Group Obvious Strategy-proofness: Definition and Characterization*

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Abstract: We introduce the concept of group obvious strategy-proofness, an extension of Li (2017)'s notion of obvious strategy-proofness, by requiring that truth-telling remains an obviously dominant strategy for any group of agents in the extensive game form implementing the social choice function. We show that this stronger condition is no more restrictive: the set of all group obviously strategy-proof social choice functions coincides with the set of all obviously strategy-proof social choice functions. Building on this equivalence result and existing results on obvious strategy-proofness, we derive further equivalence results concerning the implementability of social choice functions via round-table mechanisms: strategy-proofness, group strategy-proofness, obvious strategy-proofness, and group obvious strategy-proofness are all equivalent.

Keywords:; Group strategy-proofness; Obvious strategy-proofness.

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1 Introduction

We propose and characterize a novel implementation concept, termed *group obvious strategy-proofness*, which blends the notions of group strategy-proofness and obvious strategy-proofness. This concept imposes a stronger requirement than Li (2017)'s notion of obvious strategy-proofness because it requires that truth-telling is obviously dominant not only for individual agents but also for groups of agents who may coordinate within the extensive game form used to implement the social choice function.[1]

Our main result (Theorem 1) establishes that this seemingly stronger concept of group obvious strategy-proofness coincides with obvious strategy-proofness, implying that coalitional deviations do not impose additional restrictions.

Theorem 1 entails two interesting consequences. First, Proposition 1 in Li (2017), which states that obvious strategy-proofness implies group strategy-proofness, follows from our main result because group obvious strategy-proofness implies group strategy-proofness. Second, our result, combined with Theorem 2 in Mackenzie (2020), allow to simplify the design of extensive game forms used to implement group obviously strategy-proof social choice functions. Specifically, we argue that, without loss of generality, these extensive game forms can be assumed to be round-table mechanisms. In this case, the requirement of group obvious strategy-proofness becomes equivalent to obvious strategy-proofness, group strategy-proofness and strategy-proofness.

The paper is organized as follows. Section 2 introduces the basic notation, definitions and the extensive game forms required to define group obvious strategy-proofness, which is formally defined and characterized in Section 3. Section 4 contains two final remarks, which partially follow from our Theorem 1.

2 Preliminaries

In this section we closely follow Arribillaga, Massó and Neme (2024). We consider collective decision problems where a set of agents $N = \{1, \dots, n\}$ must select an alternative from a given set A. Each agent $i \in N$ has a (weak) preference R_i over A, which is a complete and

[1]Since Li (2017)'s seminal paper, the literature on obvious strategy-proofness has expanded rapidly and is now extensive. For a general treatment, see, for instance, Bade and Gonczarowski (2017), Mackenzie (2020), and Pycia and Troyan (2023). For analyses focusing on specific contexts and aspects of obvious strategy-proofness, see, for instance, Arribillaga, Massó and Neme (2020, 2023 and 2024), Ashlagi and Gonczarowski (2018), Tamura (2024), and Troyan (2019).

transitive binary relation on A. For a given preference R_i, we denote by P_i its induced strict preference. Let $\mathcal{R}$ denote the set of all weak preferences over A. A (preference) *profile* is an n-tuple $R = (R_1, \ldots, R_n) \in \mathcal{R}^N$, representing an ordered list of n preferences, one for each agent. Given a profile R, an agent i, and a non-empty subset of agents S, we denote by R_{-i} and R_{-S} the sub-profiles in $\mathcal{R}^{N\setminus\{i\}}$ and $\mathcal{R}^{N\setminus S}$ obtained by removing R_i and $R_S := (R_j)_{j\in S}$ from R, respectively. For agent $i \in N$, we denote by $\mathcal{D}_i \subset \mathcal{R}$ a given restricted set of i's preferences. Consequently, let $\mathcal{D} = \mathcal{D}_1 \times \cdots \times \mathcal{D}_n$ be a (Cartesian product) set of restricted preference profiles. A *social choice function* $f : \mathcal{D} \to A$ selects an alternative $f(R) \in A$ for each profile $R \in \mathcal{D}$.

A fundamental property of a social choice function f is strategy-proofness: no agent has an incentive to manipulate f by misreporting its preference. A social choice function $f : \mathcal{D} \to A$ is *strategy-proof* (SP) if for all $R \in \mathcal{D}$ and all $i \in N$, R_i is a *dominant strategy* in the direct revelation mechanism at R. Namely, for all $R \in \mathcal{D}$, all $i \in N$, and all R'_i,

$$f(R_i, R_{-i}) \; R_i \; f(R'_i, R_{-i}).$$ [2]

In other words, truth-telling is optimal for each agent regardless of other agents' preferences.

Strategy-proofness assumes that agents can engage in contingent reasoning, specifically concerning the hypothesis R_{-i} regarding other agents' behavior. However, this reasoning can become complex, even for straightforward social choice functions. To accommodate for agents who may have limited abilities in this regard, Li (2017) introduces the stronger incentive notion of obvious strategy-proofness (OSP) for general settings, where agents' types—coinciding with their preferences in our context—are considered private information. Obviously strategy-proofness transforms the hypothetical contingencies into evidences about past and common knowledge behavior in a dynamic setting where preferences are not revealed all at once, but partially as the game progresses.[3]

A social choice function $f : \mathcal{D} \to A$ is *obviously strategy-proof* (OSP) if it satisfies two main conditions. First, there must exist (i) an extensive game form Γ, played by agents in N, with outcomes corresponding to alternatives in A, and (ii) a preference-strategy profile

[2]By the revelation principal, the implementation of f in dominant strategies by the direct revelation mechanism is without loss of generality. The revelation mechanism is the normal game form where the strategy sets are the corresponding sets of restricted preferences and the outcome function coincides with the social choice function f. In this case, we say that the direct revelation mechanism SP-implements f.

[3]This description aligns with the concept of round-table mechanisms, introduced in Mackenzie (2020), which serve a role for obvious strategy-proofness akin to that of the revelation principle for strategy-proofness.

$(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ that specifies a behavioral strategy in Γ for each agent and for each of its preferences, which implement the social choice function f; that is, for every $R \in \mathcal{D}$, the outcome of playing the game Γ according to the strategy profile $\sigma^R := (\sigma_i^{R_i})_{i \in N}$ is $f(R)$. Second, for each $i \in N$ and for each $R_i \in \mathcal{D}_i$, the strategy $\sigma_i^{R_i}$ corresponding to R_i must be obviously dominant in Γ, meaning that it appears unambiguously optimal at every stage of the game (see its formal definition in the next section).

The literature contains many implementation concepts in which strategic incentives apply not only to individual agents but also to coalitions of agents.[4] Group strategy-proofness is a prominent example of such a concept. While individual manipulation is indisputable, different notions of group manipulation exist. We adopt the most common extension—strong manipulation—which requires that all members of the manipulating coalition end up strictly better off.[5]

A social choice function $f : \mathcal{D} \to A$ is *group strategy-proof* if, for all $R \in \mathcal{D}$ and all $S \subset N$, R_S is a *group dominant strategy* in the direct revelation mechanism at R. Namely, for all $R \in \mathcal{D}$, all $S \subset N$, and all $R'_S \in (\mathcal{D}_i)_{i \in S}$, there exists at least one agent $i \in S$ such that

$$f(R_S, R_{-S}) \; R_i \; f(R'_S, R_{-S}). \tag{1}$$

In words, for any potential deviation from truth-telling by a group of agents, there is always at least one agent within the deviating group who does not find the joint deviation profitable, regardless of the preferences submitted by agents outside the group. As a result, the deviation becomes invalidated. In this case, we say that the direct revelation mechanism GSP-implements f.

Barberà, Berga and Moreno (2010 and 2016) study restricted domains of preferences under which the classes of strategy-proof and group strategy-proof social choice functions coincide in public and private goods economies, respectively. They show that those domains can be highly restrictive: in general domains, the class of group strategy-proof social choice functions is a significant subset of the class of strategy-proof social choice functions. In contrast, Theorem 1 below states that, in general domains, the classes of obvious strategy-proof and group obvious strategy-proof social choice functions do coincide.

There are settings where agents can engage in pre-play communication and reach agree-

[4]Pattanaik (1970) already explored collective rationality and group decision-making in the Arrowian context.

[5]See Barberà, Berga, and Moreno (2016) for the definition of strong group strategy-proofness based on the notion of weak manipulation.

ments concerning their future actions. Although these agreements are non-enforceable, they may still serve as hypotheses about agents' anticipated behavior. It is then natural to extend Li (2017)'s concept of obvious strategy-proofness—originally based on individual incentives—to include coalitional incentives as well. We define the notion of group obvious strategy-proofness and show in Theorem 1 that it is equivalent to obvious strategy-proofness.

To formally define the stronger notion of group obvious strategy-proofness, we must deal with extensive game forms. Table 1 provides the basic notation for extensive game forms.

Table 1: Notation for Extensive Game Forms

Name	Notation	Generic element
Agents (or players)	N	i
Outcomes (or alternatives)	A	x
Histories	H	h
Nodes	Z	z
Partial order on Z	$\prec$	
Initial node	z_0	
Terminal nodes	Z_T	
Non-terminal nodes	Z_{NT}	
Nodes where i plays	Z_i	z_i
Information sets of player i	$\mathcal{I}_i$	I_i
Choices (or actions) at $z_i \in Z_{NT}$	$Ch(z_i)$	
Outcome at $z \in Z_T$	$g(z)$	

An extensive game form with set of agents (or players) N and outcomes in A (or simply, a *game*) is a seven-tuple $\Gamma = (N, A, (Z, \prec), \mathcal{Z}, \mathcal{I}, Ch, g)$, where $(Z, \prec)$ is a rooted tree. This tree is a rooted graph with the properties that any two nodes in Z are connected through a unique path and there exists a distinguished node $z_0 \in Z_{NT}$, called the root, such that $z_0 \prec z$ for all $z \in Z \setminus \{z_0\}$. Alternatively, for every node $z \in Z \setminus \{z_0\}$, there exists a *unique* node z' with the property $z' \prec z$ and no other node $z'' \in Z_{NT}$ exists such that $z' \prec z'' \prec z$; this specific node z' is referred to as the immediate predecessor of z and is denoted $IP(z)$; by convention, we set $IP(z_0) = \emptyset$.

In addition to the notation of Table 1, let $\mathcal{Z} = \{Z_1, \ldots, Z_n\}$ represent the partition of Z_{NT}, where $z \in Z_i$ indicates that agent i plays at node z. The partition of information sets is represented by $\mathcal{I} = \{\mathcal{I}_1, \ldots, \mathcal{I}_n\}$, where $z, z' \in I_i \in \mathcal{I}_i$ indicates that agent i must play at information set I_i (*i.e.*, $I_i \subseteq Z_i$) and cannot distinguish whether the game has reached node z or z'. For each $I_i \in \mathcal{I}_i$ and any pair $z, z' \in I_i$, $Ch(z) = Ch(z')$ holds, meaning that agent i cannot distinguish at I_i between nodes z and z' by observing available choices. Thus, we denote the set of available choices at I_i as $Ch(I_i)$, which is equivalent to $Ch(z)$ for any $z \in I_i$. We use $I'_i \prec I_i$ to indicate that for each $z' \in I'_i$ there exists a node $z \in I_i$ such that $z' \prec z$. Certainly, for each $z \in Z_{NT}$, there should be a one-to-one correspondence between $Ch(z)$ and the set of immediate followers of z (*i.e.*, $\{z' \in Z \mid z = IP(z')\}$). Based on this correspondence, we often identify the choice made by agent i at node $z \in Z_i$ with the subsequent node following z. A history h (of length t) is defined as a sequence $z_0, z_1, \ldots, z_t$ of $t+1$ nodes, beginning at z_0 and ending at z_t, such that, for all $m = 0, \ldots, t-1$, z_{m+1} is an immediate follower of z_m. Each history $h = z_0, \ldots, z_t$ can be uniquely identified with the node z_t, and conversely, each node z can be uniquely identified with the history $h = z_0, \ldots, z$. A history $h = z_0, \ldots, z$ is complete if $z \in Z_T$. A game Γ has *perfect recall* if $\mathcal{I}$ has the property that agents remember all of their past choices and information sets they have encountered up to any given point.[6]

Let $\mathcal{G}$ denote the class of all games with set of agents N and outcomes in A with perfect recall. For a fixed $\Gamma \in \mathcal{G}$ and $i \in N$, a (behavioral and pure) *strategy* of i in Γ is a function $\sigma_i : Z_i \to \bigcup_{z \in Z_i} Ch(z)$ such that, for each $z \in Z_i$, $\sigma_i(z) \in Ch(z)$; that is, σ_i selects one of i's available choices at each node where i must play. Additionally, σ_i is $\mathcal{I}_i$-measurable, meaning that for any $I_i \in \mathcal{I}_i$ and any pair $z, z' \in I_i$, $\sigma_i(z) = \sigma_i(z')$. Hence, we often denote the choice taken by σ_i at all nodes in I_i as $\sigma_i(I_i)$. Let Σ_i represent the set of strategies of agent i in Γ. Then, a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n) \in \Sigma := \Sigma_1 \times \cdots \times \Sigma_n$ is an ordered list of strategies, with one strategy for each agent. Given $\sigma \in \Sigma$ and $S \subseteq N$, $\sigma_S := (\sigma_i)_{i \in S} \in (\Sigma_i)_{i \in S}$ represents the strategy profile of agents in S. Let $z^\Gamma(z, \sigma)$ denote the terminal node reached in Γ when agents commence playing at $z \in Z_{NT}$ according to $\sigma \in \Sigma$.[7]

Fix a game $\Gamma \in \mathcal{G}$, a strategy profile $\sigma \in \Sigma$, and a subset of agents $S \subseteq N$. We define a history $h = z_0, \ldots, z_t$ (or node z_t) as *compatible with* σ_S if, for every $i \in S$ and each node $z_{t'} \in Z_i$ along the path from z_0 to z_t, where $0 \leq t' < t$, we have $\sigma_i(z_{t'}) = z_{t'+1}$. In other words, a history $h = z_0, \ldots, z_t$ is compatible with σ_S if, whenever an agent $i \in S$ is required to play at a node $z_{t'}$ in the path from z_0 to z_t, the choice made by agent i according to σ_i results in the node $z_{t'+1}$. It's important to note that the compatibility of $h = z_0, \ldots, z_t$ with σ_S does not rule out the possibility of agents not in S playing along the history toward z_t.

[6]For a formal definition of perfect recall, see Fudenberg and Tirole (1991) and Myerson (1991).

[7]Example 1 below illustrates all the preceding definitions.

Specifically, it's possible for a node $z_{t'} \in Z_i$ to occur at some $0 \leq t' < t$ with $i \notin S$. Given an information set I_i and σ_S, we denote by $I_i(\sigma_S)$ the set of nodes in I_i compatible with σ_S.

Note that Γ is not yet a game in extensive form because agents' preferences over alternatives (associated with terminal nodes) are not specified. However, given a game Γ and a preference profile $R \in \mathcal{D}$ over A, the pair (Γ, R) defines a game in extensive form where each agent i uses R_i to evaluate pairs of alternatives associated with pairs of terminal nodes. In the context of a given game Γ and a domain $\mathcal{D}$, a *preference-strategy* profile $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ specifies, for each agent $i \in N$ and preference $R_i \in \mathcal{D}_i$, a behavioral strategy $\sigma_i^{R_i} \in \Sigma_i$ of i in Γ. Given a preference-strategy profile $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ and a particular profile $R' \in \mathcal{D}$, we set $\sigma^{R'} := (\sigma_1^{R'_1}, \ldots, \sigma_n^{R'_n}) \in \Sigma$.

3 Group obvious strategy-proofness

3.1 Definition

This subsection introduces the concept of *group obvious strategy-proofness*, which integrates elements of both group strategy-proofness and obvious strategy-proofness. We start by providing an overview of the main ideas involved in its definition.

Let $f : \mathcal{D} \to A$ be a social choice function implemented by Γ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$; that is, for each profile R, if agents play Γ according to σ^R, the outcome of Γ is the alternative selected by f at R (see condition (GOSP.1) in Definition 3 below). Fix an arbitrary profile $R \in \mathcal{D}$. Suppose agents are considering following the strategy profile σ^R, and coalition S is evaluating a potential joint deviation from σ_S^R to σ'_S. To evaluate σ'_S, each agent $i \in S$ assumes that all agents in S will play according to the deviation, σ'_S. For σ^R to be obviously dominant over σ'_S—and thus obviously immune to this deviation—the following must hold for each agent $i \in S$. Consider any decision point in Γ, compatible with σ'_S, where agent i must choose an action that, for the first time, would differ if i follows σ'_i instead of $\sigma_i^{R_i}$ (an earliest point of departure for σ_S^R, σ'_S and i). From this point onward, i assumes that agents in $S \setminus \{i\}$ will continue with the deviation $\sigma'_{S\setminus\{i\}}$, effectively treating their future choices fixed according to $\sigma'_{S\setminus\{i\}}$. Meanwhile, agent i adopts two extreme behavioral hypotheses regarding the futures choices of agents outside the deviating coalition S: a pessimistic view for continuing with $(\sigma_i^{R_i}, \sigma'_{S\setminus\{i\}})$ and an optimistic view for the deviation to σ'_S. Then, σ_S^R group obviously dominates σ'_S if, for all $i \in S$, the least favorable alternative achievable under $(\sigma_i^{R_i}, \sigma'_{S\setminus\{i\}})$ is at least as preferred, according to R_i, as the best alternative S could attain by carrying on with the deviation σ'_S. Thus, f is group obviously strategy-proof if there exist a game $\Gamma \in \mathcal{G}$ and a preference-strategy profile $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ that implement f and, for all profiles $R \in \mathcal{D}^N$ and all coalitions $S \subset N$, σ_S^R group obviously dominates all possible deviations σ'_S.

We now turn to present the formal definitions of the main two ingredients: the extensions of an earliest point of departure from individual deviations to group deviations and of obvious dominance to group obvious dominance.

Our first extension is based on Li (2017)'s notion of earliest point of departure for $\sigma_i, \sigma'_i \in \Sigma_i$: An information set I_i is an earliest point of departure for σ_i and σ'_i if they choose different actions at I_i but chose the same action at every previous information set.

Definition 1. *Let σ_S, σ'_S, $i \in S$ and I_i be given. We say that the set $I_i(\sigma_S, \sigma'_S) \subseteq I_i$ of nodes compatible with σ'_S is an* earliest point of departure for σ_S, σ'_S and i *if, for each $z \in I_i(\sigma_S, \sigma'_S)$,*

$$\sigma_i(z) \neq \sigma'_i(z) \text{ and } \sigma_i(z') = \sigma'_i(z') \text{ for all } z' \in I'_i \prec I_i.$$

Let $\alpha_i(\sigma_S, \sigma'_S)$ be the family of all earliest points of departure for σ_S, σ'_S and i.

Remark 1. *Li (2017)'s original definition of earliest point of departure between σ_i and σ'_i coincides with our Definition 1 for $S = \{i\}$. To see that, observe that $z \in I_i(\sigma_i, \sigma'_i)$ and $\sigma_i(z') = \sigma'_i(z')$ for all $z' \in I'_i \prec I_i$ imply that z is compatible with σ'_i.*

Given σ_S, σ'_S, $i \in S$, and $I_i(\sigma_S, \sigma'_S) \in \alpha_i(\sigma_S, \sigma'_S)$, let $O(I_i(\sigma_S, \sigma'_S))$ and $O'(I_i(\sigma_S, \sigma'_S))$ be the two sets of options respectively left by $(\sigma_i, \sigma'_{S\setminus\{i\}})$ and σ'_S at the earliest point of departure $I_i(\sigma_S, \sigma'_S)$; namely,

$$O(I_i(\sigma_S, \sigma'_S)) = \{x \in A \mid \exists\, \overline{\sigma}_{-S} \in \Sigma_{-S} \text{ and } z \in I_i(\sigma_S, \sigma'_S) \text{ s.t. } x = g(z^\Gamma(z, (\sigma_i, \sigma'_{S\setminus\{i\}}, \overline{\sigma}_{-S})))\}$$

and

$$O'(I_i(\sigma_S, \sigma'_S)) = \{y \in A \mid \exists\, \overline{\sigma}_{-S} \in \Sigma_{-S} \text{ and } z \in I_i(\sigma_S, \sigma'_S) \text{ s.t. } y = g(z^\Gamma(z, (\sigma'_S, \overline{\sigma}_{-S})))\}.$$

Our second extension is based on Li (2017)'s notion of obvious dominance: Strategy σ_i obviously dominates σ'_i if, at any of their earliest points of departure, i is absolutely pessimistic when assessing the consequence of σ_i and absolutely optimistic when assessing the consequence of σ'_i and i weakly prefers the former to the latter.

Definition 2. *A joint strategy* σ_S *is* group obviously dominant *in* Γ *at* $R \in \mathcal{D}$ *if, for all* $\sigma'_S \in \Sigma_S$*, there exists* $i \in S$ *such that, for all* $I_i(\sigma_S, \sigma'_S) \in \alpha_i(\sigma_S, \sigma'_S)$*, the following holds: for all* $x \in O(I_i(\sigma_S, \sigma'_S))$ *and all* $y \in O'(I_i(\sigma_S, \sigma'_S))$*,*

$$x \, R_i \, y.$$

In words, σ_S is group obviously dominant in Γ at R if, for any joint deviation σ'_S, conditional on reaching any of the earliest points of departure for σ_S, σ'_S and $i \in S$, the best possible outcome under σ'_S is no better than the worst possible outcome under $(\sigma_i, \sigma'_{S \setminus \{i\}})$, according to R_i. When Definition 2 holds for σ_S and a particular σ'_S we say that σ_S *group obviously dominates* σ'_S. Observe that Definition 2 is the natural extension to group obvious non-manipulability of the group non-manipulability condition (1), used to define group strategy-proofness.

Definition 3. *A social choice function* $f : \mathcal{D} \to A$ *is group obviously strategy-proof* (GOSP) *if there exist a game* $\Gamma \in \mathcal{G}$ *and a preference-strategy profile* $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ *for* Γ *such that, for all* $R \in \mathcal{D}$*,*
(GOSP.1) $f(R) = g(z^{\Gamma}(z_0, \sigma^R))$ *and*
(GOSP.2) *for all* $S \subseteq N$*,* $\sigma_S^{R_S}$ *is group obviously dominant in* Γ *at* R*.*

Let Γ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ be the game and the preference-strategy profile used in Definition 3 to state that $f : \mathcal{D} \to A$ is GOSP. Then, we say that Γ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ GOSP-implement f.

When Definitions 2 and 3 are applied to a singleton set S, they yield the classic concepts of obvious dominance and obvious strategy-proofness (OSP) introduced by Li (2017). Let Γ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ be the game and the preference-strategy profile used in Definition 3 to state that $f : \mathcal{D} \to A$ is OSP. Then, we say that Γ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ OSP-implement f.

3.2 Example

Example 1 below illustrates some of the definitions introduced in Subsection 3.1 that are needed to define group obvious strategy-proofness when the set of agents S is not a singleton.

Example 1. Figure 1 depicts a game Γ where $N = \{1, 2, 3\}$, $I_1 = \{z_0\}$, $I_2^1 = \{z_1\}$, $I_3^1 = \{z_2\}$, $I_2^2 = \{z_3, z_4\}$, $I_3^2 = \{z_5, z_6\}$, $I_3^3 = \{z_7, z_8\}$, $Ch(z_2) = \{L, R\}$, $Ch(I_3^2) = \{l, r\}$, $Ch(I_3^3) = \{l', r'\}$ and $A = \{x_1, \ldots, x_{10}\}$.

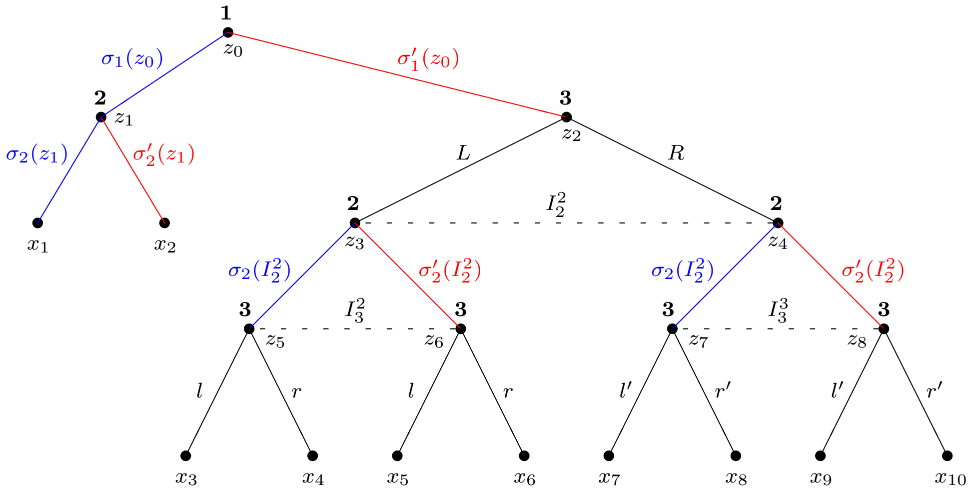

Figure 1: An extensive game form Γ that illustrates Definitions 1, 2 and 3

Consider $S = \{1, 2\}$ and the joint strategies $\sigma_S = (\sigma_1, \sigma_2)$ and $\sigma'_S = (\sigma'_1, \sigma'_2)$ depicted in Figure 1 in blue and red, respectively. To identify the earliest points of departures for σ_S, σ'_S, 1 and 2, we observe that (i) z_0 is trivially compatible with σ'_S because, at z_0, σ_1 and σ'_1 choose different actions and z_0 is the initial node; (ii) z_1 is not compatible with σ'_S, because $\sigma'_1(z_0) = z_2$, and (iii) z_3 and z_4 are both compatible with σ'_S, because $\sigma'_1(z_0) = z_2$ and $\sigma_3(z_2) = z_3$, and $\sigma'_1(z_0) = z_2$ and $\sigma'_3(z_2) = z_4$. Then, $I_1(\sigma_S, \sigma'_S) = \{z_0\}$ is the unique earliest point of departure for σ_S, σ'_S and agent 1, and $I_2(\sigma_S, \sigma'_S) = \{z_3, z_4\}$ is the unique earliest point of departure for σ_S, σ'_S and agent 2.

We identify properties of profiles $R \in \mathcal{R}^N$ for which σ_S group obviously dominates σ'_S in Γ at R. First, $O(I_1((\sigma_S, \sigma'_S)) = \{x_2\}$, because x_2 is the unique possible outcome (*i.e.*, option) if agents in S play according to (σ_1, σ'_2) (*i.e.*, $x_2 = g(z^\Gamma(z_0, (\sigma_1, \sigma'_2, \sigma_3)))$ for all $\sigma_3 \in \Sigma_3$), and $O'(I_1(\sigma_S, \sigma'_S)) = \{x_5, x_6, x_9, x_{10}\}$, because these four alternatives are possible outcomes (*i.e.*, options) if agents in S play according to σ'_S (for instance, $x_5 = g(z^\Gamma(z_3, (\sigma'_S, \sigma_3))$ if $\sigma_3(I_3^2) = l$ and $\sigma_3(I_3^3) = l'$, and $x_{10} = g(z^\Gamma(z_4, (\sigma'_S, \sigma'_3))$ if $\sigma'_3(I_3^2) = r$ and $\sigma'_3(I_3^3) = r'$). Second, $O(I_2(\sigma_S, \sigma'_S)) = \{x_3, x_4, x_7, x_8\}$ because these four alternatives are possible outcomes (*i.e.*, options) if agents in S play according to (σ'_1, σ_2) (for instance, $x_3 = g(z^\Gamma(z_3, (\sigma'_1, \sigma_2, \sigma_3)))$ if $\sigma_3(I_3^2) = l$ and $\sigma_3(I_3^3) = l'$, and $x_8 = g(z^\Gamma(z_4, (\sigma'_1, \sigma_2, \sigma'_3)))$ if $\sigma'_3(I_3^2) = r$ and $\sigma'_3(I_3^3) = r'$), and $O'(I_2(\sigma_S, \sigma'_S)) = \{x_5, x_6, x_9, x_{10}\}$, because these four alternatives are possible outcomes (*i.e.*, options) if agents in S play according to σ'_S (for instance, $x_5 = g(z^\Gamma(z_3, (\sigma'_S, \sigma_3))$ if $\sigma_3(I_3^2) = l$ and $\sigma_3(I_3^3) = l'$, and $x_{10} = g(z^\Gamma(z_4, (\sigma'_S, \sigma'_3))$ if $\sigma'_3(I_3^2) = r$ and $\sigma'_3(I_3^3) = r'$). Let $R = (R_1, R_2, R_3) \in \mathcal{R}^N$ be any profile with the property that, $x_2\, R_1\, x_k$ holds for all $k \in \{5, 6, 9, 10\}$ and $x_t\, R_2\, x_k$ holds for all $t \in \{3, 4, 7, 8\}$ and $k \in \{5, 6, 9, 10\}$. Then, σ_S

group obviously dominates σ'_S in Γ at R. $\square$

3.3 Result

We are now ready to state and prove our equivalence theorem.

Theorem 1. *Let $f : \mathcal{D} \to A$ be a social choice function. Then, f is group obviously strategy-proof if and only if f is obviously strategy-proof.*

Proof.

($\Rightarrow$) It follows directly from the two definitions that if f is GOSP, then f is OSP.

($\Leftarrow$) Assume f is OSP. Then, there exist $\Gamma \in \mathcal{G}$ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ satisfying Definition 3 for any singleton set S (*i.e.*. $\Gamma \in \mathcal{G}$ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ OSP-implement f). Therefore, since (GOSP.1) in Definition 3 is independent of S, (GOSP.1) trivially holds for any S.

We now prove by contradiction that (GOSP.2) holds. Suppose (GOSP.2) does not hold for Γ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$. Then, there exist $R \in \mathcal{D}$ and $S \subseteq N$ such that $\sigma_S^{R_S}$ is not group obviously dominant in Γ at R. Accordingly, there exist $\sigma'_S \in \Sigma_S$, $i \in S$ and $I_i(\sigma_S, \sigma'_S) \in \alpha_i(\sigma_S, \sigma'_S)$, such that

$$y \, P_i \, x \tag{2}$$

holds for some $x \in O(I_i(\sigma_S, \sigma'_S))$ and some $y \in O'(I_i(\sigma_S, \sigma'_S))$.

By Remark 1,

$$I_i(\sigma_S, \sigma'_S) \subseteq I_i(\sigma_i, \sigma'_i).$$

Then, by the definitions of the two sets of options left by σ_S and σ'_S, we have that

$$O(I_i(\sigma_S, \sigma'_S)) \subseteq O(I_i(\sigma_i, \sigma'_i))$$

and

$$O'(I_i(\sigma_S, \sigma'_S)) \subseteq O'(I_i(\sigma_i, \sigma'_i)).$$

Thus, by (2), there exist $i \in S$, $\sigma'_i \in \Sigma_i$ and $I_i(\sigma_i, \sigma'_i) \in \alpha_i(\sigma_i, \sigma'_i)$ such that

$$y \, P_i \, x$$

holds for some $x \in O(I_i(\sigma_i, \sigma'_i))$ and some $y \in O'(I_i(\sigma_i, \sigma'_i))$. This is a contradiction with the hypothesis that Γ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i, i \in N}$ OSP-implement f. $\blacksquare$

The following remark holds from the proof of Theorem 1.

Remark 2. *Let $f : \mathcal{D} \to X$ be a social choice function, and let $\Gamma \in \mathcal{G}$ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i \, i \in N}$ be the game and preference-strategy profile that* OSP*-implement f. Then, $\Gamma \in \mathcal{G}$ and $(\sigma_i^{R_i})_{R_i \in \mathcal{D}_i \, i \in N}$ also* GOSP*-implement f.*

4 Final remarks

We finish the paper with two final remarks.

First, Proposition 1 in Li (2017) establishes that obvious strategy-proofness implies group strategy-proofness. Since group obvious strategy-proofness is stronger than group strategy-proofness, Proposition 1 can be derived from our main result as follows. Let f be an obviously strategy-proof social choice function. By Theorem 1, f is group obviously strategy-proof. It then follows that f is group strategy-proof, and thus Proposition 1 in Li (2017) is recovered as a corollary of our result.

Second, given an extensive game form and a preference-strategy profile that OSP-implement a social choice function f, Mackenzie (2020) defines an algorithm that constructs a round-table mechanism which, together with the truth-telling preference-strategy profile, also OSP-implement f.[8] Moreover, by Theorem 6 in Mackenzie (2020) and a remark in Arribillaga, Massó and Neme (2020), obvious strategy-proofness is equivalent to strategy-proofness in round-table mechanisms.[9] Then, by our Theorem 1 and Remark 2, a social choice function f is GOSP if and only if there exists a round-table mechanism that OSP-implements (GOSP-implements) f with the truth-telling strategy profile. Consequently, in round table mechanisms, the notions of GOSP, OSP, and SP implementations are equivalent. Moreover, the restriction to such mechanisms is not significant for GOSP and OSP. Furthermore, by their definitions, GOSP implies GSP and GSP implies SP. Therefore, in round table mechanisms, the notions of GOSP, OSP, GSP and SP implementations are equivalent.

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[1] P. R. Arribillaga, J. Massó, and A. Neme. "On obvious strategy-proofness and single-peakedness," *Journal of Economic Theory* 186, 104992 (2020).

[8] According to Mackenzie (2020), an extensive game form is a *round-table mechanism* if the set of actions available to each agent i is the family of all non-empty subsets of preference relations, that is, $2^{\mathcal{D}_i} \setminus \{\emptyset\}$, and the following three conditions hold: (i) at any history, the set of available actions consists of disjoint subsets of preferences; (ii) when agent i plays for the first time, the set of actions is a partition of $2^{\mathcal{D}_i} \setminus \{\emptyset\}$; and (iii) at any later history h, the set of actions available to agent i is the intersection of the actions taken by i at all predecessor histories leading to h. A preference-strategy profile is called *truth-telling* if it always selects the subset of preferences that contains the agent's true preference.

[9] (Group) Strategy-proofness in a round-table mechanism means that truth-telling is a (group) dominant strategy in such mechanism.

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