# A Descriptive and Analytic Look at Marx's Own Explanations for the Falling Rate of Profit

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Abstract: This paper examines, both descriptively and analytically, Marx's arguments for the falling rate of profit from the Hodgskin section of Theories of Surplus Value, The General Law section of the recently published Volume 33 of the Collected Works and Chapter 3 of Volume III of Capital. The conclusions are as follows: First, Marx realised that his main attempt to give an intrinsic explanation of the falling rate of profit, which occurred in the General Law section, had failed; but he still hoped that he would be able to demonstrate it in the future. Second, the Hodgskin and General Law sections contain a number of subsidiary explanations, mostly related to resource scarcity, some of which are correct. Third, Part III of volume III does not contain a demonstration of the falling rate of profit, but a description of the role of the falling rate of profit in capitalist development. Forth, it also contains suppressed references to resource scarcity. And finally, in Chapter 3 of Volume III, Marx says that it is resource scarcity that causes the fall in the rate of profit described in Part III of the same volume. The key to all these conclusions in the careful analysis of the General Law section.

## I.   Introduction.

This is a paper in the history of economic thought. Its main objective is to give a descriptive and analytic portrait of Marx's own arguments for the falling rate of profit. Its secondary objective is to contrast this detailed portrait with the various interpretations that are current in the literature. Marx's own arguments have not been well understood because they are difficult, obscure and, since they appear in work that was never revised, badly constructed. Faced with this, commentators have reacted in two distinct ways: either they have given verbal summaries of Marx's arguments or they have constructed models which generally reflect Marx's position and then shown that

his conclusions were correct or, frequently, that they could not be sustained. By contrast, the approach of this paper is, first, to give a careful description of the arguments in the Hodgskin section of Theories of Surplus Valve (TSV), the General Theory section of the recently published Volume 33 of the Collected Works and, finally, Chapter 3 of Volume III of Capital which was compiled from Marx's late writings; and then, second, to use a single analytical model to, in each case, present an analytic version of Marx's arguments. The portrait that emerges from this exercise differs in important ways from the major, currently espoused interpretations of Marx's theory of the falling rate of profit.

## 1. Currently Espoused Interpretations.

Although it in no way conveys the sense of this vast literature[1], it is convenient to give a description of the current situation in terms of one commonly agreed point and four distinct positions.

### a. Part III of Volume III of Capital.

The title of this is The Law of the Tendency of the Rate of Profit of Fall, it comprises three chapters in 55 pages. Almost all commentaries on Marx's theory refer to this section. I think the statement that "this is Marx's most important explanation of why the rate of profit falls" would be upheld by all writers on the theme, regardless of what their position is on other issues.

The following four sections give four interpretations which are, to some extent, contradictory.

### b. The Falling Maximum Surplus Value Argument.

This argument can be found in many places in Marx's writings. One way of stating it is the following: The rate of profit is surplus value divided by the sum of constant and variable capital. If the sum is held constant while the labour force is sufficiently reduced, then surplus value, and thus the rate of profit, must fall.[2] I think that it is generally thought that this argument is wrong (Roemer 1981 chap. 5). But there are a number of authors, Okishio (1970) Rosdolsky (1977 pp408-10), Shaikh (1978 p240) and Tsoulfidis (2002) for example, who still hold that this line of reasoning is correct. Importantly, for the argument of this paper, they cite Marx as the authority for this.

### c. The Okishio Theorem.

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[1] Probably the best description is given in Howard and King (1992) chapters 7 and 16.

Marx claimed that capitalists would introduce techniques that lowered the rate of profit because, at the initial prices, the innovators profits would rise. A long sequence of writers, culminating with Okishio (1961) used a linear model with labour as the single basic input to show that this was false (the Okishio Theorem). Many of these authors also claimed that Marx did not believe that resource scarcity was responsible for the falling rate of profit. Examples are Roemer (1981 pp87-8) who contrasts Marx's view with that of Ricardo; Steedman (1977 p129) who qualifies his statement with a reference to TSV; and Samuelson (1957 p894) who says that Marx could not introduce land because it would conflict with his belief in the labour theory of value. These statements are important for the argument of this paper. They reflect the authors' general opinions about Marx, but they also justify the use of the one basic factor model for showing that Marx's claim was wrong.

**d. The Rising Wage.**

A considerable number of authors have connected Marx's argument for the falling rate of profit with a rising wage. The principle source for this is the opening five pages of Part III where the rate of profit is made to fall by having the rate of surplus value remain constant while the composition of capital rises. It has been know since the writings of Natalie Moszkowsha in the late 1920s that this implies a rise in the wage if, as Marx assumed, the rise in the composition of capital increases the productivity of labour. Thus, in the opening of the major explanation of the falling rate of profit, one has a long example where it is caused by a rising wage.

There have been two distinct reactions to this: Some writers like Robinson (1942 p 36) and Sweezy (1942 Chap 6) have held that Marx was unaware of this, that one of his basic assumptions was a constant wage and, thus, that the example is inappropriate since it contradicts the basic assumption. Other writers have taken the opposite tack and, based on this and other parts of his writings, have associated Marx's falling rate of profit with a rising wage. Laibman (1977), (1997), Foley (1986), Skillman (1997) and Dumnènil and Lévy (2001) have all constructed Marxian models in which the mechanism which produces the falling rate of profit involves a rising wage. In addition both Foley (pp138-9) and Duménil and Lévy have actually argued that this reflects Marx's thought.

**e. Natural Resource Arguments.**

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[2] This explanation is too short to be accessible for the unfamiliar. An account is given in Proposition 1.

There are at least six authors that have emphasised the resource scarcity element in Marx's writings on the falling rate of profit. They are Rosdolsky (1977 pp405-10) (The idea first mentioned in Rosdolsky (1956)), Meek (1967) (First appeared in (1960)), Lebowitz (1982), Perelman (1985), Moseley (1991 Chap 1), and Clarke (1994 pp214-19). These writings are diverse and justice cannot be done to them in a short summary[3]. However, first, generally these writers were aware of the weaknesses of Marx's argument and introduce resources as a way to buttress it against criticism. That is they take Marx as having a single explanation that, when thus buttressed, is correct.[4] Second, they do not attempt a formal algebraic exposition of Marx's arguments. Finally, with respect to the absence of any serious reference to resource scarcity in Part III, it is usually stated or implied that Marx assumed that readers would have read the parts of TSV which deal with this. Exceptionally Perelman holds that Marx purposely suppressed references to resource scarcity because he did not want to be dragged into a debate over how communism would resolve this type of problem.

## 2. Revisions.

The conclusions of the paper are mainly based on the analysis of the Hodgskin section and the General Law section which Marx wrote immediately afterwards. Marx had two distinct types of explanations for the falling rate of profit. The first, which I call the intrinsic explanation, is based on a one sector, one basic factor model; the second encompasses a number of different explanations which necessitate at least two sector models and involve, for the most part, resource scarcity. In the General Theory section he makes his major attempt to demonstrate the intrinsic explanation, fails and is aware of this failure. After this he retreats, as it were, and gives a number of the second type of explanations.

From this it follows that the above interpretations should be revised. First Part III is not on explanation of the falling rate of profit since it contains mere sketches of the arguments that Marx had already studied in detail and found wanting. Rather it is a description of capitalist development in which the falling rate of profit plays a leading role. Furthermore the falling maximum surplus value authors should not cite Marx as an authority since he had tried to demonstrate this, failed and knew it. The Okishio authors are wrong to say that Marx did not believe that resource scarcity would be a cause of the

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[3] See Petith (2001) for a longer summary

falling rate of profit. He did, but in an argument that was separate from the one they attacked. With regard to the rising wage, Marx saw clearly that the fixed rate of surplus value implied a rising wage. He tried to modify his argument to keep the wage fixed but failed. Thus both the authors who think he was unaware, and those who think that a rising wage is part of his logical argument have misinterpreted him. Those scarcity writers who think that this is part of a single correct explanation are wrong; rather the resource arguments are separate and their validity remains to be checked. Finally, it is likely that there is no reference to resource scarcity in Part III because Marx still thought he would be able to demonstrate the falling rate of profit without invoking it, and its mention would have taken away from the idea of the end of capitalism being due to its basic nature. These points will be clearer to the reader after he has read the body of the paper.

### 3. The Structure of the Paper.

The main theme of the paper is to set out Marx's own explanations of the falling rate of profit. Section II contains the formal structure of a one sector model which encompasses all of Marx's attempts to demonstrate the intrinsic explanation. Section III covers the Hodgskin and General Theory sections. Section IV argues in detail that the objective of Part III is not to explain why the rate of profit falls. Section V covers Chapter 3 and argues that, in his late writings, Marx was drifting towards resource scarcity based explanations. Finally Section VI concludes by emphasising the importance of the General Theory section for a correct understanding of the rate of profit aspect of Marx's writing.

## II. A Formal Model.

This section sets out the formal model and also explains Marx's methodology which is somewhat unusual.

The formal model has one sector, labour as the one basic factor, fixed input coefficients and only circulating capital (Capital is completely used up in production.). The input coefficients for capital and labour $A$ and $L$ are defined by

(1) $\qquad A \quad K/Y$

(2) $\qquad L \quad \tilde{L}/Y$

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[4] Rosdolsky is an exception.

where $K$, $\tilde{L}$ and $Y$ are capital, labour and gross output. Capital and labour productivity are thus $1/A$ and $1/L$ so that a fall in $A$ is equivalent to a rise in capital productivity etc. Profits    are given by

(3)
$$\frac{Y - K - b\tilde{L}}{K + b\tilde{L}} = \frac{1}{A + b\tilde{L}} - 1$$

where $b$ is the real wage and the last equation is from (1) and (2). In addition    is the value of the good or the amount of labour embodied in a unit of it. On the assumption that the value does not change over time, it satisfies

(4)                 $= A + L$,      $= L/(1-A)$.

Constant capital $C$, variable capital $V$ and surplus value $S$ are defined as

(5)               $C$    $K$

(6)               $V$    $b\tilde{L}$

(7)               $S$    $(Y-K-b\tilde{L}) = \tilde{L}(1- b)$

Where the last equality is from (1), (2) and (4). Two additional value concepts are the composition of capital $C/V$ and the rate of surplus value $S/V$. All together there are 11 variables $K$, $\tilde{L}$, $Y$, $A$, $L$, $b$,  ,  , $C$, $V$, and $S$; and only seven equations.[5]

There is a wide gap between Marx's intuitive and formal approaches. At the most basic level he thought that, as capitalism developed, $K$ would grow faster than $\tilde{L}$ (this was the starting point of the Hodgskin section) and that this would cause the rate of profit to fall. At an intuitive level Marx had a production function approach. He frequently asserted that an increase in the composition of capital $C/V$, which is equivalent in this model to an increase in $K/\tilde{L}$, would increase labour productivity. He also held, but with much less clarity, that it would lower capital productivity. That is, he thought that $L$ and $A$ were functions of $K/\tilde{L}$ with $L'(K/\tilde{L})<0$ and $A'(K/\tilde{L})>0$. If he had been able to continue formally in this direction, he would have seen immediately that his intuition was wrong. In particular, from (3) he would have seen that he would have to impose the additional condition on the technology, $A'(.)+bL'(.)<0$, in order to insure that the rise in $K/\tilde{L}$ would cause a fall in    .

Marx came to understand that he faced problems with his demonstration. But, since he generally thought in value terms, he saw them differently. From (3), (5), (6) and (7)

(8)                 $= \dfrac{S}{C+V} = \dfrac{S/V}{C/V+1}.$

He supposed that a rise in $K/\tilde{L}$ would cause a rise in $C/V$ and thus a fall in the rate of profit. But, because of his production function based intuition, that it would also cause $S/V$ to increase. The model shows that he was correct: (5) and (6) show that the rise in $K/\tilde{L}$ implies a rise in $C/V$ as long as $b$ remains constant; while (6), (7) and (4) give

$$(9) \qquad\qquad \frac{S}{V} = \frac{1-b}{b} = \frac{1}{b}\frac{1-A}{L} - 1$$

which shows that the rise in labour productivity $1/L$ caused by the rise in $K/\tilde{L}$ could cause $S/V$ to increase. Thus the problem Marx saw was to find a convincing way to limit the rise in $S/V$ so that    would fall.

Although Marx frequently used a production function based intuition, in his formal numerical examples he took a different approach. He would start with the basic model (1)-(7) and specify four additional relations to close the model. Three of these varied according to the type of argument but the forth was that $b=\bar{b}$, that the wage was constant, although this was not usually stated explicitly. After this he would change a variable that corresponded to a rise in $K/\tilde{L}$ and then check the effect on   . Since this strategy changes $\tilde{L}$, $K$ and $Y$, it is as if there is a production function; but Marx did not emphasise this. Rather what he was trying to do was to find four additional relations and a change that were both convincing and led to a fall in    . As we will see in the next section, these relations and this change do exist and Marx came within an ace of finding them. But this does not mean that his basic intuition was correct.

## III. The Arguments of 1862-3.

In October and November of 1862 Marx was writing out his comments on Thomas Hodgskin for the manuscript that would become TSV. Hodgskin believed that the rate of profit would be forced to fall if capital grew faster than labour. Marx tried to set out this conclusion in a logical form and appeared to be surprised to find he could not. He broke off work on the manuscript and filled two hundred notebook pages mainly attempting to deal with this. In January of 1863 he returned to the Hodgskin section and then finished the rest of the manuscript. In English, TSV became available in 1971 but the two hundred notebook pages only appeared in 1991 with the publication of Volume 33 of the Collected Works (CW). This section covers the Hodgeskin section of TSV III

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[5] The notation follows Roemer (1981).

pp.263-319 and the central section of the notebook pages, called the General Law for short, from the CW 33, pp.104-53.

## 1. Hodgskin.

Starting on p.298, Marx outlines Hodgskin's position on the falling rate of profit and then suddenly sets out his own theory. He starts with a paragraph for the case in which the rate of surplus value is constant and another for the case in which it increases because the working day increases. Then, in what are arguably the three most important paragraphs in all of Marx's writing on the falling rate of profit, he sets out his argument for the case in which the rate of surplus value rises because of the rise in the composition of capital has increased labour productivity, that is, the case of relative surplus value:

"3) If the normal working-day remains the same, surplus labour can be increased relatively by reducing the necessary labour time by[6] reducing the prices of the necessaries which the worker consumes, in comparison with the development of the productive power of labour. But this development of productive power reduces variable capital relative to constant. It is physically impossible that surplus labour-time of, say, two men, who displace twenty, can, by any conceivable increase of absolute of relative [surplus] labour-time, equal that of twenty. If each of the twenty men only work 2 hours of surplus labour a day, the total will be 40 hours of surplus labour, whereas the total life-span of two men amounts only to 48 hours in one day.

The value of labour-power does not fall in the same degree as the productivity of labour or of capital increases." (p 300).

Marx then finishes this paragraph with four reasons why the condition of the first sentence should be satisfied. These concern luxury goods, exhaustion of natural resources, monopoly in agriculture and the development of less fertile land. Their explanation requires a two sector model and is briefly discussed in section II.2.b below. The final short paragraph merely states that the rate of profit does fall.

These paragraphs are difficult to follow because Marx was trying to give form to his intuition rather then set out a formal analysis. In the first paragraph he sets out an argument for the falling rate of profit and senses a weakness. Because of this, in the

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[6] The actual text has "and" rather than "by".

second paragraph, he adds a strong condition and four reasons for its satisfaction. Finally the last paragraph indicates that the rate of profit falls because the condition is satisfied. One can use the formal model to see the initial argument, its weakness, and why the condition implies that the rate of profit will fall.

<u>Proposition 1.</u> Consider the model (1)-(7) with the additional relations $\tilde{L}=\bar{\tilde{L}}$, $b=\bar{b}$, $K=\bar{K}$ and one other relation specified arbitrarily (but not $=\bar{\ }$). The barred variables are constants. Let $d\tilde{L}$ be the change in the independent variable and $dA$ and $dL$ be the changes that this causes. If $d\tilde{L}<0$ and $dA=dL=0$ then $d\pi<0$; but if $dL<0$ is large enough relative to $d\tilde{L}$ and $dA$, then $d\pi>0$.

<u>Proof</u>: From (8) and (4)-(7)

$$\pi=\frac{S}{C+V}=\frac{1-\dfrac{L}{1-A}b\,\tilde{L}}{\dfrac{L}{1-A}K+\dfrac{L}{1-A}b\tilde{L}}=\frac{\dfrac{1}{L}-\dfrac{1}{1-A}b}{\dfrac{1}{1-A}\dfrac{K}{\tilde{L}}+b}$$

so that $\partial\pi/\partial\tilde{L}>0$ and $\partial\pi/\partial L<0$. The result follows.

The line Marx was following was that a reduction in $\tilde{L}$ would reduce surplus value and thus the rate of profit. But the weakness in this approach, which can be seen from the third expression in the sequence of equations above, is that if labour productivity rose sufficiently it might raise surplus value and lower the value of total capital sufficiently so that the rate of profit increased. Thus what Marx needed was a condition that limited the fall in $L$ relative the rise in $A$ (and the fall in $\tilde{L}$).

The first sentence of the second paragraph gives such a condition: that the value of labour power does not fall in the same degree as labour productivity rises. One can use the general framework to see this.

<u>Proposition 2.</u> Consider the model of (1)-(7) with the additional relations $\tilde{L}=\bar{\tilde{L}}$, $b=\bar{b}$, and two other relations specified arbitrarily (but not $\pi=\bar{\ }$). Let $P\equiv 1/L$ be the productivity of labour and $\tilde{V}\equiv b$ be the value of labour power. Then $\pi$ is a function of $P$ and $\tilde{V}$, i.e. $\pi(P,\tilde{V})$, with $\partial\pi(.)/\partial P<0$ and $\partial\pi(.)/\partial\tilde{V}<0$.

<u>Proof</u>: From (4)

$$A = 1 - \frac{1}{-} = 1 - \frac{b}{b \frac{1}{L}} = 1 - \frac{b}{\tilde{V}P}.$$

Substituting this into (3) gives

$$= \frac{1}{A + bL} = \frac{1}{1 - \frac{b}{P} \frac{1}{\tilde{V}} - 1} - 1 \quad \left(P, \tilde{V}\right)$$

which satisfies the conclusion of the proposition.[7]

If $d\tilde{L} < 0$ and, because of this, $dP > 0$ is sufficiently larger than $d\tilde{V} < 0$ then the rate of profit must fall; That is, if $\tilde{V}$ does not fall "in the same degree" as $P$ rises, the rate of profit will fall.

The sense is the following: Marx says the rate of profit will fall if the productivity of labour rises by more than the value of labour power falls. Think of these two events happening in sequence. The only way one of these can change while the other remains constant is if the change is compensated for by a change in the productivity of capital. Thus first when the productivity of labour rises, the productivity of capital must fall so that the value of labour power can remain constant and this last change causes the rate of profit to fall. Second when the value of labour power falls, the productivity of capital must rise so that productivity of labour can remain constant and this last change causes the rate of profit to rise. If the first effect is stronger the rate of profit will fall over all. Thus the first sentence is a complicated way of expressing a limit to the fall in $L$ relative to the rise in $A$. Marx doing partial differentiation in his head strains ones credulity but I see no other interpretation.[8]

Marx now returns to Hodgskin, attempts to describe his ideas via an example and disturbingly finds he can not do it. The example on p.304 is not carefully set out but can be reconstructed. There are initially two cases, I and II and then case III is added.

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[7] $\tilde{V} < 1$ from (1) if $> 0$ is assumed.

[8] The last two paragraphs are an example of the methodology that is used more ambitiously in the following section. It is the opposite of the strategy of using a number of quotes to convince the reader that Marx held a particular opinion. Rather Marx's argument is reconstructed so that the logic is clearly visible and then this construct is commented on. An example of this approach is Lapavitsas' (2000) analysis of Marx's "Mechanism of the Turnover". The method has the disadvantage that if the reader disagrees he/she will have to consult the original. In this case I fear that Laphavitas' "quick reflection will suffice" (fn. p229) may prove to be optimistic.

|     | C    | V   | S    |
| --- | ---- | --- | ---- |
| I   | £25  | £25 | £25  |
| II  | £175 | £25 | £25  |
| III | £175 | £25 | £125 |

The working day is 12 hours so that the worker produces £25 for his subsistence in 6 hours and £25 of surplus value in the other 6 hours. In each case the constant capital reproduces itself. Now Marx asks how many hours would have to be worked to maintain the 50% case I rate of profit in case II? As shown in case III, he would have to produce £25 of subsistence plus £100 of surplus value. Since this would take 30 hours, it is impossible and the rate of profit must fall. Then Marx notes that this is assuming constant productivity, but that if the productivity rose so that the worker could produce £125 in 12 hours then the rate of profit would not fall.[9] Marx reacts to this by supposing productivity only rises so that the worker can produce £50 in 12 hours. Then, even if he worked 24 hours he could not produce the £125 so the rate of profit must fall.[10]

Marx clearly realises this is unsatisfactory because he immediately starts going over the example again but can find no way to make the argument stronger. After this he metaphorically backs up and makes another run at the problem but can make no headway. He then refers back to the three key paragraphs (p.312), makes three and half pages of random comments and, at this point, the break occurs.

The idea that a rise in labour productivity attendant on the rise in $C/V$ may stop the rate of profit from falling is important. It may seem transparent from the table, but a little thought shows that it is not; the relation between productivity growth and the rate of profit is not part of the actual example. The general framework can be used to clarify this.

Consider the movement from case I to case III. It seems like $C, V, b,$ and $\tilde{L}$ are specified and then $C$ is changed. But since $C, V$ and $S$ are given in £ and the value of a £ is not given, it is only $C/V$ that is given. Thus an additional relation must be specified and it is convenient to make it $L = \bar{L}$.

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[9] In this case the value of labour-power falls from 5/10 of a day to 2/10 of a day, a fall of 60% rather than the 75% that Marx claims.

[10] These figures ate different from Marx's. He seems to think the rate of profit was 100% so that £200 of surplus value is needed. He allows the worker to produce £75 in 12 hours so that it is impossible to produce the required £225 in 24 hours.

<u>Proposition 3.</u>  Consider the model of (1)-(7) with the additional relations $C/V = \overline{C/V}$,

$\tilde{L} = \overline{\tilde{L}}$, $b = \overline{b}$  and  $L = \overline{L}$. If $d(C/V) > 0$, then there is a $dL < 0$ such that $d = 0$.

<u>Proof:</u> From (6) and (7)

$$= \frac{1}{b(S/V + 1)}.$$

From (8)

$$\frac{S}{V} = \frac{1}{}\ \frac{C}{V} + 1\ .$$

Thus

$$= \frac{1}{b\ \frac{1}{}\ \frac{C}{V} + 1\ + 1}.$$

From (5) and (6)

$$\frac{K}{\tilde{L}} = b\frac{C}{V}.$$

Finally from (1), (2) and (4)

$$L = \frac{1}{\frac{K}{\tilde{L}} + \frac{1}{}}.$$

Substituting for $K/\tilde{L}$ and

$$L = \frac{1}{b\ \frac{C}{V}\ 1 + \frac{1}{}\ + 1}.$$

The $dL$ is found by differentiating this equation with $b = \overline{b}$, $d = 0$ and $d(C/V) > 0$.


While this may seem like belabouring the obvious, the specification of the four additional relations is crucial. If, for example, $Y$ rather that $b$ was taken as a constant , then equation (2) shows that $L$ would not change in spite of the rise in $C/V$.

My interpretation of what happened is the following. When he started writing the section on Hodgskin, Marx thought that he could easily make the argument that the rise in the composition of capital would cause the rate of profit to fall. When he set out his position he sensed that there might be a weakness (Proposition 1) and hastily sketched the reinforcing arguments that involved elements other than the relative rise in the mass of capital (Proposition 2 and the four multi-sector indications.). But at this point he was still confident that he could demonstrate what I have called the intrinsic explanation. He

changed his strategy, set up an example in value terms and looked at the effect of a rise in the composition of capital. But again he found that he could not make a convincing argument (Proposition 3).Then, as he struggled to set out Hodgskin's argument formally, it slowly dawned on him that the problem was much deeper than he had suspected. He decided to break off the writing of TSV and sort out the problem of the falling rate of profit once and for all.

## 2. The General Law Section.

As noted, Marx has two types of explanations for the falling rate of profit: an intrinsic one which is based on the rising composition of capital and a number of subsidiary ones which involve phenomena like the worsening quality of resources. This section contains what I think is Marx's most sustained attempt to provide a justification for the intrinsic explanation. He first argues that the problem can be addressed in terms of a one sector model. After this he formulates the model first in value terms and then in terms of physical  units, thus repeating the attacks of the Hodgskin section. At last, following his deepening intuition, he tries to construct a model that combines both of these but fails because he can not handle the technical difficulties. The section ends with Marx, as it were, taking refuge in the subsidiary explanations.

### a. The Intrinsic Explanation.

In the opening paragraph of the General Law on p.104, Marx summarises the preceding section as a justification that the falling rate of profit can be dealt with in terms of a one sector model. He then sets out the problem in value terms, just as in the Hodgskin section and then appears to be surprised that, once again, he can not make the argument (pp.114-7). This impression is heightened because he then proceeds to glue in four large pages which start with the statement "let us first assemble the facts"(p.117).[11]

These four pages contain three separate attacks on the problem. The first is merely a continuation of the previous argument in terms of value categories. He sets out a number of examples in which he increases constant capital and discovers that the condition for constancy of the rate of profit is that surplus value and total capital grow at the same rate and that this implies that the rate of surplus value grows at this rate as

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[11] The transcription of these four pages is difficult to read since it has frequent gaps because the pages themselves have been damaged. Appendix 1 of Petith (2001) is a version of these pages with the gaps filled in. It is clear that this may distort what Marx wrote but there seems no other way to make the pages intelligible.

well (pp.117-9). (The analysis of why this attack does not work is given in Proposition 3 above.)

Finally convinced that this attack will not work, he shifts to the second. He lowers variable capital, either absolutely or relative to constant capital, and claims that surplus value will fall faster than total capital. He gives two reasons: first, he equates a fall in variable capital to a fall in labour which will eventually reduce surplus value and second, he claims the development of the productive power is not uniform across sectors (p.120). The first reason is incomplete since he doesn't even mention the effect on total capital, while the second is only mentioned. Once again the style of the writing gives no indication that Marx is aware of these problems but it seems likely that he is.

He now draws a horizontal line, writes and attempts to develop the second attack with more precision. He wants to construct an example in which workers are reduced and machinery increased in a way that keeps the value of total capital constant. First he implies, wrongly, that surplus value is the rate multiplied by the quantity of labour and notes that these two forces will act in opposite directions on surplus value. But then he reverts to surplus value itself rather than the product. His example shows that the reduction in the number of workers has definitely caused a fall in surplus value. The problem is that, since he has no value terms in his example, he can not calculate what has happened to the value of capital. He is forced to end with a limp statement about total capital remaining constant (pp.123-5). (The analysis of why this doesn't work is given in Proposition 1 above.)

Marx now appears to realise that he will have to have both workers and values in the same example if he wants to make his point. He writes and begins to construct the model. Since this is Marx's major attempt to show that the rate of profit must fall, I will recount what happened in detail. He starts with an example which is set out in terms of workers and hours per worker so that it is necessary to amplify the notation. Let $n$ be the number of workers and $h$ the hours each worker works. Then

$$(10) \qquad\qquad \tilde{L}=nh$$

and $bh$ is the wage per worker in value terms. The example is set out in the first four columns of the following table. In case I $C$ is 150 of raw material and 50 of

|    | $V$ | $C$ | $n$ | $bn$ | $h$ | $S$ |      | $Y$  |      | $b$ |
|----|-----|-----|-----|------|-----|-----|------|------|------|-----|
| I  | 400 | 200 | 10  | 40   | 80  | 400 | 0.66 | 1000 | 1    | 0.5 |
| II | 80  | 520 | 2   | 40   | 80  | 80  | 0.13 | 1000 | 0.68 |     |

machinery while in case II $C$ is 150 of raw material and 370 of machinery. The output in physical terms is the same in both cases. He notes that total capital is the same in both cases but that case II has a larger composition of capital. One can see where he is trying to go. The example can be finished, for example, by specifying $h=80$ and then calculating $S$ from (10) and (7) and then     from (8) for the two cases. This is done in the next three columns of the table. By specifying the model consistently in terms of both values ($V$ and $C$) and number of workers ($n$), Marx can force down surplus value by reducing the number of workers and, at the same time, keep the value of total capital constant so that the rate of profit falls. This is the argument the he wanted to make.

But there is a problem. With physical output fixed, the value of a unit has fallen. Suppose output is 1000 units, then in case I the value of a unit is 1, and in case II 680/1000=0.68. Thus the case II wage in value terms should be reduced to keep the real wage constant. The simple argument that Marx hoped to make is not viable. But at least, in terms of this framework, the basic problem can be confronted (to last paragraph p.125).

Marx is aware of exactly this problem and reacts by trying to find the values or prices so that, presumably, he will know how much he has to reduce the wage in value terms in order to keep the real wage fixed. He makes the natural assumption that machines are fixed capital, but this complicates his task in a way that he never manages to resolve. Specifically he assumes that the machine of case I lasts one year and that of case II ten years. He then calculates the price in case II as

$$326=(37+150+80+0.05\text{x}333)(1.05)$$

where 37 is the straight line depreciation of the machine, 0.05x333 is the interest payment on the un-depreciated part of the machine and (1.05) is the profit margin added by the capitalist.[12] This shows that Marx does not know how to calculate costs for the case of fixed capital[13] and he himself is aware that this method is problematic since he seems to change the method in the next case and is clearly worried, p.128 second full paragraph, by the fact that the interest payment enters into cost but not value. He ends the paragraph by noting that the price is much lower than in case I which should, by his

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[12] Marx's example, as the editors note, is riddled with errors. The text here is consistent so that the numbers differ from those of the original text.

[13] This is not that straightforward. One way to do this would be to suppose that the capitalist can borrow and lend at the rate of interest, commits the present value of total costs at time 0, reinvests all revenue, and finally sets the price so that the ratio of the accumulated revenue at time 10 to the committed funds is $(1+r)^{10}$ where $r$ is the profit margin.

method, be 600x1.05=630. It can not be emphasised too strongly that in this, Marx's main attempt to explain the falling rate of profit, he went to great lengths to keep the real wage constant (to the bottom of p.126).

He now sets out case III where $C$ is composed of 150 of raw materials and a

|      | $V$ | $C$  | $\tilde{L}$ | $bh$ |
|------|-----|------|-----|------|
| III  | 80  | 2150 | 2   | 40   |

machine which lasts either 10 or 100 years depending on the way the manuscript is interpreted. Using the same calculation as in case II, the price is either 546 or 366 depending on the interpretation.[14] There is no clear reason for the introduction of case III. If one takes the first interpretation, one possibility is that Marx was surprised by how cheap the good had become and was worried that the rate of profit would not fall so he wanted an example with a smaller price reduction. If one takes the second interpretation, then Marx wanted to see if lengthening the turnover time could raise the price, once again to get the rate of profit to fall. In any case, with case III, he has lost the line of his argument since total capital is no longer constant (to end of first paragraph on p.127).

Marx seems to realise this since he next gives a verbal discussion of the effect on price of lengthening turnover times when total capital is fixed. He says that there are two types of effects: The first merely lowers wear and tear and thus the price. The second combines this with an increase in machinery and a fall in workers in a way that causes output to fall and the price to rise. This can, perhaps, be interpreted as an attempt to redo the movement from case I to case II and then from case II to case III (second interpretation) but with total capital held constant. In any case, it is unconvincing and so far from the initial example that Marx admits that this discussion actually belongs elsewhere (to end of first full paragraph p.128).

Starting with the third paragraph of p.128, Marx makes a last attempt to revive his argument. But he can make no headway and ends, in the last full paragraph of p.130, talking about an American economist called Wayland. This is the end of what is the high water mark of all of Marx's attempts to provide a demonstration of the intrinsic explanation.

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[14] As noted, Marx calculates this case differently from the preceding one and does it in a way that is impossible to fathom. Since the method of case II is set out clearly, I have used it for this case as well.

A question with an interesting answer is what would have happened if Marx had stuck to a circulating capital model? One can imagine that by trail and error he would have found the figures that kept the real wage fixed and checked if the rate of profit fell. This can be done easily using the model: First calculate $b$ for case I from (6) and (9)

$$b=V/\ \tilde{L}=0.5.$$

Now construct Case IV with the values of $C+V$, $Y$, $n$, $h$, and $b$, given in the following table.

| $C+V$ | $Y$ | $n$ | $h$ | $b$ |
|-------|------|-----|-----|-----|
| 600 | 1000 | 2 | 80 | 0.5 |

The figures for a table like that of case I can be calculated as follows: From (2)

$$L=\tilde{L}/Y=0.16.$$

From (1), (2), (4)-(6)

$$C+V = \frac{L}{1-A}\left(AY + b\tilde{L}\right).$$

Solving for $A$ gives

(11)
$$A = \frac{C+V - b\tilde{L}^2 / Y}{\tilde{L} + C + V} = 0.77.$$

From (4)

$$= \frac{L}{1-A} = 0.7 \quad \text{and} \quad bh=28.$$

From (5) and (7)

$$V=b\ \tilde{L}=56$$

$$S=(1-b\ )\tilde{L}=104.$$

Since

$$C=V\text{-}600=544$$

we have, from (8)

$$= \frac{S}{C+V} = 0.17.$$

These are displayed in the following table.

|    | $V$ | $C$ | $n$ | $bh$ | $h$ | $S$ |      | $Y$ |     | $b$ |
|----|-----|-----|-----|------|-----|-----|------|------|-----|-----|
| IV | 56 | 544 | 2 | 28 | 80 | 104 | 0.17 | 1000 | 0.7 | 0.5 |

Thus, even though the wage in value terms falls (from 40 to 28) to keep the real wage constant, the rate of profit falls (from 0.66 to 0.17).

Thus Marx was very close to setting out an example in which the rate of profit falls. What would have happened next? Given the way he reacted previously it is probable that he would have tried other numbers to see if the rate of profit always fell. Somewhat surprisingly, as the following proposition shows, he would have found that it always did.

Proposition 4. Consider the Model (1)-(7) with the additional relations $C/V=\overline{C/V}$, $\tilde{L}=\overline{\tilde{L}}$, $b=\overline{b}$ and $Y=\overline{Y}$. Then $d\ /d\,\tilde{L}>o$ and $\lim_{\tilde{L}\ 0}S=0$.

Proof: From (11)

$$1 - A = \frac{\tilde{L}\left(1 + \tilde{L}b/Y\right)}{C + V + \tilde{L}}.$$

Thus from (2) and (4)

$$= \frac{L}{1 - A} = \frac{C + V + \tilde{L}}{Y + b\tilde{L}},$$

so that

$$S = \tilde{L}\left(1 - \quad b\right) = \frac{Y - b\left(C + V\right)}{Y/\tilde{L} + b}.$$

The result follows from this equation.

It is interesting to speculate what would have happened if Marx had avoided the fixed capital trap and had gone down this road. He would have ended with a robust example of how a fall in the number of workers with the total value of capital fixed would cause the rate of profit to fall. In a subtle way, this is not the same as the result that he had set out to demonstrate since the four additional relations impose restrictions on the technology. There are three possible continuations: In the first two Marx would have noticed the subtle difference and either convinced himself that his intuition was wrong or that it was still correct. In the third he would not have noticed the subtle difference and would have been convinced that he had a robust demonstration. In this third case I think the falling rate of profit might have played a much more prominent role in Marx's writings than it subsequently did. It is even possible that it might have appeared in Volume I.

b) The Subsidiary  Explanations.

The rest of the section contains three sketches of essentially two sector explanations of the fall in the rate of profit. The first two concern resources while the third focuses on luxury goods. The first (pp.130-1) is less detailed than the second (pp.133-6). In the latter Marx starts by explaining how an increase in labour productivity in a sector raises surplus value by cheapening the good and thus lowering the value of labour power. But he stresses that the cheapening of a single sector will only have a small effect on the economy wide value of labour power. Following on this, he notes that if productive power grew evenly in all sectors , then surplus value would also grow at the same rate. But this doesn't happen for two reasons: " The anarchy of competition" and natural conditions such as " the influence of the seasons,…, (the) exhaustion of forests, coal seams, mines and the like."  He furthermore notes that productivity in agriculture, which is the main component of workers' consumption, grows more slowly than that of industry. He concludes that the growth of surplus value is always smaller than the growth of productive power of capital in all branches of industry.

There are three problems with this as an explanation of the fall in the rate of profit. First, Marx says productive power of capital when he ought to say labour. This, I think, is just a slip; probably Marx was thinking of the growth in the productivity of labour caused by the relative increase in capital. Second, Marx does not say that this is an explanation of the fall in the rate of profit. However it can be taken to be one since, in the summary on p.148 Marx clearly refers back to this section and states that if the rate of surplus value does not rise in proportion to the growth of productive power the rate of profit will decline. Finally there is the question of whether this condition is valid. The following proposition shows that it is.

Proposition 5. Consider the model (1)-(7) with $b=\bar{b}$ and three other relations specified arbitrarily (but not $=\bar{\ }$). Let $e \equiv S/V$ be the rate of surplus value and $P \equiv 1/L$ be the productivity of labour. Then $\pi$ is a function of $e/P$ with $d\pi(.)/d(e/P)>0$.

Proof: From (1), (2) and (7)

$$e = \frac{1 - A - bL}{bL} = \frac{P}{L}(1 - A) - 1,$$

so that

$$A = 1 - \frac{b}{P}(1 + e).$$

Substituting this into (3)

$$(.) = \frac{1}{1 - \frac{e}{P}b} - 1$$

from which the conclusion follows.

Thus if $e/P$ falls so does the rate of profit. But its correctness (i.e. that the two reasons imply a fall in $e/P$) can only be accessed in the context of a formal two sector model.

The third explanation (pp.148-9) is that the increase in the composition of capital will also occur in sectors that are unrelated to worker consumption. This will not increase the rate of surplus value and thus cause the rate of profit to fall. Marx had previously made this point in the section on Hodgskin. Here he adds that the growing cheapness may expand the range of worker consumption and thus weaken the effect.

Marx does not attempt a rigorous demonstration of either of these subsidiary explanations or those of the Hodgskin section. Thus there is nothing on which an analytical look might be based. On the other hand the reader might wonder if Marx's conjectures could e supported formally. The issue is complicated both because the price and value rates of profit differ in multi-sector models and because the correctness may depend on parameter values. This is studied in detail in Petith (2001) which provides the first mathematical treatment of these issues. The quick answer is that, independent of parameter values, three out of four of Marx's subsidiary explanations can be supported for at least one of the rates of profits.

## 3. Conclusion.

One can speculate about how Marx felt about resource scarcity explanations and the intrinsic explanation of the falling rate of profit after he had finished the work on TSV.

With respect to the first, reference must be made to the section on Cherbulitz which Marx wrote as one of the final sections of TSV III. In this, Marx's explanation of the rise in the composition of capital partly consisted of a page long detailed description of growing resource scarcity (p.368). I think it is incontestable that at this point Marx thought that resource scarcity would cause the rate of profit to fall. Its mention in the comments on Hodgskin, and in the sections in the General Law, and finally its emphasis in the Cherbulitz piece allow no other conclusion.

With respect to the intrinsic explanation, on the one hand this argument fitted better with Marx's general view since it was a "barrier" that arose from the nature of capitalism itself. On the other hand Marx had just failed in his protracted attempt to

demonstrate it. At least two positions are possible: first that he had changed his mind and now thought the proposition was incorrect and second, that he still thought the proposition was correct and hoped to demonstrate it in the future. I think the second is more likely for a number of reasons: first his problems were technical so that he could hope to resolve them, second when he came to write Part III of Volume III of Capital he gave the impression that he thought the intrinsic explanation was correct, and finally in the 1870s he had another go at demonstrating it.

## IV.    Part III of Volume III of Capital.

Part III, is generally taken to be the main source for Marx's explanation for the falling rate of profit. This section of the present paper gives a somewhat different interpretation. I will argue 1) that Part III does not contain an explanation of why the rate of profit falls, 2) that it contains clear references to previous work in which resource scarcity figured as a cause of the falling rate of profit and 3) that its purpose is to describe the role of the rate of profit in capitalist development. The section finishes with a hypothesis about Marx's beliefs that makes these three points part of a consistent whole.

### 1.   The Absence of Explanations.

At first glance, the point that the section contains no explanations of the falling rate of profit is hard to sustain since there are three clear candidates. First, the rise in the rate of profit of the innovator and the eventual fall of the general rate that  is mentioned in the last paragraph of p.233 and set out clearly starting on p.264. Second, the constant rate of surplus value, rising composition of capital argument that opens the section on pp.211-6. And, finally, the falling maximum surplus value argument which is mentioned many places like lines 7-13 p.235 and stated forcefully on p.247.

The first candidate, that of the rising profit of the innovator, is not an explanation but rather a defence against a possible attack. This is the argument referred to in the section on the Okishio Theorem. Marx does not explain why he inserts it but it is probable that he saw it as a defence against the point that no capitalist would introduce a technique tht would lower his rate or profit and thus the rate of profit could not fall. The important point here is that Marx gives no argument for its validity. This is not a defect when the claim is used as a defence (since it puts the ball in the attacker's court), but it certainly disqualifies it as an explanation of the falling rate of profit.

The second candidature, that of the rising constant capital which opens Part III, seems like a formal demonstration of the falling rate of profit but it is not for two distinct reasons. First, (p213) Marx admits that the rate of surplus value may rise and then claims that the rate of profit will still fall. Thus the demonstration is reduced to a claim. Second, this is one of the arguments at which Marx looked carefully both in the Hodgskin section (See Proposition 3) and in the General Law section. His dissatisfaction with this was what drove him, in the first instance to write the General Law section; and in the second, to try to construct the argument there that used both value and physical terms simultaneously. After all this it is unthinkable that he would have presented the example as a valid demonstration. Rather, I think that he meant it merely as an introduction that would give the reader the feeling of why the rate of profit might be expected to fall.

The third candidature, that of the falling maximum surplus value, is harder to dismiss since Marx fills a full page with a careful explanation of how a fall in labour must eventually lower surplus value and thus the rate of profit. however it seems unlikely that he could have meant this as an explanation since, as with the second candidate, he had already examined this argument in both the Hodgskin and the General Law sections and had seen its weakness. (See Proposition 1.) Rather I think that he inserted it because it fitted with the theme of the section which was contradictions, or really, tendencies which go in opposite directions. Thus he emphasised that the rise in the rate of surplus value and the fall in variable capital have opposite effects on surplus value, but that the second must dominate so the surplus value must eventually fall. But he says not a word about the sticking point in the falling rate of profit argument, that the value of total capital may also fall. Thus I think this is best regarded as one of the examples of contradictions rather than am attempt to explain why the rate of profit will fall.

## 2.  References to Resource Scarcity.

There are a number of sections of Part III that are taken virtually verbatim from longer sections in either TSV or the General Law section where the longer sections are arguments for the falling rate of profit based, usually, on resource scarcity.

First, in Part III Marx writes "Outside a few cases (for instance, if the productiveness of labour uniformly cheapens all elements of constant and variable capital) the rate of profit will fall, in spite of the higher rate of surplus value" p.226. But in the General Law he wrote "Only if productive power were to increase evenly in all branches of industry which directly or indirectly provide products for workers consumption could a

proportional growth in surplus value correspond to a proportional increase in productive power" p.135. A paraphrase makes the parallelism clearer. In the first Marx states that only if all elements of constant and variable capital are uniformly cheapened will the rate of profit not fall. In the second he states that only if the same condition holds will a proportional increase in surplus value correspond to a proportional increase in productive power, which implies, as Marx later correctly states, that the rate of profit will not fall. (See Proposition 5.) Thus the content of the two statements are identical. Why will all elements of constant and variable capital not be cheaped uniformly? In Part III he cites the two failed arguments of Propositions 1 and 3, while in the General Theory section there is a long reference mainly to resource scarcity,

Next, in Part III on p.236 Marx wants to argue that the composition of capital will increase. Certainly the mass of capital increases relative to the number of workers but this may be outweighed by the fall in value of the constant capital. Specifically he takes the case of the modern spinner who works up more cotton with more machinery than his predecessor with a spinning wheel. However both the cotton and the machinery now have lower value. Marx states without argument that this will only slow the rise in the composition of capital. But on pp.365-9 of TSV III Marx deals with exactly the same spinners in much greater detail. He gives a complicated explanation about why the mass of machinery grows faster than the value falls. And then goes on for a full page about how growing resource scarcity means that the value of the raw materials can not fall as fast as their mass rises.

Finally there is a long section in Part III p.260, lines 7-19, that has been copied virtually verbatim from the General Law section, p.135, lines 10-22. The quote from Part III appears as the first paragraph of section IV, supplementary remarks. It is followed by another unconnected paragraph and then a long interposition by Engels. On the other hand, as I argued above in III.2.b and Proposition 5, the section in the General Law is part of a coherent argument that resource scarcity is one of the causes of the fall in the rate of profit.

In general it is difficult to escape the feeling that Marx made a conscious effort to avoid emphasising the role of resource scarcity.

## 3.  The Role of the Rate of Profit

I support the claim that Part III is a description of the role of the rate of profit in capitalist development with a compressed description of the content of the three chapters.

### a. Chapter 13. The Law as Such.

This chapter is about the relation between the falling rate of profit and the movement of aggregates during capitalist development. It starts with what appears to be a five page explanation of why the rate of profit falls (pp 211-6). But, as I argued above, this is more of an attempt to give the reader a reason why he might expect it to fall. After this Marx discusses the following issues: The mass of surplus value will grow in spite of the falling rate of profit (pp 216-7 and 220-5). Capital will grow faster than labour (pp 217-20). The fall in the value of each unit and thus the fall in surplus value from each unit will not mean that total surplus value falls (pp 225-31). In the middle of this last (pp 226-7) he states that this fall will cause $S/V$ to rise but that the rate of profit will not rise. He justifies this by brief references to, first, the resource scarcity argument referred to in the proceeding section III.2.b and then to the two failed arguments of Propositions 1 and 3. He then finishes the argument that the surplus will not fall. Thus the point is to explain the movement of the main aggregates more of less in spite of the falling rate of profit.

### b. Chapter 14. Counteracting Influences.

This chapter is concerned with the speed of the fall of the rate of profit. It is divided into six sections of which only the first, "Increasing Intensity of Exploitation" appears to contain an explanation of the fall. Marx gives examples of both absolute and relative surplus value. Generally increasing exploitation increases the quantity of constant capital used as well as that of labour. Since Marx wants to keep total capital constant, this implies a reduction in labour. The rate of surplus value $S/V$ increases because of the rise in intensity of exploitation but variable capital $V$ falls because of the fall in labour. The first is the counteracting influence, the second is what ultimately causes the fall in the rate of profit. He gives some examples like the introduction of female labour and improvements in agricultural techniques which don't increase constant capital and thus don't imply a fall in variable capital so that they actually cause a rise in the rate of profit. But the only place where he gives an indication of why generally the fall in variable capital dominates and the rate of profit falls is the sentence starting with "Moreover, it has already been demonstrated… ." on p 233 which refers to the failed explanation of Proposition 1. The rest of the chapter gives four more counter tendencies and closes with the wonderful and unexplained statement that the rate of profit falls not because labour productivity falls, but because it rises. Thus the point here is to explain why the rate of profit falls so slowly.

**c. Chapter 15. Exposition of the Internal Contradictions of the Law.**

The theme of this chapter is the broad theme of contradictions or opposing tendencies in capitalism. The opening section gives a general description of many of them. In the second section Marx concentrates on production. First with regard to the falling rate of profit, he emphasises (as I noted above) that, with capital fixed, the rate of surplus value and variable capital move in opposite directions. Next, the rise in productivity with its contradictory effects on the rate of profit, affects the accumulation of capital in contradictory ways. He concludes with a few more examples including the following: capitalism both increases the demand for labour and creates surplus population; and, the devaluation of capital in crises checks the fall in the rate of profit and allows capital accumulation to start a fresh. The penultimate section is mainly about crises. (The ultimate only contains the rising innovational profit argument.) It starts with an explanation of how there can be excess supply of capital and labour simultaneously and goes on to describe the movement of wages and the devaluation of capital. The fall in the rate of profit seems to have a causal role. He summarises by noting the general contradictions that production is for profit, not consumption and that the rate of profit falls because of the attempt to increase profits. This section ends in a somewhat raised tone by stating that the falling rate of profit is the characteristic that shows that capitalism "is not an absolute but only a limited historical mode of production corresponding to a limited epoch." In general, the falling rate of profit plays the leading role in the contradictions that occur during capitalist development.

**d. Conclusion.**

This examination of the content of Part III show that its focus is not on explaining why the rate of profit falls. Rather Part III explains why the rate falls only slowly, why the aggregates grow in spite of this fall, and how many of the contradictions stem from this fall. That is, Part III is actually a description of capitalist development in which the falling rate of profit plays a leading role.

**4. Conclusion.**

If one supposes that Marx still hoped to be able to demonstrate the intrinsic explanation satisfactorily then these three aspects coalesce into a consistent view of Part III. First the absence of a demonstration and the frequent reassurances follow immediately from the supposition. Second, the role of the falling rate of profit in capitalist development was sufficiently important for Marx to describe it before he had a firm demonstration of the reason for the fall. This is shown by his final conclusion that

capitalism is a limited historical mode. Finally, he occasionally drifted toward explanations which involved resource scarcity but pulled back since this explanation would have considerably weakened his main conclusion. Moreover this was not dishonest since Marx believed that he would be able to give a proper demonstration of the intrinsic explanation. All this, it must be stressed, is based on supposition; but it does afford a coherent explanation of the main characteristics of Part III.

## V.  The Argument of the 1870s.

Marx made another attack on the falling rate of profit in his Mathematical Investigations of the 1870s. These were left in a disordered state and contained a large quantity of mathematical calculation.[15] Engels got a Cambridge mathematician friend to put the parts that were relevant in order and inserted them as Chapter 3 of Volume III of Capital. In it Marx takes the formula $=S/(C+V)$ and for twenty pages systematically varies individual variables or combinations of them and, in each case, observes the effect on the rates of profit and surplus value. The tediousness of the exercise makes it understandable that the chapter has lain un-discussed for over a hundred years, but there is gold buried beneath the detail. It is possible to interpret Marx as saying that the constancy of the rate of surplus value while constant capital is increasing in the opening section of Part III is due to growing resource scarcity.

The key example is on pp.56-8.[16]

"Now, the variable capital may either rise of fall. Let us first take an example in which it rises. Let a certain capital be originally constituted and employed as follows:

$$I. \qquad 100_c + 20_v + 10_s; \ C = 120, \ s = 50\%, \ p = 8\tfrac{1}{3}\%.$$

Now let the variable capital rise to 30. In that case, according to our assumption, the constant capital must fall from 100 to 90 so that total capital remains unchanged at 120. The rate of surplus-value remaining constant at 50%, the surplus-value produced will then rise from 10 to 15. We shall then have:

$$II. \qquad 90_c + 30_v + 15_s; \ C = 120, \ s = 50\%, \ p = 12\tfrac{1}{2}\%.$$

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[15] The original work is available in German and French but not English. See Marx (1974) and Alcouffe (1985).

[16] Marx's notation is different from that used in the paper: c, v, and s are constant and variable capital and surplus value; s' and p' are the rates of surplus value and profit and C=c+v.

Let us first proceed from the assumption that wages remain unchanged. Then the other factors of the rate of surplus-value, i.e., the working-day and the intensity of labour, must also remain unchanged. In that event the rise of $v$ (from 20 to 30) can signify only that another half as many labourers are employed. Then the total value produced also rises one-half, from 30 to 45, and is distributed, just as before, 2/3 for wages and 1/3 for surplus-value. But at the same time, with the increase in the number of labourers, the constant capital, the value of the means of production, has fallen from 100 to 90. We have, then, a case of shrinkage of constant capital. Is such a case economically possible?

In agriculture and the extractive industries, which a decrease in labour productivity and, therefore, an increase in the number of employed labourers is quite comprehensible, this process is-on the basis and within the scope of capitalist production-attended by an increase, instead of a decrease, of constant capital. Even if the above fall of $c$ were due merely to a fall in prices, an individual capital would be able to accomplish the transition from I to II only under very exceptional circumstances. But in the case of two independent capitals invested in different countries, or in different branches of agriculture or extractive industry, it would be nothing out of the ordinary if in one of the cases more labourers (and therefore more variable capital) were employed and worked with less valuable or scantier means of production than in the other case."

Marx now supposes the rise in $v$ is due to a rise in wages and at the same time the working day increases proportionally. Then he returns to the original situation. " Now let us assume that the variable capital falls, instead of rising. Then we have but to reverse our example, taking II as the original capital, and passing from II to I.

*II*. $90_c + 30_v + 15_s$ then changes into

*I*. $100_c + 20_v + 10_s$ , and it is evident that this transposition does not in the least alter any of the condition regulating the respective rates of profit and their mutual relation.

If $v$ falls from 30 to 20 because 1/3 fewer labours are employed with the growing constant capital, then we have before us the normal case of modern industry, namely, an increasing productivity of labour, and the operation of a larger quantity of means of production but fewer labourers. That this movement is necessarily connected with a simultaneous drop in the rate of profit will be developed in the third part of this book."

In the first half Marx worries about whether the case he has described is economically possible. It is not clear what he is worried about or why he has to invoke agriculture and extractive industries, and comparisons between countries to resolve his doubts. A possible explanation is that he thought the increase in labour which lowers labour productivity (the intuitive level) would cause the rate of surplus value to fall (equation (9)) thus making the assumption that it was constant impossible. The resolution for the example of agriculture is that, in the country with more labourers and scantier means of production (less tractors) labour productivity and the rate of surplus value does not fall because the soil is richer.

This is important because he says that the movement of the second half is connected with the fall in the rate of profit that will be developed in the third part to the book. In the opening five pages of Part III, just as in the second half of the argument here, the rate of profit falls because the rate of surplus value remains fixed when constant capital is increased relative to variable capital. The implication is that the fall in the rate of profit described in Part III is, just as the one described here, caused by increasing resource scarcity. This is the main point.

It may be objected that there is nothing very strange about a constant rate of surplus value when labour rises relative to capital. Looking at equation (9), the rise in $L$ (the fall in labour productivity) can easily be compensated for by a fall in $A$ (a rise in capital productivity) so that, while it is not clear what was worrying Marx, it could not have been this.

In answer it can be shown that, in terms of net productivities, the assumption that the rate of surplus value is constant implies that labour productivity does not fall as labour is increased. Thus the assumption violates Marx's basic intuition and he was right to be preoccupied.

First consider the net labour and capital productivities $P_L$ and $P_K$. Because the model has only circulating capital which is used up in production $P_L$ $(Y-K)/\tilde{L}$ and $P_K$ $(Y-K)/K$. From (1) and (2)

$$P_L = \frac{1}{L} - \frac{K}{\tilde{L}} = \frac{1}{L}(1 - A)$$

$$P_K = \frac{1}{A} - 1.$$

If one thinks in terms of a gross production function with $A'(K/\tilde{L})>0$ and $L'(K/\tilde{L})<0$, then this implies a net one with $P_L'(K/\tilde{L})>0$ and $P_K'(K/\tilde{L})<0$ and visa versa. Thus if

one only uses the signs of the derivatives and assumes they are non-zero, then it doesn't matter if one thinks in terms of gross or net productivities. Since Marx does not seem to have made the distinction, it seems fair to describe his intuition in terms of either.

Now consider formalising Marx's example. He seems to specify five additional relations, i.e. he takes as givens $C+V$, $V$, $S/V$, $V/\tilde{L}$ and, by our convention, $b$. But these are not independent. $S/V=(1-b)/b$ from (9) so that is constant. $V/\tilde{L}=b$ so that the constancy of $V/\tilde{L}$ is not independent.

Proposition 6. Consider the model of (1)-(7) with the additional relations $C+V=\overline{C/V}$, $V=\overline{V}$, $S/V=\overline{S/V}$, and $b=\overline{b}$. For this model $dP_L/dV=0$.

Proof: There is a gross production function implicit in the model. From (1) and (2)

$$\frac{A}{L} = \frac{K}{\tilde{L}}.$$

Solving for $L$ from this and (4)

$$L = \frac{}{1 + \dfrac{K}{\tilde{L}}}.$$

From this and (2) the gross production function is

$$\frac{Y}{\tilde{L}} = \frac{1}{L} = \frac{1}{} + \frac{K}{\tilde{L}}.$$

The net production function is thus

$$P_L = \frac{Y-K}{\tilde{L}} = \frac{1}{L} - \frac{K}{\tilde{L}} = \frac{1}{}.$$

Since does not depend on $V$ ($S/V=(1-b)/b$), the conclusion follows.

That is, if variable capital (or $\tilde{L}$ since $V/\tilde{L}=\overline{V/\tilde{L}}$) increases while total capital is held constant, the net productivity of labour does not fall.

The point that gives this section its importance is that Marx associates the example with the fall in the rate of profit described in Part III. If one accepts the interpretation given here, it means that by the 1870s Marx interpreted, for example, the fall in the rate of profit portrayed in the opening five pages of Part III as being caused by growing resource scarcity. And by implication, that he accepted that there was no intrinsic reason for the rate of profit to fall. I think this is overstated, but it hard to avoid the impression that Marx was drifting in this direction.

## VI. Conclusion.

The view expressed in this paper is different from those that are held by virtually everyone who has thought about Marx's writings on the falling rate of profit. The root of the difference is the careful analysis of the General Law section. The general opinion is: first, that Marx's main demonstration of the falling rate of profit occurs in Part III of Volume III; and second, that Marx thought his demonstration was successful and did not realise that it had failed. This, I think is to do Marx an injustice. The careful study of the General law section shows that Marx both knew what he had to do to demonstrate that the rate of profit would fall, and understood that he had not successfully done it.

Not appreciating this and a general unawareness of the General Law section leads to a number of errors of interpretation. First, analysis of Marx's argument for the falling rate of profit should not be based on Part III since this was meant rather as a description of the role of the falling rate of profit in capitalist development. Second, the absence of direct references to resource scarcity there should not be taken to imply that Marx that this was unimportant since there are numerous suppressed ones. Third, it is wrong to cite Marx as an authority for such arguments as the falling maximum rate of surplus value since Marx, himself, knew that there were problems with these. Third, unawareness of the effort Marx made to keep the real wage constant has made it easier to call theories based on rising wages Marxian. And forth, lack of knowledge of Marx's frequent appeals to and late emphasis on resource scarcity has led to mainstream neglect of this aspect of his thought. In sum, the careful study of the General Law section adds greatly to the understanding of Marx's work on the falling rate of profit.

## Bibliography

Alcouffe, A. (1985). Les Manuscrits Mathématiques de Marx. Étude et présentation. Paris: Economica.

Clarke, S. (1994). Marx's Theory of Crisis. London: The MacMillan Press, Ltd.

Duménil, G. And D. Lévy. (2000). "Technology and Distribution: Historical Trajectories a la Marx." Typescript, CEPREMAP, Paris. Forthcoming in Journal Economic Behaviour and Organisation.

Foley, D. (1986). Understanding Capital. Cambridge, Mass: Harvard University Press.

Laibman, D. (1977). "Toward a Marxian Model of Economic Growth." American Economic Review, 387-92.

Laibman, D, (1997). Capitalist Macrodynamics. London: Macmillan Press.

Lapavitsas, C. (2000). "On Marx's analysis of Money Hoarding in the Turnover of Capital." Review of Political Economy 12, 219-35.

Lebowitz, M. (1982). "The General and the Specific in Marx's Theory of Crisis." Studies in Political Economy 7, 5-25.

Marx, K. (1959). Capital, Volume III. London; Lawrence and Wishart.

Marx, K. (1969,72). Theories of Surplus Value, Parts II and III. London: Lawrence and Wishart.

Marx, K. (1974). Mathematische Manuskripte, ed., introduced and commented on by W. Endemann. Kronberg:Scriptor Verlag.

Marx. K. and F. Engels, (1991). Collected Works, Volume 33. London: Lawrence and Wishart.

Meek, R. (1967). The Falling Rate of Profit in R. Meek, Economics and Ideology and Other Essays. London: Chapman and Hall, Ltd.,129-42.

Meek, R. (1960). "The Falling Rate of Profit." Science and Society, Winter.

Moseley, G. (1991). The Falling Rate of Profit in the Post-War US Economy. London: MacMillan.

Okishio, N. (1961)." Technical Change and the Rate Profit." Kobe University Economic Review 7, 86-99.

Okishio, N. (1977). " Notes on Technical Progress and Capitalist Society." Cambridge Journal of Economics 1, 93-100.

Perelman, M. (1985)." Marx, Malthus and the Organic Composition of Capital." History of Political Economy 17,3, 461-90.

Petith H. (2001). A Descriptive and Analytic Look at Marx's Own Explanations for the Falling Rate of Profit (Long Version). UAB working paper available at http:// Pareto.uab.es/wp

Robinson, J. (1942). An Essay on Marxian Economics. London: Macmillan.

Roemer, J. (1981). Analytical Foundation of Marxian Economic Theory. Cambridge: Cambridge University Press.

Rosdolsky, R. (1977). The Making of Marx's Capital. London: Pluto Press.

Rosdolsky, R. (1956). "Zur neuren Kritik des Marxschen Gesetzes der fallenden Profitrate." Kyklos, 9, no. 2.

Samuelson, P. (1957). "Wages and Interest : A Modern Dissection of Marxian Models." American Economic Review 47, 884-912.

Shaikh, A. (1978). "Political Economy and Capitalism: Notes on Dobb's Theory of Crisis." Cambridge Journal of Economics 2, 233-51

Skillman, G. (1997). "Technical Change and the Equilibrium Profit Rate in a Market with Sequential Bargaining," Metroeconomica 48, 238-61.

Steedman, I. (1977). Marx in the Light of Sraffa. Old Woking, Surrey: The Gresham Press.

Sweezy. P. (1942). The Theory of Capitalist Development. New York: Monthly Review Press.

Tsoulfidis, L. (2002). "Classical, Neo-classical, and Marxian Perspectives on the Falling Rate of Profit and the Tendency to Crisis." Paper presented at ESHET 2002.