POLITICAL COMPROMISE:
PLATFORMS AND POLICIES

Humberto Llavador

Universitat Pompeu Fabra

The Institut de Ciències Polítiques i Socials (ICPS) was created by the Universitat Autònoma de Barcelona and the Diputació de Barcelona in 1988. The ICPS is attached to the Universitat Autònoma de Barcelona.

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Abstract

It is frequently believed, and empirically observed, that the outcome of an election determines government action to at least some extent. However, the standard two-party spatial model of political competition trivializes any post-electoral policy-making process by assuming that the party that obtains more than fifty percent of the vote adopts the policy announced during the electoral campaign. Its best known implication is that, under an environment of perfect information, both candidates announce the same policy, which coincides with the policy preferred by the median voter.

In this paper, we enrich the theoretical framework of party competition to allow for a non-trivial policy-setting process and for sophisticated voters who care not only about the policy implemented but also about the platform they support with their vote. First, we show that platform convergence is a non-robust feature created by the winner-takes-all assumption. The lightest influence of the opposition in the policy-making process provokes a divergent tendency in platform writing. Without abstention, an equilibrium is characterized by polarized platforms and a *moderate* implemented policy that consistently differs from the median voter ideal policy. When abstention is introduced into the analysis, parties announce differentiated yet non-extreme policies, voters concentrate around the platforms, and substantial turnout rates are generically obtained where abstention occurs among voters with extreme views as well as with moderate views.
Key bords: Political competition, compromise, platforms, abstention.

"It was not the kind of speech Mr. Bush would have delivered had he won the large victory his aides were predicting on election night. He offered nothing to the conservative wing of his party, and evoked none of the cultural issues that often divide the two parties".

The New York Times. December 14, 2000

1. **INTRODUCTION**

It is frequently believed that the outcome of an election determines government action to at least some extent[1]. Many empirical studies support this view. Papers like Budge and Hofferbert (1990, 1992), Hofferbert and Klingemann (1990), Klingemann et al. (1994), and King et al. (1993) analyze the relationship between party platforms and policies. They find that, while platforms and governmental outcomes consistently differ, there exist strong links between them. A second group of studies (Poole and Rosenthal, 1984b; Fiorina, 1974; Poole and Rosenthal, 1984a; Poole and Daniels, 1985; Poole and Rosenthal, 1991, 1997; Snyder, 1996; Alesina and Rosenthal, 1995) compare the perceived location of platforms and implemented policies, and conclude that while platforms tend to be polarized, policies are moderate or centrist[2].

But the standard two-party spatial model of political competition does not differentiate between platforms and policies. It assumes that the party that obtains more than fifty percent of the vote adopts the policy announced during the electoral campaign. As a consequence, any post-electoral policy-making process becomes a triviality.

On theoretical grounds, this assumption has been rejected by several authors as an unrealistic way to model the outcome of the political competition in many modern democratic societies (Ortuño-Ortín, 1997; Gerber and Ortuño-Ortín, 1998; Dixit et al., 2000; Alesina and Rosenthal, 1995, 1996; Austen-Smith and Banks, 1998; Grossman and Helpman, 1996). Ortuño-Ortín argues that a democratic society is integrated by different institutions and groups. Some of them will favor the policies announced by the winning party, some will favor the opposition's platforms, but all may have some influence on the actions taken by the government, and their influence will be a non-decreasing function of the support received by the corresponding party: "Clearly a party winning 51% of the votes will have more difficult to carry out its proposal than one winning 80% of the votes" (Ortuño-Ortín, 1997, p. 428). Dixit, Grossman and Gul (2000) obtain tacit collusion in a plurality political environment. They show that the ruling party will go to some extent towards the interests of the opposition expecting the same treatment when it itself becomes the opposition party. Of course, the party in power must know that its term will reach an end and must expect the other party to gain power sometime in the future.

In this paper we follow this view and, in the spirit of the empirical results, let the implemented policy differ to some extent from the electoral platform of the winning party. In particular, we let the implemented policy depend on the electoral outcome and on the platforms announced by each party, in such a way that the higher the fraction of votes obtained by a party, the closer the implemented policy is to its proposal. This post-electoral policymaking process is what we call *political compromise*.

Modifying the policy-setting process has important implications on voting behavior often overlooked by the literature on political compromise. In the winner-takes-all case, when the implemented policy is the platform announced by the winner, voting for the preferred candidate is always a

dominant strategy for all voters. The intuition is simple. Suppose that an agent prefers the policy proposed by party L to the policy proposed by party R. By voting, either he does not change the outcome or he increases the probability of winning for that party. Thus, he will always vote for his favorite platform. However, when electoral platforms and implemented policies may differ, voting for the preferred platform is not necessarily a dominant strategy. Consider, for instance, a voter with moderate views. She finds the proposal of the left party more appealing than the one from the right party. However, she expects that, due to an overwhelming support for the left party, the implemented policy will be "too left" for her. Then, she may decide to vote for the right party. That is, a voter who expects party L to win and who prefers the alternative offered by L to the one announced by party R may, nevertheless, vote for R to "moderate" L's policy.

On different grounds, when platforms and policies do not necessarily coincide, voters will base their decision not only on the implications over final policies, but also on the platforms announced by the parties. For example, many people would feel reluctant to vote for a *nazi* party regardless the impact that a larger support for that *nazi* party would have on the implemented policy. Their reluctance comes from the platform represented by such a party and not from the influence over the policy. If the *nazi* party were the only alternative to obtain a more conservative policy, we may expect many of those voters to abstain.

Therefore, we distinguish two components in voting behavior. On the one hand, insofar as voters care about the policy outcome, they will choose to vote for one party or the other based on the effect that a larger support for each party have over the implemented policy. We refer to this component as the *pragmatic* component of preferences. On the other hand, citizens also consider the platform they support with their vote when deciding for whom

to vote. Because platforms and policies do not necessarily coincide, voters who would support a party from a purely pragmatic point of view may decide not to do it because they strongly disagree with the party's platform. This is the *ideological* component of preferences. The final voting decision will depend then on both the *pragmatic* and the *ideological* components of preferences.

We study an unidimensional political game with a dynamic structure. At period one, parties announce their campaign platforms. At period two, citizens observe the platforms and decide whether to vote for one of the two candidates, or to abstain. Their decisions are based on their preferences and their expectations about the outcome of the election. Finally, after the election, a policy-setting process takes place from which a policy arises as a function of both the platforms of each party and their electoral support.

Our first result shows that platform convergence is a non-robust feature created by the winner-takes-all assumption. The lightest influence of the opposition in the policy making process provokes a divergent tendency in platform writing. Second, we find that, under reasonable assumptions, an equilibrium always exists. Without abstention, an equilibrium is characterized by polarized platforms and a moderate implemented policy that consistently differs from the median voter ideal policy. When abstention is incorporated into the analysis, parties announce differentiated, non-extreme policies, voters concentrate around the platforms, and abstention occurs among voters with extreme views as well as with moderate views.

The reader is encouraged to look at Figure 4 where we show two typical examples of political equilibrium.

2. THE MODEL

The electoral arena is assumed to be represented by an unidimensional policy space

$$T = [\underline{t}, \overline{t}] \subset \nabla. \text{ Let } T^2 = T \times T.$$

We model the electoral process as a political game between parties and voters with a dynamic structure. Two parties, L and R, announce their campaign platform by choosing a location on the policy space. Citizens observe the platforms and vote for one of the two candidates, or abstain. Finally, after the election, a policy formation process takes place during which both parties play a role in shaping the implemented policy: one as the governing party, the other as the opposition.

The following sections describe the three steps in the political game: platform announcement, voting, and policy formation. Following the standard *backward induction* methodology, we start from the last stage, and then work backward to describe the behavior of the players before.

2.1 Policy Formation

A key feature of the present model is that the implemented policy may differ from the electoral platform of the winner. Policy is the result of a post-electoral process where both parties play a role (relative to their electoral support). For the purpose of this paper, it is convenient to model this post-election process in its reduced form by directly specifying a map from platforms and electoral outcomes to final policies.

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A datum of the model is a *weight function*, $g : [0,1] \rightarrow [0,1]$. If v is the fraction of active voters that vote for L, then $g(v)$ captures the weight of party L's proposal in the implemented policy.

Assumption 1 (A1)   a) $g \in C^1$;
                           b) $g(0) = 0$, $g(1) = 1$;
                           c) $g'(v) > 0$, $\forall\ v \in (0,1)$.

Parts (b) and (c) are very natural assumptions. (b) says that if one of the parties obtains all the votes, its platform is then adopted. By (c), the larger the fraction of the vote a party receives, the stronger its influence in the determination of the policy. Finally, (a) introduces continuity: small perturbations in the electoral outcome cannot produce big changes in the implemented policy.

We assume that the relevant properties of the outcome mapping are captured by an implemented policy function, which associates to each electoral result a final policy outcome. Formally[3]:

Definition 2.1 *Define the **implemented policy function*** $\hat{t} : T^2 \times [0, 1] \rightarrow T$ *as*
$$\hat{t}\ (t_L, t_R, v) = g(v)\ t_L + (1 - g(v))\ t_R \tag{2.1}$$

The implemented policy function $\hat{t}$ is general enough to represent a wide variety of scenarios. For instance, although the continuity of g rules out a winner-takes-all structure, it is easy to find a function g that satisfies A1 and for which the implemented policy is *almost* discontinuous at $v = \frac{1}{2}$ (see Figure 1(a)). In that example, the winner party would adopt a policy arbitrarily close to its electoral platform if it obtained a plurality larger than just $\frac{1}{2} + \varepsilon$.

As a further support of our claim that $\hat{t}$ is very general, Figure 1(b) represents a function that is asymmetric but yet it satisfies A1. That is, $\hat{t}$ may

very well represent a case where one of the parties is more inclined to compromise than the other one.

Figure 1
The Weight-of-Votes Function. The function g assigns to the share of votes received by a party the weight of that party in shaping the policy. Assumption 1 allows for *almost* discontinuous functions *(a)* and non-symmetric functions *(b)*.
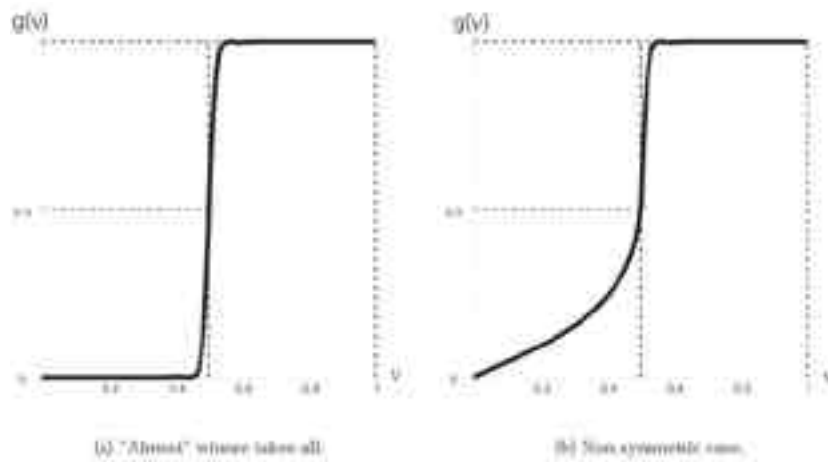
2.2 **Voting**

In this section we describe the actual electoral process: we model voting behavior and define the set of possible electoral outcomes.

Consider a continuum of citizens distributed according to their ideal policy. Let $F : \nabla \rightarrow [0,1]$ describe the distribution of citizens, and let μ be the probability measure associated to T^4. That is, for any Borel subset B of T, $\mu(B) = \int_B dF(\tau)$ represents the measure of citizens with ideal policies in B.

2.2.1 *The Pragmatic Component and the Ideological Component of Preferences*

Citizens observe the platforms announced by the parties, t_L and t_R in T, and anticipate the policy-making process (i.e. they know the implemented

policy function $\hat{t}$ and they have expectations regarding the support that each party will receive). With this information in mind, each citizen has to decide between voting for L, voting for R, or abstaining. Let S = {L, R, A} be the set of actions for a particular citizen, with typical element s: s = L and s = R denote that the citizen votes for party L and party R, respectively; s = A when the citizen abstains.

A citizen takes t_L, t_R, and n as given. We postulate that she chooses an action $s \in S$ in order to maximizes her preferences, represented by an utility function $v: S \times T^2 \times [0,1] \times t \to \nabla$. We interpret $v(s; t_L, t_R, n, \tau)$ as the utility of a citizen with ideal policy t who chooses s when the platforms are t_L and t_R, and she expects a fraction n of the electorate to support L.

As explained in the Introduction, a citizen has two concerns when casting her vote: the impact on the policy and the platform that she is supporting with her vote. In fact, the utility function *v* will be defined as the sum of two components, one for each of the two concerns.

The first component is derived as follows. Insofar voters care about the policy outcome, they will choose to vote for one party or the other based on the effect that a larger support for that party has on the implemented policy[5]. We refer to this component as *pragmatic voting*. Thus, purely pragmatic voting must be derived from the preferences of the voter over policies[6]. Let preferences over policies be represented by the single-peaked utility $u: T \times T \to \nabla$, where $u(t; \tau)$ represents the utility that a citizen t obtains when t is implemented. Let $\tau = \arg\max \{ u(t; \tau): t \in T \}$, i.e. τ represents the ideal policy of citizen τ. Because the policy is a function of the electoral platforms and the allocation of votes, it is convenient to define the reduced form of the utility over implemented policies as

$$\hat{u}\left(t_L, t_R, v; \tau\right) = u\left(\hat{t}\left(t_L, t_R, v\right); \tau\right) \qquad (2.2)$$

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Thus, citizen τ's *pragmatic* or *indirect-effect* component of voting for L is the utility change implied by a larger support for party L. More precisely, for each pair of policies (t_L, t_R) and given the expected electoral outcome ν, τ's pragmatic utility of voting for L will be the partial derivative of τ's utility over policies with respect to a change in the fraction of the vote received by party L [7]. Observe that, because the fraction of voters for R is one minus the fraction of voters for L, an increase in the support for R is equivalent to an equal decrease in the support for L. Finally, if a citizen abstains, we set the utility equal to zero. Consequently, we define the **pragmatic component of voting**, $w: S \times T^2 \times [0,1] \times T \to \nabla$, as [8]

$$w(s; t_L, t_R, \nu, \tau) = \begin{cases} \dfrac{\partial \hat{u}(t_L, t_R, \nu; \tau)}{\partial \nu} & \text{if } s = L \\ -\dfrac{\partial \hat{u}(t_L, t_R, \nu; \tau)}{\partial \nu} & \text{if } s = R \\ 0 & \text{if } s = A \end{cases} \quad (2.3)$$

The second component of voting reflects the concern of a voter with the platform she supports with her vote, beyond the impact on the implemented policy. Let $\hat{\eta}: S \times T^2 \times T \to \nabla$ be the *non-pragmatic* or **ideological component of voting**, such that $\hat{\eta}(L; t_L, t_R, \tau) \equiv \eta(t_L; \tau)$ represents the utility of agent t from voting for the platform t_L and, similarly $\hat{\eta}(R; t_L, t_R, \tau) \equiv \eta(t_R; \tau)$ is the utility of voting for platform t_R. Observe that non-pragmatic utility depends only on the platform supported by the vote, and not on the implemented policy. In particular, it does not involve any expectations about what the rest of the electorate will choose to do. We normalize the ideological utility of abstention to zero. Then,

$$\hat{\eta}(s; t_L, t_R, \tau) = \begin{cases} \eta(t_L; \tau) & \text{if } s = L \\ \eta(t_R; \tau) & \text{if } s = R \\ 0 & \text{if } s = A \end{cases} \quad (2.4)$$

Finally, let the pragmatic and the non-pragmatic components of voting enter additively into the utility function:

$$v(s; t_L, t_R, v, \tau) = w(s; t_L, t_R, v, \tau) + \hat{\eta}\ (s; t_L, t_R, \tau). \qquad (2.5)$$

2.2.2 *Consistent Vote*

Given (t_L, t_R, v), we partition the electorate into those citizens who vote for L, those who vote for R, and those who abstain according to

$$L(t_L, t_R, v) = \{\ \tau \in T : v(L; t_L, t_R, v, \tau) \geq \max\ \{\ 0, v(R; t_L, t_R, v, \tau)\ \}\ \},$$
$$R(t_L, t_R, v) = \{\ \tau \in T : v(R; t_L, t_R, v, \tau) > \max\ \{\ 0, v(L; t_L, t_R, v, \tau)\ \}\ \},$$
$$A(t_L, t_R, v) = \{\ \tau \in T : \max\ \{\ v(L; t_L, t_R, v, \tau), v(R; t_L, t_R, v, \tau)\ \} < 0\ \}.$$

Then, a fraction $V_L(t_L, t_R, v) = \mu(L(t_L, t_R, v))$ of the citizenry votes for L, while party R receives $V_R(t_L, t_R, v) = \mu(R(t_L, t_R, v))$. Finally, a fraction $\mu(A(t_L, t_R, v))$ of the citizenry abstains.

We have described how citizens vote (or abstain) as a function of the electoral platforms and the expected support for each party. However, our goal is to find a correspondence that assigns to each pair of platforms the set of all *consistent* or *rational-expectations* electoral outcomes.

Definition 2.2 *We say that* n *is a* **consistent vote** *for the pair of policies* t_L, t_R *in* T *if*

$$\frac{V_L\ (t_L, t_R, v)}{V_L\ (t_L, t_R, v) + V_R\ (t_L, t_R, v)} = v, \qquad (2.6)$$

that is, if the allocation of votes implied when v *is expected gives rise to a fraction of votes for L equal to* v.

Thus, we can define the **electoral outcome correspondence** $\chi: T^2 \rightrightarrows 2^{[0,1]}$ as,

$$\chi(t_L, t_R) = \{ v \in [0,1] : v \text{ is a } \textit{consistent vote} \text{ for } (t_L, t_R) \}.$$

Perhaps abusing language, and without further specifications, this definition admits the possibility that $\chi(t_L, t_R) = \varnothing$ for some t_L and t_R. However, in what follows, we will restrict the analysis to environments where $\chi(t_L, t_R) \neq \varnothing, \forall (t_L, t_R)$.

2.3 **Political Equilibrium**

Parties are the actual players of the political game. There are two parties that run for election, and have single-peaked preferences over policies represented by the utility function $\pi_J: T \to \nabla$, $J = L, R$. Let $\tau_J = \arg \max \{ \pi_J(t) : t \in T \}$ be the ideal policy of party J [9]. Assume, without loss of generality, that $t_L < t_R$.

It is convenient to write the utility of a party as a function of the platforms and the vote allocation by using the definition of the implemented policy function. Define $\hat{\pi}_J : T^2 \times [0,1] \to \nabla$ as

$$\hat{\pi}_J (t_L, t_R, v) = \pi_J(\hat{t} (t_L, t_R, v)).$$

For illustrative purposes, assume that there exists a unique consistent electoral outcome for each pair of policies, that is, χ is a function. Then we can write $\tilde{\pi}_J (t_L, t_R) = \hat{\pi}_J (t_L, t_R, \chi(t_L, t_R))$ and a political equilibrium is simply a Nash equilibrium of the two-party game where parties choose platforms in T to maximize their payoffs $\tilde{\pi}_J$. In general, however, χ may not be single-valued and non-empty for all pair of policies. Therefore we introduce the following more general definition of a political equilibrium:

Definition 2.3 We say that $(t_L, t_R, \overline{v})$ is a ***political equilibrium*** *if*

(i) $\overline{v} \in \chi(t_L, t_R))$

(ii) $\hat{\pi}_L (t_L, t_R, \overline{v}) \geq \hat{\pi}_L (t, t_R, v) \ \forall t \in T$ *and* $\forall v \in \chi(t, t_R)$, *and*

(iii) $\hat{\pi}_R (t_L, t_R, \overline{v}) \geq \hat{\pi}_R (t_L, t, v) \ \forall t \in T$ *and* $\forall v \in \chi(t_L, t)$.

3. EXAMPLE

We anticipate the main results with an illustrative example, before presenting their formal proofs.

Consider a political process where two ideological parties, L and R, compete over a single issue, a tax rate, for example. Let T= [0,1] represent the policy space. The weight-of-votes function takes the following form (see Figure 2):

$$g(v) = \frac{1 - \cos(\pi v)}{2} \tag{3.1}$$

Observe that, because g is concave for $v > \frac{1}{2}$ and convex for $v < \frac{1}{2}$, winning the election makes a difference: the weight of the platform of the winner party in the policy-setting process is more than proportional to the share of votes received by that party.

Figure 2
Weight of Votes Function
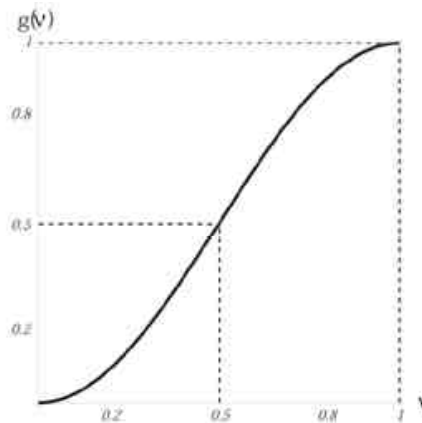
Voters care about the implemented policy (the tax rate). Let the preferences of voters over tax rates be represented by the following Euclidean utility function:

$$u(t; \tau) = -\frac{1}{2}(t - \tau)^2 \qquad (3.2)$$

Then, the pragmatic component of voting is given by (see(2.3)).

$$w(L; t_L, t_R, v, \tau) = (\hat{t}(t_L, t_R, v) - \tau) \, g'(v)(t_R - t_L)$$
$$w(R; t_L, t_R, v, \tau) = (\tau - \hat{t}(t_L, t_R, v)) \, g'(v)(t_R - t_L)$$

On the other hand, voters also care about the platform they are supporting with their vote. Let the ideological component of voting represent the dis-utility of supporting a platform far from one's ideal policy. In particular, we take ideological voting to be a linear function of the distance between the "ideology" of the voter and the platform of the party:

$$\eta(t; \pi) = -2|t - \pi|.$$

Finally, if a citizen abstains, her utility of voting is zero, since both the pragmatic component and the ideological component are zero.

In summary, the utility of voting (the sum of the pragmatic and the ideological components of voting) of a citizen τ is

$$v(L; t_L, t_R, v, \tau) = (\hat{t}(t_L, t_R, v) - \tau) \, g'(v)(t_R - t_L) - 2|t - \pi|$$
$$v(R; t_L, t_R, v, \tau) = (\tau - \hat{t}(t_L, t_R, v)) \, g'(v)(t_R - t_L) - 2|t - \pi|$$
$$v(A; t_L, t_R, v, \tau) = 0$$

Let the electorate be distributed according to a triangular distribution with mode m (see Figure 3 for a few members of this family of density functions).

Figure 3
Family of Triangular Density Functions
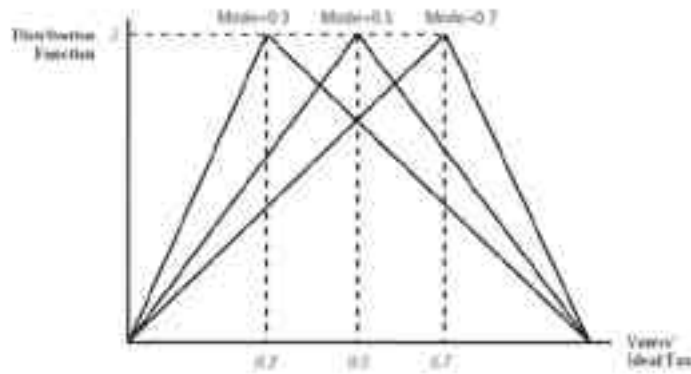
Finally, parties also have Euclidean preferences represented by

$$\pi_J(t) = -\frac{1}{2}\,(t - \tau_J)^2.$$

We take the ideal policy of the left party to be $\tau_L = 0.3$, while the right party's ideal policy is $\tau_R = 0.8$.

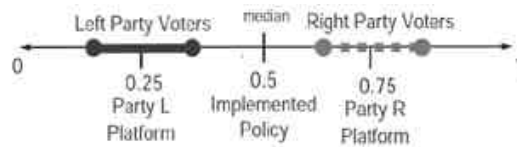After several manipulations, it is easy to show that, given (t_L, t_R, v), active voting occurs on two intervals around the electoral platforms: $[\lambda_1, \lambda_2]\ni t_A$ and $[\rho_1, \rho_2]\ni t_B$. That is[10],

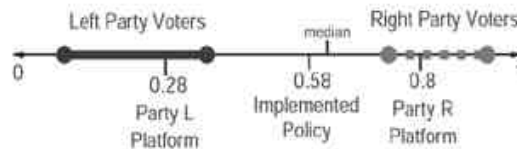$$V_L(t_L, t_R, v) = F(\lambda_2) - F(\lambda_1),$$
$$V_R(t_L, t_R, v) = F(\rho_2) - F(\rho_1).$$

First, we analyze the case when the electorate is symmetrically distributed: $m = \frac{1}{2}$.

Figure 4
Display of Typical Equilibria for Different Distributions of the Electorate

(a) Symmetric Distribution

(b) Right-skewed Distribution

Figure 4(a) depicts the political equilibrium. The following implications are worth emphasizing (we will prove in Section 5 that they are not particular to the present example, but characteristic of a symmetrically distributed electorate):

(i)   parties announce differentiated, non-extreme, symmetric platforms: $t_L^* = .25$ and $t_R^* = .75$;

(ii)   the equilibrium policy coincides with the mean and the median: $t_R^* = .5$;

(iii)   each party receives 50% of the vote cast;

(iv)   a substantial fraction of the population prefers voting to abstaining: turnout $\approx 37\,\%$; and

(v)   abstention occurs for voters with extreme views as well as with moderate views.

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Observe that the symmetric features of the equilibrium are a consequence of the symmetry imposed on the electorate, since parties are not symmetric: $\tau_L = .3$ and $\tau_R = .8$.

How important is the symmetry assumption in the previous case? In a second exercise we consider a right skewed electorate: m = 0.7. The equilibrium is presented in Figure 4(b). Not surprisingly, symmetry vanishes. However, and more interestingly, the substance of the equilibrium does not change: parties announce differentiated platforms, abstention occurs for extreme and moderate voters, and there is a substantial turnout (40.5%). Unlike in the symmetric case, the equilibrium policy differs from the median ($t^* = 0.58 < 0.61 =$ median). Finally, one party wins (party R receives 55% of the vote). This is an important finding since most models of political competition, even those with uncertainty built in, *unrealistically* predict that each party always obtains (or expects to obtain) fifty percent of the vote.

In this section, we have worked out an example that anticipates the most relevant results of the paper. Section 5 presents the formal analysis of the existence and characterization of equilibria. First, however, we find interesting to consider the particular case where voting is purely pragmatic.

4. **PURELY PRAGMATIC VOTING**

By abstracting from non-pragmatic voting we are able to recreate the *standard* political competition model with perfect information, except that here the implemented policy function substitutes a winner-takes-all assumption. Therefore, this particular case isolates the impact that incorporating political compromise has in the study of electoral politics.

A standard result in the political competition literature is that, in an environment of perfect information, the parties' platforms converge (Roemer, 1996). Moreover, no citizen has an incentive to vote: because both parties announce identical platforms, the probability of being pivotal is zero for all citizens.

We obtain completely opposite results when political compromise replaces the winner-takes-all postulate: parties propose extreme platforms and "everybody" prefers active voting to abstaining[11]. Moreover, it is notable that, without additional assumptions beyond single-peakedness, an equilibrium always exists and is unique for any distribution of the electorate. This is an important finding in a field where existence of equilibrium is a permanent struggle (Roemer, 1997).

For the rest of this section we assume that voting behavior is fully driven by its pragmatic component.

Assumption 2 (A2) $\eta(t;\tau) = 0$ *for all* $t \in T$ *and for all agent* τ.
Observe that A2 is equivalent to

$$v(s; t_L, t_R, \nu; \tau) = w_\tau(s; t_L, t_R, \nu).$$

Lemma 4.1 *Let* A1 *and* A2 *hold. Then:*

   (i)   *Given* $t_L, t_R \in T$ *and* $\nu \in [0,1]$, *everybody votes.*
        Moreover, for $t_L < T_R$, *everybody to the left of* $\hat{t}$ (t_L, t_R, ν) *votes*
        for L *while everybody to the right of* $\hat{t}$ (t_L, t_R, ν) *votes for* R.
        That is, $V_L(t_L, t_R, \nu) = [\underline{t}, \hat{t}$ $(t_L, t_R, \nu))$, *and* $V_R(t_L, t_R, \nu) = (\hat{t}$ $(t_L, t_R, \nu), \bar{t}$
   (ii) $\chi(t_L, t_R)$ *is non-empty and single-valued for all* $(t_L, t_R) \in T^2$.
   (iii) χ *is continuous, except may be at* $t_L = t_R$.

     Proof: All the proofs are presented in the Appendix.   ◆

Because χ is a function, we are able to formulate the implemented policy and the parties' utilities in their *reduced* form. Let $\tilde{t} : T^2 \rightarrow T$ be defined by

$$\tilde{t}(t_L, t_R) = \hat{t}(t_L, t_R, \chi(t_L, t_R)). \tag{4.1}$$

And, for J = L, R, let $\tilde{\pi}_J : T \rightarrow T$ be defined by

$$\tilde{\pi}_J(t_L, t_R) = \pi_J(\tilde{t}(t_L, t_R)). \tag{4.2}$$

Lemma 4.2 *The functions $\tilde{t}$ and $\tilde{\pi}_J$ are continuous on T^2.*

We follow the standard procedure to prove the existence of a Nash equilibrium: first, we construct the best-response correspondences, then we look for a fixed point.

The **best-response correspondence** of party J = L, R assigns to each alternative the set of utility maximizers. That is,

$$BR_L(t_R) = \arg\max\{\tilde{\pi}_L(t, t_R) : t \in T\}$$
$$BR_R(t_L) = \arg\max\{\tilde{\pi}_R(t_L, t) : t \in T\}$$

Before the formal statement of the existence theorem, Lemma 4.3 shows that if a party cannot ensure the implementation of its ideal policy, then its best choice is to propose an extreme platform, namely, $\underline{t}$ for L or $\bar{t}$ for R.

Lemma 4.3 *Let A1 and A2 hold.*

(i) *The best-response correspondence of party J, BR_J, is single-valued and continuous (thus it can be viewed as a continuous function), J = L,R.*

(ii) *Given $t_R \geq \tau_L$, if there exists a t_L^0 such that $\tilde{t}(t_L^0, t_R) = \tau_L$, then $BR_L(t_R) = \{t_L^0\}$; otherwise, $BR_L(t_R) = \{\underline{t}\}$.*

(iii) *Given $t_L \leq \tau_L$, if there exists a t_R^0 such that $\tilde{t}(t_L, t_R^0) = \tau_R$, then $BR_R(t_L) = \{t_R^0\}$; otherwise, $BR_R(t_L) = \{\bar{t}\}$.*

Theorem 4.1 *Let A1 and A2 hold. Then:*

(i)   *a political equilibrium exists, and*
(ii)   *if* (t_L^*, t_R^*) *are the platforms at equilibrium,*
         either $(t_L^*, t_R^*) = (\underline{t}, \overline{t})$
         or $\tilde{t}$ $(t_L^*, t_R^*) = \tau_J$, *for some* J=L,R.

This section has recreated the standard political competition model with perfect information, substituting the winner-takes-all assumption for a political compromise policy-setting process. The main observation is the disappearance of the convergence tendency of parties' proposals. We obtain just the opposite result: the smallest influence of the opposition's platform in the determination of the implemented policy provokes a divergence tendency that takes parties to radicalize their platforms.

In the following section we expand the model to allow for ideological voting. We obtain that typically parties announce differentiated, but non-extreme platforms. The key element to understand the intuition is abstention. By announcing an extreme platforms, a party alienates part of its constituency (who cares now about the platform.) If this alienation effect is strong enough, it will drive parties away from radicalizing their platforms.

5. **PRAGMATIC AND IDEOLOGICAL VOTING**

The main conclusion from the previous section appears in Theorem 4.1: convergence of parties' platforms, traditionally associated with political competition, is an artifact created by the winner-takes-all assumption. As soon as we perturb this assumption a little bit and let the opposition intervene (to some extent) into policy-setting, a tendency towards divergence appears. Recall that an *almost* winner-takes-all implemented policy function (like the one in Figure 1(a)) would be enough to make parties announce radical

platforms. One may object that this result is as unrealistic as the standard Median Voter Theorem, where parties propose exactly the same policy and everybody votes. However, we claim that the "pathological" behavior found here is the consequence of an unrealistic feature of the model studied in Section 4, namely the absence of ideological voting. We will see that under a more realistic (and more complicated) structure, already presented in Section 2, parties' and voters' behavior resemble closer what we observe in reality.

The inclusion of a non-pragmatic component of voting that depends on the platforms of the parties opens the possibility to abstention. Because abstention notably complicates the analysis, we sacrifice some generality and work within a simpler framework. First, we substitute Assumption 1 with the following assumption:

Assumption 3 (A3) $\tilde{t}\left(t_L t_R, v\right) = v\, t_L + \left(1 - v\right) t_R$, *for all* $\left(t_L t_R, v\right)$.

That is, we take the implemented policy to be equivalent to proportional representation[12].

Second, consider a continuum of voters with Euclidean preferences over policies represented by the following utility function:

Assumption 4 (A4) $u\left(t; \tau\right) = -\dfrac{1}{2}\left(t - \tau\right)^2$

Third, let the ideological component of voting be a non-increasing and concave function of the distance between the platform of the party and the "ideology" of the voter:

Assumption 5 (A5) $\eta(t:\tau) = \zeta(|t-\tau|)$, *with* $\zeta' < 0$ *(decreasing),* $\zeta'' < 0$ *(concave), and* $\zeta(0) = 0$.

Finally, it is convenient for the exposition to adopt the convention $T = [0,1]$.

We want to emphasize that all specifications have been imposed on citizen's preferences and none on parties' preferences, which are only assumed to be single-peaked.

A first result (Theorem 5.1) proves what we already observed in the examples presented in Section 3: the support for each party is an interval around the platform announced by that party. It will follow (Theorem 5.2) that there exists one and only one consistent electoral outcome associated to each pair of platforms.

Theorem 5.1 *Let A3, A4, and A5 hold. Given* t_L, t_R, *and* n, *with* t_L ¹ t_R, *there exist two intervals*

$$I_\lambda(t_L,t_R,v) = [\lambda_1(t_L,t_R,v), \lambda_2(t_L,t_R,v)], \ and$$
$$I_\rho(t_L,t_R,v) = [\rho_1(t_L,t_R,v), \rho_2(t_L,t_R,v)]$$

such that if τ *votes for* L, *then* $\tau \in I\lambda$, *and if* τ *votes for* R, *then* $\tau \in I\rho$. *Thus*

$$V_L(t_L,t_R,v) = \mu(I_\lambda(t_L,t_R,v)) \ and$$
$$V_R(t_L,t_R,v) = \mu(I_\rho(t_L,t_R,v)).$$

Moreover, $V_L(t_L,t_R,n) + V_R(t_L,t_R,n) > 0$, V_L *and* V_R *are continuous,* V_L *is non-increasing in* v, *and* V_R *is non-decreasing in* v.

Lemma 5.2 *Let A3, A4, and A5 hold. The consistent electoral outcome correspondence* c *is non-empty, single-valued, and continuous for all* $t_L \neq t_R$.

The previous lemma shows that there exists a unique electoral outcome associated to each pair of differentiated platforms. When both parties propose identical platforms, the implemented policy is unmistakably the common

platform, independent of the electoral outcome. Therefore, we can write the implemented policy as:

$$\tilde{t}(t_L, t_R) = \begin{cases} \hat{t}(t_L, t_R, \chi(t_L, t_R)) & \text{if } t_L \neq t_R, \\ t & \text{if } t_L = t_R = t. \end{cases}$$

We argued at the end of the previous section that parties proposed extreme platforms because voters were only concerned about pragmatic voting. In that context, everybody voted and therefore, parties did not worry about alienating part of the electorate by becoming too extreme in their platforms. In order to validate our claim we need to show that when a non-pragmatic component of voting is present, parties do not take extreme positions (at least not necessarily). We start by studying a particular case: a symmetrically distributed electorate[13].
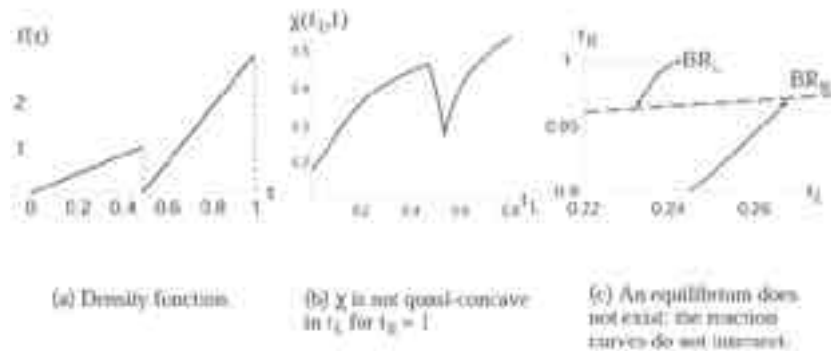
The following theorem shows that for the symmetric case an equilibrium always exists and, not surprisingly, it is symmetric: parties propose differentiated and symmetric policies, the vote is equally split, and the implemented policy is the median policy, which, in this case, coincides with the mean. It is easy to construct examples where platforms are not extreme. We only need a sufficiently relevant non-pragmatic component of voting and a fairly concentrated distribution (refer to the example 1 in Section 3).

Theorem 5.1 (Symmetry) *Let A3, A4, and A5 hold. Let the electorate be distributed symmetrically around ½:* $f(x) = f(1 - x)$. *If* $\tau_L \leq \frac{1}{2} \leq \tau_R$, $\tau_L \neq \tau_R$, *then:*

(i)   *there exists a political equilibrium* $(t_L{}^*, t_R{}^*, v^*)$, *with* $t_L{}^* = \frac{1}{2} - k$,
        $t_R{}^* = \frac{1}{2} + k$, *for some* $k \in [0, \frac{1}{2})$;
(ii)   *the implemented policy is* $t^* = \frac{1}{2}$, *and both parties receive the
        same share of votes* $v^* = \frac{1}{2}$.

The results found under symmetry extend to a more generic case where the electorate is not symmetrically distributed. However, when the distribution is not symmetric one may construct examples where an equilibrium fails to exist[14]: Figure 6 illustrates indeed such an example (details available from the author). These examples are not easy to find and involve very strong asymmetries. The example here shows that, in the asymmetric case with an ideological component of the vote, existence requires some assumptions beyond the ones discussed so far. Ideally, such assumptions would be imposed on the primitives of the model (g, F, u, and η). But, because χ is implicitly derived from a complex computation, this would be an arduous task, and we choose to postulate a condition on χ, namely the log-concavity of χ, a premise also known in the literature as decreasing *hazard rate*[15]. As Figure 6(b) shows, this condition is violated in our example of non-existence of equilibrium.

Figure 6
Example of non-existence of equilibrium for very asymmetric distributions.
The density function is bi-triangular and $\eta(t;\tau) = -5|t-\tau|$

(a) Density function.   (b) χ is not quasi-concave in t_L for $t_R = 1$ (c) An equilibrium does not exist: the reaction curves do not intersect.

Theorem 5.2 *Let A3, A4, and A5 hold. Let χ be log-concave in t_L and let $1-\chi$ be log-concave in t_R. Then:*

  (i)   *a political equilibrium exists,*

  (ii)   *at equilibrium parties propose differentiated but not necessarily extreme policies, and*

  (iii)   *the implemented policy is the same in all equilibria.*

6. CONCLUDING REMARKS

The well known and widely used median voter theorem (at equilibrium, both candidates announce the same policy which coincides with the policy preferred by the median voter) is the standard result in political competition under perfect information or certainty. This elegant theoretical proposition is clearly at odds with the empirical evidence. In this paper we enrich the theoretical framework to allow for non-trivial policy-setting processes and for sophisticated voters who may abstain. First, we show that platform convergence is a non-robust feature created by the winner-takes-all assumption (Section 4). A divergent tendency in platform writing arises when the policy implemented by the winner party is sensitive to the electoral margin of victory. And this is true for any positive degree of sensitivity. Second, we show that, under reasonable assumptions, an equilibrium always exists (Section 5). At equilibrium, parties announce differentiated but non-extreme platforms, the implemented policy consistently differs from the ideal policy of the median voter (thus the median voter ceases to be decisive), voters concentrate around the announce platforms and abstention occurs among voters with extreme views as well as with moderate views.

The results provided in our model present a more accurate description of political competition. Consider, for example, two countries identical in

everything but on the distribution of their electorate. While country A's electorate follows a concentrated-in-the-middle, unimodal distribution, the electorate of country B is fractured into two radical groups (its density function presents two peaks with the majority of the population located around them). According to our model, parties in country A would move towards the center to avoid the alienation of the core of their constituency, while parties in country B will radicalize their positions[16]. Suppose that the median voter is the same in both countries: a moderate citizen. Our conclusions would not change. However, those models based on the median voter would be insensitive to the radicalization of the electorate in country B: not only the implemented policy would be the same moderate policy in country A and country B, but also parties would take moderate positions in both countries.

Thus, according to our findings, parties move towards the center of the political spectrum because *(i)* the mass of the electorate is concentrated around moderate views, and *(ii)* a party alienates the core of its constituency by radicalizing its platforms. This is in contrast with the standard claim -originated in Downs' (1957) seminal book- that parties tend to the middle because they compete for the vote of the median voter, even when the electorate is concentrated in radical positions.

The description of the political process can be enriched by extending some of its stylizations. Perhaps the most interesting extension would be to make the level of compromise endogenously determined by letting parties announce not only the platform they stand for, but also how much they will *compromise* if they win the election. An analysis of these possibilities is left for future research.

NOTES

1. The recent U.S. presidential election is the latest evidence. Several newspapers have pointed out that the narrow victory of Bush over Gore will have the moderation of his political and economic positions as the most likely consequence.

2. Poole and Rosenthal (1984b) use thermometer scores to measure the perceived location of candidates and policies; Fiorina (1974), Poole and Rosenthal (1984a), Poole and Daniels (1985) use interest groups ratings; Poole and Rosenthal (1991, 1997), Snyder (1996), Alesina and Rosenthal (1995) discover the same pattern using roll call votes in congress.

3. The implemented policy function described here is equivalent to *the legislative outcome function* in Austen-Smith (1989), the *implemented policy function* in Ortuño-Ortín (1997), and the *outcome function* F in Gerber and Ortuño-Ortín (1998).

4. Since F is a distribution function, there exists a unique probability measure μ such that $\mu([a,b]) = F(b) - F(a)$ for all a, b (see Durrett [1996], Corollary 1.3, p. 6).

5. Recall that policies are determined not only by the platforms announced but also by the relative support that each party receives.

6. Because, unlike under the winner-takes-all assumption, the implemented policy will generally differ from the platforms, it is natural to differentiate between voting preferences and the utility from implemented policies. Perhaps a similar distinction should be incorporated in winner-takes-all models when uncertainty is present. In those cases, the *expected* policy consistently differs from the platform of the winning party.

7. The use of the derivative comes from the assumption that there exists a continuum of citizens. However, our results do not depend on that assumption. We could consider instead a sufficiently large number of citizens whose distribution is represented by a density function. In that case, votes would have a positive, but tiny effect on the implemented policy, and the pragmatic component of voting would be defined by the change in the utility of the voter implied by the effect of his action on the implemented policy. The use of the continuum simplifies the presentation and is standard in the literature of mass

elections. Likewise, we could replace n, the fraction of active voters who vote for L, by the fraction of citizens who vote for L without affecting the results.

8. The pragmatic component of voting is similar to what Alesina and Rosenthal (1995) define as voting with *conditional sincerity*. They say that a voter votes with conditional sincerity if he prefers an increase in the expected vote for the party he votes for.

9. Note that a party's preferences do not need to coincide with those of a particular voter, even if they share the same ideal point.

10. Of course, λ_1, λ_2, ρ_1, and ρ_2 are functions of t_L, t_R, and v. In particular, for $t_L < t_R$,

$$\lambda_2 = \alpha\,\hat{t} + (1 - \alpha)\,t_L; \quad \lambda_1 = \max\{\beta\hat{t} + (1 - \beta)\,t_L, 0\};$$

$$\rho_1 = \alpha\,\hat{t} + (1 - \alpha)\,t_R; \quad \rho_2 = \min\{\beta\hat{t} + (1 - \beta)\,t_R, 1\};$$

$$\text{where,}\quad \alpha = -\frac{g'(\chi(t_L, t_R))(t_R - t_L)}{g'(\chi(t_L, t_R))(t_R - t_L) + 2},\quad \beta = \frac{g'(\chi(t_L, t_R))(t_R - t_L)}{g'(\chi(t_L, t_R))(t_R - t_L) - 2},$$

$$\text{and } \hat{t} = g(v)\,t_L + (1 - g(v))\,t_R.$$

11. To be precise, those voters with ideal policy equal to the implemented policy are indifferent between voting and abstaining. But everybody else (a subset of the electorate of measure one) prefers voting to abstaining.

12. Ortuño-Ortín (1997) and Gerber and Ortuño-Ortín (1998) restrict themselves to this type of weight function.

13. One may argue that symmetry facilitates that both parties locate at the extremes. Therefore, finding non-extreme platforms for symmetrically distributed populations strongly supports our claim.

14. In a previous paper (Llavador 2000), we showed that an equilibrium may fail to exist also in the classic winner-takes-all model when abstention is included and the distribution has several peaks.

15. In a similar context, Roemer (1997, p. 492) also requires the log-concavity of a compound function.

16. The implemented policy in country B may or may not be radical, depending on the relative weight of each group and on the "conciliatory" spirit of the winning party (the function g in the model).

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REFERENCES

ALESINA, Alberto; ROSENTHAL, Howard: *Partisan Politics.* Cambridge, Cambridge University Press, 1995.

ALESINA, Alberto; ROSENTHAL, Howard: "A theory of divided government", *Econometrica*, 64(6), 1995, p. 1311-1341.

AUSTEN-SMITH, David: "Sincere voting in models of legislative elections", *Social Choice and Welfare*, 6(4), 1989, p. 287-299.

AUSTEN-SMITH, David;: BANKS, Jeffrey: "Elections, coalitions and legislative outcomes", *American Political Science Review*, 82, 1998, p. 405-422.

BUDGE, Ian; HOFFERBERT, Richard I.: "Mandates and policy outputs: U.S. party platforms and federal expenditure", *American Political Science Review*, 84(1), 1990, p. 111-131.

BUDGE, Ian; HOFFERBERT, Richard I.: "The party mandate and the Westminster model: Election programs and government spending in Britain", *British Journal of Political Science*, 22, 1992, p. 151-182.

DIXIT, Avinash; GROSSMAN, Gene M.; GUL, Faruk: "A theory of political compromise", *Journal of Political Economy*, 108(3), 2000, p. 531-568.

DOWNS, Anthony: *An Economic Theory of Democracy.* New York, HarperCollins, 1957.

DURRETT, Richard: *Probability: Theory and Examples.* Belmont, Duxbury Press, second edition, 1996.

FIORINA, M.: *Representatives, Roll Calls, and Constituencies.* Lexington, MA., D.C. Heath, 1974.

GERBER, Anke; ORTUÑO-ORTÍN, Ignacio: "Political compromise and endogenous formation of coalitions", *Social Choice and Welfare*, 13(3), 1998, p. 445-454.

GROSSMAN, Gene M.; HELPMAN, Elhanan: "Electoral competition and special interest politics", *Review of Economic Studies*, 63(2), 1996, p. 265-286.

HOFFERBERT, Richard I.; KLINGEMANN, Hans-Dieter: "The policy impact of party programs and government declarations in the Federal Republic of Germany", *European Journal of Political Research*, 18, 1990, p. 277-304.

KING, Gary; LAVER, Michael; HOFFERBERT, Richard I.; BUDGE, Ian; McDONALD, Michael D.: "Party platforms, mandates, and government spending", *American Political Science Review*, 87(3), 1993, p. 744-750.

KLINGERMANN, Hans-Dieter; HOFFERBERT, Richard I.; BUDGE, Ian; KEMAN, Hans; PÉTRY, François; BERGMAN, Torbjorn; STROM, Kaare: *Parties, Policies and Democracy*. Boulder, Colorado, Westview Press, 1994.

LLAVADOR, Humberto G.: "Abstention and political competition", *Review of Economic Design*, 5(4), 2000, p. 411-432.

ORTUÑO-ORTÍN, Ignacio: "A spatial model of political competition and proportional representation", *Social Choice and Welfare*, 14, 1997, p. 427-438.

POOLE, Keith T.; STEVEN, Daniels: "Ideology, party, and voting in the U.S. Congress, 1959-80", *American Political Science Review*, 79, 1985, p. 373-399.

POOLE, Keith T.; ROSENTHAL, Howard: "The polarization of American politics", *Journal of Politics*, 46, 1984a, p. 1061-1079.

POOLE, Keith T.; ROSENTHAL, Howard: "U.S. presidential elections 1968-1980: A spatial analysis", *American Journal of Political Science*, 28, 1984b, p. 283-312.

POOLE, Keith T.; ROSENTHAL, Howard: "Patterns of congressional voting", *American Journal of Political Science*, 35, 1991, p. 228-278.

POOLE, Keith T.; ROSENTHAL, Howard: *Congress: A Political-economic History of Roll Call Voting*. New York and Oxford, Oxford University Press, 1997.

ROEMER, John E.: *Ideological parties and Downsian candidates: A synthesis of two perspectives*. Working paper, University of California, Davis, Department of Economics, 1996.

ROEMER, John E.: "Political-economic equilibrium when parties represent constituents: The unidimensional case", *Social Choice and Welfare*, 14, 1997, p. 479-502.

SNYDER, James M.: "Constituency preferences: California ballot propositions, 1974-1990", *Legislative Studies Quarterly*, 21, 1996, p. 463-488.

Appendix

Proof of Lemma 4.1

(i) That everybody votes follows directly from the definition of w_τ. Because, for all (t_L, t_R, v) and for all τ, $w(L; t_L, t_R, v, \tau) = - w(R; t_L, t_R, v, \tau)$ and $w(A; t_L, t_R, v, \tau) = 0$, then either voting for L or voting for R is at least as preferred as abstaining.

Now, let $t_L < t_R$ and $t^0 = \hat{t}(t_L, t_R, v)$. From (2.1), (2.2) and (2.3):

$$w(L; t_L, t_R, v, \tau) = u(t^0; \tau) \cdot g'(v) \cdot (t_L - t_R).$$

Thus, $w(L; t_L, t_R, v, \tau) > 0$ if and only if $\dfrac{\partial u}{\partial t}(t^0; \tau) < 0$. By single-peakedness,

$\dfrac{\partial u}{\partial t}(t^0; \tau) < 0$ if and only if $\tau < t^0$. Hence, everybody to the left of t^0 votes for L, while everybody to the right of t^0 votes for R.

(ii) From *(i)*, $V_L(t_L, t_R, v) = F(t^0)$ and $V_L(t_L, t_R, v) + V_R(t_L, t_R, v) = 1$. Thus, we only need to show that there exists a unique solution to the following equation in v:

$$v = F(g(v)\, t_L + (1-g(v))\, t_R) \qquad \text{if } t_L \leq t_R$$

$$\text{or} \qquad v = 1 - F(g(v)\, t_L + (1-g(v))\, t_R) \qquad \text{if } t_L > t_R$$

Consider 3 cases:

<u>Case 1</u>: $t_L < t_R$. Define $\Psi_1(v) = F(g(v)\, t_L + (1-g(v))\, t_R) - v$. Note that $g \in C^1$ (A1), and $F \in C^1$. Then $\Psi_1 \in C^1$, and

$$\Psi'_1(v) = f(\hat{t}(t_L, t_R, v)) \cdot g'(v) \cdot (t_L - t_R) - 1 < 0,$$

since $f \geq 0$, $g' \geq 0$, and $(t_L - t_R) < 0$. Thus, Ψ_1 is a continuous, strictly decreasing function. Because $\Psi_1(0) = F(t_R) > 0$ and $\Psi_1(1) = F(t_L) - 1 < 0$, there exists a unique v in $(0,1)$ that solves the equation $\Psi_1(v) = 0$.

<u>Case 2</u>: $t_L > t_R$. Define $\Psi_2(v) = 1 - F(g(v)\, t_L + (1-g(v))\, t_R) - v$. Similarly, $\Psi_2 \in C^1$, and

$$\Psi'_2(v) = -f(\hat{t}(t_L, t_R, v)) \cdot g'(v) \cdot (t_L - t_R) - 1 < 0, \qquad \text{(P.1)}$$

since now $(t_L - t_R) > 0$. Thus, Ψ_2 is a continuous, strictly decreasing function with $\Psi_2(0) = 1 - F(t_R) > 0$ and $\Psi_2(1) = -F(t_L) < 0$. Therefore there exists a unique solution v in $(0,1)$ to the equation $\Psi_2(v) = 0$.

<u>Case 3</u>: $t_L = t_R = t$. For any $t \in T$, $\hat{t}(t,t,v) = t$. Thus $v = F(t)$ is the unique solution we are looking for.

(iii) Let $t_L < t_R$. We know from *(ii)* that $\Psi'_1(v) \neq 0$. By the Implicit Function Theorem there exists an open ball $B_\varepsilon(\bar{t})$ of radius ε and center $\bar{t} = (\bar{t}_L, \bar{t}_R)$, and a differentiable function $\psi_{\bar{t}} : B_\varepsilon(\bar{t}) \to \nabla$, such that if $(t_L, t_R) \in B_\varepsilon(\bar{t})$, then

$$\Psi_1(\psi_{\bar{t}}(t_L, t_R); t_L, t_R) = 0. \tag{P.2}$$

Because, from *(ii)*, the solution to "$\Psi(v, t_L, t_R) = 0$" is unique, $\chi(t_L, t_R)$ is a singleton and, thus, can be viewed as a function. Then, (P.2) implies that $\chi(t_L, t_R) = \psi_{\bar{t}}(t_L, t_R)$ for all $(t_L, t_R) \in B_\varepsilon(\bar{t})$, i.e., $\psi_{\bar{t}}$ is the restriction of χ to $B_\varepsilon(\bar{t})$. Thus, χ is continuous on $B_\varepsilon(\bar{t})$ and, in particular, at $\bar{t}$. Since this is true for all $(\bar{t}_L, \bar{t}_R)$, with $\bar{t}_L < \bar{t}_R$, we conclude that χ is continuous on $\{(t_L, t_R) \in T^2 : t_L < t_R\}$. A similar argument applies for $t_L > t_R$. ◆

Proof of Lemma 4.2

Recall that $\widetilde{t}(t_L, t_R) = g(\chi(t_L, t_R)) t_R + (1 - g(\chi(t_L, t_R))) t_R$. By A1, g is a continuous function and, by Lemma 4.1*(iii)*, χ is continuous for $t_L \neq t_R$. Thus, $\widetilde{t}$ is continuous for $t_L \neq t_R$.

Let $\bar{t} \in T$, and $\left(t_L^k, t_R^k\right) \to (\bar{t}, \bar{t})$ as $k \to \infty$. It is easy to show that $\widetilde{t}\left(t_L^k, t_R^k\right) \to \widetilde{t}(\bar{t}, \bar{t})$. Observe that, $\lim_{k \to \infty} \chi\left(t_L^k, t_R^k\right) = F(\bar{t})$, and $\widetilde{t}\left(t_L^k, t_R^k\right) = t_R^k + g(\chi\left(t_L^k, t_R^k\right))\left(t_L^k - t_R^k\right)$. Then, $\lim_{k \to \infty} \widetilde{t}\left(t_L^k, t_R^k\right) = \bar{t} = \widetilde{t}(\bar{t}, \bar{t})$. It follows that $\widetilde{t}$ is continuous for $t_L = t_R$ as well. Hence, $\widetilde{t}$ is continuous on T^2.

Finally, since $\widetilde{t}$ and π_J are continuous, we have that $\widetilde{\pi}_J(t_L, t_R) = \pi_J(\widetilde{t}(t_L, t_R))$ is also a continuous function on T^2. ◆

Proof of Lemma 4.3

(i) We only prove the lemma for party L. The proof for party R is symmetric.

1. Claim: If t', t" $\in BR_L(t_R)$, then $\widetilde{t}(t', t_R) = \widetilde{t}(t'', t_R)$. Suppose not, say $\widetilde{t}(t', t_R) < \widetilde{t}(t'', t_R)$. By the strict quasi-concavity of π_L: $\widetilde{t}(t', t_R) < \tau_L < \widetilde{t}(t'', t_R)$. But $\widetilde{t}$ is a continuous function (Lemma 4.2). Thus, there exists a

t''' between t' and t'' such that $\tilde{t}(t''',t_R) = \tau_L$, i.e. $\tilde{\pi}_L(t''',t_R) = \pi_L(\tau_L) >$ $\tilde{\pi}_L(t',t_R)$, contradicting the fact that t' is a best response.

2. It is easy to see that $BR_L(\tau_L) = \{\tau_L\}$, since $\tilde{t}(t,\tau_L) \neq \tau_L$ for all $t \neq \tau_L$.

3. Let $t_R > \tau_L$ and assume that t' and t'' are best responses for party L to t_R, with $t' \neq t''$. Assume, without loss of generality, that $t' < t''$. Then $t'' < t_R$. Otherwise party L would be better-off by choosing t_R: $\tau_L < \tilde{t}(t_R,t_R) = t_R < \tilde{t}(t'',t_R)$.

We compute the derivative of $\tilde{t}$ w.r.t. t_L as

$$\frac{\partial \tilde{t}}{\partial t_L}(t_L,t_R) = -g'(\chi(t_L,t_R)) \frac{\partial \chi(t_L,t_R)}{\partial t_L}(t_R-t_L) + g(\chi(t_L,t_R)),$$

where χ is implicitly defined as the unique solution of $F(g(v)\,t_L+(1-g(v))\,t_R) - v = 0$ (see the proof of Lemma 4.1 (ii)). Then, using the fact that

$$\frac{\partial \chi(t_L,t_R)}{\partial t_L} = \frac{f(\tilde{t})g(\chi(t_L,t_R))}{f(\tilde{t})g'(\chi(t_L,t_R))(t_R - t_L)+1}$$

we can conclude that

$$\frac{\partial \tilde{t}}{\partial t_L}(t_L,t_R) = \frac{g(\chi(t_L,t_R))}{f(\tilde{t}(t_L,t_R))g'(\chi(t_L,t_R))(t_R - t_L)+1} > 0$$

But this implies that $\tilde{t}(t',t_R) < \tilde{t}(t'',t_R)$, a contradiction with step 1.

4. A similar argument applies to $t_R < \tau_L$. Thus, the best-response correspondence is single-valued.

5. Continuity is a direct implication of the Maximum Theorem.

(ii) If t_L^0 exists, then it is a global maximizer of $\tilde{\pi}_L(t,t_R)$ and clearly the best choice for party L. If there is no $t \in T$ such that $\tilde{t}(t,t_R) = \tau_L$, then it follows from the continuity of $\tilde{t}$ that $\tilde{t}(t,t_R) > \tau_L$ for all $t \in T$ (recall that $\tilde{t}(t_R,t_R) = t_R > \tau_L$). Because $\tilde{t}$ is monotone increasing in t_L (see step 3 above) and π_L is single-peaked, it follows that $\tau_L < \tilde{t}(\underline{t},t_R) < \tilde{t}(t,t_R)$ for all $t > \underline{t}$. Thus, $BR_L(t_R) = \underline{t}$.

(iii) The proof is a symmetric replica of (ii). ◆

Proof of Theorem 4.1

(i) Existence. Let BR: $T^2 \to T^2$ defined by $BR(t_L, t_R) = (BR_L(t_R), BR_R(t_L))$. From Lemma 4.3, BR is a continuous function. Thus, by Brouwer's fixed point theorem, there exists $(t_L^*, t_R^*) \in T^2$ such that $BR(t_L^*, t_R^*) = (t_L^*, t_R^*)$. It follows that (t_L^*, t_R^*) is an equilibrium.

(iii) Let (t_L^*, t_R^*) be an equilibrium and suppose that $(t_L^*, t_R^*) \neq (\underline{t}, \bar{t})$. If $t_L^* \neq \underline{t}$, then $\tilde{t}(t_L^*, t_R^*) = \tau_L$ (Lemma 4.3). If $t_L^* = \underline{t}$, then $t_R^* \neq \bar{t}$ and $\tilde{t}(t_L^*, t_R^*) = \tau_R$ (Lemma 4.3, once more).

(ii) Uniqueness. Suppose that (t_L^1, t_R^1) and (t_L^2, t_R^2) are two different equilibria. From *(iii)*, at least one of them is not $(\underline{t}, \bar{t})$ and it implements the ideal policy of one party. Without loss of generality, take $(t_L^1, t_R^1) \neq (\underline{t}, \bar{t})$, and suppose that $\tilde{t}(t_L^1, t_R^1) = \tau_L$ (nothing would change had we taken τ_R instead).

Because τ_L is implemented, $BR_R(t_L^1) = \bar{t}$ (Lemma 4.3). That is, $(t_L^1, \bar{t})$ is an equilibrium and, therefore, $t_L^1 = BR_L(\bar{t})$. It follows that $(t_L^2, t_R^2) \neq (\underline{t}, \bar{t})$ as well, since BR_L is a function and $(t_L^2, t_R^2) \neq (t_L^1, \bar{t})$. Thus it must be that $\tilde{t}(t_L^2, t_R^2) = \tau_R$ and $t_L^2 = \underline{t}$, by a similar reasoning as above.

Therefore, we have shown that $\tilde{t}(t_L^1, t_R^1) = \tau_L$, $\tilde{t}(t_L^2, t_R^2) = \tau_R$, $t_R^1 = \bar{t}$ and $t_L^2 = \underline{t}$.

From Lemma 4.1, $\chi(t_L^1, t_R^1) = F(\tau_L)$ and $\chi(t_L^2, t_R^2) = F(\tau_R)$. Then, using the definition of $\tilde{t}$,

$$\tau_L = \tilde{t}(t_L^1, \bar{t}) = g(F(\tau_L)) \, t_L^1 + (1 - g(F(\tau_L))) \, \bar{t}, \text{ and}$$

$$\tau_R = \tilde{t}(\underline{t}, t_R^2) = g(F(\tau_R)) \, \underline{t} + (1 - g(F(\tau_R))) \, t_R^2.$$

Solving for t_L^1 and t_R^2, we obtain

$$t_L^1 = \frac{\tau_L - (1 - g(F(\tau_L)))\bar{t}}{g(F(\tau_L))} \geq \underline{t},$$

$$t_R^2 = \frac{\tau_R - g(F(\tau_R))\underline{t}}{1 - g(F(\tau_L))} \leq \bar{t}.$$

Rearranging terms, we obtain

$$\tau_L \geq g(F(\tau_L))\,\underline{t} + (1 - g(F(\tau_L)))\,\bar{t},$$

$$\tau_R \leq g(F(\tau_R))\,\underline{t} + (1 - g(F(\tau_R)))\,\bar{t}.$$

But this contradicts the condition $\tau_L < \tau_R$, since g is a non-decreasing function. Therefore, the equilibrium is unique. ♦

Proof of Lemma 5.1

Intervals and Positive Turnout. (Please refer to Figure 5 for intuition). Take $t_L, t_R \in T$ and $\nu \in [0,1]$. Let $t_L < t_R$ (the proof can be easily replicated for $t_L > t_R$). Write $\tilde{w}(s;\tau) = w(s; t_L, t_R, \nu, \tau)$, $\tilde{\eta}(s;\tau) = \hat{\eta}(s; t_L, t_R, \tau)$, $\tilde{v}(s;\tau) = \tilde{w}(s;\tau) + \tilde{\eta}(s;\tau)$, and $t^* = \hat{t}(t_L, t_R, \nu)$.

From (2.3) and A4, $\tilde{w}(L,\tau) = (t^* - \tau)(t_R - t_L)$ and $\tilde{w}(R,\tau) = -\tilde{w}(L,\tau)$. Because $t_R > t_L$, every $\tau < t^*$ prefers L to R, while every $\tau > t^*$ prefers R to L. Thus, every citizen to the left (right) of t^* either votes for L(R) or abstains.

Figure 5

Analysis of vote allocation. Voters in the interval $[\lambda_1, \lambda_2]$ vote for L. Voters in the interval $[\rho_1, \rho_2]$ vote for R. The rest of the electorate abstains.
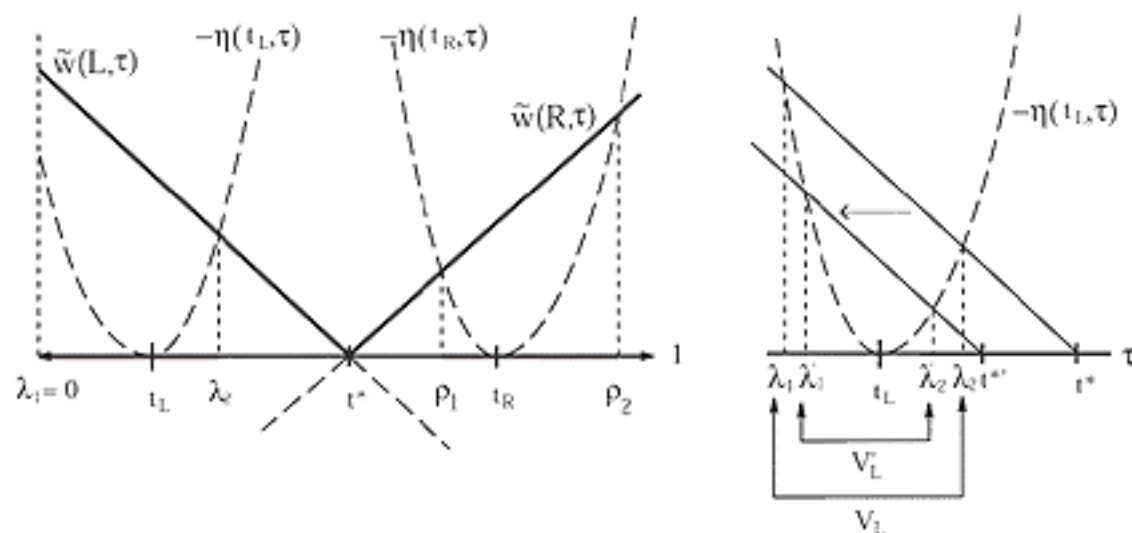
(a) Voters concentrate in intervals   (b) As ν increases, the support

Observe that $\widetilde{W}(L,\tau)$ is an affine and decreasing function of τ. On the other hand, η is concave in τ and $\eta(t_L;t_L) = 0 \geq \eta(t_L;\tau)$ for all τ. Because $\widetilde{W}(L,t_L) \geq \eta(t_L,t_L) = 0$ and $\widetilde{W}(L,t^*) = 0 \geq \eta(t_L,t^*)$, we can define λ_1 and λ_2 as follows (see Figure 5(a)):

(i)   If $\widetilde{V}(L,\tau) > 0$ for all $\tau < t_L$, take $\lambda_1 = 0$. Otherwise, let λ_1 be the unique $\tau \leq t_L$ that solves $\widetilde{V}(L,\tau) \equiv \widetilde{W}(L,\tau) + \eta(t_L,\tau) = 0$. (Uniqueness is obtained from the linearity of $\widetilde{W}$ and the concavity of η.)

(ii)   Take λ_2 to be the unique $\tau \in [t_L, t^*]$ satisfying $\widetilde{V}(L,\tau) = 0$. Observe that $\widetilde{V}$ is monotone decreasing in τ over $[t_L, t^*]$ (both $\widetilde{W}(L,\tau)$ and $\eta(t_L,\tau)$ are decreasing), and $\widetilde{V}(L, t_L) \geq 0 \geq \widetilde{V}(L, t^*)$.

It follows that for $\tau \in [\lambda_1, \lambda_2]$, $\widetilde{V}(L,\tau) \geq \max\{0, \widetilde{V}(R,\tau)\}$. Hence $\tau \in L(t_L, t_R, v)$. A symmetric argument obtains ρ_1 as the unique $\tau \leq t_R$ that solves $\widetilde{V}(R,\tau) = 0$; and $\rho_2 = 1$ if $\widetilde{V}(R, \tau) > 0$ for all $\tau \geq t_R$, otherwise ρ_2 equals the unique $\tau \geq t_R$ that solves $\widetilde{V}(R,\tau) = 0$.

Note that $\lambda_1 = \lambda_2$ only if $t^* = t_L$ (i.e. $v = 1$), and $\rho_1 = \rho_2$ only if $t^* = t_R$ (i.e. $v = 0$). Therefore, $V_L(t_L, t_R, v) + V_R(t_L, t_R, v) > 0$.

Continuity and monotonicity. Because party L's voters concentrate in the interval I_λ, we can write

$$V_L(t_L, t_R, v) = F(\lambda_2(t_L, t_R, v)) - F(\lambda_1(t_L, t_R, v)),$$

where λ_2 is implicitly defined as the unique $\tau > t_L$ that solves $v(L; t_L, t_R, v, \tau) = 0$. Then, by the Implicit Function Theorem, λ_2 is continuous and

$$\frac{\partial \lambda_2}{\partial v}(t_L, t_R, v) = -\frac{\dfrac{\partial v}{\partial v}(L; t_L, t_R, v, \tau)}{\dfrac{\partial v}{\partial \tau}(L; t_L, t_R, v, \tau)}$$

evaluated at $\tau = \lambda_2$. Because, at $\tau = \lambda_2$,

$$\frac{\partial v}{\partial v}(L; t_L, t_R, v, \tau) = \frac{\partial w}{\partial v}(L; t_L, t_R, v, \tau) =$$

$$(t_R - t_R) \frac{\partial \hat{t}(t_L, t_R, v)}{\partial v} < 0, \text{ and}$$

$$\frac{\partial v}{\partial \tau}(L; t_L, t_R, v, \tau) = \frac{\partial w}{\partial \tau}(L; t_L, t_R, v, \tau) + \frac{\partial \eta}{\partial \tau}(t_L; \tau) < 0,$$

then $\frac{\partial \lambda_2}{\partial v}(t_L, t_R \ v) < 0$ (see Fig 5(b).) That is, λ_2 decreases as v increases.

If $\lambda_1 = 0$, it cannot decrease any further. If $\lambda_1 > 0$, then it is continuous and, for $\tau = \lambda_1$,

$$\frac{\partial \lambda_1}{\partial v}(t_L, t_R, v) = \frac{\dfrac{\partial w}{\partial v}(L; t_L, t_R, v, \tau)}{\dfrac{\partial w}{\partial \tau}(L; t_L, t_R, v, \tau) + \dfrac{\partial \eta}{\partial \tau}(t_L; v)} > 0.$$

Since, first, the numerator is positive: $\frac{\partial w}{\partial v} = (t_R - t_L) \frac{\partial \hat{t}}{\partial v} < 0$; and, second,

the denominator is negative $\frac{\partial w}{\partial \tau} = -(t_R - t_L) < 0$, $\frac{\partial \eta}{\partial \tau} = -\zeta > 0$ and

$\left| \frac{\partial w}{\partial \tau} \right| < \left| \frac{\partial \eta}{\partial \tau} \right|$ because w is linear in τ, $-\eta$ is convex in τ and $w(t_L, \lambda_1) = \eta(t_L, \lambda_1)$ (see Fig5(b)).

It follows that

$$\frac{\partial V_L}{\partial v} = f(\lambda_2) \frac{\partial \lambda_2}{\partial v} - f(\lambda_1) \frac{\partial \lambda_1}{\partial v} \leq 0.$$

Similar calculations for V_R obtain that V_R is non-decreasing in v. ◆

Proof of Lemma 5.2

Take $t_L \neq t_R$. From Lemma 5.1 V_L and V_R are continuous and $V_L(t_L,t_R,\nu)+ V_R(t_L,t_R,\nu) > 0$. Define

$$\Psi(\nu\,;\,t_L,\,t_R) = \frac{V_L(t_L,\,t_R,\,\nu)}{V_L(t_L,\,t_R,\,\nu) + V_R(t_L,\,t_R,\,\nu)} - \nu$$

We know that $\nu \in \chi(t_L,t_R)$ if and only if $\Psi(\nu\,;\,t_L,t_R) = 0$ (see (2.6)), that Ψ is continuous and decreasing (Lemma 5.1), that $\Psi(0;t_L,t_R) \geq 0$, and that $\Psi(1;t_L,t_R) \leq 0$. Thus, there exists a unique ν such that $\Psi(\nu\,;\,t_L,t_R) = 0$. That is, χ is non-empty and single-valued. Continuity follows directly from the Implicit Function Theorem. $\blacklozenge$

Proof of Theorem 5.1 (Symmetry)

Consider the game $\Gamma = (2,T,\pi_L,\pi_R)$, where the payoffs of the parties are $\pi_L(t_L,t_R) = -\bar{t}(t_L,t_R)$ and $\pi_R(t_L,t_R) = \bar{t}(t_L,t_R)$, respectively. This is a two-party zero-sum game. Let

$$\tilde{t}_R^0 = \max_{t_R} \min_{t_L} \tilde{t}(t_L,t_R), \text{ and}$$

$$\tilde{t}_L^0 = \max_{t_L} \min_{t_R} \tilde{t}(t_L,t_R).$$

Because party R can surely implement at least $\tilde{t}_R^0$ and party L can keep the policy from being more than $\tilde{t}_L^0$ then it cannot be that $\tilde{t}_R^0$ is larger than $\tilde{t}_L^0$, i.e., $\tilde{t}_R^0 \leq \tilde{t}_L^0$. On the other hand, a party can always impose a policy $t^* = \frac{1}{2}$ by announcing asymmetric platform to its opponent's (i.e. if R announces t_R, L could announce $1-t_R$, obtain the same fraction of votes than R, and set $\tilde{t}(1-t_R, t_R) = \frac{1}{2}(1-t_R) + \frac{1}{2}t_R = \frac{1}{2}$.) It follows that $\tilde{t}_L^0 \leq \frac{1}{2}$ and $\tilde{t}_R^0 \geq \frac{1}{2}$. Therefore, $\tilde{t}_L^0 = \tilde{t}_R^0 = \frac{1}{2}$.

That is, an equilibrium exits and the implemented policy is $\frac{1}{2}$.

Let (t_L^*, t_R^*) be an equilibrium of Γ. Then $\tilde{t}(t_L^*,t) \leq \tilde{t}(t_L^*,t_R^*) \leq \tilde{t}(t,t_R^*)$ for all t, i.e., party L cannot lower the implemented policy, neither can party R increase it. Therefore, (t_L^*, t_R^*) is a

political equilibrium. Furthermore, because the political game is a strictly competitive game, all equilibria yield the same payoffs for the parties: $t^* = \dfrac{1}{2}$ ♦

Proof of Theorem 5.2

(i) We follow the steps of the symmetric case. First, we construct a zero-sum game and prove that an equilibrium always exists. Second, we show that if the zero-sum game equilibrium policy locates between the ideal policies of the parties, then it is an equilibrium of the political game. On the other hand, if the equilibrium policy is more extreme than a party's bliss point, there exists a political equilibrium where that party is able to implement its ideal policy.

Consider the two-party, zero-sum game $\Gamma = (T, \Pi_L, \Pi_R)$, with payoff functions $\Pi_L(t_L, t_R) = -\hat{t}(t_L, t_R, \chi(t_L, t_R))$ and $\Pi_R(t_L, t_R) = \hat{t}(t_L, t_R, \chi(t_L, t_R))$, respectively. Use the implemented policy function (A3) to write:

$$\Pi_L(t_L, t_R) = -t_R + \chi(t_L, t_R)\,(t_R - t_L), \text{ and}$$
$$\Pi_R(t_L, t_R) = t_L + (1 - \chi(t_L, t_R))\,(t_R - t_L).$$

Therefore, maximizing Π_L (with respect to t_L) and Π_R (with respect to t_R) is equivalent to maximizing

$$\kappa_L(t_L, t_R) \equiv \chi(t_L, t_R)\,(t_R - t_L) \text{ and}$$
$$\kappa_R(t_L, t_R) \equiv (1 - \chi(t_L, t_R))\,(t_R - t_L).$$

Next we show that the log-concavity of χ implies the quasi-concavity of κ_L. The function κ_L is quasi-concave in t_L if $\dfrac{\partial^2 \kappa_L}{\partial t_L^2}\left(t_L, t_R\right) \le 0$ for all t_L such that $\dfrac{\partial \kappa_L}{\partial t_L}\left(t_L, t_R\right) \le 0$. Write $\chi_L = \dfrac{\partial \chi}{\partial t_L}$ and $\chi_{LL} = \dfrac{\partial^2 \chi}{\partial t_L^2}$. We compute:

$$\frac{\partial \kappa_L}{\partial t_L}\left(t_L, t_R\right) = \chi_L(t_L, t_R)\,(t_R - t_L) - \chi(t_L, t_R), \text{ and} \qquad (P.3)$$

$$\frac{\partial^2 \kappa_L}{\partial t_L^2}\left(t_L, t_R\right) = \chi_{LL}(t_L, t_R)\,(t_R - t_L) - 2\,\chi_L(t_L, t_R). \qquad (P.4)$$

Let $t_L \in T$ such that $\dfrac{\partial \kappa_L}{\partial t_L} = 0$, then from (P.3) we get $(t_R - t_L) = \dfrac{\chi(t_L, t_R)}{\chi_L(t_L, t_R)}$ and, substituting into (P.4), we obtain (all functions are evaluated at (t_L, t_R)):

$$\left.\frac{\partial^2 \kappa_L}{\partial t_L^2}\right|_{\frac{\partial \kappa_L}{\partial t_L} = 0} = \chi_{LL}\big|_{\frac{\partial \kappa_L}{\partial t_L}=0} \cdot \frac{\chi}{\chi_L\big|_{\frac{\partial \kappa_L}{\partial t_L}=0}} - 2 \cdot \chi_L\big|_{\frac{\partial \kappa_L}{\partial t_L}=0}.$$

Thus,

$$\left.\frac{\partial^2 \kappa_L}{\partial t_L^2}\right|_{\frac{\partial \kappa_L}{\partial t_L}=0} \geq 0 \Leftrightarrow \chi_{LL}\big|_{\frac{\partial \kappa_L}{\partial t_L}=0} \cdot \chi - 2 \cdot \left(\chi_L\big|_{\frac{\partial \kappa_L}{\partial t_L}=0}\right)^2 \leq 0, \text{(P.5)}$$

since $\chi_L\big|_{\frac{\partial \kappa_L}{\partial t_L}=0} = \cdot \dfrac{\chi}{t_R - t_L} > 0.$

The log-concavity of χ implies that $\chi_{LL}\,\chi - (\chi_L)^2 \leq 0$. It follows from (P.5) that κ_L is quasi-concave. A symmetric argument shows that the log-concavity of $(1-\chi)$ implies that κ_R is quasi-concave in t_R.

Let $BR_L(t_R)$ be the set of maximizers of $\Pi_L(t_L, t_R)$ for a given t_R (i.e., the best response of L to t_R). Since κ_L is quasi-concave, BR_L is convex-valued. From the Theorem of the Maximum, BR_L is non-empty and upper hemi-continuous. Similarly, BR_R is also non-empty, convex-valued and upper hemi-continuous. Then, the correspondence $BR:T^2 \to T^2$: $BR(t_L, t_R) = (BR_L(t_R), BR_R(t_L))$ is non-empty, convex-valued and upper hemi-continuous. By Kakutani's fixed point theorem there exists a (t_L^*, t_R^*) in $BR(t_L^*, t_R^*)$. Therefore, (t_L^*, t_R^*) is an equilibrium of Γ.

Let $t^* = \tilde{t}(t_L^*, t_R^*)$. If $\tau_L \leq t^* \leq \tau_R$, then neither party can pull the implemented policy towards its ideal point given the platform announced by the other party. Hence, (t_L^*, t_R^*) is a political equilibrium. Suppose, on the other hand, that $t^* \notin [\tau_L, \tau_R]$. Take, for example, $t^* < \tau_L$. Since Γ is a zero-sum game, it follows that L can secure a policy as to the left as t^*. Because $\tau_L > t^*$, t^* represents a lower payoff for L in the zero-sum game, and party L can definitely secure τ_L. Thus, if $t^* < \tau_L$, there exists a political equilibrium where the implemented policy is τ_L. Similarly, if $t^* > \tau_R$, there exists a political equilibrium with τ_R as the

implemented policy.

(ii) Because the political game is a strictly competitive game, the implemented policy is the same in all equilibria.

(iii) Finally, observe that one can never have an equilibrium where both parties propose the same policy. At least one party can always pull the policy a bit closer to its ideal point by differentiating its platform and capturing a small fraction of the vote. As the example in Section 3 shows, platforms may not be extreme at equilibrium. ◆