Trade Effects on the Personal Distribution of Wealth *

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Abstract

This paper develops a dynamic Heckscher-Ohlin model and studies the interaction between international trade and the dynamics of the wealth distribution in a small open economy. I prove that trade generates a permanent decline in inequality (relative to the level under autarky) if the economy opens to trade with a stock of capital sufficiently close to its steady state level. I then use numerical simulations to study wealth distribution dynamics when the economy opens to trade while far away from the steady state. My results suggest that trade always helps to reduce inequality in wealth.

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1 Introduction

In the theoretical literature on international trade, the Stolper-Samuelson theorem stands as one of the main results about income distribution. Roughly speaking, the theorem states that with the opening to international trade, prices of relatively abundant factors increase and prices of relatively scarce factors decrease. This change in factor prices is the consequence of a more specialized production in goods that use intensively the relatively abundant factors.[1] Thus, the theorem describes the changes in the *functional distribution* of income of an economy that opens to international trade.[2] However, it is not obvious how these changes may affect personal inequality in income and/or wealth. Perhaps surprisingly, the effects of international trade on the personal distribution of wealth have received little attention in the theoretical literature.

In this paper, I bring together the literatures on optimal paths of capital accumulation and on inequality, in the context of a small open economy. Specifically, I extend the study by Chatterjee (1994) of the dynamics of the distribution of income and wealth in a standard one-sector neoclassical model of growth to the case of a dynamic Heckscher-Ohlin model of international trade similar to that in Atkeson and Kehoe (2000). Chatterjee (1994) showed that in an economy where agents differ only in their initial wealth,

[1] There are several empirical applications of the Stolper-Samuelson theorem as an explanation for observed wage differentials between skilled and unskilled workers in trading economies. The empirical evidence supporting the theorem is mixed. See, among others, Wood (1997) and Robertson (2001).

[2] Ripoll (2000) develops a three-good, three-factor, dynamic model and shows that in addition to relative abundance of factors, the timing of the opening to trade has sizable effects on steady states and on the dynamic path of factor prices.

the transition towards the steady state from below has a negative effect on the degree of lifetime wealth/income inequality prevailing in the economy.[3] His findings are relevant in the study of the effects of international trade because in a dynamic model, trade is likely to give rise to a different steady state than under autarky. Following Atkeson and Kehoe (2000), I analyze the case of a small open-economy that trades intermediate goods with the rest of the world. In particular, I assume that all economies are identical and that the only difference among them is that the rest of the world is already at the steady state. Because of the small open-economy assumption, trade has no effects on the distribution of wealth in the rest of the world. Nevertheless, the distribution of wealth in the small economy does change over the transition to the steady state. Over a transition to the steady state from below (when the initial stock of capital is smaller than its long run level), these changes in the small open-economy are the result of two conflicting effects: an "international trade" effect, which tends to reduce inequality in the functional distribution of income (through the Stolper-Samuelson theorem familiar from static models), and a "transition" effect that tends to increase inequality in the personal distribution of income and wealth.

I state necessary and sufficient conditions for inequality in wealth to fall – relative to the level under autarky – both at the moment when the economy opens to international trade, and in the long run. I prove that these conditions are satisfied in several cases. In particular, they are satisfied when the

[3] Caselli and Ventura (2000) extend these results in a continuous time model with additional sources of heterogeneity (preferences and labor productivity); see also Sorger (2000) where the effect of a leisure/labor decision is studied. Relatedly, Obiols-Homs and Urrutia (2003) study the dynamics of the distribution of assets.

opening to trade occurs once the capital stock of the small open-economy is sufficiently close to its steady state level under trade. I extend these results using numerical methods and compare the dynamics of inequality under trade and under autarky when the economy opens to international trade far away from the steady state (i.e., starting from arbitrary levels of capital). I find that inequality in wealth under trade is always smaller than under autarky. I also find that the sooner the economy opens to international trade, the smaller will be the level of inequality in the long run. These results suggest that the "international trade" effect dominates the "transition" effect.

The intuition to explain the results comes from the Stolper-Samuelson theorem. When the small economy opens to international trade, labor is relatively more abundant than in the rest of the world. Since the assumptions of the Heckscher-Ohlin model are satisfied, the economy then finds it optimal to specialize production in the labor intensive good, thus labor income increases and capital income decreases. These changes in factor prices benefit more those agents for whom labor income represents a larger fraction of their wealth portfolio, i.e., agents with relatively fewer units of capital. As a consequence of trade, therefore, inequality shrinks.

The results in this paper are related to other studies about the relationship between trade and inequality. For instance, Fisher and Serra (1996) develop a static model in which the income of the median voter determines whether an economy opens to free trade or not with other richer/poorer economies. More related to this paper, Das (2000) studies the effects of trade among similar economies on the personal distribution of income and wealth in an overlapping generations model where agents live for one period (and there is a bequest motive), markets are imperfectly competitive, and where capital is tradable. In my model I relax these assumptions and obtain similar results to Das. Also, Wynne (2003) follows a different approach and studies how the distribution of wealth affects the pattern of trade in a model where firms in different sectors have differential access to credit. In Wynne's model the distribution of wealth determines comparative advantages, thus the distribution affects the pattern of trade and trade affects the distribution, and it is able to explain Trefler's missing trade mystery. With respect to applied work, the empirical evidence regarding the effects of international trade on inequality is inconclusive. For instance, Edwards (1997) reports that trade reforms do not seem to affect income distribution, Litwin (1998) finds that trade openness in general worsen income distribution, and Wei and Wu (2001) find that openness to trade and urban-rural inequality are negatively associated in Chinese cities. In this respect, my results suggest that to understand trade effects on personal inequality we need to look at the pattern of production and specialization.

The paper continues as follows: Section 2 introduces a world with many competitive economies, section 3 describes equilibrium dynamics under autarky and shows that the results in Chatterjee (1994) can be extended to two-sector economies. Section 4 studies the effects of trade on the distribution of wealth of a small open-economy. Section 5 extends the previous results using numerical methods, and section 6 concludes. An appendix at the end of the paper contains proofs and a description of the numerical methods used in section 5.

2 The model

There is a large number of small economies. These economies are identical in all respects except perhaps in the initial distribution of capital among agents. A typical economy is described below. To fix notation, a variable x_t^i denotes the value of x corresponding to agent i in a period t, and x_t denotes the average over agents. These variables under international trade are denoted $\hat{x}_t^i$, and $\hat{x}_t$. Long run values under autarky and trade are denoted respectively x^* and $\hat{x}^*$.

2.1 Production

In each economy production is organized in two sectors, one producing a final good that can be devoted to consumption and investment, and the other producing intermediate goods which are used as inputs in the final goods sector. In the intermediate goods sector there are two industries producing goods x and y using capital and labor as primary factors. Technologies for production display constant returns to scale and the only difference between them is that they use primary inputs in different intensities: $x = k_x^{\theta} l_x^{1-\theta}$, $y = k_y^{\eta} l_y^{1-\eta}$, with $\theta, \eta \in (0,1)$. In the previous equations the subindices x and y of the primary factors indicate amounts used in the production of each good. Assuming $\theta > \eta$, the production of x is capital intensive. Furthermore, by assuming Cobb-Douglas technologies I am also ruling out factor intensity reversals. The technology in the final goods sector also displays constant returns to scale and uses as inputs intermediate goods only: $z = x^{\gamma}y^{1-\gamma}$, with $\gamma \in (0,1)$. Finally, in each sector there is a large number of firms and markets are perfectly competitive.

2.2 Consumers, preferences, and endowments

Each economy is inhabited by N agents indexed by $i = 1, 2, 3, ...N$. Each of these agents behaves so as to maximize the present value of the utility derived from the consumption of the homogeneous final good over an infinite horizon:

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i), \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, which is taken to be the same for all agents. In the rest of the paper it will be assumed that preferences take the form of $u(c_t^i) = \log(c_t^i - \bar{c})$, where $\bar{c} \geq 0$ is a real number (the same for all agents). If $\bar{c} > 0$, the marginal utility of consumption can be arbitrarily large even for strictly positive levels of consumption. The interpretation in this case is that there is a minimum consumption level and it will be required that $c_t^i - \bar{c} \geq 0$. As shown in Chatterjee (1994), the implications of trade on the personal distribution of wealth I derive in this paper will also hold under a more general class of utility functions.[4]

Agents are endowed with k_0^i units of productive capital in the first period.

[4]This class includes $u(c) = \rho(\bar{c} + \psi c)^{\sigma}$ with a) $\sigma < 1$ but different from zero, $\rho = 1/\sigma$, $\psi = 1$ and $\bar{c}$ a real number, or b) $\sigma = 2$, $\rho = -1/2$, $\psi = -1$ and $\bar{c} > 0$. It also includes the case of $u(c) = -\bar{c}\exp(-\psi c)$ with both $\bar{c}$ and ψ strictly positive.

The initial endowment of capital is the only difference among agents. To transfer capital across periods agents have access to the following investment technology:

$$k_{t+1}^i = i_t^i + (1-\delta)k_t^i. \tag{2}$$

In the previous equation $\delta \in [0,1]$ is the depreciation rate of capital. In addition to the initial endowment of capital, in the beginning of each period agents receive one unit of time which they inelastically supply as labor. Both capital and labor are freely mobile in the intermediate goods sector.

2.3 An agent's problem

The utility maximization problem a given agent i solves can be written formally as follows:

$$\begin{array}{ll} \max & \sum_{t=0}^{\infty} \beta^t \log(c_t^i - \bar{c}) \\ \text{s. to} & c_t^i + i_t^i = w_t + r_t k_t^i, \\ & k_{t+1}^i = i_t^i + (1-\delta)k_t^i, \\ & c_t^i \geq \bar{c},\ k_t^i \geq 0,\ \forall t \geq 0,\ \text{given } k_0^i, \end{array} \tag{3}$$

where we have used the fact that free mobility of primary factors and perfect competition in factor markets imply a unique equilibrium rental rate of capital, r_t, and labor, w_t. Assuming the initial condition for capital is large enough so that the solution to the problem is interior, the first-order necessary condition for optimality is given by

$$\frac{1}{c_t^i - \bar{c}} = \beta R_{t+1} \frac{1}{c_{t+1}^i - \bar{c}}, \tag{4}$$

where $R_{t+1} = r_{t+1} + 1 - \delta$ is the interest factor. Following Chatterjee (1994), let lifetime wealth of agent i in a period t be given by

$$\omega_t^i = R_t \left[k_t^i + W_t \right], \tag{5}$$

where $W_t = \sum_{j=0}^{\infty} (w_{t+j} / (\prod_{s=0}^{j} R_{t+s}))$. Notice that ω_t^i is composed of the real value of capital at the end of the period plus the real present value of labor. This measure of wealth is useful for the purposes of this paper because it summarizes the changes in factor-prices, which is a central issue in international trade theory.

The measure of inequality I use is the coefficient of variation (standard deviation divided by the mean) in ω_t. In order to study the dynamics of inequality it is convenient to substitute repeatedly Equation (4) in the budget constraint in (3) and use the definition of ω_t^i to obtain:

$$c_t^i = (1 - \beta)\omega_t^i + P_t, \tag{6}$$

where $P_t = \bar{c} \sum_{j=0}^{\infty} ((\beta R_{t+1+j} - 1) / (\prod_{s=0}^{j} R_{t+1+s}))$. Thus an agent's consumption is a linear function of wealth, and hence, of capital. From the budget constraint of the agent and the definition of wealth it is easy to see that wealth evolves over time according to $\omega_{t+1}^i = R_{t+1}(\omega_t^i - c_t^i)$. Finally, using Equation (6) to substitute out consumption in the previous expression we obtain $\omega^i_{t+1} = \beta R_{t+1}\omega^i_t - R_{t+1}P_t$. It follows that

$$cv(\omega_{t+1}) = cv(\omega_t)\frac{\omega_t}{\omega_{t+1}}\beta R_{t+1}. \tag{7}$$

Thus equation (7) states that wealth dynamics are determined by the growth rate of wealth relative to the interest factor. Given that wealth is essentially a combination of all future prices of capital and labor, the focus of the paper is on the differences between autarky and international trade with respect to competitive prices of primary factors.

In the following section I restate the result in Chatterjee (1994) about wealth dynamics in a closed economy using the coefficient of variation, and later I turn to the effects of international trade.

3 Wealth dynamics under autarky

The following result establishes that For any of the previous economies, a competitive equilibrium under autarky is a list of sequences $\{p^x_t, p^y_t, w_t, r_t\}$ such that markets for primary factors and intermediate goods clear and such that the aggregation of optimal decisions of agents satisfy market clearing for final goods. The following result establishes that this market clearing condition can be written as in the one-sector neoclassical growth model.

Lemma 1. Let ν_t and l_t denote the fractions of capital and labor, respectively, devoted to the production of good x. Under the maintained assumptions, $\nu_t = \nu^*$ and $l_t = l^*$, thus they are independent of k_t. Market clearing for the *final goods can be written as:*

$$\frac{\sum_N c_t^i + k_{t+1}^i}{N} = Ak_t^\xi + (1-\delta)\frac{\sum_N k_t^i}{N}, \tag{8}$$

where $\xi = \theta\gamma + \eta(1-\gamma)$ and $A = (\nu^*/l^*)^{\theta\gamma}\left(\frac{1-\nu^*}{1-l^*}\right)^{\eta(1-\gamma)} l^{*\gamma}(1-l^*)^{1-\gamma}$.

Proof: See the Appendix. ■

Remember that consumption in Equation (6) is linear in wealth, and wealth in Equation (5) is linear in capital. It follows from the market clearing equation (8) that average capital depends only on the consumption of an agent that has the average capital in the economy. Therefore the evolution of capital over time can be studied by means of the problem of a central authority that solves

$$\begin{aligned} \max \quad & \textstyle\sum_{t=0}^{\infty} \beta^t \log(c_t - \bar{c}) \\ \text{s. to} \quad & c_t + k_{t+1} = Ak_t^\xi + (1-\delta)k_t \\ & c_t \geq \bar{c},\ k_t \geq 0,\ \forall t \geq 0,\ \text{given } k_0. \end{aligned} \tag{9}$$

This problem is a version of the neoclassical model of growth studied at length in the literature. Since $\bar{c}$ can be positive I will assume that the initial stock of capital is larger than some lower bound $\underline{k} \geq 0$ so that the feasible set is not empty. I will also assume that $\underline{k} < k^*$.[5] The following proposition states some well known properties of the solution to the previous problem (see theorem 1 in Obiols-Homs and Urrutia (2003) for a proof). For future

[5] If $\bar{c} \leq 0$, then $\underline{k} = 0$. For $\bar{c} > 0$ the feasible set will be empty unless there exists a solution to $Ak^\xi - \bar{c} - \delta k = 0$. I will assume that the previous equation has two solutions an that the smaller one satisfies $\underline{k} < k^*$, where k^* is the unique k satisfying condition *(i)* in Proposition 1.

reference, the theorem introduces a version of the welfare theorems to state the connection between the optimal allocation and competitive factor prices.

Proposition 1. Under the maintained assumptions, if $k_0 > \underline{k}$ *then there exists a sequence* $\{c_t, k_{t+1}\}$ *that solves the planner's problem. The sequence* $\{c_t, k_{t+1}\}$ *monotonically converges to stationary values* $\{c^*, k^*\}$ *satisfying (i)* $A\xi(k^*)^{\xi-1} - \delta = (1-\beta)/\beta$, *and (ii)* $c^* = A(k^*)^{\xi} - \delta k^*$. *Furthermore,* $R_t = A\xi k_t^{\xi-1} + 1 - \delta$ *and* $w_t = A(1-\xi)k_t^{\xi}$ *for all* t.

I will refer to a situation where $k_0 < k^*$ –and thus the stock of capital will be growing over time– as a *transition from below.* We are now ready to describe wealth dynamics:

Proposition 2 (Chatterjee 1994). Over a transition from below under autarky: a) inequality monotonically increases over time if $\bar{c} > 0$*; and b) inequality remains constant over time if* $\bar{c} = 0$.

Proof: From $\omega_{t+1} = R_{t+1}(\omega_t - c_t)$, Equation (6), and the definition of ω_t we obtain

$$\frac{\omega_t}{\omega_{t+1}}\beta R_{t+1} = 1 + \frac{P_t}{k_{t+1} + W_{t+1}}.$$

Next, $k_t < k^*$ implies that $\beta R_t > 1$ for all t. For the first part, use the previous result together with $\bar{c} > 0$ to get that $P_t > 0$ for all t. It follows that $\beta R_{t+1}\omega_{t+1}/\omega_t > 1$, for all t. The desired conclusion is obtained using this observation in Equation (7). The second part follows directly from the same reasoning because with $\bar{c} = 0$, then $P_t = 0$ for all t. ■

In the following section I study the implications of the previous proposition once the economies engage in international trade in intermediate goods.

4 Wealth dynamics in a small open-economy

Following Atkeson and Kehoe (2000), I study the effects of trade on the distribution of lifetime wealth of a small economy that starts its development process once the rest of the world has reached the steady state. In particular I assume that all but one economy started growing towards the steady state at the same time and with the same level of initial capital. This means that whether these economies were allowed to trade in intermediate goods over the transition to the steady state is of no consequence. Therefore the equilibrium dynamics for these economies are described by Propositions 1 and 2 above: all economies converge to the same steady state independently of the initial distribution of wealth, and at the steady state the only difference among them is in the stationary distribution of wealth.

Consider now the equilibrium dynamics of the economy that starts its process of development once the rest of the world has reached the steady state. This economy is allowed to trade intermediate goods at the international equilibrium prices. Since aggregate dynamics do not depend on the initial distribution of wealth, the evolution of this economy can be described by the decisions of a central authority that buys and sells intermediate goods in international markets and that organizes efficiently domestic production. As in the preceding section, it is useful to write the problem in per capita terms so that the planner chooses the fraction of available capital and labor to be devoted to the production of each intermediate good. Using the notation introduced above the problem of the central authority can be described as

$$
\begin{aligned}
\max \quad & \textstyle\sum_{t=\tau}^{\infty} \beta^t \log(\hat{c}_t - \bar{c}) \\
\text{s. to} \quad & \hat{c}_t + \hat{k}_{t+1} = (x_t^d)^{\gamma} (y_t^d)^{1-\gamma} + (1-\delta)\hat{k}_t \\
& x_t^s = (\nu_t \hat{k}_t)^{\theta} \hat{l}_t^{1-\theta},\ y_t^s = ((1-\hat{\nu}_t)\hat{k}_t)^{\eta} (1-\hat{l}_t)^{1-\eta}, \\
& p_x^* x_t^d + p_y^* y_t^d = p_x^* x_t^s + p_y^* y_t^s, \\
& \hat{c}_t \geq \bar{c},\ \hat{k}_t \geq 0,\ \hat{l}_t \in [0,1], \hat{\nu}_t \in [0,1]\ \forall t \geq \tau, \\
& \text{given } k_\tau.
\end{aligned}
\tag{10}
$$

The first equation is the feasibility constraint that restricts consumption and capital accumulation to the sum of current output and undepreciated capital; the second and third equations simply relate the domestic supply of intermediate goods to the technology constraints; the forth equation is the condition for balance in international trade. The following Proposition 3 collects a number of useful results about the solution of the previous planner's problem (the proof is omitted because the arguments in Atkeson and Kehoe (2000), given in a model with continuous time, apply without change to the present context).

Proposition 3 (Atkeson and Kehoe 2000). Let τ *denote the period when the late-bloomer opens to international trade. There exist*

$$
k_y = \frac{(1-\theta)\eta}{\theta(1-\eta)} \left(\frac{p_x^*}{p_y^*} \left(\frac{\theta}{\eta}\right)^{\theta} \left(\frac{1-\theta}{1-\eta}\right)^{1-\theta} \right)^{1/(\eta-\theta)},
$$

with $0 < k_y < k^*$ *such that: a) if* $k_\tau < k_y$*, then* $\hat{\nu}_t = 0 = \hat{l}_t\ \forall t \geq \tau$ *and* $\hat{k}$ *is monotonically increasing and converges to* $\hat{k}^* = k_y$*; and b) if* $k_y \leq k_\tau \leq k^*$*, then* $\hat{\nu}_t = \hat{\nu}$*,* $\hat{l}_t = \hat{l}$ *(with* $\hat{\nu} \in [0, \nu^*]$ *and* $\hat{l} \in [0, l^*]$*), and* $\hat{k}_t = k_\tau = \hat{k}^*\ \forall t \geq \tau$*.*

The following lemma is a version of the factor-price equalization theorem and is introduced for future reference.

Lemma 2. Assume $k_t \in [k_y, k^*]$*. Then* $\hat{w}_t = w^*$ *and* $\hat{R}_t = R^*$ *for all* t*.*

Proof: See the Appendix. ■

Proposition 3 states conditions such that the late-bloomer specializes production completely after the opening to international trade. Specifically, with $k_\tau < k_y$ the economy specializes in the labor intensive good, and the stock of capital grows over time converging to $\hat{k^*} = k_y$. Interestingly, over such a transition Proposition 2 applies, thus after the opening to trade the distribution of wealth becomes more unequal. With $k_y \leq k_\tau < k^*$ there is only a reallocation of primary factors across sectors, that is, the economy immediately "jumps" to the new steady state. The interesting question is whether wealth inequality with trade is larger or smaller than under autarky. The following proposition states a necessary and sufficient condition for inequality to be smaller in the moment of opening the economy to international trade than under autarky.

Proposition 4. Assume the late-bloomer opens to international trade in a period τ*. Then* $cv(\hat{\omega}_\tau) \leq cv(\omega_\tau) \iff \hat{W}_\tau \geq W_\tau$*.*

Proof: From Equation (5) we have that $sd(\omega_\tau^i) = R_\tau sd(k_\tau^i)$ holds under autarky, whereas $sd(\hat{\omega}_\tau^i) = \hat{R}_\tau sd(k_\tau^i)$ holds with international trade. Notice that the distribution of k_τ was decided in period $t-1$ and thus, it is the same under autarky and under trade. The result follows directly once we divide the previous equations respectively by ω_τ and $\hat{\omega}_\tau$, and we obtain $cv(\omega_\tau) = cv(k_\tau)k_\tau/(k_\tau + W_\tau)$ and $cv(\hat{\omega}_\tau) = cv(k_\tau)k_\tau/(k_\tau + \hat{W}_\tau)$. ■

The intuition for Proposition 4 is as follows. Since labor income is the same for all agents, an increase in its present value benefits more those agents for whom labor income represents a larger fraction of their lifetime wealth. Thus wealth inequality necessarily declines if the present value of labor income increases with trade. This latter condition is a dynamic version of the Stolper-Samuelson theorem about the functional distribution of income that holds in static Heckscher-Ohlin models: trade promotes gains in efficiency by specializing production in goods that use intensively relatively abundant factors, and as a consequence, income rewarding those factors tends to increase and the income rewarding relatively scarce factors tends to decrease.

The next result collects two situations where the previous condition is satisfied. In those situations the same condition is also sufficient to provide a decline in inequality in all future periods after the opening to trade.

Corollary 1. Assume the late-bloomer opens to international trade in a period τ. International trade promotes a permanent reduction in wealth inequality when: a) $k_y \leq k_\tau < k^*$; or b) $k_\tau < k_y$, and $\bar{c} = 0$, $\delta = 1$.

Proof: To see part a), notice that since $k_\tau < k^*$, then Proposition 2a implies inequality will increase under autarky. Furthermore, Proposition 1 implies that w_t is monotonically increasing and converges to w^*, and that R_t is monotonically decreasing and converges to R^*. Under trade, however, the economy jumps to a steady state where $\hat{w}_t = w^*$ and $\hat{R}_t = R^*$, $\forall \tau \geq t$ (by Lemma 2). Therefore $\hat{W}_\tau > W_\tau$ and Proposition 4 implies that $cv(\hat{\omega}_\tau) < cv(\omega_\tau)$. This completes the argument because $cv(\hat{\omega}_{\tau+j}) = cv(\hat{\omega}_\tau) < cv(\omega_\tau) \leq cv(\omega_{\tau+j})$ for all $j > 0$. To see part b), notice that

Proposition 3 implies that the planner's problem with trade can be written without loss of generality as

$$\begin{array}{ll} \max & \sum_{t=\tau}^{\infty} \beta^t \log(\hat{c}_t - \bar{c}) \\ \text{s. to} & \hat{c}_t + \hat{k}_{t+1} = \hat{A}(\hat{k}_t)^{\hat{\xi}} + (1-\delta)\hat{k}_t \\ & \hat{c}_t \geq \bar{c},\ \hat{k}_t \geq 0,\ \forall t \geq \tau,\ \text{given } \hat{k}_\tau, \end{array} \tag{11}$$

where $\hat{\xi} = \eta$ and $\hat{A} = A\left(\frac{1-l^*}{1-\nu^*}\right)^{\eta} \frac{(1-\gamma)}{(1-l^*)}(k^*)^{\gamma(\theta-\eta)}$. It is well known that with $\bar{c} = 0$ and $\delta = 1$, then $k_{t+1} = \beta A \xi k_t^{\xi}$ and $\hat{k}_{t+1} = \beta \hat{A} \hat{\xi} k_t^{\hat{\xi}}$ for all $t \geq \tau$. It is straight forward to check that $\hat{W}_\tau = k_\tau(1-\hat{\xi})/(\hat{\xi}(1-\beta))$ and $W_\tau = k_\tau(1-\xi)/(\xi(1-\beta))$. Thus $\hat{W}_\tau > W_\tau$ because $\hat{\xi} = \eta < \gamma\theta + (1-\gamma)\eta = \xi$. Therefore the result follows from Propositions 2b and 4. ■

The previous corollary states that when there is no transition, either because $k_\tau \geq k_y$ and thus the stock of capital does not change over time, or because $\delta = 1$ and $\bar{c} = 0$ in which case the distribution of wealth remains constant over time, the distribution of wealth improves with the opening to international trade. When $k_\tau < k_y$ and $\bar{c} > 0$ the late-bloomer observes a transition towards the new steady state with trade. Over this transition there will be the "international trade" effect discussed after Proposition 4, and the "transition" effect described in Proposition 2. These two effects work in opposite directions. In particular, even though capital is the relatively scarce factor in the late-bloomer economy, agents that own an amount of capital larger (smaller) than the average capital will observe a larger (smaller) increase in their lifetime wealth over the transition to the new steady state with trade. Is it possible that the long run level of inequality with international trade is larger than the level it would have prevailed had the economy remained closed? Proposition 5 below states necessary and sufficient conditions for the long run level of inequality under trade to be smaller than under autarky.

Proposition 5. Assume $\bar{c} > 0$ *and that the late-bloomer opens to international trade in a period* τ *with* $k_\tau < k^*$. *Then* $cv(\hat{\omega}^*) < cv(\omega^*) \iff \hat{R}_\tau u'(\hat{c}_\tau)/(\hat{\omega}^* u'(\hat{c}^*)) < R_\tau u'(c_\tau)/(\omega^* u'(c^*))$.

Proof: The proof proceeds in two steps. In the first step, we write

$$cv(\omega^*) = cv(k_\tau)\frac{k_\tau}{k^*}\frac{k^*}{k^* + W^*}\beta R_\tau \frac{u'(c_\tau)}{u'(c^*)}, \tag{12}$$

and a similar expression for $cv(\hat{\omega}^*)$. The previous expression follows from the fact that $cv(\omega^*) = cv(k^*)k^*/(k^* + W^*)$, and that from the budget constraint in period τ, together with the expression for consumption in Equation (6) and the definition of ω_τ we get that $k^i_{\tau+1} = \beta R_\tau k^i_\tau + w_\tau - (1-\beta)R_\tau W_\tau - P_\tau$. Therefore $cv(k_{\tau+1}) = cv(k_\tau)k_\tau/k_{\tau+1}\beta R_\tau$, and iterating forward, we get that $cv(k^*) = cv(k_\tau)k_\tau/k^* \lim_{j\to\infty}\prod_{s=0}^{j-1}(\beta R_{\tau+s})$. Next, use the first order condition for optimality in the consumer's problem in (4) to get that $\lim_{j\to\infty}\prod_{s=0}^{j-1}(\beta R_{\tau+s}) = \beta R_\tau u'(c_\tau)/u'(c^*)$. The expression in (12) is the result of combining the previous equations. Moreover, the present value of labor income (W^*) is the same under both trade and autarky because of Lemma 2. Likewise, the term $cv(k_\tau)k_\tau$ is also the same under trade and autarky because it was determined in period $\tau - 1$. For the second step, define

$$\Lambda(k_\tau) \equiv cv(k_\tau)k_\tau\beta\left(\frac{\hat{R}_\tau}{\hat{k}^*+W^*}\frac{u'(\hat{c}_\tau)}{u'(\hat{c}^*)}-\frac{R_\tau}{k^*+W^*}\frac{u'(c_\tau)}{u'(c^*)}\right).$$

The function $\Lambda(k_\tau)$ measures the difference in the long run level of inequality between trade and autarky as a function of the stock of capital in the moment of the opening to international trade. The conclusion follows multiplying $\Lambda(k_\tau)$ by R^* and dividing the terms in parenthesis by $\hat{R}^*$ and R^* (which are identical by Lemma 2). ■

The last result in this section uses Proposition 5 to state that there is an open set of levels of capital smaller than $\hat{k}^*$ such that if the economy opens to trade with a capital level in that set, then inequality in the long run is smaller than under autarky.

Corollary 2. Assume $\bar{c} > 0$ and that the late-bloomer opens to international trade in a period τ with $k_\tau < \hat{k}^*$ but close to $\hat{k}^*$. Then $cv(\hat{\omega}^*) < cv(\omega^*)$.

Proof: Part a) of Corollary 1 implies that $\Lambda(\hat{k}^*) < 0$. This concludes the proof, since $\Lambda(k)$ is continuous for $k_\tau > \underline{k}$. ■

It is difficult to generalize the result of the previous corollary for capital levels arbitrarily far away from the steady state. Intuitively, in the class of economies studied in this paper the changes in the distribution of wealth are directly related to the speed of convergence to the steady state through the changes in factor prices. Then, a neoclassical economy converges faster to its steady state the further away it is from it. This suggests that convergence under autarky should be faster than under trade because $\hat{k}^* < k^*$. However, convergence is also faster the smaller is the capital share in the technology to produce final goods (see King and Rebelo (1993)). Since $\hat{\xi} < \xi$, this suggests a faster convergence under trade than under autarky. In the following section I resort to numerical simulations to investigate these issues.

5 Simulations

I simulate several examples of the evolution of inequality for the late-bloomer economy under autarky and under trade.[6] The parameter values used in the benchmark simulations are as follows. I use $\beta = .99$ and $\delta = .025$ which are commonly used in applied work for the U.S. economy simulating quarterly data. For the parameters in the technologies I use $\gamma = .5$, $\theta = .38$ and $\eta = .34$. These parameter values produce a capital share about .36 which is again standard in the quantitative literature. I also assume $\bar{c} = 0.1$, and I fix an arbitrary distribution of capital which then implies the initial distribution of wealth. The distribution of capital under autarky in the opening period τ is used to determine the initial distribution of wealth under trade.

Figure 1 displays the evolution of inequality in lifetime wealth over the transition to the steady state for an economy under autarky. It also displays the evolution of inequality under trade starting with a capital level about 90% of that at the steady state. As predicted by Proposition 2, inequality is clearly increasing over the transition under both regimes. However, inequality initially falls after the opening to international trade, then it starts

[6] A brief description of the numerical method used to solve the model can be found in the Appendix.

increasing and it converges to a lower level than it would under autarky. This transition can be seen as an example where both Corollaries 1 and 2 hold. Furthermore, in this example inequality under trade is smaller than under autarky in all periods over the transition to the steady state.

Figure 2 displays the evolution of inequality under autarky and under trade for the economy opening with a stock of capital equal to 10% and 50% of the stock of capital at the steady state with trade. The figure suggests that the result in Corollary 2 applies even with a capital level very far away from the level at the steady state with trade. Furthermore, it is clear from Figure 2 that the later the opening to international trade occurs, the smaller the reduction in inequality relative to that under autarky will be and the larger will be in the long run. I have obtained the same sort of results under several initial distributions of wealth, minimum consumption requirements, and different combinations for the parameter values in the technologies (these results are not reported for reasons of space). These experiments suggest that the "international trade" effect from the Stopler-Samuelson theorem always dominates the "transition" effect, thus trade effectively helps to reduce wealth inequality.

6 Conclusion

In this paper I develop a dynamic Heckscher-Ohlin model to study international trade effects on personal wealth inequality in a small open economy. Over a transition to the steady state from below inequality in wealth tends to increase. This effect due to the transition may be counterbalanced once the economy opens to international trade because the Stolper-Samuelson theorem holds and thus income accruing to relatively abundant factors increases and income accruing to relatively scarce factors decreases. I formally show that international trade promotes a decline in wealth inequality when the economy opens to trade with a level of capital sufficiently large (i.e., close to the steady state level with trade). I then use numerical simulations to investigate the effects of international trade starting from arbitrary initial levels of capital. In the simulations I find that trade always helps reduce inequality in wealth relative to the level of inequality under autarky.

My results suggest that the effect of trade in factor prices dominates the effect of the transition and therefore, the changes in the personal distribution of wealth are directly related to changes in the functional distribution of income. Like in the static Heckscher-Ohlin model, the changes in the functional distribution of income in the dynamic model are linked to the pattern of trade. This observation should be useful in empirical work trying to asses the effects of international trade on personal inequality. Finally, it would be interesting to investigate the predictions of a similar model but abandoning the small open-economy assumption. This extension is left for future work.

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Appendix

Proofs

Proof of Lemma 1: Since both intermediate goods are fundamental in the production of the final good, in equilibrium they will be produced in all periods. It follows from the profit maximizing problem of final goods producers that

$$\frac{p_{x,t}}{p_{y,t}} = \frac{\gamma}{1-\gamma}\frac{y_t}{x_t}. \tag{13}$$

The stock of capital in per capita terms used in the production of each intermediate good can be written as $k_{x,t} = \nu_t k_t$ and $k_{y,t} = (1-\nu_t)k_t$. Since primary factors are perfectly mobile across firms and industries, in equilibrium it must be the case that $w_{x,t} = w_{y,t}$ and that $r_{x,t} = r_{y,t}$. Using Equation (13) in the equilibrium condition $r_{x,t} = r_{t,y}$, and using the fact that in equilibrium factor prices coincide with the value of their marginal productivity, a few manipulations produce $\eta/\theta(1-\gamma)/\gamma = (1-\nu_t)/\nu_t$, thus $\nu^* = \theta\gamma/(\eta(1-\gamma)+\theta\gamma)$. Using the fact that prices of primary factors are the same in both industries and dividing the FOC of intermediate goods produces provides $\theta/(1-\theta)l_t^x/\nu^* = \eta/(1-\eta)(1-l_t^x)/(1-\nu^*)$, thus $l^* = (1-\theta)\gamma/((1-\eta)(1-\gamma)+(1-\theta)\gamma)$. The market clearing condition for final goods stated in the text is obtained after substituting the expressions for l^* and ν^* into $\sum_N (c_t^i + k_{t+1}^i)/N = x_t^\gamma y_t^{1-\gamma} + (1-\delta)\sum_N k_t^i/N$. ■

Proof of Lemma 2: The welfare theorems apply too in the environment with trade. Thus it suffices to show that equilibrium factor prices with trade coincide with those under autarky when $k_t \in [k_y, k^*]$. For $k_t = k^*$ this is obvious. For $k_t \in (k_y, k^*)$ the result follows from the factor price equalization theorem (see for instance Samuelson (1996)), since in both the late-bloomer and the rest of the world goods prices are the same, there is no perfect specialization, technologies are the same in all economies, and they do not allow factor intensity reversals. For $k_t = k_y$, notice that when the late-bloomer specializes completely in the production of y the corresponding first order condition implies that $r = p_y^* \eta k_y^{\eta-1}$. For the early-bloomers the corresponding expression is given by $r = p_y^* \eta (k^*(1-\nu^*)/(1-l^*))^{\eta-1}$. Since early-bloomers produce both intermediate goods, at the steady state $p_x^*/p_y^* = \gamma/(1-\gamma)Bk^{*\eta-\theta}$, where $B = ((1-\nu^*)/(1-l^*))^\eta (l^*/\nu^*)^\theta (1-l^*)/l^*$. Using the definitions for l^*, ν^* and k_y, it follows that $k_y = k^*(1-\nu^*)/(1-l^*)$. Therefore the factor price equalization theorem also applies to the limiting case of $k_t = k_y$. ■

Computation

The numerical method is based on dynamic programming: starting from an arbitrary function v_0 of the state k for the value function, perform iterations on:

$$\frac{1}{Ak^\xi + (1-\delta)k - k' - \bar{c}} = \beta v_0'(k).$$

The equation above is the first order condition associated to the corresponding planner's problem, and it is evaluated on a grid of points. The decision rule for capital accumulation is approximated with piecewise linear functions between grid points (see Obiols-Homs and Urrutia (2003) for further details).[7] Once the decision rule for capital has approximately converged I

[7] In practice I use 1,300 evenly spaced points in the grid. Computing time is small because there is only one state variable.

simulate a transition of capital and factor prices towards the steady state over 1,000 periods and I compute the objects of interest. To gain accuracy in the computations, I first compute lifetime wealth of the representative agent over the transition. Then I fix an initial distribution of capital and I simulate the transition of 10 agents as follows. From Equation (6) it follows that $c_t^i = c_t + (1-\beta)R_t(k_t^i - k_t)$, and k_{t+1}^i is determined using the budget constraint of each agent. The same Equation (6) implies that

$$\omega_t^i = \omega_t + \frac{c_t^i - c_t}{1-\beta}.$$

Using the ω_t^i's to compute the coefficient of variation at each period is much more accurate than using the recursive expression in Equation (7).

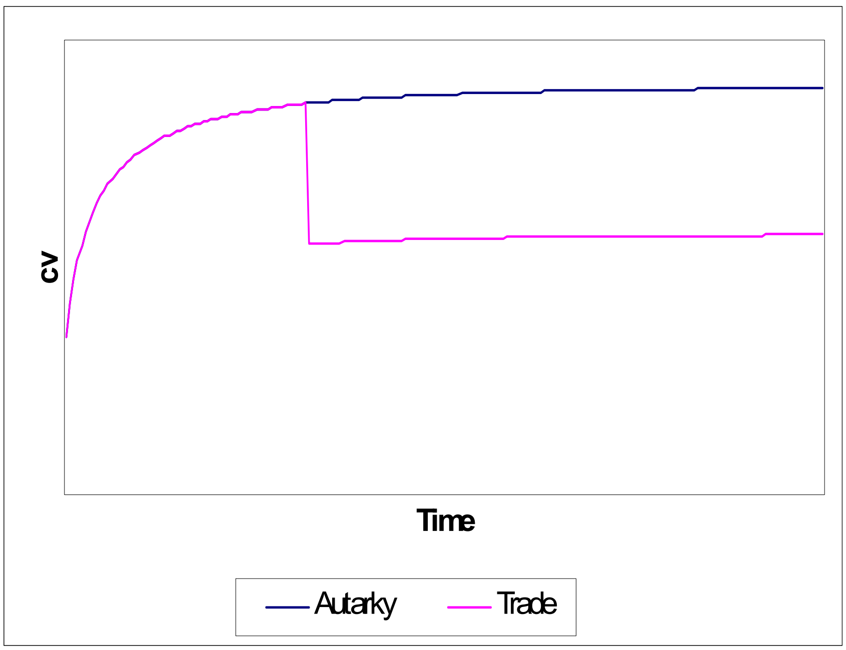

Figure 1: The evolution of inequality when the economy is close to the steady state.

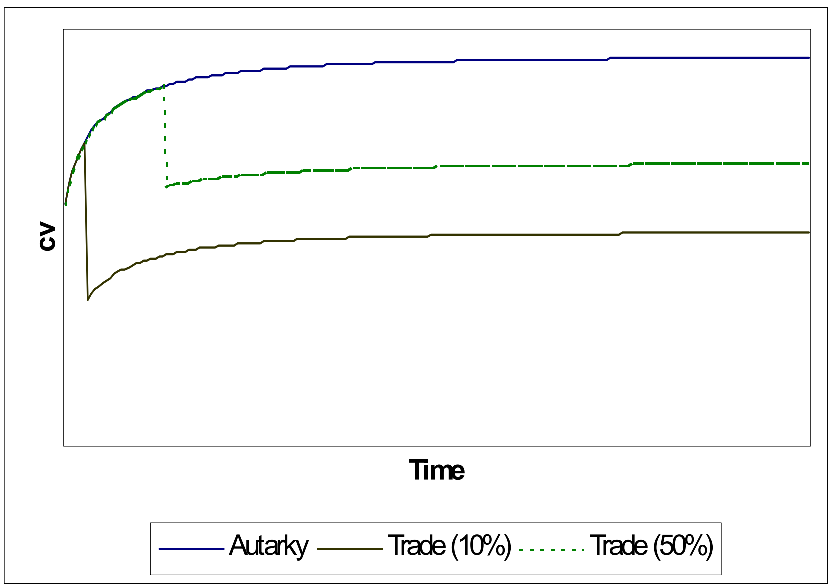

Figure 2: The evolution of inequality when the economy opens far away from the steady state.