Technical appendix

Here we complete the steps missing in the proofs and provide the expressions that have been left out. Second, we give the expressions for the expected profits in the private values and common value model that have been used for the examples in sections 6.1 and 6.2.

Proof of Corollary 3

We show here that $\frac{\partial}{\partial n}\Delta w^D > 0$. This is equivalent to prove that $\frac{\partial}{\partial n}\frac{G(n)}{S^2(n,k)(n+\lambda+1)} > 0$ and that $R(n) \equiv G'(n)S(n,k)(n+\lambda+1)-2G(n)[(k+\lambda)(n+\lambda+1)+S(n,k)] > 0$. But $R(n)$ is a polynomial function of degree 3 with $R'''(n) = 12k(k-1)^2\lambda(k+\lambda) > 0$ for all n and $R''(k) = 6k^2(k-1)(k+\lambda)(\lambda k+2\lambda^2+3\lambda+2) > 0$, $R'(k) = k(k-1)[\lambda k^4+2(3\lambda^2+5\lambda+3)k^3+(12\lambda^3+42\lambda^2+49\lambda+18)k^2+6\lambda(1+\lambda)^2(4+\lambda)k+6\lambda^2(1+\lambda)^2] > 0$ and $R(k) = k(k-1)(1+\lambda)[2k^4+(3\lambda^2+14\lambda+8)k^3+(5\lambda^3+34\lambda^2+42\lambda+14)k^2+(2\lambda^3+23\lambda^2+37\lambda+16)\lambda k+4\lambda^2(1+\lambda)^2] > 0$. Therefore $R(n) > 0$ for all n and $\frac{\partial}{\partial n}\Delta w^D > 0$.

Proof of Proposition 6

We need to prove that $J^{iv}(n) > 0$ for all n , $J'''(k) > 0$, $J''(k) > 0$, $J'(k) > 0$ and $J(k) > 0$. Computing and rearranging we get that

$J^{iv}(n) = 24[(k+\lambda)^2(2k+\lambda)\phi_2(1)-(1+\lambda)^2(2k+\lambda)\phi_1(k)] = 24(k-1)[(4\lambda+6)k^2+(10\lambda^2+19\lambda+6)k+(5\lambda^2+10\lambda+4)\lambda] > 0$,

$J'''(k) = 6(k-1)[(2\lambda+3)4k^3+(21\lambda^2+45\lambda+20)2k^2+(53\lambda^3+136\lambda^2+98\lambda+20)k+(3\lambda+2)^2(2\lambda+3)\lambda] > 0$,

$J''(k) = 2(k-1)[(4\lambda+6)k^4+(48\lambda^2+111\lambda+58)k^3+(58\lambda^3+164\lambda^2+137\lambda+33)2k^2+(94\lambda^4+306\lambda^3+332\lambda^2+143\lambda+22)k+(12\lambda^4+43\lambda^3+54\lambda^2+29\lambda+6)2\lambda] > 0$,

$J'(k) = (k-1)(1+\lambda)(2k+\lambda)[(2\lambda+3)4k^3+(19\lambda^2+42\lambda+20)2k^2+(43\lambda^3+117\lambda^2+88\lambda+16)k+14\lambda^4+45\lambda^3+47\lambda^2+20\lambda+4] > 0$ and

$J(k) = (2k+\lambda)(\phi_1(1)(k-1)+\phi_2(1)(k-1)^2)S^2 > 0$.

Proof of Proposition 7

We need to prove that $\Delta w^U - \Delta w^{U,F}$ is positive for all $n \geq k$. This expression is positive whenever $L(n)$ is positive, where $L(n) = 2k(k-1)(4k+k\lambda+\lambda)2\lambda(1+\lambda)^2S(n,k)^2S(n,1)^2 - \lambda(\lambda+2)^2(2k+\lambda)^2[S(n,1)^2\sum_{r=0}^{3}\eta_r(k)(n-k)^r - S(n,k)^2\sum_{r=0}^{3}\eta_r(1)(n-1)^r]$. This is a polynomial function of degree four. Computing and rearranging,

$L^{iv}(n) = 48k(k-1)\lambda^3(1+\lambda)^2[4(\lambda^2+5\lambda+5)k^3+4(3\lambda^3+16\lambda^2+20\lambda+5)k^2+(8\lambda^3+47\lambda^2+64\lambda+20)\lambda k+(\lambda^3+8\lambda^2+12\lambda+4)\lambda^2] > 0$.

$L'''(k) = 24k(k-1)\lambda^3(1+\lambda)^2[4(\lambda^2+5\lambda+5)k^4+8(3\lambda^3+17\lambda^2+25\lambda+10)k^3+(36\lambda^4+227\lambda^3+404\lambda^2+252\lambda+44)k^2+(17\lambda^4+121\lambda^3+241\lambda^2+176\lambda+40)\lambda k+2(\lambda^4+9\lambda^3+20\lambda^2+16\lambda+4)\lambda^2] > 0$.

$L''(k) = 4k(k-1)\lambda^3(1+\lambda)^2[4(\lambda^2+5\lambda+5)k^5+4(13\lambda^3+76\lambda^2+120\lambda+55)k^4+(144\lambda^4+955\lambda^3+1916\lambda^2+1480\lambda+380)k^3+(141\lambda^5+1042\lambda^4+2422\lambda^3+2364\lambda^2+960\lambda+116)k^2+(53\lambda^5+439\lambda^4+1133\lambda^3+1255\lambda^2+608\lambda+100)\lambda k+(6\lambda^3+47\lambda^2+68\lambda+20)(1+\lambda)^2\lambda^2] > 0$.

$L'(k) = 2k(k-1)\lambda^3(1+\lambda)^3(2k+\lambda)[8(\lambda^2+5\lambda+5)k^4+(42\lambda^3+250\lambda^2+392\lambda+176)k^3+(57\lambda^4+389\lambda^3+770\lambda^2+564\lambda+128)k^2+(28\lambda^5+217\lambda^4+505\lambda^3+480\lambda^2+188\lambda+24)k+2(2\lambda^5+18\lambda^4+47\lambda^3+52\lambda^2+24\lambda+4)\lambda] > 0$.

$L(k) = 2k(k-1)\lambda^3(1+\lambda)^4(2k+\lambda)^4[(\lambda^2+5\lambda+5)k+\lambda^3+7\lambda^2+13\lambda+7)] > 0$.

They are all positive because all the terms are positive. Thus, $L(n)$ and $\Delta w^U - \Delta w^{U,F}$ are positive for all $n \geq k$.

Proof of Proposition 9

We have that $L(k,t) = k^2t^2(k-1)^2[1+t(k-1)] > 0$ for all t and $\frac{\partial^2 L(n,t)}{\partial^2 n} = -2k^2(k-1)(k+1-t)t^4 < 0$ for all n and t. On the other hand, $M(n,t) = \frac{\partial^2 L(n,t)}{\partial^2 t}$ and $M(n,0) = 8k^2(k-1)^2 > 0$ and $M(n,1) = -4k^2(k-1)[(n-k)^2(3k-2)+(k-1)(2n-k)] < 0$. Moreover, $\frac{\partial^3 M(n,t)}{\partial^3 t} = 120k^2(k-1)(n-1)^2 > 0$ for all t and $\frac{\partial^2 M(n,0)}{\partial^2 t} = -24k^2(k-1)[k(n-k)^2+n[n-2+2(k-2)]+(k-2)^2+k+1] < 0$.

Expected profits in the private values model

Substituting the output decisions of the general model with $\rho = 1$ into the expected profits, we have that $E(\pi_I^P) \equiv g_I^{D,P} + g_I^{U,P}$ where $g_I^{D,P} = g_I^D$ and

$$g_I^{U,P}(n,k) = \frac{1}{2\lambda V(n,k,\rho,1)}\left[\sum_{r=0}^{2} \nu_r (n-k)^r\right]\sigma_\theta^2,$$

where $\nu_0 = (2k+\lambda)(2+\lambda-\rho)^2[1+\rho(k-1)]^2[2(k-1)(1-\rho)+\lambda]$, $\nu_1 = 2(2k+\lambda)(k-1)\rho(1-\rho)(2+\lambda-\rho)[1+\rho(k-1)][2(1-\rho)+k\rho]$ and $\nu_2 = (k-1)\rho^2(1-\rho)[[2(1-\rho)+\lambda](2k+\lambda)[1+\rho(k-1)]+k^3\rho^2]$. The expected profits when firms remain independent are $E(\pi_N^P) \equiv g_N^{D,P} + g_N^{U,P}$ where $g_N^P(n) = g_I^P(n,1)$. In the example of Section 6.1, when $k=2$, $n=4$ and $\lambda=1$,

$$\Delta g_I^{U,P} = \frac{180+840\rho+965\rho^2-630\rho^3-1398\rho^4-204\rho^5+189\rho^6-6\rho^7}{6(1+\rho)^2(-15-20\rho+3\rho^2)^2}\sigma_\theta^2.$$

This is positive whenever $\rho < 0.985$.

Expected profits in the common value model

Substituting the output decisions of the general model with $t = 1$ into the expected profits, we have that $E(\pi_I^C) \equiv g_I^{D,C} + g_I^{U,C}$ where $g_I^{D,C} = g_I^D$ and

$$g_I^{U,C}(n,k) = \frac{tk(2k+\lambda)(2+\lambda-t)^2[1+t(k-1)]}{2\lambda V(n,k,1,t)}\,\sigma_\theta^2.$$

The expected profits when firms remain independent are $E(\pi_N^C) \equiv g_N^{D,C} + g_N^{U,C}$ where $g_N^C(n) = g_I^C(n,1)$. In the example of Section 6.2, when $k = 2$, $n = 4$ and $\lambda = 1$,

$$\Delta g_I^{U,C} = \frac{t(45+30t-10t^2-60t^3-99t^4+30t^5)}{6(1+t)^2(-15-20t+3t^2)^2}\,\sigma_\theta^2.$$

This is positive whenever $t < 0.8$.