When does universal peace prevail? Secession and group formation in rent seeking contests and policy conflicts

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Abstract

This paper analyzes secession and group formation in a general model of contest inspired by Esteban and Ray (1999). This model encompasses as special cases rent seeking contests and policy conflicts, where agents lobby over the choice of a policy in a one-dimensional policy space. We show that in both models the grand coalition is the efficient coalition structure and agents are always better off in the grand coalition than in a symmetric coalition structure. Individual agents (in the rent seeking contest) and extremists (in the policy conflict) only have an incentive to secede when they anticipate that their secession will not be followed by additional secessions. Incentives to secede are lower when agents cooperate inside groups. The grand coalition emerges as the unique subgame perfect equilibrium outcome of a sequential game of coalition formation in rent seeking contests. *Journal of Economics Literature Classification Numbers: D72, D74. Keywords:* secession, group formation, rent seeking contests, policy conflicts.

1 Introduction

Why doesn't universal peace prevail? The world is riddled with conflicts: states fight over territories, firms over markets, individuals over honors and prizes, political parties and interest groups over policies. In each of these situations, agents are willing to waste valuable resources in order to compete while they could enter into an efficient peaceful agreement.

There is of course a distinguished literature in peace and conflict theory (and its natural extension in economics– the rent seeking theory pioneered by Tullock (1967)) whose objective is precisely to understand how conflicts emerge and can be resolved.[1] The focus of the theory of rent seeking has always been on the level of resources spent in contests. For example, in a recent article, Esteban and Ray (1999) analyze how the total amount of resources spent in contests depends on the distribution of a population with heterogeneous characteristics. But while the theory of contests has been extended in a number of directions, it is still almost silent on one important issue: why do agents form groups, or engage in contests when they could agree to a universal agreement?

Our objective in this paper is to shed light on this issue, by studying the incentives to secede from a universal agreement and to form groups in a general model of contest. More precisely, we consider the following set of questions. Given that the efficient structure is universal peace, where all agents form a single group to divide rents or choose policy, why do we observe conflict among agents or groups of agents? Which agents have an incentive to secede from the universal agreement? What conjectures should they form on the reaction of other agents to make the secession profitable? Alternatively, if agents are initially isolated, what is the process by which

[1] For an introduction to conflicts and collective action, see the classical book of Olson (1965) and the book by Sandler (1992)).)

they end up forming a single, efficient group?

To answer these questions, we rely on the recent noncooperative models of coalition formation developed, among others, by Hart and Kurz (1983), Bloch (1996), Yi (1997) and Ray and Vohra (1999) . (See Bloch (1997) for a survey.) These models, which to the best of the knowledge have not yet systematically been applied to the study of conflicts, enable us to obtain sharp, general conclusions on the viability of universal agreements and the formation of groups. Another distinguishing feature of our approach is that we consider a general model of conflict, which admits as special cases the traditional rent-seeking model as well as models of policy conflicts, where interest groups located on a one-dimensional space lobby for the adoption of a policy. Our analysis sheds light on the common structure of conflict models as well as on the specific features of rent-seeking contests and policy conflicts.

Our analysis starts with a description of a general model of conflict, adapted from Esteban and Ray (1999). In this model, we explicitly allow for the formation of groups and the existence of external effects across groups. This general model encompasses as specific cases pure rent seeking (with a collective or private divisible good), as well as policy conflicts where the choice of a policy by the winning group induces external effects on all the agents. Our first results show that the efficient coalition structure is always the grand coalition, where no resources are wasted on conflict and agents divide rents or choose policy inside a single group. While this result is well known and obvious in the case of rent seeking, it is not immediately obtained in the case of policy conflict, and requires some qualification. We show that, as long as the utility loss is a convex function of the distance between an agent's ideal point and the policy chosen, universal agreement will always be the efficient coalition structure in the model of policy conflict.

Our study then focuses on the incentives to secede from the grand coalition. For the rent seeking contests and policy conflicts, we construct a valuation, expressing the utility of every player in every coalition structure. In the rent seeking model, this valuation can be explicitly computed while in the policy conflict, we can only construct the valuation for a small number of players. However, our analysis shows that, generally, the payoffs obtained by every player in a symmetric coalition structure is lower than the payoff obtained in the grand coalition. This result suggests that if agents want to secede, they only have an incentive to do so if the resulting coalition structure is asymmetric. In fact, we establish that a single player (in the rent seeking contest) or a single extremist (in the policy conflict) always has an incentive to secede when all other players form a single group. Hence, it appears that individual players have no incentive to secede when their secession results in a complete collapse of the universal agreement (a symmetric coalition structure where all groups are singletons), but are always willing to secede when their secession is not followed by any additional change. We formalize this observation, using the terminology introduced by Hart and Kurz (1983). In the γ model (where a secession is followed by the collapse of the group), the grand coalition is an equilibrium, whereas it is not an equilibrium in the δ model (where after a secession, members of a group remain together). In the rent seeking model, we are able to go one step further, and endogenize the reaction of other players to a secession. Considering the sequential model of coalition formation proposed in Bloch (1996) and Ray and Vohra (1999), we show that the grand coalition is indeed the unique equilibrium outcome of the process of coalition formation.

While the previous results were obtained under the assumption that every agent chooses noncooperatively the amount of resources spent in the conflict, we also consider a cooperative model where members of a group coordinate their investments in the contest. Our main finding is that incentives to secede are lower in the cooperative model, as seceding players face a higher level of conflict than in the noncooperative model. In fact, while an individual still has an incentive to secede in the cooperative rent seeking contest when all other agents remain together, in the policy conflict, an extremist no longer has an incentive to secede, once she knows that all other agents will choose their outlays cooperatively in the remaining group.

Our paper draws its inspiration from recent studies by Esteban and Ray (Esteban and Ray (1999), (2001a) and (2001b)). Esteban and Ray (1999) introduce the general model of conflict that we use. Their analysis focuses on the relation between distribution and the level of conflict, and shows that this relation is nonmonotonic and usually quite complex. We encounter the same complexity in our study, but focus our attention to a different problem: the endogenous formation of groups in models of conflicts. By simplifying their model in some dimensions (considering a specific contest technology and assuming that agents are uniformly distributed along the line in the policy conflict), we are able to obtain new results on the incentives to secede and form groups in models of conflicts, thereby progressing on a research agenda which is implicit in their analysis (Section 4.3.2 on group mergers in Esteban and Ray (1999), pp. 396-397.) Esteban and Ray (2001b) study explicitly the effect of changes in group sizes in a model of rent seeking with increasing marginal cost and prizes having both private and collective components. Again, they focus their attention on the global level of conflict, and do not discuss incentives to form groups or secede from the grand coalition.

In the rent-seeking literature, the issue of group and alliance formation has received some attention since the early 80's (See Tullock (1980), Katz, Nitzan and Rosenberg (1991), Nitzan (1991), and the survey by Sandler (1993).) The early literature treated groups and alliances as exogenous, and did not consider incentives to form groups in contests. Baik and Shogren (1995), Baik and Lee (1997) and Baik and Lee (2001) obtain partial results on group formation in rent seeking models with linear costs. They consider a three-stage model, where players form groups, decide on a sharing rule, and then choose noncooperatively the resources they spend on conflict. Baik and Shogren (1995) analyze a situation where a single group faces isolated players, Baik and Lee (1997) consider competition between two groups and Baik and Lee (2001) analyze a general model with an arbitrary number of groups. In all three models, it appears that the group formation model leads to the formation of groups containing approximately one half of the players. Our paper is closest to Baik and Lee (2001) because we consider the formation of arbitrary groups. Our analysis differs from theirs in two important respects: we consider very different models of group formation, where players can choose to exclude other players from the group (they only consider open membership games), and we analyze a variety of models of conflicts, whereas they focus on a pure rent seeking model with linear costs. A recent strand of the literature (Skaperdas (1998) and Tan and Wang (1999)) analyzes the formation of alliances in models with continuing conflict: once an alliance has won a contest, a new contest is played among members of the winning alliance. Tan and Wang (1999) consider a general model with asymmetric players, but suppose that the amount resources spent of conflict is exogenous. Skaperdas (1998) allows for an endogenous choice of fighting expenses, but limits his analysis to three players. The main distinction between these models and ours is that we only consider one conflict: once a group has won the contest, either it obtains the right to decide collectively on the policy, or it shares the prize between its members according to a fixed sharing rule.

Finally, our analysis of policy conflicts bears some resemblance to the study of country formation and secession in local public goods games. (Alesina and Spolaore (1997) and Le Breton and Weber (2000).) As in these models, we analyze incentives to form groups for agents located on a line and whose utility depends on the distance between their location and the location of the local public good (or policy). There are two important differences between local public goods economies and policy conflicts, which make the comparison between the two models difficult to interpret. First, in local public goods economies, it may be efficient to divide the population into different groups (when the cost of providing the public good is low with respect to the utility loss due to distances between the location of the agent and of the public good), whereas in the policy conflict the grand coalition is always efficient. Second, in local public goods economies, as agents do not benefit from public goods offered outside their jurisdiction, there are no externalities across groups, whereas in the policy conflict, an agent's utility depends on the entire coalition structure, as it determines both the location of the policies and the winning probabilities of the different groups.

The remainder of the paper is organized as follows. Section 2 describes the model and preliminary results on the equilibrium of the games of conflict. Section 3 focuses on rent seeking contests, and Section 4 discusses policy conflicts. Section 5 contains our conclusions and discussion of the limitations of the analysis and future research.

2 A Model of Conflicts and Contests

We borrow the model of conflicts and contests from Esteban and Ray (1999), and extend it to allow for the formation of groups of agents. This is a general model encompassing as special cases the pure rent seeking contest and conflict among lobbyists over the choice of social policies. We assume that there are $n+1$ players, indexed by $i = 0, 1, 2, ..., n$. The set of all players (with cardinality $n+1$) is denoted N. A coalition C_j is a nonempty subset of N, and a coalition structure $\pi = \{C_1, C_2, .., C_m\}$ is a partition of the set of players into coalitions. Once a group of players C_j is formed, its members spend effort (or invest resources) in order to make the group win the contest. We adopt the simple contest technology initially advocated by Tullock (1967), and axiomatized by Skaperdas (1996). The probability that group C_j wins is given by

$$p_j = \frac{\sum_{i \in C_j} r_i}{R},$$

where r_i denotes the resources spent by agent i, and $R = \sum_{i \in N} r_i$ the total amount of resources spent on conflict by all the agents. Resources are costly to acquire, and each agent faces an identical quadratic cost function,

$$c(r_i) = \frac{1}{2} r_i^2.$$

This specification of the cost function differs from the linear function usually assumed in the rent seeking literature, and is adapted from the general cost functions analyzed by Esteban and Ray (1999).[2] We depart from the usual linear specification because, with heterogeneous agents and groups, the cost function must satisfy $c'(0) = 0$ to guarantee the existence of an interior equilibrium.

Upon winning the contest, the group C_j either obtains a fixed prize (in the case of rent seeking contests) or the right to choose the policy implemented for all agents (in the case of policy conflicts). We denote by u_{ij} the

[2]Esteban and Ray (1999) conduct their analysis for cost functions satisfying $c'(0) = 0, c' > 0, c'' \geq 0$ and $c''' \geq 0$. The quadratic cost is a special case of their general family of cost functions.

utility obtained by agent i when group C_j wins the contest. With all these notations in mind, the utility of agent i can be written as

$$U_i = \sum_{j=1}^{m} p_j u_{ij} - c(r_i).$$

As in Esteban and Ray (1999), this formulation is general enough to cover the case of pure rent seeking contests (where agents only derive positive utility when their group wins the contest), and conflicts (or contests with externalities), where agents derive different utilities, when losing the contest, according to the identity of the winning group. However, as opposed to Esteban and Ray (1999), we do not suppose that all agents inside a group obtain the same utility level, (u_{ij} may be different from $u_{i'j}$ for two agents i and i' in group C_j), nor that agents systematically favor the group they belong to (u_{ij} may be smaller than $u_{ij'}$ for two disjoint coalitions C_j and $C_{j'}$ where $i \in C_j$). However, we will maintain Esteban and Ray (1999)'s assumption that the *total utility* obtained by group C_j is higher when the group wins than when any other group wins the contest, i.e.

$$\sum_{i \in C_j} u_{ij} > \sum_{i \in C_j} u_{ik} \quad \text{for all } k \neq j$$

We distinguish between two models of interaction between members of a group. In the *noncooperative* model, every agent chooses her contribution r_i individually. In the *cooperative* model, total contributions are chosen cooperatively (and denoted R_j for the coalition C_j). Hence, in the cooperative model, we can collapse the game into a game played by representatives of each group, where each representative has a utility function given by

$$U_j = \sum_{j=1}^{m} p_j \sum_{i \in C_j} u_{ij} - \sum_{i \in C_j} c(r_i).$$

We start our analysis by deriving, for any coalition structure π, the Nash equilibrium of the game of conflict and contest, where players choose (either noncooperatively or cooperatively) the level of resources they spend on conflict. It is easy to see that the cooperative conflict game is formally identical to the game considered by Esteban and Ray (1999). Hence, we refer to their Propositions 3.2 and 3.3 (Esteban and Ray (1999), p. 386) to state:

Proposition 1 *(Esteban and Ray (1999)). The cooperative game of conflict admits a unique equilibrium* $(R_1^*, R_2^*, ..., R_m^*)$, *characterized by the interior first order conditions:*

$$\frac{\sum_{k\neq j} R_k(\sum_{i\in C_j} u_{ij} - \sum_{i\in C_k} u_{ik})}{R^2} = \frac{R_i}{|C_i|}$$

Proof. See Esteban and Ray (1999). Our model is a special case of their model, with a quadratic cost of acquiring conflict resources. ■

Using Proposition 1, we derive the indirect utility function of each agent as

$$v_i = \sum_{j=1}^{m} \frac{R_j^*}{R^*} u_{ij} - \frac{1}{2} \frac{R_j^{*2}}{|C_j|^2}.$$

This indirect utility function assigns to each coalition structure π a vector of payoffs for all the agents. It enables players to evaluate the coalition structures they form, and has been labeled a "valuation" in the literature. (See Hart and Kurz (1983) for an early example and Bloch (1997) for a general discussion.) We denote this valuation by $v_i^C(\pi)$.

We now turn to the noncooperative game of conflict, which was not considered by Esteban and Ray (1999), but retains close similarities with the cooperative game. We obtain the first order condition:

$$\sum_{k\neq j}\frac{R_k}{R^2}(u_{ij}-u_{ik})-r_i=0. \tag{1}$$

Notice that condition (1) does not guarantee that individual contributions to the contest will always be positive. If in fact,

$$\sum_{k\neq j}R_k(u_{ij}-u_{ik})<0,$$

the agent will prefer to see her group lose, and will make *negative* contributions to the contest.[3]

Following the same lines as Esteban and Ray (1999), we can prove:

Proposition 2 *The noncooperative game of conflict admits a unique Nash equilibrium* $(r_1^*, r_2^*, ..., r_n^*)$ *characterized by the interior first order conditions:.*

$$\sum_{k\neq j}\frac{R_k}{R^2}(u_{ij}-u_{ik})-r_i=0.$$

Proof. To prove existence, note that the first order condition (1) define a *unique* best response of player i for all vectors of contributions (r_{-i}). Furthermore, this best response is a continuous function of the contributions (r_{-i}). As $(u_{ij}-u_{ik})$ is finite, condition (1) guarantees that r_i is bounded above by some positive real number $\overline{R}$. Now consider the function $\Phi_i(r_{-i})$ defined over $[0,\overline{R}]$ by the first order conditions. Let $\Phi=\times_i\Phi_i$.The function Φ is a continuous map from a compact space into itself, and hence admits a fixed point by Brouwer's fixed point theorem. The fixed point of the function Φ is clearly a Nash equilibrium of the game of the noncooperative game of conflict.

[3] Negative contributions have to be understood as investments undermining the probability of success of the group. An alternative model could be considered, where players make nonnegative contributions. The analysis and results would not be altered by placing a positivity constraint on investments.

To prove that the equilibrium is unique, suppose by contradiction that there exist two equilibria r and r'. Without loss of generality, suppose that $R' \leq R$. Pick the index j for which the ratio $\frac{p_k}{p'_k}$ is maximal. Consider the total contributions made by players in group C_j equilibrium r. A simple summation of the individual first order conditions gives:

$$\begin{aligned} R_j &= \frac{1}{R} \sum_{i \in C_j} \sum_{k \neq j} (u_{ij} - u_{ik}) p_k \\ &= \frac{1}{R} \sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p_k \end{aligned}$$

As $\sum_{i \in C_j} (u_{ij} - u_{ik}) > 0$, total contributions of group j are positive. Now comparing total contributions made in the two equilibria r and r' we obtain

$$\begin{aligned} \frac{R_j}{R'_j} &= \frac{R'}{R} \frac{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p_k}{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p'_k} \\ &= \frac{R'}{R} \frac{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p'_k (p_k / p'_k)}{\sum_{k \neq j} \sum_{i \in C_j} (u_{ij} - u_{ik}) p'_k} \\ &< \frac{R'}{R} \frac{p_j}{p'_j} = (\frac{R'}{R})^2 \frac{R_j}{R'_j} \leq \frac{R_j}{R'_j}, \end{aligned}$$

yielding a contradiction. ■

Again, we define the valuation for each agent in the noncooperative model as the indirect utility function

$$v_i^N(\pi) = \sum_{j=1}^{m} \frac{R_j^*}{R^*} u_{ij} - r_i^*$$

3 Rent Seeking Contests

In this Section, we analyze a first model of contest, where agents fight over a fixed prize V. The literature on group rent seeking discusses various alternatives for the sharing of the prize among members of the winning group (see Nitzan (1991) and Baik and Shogren (1995)). Typically, one considers a sharing rule which is a weighted combination of equal sharing and sharing proportional to individual investments in the group. Equal sharing induces group members to free-ride on the contribution of other members, and results in lower investments in the contest ; proportional sharing, on the other hand, induces a "rat race" effect, and results in higher investments in the contest. While the role of various sharing rules and the endogenous determination of the optimal rule have been emphasized in the literature on group rent seeking, we focus in this paper on a different issue, and simply assume that the prize is equally shared among members of the winning group. Hence, the utility of an agent is given by

$$\begin{aligned} u_{ij} &= V/|C_j| \text{ if } i \in C_j, \\ u_{ij} &= 0 \text{ if } i \notin C_j. \end{aligned}$$

In this simple group rent seeking model, it is well known that the efficient coalition structure is the grand coalition. Formally, a coalition structure π is *efficient* (in the cooperative or noncooperative sense) if there exists no coalition structure π' such that $\sum_{i \in N} v_i(\pi') > \sum_{i \in N} v_i(\pi)$, where the valuation v is defined respectively in the cooperative or noncooperative model. We can state:

Lemma 3 *In the rent seeking contest, the efficient coalition structure is the grand coalition both in the noncooperative and cooperative models.*

Proof. The proof is obvious. In the grand coalition, no resources are dissipated and the sum of utilities is equal to the prize. Any model of conflict (cooperative or noncooperative) with at least two groups results in rent dissipation, and yields a smaller total payoff. ■

Our next result shows that the payoff received by any agent in a symmetric coalition structure is always lower than the payoff received in the grand coalition. Formally, a coalition structure is *symmetric* if and only if $|C_j| = |C_k|$ for all groups C_j and C_k in π.

Lemma 4 *In the rent seeking contest, both in the cooperative and noncooperative models, for any symmetric coalition structure* $\pi, v_i(\pi) < v_i(\{N\})$ $\forall i \in N$.

Proof. Consider first the rent seeking contest. In any regular coalition structure with m groups of $(n+1/m)$ players, the expected utility of a player is:

$$\frac{V}{m(n+1/m)} - c(r_i) < \frac{V}{n+1}.$$

Hence, any player gets a smaller payoff in a regular coalition structure than in the grand coalition. ■

The intuition underlying Lemma 4 is easily grasped. In a symmetric coalition structure, all agents are symmetric, and obtain the same expected gain than in the grand coalition, but must also incur the cost of conflict. While this Lemma is very simple, it will prove helpful in the analysis of secession and group formation.

3.1 Valuations in rent seeking contests

We now derive explicitly the valuations in the noncooperative and cooperative models of rent seeking contests. In the noncooperative model, we are able to derive an explicit analytical formula for the valuation. The interior first order condition gives

$$\frac{V}{|C_j|}\frac{\sum_{k\neq j} R_k}{R^2} = r_i \ \forall i \in C_j$$

Summing over all members of group C_j,

$$V\frac{\sum_{k\neq j} R_k}{R^2} = R_j.$$

Notice that this last expression is symmetric for all groups. Hence, in equilibrium, every group will spend the same resources in the conflict, and the winning probability is identical across groups. Straightforward computations then show that the total level of conflict and individual expenses can be computed as:

$$\begin{aligned} R &= \sqrt{V(m-1)} \\ r_i &= \frac{\sqrt{V(m-1)}}{m|C_j|} \end{aligned}$$

The valuation is thus given by

$$v_i^N(\pi) = V\{\frac{1}{m|C_j|} - \frac{1}{2}\frac{m-1}{m^2|C_j|^2}\} \tag{2}$$

In the noncooperative model of rent seeking contest, the valuation thus takes a particularly easy form. It only depends on the total number of groups formed (m) and on the size of the group to which player i belongs ($|C_j|$). The valuation is independent of the size distribution of coalitions to which the player does not belong, and of the total number of agents in the society.[4] We use the analytical expression to compute the valuation for

[4] This very simple expression is of course only obtained under very specific assumptions on the contest technology, and would not obtain for alternative specifications. It is however illustrative of the qualitative properties of the valuation in rent seeking contests. Notice that a similar simple expression can be found in a very different context – cartel formation in linear Cournot oligopolies studied in Bloch (1996) and Ray and Vohra (1999).

small numbers of players. (Tables only report the values for some of the partitions. The values for partitions which can be obtained by permutation of the players are not given here.)

Player/Coalition Structure	0	1	2
012	$V/3$	$V/3$	$V/3$
0|12	$3V/8$	$7V/32$	$7V/32$
0|1|2	$2V/9$	$2V/9$	$2V/9$

TABLE 1: VALUATIONS FOR THE NONCOOPERATIVE RENT SEEKING CONTEST (3 PLAYERS).

Player/Coalition Structure	0	1	2	3
0123	$V/4$	$V/4$	$V/4$	$V/4$
0|123	$3V/8$	$11V/72$	$11V/72$	$11V/72$
01|23	$7V/32$	$7V/32$	$7V/32$	$7V/32$
01|2|3	$5V/36$	$5V/36$	$2V/9$	$2V/9$
0|1|2|3	$5V/32$	$5V/32$	$5V/32$	$5V/32$

TABLE 2: VALUATIONS FOR THE NONCOOPERATIVE RENT SEEKING CONTEST (4 PLAYERS).

Tables 1 and 2 illustrate some important properties of the valuation. First of all, it appears that the payoff a players receives in the grand coalition is only dominated by the payoff she receives when she is an isolated player, facing a group of size $(n-1)$. Any other coalition structure results in lower payoffs for all the players. Furthermore, it appears that the formation of a group (or the merger between groups) always creates *positive spillovers* to the other players. (As can be seen from the analytical expression for the valuation, a decrease in the total number of groups m induces an increase in the payoff for any player not affected by the merger.) This positive externality is the source of a free-riding problem, which leads any player to prefer to let the other players form groups. This free-riding problem is highlighted by the fact that the only case where a player obtains a higher payoff than in the grand coalition is when it faces a group formed by all the other players.[5]

When players choose cooperatively their contributions, an analytical expression for the pure rent seeking contest cannot be obtained. Instead, we compute below the valuation for small numbers of players

Player/Coalition Structure	0	1	2
012	$V/3$	$V/3$	$V/3$
0|12	$0.29V$	$0.21V$	$0.21V$
0|1|2	$2V/9$	$2V/9$	$2V/9$

TABLE 3: VALUATIONS FOR THE COOPERATIVE RENT SEEKING CONTEST (3 PLAYERS)

Player/Coalition Structure	0	1	2	3
0123	$V/4$	$V/4$	$V/4$	$V/4$
0|123	$V/4$	$0.14V$	$0.14V$	$0.14V$
01|23	$0.19V$	$0.19V$	$0.19V$	$0.19V$
01|2|3	$0.16V$	$0.16V$	$0.18V$	$0.18V$
0|1|2|3	$5V/32$	$5V/32$	$5V/32$	$5V/32$

TABLE 4: VALUATION FOR THE COOPERATIVE RENT SEEKING CONTEST (4 PLAYERS)

Tables 3 and 4 show that the valuation in the cooperative model displays the same qualitative properties as the valuation in the noncooperative

[5] A similar free-riding problem appears in the study of cartel formation. The cartel game is also a game with positive spillovers. See Bloch (1997) and Yi (1997) for a general discussion of games with positive spillovers.

model. In the cooperative model, the payoff received in the grand coalition dominates the payoff received in any other coalition structure.(The two payoffs are equal when one agent faces a group of three other agents). One can also check that, for a small number of players, the cooperative model displays positive spillovers. Finally, the payoffs are typically lower in the cooperative model than in the noncooperative model. This observation (which may seem counterintuitive at first glance) is due to the fact that the total level of conflict is higher in the cooperative model, as members of a group coordinate their choices of investments in contest, and do not face free-riding from other group members.

3.2 Secession in rent seeking contests

Given that the efficient coalition structure is the grand coalition, we now analyze under which conditions the grand coalition is immune to secession. Our analysis will be centered around *individual* deviations, and we ask: When does an individual agent have an incentive to leave the group and initiate a contest? The previous tables show that the answer to this question depends on the anticipated reaction of the other players to the initial secession. As a first step, we analyze individual incentives to secede, with an exogenous description of the reaction of other agents.

Borrowing from Hart and Kurz (1983), we define two possible reactions of the external players. In the γ model, the grand coalition dissolves, and all the players become singletons. In the δ model, after the secession of a player, all other players remain together in a complementary coalition.[6] We

[6] In Hart and Kurz (1983)'s original formulation, the γ and δ models were defined in terms of noncooperative games of coalition formation. In the γ model, a coalition is formed if all its members unanimously agree on the coalition ; in the δ model, a coalition

thus say that the grand coalition is $\gamma-$*immune to secession* by player i if $v_i(\{N\}) \geq v_i(\{\{0\},...\{n\}\}$. (As the valuations obtained by a player in the grand coalition and in the coalition structure formed of singletons are identical in the cooperative and noncooperative models, we do not need to specify the model we use in the γ case). The grand coalition is $\delta-$*immune to secession* by player i in the noncooperative (respectively cooperative) models if $v_i(\{N\}) \geq v_i^N(\{\{i\}, N\backslash\{i\}\})$ (respectively $v_i(\{N\}) \geq v_i^C(\{\{i\}, N\backslash\{i\}\})$

Proposition 5 *In the rent seeking contest, the grand coalition is $\gamma-$immune to secession for all the players. The grand coalition is not $\delta-$immune to secession in the noncooperative model for $n \geq 2$ and it is not δ-immune to secession in the cooperative model for $n \geq 4$.*

Proof. In the γ model, Lemma 4 immediately shows that the value of every player in a coalition structure formed of singletons is lower than in the grand coalition.

In the δ model, for the noncooperative case, a direct computation gives the value $v_i^N(\{\{i\}, N\backslash\{i\}\}) = V(\frac{1}{2} - \frac{1}{8}) = \frac{3V}{8} > \frac{V}{n+1}$ for $n \geq 2$.

In the cooperative case, a simple computation shows that

$$v_i^N(\{\{i\}, N\backslash\{i\}\}) = V\frac{2+\sqrt{n}}{2(1+\sqrt{n})^2} > \frac{V}{n+1} \text{ for } n \geq 4.$$

■

Proposition 5 shows that the profitability of a secession depends on the anticipated reaction of the other players. If the other players react by breaking into singletons, the deviation is not profitable ; if, on the other hand,

is formed by all players who have announced the same coalition. A coalition structure is then γ (respectively δ) immune to secession if and only if it is a Nash equilibrium outcome of the γ (respectively δ) game of coalition formation.

they react by staying into a single group, an individual deviation becomes profitable. Furthermore, payoffs obtained in a cooperative contest are lower than the payoffs obtained in a noncooperative contest, so that the incentive to secede is *lower* in the cooperative model.

3.3 Group formation in rent seeking contests

The analysis of the previous subsection relies on an exogenous specification of the behavior of players following a secession. We now turn to a group formation model where the reaction of players to a secession is endogenized. Bloch (1996) and Ray and Vohra (1999) propose a sequential model of coalition formation, where every player acts optimally, anticipating the behavior of subsequent players. This forward looking game of coalition formation is formalized as follows. At each period t, one player is chosen to make a proposal (a coalition to which it belongs), and all the prospective members of the coalition respond in turn to the proposal. If the proposal is accepted by all, the coalition is formed and another player is designated to make a proposal at $t+1$; if some of the players reject the proposal, the coalition is not formed, and the first player to reject the offer makes a counteroffer at period $t+1$. The identity of the different proposers and the order of response are given by an exogenous rule of order. There is no discounting in the game but all players receive a zero payoff in case of an infinite play. As the game is a sequential game of complete information and infinite horizon, we use as a solution concept *stationary perfect equilibria*.

When players are ex ante identical, it can be shown that the coalition structures generated by stationary perfect equilibria can also be obtained by analyzing the following simple finite game. The first player announces an integer k_1, corresponding to the size of the coalition she wants to see formed, player k_1+1 announces an integer k_2, etc.;, until the total number

n of players is exhausted. An equilibrium of the finite game determines a sequence of integers adding up to n, which completely characterizes the coalition structure as all players are ex ante identical.

The characterization of the subgame perfect equilibrium outcome of the sequential game of group formation requires an explicit analytical expression for the valuation, and hence can only be done in the noncooperative rent seeking contest. We obtain

Proposition 6 *In the rent seeking contest, the grand coalition is the unique equilibrium coalition structure of the sequential game of coalition formation.*

Proof. To prove the Proposition, we consider the finite game of announcement of coalition sizes, and compute by backward induction the unique subgame perfect equilibrium. The proof of the Proposition relies on the following Lemma.

Lemma 7 *Suppose that $K \geq 1$ coalitions have been formed and that there are j remaining players in the game, with $j \geq 2$. Then player $(n+1-j)$ optimally chooses to form a coalition of size 1 when she anticipates that all subsequent players form singletons.*

To prove the Lemma, we compute the payoff of player $n+1-j$ as a function of the size μ of the coalition she forms, anticipating that all subsequent $j-\mu$ players form singletons.

$$F(\mu) = \frac{1}{(K+j-\mu+1)\mu} - \frac{1}{2}\frac{K+j-\mu}{(K+j-\mu+1)^2\mu^2}$$

Let $a = K+j$ and define

$$G(\mu) = \frac{F(\mu)}{F(1)} = \frac{a^2\left[-2\mu^2+\mu(2a+3)-a\right]}{(a-\mu+1)^2\mu^2\,(a+1)}$$

and

$$h(\mu) = (a - \mu + 1)^2 \mu^2 (a+1) - a^2 \left[-2\mu^2 + \mu(2a+3) - a\right].$$

We will show that $h(\mu) > 0$ for all $j \geq \mu > 1$, thus establishing that the optimal choice of player $n+1-j$ is to choose a coalition of size 1. We first note that $h(1) = 0$ and

$$h(j) = j[(j + \frac{1}{j} - 2)K^3 + j(j-1)\left(K^2 - 1\right)] > 0 \quad \text{as } K \geq 1 \text{ and } j \geq 2.$$

Next we compute

$$h'(\mu) = 2(a+1)(a+1-\mu)(a+1-2\mu)\mu - a^2 [2a+3-4\mu]$$

and obtain

$$h'(1) = 2a(a-2) \geq 0 \text{ as } a \geq 2,$$

$$h'(j) = 2(K+1-j)[(j-1)K^2 + j^2K + j)] - (K+j).$$

Finally, we compute the second derivative

$$h''(\mu) = 2(a+1)[6\mu^2 - 6\mu(a+1) + (a+1)^2] + 4a^2$$

The second derivative h'' is a quadratic function, and the equation $h''(x) = 0$ admits two roots given by

$$\begin{aligned} x_1 &= \frac{a+1}{2} - \sqrt{\Delta}, x_2 = \frac{a+1}{2} + \sqrt{\Delta} \\ \text{with } \Delta &= 48\left[(a+1)^4 - 4a^2(a+1)\right] \end{aligned}$$

We conclude that the function h' is increasing over the interval $[-\infty, x_1]$, decreasing over the interval $[x_1, x_2]$ and increasing over the interval $[x_2, +\infty]$.

We now distinguish between two cases. If $h'(j) < 0$, as the function h' is continuous over $[1, j]$, and $h'(1) > 0 > h'(j)$, there exists a value x for which $h(x) = 0$.We show that this value is unique. Suppose by contradiction that $h'(x) = 0$ admits multiple roots over the interval $[1, j]$. As $h'(1) > 0$ and $h'(j) < 0$, there must exist at least three values $y_1 < y_2 < y_3$ with $h'(y_1) = h'(y_2) = h'(y_3) = 0$ and $h''(y_1) < 0, h''(y_2) > 0, h''(y_3) < 0$. However, our earlier study of the second derivative established that there exist no values satisfying these conditions. Hence, there exists a unique root $x^*$of the equation $h'(x) = 0$ in the interval $[1, j]$ and $h'(x) \geq 0$ for all $x \in [1, x^*], h'(x) \leq 0$ for all $x \in [x^*, j]$. Hence, the function h attains its minimum either at $\mu = 1$ or $\mu = j$ and as $h(j) > h(1) = 0, h(\mu) > 0$ for all $j \geq \mu > 1$.

If now $h'(j) > 0$, we necessarily have $j < K + 1$. Hence, $j < \frac{a+1}{2} < x_2$. In that case, we show that there is no value $x \in [1, j]$ for which $h'(x) = 0$. Suppose by contradiction that the function crosses the horizontal axis. Then there exists at least two values $y_1 < y_2 < x_2$ for which $h'(y_1) = h'(y_2) = 0$ and $h''(y_1) < 0, h''(y_2) > 0$. Our earlier study of the second derivative h'' shows that there exist no values satisfying those conditions. Hence $h'(\mu) > 0$ for all $\mu \in [1, j]$ and as $h(1) = 0, h(\mu) > 0$ for all $j \geq \mu > 1$, completing the proof of the Lemma.

We now use the preceding Lemma to finish the proof. We first claim that, in a subgame perfect equilibrium, after any coalition has been formed, all players choose to form singletons. The proof of this claim is obtained by induction on the number j of remaining players. If $j = 1$, the result is immediate. Suppose now that the induction hypothesis is true for all $t < j$. By the induction hypothesis, in equilibrium, all players following player

$(n-j+1)$ form singletons. By the preceding Lemma, player $(n-j+1)$ optimally chooses to form a coalition of size 1.

Finally, consider the first player. In a subgame perfect equilibrium, she knows that players form singletons after she moved. Hence, she computes her expected profit as

$$\begin{aligned} F(\mu) &= \frac{1}{(n-\mu+2)\mu} - \frac{1}{2}\frac{n-\mu+1}{(n-\mu+2)^2\mu^2} \\ &= \frac{(n-\mu+1)(2\mu-1)+2\mu}{2(n-\mu+2)^2\mu^2}. \end{aligned}$$

To show that $F(\mu) < F(n+1)$ for all $\mu < n+1$, notice first that

$$n+1 \leq \mu(n-\mu+2),$$

as the left hand side of this inequality defines a concave function of μ, which is increasing until $\mu = \frac{n}{2}+1$, then decreasing and attains the values $n+1$ for $\mu = 1$ and $\mu = n+1$. We thus have:

$$\begin{aligned} \frac{(n-\mu+1)(2\mu-1)+2\mu}{2(n-\mu+2)^2\mu^2} &\leq \frac{(n-\mu+1)(2\mu-1)+2\mu}{2(n-\mu+2)\mu(n+1)} \\ &< \frac{2\mu(n-\mu+2)}{2(n-\mu+2)\mu(n+1)} = \frac{1}{n+1}, \end{aligned}$$

establishing that the first player chooses to form the grand coalition. ■

4 Policy conflicts

The second model we consider is a model of *policy conflict* inspired by Esteban and Ray (1999). In this model, agents lobby for a policy and each agent receives utility from the policy chosen in the contest. We take the policy space to be the segment $[0,1]$ and suppose that the $n+1$ are equally spaced along the line. The location of agent i (which corresponds to the point i/n

on the segment) represents her optimal policy. We suppose that agents have Euclidean preferences and suffer a loss from the choice of a policy different from their bliss point. The primitive utility of agent i is thus a decreasing function of the distance between the policy x and her ideal point i/n. More precisely, we describe the primitive utility of agent i as

$$u_i = V - f(|i/n - x|),$$

where V denotes a common payoff for all agents, and f is a strictly increasing and convex function of the distance between agent i and the implemented policy x, with $f(0) = 0$.[7]

We restrict our attention to the formation of *consecutive* groups of agents, i.e. groups which contain all the players in the interval $[i, k]$ whenever they contain the two agents i and k. If a group $C_j = [i, k]$ wins the contest, we suppose that the policy chosen is at the mid-point of the interval $[i, k]$. Whenever the group C_j contains an odd number of players, this point is the policy chosen by the median voter. If the group C_j contains an even number of players, this point can be understood as a random draw between the optimal policies of the two middle voters.[8] Furthermore, it is clear that this policy choice is the one which maximizes the sum of payoffs of all the group members.

Hence, letting m_j denote the midpoint of group C_j, the utility of an agent i is given by

[7] In some of the computations to follow, we will focus on *linear utilities*, and assume that the function f is the identity.

[8] We are of course aware of the fact that, with an even number of group members, the choice of this policy cannot be rationalized by a voting model. However, we have chosen to make this assumption in order to keep the model simple, and allow us to derive results independently of the fact that the number of agents is a group is odd or even.

$$u_{ij} = V - f(|i/n - m_j|).$$

The policy conflict is thus a contest model with externalities: the payoff of a losing agent depends on the identity of the winning group. An added complexity of the model stems from the fact that agents are ex ante asymmetric. It thus appears that policy conflicts are much more complex to analyze than rent seeking contests. However, in spite of these complexities, we are able to obtain results which parallel the results obtained for the rent seeking model. In particular, we can show:

Proposition 8 *In the policy conflict, the efficient coalition structure is the grand coalition both in the cooperative and noncooperative models..*

Proof. The proof of the proposition amounts to showing that the sum of utilities of all agents is higher in the grand coalition than in any efficient structure and does not distinguish between the cooperative and noncooperative cases. In fact, both in the cooperative and noncooperative cases, for any coalition structure π,

$$\begin{aligned}\sum_i v_i(\pi) &= nV - \sum_i \sum_j p_j f(|i/n - m_j|) - \sum_i c(r_i) \\ &\leq nV - \sum_i \sum_j p_j f(|i/n - m_j|).\end{aligned}$$

Now, reversing the order of summation,

$$\sum_i \sum_j p_j f(|i/n - m_j|) = \sum_j p_j \sum_i f(|i/n - m_j|)$$

We will show that for any median midpoint m_j,

$$\sum_i f(|i/n - m_j|) - \sum_i f(|i/n - 1/2|) \geq 0,$$

so the highest sum of utilities is obtained when the grand coalition is formed, the policy chosen is $1/2$ and no resources are dissipated in the conflict.

The computation of the sum of utilities depends on the parity of the cardinal of the coalition C_j and the total number of players, $n+1$. A straightforward computation shows that

$$\begin{aligned}
\sum_i f(|i/n - m_j|) &= \sum_{i \le m_j} f(m_j - i/n) + \sum_{i \ge m_j} f(i/n - m_j) \\
&= \sum_{t=1}^{m_j} f(t/n) + \sum_{t=1}^{n-m_j} f(t/n) \text{ if } |C_j| \text{ is odd} \\
&= \sum_{t=0}^{m_j-1/2} f(\frac{2t+1}{2n}) + \sum_{t=0}^{n-1/2-m_j} f(\frac{2t+1}{2n}) \text{ if } |C_j| \text{ is even.}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\sum_i f(|i/n - 1/2|) &= 2\sum_{t=1}^{n/2} f(t/n) \text{ if } n \text{ is even} \\
&= 2\sum_{t=0}^{(n-1)/2} f(\frac{2t+1}{2n}) \text{ if } n \text{ is odd.}
\end{aligned}$$

Without loss of generality, we suppose that $m_j \le 1/2$. If $|C_j|$ and $n+1$ are odd, we compute

$$\begin{aligned}
\sum_i f(|i/n - m_j|) - \sum_i f(|i/n - 1/2|) &= 0 \text{ if } m_j = 1/2 \\
&= \sum_{t=n/2+1}^{n-nm_j} f(t/n) - \sum_{t=nm_j+1}^{n/2} f(t/n) \ \ge 0 \\
\text{if } m_j &< 1/2
\end{aligned}$$

where the last inequality is obtained because f is increasing. If $|C_j|$ and

$n+1$ are even, we obtain

$$\begin{aligned}
\sum_i f(|i/n-m_j|)-\sum_i f(|i/n-1/2|) &= 0 \text{ if } m_j = 1/2\\
&= \sum_{t=n/2+1/2}^{n-1/2-nm_j} f(\frac{2t+1}{2n}) - \sum_{t=nm_j-1/2}^{n/2-1/2} f(\frac{2t+1}{2n}) \geq 0\\
\text{if } m_j &< 1/2.
\end{aligned}$$

Next suppose that $|C_j|$ is odd and $n+1$ is even. By convexity of the function f,

$$2f(\frac{2t+1}{2n}) \leq f(t/n) + f((t+1)/n).$$

Hence,

$$2\sum_{t=0}^{(n-1)/2} f(\frac{2t+1}{2n}) \leq f(0) + 2\sum_{t=1}^{(n-1)/2} f(t/n) + f((n+1)/2n).$$

and as $f(0)=0$,

$$\sum_i f(|i-n/2|) \leq 2\sum_{t=1}^{(n-1)/2} f(t/n) + f((n+1)/2n)$$

As nm_j is an integer and $n/2$ is not, the condition $m_j \leq 1/2$ implies that $nm_j \leq (n-1)/2$. Then,

$$\begin{aligned}
\sum_i f(|i/n-m_j|)-\sum_i f(|i/n-1/2|) &\geq \sum_{t=1}^{nm_j} f(t/n) + \sum_{t=1}^{n-nm_j} f(t/n)\\
&\quad -2\sum_{t=1}^{(n-1)/2} f(t/n) - f((n+1)/2n)\\
&= 0 \text{ if } nm_j = (n-1)/2\\
&= \sum_{t=(n+3)/2}^{n-nm_j} f(t/n) - \sum_{t=nm_j+1}^{(n-1)/2} f(t/n) \geq 0\\
\text{if } nm_j &< (n-1)/2.
\end{aligned}$$

Finally, suppose that $|C_j|$ is even and $n+1$ is odd. By convexity of the function f, for any $t \geq 1$

$$2f(t/n) \leq f(\frac{2t-1}{2n}) + f(\frac{2t+1}{2n}).$$

Hence,

$$\begin{aligned}
\sum_i f(|i/n - 1/2|) &= 2\sum_{t=1}^{n/2} f(t/n) \leq f(0) + 2\sum_{t=0}^{n/2-1} f(\frac{2t+1}{2n}) + f((n+1)/2n) \\
&= 2\sum_{t=0}^{n/2-1} f(\frac{2t+1}{2n}) + f((n+1)/2n).
\end{aligned}$$

As $n/2$ is an integer and nm_j is not, the condition $m_j \leq 1/2$ implies $nm_j \leq (n-1)/2$. Hence,

$$\begin{aligned}
\sum_i f(|i/n - m_j|) - \sum_i f(|i/n - 1/2|) &\geq \sum_{t=0}^{nm_j-1/2} f(\frac{2t+1}{2n}) + \sum_{t=0}^{n-1/2-nm_j} f(\frac{2t+1}{2n}) \\
&\quad -2\sum_{t=0}^{n/2-1} f(\frac{2t+1}{2n}) - f((n+1)/2n) \\
&= 0 \text{ if } nm_j = (n-1)/2 \\
&= \sum_{t=n/2}^{n-1/2-nm_j} f(\frac{2t+1}{2n}) - \sum_{t=nm_j-1/2}^{n/2-1} f(\frac{2t+1}{2n}) \geq 0 \\
\text{if } nm_j &< (n-1)/2
\end{aligned}$$

■

Proposition 8 shows that the grand coalition is also the efficient structure in the policy conflict game. A careful reading of the proof shows that this result is independent of the contest technology, and only relies on the convexity of the distance function. Because the distance function is convex, the sum of utility losses incurred by the agents is minimized when the grand coalition is formed, and the policy 1/2 is chosen with certainty.

The next Proposition parallels Lemma 4 and shows that every player obtains a lower payoff in a symmetric coalition structure than in the grand coalition. Given that players are ex ante asymmetric, we define a symmetric coalition structure as a partition which is symmetric around the point 1/2. Formally, a coalition structure π is *symmetric* if, whenever two players i and j belong to the same coalition in π, players $n-i$ and $n-j$ also belong to the same coalition in π.

Proposition 9 *In the policy conflict, both in the cooperative and noncooperative models, for any symmetric coalition structure* $\pi, v_i(\pi) < v_i(\{N\})$ $\forall i \in N$.

Proof. For a symmetric coalition structure, consider the coalitions to the left of 1/2, $C_1, ..., C_J$, and let $p_1, .., p_J$ denote the winning probabilities of the corresponding groups. We distinguish between two cases. (i) If C_J contains players to the right of 1/2, there are in total $2J-1$ coalitions in π, $2\sum_{j=1}^{J-1} p_j + p_J = 1$, and the coalition C_J is centered around 1/2 (ii) If C_J does not contain any player to the right of the 1/2, then there are $2J$ coalitions in π and $2\sum_{j=1}^{J} p_j = 1$. In both cases, we compute the payoff of any player $i \leq n/2$ in the coalition structure π. It turns out that the computation does not rely on a specification of the resources spent in rent seeking and hence is identical in the cooperative and noncooperative cases.

Case (i) $v_i(\pi) = V - \sum_{j=1}^{J-1} p_j(f(|i/n - m_j|) + f(1-m_j-i/n)) - p_J f(1/2 - i/n) - c(r_i)$. Now, if $i/n \leq m_j$, by convexity of the function f,

$$f(m_j - i/n) + f(1 - m_j - i/n) \geq 2f(1/2 - i/n).$$

If $i/n \geq m_j$, by convexity of the function f,

$$f(i/n - m_j) + f(1 - m_j - i/n) \geq 2f(1/2 - m_j) \geq 2f(1/2 - i/n).$$

Hence,

$$v_i(\pi) \leq V - 2\sum_{j=1}^{J-1} p_j f(1/2 - i/n) - p_J f(1/2 - i/n) - c(r_i).$$

As $2\sum_{j=1}^{J-1} p_j + p_J = 1$,

$$v_i(\pi) \leq V - f(1/2 - i/n) - c(r_i) < V - f(1/2 - i/n) = v_i(\{N\}).$$

Case (ii). By a similar computation, we obtain:

$$\begin{aligned} v_i(\pi) &\leq V - 2\sum_{j=1}^{J} p_j f(1/2 - i/n) - c(r_i) = V - f(1/2 - i/n) - c(r_i) \\ &< V - f(1/2 - i/n) = v_i(\{N\}). \end{aligned}$$

■

Proposition 9 again is independent of the contest technology and only relies on the convexity of the distance function. The proof of the Proposition exploits the fact that, in a symmetric coalition structure, the winning probabilities of two coalitions which are symmetric around 1/2 are equal. Hence, for any player i, the expected distance to the chosen policy point is equal to $|i/n - 1/2|$. However, since the distance function is convex, the total utility loss is necessarily at least as large as the loss incurred in the grand coalition where the policy 1/2 is chosen with certainty.

4.1 Valuations in policy conflicts

We now turn to a computation of the valuation for the policy conflict. Not surprisingly, we have been unable to obtain an analytical expression for the valuation, and derive below the valuations in the noncooperative policy conflict for 3 and 4 players and linear utilities. Again, we have omitted from the tables those coalition structures which can be obtained by a permutation of the agents.

Player/Coalition Structure	0	1	2
012	$V - 0.5$	V	$V - 0.5$
0|12	$V - 0.49$	$V - 0.37$	$V - 0.74$
0|1|2	$V - 0.59$	$V - 0.41$	$V - 0.59$

TABLE 5: VALUATION FOR THE NONCOOPERATIVE POLICY CONFLICT, (3 PLAYERS)

Player/Coalition Structure	0	1	2	3
0123	$V - 0.5$	$V - 0.167$	$V - 0.167$	$V - 0.5$
0|123	$V - 0.47$	$V - 0.33$	$V - 0.32$	$V - 0.65$
01|23	$V - 0.55$	$V - 0.34$	$V - 0.34$	$V - 0.55$
01|2|3	$V - 0.61$	$V - 0.38$	$V - 0.36$	$V - 0.51$
0|12|3	$V - 0.58$	$V - 0.41$	$V - 0.41$	$V - 0.58$
0|1|2|3	$V - 0.57$	$V - 0.40$	$V - 0.40$	$V - 0.57$

TABLE 6: VALUATION FOR THE NONCOOPERATIVE POLICY CONFLICT, (4 PLAYERS)

Tables 5 and 6 clearly demonstrate the complexity of the structure of the valuation in the policy conflict. It appears that, as in the case of the rent seeking contest, the only case where an agent obtains a higher payoff than in the grand coalition is when an extremist individual breaks away from the grand coalition while all other players remain together. Furthermore, notice that the spillovers due to the formation of a group are either positive or negative depending on the coalition structure. In the four player case, when players 0 and 1 have formed a group, they obtain a higher payoff when 2 and 3 merge than when 2 and 3 are independent agents. On the other hand, it turns out that player 0 obtains a higher payoff in the coalition structure

0|1|2|3 than in the coalition structure 0|12|3. There does not seem to be any regularity in the direction of externalities induced by mergers between groups of agents!

Player/Coalition Structure	0	1	2
012	$V-0.5$	V	$V-0.5$
0|12	$V-0.55$	$V-0.405$	$V-0.595$
0|1|2	$V-0.59$	$V-0.41$	$V-0.59$

TABLE 7: VALUATION FOR THE COOPERATIVE POLICY CONFLICT, (3 PLAYERS)

Player/Coalition Structure	0	1	2	3
0123	$V-0.5$	$V-0.167$	$V-0.167$	$V-0.5$
0|123	$V-0.54$	$V-0.38$	$V-0.24$	$V-0.57$
01|23	$V-0.56$	$V-0.39$	$V-0.39$	$V-0.56$
01|2|3	$V-0.56$	$V-0.40$	$V-0.39$	$V-0.58$
0|12|3	$V-0.57$	$V-0.40$	$V-0.40$	$V-0.57$
0|1|2|3	$V-0.57$	$V-0.40$	$V-0.40$	$V-0.57$

TABLE 8: VALUATION FOR THE COOPERATIVE POLICY CONFLICT (4 PLAYERS)

Tables 7 and 8 again illustrate the complexity of the valuation in the policy conflict, which does not seem to display any regularity. Notice that, as opposed to the noncooperative case, the grand coalition dominates all coalition structures: an extremist never benefits from breaking away. A comparison between Tables 6 and 8 shows that agents do not necessarily benefit from choosing their resources collectively. This is due to the fact that we do not allow transfers among agents in a group. Hence, even though agents collectively benefit from cooperating in their choices of investment, some agents may end up with a lower utility in the cooperative model.

4.2 Secession in policy conflicts

As in the rent seeking contests, we now investigate whether the grand coalition is immune to secession in the policy conflicts.

Proposition 10 *In the policy conflict, the grand coalition is $\gamma-$immune to secession for all the players. The grand coalition is not $\delta-$immune to secession by an extremist player in the noncooperative model with linear utilities for $n \geq 2$. However, the grand coalition is $\delta-$immune to secession by an extremist player in the cooperative model with linear utilities.*

Proof. The fact that the grand coalition is γ immune to secession is a direct consequence of Proposition 9, as the coalition structure formed of singletons is symmetric.

To show that an extremist benefits from breaking away in the noncooperative model with linear utilities, we compute the equilibrium payoffs. Let C denote the coalition $\{1, .., n\}$. We denote by r_0 the equilibrium investment of agent 0 and by R_C the total equilibrium investments of group C. The distance between 0 and the midpoint of C is $\frac{n+1}{2n}$. Hence the first order condition for player 0 is:

$$\frac{n+1}{2n}\frac{R_C}{R^2} = r_0.$$

Now consider players in C. As long as $i \leq \frac{n+1}{4}$, player i prefers the policy choice of player 0 to the policy choice of the coalition C and contributes a negative amount:

$$r_i = \frac{r_0}{R^2}\frac{(4i - (n+1))}{2n}.$$

For $\frac{n+1}{4} \leq i \leq \frac{n+1}{2}$, player i contributes a positive amount:

$$r_i = \frac{(4i-(n+1))}{2n}\frac{r_0}{R^2}.$$

For players to the right of $\frac{n+1}{2}$, the difference in distances is

$$(\frac{n+1}{2n}) - \frac{i}{n} + \frac{i}{n} = \frac{n+1}{2n},$$

and the contribution is given by the first order condition

$$\frac{n+1}{2n}\frac{r_0}{R^2} = r_i.$$

Let r_C denote the solution to this last equation. Then

$$R_C = \sum_{i>0} r_i = r_c(Card\{i, i > \frac{n+1}{2}\} + \sum_{1 \leq i \leq \frac{n+1}{2}} \frac{4i-(n+1)}{n+1}).$$

We define

$$A(n) = Card\{i, i > \frac{n+1}{2}\} + \sum_{1 \leq i \leq \frac{n+1}{2}} \frac{4i-(n+1)}{n+1},$$

and the Nash equilibrium of the game of individual contributions can be obtained by solving the system of two equations:

$$\frac{r_c A(n)}{R^2}\frac{n+1}{2n} = r_0, \tag{3}$$

$$\frac{r_0}{R^2}\frac{n+1}{2n} = r_c. \tag{4}$$

Dividing the two equations, we obtain $r_0 = \sqrt{A(n)}\, r_c$, and equation 3 yields:

$$r_0^2 = \frac{n+1}{2n}\frac{\sqrt{A(n)}}{(1+\sqrt{A(n)})^2}.$$

Hence,

$$\begin{aligned} U_0 &= V - \frac{\sqrt{A(n)}}{1+\sqrt{A(n)}}\frac{n+1}{2n} - \frac{n+1}{4n}\frac{\sqrt{A(n)}}{(1+\sqrt{A(n)})^2} \\ &= V - \frac{n+1}{2n}\frac{\sqrt{A(n)}(3+2\sqrt{A(n)})}{2(1+\sqrt{A(n)})^2}. \end{aligned}$$

To show that player 0 obtains a higher profit than in the grand coalition, it thus suffices to show

$$\frac{n+1}{2n}\frac{\sqrt{A(n)}(3+2\sqrt{A(n)})}{2(1+\sqrt{A(n)})^2} < \frac{1}{2}. \tag{5}$$

Inequality 5 is equivalent to

$$-2A(n) + (n-3)\sqrt{A(n)} + 2n > 0.$$

As $A(n) < n$, this inequality is always satisfied for $n \geq 3$. A direct computation (Table 5) shows that the inequality is also satisfied for $n = 2$.

In the cooperative model, two cases must be considered according to the parity of the number of elements in the set $C = \{1, ..., n\}$. The first order condition for the extremist remains

$$\frac{R_C}{R^2}\frac{n+1}{2n} = r_0$$

If n is odd, the first order condition for the complement coalition is

$$\frac{r_0}{R^2}\frac{(n+1)^2}{4n} = \frac{R_C}{n}$$

and if n is even,

$$\frac{r_0}{R^2}\frac{n+2}{4} = \frac{R_C}{n}$$

In the latter case,

$$\begin{aligned} r_0 &= \frac{(2(n+1)(n+2))^{\frac{1}{4}}}{\sqrt{2(n+1)} + n\sqrt{(n+2)}}\sqrt{\frac{n+1}{2}} \\ R &= \frac{1}{2}(2(n+1)(n+2))^{\frac{1}{4}} \end{aligned}$$

and the individual payoff is

$$u_0^e = V - \frac{n+1}{4}\frac{3\sqrt{2(n+1)(n+2)} + 2n(n+2)}{(\sqrt{2(n+1)} + n\sqrt{(n+2)})^2}.$$

When n is odd, an analogous computation shows:

$$u_0^o = V - \frac{n+1}{4n}\frac{3\sqrt{2n(n+1)}+2n(n+1)}{(\sqrt{2}+\sqrt{n(n+1)})^2}$$

It can be checked that

$$u_0^o < x_0 \Leftrightarrow \frac{n+1}{4n}\frac{3\sqrt{2n(n+1)}+2n(n+1)}{(\sqrt{2}+\sqrt{n(n+1)})^2} > \frac{1}{2} \Leftrightarrow (3-n)\sqrt{(n+1)}+\sqrt{2n}(n-1) > 0$$

The latter expression is increasing in n and positive for $n = 1$. Hence it is always positive. In the even case

$$u_0^e < x_0 \Leftrightarrow \frac{n+1}{4}\frac{3\sqrt{2(n+1)(n+2)}+2n(n+2)}{(\sqrt{2(n+1)}+n\sqrt{(n+2)})^2} > \frac{1}{2} \Leftrightarrow (3-n)\sqrt{(n+1)(n+2)}+\sqrt{2}(n^2-2) > 0 \tag{6}$$

Again the last term is increasing in n and positive for $n = 2$. We conclude that an extremist never has an incentive to break away from the grand coalition in the cooperative model. ■

Proposition 9 establishes a close parallel between incentives to secede in rent seeking contests and policy conflicts. An extremist agent has an incentive to secede in the noncooperative policy conflict only when she anticipates that all other agents remain in a single group (the δ model). If she believes that her secession will lead to a dissolution of the group, an extremist agent has no incentive to break away from the grand coalition. Interestingly, in the cooperative policy conflict, an extremist agent does not have an incentive to secede from the grand coalition, even when all other agents remain together. This result is due to the fact that, by cooperating inside a group, all other agents are able to increase the amount of resources spent on the contest, so that the payoff of a seceding extremist is always lower in the cooperative model than in the noncooperative model. Finally, note that we have been unable to characterize the incentives to secede by agents who are not at the extreme points of the segment. While we strongly believe that these agents have less incentive to secede than extremists, we have not been able to prove it formally.

5 Conclusion

This paper analyzes secession and group formation in a general model of contest inspired by Esteban and Ray (1999). This model encompasses as special cases rent seeking contests and policy conflicts, where agents lobby over the choice of a policy in a one-dimensional policy space. We show that in both models the grand coalition is the efficient coalition structure and that agents are always better off in the grand coalition than in a symmetric coalition structure. As a consequence, individual agents only have an incentive to secede if their secession results in an asymmetric structure. We show that individual agents (in the rent seeking contest) and extremists (in the policy conflict) only have an incentive to secede when they anticipate that their secession will not be followed by additional secessions. Furthermore, if group members choose cooperatively their investments in conflict, incentives to secede are lower. In the policy conflict, an extremist never has an incentive to secede when she faces a group of agents coordinating the amount they spend in the conflict.

We should stress that our analysis suffers from severe limitations. We have only considered *individual* incentives to secede, and do not consider joint secessions by groups of agents. This focus on individual deviations is motivated by the analysis of valuations with small numbers of players, where it appears that the most favorable cases for secessions are secessions by individual players (in the rent seeking contest) or individual extremists (in the policy conflict). However, a complete analysis of group secessions is still needed to analyze the stability of the grand coalition. We have also limited our analysis by forbidding transfers across group members. Allowing for transfers in a model with individual secessions can only bias the analysis in favor of the grand coalition, as the grand coalition could implement a transfer scheme to prevent deviations by individuals. In a model with group secession, the effect of transfers is less transparent, as transfers would simultaneously increase the set of feasible utility allocations in the grand coalition and in deviating groups. This is an issue that we plan to tackle in future research.

Finally, the main findings of our analysis leave us somewhat dissatisfied. We have found that the grand coalition is surprisingly resilient. In the rent seeking contest, it is the only outcome of a natural procedure of group formation. In the policy conflict, the grand coalition is immune to secession when group members coordinate their choice of investments. This suggests that the level of conflict, and the formation of groups and alliances that we observe in reality cannot be justified purely on strategic grounds. In order to explain conflict, we probably need to resort to other elements – group identity, ethnic belonging– which are not easily incorporated in an economic model.

6 References

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