BIASED TECHNICAL CHANGE AND THE MALMQUIST PRODUCTIVITY INDEX

R. Färe*, E. Grifell-Tatjé**, S. Grosskopf* and C. A. K. Lovell***

Abstract

The Malmquist productivity index has many attractive features. One is that it decomposes into a technical efficiency change index and a technical change index. Under constant returns to scale, its technical efficiency change index has been decomposed into a "pure" technical efficiency change index, a scale efficiency change index, and a congestion change index. Here we maintain the same assumption, and we decompose its technical change index into a magnitude index and a bias index. We then decompose the bias index into an output bias index and an input bias index, and we state conditions under which either bias index makes no contribution to productivity change.

JEL codes: G2, O3
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* Department of Economics, Southern Illinois University, Carbondale, IL 62901, USA
** Departament Economia de l'Empresa, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), SPAIN
*** Department of Economics, University of Georgia, Athens, GA 30602, USA [phone (706)542-3689, Fax (706)542-3376, email: “knox@rigel.econ.uga.edu”)

Biased Technical Change and the Malmquist Productivity Index*

1. Introduction

Over forty years ago Malmquist (1953) proposed a quantity index for use in consumption analysis. The index scales consumption bundles up or down, in a radial fashion, to some arbitrarily selected indifference surface. In this context Malmquist's scaling factor turns out to be Shephard's (1953) input distance function, and Malmquist quantity indexes for pairs of consumption bundles can be constructed from ratios of corresponding pairs of input distance functions.[1] Although it was developed in a consumer context, the Malmquist quantity index recently has enjoyed widespread use in a production context, in which multiple but cardinally measurable outputs replace scalar-valued but ordinally measurable utility. In producer analysis Malmquist indexes can be used to construct indexes of input, output or productivity, as ratios of input or output distance functions.

Malmquist indexes have a number of desirable features. They do not require input prices or output prices in their construction, which makes them particularly useful in situations in which prices are distorted or non-existent. They do not require a behavioral assumption such as cost minimization or profit maximization, which makes them useful in situations in which producers' objectives differ, or are unknown or are unachieved. They are easy to compute, as Färe, Grosskopf, Lindgren and Roos (FGLR) (1995) have demonstrated. Under certain conditions they can be related to the superlative Törnqvist (1936) and Fisher (1922) ideal quantity indexes, as Caves, Christensen and Diewert (CCD) (1982), Färe and Grosskopf (1992) and Balk (1993) have shown.

An attractive feature of the Malmquist productivity index is that it decomposes. This was first demonstrated by FGLR (1995) using the geometric mean formulation of the Malmquist index.[2] Following FGLR (1995), Førsund (1991) derived the decomposition of the simple version of the Malmquist productivity index into technical change and efficiency change. Färe, Grosskopf and Lovell (FGL) (1994) showed that the *technical efficiency change* index of the geometric mean of adjacent-period Malmquist productivity indexes, derived under the assumption of constant returns to scale, can be expressed as the product of an index of pure technical efficiency change, an index of scale efficiency change, and an index of congestion change. The value of each of these decompositions is that they provide insight into the sources of productivity change.

Our objective in this paper is to extend the decomposition literature by decomposing the *technical change* index of the geometric mean of adjacent-period Malmquist productivity indexes. Our motivation for decomposing the technical change index comes from recent experience in applying the FGLR (1995) decomposition to panel data on Spanish savings banks, reported in Grifell and Lovell (1995). There we found many instances of intersecting annual technologies, which suggested the presence of non-neutral technical change. Earlier, Førsund (1993) found similar results in a study of productivity change in Norwegian ferries. However, the FGLR (1995) decomposition contains no index reflecting the contribution to productivity change of the bias of technical change. In this paper we provide a pair of decompositions of the technical change index of the geometric mean of adjacent-period Malmquist productivity indexes into a pure magnitude index, an output bias index, and an input bias index. We also state conditions under which the two bias indexes make no contribution to productivity change.

We conduct our analysis on the geometric mean of adjacent-period Malmquist productivity indexes, because this is the only version of the Malmquist productivity index which has a natural relationship to the Törnqvist and Fisher ideal productivity indexes.[3] However a similar analysis can be conducted on either of the component adjacent-period Malmquist productivity indexes. In order to link our decomposition of the technical change index with the FGL (1994) decomposition of the technical efficiency change index, we derive our Malmquist productivity indexes under the assumption of constant returns to scale.

Our paper unfolds as follows. In Section 2 we describe a pair of adjacent-period Malmquist productivity indexes, and we decompose each index into its technical efficiency change index and its technical change index. We then obtain the geometric mean of the two adjacent-period Malmquist productivity indexes; the technical efficiency change index and the technical change index are each geometric means of the two adjacent-period indexes. In Section 3 we decompose the technical change index into a magnitude index, an input bias index, and an output bias index. This decomposition provides additional insight into the sources of productivity change, and provides the analytical foundation for empirical analyses of the contributions of both the magnitude and the separate biases of technical change to productivity change. We also state conditions under which each of the bias indexes make no contribution to productivity change. Section 4 concludes.

2. Malmquist Productivity Indexes

Let $x^t = (x_1^t,\dots,x_N^t) \in \Re_+^N$ and $y^t = (y_1^t,\dots,y_M^t) \in \Re_+^M$ denote an input vector and an output vector in period t, t = 1,...,T. The graph of production technology,

$$GR^t = \left\{(x^t, y^t) : x^t \text{ can produce } y^t\right\},\ t = 1,\dots,T, \tag{1}$$

is the set of all feasible input-output vectors. The output sets are defined in terms of GR^t as

$$P^t(x^t) = \left\{y^t : (y^t, x^t) \in GR^t\right\},\ t = 1,\dots,T. \tag{2}$$

The output sets are assumed to be closed, bounded, convex, and to satisfy strong disposability of outputs. A functional representation of production technology is provided by Shephard's (1970) output distance function[4]

$$D_o^t(x^t, y^t) = \inf\left\{\theta : (y^t/\theta) \in P^t(x^t)\right\},\ t = 1,\dots,T. \tag{3}$$

Among other properties, the output distance function satisfies the inequality $D_o^t(x^t, y^t) \leq 1,$ with $D_o^t(x^t, y^t) = 1$ if, and only if, $y^t \varepsilon$ Isoq $P^t(x^t) = \left\{y^t : y^t \varepsilon P^t(x^t),\ \theta y^t \notin P^t(x^t), \theta > 1\right\}$. Moreover, Shephard's output distance function is the reciprocal of Farrell's (1957) output-oriented measure of technical efficiency. We refer to $D_o^t(x^t, y^t)$ as a within-period output distance function; adjacent-period output distance functions $D_o^t\left(x^{t+1}, y^{t+1}\right)$ and $D_o^{t+1}(x^t, y^t)$ are defined analogously. Both within-period and adjacent-period output distance functions are used in the definition and the decomposition of the output-oriented Malmquist productivity index.

The following pair of definitions can be found in CCD (1982).

Definition 1: The period t output-oriented Malmquist productivity index is

$$M_o^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = D_o^t\left(x^{t+1}, y^{t+1}\right) / \ D_o^t(x^t, y^t). \tag{4}$$

$M_o^t(x^t,y^t,x^{t+1},y^{t+1})$ compares (x^{t+1},y^{t+1}) to (x^t,y^t) by scaling y^{t+1} to Isoq $P^t(x^{t+1})$, that is, by using period t technology as a reference. Although $D_o^t(x^t, y^t) \leq 1$, it is possible that $D_o^t(x^{t+1},y^{t+1}) > 1$, since period t+1 data may not be feasible with period t technology.

Thus $M_o^t(x^t,y^t,x^{t+1},y^{t+1}) \begin{matrix} > \\ = \\ < \end{matrix} 1$ according as productivity change is positive, zero or negative between periods t and t+1, from the perspective of period t technology.

Decomposition 1: The period t output-oriented Malmquist productivity index decomposes as

$$M_o^t\left(x^t,y^t,x^{t+1},y^{t+1}\right) = \Delta TE\left(x^t,y^t,x^{t+1},y^{t+1}\right) \bullet \Delta T^t\left(x^t,y^t,x^{t+1},y^{t+1}\right) \tag{5}$$

$$= \frac{D_o^{t+1}\left(x^{t+1},y^{t+1}\right)}{D_o^t(x^t,y^t)} \; \frac{D_o^t\left(x^{t+1},y^{t+1}\right)}{D_o^{t+1}\left(x^{t+1},y^{t+1}\right)},$$

where $\Delta TE(\cdot)$ refers to technical efficiency change and $\Delta T^t(\cdot)$ refers to technical change.

Definition 2: The period t+1 output-oriented Malmquist productivity index is

$$M_o^{t+1}\left(x^t,y^t,x^{t+1},y^{t+1}\right) = D_o^{t+1}\left(x^{t+1},y^{t+1}\right) / \; D_o^{t+1}(x^t,y^t). \tag{6}$$

$M_o^{t+1}(x^t,y^t,x^{t+1},y^{t+1})$ compares (x^{t+1},y^{t+1}) to (x^t,y^t) by scaling y^t to $P^{t+1}(x^t)$, that is, by using period t+1 technology as a reference. Although $D_o^{t+1}(x^{t+1}, y^{t+1}) \leq 1$, it is possible that $D_o^{t+1}(x^t, y^t) > 1$, since data from period t may not be feasible with period t+1 technology.

Decomposition 2: The period t+1 output-oriented Malmquist productivity index decomposes as

$$M_o^{t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \Delta TE\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \bullet \Delta T^{t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \tag{7}$$
$$= \frac{D_o^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^t(x^t, y^t)} \frac{D_o^t\left(x^t, y^t\right)}{D_o^{t+1}\left(x^t, y^t\right)}$$

Since $M_o^t(x^t,y^t,x^{t+1},y^{t+1})$ and $M_o^{t+1}(x^t,y^t,x^{t+1},y^{t+1})$ use different reference technologies, they can generate qualitatively as well as quantitatively different evidence concerning productivity change. The two indexes yield the same values if, and only if, the output distance function is of the form

$$D_o^\tau(x^\tau, y^\tau) = A(\tau) \hat{D}_o(x^\tau, y^\tau), \ \tau = t, t+1. \tag{8}$$

(The proof is given in the Appendix). To avoid the ambiguity of choosing one of the above two indexes, FGLR (1995) suggested the geometric mean of $M_o^t(x^t,y^t,x^{t+1},y^{t+1})$ and $M_o^{t+1}(x^t,y^t,x^{t+1},y^{t+1})$ as a third Malmquist-type productivity index.

Definition 3: The geometric mean of adjacent-period output-oriented Malmquist productivity

indexes is

$$M_o^G\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \left[M_o^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \bullet M_o^{t+1}\left(x^y, y^t, x^{t+1}, y^{t+1}\right)\right]^{1/2}$$
$$= \left[\frac{D_o^t\left(x^{t+1}, y^{t+1}\right)}{D_o^t(x^t, y^t)} \frac{D_o^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^{t+1}(x^t, y^t)}\right]^{1/2} \tag{9}$$

$M_o^G(x^t,y^t,x^{t+1},y^{t+1}) \underset{<}{\overset{>}{=}} 1$ according as productivity change is positive, zero or negative between periods t and t+1.

Decomposition 3: The geometric mean of two adjacent-period output-oriented Malmquist productivity indexes decomposes as

$$M_o^G\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \Delta TE\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \bullet \Delta T^G\left(x^t, y^t, x^{t+1}, y^{t+1}\right)$$
$$= \frac{D_o^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^t(x^t, y^t)} \left[\frac{D_o^t\left(x^{t+1}, y^{t+1}\right)}{D_o^{t+1}\left(x^{t+1}, y^{t+1}\right)} \frac{D_o^t(x^t, y^t)}{D_o^{t+1}(x^t, y^t)}\right]^{1/2} \tag{10}$$

Note that all three technical efficiency change indexes are the same, but that the three technical change indexes differ. The latter coincide if, and only if, the distance functions are of the form (8).

In order to provide additional insight into the sources of productivity change, FGL (1994) provided a decomposition of $\Delta TE(x^t,y^t,x^{t+1},y^{t+1})$, derived under the assumption of constant returns to scale, into indexes of "pure" technical efficiency change, scale efficiency change, and congestion change. In the next Section we provide a decomposition of $\Delta T^G(x^t,y^t,x^{t+1},y^{t+1})$, under the same assumption.

3. Decompositions of the Technical Change Component of the Geometric Mean Version of the Malmquist Productivity Index

Partly because only the geometric mean formulation of the Malmquist productivity index can be related to the Törnqvist and Fisher ideal productivity indexes, and partly to conserve space in our exposition, we undertake our decomposition only for the geometric mean formulation of the Malmquist productivity index. Analogous decompositions of the two component adjacent-period indexes follow trivially.

Decomposition 4: The technical change index $\Delta T^G(x^t,y^t,x^{t+1},y^{t+1})$ decomposes as

$$\begin{aligned}\Delta T^G\left(x^t,y^t,x^{t+1},y^{t+1}\right) &= \Delta T(x^t,y^t) \bullet B\left(x^t,y^t,x^{t+1},y^{t+1}\right) \\ &= \frac{D_o^t(x^t,y^t)}{D_o^{t+1}(x^t,y^t)}\left[\frac{D_o^{t+1}(x^t,y^t)}{D_o^t(x^t,y^t)}\;\frac{D_o^t(x^{t+1},y^{t+1})}{D_o^{t+1}(x^{t+1},y^{t+1})}\right]^{1/2}\end{aligned} \tag{11a}$$

or, alternatively, as

$$\begin{aligned}\Delta T^G(x^t,y^t,x^{t+1},y^{t+1}) &= \Delta T(x^{t+1},y^{t+1}) \bullet \left[B(x^t,y^t,x^{t+1},y^{t+1})\right]^{-1} \\ &= \left[\frac{D_o^t(x^{t+1},y^{t+1})}{D_o^{t+1}(x^{t+1},y^{t+1})}\right]\left[\frac{D_o^t(x^t,y^t)\;\;D_o^{t+1}(x^{t+1},y^{t+1})}{D_o^{t+1}(x^t,y^t)\;\;D_o^t(x^{t+1},y^{t+1})}\right]^{1/2}\end{aligned} \tag{11b}$$

Consider decomposition (11a). $\Delta T(x^t,y^t)$ measures the magnitude of technical change along a ray through period t data. $B(x^t,y^t,x^{t+1},y^{t+1})$ measures the bias of technical change between periods t and t+1. It is the geometric mean of the ratio of the magnitude of technical change along a ray through period t+1 data to the magnitude of technical change along a ray through period t data. The bias index makes no contribution to productivity change if the magnitude of technical change is the same when measured along the two rays, i.e., if, and only if, the distance functions are of the form (8). The contribution is positive or negative when the magnitude of technical change measured along a ray through period t+1 data exceeds or falls short of the magnitude of technical change measured along a ray through period t data. It is in this sense that we refer to the second index as a bias index. Decomposition (11b) is interpreted in exactly the same way, except that $\Delta T(x^{t+1},y^{t+1})$ measures the magnitude of technical change along a ray through period t+1 data, and the bias index is the reciprocal of the bias index of decomposition (11a). The two measures of the magnitude of technical change $\Delta T(x^{t+1},y^{t+1})$ and $\Delta T(x^t,y^t)$ are equal if, and only if, the distance functions are of the form (8). (The proof is the same as that leading to expression (8)). The same is true for the bias term to equal its reciprocal.

It is possible to gain additional insight into the nature of the bias index, by decomposing it into an input bias index and an output bias index. There are two possible decompositions.

Decomposition 5: The bias index $B(x^t,y^t,x^{t+1},y^{t+1})$ decomposes as

$$\begin{aligned} B(x^t,y^t,x^{t+1},y^{t+1}) &= OB\Delta T^{t+1}(y^t,x^{t+1},y^{t+1}) \bullet IB\Delta T^t(x^t,y^t,x^{t+1}) \\ &= \left[\frac{D_o^{t+1}(x^{t+1},y^t)\ D_o^t(x^{t+1},y^{t+1})}{D_o^t(x^{t+1},y^t)\ D_o^{t+1}(x^{t+1},y^{t+1})}\right]^{1/2} \left[\frac{D_o^{t+1}(x^t,y^t)\ D_o^t(x^{t+1},y^t)}{D_o^t(x^t,y^t)\ D_o^{t+1}(x^{t+1},y^t)}\right]^{1/2} \end{aligned} \tag{12a}$$

or, alternatively, as

$$B(x^t, y^t, x^{t+1}, y^{t+1}) = OB\Delta T^t(x^t, y^t, y^{t+1}) \bullet IB\Delta T^{t+1}(x^t, x^{t+1}, y^{t+1}) \tag{12b}$$

$$= \left[\frac{D_o^{t+1}(x^t, y^t)\ \ D_o^t(x^t, y^{t+1})}{D_o^t(x^t, y^t)\ \ D_o^{t+1}(x^t, y^{t+1})}\right]^{1/2} \left[\frac{D_o^{t+1}(x^t, y^{t+1})\ \ D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^{t+1})\ \ D_o^{t+1}(x^{t+1}, y^{t+1})}\right]^{1/2}$$

The decomposed bias index in (12a) is itself the product of two indexes. $OB\Delta T^{t+1}(y^t, x^{t+1}, y^{t+1})$ involves the input vector from period t+1 and the output vector from both periods. Holding the input vector constant at x^{t+1}, it compares the magnitude of technical change along a ray through y^{t+1} with the magnitude of technical change along a ray through y^t. Consequently we refer to $OB\Delta T^{t+1}(y^t, x^{t+1}, y^{t+1})$ as a period t+1 output bias index. This index is the geometric mean of the two output quantity indexes $D_o^{t+1}(x^{t+1}, y^t)$ / $D_o^{t+1}(x^{t+1}, y^{t+1})$ and $D_o^t(x^{t+1}, y^{t+1})$ / $D_o^t(x^{t+1}, y^t)$.[5] $IB\Delta T^t(x^t, y^t, x^{t+1})$ involves the output vector from period t and the input vector from both periods. Holding the output vector constant at y^t, it compares the magnitude of technical change along a ray through x^{t+1} with the magnitude of technical change along a ray through x^t. Consequently we refer to $IB\Delta T^t(x^t, y^t, x^{t+1})$ as a period t input bias index.[6]

Although we have maintained the assumption of constant returns to scale in defining and decomposing all of our indexes, this assumption is not necessary for any of the decompositions. However the assumption does help provide intuition for the input bias indexes in equations (12a) and (12b), where input biases are expressed unnaturally in terms of output distance functions. The assumption of constant returns to scale enables us to express the input bias indexes more naturally in terms of input distance functions.[7]

Input distance functions are related to output distance functions by (Färe and Primont (1995))

$$D_i^t(y^t, x^t) = \sup\left\{\theta : D_o(x^t/\theta, y^t) \le 1\right\}. \tag{13}$$

$D_i(y^t, x^t) \ge 1$, and $D_i(y^t, x^t) = [D_o(x^t, y^t)]^{-1}$ if, and only if, constant returns to scale obtain. Thus under constant returns to scale the input bias indexes in equations (12a) and (12b) can be expressed more naturally in terms of input distance functions as

$$IB\Delta T^t(x^t, y^t, x^{t+1}) = \left[\frac{D_i^t(y^t, x^t)\ D_i^{t+1}(y^t, x^{t+1})}{D_i^{t+1}(y^t, x^t)\ D_i^t(y^t, x^{t+1})}\right]^{1/2} \tag{14a}$$

and

$$IB\Delta T^{t+1}(x^t, x^{t+1}, y^{t+1}) = \left[\frac{D_i^t(y^{t+1}, x^t)\ D_i^{t+1}(y^{t+1}, x^{t+1})}{D_i^{t+1}(y^{t+1}, x^t)\ D_i^t(y^{t+1}, x^{t+1})}\right]^{1/2} \tag{14b}$$

respectively. The period t input bias index in equation (14a) holds the output vector constant at y^t and compares the magnitude of technical change along a ray through x^{t+1} with the magnitude of technical change along a ray through x^t. The period t+1 input bias index in equation (14b) is interpreted similarly, but it holds the output vector constant at y^{t+1}. When a single output is produced, expressions (14a) and (14b) are equal. This is a consequence of the fact that under constant returns to scale the input distance function has the property that

$$D_i(\theta y, x) = (1/\theta)\ D_i(y, x),\ \theta > 0. \tag{15}$$

In the multiple output case Färe and Primont (1995) have shown that the two expressions are also equal, provided that the technology is inversely homothetic, i.e.,

$$D_i(y, x) = D_i(1, x)\ /\ J^{-1}(D_o(1, y)), \tag{16}$$

which under constant returns to scale becomes

$$D_i(y, x) = D_i(1, x)\ /\ D_o(1, y). \tag{17}$$

The next two propositions establish conditions under which either the output bias index or the input bias index make no contribution to productivity change. Proofs appear in the Appendix.

Proposition 1a: The output bias indexes (12a) and (12b) equal unity, and make no contribution to productivity change, if either (i) $(y^t / |y^t|) = (y^{t+1} / |y^{t+1}|)$ or (ii) the technology exhibits implicit Hicks output-neutral technical change.

The technology is said to exhibit implicit Hicks output-neutral technical change if the output distance function can be written as (see Chambers and Färe (1994))

$$D_o^t(x^t, y^t) = B(x^t, t)\ \hat{D}_o(x^t, y^t). \tag{18}$$

A partial converse of Proposition 1a can now be stated.

Proposition 1b: If $OB\Delta T^{t+1}(y^t, x^{t+1}, y^{t+1}) = 1$ or if $OB\Delta T^t(x^t, y^t, y^{t+1}) = 1$, then the technology is implicit Hicks output-neutral.

The input bias indexes equal one under conditions similar to those for the output bias indexes. In particular, we say that the technology exhibits implicit Hicks input-neutral technical change if the input distance function can be written as

$$D_i^t(y^t, x^t) = A(y^t, t)\ \hat{D}_i(y^t, x^t). \tag{19}$$

Proposition 2a: The input bias indexes (14a) and (14b) equal unity, and make no contribution to productivity change, if either (i) $(x^t / |x^t|) = (x^{t+1} / |x^{t+1}|)$ or (ii) the technology exhibits implicit Hicks input-neutral technical change.

Proposition 2b: If $IB\Delta T^{t}(x^{t}, y^{t}, x^{t+1}) = 1$ or if $IB\Delta T^{t+1}(x^{t}, x^{t+1}, y^{t+1}) = 1$, then the technology is implicit Hicks input-neutral.

4. Summary and Conclusions

Measuring productivity change is an important exercise. Decomposing measured productivity change into its sources is an equally important exercise, since the enhancement of productivity growth requires a knowledge of the relative importance of its sources. In this regard the Malmquist productivity index is particularly enlightening, since it decomposes naturally into a technical efficiency change index and a technical change index. Building on this elementary decomposition, FGL (1994) obtained a further decomposition of the technical efficiency change index into a "pure" technical efficiency change index, a scale efficiency change index, and a congestion change index. In this paper we have provided a decomposition of the technical change index into a magnitude index, an output bias index, and an input bias index. We have also provided sets of conditions under which each bias index makes no contribution to productivity change. The fact that these conditions are stringent makes it likely that the bias indexes do contribute to productivity change, thereby making it important to have such a decomposition.

Appendix

Proof of Proposition 1a: First let $a = (y^t / |y^t|) = (y^{t+1} / |y^{t+1}|)$, then

$$OB\Delta T^{t+1}(y^t, x^{t+1}, y^{t+1}) = \left(\frac{D_o^{t+1}(x^{t+1}, a)}{D_o^t(x^{t+1}, a)} \; \frac{D_o^t(x^{t+1}, a)}{D_o^{t+1}(x^{t+1}, a)} \right)^{1/2} = 1.$$

The same holds for $OB\Delta T^t(x^t, y^t, y^{t+1})$. Second, let the technology be implicit Hicks output-neutral, then

$$OB\Delta T^{t+1}(y^t, x^{t+1}, y^{t+1}) = \left(\frac{B(x^{t+1}, t+1) \; \hat{D}_o(x^{t+1}, y^t) \; B(x^{t+1}, t) \; \hat{D}_o(x^{t+1}, y^{t+1})}{B(x^{t+1}, t) \; \hat{D}_o(x^{t+1}, y^t) \; B(x^{t+1}, t+1) \; \hat{D}_o(x^{t+1}, y^{t+1})} \right)^{1/2} = 1.$$

The same holds for $OB\Delta T^t(x^t, y^t, y^{t+1})$.

Q.E.D.

Proof of Proposition 1b: Assume that $OB\Delta T^{t+1}(y^t, x^{t+1}, y^{t+1}) = 1$, then we have

$$D_o^{t+1}(x^{t+1}, y^{t+1}) = \frac{D_o^{t+1}(x^{t+1}, y^t) \; D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^{t+1}, y^t)}.$$

Next, fix $y^t = \bar{y}^t$ and $t = \bar{t}$ and define

$$D_o^{t+1}(x^{t+1}, \bar{y}^t) = B(x^{t+1}, t+1), \text{ and } \hat{D}_o(x^{t+1}, y^{t+1}) = \frac{D_o^{\bar{t}}(x^{t+1}, y^{t+1})}{D_o^{\bar{t}}(x^{t+1}, \bar{y}^t)}, \text{ then}$$

$D_o^{t+1}(x^{t+1}, y^{t+1}) = B(x^{t+1}, t+1) \; \hat{D}_o(x^{t+1}, y^{t+1})$. The same holds for $OB\Delta T^t(x^t, y^t, y^{t+1})$.

Q.E.D.

Proof of (8): Suppose that the distance is of the form (8), then (4) becomes

$$\frac{A(t) \; \hat{D}_o(x^{t+1}, y^{t+1})}{A(t) \; \hat{D}_o(x^t, y^t)}, \text{ and (6) becomes } \frac{A(t) \; \hat{D}_o(x^{t+1}, y^{t+1})}{A(t) \; \hat{D}_o(x^t, y^t)},$$

and the two definitions coincide.

Conversely, assume that (4) equals (6), then

$$D_o^{t+1}(x^{t+1}, y^{t+1}) \;=\; D_o^{t+1}(x^t, y^t)\,\frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}.$$

Now fix $t = \bar{t},\ x^t = \bar{x}^t$ and $y^t = \bar{y}^t$, then

$$D_o^{t+1}(x^{t+1}, y^{t+1}) \;=\; A(t+1)\ \bar{D}_o(x^{t+1}, y^{t+1}).$$

Similarly one can show that

$$D_o^t\ (x^t, y^t) \;=\; A(t)\ \bar{D}_o(x^t, y^t).$$

Q.E.D.

ENDNOTES

*. This paper is a merged revision of "Malmquist Productivity Indexes and Biased Technical Change," by Färe and Grosskopf, and "A New Decomposition of the Malmquist Productivity Index," by Grifell and Lovell. We are indebted to the Editor and the referees for their guidance in the preparation of this revision. Grifell acknowledges financial support from CIRIT, Generalitat de Catalunya.

1. See Deaton (1979) and Chambers, Färe and Grosskopf (1994) for analyses of the use of distance functions and Malmquist quantity indexes.

2. Nishimizu and Page (1982) used a calculus approach to decompose productivity growth into technical change and efficiency change components. Although they mention the Malmquist productivity index, they estimate a frontier production function. FGLR (1995) showed how to compute the Malmquist index using noncalculus, nonparametric methods, and how to decompose it into technical change and efficiency change in a 1989 working paper which appeared as FGLR (1995).

3. CCD (1982) showed that the geometric mean formulation can be related to the Törnquist productivity index. Färe and Grosskopf (1992) showed that it can also be related to the Fisher ideal productivity index. These relationships do not hold for the adjacent-period Malmquist productivity indexes given in Definitions 1 and 2 below.

4. Throughout most of the paper we use output distance functions to construct output-oriented Malmquist productivity indexes. It is also possible to use Shephard's (1953) input distance functions to construct input-oriented Malmquist productivity indexes. CCD (1982) and Førsund (1991) discuss the relationship between the two orientations.

5. This was pointed out to us by a referee.

6. Some of the distance functions appearing in decompositions (12a) and (12b) involve inputs from one year and outputs from the adjacent year. CCD (1982) and Bjurek (1994) also employ $D_o^t(x^t, y^{t+1})$ and $D_o^{t+1}(x^{t+1}, y^t)$, to construct Malmquist output quantity indexes. This was pointed out to us by a referee.

7. The assumption of constant returns to scale is significant in another sense. $M^G(x^t,y^t,x^{t+1},y^{t+1})$ can be interpreted as a productivity measure generalizing the single output-single input ratio $[(y^{t+1}/x^{t+1})/(y^t/x^t)]$ if, and only if, constant returns to scale are imposed in the construction of the index. Sufficiency was noted by Berg, Førsund and Jansen (1992), and a proof of necessity and sufficiency appears in Färe and Grosskopf (1995). Grifell and Lovell (1994) noted that it is possible to generalize a

variable returns to scale Malmquist productivity index by incorporating an index which accounts for the effect of non-constant returns to scale on productivity change.

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