or are perpendicular to the direction of propagation of the wave.

It is true that $1/\sqrt{\mu_0\epsilon_0}$ gives a speed that is equal to the speed of a photon but if we desire to have an interpretation the solution of Equation(3.84) as wave equation, the electric field should be simply replaced by a mathematical function Φ(r,t). And in the solution given by Equation(3.93) the function $\vec{E}(x,t)$ should be replaced by an arbitrary function ψ(x,t).

When we consider the modes of production of electromagnetic waves cited in reference [16] , I quote:

“ 1-Electromagnetic waves are produced when free charges accelerate or when electrons bound to the atoms or molecules make transitions to lower energy states,

2-A continuum x-ray spectrum is produced by the deceleration of electrons when they crash into a metal,

3- …. synchrotron radiation from circular motion of charged particles (usually electrons and protons) in nuclear accelerators called synchrotron.”,

it is indisputable that all the three examples cited above produce photons not directly from electric energy or magnetic energy, but mostly from the conversion of the kinetic energy of the electrons. This was what was also proven in **section3.1.2.2**. When electrons in atoms for example make transitions to lower energy states, they emit photons. Also the bremsstrahlung radiation is produced when electrons are suddenly decelerated as the crash into a metal thereby converting some of their kinetic energies into photons because of the conservation of energy.

It was however true that to accelerate charged particles, we take them through a potential difference as can be seen from the unit of *electron volt* which is a commonly used unit in atomic and nuclear physics. An electron volt is the change in the kinetic energy of an electron when it is taken through a potential difference of one volt. Thus,

$$1\,eV = |charge\ of\ an\, electron| \times 1\, volt$$

$$1\text{eV} = 1.6\times 10^{-19}\, coulomb \times 1\, volt$$

$$1\,eV = 1.6\times 10^{-19}\, Joule \tag{3.113}$$

In fact why did Maxwell believed in the existence of oscillating electromagnetic wave? Maxwell and his contemporaries believed that the amplitudes electromagnetic wave should be oscillating because of the fact that the (classical) waves that propagate on the string and sound waves that propagate through the air cause the medium to oscillate. They also believed so because at that time they believed in the existence of the aether drift that must sustain the propagation of the electromagnetic wave and it would be this aether that would be oscillating as the wave propagates through the medium. This can be seen from some of the early literatures such as : Statistical Mechanics by R. H. Fowler ,second edition(1936) [first edition(1929)]. In it, at page 115, section 4.3 is titled: **Radiation as Vibrations of Normal modes of the Aether.** We can also read sentences such as “ We shall assume that the zero of energy for the vibrations of the aether is the state in which every normal mode has its lowest possible quantum number”. At page 114 we can also read “For the aether, the number of normal modes with frequencies between ν and ν +dν is $\frac{8\pi}{c^3}\nu^2 d\nu$, where c is the speed of light”.

The above statements really confirm that they believed in the oscillating electromagnetic wave because of the fact that they believed that the propagation electromagnetic wave is sustained by a medium called aether. It also shows that the zero-point of energy is deeply rooted in the existence of the aether drift. After Einstein brought The Special Theory of Relativity to banish the existence of the aether drift, what was remaining was to think carefully about the oscillating amplitudes of electromagnetic wave. But no body thought about it because of the fact that everybody also believed that the electric and magnetic fields vectors extend from one point to another point in space (that is a mathematical idea) exist physically. Everybody further believe that the solution of Maxwell equations in vacuum is a wave function with electric and magnetic field vectors associated to it, that extend from point to point, oscillating up and down or left and right. But I have also proved that that the idea of electric field vector existing in space and extending from point to point is a mathematical idea that do not exist physically in reality.

To conclude this chapter, I would say that the electric and magnetic fields do not store energy or have no energy density. Also the electromagnetic wave with oscillating electric and magnetic field vectors cannot not exist. A single electromagnetic wave is the photon or alternatively the photon is a wave, and light is made up of a huge collection of waves that are photons propagating as wave trains.

CHAPTER FOUR

4.0 Quantization of em fields & the Blackbody radiation

This chapter starts with the quantization of EM fields (a standard approach), followed with one of the standard approaches of the derivation of the blackbody radiation formula, and ends with my own approach.

4.1 Quantization of EM fields(standard Approach).

Photons are quanta of radiation field. As a similar system, we may consider an assembly of phonons, or quanta of oscillation of a perfect crystal lattice. According to standard theory, a radiation field is the same as an electromagnetic field, and an electromagnetic field in vacuum can be described as an assembly of electromagnetic waves, with electric and magnetic vectors perpendicular to the direction of propagation of the wave that carry quanta of energy that are photons . But an electromagnetic wave with frequency ν is equivalent to an oscillator with frequency ν . The amplitude of the electromagnetic wave corresponds to the amplitude of the oscillator, and the nth excited level of the oscillator corresponds to the state of the electromagnetic wave with n photons, each of which has energy $\epsilon = h\nu$.

according to reference[10], The word 'mode' characterizes a particular oscillation amplitude in the cavity or in the solid.

This section contains a part of a lecture notes given by G.M. Wysin of the Physics Department of the Kansas State University [11], with almost no modification of the sentences , but it contains only some parts that I think are important for my discussion later on.

All the boldface letters in this section stand for vectors.

4.1.1 Maxwell's equations and the Lagrangian and Hamiltonian Densities.

The Lagrangian density for the EM fields is

$$\mathscr{L} = \frac{1}{8\pi}[\boldsymbol{E}^2 - \boldsymbol{B}^2] \quad . \qquad (4.1)$$

The differential form of Maxwell's equations in free space, written for the electric(**E**) and magnetic(**B**) fields in CGS units are

$$\nabla\cdot\boldsymbol{B} = 0 \quad , \quad \nabla\times\boldsymbol{E} + \frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t} = 0 \quad , \quad \nabla\cdot\boldsymbol{E} = 0 \quad , \quad \nabla\times\boldsymbol{B} - \frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t} = 0 \quad . \qquad (4.2)$$

The divergence of **B** and the Faraday's law allow the introduction of the vector and scalar potentials, **A** and ϕ, respectively, that gives the fields:

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} \quad , \quad \boldsymbol{E} = \nabla\Phi - \frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t} \quad . \qquad (4.3)$$

The problem is considered far from sources. If sources were present, they would appear in the last two equations in Equation4.2, so the potentials could still apply. The potentials are not unique and have a gauge symmetry. They can be shifted using some gauge transformation (f) without changing the electric and magnetic fields:

$$\boldsymbol{A}'=\boldsymbol{A}+\nabla f \quad , \qquad \Phi'=\Phi-\frac{1}{C}\frac{\partial f}{\partial t} \quad . \tag{4.4}$$

The Euler-Lagrange variation of the Lagrangian w.r.t the coordinates $q=(\Phi, A_x, A_y, A_z)$ gives back Maxwell's equations. To approach the quantization, the canonical momenta p_i need to be identified. But there is no derivative of the ϕ in $\mathscr{L}$, so there is no p_ϕ and ϕ should be eliminated as coordinate, in some sense. There are time derivatives of **A**, hence their canonical momenta are found as

$$p_i=\frac{\partial \mathscr{L}}{\partial \dot{A}_i}=\frac{1}{4\pi c}\left(\frac{\partial \Phi}{\partial x_i}+\frac{1}{c}\frac{\partial A_i}{\partial t}\right)=\frac{-1}{4\pi c}E_i \quad , \ \text{i=1,2,3.} \tag{4.5}$$

the transformation to the Hamiltonian energy density is the Legendre transform,

$$\aleph=\sum_i p_i q_i-\mathscr{L}=\boldsymbol{p}.\frac{\partial \boldsymbol{A}}{\partial t}-\mathscr{L}=2\pi c\,\boldsymbol{p}^2+\frac{1}{8\pi}(\nabla\times\boldsymbol{A})^2-c\,\boldsymbol{p}\cdot\nabla\Phi \quad . \tag{4.6}$$

When integrated over all space, the last term gives nothing , because $\nabla\cdot\boldsymbol{E}=0$, but the first two terms gives a well known result for the effective energy density, in terms of the EM fields,

$$\aleph=\frac{1}{8\pi}(\boldsymbol{E}^2+\boldsymbol{B}^2) \quad . \tag{4.7}$$

We might keep the actual Hamiltonian in terms of coordinates **A** and their conjugate momenta **P**, leading to the classical EM energy,

$$H=\int d^3r\,[2\pi c^2\boldsymbol{p}^2+\frac{1}{8\pi}(\nabla\times\boldsymbol{A})^2] \quad . \tag{4.8}$$

Now it is useful to apply the coulomb gauge, where $\Phi=0$ and $\nabla\cdot\boldsymbol{A}=0$.

We can use either the Lagrangian or the Hamiltonian Equations of motions to see the dynamics. For instance, by the Hamiltonian method, we have

$$\dot{q}_i=\frac{\delta\aleph}{\delta p_i} \quad , \qquad \dot{p}_i=-\frac{\delta\aleph}{\delta q_i} \quad . \tag{4.9}$$

Recall that the variation of a density like this means

$$\frac{\delta\aleph}{\delta f}=\frac{\partial\aleph}{\partial f}-\sum_i\frac{\partial}{\partial x_i}\frac{\partial\aleph}{\partial(\frac{\partial f}{\partial x_i})}-\frac{\partial}{\partial t}\frac{\partial\aleph}{\partial(\frac{\partial f}{\partial t})} \quad . \tag{4.10}$$

The variation for example with respect to coordinate $q_i=A_x$ gives the results

$$\frac{\partial A_x}{\partial t}=4\pi c^2 p_x \quad , \quad \frac{\partial p_x}{\partial t}=\frac{1}{4\pi}\nabla^2 A_x \quad . \tag{4.11}$$

By combining these, we see that all the components of the vector potential (and the conjugate momentum, which is proportional to **E**) satisfy a wave equation, as could be expected!

$$\nabla^2-\frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial^2 t}=0 \quad . \tag{4.12}$$

Wave motion is essentially oscillatory, hence strong connection of this problem with the harmonic oscillator solutions.

The above equation has plane wave solutions $\mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{r}-\omega_k t}$ at angular frequency $\omega_{\boldsymbol{k}}$ and wave vector **k** that have a linear dispersion relation, $\omega_{\boldsymbol{k}}=ck.$ For the total field in some volume V, we can try a Fourier expansion over a collection of modes, supposing periodic boundary conditions.

$$\boldsymbol{A}(\boldsymbol{r},t)=\frac{1}{\sqrt{V}}\sum_{\boldsymbol{k}}\boldsymbol{A}_{\boldsymbol{k}}(t)\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}} \quad . \tag{4.13}$$

The components of the vector **k** have discrete values (a consequence of the boundary condition that **A** has the same value on opposite walls of the box),

$$k_x=\frac{2\pi n_x}{L}\quad,\quad k_y=\frac{2\pi n_y}{L}\quad,\quad k_z=\frac{2\pi n_z}{L}\quad,\quad n_x, n_y, n_z = 0, \pm1, \pm2, \ldots \tag{4.14}$$

Each coefficient $\mathbf{A_k}$ (t) is an amplitude for a wave at the stated wave vector. The different modes are orthogonal (or independent) due to the normalization condition

$$\int d^3\boldsymbol{r}\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}\,\mathrm{e}^{-i\boldsymbol{k}'\cdot\boldsymbol{r}}=V\,\delta_{\boldsymbol{k}\boldsymbol{k}'} \quad . \tag{4.15}$$

The gauge assumption $\nabla\cdot\boldsymbol{A}=0$ then is the same as $\mathbf{k.A_k}$ =0, which shows that the wave is transverse. For any **k**, there are two transverse directions , hence two independent polarizations directions, identified by the unit vectors $\mathbf{e}_{\mathbf{k}\alpha}$, α=1,2. Thus the total amplitude looks like

$$\boldsymbol{A}_{\boldsymbol{k}}=\hat{\boldsymbol{\epsilon}}_{\boldsymbol{k1}}A_{\boldsymbol{k1}}+\hat{\boldsymbol{\epsilon}}_{\boldsymbol{k2}}A_{\boldsymbol{k2}}=\sum_{\alpha}\hat{\epsilon}_{\boldsymbol{k}\alpha}A_{\boldsymbol{k}\alpha} \quad . \tag{4.16}$$

Yet, from the wave equation, both polarizations are found to oscillate identically, except perhaps not in phase,

$$\boldsymbol{A}_{\boldsymbol{k}}(t)=\boldsymbol{A}_{\boldsymbol{k}}\,\mathrm{e}^{-i\omega_k t} \quad . \tag{4.17}$$

Now the amplitudes $\mathbf{A_k}$ are generally complex, whereas we want to have the actual field being quantized to be real. This is accomplished by combining these waves appropriately with their complex conjugates (c.c). Let us try to write **A** in Equation4.13 in a different way that exhibits the positive and negative wave vectors together,

$$\boldsymbol{A}(\boldsymbol{r},t)=\frac{1}{2\sqrt{V}}\sum_{\boldsymbol{k}}\left[\boldsymbol{A}_{\boldsymbol{k}}(t)\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}+\boldsymbol{A}_{-\boldsymbol{k}}(t)\,\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\right] \quad . \tag{4.18}$$

[The sum over **k** here includes wave vectors in all directions. Then both **k** and -**k** are included twice. It is divided by 2 to avoid double counting.] In order for this to be real, a little consideration shows that the 2nd term must be the c.c. of the the first term,

$$\boldsymbol{A}_{-\boldsymbol{k}}=\boldsymbol{A}_{\boldsymbol{k}}^{*} \quad . \tag{4.19}$$

A wave need to be identified by both $\mathbf{A_k}$ and its complex conjugate (or equivalently, two real constants). So the vector potential is written in Fourier space as

$$\boldsymbol{A}(\boldsymbol{r},t)=\frac{1}{2\sqrt{V}}\sum_{\boldsymbol{k}}\left[\boldsymbol{A}_{\boldsymbol{k}}(t)\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}+\boldsymbol{A}_{\boldsymbol{k}}^{*}(t)\,\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\right] \quad . \tag{4.20}$$

Note that the c.c. reverses the sign on the frequency also, so the first term oscillates at positive frequency and the second term at negative frequency. But curiously, both gives a real wave traveling along the direction of **k**. Based on this expression, the fields are easily determined by applying (4.3), with $\nabla\rightarrow\pm i\boldsymbol{k}$,

$$\boldsymbol{E}(\boldsymbol{r},t)=\frac{i}{2c\sqrt{V}}\sum_{\boldsymbol{k}}\omega_{\boldsymbol{k}}[\boldsymbol{A}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}-\boldsymbol{A}^{*}_{\boldsymbol{k}}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}] \quad , \tag{4.21}$$

$$\boldsymbol{B}(\boldsymbol{r},t)=\frac{i}{2\sqrt{V}}\sum_{\boldsymbol{k}}\boldsymbol{k}\times[\boldsymbol{A}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}-\boldsymbol{A}^{*}_{\boldsymbol{k}}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}] \quad . \tag{4.22}$$

Now let's evaluate the total energy, that is the Hamiltonian. There are direct terms (btwn **k** and itself) and indirect terms (btwn **k** and **-k).**

$$\int d^3\boldsymbol{r}|\boldsymbol{E}|^2=\frac{1}{4cV}\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'}\omega_k\omega_{k'}\int d^3\boldsymbol{r}[\boldsymbol{A}_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}-\boldsymbol{A}^{*}_{\boldsymbol{k}}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}][\boldsymbol{A}^{*}_{\boldsymbol{k}'}e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}}-\boldsymbol{A}_{\boldsymbol{k}'}e^{i\boldsymbol{k}'\cdot\boldsymbol{r}}] \quad . \tag{4.23}$$

Upon integration over the volume, the orthogonality relation will give 2 terms with **k'=k** and 2 terms with **k'=-k**, for 4 equivalent terms in all. The same happens for the calculation of the magnetic energy. Also one can't forget that $\mathbf{A_k}$ is the same as $\mathbf{A_{-k}^*}$. These become

$$\frac{1}{8\pi}\int d^3\boldsymbol{r}|\boldsymbol{E}|^2=\frac{1}{8\pi}\sum_{\boldsymbol{k}}\frac{\omega_k^2}{c^2}|\boldsymbol{A}_{\boldsymbol{k}}(t)|^2 \quad , \tag{4.24}$$

$$\frac{1}{8\pi}\int d^3\boldsymbol{r}|\boldsymbol{B}|^2=\frac{1}{8\pi}\sum_{\boldsymbol{k}}\boldsymbol{k}^2|\boldsymbol{A}_{\boldsymbol{k}}(t)|^2 \quad . \tag{4.25}$$

It is obvious that the magnetic energy is equal to the electric energy since $\boldsymbol{k}^2=\omega_{\boldsymbol{k}}^2/c^2$. The total energy is simply,

$$H=\frac{1}{8\pi}\int d^3\boldsymbol{r}(\boldsymbol{E}^2+\boldsymbol{B}^2)=\frac{1}{4\pi}\sum_{\boldsymbol{k}\alpha}k^2|A_{\boldsymbol{k}\alpha}|^2 \quad . \tag{4.26}$$

The last form recalls that each wave vector is associated with two independent polarizations. They are orthogonal, so there are no cross terms between them from squaring.

The Hamiltonian shows that the modes do not interfere with each other, hence we just need to quantize the modes as a collection of independent harmonic oscillators. To do that, we need to transform the expression into the language of coordinates and momenta. **It would be good to see H is expressed through squared of coordinate (potential energy term) and squared momentum (kinetic energy term).**

The electric field is proportional to the canonical momentum, $\boldsymbol{E}=-4\pi c\,\boldsymbol{p}$. So really, the electric field energy term already look like a sum of squared momenta. Similarly, the magnetic field energy is determined by the curl of the vector potential, which is the basic coordinate here. So we have some relations,

$$\boldsymbol{P}(\boldsymbol{r},t)=-\frac{1}{4\pi c}\boldsymbol{E}(\boldsymbol{r},t)=\frac{-i}{8\pi c^2\sqrt{V}}\sum_{\boldsymbol{k}}\omega_{\boldsymbol{k}}[\boldsymbol{A}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}-\boldsymbol{A}^{*}_{\boldsymbol{k}}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}] \quad . \tag{4.27}$$

This suggests the introduction of the momenta at each wave vector, i.e. , analogous with the Fourier expansion for the vector potential(i.e. , the generalized coordinates),

$$\boldsymbol{P}(\boldsymbol{r},t)=\frac{1}{2\sqrt{V}}\sum_{\boldsymbol{k}}[\boldsymbol{p}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}+\boldsymbol{p}^{*}_{\boldsymbol{k}}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}] \quad . \tag{4.28}$$

Then we can make an important identifications,

$$\boldsymbol{p_k}(t)=\frac{-i\omega_{\boldsymbol{k}}}{4\pi c^2}\boldsymbol{A_k}(t) \quad . \tag{4.29}$$

Even more simply, just write the electric field (and its energy) in terms of the momenta now.

$$\boldsymbol{E}(\boldsymbol{r},t)=-4\pi c\,\boldsymbol{p}(\boldsymbol{r},t)=\frac{-2\pi c}{\sqrt{V}}\sum_{\boldsymbol{k}}[\boldsymbol{p_k}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}+\boldsymbol{p}_{\boldsymbol{k}}^{*}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}] \quad . \tag{4.30}$$

When squared , the electric energy involves four equivalent terms, and there results

$$\frac{1}{8\pi}\int d^3\boldsymbol{r}|\boldsymbol{E}|^2=\frac{16\pi^2c^2}{8\pi}\sum_{\boldsymbol{k}}\boldsymbol{p_k}.\boldsymbol{p}_{\boldsymbol{k}}^{*}=2\pi c^2\sum_{\boldsymbol{k}}\boldsymbol{p_k}.\boldsymbol{p}_{\boldsymbol{k}}^{*} \quad . \qquad 4.31)$$

Let's also rewrite the magnetic energy. The generalized coordinates are the components of **A ,** i.e. ,let's write $\boldsymbol{q_k}=\boldsymbol{A_k}$. (4.32)

Let us also consider the magnetic field written as

$$\boldsymbol{B}(\boldsymbol{r},t)=\frac{i}{2\sqrt{V}}\sum_{\boldsymbol{k}}\boldsymbol{k}\times[\boldsymbol{q_k}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}-\boldsymbol{q}_{\boldsymbol{k}}^{*}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}] \quad , \tag{4.33}$$

and the associated energy is written,

$$\frac{1}{8\pi}\int d^3\boldsymbol{r}|\boldsymbol{B}|^2=\frac{4}{4\times 8\pi}\sum_{\boldsymbol{k}}\boldsymbol{k}^2\boldsymbol{q_k}.\boldsymbol{q}_{\boldsymbol{k}}^{*}=\frac{1}{8\pi c^2}\sum_{\boldsymbol{k}}\omega_{\boldsymbol{k}}\boldsymbol{q_k}.\boldsymbol{q}_{\boldsymbol{k}}^{*} \quad . \tag{4.34}$$

This gives the total canonical Hamiltonian, expressed in Fourier modes, as

$$H=2\pi c^2\sum_{\boldsymbol{k}}\boldsymbol{p_k}.\boldsymbol{p}_{\boldsymbol{k}}^{*}+\frac{1}{8\pi c^2}\sum_{\boldsymbol{k}}\omega_{\boldsymbol{k}}\boldsymbol{q_k}.\boldsymbol{q_k} \quad . \tag{4.35}$$

4.1.2 Quantization of modes: Simple Harmonic oscillator example.

Next the quantization of each mode needs to be accomplished. But since each mode analogous to a harmonic oscillator, as we'll show, the quantization is not too difficult. We already can see that the modes are independent. So we need to proceed essentially on the individual modes, at a given wave vector and polarization. But I won't for now be writing any polarization indices, for simplicity.

Recall the quantization of a simple harmonic oscillator. The Hamiltonian can be re-expressed in the rescaled operators:

$$H=\frac{p^2}{2\mathrm{m}}+\frac{m\omega^2}{2}q^2=\frac{h\omega}{2}(P^2+Q^2) \quad ; \quad Q=\sqrt{\frac{m\omega}{\hbar}}q \quad , \quad P=\frac{1}{\sqrt{m\hbar\omega}}p \quad . \tag{4.36}$$

Then if the original commutator is $[x,p]=i\hbar$, we have a unit commutator here,

$$[Q,P]=\sqrt{\frac{m\omega}{\hbar}}\frac{1}{\sqrt{m\hbar\omega}}[x.p]=i \quad . \tag{4.37}$$

The Hamiltonian can be expressed in a symmetrized form as follows :

$$H=\frac{\hbar\omega}{2}\frac{1}{2}[(Q+iP)(Q-iP)+(Q-iP)(Q+iP)] \quad . \tag{4.38}$$

This suggest defining annihilation and creation operators

$$a=\frac{1}{\sqrt{2}}(Q+iP), \qquad a^{\dagger}=\frac{1}{\sqrt{2}}(Q-iP), \tag{4.39}$$

Their commutation relation is then conveniently unity,

$$[a,a^{\dagger}]=\left(\frac{1}{\sqrt{2}}\right)^{2}[Q+iP,Q-iP]=\frac{1}{2}(-i[Q,P]+i[P,Q])=1 \quad . \tag{4.40}$$

The coordinate and the momentum are expressed

$$Q=\frac{1}{\sqrt{2}}(a+a^{\dagger}), \qquad P=\frac{1}{i\sqrt{2}}(a-a^{\dagger}). \tag{4.41}$$

The Hamiltonian becomes

$$\frac{\hbar\omega}{2}(aa^{\dagger}+a^{\dagger}a)=\hbar\omega(a^{\dagger}a+\frac{1}{2}) \quad , \tag{4.42}$$

where the last step used the commutation relation in the form , $aa^{\dagger}=a^{\dagger}a+1.$ The operator $\hat{n}=a^{\dagger}a$ is the number operator that count the quanta of excitation. The number operator can be easily shown to have the following commutation relations:

$$[\hat{n},a]=[a^{\dagger}a,a]=[a^{\dagger},a]a=-a \quad , \quad [\hat{n},a^{\dagger}]=[a^{\dagger}a,a^{\dagger}]=a^{\dagger}[a^{\dagger},a]=+a \quad . \tag{4.43}$$

These shows that $a^{\dagger}$ creates or adds one quantum of excitation to the system, while a destroy or removes one quantum. The Hamiltonian famously shows that the system has a zero-point energy of $\hbar\omega/2$ each quantum of excitation adds an additional $\hbar\omega$ of energy.

The eigenstates of the number operator are also the eigenstates of the Hamiltonian(H). And while a lowers and $a^{\dagger}$ rises number of quanta present, the eigenstates of the Hamiltonian are not their eigenstates. But later we need some matrix elements, hence it is good to summarize here exactly the operations of a or $a^{\dagger}$ on the number eigenstates, $|n\rangle$, which are assumed to be unit normalized.

If the state $|n\rangle$ is a normalized eigenstate of $\hat{n}$, with eigenvalue n, then we must have

$$a^{\dagger}\ |n\rangle = c_n\ |n+1\rangle \quad , \quad \langle n|\, a = c_n^{*}\ \langle n+1| \quad , \tag{4.44}$$

where c_n is a normalization constant. Putting these together, and using the commutation relation, gives

$$1=\langle n|aa^{\dagger}|\,n\rangle=|c_n|^2\langle n+1|n+1\rangle \;\Rightarrow\; |c_n|^2=\langle n|aa^{\dagger}|n\rangle=\langle n|a^{\dagger}a+1|n\rangle=n+1 \quad . \tag{4.45}$$

In the same fashion, let's consider the action of the lowering operator,

$$a\ |n\rangle = d_n\ |n-1\rangle \quad , \quad \langle n|\, a^{\dagger} = d_n^{*}\ \langle n-1| \quad , \tag{4.46}$$

$$1=\langle n|a^{\dagger}a|\,n\rangle=|d_n|^2\langle n-1|n-1\rangle \;\Rightarrow\; |d_n|^2=\langle n|a^{\dagger}a|n\rangle=n \quad . \tag{4.47}$$

Therefore when these operators act, they change the normalization slightly, and we can write

$$a^{\dagger}\ |n\rangle = \sqrt{n+1}\ |n+1\rangle \quad , \quad a\ |n\rangle = \sqrt{n}\ |n-1\rangle \quad . \tag{4.48}$$

4.1.3 Fundamental commutation relations for the EM modes.

Now how do we relate what we know to EM field Hamiltonian, Eqn.(4.35)? The main difference here is the presence of operators together with their complex conjugates in the classical Hamiltonian. How to decide their fundamental commutators? That is based on the fundamental commutation relation in real space (for one component only):

$$[A_i(\boldsymbol{r},t),p_i(\boldsymbol{r}',t)]=i\hbar\,\delta(\boldsymbol{r}-\boldsymbol{r}')\,. \tag{4.49}$$

The fields are expressed as in Eqns. (4.20) and (4.28). Using these expressions to evaluate the LHS of Eqn. (4.49) , we get,

$$[A_i(\boldsymbol{r},t),p_i(\boldsymbol{r}',t)]=\frac{1}{4V}\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'}[\boldsymbol{A}_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}+\boldsymbol{A}_{\boldsymbol{k}}^{*}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}},\boldsymbol{p}_{\boldsymbol{k}'}e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'}+\boldsymbol{p}_{\boldsymbol{k}'}^{*}e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}'}]\quad. \tag{4.50}$$

In a finite volume, however, the following is representation of a delta function:

$$\delta(\boldsymbol{r}-\boldsymbol{r}')=\frac{1}{V}\sum_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\quad. \tag{4.51}$$

We can see that Eqns. (4.49) and (4.50) match if modes operators have the following commutation relations (for each component):

$$[\boldsymbol{A}_{\boldsymbol{k}},\boldsymbol{p}_{\boldsymbol{k}'}^{*}]=i\hbar\,\delta_{\boldsymbol{k},\boldsymbol{k}'}\quad,\quad[\boldsymbol{A}_{\boldsymbol{k}},\boldsymbol{p}_{\boldsymbol{k}'}^{*}]=i\hbar\,\delta_{\boldsymbol{k},\boldsymbol{k}'}\quad,\quad[\boldsymbol{A}_{\boldsymbol{k}},\boldsymbol{p}_{\boldsymbol{k}'}]=i\hbar\,\delta_{\boldsymbol{k},-\boldsymbol{k}'}\quad,\quad[\boldsymbol{A}_{\boldsymbol{k}}^{*},\boldsymbol{p}_{\boldsymbol{k}'}^{*}]=i\hbar\,\delta_{\boldsymbol{k},-\boldsymbol{k}'}\quad. \tag{4.52}$$

These together with the delta function representation, give the result for the RHS of Eqn.(4.50) ,

$$[A_i(\boldsymbol{r},t),p_i(\boldsymbol{r}',t)]=\frac{1}{4V}\sum_{\pm\boldsymbol{k}}i\hbar[2e^{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}+2e^{-i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}]=i\hbar\,\delta(\boldsymbol{r}-\boldsymbol{r}')\quad. \tag{4.53}$$

Thus the basic commutators modes are those in (4.52). We can now apply them to quantize the modes.

4.1.4 Quantization of the EM fields

At some point, one should keep in mind that these canonical coordinates are effectively scalars, once the polarization is counted for:

$$\boldsymbol{q}_{\boldsymbol{k}}=\sum_{\alpha}\hat{\epsilon}_k q_{k\alpha}\quad,\qquad\boldsymbol{p}_{\boldsymbol{k}}=\sum_{\alpha}\hat{\epsilon}_k p_{k\alpha}\quad. \tag{4.54}$$

The polarizations are decoupled, so mostly its effects can be ignored. But the Hamiltonian (4.35) has really two terms at each wave vector, one for each polarization. For simplicity we will not be writing the polarization index, but just write the scalar $q_{\mathbf{k}}$ and $p_{\mathbf{k}}$ for each mode's coordinate and momentum. For any scalar coordinate and its momentum, we postulate from (4.52) that

$$[\boldsymbol{A}_{\boldsymbol{k}},\boldsymbol{p}_{\boldsymbol{k}'}^{*}]=i\hbar\,\delta_{\boldsymbol{k},\boldsymbol{k}'}\quad,\quad[q_k^{\dagger},p_{k'}]=i\hbar\,\delta_{\boldsymbol{k},\boldsymbol{k}'}\quad,\quad[q_k,p_{k'}]=i\hbar\,\delta_{\boldsymbol{k},-\boldsymbol{k}'}\quad[q_k^{\dagger},p_{k'}^{\dagger}]=i\hbar\,\delta_{\boldsymbol{k},-\boldsymbol{k}'}\quad. \tag{4.55}$$

Let's look at some algebra that hopefully leads to creation and annihilation operators. First, let's get some coordinates and the momenta with unit normalized commutators. Supposed that a given mode **k**α has a Hamiltonian from (4.35). let's consider first one term at one wave vector: [Even though, classically, the terms at **k** and **-k** in the sum give equal contributions] making a transformation to $Q_{\mathbf{k}}$ and $P_{\mathbf{k}}$,

$$H_{+k\alpha}=2\pi c^2 p_k^\dagger P_k+\frac{\omega_k}{8\pi c^2}q_k^\dagger q_k=\frac{\hbar\omega_k}{2}(P_k^\dagger P_k+Q_k^\dagger Q_k) \quad . \tag{4.56}$$

Here because it is a quantum problem, we supposed that the terms from **k** and **-k** modes are really not the same. Thus there is a similar term for the negative :

$$H_{-k\alpha}=2\pi c^2 p_{-k}^\dagger P_{-k}+\frac{\omega_k}{8\pi c^2}q_{-k}^\dagger q_{-k}=\frac{\hbar\omega_k}{2}(P_{-k}^\dagger P_{-k}+Q_{-k}^\dagger Q_{-k}) \quad . \tag{4.57}$$

The right hand sides are the same as the energies for the SHO's in the normalized coordinates and momenta. However, we have the relations like $q_{-k}^\dagger=q_k$, $q_{-k}=q_k^\dagger$, and we suppose they should apply to the new rescaled coordinates and momenta. So this relation also takes the form

$$H_{-k\alpha}=2\pi c^2 p_k p_k^\dagger+\frac{\omega_k}{8\pi c^2}q_k q_k^\dagger=\frac{\hbar\omega_k}{2}(P_k P_k^\dagger+Q_k Q_k^\dagger) \quad , \tag{4.58}$$

In the quantum problem, the order in which conjugate operators act is very important and should not be modified. So $H_{+k\alpha}$ and $H_{-k\alpha}$ are not the same. To match the sides, let's try the identifications,

$$P_k=\sqrt{\frac{4\pi c^2}{\hbar\omega_k}}p_k \quad , \qquad Q_k=\sqrt{\frac{\omega_k}{4\pi c^2\hbar}}q_k \quad . \tag{4.59}$$

The basic commutator that results between them is unit normalized,

$$[Q_k,P_k^\dagger]=\sqrt{\frac{\omega_k}{4\pi c^2\hbar}}\sqrt{\frac{4\pi c^2}{\hbar\omega_k}}[q_k,p_k^\dagger]=\frac{1}{\hbar}i\hbar=i \quad . \tag{4.60}$$

It is obvious also that

$$[Q_k^\dagger,P_k]=i \quad . \tag{4.61}$$

Let's also introduce the creation and annihilation operators, for both the positive and negative wave vectors:

$$a_k=\frac{1}{\sqrt{2}}(Q_k+iP_k) \quad , \quad a_k^\dagger=\frac{1}{\sqrt{2}}(Q_k^\dagger-iP_k^\dagger) \quad . \tag{4.62}$$

$$a_{-k}=\frac{1}{\sqrt{2}}(Q_k^\dagger+iP_k^\dagger) \quad , \quad a_{-k}^\dagger=\frac{1}{\sqrt{2}}(Q_k-iP_k) \quad . \tag{4.63}$$

By their definition they must have unit real commutators, e.g

$$[a_k,a_k^\dagger]=1 \quad . \tag{4.64}$$

On the other hand, a commutator between different modes (or with different polarizations at one wave vector) gives zero. The individual term in the Hamiltonian sum is

$$H_{+k\alpha}=\frac{\hbar\omega_k}{2}\frac{1}{2}[a_k a_k^\dagger+a_k^\dagger a_k+a_{-k}a_{-k}^\dagger+a_{-k}^\dagger a_{-k}] \quad . \tag{4.65}$$

So the total field Hamiltonian is the sum

$$H=\sum_k\frac{\hbar\omega_k}{2}[a_k^\dagger a_k+\frac{1}{2}+a_{-k}^\dagger a_{-k}+\frac{1}{2}] \quad . \tag{4.66}$$

The sum over all wave vectors, and the positive and negative terms give the same total, so

$$H=\sum_{\mathbf{k}\alpha}\hbar\omega_{\mathbf{k}}[a^{\dagger}_{\mathbf{k}\alpha}a_{\mathbf{k}\alpha}+\frac{1}{2}] \quad . \tag{4.67}$$

Then each mode specified by a wave vector and polarization contributes $\hbar\omega[a^{\dagger}_{\mathbf{k}\alpha}a_{\mathbf{k}\alpha}+\frac{1}{2}]$ to the Hamiltonian. Every mode is equivalent mathematically to simple harmonic oscillator.

4.1.5 Application: Calculation of blackbody radiation energy

The approach below is mostly taken from reference[12].

For the purposes of calculations, it is convenient to think of black body as a cubic cavity of side L. A hole in one of the faces allows radiation to enter the cavity. Due to multiple reflections inside the cavity the radiation is nearly completely absorbed. Since the black body is in equilibrium at temperature T , the energy content of the black body is a constant except for small thermal fluctuations. This situation can be modeled by standing waves inside the black body cavity. Note that standing waves do not transmit any energy. This crucial idea is due to Lord Rayleigh who first attempted to explain the black body spectrum 1890s.

The total energy of the black body is stored in the form of standing waves. The technique is to count the number of standing waves with frequencies in the range ν and ν + dν. Thus, we have the energy in the frequency range to be

$$E(\nu)d\nu=(No.of\ standing\ waves)\bar{\epsilon}\,d\nu \quad , \tag{4.68}$$

where, $\bar{\epsilon}$ is the average energy of each standing wave. Since we do not want our energy to be dependent on the volume of the cavity, we define energy density (energy per unit volume) in the frequency range ν and ν + dν) to be,

$$U(\nu)d\nu=\frac{1}{L^3}(No.of\ standing\ waves)\bar{\epsilon}\,d\nu \quad . \tag{4.69}$$

Now, the calculation boils down to determining the number of standing waves and the average energy of standing wave.

4.1.5.1 Number of standing waves

In principle, standing waves of all possible wavelengths should be present. However, the boundary conditions (waves should have a node at the walls of the cavity) of the cavity allow only modes of certain wavelengths to be present inside the cavity. The allowed wavelengths are obtained from the condition for standing waves in a cavity in one dimension to be n = 2L/λ, where λ is the wavelength of the standing wave and n is the number of half-wavelengths. In a 3D cavity, this condition is generalized to,

$$n_x^2+n_y^2+n_z^2=\left(\frac{2L}{\lambda}\right)^2 \quad ,, \quad n_x,n_y,n_z=1,2,3,\ldots \quad . \tag{4.70}$$

In a 3D, each triplet of integers (n_x,n_y,n_z) corresponds to one possible mode of standing wave inside the cavity. Let's consider a 3D space of integers (n_x,n_y,n_z) and every point in this space

corresponds to one possible mode of standing wave. The number of independent modes that lie in the range of wavelengths λ and λ + dλ is given as

$$dN=2\left(\frac{1}{8}\right)n^2\,dn \quad . \tag{4.71}$$

The factor 2 comes from two possible states of polarization for each standing wave. The result in terms of frequency is found by making the substitution,

$$n=\frac{2\mathrm{L}}{\lambda}=\frac{2\mathrm{L}\nu}{c} \quad \rightarrow \quad dn=\frac{2\mathrm{L}}{c}\,d\nu \quad . \tag{4.72}$$

Substituting for n from Eqn.4.72 in Eqn.4.71 we get the result for number of standing waves in the cavity in [ν, ν + dν] to be,

$$No.\,of\ standing\,waves\,[\nu\,,\nu+d\nu]=\frac{8\pi L^3}{c^3}\nu^2\,d\nu \quad . \tag{4.73}$$

4.1.5.2 Average energy

The other thing we need to compute the energy density is the average energy of each mode of standing wave.

a. Rayleigh-Jeans Formula

Classically, this is obtained from the theorem of equipartition of energy . According to the explanation given in reference[13] it is as follows: The equipartition law predicts that the energy per degree of freedom is kT/2 for an oscillator - and the modes of the electromagnetic fields are harmonic oscillators - a contribution of kT/2 from kinetic energy is matched by contribution from the potential energy kT/2, giving $\bar{\epsilon}=kT$, which is the average energy of the electromagnetic oscillator.

Substituting all the known values in Eqn. 4.71, we get the required energy density to be,

$$U(\nu\,,T)d\nu=\frac{8\pi\nu^2}{c^3}kT\,d\nu \quad . \tag{4.74}$$

This is the Rayleigh-Jeans formula. The obvious problem with this relation is that the total energy integrated from ν = 0 to ν = ∞ gives infinity.

b. Planck's Radiation law

Max Planck assumed that the energy exchange between the oscillators on the walls of the cavity and the standing waves takes place in discrete quanta given by,

$$E=nh\nu \quad . \tag{4.75}$$

Alternatively, the energy of an electromagnetic oscillator(mode) is given as

$$E'=h\nu\left(n+\frac{1}{2}\right) \quad , \tag{4.76}$$

where h is the Planck’s constant. Using this energy relation in Boltzmann distribution, he obtained the average energy to be,

$$\bar{\epsilon}=\frac{h\nu}{\mathrm{e}^{h\nu/kT}-1} \quad . \tag{4.77}$$

Substituting this in Eqn. 4.69, we get the energy density to be,

$$U(\nu,T)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1} \quad . \tag{4.78}$$

This is the Planck's radiation law and displays excellent agreement with the measured black body spectrum.

4.2 My derivation of the blackbody spectral energy density

4.2.1 Is “the standing wave” derived from Maxwell's equations equivalent to a simple harmonic oscillator.

Before proposing an alternative derivation of the blackbody radiation law or of the quantization of the black body radiation, let's first investigate whether the similarity made between a standing-wave of an electromagnetic wave given by the function below, and the simple harmonic oscillator, is valid.

$$\vec{E}(\boldsymbol{r},t)=\vec{E}_0\sin(\omega t)\sin(n_x\pi x/L)=E_0\sin(\omega t)\sin(n_x\pi x/L)\,j \quad , \qquad (4.79a)$$

or if we want to replace $\vec{E}_0$ by a specified value, we can write,

$$\vec{E}(\boldsymbol{r},t)=(400\mathrm{N}/C)\sin(\omega t)\sin(n_x\pi x/L)\,j \quad . \qquad (4.79b)$$

This function, as already explained, gives the variation of field strength along the x-axis. If we take for example $n_x=1$, Equation4.79b becomes

$$\vec{E}(\boldsymbol{r},t)=(400\mathrm{N}/C)\sin(\omega t)\sin(\pi x/L)\,j \quad . \qquad (4.80)$$

And for $x=l/2$, we have

$$\vec{E}(\boldsymbol{r},t)=(400\mathrm{N}/C)\sin(\omega t)\sin(\pi L/2L)\,j=(400\mathrm{N}/C)\sin(\omega t)\sin(\frac{\pi}{2})\,j=(400\mathrm{N}/C)\sin(\omega t)\,j \quad . \qquad (4.81)$$

This function does not tell us, physically, that the electric field vector equals $\vec{E}_0=400\mathrm{N}/C\,j$ oscillates up and down , with a frequency ω. But, it does tell us that at point $x=L/2$ the field strength is 400N/C and that a charged particle found at that point will be accelerated either along the j-direction or -j-direction, and that the field strength at the point keep changing with a frequency equals ω. This can be likened to the polarity of an AC voltage that keep changing (if we want we can use the term oscillating) with a frequency ω . But it would be completely wrong to say that the polarity keep moving up and down or to try associate any kinetic energy or a potential energy it.

As explained in section3.3.3, there is mathematical idea that we use to solve problems in physics. The idea of the electric field vector at point $x=l/2$ pointing upwards or downwards (or extending from one point to another point) is a mathematical idea that helps us solve problems in physics. Once I obtain my answer I should be able to interpret it, physically, as meaning that the field strength at the point is given by a particular value and that a charged particle if found at that point will be accelerated in the direction of the vector. When we represent the electric field vector mathematically at a point, the length of the vector gives us an information about the field strength at the point, and the direction in which the electric field vector points also gives us the direction in which a charged particle will experience acceleration, if found at that point.

So considering that the function given by Equation4.79 is a standing-wave function, with electric field vectors (amplitudes) oscillating up and down or along the the y-axis is just a pure imagination.

Let's also consider the standing wave given by

$$\vec{E}(r,t)=\vec{E}_0\sin(\omega t)\sin(n_x\pi x/L)\sin(n_y\pi y/L)\sin(n_z\pi z/L) \quad , \qquad (4.82)$$

upon substitution of this into the wave equation given by

$$c^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\vec{E}=\frac{\partial^2\vec{E}}{\partial t^2} \quad , \tag{4.83}$$

we get

$$-c^2(k_x^2+k_y^2+k_z^2)\vec{E}(r,t)=\frac{\partial^2\vec{E}(r,t)}{\partial t^2} \quad . \tag{4.84}$$

This Equation is of course mathematically equivalent to the equation for one-dimensional harmonic oscillator. If $\vec{E}$ is replaced by x and $(k_x^2+k_y^2+k_z^2)$ by κ(kappa) , and 1/c^2 is replaced by m, then the equation becomes

$$-\kappa x=m\frac{\partial^2 x}{\partial t^2} \quad . \tag{4.85}$$

Equations (4.84)and (4.85) are mathematically equivalent but physically different. They are mathematically equivalent meaning that they have the same form of mathematical solutions :

$$x(t)=A\sin(\omega t) \quad \text{, for Equation(4.85)} \tag{4.86}$$

where A is the maximum displacement from the equilibrium position called the amplitude.

And $$\vec{E}(t)=\vec{E}_0\sin(\omega t) \quad \text{, for Equation(4.84)} \tag{4.87}$$

where $\vec{E}_0$ is the maximum value of the field strength at the point it is being evaluated.

The two solutions are physically different in the sense that Equation(4.86) tells us that the displacement of the mass m from the equilibrium position varies from a maximum positive value (A) to a minimum negative value(-A) harmonically. That is the mass oscillates from an equilibrium position. Whereas Equation4.87 tells us the the field strength, at the point it is being evaluated, keep increasing and decreasing harmonically with time. And that a charged particle if found at that point would be accelerated either along the direction of the vector $\vec{E}_0$ or in the direction of the vector $-\vec{E}_0$. Equation (4.87)has no physical meaning of oscillation up and down, or left and right ,or about any equilibrium position as does the harmonic oscillator or the amplitudes of harmonic waves on a string. We cannot therefore say that the physical solutions of the two equations are equal. Consequently, we cannot say that the quantum harmonic oscillator solution can also be used for Equation(4.87).

4.2.2 Derivation of the number of permitted full-wavelengths within a given wavelength range in the cavity.

In the remaining of this work, I will continue to use the term electromagnetic wave, but this wave will have nothing to do with electric and magnetic fields or energies.

Having proved or demonstrated that the electric field has no energy density or do not store energy (which applies to the magnetic field also) but can do work on a charged particle found in it, and that the wave with electric and magnetic field vectors is purely mathematical idea, and can not be likened to a harmonic oscillator to be able to quantize it, an electromagnetic wave is to be regarded as a photon in the remaining of this work. Also in the remaining of this work a photon is to be regarded as a single wave. It can nevertheless also be interpreted as a particle when we consider it interactions with matter (eg photoelectric effect and Compton scattering). The idea that photons create electric and magnetic fields and it is those fields that are waves and oscillate is also banished in this work. Alternatively, the idea that the electromagnetic field is made of photons is not valid in this work. I do not see any link between electromagnetic field or electric energy and photons. The electromagnetic field might be a wave field as some people claim, but I do not think it is the wave that describe photons or light. The photons are the waves in the cavity. **It is important to note that energy of electromagnetic waves(photons) that make up thermal radiation or light; or that is emitted when electrons in atoms or molecules make transitions to lower energy levels depends on their wavelengths (or of the frequencies) and not on the amplitude of the wave. The amplitude of a wave is immaterial in this situation**.

Me holding the idea that the photon is the wave in this work is not in contradiction with the standard theories as can be seen from reference [14](an excellent graduate book written by Donald A. McQuarrie from Indiana University.) I quote:

The quantum mechanical theory of electromagnetic radiation tells us that an electromagnetic wave may be regarded as a massless particle of spin angular momentum $\hbar = h/2\pi$ and with momentum and energy that that are functions of the wavelength.

He is also saying or confirming that the photon is an electromagnetic wave .

The fact we have to know is that certain objects obey Quantum mechanics rules whereas others obey Classical mechanics'. Waves on a string or sound waves obey classical mechanics, that is their energies depend on the the amplitude of the oscillating medium; but single electromagnetic wave that is a photon obey quantum mechanics rules, its energy depending of its wavelength; not being sustained by any medium its amplitude does not oscillate up and down.

Concerning light waves that make up light and that are used to describe the phenomenon of interference pattern and the superposition of waves that describe the wave property of light are made up of many photons that propagate as wave train in a continuous way as given in Figure 4.1. An assembly of those wave trains propagating as a beam of light gives us a light visible to the naked eye. Light rays that are used to describe certain properties of light are also made of infinitely many photons that propagate as one in a continuous way. It is only when light interacts with matter that the individual photons will interact with the atoms or electrons constituting the matter, because interaction with matter happen at the quantum level.

Considering the wave particle duality of light, I think that it should be applied to the photons as the wave particle duality of matter is applied to the electron. Instead of saying that light propagates as wave and interacts as particle, I would suggest that, to be in line with this new approach, we say that: the photon propagates as a wave but interacts as a particle.

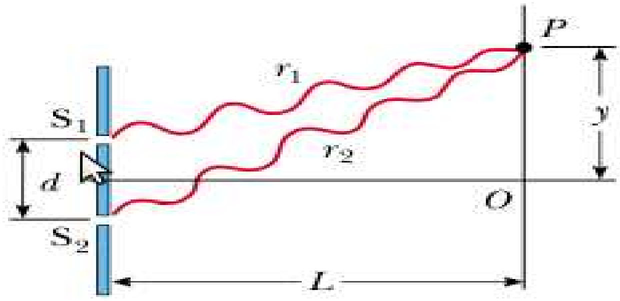


Figure 4.1 Construction for analyzing the double-slit interference pattern. A bright fringe, or intensity maximum, is observed at *O*.

A wave function to describe light waves in one dimension -propagating along the +x-axis- for example should be of the form

$$\Phi(x,t)=\Phi_0 \sin(kx-\omega t) \qquad (4.88)$$

where Φ is the amplitude of the light wave that is not the electric or the magnetic field vectors. The general fact that the intensity of light waves is proportional to the square of the amplitude of the wave remains questionable to me. But I would not want to take a stand on it till I have the chance one day to verify it experimentally.

We would use the standing waves condition as procedure (that is we would focus on the mathematical meaning of this condition) to go about in the the derivation of the spectral energy density, since as the volume of the cavity becomes large, the boundary condition chosen has no effect on the thermodynamic properties. But the idea that the (standing) waves' amplitudes oscillate up and down is completely banished because the photons that I consider as single waves need not form standing waves. It is the incorporation of this idea and the idea of electric field vector extending from point to point that has led to perception that waves contain modes of electric and magnetic field vectors which carry electric or magnetic energy that are photons; meaning that it is the modes(amplitudes) that we need to quantize and that the magnitude of an amplitude of the oscillating electric or magnetic field vector depends on the number of photons it carries. That idea has led to the zero-point energy $E_0=\frac{1}{2}h\nu$; meaning the modes with energies less than the energy of a photon will have smaller amplitudes of oscillation and therefore do not carry photons.

Having banished the idea of oscillation of the electromagnetic wave (that is a photon) in the whole of this work, the principal feature that characterizes a given electromagnetic wave or photon is its wavelength and not the frequency of the supposed oscillating electric and magnetic field vectors that make up the wave. The frequency of the wave here has nothing to do with the the frequencies of the oscillating field vectors of the standard version. The **period** is therefore the time required for one wave to pass a given point, and the **frequency** of the wave is the number of waves passing a given point per unit time.

Before continuing with the blackbody radiation, let us also establish certain facts about classical standing waves though it is not important for this work. Having proved in section 2.2 that a wave on string(classical wave) does not oscillate but the medium does as the wave propagates through it, the same argument can be applied to standing waves also. This can be seen from the statements made in reference[15] . I quote: *The modes of vibration associated with resonance in extended objects like strings and air columns have characteristic patterns called standing waves. These standing wave modes arise from the combination of reflection and interference such that the reflected waves interfere constructively with the incident waves. An important part of the condition for this constructive interference for stretched strings is the fact that the waves change phase upon reflection from a fixed end. Under these conditions,* ***the medium appears to vibrate in segments or region and the fact that these vibrations are made up of traveling waves is not apparent – hence the term "standing wave"****.*

This statement also demonstrates that the vibrations of standing waves are in reality traveling waves.

The term mode will not be used in the following derivations.

Consider a large (Meaning side length $L \gg \lambda$, where λ is the longest wavelength of interest) cubical cavity filled with radiation in thermal equilibrium. Since the radiation is in thermal equilibrium with the cavity, the photons are reflected back and forth of the walls of the cavity.

Let's introduce a Cartesian coordinate system at one of the corners of our cavity. Let's now consider a wave whose normal is in the x-direction. Finally, let's impose our boundary conditions such that the amplitude(A) of the wave be zero at the walls of the cavity A=0 at x=0 and at x= L.

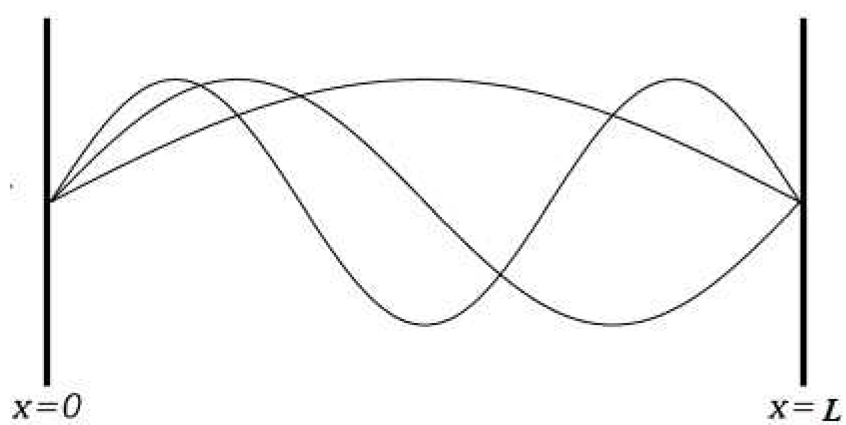


Figure 4.2 *Permitted half-wavelengths corresponding to* $n(\lambda_x/2)=L$ *for n= 1, 2, 3. Horizontal axis: X-axis of the cavity, bounded by walls at x=0 and x=L. Vertical axis: Amplitude of the waves that is considered here as photons*

$$\frac{\lambda_x}{2}=L \quad , \quad 2\frac{\lambda_x}{2}=L \quad , \quad 3\frac{\lambda_x}{2}=L$$, ….... can be permitted. (4.89)

Thus $$n\frac{\lambda_x}{2}=L$$ where n= 1, 2, 3, ….. (4.90)

Which implies that $$\frac{\lambda_x}{2}=\frac{L}{n}$$, n=1,2,3 …., (4.91)

according to the boundary conditions given above. If we focus our attention on the boundary condition of standing and forget about our knowledge that standing waves on string or sound waves in air column oscillate; the true statement of this condition is that each value of n corresponds to a given half-wavelength as given in Equation(4.91). This is the mathematical and the physical meaning of the condition. This means that each value of n corresponds to a given value of $\frac{\lambda}{2}$. So when we used this condition in the calculation of the number of independent triplets(n_x ,n_y, n_z) in 3D-space where each triplet corresponds to a given n the result that we obtain is the number of independent half-wavelengths. I say half-wavelengths not half-waves. The condition in one dimension stated in Equation(4.91) and in Figure 4.2 tells us about values half-wavelengths. It is also true that n is equal to the number of half-wavelengths for a given standing wave, but I would not want to focus my attention on that fact because careful observation of the boundary condition tells me that each n value determines a given value of $\frac{\lambda}{2}$. It does not matter how many half-wavelengths are there corresponding to a given value of n. Like I mentioned earlier, the boundary conditions are just a procedure to allow us perform the calculations. It does not necessarily mean that the photons that I consider as waves here really form standing waves. The validity of this statement comes from the fact that the spectrum derived describes any blackbody spectrum in the nature such the sun, the earth and the stars which can be assumed to be blackbodies to the first approximation. If we want to make it real, it does not even make sense to have half-wavelengths in the cavity.

As an example, putting indices on λ we have for example for L=1mm,

$$\frac{\lambda_1}{2}=1\text{mm} \quad , \quad \frac{\lambda_2}{2}=0.5\text{mm} \quad , \quad \frac{\lambda_{20}}{2}=0.05\text{mm}$$, etc. (4.92)

From Equation(4.91), we can also write in terms of the frequency by substituting $\lambda=\frac{c}{v}$ into that equation. Doing this we obtain

$$\frac{c}{2v}=\frac{L}{n} \quad \rightarrow \quad 2v=\frac{cn}{L} \quad .$$ (4.93)

This also implies that each value of n corresponds to a given value of $2v$, since dividing the wavelength by two(2) is equivalent to multiplying the frequency by two(2)

Now returning to our boundary conditions, we similarly have

$$\frac{\lambda_x}{2}=\frac{L}{n_x} \quad , \quad \frac{\lambda_y}{2}=\frac{L}{n_y} \quad , \quad \frac{\lambda_z}{2}=\frac{L}{n_z} \quad ,$$ for waves normal in the x, y, z directions. (4.94)

For a wave whose normal is in an arbitrary direction, let's say a wave whose normal makes angles of α, β, δ with the x, y, and z axes respectively; we have the following:

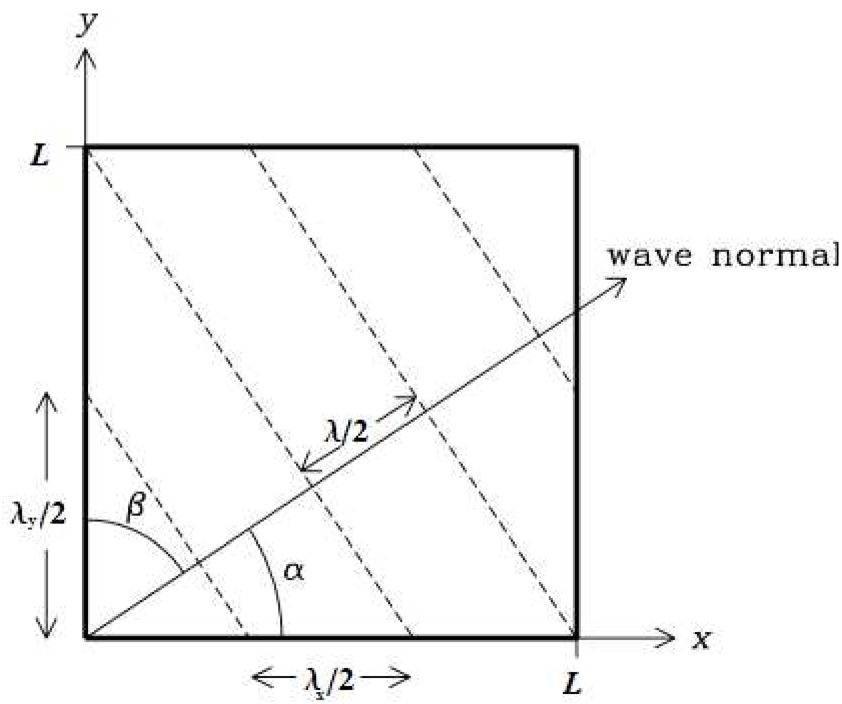


Figure 4.3 *This figure illustrates permitted half-wavelengths for waves propagating in the cavity with wave normals at angles α and β from the x and y axes respectively. Example of nodes are indicated by dashed lines for* $n_x=3$ *and* $n_y=2$

From the figure we can infer that

$\frac{\lambda}{2}=\frac{\lambda_x}{2}\cos(\alpha)$, where λ is the wavelength measured in the direction of the wave normal. And $\frac{\lambda_x}{2}$ is the spacing between the wave nodes measured along the x-axis.

Thus, $\lambda_x=(\lambda)/\cos(\alpha)$.

Similarly,

$\lambda_y=\lambda/\cos(\beta)$, $\lambda_z=\lambda/\cos(\delta)$.

Hence, our boundary conditions (4.94) become

$$n_x=\frac{2L\cos(\alpha)}{\lambda} \quad (a) \qquad n_y=\frac{2L\cos(\beta)}{\lambda} \quad (b) \qquad n_z=\frac{2L\cos(\delta)}{\lambda} \quad (c) \quad . \tag{4.95}$$

Squaring equations (4.95) and summing, we obtain

$$n_x^2+n_y^2+n_z^2=(\frac{2L}{\lambda})^2(\cos(\alpha)^2+\cos(\beta)^2+\cos(\delta)^2) \quad .$$

Since $(\cos(\alpha)^2+\cos(\beta)^2+\cos(\delta)^2)=1$,

we obtain

$$n_x^2+n_y^2+n_z^2=(\frac{2L}{\lambda})^2 \quad , \qquad (4.96a)$$

or

$$\frac{\lambda}{2}=\frac{L}{\sqrt{n_x^2+n_y^2+n_z^2}} \quad , \qquad (4.96b)$$

these are the permitted half-wavelengths. Each triplet of integers (n_x ,n_y, n_z) corresponds to a possible half-wavelength in the cavity.

In terms of frequencies we have

$$\frac{2v}{c}=\frac{1}{L}\sqrt{n_x^2+n_y^2+n_z^2} \quad , \qquad (4.97)$$

where n_x ,n_y ,n_z are positive integers.

Now consider a 3D space of three integers (n_x ,n_y, n_z) and every point in this space corresponds to one possible half-wavelength.

We can represent the independent triplets of integers *a*s a lattice in the positive octant of a space with axes n_x, n_y, and n_z , where each point of the lattice represents one possible half-wavelength of equilibrium cavity radiation.

Let $n_x^2+n_y^2+n_z^2=n^2$,

then, the number of independent triplets in the volume of one octant of a spherical shell (Since we are dealing with positive integers alone) between n and n+dn, where n is radius of the sphere, is $dN=N'(n)dn$. Because the space density of points in this lattice is unity, so the number of points in any volume is equal to that volume.

This is equal to

$$dN=\frac{4\pi n^2 dn}{8} \quad . \qquad (4.98)$$

Since each triplet corresponds to a given half-wavelength(ie $\frac{\lambda}{2}$) the number of permitted half-wavelengths in the wavelength range λ to $\lambda+d\lambda$ is equal to

$$dN(\lambda)=\frac{4\pi L^3}{\lambda^4}d\lambda \quad , \qquad (4.99)$$

where we have use from Equation (4.96a) that

$$dn=\frac{-2L}{\lambda^2}d\lambda \quad , \qquad (4.100)$$

and taking the absolute values of both sides we obtain positive value.

In terms of the frequency we have

$$dN(v)=\frac{4\pi L^3}{c^3}v^2\,dv \quad . \tag{4.101}$$

From Equation(4.97) we can write

$$2v=\frac{c\,n}{L} \quad , \tag{4.102a}$$

and multiplying both sides by Planck's constant, we obtain

$$2hv=\frac{hc\,n}{L} \quad . \tag{4.103}$$

The permitted energy in the cavity in the frequency range ν to ν+ dν is found by multiplying the right hand of Equation(4.101) by the left hand side of Equation(4.103). Performing the operation, we obtain

$$dU(v)=\frac{8\pi hL^3}{c^3}v^3\,dv \quad . \tag{4.104}$$

This is an important result here that I have arrived at. We have a factor 8 in the Equation(4.104) which is the same factor that one would obtain from the standard derivation by considering that the electromagnetic wave has two modes of polarization. In this derivation I did not make use of the two modes of polarization of electromagnetic wave since I have already proven in the previous chapter that the electromagnetic field has no energy density or do not store energy, I have also tried to explain that there is no direct relation between the electric energy and photons.

To obtain the actual energy in the cavity in the frequency range from ν to ν+ dν, we need to multiply Equation(4.104) by the probability of finding (or obtaining) a given frequency in the cavity. This probability can be stated (even though I will derive it in the next section) to be

$$f(\nu)=\frac{1}{\exp(h\nu/kT)-1} \quad . \tag{4.105}$$

Hence the actual energy density in the frequency range from ν to ν+ dν ; which is called simply the energy density in the frequency range from ν to ν+ dν is equal to

$$du(v)=\frac{dU(v)}{L^3}=\frac{8\pi h}{c^3}\frac{v^3}{\exp(hv/kT)-1}dv \quad . \tag{4.106}$$

An alternative explanation of the reason why we should multiply by a factor two is that Equation(4.99) gives as the number of half-wavelengths($\frac{\lambda}{2}$) in the cavity within a certain wavelength range. If we want to have λ 's that is full-wavelengths, we need to multiply the result by two. This can be seen clearly by taking concrete values. Let for example $\frac{\lambda}{2}=10\text{nm}$. If the number of 10nm's within a certain infinitesimal wavelength range is given by Equation(4.99), then to increase the length to full-wavelengths $\lambda=20\text{nm}'s$ we need to multiply the number of half-wavelengths (ie the right-hand side of Equation4.99) by two. And this can be seen even from the figure below which is a representation of standing waves.

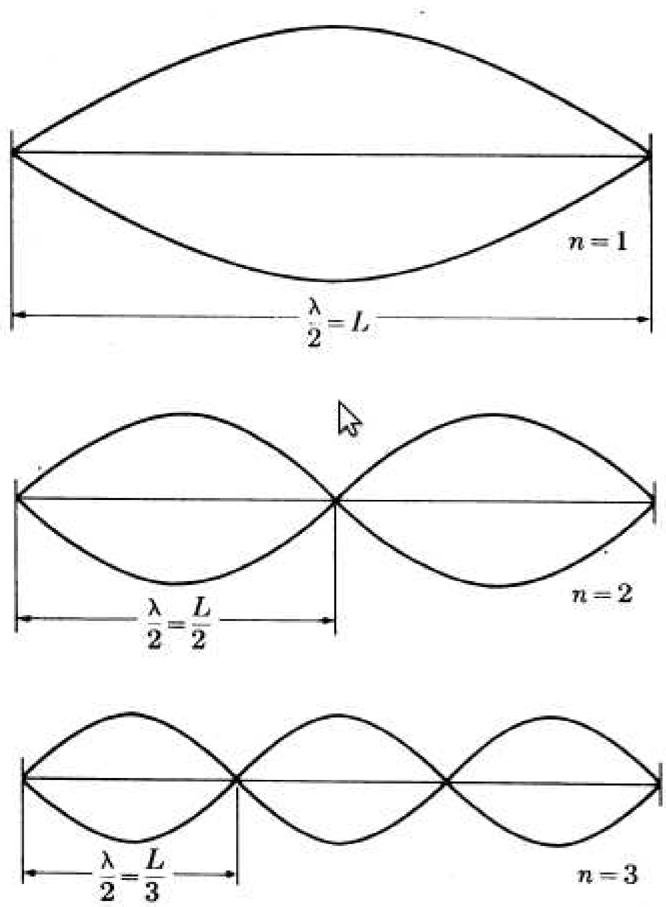


Figure 4.4 *Waves which can fit into a box with perfectly conducting walls.*

So multiplying Equation 4.99 by two gives us **number of permitted full-wavelengths** within a given wavelength range. This equal to

$$dN(\lambda)=\frac{8\pi L^3}{\lambda^4}d\lambda \quad . \tag{4.107}$$

This multiplication by a factor of two is not due to the polarization of the wave but it is due to the fact that we calculated the number of permitted half-wavelengths and we need to multiply by two to obtain the number of permitted full-wavelengths .

It is important that I state clearly that we are not calculating the number of half-wavelengths corresponding to a given triplet (n_x, n_y n_z) but we are calculating the number of half-wavelengths corresponding to independent triplets. To be specific for example in Figure4.4 , for n=3, I am interested only in one $\frac{\lambda}{2}=\frac{L}{3}$; and the up and down half-wavelengths make a full-wavelength that can be regarded as the length (wavelength) of a photon. As already stated above the boundary condition that we should have standing waves is just a procedure, the photons in the cavity need not form standing waves.

4.2.3 Comparison with the standard method

The result given by Equation(4.106) is also the result one obtains from the standard approach which consist of transforming Equation(4.102a) to obtain

$$v=\frac{cn}{2\mathrm{L}} \quad , \tag{4.102b}$$

and using the modes of polarization of the electromagnetic wave by multiplying Equation(4.101) by two to account for the supposed two modes of polarization . From Equation(4.102b) when we multiply it by Planck's constant to find the energy, we obtain

$$hv=\frac{hcn}{2\mathrm{L}} \quad , \tag{4.108}$$

which is the energy of a single photon according to the standard approach.

Using the polarization also, we obtained that the number of standing-wave modes is

$$dN(v)=\frac{8\pi L^3}{c^3}v^2 dv \quad . \tag{4.109}$$

Hence using the average energy in a mode with a given frequency, we obtain the energy density in the frequency range from ν to ν+ dν is equal to

$$du(v)=\frac{8\pi h}{c^3}\frac{v^3}{\exp(hv/kT)-1}dv \quad . \tag{4.110}$$

Now let's substitute hv in Equation(4.110) by it alternative value given in equation(4.108), we obtain

$$du(v)=\frac{8\pi}{c^3}\frac{v^2}{\exp(hv/kT)-1}[\frac{chn}{2\mathrm{L}}]dv=\frac{4\pi}{c^2}\frac{\nu^2}{\exp(hv/kT)-1}\cdot[\frac{n}{L}]d\nu \quad . \tag{4.111}$$

Using my approach which consist of calculation the number of half-wavelengths and not the number of independent oscillation of standing waves (comparison with standing waves on string) , we obtained from Equation(4.103) that

$$2hv=\frac{hcn}{L} \quad . \tag{4.103 b}$$

Now rewriting Equation(4.106) we have

$$du(v)=\frac{4\pi}{c^3}\frac{v^2}{\exp(hv/kT)-1}(2\mathrm{h}v)dv \quad , \tag{4.112a}$$

substituting $(2\mathrm{h}v)$ by it value in Equation(4.103b) we also obtain

$$du(v)=\frac{4\pi}{c^3}\frac{v^2}{\exp(hv/kT)-1}[\frac{chn}{L}]dv=\frac{4\pi}{c^2}\frac{\nu^2}{\exp(hv/kT)-1}\cdot[\frac{n}{L}]d\nu \quad , \tag{4.112b}$$

which is the same result as obtained in Equation(4.111) by using just the standard approach.

This result confirms the validity of my approach. And after all, having proven in the previous chapter that the electromagnetic field do not store energy and that that wave function, with electric and magnetic field vectors perpendicular to the direction of propagation of the wave, derived by Maxwell is mathematical idea, and hence has no physical meaning of wave function; it is clear that the standard approach which consists of calculating the number of actual oscillating standing electromagnetic waves (made up of electric and magnetic components) need to be reconsidered.

4.2.4 Derivation of the average energy per each set of wavelengths.

To start the derivation of the average energy of a set of "standing-waves" satisfying the boundary conditions, it is important to state that due to our boundary conditions not all permissible frequencies inside the cavity are independent. This statement will be proven in the next paragraphs.

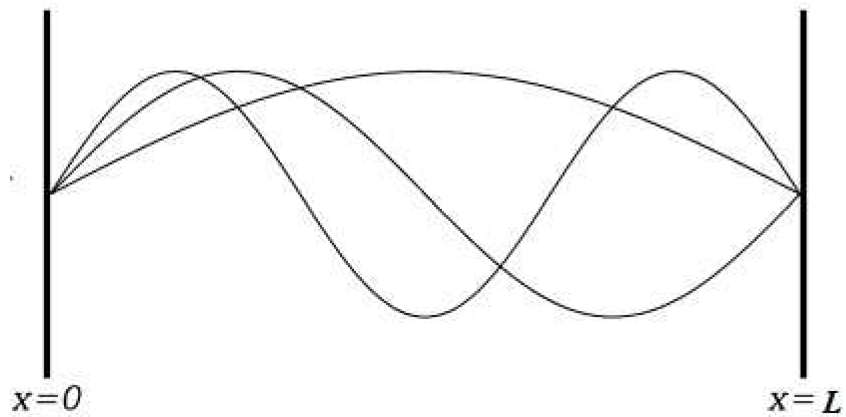


Figure 4.5 "*Standing-waves" ie permitted half-wavelengths corresponding to* $n\lambda_n/2=L$ *for n= 1, 2, 3. Horizontal axis: X-axis of the cavity, bounded by the walls at x=0 and x=L. Vertical axis: Amplitude(A)(immaterial for the energy of the photon.) Waves satisfying the boundary conditions A=0 at x=0 and x=L*

Let us number the permitted wavelengths with normal along the x-axis from the longest wavelength to infinitesimally short wavelengths as: λ_1 , λ_2 ,... λ_n ,........ This an infinite series.

From our boundary conditions that the permitted half-wavelengths must form nodes at the walls of the cavity; it implies that we can only have standing-waves satisfying the following equations:

$$\begin{aligned}
&\frac{\lambda_1}{2}=L \\
&\frac{\lambda_2}{2}=\frac{L}{2} \quad \rightarrow \quad \lambda_2=\frac{\lambda_1}{2} \\
&\frac{\lambda_3}{2}=\frac{L}{3} \quad \rightarrow \quad \lambda_3=\frac{\lambda_1}{3} \\
&\quad . \qquad\qquad\qquad . \\
&\quad . \qquad\qquad\qquad . \\
&\frac{\lambda_n}{2}=\frac{L}{n} \quad \rightarrow \quad \lambda_n=\frac{\lambda_1}{n} \\
&\quad . \qquad\qquad\qquad . \\
&\quad . \qquad\qquad\qquad .
\end{aligned} \tag{4.113}$$

Or in terms of the frequencies, we have

$$\nu_n = n\nu_1 \quad \text{or} \quad \omega_n = n\,\omega_1 \quad . \tag{4.114}$$

This means that in the cavity, the energy of a photon or a electromagnetic wave with frequency ν_n corresponds to n times energy of the longest wavelength with frequency ν_1 satisfying the boundary conditions. This is so because the frequency of any permitted wave inside the cavity should be an integer multiple of the frequency of a wave with the longest wavelength satisfying the boundary conditions and propagating in the same direction . Alternatively, any permitted wave must satisfy the boundary conditions up to waves with infinitesimally short wavelengths or infinitely high frequencies.

I will call any of the smallest frequencies (or longest wavelengths) satisfying the boundary conditions “fundamental-frequency”.

This means the frequency of any permitted wave propagating in any arbitrary direction must be a multiple of the fundamental-frequency propagating in the same direction.

The frequencies of the permitted electromagnetic waves propagating, let's say in the x-direction corresponding to our example, form a series similar to the series of simple harmonic oscillator's energy levels relative to the ground state energy level (or quantum states, since the energy levels of the simple harmonic oscillator are non-degenerate) . This series can be written as

$$h\nu_1 \;,\quad 2\mathrm{h}\nu_1 \;, \ldots\ldots ,\; nh\nu_1 \;, \ldots\ldots \tag{4.115}$$

We can then write the partition function corresponding to these set of photons (or if we want to make the analogy with oscillators: the partition function of those series of energy levels of the oscillator (measured relative to the ground state energy level of the oscillator).

The main reason why we need to find the average energy is not due to the average energy of oscillating modes but is due to the fact that in the cavity different permitted waves (each with a defined frequency) have their frequencies related; and the energies of those waves form a series corresponding to the energy levels of an oscillator; measured relative to the ground state energy level.

Another alternative approach is to view the problem in terms of probability. Because all frequencies are permitted, the actual presence of a given frequency or wavelength will depend on the probability to obtain that frequency (ie a photon with a particular energy) in the cavity. This probability is given by the Boltzmann factor.

Going back to our example of waves with normals in the x-direction, we can write the partition function of these set of permissible waves as

$$\boldsymbol{Z} = \sum_{n=0}^{n=\infty} \exp(-nh\nu_1/kT) \quad . \tag{4.116}$$

The summation starts from zero of course because $\epsilon_0 = 0$ corresponds to the case where there is no wave or radiation. This is trivial because zero wave is permitted.

The probability of having a photon with frequency $\nu_n = n\nu_1$ corresponds to the probability that an oscillator with frequency ν_1 is the n^{th} excited state or alternatively, it is the probability that the oscillator has the energy $\epsilon_n = nhv_1$ and it is given as

$$\boldsymbol{P(n) = \frac{\exp(-nhv_1/kT)}{Z}} \quad . \tag{4.117}$$

The average energy of those set of permissible wavelengths satisfying our boundary conditions along the x-axis is then

$$\boldsymbol{\langle E \rangle = \sum_{n=0}^{n=\infty} nhv_1 P(n)} \quad , \tag{4.118}$$

$$\langle E \rangle = \frac{\sum_{n=0}^{n=\infty} nhv_1 \exp(-nhv_1/kT)}{\sum_{n=0}^{n=\infty} \exp(-nhv_1/kT)} \quad . \tag{4.119}$$

Evaluating this expression, we obtain

$$\boldsymbol{\langle E \rangle = \frac{hv_1}{\exp(hv_1/kT) - 1}} \quad . \tag{4.120}$$

It is very important to emphasize that this average energy is not due to the average energy of oscillating electromagnetic mode inside the cavity nor it is due the average energy of the number of photons in a given mode but it is due to the average energy of a set of permissible electromagnetic waves satisfying our boundary conditions along the x-axis.

Similar reasoning applies to set of permissible waves with normals in the y , z ,and any arbitrary direction and that satisfy our boundary conditions in those directions. Since we can not number all longest wavelengths satisfying the boundary conditions, we drop the subscript one(1) from the expression of the average energy of the set of permissible waves whose frequencies are related (that is they form a series similar to that of an oscillator's energy levels relative to the ground state energy) and write the average energy simply as

$$\boldsymbol{\langle E \rangle = \frac{hv}{\exp(hv/kT) - 1}} \quad . \qquad 4.121)$$

Before computing the spectral energy density of the radiation inside the blackbody cavity, it is necessary to mention that the number of permitted wavelengths within a certain wavelength range (the number of photons with wavelengths in that wavelength range) inside the cavity is not equal to the **actual number of photons** present in the cavity. We find the **actual number of photons** at any given frequency range by multiplying the **number of permissible wavelengths** in that wavelength or frequency range by the expression

$$f(\nu) = \frac{1}{\exp(h\nu/kT) - 1} \quad , \tag{4.122}$$

and this term is to be interpreted as the probability of finding a photon with frequency ν inside the cavity.

The spectral energy density inside the blackbody cavity is found by transforming the right-hand side of Equation (4.107) in terms of frequency and, multiplying it with the right-hand side of Equation(4.120) and dividing by the volume of the cavity which is L^3 . It is equal to

$$\mu(\boldsymbol{v}) = \frac{\boldsymbol{dN(v)}\langle \boldsymbol{E} \rangle}{\boldsymbol{L}^3}, \tag{4.123}$$

$$\mu(\boldsymbol{v}) = \frac{8\pi \boldsymbol{h}}{\boldsymbol{c}^3} \frac{\boldsymbol{v}^3}{\exp(\boldsymbol{hv}/\boldsymbol{kT}) - 1} \quad . \tag{4.124}$$

This is the **Planck's blackbody radiation law**.

The main difference between my approach and that of the standard approach is the I view radiation as made of waves that are photons. I do not view electromagnetic radiation as made of waves that are made up of electric and magnetic field vectors that oscillate up and down, and left and right with oscillating amplitudes carrying energy; where a quantum of this energy is a photon. In this approach radiation is quantized without modes of oscillating electric and magnetic fields. If the Heisenberg Uncertainty principle should be applied to radiation, it should be applied to a photon which is a quantum of energy as well as a single wave. In this approach, we also made a difference between the electromagnetic field and the electromagnetic radiation. We did not consider the electromagnetic radiation as made of electric and magnetic field.

CHAPTER FIVE

5.0-Application to the calculation of the energy density of the vacuum

The remaining of this work is done assuming that Einstein Theory of General Relatively actually describes the universe. In this work, instead of focusing on the modification of Einstein Equations as it is the case of all the solutions proposed so far, we rather consider that the problem of the energy of the vacuum is a quantum problem that originated in classical physics. It is a quantum problem in the way we view radiation to be made up of oscillating electric and magnetic field vectors that can be likened to a simple harmonic oscillator, with oscillating vectors carrying energy that is quantized. The fact whether Einstein's theory actually describes the universe or not would not be discussed in this thesis, for future investigations will tell us whether the theory is valid or need to be modified.

Having banished completely the idea that photons create electromagnetic(em) field (in this work), I would hold the same principle for any type of particle, that is I also banish the idea that for example that spinless boson would create a scalar field, because the existence of oscillating modes or wave vectors in vacuum is purely mathematical idea that has no physical meaning. I also do not see any direct relation between the photons or thermal radiation and electric and magnetic fields or electric and magnetic energies. **If I have banished the modes it indirectly implies also that I have banished the zero-point of energy of radiation in this dissertation.** It should be made clear that in this thesis when I banish the zero-point of energy of electromagnetic radiation, I banish the idea that consists of considering an electromagnetic wave with oscillating electric and magnetic field vectors and not the fact that the vacuum might contain energy. The energy of the quantum vacuum (the energy of the vacuum states or the zero-point of energy) is equal to zero in this novel approach of the quantization of the electromagnetic radiation. To explain the accelerating expansion of the universe then requires that we think of alternative possibility instead of zero-point of electromagnetic field that gives an infinite energy density of the quantum vacuum. For that reason, I suggest that we think of the vacuum as meaning empty or free space: that is when we remove matter and radiation. An approximation of this can be viewed as the interstellar space or the intergalactic media that has low density and pressure: that is few hydrogen atoms per cubic meter. I will consider this energy as being due to virtual photons. I also do not consider these virtual photons as emitted by atoms or charges that make up the normal matter, but I consider them as arising out of the vacuum (empty space) naturally, and disappearing into the vacuum again. I will call them *vacuum-electromagnetic-waves.* And that exists by fiat as does the CMB radiation. Though I consider that the energy of the vacuum might be due to virtual photons in this interpretation, I do not reject the fact that it can be due to something else such as particle-antiparticle pairs or the possibility that both the quantum vacuum (ground states of fields) and the the classical vacuum (empty or free space) might not contain energy and the solution to the dark energy problem might require us thinking about something else.

Whether zero-point electromagnetic field exists or not is not important in this work. If it exists, the only important thing is the fact that it stores energy is very questionable given that it has been explained that the actual electromagnetic field itself does not store energy.

5-1. The energy density of the vacuum due to virtual photons.

The vacuum-electromagnetic-waves are not waves of anything substantive, but are ripples in a state of a theoretically defined space. However, these waves carry energy and momentum. Each wave has a specific direction of propagation and wavelength.

When we consider the whole universe , we can take an imaginary cavity in a portion of the universe , and apply the boundary conditions discussed in the previous section. As already stated in the previous chapter the boundary is way to help us make the calculations and is does not mean that that waves form necessary standing waves. The number density of permissible frequencies in the frequency range ν to ν+dν is given by:

$$\lim_{L\to\infty}\frac{N(\nu)d\nu}{L^3}=n(\nu)d\nu=\frac{8\pi}{c^3}\nu^2 d\nu \quad . \tag{5.1}$$

The average energy corresponding to a given set of permissible waves satisfying our boundary conditions with related frequencies (integer multiples of the fundamental-frequency) will be given by

$$\langle \boldsymbol{E}\rangle=\frac{\boldsymbol{h\nu}}{\exp(\boldsymbol{h\nu/kT})-1} \quad , \tag{5.2}$$

where T is the average temperature of the universe on a large scale.

Is really the actual average temperature of the universe equal to the temperature of the CMB as it is always assumed? According to the standard physics, we consider that the background temperature of the deepest outer space is equal to the temperature of the CMB, which is about 2.73 K. Let's consider that the whole universe was at the same temperature before and just after the last scattering. At the last scattering(Temperature ~ 3000k), when free electrons combined with the ions to form neutral atoms, the black body photons started streaming freely throughout the universe without further scattering off free electrons. The expansion still continue, and some of the hydrogen and helium atoms started crowding to form clouds and those clouds collapsed to form galaxies and stars, under the force of gravity of course. It is true that the temperature of a black body in an expanding universe decreases with the scale factor as $T\propto a^{-1}$. Alternatively, the wavelength of the blackbody photons increases with the scale factor. But the question that I am raising is: Is the CMB radiation still in thermal equilibrium with the surrounding environment? Alternatively, when we measure the energy of CMB radiation coming here on the earth, is this radiation in thermal equilibrium with our environment? The CMB photons ceased to be in thermal equilibrium with its environment at the time of last scattering when the photons were allowed to move freely throughout the universe without scattering off free electrons. The fact that the spectrum is a black body spectrum does not mean that the the CMB photons that are in the outer space are in thermal equilibrium with their medium. They are not scattered constantly to be in thermal equilibrium, but they are moving freely throughout the whole universe, traversing both hot and cold areas in the universe. It is nevertheless normal that the CMB radiation should have a black body spectrum because before the last scattering, the whole universe was or could be approximated to be a perfect black body. Thus, the radiation it contained fitted a black body spectrum. It is trivial that this radiation continues to be that of a black body since the expansion of the universe is homogeneous and isotropic. All the CMB photons that fitted the spectrum of a black body will continue to fit a black body spectrum because they (photons) all suffer expansion (increase) of their wavelengths. To further elucidate this point, let's consider the radiation emitted from a distant a star. This radiation is that of a black body to the first approximation. As this radiation travels from the star to us, the photons will suffer cosmological redshift due to the expansion of the universe. Let's assume that the surface of the star is 6000K. When this radiation arrives here on the earth, it should still fit a black body spectrum but the peak of the spectrum would be of course shifted to longer wavelengths. That is the radiation would still be that of a black body but with a temperature that is lower that the temperature of the surface of the star due to the cooling of the radiation because of the universe's expansion.

The CMB radiation continues to cool due to expansion and does not have possibility to reheat up itself. On the contrary the universe (the space) though it cools because of expansion, it is continually heated due to the presence and of the formation of galaxies, stars and other structures , and of violent phenomena such as the supernovae explosions that eject extremely hot matter and radiation into space. Heat energy emitted by the stars and galaxies in form of radiation is also continually heating the universe. I would not have any objection to the assumption that the temperature of the deepest outer space is that of the CMB if the universe was filled with nothing but the CMB radiation, but this is not actually the case. The baryonic matter that makes up the galaxies and the gaseous nebulae, though they are non-relativistic matter, but they are not also extremely cold. They do also radiate heat energy into the interstellar or intergalactic media. The rate of expansion is not also grater that the rate of transfer of heat through radiation, hence we cannot completely reject the possibility that during the formation of galaxies the heat ejected could make the temperature of the intergalactic medium a bit higher that the temperature of the CMB at that time. For these reasons it would not be a blatant statement to say that the temperature of intergalactic media, if considered to be the coldest regions in the universe, could be a bit higher that the temperature of the CMB.

The energy density of the vacuum filled with vacuum-em-waves with frequencies in the range ν to ν+dν is therefore

$$\mu'(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{\exp(h\nu/kT)-1}\,d\nu \quad . \qquad (5.3)$$

The total energy density is

$$U_T = \int_0^\infty \frac{8\pi h}{c^3}\frac{\nu^3}{\exp(h\nu/kT)-1}\,d\nu \quad . \qquad (5.4)$$

Evaluating this integration, we obtain

$$U_T = \frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^3} \quad . \qquad (5.5)$$

This would be the energy density of the vacuum filled with vacuum EM waves.

It is just similar to energy density of the blackbody at at given temperature. The temperature T here can be regarded as the background temperature of the the empty space.

CHAPTER SIX

6.0 Conclusion

This Chapter summarizes the discussions made in the previous chapters concerning the electric and magnetic fields, the electromagnetic radiation, the quantization of the electromagnetic fields, the blackbody radiation and the energy of the vacuum. It ends by giving a future outlook of the research to be carried on.

6.1 Conclusion

In this work , an attempt was made to give an alternative possible solution the problem of energy of the vacuum, which is always considered as the energy of the quantum vacuum due to zero-point of energy or the energy of the vacuum states in quantum theory of fields. We considered the problem to be a quantum problem that has its roots in classical physics. For that reason, we tried to establish whether the electric field could store energy. It was found in this new approach that the electric field could be interpreted as a force field that does not store energy or have an energy density. We also explained that the electrostatic potential energy of a discrete charge distribution or continuous charge distribution is be stored by the charges or might be the energy possessed by the charges. The same idea was applied to the electrostatic energy of the capacitor, and it is also found that the electrostatic potential energy of the capacitor would be the energy the charges of the capacitor have. It is also equal to the work that the capacitor will deliver when it discharges; meaning that there is a possibility that the electric field between the plates of the capacitor or any electric field does not store energy too. It is found that there is no direct relation between electric energy and photons.

We finally proposed an alternative possible derivation of the black body radiation formula not based on modes of oscillating electric or magnetic field vectors. In this approach, we considered the photon to be a single electromagnetic wave. We also did consider both the mathematical and the physical meaning of the standing-wave condition from which the number of modes per unit volume are derived in the standard approach. In this new approach, it is found that the results of the black body radiation can be interpreted without the two modes of polarization of the electromagnetic wave. The average energy calculated, in this approach, is not also the average energy of each electromagnetic oscillator, but the average energy of a given set of permitted waves (photons) satisfying the boundary conditions. Each set of permitted waves have their energies forming a series that is similar to the energies of an oscillator's energy levels forgetting the energy of the vacuum state of the oscillator. In this approach, the black body radiation was quantized without the modes of oscillation. Since we did not make use of the modes of oscillating electric and magnetic fields, the zero-point of energy did not appear in our interpretation. Alternatively, we interpreted the results in a way that the zero-point of energy was banished. We used the same approach to derive the energy of the vacuum (empty space here and not the ground state of fields). In this derivation, it was assumed that the energy of the vacuum might be due to virtual photons. In this novel approach, assuming Einstein Equations actually describe the universe, the energy of the vacuum or the missing energy cannot be due to zero-point of energy of either the electromagnetic radiation or the zero-point of energy of something else, since we have explained that there is a possibility that the electromagnetic radiation does have no zero-point of energy.

I would honestly say that this thesis does not provide an ultimate solution to the problem of the energy of the vacuum, but it could nevertheless serve as a hint for us to think of different possibility to the dark energy problem instead of directing all our attention to the zero-point of energy of the electromagnetic field or any other field; that gives an infinite energy density. I also hope to continue with the current work to be able to provide a conclusive solution to the problem.

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