Information-Sharing Implications of Horizontal Mergers *

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Abstract

We analyze the effects of uncertainty and private information on horizontal mergers. Firms face uncertain demands or costs and receive private signals. They may decide to merge sharing their private information. If the uncertainty parameters are independent and the signals are perfect, uncertainty generates an informational advantage only to the merging firms, increasing merger incentives and decreasing free-riding effects. Thus, mergers become more profitable and stable. These results generalize to the case of correlated parameters if the correlation is not very severe, and for perfect correlation if the firms receive noisy signals. From the normative point of view, mergers are socially less harmful compared to deterministic markets and may even be welfare enhancing. If the signals are, instead, publicly observed, uncertainty does not necessarily give more incentives to merge, and mergers are not always less socially harmful.

1 Introduction

Business commentators argue that market volatility is an important determinant of merger activity. For instance, some claim that the recent wave of consolidation in the oil industry is due to the high uncertainty in the cost of retail production. An increased risk of exploration and a high volatility of crude oil prices are cited as important reasons for this sizable merger activity.[1] Similarly, the recent deals in the biotechnology industry have been related to large uncertainty in drug development and therefore, in the resulting product demand.[2] The goal of this paper is to analyze the effects of demand and cost uncertainty on horizontal mergers.

Firms often take long term decisions without the knowledge about all the short term market conditions. In practice, merger decisions are taken while costs and demands are still uncertain. For example, oil companies might find more cost-efficient oilfields or, biotechnology companies might discover highly demanded new products. Accordingly, we construct a model in which firms fully observe their costs and demands after taking merger decisions. Moreover, firms may or may not observe the outcomes of their rivals. For example, institutional changes affecting costs or demands are publicly observed. In contrast, the quality of new oilfields or the last R&D results are likely to be only privately known. Our model considers both types of uncertainty and shows that this distinction plays a crucial role.

This paper analyzes the incentives to merge and the welfare effects of mergers in uncertain markets. This is contrasted with the benchmark case of deterministic markets. In the absence of uncertainty, mergers are profitable only if the industry is already very concentrated (Salant et al., 1983 and Perry and Porter, 1985).[3] The reduction in the combined production of the merged firms is compensated by an increase in price only if there are few firms in the market. However, even when industry profits rise, the decrease in total production and therefore in consumer welfare are such that mergers in symmetric

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[1]See for example *The Economist*: "Thanks to the rising cost and risk of exploration in ever more remote areas, life has got harder for oil companies. That explains the wave of consolidation of the past three years...big companies are better able to weather the increasing volatility in oil markets." 24/11/01, p. 66.

[2]"There remains a large element of uncertainty in drug development, and to help reduce this the industry is forming alliances and partnerships. Mergers between biotech firms are creating organisations that start to rival the old pharmaceutical companies in scale." *The Economist*, 29/06/02, p. 69.

[3]Salant et al. (1983) study the private incentives in a Cournot model with constant marginal costs whereas Perry and Porter (1985), as the model presented here, assume linear increasing marginal costs.

industries always result in a lower social welfare (McAffee and Williams, 1992).

The first contribution of this paper is that firms always have more incentives to merge in uncertain environments if the uncertainty is privately observed. Firms in a merged entity take into account the information revealed by the other insiders while making production decisions. This increases the volatility of their individual production generating losses. However, firms are more aggressive when they belong to a merged entity, cutting production more sharply in the event of a bad signal (and expanding it more in case of a good outcome). In addition, their output is more correlated with market prices. We show that the last two effects dominate the first one and hence, firms have more incentives to merge in uncertain environments. Moreover, higher volatility in the market conditions enhances merger incentives and therefore, industry concentration. In contrast to the deterministic case, mergers are also profitable in unconcentrated markets.

The second contribution is that mergers in uncertain environments are socially less harmful if the uncertainty is privately observed. Mergers generate an efficiency gain due to the pooling of information. Total output is produced in a more cost-efficient way. Merging firms react more aggressively inducing low cost plants to produce relatively more than high cost ones. Moreover, when firms merge total output variability reduces and consequently social welfare increases (Shapiro, 1986). To summarize, mergers generate informational gains thanks to the information aggregation but also welfare losses due to enhanced market power. We show that when the uncertainty is high and the market is not very concentrated, mergers boost social welfare. Indeed, in a symmetric industry, Vives (2002) shows that for a relatively large number of firms and high levels of uncertainty, the welfare losses due to private information are larger than the ones due to market power.

On the other hand, if the uncertainty is publicly observed, firms do not always have more incentives to merge than in deterministic markets. Although they are more aggressive and their output is more correlated with market prices when they merge, the change is lower than in the private information case. Even if the merger does not go ahead, the firms will condition the readjustment to the observed signals of the others. Therefore, the rationalization of output following the merger is weaker and does not always compensate for the loss derived from increased volatility of individual production. Moreover, mergers may be socially worse than in deterministic environments.

Our paper combines elements of two strands of the literature in Cournot oligopoly. On the one hand, the merger literature identifies few merger incentives and negative consequences for social welfare. On the other, the literature in information sharing literature

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shows that there are strong incentives for competing firms to share private idiosyncratic information (e.g., cost) and favorable consequences on welfare (see Shapiro, 1986 and Raith, 1996). In our paper, merging firms share private information and this information can only be shared by merging.[4]

Merger incentives under uncertainty and private information have been studied by Gal-Or (1988). She shows that when firms face a common stochastic demand and receive noisy private signals, they may have less incentives to merge compared to deterministic environments.[5] Our model shows, in contrast, that when firms receive perfect signals about different parameters they always have more incentives to merge. Generalizing this model, we recover Gal-Or's result and show that when firms have less incentives to merge in an uncertain environment, they would not have merged anyway in a deterministic one. Therefore, even in Gal-Or's context, more mergers would take place in uncertain markets.

Non-academic press often claim that an increase in market volatility leads to a more concentrated industry. Our results consolidate this view by showing that there are higher incentives to merge in more uncertain industries, but only if this uncertainty is characterized by private information. As examples of mergers driven by motives of sharing private information one can cite oil or biotechnology industries where the firms obtain more information by observing others' oilfields or R& D activities. This may explain the merger activity in these industries and more importantly, why investors rewarded oil and biotechnology companies for getting bigger, while in many other industries mergers and takeovers destroyed value for shareholders.[6]

The normative findings call for a more careful interpretation of the claims that competition authorities should be more lenient in more uncertain markets.[7] In uncertain markets, mergers unambiguously generate efficiency gains only if there is private information. When the information is public, it is no longer necessary for a firm to merge in

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[4]We assume that the formation of informational coalitions among competing firms is not possible. In practice, such alliances are often illegal (see Kühn and Vives, 1995) or the firms' revelation of information can not be verified. In addition, in a later part of the paper, we show that in uncertain environments firms prefer to merge rather than form informational coalitions.

[5]Qiu and Zhou (2002) in a model inspired in Gal-Or (1988), studies the private incentives and the welfare consequences of a merger between a domestic firm and a foreign firm when the domestic firm knows the demand while the foreign firm is completely uninformed. From the normative point of view, Stenneck (2001) considers a model where two duopolists having private cost information merge and studies the effects on price and on consumer welfare.

[6]See *The Economist*, 24/11/01, p. 66 and 27/03/99, p. 76.

[7]For instance, in the European market for third generation (3G) mobile services.

order to condition its production on the information possessed by others.

The remainder of the paper is organized as follows. Section 2 introduces the basic model and the game for both the private and public information cases. The next section studies the incentives to merge and the welfare consequences of mergers within uncertain markets when the uncertainty is privately observed. Section 4 analyzes the public information case and compares with the private one. The following section compares sharing information mergers with informational alliances. Section 6 extends the basic model with private information to an information structure encompassing the private values and common value models. Finally, Section 7 concludes. All proofs are provided in the Appendix.

2 Basic model

Setup

Consider a market for a homogeneous product with linear demand, $D(X) = a - X$, where a is a positive constant and X the consumption level. Let there be n risk-neutral firms, $j = 1, \ldots, n$, producing at cost $C_j(x_j, \theta_j) = \theta_j x_j + \frac{e}{2\kappa} x_j^2$, where x_j is the production, θ_j a random parameter, e a positive constant and κ the firm's capital investment. That is, marginal costs' curves are (linear) strictly increasing and their slopes are reduced by a larger amount of capital, $MC_j(x_j, \theta_j) = \theta_j + \frac{e}{\kappa} x_j$. As noted by Perry and Porter (1985), increasing marginal costs are crucial for yielding sensible descriptions of mergers. The merged firm is bigger than either of the merging firms because it combines the assets of these firms.[8] In our model marginal costs have random intercepts, $\theta_1, \ldots, \theta_n$. In the basic setup, they are independently and identically distributed with mean $\overline{\theta}$ and variance σ_θ^2 in the support $[\theta_{min}, \theta_{max}]$.

Merger

We assume that when firms merge, they set up a new entity that manages these firms as plants. This is equivalent to setting up a bigger firm that owns the capital of the merging firms. Take for example an entity that manages two identical firms, j and l,

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[8]With constant (or decreasing) average costs, possibly varying across firms, mergers would lead to the shutdown of all but one firm(s), which is almost never observed in real mergers. Nevertheless, these models are often used for their simplicity (e.g. Stennek, 2001).

with marginal costs $MC_j(x, \theta) = MC_l(x, \theta) = \theta + \frac{e}{\kappa}x$. The multiplant firm is identical to a single plant firm with marginal costs $MC_{jl}(x, \theta) = \theta + \frac{e}{2\kappa}x$ (Figure 1). Next, take an entity that manages two different firms, q and r, with marginal costs $MC_q(x, \theta_q) = \theta_q + \frac{e}{\kappa}x$ and $MC_r(x, \theta_r) = \theta_r + \frac{e}{\kappa}x$ with $\theta_q \leq \theta_r$. This is equivalent to a firm with marginal costs $MC_{qr}(x, \theta_q, \theta_r) = \min\{\theta_q + \frac{e}{\kappa}x, \frac{\theta_q+\theta_r}{2} + \frac{e}{2\kappa}x\}$ (Figure 2). If the new firm produces a small amount, it will use the most efficient plant. If it produces a larger amount, it will produce with the two plants or with a plant using the double of capital of any of the merging firms.
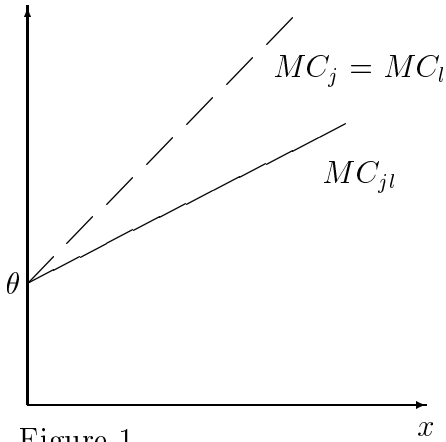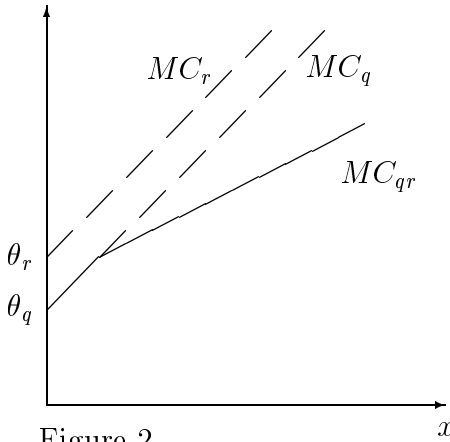

Figure 1.   Figure 2.

Denoting $\lambda \equiv \frac{e}{\kappa}$, the profits of an independent firm are

$$\pi_j = (a - X)x_j - (\theta_j + \frac{\lambda}{2}x_j)x_j, \tag{1}$$

while the profits of a merger of k firms are

$$\pi_M = (a - X)(x_1 + \cdots + x_k) - \sum_{i=1}^{k}(\theta_i + \frac{\lambda}{2}x_i)x_i. \tag{2}$$

Timing of events

Cost uncertainty is either privately or publicly observed. In order to facilitate the comparison, both cases are studied within the same framework. On the basis of the information sharing literature (see e.g. Shapiro, 1986), consider the following three stage game for the *private information* case: [1] k ($\leq n$) firms decide whether to merge; [2] each firm observes its own costs (a merged firm observes the costs of its insiders); [3] each firm chooses an output level (a merged entity chooses an output level for each plant).[9]

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[9]Firstly, we assume that firms take merger decisions before the realization of the uncertainty because they are long term decisions whereas market conditions change rapidly. Moreover, uncertainty outcomes

The only commitment device to share private information is a merger.[10] Merger decisions are taken prior to the acquisition of information, so issues of incentives to merge when a firm is already aware of its own costs are not considered.[11]

For the *public information* case, we use the same game with a slight variation in the second stage. Namely, each firm observes the costs of all the firms (and plants) instead of only their own.

Assumptions

Due to the symmetry at the first stage, it is assumed that the gains of the merged entity are split equally among the insiders.[12] In addition, similar to Shapiro (1986) and Vives (2002), in order to avoid boundary problems in which some firms or plants are inactive, it is assumed that the underlying parameters are such that for all realizations of the cost parameters each firm finds optimal to produce a positive amount in each of its plants. This means that the relative inefficiency of firms and plants is never so large as to imply the shutdown of the inefficient producers.[13]

Reinterpretations

The basic model presented admits other interpretations. It can also be interpreted as a Cournot market with product differentiation and constant marginal costs (Vives, 2002). The profits of a single-plant firm j can be rewritten as $\pi_j = \left(a - (1 + \frac{\lambda}{2})x_j - X_{-j}\right)x_j -$

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and output choices are likely to be repeated several times in a given period, for example each quarter in a year, whereas decisions to merge are taken once for all. Secondly, we assume that firms set output decisions after uncertainty realization because we want to study the effects of the asymmetric information. In addition, the uncertainty would not change the incentives to merge (under risk neutrality) if it was realized or observed before.

[10]Insiders have no strategic incentives to conceal or misrepresent information.

[11]In an international context, Das and Sengupta (2002) study the incentives to merge with private information but no uncertainty: the domestic firm is already aware of the demand whereas the foreign firm already knows the production costs.

[12]Although it is natural when firms are ex-ante symmetric, this assumption is not necessary for the results. If the merger is globally profitable, the firms will find a sharing rule that satisfies participation constraints.

[13]This condition will impose a restriction in the length of the support of the random variables with respect to the net demand $(a - \overline{\theta})$ for a given number of firms. In turn this will fix an upper bound to the variance. The upper bounds for the private information case are important in our analysis and are derived in the Appendix A.

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$\theta_j x_j$. Its inverse demand function is $d_j = \left(a - (1 + \frac{\lambda}{2})x_j - X_{-j}\right)$, with $(1 + \frac{\lambda}{2})$ and 1, respectively, the own and the cross demand effects. Here λ represents the (symmetric) degree of differentiation between the products. When $\lambda = 0$ firms produce homogeneous products and as λ increases brands are less related. In this model, the merged entity, instead of producing an homogeneous good with several plants, produces several differentiated products.

Alternatively, the model can be reinterpreted as a model with (idiosyncratic) demand uncertainty. Indeed, the profits of a single-plant firm j can be rewritten as $\pi_j = (a - \theta_j - (1 + \frac{\lambda}{2})x_j - X_{-j})x_j$. Here θ_j is a random shock to the demand and the firms have constant marginal costs normalized to 0 for simplicity. However, in order to clarify the arguments, the discussion for most of the paper is based on the original interpretation of the model.

3 Uncertainty and private information

This section examines the incentives to merge and the welfare consequences of mergers when the uncertainty is privately observed or realized. The first objective is to determine in which market structures mergers will take place. The second one is to assess their impact on social welfare.

3.1 Production and profits

In the third stage, either all firms behave independently or a subset of firms has merged. In both cases, the linear-quadratic model yields unique Bayesian Nash equilibria and the unique equilibrium strategies are affine in the information firms have.

Consider first the case where no merger has occurred. Firm j selects its output x_j after observing its cost θ_j in order to maximize its profits (1). Thus, each firm chooses a linear decision rule of the form

$$x_j(\theta_j) = r_N^D (a - \overline{\theta}) - r_N^U (\theta_j - \overline{\theta}) \qquad j = 1, \ldots, n, \tag{3}$$

where $r_N^D = \frac{1}{n+\lambda+1}$ and $r_N^U = \frac{1}{2+\lambda}$.[14] In equilibrium, firms follow identical decision rules and in particular, $\overline{x}_j = \overline{x}_l \equiv \overline{x}$ for any j, l.[15] Notice that the reaction of each firm to the realization of the random cost, the responsiveness r_N^U, is independent of the number

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[14]r_N^D and r_N^U stand for reaction to the deterministic and uncertain margins when no firm has merged.

[15]As usual, $\overline{s}$ denotes the mean of the variable s.

of firms in the market. Since the shocks are independent, the knowledge of the own cost does not give any additional information about the others. Therefore each firm reacts equally in the presence of more firms.

Consider now the case where k firms merge. The resulting entity, after observing the costs of its plants $\theta_1, \ldots, \theta_k$, selects its output in order to maximize its profits (2) whereas the outsiders solve the same problem as before, they maximize (1). The merged entity produces in each plant i,

$$x_i(\theta_1, \ldots, \theta_k) = r_I^D (a - \overline{\theta}) - r_I^{U,O}(\theta_i - \overline{\theta}) + r_I^{U,P} \sum_{p=1, p \neq i}^{k} (\theta_p - \overline{\theta}) \qquad i = 1, \ldots, k, \qquad (4)$$

where $r_I^D = \frac{1+\lambda}{(2k+\lambda)(1+\lambda)+(n-k)(k+\lambda)}$, $r_I^{U,O} = \frac{2(k-1)+\lambda}{\lambda(2k+\lambda)}$ and $r_I^{U,P} = \frac{2}{\lambda(2k+\lambda)}$.[16] The pooling of the information allows the new entity to rationalize its production among its plants. Indeed, each plant increases production when it receives a low cost $(r_I^{U,O} > 0)$ and when a partner receives a high one $(r_I^{U,P} > 0)$. However, the reaction to its own shock is more aggressive than the reaction to the others' $(r_I^{U,O} > r_I^{U,P})$. Each outsider o produces

$$x_o(\theta_o) = r_O^D (a - \overline{\theta}) - r_O^U (\theta_o - \overline{\theta}) \qquad o = k+1, \ldots, n, \qquad (5)$$

where $r_O^D = \frac{k+\lambda}{(2k+\lambda)(1+\lambda)+(n-k)(k+\lambda)}$ and $r_O^U = \frac{1}{2+\lambda}$.

The previous expressions allow us to compute the expected profits for each firm. In both market structures, they can be broken up into two parts. When the merger does not take place in the first stage, expected profits are identical, $E(\pi_j) = E(\pi_l) \equiv E(\pi_N)$ for any j, l. Taking expectations in (1) and rewriting, $E(\pi_N) \equiv g_N^D + g_N^U$ where

$$g_N^D = (a - \overline{X})\overline{x} - (\overline{\theta} + \frac{\lambda}{2} \overline{x}) \overline{x}, \qquad (6)$$

and

$$g_N^U = -E[(x_j - \overline{x})(X - \overline{X})] - E[(x_j - \overline{x}) (\theta_j - \overline{\theta})] - \frac{\lambda}{2} E[(x_j - \overline{x})^2]. \qquad (7)$$

g_N^D represents the expected profits that would arise in the equivalent deterministic market i.e. if the output of each firm was always $\overline{x}$ and produced at cost $\overline{\theta}$. g_N^U represents the extra expected profits derived from the uncertainty. The first term in g_N^U is the negative covariance between total output and firm's output. As correlation decreases expected profits rise. Indeed, when the ex-post individual production turns to be large, a lower correlation implies that the total output is lower and the price higher. On the contrary,

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when the ex-post individual production turns to be low the price is lower. Ex-ante, the expected price per unit is higher as correlation falls. The second term is the negative covariance between firm's production and firm's marginal costs intercept. Expected profits increase when the reaction to the random shock is stronger because the cost per unit is lower. Finally, the last term is the negative variability of firm's output. Firms have increasing marginal costs and therefore higher costs when they produce large quantities.

Similarly, when the merger takes place in the first stage, the expected profits of the merged firm $E(\pi_M)$ and of the outsiders $E(\pi_O)$ can also be decomposed in two parts.

3.2   Merger incentives

Since firms are ex-ante symmetric and the profits of the merged entity are shared equally among the k insiders, $E(\pi_I) = \frac{E(\pi_M)}{k}$, there is no conflict of interest regarding the desirability of a merger. Each firm finds it profitable whenever $\Delta E(\pi_I) = E(\pi_I) - E(\pi_N) \geq 0$.

Lemma 1 *The expected gains of a merger can be broken up into parts $\Delta E(\pi_I) \equiv \Delta g_I^D + \Delta g_I^U$, where $\Delta g_I = g_I - g_N$.*

Using the decomposition of the expected profits (see (6) and (7)), merger incentives within uncertain markets can be studied in two steps. Firstly, the term Δg_I^D measures the change that would arise in the equivalent deterministic market, where $\theta_j = \bar{\theta}$ for $j = 1, \ldots, n$ with probability one. This expression is identical to the one derived by Perry and Porter (1985). Secondly, the term Δg_I^U measures the change in profits derived from the *introduction* of the uncertainty. The results are stated in the following two propositions.

Proposition 1 *In the equivalent deterministic market (where $\theta_j = \bar{\theta}$ for $j = 1, \ldots, n$), mergers are profitable only if the market is very concentrated.*

The incentives for k firms to merge in a deterministic environment depend on a trade-off. Even though merging firms' production is lower ($r_I^D < r_N^D$) the market price is higher. Indeed, although the outsiders increase production ($r_O^D > r_N^D$), they do not compensate the decrease of the insiders ($k r_I^D + (n-k) r_O^D < n r_N^D$). When the industry is very concentrated (n very small) the decrease in joint production is compensated by the increase in price. In particular, a merger to monopoly (i.e. $n = k$) is always profitable (for any λ and k).

When the industry is less concentrated, the merging firms influence less the price, and the reduction-in-production effect dominates.[17]

Proposition 2 *Under uncertainty and private information, firms have more incentives to merge than in deterministic markets.*

The revelation of information affects production decisions. If an insider receives a bad signal, the other insiders increase their production shifting down the first insider's residual demand curve. Thus, the insider reduces even more its production. In the same way if an insider receives a good outcome, it expands output even more. Thus, merging firms react more aggressively to the realization of the uncertainty $(r_I^{U,O} > r_N^U)$ and yet their output is less correlated with total output $(r_I^{U,O} + (k-1)r_I^{U,P} < r_N^U)$. Thus, the first two terms in (7) increase following the merger whereas the third decreases (output rationalization also increases production volatility). However, the last term is dominated and the expected profits derived from uncertainty increase.

The previous two propositions lead to the following corollary, stated in terms of the importance of the uncertain (σ_θ^2) with respect to the deterministic parts $(a - \overline{\theta})$.

Corollary 1 *In the basic model under private information,*

(a) When uncertainty is low with respect to demand, mergers are profitable only if the market is very concentrated.

(b) When uncertainty is high with respect to demand, mergers are always profitable.

(c) For intermediate levels of uncertainty, mergers are profitable when the market is concentrated or very unconcentrated.

For low values of n, the deterministic and uncertain parts move in the same direction and hence mergers are profitable. For higher values, the deterministic part produces losses (Proposition 1) whereas the uncertain generates gains (Proposition 2). When uncertainty is high these gains compensate any loss of the deterministic part. For intermediate levels of uncertainty, these gains compensate only when these losses are small. These results are illustrated in Figure 3.

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[17]Even though our discussion is centered in the effects of the number of firms, comparative statics also show that firms have more incentives to merge as λ increases.
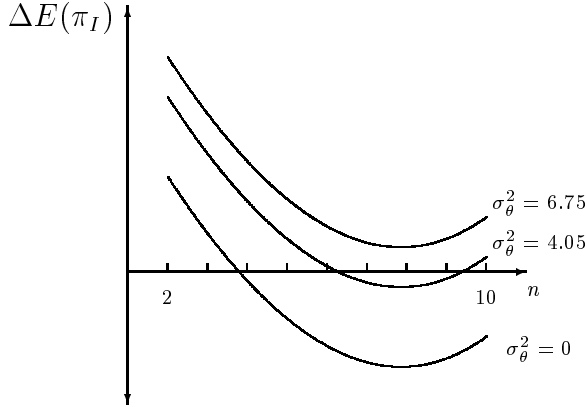
Figure 3: Merger incentives for different levels of uncertainty when $(a - \overline{\theta}) = 10$, $\lambda = 4$ and $k = 2$ taking into account the positive production assumption (see the Appendix A).

The effects of mergers to non-participating firms are stated in the following proposition.

Proposition 3 *Outsiders always benefit from mergers.*

Although outsiders' expected production increases, expected total production decreases when mergers take place. Therefore, in the equivalent deterministic market, outsider profits raise. This, coupled with the fact that expected profits derived from uncertainty do not change, implies that mergers within uncertain markets are always positive for non-participating firms.

3.3 Stability

When three or more firms are considering the possibility to merge $(k \geq 3)$, the following interesting question might also be posed: provided that firms have incentives to merge, do they have incentives to break the agreement leaving the remaining insiders together? Define a stable merger as an agreement in which none firm has incentives to break and become an outsider provided that if it left the other insiders would stay together. According to this definition, mergers in deterministic markets are in general not stable, given that non-participating firms are the ones that greatly benefit from the mergers (free-riding effect).

Similarly, in uncertain markets where firms have private information about a common demand parameter, Gal-Or (1988) shows that a merger generates an informational advantage to each outsider firm that exceeds the informational advantage of each insider.

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Thus mergers are even less stable than their deterministic counterparts. In our setting, when firms have private information about (independent) cost intercepts, the opposite result holds.

Corollary 2 *Mergers under uncertainty and private information are more stable than their deterministic counterparts.*

This corollary follows straightforwardly from the two previous propositions. The uncertainty generates extra-profits for the merging firms (Proposition 2) but not for the outsiders (Proposition 3). Therefore, a bigger share of the industry profits is earned by the insiders, alleviating the free-riding effect.

3.4 Welfare consequences

Social welfare is represented by

$$W = \int_0^X D(z)\mathrm{d}z - \sum_{j=1}^n C(x_j).$$

Income effects are ignored and the standard welfare measure of consumer plus producer surplus is employed. In our case, with linear demand and linear marginal costs, social welfare can be written as

$$W = (a - \frac{X}{2})X - \sum_{j=1}^n (\theta_j + \frac{\lambda}{2}x_j)x_j.$$

Taking expectations, we can also rewrite expected welfare in terms of a deterministic and uncertain parts, $E(W) \equiv w^D + w^U$, where

$$w^D = (a - \frac{\overline{X}}{2})\overline{X} - \sum_{j=1}^n (\overline{\theta} + \frac{\lambda}{2}\overline{x}_j)\overline{x}_j, \tag{8}$$

and

$$w^U = -\sum_{j=1}^n E[(x_j - \overline{x}_j)(\theta_j - \overline{\theta})] - \frac{1}{2}E[(X - \overline{X})^2] - \frac{\lambda}{2}\sum_{j=1}^n E[(x_j - \overline{x}_j)^2]. \tag{9}$$

w^D represents the expected welfare that would arise in the equivalent deterministic market. The first term in w^U represents the extra benefits arising from a more efficient distribution of output across firms (or plants). If one firm increases the responsiveness to the shock, not only the firm's profits are increased, but also total welfare is enhanced. The

second term is the variance of aggregate output. Variability of output, *ceteris paribus*, reduces expected welfare. Although it increases consumer welfare (consumer receive low prices when they consume more), it reduces by more the industry profits. The last term, the sum of the variance of the output of each plant, reduces expected profits of the firms and expected social welfare.

Substituting the output decisions of Section 3.1 into (8) and (9), we obtain the expected welfare when the merger has occurred and when not. We can state the following two propositions.

Proposition 4 *In the equivalent deterministic market, mergers always reduce social welfare.*

In the absence of uncertainty, although mergers generate always more profits for the outsiders and sometimes for the insiders, they reduce sharply total production. As a consequence, consumer welfare is so largely reduced that social welfare is lower.

Proposition 5 *Under uncertainty and private information, mergers are less harmful for social welfare.*

To understand this proposition let us refer to (9). Following the merger, the first term increases because total output is produced in a more cost efficient way. As explained in Section 3.2, merging firms react more aggressively to the cost realization and therefore low cost plants produce relatively more than high cost ones. In addition, non-participating firms do not change their reaction. This leads us to the second effect. The second term in (9) increases because total production volatility is reduced. This is due to the fact that the merged firm has less variability in output than if the firms were independent. The third term, in contrast, decreases after the merger because the production volatility of each plant increases. Summing up, the last term is always dominated by the first two and hence social welfare increases.

The previous two propositions form the basis of the welfare trade-off. On the one hand mergers increase market power and decrease social welfare. On the other hand, mergers aggregate information and increase welfare. Two cases may arise:

Corollary 3 *In the basic model under private information,*
(a) When uncertainty is low with respect to demand, mergers always reduce social welfare.

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(b) When uncertainty is higher, mergers increase social welfare if the market is not concentrated.

If uncertainty is low with respect to demand the market power loss dominates and therefore when k firms merge expected welfare is always reduced. If uncertainty is higher, the aggregation information gain dominates when the market power loss is small, i.e. when n is high. These results are illustrated in Figure 4.
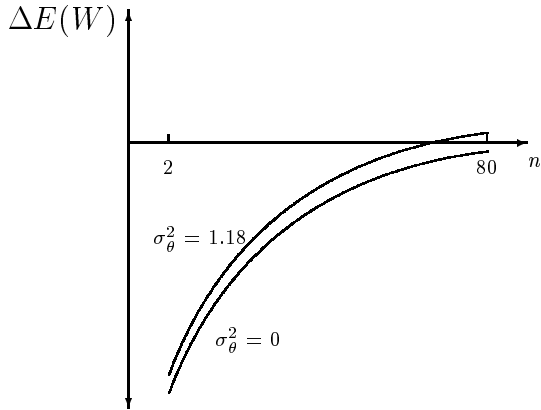

Figure 4: Merger consequences for social welfare in deterministic and uncertain markets when $(a - \overline{\theta}) = 10$, $\lambda = 8$ and $k = 2$ taking into account the positive production assumption (see the Appendix A).

4   Uncertainty and public information

In the previous section we have showed that the introduction of random private shocks increases the incentives to merge and lessens the impact of mergers on social welfare. These results were driven by the aggregation of information following the merger and the subsequent rationalization of output among merging firms.

This and the following sections isolate these two effects in turn. In this section firms receive public shocks instead of private ones. Here, there is rationalization of production but no information aggregation since all the information is public. In the next section firms may form informational coalitions instead of mergers. A group of firms may commit to share their private information but will compete in the product market afterwards. There is information aggregation but no rationalization of output.

4.1 Merger incentives

Similar to Section 3, a group of firms are considering the possibility to merge in an uncertain market. In this case each firm receives perfect information about the parameter of all the firms in the market.

Under uncertainty and public information firms do not always have more incentives to merge than in the equivalent deterministic market. Consider for example a n-firm market where two may merge ($k = 2$) and $\lambda = 5$. If this market has more than 17 firms ($n > 17$), the introduction of public random shocks would reduce the incentives to merge. Thus, the introduction of a (highly volatile) random shock may turn a previously profitable merger, unprofitable.

Remark 1 *Under uncertainty and public information, firms may have less incentives to merge than in deterministic markets.*

Similar to the private information case, the merging firms rationalize output. Each plant reacts more aggressively to its own shock and the total firm output is less correlated with total output even if the volatility of each plant increases. However, when there is no informational advantage, the first two effects are weaker and do not always compensate the third one. The rationalization of production alone does not necessarily provides more incentives to merge.

Furthermore, even when the introduction of random public shocks increases the incentives to merge, these incentives are lower than if the shocks were private.

Proposition 6 *Firms have always more incentives to merge under private than under public information.*

Therefore we should observe many more mergers in uncertain markets when the realization of the uncertainty is private rather than public, *ceteris paribus*.

4.2 Welfare consequences

The implications for social welfare run parallel to the ones obtained for merger incentives.

Under uncertainty and public information mergers are not necessarily socially less harmful than in deterministic markets. Take for example a n-firm industry where two

($k = 2$) may merge and $\lambda = 5$. If this market has more than two firms ($n > 2$), mergers are even less desirable. Greater volatility makes the consequences of mergers even worse.[18]

Remark 2 *Under uncertainty and public information, mergers may be even worse than in deterministic markets.*

As mentioned above, here the rationalization of output is less strong. Although total output is produced in a more cost-efficient way, its total variance may increase. Mergers may be even less desirable than in deterministic markets.

Proposition 7 *Mergers are always better for social welfare under private than under public information.*

Although mergers in uncertain environments may be better independent of the information type, they are always more desirable when there is private information.

5   Informational coalitions

The framework of this paper allows the comparison between mergers and alliances to share information, an intermediate form of association. Modifying the basic model under private information, k firms envisages forming a coalition that will exchange their private information but not coordinate their actions in the product market.

Shapiro (1986) shows that firms always have incentives to form an informational coalition. The next proposition states in which cases, if they could choose, firms would prefer to merge rather to form informational coalitions.

Proposition 8 *(a) When uncertainty is low with respect to demand, firms will prefer to merge rather than form an informational coalition only if the market is very concentrated.*
(b) When uncertainty is high with respect to demand, they will always prefer to merge.
(c) For intermediate levels of uncertainty, they will prefer to merge when the market is concentrated or when it is very unconcentrated.

With respect to the deterministic part, the comparison between a merger and an informational coalition is the same as when studying the incentives to merge. In the

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[18]Note that when the uncertainty is very high and $2 < n \leq 17$, firms have incentives to merge but mergers would imply social losses.

absence of uncertainty, informational coalitions have no value. The profits derived from the uncertainty are always larger when firms merge. If they form a coalition, they do not get the full benefits of cutting their production in the event of a bad signal. Their rationalization is less strong and hence their expected profits lower.

However, Shapiro (1986) shows that when a group of firms share their private cost information, social welfare increases. Surprisingly, the informational part is not always better when a merger is formed. Even if the merging firms are better-off with a merger, the consumers will prefer an informational coalition because the output variance is higher. In some cases, an informational coalition will be socially preferred.

Remark 3 *The welfare derived from uncertainty may be greater when firms form an informational coalition instead of a merger.*

Therefore, even when uncertainty is high the informational coalition may be socially preferred than a merger.

6 Uncertainty and private information: a generalized information structure

This section generalizes the information structure of the basic model. The generalization is two-fold. Firstly, the random marginal cost intercepts (or the idiosyncratic random demand intercepts) may be correlated. Secondly, the firms receive private signals about their parameters that may be noisy.

The joint distribution of the random variables has to be restricted. The information structure should yield affine conditional expectations. In that case, the linear-quadratic model has a unique (and affine in information) Bayesian Nash equilibria. In what follows, for convenience, the random variables are assumed to be jointly normally distributed.[19]

Before uncertainty is realized, all firms face the same prospects. The vector of random variables $(\theta_1, \ldots, \theta_n)$ is jointly normally distributed with $E(\theta_j) = \overline{\theta}$, $Var(\theta_j) = \sigma_\theta^2$ and $Cov(\theta_j, \theta_l) = \rho \sigma_\theta^2$, for $j \neq l$, $0 \leq \rho \leq 1$. ρ is the correlation coefficient between θ_j and θ_l.[20]

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[19]There are other pairs of prior distribution and likelihood that result in affine conditional expectations and do not require unbounded support for the uncertainty (see Vives, 1988). Normal distributions, however, make the analysis simpler.

[20]ρ is assumed to be positive because it is the natural case in practice.

Each firm receives a signal $s_j = \theta_j + \epsilon_j$, where $\epsilon_j \sim N(0, \sigma_\epsilon^2)$, $Cov(\epsilon_j, \epsilon_l) = 0$ for $j \neq l$ and $Cov(\theta_j, \epsilon_l) = 0$ for all j and l. Signals can range from perfect ($\sigma_\epsilon^2 = 0$ or infinite precision) to pure noise ($\sigma_\epsilon^2 = \infty$ or zero precision). The precision of a signal s_i is denoted by $\tau_\epsilon = \frac{1}{\sigma_\epsilon^2}$.[21]

Under the normality assumption, conditional expectations are affine. Each single-plant firm receives a signal (s_j) and needs to estimate its own realization (θ_j) and the signals the other firms (s_w). We have $E(\theta_j - \overline{\theta}|s_j) = t(s_j - \overline{\theta})$ and $E(s_w - \overline{\theta}|s_j) = E(\theta_w - \overline{\theta}|s_j) = \rho t(s_j - \overline{\theta})$, where $t = \frac{\tau_\epsilon}{\tau_\theta + \tau_\epsilon}$. Note that $0 \leq t \leq 1$. In particular when signals are perfect: $t = 1$, $E(\theta_j - \overline{\theta}|s_j) = (s_j - \overline{\theta})$ and $E(s_w - \overline{\theta}|s_j) = \rho(s_j - \overline{\theta})$. When they are uninformative: $t = 0$, $E(\theta_j - \overline{\theta}|s_j) = E(s_w - \overline{\theta}|s_j) = 0$.

The k-plant firm receives k signals $(s_1, \ldots, s_k)$ and needs to estimate the realization of each of its plants

$$E(\theta_i - \overline{\theta}|s_1, \ldots, s_k) = \frac{t[1 - \rho t + \rho t(k-1)(1-\rho)]}{(1-\rho t)[1 + \rho t(k-1)]}(s_i - \overline{\theta})$$
$$+ \frac{\rho t(1-t)}{(1-\rho t)[1 + \rho t(k-1)]}\Sigma_{p=1, p \neq i}^{k}(s_p - \overline{\theta}),$$

and the signal of the outsiders

$$E(s_o - \overline{\theta}|s_1, \ldots, s_k) = \frac{\rho t}{1 + \rho t(k-1)}\Sigma_{i=1}^{k}(s_i - \overline{\theta}).$$

The following two lemmas characterize the equilibria when a merger has occurred and when has not.

Lemma 2 *When no merger has occurred, the strategies followed by each firm are given by $x_j(s_j) = r_N^D(a - \overline{\theta}) - r_N^U(s_j - \overline{\theta})$ for $j = 1, \ldots, n$, where r_N^D is defined in (3) and*

$$r_N^U = \frac{t}{2 + \lambda + \rho t(n-1)}. \tag{10}$$

In contrast to the basic model, the market structure affects the responsiveness when the parameters are positively correlated. Intuitively, when a firm receives a good signal, it is likely that the other firms have also receive good ones. In consequence the reaction is less strong as the number of firms increases since more rivals are likely to push up production. The reaction is also less strong when the signals are more correlated or less precise.

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[21]Similarly, the inverse of σ_θ^2 is denoted by τ_θ.

Lemma 3 *When a merger takes place, the aggregated production of the merged firm is* $x_M(s_1, \ldots, s_k) = k r_I^D (a - \overline{\theta}) - r_M^U \Sigma_{i=1}^k (s_i - \overline{\theta})$, *where* r_I^D *is defined in (4) and*

$$r_M^U = \frac{t[(2 + \lambda - \rho t)[1 + \rho(k-1)] - (n-k)\rho t(1-\rho)(k-1)]}{V(n, k, \rho, t)}, \tag{11}$$

meanwhile, the outsiders will produce $x_o(s_o) = r_O^D(a - \overline{\theta}) - r_O^U(s_o - \overline{\theta})$ *for* $o = k+1, \ldots, n$, *where* r_O^D *is defined in (5) and*

$$r_O^U = \frac{t[(2k + \lambda)[1 + \rho t(k-1)] - \rho t k[1 + \rho(k-1)]]}{V(n, k, \rho, t)}, \tag{12}$$

where

$$
\begin{aligned}
V(n, k, \rho, t) =& (2k + \lambda)[1 + \rho t(k-1)](2 + \lambda - \rho t) \\
&+ (n-k)\rho t[(2k + \lambda)[1 + \rho t(k-1)] - \rho t k^2].
\end{aligned}
$$

This information structure encompasses the three cases analyzed by the information sharing literature: independent values, private values and common value. The independent values model has been analyzed in the previous sections. Firms receive perfect signals ($t = 1$) and the random parameters are independent ($\rho = 0$). In the following subsections the latter two models are studied. In the private values model, firms also receive perfect signals ($t = 1$) but the random parameters are not independent ($0 < \rho < 1$). In the common value model, firms receive noisy signals ($t < 1$) of the same random parameter, or equivalently, of different parameters that are perfectly correlated ($\rho = 1$).

6.1 Private values model

In the private values model, firms receive perfect signals ($t = 1$) but the random parameters are not independent ($0 < \rho < 1$). This can be applied to an industry in which firms, once they know their costs (or demands), they can partially infer the costs (or demands) of the other firms.[22]

When the parameters are correlated firms may have less incentives to merge than in deterministic environments. Take for example a four-firm industry ($n = 4$) where two decide whether to merge ($k = 2$) and $\lambda = 1$. These firms have more incentives to merge than in deterministic markets only if $\rho < 0.985$.[23] For larger values, they have less incentives to merge.

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[22]Suppose for example that firms have a raw material in common. By observing its cost, each firm can guess the costs of the rivals.

[23]Note that when $\rho \to 0$ the results tend to the ones obtained in the independent values model. Firms should always have more incentives to merge when $\rho \to 0$.

Remark 4 *In the private values model,*

(a) When the correlation of the parameters is weak, firms have more incentives to merge than in deterministic markets.

(b) When the correlation is strong, firms may have less incentives to merge.

When the parameters are correlated, the expected profits due to a more efficient distribution of the production among the insiders not only depend on the reaction towards the own parameter, but also on the reaction towards the signals of the other insiders.

$$-E[(x_i - \overline{x}_i)(\theta_i - \overline{\theta})] = \left[r_I^{U,O} - \rho(k-1)r_I^{U,P} \right] \sigma_\theta^2.$$

Intuitively, a positive signal for a given insider is likely to be related to positive signals for the other insiders. The responsiveness towards the own signal is likely to be restrained by the reaction towards the signals of the others. As ρ increases, the more often insiders receive similar signals. For very large values of ρ, the correlation between production and the signal is higher (and the second term in (7) lower) after the merger ($r_I^{U,O} - (k-1)r_I^{U,P} = r_M^U < r_N^U$). Hence, the expected profits derived from the uncertainty may be greater when firms remain independent.

6.2 Common value model

In the common value model, firms receive noisy signals ($t < 1$) of the same random parameter, or equivalently, of different parameters that are perfectly correlated ($\rho = 1$). This can be applied to an industry in which equally efficient firms receive some imperfect (and private) information about the uncertain market demand.

Similarly to the previous subsection, firms do not always have more incentives to merge under uncertainty. Take again a four-firm industry ($n = 4$) where two may merge ($k = 2$) and $\lambda = 1$. These firms have more incentives to merge only if $t < 0.8$. For larger values, they have less incentives to merge than in deterministic environments.

Remark 5 *In the common value model,*

(a) When signals are not very precise, firms have more incentives to merge than in deterministic markets.

(b) When signals are very precise, firms may have less incentives to merge.

In the independent and private values models the important feature is the reaction to each parameter. The merged firm receives perfect signals about different parameters and

produces different quantities in each plant. In contrast, in the common value setup, each merging firm produces the same amount ($r_I^U = \frac{r_M^U}{k}$). Here, the importance hinges upon the accurateness of the prediction and upon the responsiveness to this prediction. The merged firm collects several signals and thus has a more accurate prediction than single plant firms. Specifically, denoting $\theta \equiv \theta_j$ ($j = 1, \ldots, n$),

$$Var(\theta|s_1, \ldots, s_k) = \frac{1-t}{1+t(k-1)} \, \sigma_\theta^2 < (1-t) \, \sigma_\theta^2 = Var(\theta|s_j).$$

Concerning production decisions, however, insiders may be less aggressive than outsiders ($r_I^U < r_O^U$). This happens when $t \geq t^* = \frac{\lambda}{k+\lambda}$. In particular, as showed by Gal-Or (1988), when $\lambda = 0$ the insiders are always less responsive than the outsiders.[24] The reason relates to the implication of collusion on the Cournot behaviour of oligopolistic firms. Collusion among equally efficient firms means reduced production by each in order to accommodate the others. When λ increases, the accommodation effect is alleviated: if the signals are not very precise ($t < t^*$), the merged firm responds more aggressively because it has a better prediction than the single plant firms.

When the signals are not very precise (t low), the insiders respond more aggressively and predict much better following the merger. Hence firms have more incentives to merge. In contrast, when the signals are very precise (t large), the insiders are less aggressive and their accurateness almost does not increase. Hence, the merger may even generate an informational disadvantage to the merging firms.

Note however that the previous observation leaves unanswered whether, under common values, we will observe more or less mergers than in deterministic environments. It may be that the informational disadvantage arises *only* when firms already had no incentive to merge in the deterministic framework and hence more mergers should be observed. The following proposition, restricted to the case $\lambda = 0$, shows that this is in fact the case.

Proposition 9 *When firms sell homogeneous goods and marginal costs are constant ($\lambda = 0$), more mergers take place in the common value model than in the equivalent deterministic market.*

Thus, complementing the work of Gal-Or (1988), even if the merger generates an informational disadvantage to the merging firms, this informational disadvantage never

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[24]In the common value model, the positive production assumption does not imply $\lambda > 0$. If $\lambda = 0$ the merged firm is indifferent between closing some plants or keeping all of them producing. Assuming that the latter happens, the results of this section for the particular case $\lambda = 0$ are equivalent to the ones obtained by Gal-Or (1988).

discourages a merger. On the contrary, the informational advantage may arise in cases where the firms had no incentives to merge and therefore may encourage mergers.

7   Concluding remarks

Motivated mainly by the needs of competition authorities, a large strand of the literature studies the positive and normative aspects of horizontal mergers. However, this literature generally assumes that firms have perfect knowledge about the market conditions at the instance of taking merger decisions. In practice, this is rarely the case. In this paper, we analyze the effects of uncertainty and private information on horizontal mergers.

Common wisdom suggests that more uncertain markets are related to more incentives to merge. We show formally that this is not always the case. When the uncertainty is publicly observed, firms may have less incentives to merge than in deterministic markets. Indeed, uncertainty *per se* does not lead to more concentrated industries and may even discourage mergers that would have been profitable. In contrast, when the uncertainty is privately observed, firms always have more incentives to merge and more uncertainty results in more concentrated industries.

The private information results are firstly derived for the case in which firms receive perfect signals about their uncertain and independent characteristics. We show afterwards that firms also have more incentives to merge when the characteristics are mutually related and when the signals are not perfect. However, when the signals are precise and the characteristics very correlated, firms may have less incentives to merge than in deterministic markets. Indeed, in this case they can infer all the information about their rivals and we are close to the situation in which public information is received. Thus, mergers in uncertain markets are more profitable as long as firms retain some degree of private information.

The consequences of mergers on social welfare in uncertain markets also depend on the type of information. When the uncertainty is publicly observed, mergers may be socially worse than in deterministic markets. In the presence of private information, mergers in uncertain markets not only take place more frequently but are also better for social welfare. We show that in this case the aggregation of disseminated information results in social gains.[25] In markets with high volatility, these gains can compensate the anti-competitive

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[25]In this sense, our paper provides a new argument for the so-called efficiency defense of mergers. Even if some OECD merger control systems already take into account the positive sides of mergers, e.g. in the

effects of mergers and hence mergers under uncertainty and private information enhance welfare.

Appendix A: Maximum variance

It has been assumed throughout the paper that no firm is driven out of the market and that the merged firm does not close any of its plants. This assumption is not innocuous. For any combination of the parameters, this condition fixes an upper bound to the variance. In this section I identify the maximum variance for the private information part of the basic model.

If no firm has merged, from (3), no firm will close if it is assumed that $x_j(\theta_{max}) \geq 0$. Similarly, if the merger has taken place, from (5), no outsider is driven out of the market if $x_o(\theta_{max}) \geq 0$ whereas from (4), the merged entity will not close any of its plants if $x_1(\theta_{max}, \theta_{min}, \ldots, \theta_{min}) \geq 0$. The latter condition is tighter than the former two. Thus, if the parameters satisfy this condition, it is ensured that, for any realization of the random variables and independently of the merger decision, all firms (and plants) will produce a positive amount. Rearranging this restriction and defining $T \equiv \theta_{max} - \theta_{min}$ and $qT \equiv \theta_{max} - \overline{\theta}$, the maximum length of the support is

$$T_{max}(q) = \frac{\lambda(1+\lambda)(2k+\lambda)}{[2(k-1)+q\lambda]S}(a-\overline{\theta}), \tag{13}$$

where for notational simplicity

$$S = (2k+\lambda)(1+\lambda) + (n-k)(k+\lambda).$$

By definition, the maximum variance of a random variable defined on a support of length T is $\sigma_{max}^2 = q(1-q)T^2$, where q is defined above. Substituting (13) in this expression, and maximizing with respect to q, we have that

$$\sigma_{max}^2 = \frac{\lambda^2(1+\lambda)^2(2k+\lambda)^2}{8(k-1)[2(k-1)+\lambda]S^2}(a-\overline{\theta})^2.$$

In the example depicted in Figure 3, where $(a-\overline{\theta}) = 10$, $\lambda = 4$ and $k = 2$, we have that $\sigma_{max}^2 = 0.689$ for industries with ten or less firms. Similarly in Figure 4, where $(a-\overline{\theta}) = 10$, $\lambda = 8$ and $k = 2$, we have that $\sigma_{max}^2 = 0.183$ for industries with eighty firms.

Appendix B: Proofs

Proof of Lemma 1

Taking expectations in (2), the expected profits of the merged entity can be written as $E(\pi_M) \equiv g_M^D + g_M^U$, where substituting (4) and (5),

$$g_M^D(n,k) = \frac{(1+\lambda)^2(2k+\lambda)k}{2S(n,k)^2}(a-\overline{\theta})^2, \tag{14}$$

United States, the role of the efficiencies is still not considered in the European merger policy (Roller, Stennek and Verboven, 2000).

and

$$g_M^U(n,k) = \frac{[2(k-1)+\lambda]k}{2\lambda(2k+\lambda)}\,\sigma_\theta^2. \tag{15}$$

The expected profits when firms remain independent are $E(\pi_N) \equiv g_N^D + g_N^U$ where $g_N(n) = g_M(n,1)$. Straightforwardly, since $g_I = \frac{g_M}{k}$,

$$\Delta g_I^D = \frac{1}{2}\left[\frac{(1+\lambda)^2(2k+\lambda)}{S^2} - \frac{2+\lambda}{(n+\lambda+1)^2}\right](a-\overline{\theta})^2, \tag{16}$$

and

$$\Delta g_I^U = \frac{2(k-1)}{\lambda(2+\lambda)(2k+\lambda)}\,\sigma_\theta^2. \tag{17}$$

Proof of Proposition 1

The change in expected profits in the equivalent deterministic market ($\sigma_\theta^2 = 0$) is given by (16). This expression has the same zeros (and the same sign) in n as $D(n) = (1+\lambda)^2(2k+\lambda)(n+\lambda+1)^2 - (2+\lambda)S(n,k)^2$. Since $D(n)$ is a quadratic function with $D''(n) = -2(k-1)[(2+\lambda)k+\lambda] < 0$ and $D(k) = (k-1)^2(1+\lambda)^2(2k+\lambda) > 0$, there exists a unique n_d, $n_d > k$, such that $D(n_d) = 0$. If $n \leq n_d$, $D(n) \geq 0$ and $\Delta g_I^D(n,k) \geq 0$ whereas if $n > n_d$, $D(n) < 0$ and $\Delta g_I^D(n,k) < 0$.

Proof of Proposition 2

The proof follows directly from (17) since all the terms are positive when $k \geq 2$.

Proof of Corollary 1

We have seen in the proof of Proposition 1 that the expression $\Delta g_I^D(n,k)$ is positive and then negative as n increases. Clearly, $\lim_{n\to\infty}\Delta g_I^D(n,k) = 0$. Now we prove that this function is U-shaped in n, that is, it has a single minimum n_m. Take $\frac{\partial}{\partial n}\Delta g_I^D(n,k) = -\left[\frac{(1+\lambda)^2(2k+\lambda)(k+\lambda)}{S^3} - \frac{2+\lambda}{(1+\lambda+n)^3}\right](a-\overline{\theta})^2$. This function has the same zeros as $P(n) = (2+\lambda)S(n,k)^3 - (1+\lambda)^2(2k+\lambda)(k+\lambda)(1+\lambda+n)^3$. Since $P'''(n) = 6(k-1)(k+\lambda)[(2+\lambda)k+\lambda] > 0$ for all n and $P''(k) = 6(k-1)(1+\lambda)(k+\lambda)(2k+\lambda) > 0$, then $P''(n)$ is positive for all n and hence $P(n)$ is convex. Finally since rewriting we get that $P(k) = -(k-1)(1+\lambda)^2(2k+\lambda)[\lambda^2(2k-1)+\lambda(4k^2+k-1)+k(k^2+4k-1)] < 0$ and $\lim_{n\to\infty}P(n) = \infty$, there exists a unique n_m such that $P(n_m) = 0$ and therefore $\frac{\partial}{\partial n}\Delta g_I^D(n_m,k) = 0$. If $n < n_m$, $P(n) < 0$ and $\frac{\partial}{\partial n}\Delta g_I^D(n,k) < 0$ whereas if $n \geq n_m$, $P(n) \geq 0$ and $\frac{\partial}{\partial n}\Delta g_I^D(n,k) \geq 0$. Obviously, $\Delta g_I^D(n_m,k) < 0$.

Thus, on the one hand the deterministic curve $\Delta g_I^D(n,k)$ is U-shaped with $\Delta g_I^D(k,k) > 0$ and $\lim_{n\to\infty}\Delta g_I^D(n,k) = 0$. On the other hand since, from (17), Δg_I^U is positive, independent of n and increasing in σ_θ^2, the deterministic curve is shifted up as the uncertainty increases and we have the conclusions of the text.

Proof of Proposition 3

The profits of each outsider are $E(\pi_O) \equiv g_O^D + g_O^U$, where

$$g_O^D(n,k) = \frac{(k+\lambda)^2(2+\lambda)}{2S(n,k)^2}\,(a-\overline{\theta})^2, \tag{18}$$

and

$$g_O^U(n,k) = \frac{1}{2(2+\lambda)}\,\sigma_\theta^2.\tag{19}$$

Subtracting the expected profits when the merger does not take place (see proof of Lemma 1), we get that $\Delta E(\pi_O) = \Delta g_O^D$ and

$$\Delta g_O^D = \frac{(2+\lambda)}{2}\left[\frac{(k+\lambda)^2}{S^2} - \frac{1}{(n+\lambda+1)^2}\right](a-\overline{\theta})^2.$$

The previous expression has the same zeros in n as $F(n) = (k+\lambda)^2(n+\lambda+1)^2 - S(n,k)^2$. Since $F(n)$ is a linear function with $F'(n) = 2k(k-1)(k+\lambda) > 0$ and $F(k) = k(k-1)[k^2 + k(3+4\lambda) + 2\lambda(1+\lambda)] > 0$, $F(n) > 0$ and therefore $\Delta g_O^D(n,k) > 0$ for all $n \geq k$.

Proof of Proposition 4

Consider, firstly, the case in which k insiders merge into a single firm. Substituting (4) and (5) in (8) we get that

$$w_M^D(n,k) = \frac{1}{2\lambda S(n,k)^2}\left[\sum_{r=0}^2 \upsilon_r(k)(n-k)^r\right](a-\overline{\theta})^2,$$

where $\upsilon_0 = k(1+\lambda)^2(3k+\lambda), \upsilon_1 = (k+\lambda)[\lambda^2 + \lambda(3k+2) + 4k]$ and $\upsilon_2 = (k+\lambda)^2$.

Next, the expected welfare in the equivalent deterministic environment if the firms decide not to merge is $w_N^D(n) = w_M^D(n,1)$. Hence,

$$\Delta w^D = \frac{1}{2\lambda}\left[\frac{\sum_{r=0}^2 \upsilon_r(n-k)^r}{S^2} - \frac{n(n+\lambda+2)}{(n+\lambda+1)^2}\right](a-\overline{\theta})^2.$$

This expression is negative whenever $G(n) = (n+\lambda+1)^2\sum_{r=0}^2 \upsilon_r(k)(n-k)^r - n(n+\lambda+2)S(n,k)^2$ is negative. We have that $G(n)$ is quadratic polynomial function and $G''(n) = -2k\lambda(k-1)^2 < 0$. Since $G'(k) = -k^2(k-1)[2\lambda^2 + \lambda(k+3) + 2] < 0$ and $G(k) = -k(k-1)(1+\lambda)^2[k^2 + (3+\lambda)k + \lambda] < 0$, it follows that $G(n)$ and, consequently, $\Delta w^D(n,k)$ are negative for all $n \geq k$.

Proof of Proposition 5

The expected social welfare derived from the uncertainty if the merger goes ahead, substituting (4) and (5) in (9), is given by

$$w_M^U(n,k) = \left[\frac{(\lambda+3)n}{2(\lambda+2)^2} + \frac{2k(k-1)(4k+k\lambda+\lambda)}{\lambda(\lambda+2)^2(2k+\lambda)^2}\right]\sigma_\theta^2.$$

If the firms decide not to merge we have that $w_N^U(n) = w_M^U(n,1)$ and therefore

$$\Delta w^U = \frac{2k(k-1)(4k+k\lambda+\lambda)}{\lambda(\lambda+2)^2(2k+\lambda)^2}\,\sigma_\theta^2.$$

Clearly this function is always positive when $k \geq 2$.

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Proof of Corollary 3

We have seen in the proof of Proposition 4 that $\Delta w^D(n,k) < 0$ and clearly $\lim_{n\to\infty} \Delta w^D(n,k) = 0$. Through tedious but straightforward algebra, we can show that $\frac{\partial}{\partial n} \Delta w^D > 0$ for all $n \geq k$. Since Δw^U is always positive, independent of n and increasing in σ_θ^2, we have the conclusions in the text.

Proof of Proposition 6

If a merger has been produced in the first stage, then, defining $\widetilde{\theta} = \frac{1}{n}\sum_{i=1}^n \theta_i$, the production of each of the insiders will be

$$x_i(\theta_1,\ldots,\theta_n) = r_I^D(a - \widetilde{\theta}) - r_{I,O}^{U,F}(\theta_i - \widetilde{\theta}) + r_{I,P}^{U,F}\sum_{p=1,p\neq i}^k (\theta_p - \widetilde{\theta}), \tag{20}$$

where $r_{I,O}^{U,F} = \frac{(\lambda+k-1)S+\lambda(k-1)}{\lambda(k+\lambda)S}$, $r_{I,P}^{U,F} = \frac{S-\lambda(k-1)}{\lambda(2+\lambda)S}$ and those of the outsiders

$$x_o(\theta_1,\ldots,\theta_n) = r_O^D(a - \widetilde{\theta}) - r_{O,O}^{U,F}(\theta_o - \widetilde{\theta}) + r_{O,P}^{U,F}\sum_{p=k+1,p\neq o}^n (\theta_p - \widetilde{\theta}), \tag{21}$$

where $r_{O,O}^{U,F} = \frac{S-(k-1)}{(1+\lambda)S}$, $r_{O,P}^{U,F} = \frac{k-1}{(1+\lambda)S}$. Substituting $k = 1$, the case in which no merger has been produced in the first stage is obtained. The expected productions, when the merger has been produced as well as when it has been not, are the same as in the private information case.

It is again possible to break expected profits up in two parts, $E(\pi^F) = g^{D,F} + g^{U,F}$. On the one hand, since the expected production is the same, the first term will be the same as in the private information case, $g^{D,F} = g^D$. On the other hand, proceeding in the same way as in the private information case we can obtain the expected profits derived from the uncertainty for the insiders. Substituting (20) and (21),

$$g_I^{U,F} = \frac{1}{2\lambda S^2}\left[\sum_{r=0}^2 \varphi_r(n-k)^r\right]\sigma_\theta^2,$$

where $\varphi_0 = (1+\lambda)^2(2k+\lambda)[2(k-1)+\lambda]$, $\varphi_1 = (2k+\lambda)[2(k-1)(1+\lambda)+\lambda(2\lambda+3)]$ and $\varphi_2 = (k-1)(k+\lambda)+\lambda(k+1+\lambda)$. Computing, we get that

$$g_I^{U,F} - g_I^U = \frac{1}{2(2k+\lambda)S^2}\left[\sum_{r=1}^2 \phi_r(n-k)^r\right]\sigma_\theta^2, \tag{22}$$

where $\phi_1 = (2k+\lambda)(2(k+1)+3\lambda)$, $\phi_2 = (3k+2\lambda)$. Clearly this difference is positive. Therefore the profits derived from uncertainty are greater for the insiders and for the non-merging firms (taking $k = 1$).

We have that $\Delta g_I^U - \Delta g_I^{U,F} = (g_N^{U,F} - g_N^U) - (g_I^{U,F} - g_I^U)$. From (22), this expression is positive whenever $J(n) = (2k+\lambda)S(n,k)^2\sum_{r=1}^2 \phi_r(1)(n-1)^r - (2+\lambda)S(n,1)^2\sum_{r=1}^2 \phi_r(k)(n-k)^r$ is positive. Following a similar procedure as before, we have that $J(n)$ is a polynomial function of degree four with $J^{iv}(n) > 0$ for all n, $J'''(k) > 0$, $J''(k) > 0$, $J'(k) > 0$ and $J(k) > 0$. Therefore $J(n) > 0$ and consequently $\Delta g_I^U - \Delta g_I^{U,F} > 0$.

Proof of Proposition 7

It is also possible to decompose $E(W^F) = w^{D,F} + w^{U,F}$. Similar to before, $w^{D,F} = w^D$ and

$$w_M^{U,F} = \frac{1}{2\lambda(1+\lambda)^2 S^2} \left[\sum_{r=0}^{3} \eta_r (n-k)^r \right] \sigma_\theta^2,$$

where $\eta_0 = k(1+\lambda)^4[4k^2+(-1+\lambda)4k+(-1+\lambda)\lambda]$, $\eta_1 = (1+\lambda)[(1+\lambda)^2 4k^3+(10\lambda^3+19\lambda^2+\lambda-4)k^2+(\lambda+2)6\lambda^3 k+(-1+\lambda)\lambda]$, $\eta_2 = (1+\lambda)^2 k^3+(6\lambda^3+14\lambda^2+6\lambda-1)k^2+(7\lambda^2+18\lambda+9)\lambda^2 k+(\lambda^2+5\lambda+2)\lambda^3$ and $\eta_3 = \lambda(2+\lambda)(k+\lambda)^2$. Repeating the same process as in Proposition 6, we can prove that $\Delta w^U - \Delta w^{U,F} \geq 0$.

Proof of Proposition 8

The output of a firm that belongs to the coalition is given by

$$x_i(\theta_1, \ldots, \theta_k) = r_C^D (a - \overline{\theta}) - r_C^{U,O}(\theta_i - \overline{\theta}) + r_C^{U,P} \sum_{p=1, p \neq i}^{k} (\theta_p - \overline{\theta}), \tag{23}$$

where $r_C^D = r_N^D$, $r_C^{U,O} = \frac{(k+\lambda)}{(1+\lambda)(1+\lambda+k)}$ and $r_C^{U,P} = \frac{1}{(1+\lambda)(1+\lambda+k)}$, whereas the outsiders produce following the same rule as in the case where no merger or no coalition has been formed.

The expected profits of a coalition member is given by $E(\pi_C) = g_C^D + g_C^U$. The deterministic part is independent of the formation of the coalition and it is the same as when no merger had been formed in the first stage, $g_C^D = g_N^D$. The second part is, substituting (23),

$$g_C^U(n,k) = \frac{(2+\lambda)[(k-1)+(k+\lambda)^2]}{2(1+\lambda)^2(1+\lambda+k)^2} \sigma_\theta^2.$$

Of course, $g_C^U(n,1) = g_N^U(n)$ and $g_C^U(n,n) = g_N^{U,F}(n)$. Computing we get that,

$$\Delta g_C^U(n,k) = \frac{(k-1)[3\lambda^2+(8+2k)\lambda+3k+5]}{2(2+\lambda)(1+\lambda)^2(1+\lambda+k)^2} \sigma_\theta^2 \geq 0. \tag{24}$$

We only need to prove that the gains derived from the uncertainty are larger when firms merge than when they form a coalition. From (17) and (24),

$$\Delta g_I^U - \Delta g_C^U = \frac{(k-1)[\lambda^3+6\lambda^2+(5k+7)\lambda+2(k+1)^2]}{2\lambda(2k+\lambda)(1+\lambda)^2(1+\lambda+k)^2} \sigma_\theta^2 \geq 0.$$

Proof of Lemma 2

Firms solve $Max_{x_j} E(\Pi_j | s_j)$ or substituting,

$$Max_{x_j} \left(a - \overline{\theta} - x_j - \sum_{l=1, l \neq j}^{n} E(x_l | s_j) \right) x_j - \left(E(\theta_j - \overline{\theta} | s_j) + \frac{\lambda}{2} x_j \right) x_j. \tag{25}$$

From the first order conditions, we posit that this problem has a linear symmetric equilibrium, $x_j = \overline{x} - r_N^U(s_j - \overline{\theta})$ and solve for r_N^U. This equilibrium is unique (see Vives, 1999).

Proof of Lemma 3

The merged firm solves $Max_{x_1,\ldots,x_k} E(\Pi_M|s_1,\ldots,s_k)$ or substituting,

$$Max_{x_1,\ldots,x_k}\left(a-\overline{\theta}-\sum_{i=1}^{k}x_i-\sum_{l=k+1}^{n}E(x_l|s_1,\ldots,s_k)\right)\sum_{i=1}^{k}x_i-$$
$$\sum_{i=1}^{k}\left(E(\theta_i-\overline{\theta}|s_1,\ldots,s_k)+\frac{\lambda}{2}x_i\right)x_i.$$

The outsiders solve $Max_{x_o} E(\Pi_o|s_o)$, the analogous expression of (25). Similar to the previous proof, we posit a linear equilibrium and solve for $r_I^{U,O}$, $r_I^{U,P}$ and r_O^U. Again this equilibrium is unique.

Proof of Proposition 9

The gains derived from the uncertainty when $\lambda = 0$, substituting the corrresponding output choices of Lemma 2 and Lemma 3 in the expected profits derived in Section 3.1,

$$\Delta g_I^U(n,k,t)=\left[\frac{tk^2(2-t)^2[1+t(k-1)]}{V(n,k,1,t)^2}-\frac{t}{[2+t(n-1)]^2}\right]\sigma_\theta^2.$$

This function is positive whenever $L(n,t) = k^2(2-t)^2[1+t(k-1)][2+t(n-1)]^2 - V(n,k,1,t)^2$ is positive. $L(n,t)$ is a quadratic polynomial function in n, $\frac{\partial^2 L(n,t)}{\partial^2 n} < 0$ and $L(k,t) > 0$ for all n and t. Hence, for any t^* there exists n^* ($n^* > k$) such that $L(n^*,t^*) = 0$ and $\frac{\partial L(n^*,t^*)}{\partial n} < 0$. If we prove that $\frac{\partial L(n^*,t^*)}{\partial t} < 0$, by the implicit function theorem we have that $\frac{\partial n^*(t^*)}{\partial t} < 0$ and therefore $n^*(t^*) \geq n^*(1)$ for any t^* ($\in [0,1]$). Going back to the initial function we have that if $\Delta g_I^U(n,k,t) < 0$ (i.e. $n > n^*(t)$) then $\Delta g_I^U(n,k,1) < 0$ (because $n > n^*(t) > n^*(1)$). However, computing we have that $\Delta g_I^U(n,k,1) < 0$ if and only if $\Delta g_I^D(n,k) < 0$. Therefore if $\Delta g_I^U(n,k,t)$ is negative then $g_I^D(n,k)$ is also negative.

Thus, we only need to show that $\frac{\partial L(n^*,t^*)}{\partial t} < 0$. We know that for a given n^* there exists at least one t^* such that $L(n^*,t^*) = 0$. If we show that it is unique, then since $L(n,0) = 0$, $\frac{\partial L(n,0)}{\partial t} = 0$ and $\frac{\partial^2 L(n,0)}{\partial^2 t} > 0$ for all n and in particular for n^*, we have that $\frac{\partial L(n^*,t^*)}{\partial t} < 0$.

Define $M(n,t) = \frac{\partial^2 L(n,t)}{\partial^2 t}$. Since it is a polynomial function of degree 3 in t and $M(n,0) > 0$ and $M(n,1) < 0$, it can only have one or three zeros in t between 0 and 1. But since $\frac{\partial^3 M(n,t)}{\partial^3 t} > 0$ and $\frac{\partial^2 M(n,0)}{\partial^2 t} < 0$, $M(n,t)$ is concave and may be convex as t increases. Therefore, since $M(n,0) > 0$, $M(n,t)$ has no more than two zeros in t and thus it should have only one. Hence $\frac{\partial^2 L(n,t)}{\partial^2 t}$ is positive firstly and then negative as t increases. In consequence, $L(n,t)$ is first convex and concave after. This, together with $L(n,0) = 0$ and $\frac{\partial L(n,0)}{\partial t} = 0$, proves that $L(n,t)$ can have only one zero in t for a given n.

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