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Abstract

It is a general belief in the market that the behaviour of stock prices in a bull market is different from that in a bear market; the magnitude of returns could be different when the return is positive or when it is negative. This asymmetric pattern shows a form of non-linearity. The aim of this paper is to model the possible asymmetry of the daily returns series of IBEX-35 composite index from 1988 to 1996 with the self-exciting threshold autoregressive (SETAR) model.

Keywords: Non-linear time series, non-linearity test, SETAR model, skewness.

Introduction.

The framework of this research is the elaboration of the PhD. Dissertation: "*Volatility of return time series with SETAR models: The improvement of the algorithm of identificacion*". This thesis is focused on the study of the volatility of the IBEX-35 returns with a non-linear model, the *self-exciting threshold autoregressive SETAR models* and the improvement of the identification process.

There can be no doubt that the modelization of the volatility requires a previous study of time series returns. In this paper, following the work of Li and Lam (1995) on the asymmetric behaviour of the Hong Kong Hang Seng Index from 1970 to 1991, we examine the asymmetry of the daily returns series of the IBEX-35 composite index from 1988 to 1996.

It is a general belief in the market that, the behaviour of stockmarket prices could be different according to whether the magnitude of the return is positive or negative. We can observe a non-linear behaviour in the return time series of the IBEX-35 and, also, that the return distribution is skewed. If the skewness in the return distribution is an evidence of the asymmetry in the return generation mechanism, then we can consider that the asymmetry can be one possible non-linearity.

The primary purpose of this paper is to investigate the existence and source the non-linearity in the return time series. The secondary purpose is to explore the use of threshold autoregressive TAR models (Tong 1978, 1983, and Tong and Lim 1980) in describing the asymmetry of the daily return series of the IBEX-35 composite index from 1988-1996. The TAR model has certain features -such as asymmetric limit cycles behaviour, amplitude dependent on frequencies and jump phenomena- that cannot be captured by linear time series models. For instance, Tong and Lim (1980) showed that the TAR model is capable of producing the asymmetric, periodic behaviour exhibited in the annual Wolf sunspot data and the Canadian Lynx data. The TAR model has also been applied in financial and economic time series. For example: In "Study of IBM daily common stock closing prices from 1959 to 1962" (Tyssedal and Tjostheim, 1988), in the analysis of Hang Seng Index[1] from 1984 to 1987 (Tong, 1990), also Pope and Yadav (1990) employed a TAR model to characterise the mispricing behaviour of FTSE 100 index futures, Cao and Tsay (1993) described monthly volatility series of the S&P[2], Tiao and Tsay (1994) used a TAR model, in studying the cyclical properties of real U.S. GNP quartily series, and Gao and Wang (1999) represented non-linear dynamics of S&P 500 with a TAR model.

The paper is structured as follows: In Section 1, the behaviour of time series return is introduced. In Section 2, the *self-excited threshold autoregressive SETAR (2; p_1, p_2) model* is introduced as a tool to model asymmetry. Empirical results for data are reported in Section 3. Finally, in Section 4, the conclusions are presented.

[1] Hang Seng index is compiled from 33 major stocks of Hong Kong.
[2] Standard & Poor' acts as an indicator of Nueva York stocks exchange.

Section 1: Statistical tests in time series return.

Return series are usually defined as the log-difference of the index:

$$Y_t = \ln \frac{x_t}{x_{t-1}}$$

where $x_1, x_2, \cdots, x_n$, are the daily observations of the index.

When we consider the modelisation of the return series, we can choose between linear and non-linear models. The skewness of the return distribution is a sign of one possible non-linearity form. We think that in the real world, the true model that generates observed time series is unknown. In order to know the main features of the return, firstly, we study the characteristic parameters of distribution: mean, standard deviation, skewness, kurtosis. Also, we test the normality of the series with the Lin-Mudholkar test (1980).

We can calculate the Lin -Mudholkar statistic, if we assume the normality of the return as null hypothesis, as follows:

$$Lin - Mudholkar = \frac{1}{2}\sqrt{\frac{T}{3}}\log\left(\frac{1+R}{1-R}\right)$$

follows a Normal (0, 1), where

$$R = \frac{\sum_{i=1}^{T}\left(\hat{\varepsilon}_i - \bar{\varepsilon}\right)\left(Z_i - \bar{Z}\right)}{\left[\sum_{i=1}^{T}\left(\hat{\varepsilon}_i - \bar{\varepsilon}\right)^2 \sum_{i=1}^{T}\left(Z_i - \bar{Z}\right)^2\right]^{1/2}}$$

and

$$Z_i = \left[\frac{1}{T}\left(\sum_{j \neq i}^{T}\hat{\varepsilon}_j^2 - \frac{1}{T-1}\left(\sum_{j \neq i}^{T}\hat{\varepsilon}_j\right)^2\right)\right]^{1/3} \quad , \quad i = 1, 2, \cdots, T$$

Secondly, we test the linearity with two tests, the Likelihood ratio test of Chan (1989), and the TAR-F test of Tsay (1989). The last one is a specific test for a threshold non-linearity based on the arranged autoregression on the study of the predictive residuals.

The *Likelihood Ratio Test* (Chang and Tong) for a SETAR model uses the following statistic:

$$\lambda_r = \left[\frac{\hat{\sigma}^2(NL, r_1)}{\hat{\sigma}^2(L)} \right]^{-(n-p+1)/2}$$

where N is the sample size, $\hat{\sigma}^2(NL, r)$ is the usual average residual sum of squares under the hypothesis:

$$H_1: \quad \Phi_i^{(1)} \neq \Phi_i^{(2)} \quad \text{for } 0 \le i \le p$$

$\hat{\sigma}^2(L)$ is the usual average residual sum of squares under the hypothesis:

H_0 : this statistic, $-2\log\lambda_r$, is asymptotically like a χ^2_{p+1} distribution.

$\hat{r}_1$ is the least square estimator of r, assuming that the residuals are normally distributed.

The TAR–F statistic developed by Tsay (1989) is intended to detect the threshold autoregressive (TAR) type of nonlinearity in the time series. That is, the TAR-F statistic can be used to detect whether the time series follows a different linear process when a threshold variable falls into different regions. The TAR test of Tsay is based on the next theorem.

Theorem:

Suppose that X_t is a linear stationary AR process of order p. Then, for large n the statistics $\hat{F}(p,d)$ defined as:

$$\hat{F}(p,d) = \frac{\left(\sum \hat{e}_t^2 - \sum \hat{\varepsilon}_t^2\right) \Big/ (p+1)}{\sum \hat{\varepsilon}_t^2 \Big/ (n-d-b-p-h)} \qquad \text{with}$$

$$b = \frac{n}{10} + p, \qquad h = max\{1,\ p+1-d\}$$

follow approximately an F distribution with $p+1$ and $(n-d-b-p-h)$ degrees of freedom. Furthermore, $(p+1)\hat{F}(p,d)$ is asymptotically a chi-squared random variable with $p+1$ degrees of freedom.

Tsay uses the consistency property of least squares estimates of a linear AR and a martingalle central limit theorem of Billingsley (1961) to prove this theorem.

Tsay (1989) and Gao (1994) have studied the finite sample properties of TAR-F statistic. Gao (1994) found that the TAR-F test is effective in detecting non-linear time series such as non-linear AR, non-linear MA, and TAR process.

The TAR-F test enables one to estimate the delay parameter in TAR models. To determine the delay parameter *d*, the TAR-F test should be run for different threshold lags with an AR order *p* that is not too small. The delay parameter is determined by the threshold lag, which corresponds to the value of the highest statistic.

Section 2: Threshold Autoregressive model.

The asymmetric pattern of the stock return can be captured by using the self-exciting threshold autoregressive SETAR model proposed by Tong (1978) and Tong and Lim (1980). The SETAR model and TAR model in general can be interpreted as one member of the switching linear regression models. The switching mechanism is controlled by threshold variable Y_{t-d}, not by the time index t.

A time series Y_t is a SETAR process if it satisfies the model below:

$$Y_t = \phi_0^{(i)} + \phi_1^{(i)} Y_{t-1} + \cdots + \phi_p^{(i)} \phi_1^{(i)} Y_{t-p} + \varepsilon_t^{(i)} \quad if \quad Y_{t-d} \in L_i, \quad i = 1,2,\cdots,k$$

where the L_i forms a non-overlapping partition of the real line i.e.

$\bigcup_{i=1}^{k} L_i = R$ and $L_i \cap L_j = 0$ if $i \neq j$, k is the number of threshold regimes, d is the delay parameter (or threshold lag), p is the AR order; and $\{\varepsilon_t^{(j)}\}$ is a sequence of i.i.d. normal random variables with zero mean and variance σ_i^2 such that $\{\varepsilon_t^{(i)}\}$ and $\{\varepsilon_t^{(j)}\}$are independent if $i \neq j$. A self-exciting threshold autoregressive model is a piecewise linear model in the space of Y_{t-d} (not in time) and is capable of providing accurate "local approximations" in this space. The delay variable Y_{t-d} is the switch variable of the system. Under such a model, the return generating mechanism depends on the behaviour of the price in previous period.

The key features of SETAR models include time irreversibility; asymmetric limit cycles and jumps phenomenon. One major advantage of this model is that the parameter can be readily estimated by the least squares method.

The TAR model uses threshold values to partition a non-linear time series into piecewise linear autorregresive models within each threshold region. This mechanism makes TAR a useful model to represent the empirical dynamic of financial return past series evolving with past return time series. This dynamic return pattern often refers to large returns following large returns and small returns following small returns. Furthermore, the TAR model enables one to estimate varying autoregressive coefficients within different threshold regions.

The determination of the "structural parameters", namely the "delay parameter" d, the threshold region L_i, and the individual model orders, $p_1, \cdots, p_k$, is a difficult problem in the estimation process.

The algorithm proposed by Tong is based on the use of Akaike's AIC criterion (1978).

For simplicity we consider a SETAR $\left(2; k_1, k_2\right)$, its expression is:

$$Y_t = \Phi_0^{(1)} + \sum_{i=1}^{k_1} \Phi_i^{(1)} Y_{t-i} + \varepsilon_t^{(1)} \qquad \text{if} \quad Y_{t-d} < r$$

$$Y_t = \Phi_0^{2)} + \sum_{i=1}^{k_2} \Phi_i^{(2)} Y_{t-i} + \varepsilon_t^{(2)} \qquad \text{if} \quad Y_{t-d} \geq r$$

The algorithm for a SETAR $\left(2; k_1, k_2\right)$ proceeds as follows (Priestley, 1991):

> "**Stage 1**. For given values of *d* and *r*, fit separate AR models to the appropriate subset of the data. Let $AIC\left(k_1\right)$, $AIC\left(k_2\right)$, denote the usual AIC criteria for individual models, and let $\hat{k}_1, \hat{k}_2$ denote these values which minimise $AIC\left(k_1\right), AIC\left(k_2\right)$ respectively. Write:

$$AIC(d,r) = AIC(\hat{k}_1) + AIC(\hat{k}_2)$$

Stage 2. Consider a set of possible value for r, say, $r^{(1)}$, $r^{(2)}$, $\cdots$ $r^{(q)}$. Repeat the stage 1 for $r = r^{(i)}$, $i = 1, 2, \cdots, q$, with d remaining fixed. Choose that value, $\hat{r}$, say, for which $AIC\,(d, r)$ attains its minimum value, and write

$$AIC\,(d) = AIC\,(d, \hat{r})$$

Stage 3. Now search for the "best" value of d over a range of possible values, $d_1, d_2, \ldots d_p$, say, by repeating both stages 1 and 2 for $d = d_i$ $i = 1, 2, \cdots, p$. Select the value of d for which $NAIC\,(d)$ attains its minimum value."

The total implementation of the Tong algorithm in computer languages is not possible, this is the main problem of the process.

Another way to estimate the structural parameters is the SETAR modelling procedure proposed by Tsay (1989). In summary this procedure consists of the following steps:

1. Select a tentative AR order k and a set of possible threshold variables.
2. For each threshold variable considered, perform non-linearity tests, especially a threshold non-linear test.
3. Select the threshold variable Y_{t-d} based on the results of step 2. The performance of the TAR-F statistic allows the selection of d before locating the threshold values. It assumes that the AR order p is given. For a given TAR process and an AR order p, one selects an estimate of the delay parameter, say d_p such that

$$\hat{F}(p, d_p) = \max_{v \in S} \{\hat{F}(p, v)$$

where S is a prespecified positive integer, that is a collection of possible values of d.

4. Perform an arranged autoregression to locate the possible threshold values.
5. Estimate the specified SETAR model by conditional least squares.
6. Check the estimated SETAR model and refine if it is necessary.

The implementation of some steps of Tsay procedure is very difficult, specifically the location of threshold values.

In this work we use firstly, the stages 1, 2, and 3 of Tsay procedure and then we use Tong's algorithm. In this way the algorithm works faster because the delay parameter is fixed. Another specificity of estimation process is the choice of only two possible regimes, this is motivated by the belief in the market that the behaviour of stocks prices could be different according to whether the magnitude of the return is positive or negative. Once the structural parameters were estimated, we estimated the coefficients of the AR process by conditional least squares (as Stage 5 of Tsay procedure).

In order to check the entertained SETAR model we consider the standardised residuals series. We study the autocorrelation and the partial autocorrelation functions and we also test the whiteness of these standardised residuals. We use the Ljung-Box test to check that the standardised residuals are a white noise.

The Ljung-Box statistic is

$$Q^* = T(T+2)\sum_{j=1}^{M}\frac{\hat{\rho}_j^2(\hat{\varepsilon}_t)}{T-j}$$

where M is the greatest integer less than or equal to $min\left(T/2, 3\sqrt{T}\right)$.

Q^* is asymptotically a chi-squared random variable with *(M-k)* degrees of fredom, k is the parameter's number in the model.

Section 3: Empirical results.

We have 2349 observations of the daily closed price of IBEX-35 from 1988 to 1996 (Figure 1). The return series is defined as the log-difference of the index. During the period taken into account, the market has undergone many structural changes. So, it is appropriate to divide the data into non-overlapping periods, a natural way is to consider

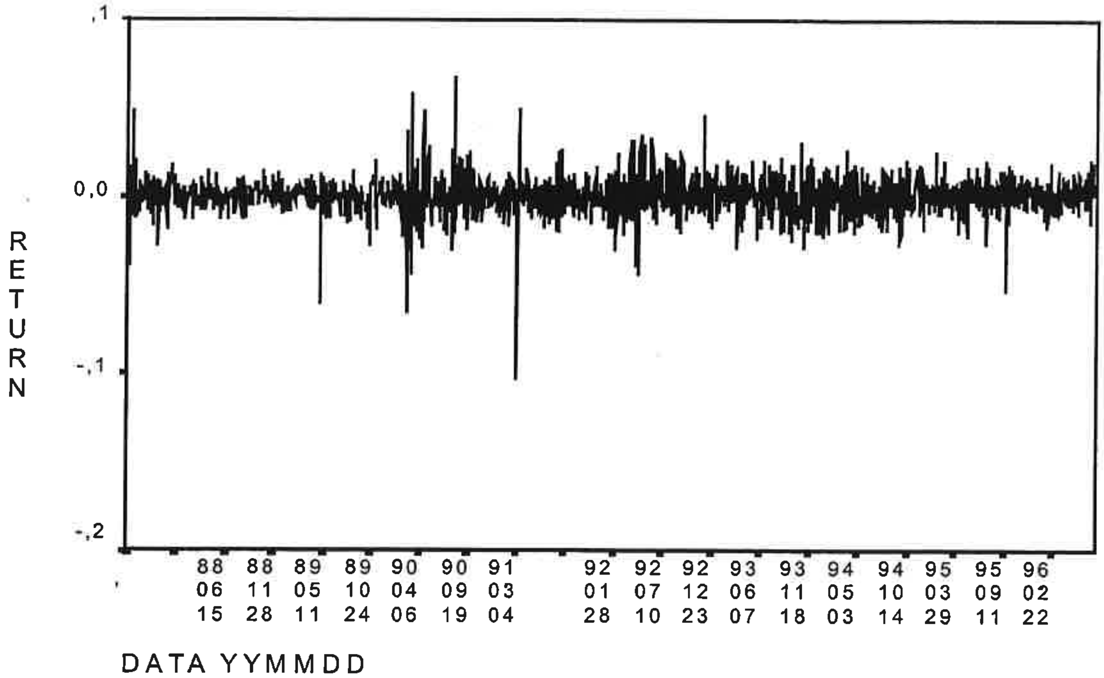

Figure 1

each single year, there are about 260 observations for each year. We consider 11 different daily time series, nine each for a single year and two time series for a long period, one of daily data, and one of weekly data.

The next table shows (Table 1), the mean, standard deviation, skewness and kurtosis for each time series. We can observe that most of the time series are skewness (except return time series for 1992 and 1994).

Table 1. Summary statistics. (* the test rejects the normality $\alpha = 0.05$).

Period	N° obs.	Mean	St.deviation	Skewness	Kurtosis	L-M test
1988	261	0.0005	0.0083	0.856	9.129	-2.49 *
1989	261	0.0004	0.0063	-3.482	33.534	6.13 *
1990	261	-0.0010	0.0130	-0.2270	5.914	0.76
1991	261	0.0010	0.0120	-1.435	25.175	2.84 *
1992	261	0.0000	0.0110	-0.025	1.965	0.13
1993	261	0.0020	0.0090	0.333	2.104	-1.58
1994	260	-0.0010	0.0110	0.002	-0.500	-0.02
1995	261	0.0006	0.0001	-0.0229	0.898	-1.27
1996	261	0.0013	0.0080	-1.2070	-0.500	3.83 *
88-96 (d)	2,348	0.0000	0.0100	-0.4950	9.833	4.05 *
88-96(w)	469	0.0003	0.0049	-0.267	1.9362	1.69

After to study the skewness and in order to guarantee the non-normality and non-linearity of the returns distribution, we apply, for each year and for the whole period, the Lin-Mudholkar test to detect the non-normality and the Likelihood ratio test and TAR – F test both of them to detect non-linearity. The specificity of TAR-F test enable one to capture the non-linearity in periods like 1989 and 1993, where the Likelihood ratio test cannot reject the null hypothesis of linearity (Table 2). Both tests fail to reject the linearity of the time series return for 1994.

Table 2 Test of linearity (* the test no reject the linearity $\alpha = 0.05$).

Period	TAR-F $\nu_1 ; \nu_2$	Test F	Likelihood Ratio Test
1988	4; 226	2.61	17.30
1989	4; 226	7.70	13.00 *
1990	4; 226	6.24	27.09
1991	4; 226	3.78	27.09
1992	4; 226	5.47	17.41
1993	4; 226	3.32	11.43 *
1994	3; 228	2.20 *	6.27 *
1995	12; 210	1.73	34.86
1996	12; 209	1.43	2.07

The TAR-F test is able to specify the delay parameter *d*. We can observe (Table 3) that for most of the periods, *d* is equal to one. Few periods show a different value of *d*: 1992, 1993 and 1996. In 1992, the time series return exhibits a symmetric behaviour, this can influence the following year 1993, which is placed between two symmetric periods. The value of the delay parameter in 1996 is rare, and it is necessary to study this fact. However, the *d* value in the long period (1988-1996) for daily and weekly data is equal to one. Then, we can say that *d* = *1* is the normal value of the delay parameters for the IBEX-35 returns.

In the algorithm proposed by Tong the determination of the structural parameters *d, k, r and p* is based on a grid search. In our case the *delay parameter d* has been specified to the TAR-F test, and we consider this value as fixed in the algorithm. The parameter *k* has been fixed too, because we consider only two possible regimes (the rise and fall). After the estimation process of Tong we obtain the estimates of r, p_1, p_2, and the AR coefficients $\phi_i^{(j)}$ by the conditional least squares.

The best fitting SETAR $(2;\ p_1,\ p_2)$ is reported in Table 3 and the estimate of the parameters d and r. We can observe that in the most of the estimate models r is near to 0. This value explains the asymmetric behaviour of the return series by conditioning the previous rise ($Y_{t-d} \geq 0$) and fall ($Y_{t-d} < 0$). Finally, for each period we have estimated a SETAR model, all of these models are SETAR $(2;\ p_1,\ p_2)$, where the orders of AR regimes are generally low.

To check the goodness of the model, we analyse the standardized residuals. In order to guarantee the normality and the independence of residuals we use the Lin-Mudholkar test (1980) and Ljung-Box test (1978). In general, the standardised residuals are normally distributed (except for the year 1989 and for the daily data for the whole period). All the residuals are white noise, with a significance of 0.05, except for the daily time series in the whole period (with significance of 0.1). The results obtained for each year and the whole period (daily and weekly) are in Table 3.

We can observe that for 1992 the TAR-F test rejects linearity and it is possible to fit a SETAR model with a cyclical structure. This is the reason for the symmetry of the return distribution.

The TAR-F test and the Likelihood Ratio test are not able to reject the linearity for the year 1994. In this case a SETAR model may not be the best fitting of the return distributions.

Table 3. Summary of the results.

Period	Skewness	TAR-F	d	R	SETAR ($l;p_1p2$)	Stand. resid. L-M test	Stand. resid. L-B test
1988	0.857	2.61	1	0.0043	(2; 3, 0)	1.34	24.54
1989	-3.482	7.70	1	-0.0019	(2; 1, 3)	3.92 [c]	21.71
1990	-0.227	6.24	1	0.000	(2; 0, 3)	1.75	13.02
1991	-1.435	3.78	1	0.000	(2; 1, 2)	2.55 [d]	15.51
1992	-0.0025 [a]	5.47	3	0.000	(2; 2, 1)	-1.50	20.87
1993	0.333	3.32	2	0.0103	(2; 1, 1)	-1.53	9.76
1994	0.002 [a]	2.20 [b]	1	--	-	-	-
1995	-0.229	1.73	1	0.0006	(2; 10, 0)	0.07	8.28
1996	-1.207	1.43	5	0.000	(2; 4, 1)	2.42 [d]	13.90
88-96(d)	-0.495	17.89	1	0.000	(2; 0, 1)	4.45 [c]	171.04
88-96(w)	-0.267	4.485	1	0.000	(2; 4, 2)	1.76	9.26

(a) Distribution not skewness
(b) The TAR-F test does not reject linearity with $\alpha = 0.05$
(c) Significant with $\alpha = 0.05$
(d) Significant with $\alpha = 0.10$

If we analyse the estimated coefficients of the AR process, we can observe opposite signs for the significant coefficients (Table 4). If a SETAR model is not taken into account, we obtain an insignificant sample autocorrelation in the return series.

Table 4 Significant coefficient.

Period	$Y_{t-d} \leq r$		$Y_{t-d} > r$		
1988	ϕ_0 = -0.0010 (0.0005)	ϕ_3 = 0.1326 (0.0575)	ϕ_0 = 0.0038 (0.0013)		
1989	ϕ_0 = -0.0042 (0.0012)	ϕ_1 = -0.2778 (0.1320)	ϕ_1 = -0.3128 (0.0932)	ϕ_2 = -0.0870 (0.0495)	ϕ_3 = -0.1362 (0.0513)
1990	ϕ_0 = -0.0043 (0.0011)		ϕ_1 = 0.3431 (0.1192)	ϕ_3 = 0.2573 (0.0889)	
1991	ϕ_0 = -0.0042 (0.0015)	ϕ_1 = -0.4786 (0.1202)	ϕ_0 = 0.0022 (0.0011)	ϕ_1 = -0.1445 (0.0607)	
1992	ϕ_2 = 0.3365 (0.0982)		ϕ_1 = 0.3802 (0.0772)		
1993	ϕ_0 = 0.0016 (0.0007)	ϕ_1 = 0.1704 (0.0679)	ϕ_0 = 0.2673 (0.1035)		
1994					
1995	ϕ_5 = -0.2888 (0.0926)	ϕ_{10} = 0.3660 (0.1048)	ϕ_0 = 0.0016 (0.0006)		
1996	ϕ_3 = -0.2598 (0.1228)		ϕ_1 = 0.2068 (0.0638)		
88-96 (d)	ϕ_0 = -0.0017 (0.0004)		ϕ_0 = 0.0008 (0.0004)	ϕ_1 = 0.1467 (0.0387)	
88-96 (w)	ϕ_2 = 0.1542 (0.0722)	ϕ_4 = -0.1479 (0.0734)	ϕ_0 = 0.0010 (0.0005)	ϕ_2 = 0.0934 (0.0591)	

Now we analysed the two models (daily and weekly data) obtained for a whole period 1988-1996. The model estimated for daily data is a SETAR (2; 0,1). The numerical expression of the model is the following:

$$Y_t = \underset{(0.0004)}{-0{,}0017} + \varepsilon_t^{(1)} \qquad \text{if} \quad Y_{t-1} < 0{,}0000$$

$$Y_t = \underset{(0{,}0004)}{0{,}0008} + \underset{(0{,}0387)}{0{,}1467}\, Y_{t-1} + \varepsilon_t^{(2)} \qquad \text{if} \quad Y_{t-1} \geq 0{,}0000$$

$$\text{var}\,(\varepsilon_t^{(1)}) = 0{,}001 \qquad \text{var}\,(\varepsilon_t^{(2)}) = 0{,}0000 \qquad \textit{pooled} \quad \text{var} = 0{,}0000$$

We can observe that the first regime is constant, while the second regime depends on the previous return only. This model doesn't seem a realistic.

When we consider the weekly data, the goodness of the model improves. The data passes the Ljung-Box test with 5% significance and the estimated model seems more realistic than the previous one. The estimated model for weekly data is a SETAR (2; 4, 2). This is the expression of the model:

$$Y_t = \underset{(0.0004)}{-0.0003} \underset{(0.1057)}{- 0.0560}\, Y_{t-1} \underset{(0.0722)}{+ 0.1542}\, Y_{t-2} \underset{(0.0684)}{- 0.0215}\, Y_{t-3} \underset{(0.0734)}{- 0.1479}\, Y_{t-4} + \varepsilon_t^{(1)} \qquad \text{if} \quad Y_{t-1} < 0{,}0000$$

$$Y_t = \underset{(0{,}0005)}{0{,}0010} \underset{(0{,}0946)}{- 0{,}1057}\, Y_{t-1} \underset{(0.0591)}{+ 0.0934}\, Y_{t-2} + \varepsilon_t^{(2)} \qquad \text{if} \quad Y_{t-1} \geq 0{,}0000$$

$\mathrm{var}(\varepsilon_t^{(1)}) = 0{,}00003$ $\quad$ $\mathrm{var}(\varepsilon_t^{(2)}) = 0{,}00002$ $\quad$ *pooled* $\mathrm{var} = 0{,}00000$

The first regime has two significant coefficients Y_{t-2} and Y_{t-4} with opposite signs. We can think that when the return is negative in the previous period, the time series auto-regulates itself. This behaviour allows the fall of the return to stop. Also we observe that the past information, until lag four, is necessary to guarantee the stability.

In the second regime, when the past return is positive, the AR process depends on the two past periods only. The constant coefficient is positive and significant, the same as the coefficient of the Y_{t-2}.

The weekly model is better than the daily one. If we compare the results of the Lin-Mudholkar test and the Ljung-Box test for standardised residuals (Table 3), we observe that the standardised residuals of daily models are not normal and then pass the Ljung-Box test of whiteness with $\alpha = 0.1$; while the standardised residuals of weekly data are normally distributed and pass the test of whiteness with $\alpha = 0.05$. The goodness of the fit improves if we consider weekly data.

We can note the importance of the data frequency in these analyses. In this work we cannot use monthly data because we have only a hundred monthly observations. The same reason makes us to do the yearly study with weekly data

Section 4: Conclusions and comments.

After the analyse we can conclude that:

- The series for the whole period and each single year (except for 1992 and 1994) are asymmetric.
- The self-excited autoregressive SETAR model is useful to capture asymmetry in the return series.
- For 1992, the TAR-F test rejects linearity and it is possible to fit a SETAR model with a cyclical structure. This is a reason for symmetry.
- The TAR-F test and Likelihood Ratio test are not able to reject the linearity for 1994. In this case a SETAR model may not be the best fitting of the returns distribution.
- In summary, when the skewness is present, the SETAR models are the best fitting of the return distribution. In this case d is equal to 1 in general (without 1993 and 1996), and the threshold value r is near 0.
- If the distribution is asymmetric, the return generating mechanism for today depends, in general, on whether the prices rose or fell ($r = 0$) on a previous day ($d = 1$). The delay variable Y_{t-d} governs the dynamic pattern of the stock return.

Finally we would like to remark that the previous results are in accordance with the conclusions of Li and Lam (1995) in their paper about the asymmetric behaviour of the Hang Seng Index. But they make the assumption that the threshold value r is equal to zero, and the delay parameter d is equal to one. In our paper, we don't make any assumption about these parameters. We estimate, using the TAR-F test of Tsay and a grid search in Tong's procedure, the structural parameters of the SETAR models, specifically d and r. We have obtained with statistical methods the same results as Li and Lam, in general for the IBEX-35 return $d = 1$ and $r = 0$. We think that this is an important aspect of the work.

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